Frekans Filtreleme

Frequency Filtering

FT'nin konvolüsyon özelliği

Let functions $f(r,c)$ and $g(r,c)$ have Fourier Transforms $F(u,v)$ and $G(u,v)$.

Then,

$$\mathbf{F}\{f * g\} = F \cdot G.$$

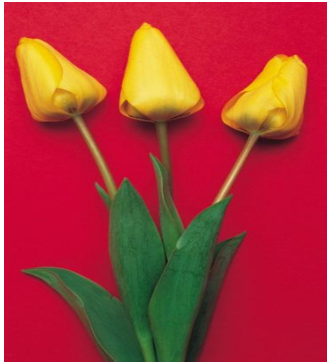
Moreover,

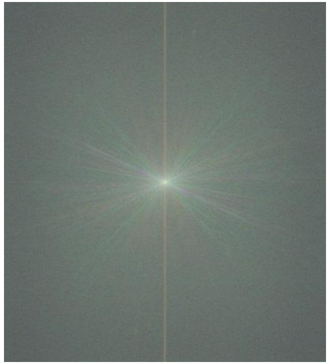
$$\mathbf{F}\{f \cdot g\} = F * G.$$

$*$ = konvolüsyon
$\cdot$ = çarpma

Bir konvolüsyonun FT si FT lerin çarpımına eşit.

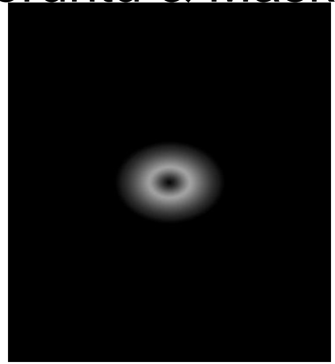
FT ile konvolüsyon

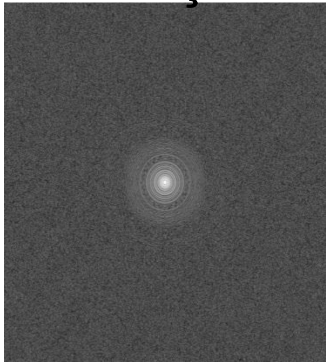


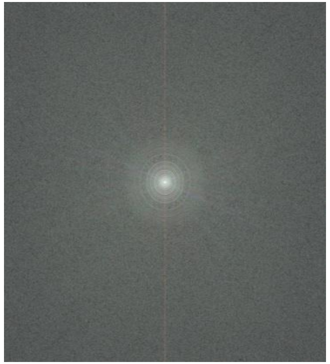


Görüntü & Maske

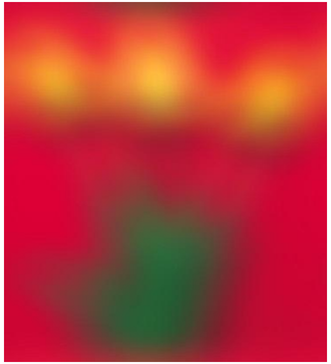
Dönüşüm







Noktasal Çarpım



Ters dönüşüm

Renkli görüntülerde, bu işlem her bir bant için ayrı yapılmalı.

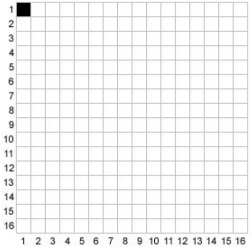
Matlab : FT ile konvolüsyon

1. Görüntüyü oku, `I`.
2. Maskeyi hazırla, `h (5x5)`. Maske genellikle tek banttır
3. Maske toplamını hesapla: `s = sum(sum(h));`
4. If `s == 0,` set `s = 1;`
5. `H = zeros(size(I));`
6. `h` maskesini `H`'in ortasına kopyala
7. `H`' kaydır. `H = ifftshift(H);`
8. `I` ve `H`'ın FFTsini hesapla: `FI=fft2(I); FH=fft2(H);`
9. Noktasal çarpımı yap: `FJ=FI.*FH;`
10. Ters FT'yi hesapla: `J = real(ifft2(FJ));`
11. Sonucu normalize et: `J = uint8(J/s);`

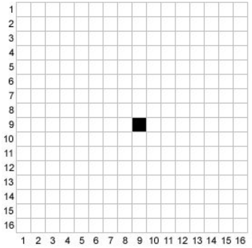
FFT’nin koordinat merkezi

merkez=
(floor(R/2)+1, floor(C/2)+1)

Çift

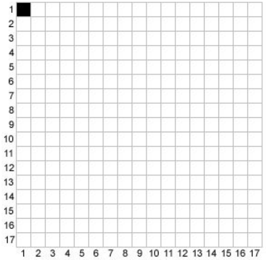


Görüntü Merkezi

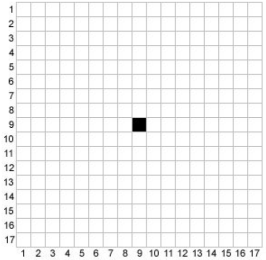


FFT shift

Tek

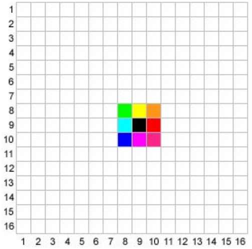


Görüntü Merkezi

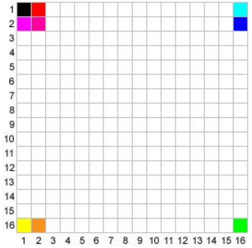


FFT shift

Çift

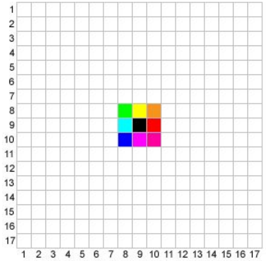


Weight Matrix Origin

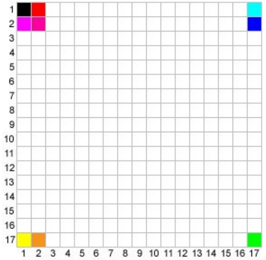


IFFT shift

Tek



Weight Matrix Origin

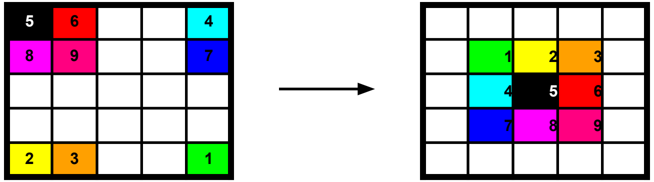


IFFT shift

Matlabda `fftshift` ve `ifftshift`

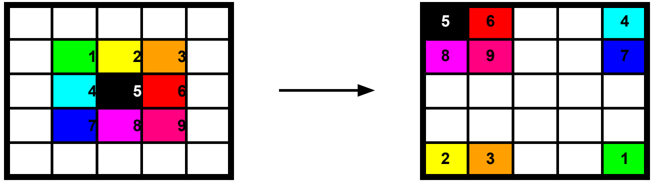
`J = fftshift(I):`

$$I\,(1,1) \rightarrow J\,(\,\lfloor R/2 \rfloor + 1,\, \lfloor C/2 \rfloor + 1)$$



`I = ifftshift(J):`

$$J\,(\,\lfloor R/2 \rfloor + 1,\, \lfloor C/2 \rfloor + 1) \rightarrow I\,(1,1)$$



$\lfloor x \rfloor$ = `floor(x)` $= x$ den daha küçük en büyük tamsayı

Bulanıklaşma: Ortalama / Düşük Geçiren Filtre

Bulanıklaşma kaynaklanır:

- Uzaysal alanda piksel ortalamasından
  - Her bir çıkış pikseli komşuların ağırlıklandırılmış bir ortalamasına sahiptir.
  - Ağırlık matrisin toplamı birdir ve yapılan bir konvolüsyon işlemidir.
- Frekans alanında düşük geçiren filtre:
  - Yüksek frekanslar elimine edilir.
  - Bağımsız frekans bileşenleri ω'nin artış olmayan bir fonksiyonla çarpılmasıyla. $1/\omega = 1/\sqrt{(u^2+v^2)}$.

Keskinleştirme: Çıkarım / Yüksek Geçiren Filtre

Keskinleştirme görüntüye bir kopyasının eklenmesiyle olur ki, bu kopya

- Uzaysal alanda piksel fark
  - Her bir çıkış pikseli, kendisi ve komşularının ağırlıklandırılmış bir ortalamasının farkına eşittir.
  - Ağırlık matrisinin toplamı sıfırdır ve yapılan işlem bir konvolüsyondur.
- Frekans alanında yüksek geçiren filtre:
  - Yüksek frekanslar güçlendirilir veya iyileştirilir.
  - Bağımsız frekans bileşenleri ω'nun artan bir fonksiyonuyla çarpılır. $\alpha\omega = \alpha\sqrt{(u^2+v^2)}$, burada α sabittir.

Hatırlayalım ki:

FT'nin konvolüsyon özelliği

Let functions $f(r,c)$ and $g(r,c)$ have Fourier Transforms $F(u,v)$ and $G(u,v)$.

Then,

$$\mathsf{F}\{f * g\} = F \cdot G.$$

Moreover,

$$\mathsf{F}\{f \cdot g\} = F * G.$$

Thus we can compute $f * g$ by

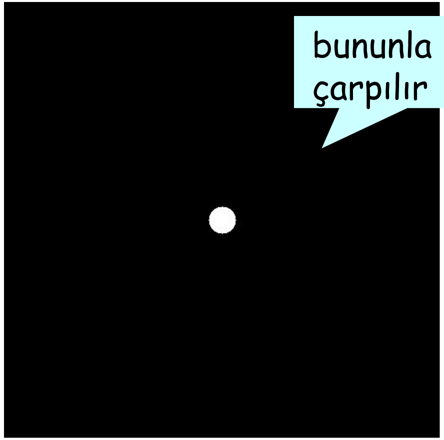
$$f * g = \mathsf{F}^{-1}\{F \cdot G\}.$$

$*$ = konvolüsyon
$\cdot$ = çarpma

Bir konvolüsyonun FT si FT lerin çarpımına eşit.

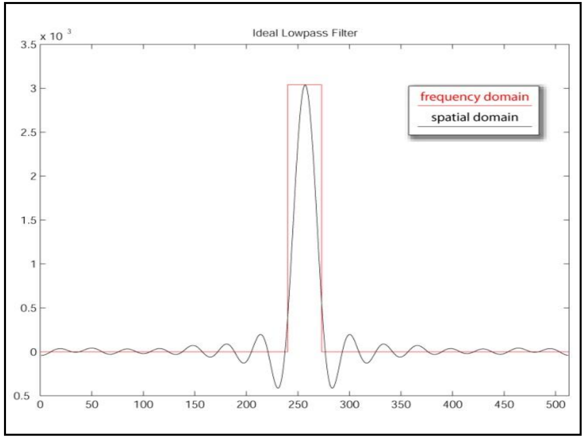
İdeal Düşük Geçiren Filtre

Görüntü: 512x512
FD filtre çapı: 16



FT sunumu

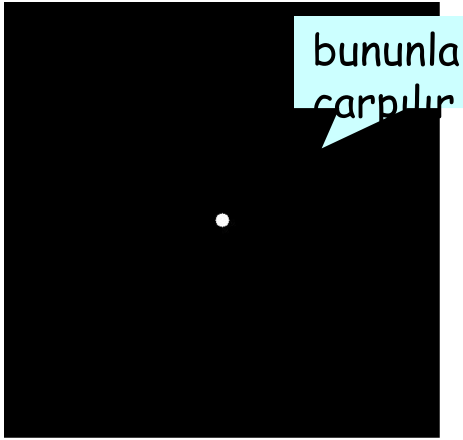
Uzaysal sunum



Merkez profili

İdeal Düşük Geçiren Filtre

Görüntü: 512x512
FD filtre çapı: 8

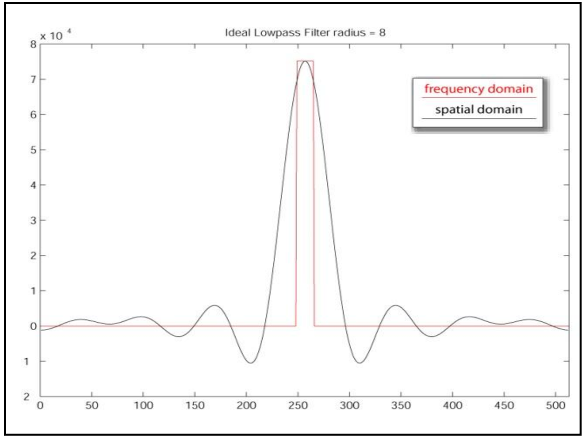


bununla çarpılır

FT sunumu

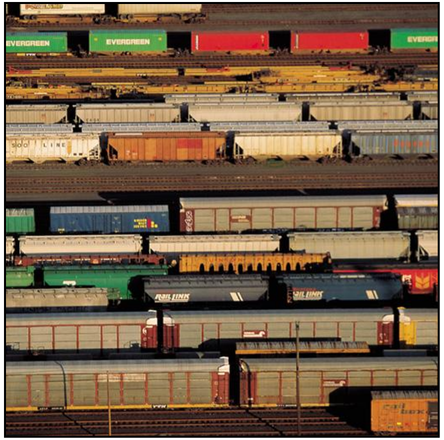
bununla konvolüsyon alınır

Uzaysal sunum

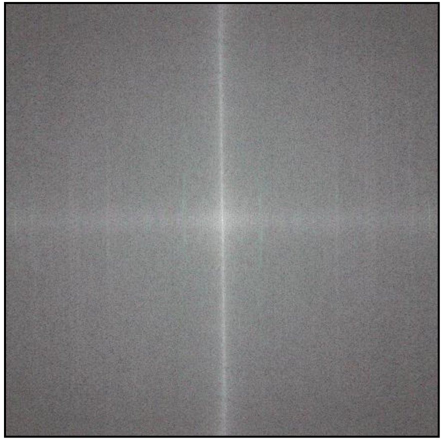


Merkez profili

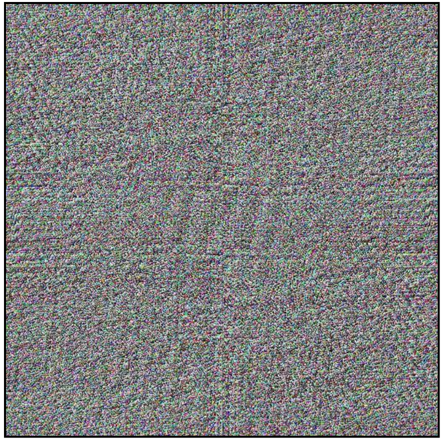
Güç spektrumu ve Faz



Orijinal Görüntü



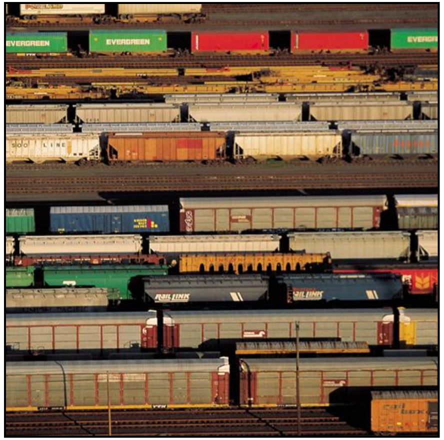
Güç spektrumu



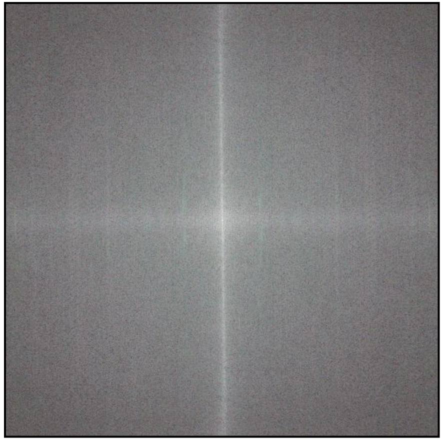
Faz

İdeal Düşük Geçiren Filtre

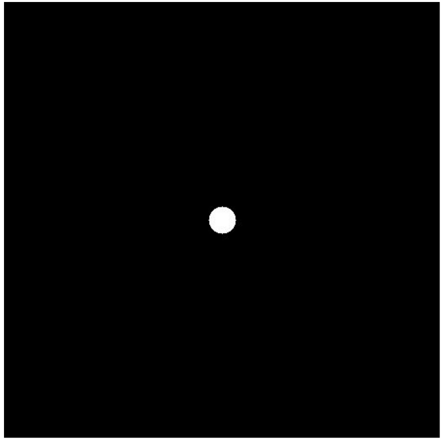
Görüntü: 512x512
FD filtre çapı: 16



Orijinal Görüntü



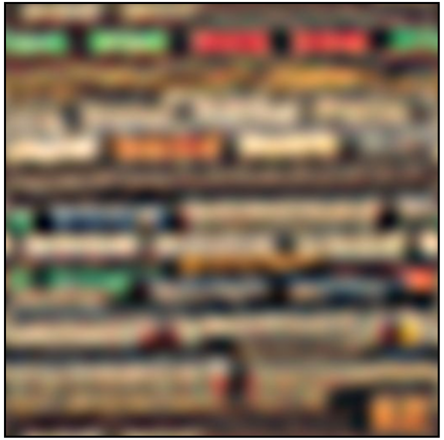
Güç spektrumu



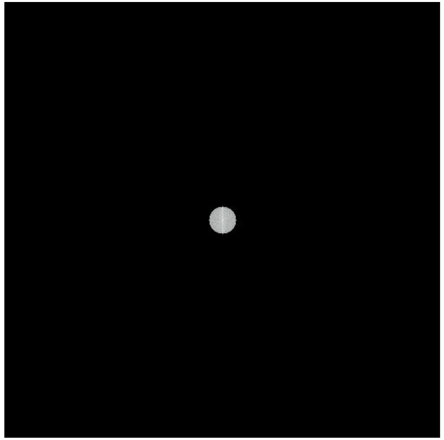
FD'de Ideal LPF

İdeal Düşük Geçiren Filtre

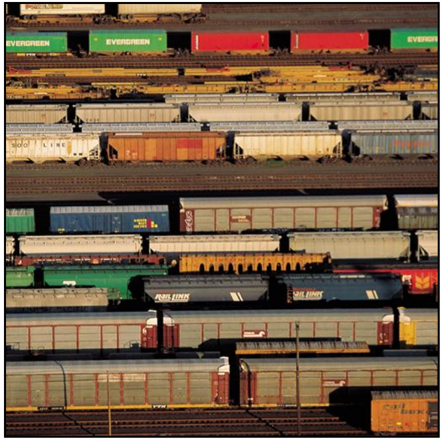
Görüntü: 512x512
FD filtre çapı: 16



Filtrelenmiş Görüntü



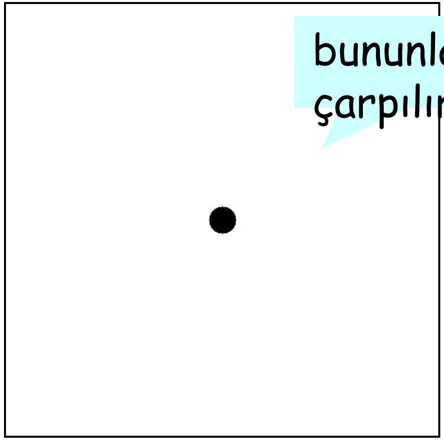
Filtrelenmiş PS



Orijinal Görüntü

İdeal Yüksek Geçiren Filtre

Görüntü: 512x512
FD dar geçit çapı: 16

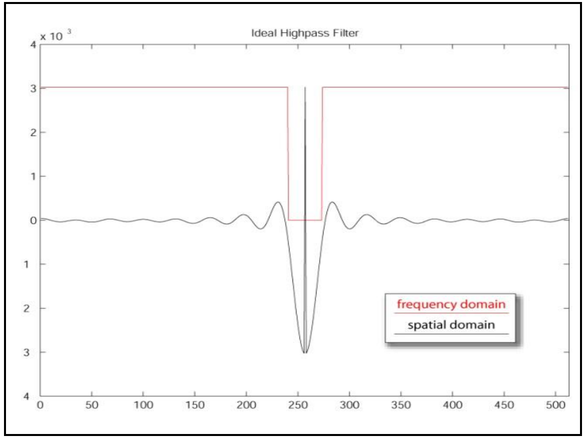


bununla çarpılır

FD sunumu

bununla konvolüsyon alınır

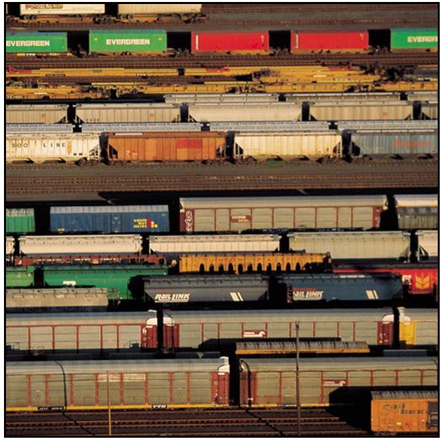
Uzaysal sunum



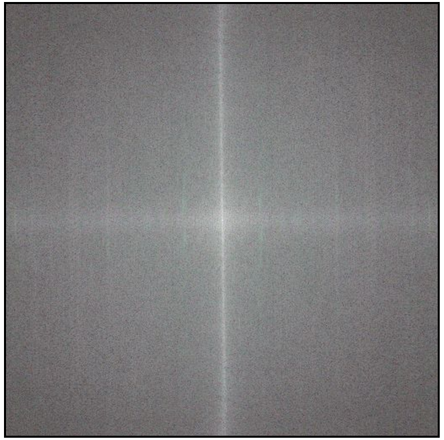
Merkez Profil

İdeal Yüksek Geçiren Filtre

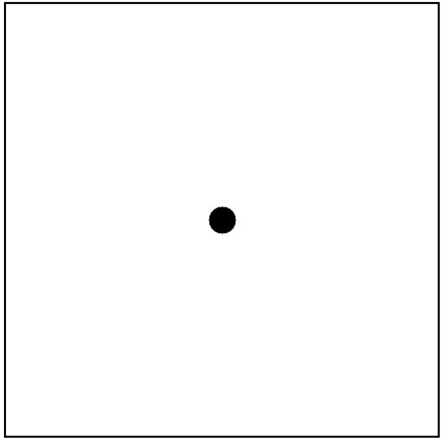
Görüntü: 512x512
FD dar geçit çapı: 16



Görüntü



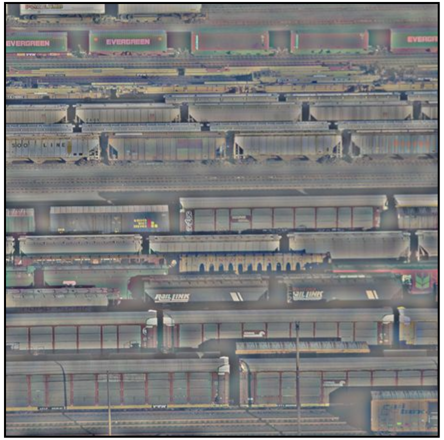
PS



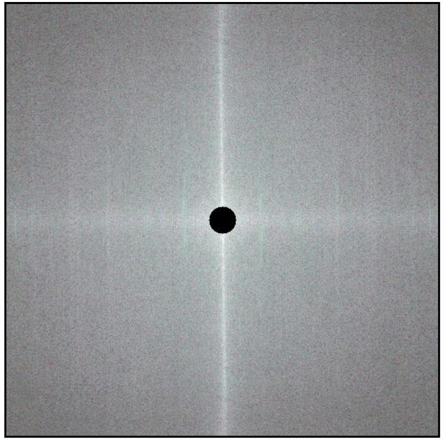
FD'de ideal HPF

İdeal Yüksek Geçiren Filtre

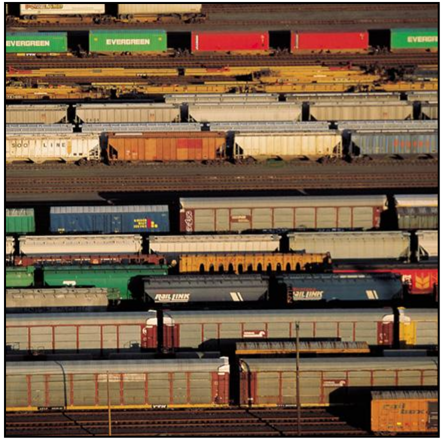
Görüntü: 512x512
FD dar geçit çapı: 16



Filtrelenmiş Görüntü



Filtrelenmiş PS

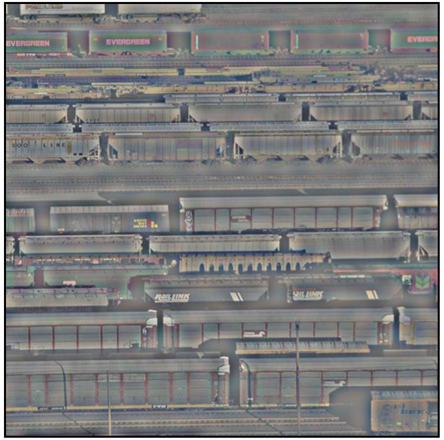


Orijinal Görüntü

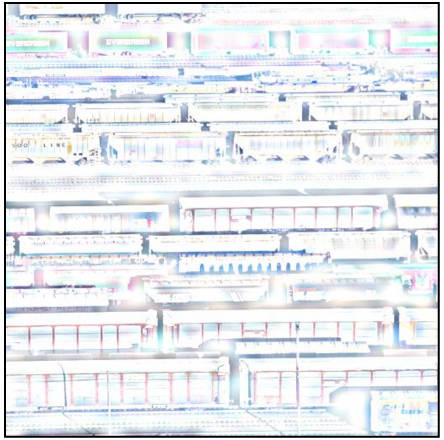
İdeal Yüksek Geçiren Filtre

Görüntü: 512x512
FD dar geçit çapı: 16

Pozitif pikseller

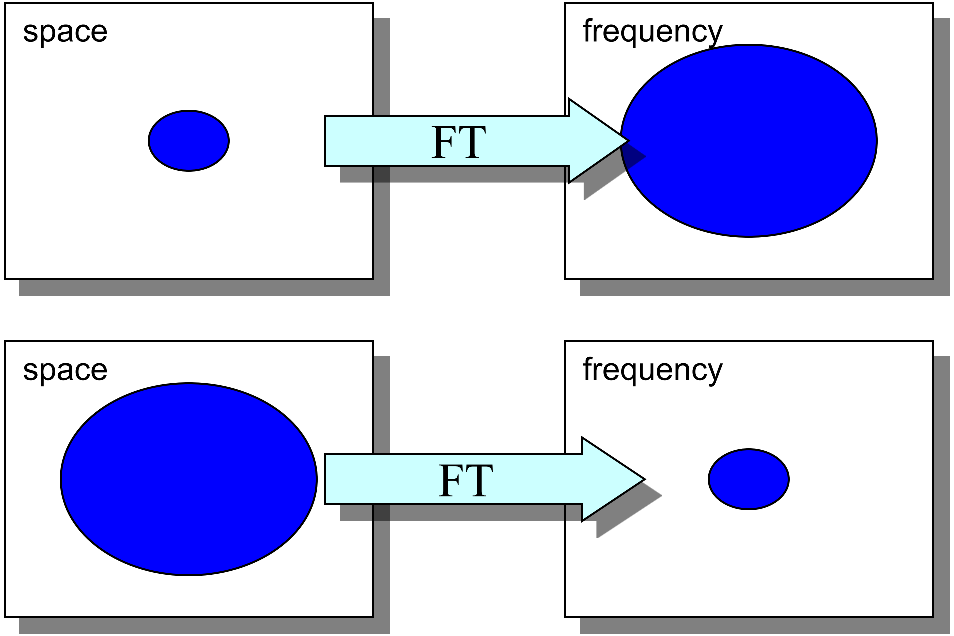


Filtrelenmiş Görüntü



Negatif pikseller

Belirsizlik İlişkisi

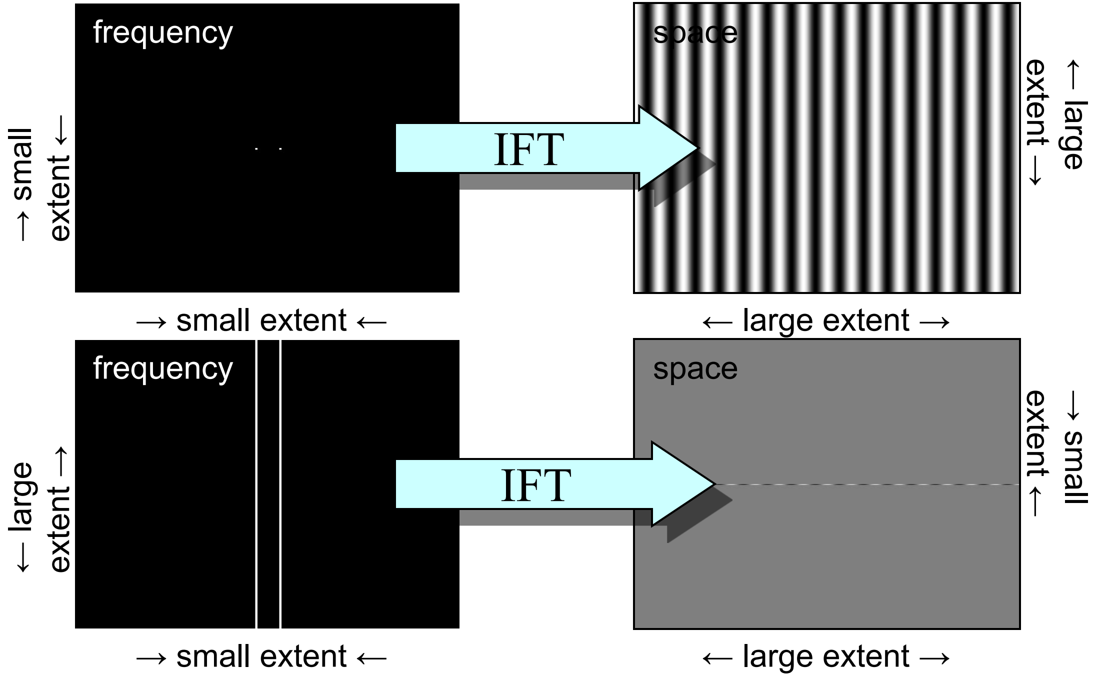


If $\Delta x \, \Delta y$ is the extent of the object in space and if $\Delta u \, \Delta v$ is its extent in frequency then,

$$\Delta x \, \Delta y \cdot \Delta u \, \Delta v \geq \frac{1}{16\pi^2}$$

Uzaysal alandaki küçük bir nesne, frekans alanında büyük bir miktara sahiptir.

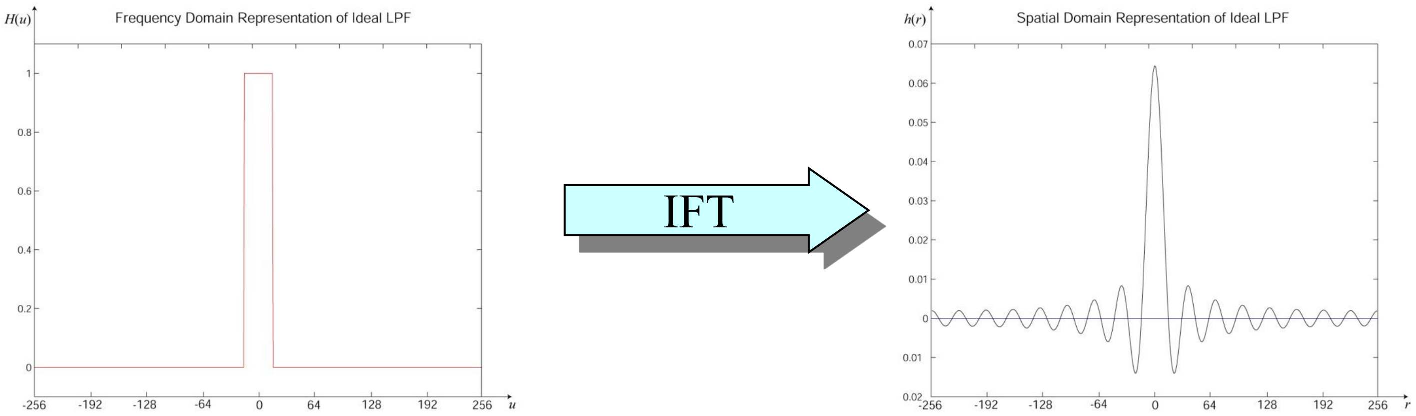
Belirsizlik İlişkisi



Hatırlaki, FD'deki bir çift impulse uzaysalda bir sinüzoid olur.

FD'deki simetrik bir çizgi uzaysalda sinüzoidal bir çizgi olur.

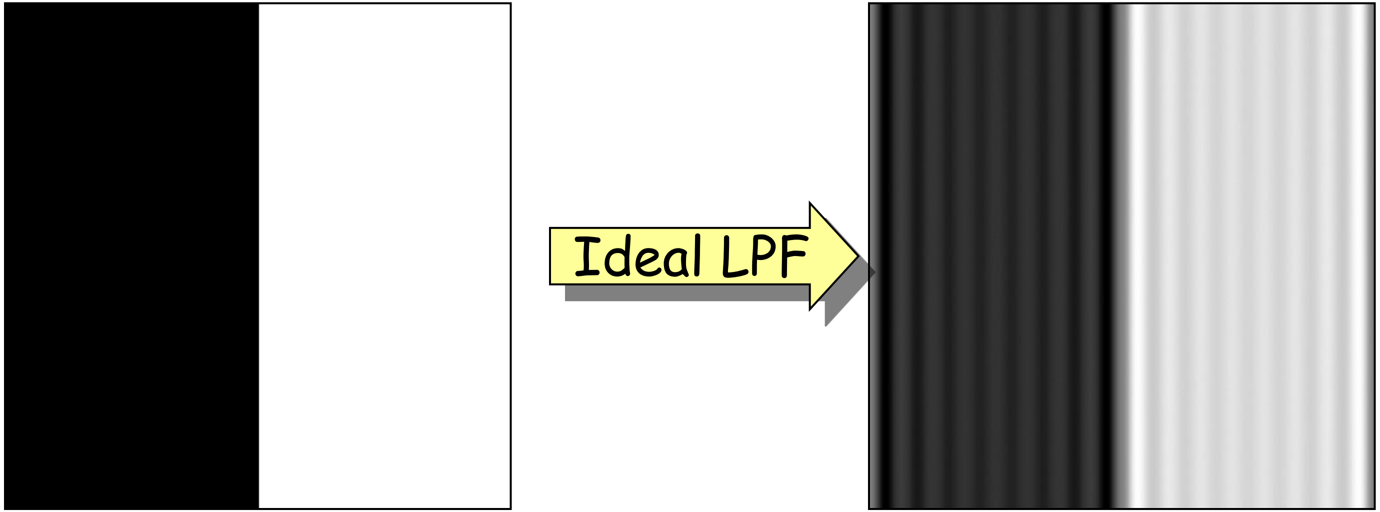
İdeal Filtreler İdeal Sonuçlar Üretmez



FD'de keskin bir cuttoff..

...uzaysalda dalgalanmaya neden olur.

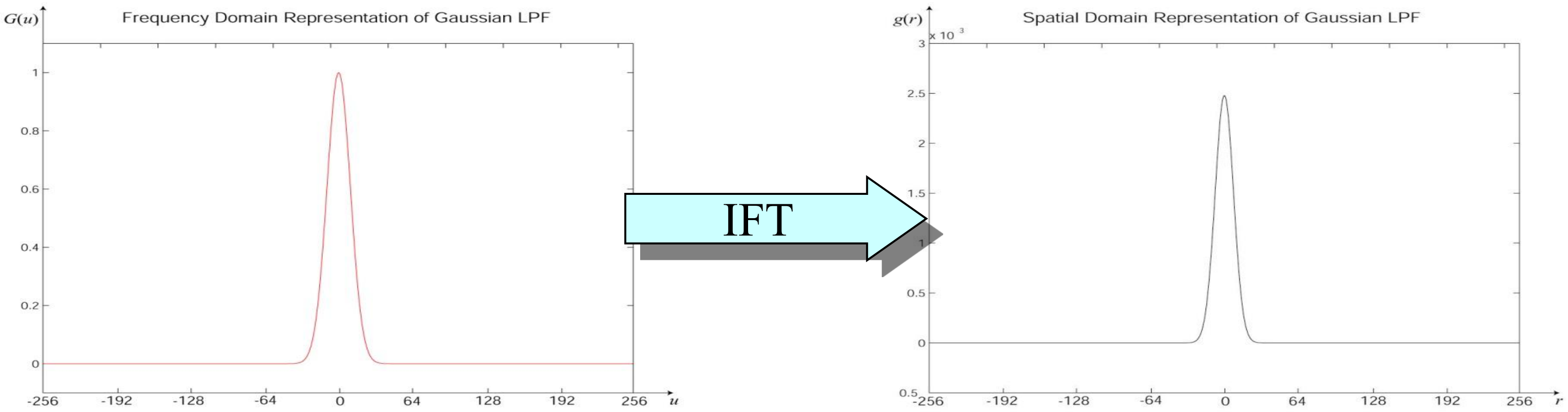
İdeal Filtreler İdeal Sonuçlar Üretmez



ILPF ile görüntüyü bulanıklaştırma

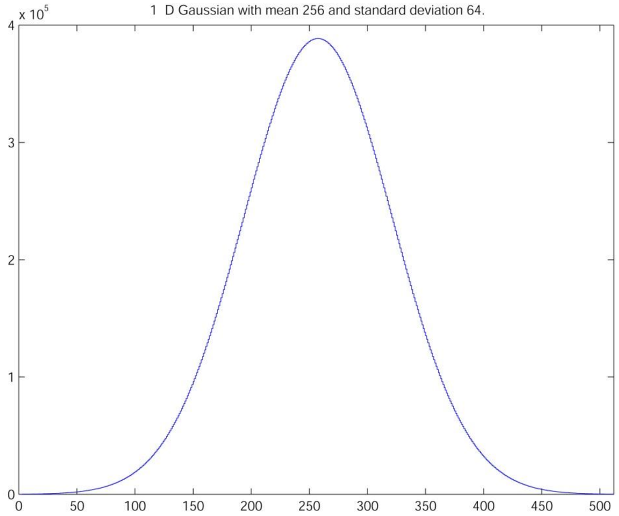
…sonucu dalgalanma ve gölgelenmeyle bozar

Optimal Filtre: Gaussian



Gaussian filtre belirsizlik ilişkisini optimize eder. Bu fonksiyon en keskin cutoff ve en az dalgalanma sağlar.

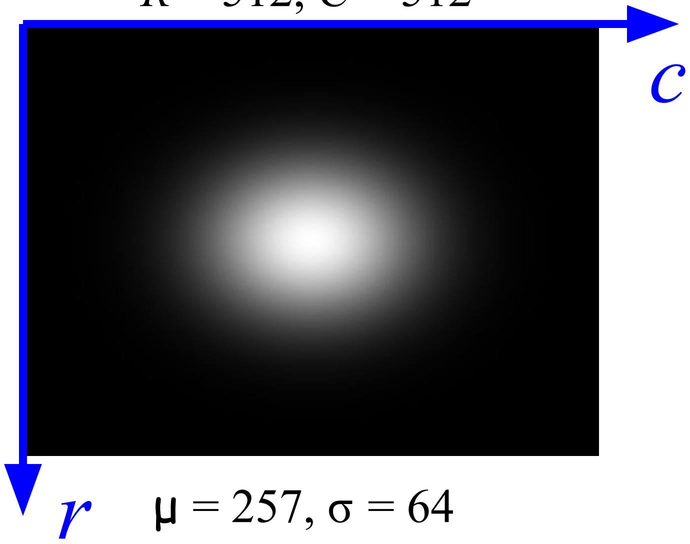
1d Gaussian



$$g(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

2d Gaussian

$R = 512, C = 512$



$\mu = 257, \sigma = 64$

Eğer r & c için μ ve σ farklıysa ...

$$g(r,c) = g(r)g(c)$$

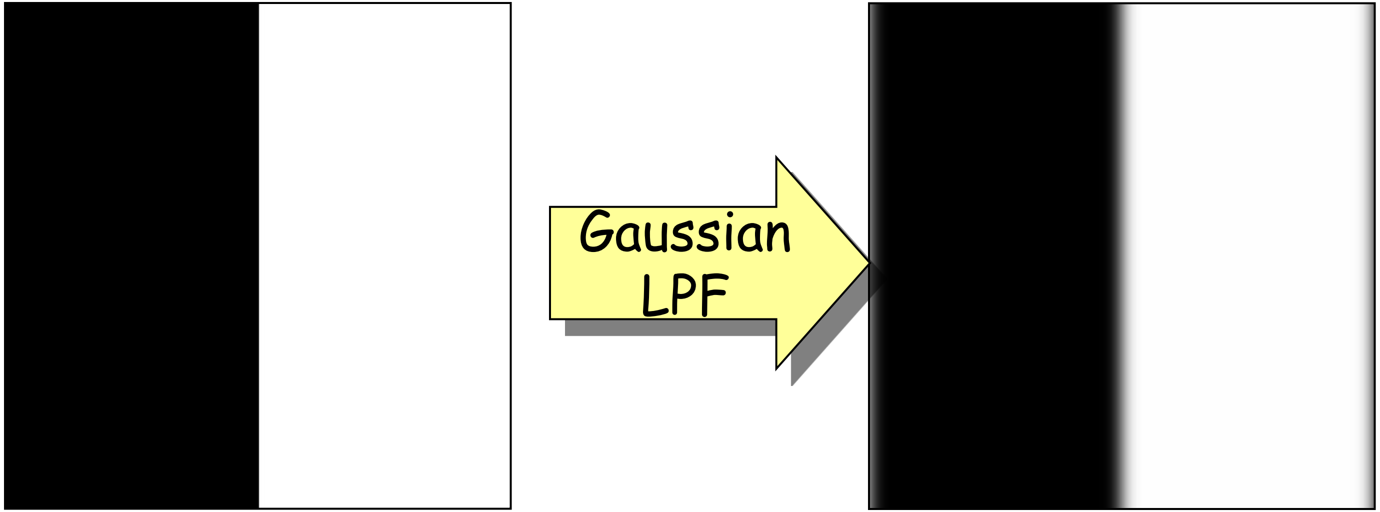
$$= \frac{1}{\sigma_r \sigma_c 2\pi} e^{-\frac{(r-\mu_r)^2}{2\sigma_r^2} - \frac{(y-\mu_c)^2}{2\sigma_c^2}}$$

$$= \frac{1}{\sigma_r \sigma_c 2\pi} e^{-\frac{\sigma_c^2 (x-\mu_r)^2 + \sigma_r^2 (y-\mu_c)^2}{2\sigma_r^2 \sigma_c^2}}$$

...veya eğer r & c için μ ve σ aynıysa

$$g(r,c) = \frac{1}{\sigma^2 2\pi} e^{-\frac{(r-\mu)^2 + (c-\mu)^2}{2\sigma^2}}$$

Optimal Filtre: Gaussian

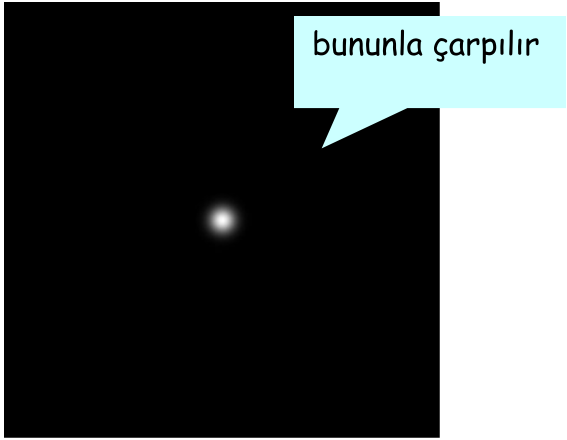


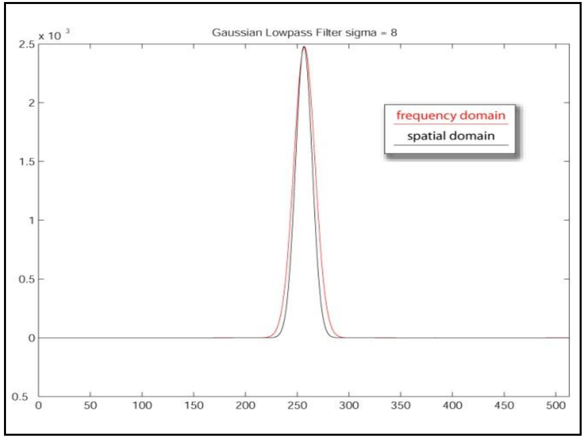
Gaussian düşük geçiren filtreyle…

… dalgalanma ve gölgelenme olmadan yumuşatır.

Gaussian Düşük Geçiren Filtre

Görüntü: 512x512
SD filtre sigma = 8





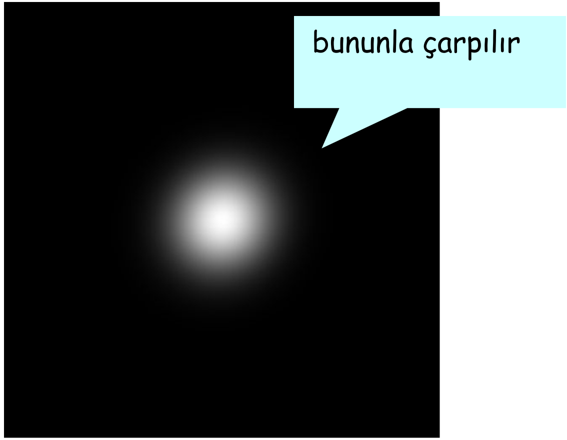
Frekans Domain (FD)

Uzaysal Domain (SD)

Merkez Profil

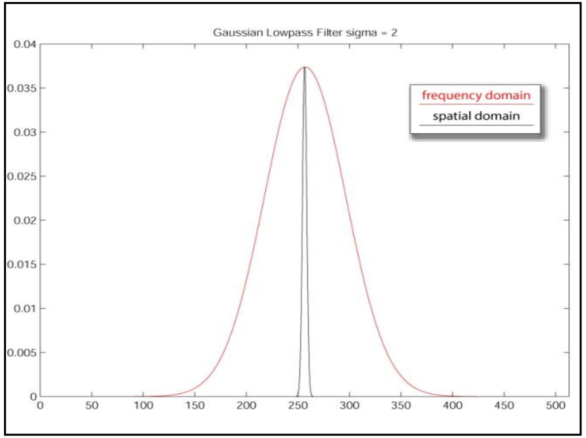
Gaussian Düşük Geçiren Filtre

Görüntü: 512x512
SD filtre sigma = 2



Frekans Domain (FD)

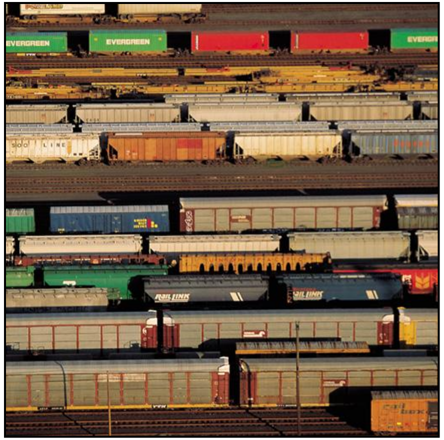
Uzaysal Domain (SD)



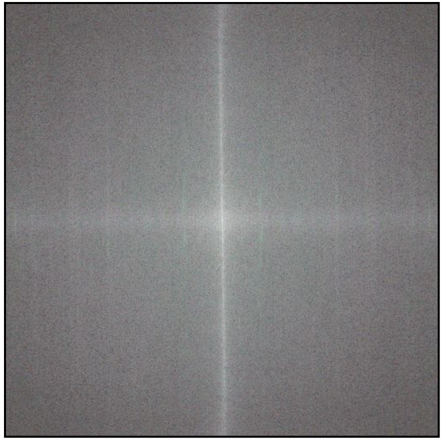
Merkez Profil

Gaussian Düşük Geçiren Filtre

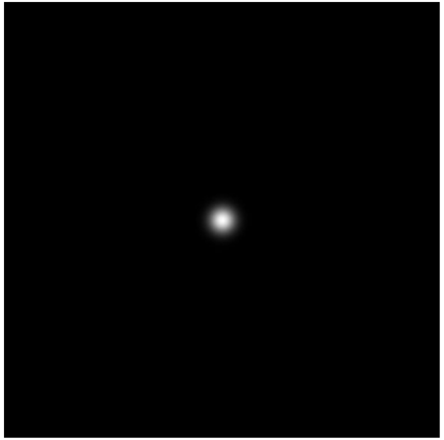
Görüntü: 512x512
SD filtre sigma = 8



Orijinal Görüntü



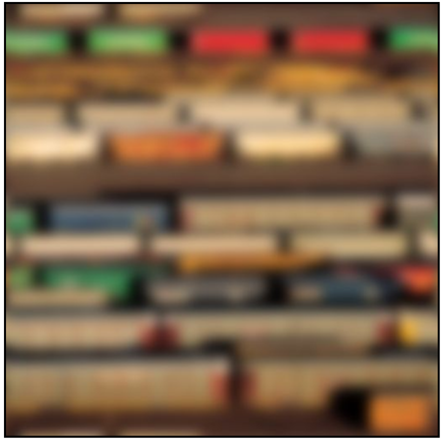
PS



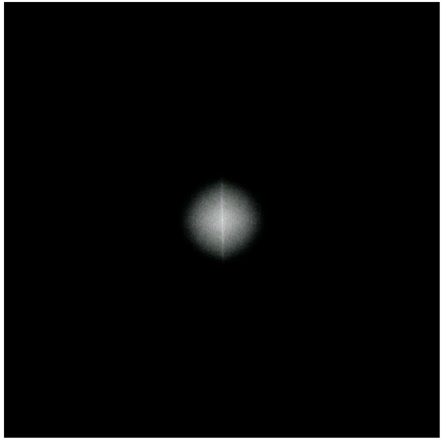
FD'de Gaussian LPF

Gaussian Düşük Geçiren Filtre

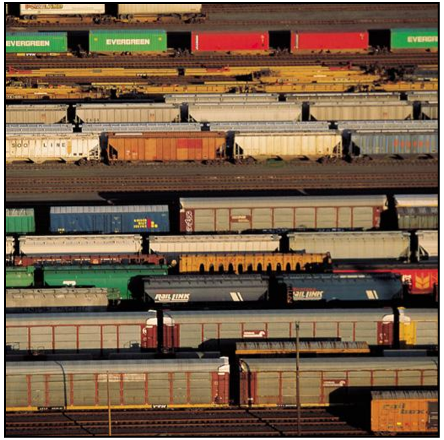
Görüntü: 512x512
SD filtre sigma = 8



Filtrelenmiş Görüntü



Filtrelenmiş PS

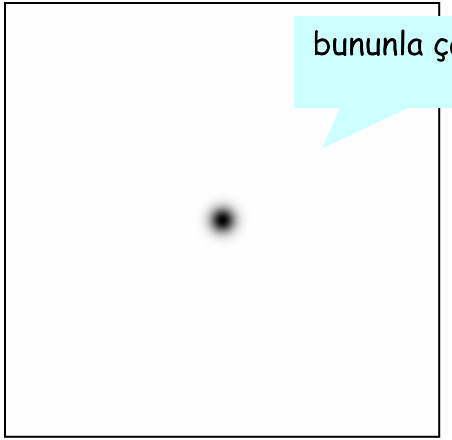


Orijinal Görüntü

Gaussian Yüksek Geçiren Filtre

Görüntü: 512x512
FD'deki daire sigma = 8

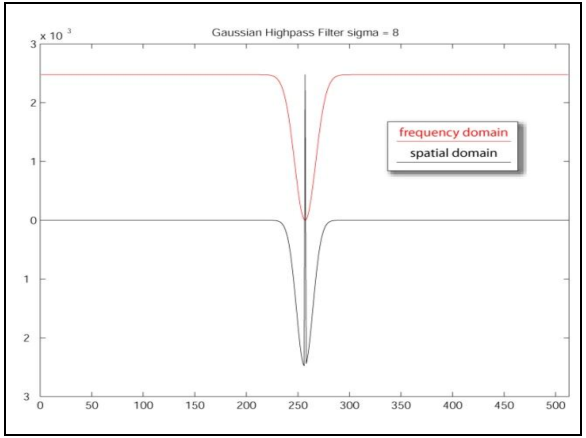
bununla çarpılır



FD

bununla konvolüsyon alınır

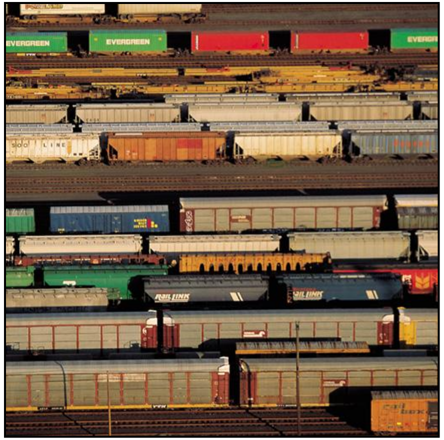
SD



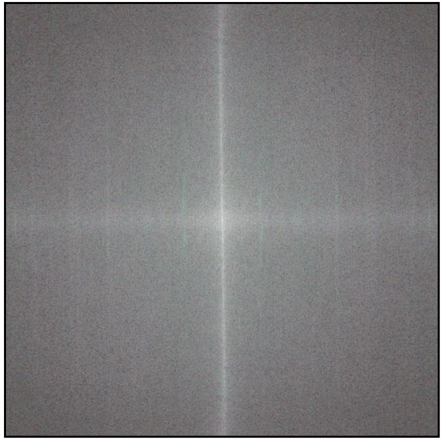
Merkez Profil

Gaussian Yüksek Geçiren Filtre

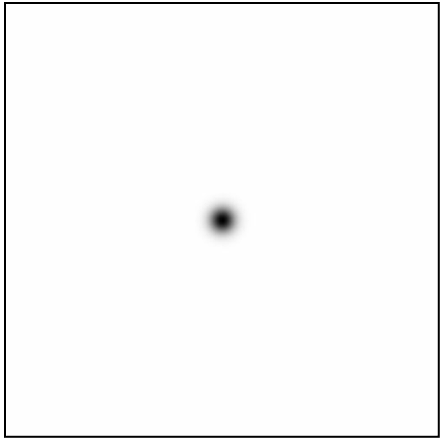
Görüntü: 512x512
FD'deki daire sigma = 8



Orijinal Görüntü



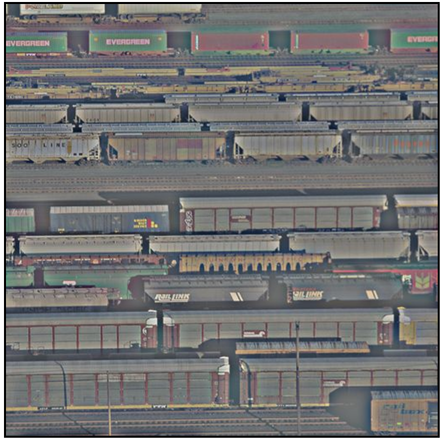
PS



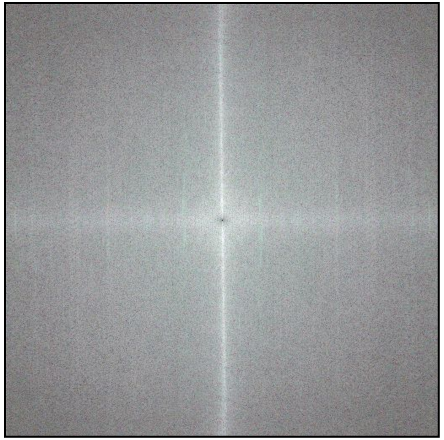
FD'de Gaussian HPF

Gaussian Yüksek Geçiren Filtre

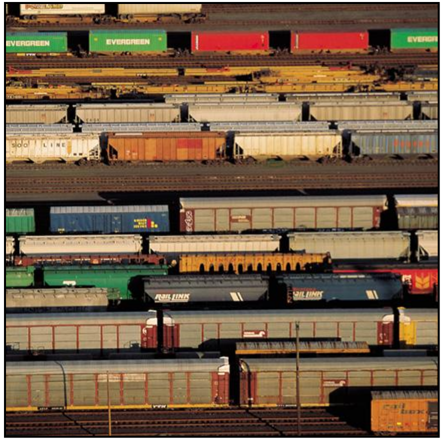
Görüntü: 512x512
FD'deki daire sigma = 8



Filtrelenmiş Görüntü



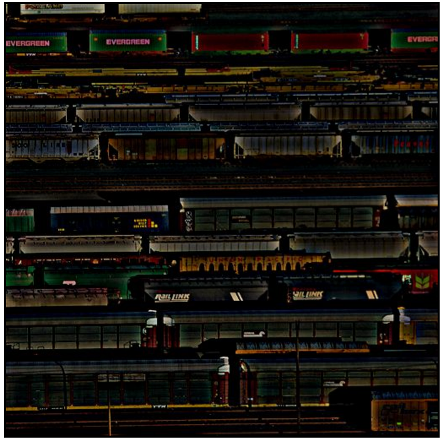
Filtrelenmiş PS



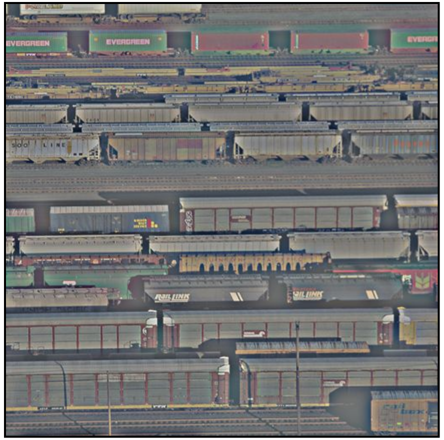
Orijinal Görüntü

Gaussian Yüksek Geçiren Filtre

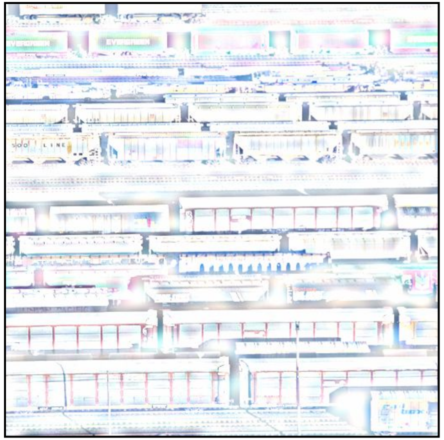
Görüntü: 512x512
FD'deki daire sigma = 8



Pozitif pikseller

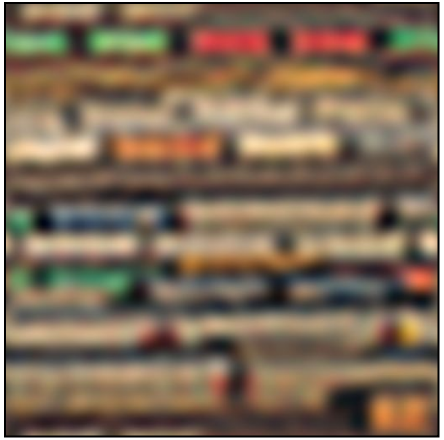


Filtrelenmiş Görüntü

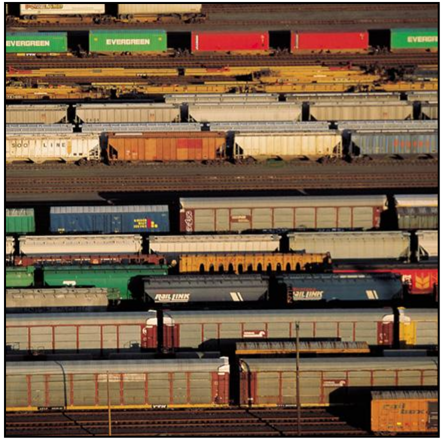


Negatif pikseller

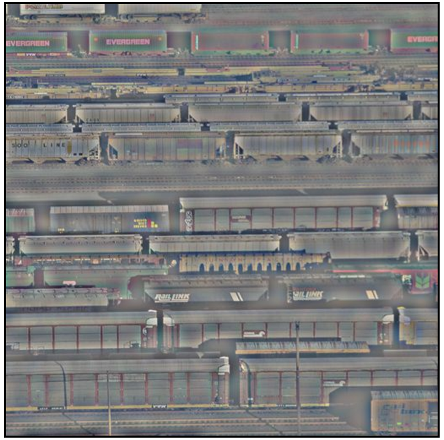
Karşılaştırma: Gaussian – İdeal Filtre



Ideal LPF

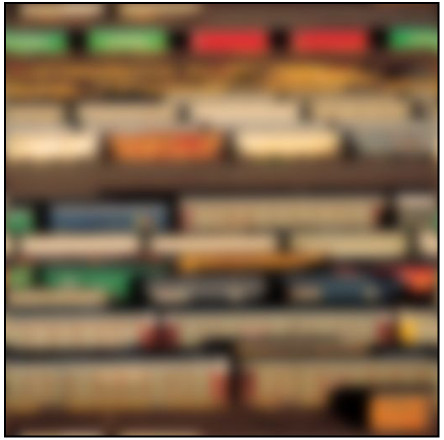


Orijinal Görüntü

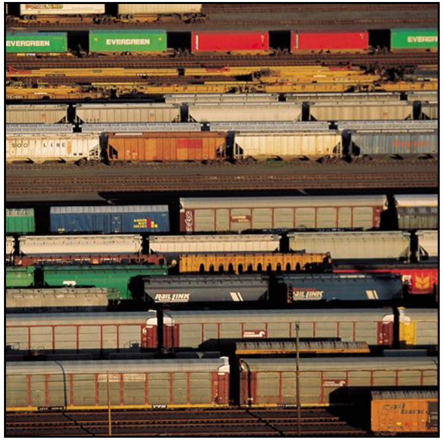


Ideal HPF

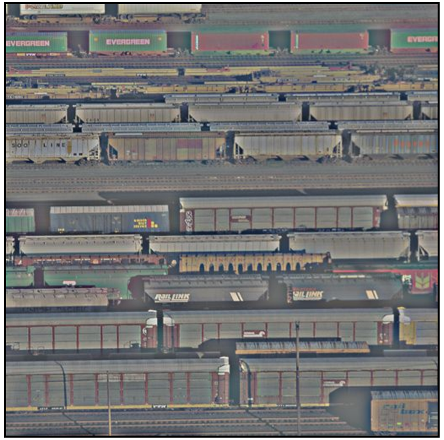
Karşılaştırma: Gaussian – İdeal Filtre



Gaussian LPF

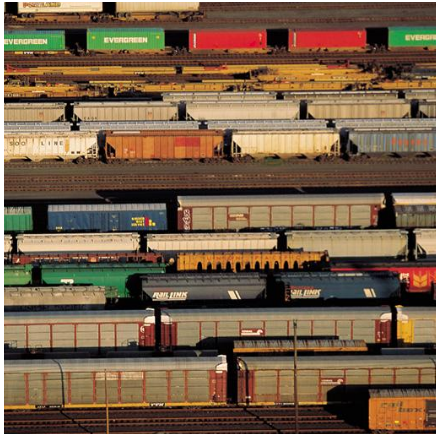


Orijinal Görüntü

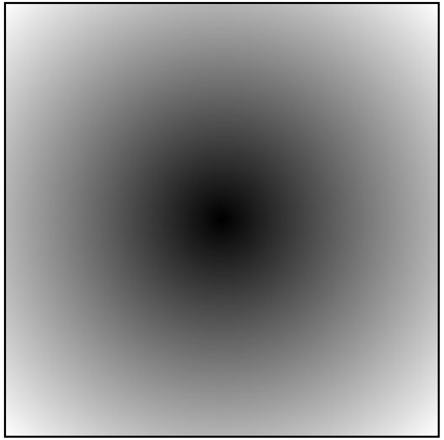


Gaussian HPF

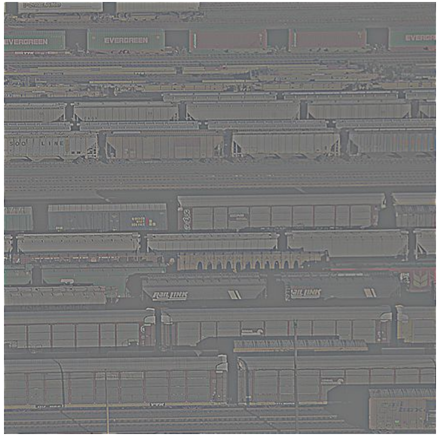
Başka bir yüksek geçiren filtre



Orijinal görüntü

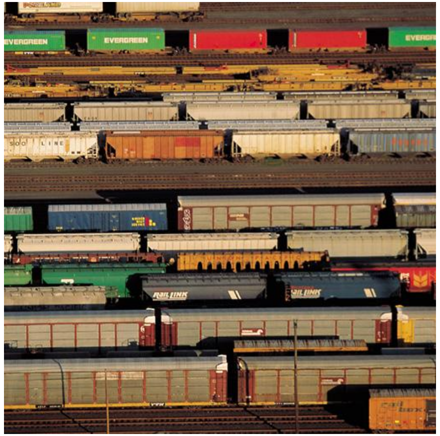


Filtre PS

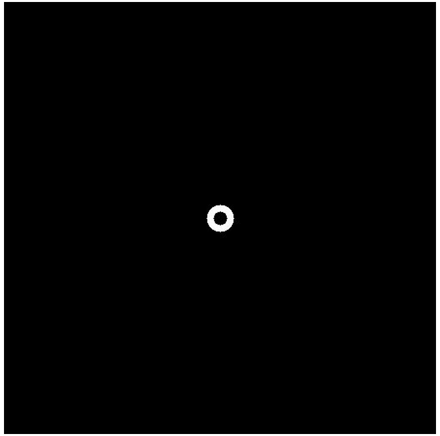


Filtrelenmiş görüntü

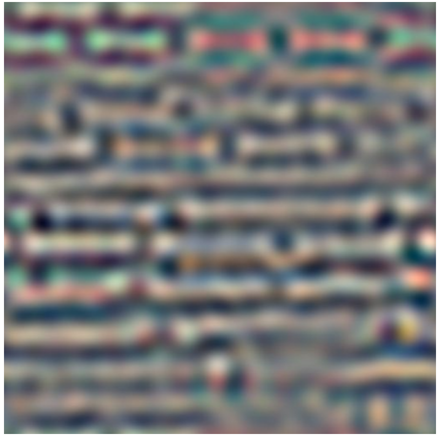
Ideal Band geçiren filtre



Orijinal görüntü



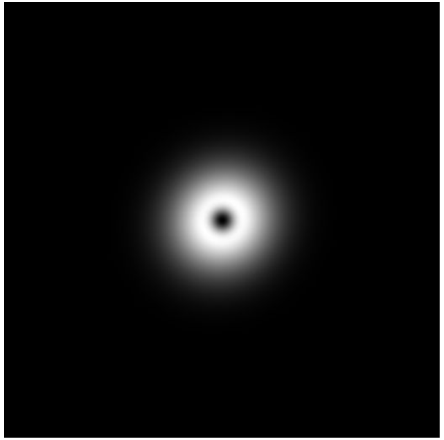
Filtre PS



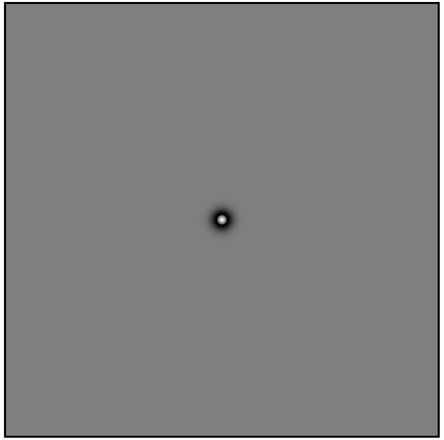
Filtrelenmiş görüntü

Gaussian Band Geçiren Filtre

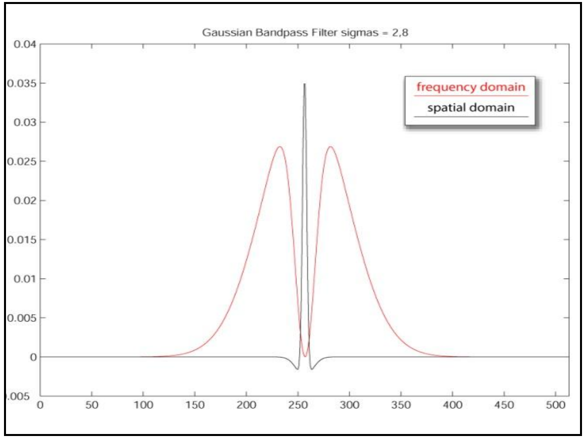
Görüntü: 512x512
sigma = 2 - sigma = 8



FD



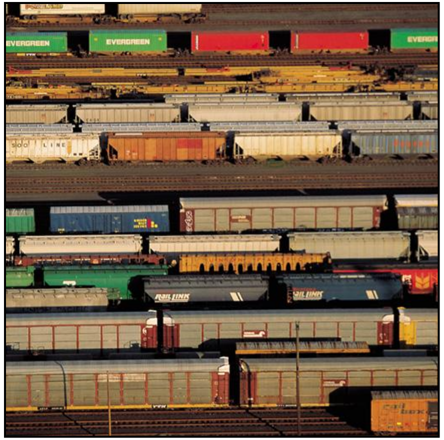
SD



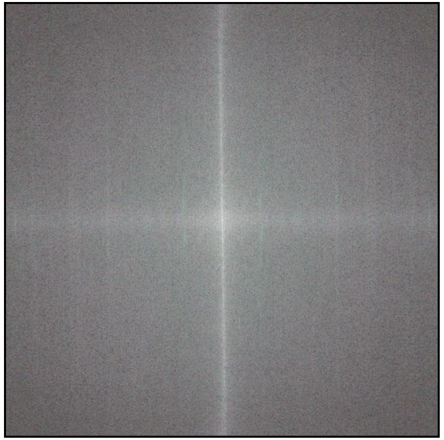
Merkez profil

Gaussian Band Geçiren Filtre

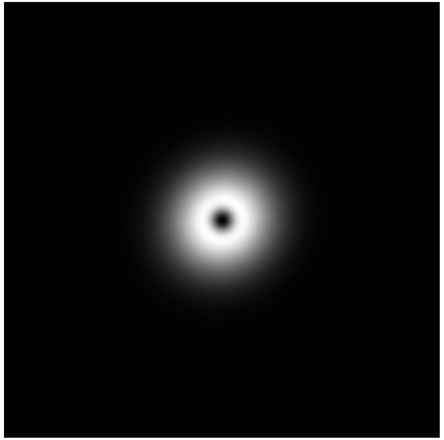
Görüntü: 512x512
sigma = 2 - sigma = 8



Orijinal Görüntü



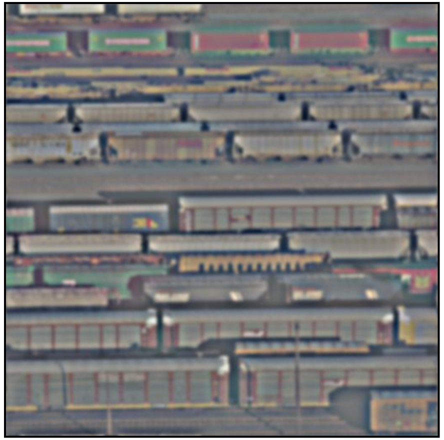
PS



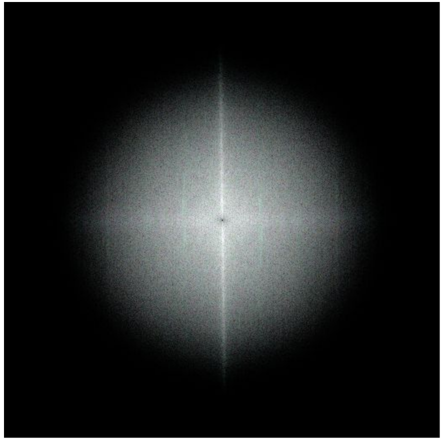
FD'de Gaussian BPF

Gaussian Band Geçiren Filtre

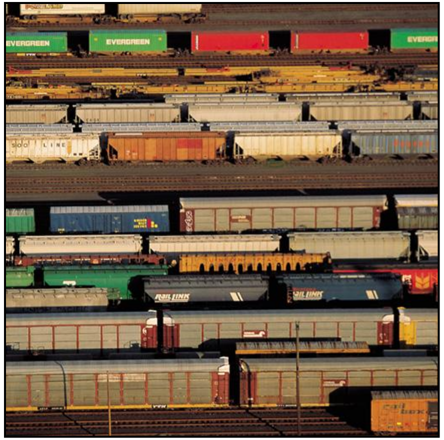
Görüntü: 512x512
sigma = 2 - sigma = 8



Filtrelenmiş görüntü



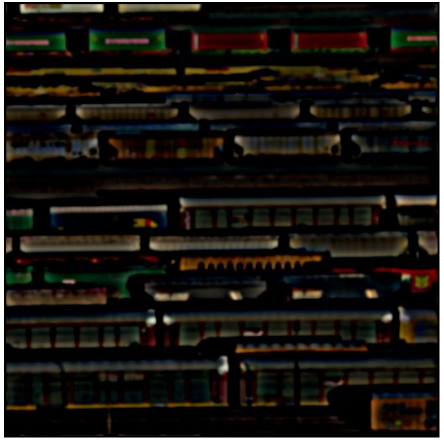
Filtre PS



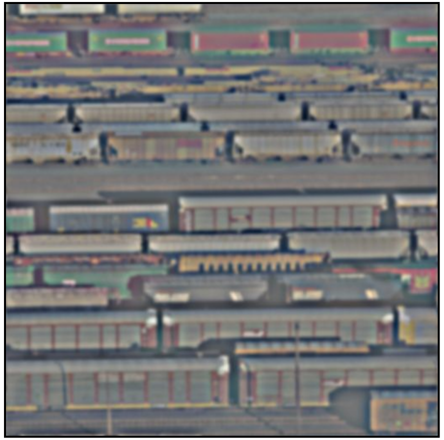
Orijinal görüntü

Gaussian Band Geçiren Filtre

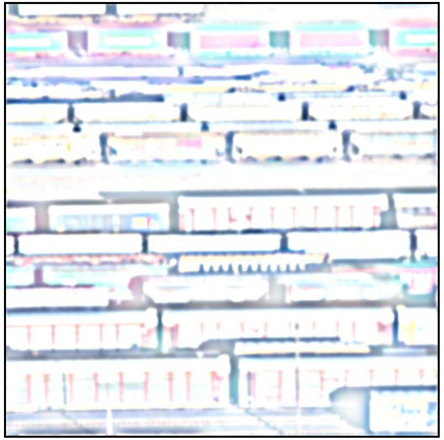
Görüntü: 512x512
sigma = 2 - sigma = 8



Pozitif Pikseller

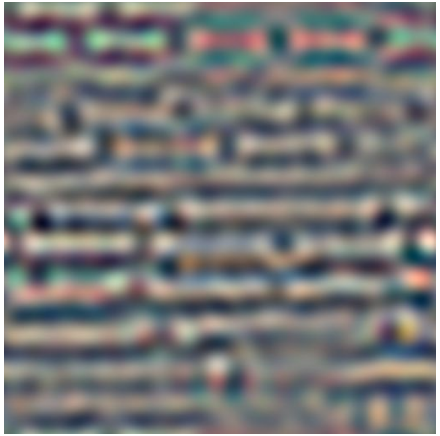


Filtrelenmiş Görüntü

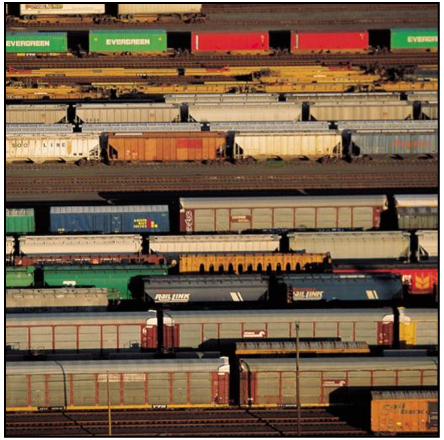


Negatif Pikseller

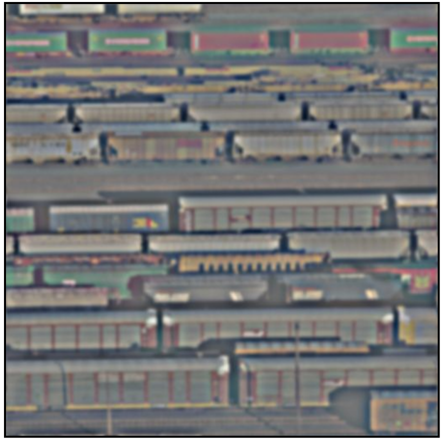
Karşılaştırma: İdeal - Gaussian



Ideal BPF

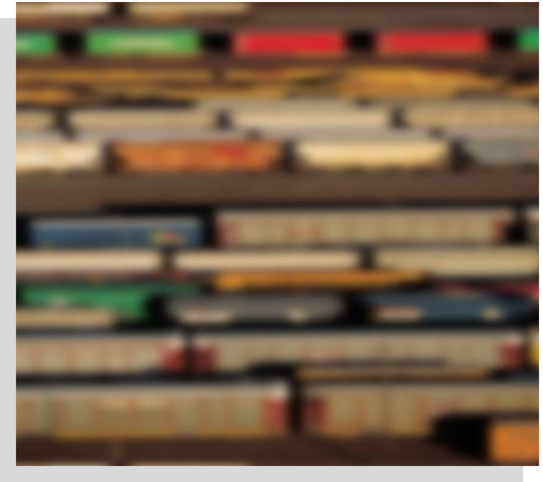


Orijinal Görüntü



Gaussian BPF

Bulanıklaşmış Görüntünün Güç Spektrumu ve Fazı

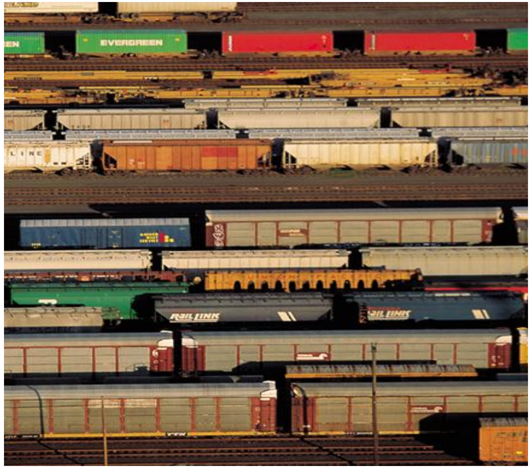


Bulanık Görüntü

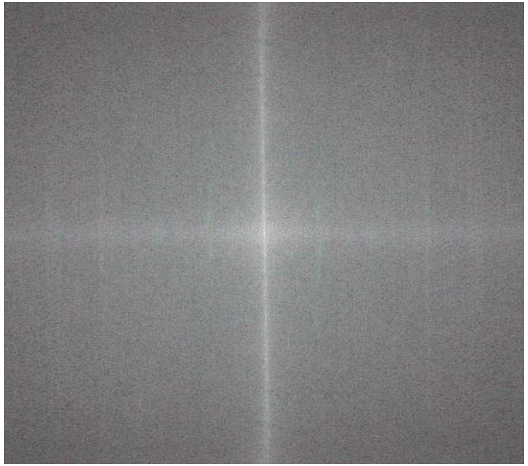
PS

Faz

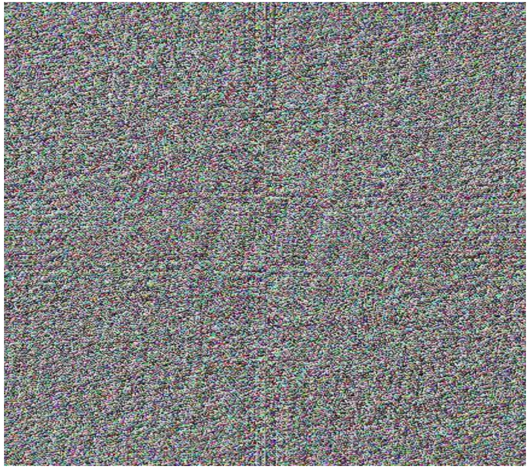
Orijinal Görüntünün Güç Spektrumu ve Fazı



Orijinal görüntü

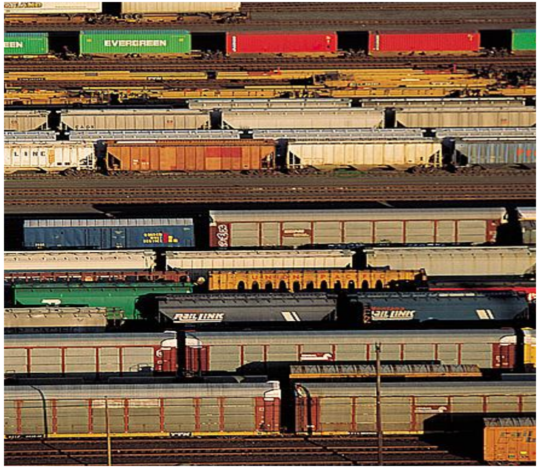


PS

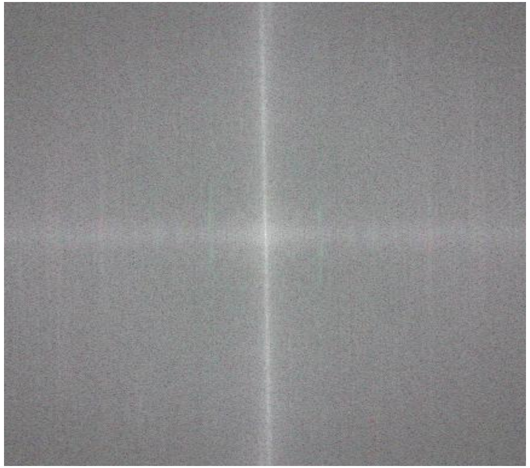


Faz

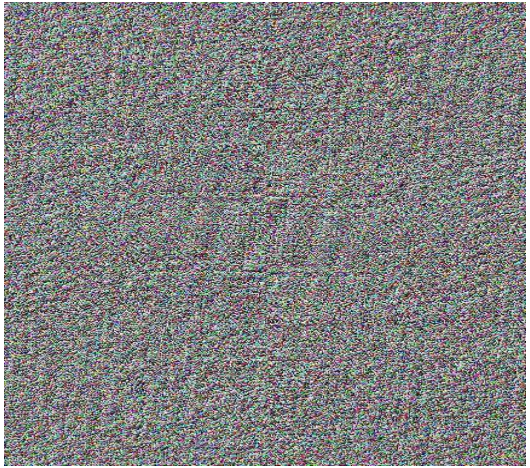
Keskin Görüntünün Güç Spektrumu ve Fazı



Keskin Görüntü



PS



Faz