CAM DYNAMICS

Cam Dynamics Problems

- **Dynamic Force Analysis**
- **Analysis and Prevention of Follower Jump**
- **Dynamic Motion Analysis**
- **Vibration Analysis and Control**

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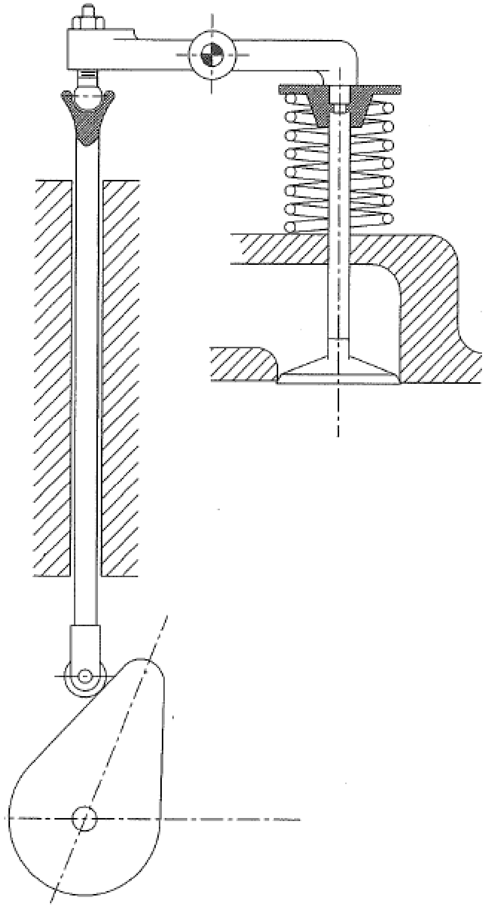
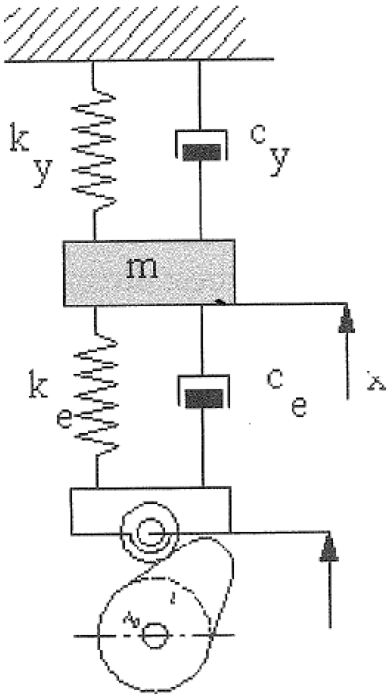
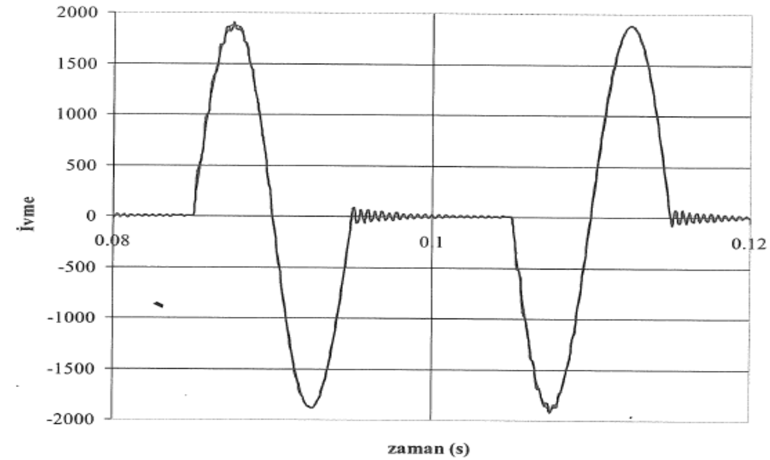
- ***Dynamic Force Analysis***:
  - Motion is assumed and driving force against the known follower resistance is calculated along with the accompanying reaction forces
  - Typically, cam shaft is assumed to rotate at a constant angular speed, and the required cam-shaft torque, cam-follower contact force, and other reaction forces are calculated
- ***Follower Jump***:
  - Separation of cam and follower, and subsequent impact
  - Calculation of required force to maintain the contact between the cam and the follower at all times
  - Typically the spring constant and its initial deformation

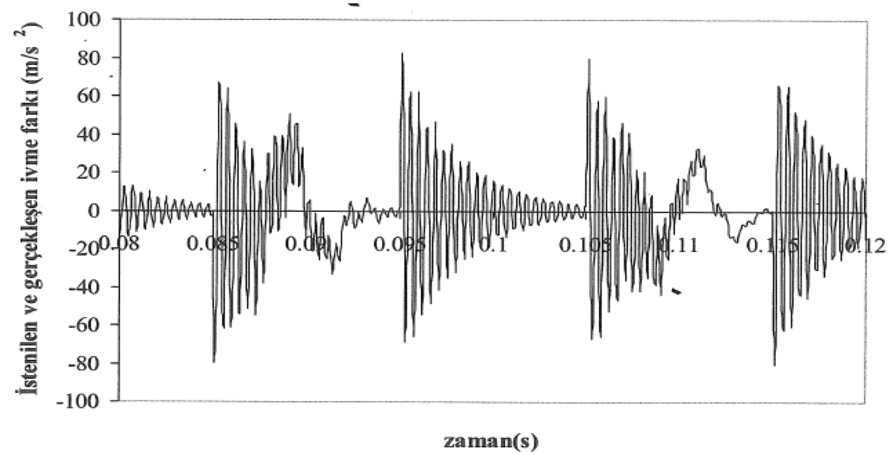
- ***Dynamic Motion Analysis***:
  - Calculation of follower motion, given all the external forces acting on the system, including the driving torque (or force), follower resistance, spring forces, etc.
  - Typically, a model of the prime-mover, such as an electric motor, is coupled with the cam-follower system, and resulting diffrential equation of motion is integrated
- ***Vibration Analysis and Control*** :
  - Analysis of vibratory motion of follower (superimposed on its gross-motion) based on a vibration model
  - Typically torsional elasticity of the cam-shaft is the main source of the potential energy storage
  - Vibration models of varying DOF are employed

Vibration Models

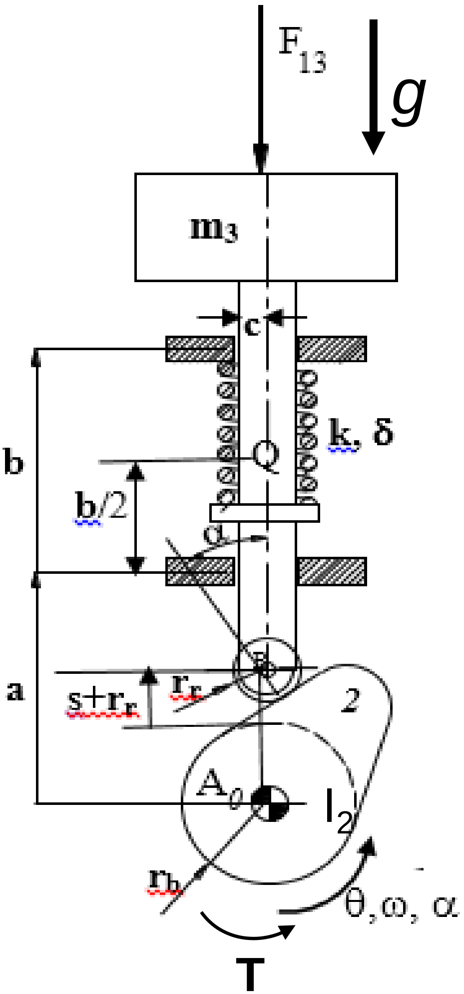








DYNAMIC MOTION ANALYSIS OF CAM - FOLLOWER SYSTEMS



A force-closed radial cam imparts following cycloidal motion to the in-line translating roller follower: Rise by 50 mm for 120^0 cam-shaft rotation, followed by a 60^0 dwell, then return back to the initial position for 120^0 followed by a 60^0 dwell

A constant follower resistance F_{13} = 1200 N acts downwards during the rise period only. Driving torque T is specified

The base circle radius of the cam is r_b = 40 mm, roller radius is r_r=10 mm

The spring constant is 60000N/m, and is pre-compressed by 20 mm when the follower is at its lowest position. Assume contact is maintained

m_3 = 4 kg ; I_2 = 0.01 kgm^2 (about A_0), ignore rotary inertia of the roller and friction

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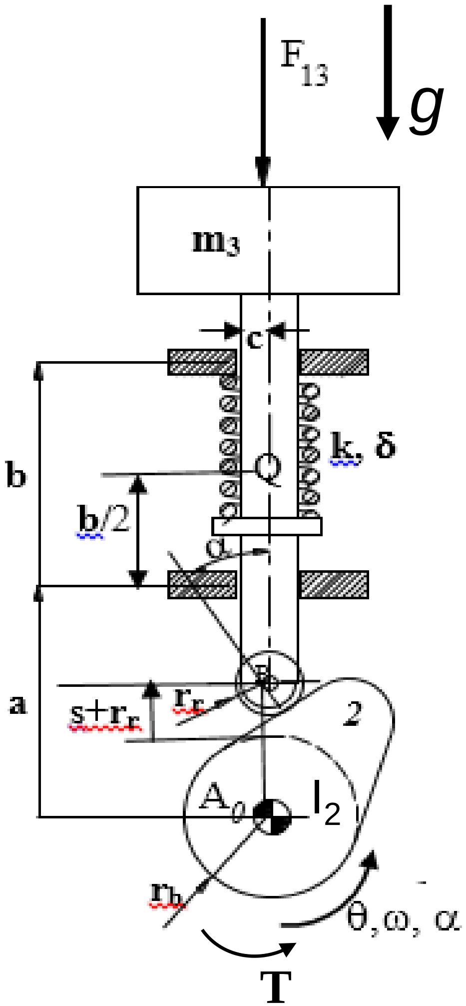


Single DOF system

Generalized coordinate $q \rightarrow \theta$

Dependent coordinate $\emptyset \rightarrow s$

Constraint equation and its derivatives:

$$s(\theta), s'(\theta), s''(\theta)$$

Then:

$$\frac{d\emptyset}{dq} = g = \frac{ds}{d\theta} = s'$$

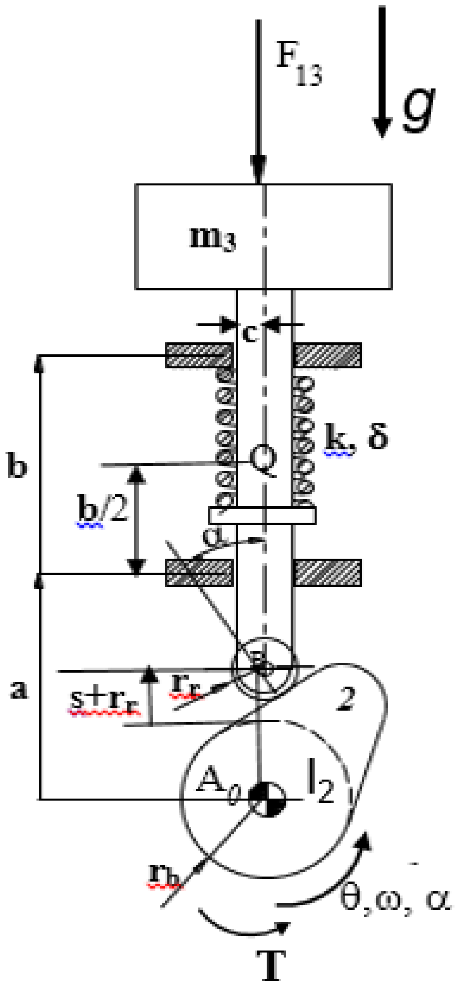
$$\frac{d^2\emptyset}{dq^2} = g' = \frac{d^2s}{d\theta^2} = s''$$

Follower velocity: $v = \omega s'$

Follower Acceleration: $a = \alpha s' + \omega^2 s''$

Equation of Motion

$$J\ddot{\theta} + \frac{1}{2}\frac{dJ}{d\theta}\dot{\theta}^2 = Q$$



Generalized and Centripetal Inertia

- $J(\theta) = I_2 + m_3(s')^2$
- $C(\theta) = \frac{1}{2}\frac{dJ}{d\theta} = m_3 s' s''$

Generalized Force

- $Q = Q_T + Q_{m_3 g} + Q_{F_{13}} + Q_{spr}$
- $Q = T - [m_3 g + F_{13} + k(s + \delta)]s'$

If Cam is Driven by an Electric Motor: $\boldsymbol{T = T(\omega)}$

- Equation of motion $J\ddot{\theta} + C\dot{\theta}^2 = Q$ is integrated for initial conditions (IC): $\theta_o, \dot{\theta}_o$ at $t = 0$
- Solution will yield: $\theta(t)$, $\dot{\theta}(t)$, $\ddot{\theta}(t)$
- Constraint equations (standard cam motion curves) will give (with θ as the parameter):

$$s, s', s''$$

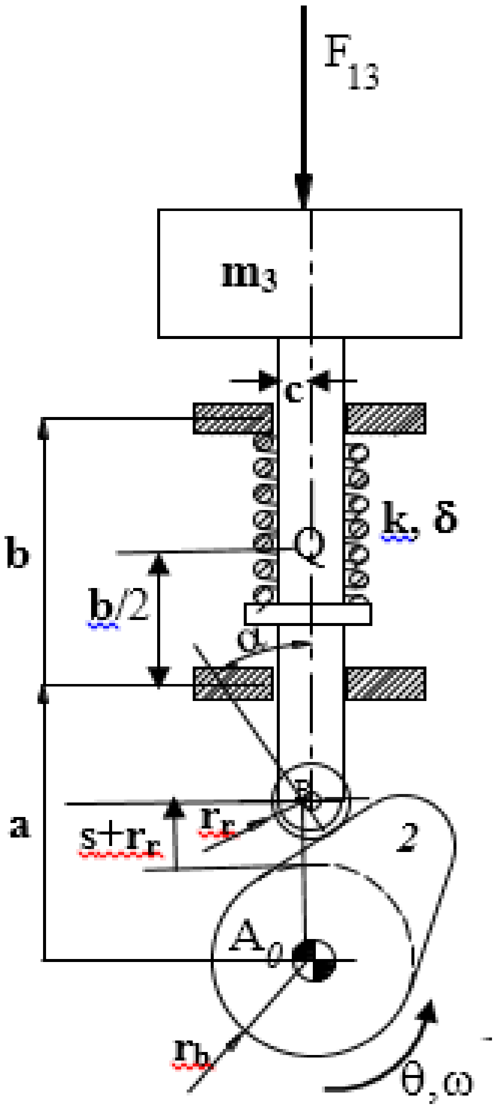
- Velocity and acceleration of the follower:

$$v(t) = \dot{s}(t) = \dot{\theta}(t)s'$$

$$a(t) = \ddot{s}(t) = \ddot{\theta}(t)s' + \dot{\theta}^2(t)s''$$

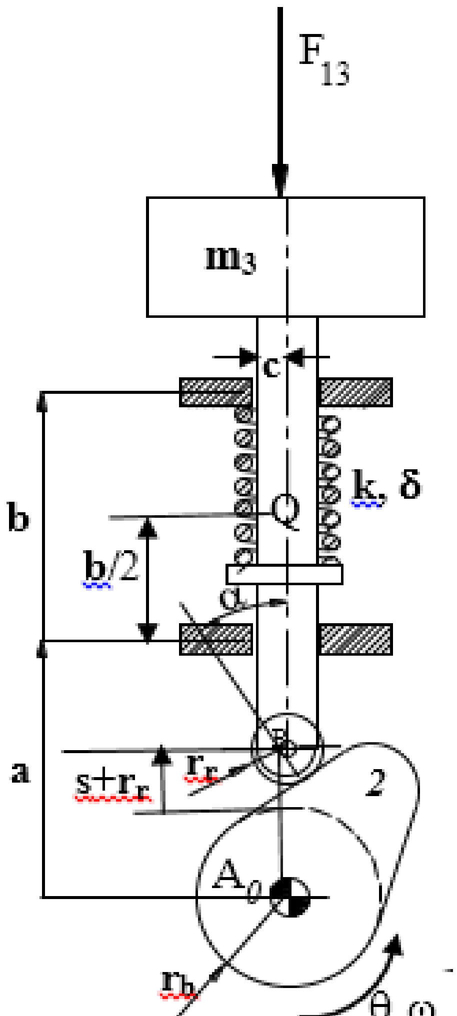
Force Analysis Example

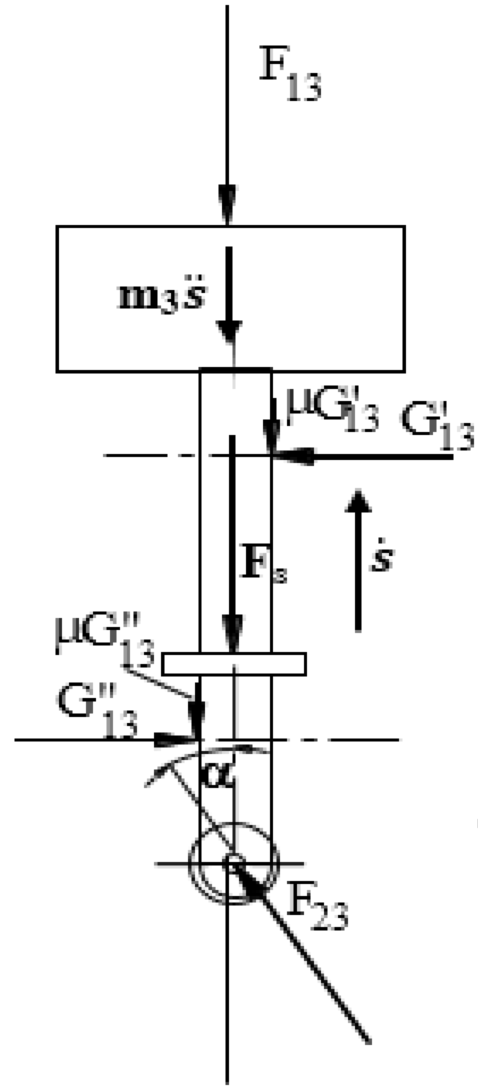
(Example 4.1 of "Makina Dinamiği" by E. Söylemez)



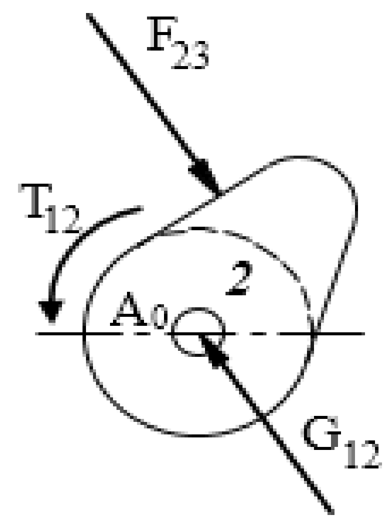
- The same follower motion and follower force F_{13} as in the previous example
- Horizontal plane (no gravity)
- The cam shaft rotates at a constant speed of 1000 rpm
- The follower mass is 4 kg
- The base circle radius is r_b = 40 mm, roller radius is r_r=10 mm. The bearing locations and follower stem width are: a=200 mm, b=250 mm, c=20 mm
- The coefficient of dry friction is 0.1 between the follower and its bearings
- The spring constant is 60000N/m, and is pre-compressed by 20 mm when the follower is at its lowest position

FBD's of Cam and Follower during RISE

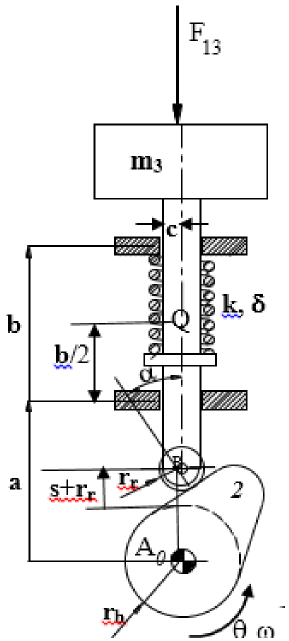


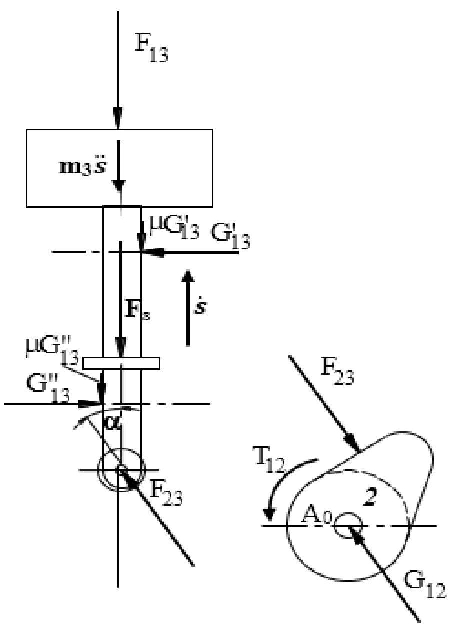


$$F_s = k(s + \delta)$$



During Rise:





Link-2:

$$\sum M_{A_0} = 0 \;\; \rightarrow \;\; T_{12} - F_{23}(s + r_r + r_b)\sin\alpha = 0 \quad (1)$$

Link-3:

$$\sum F_x = 0 \rightarrow G''_{13} - G'_{13} - F_{23}\sin\alpha = 0 \qquad (2)$$

$$\sum F_y = 0 \rightarrow F_{23}\cos\alpha - F_s - F_{13} - \mu(G'_{13} + G''_{13}) - m_3\ddot{s} = 0 \;(3)$$

$$\sum M_Q = \;0 \quad \rightarrow \quad \frac{b}{2}(G'_{13} + G''_{13}) + c\mu(G''_{13} -$$

During Rise:

Solve $G''_{13} - G'_{13}$ from (2) and $G''_{13} + G'_{13}$ from (3), and substitute into (4) to get:

$$F_{23} = \frac{b}{b\cos\alpha + \mu\sin\alpha\big(2\mu c - 2a - b + 2(r_b + r_r + s)\big)}(F_s + F_{13} + m_3\ddot{s}) \quad (5)$$

Substitute (5) into (1) to get:

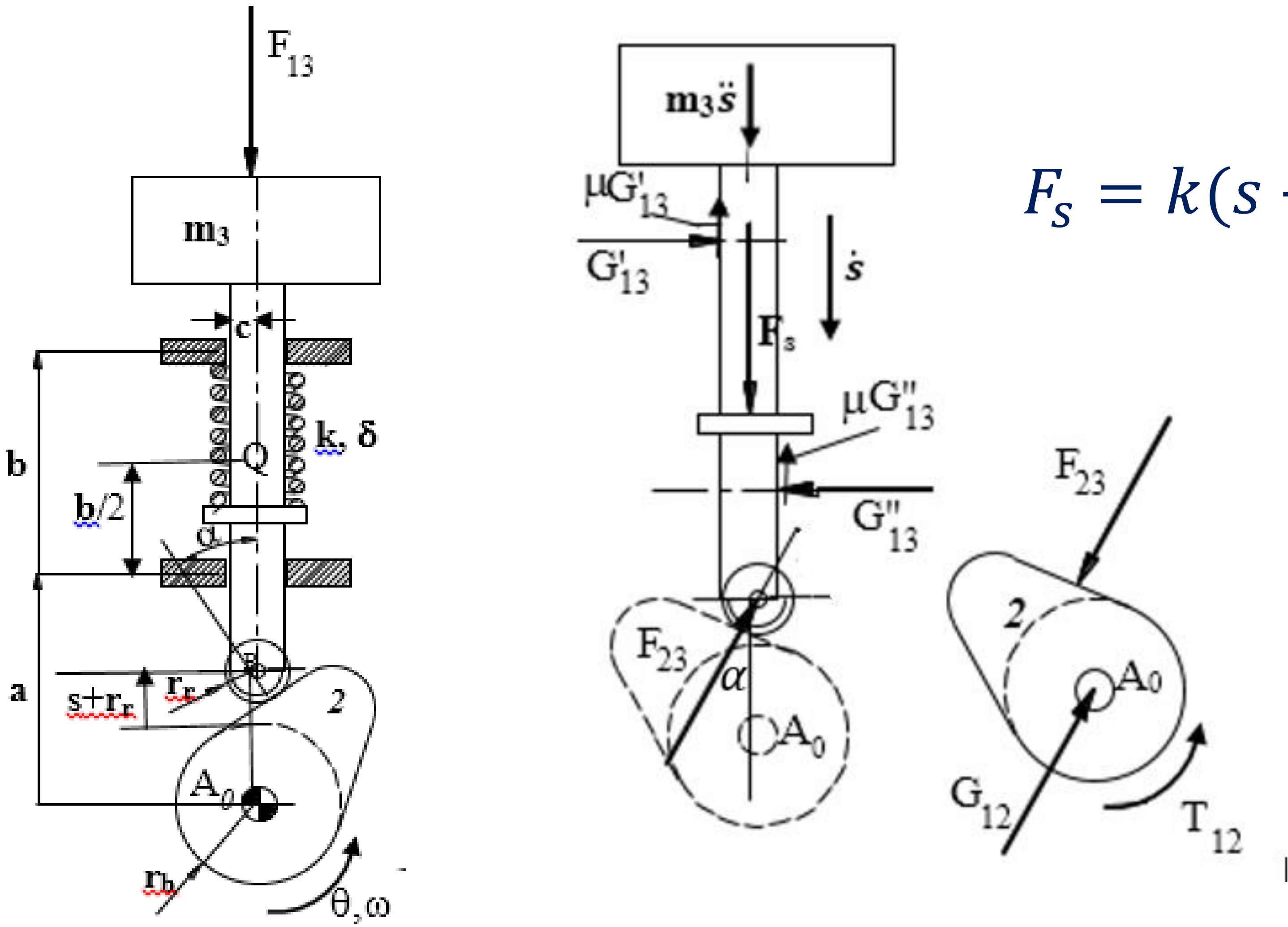
$$T_{12} = F_{23}(s + r_r + r_b)\sin\alpha$$

Substitute (5) into (2) and (3) and solve to get:

$$G'_{13} = \frac{\sin\alpha\big(a - \mu c - (r_b + r_r + s)\big)}{b\cos\alpha + \mu\sin\alpha\big(2\mu c - 2a - b + 2(r_b + r_r + s)\big)}(F_s + F_{13} + m_3\ddot{s})$$

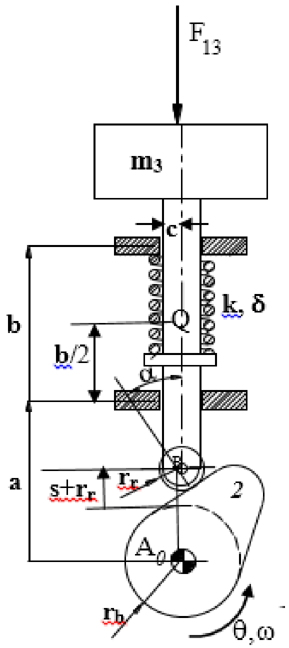
$$G''_{13} = \frac{\sin\alpha\big(a + b - \mu c - (r_b + r_r + s)\big)}{b\cos\alpha + \mu\sin\alpha\big(2\mu c - 2a - b + 2(r_b + r_r + s)\big)}(F_s + F_{13} + m_3\ddot{s})$$

FBD's of Cam and Follower during RETURN



$$F_s = k(s + \delta)$$

During Return:



Link-2:

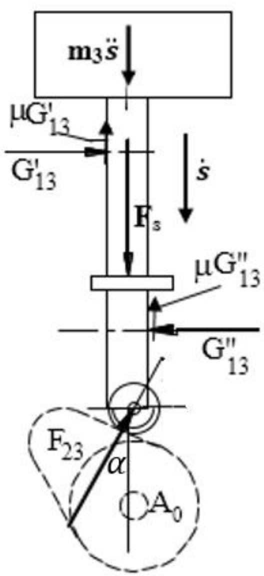
$$\sum M_{A_0} = 0 \rightarrow T_{12} + F_{23}(s + r_r + r_b)\sin\alpha = 0 \ (1)$$

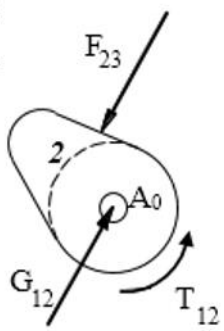
Link-3:

$$\sum F_x = 0 \rightarrow \ G'_{13} - G''_{13} + F_{23}\,\sin\alpha = 0 \qquad (2)$$

$$\sum F_y = 0 \rightarrow F_{23}\cos\alpha + \mu(G'_{13} + G''_{13}) - F_s - m_3\ddot{s} = 0 \ (3)$$

$$\sum M_Q = \ 0 \rightarrow \ -\frac{b}{2}(G'_{13} + G''_{13}) + c\mu(G''_{13} -$$





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During Return:

Solve $G''_{13} - G'_{13}$ from (2) and $G''_{13} + G'_{13}$ from (3), and substitute into (4) to get:

$$F_{23} = \frac{b}{b\cos\alpha + \mu\sin\alpha\big(2\mu c + 2a + b - 2(r_b + r_r + s)\big)}(F_s + m_3\ddot{s}) \quad (5)$$

Substitute (5) into (1) to get:

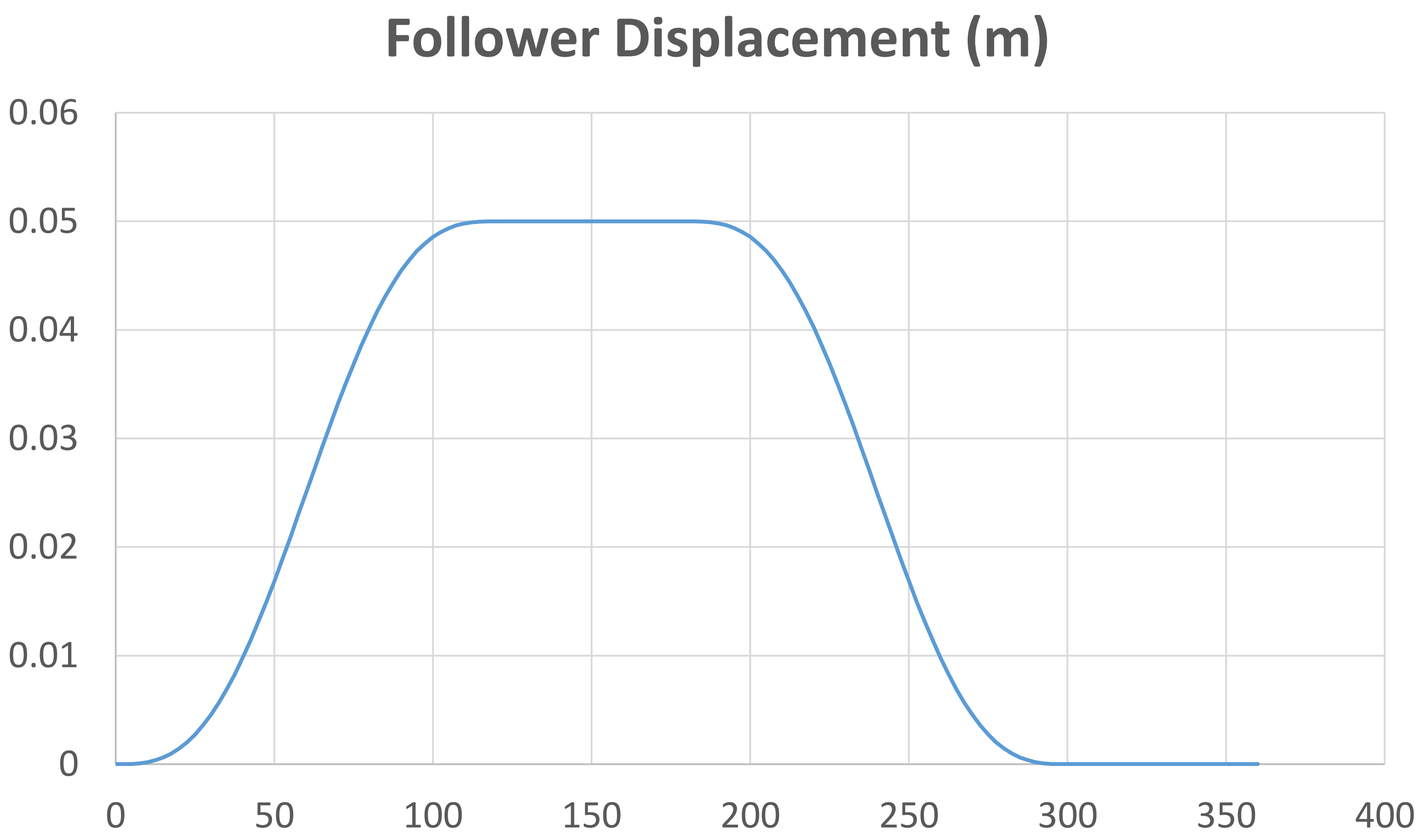
$$T_{12} = -F_{23}(s + r_r + r_b)\sin\alpha$$

Substitute (5) into (2) and (3) and solve to get:

$$G'_{13} = \frac{\sin\alpha\big(a + \mu c - (r_b + r_r + s)\big)}{b\cos\alpha + \mu\sin\alpha\big(2\mu c + 2a + b - 2(r_b + r_r + s)\big)}(F_s + m_3\ddot{s})$$

$$G''_{13} = \frac{\sin\alpha\big(a + b + \mu c - (r_b + r_r + s)\big)}{b\cos\alpha + \mu\sin\alpha\big(2\mu c + 2a + b - 2(r_b + r_r + s)\big)}(F_s + m_3\ddot{s})$$

RESULTS
Follower Displacement



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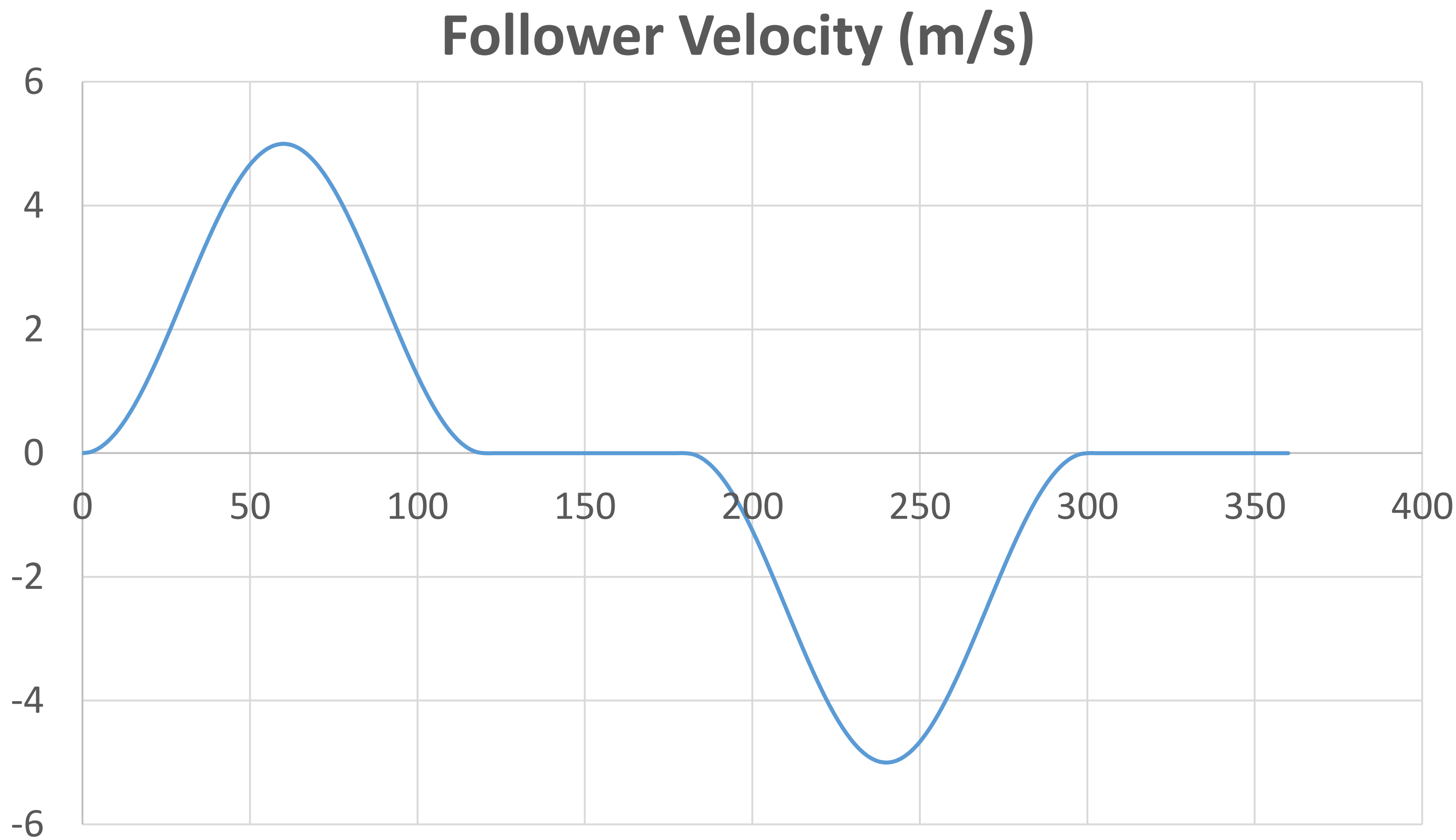
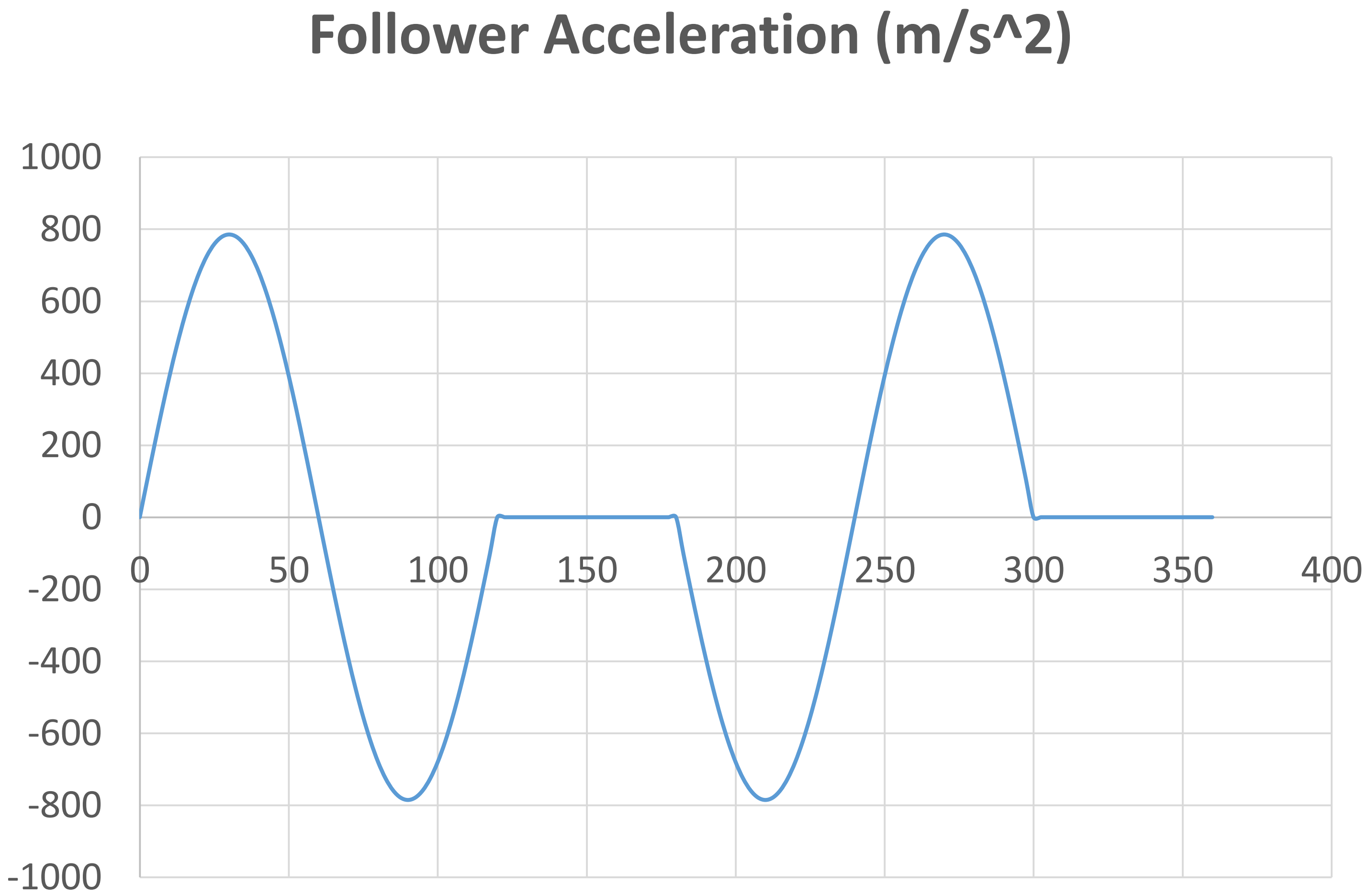
RESULTS
Follower Velocity

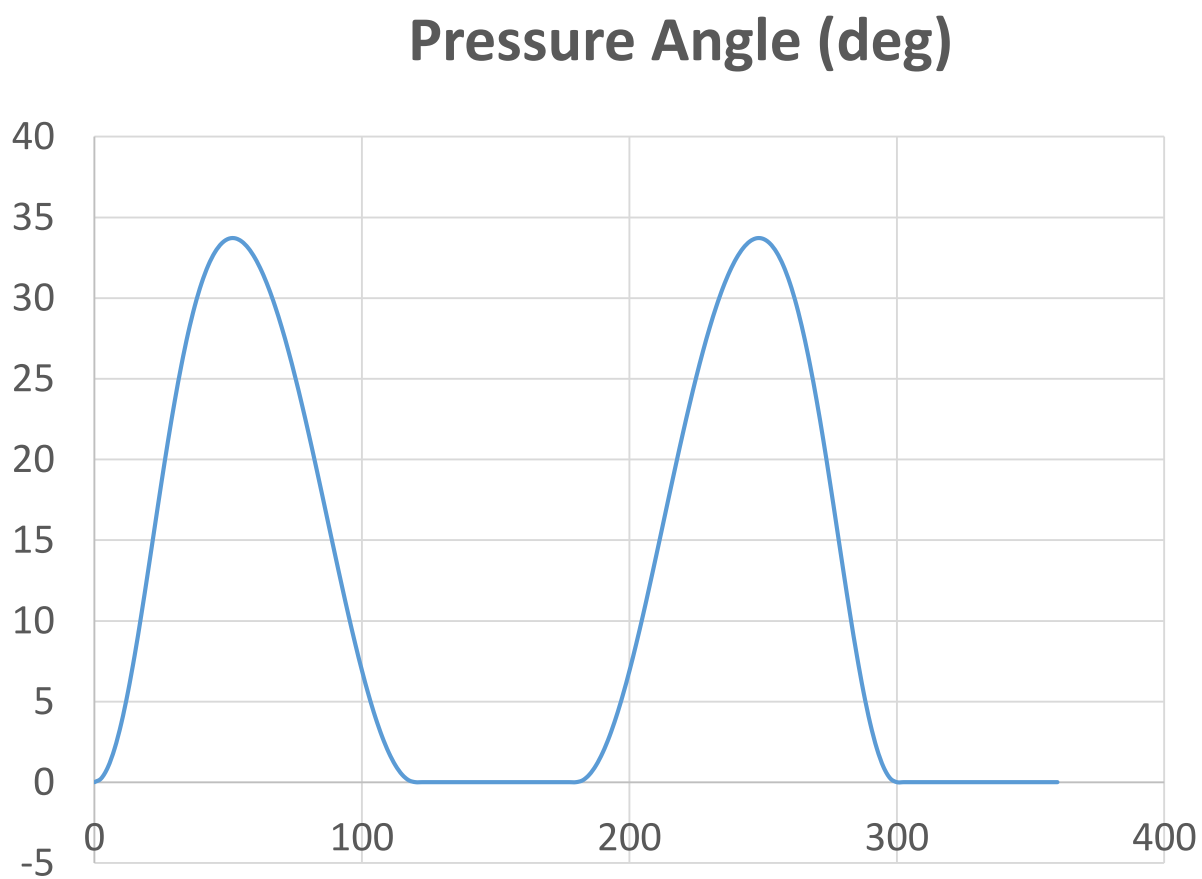


RESULTS
Follower Acceleration



RESULTS
Pressure Angle

$$\tan\alpha = \frac{s'}{r_b + r_r + s}$$



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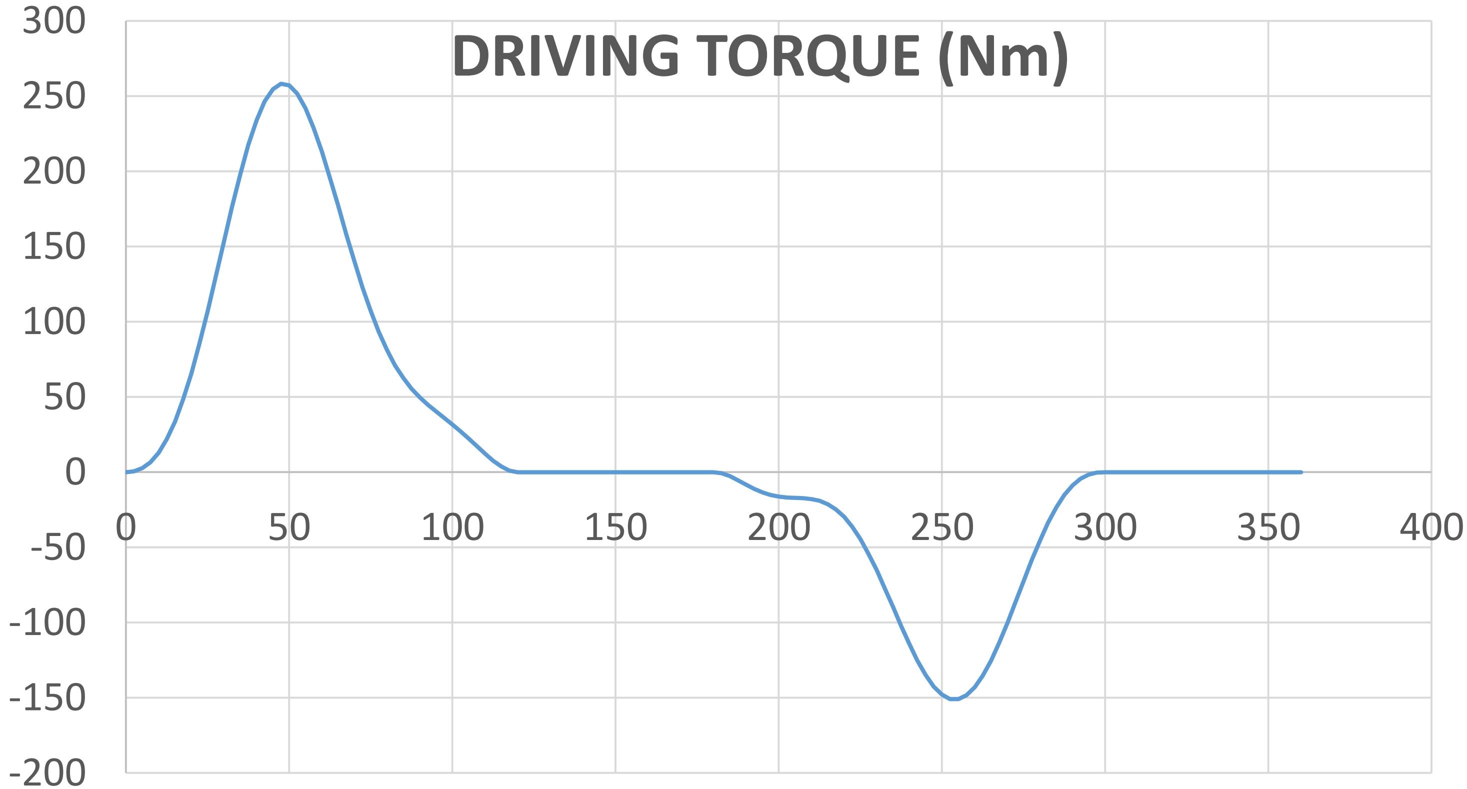
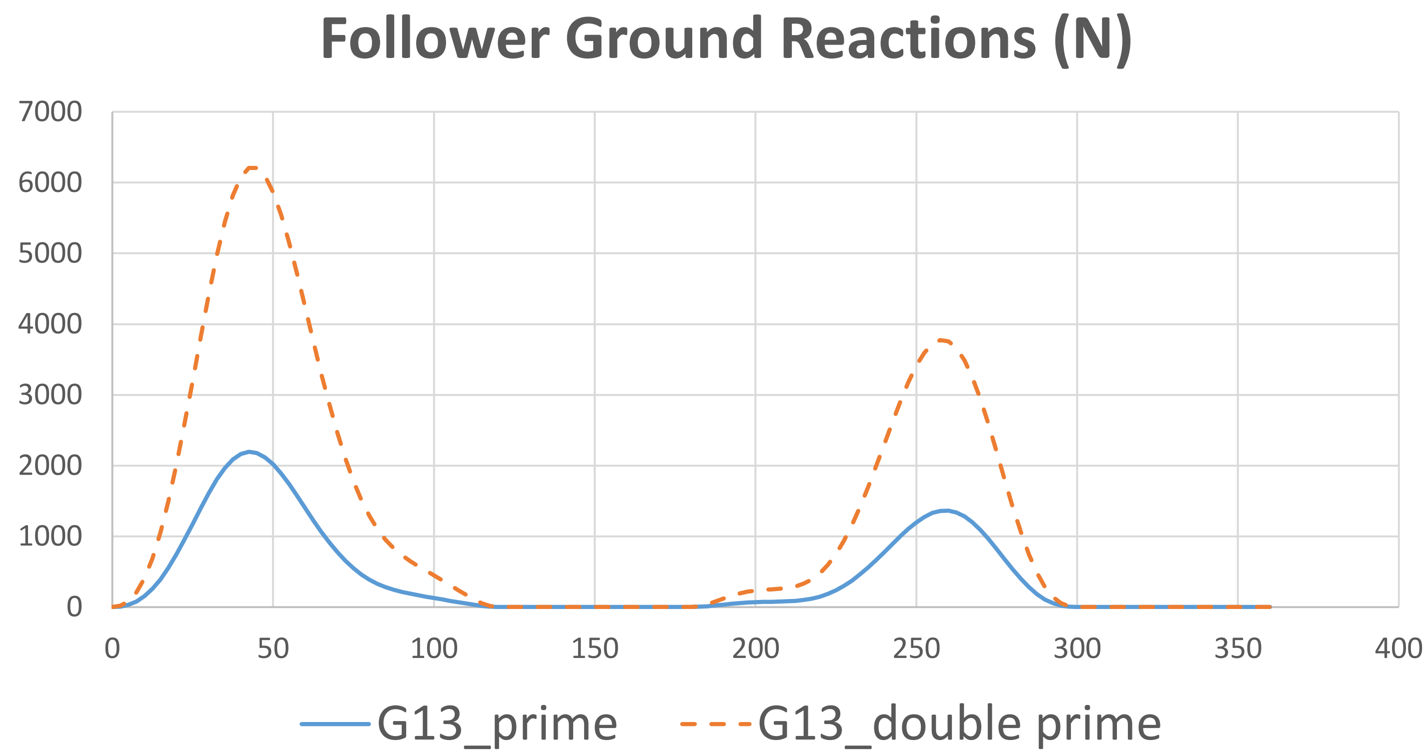
RESULTS
Driving Torque



RESULTS
Ground Reactions of the Follower



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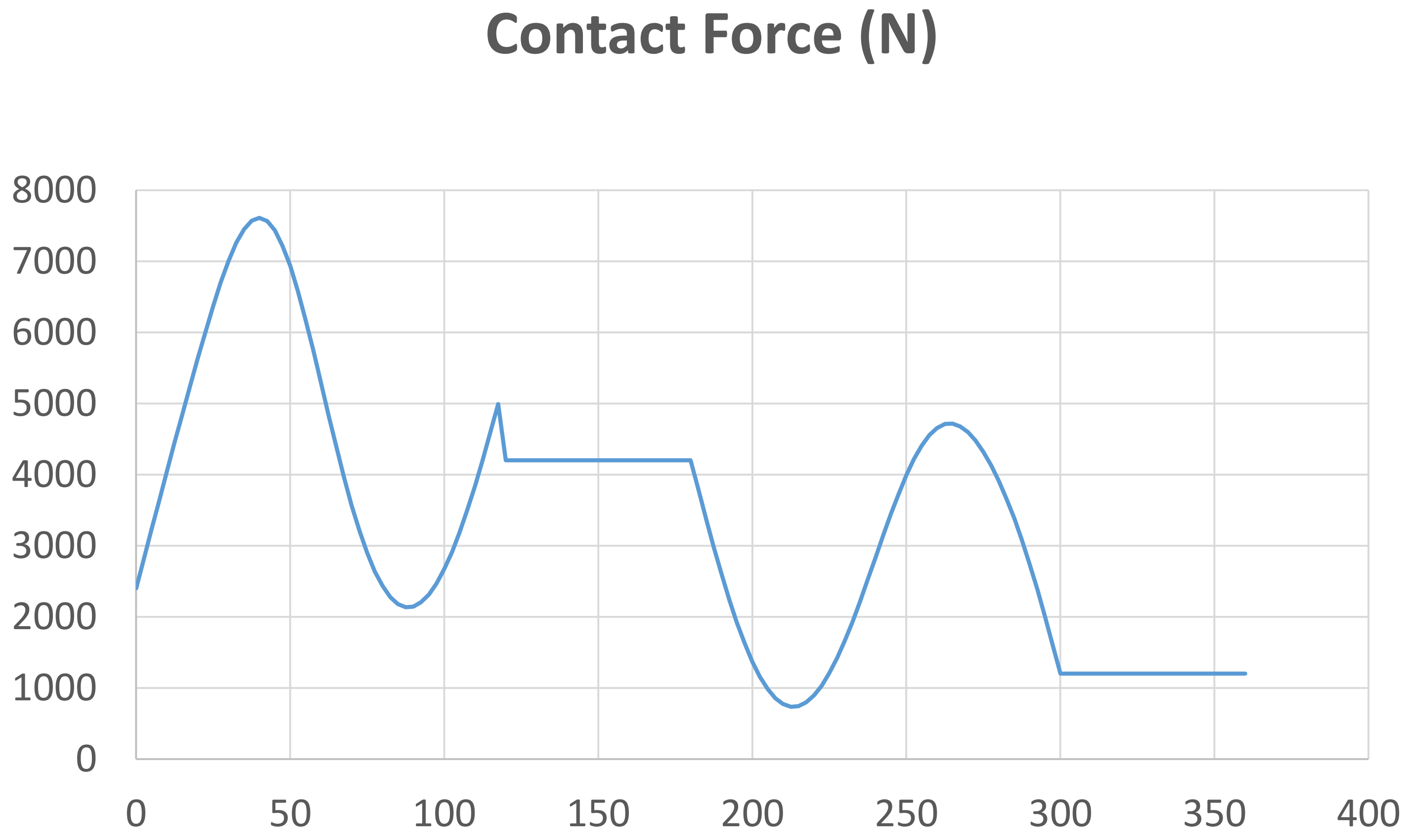
RESULTS
Contact Force



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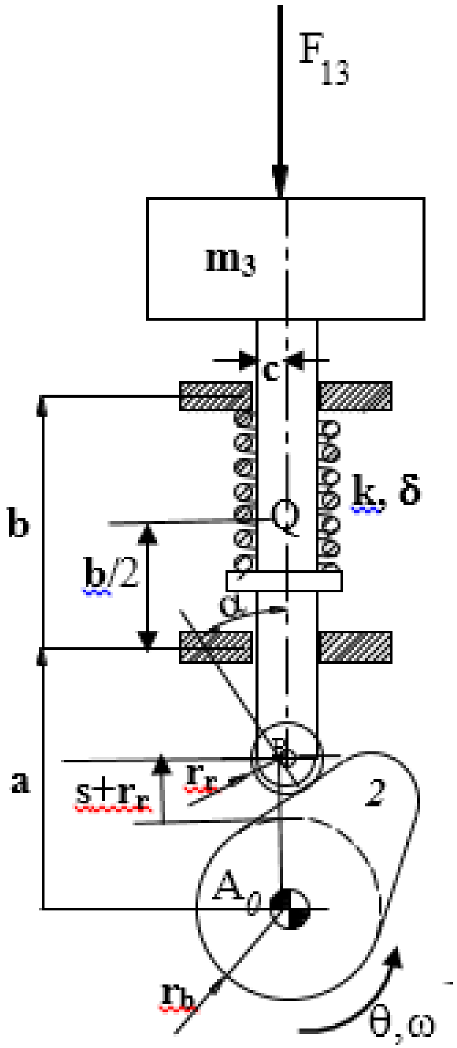
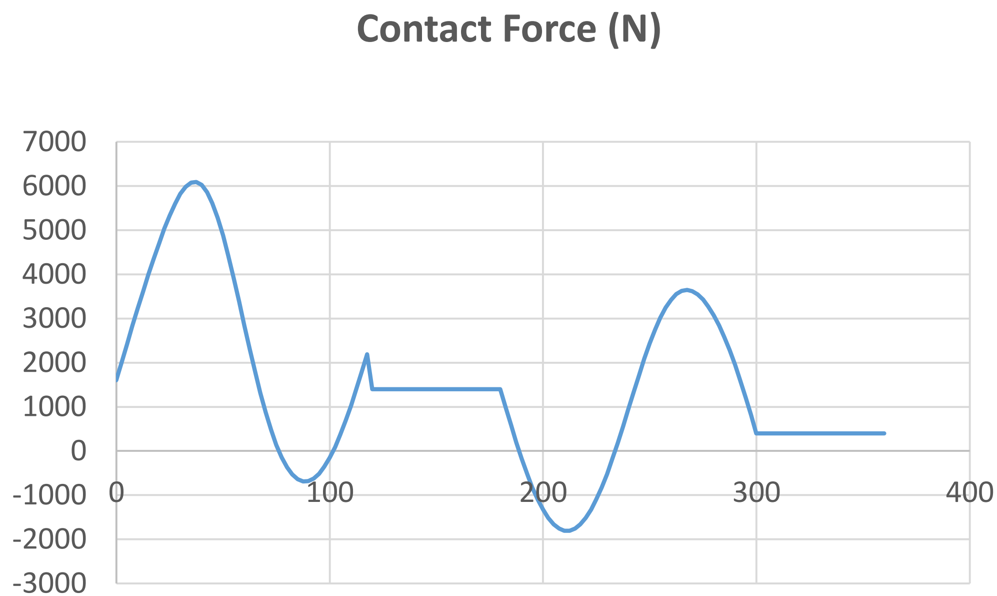
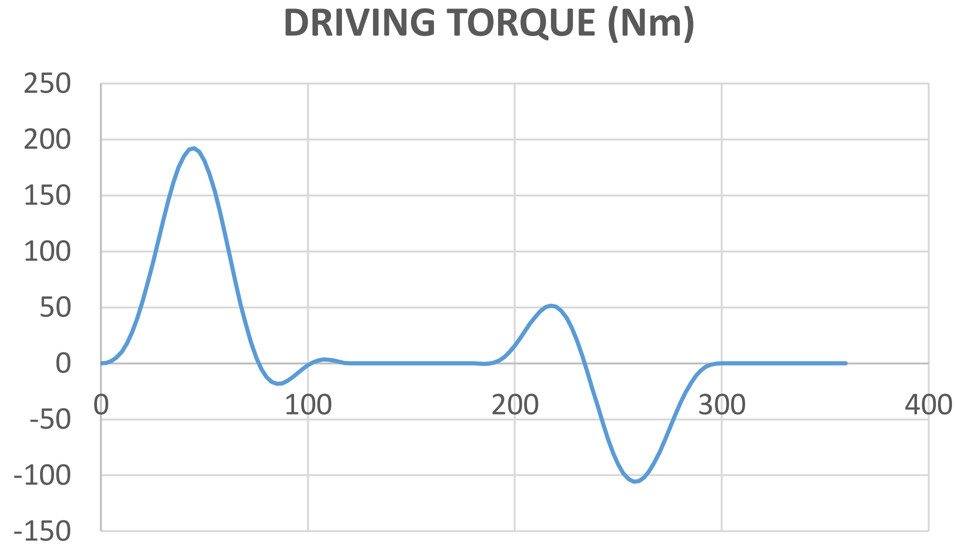
Consider the previous example with revised spring constant (k = 60000 → 20000 N/m)







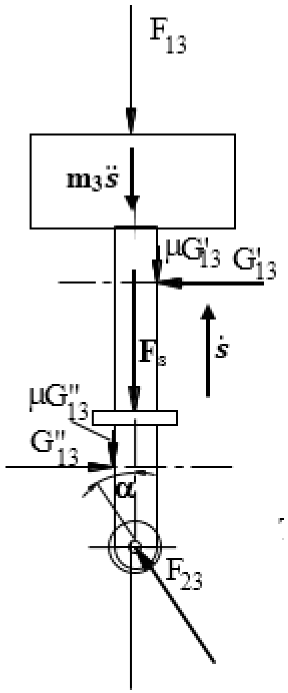
Consider the previous example: Ignore friction and calculate the spring stiffness in order that minimum contact force is 50N.

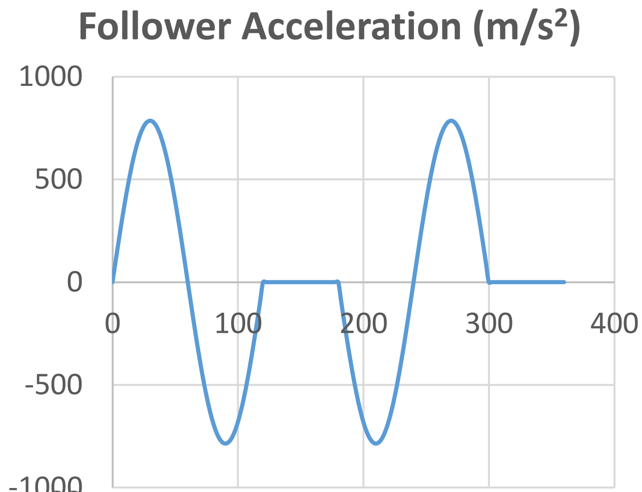
During Rise

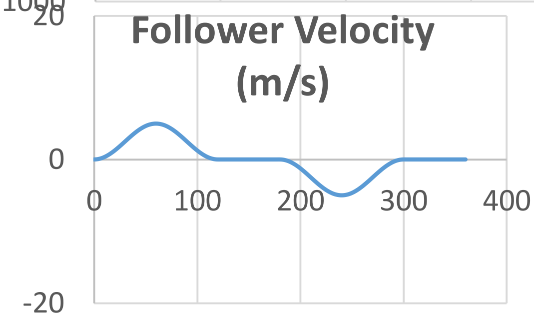
$$F_{23} = \frac{1}{\cos\alpha}(F_s + F_{13} + m_3\ddot{s})$$

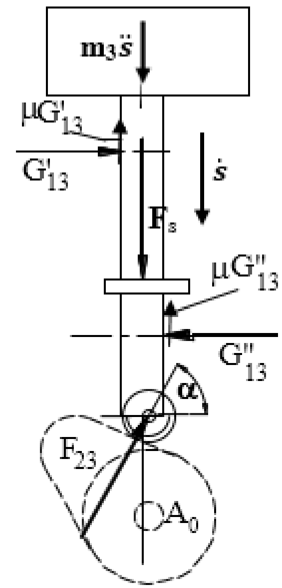
During Return

$$F_{23} = \frac{1}{\cos\alpha}(F_s + m_3\ddot{s})$$









Critical (minimum) F_{23} clearly occurs during return when $\ddot{s}$ is at its largest negative value, i.e. at $\theta = 210^0$ ($7\pi/6$ rad.)

$$F_{23} = \frac{1}{\cos\alpha}(F_s + m_3\ddot{s})$$

where; $(F_{23})_{min} = 50\ N$ and $F_s = k(s+\delta)$ when $\theta = {}^{7\pi}/_{6}$

We therefore need to calculate $\alpha, \ddot{s}$, and s when $\theta = {}^{7\pi}/_{6}$:

$180^0 \leq \theta \leq 300^0$ RETURN: H = 0.05m, $\beta = 2\pi/3$, $\gamma = \pi$ for $\theta = {}^{7\pi}/_{6}$

$$s(\theta) = 0.05 - 0.05\left(\frac{{}^{7\pi}/_{6} - \pi}{3} - \frac{1}{2\pi}\sin\frac{2\pi\left({}^{7\pi}/_{6} - \pi\right)}{{}^{2\pi}/_{3}}\right) = 0.0455\text{m}$$

$$s'(\theta) = -\frac{0.05}{{}^{2\pi}/_{3}}\left(1 - \cos\frac{2\pi\left({}^{7\pi}/_{6} - \pi\right)}{{}^{2\pi}/_{3}}\right) = -0.0239\ \text{m/rad}$$

$$s''(\theta) = -0.05\frac{2\pi}{\left({}^{2\pi}/_{3}\right)^2}\sin\frac{2\pi\left({}^{7\pi}/_{6} - \pi\right)}{{}^{2\pi}/_{3}}) = -C_a\frac{H}{\beta^2} = -0.0716\ \text{m/rad}^2$$

Substituting:

$$\tan\alpha = \frac{s'}{r_b + r_r + s} = \frac{-0.0239}{0.04+0.01+0.0455} \rightarrow \boldsymbol{\alpha = 14^0}$$

$$\ddot{\boldsymbol{s}} = s''\omega^2 = -0.0716\left(1000\,{}^{\pi}/_{30}\right)^2 = \boldsymbol{-785.4\, m/s^2}$$

$$\boldsymbol{F_s} = k(s+\delta);\; \boldsymbol{s = 0.0455\, m}\,;\; \boldsymbol{\delta = 0.0\,2m;\, m_3 = 4\, kg}$$

into:

$$(F_{23})_{min} = 50N = \frac{1}{\cos\alpha}(F_s + m_3\ddot{s})$$

$$50 = \frac{1}{\cos 14^0}[k(0.0455 + 0.02) + 4(-785.4)]$$

$$\boldsymbol{k = 48735\ \mathrm{N/m}}$$

Thank you

for your attention…