MECHANICAL ENGINEERING FACULTY

System and Process Control

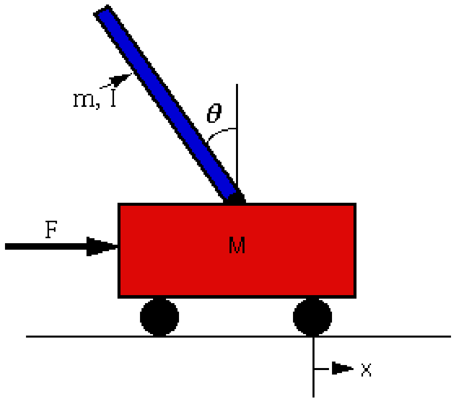
Semestral Work

Linear Servo Base Unit with Inverted Pendulum

Teachers: Prof. Ing. Tomáš Vyhlídal Ph.D.
Ing. Stanislav Vrána, DiS., Ph. D.
Ing. Jaroslav Bušek, Ph.D.
Student: Caner Emrecan

CONTENTS

1.ASSIGMENT



where is $M = 0.38kg$ - mass of the cart, $m = 0.127kg$ - mass of the rod, g - acceleration due to gravity, $l = 0.6413m$ – distance to the center of the rod, θ – angle of the rod, F – force as input, x – displacement.

2.1 Linearized Model

The inverted pendulum, , is described by the following nonlinear mathematical model

$$(M + m)\ddot{x} - ml\ddot{\theta}cos + ml\dot{\theta}^2 sin\theta = F$$
$$ml^2\ddot{\theta} - ml\ddot{x}cos\theta - mglsin\theta = 0$$

$$X_1 = x, X_2 = \dot{x}, X_3 = \theta \; X_4 = \dot{\theta}$$

$$f_1 = X_2$$

$$f_2 = \frac{u - ml(sinX_3){X_4}^2 + mgcosX_3 sinX_3}{[(M + m) + mcos^2 X_3]}$$

$$f_3 = X_4$$

$$f_4 = \frac{u.cosX_1 + (M+m)sinX_1 g - ml(cosX_1 sinX_1){X_2}^2}{(M+m)l - mlcos^2 X_1}$$

2.2 State-Space Model

$\dot{X}_1$=X_2 ,

$$\dot{X}_{2=} -\frac{1}{M}X_2 + \frac{mg}{M}X_3 + \frac{1}{M}F$$

$\dot{X}_3 = X_4$

$$\dot{X}_3 = -\frac{1}{Ml}X_2 + \frac{g}{l}\left(1+\frac{m}{M}\right)X_3 + \frac{1}{MR}F$$

$$\frac{d\delta x}{dt} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{1}{M} & \frac{mg}{M} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{1}{Ml} & \frac{g}{l}\left(1+\frac{m}{M}\right) & 0 \end{bmatrix} \delta_x + \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ \frac{1}{Ml} \end{bmatrix} \delta_u$$

3. Checking Controllability and Observability

State feedback can be applied with all states available.
Supposing that we can check controllability

```
rR=rank(ctrb(A,B))
P=-3*[1 1 1 1];


K=acker(A,B,P)
```

Build observer
Check observability matrix

```
rP=rank(obsv(A,C))
Po=-8+[-1 0 0+j 0-j];
L=(place(A',C',Po))'
```

Po is placed observer poles

4.Design a ectended state-feedback with observer of chosen system using advanced algorithms of control design

Application state feedback with the observer for linear system

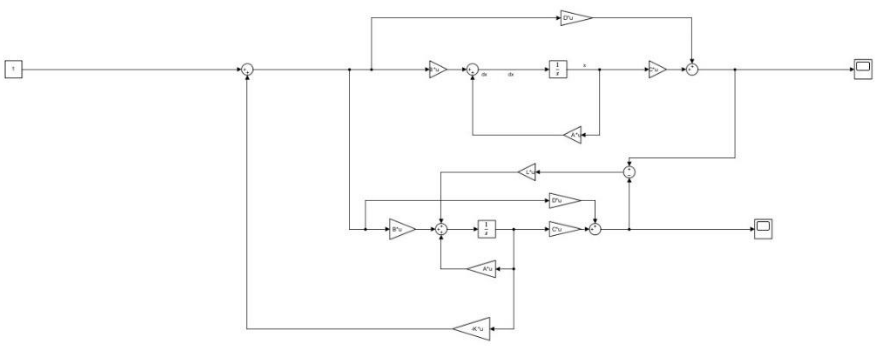


Figure 1. The state-feedback ,state-observer and state extended have structure depiced in Matlab/Simulink

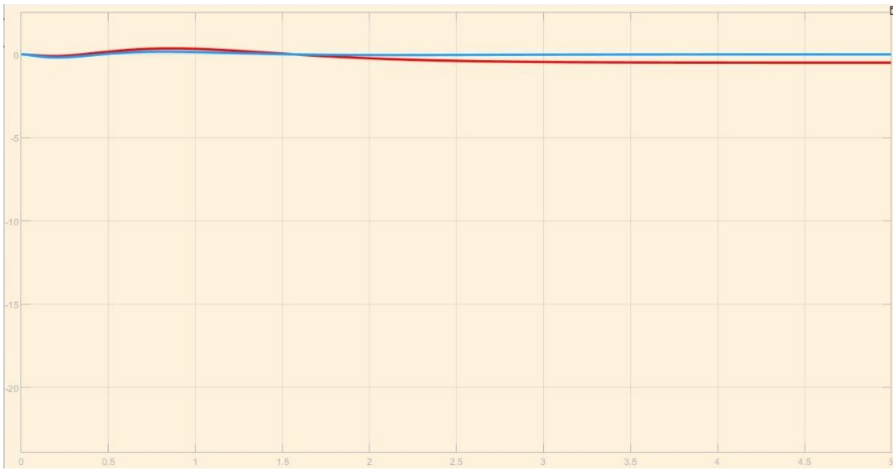


Figure 2.Set-Point Response

5. Design a extended state-feedback with observer of non-linear system using advanced algorithms of control design

Extended State feedback conroller design

```
Pe=-5*[1 1 1 1 1];
Aext=[A zeros(4,1);[1 0 0 0] 0];
Bext=[B;0];
Kext=acker(Aext,Bext,Pe)
Kp=Kext(1:4)
Ki=Kext(5)
```

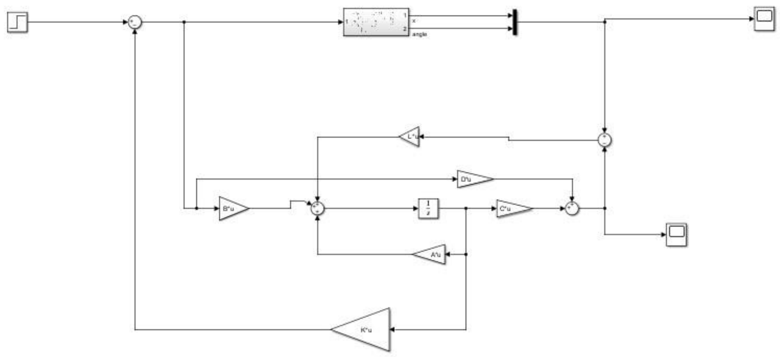


Figure 3. The state-feedback with observer have structure depiced for non-linear in Matlab/Simulink

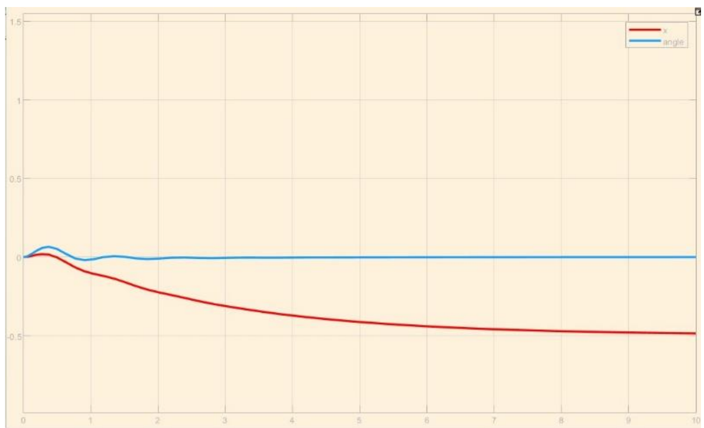


Figure 4.Set-Point Response of non-linear form state feedback controller with observer

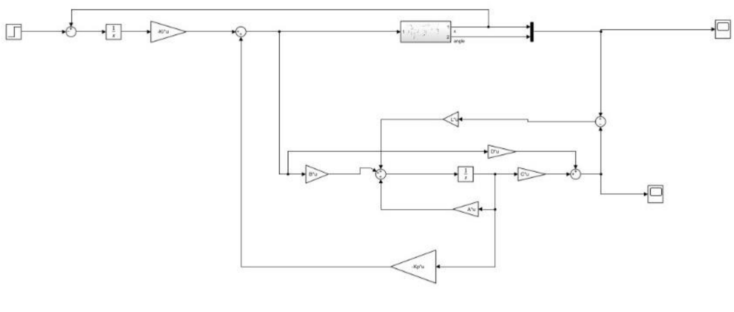


Figure 5. . The extended state-feedback with observer have structure depiced for non-linear in Matlab/Simulink

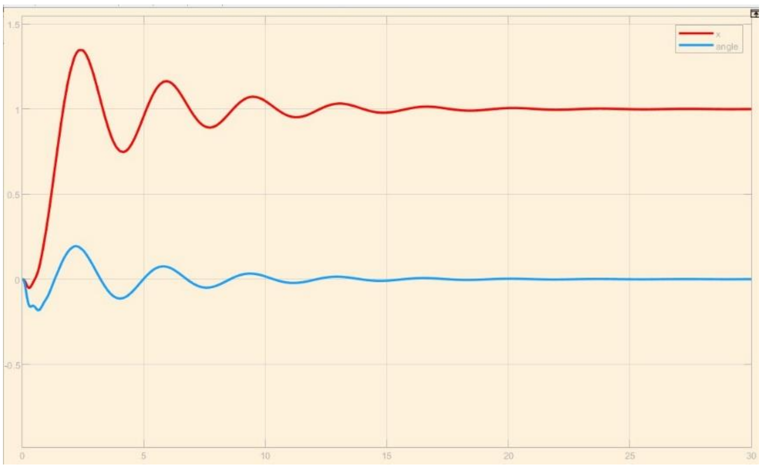


Figure 6.Set-Point Response of non-linear form extended state feedback controller with observer

6.Conclusion

Firstly,the nonlinear model for the inverted pendulum-cart dynamic system, it is required to represent the nonlinear equations into the standard state space form.After that,since the goal of this particular system is to keep the inverted pendulum in the upright position around θ=0.Lastly,extended state-feedback with the observer is an optimal control technique to make the optimal control decisions have been implemented to control the nonlinear inverted pendulum-cart system without and with continuous disturbance input.