# Homework-1 Report

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BGK-516

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# 1. Question

## Answer (1.a)

A 350-digit number can be found between the next two digit numbers.

$$\mathbf{10^{349} \leq \boldsymbol{x} < 10^{350}}$$

We compute this formula for two limit values for 350-digit prime numbers, then take the difference between them to get the result:

$$\mathbf{\frac{10^{350}}{\ln(10^{350})} - \frac{10^{349}}{\ln(10^{349})}}$$

The following is how we can compute these values,

$$\begin{gathered}
\boldsymbol{ln}(\mathbf{10^{350}}) = \mathbf{350 \cdot \ln(10)} \\
\boldsymbol{ln}(\mathbf{10^{349}}) = \mathbf{349 \cdot \ln(10)} \\
\boldsymbol{ln}(\mathbf{10}) \approx \mathbf{2.302585} \\
\boldsymbol{ln}(\mathbf{10^{350}}) = \mathbf{350 \times 2.302585 \approx 805.905} \\
\boldsymbol{ln}(\mathbf{10^{349}}) = \mathbf{349 \times 2.302585 \approx 803.603}
\end{gathered}$$

Let's now compute each term:

$$\begin{gathered}
\mathbf{\frac{10^{350}}{805.905} \approx 1.2404 \times 10^{347}} \\
\mathbf{\frac{10^{349}}{803.603} \approx 1.2444 \times 10^{346}}
\end{gathered}$$

Finnaly we observe that,

$$\mathbf{1.2404 \times 10^{347} - 1.2444 \times 10^{346} \approx 1.11596 \times 10^{347}}$$

Thus, the number of 350-digit prime numbers is roughly $\mathbf{1.11596 \times 10^{347}}$

## Answer (1.b)

The Binomial distribution can be used to determine the likelihood of selecting the same prime number twice or more when selecting at random from a pool of 2 million 350-digit prime numbers. Although there are other approaches, I preferred to use the binomial method. Here, $\boldsymbol{N}$ is the total of all prime numbers with 350 digits. Out of these numbers, we selected $\boldsymbol{n}$ prime numbers.

$$\boldsymbol{N \approx 1.11596 \times 10^{347}}$$

$$\boldsymbol{n = 2,000,000}$$

$$\boldsymbol{p = \frac{1}{N} \approx \frac{1}{1.11596 \times 10^{347}}}$$

The following formula can be used to represent the likelihood of choosing at least two identical primes.

$$\boldsymbol{P(X \geq 2) = 1 - P(X = 0) - P(X = 1)}$$

The following equation represents the scenario in which no prime number is chosen once again.

$$\boldsymbol{P(X = 0) = (1 - p)^n \approx e^{-np}}$$

We determine the likelihood that no prime number is selected after solving the equation and changing the pertinent values.

$$\boldsymbol{np = 2,000,000 \times \frac{1}{1.11596 \times 10^{347}} \approx 1.792 \times 10^{-341}}$$

$$\boldsymbol{P(X = 0) \approx 1}$$

Let's now look at the scenario in which each prime number is chosen no more than once.

$$\boldsymbol{P(X = 1) = n \cdot p \cdot (1 - p)^{n-1}}$$

We determine the likelihood that each prime number is chosen no more than once after solving the equation and changing the pertinent values.

$$\boldsymbol{P(X = 1) \approx 2{,}000{,}000 \times \frac{1}{1.11596 \times 10^{347}} \approx 0}$$

As a result, we will obtain the following result regarding the probability of selecting at least two identical primes. Because the result we calculated for 1 is close to 0, this value is expected to be much lower for 2.

$$\boldsymbol{P(X \geq 2) \approx 0}$$

I used the gmp library to test the expressions mentioned earlier in the C programming language. Below are the results of the program and the values I acquired.

$$\boldsymbol{P(X = 0) = 1}$$

$$\boldsymbol{P(X = 1) = 1.7921789311444854 6543 \times 10^{-309}}$$

$$\boldsymbol{P(X = 2) = -1.79217893114448546543 \times 10^{-309}}$$

The values for 1 and 2 can be regarded as zero since they are so near to it. The answer I obtain does not indicate that the outcome is incorrect, but rather how sensitive the necessary computation is, even though the value for 2 is negative because it is so near to zero. Only with the hardware I had could I get such an exact value.

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C:\Users\emrecan\CLionProjects\question1\cmake-build-debug\question1.exe
P(X = 0): 1
P(X = 1): 0.000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000017921789
3114448546543
P(X >= 2): -0.0000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000179217
893114448546543
```