ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

DEPARTMENT OF MECHANICAL ENGINEERING

ME418
Dynamics of Machinery
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Properties of selected motor and gearbox are tabulated as follows.

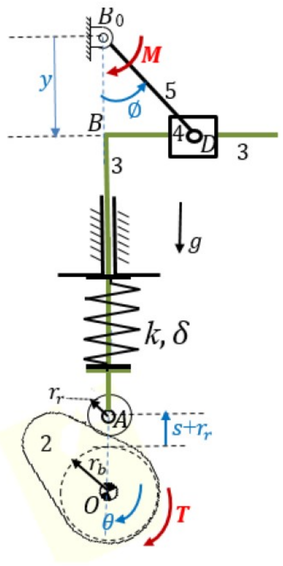
Motor Chosen: NM- 90L-4 ; **Reduction Ratio of Gear Box:** 3.5525			
Initial Cost Measure	Power	Weight	Flywheel Size
	1.5 kW	22 kg	0.06 kgm^2
Stability Measure	84.04 %		
Speed Fluctuation Measure	5.8 %		
Running Cost Measure	9.5 %		
Service Life Measure	0.65 s		

Introduction

In this report, a mechanism containing roller follower, force closed, inline cam is analyzed. Then, a suitable AC induction motor, a gearbox and a flywheel inertia is selected. Undercutting of the the cam and the pressure angle of the cam mechanism is checked for an acceptable operation. For the motor, various parameters are considered to make sure that the choice is reasonable in terms of cost, performance and safety of the motor.

Dynamic Modeling and Design of the Drive

Physical parameters of the mechanism are given as follows.



$H := 0.06\ m$ $\quad \beta := \frac{5\ \pi}{6}$ $\quad \gamma := \beta$ $\quad \delta := 10\ mm$

$r_r := 12\ mm$ $\quad r_b := 40\ mm$ $\quad k := 20000\ \frac{N}{m}$

$I_{O2} := 0.02\ kg \cdot m^2$ $\quad I_{Bo5} := 0.05\ kg \cdot m^2$ $\quad a_2 := 0.18\ m$

$a_1 := 0.3\ m$ $\quad m_3 := 7\ kg$ $\quad a_3 := 0.1\ m$

$M := 150\ N \cdot m$

a) Equations for dynamic modelling of the system

<u>During rise period,</u>

$$s = \frac{H}{2} \cdot \left(1 - \cos\left(\frac{\pi \cdot \theta}{2\ \pi - \frac{5}{6}\ \pi}\right)\right)$$

$$s' = \frac{H}{2} \cdot \left(\frac{\pi}{2\ \pi - \frac{5}{6} \cdot \pi}\right) \cdot \sin\left(\frac{\pi \cdot \theta}{2\ \pi - \frac{5}{6}\ \pi}\right)$$

$$s'' = \frac{H}{2} \cdot \left(\frac{\pi}{2\ \pi - \frac{5}{6} \cdot \pi}\right)^2 \cdot \cos\left(\frac{\pi \cdot \theta}{2\ \pi - \frac{5}{6}\ \pi}\right)$$

<u>During return period,</u>

$$s = H - \frac{H}{2} \cdot \left(1 - \cos\left(\pi \cdot \frac{(\theta - \gamma)}{2\ \pi - \frac{5}{6}\ \pi}\right)\right)$$

$$s' = \frac{-H}{2} \cdot sin\left(\pi \cdot \frac{(\theta - \gamma)}{2\ \pi - \frac{5}{6}\ \pi}\right) \cdot \left(\frac{\pi}{2\ \pi - \frac{5}{6} \cdot \pi}\right)$$

$$s'' = \frac{-H}{2} \cdot \left(\frac{\pi}{2\ \pi - \frac{5}{6} \cdot \pi}\right)^2 \cdot cos\left(\pi \cdot \frac{(\theta - \gamma)}{2\ \pi - \frac{5}{6} \cdot \pi}\right)$$

The above equations are plotted in Matlab. The plot can be seen in Figure 1. below. The script is provided in appendix part.

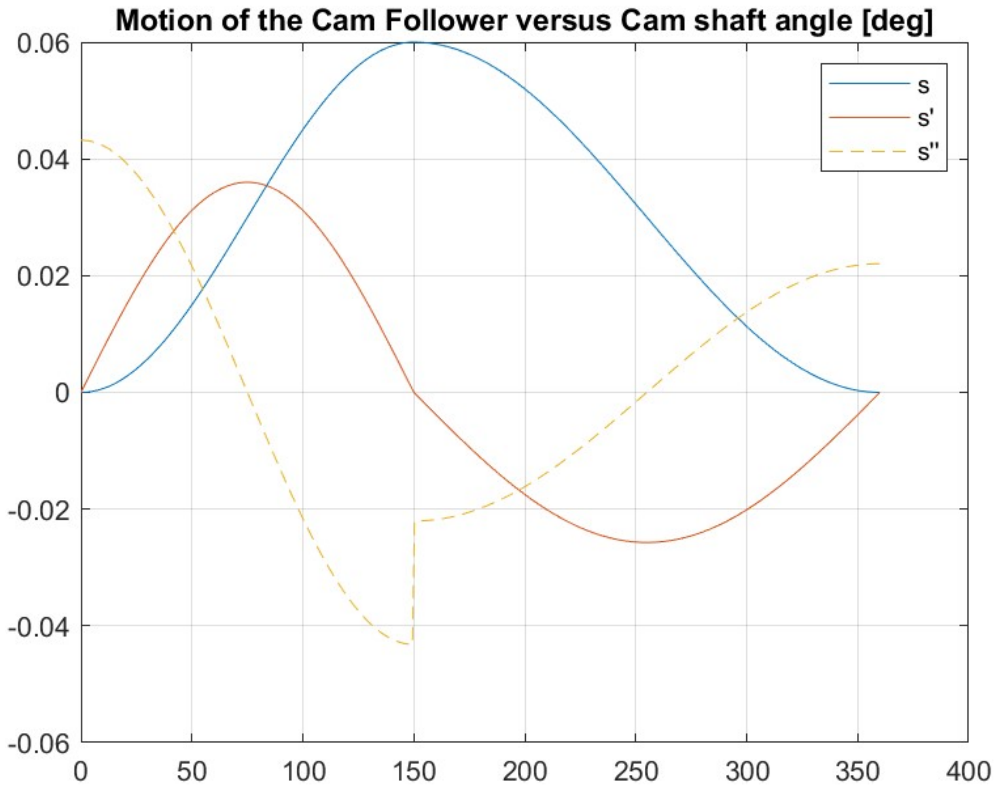


Figure 1. Motion of the cam follower versus cam shaft angle

Let θ be the generalized coordinate q. Then, ϕ, y and s are the dependent parameters.

$y = a_2 \cdot cos(\phi)$ & $y = a_1 - (r_b + r_r + s) - a_3$

Combining these two equations. One can obtain that

$$\phi = acos\left(\frac{a_1 - (r_b + r_r + s) - a_3}{a_2}\right)$$

$$a_1 - (r_b + r_r + s) - a_3 = a_2 \cdot cos(\phi)$$

Take the derivative of above expression with respect to generalized coordinate q

$$-s' = -a_2 \cdot g_1 \cdot sin(\phi)$$ Then, $$g_1 = \frac{s'}{-a_2 \cdot sin(\phi)}$$

Taking the derivative of the g_1 with respect to q

$$g_1' = \frac{s'' \cdot a_2 \cdot sin(\phi) - a_2 \cdot cos(\phi) \cdot s'}{(a_2 \cdot sin(\phi))^2}$$

The generalized inertia of the mechanism J can be written as follows.

$$J = I_{O2} + I_{BO5} \cdot {g_1}^2 + m_3 \cdot (s')^2$$

The centripetal inertia can be written as follows.

$$C = \frac{1}{2} \cdot \frac{dJ}{dq} = I_{BO5} \cdot g_1 \cdot g_1' + m_3 \cdot (s') \cdot s''$$

Generalized forces can be written as follows.

$$Q = Q_T + Q_M + Q_W + Q_{spr}$$

where $Q_T = T$ $\quad Q_M = -M \cdot g_1$ $\quad Q_W = -m_3 \cdot g \cdot s'$ $\quad Q_{spr} = -k \cdot (s + \delta)$

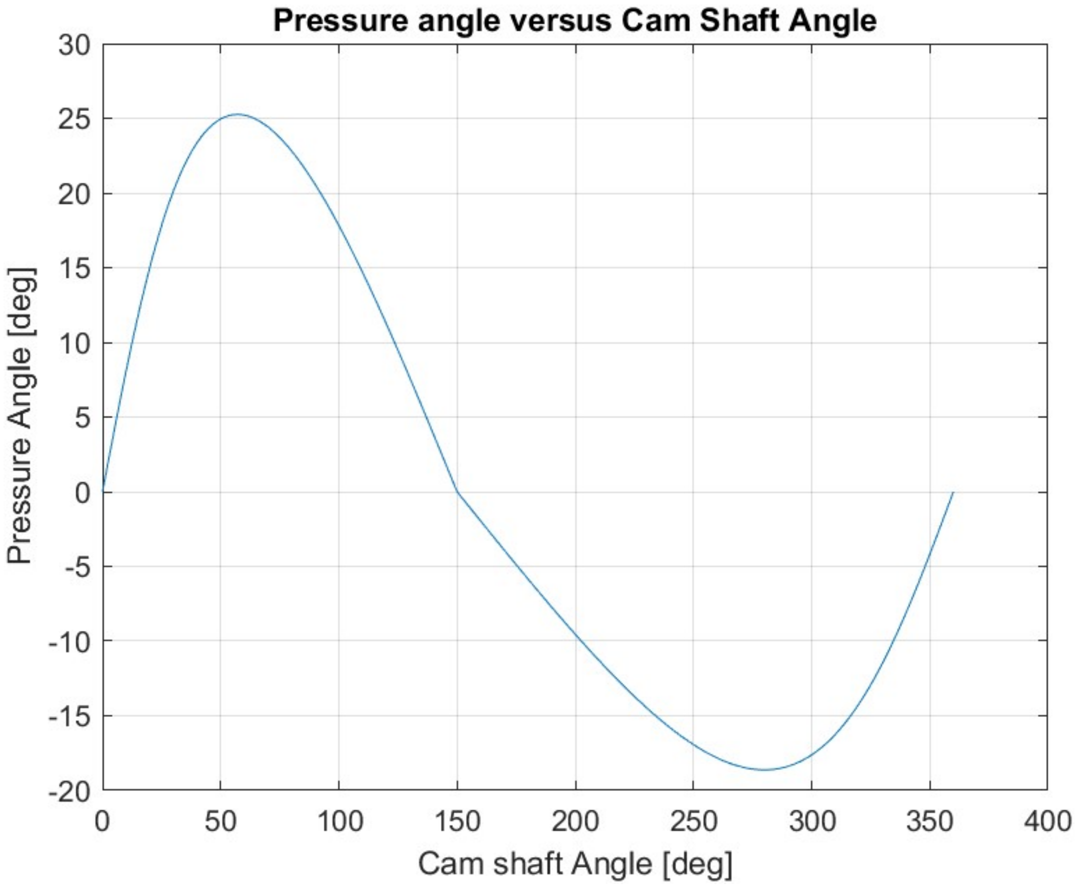
b) Cheking the pressure angle and undercutting of the mechanism using the following aapproach.

$$\tan \alpha = \frac{v_{B3}}{v_{B2}} = \frac{s'}{r_b + r_r + s}$$

Then, $\alpha = atan2(s', r_b + r_r + s)$

The variation of pressure angle with respect to generalized coordinate is plotted in Matlab. The plot can be seen as follows.

The maximum pressure angle is found as $\alpha_{max} := 25.3\ deg$, which is in allowable range for in-line translating roller follower.

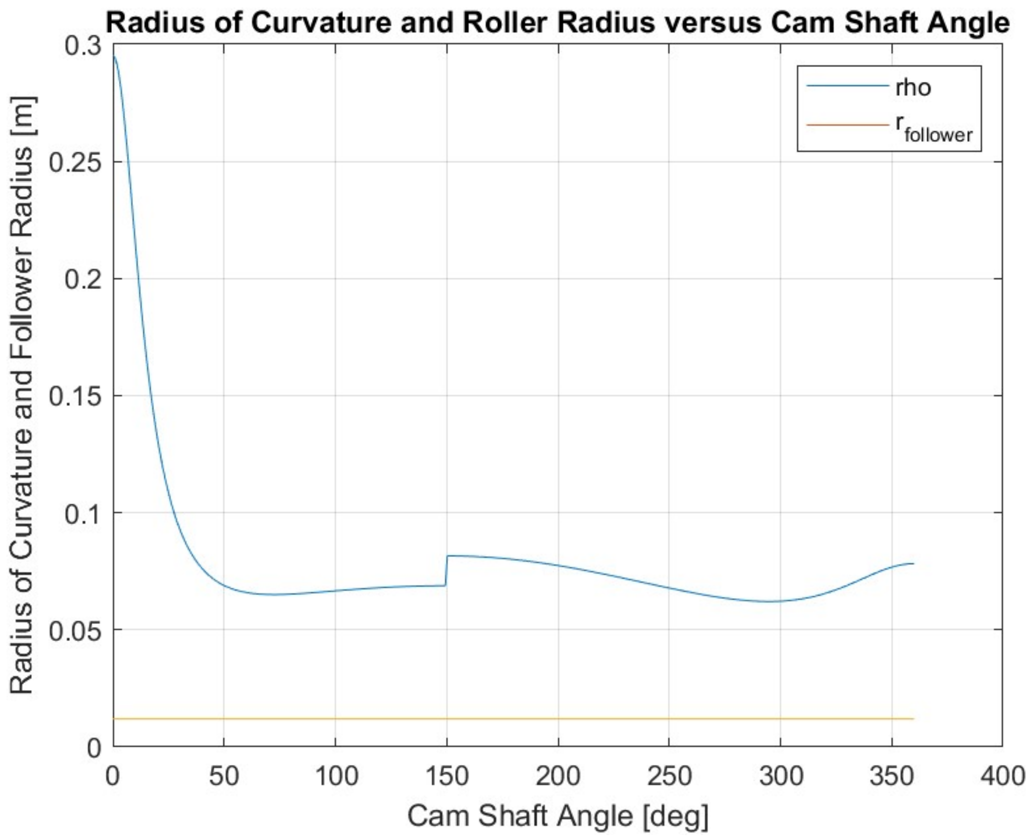


Checking if there is undercutting using the following formula,

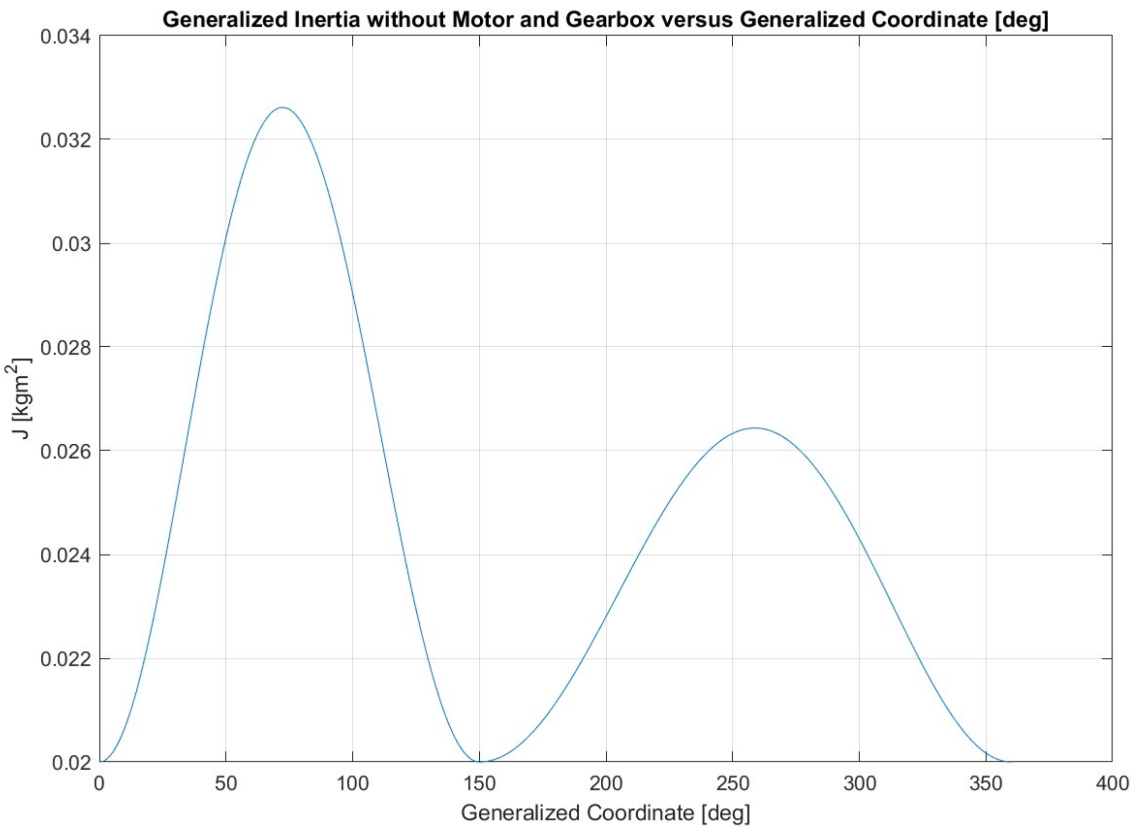
$$\rho = \frac{\left[(r_b + r_r + s)^2 + (s')^2\right]^{3/2}}{(r_b + r_r + s)^2 + 2(s')^2 - (r_b + r_r + s)s''} - r_r$$

The above equation is solved numerically for a complete cycle and the radius of curvature is plotted as follows.

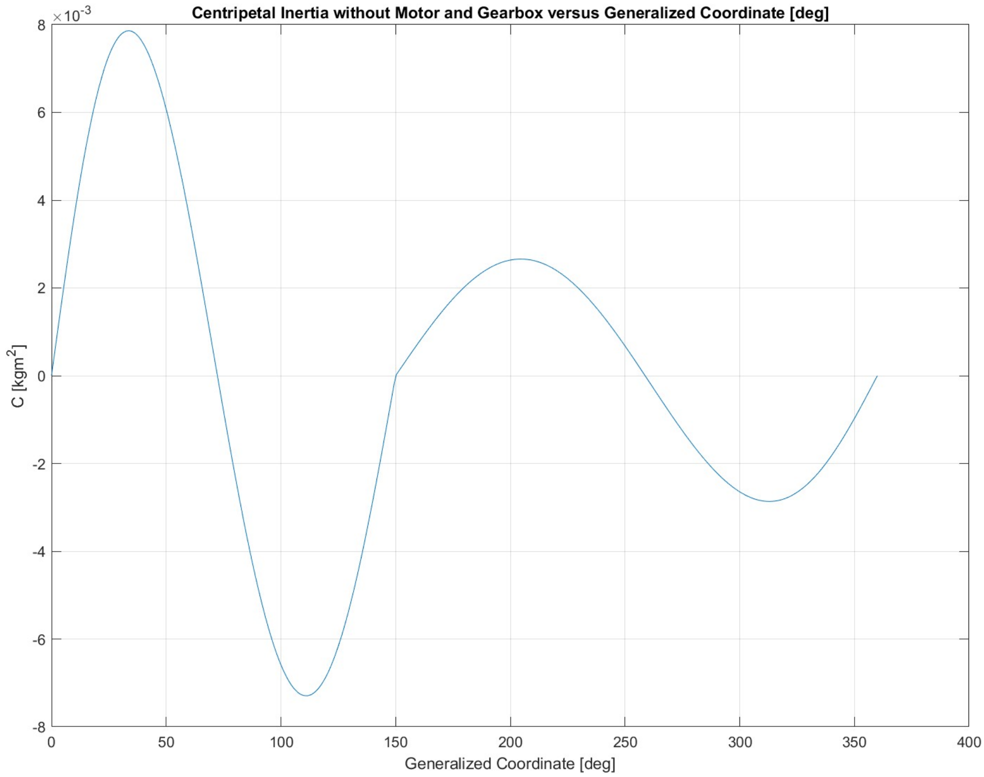
As can be seen from the below plot, the radius of curvature is much higher than the radius of follower for any position of the mechanism. Therefore, there is no undercutting.



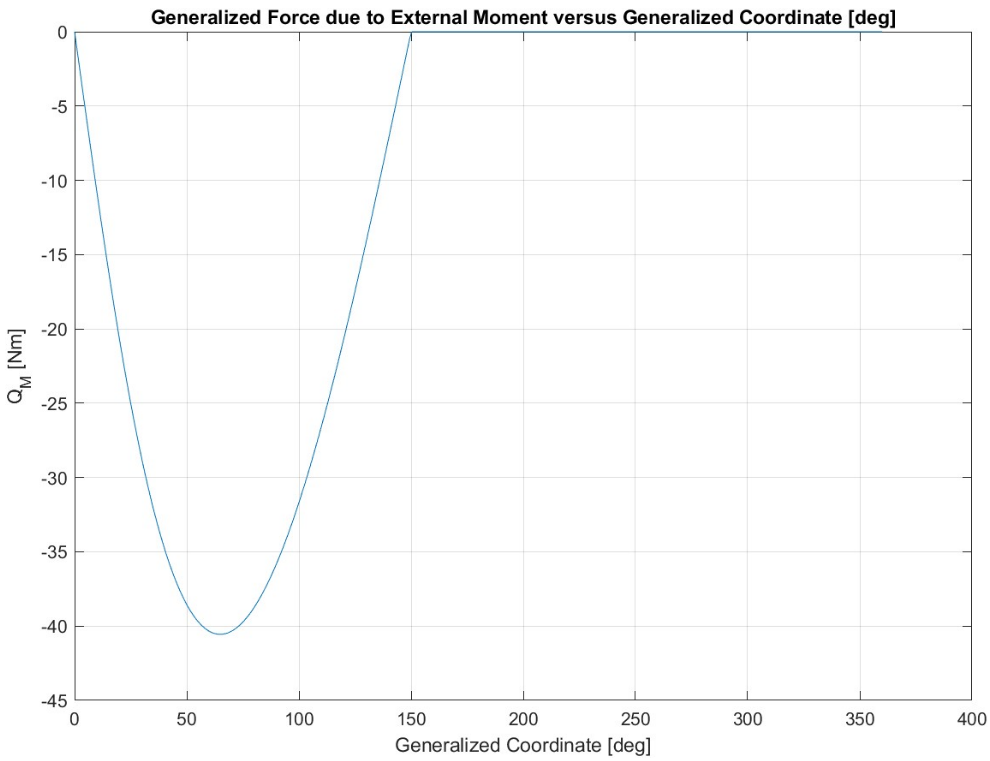
c) Generalized inertia without motor or gearbox versus the generalized coordinate θ

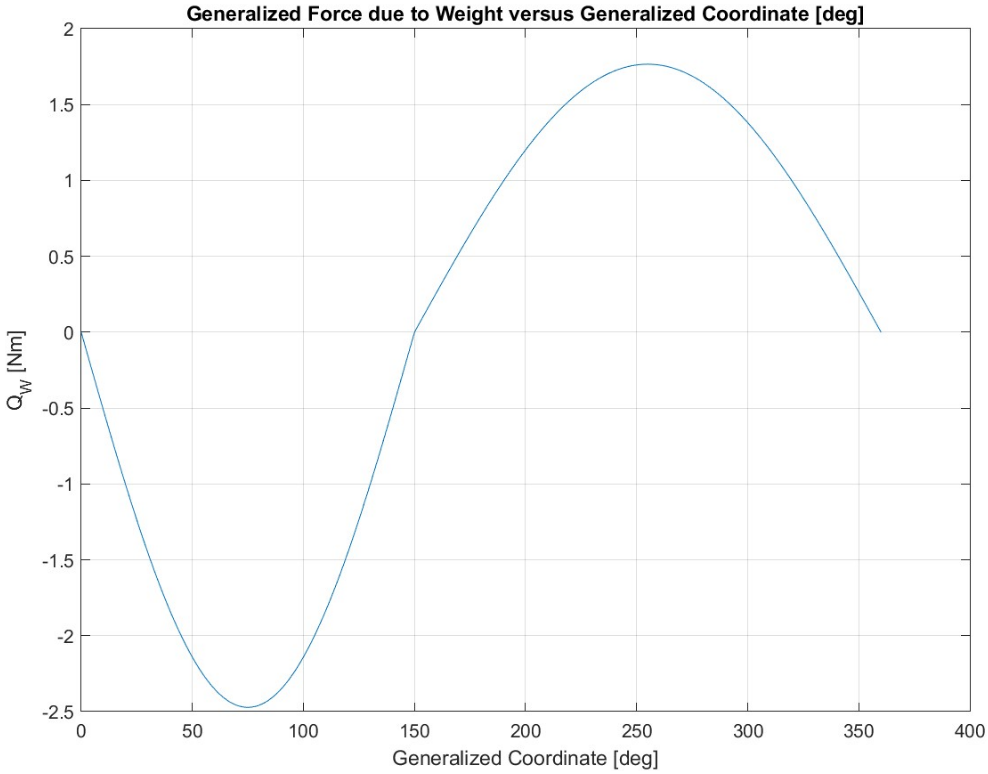


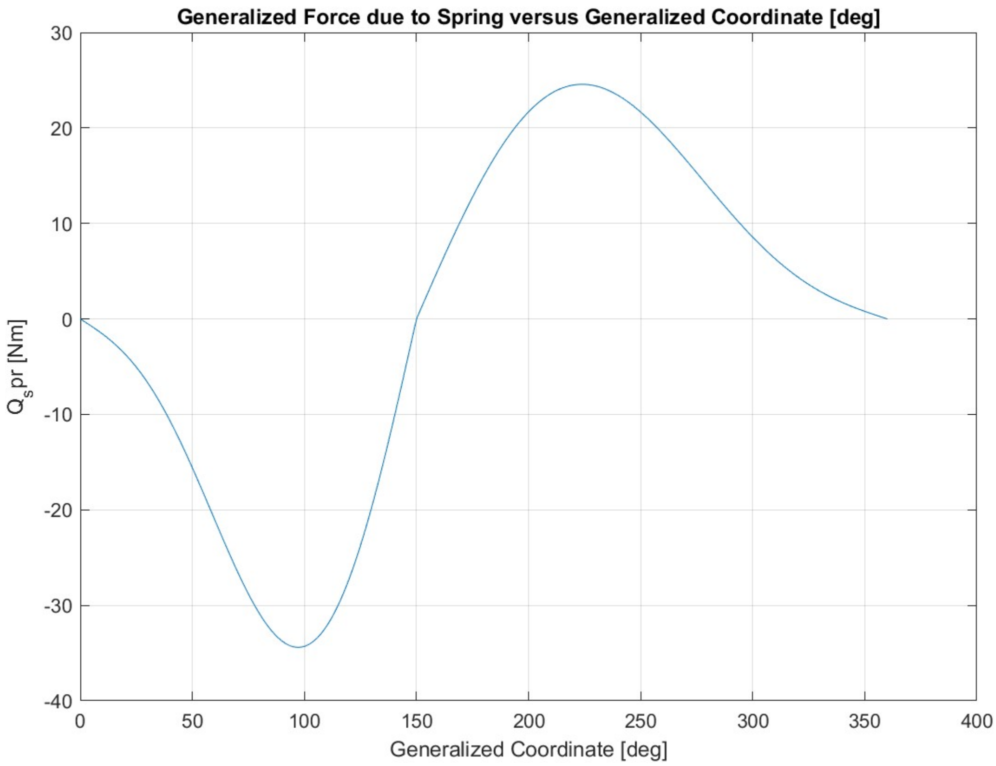
d) Centripetal inertia without motor or gearbox versus the generalized coordinate θ



e) Variation of generalized forces with respect to θ




Generalized Force due to Weight versus Generalized Coordinate [deg]
Q_W [Nm]
Generalized Coordinate [deg]
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
0
50
100
150
200
250
300
350
400


Generalized Force due to Spring versus Generalized Coordinate [deg]
Q_spr [Nm]
Generalized Coordinate [deg]
30
20
10
0
-10
-20
-30
-40
0
50
100
150
200
250
300
350
400

f) Dynamic simulation of the mechanism is performed in the Matlab script. The explanations are given in the related sections of the report.

g) Selecting the motor, it is desired that the motor operates at its rated speed and power. For this reason, a gearbox with a definite gear ratio is chosen to accomplish this goal. Also, several motors have been tested if the properties of the motor fits the mechanism and the conditions of stability, initial cost, speed fluctuation, running cost and service life.

While selecting the motor, it is critical to have a motor that has the break down moment which is higher than the maximum torque needed for the operation of mechanism. The maximum torque in the mechanism can be estimated by assuming constant cam shaft speed. The calculation of maximum torque can be carried out as follows.

EOM: $$J \cdot \theta_{dd} + C \cdot {\theta_d}^2 = T + Q_M + Q_{spr} + Q_W$$

Let $\theta_{dd} = 0$. Then,

$$T = C \cdot {\theta_d}^2 - \left(Q_M + Q_{spr} + Q_W\right)$$ where $\theta_d = 400\ rpm$

Solving the above formula gives maximum torque of 70.7 at the cam shaft. This value is used to select a proper motor. If the breakdown moment of a motor can not sustain this amount of torque with the help of the gearbox, the motor can stall and heats up resulting permanent damaging of itself.

Selecting NM90L-4 induction motor from the EMTAŞ Catalogue. This is an 4-pole motor having synchronous speed of 1500 rpm. Other properties of the motor are as follows.

$n := 400\ \mathbf{rpm}$ $\quad P := 1.5\ \mathbf{kW}$ $\quad n_n := 1421\ \mathbf{rpm}$ $\quad \eta := 0.78$ $\quad T_n := 10.1\ \mathbf{N \cdot m}$

$T_b := 2.7 \cdot T_n = 27.27\ \mathbf{N \cdot m}$ $\quad J_{motor} := 0.0028\ \mathbf{kg \cdot m^2}$ $\quad m_{motor} := 22\ \mathbf{kg}$

$s_d := 0.315$ $\quad a := 1.587$ $\quad b := 1.416$

Gear ratio, $$r := \frac{n_n}{n} = 3.553$$

Checking the effect of breakdown moment on the cam shaft,

$$T_{b\_cam} := T_b \cdot r = 96.877\ (\mathbf{N \cdot m})$$

$T_{b\_cam} > 70.7\ \mathbf{N \cdot m} = 1$ Therefore, this motor can sustain the required torque.

Torque-speed characteristic of the motor can be derived using Eres Söylemez's formula for EMTAŞ AC induction motors taken from the lecture notes.

$$T = \frac{T_b}{1 + (s_b - s)^2(\frac{a}{s} - bs^2)}$$

where; T_b is breakdown torque,
s_b is slip at breakdown,
a and b are motor constants

At this point, it may be wise to define efficiency value for the gearbox. Since some of the rotational energy is dissipated as heat in the gearbox, the torque in not linearly amplified by the gear ratio. Typical efficiency of a spur gear pair ranges from 0.8 to 0.98. Assuming 2 set of gear pair is used, I will assume transmission efficiency of 0.9 for this particular problem. Then, torque on the cam shaft due to motor can be written as follows.

$$Q_T = T \cdot r \cdot \eta_{gearbox}$$

At this point, the equation of motion of the mechanism can be fully calculated since all the inertias and forces are known. This is a single degree of freedonm mechanism. Therefore, the following equation can be used for EoM.

$$J\ddot{q} + C\dot{q}^2 = Q \text{ ; where, } C = \frac{1}{2}\frac{dJ}{dq}$$

The Generalized inertia can be written with the motor, flywheel and gearbox inertia added. The inertia of the gearbox having below value is added directly to the cam shaft. The inertia of the motor is converted by using gear ratio since the speed of motor is proportional by gear ratio. Finally, a flywheel is located at the motor side of the gearbox. As a rule of thumb, it is logical to put flywheel where speed is maximum. This enables designer to use less weight of flywheel and deal with less loads on the bearings and shafts.

$$J = I_{O2} + I_{BO5} \cdot {g_1}^2 + m_3 \cdot (s')^2 + J_{gearbox} + J_{motor} + J_{flywheel}$$

where

$$J_{gearbox} = (1 + 0.1 \cdot r) \cdot J_{emtaş} \qquad J_{motor} = J_{emtaş} \cdot r^2 \qquad J_{flywheel} = I_{flywheel} \cdot r^2$$

Centripetal inertia is still the same since the added terms in generalized inertia are constant.

The equation of motion should be solved in terms of time using numerical methods. Numerical integration should be carried out. There are several methods such as Euler, Runge Kutta with several orders. I prefer using Euler Method due to its easy implementation. Although Euler Method introduces much errors in the results compared to the Runge Kutta, this can be overcomed utilizing small time step values.

Euler Method can be defined as follows.

$$q^{n+1} = q^n + h\dot{q}^n \qquad \text{where; } \ddot{q} = \frac{Q - C\dot{q}^2}{J}$$
$$\dot{q}^{n+1} = \dot{q}^n + h\ddot{q}^n$$

The method is implemented in the Matlab script.

Having no flywheel causes high fluctuations. For example, it is around 20% for this case. Therefore, several inertia values for the flywheel is swept using the code to have fluctuations at an acceptable level. I will try to have fluctuation at steadt state around 5%. Having high inertia reduces the fluctuation massively. However, it results in an increase in the time where motor reaches steady state operation. The increase in time can cause damage in the motor. At this point, a trade off needs to be carried out. To make a wise decision between the parameters, it would be great to know the purpose of this machine. Therefore, we may decide which parameter to compromise.

Selecting an inertia of $0.06\ kg \cdot m^2$ provides satisfactory results.

The performance characteristics of the motor are calculated in the Matlab script and the results are presented as follows.

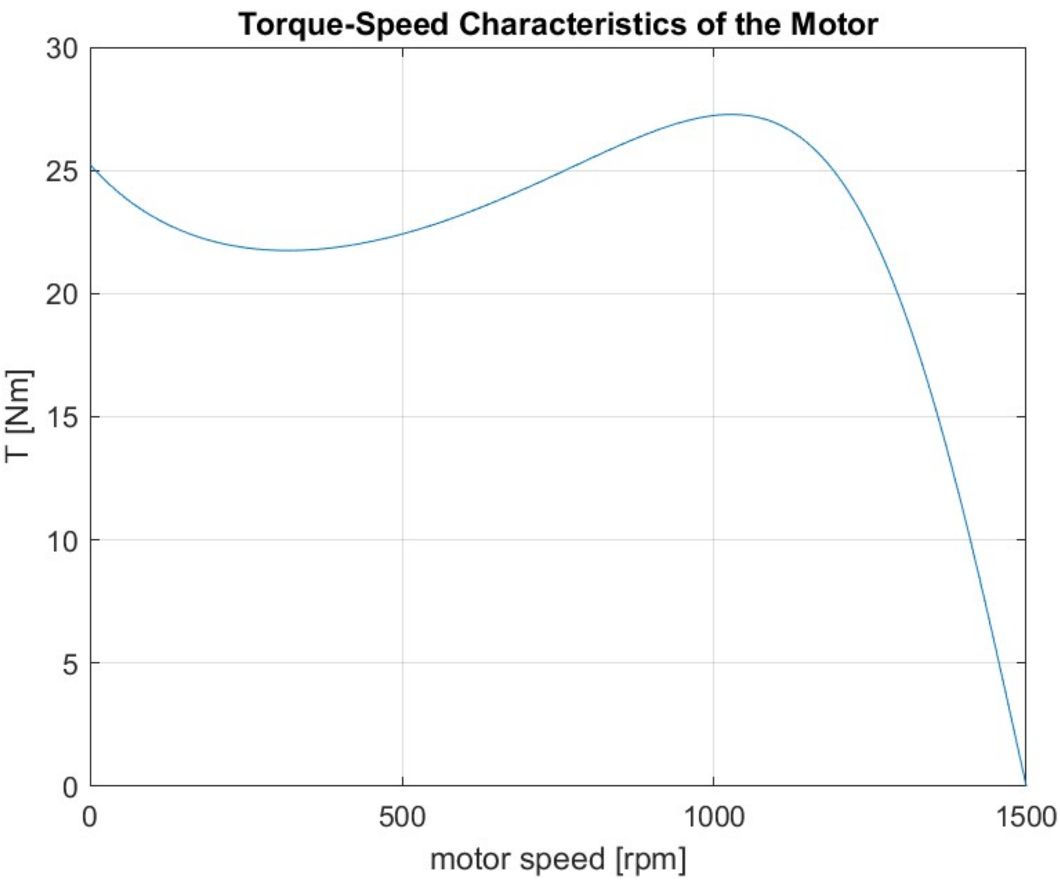
$steadyStateTime := 0.65\ s$

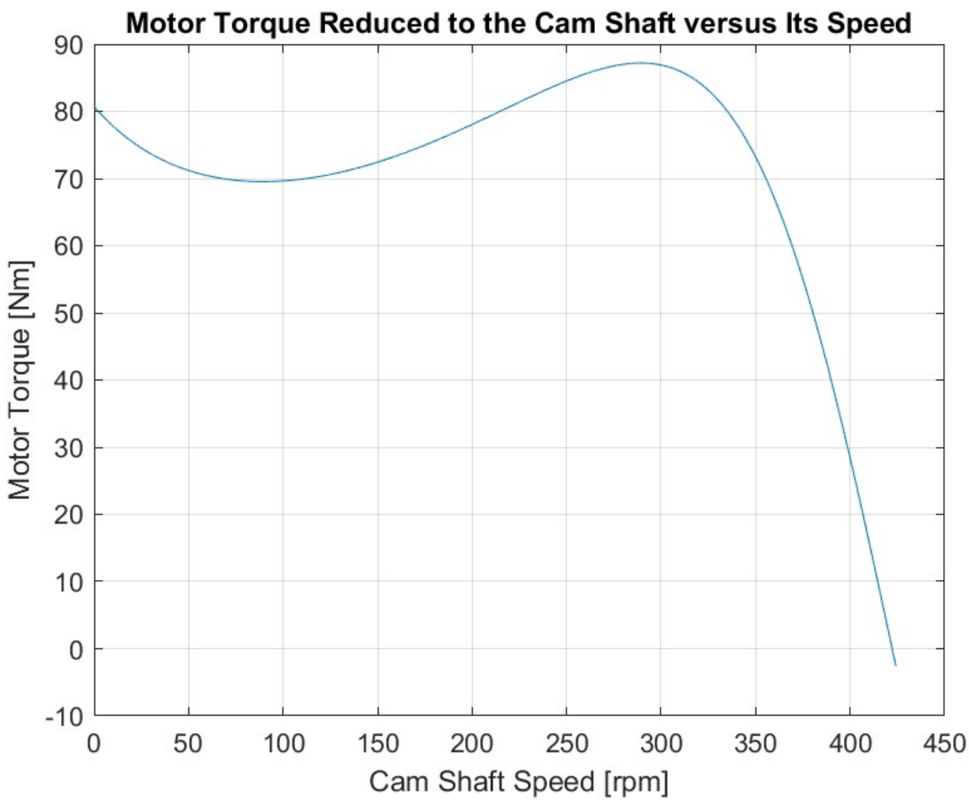
$stability := 84.04\%$

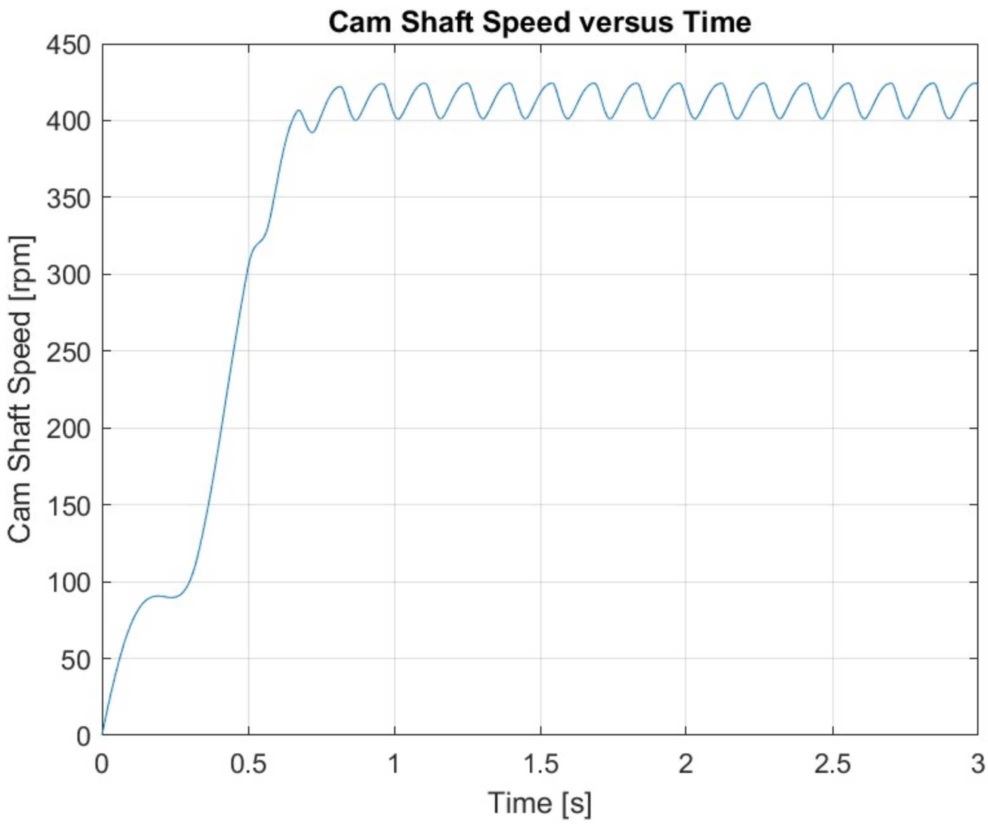
$fluctuation := 5.8\%$

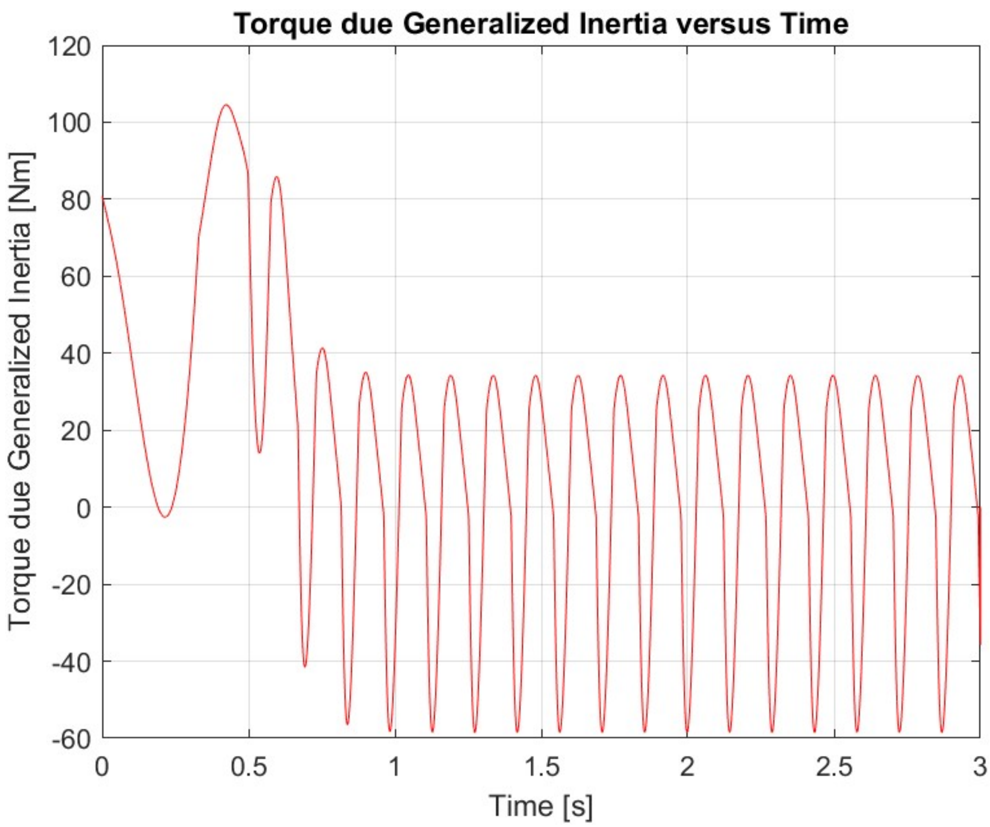
$runningCost := 9.5\%$

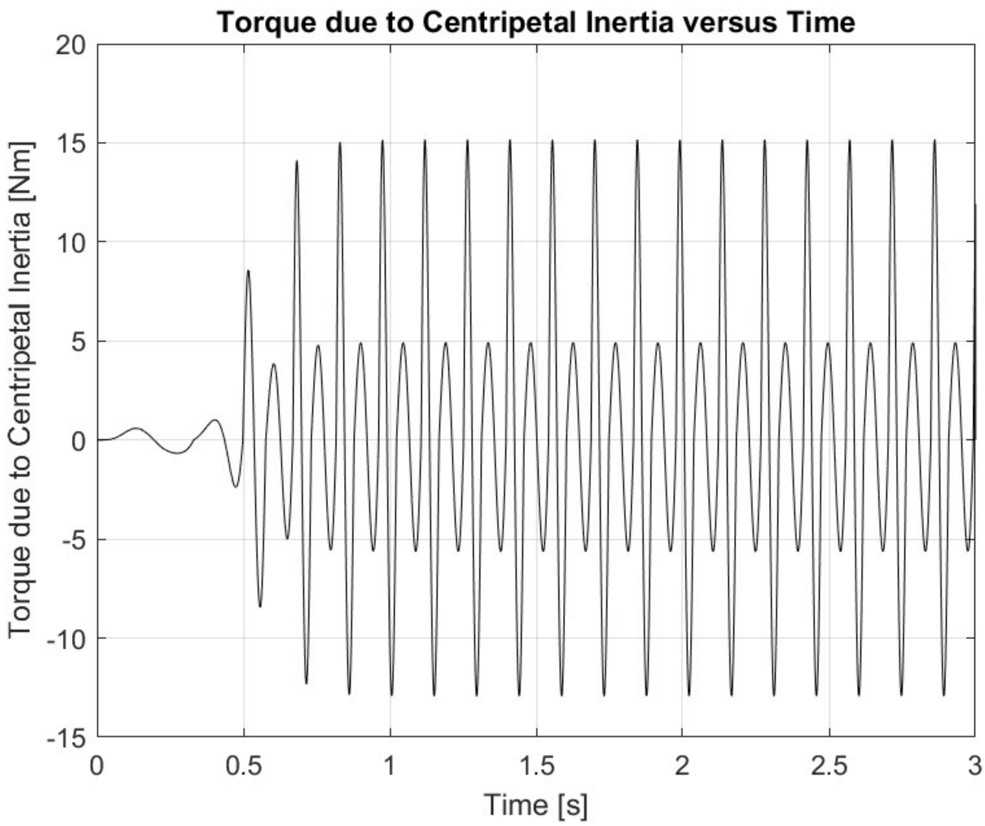
h) The required plots are presented as follows.





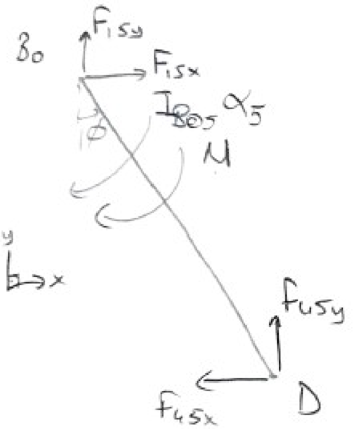

Cam Shaft Speed versus Time
Cam Shaft Speed [rpm]
Time [s]
0
50
100
150
200
250
300
350
400
450
0.5
1
1.5
2
2.5
3


Torque due Generalized Inertia versus Time
Torque due Generalized Inertia [Nm]
Time [s]
-60
-40
-20
0
20
40
60
80
100
120
0.5
1
1.5
2
2.5
3



i) Contact force is calculated to check if jump occurs

Link 5



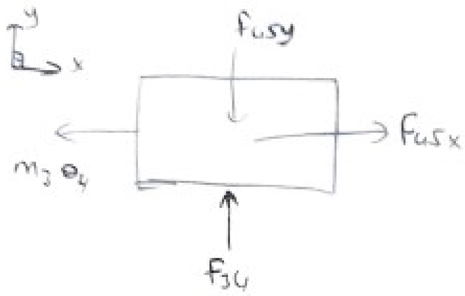
$$\sum M_{B_0} = f_{45y}\, a_2 \sin\phi - \underbrace{f_{45x}\, a_2 \cos\phi}_{0} - M - I_{B_05}\alpha_5 = 0$$

$$\alpha_5 = \ddot{\phi}$$

$$\Rightarrow f_{45y} = \frac{I_{B_05}\ddot{\phi} + M}{a_2 \sin\phi} \quad \ldots\ldots (1)$$

$$\ddot{\phi} = g_1 \ddot{\theta} + g_1' \dot{\theta}^2$$

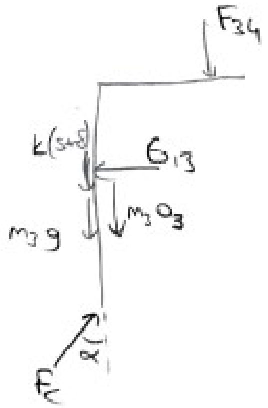
Link 4



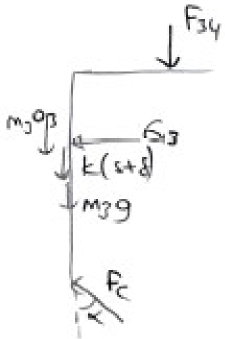
$$\sum F_y = 0 \Rightarrow F_{45y} = F_{34} \quad \ldots\ldots (2)$$

Link 3

Rise



Return



$$\sum F_y = 0 = -F_{34} - k(s+\delta) - m_3 g - m_3 a_3 + F_c \cos\alpha = 0 \quad \ldots\ldots (3)$$

inserting (2) into (3),

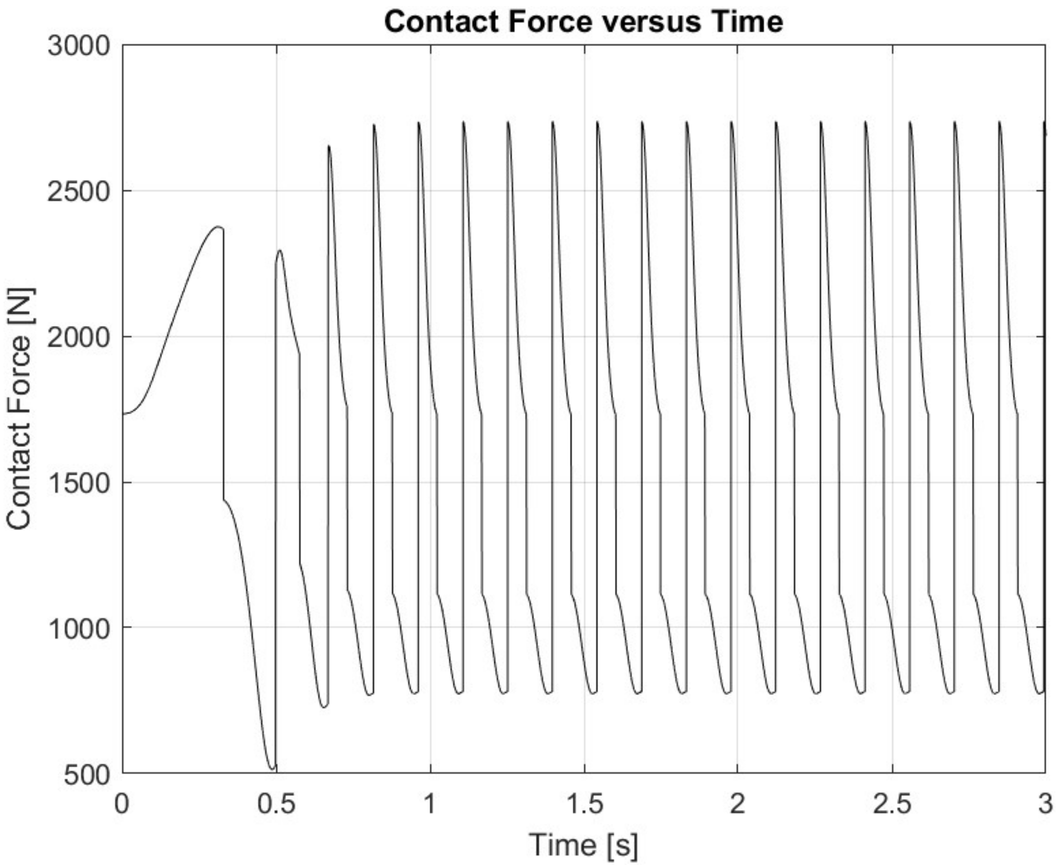
$$-F_{45y} - k(s+\delta) - m_3 g - m_3 \ddot{s} + F_c \cos\alpha = 0$$

where $\ddot{s} = s'\ddot{\theta} + s''\dot{\theta}^2$

$$\Rightarrow F_c = \frac{F_{45y} + k(s+\delta) + m_3 g + m_3(s'\ddot{\theta} + s''\dot{\theta}^2)}{\cos\alpha} \quad \ldots\ldots (4)$$

For the return, $F_c \cos\alpha$ is replaced by $F_c \cos(-\alpha)$ in (3), which results in the same expression.

Calculating the contact force, one obtains that the minimun contact force $514.2\ N$, which implies that there is no jump in the mechanism. Also, contact force versus time plot of the mechanism can be seen below.



Concluding Remarks

In this report, a mechanism containing roller follower in-line cam is analyzed. Then, a motor, gearbox and flywheel is selected for the mechanism. Various parameters are considered during the selection of these components. The motor should be capable of providing necessary torques for the mechanism to perform desired motion. Then, 5 other design criterias are emposed for motor selection. These are initial cost, stability, speed fluctuation, running cost and service life measures.

The selected motor is NM-90L-4, one of the cheapest motor among the listed motors in Emtaş Catalogue. For this particular mechanism, the motor has the stability of about 84%. It is quite acceptable depending on the application where the mechanism is used. If the timing of the valve is critical this value may be further reduced, which adversely affects the time it takes to steady-state operation condition. The fluctuation of the motor speed is 5.8 %. This is a typical value, so it is in an acceptable range. The running cost of the motor is 9.5 %. The time it takes the motor to reach steadt state operation is around 0.65 second. This time is critical for the lifespan of the motor. Higher the time more the motor heats up during start-up, which may damage the motor. However, 0.65 second is quite less.

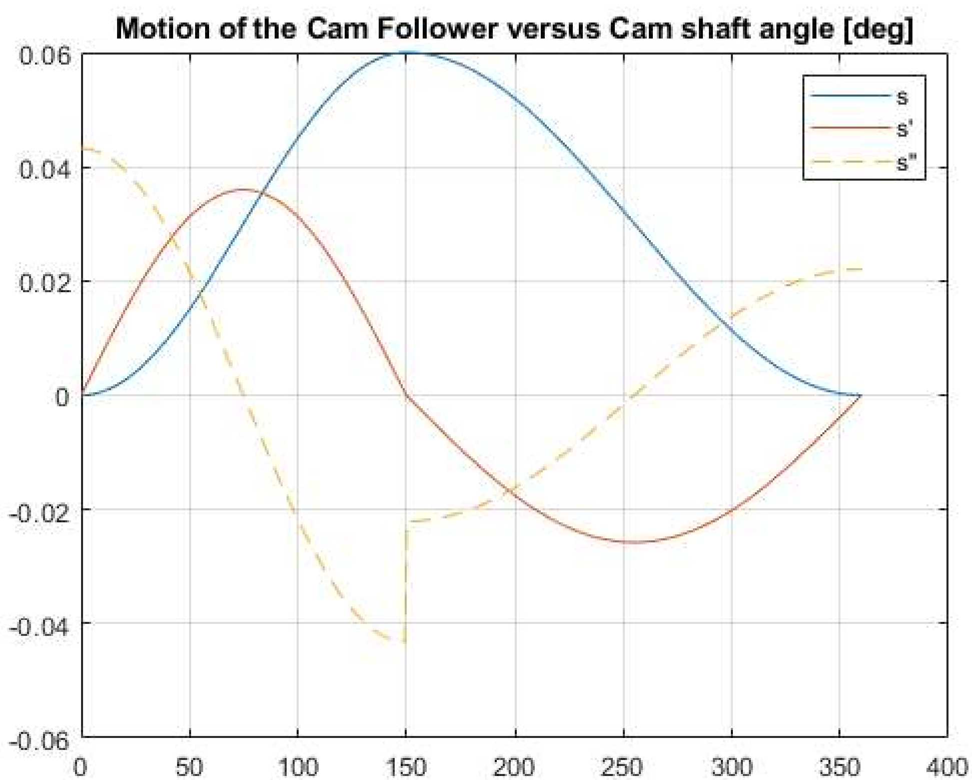
An inertia is needed to have the speed fluctuation in an acceptable range. The flywheel is placed at the input side of the gearbox. The reason for this is to use a weight. This property provides many advantages such as less load on the bearings and shafts and small space allocation for the flywheel.

For the gearbox, an efficiency value is assumed by making an educated guess. This efficiency value has the effect on proportion of torque transmission.

Contents


```
clear
close all
clc

%in the following line, simple harmonic motion is defined.
theta=0:pi/200:2*pi;
H=0.06; % max rise in [mm]
beta=5*pi/6; %rise up to beta
gama=beta; %start of return
r_r=12/1000; % roller radius in [m]
r_b=40/1000; %base radius in [m]
k=20000; %spring constant in [N/m]
delta=10/1000; % spring precompression at s=0 in [m]

Io2=0.02; %mass moment inertia of link 2 in [kgm^2]
Ibo5=0.05; %mass moment inertia of link 5 in [kgm^2]
a2=0.18; %crank length in [m]
a1=0.3; % distance between two fixed revolute joints [m]
m3=7; % in [kg]
a3=0.1; %in [m]

g=9.81; % in [m/s^2]
M=150; % during rise period in [Nm]

%defining s
for i=1:length(theta)

if theta(i)<=5*pi/6

s(i)=H/2*(1-cos(pi*theta(i)/beta));
else
    s(i)=H-H/2*(1-cos(pi*(theta(i)-gama)/(2*pi-5*pi/6)));
end
end

%defining s'
for i=1:length(theta)
    if theta(i)<=5*pi/6
    s_p(i)=pi*H/(2*beta)*sin(pi*theta(i)/beta);
    else
        s_p(i)=-H/2*(pi/(2*pi-5*pi/6))*sin(pi*(theta(i)-gama)/(2*pi-5*pi/6));
    end
end


%defining s''
```

```
for i=1:length(theta)
    if theta(i)<=5*pi/6
        s_pp(i)=pi^2*H/(2*beta^2)*cos(pi*theta(i)/beta);
    else
        s_pp(i)=-H/2*(pi/(2*pi-5*pi/6))^2*cos(pi*(theta(i)-gama)/(2*pi-5*pi/6));
    end
end

%plot s, s' and s'' in the same plot
figure;
plot(theta*180/pi, s);
grid on
hold on
plot(theta*180/pi, s_p,"-");
plot(theta*180/pi, s_pp,"--");
legend("s","s'", "s''");
title("Motion of the Cam Follower versus Cam shaft angle [deg]");
hold off
```

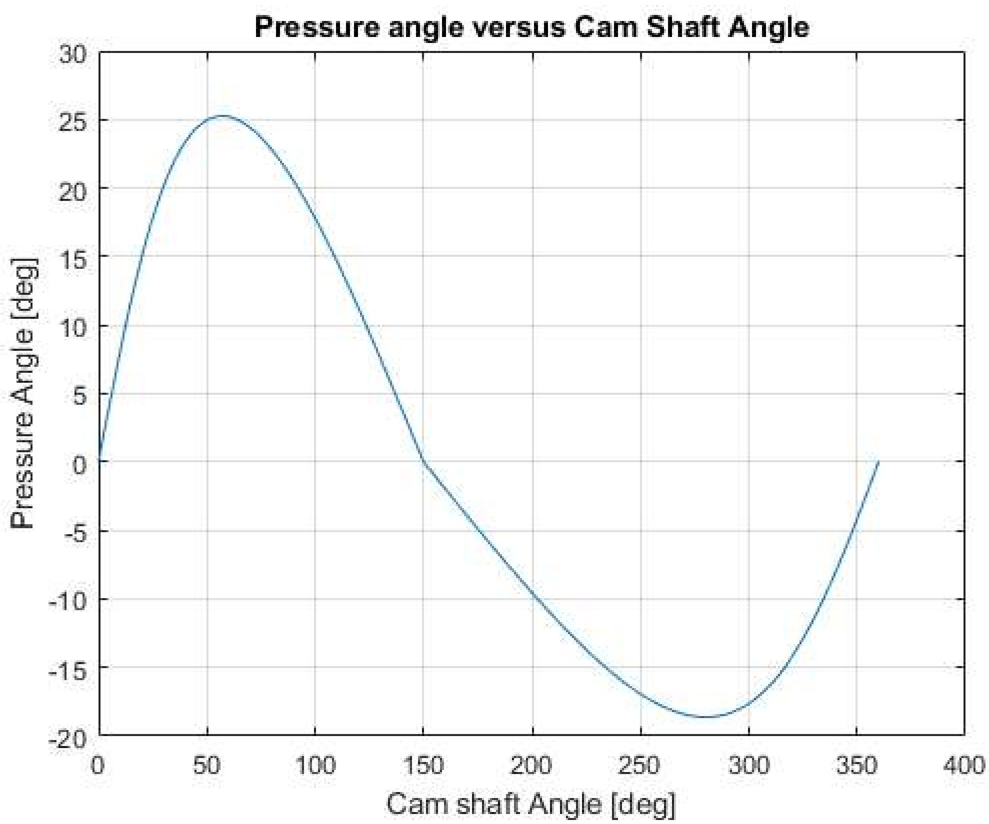


check the pressure angle

```
alpha= atan2(s_p, r_b+r_r+s);
max_alpha=max(alpha);

maxPressureAngle=max(alpha)*180/pi

%plot the pressure angle value
figure
plot(theta*180/pi,alpha*180/pi)
title("Pressure angle versus Cam Shaft Angle")
ylabel("Pressure Angle [deg]")
```

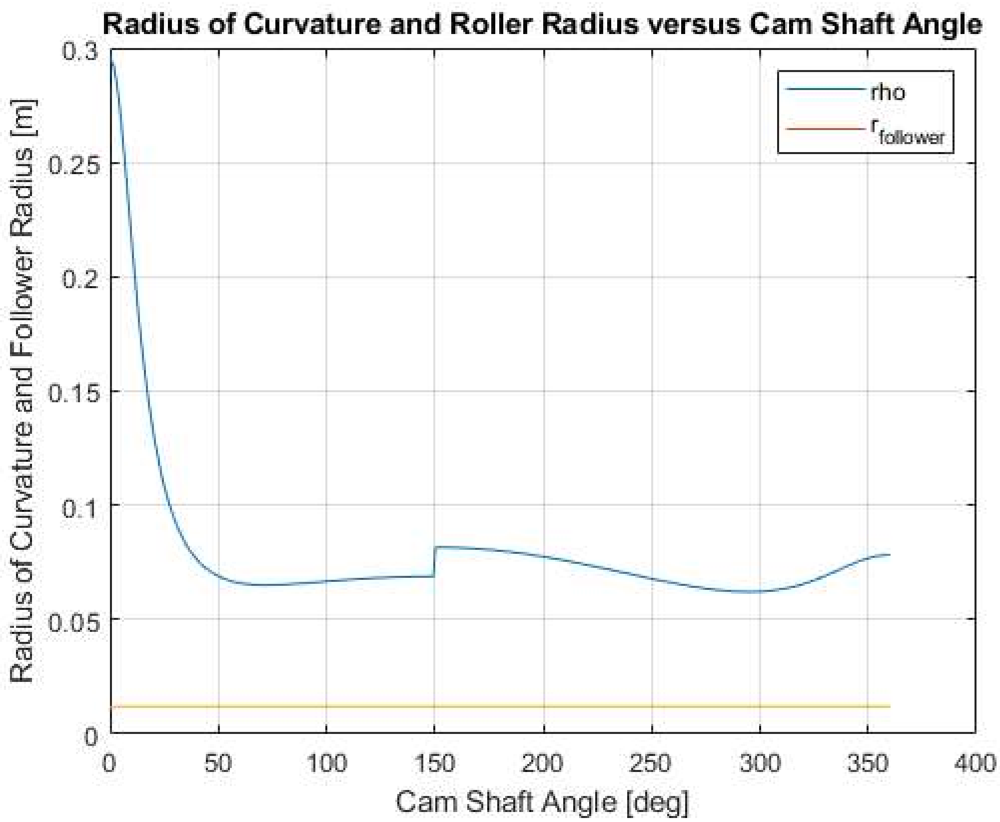
```
xlabel("Cam shaft Angle [deg]")
grid on
```

```
maxPressureAngle =

   25.2536
```



Check undercutting !!!!!

```
rho= ((r_b+r_r+s).^2+s_p.^2).^(3/2)./((r_b+r_r+s).^2+2*s_p.^2-(r_b+r_r+s).*s_pp)-r_r;
r_follower=ones(length(theta))*r_r;

figure
plot(theta*180/pi, rho)
hold on
plot(theta*180/pi, r_follower)
title("Radius of Curvature and Roller Radius versus Cam Shaft Angle")
ylabel("Radius of Curvature and Follower Radius [m]")
xlabel("Cam Shaft Angle [deg]")
grid on
legend("rho", "r_{follower}")
hold off
```

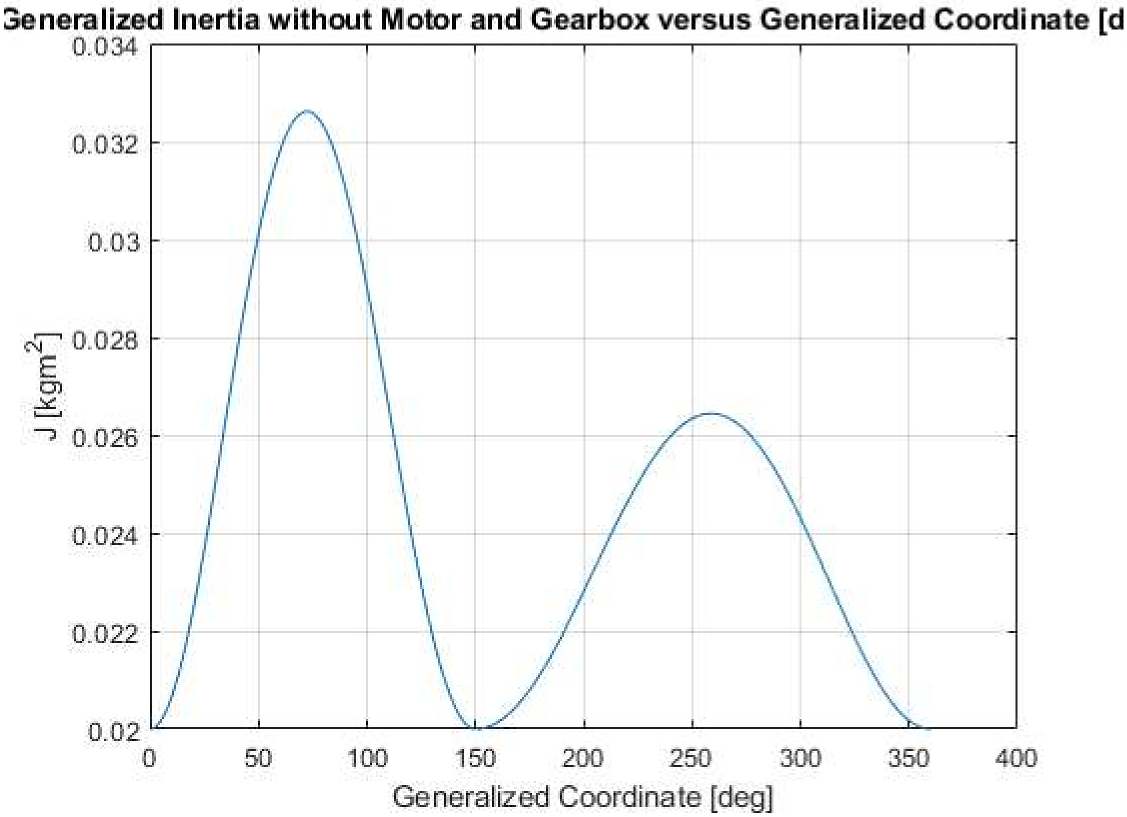


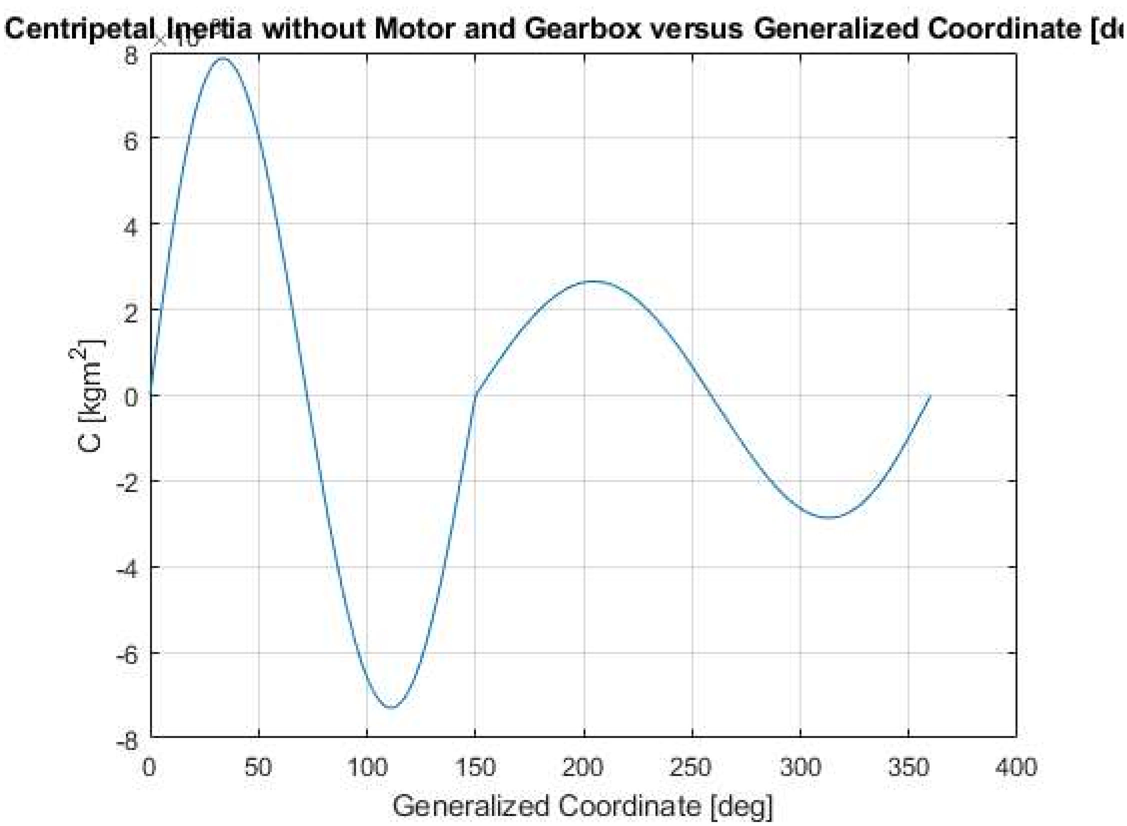

```
phi=acos((a1-a3-r_b-r_r-s)/a2);
g1=s_p./(a2*sin(phi));

g1_p=(a2*s_pp.*sin(phi)-a2*s_p.*cos(phi).*g1)./(a2*sin(phi)).^2;

J=Io2+Ibo5*g1.^2+m3*s_p.^2;
C= Ibo5.*g1.*g1_p+m3*s_p.*s_pp;


for x=1:length(theta)
    if theta(x)<beta
        Q_M(x)=-M.*g1(x);
    else
        Q_M(x)=0;
    end
end
Q_W=-m3*g*s_p;
Q_spr= -k*(s+delta).*s_p;

figure
plot(theta*180/pi, J)
title("Generalized Inertia without Motor and Gearbox versus Generalized Coordinate [deg]")
ylabel("J [kgm^2]")
xlabel("Generalized Coordinate [deg]")
grid on

figure
plot(theta*180/pi, C)
title("Centripetal Inertia without Motor and Gearbox versus Generalized Coordinate [deg]")
ylabel("C [kgm^2]")
xlabel("Generalized Coordinate [deg]")
grid on
```

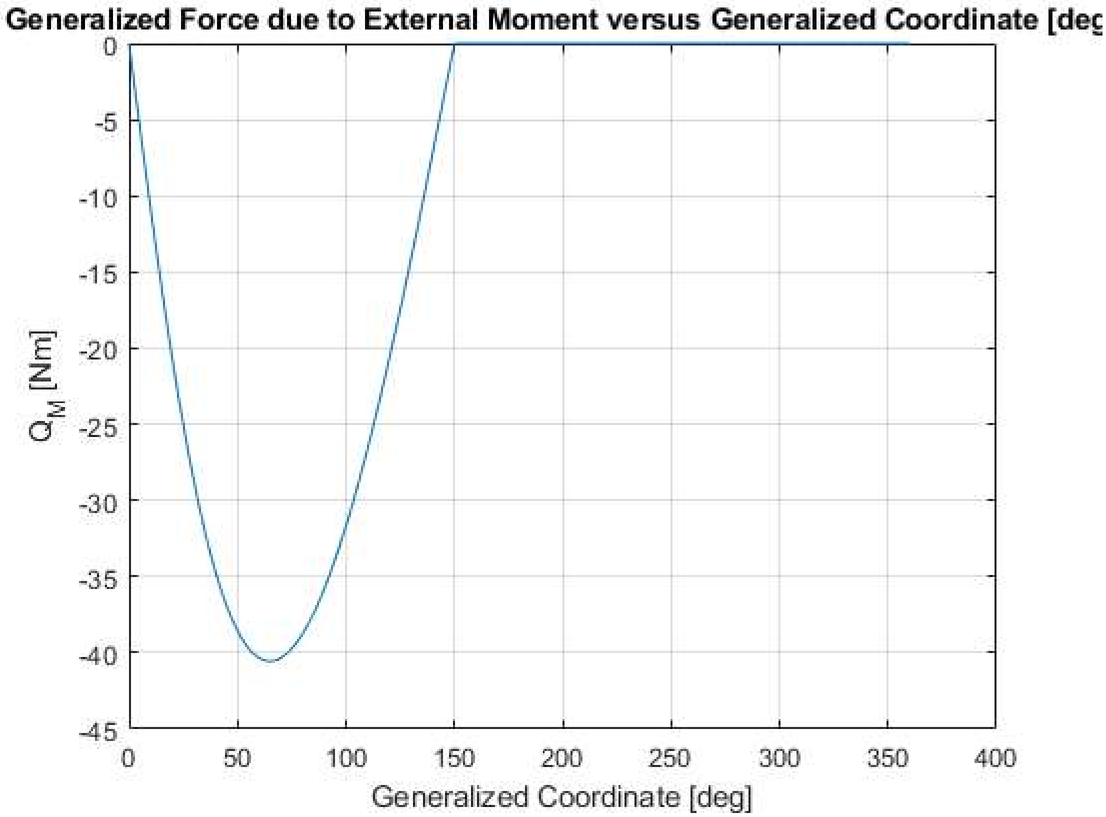
```
figure
plot(theta*180/pi, Q_M)
title("Generalized Force due to External Moment versus Generalized Coordinate [deg]")
ylabel("Q_M [Nm]")
xlabel("Generalized Coordinate [deg]")
grid on

figure
plot(theta*180/pi, Q_W)
title("Generalized Force due to Weight versus Generalized Coordinate [deg]")
ylabel("Q_W [Nm]")
xlabel("Generalized Coordinate [deg]")
grid on

figure
plot(theta*180/pi, Q_spr)
title("Generalized Force due to Spring versus Generalized Coordinate [deg]")
ylabel("Q_spr [Nm]")
xlabel("Generalized Coordinate [deg]")
grid on

slip=1:-0.001:0;
n= 1500*(1-slip); %


%estimating the maximum torque in the mechanism
%by assuming constant speed at the cam shaft
hiz= 400*pi/30;
T_estimation=C*hiz^2-Q_M-Q_spr-Q_W;
maxTorqueEstimated=max(T_estimation) % estimated maximum torque


%Defining the parameters of the motor
n_n=1421;
T_b=27.27;
s_b=0.315;
a=1.587;
b=1.416;
J_motor=0.0028;
T_motor=T_b./(1+(s_b-slip).^2.*(a./slip-b*slip.^2));

%defining gearbox parameters
r=n_n/(hiz*30/pi); %gear ratio
J_gearbox=(1+0.1*r)*J_motor; %inertia of the gearbox


figure
plot(n, T_motor)
title("Torque-Speed Characteristics of the Motor")
ylabel("T [Nm]")
xlabel("motor speed [rpm]")
grid on

% defining time step and span for Euler method
h=0.00001; %time-step in [s]
t=0:h:3; %span from 0 to 3 seconds

q=zeros(1,length(t));
q_d=zeros(1,length(t));
q_dd=zeros(1,length(t));
J=zeros(1,length(t));
```

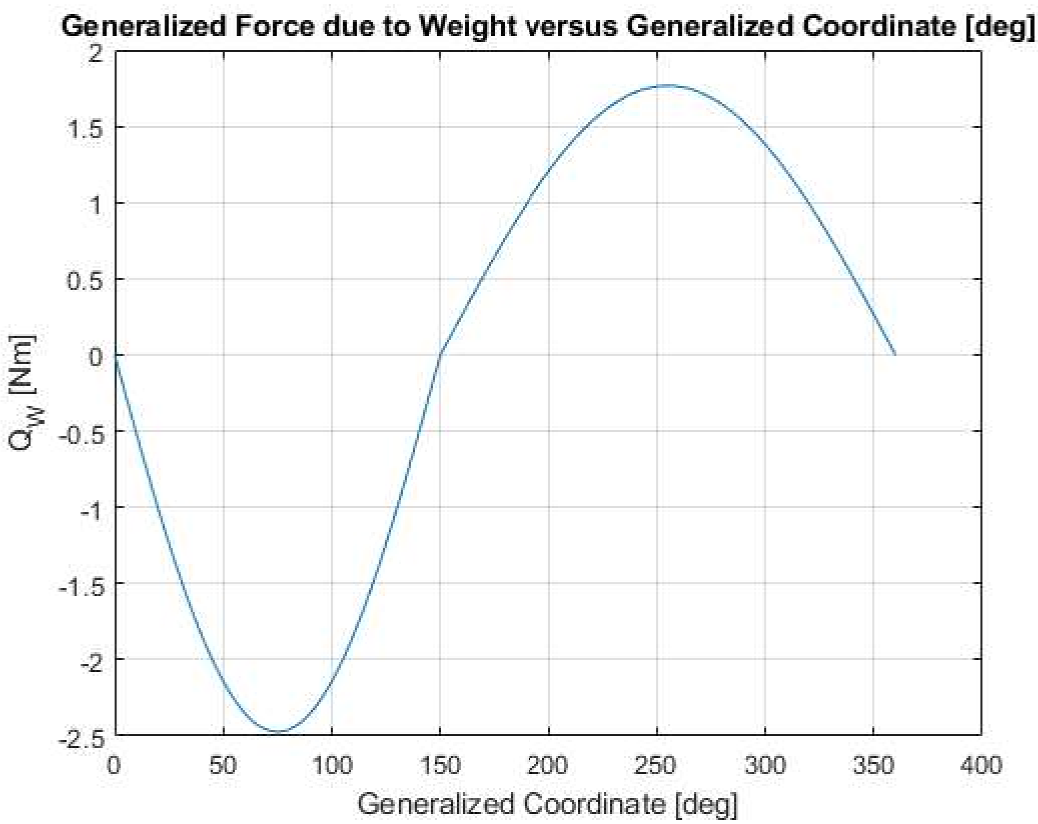
```
C=zeros(1,length(t));
s=zeros(1,length(t));
s_p=zeros(1,length(t));
s_pp=zeros(1,length(t));
M=zeros(1,length(t));
phi=zeros(1,length(t));
g1=zeros(1,length(t));
g1_p=zeros(1,length(t));

Flywheel=0.06;  % in [kgm^2]
eff=0.9; % gearbox efficiency
counter=1;

for i=1:length(t)-1
    if mod(q(i), 2*pi) <= 5*pi/6
        s(i)=H/2*(1-cos(pi*mod(q(i), 2*pi)/beta));
        s_p(i)=pi*H/(2*beta)*sin(pi*mod(q(i), 2*pi)/beta);
        s_pp(i)=pi^2*H/(2*beta^2)*cos(pi*mod(q(i),2*pi)/beta);
        M(i)=150;

    else

        s(i)=H-H/2*(1-cos(pi*(mod(q(i), 2*pi)-gama)/(2*pi-5*pi/6)));
        s_p(i)=-H/2*(pi/(2*pi-5*pi/6))*sin(pi*(mod(q(i), 2*pi)-gama)/(2*pi-5*pi/6));
        s_pp(i)=-H/2*(pi/(2*pi-5*pi/6))^2*cos(pi*(mod(q(i), 2*pi)-gama)/(2*pi-5*pi/6));
        M(i)=0;
    end


    phi(i)=acos((a1-a3-r_b-r_r-s(i))/a2);
    g1(i)=s_p(i)./(a2*sin(phi(i)));
    g1_p(i)=(a2*s_pp(i).*sin(phi(i))-a2*s_p(i).*cos(phi(i)).*g1(i))./(a2*sin(phi(i))).^2;
    J(i)=Io2+Ibo5*g1(i).^2+m3*s_p(i).^2+J_gearbox+J_motor*r^2+Flywheel*r^2;
    C(i)=Ibo5.*g1(i).*g1_p(i)+m3*s_p(i).*s_pp(i);
    Q_M=-M(i).*g1(i);
    Q_W=-m3*g*s_p(i);
    Q_spr=-k*(s(i)+delta).*s_p(i);
    slip=1-abs(q_d(i))*r/(1500*pi/30);
    Q_motor(i)= T_b./(1+(s_b-slip).^2.*(a./slip-b*slip.^2))*r*eff;
    Q(i)=Q_motor(i)+Q_M+Q_W+Q_spr;

    alpha(i)=atan2(s_p(i),r_b+r_r+s(i));

    q(i+1)=q(i)+h*q_d(i);
    q_dd(i)= (Q(i)-C(i)*q_d(i)^2)/J(i);
    q_d(i+1)= q_d(i)+h*q_dd(i);

    if q_d(i)>400*pi/30 & counter==1
        steadtStateTime=t(i);
        counter=71;
    end
end
```

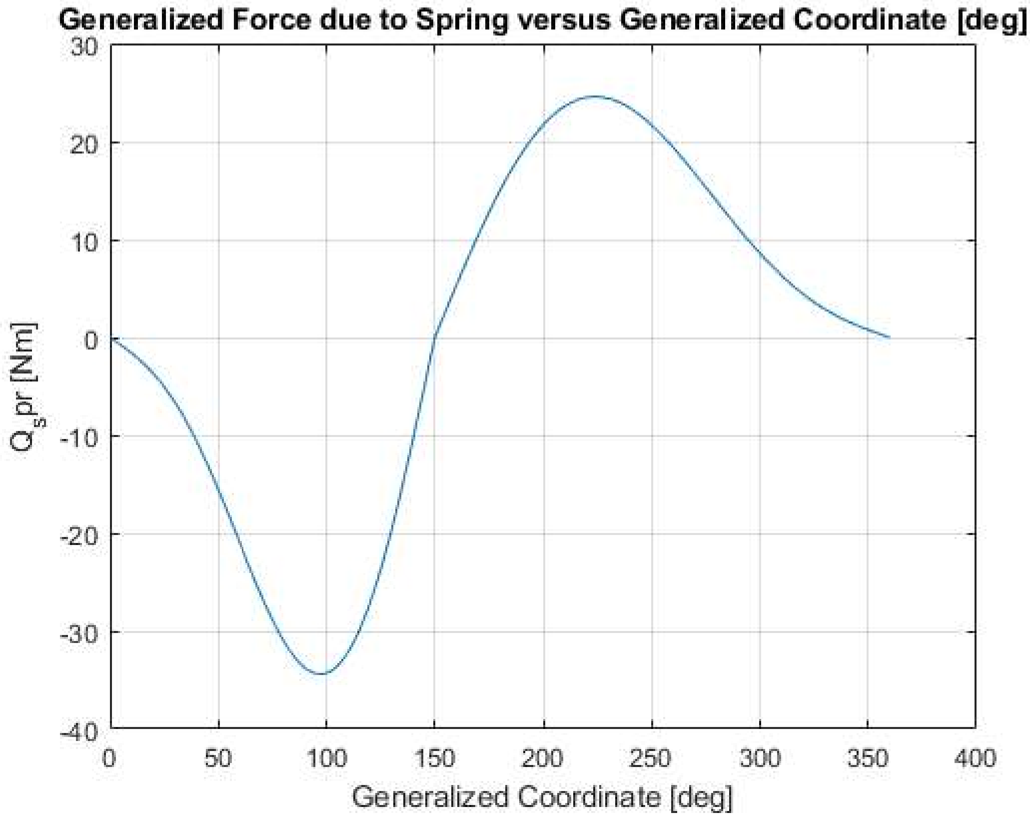
```
maxTorqueEstimated =

   70.6292
```

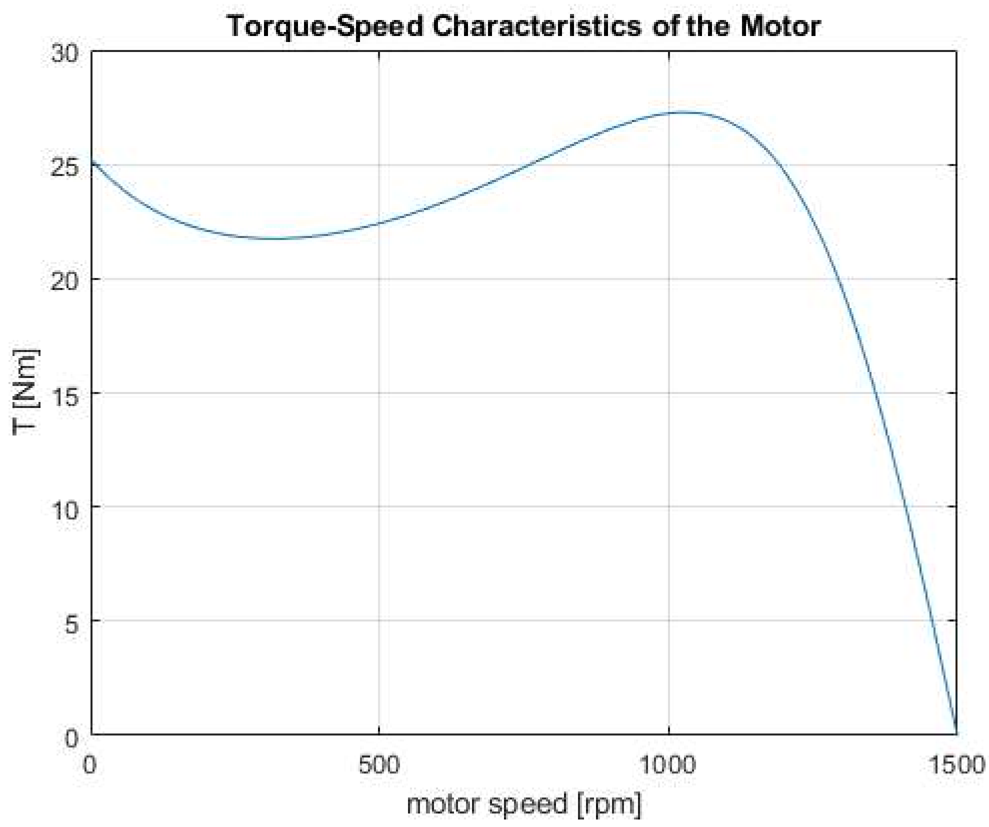

Generalized Inertia without Motor and Gearbox versus Generalized Coordinate [d
J [kgm^2]
0.034
0.032
0.03
0.028
0.026
0.024
0.022
0.02
0
50
100
150
200
250
300
350
400
Generalized Coordinate [deg]


Centripetal Inertia without Motor and Gearbox versus Generalized Coordinate [d
C [kgm^2]
8
6
4
2
0
-2
-4
-6
-8
0
50
100
150
200
250
300
350
400
Generalized Coordinate [deg]


Generalized Force due to External Moment versus Generalized Coordinate [deg
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
Q_M [Nm]
0
50
100
150
200
250
300
350
400
Generalized Coordinate [deg]


Generalized Force due to Weight versus Generalized Coordinate [deg]
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
Q_W [Nm]
0
50
100
150
200
250
300
350
400
Generalized Coordinate [deg]





creating expression for fluctuation

```
q_dd_avg=400*pi/30;
fluctuation=(max(q_d(ceil(length(t)/3):length(t)-1))-min(q_d(ceil(length(t)/3):length(t)-1)))/q_dd_avg*100;
```

creating expression for stability

```
omega_bd=(1-s_b)*max(n);
omega_min=min(q_d(ceil(length(t)/3):length(t)-1))*30/pi; %[rpm]
omega_max=max(q_d(ceil(length(t)/3):length(t)-1))*30/pi; %[rpm]
stability= (r*omega_min-omega_bd)/(max(n)-omega_bd)*100;
```

running cost

```
omega_Avg=(omega_min+omega_max)/2;
runningCost= abs(omega_Avg*r-n_n)/(max(n)-omega_bd)*100;
```

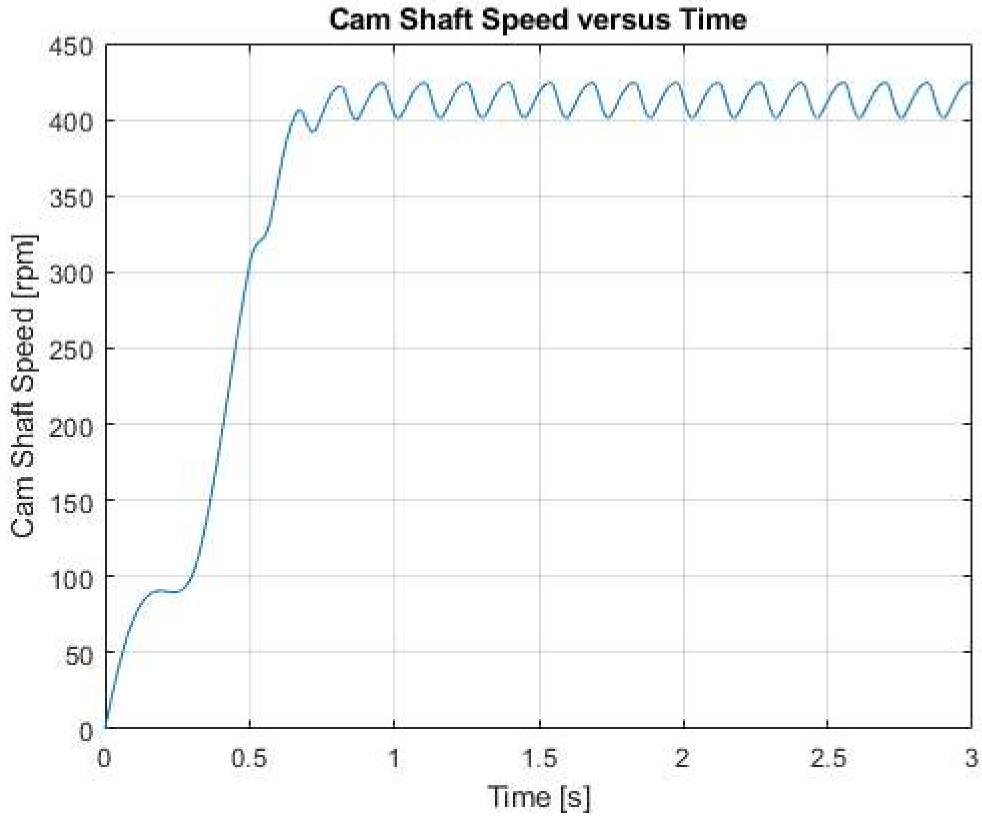
performance parameters

```
steadtStateTime,stability, fluctuation,runningCost
```

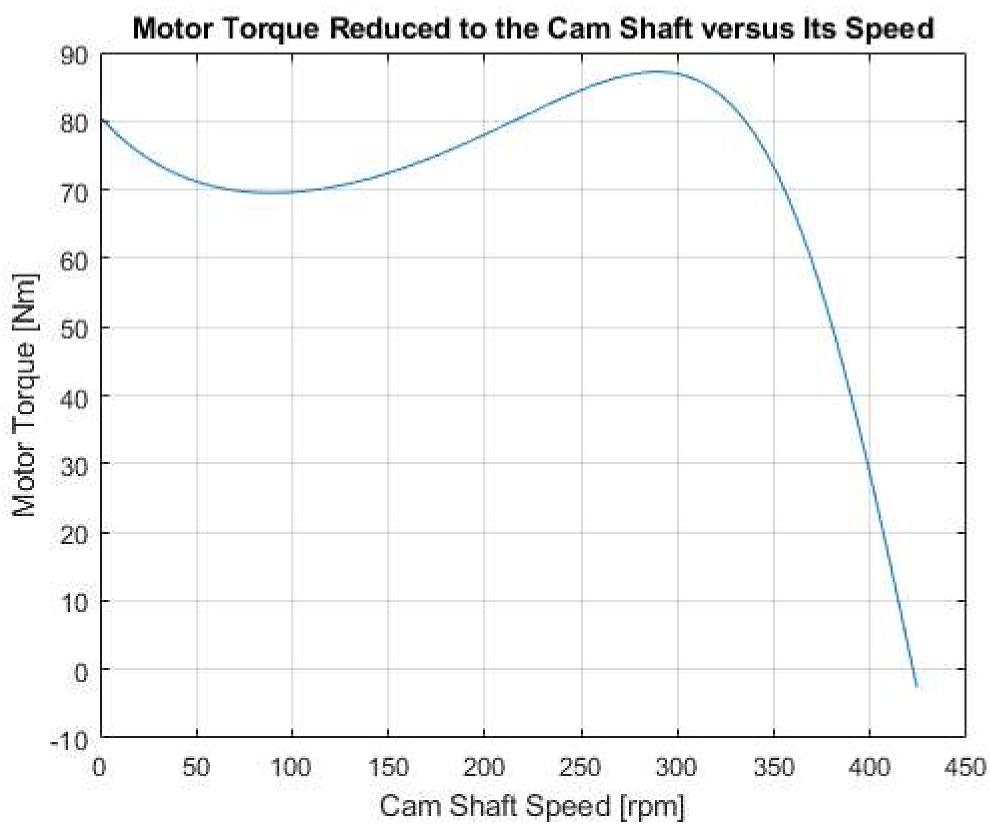
```
steadtStateTime =

    0.6513


stability =

   84.0363


fluctuation =

    5.8038


runningCost =

    9.4831
```

```
figure
plot(t,abs(q_d)*30/pi)
title("Cam Shaft Speed versus Time")
ylabel("Cam Shaft Speed [rpm]")
xlabel("Time [s]")
grid on

figure
plot(q_d(1:length(Q_motor))*30/pi, Q_motor)
title("Motor Torque Reduced to the Cam Shaft versus Its Speed")
ylabel("Motor Torque [Nm]")
xlabel("Cam Shaft Speed [rpm]")
grid on

%plotting the torque due to generalized inertia versus time
figure
plot(t, J.*q_dd,"r")
title("Torque due Generalized Inertia versus Time")
```

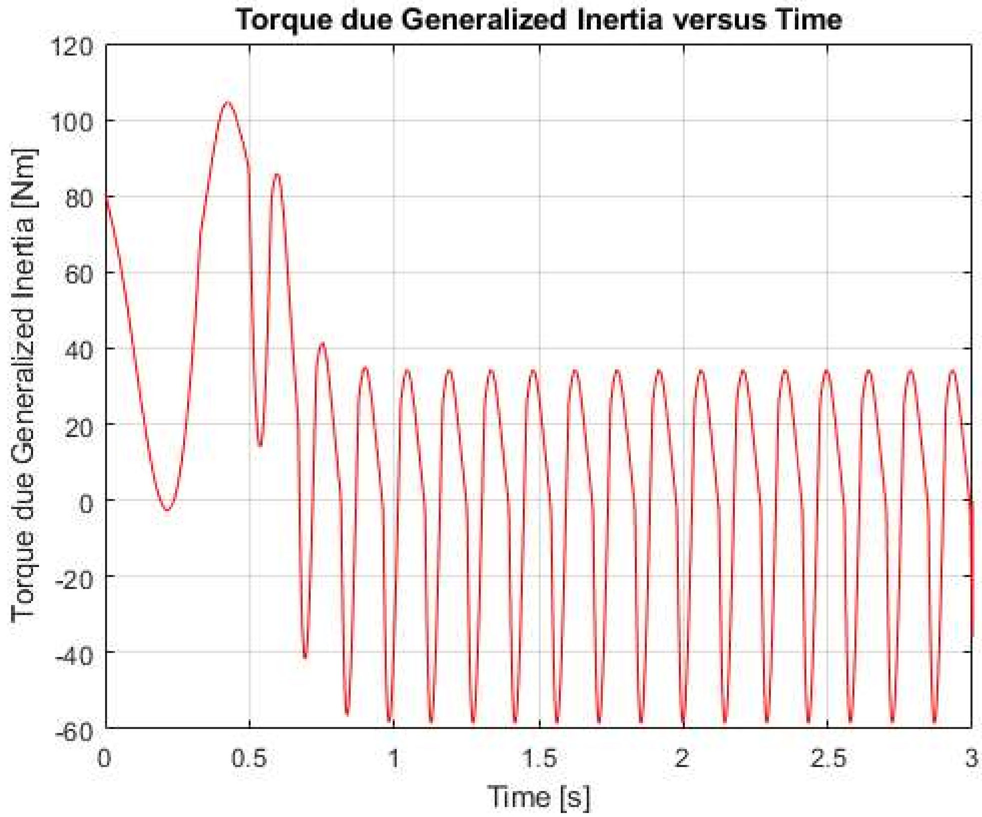
```
ylabel("Torque due Generalized Inertia [Nm]")
xlabel("Time [s]")
grid on

%plotting torque due to centripetal inertia
figure
plot(t, C.*q_d.^2,"k")
title("Torque due to Centripetal Inertia versus Time")
ylabel("Torque due to Centripetal Inertia [Nm]")
xlabel("Time [s]")
grid on
```

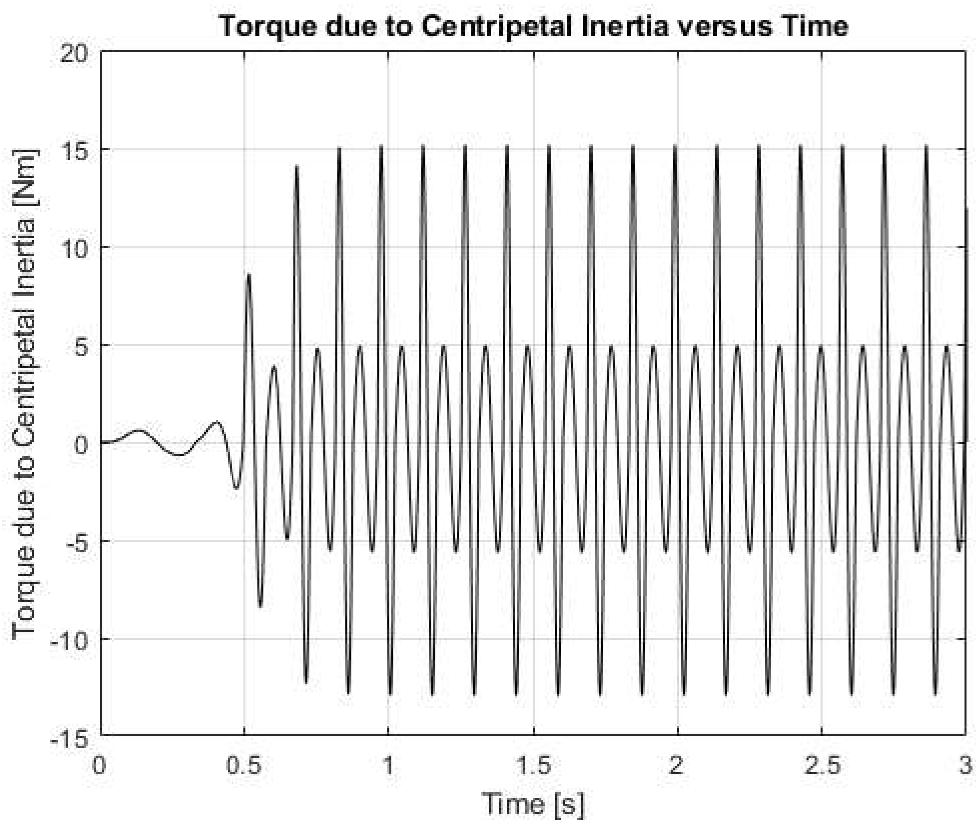


Cam Shaft Speed versus Time
Cam Shaft Speed [rpm]
Time [s]
0
50
100
150
200
250
300
350
400
450
0
0.5
1
1.5
2
2.5
3



Motor Torque Reduced to the Cam Shaft versus Its Speed
Motor Torque [Nm]
Cam Shaft Speed [rpm]
-10
0
10
20
30
40
50
60
70
80
90
0
50
100
150
200
250
300
350
400
450



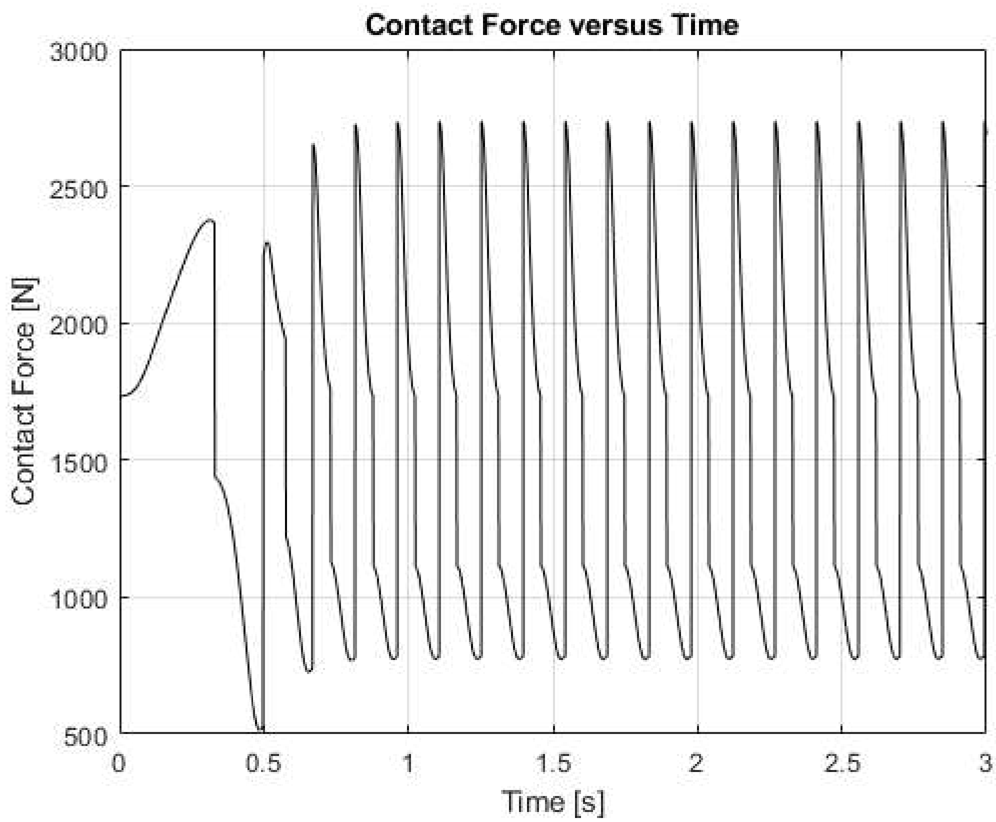


contact force

```
alpha(length(alpha)+1)=alpha(length(alpha));
F45y=(Ibo5*(g1.*q_dd+g1_p.*q_d.^2)+M)./(a2*sin(phi));
F_contact=(m3*(s_p.*q_dd+s_pp.*q_d.^2)+m3*g+k*(s+delta)+F45y)./(cos(alpha));
```

```
minimumContactForce=min(F_contact)

figure
plot(t, F_contact,"k")
title("Contact Force versus Time")
ylabel("Contact Force [N]")
xlabel("Time [s]")
grid on
```

```
minimumContactForce =

  514.2013
```



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