HW1 (2,3)

2-) Use a fixed-point iteration method to determine a solution accurate to within $10^{(-2)}$ for $x^4 - 3x^2 - 3 = 0$ on [1, 2]. $Use\, p_0 = 1$.

$$x^4 = 3x^2 + 3$$
$$x = \sqrt[4]{(3x^2+3)} \rightarrow x = g(x)$$

$$Iter\,1 \rightarrow p_0 = x = 1 \rightarrow (3*1^2+3)^{(1/4)} = p_1 = 1{,}565084580 \rightarrow p_1 - p_0 = 0{,}565 > 0.01$$

$$Iter\,2 \rightarrow p_1 = x = 1{,}565084580 \rightarrow (3*1{,}565084580^2+3)^{(1/4)} = p_2 = 1{,}793572879 \rightarrow p_2 - p_1 = 0{,}228 > 0.01$$

$$Iter\,3 \rightarrow p_2 = x = 1{,}793572879 \rightarrow (3*1{,}793572879^2+3)^{(1/4)} = p_3 = 1{,}885943743 \rightarrow p_3 - p_2 = 0{,}092 > 0.01$$

$$Iter\,4 \rightarrow p_3 = x = 1{,}885943743 \rightarrow (3*1{,}885943743^2+3)^{(1/4)} = p_4 = 1{,}922847844 \rightarrow p_4 - p_3 = 0{,}036 > 0.01$$

$$Iter\,5 \rightarrow p_4 = x = 1{,}922847844 \rightarrow (3*1{,}922847844^2+3)^{(1/4)} = p_5 = 1{,}937507540 \rightarrow p_5 - p_4 = 0{,}0146 > 0.01$$

$$Iter\,6 \rightarrow p_5 = x = 1{,}937507540 \rightarrow (3*1{,}937507540^2+3)^{(1/4)} = p_6 = 1{,}943316930 \rightarrow p_6 - p_5 = 0{,}058 < 0.01$$

kök : `1,943316930 , 6. iterasyonda bulundu`

Teorik olarak gerekli iterasyon sayısını bulabilmek için ; fonksiyonun türevini alalım $\rightarrow$

$$g'(x) = \frac{3}{2} \cdot \frac{x}{\sqrt[4]{(3x^2+3)^3}}\, ve$$

$$g'(x) < k < 1, [1,2]\, uç\, degerleri\, için : g'(1) = 0{,}39127\; g'(2) = 0{,}393598$$

$$corollary : |p_n - p_{(n-1)}| < \frac{k^n}{1-k} |p_1 - p_0| < 10^{-2}$$

$$k = 0{,}393598 , p_0 = 1 , p_1 = 1{,}565$$

$$0{,}565 . \frac{0{,}393598^n}{1 - 0{,}393598} < 10^{-2}$$

$$n \log(0{,}393598) < \log(0.0107)$$

$$n > 4.877 \rightarrow n = 5$$

3-)
EXERCISE 4

Let f (x) = $-x^3 - \cos x$. With p 0 = −1 and p 1 = 0, find p 3 .

a. Use the Secant method.

b. Use the method of False Position.

$$Secant\ Methodu\ formülü: p_{(n-1)}-\frac{f(p_{(n-1)})(p_{(n-1)}-p_{(n-2)})}{f(p_{(n-1)})-f(p_{(n-2)})}$$

$Elimizde\ p_0\ ve\ p_1\ olduğuna\ göre\ önce\ p_2\ yi, sonra\ p_2\ ve\ p_1\ i\ kullanarak\ p_3\ ü\ bulabiliriz.$

a.

$$p_2=p_1-\frac{f(p_1)(p_1-p_0)}{f(p_1)-f(p_0)}=0-\frac{f(0).1}{f(0)-f(-1)}=-\frac{-1}{-1-(1-\cos(-1))}$$

$$p_2=\frac{1}{-2+0.5403}=-0.6851$$

$$p_3=p_2-\frac{f(p_2)(p_2-p_1)}{f(p_2)-f(p_1)}=-0.6851-\frac{f(-0.6851)(-0.6851-0)}{f(-0.6851)-f(0)}$$

$$p_3=-0.6851-\frac{-0.6851(0.32156-(-0.687))}{-(-0.32156-(-0.687)+1)}=-1.252$$

b.

$$False\ Position\ Methodu\ Formülü: p_3=p_2-(\frac{f(p_2)(p_2-p_0)}{f(p_2)-f(p_0)})$$

$$f(p_2)=-0.454, f(p_0)=0.46$$

$$p_3=-0.6851-(\frac{(-0.454)(-0.6851+1)}{(-0.454-0.46)})$$

$$p_3=0.8411$$

EXERCISE 5

Use Newton's method to find solutions accurate to within $10^{(-4)}$ for the following problems.

a. $x^3-2x^2-5=0,[1,4]$

b. $x^3+3x^2-1=0,[-3,-2]$

c. $x-\cos x=0,[0,\pi/2]$

d. $x-0.8-0.2\sin x=0,[0,\pi/2]$

a.

$$Newton\ Methodu\ Formulu: p_{(n+1)}=p_n-\frac{f(p_n)}{f'(p_n)}$$

$$f'(x)=3x^2-4x$$

$$p_0 \rightarrow 3\ seçersek$$

$$p_1=3-4/15=2{,}7 \rightarrow 3-2{,}7>10^{(-4)}$$

$$p_2=2{,}7-0.103/11.07=2{,}7-0{,}00930442637=2.690695 \rightarrow 2{,}7-2.690695<10^{(-4)}$$

$$p_2=2.690695, 2.\ iterasyonda\ degere\ ulasmıs\ oluruz.$$