

CAM MECHANISMS

Cam and linkage mechanisms are alternatives, in that motion transmission is non-linear in both





Comparison: (Favorable + ; Unfavorable -)

Factor	Cam Mech.	Linkage	Remark
Ease of design	+	-	Dwell so easy
Compactness	+	-	Few parts
Ease of adjustment	+	-	Just change cam
Cost	-	+	
Wear resistance, life	-	+	
High speed performance	-	+	Jump phenomenon

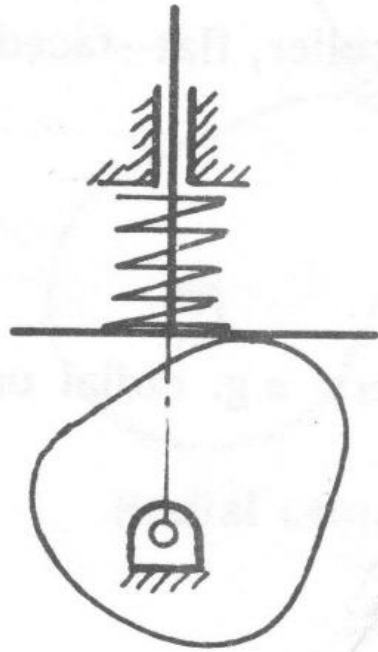
In cam mechanisms (generally):

- Input link is *cam*
- Output link is *follower*

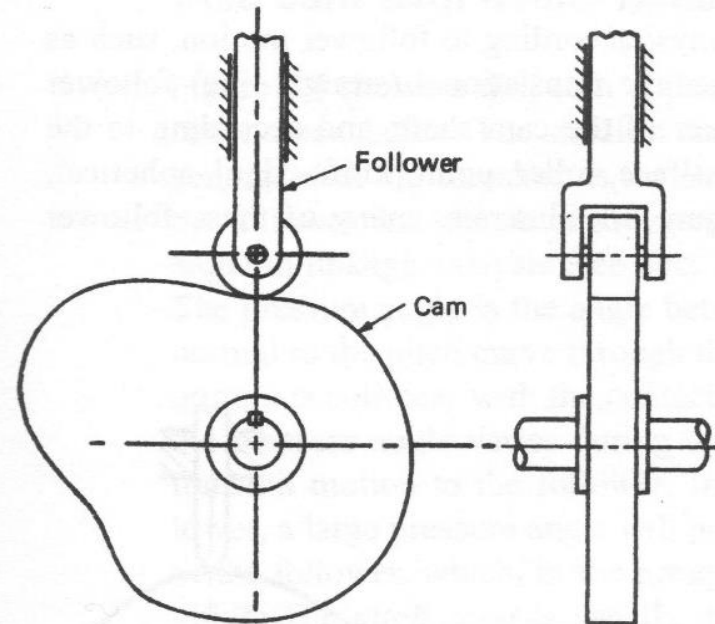
CLASSIFICATION OF CAM MECHANISMS

- According to the shape of the cam 
 - Disk (Radial, Plate), Wedge, Cylindrical, Face, Conical
- According to the shape of the follower 
 - Knife-edge, Flat-faced, Roller, Spherical followers
- According to the type of follower motion 
 - Oscillating follower, translating follower (in-line/offset)
- According to the way contact is maintained between cam and follower 
 - Force closed (spring, gravity, etc.), Form closed

Cam Mechanisms

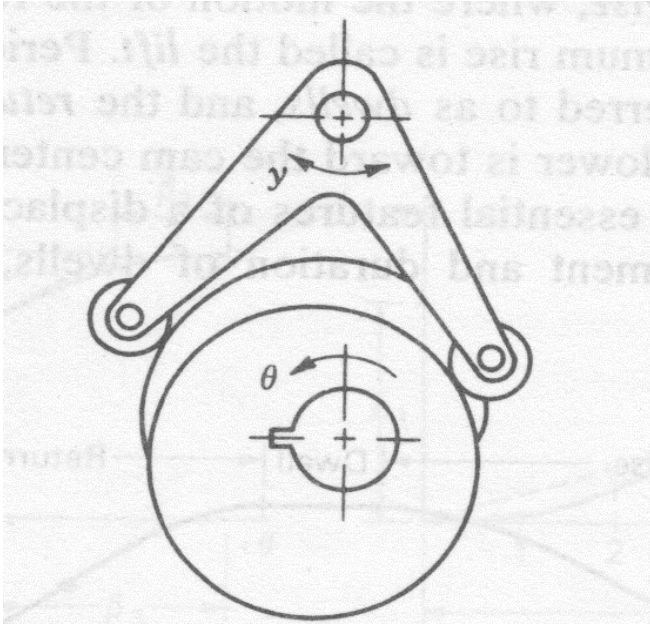


Force-closed radial (disk) cam with in-line translating flat-faced follower

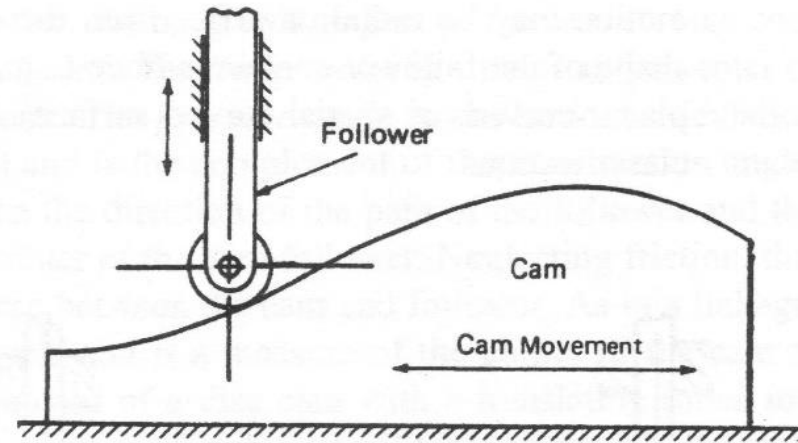


Radial (disk) cam with in-line translating roller follower

Cam Mechanisms

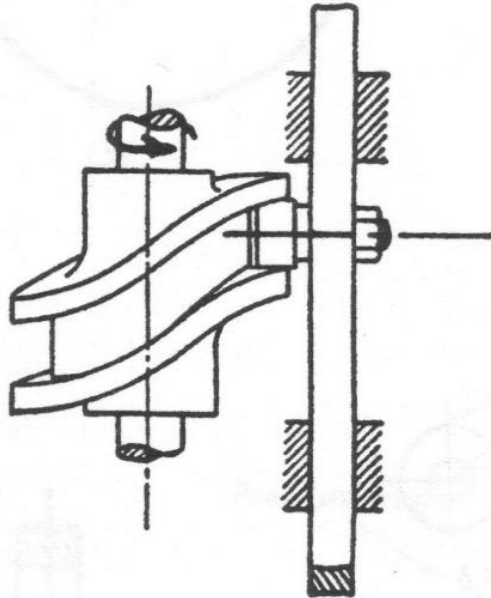


*Conjugate disk cams
with oscillating roller
follower*

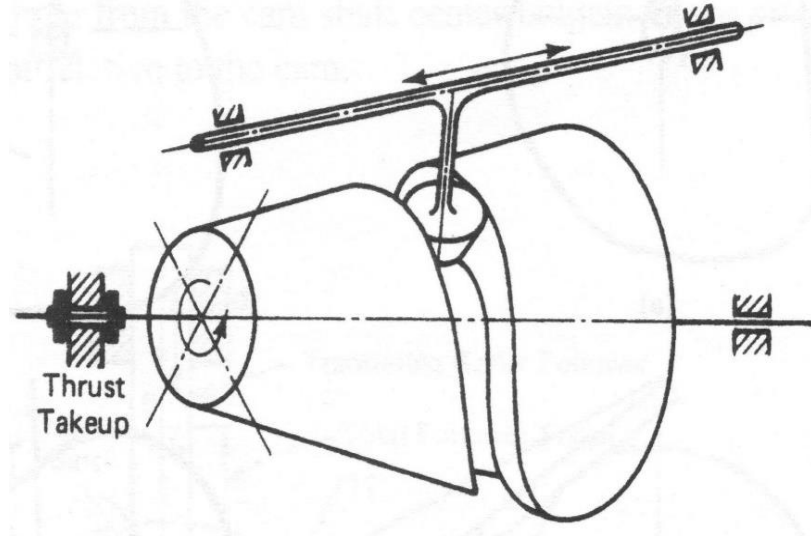


*Wedge cam with
translating roller
follower*

Cam Mechanisms

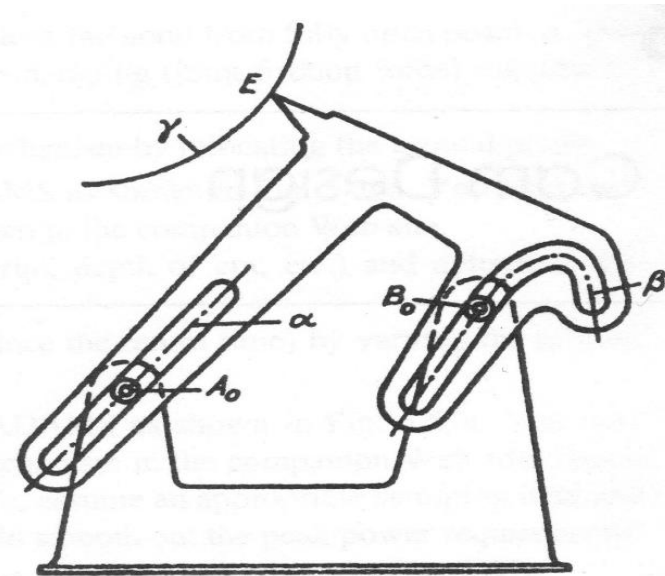
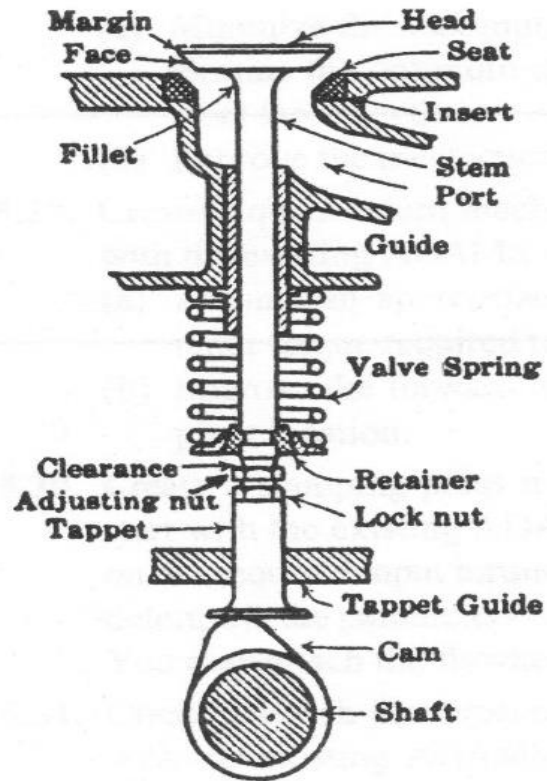


*Cylindrical cam with
translating roller
follower*



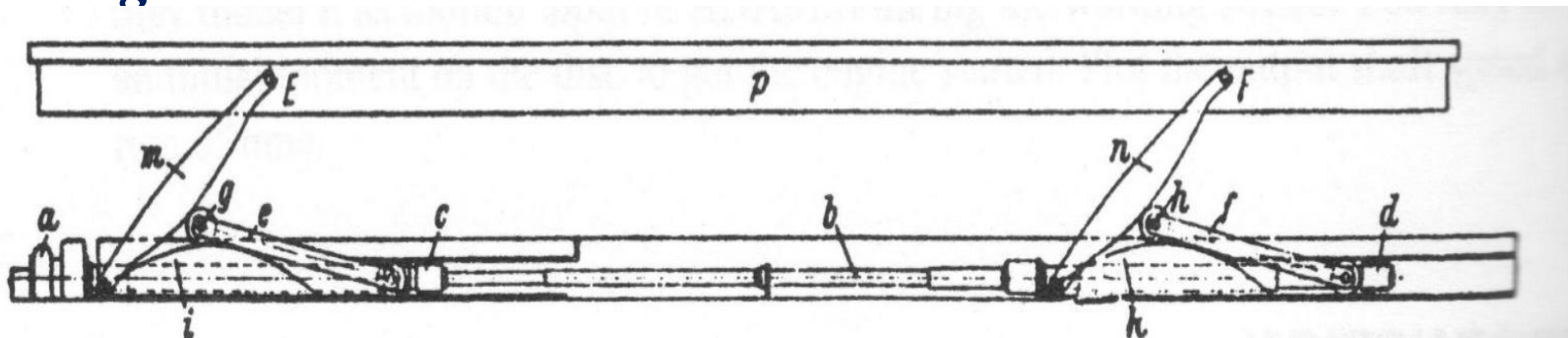
*Conical cam with
translating follower*

Applications of Cam Mechanisms



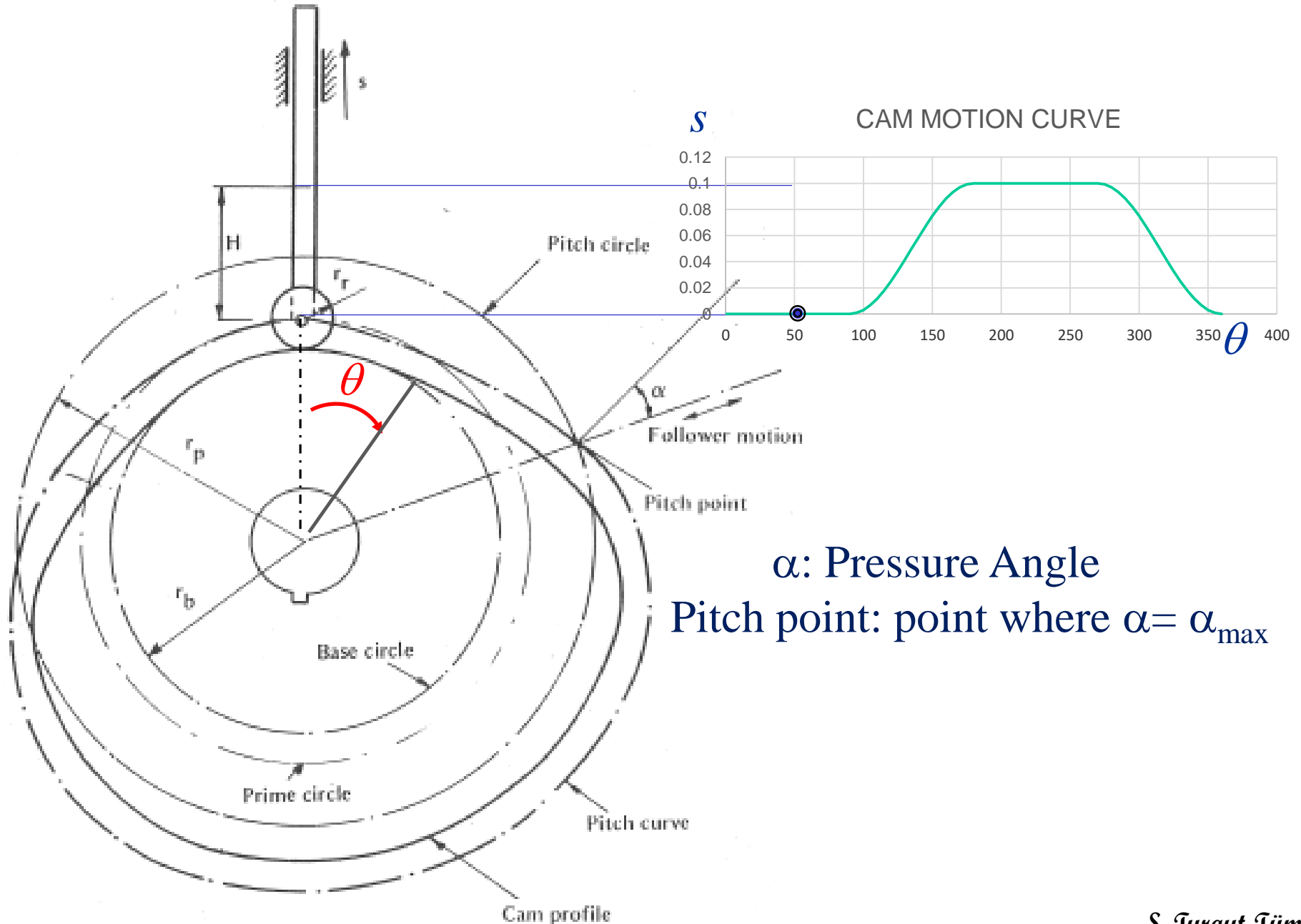
Path generation cams

Function generation cams



Motion generation cams

CAM NOMENCLATURE



α : Pressure Angle

Pitch point: point where $\alpha = \alpha_{\max}$

Global Follower Motion Characteristics

- **Rise:** Motion of follower away from cam shaft centre
- **Return:** Motion of follower towards cam shaft centre
- **Dwell:** Follower at rest

Types of Follower Motion

- R-R (*Rise-Return*)
- R-D-R (*Rise-Dwell-Return*)
- R-R-D (*Rise-Return-Dwell*)
- R-D-R-D (*Rise-Dwell-Return-Dwell*)

STANDARD MOTION CURVES

For Rise and Return

- Controlled motion characteristics (low inertia forces, avoiding shock etc.)
- Reproducibility
- Ease of layout

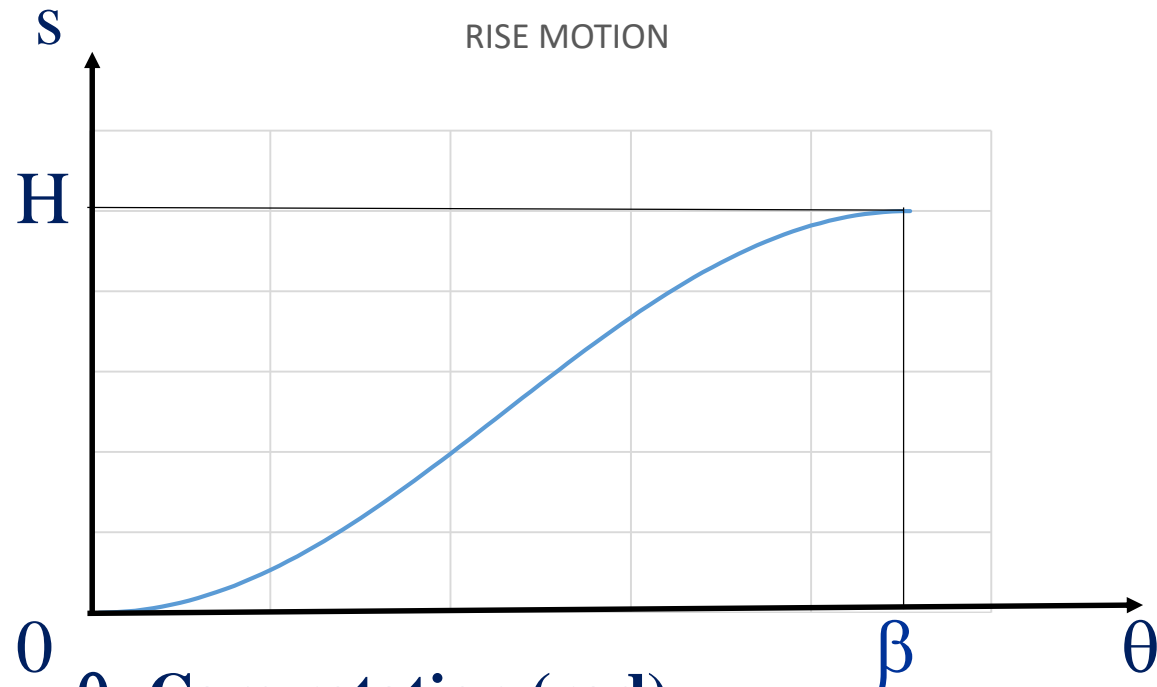
Cams are also known by the name of the motion they impart on to the follower
(e.g. Cycloidal Cam, Parabolic Cam, etc.)

STANDARD CAM MOTION CURVES

- **Basic Trigonometric Curves**
 - ✓ **Simple Harmonic Motion**
 - ✓ **Cycloidal Motion**
 - ✓ **Modified Harmonic Motion**
- **Basic Polynomial Curves**
 - ✓ **Uniform Motion**
 - ✓ **Parabolic (Constant Acceleration) Motion**
 - ✓ **Cubic #1**
 - ✓ **Cubic #2**
 - ✓ **Higher Order Polynomial Curves (3-4-5, 5-6-7, etc.**
- **Combination Curves**
 - ✓ **Trapezoidal Acceleration**
 - ✓ **Modified Trapezoidal Acceleration**
 - ✓ **Spline**

Equations of Standard Motion Curves

Equations of standard motion curves are given for a rise section starting from $\theta = 0$



θ : Cam rotation (rad)

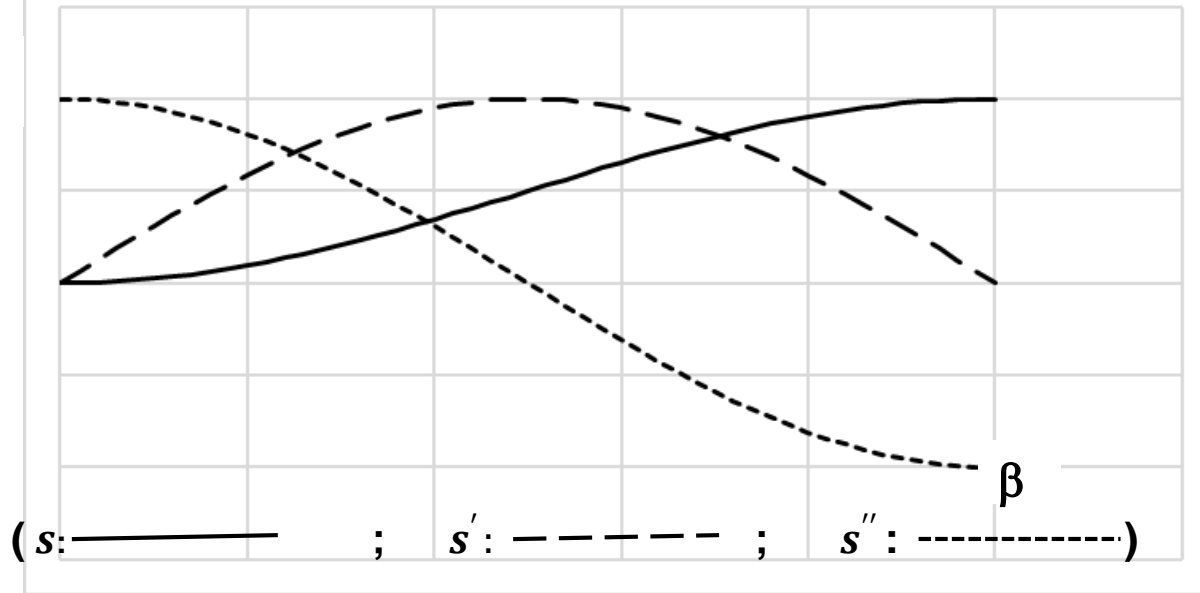
s : Follower displacement

H : Follower Lift

β : Rise Period (rad)

SIMPLE HARMONIC MOTION

H, C_v, C_a

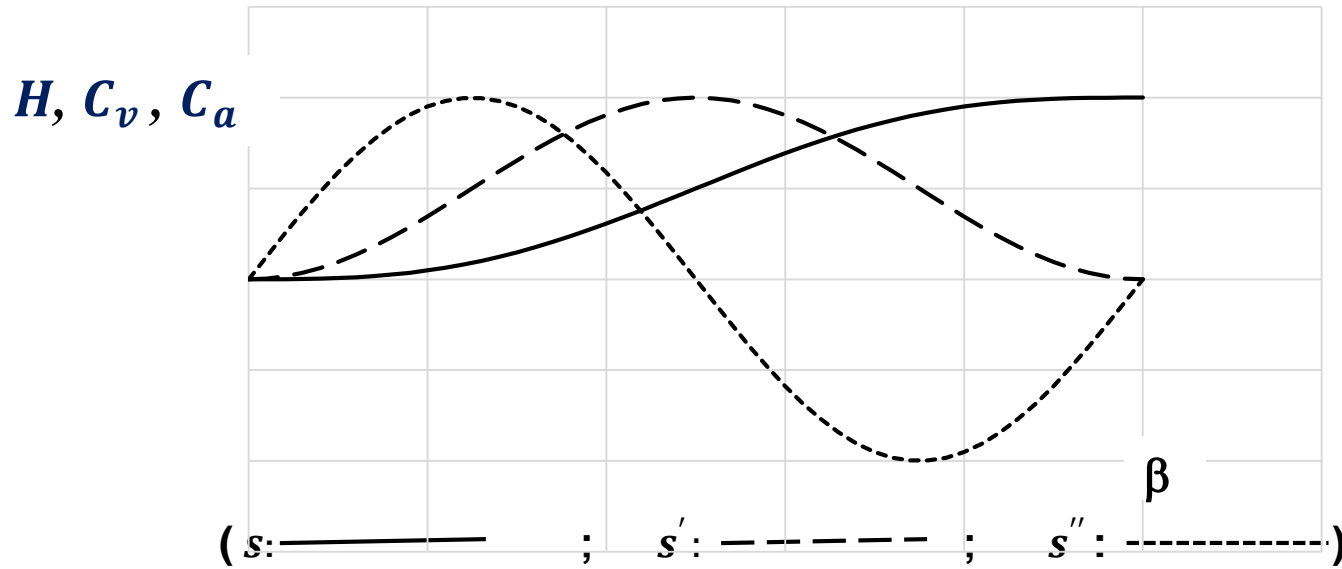


$$s = \frac{H}{2} \left(1 - \cos \frac{\pi \theta}{\beta} \right) ; s' = \frac{\pi H}{2\beta} \sin \frac{\pi \theta}{\beta} ; s'' = \frac{\pi^2 H}{2\beta^2} \cos \frac{\pi \theta}{\beta}$$

$$s'_{max} = \frac{C_v H}{\beta} ; C_v = \pi/2$$

$$s''_{max} = \frac{C_a H}{\beta^2} ; C_a = \pi^2/2$$

CYCLOIDAL MOTION



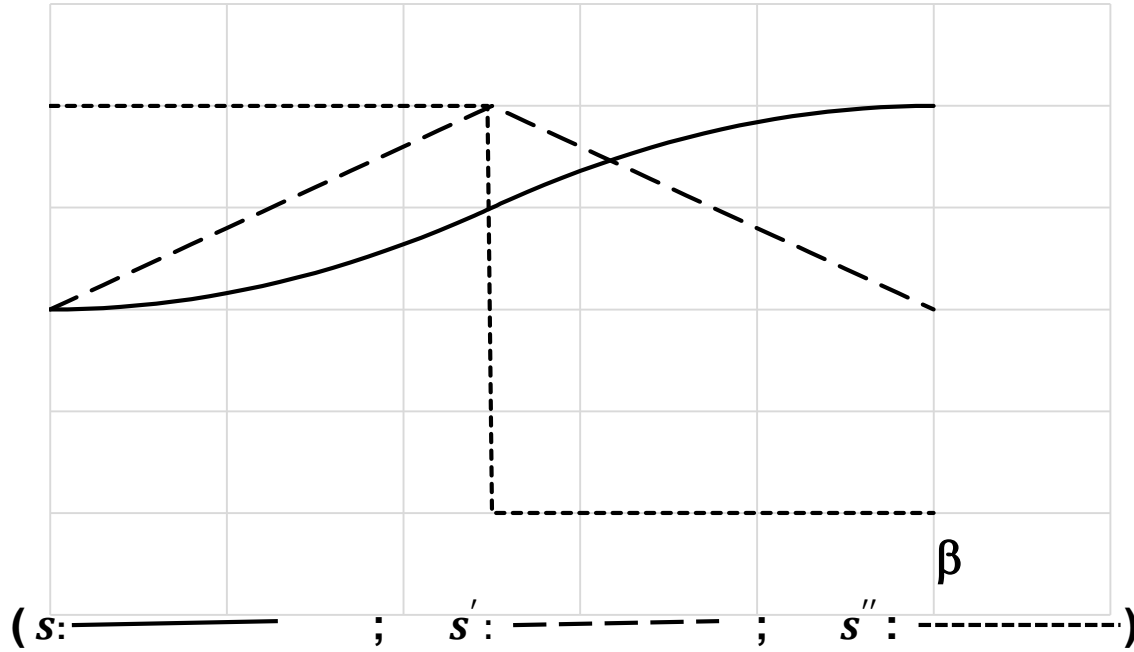
$$s = H\left(\frac{\theta}{\beta} - \frac{1}{2\pi} \sin \frac{2\pi\theta}{\beta}\right) ; s' = \frac{H}{\beta} (1 - \cos \frac{2\pi\theta}{\beta}) ; s'' = \frac{2\pi H}{\beta^2} \sin \frac{2\pi\theta}{\beta}$$

$$s'_{max} = \frac{C_v H}{\beta} ; C_v = 2$$

$$s''_{max} = \frac{C_a H}{\beta^2} ; C_a = 2\pi$$

PARABOLIC MOTION

H, C_v, C_a



$$0 \leq \theta \leq \frac{\beta}{2} \rightarrow s = 2H(\theta/\beta)^2 ; \quad s' = \frac{4H}{\beta} \frac{\theta}{\beta} ; \quad s'' = \frac{4H}{\beta^2}$$

$$\frac{\beta}{2} \leq \theta \leq \beta \rightarrow s = H[1 - 2(1 - \theta/\beta)^2] ; \quad s' = \frac{4H}{\beta} (1 - \frac{\theta}{\beta}) ; \quad s'' = -\frac{4H}{\beta^2}$$

$$s'_{max} = \frac{C_v H}{\beta} ; \quad C_v = 2$$

$$s''_{max} = \frac{C_a H}{\beta^2} ; \quad C_a = 4$$

Relationship Between Derivatives of Displacement Diagram and Follower Motion

Let; $s' = \frac{ds}{d\theta}$; $s'' = \frac{d^2s}{d\theta^2}$; $s''' = \frac{d^3s}{d\theta^3}$; etc...

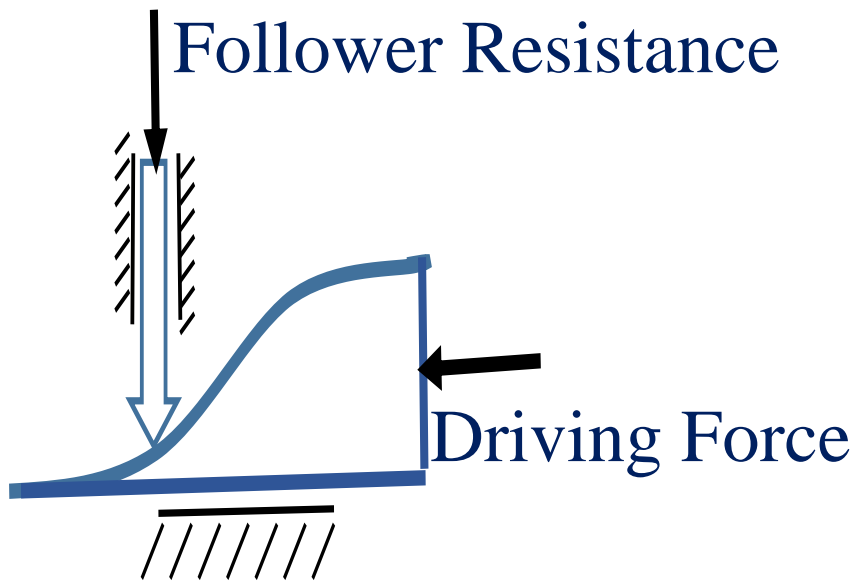
If cam-shaft speed ω (rad/s) is constant, then:

- Follower velocity: $v = \dot{s} = \omega s'$
- Follower Acceleration: $a = \ddot{s} = \omega^2 s''$
- Follower Jerk: $j = \dddot{s} = \omega^3 s'''$

Why higher derivatives of $s(\theta)$ are critical?

- We want low $s' = \frac{ds}{d\theta}$: If it is high, then the force transmission is poor; large contact forces are developed to overcome small follower resistance

This relation can readily be observed for a wedge cam with translating knife-edge follower



We will later establish the relation between force transmission characteristics (*pressure angle*) and s' for a disk cam with reciprocating inline roller follower

Why higher derivatives of $s(\theta)$ are critical?

➤ We want low $s'' = \frac{d^2s}{d\theta^2}$: If it is high, then:

- Inertia force and resulting reactions are high, due to high *follower acceleration*
- Radius of curvature of cam contour is small, resulting in high contact (Hertz) stresses (If s'' is very large, cam surface becomes pointed!)
- Larger cam size is required in order to avoid a phenomenon called *undercutting*, resulting in large space occupied and large unbalanced mass.

Undercutting will be discussed later on.

Why higher derivatives of $s(\theta)$ are critical?

- We want low $s''' = \frac{d^3 s}{d\theta^3}$: If it is high, then acceleration and consequently forces show sharp changes (shock!)

We even want higher derivatives of $s(\theta)$ beyond third derivative as low as possible, at least finite, for a smooth operation...

Normalization

Recall expressions for maximum follower velocity, acceleration and jerk, for a rise period β , lift H and cam-shaft speed ω :

$$v_{max} = \frac{C_v H}{\beta} \omega ; a_{max} = \frac{C_a H}{\beta^2} \omega^2 ; j_{max} = \frac{C_j H}{\beta^3} \omega^3$$

To compare various motion curves, let:

$$H = 1\text{m(or rad)} ; \beta = 1 \text{ rad} ; \omega = 1 \text{ rad/s}$$

Then:

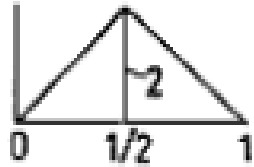
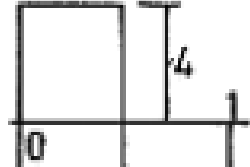
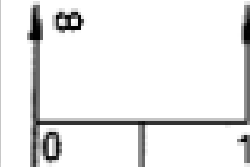
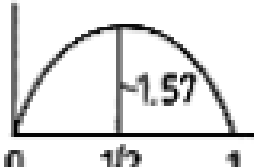
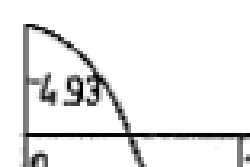

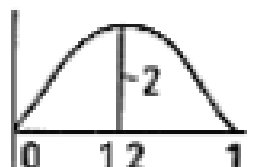
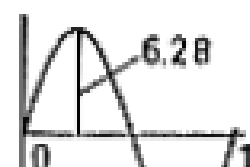
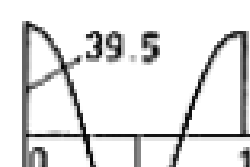
C_v is indicative of maximum follower velocity

C_a is indicative of maximum follower acceleration

C_j : is indicative of maximum follower jerk

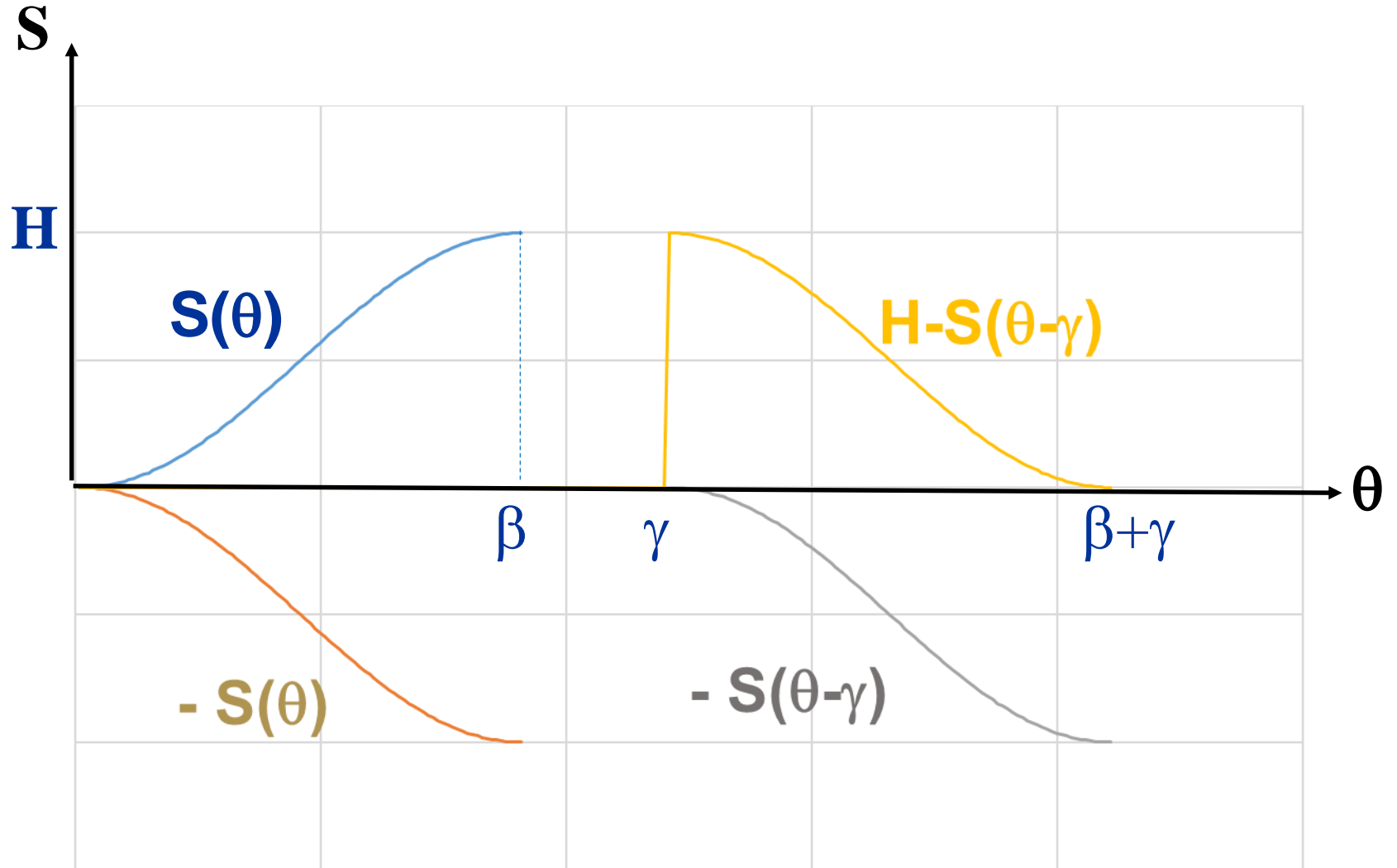
Therefore, low values of C_v, C_a, C_j are indicative of smooth operation

Normalized Cam Motion Curves

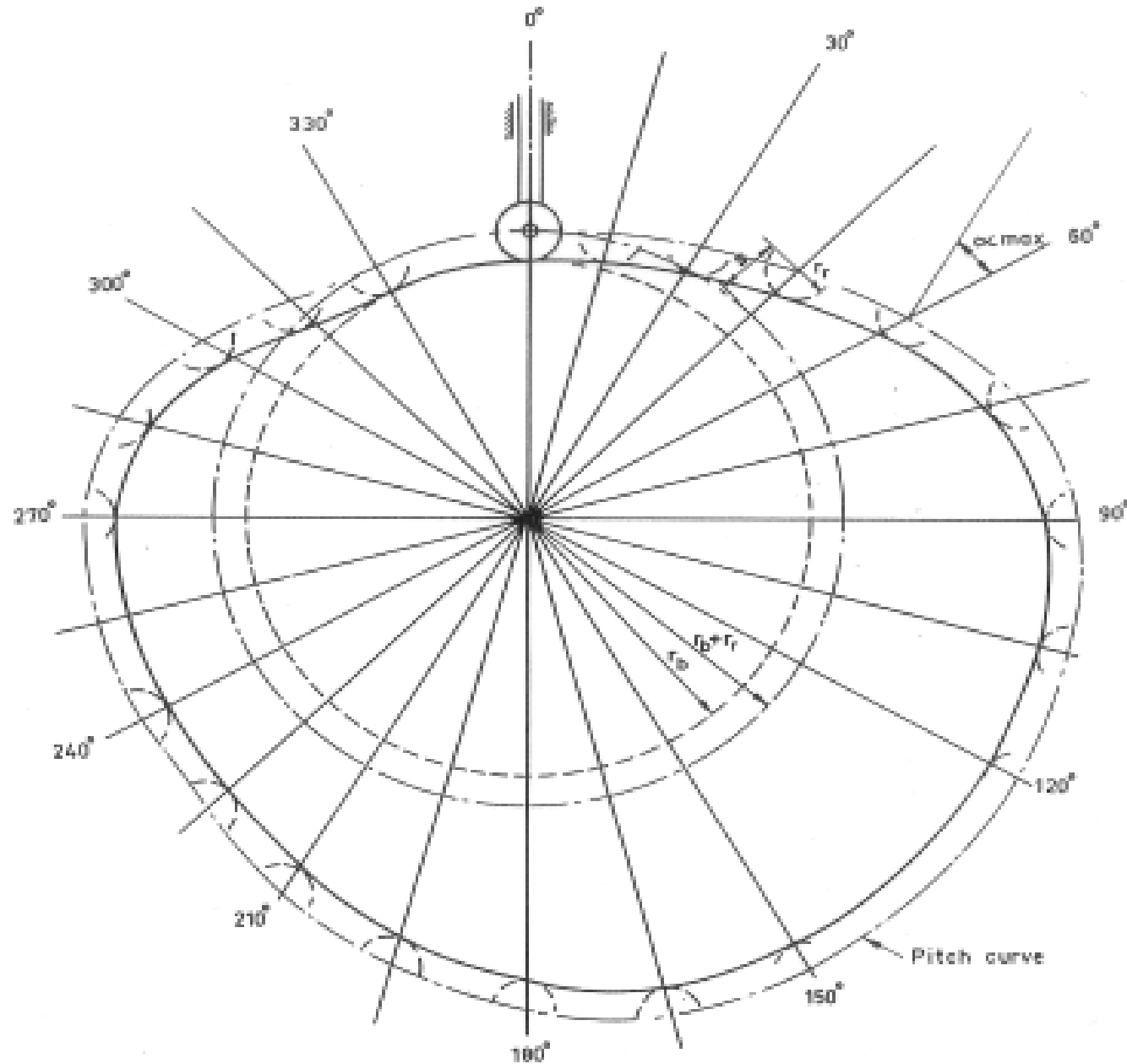
Displacement	Velocity	Acceleration	jerk	Comments
<p>Parabolic</p> $0 < \theta < \frac{1}{2}$ $s = 2\theta^2$ $\frac{1}{2} < \theta < 1$ $s = 1 - 2(1 - \theta)^2$	 <p>$C_v = 2$</p>	 <p>$C_a = 4$</p>	 <p>$C_j = \infty$</p>	Backlash serious
<p>Simple Harmonic</p> $s = \frac{1}{2} (1 - \cos \pi \theta)$	 <p>$C_v = 1.57$</p>	 <p>$C_a = 4.93$</p>	 <p>$C_j = \infty$</p>	Moderate
<p>Cycloidal</p> $s = \frac{1}{\pi} (\pi \theta - \frac{1}{2} \sin 2\pi \theta)$	 <p>$C_v = 2$</p>	 <p>$C_a = 6.28$</p>	 <p>$C_j = 39.5$</p>	Excellent for high speeds

Displacement	Velocity	Acceleration	jerk	Comments
Parabolic $0 < \theta < \frac{1}{2}$ $s = 2\theta^2$ $\frac{1}{2} < \theta < 1$ $s = 1 - 2(1 - \theta)^2$	 $C_v = 2$	 $C_a = 4$	 $C_j = \infty$	Backlash serious
Cubic #1 $0 < \theta < \frac{1}{2}$ $s = 4\theta^3$ $\frac{1}{2} < \theta < 1$ $s = 1 - 4(1 - \theta)^3$	 $C_v = 3$	 $C_a = 12.5$	 $C_j = \infty$	Not suggested
Cubic #2 $s = 3\theta^2 - 2\theta^3$	 $C_v = 1.5$	 $C_a = 6$	 $C_j = \infty$	Not suggested for high speeds
Simple Harmonic $s = \frac{1}{2}(1 - \cos \pi \theta)$	 $C_v = 1.57$	 $C_a = 4.93$	 $C_j = \infty$	Moderate
Cycloidal $s = \frac{1}{\pi}(\pi \theta - \frac{1}{2} \sin 2\pi \theta)$	 $C_v = 2$	 $C_a = 6.28$	 $C_j = 39.5$	Excellent for high speeds
Double Harmonic $s = \frac{1}{4} - \frac{1}{2} \cos \pi \theta + \frac{1}{4} \cos 2\pi \theta$	 $C_v = 2$	 $C_a = 5.5$	 $C_j = \infty$	Best for D-R-R cam
Trapezoidal acceleration	 $C_v = 2$	 $C_a = 5.3$	 $C_j = 4.4$	Excellent for high speeds. May have machining difficulty.

Coordinate Transformation



GRAPHICAL CONSTRUCTION OF CAM PROFILE

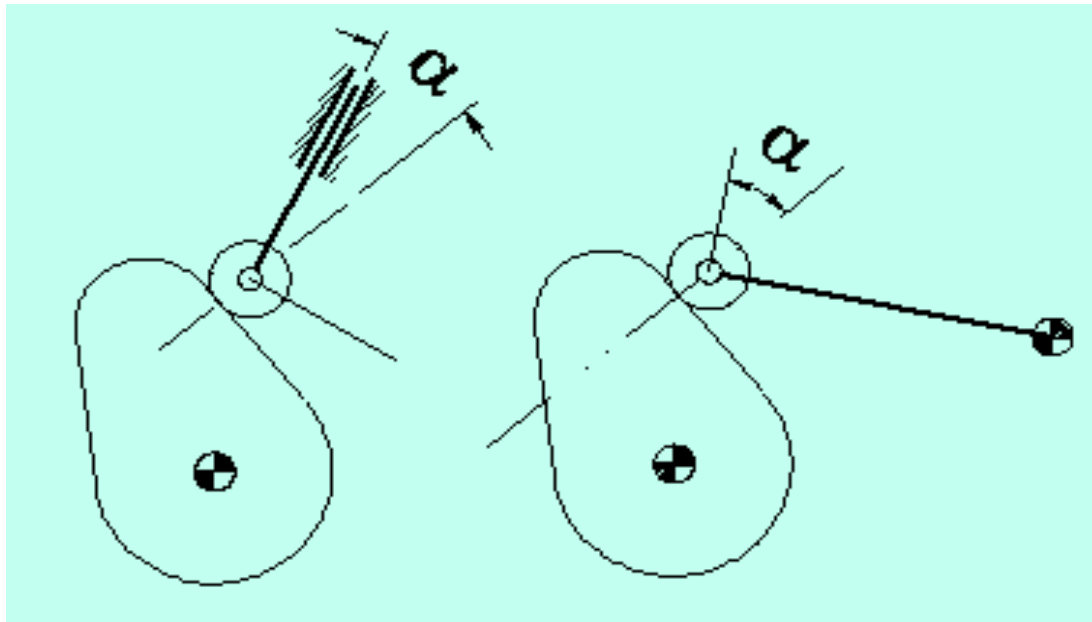


Determination of the Cam Size (r_b)

- Factors imposing the use of small cams:
 - Space occupied
 - Unbalanced mass
 - High surface velocity (wear)
- Factors imposing the use of large cams:
 - Poor force transmission (pressure angle)
 - Small radius of curvature (high contact stresses, undercutting)
 - Size of cam shaft

Pressure Angle (1/2)

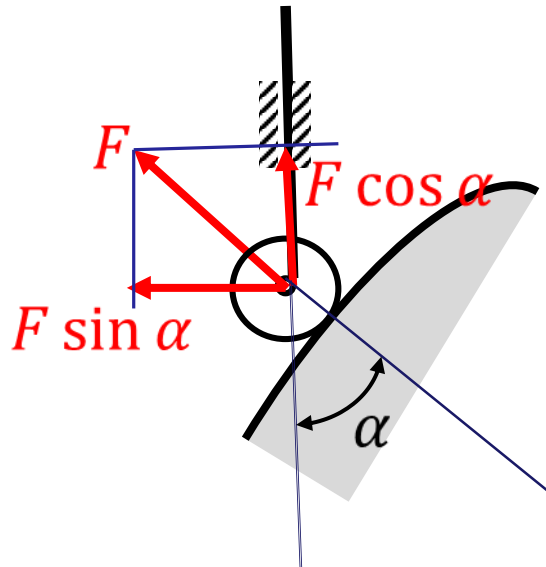
Angle between the common normal to the cam and follower surfaces (direction of contact force) and the direction of follower motion:



α changes as
cam rotates

For flat-faced followers α is zero all over...

Pressure Angle (2/2)



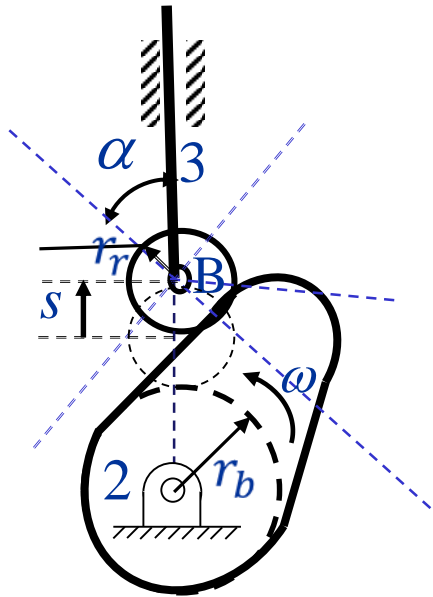
$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha} = \frac{\text{Bearing Force}}{\text{Driving Force}}$$

We want α as small as possible

As a RULE OF THUMB:

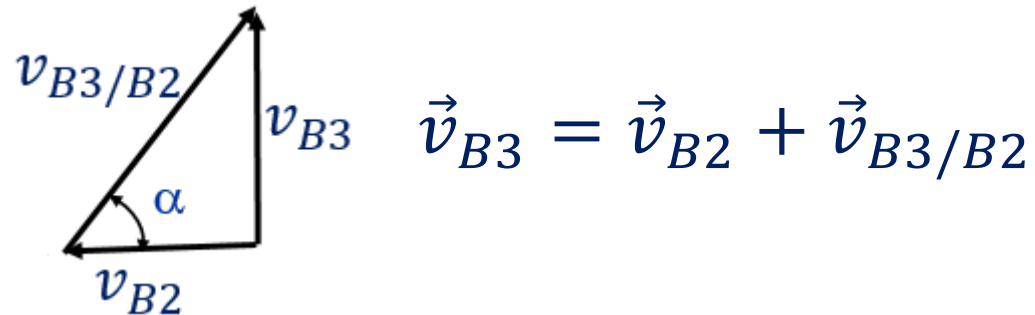
	Translating Roller follower	Oscillating Roller Follower
Allowable α_{\max}	30-40 degrees	50-60 degrees

Pressure Angle of a disk cam with in-line reciprocating roller follower



$$v_{B3} = \dot{s} = \frac{ds}{d\theta} \omega = s' \omega (\uparrow)$$

$$v_{B2} = (r_b + r_r + s) \omega (\leftarrow)$$



$$\tan \alpha = \frac{v_{B3}}{v_{B2}} = \frac{s'}{r_b + r_r + s}$$

$$\tan \alpha_{max} \cong \frac{(s')_{max}}{r_b + r_r + s_{at (s')_{max}}}$$

Pressure Angle of a disk cam with in-line reciprocating roller follower

$$\tan \alpha_{max} \cong \frac{(s')_{max}}{r_b + r_r + s_{at (s')_{max}}}$$

In order to have small α_{max} , we need:

- Small s'_{max}
- Large $r_b + r_r$; which leads to large cam!

Recall: $s'_{max} = C_v \frac{H}{\beta}$; and

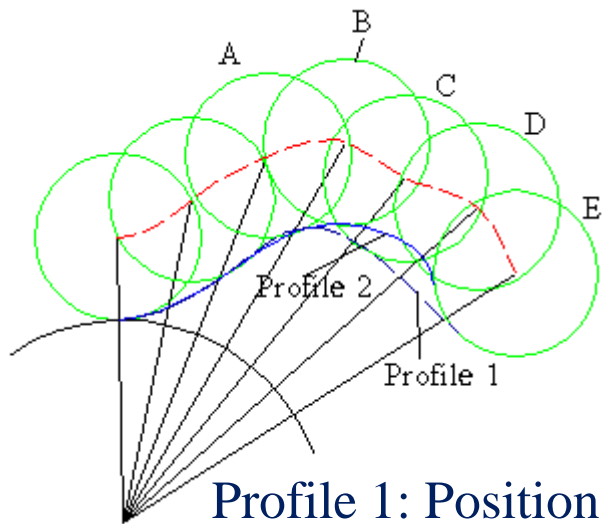
$C_v=1.57$ for SHM and $C_v= 2$ for CYC and PARAB motions

For the same rise H , period β , and pressure angle α_{max} ;
SHM cam will be smaller than corresponding
CYC and PARAB cams...

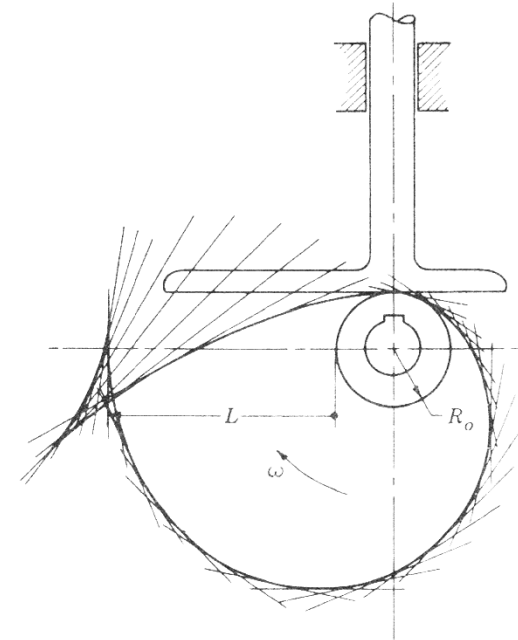
Undercutting

Recall the graphical construction of cam profile:

- Cam is considered fixed and follower moving around the cam;
- Cam profile is the smooth curve tangent to successive positions of the circle or straight line; depending on the type of follower




Profile 1: Positions B-D are missed
Profile 2: Positions C-D are missed



The situation when a smooth cam profile tangent to all successive positions of the follower can not be constructed is called **undercutting**

Undercutting of Cams with Roller Follower

- Cam becomes undercut when the radius of curvature of cam profile is less than the roller radius
- At the verge of undercutting, radius of curvature of pitch curve is zero, in which case all positions of follower are satisfied but the cam becomes pointed! 
- To avoid undercutting within a safe margin, and also to reduce contact stresses, radius of curvature at every point of the cam profile must be greater than an allowable value:

$$\rho_{min} \geq \rho_{all}$$

- On the other hand, large radius of curvature means large cam size (r_b), which is also undesirable
- Relationship between cam parameters and radius of curvature is needed to make a compromise

Undercutting

IN-LINE TRANSLATING ROLLER FOLLOWER

- Cam size effects both pressure angle and radius of curvature
- The following equation gives the radius of curvature at any point on the cam profile:

$$\rho = \frac{[(r_b + r_r + s)^2 + (s')^2]^{3/2}}{(r_b + r_r + s)^2 + 2(s')^2 - (r_b + r_r + s)s''} - r_r$$

It should be noted that to get large radius of curvature,
we need large r_b but small r_r

Undercutting

IN-LINE TRANSLATING ROLLER FOLLOWER

- To find the position at which critical value of ρ occurs, (i.e. solving $d\rho/d\theta=0$) for different cam motions is far from being easy!
- Instead, one can plot the radius curvature for discrete values of $0 \leq \theta \leq 2\pi$ for small intervals from:

$$\rho = \frac{[(r_b + r_r + s)^2 + (s')^2]^{3/2}}{(r_b + r_r + s)^2 + 2(s')^2 - (r_b + r_r + s)s''} - r_r$$

- Determine the minimum radius of curvature ρ_{min} from ρ versus θ plot for a specified cam size (r_b, r_r)

Procedure to find base circle radius (r_b) and roller radius (r_r) for disk cams with in-line translating roller follower

- Find minimum permissible $r_b + r_r$ based on a specified maximum allowable pressure angle using:

$$\tan \alpha_{max} \cong \frac{(s')_{max}}{r_b + r_r + s_{at (s')_{max}}}$$

- Determine the minimum radius of curvature ρ_{min} , and try different values of r_r until $\rho_{min} \geq \rho_{all}$ is satisfied

Undercutting

IN-LINE TRANSLATING FLAT-FACED FOLLOWER

- As pressure angle is zero all over the cam profile, cam size is determined considering radius of curvature, i.e. undercutting and contact stresses
- The following equation gives the radius of curvature at any point on the cam profile:

$$\rho = r_b + s + s''$$

- Critical value of ρ occurs normally when s'' is at its largest negative value, i.e. $(s'')_{min}$, therefore:

$$\rho_{min} = r_b + s_{at (s'')_{min}} + (s'')_{min} \geq \rho_{all}$$

$$r_b \geq \rho_{all} - (s'')_{min} - s_{at (s'')_{min}}$$

Example

A force-closed radial (disk) cam imparts following motion to the in-line translating roller follower:

- i. Cycloidal rise by 120 mm during $0 \leq \theta \leq 60^\circ$
- ii. Dwell at 120 mm during $60^\circ \leq \theta \leq 180^\circ$
- iii. Cycloidal return by 120 mm during $180^\circ \leq \theta \leq 270^\circ$
- iv. Dwell at 0 mm during $270^\circ \leq \theta \leq 360^\circ$;

where θ is the cam shaft angle, and $\theta=0$ at the beginning of rise.

The cam shaft rotates at a constant speed of 25 rpm.

a) Write down equations for $s(\theta)$, $s'(\theta)$, $s''(\theta)$ for $0^\circ \leq \theta \leq 360^\circ$

b) Sketch the follower displacement (s), follower velocity ($v=ds/dt$), and follower acceleration ($a=dv/dt$) as functions of θ for $0^\circ \leq \theta \leq 360^\circ$.

Example (2/2)

- c) Calculate the peak values (i.e. minimum and maximum) of follower velocity and acceleration, and indicate all the peak values and corresponding θ values on $s(\theta)$, $v(\theta)$, $a(\theta)$ sketches above.
- d) Calculate the position, velocity and acceleration of the follower 1.5s after it starts to rise.
- e) Calculate the minimum prime circle radius ($R_0 = r_b + r_r$) in order that the pressure angle does not exceed 40° over the whole cam profile.
- f) Calculate the maximum roller radius (r_r), so that the minimum radius of curvature of the cam profile is greater than 90 mm.
- g) Calculate the pressure angle (α) and radius of curvature of the cam profile (ρ) for the position of part (d), and using the cam size found in parts (e) and (f)

(a)

$$0 \leq \theta \leq 60^\circ$$

$$\text{RISE: } H = 0.12\text{m} , \beta = \pi/3$$

$$s(\theta) = 0.12 \left(\frac{\theta}{\pi/3} - \frac{1}{2\pi} \sin \frac{2\pi\theta}{\pi/3} \right)$$

$$s'(\theta) = \frac{0.12}{\pi/3} \left(1 - \cos \frac{2\pi\theta}{\pi/3} \right)$$

$$s''(\theta) = 0.12 \frac{2\pi}{(\pi/3)^2} \sin \frac{2\pi\theta}{\pi/3}$$

$$60 \leq \theta \leq 180^\circ \quad \text{DWELL: } H = 0.12 \text{ m}$$

$$s(\theta) = 0.12 ; s'(\theta) = 0 ; s''(\theta) = 0$$

$180 \leq \theta \leq 270^\circ$ RETURN: $H = 0.12\text{m}$, $\beta = \pi/2$, $\gamma = \pi$

$$s(\theta) = 0.12 - 0.12 \left(\frac{\theta - \pi}{\pi/2} - \frac{1}{2\pi} \sin \frac{2\pi(\theta - \pi)}{\pi/2} \right)$$

$$s'(\theta) = -\frac{0.12}{\pi/2} \left(1 - \cos \frac{2\pi(\theta - \pi)}{\pi/2} \right)$$

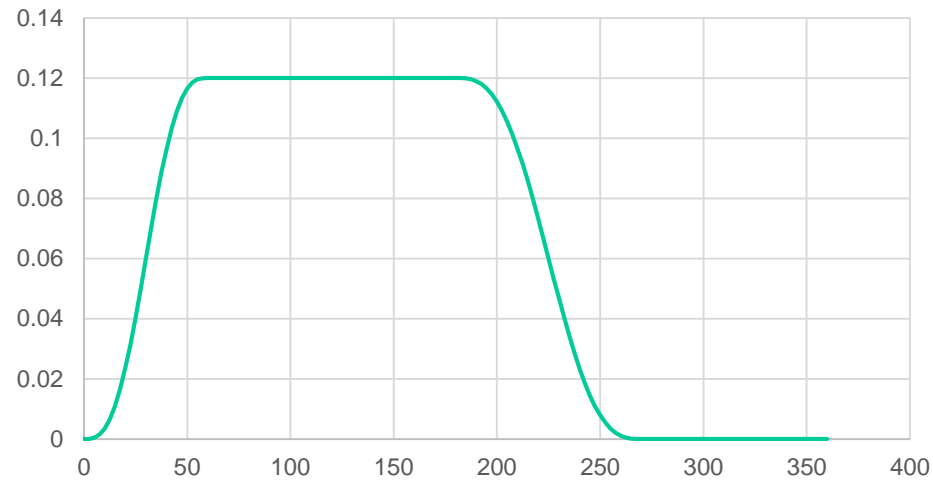
$$s''(\theta) = -0.12 \frac{2\pi}{(\pi/2)^2} \sin \frac{2\pi(\theta - \pi)}{\pi/2}$$

$270^\circ \leq \theta \leq 360^\circ$ DWELL: $H = 0 \text{ m}$

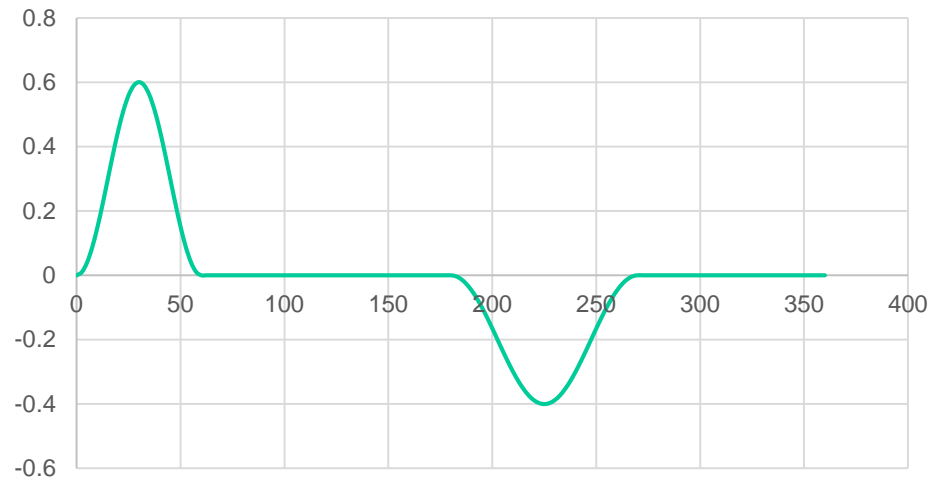
$$s(\theta) = 0; \quad s'(\theta) = 0; \quad s''(\theta) = 0$$

(b)

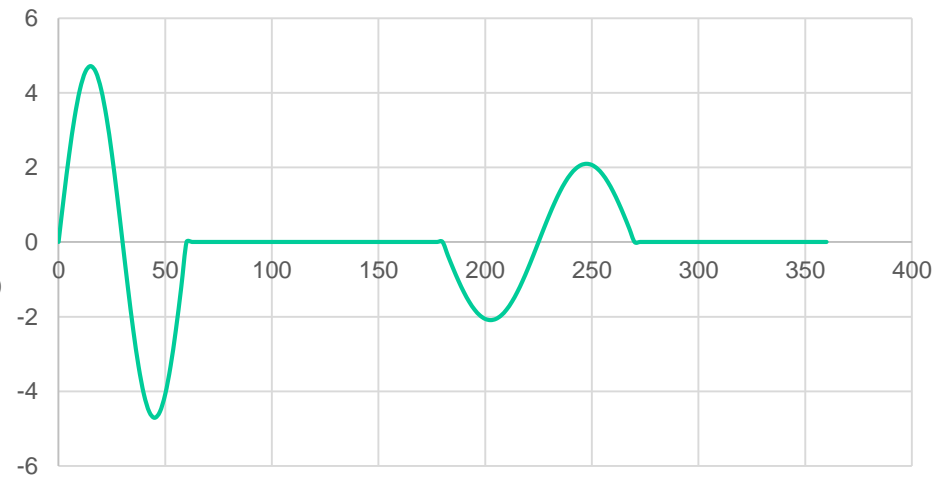
displacement



velocity



acceleration



(c) Rise: $\beta=60^0=\pi/3$ rad ; $H=0.12\text{m}$

$$v_{max} = (C_v H / \beta) \omega = (2 * 0.12 * 3 / \pi) (25 * \pi / 30) = 0.229 * 2.62 = 0.6 \text{ m/s}$$

$$a_{max} = (C_a H / \beta^2) \omega^2 = (2\pi * 0.12 * 9 / \pi^2) (25 * \pi / 30)^2 = 0.688 * 2.62^2 = 4.72 \text{ m/s}^2$$

Return: $\beta=90^0=\pi/2$ rad ; $H=0.12\text{m}$

$$v_{max} = (C_v H / \beta) \omega = (2 * 0.12 * 2 / \pi) (25 * \pi / 30) = 0.153 * 2.62 = 0.4 \text{ m/s}$$

$$a_{max} = (C_a H / \beta^2) \omega^2 = (2\pi * 0.12 * 4 / \pi^2) (25 * \pi / 30)^2 = 0.306 * 2.62^2 = 2.1 \text{ m/s}^2$$

(d) $t=1.5\text{s} \rightarrow \theta=\omega t=(25 * \pi / 30) * 1.5=0.393$ rad $\rightarrow 225^0$

From the sketches of s , v and a ; one can see that when $\theta = 225^0$

$$s = 0.06\text{m}, v = -0.4 \text{ m/s}, \text{ and } a = 0$$

Alternatively substituting $\theta = 225^0$ into the relevant equations will give the same results.

$$(e) \quad \tan \alpha_{max} \cong \frac{(s')_{max}}{r_b + r_r + s_{at (s')_{max}}}$$

$$\tan 40^0 \cong \frac{(C_v \frac{H}{\beta})_{max}}{r_b + r_r + \frac{H}{2}} \quad 0.839 = \frac{2^{\frac{0.12}{\pi/3}}}{r_b + r_r + 0.06} \quad r_b + r_r > 0.213m$$

Chose: $r_b + r_r = 0.215 \text{ m}$

(f) Determine the minimum radius of curvature ρ_{min} from ρ versus θ plot with $r_b + r_r = 0.215 \text{ m}$, and try different values of r_r until $\rho_{min} \geq 90 \text{ mm}$ is satisfied:

$$r_r = 0.025 \text{ m} ; r_b = 0.19m$$

(g)

At $t=1.5\text{s} \rightarrow \theta=\omega t=(25*\pi/30)*1.5=0.393 \text{ rad}=225^\circ$

$s' = -C_v H/\beta = -2*0.12*2/\pi = -0.153\text{m}$; $s=H/2=0.06\text{m}$

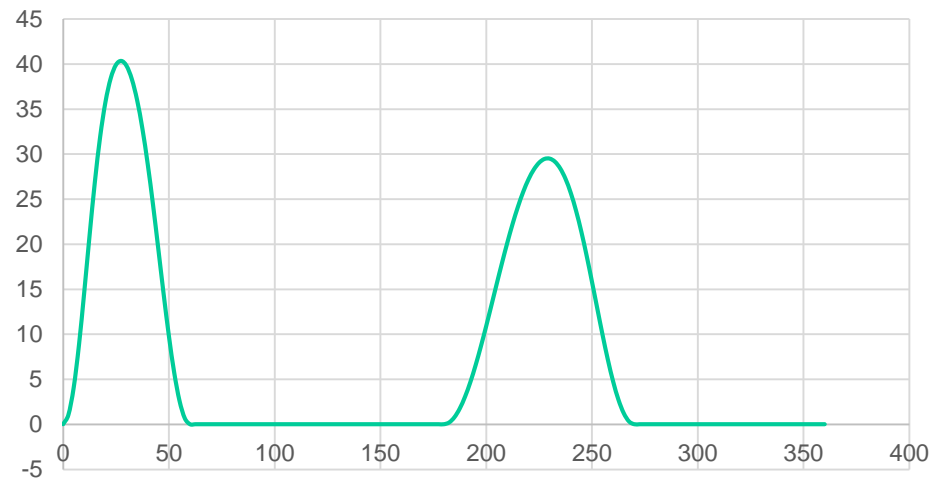
$$\tan \alpha = \frac{s'}{r_b + r_r + s} = \frac{-0.153}{0.215 + 0.06} \rightarrow \alpha = 29^\circ$$

$$\rho = \frac{[(r_b + r_r + s)^2 + (s')^2]^{3/2}}{(r_b + r_r + s)^2 + 2(s')^2 - (r_b + r_r + s)s''} - r_r$$

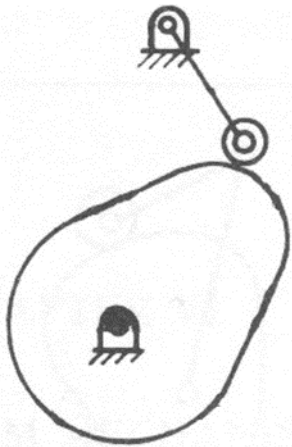
$r_b = 0.19\text{m}$; $r_r = 0.025\text{m}$; $s = 0.06\text{m}$; $s' = -0.153\text{m}$; $s'' = 0$

$$\rho = (0.0312/0.1224) - 0.025 \rightarrow \rho = 0.229\text{m}$$

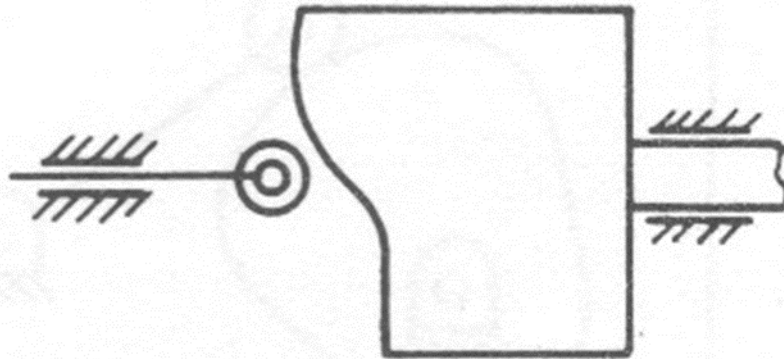
Pressure Angle



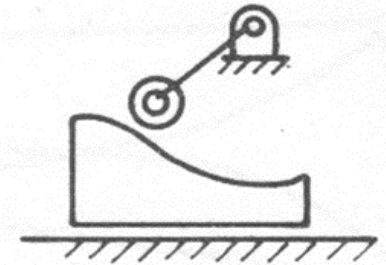
*Thank you
for your attention...*



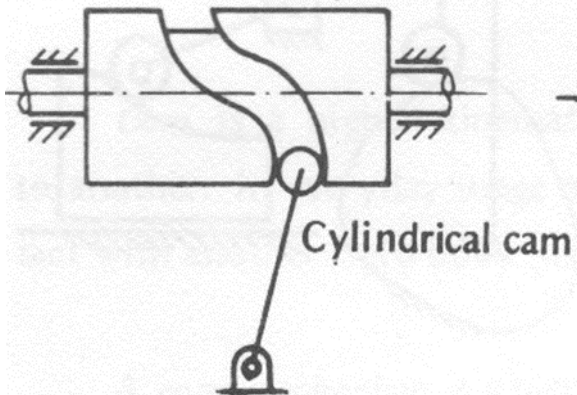
Radial cam



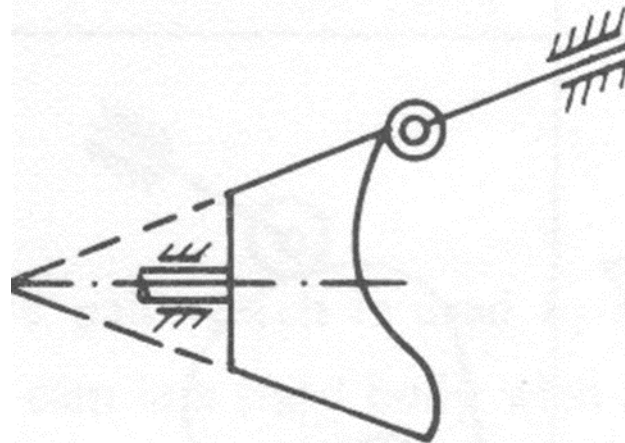
Face cam



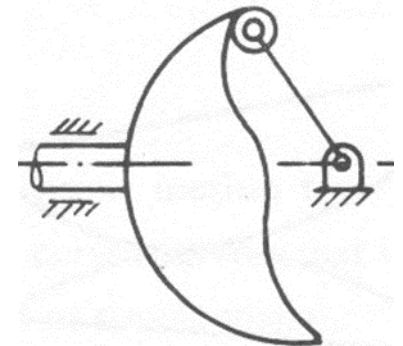
Wedge cam



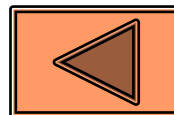
Cylindrical cam

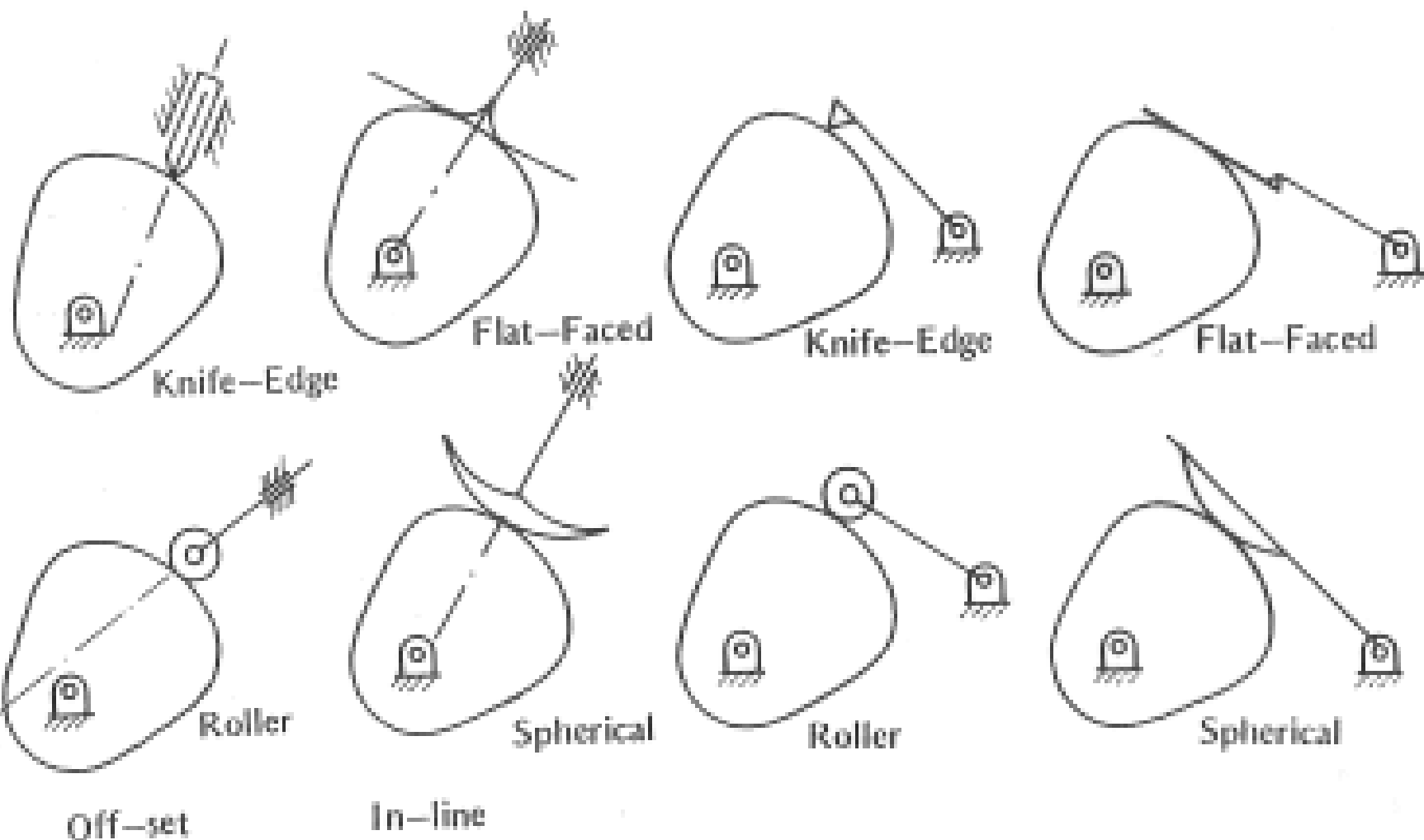


Conical cam



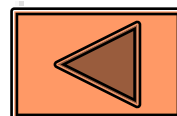
Spherical cam

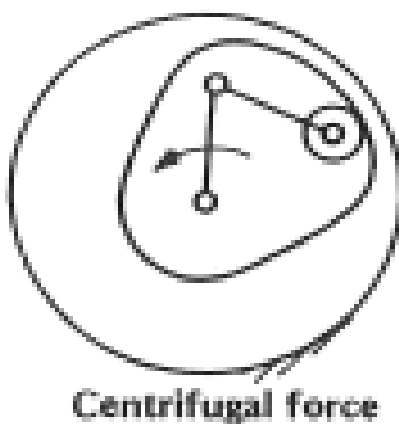
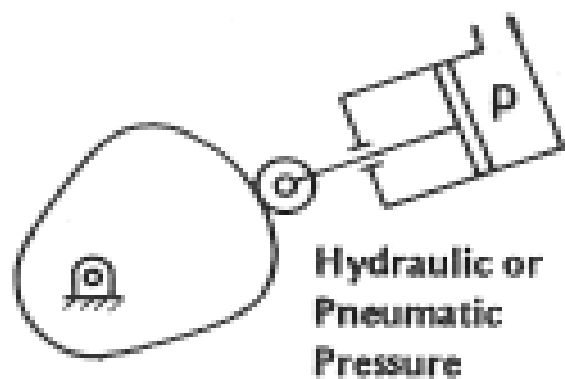
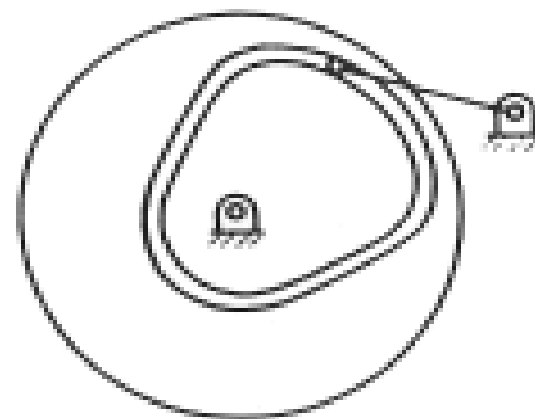
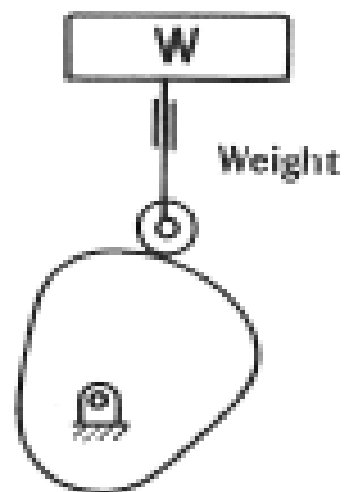
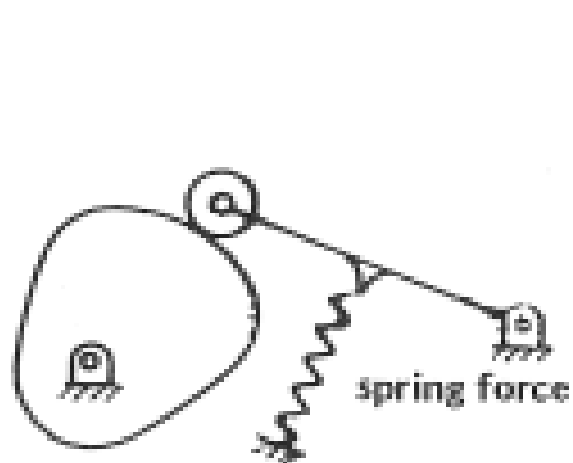




Translating followers

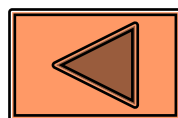
Oscillating followers

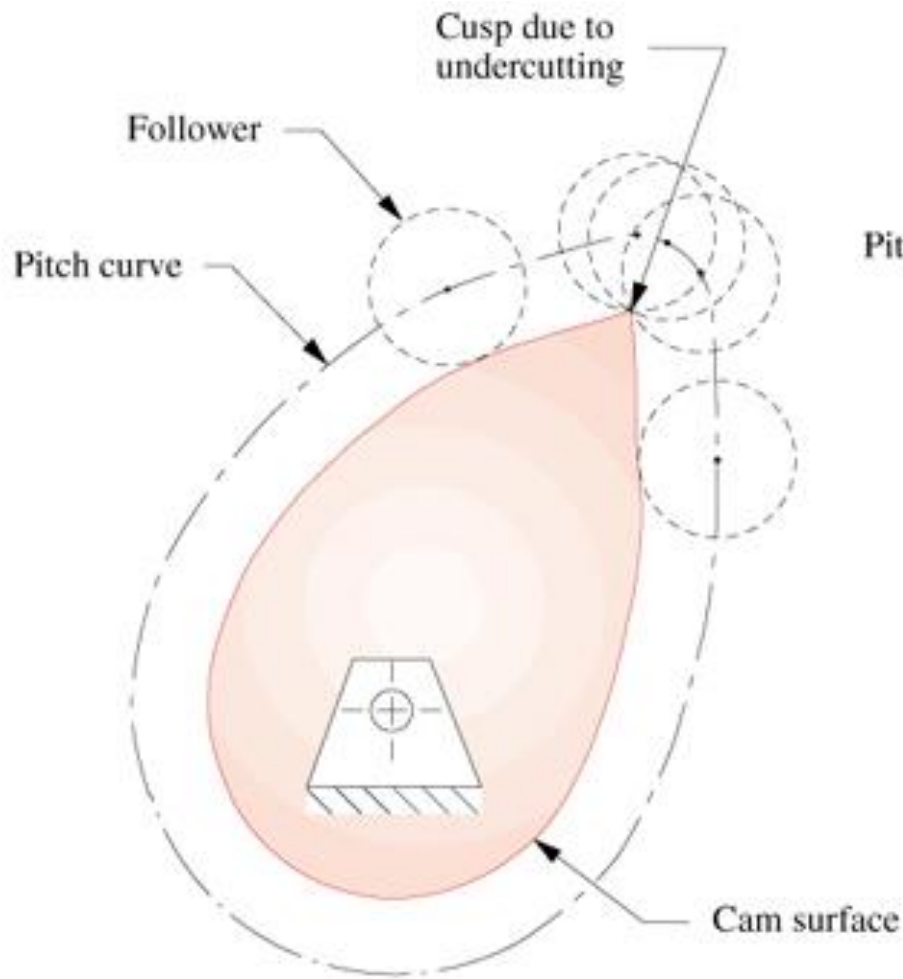




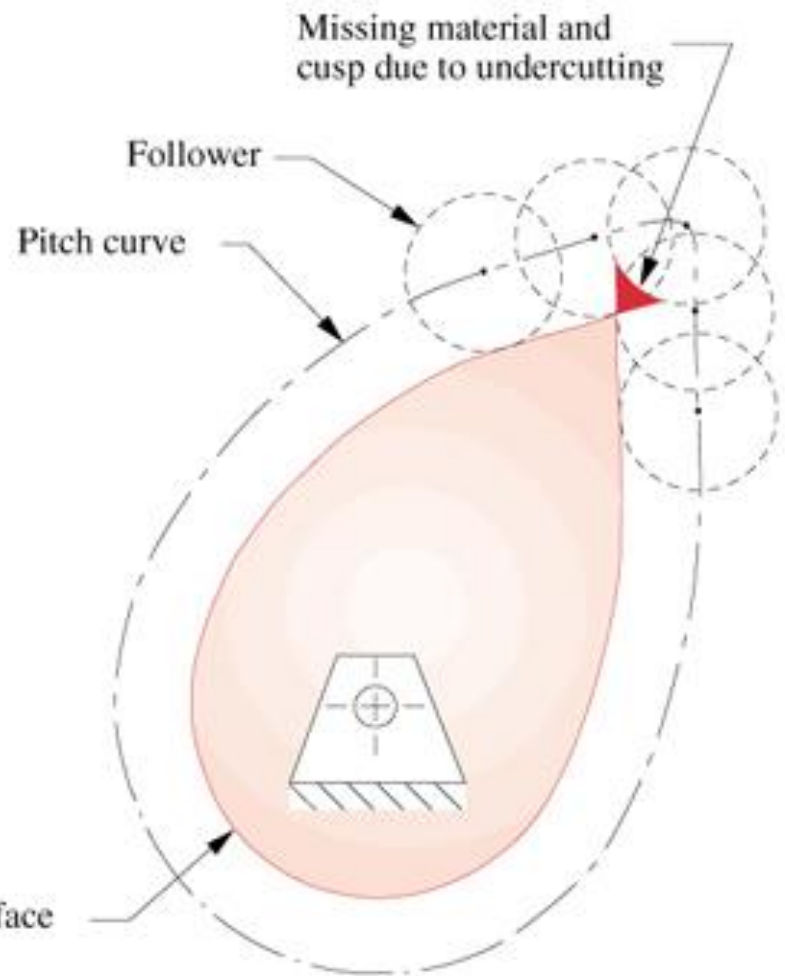
Force—Closed Cam Pairs :

Form—Closed Cam pairs





(a) Radius of curvature of pitch curve equals the radius of the roller follower



(b) Radius of curvature of pitch curve is less than the radius of the roller follower

