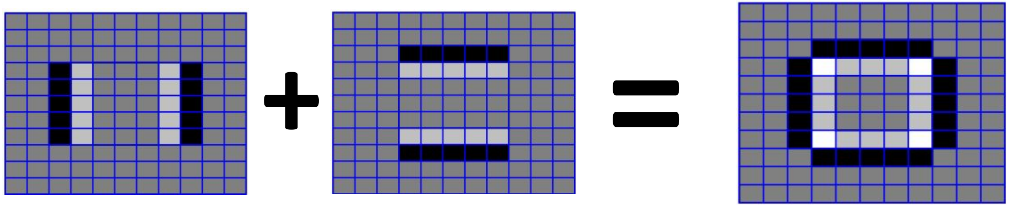
Yönsel Türev

Directional Derivative

Yönsel türev (directional derivative)

Klasik Türev

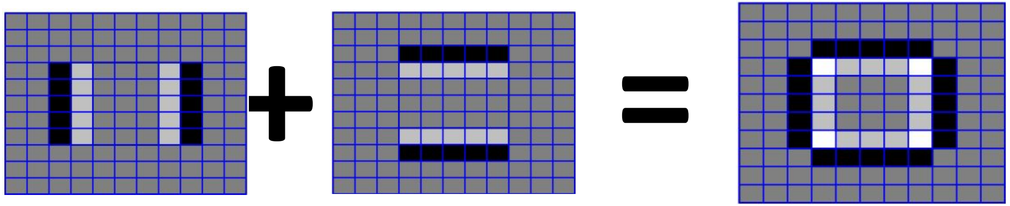
∂f / ∂x ile +x yönünde, **∂f / ∂y** ise +y yönündeki parlaklık değişimini verir.



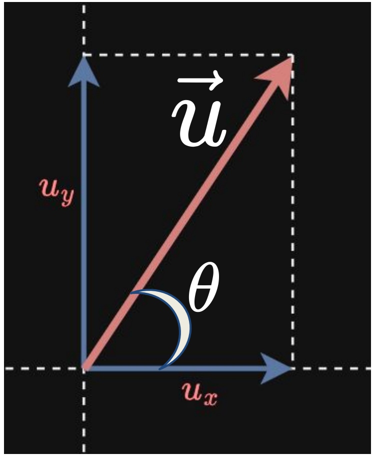
$$\frac{\partial f}{\partial x} \qquad \frac{\partial f}{\partial y} \qquad \nabla f$$

Yönsel Türev

Belirli bir yöndeki, örneğin 45 derecedeki türevi hesaplamak için yönsel türev kullanılır.



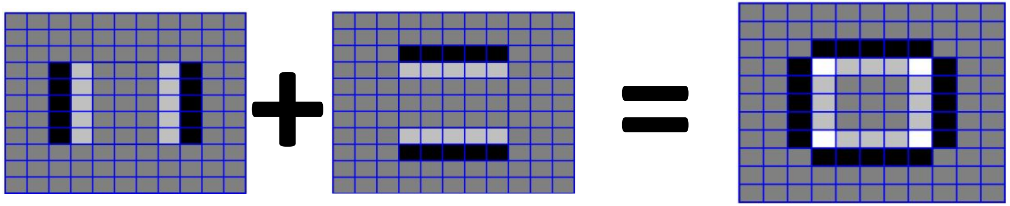
$$\frac{\partial f}{\partial x} \circ u_x \qquad \frac{\partial f}{\partial y} \circ u_y \qquad \nabla_{\vec{u}} f$$



Yönsel türev (directional derivative)

$\vec{u}$ vektörü birim seçilirse $\begin{aligned} |u_x| &= cos(\theta) \\ |u_y| &= sin(\theta) \end{aligned}$ olur

Böylece yönsel türev:



$$\frac{\partial f}{\partial x} \cdot \cos\theta \qquad \frac{\partial f}{\partial y} \cdot \sin\theta \qquad \nabla_{\vec{u}} f$$

Bu harika bir sonuçtur. Çünkü dilediğimiz yöndeki türevi yatay ve dikey türevden elde edebileceğimizi gösterir!!

Yüksek dereceli yönsel türev

$$f_\theta^{'}(x,y) = \frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta$$

$$f_\theta^{''}(x,y) = \frac{\partial^2 f}{\partial x^2}\cos^2\theta + 2\frac{\partial^2 f}{\partial x \partial y}\cos\theta\sin\theta + \frac{\partial^2 f}{\partial y^2}\sin^2\theta$$

$$f_\theta^{'''}(x,y) = \frac{\partial^3 f}{\partial x^3}\cos^3\theta + 3\frac{\partial^3 f}{\partial x^2 \partial y}\cos^2\theta\sin\theta + 3\frac{\partial^3 f}{\partial x \partial y^2}\cos\theta\sin^2\theta + \frac{\partial^3 f}{\partial y^3}\sin^3\theta$$

İkinci yönsel türev (gradient yönü boyunca)

$$\boxed{f_\theta''(x,y) = \frac{\partial^2 f}{\partial x^2}\cos^2\theta + 2\frac{\partial^2 f}{\partial x \partial y}\cos\theta\sin\theta + \frac{\partial^2 f}{\partial y^2}\sin^2\theta}$$

Gradient Yönü

$$\boxed{\theta = \begin{pmatrix} \dfrac{\partial f}{\partial x} \\ \dfrac{\partial f}{\partial y} \end{pmatrix} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}}$$

$$\frac{\partial^2 f}{\partial n^2} \equiv \frac{f_x{}^2 f_{xx} + 2 f_x f_y f_{xy} + f_y{}^2 f_{yy}}{f_x{}^2 + f_y{}^2}$$

İkinci yönsel türev kullanarak kenar yakalama

Laplacian: $$\nabla^2 f(x,y) \equiv \frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial y^2}$$

veya $$\nabla^2 f \equiv f_{xx} + f_{yy}$$

Gradyan yönü boyunca ikinci yönsel türev

$$\frac{\partial^2 f}{\partial n^2} \equiv \frac{{f_x}^2 f_{xx} + 2 f_x f_y f_{xy} + {f_y}^2 f_{yy}}{{f_x}^2 + {f_y}^2}$$

Kenar noktasında;

(i) İkinci yönsel türev sıfıra eşittir.

(ii) Üçüncü yönsel türev negatiftir.

İkinci Yönsel Türevin Özellikleri

Mathematical:

1. $\dfrac{\partial^2}{\partial n^2}$ is non-linear
2. $\dfrac{\partial^2}{\partial n^2}$ neither commutes nor associates with convolution

$$
\begin{aligned}
\frac{\partial^2}{\partial n^2}(g*f) &\neq \left(\frac{\partial^2 g}{\partial n^2}\right)*f \\
\left(\frac{\partial^2 g}{\partial n^2}\right)*f &\neq g*\left(\frac{\partial^2 f}{\partial n^2}\right)
\end{aligned}
$$

3. $\dfrac{\partial^2}{\partial n^2}$ is not everywhere defined (i.e., require ${f_x}^2 + {f_y}^2 \neq 0$)

Experimental:

4. $\dfrac{\partial^2}{\partial n^2}$ provides better localization, especially at corners

Yönsel Türevler

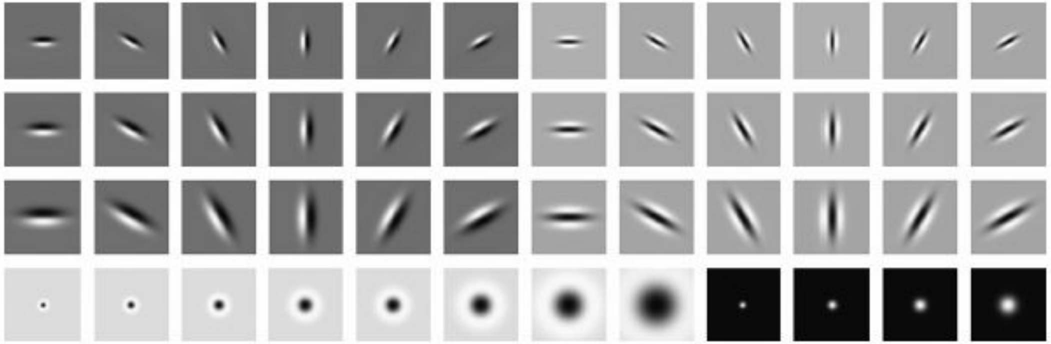


Fig. 1. LM filter set.

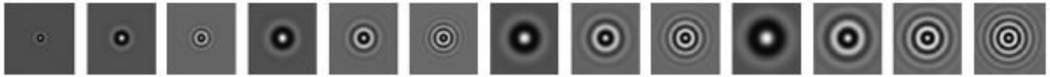


Fig. 2. S filter set.

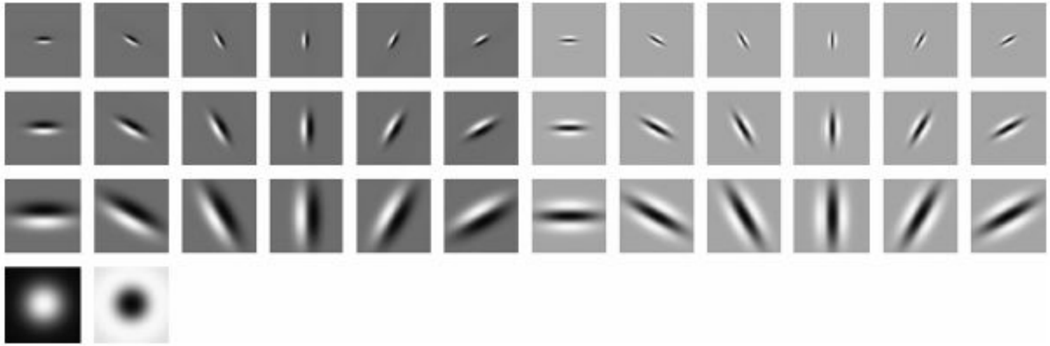


Fig. 3. MR8 filter bank.

Yönsel Türevler

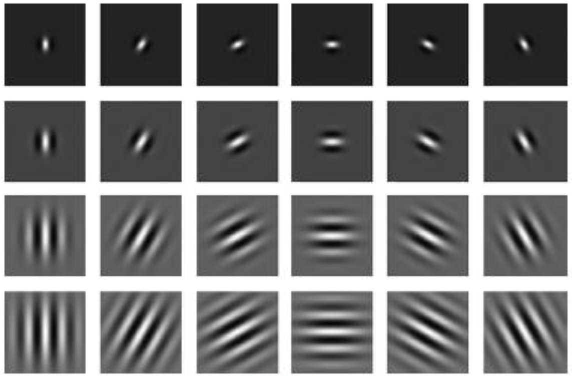


Fig. 4. Gabor filter set.

$$\begin{bmatrix} & & \\ 1 & -1 & \\ & & \end{bmatrix} \quad \begin{bmatrix} 1 & & \\ & -1 & \\ & & \end{bmatrix} \quad \begin{bmatrix} & 1 & \\ & -1 & \\ & & \end{bmatrix} \quad \begin{bmatrix} & & 1 \\ & -1 & \\ & & \end{bmatrix}$$

$$\begin{bmatrix} & & \\ & -1 & 1 \\ & & \end{bmatrix} \quad \begin{bmatrix} & & \\ & -1 & \\ & & 1 \end{bmatrix} \quad \begin{bmatrix} & & \\ & -1 & \\ & 1 & \end{bmatrix} \quad \begin{bmatrix} & & \\ & -1 & \\ 1 & & \end{bmatrix}$$

Fig. 5. Local derivative filter set.

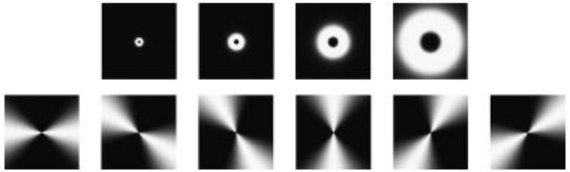
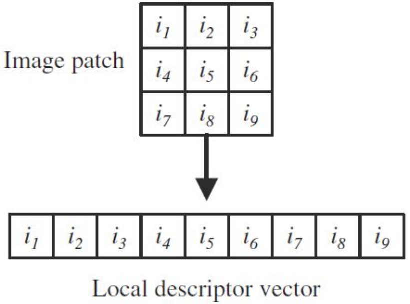


Fig. 6. Ring and wedge filter set.The first row are 4 ring filters and the second row are six wedge-shaped orientation filters.

Tek boyutlu maske kullanımı

1. İlk olarak 2d → 1d dönüşüm yapılır



2. Tek boyutlu bir f maske fonksiyonunun türevleri elde edilir:

$$f_1 = \frac{-2t}{\sigma^2} e^{-\frac{t^2}{\sigma^2}},$$

$$f_2 = e^{-\frac{t^2}{\sigma^2}},$$

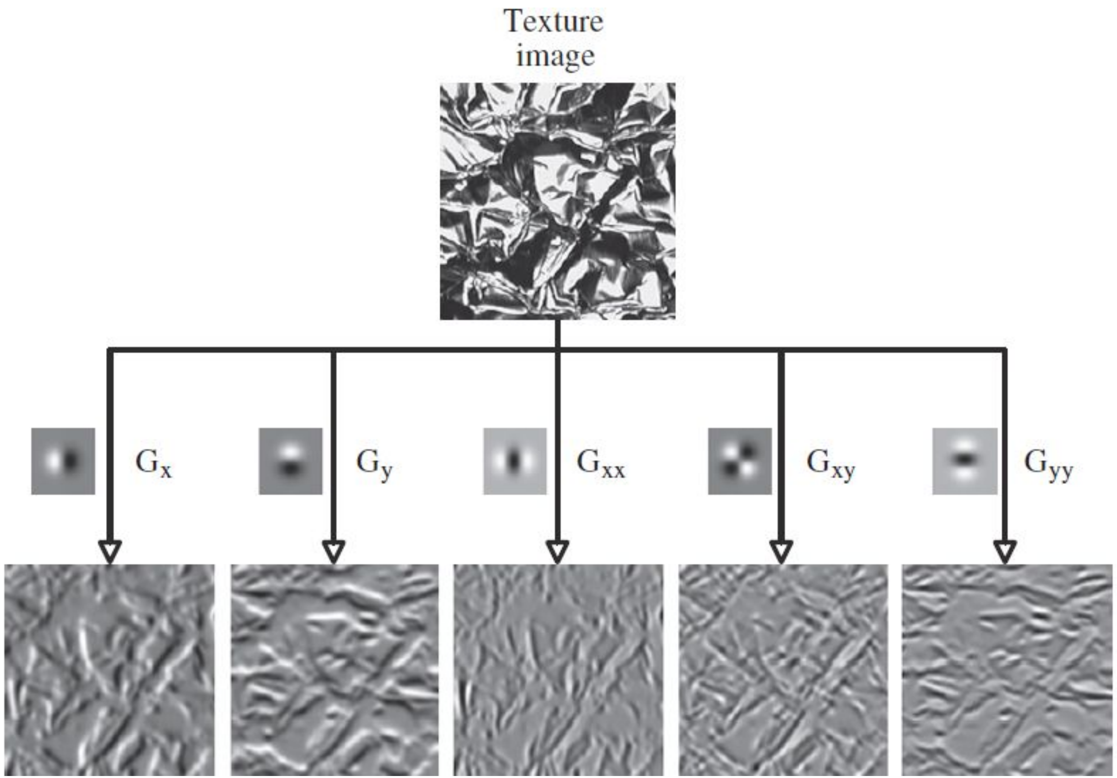
$$f_3 = \frac{2}{\sigma^2}\left(\frac{2t^2}{\sigma^2} - 1\right) e^{-\frac{t^2}{\sigma^2}},$$

$$f_4 = \frac{2t}{\sigma^2} e^{-\frac{t^2}{\sigma^2}}.$$

Basic filters	Filter in x	Filter in y
G_x	f_1	f_2
G_y	f_2	f_1
G_{xx}	f_3	f_2
G_{xy}	f_4	f_4
G_{yy}	f_2	f_3

2b filtreye örnek

1. Filtre fonksiyonumuzun Gauss olduğunu farz edelim: $G(x,y) = e^{\frac{-(x^2+y^2)}{\sigma^2}}$
2. Bu durumda görüntülerimizin birinci ve ikinci türevleri aşağıdaki gibi elde edilir:

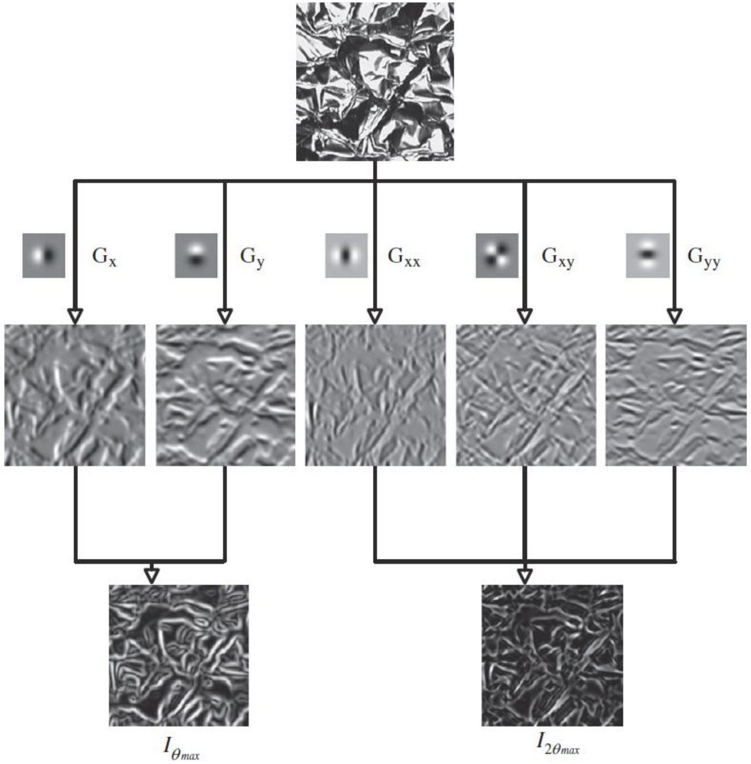


Maksimum birinci yönsel türev

$$I_\theta = \cos(\theta) I * G_x + \sin(\theta) I * G_y$$
$$= \sqrt{(I*G_x)^2 + (I*G_y)^2}\left(\cos(\theta)\frac{I*G_x}{\sqrt{(I*G_x)^2+(I*G_y)^2}} + \sin(\theta)\frac{I*G_y}{\sqrt{(I*G_x)^2+(I*G_y)^2}}\right)$$
$$= \sqrt{(I*G_x)^2 + (I*G_y)^2}\sin(\theta+\phi),$$

where $\phi = \arctan\frac{I*G_x}{I*G_y}$. Thus, when $\theta = \frac{\pi}{2} - \phi$, the maximum value of I_θ obtains

$$I_{\theta max} = \sqrt{(I*G_x)^2 + (I*G_y)^2}.$$



Maksimum ikinci yönsel türev-Hessian özdeğeri ile ilişkisi

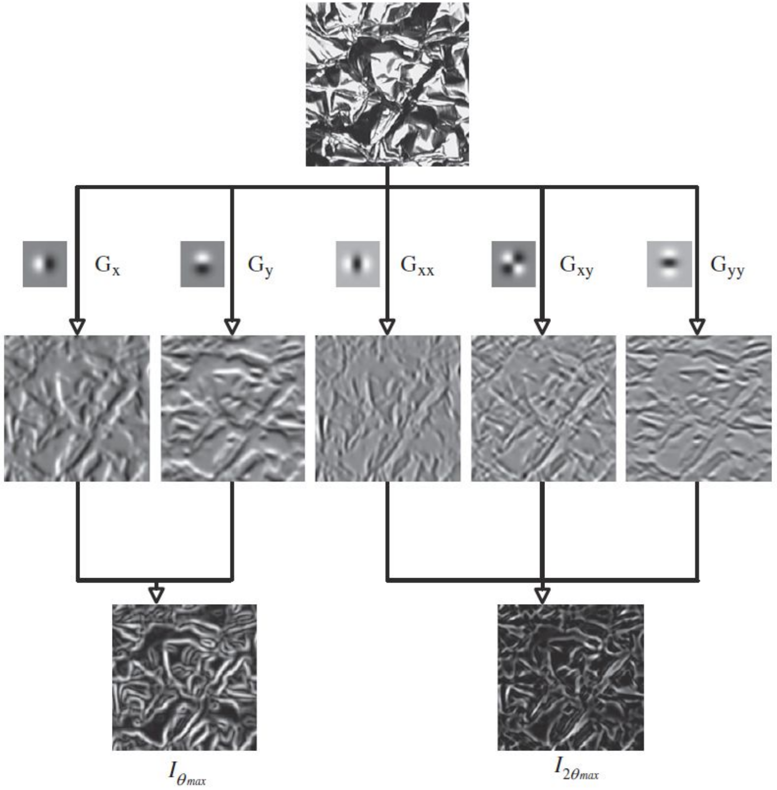
$$
\begin{aligned}
I_{2\theta} &= \cos^2(\theta) I * G_{xx} - 2\cos(\theta)\sin(\theta) I * G_{xy} + \sin^2(\theta) I * G_{yy} \\
&= \frac{2cos^2(\theta) I * G_{xx} - I_{xx}}{2} - 2cos(\theta)sin(\theta) - \frac{I_{yy} - 2sin^2(\theta) I_{yy}}{2} \\
&\quad + \frac{I * G_{xx} + I * G_{yy}}{2} \\
&= \frac{cos(2\theta) I_{xx}}{2} - sin(2\theta) I_{xy} - \frac{cos(2\theta) I_{yy}}{2} + \frac{I * G_{xx} + I * G_{yy}}{2} \\
&= \sqrt{\frac{(I * G_{xx} - I * G_{yy})^2}{4} + (I * G_{xy})^2} cos(2\theta - \phi) \\
&\quad + \frac{I * G_{xx} + I * G_{yy}}{2},
\end{aligned}
$$

where $\phi = \arctan \frac{2I * G_{xy}}{I * G_{xx} + I * G_{yy}}$. Thus, when $\theta = \pi + \frac{\phi}{2}$, the maximum value of $I_{2\theta}$ obtains,

$$
I_{2\theta max} = \sqrt{\frac{(I * G_{xx} - I * G_{yy})^2}{4} + (I * G_{xy})^2} + \frac{I * G_{xx} + I * G_{yy}}{2}
$$

$$
H = \begin{bmatrix} I * G_{xx} & I * G_{xy} \\ I * G_{xy} & I * G_{yy} \end{bmatrix}
$$

$$
\lambda = \pm \sqrt{\frac{(I * G_{xx} - I * G_{yy})^2}{4} + (I * G_{xy})^2} + \frac{I * G_{xx} + I * G_{yy}}{2}
$$



Link

Temel eğrilikler (Principal curvatures - k1,k2)

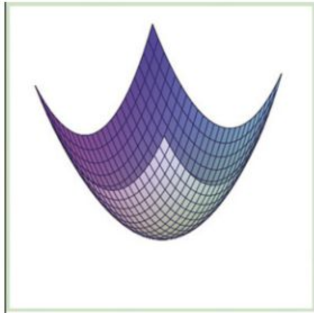
$$K = \frac{I_{xx}I_{yy} - I_{xy}^2}{\left(1 + I_x^2 + I_y^2\right)^2},$$

$$H = \frac{I_{xx}\left(1 + I_y^2\right) + I_{yy}\left(1 + I_x^2\right) - 2I_xI_yI_{xy}}{\left(1 + I_x^2 + I_y^2\right)^{3/2}}$$

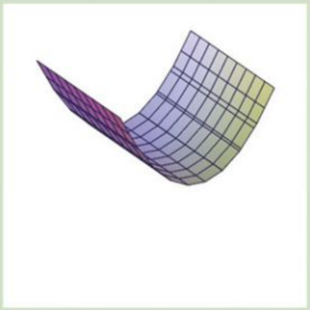
the principal curvatures (k_1, k_2) are

$$k_1 = H + \sqrt{H^2 - K},$$

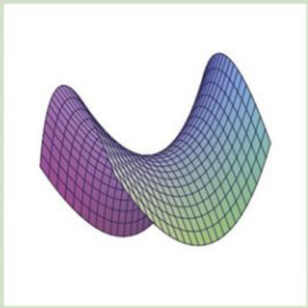
$$k_2 = H - \sqrt{H^2 - K}.$$



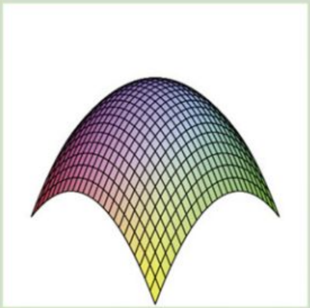
$k_1<0,\ k_2<0$



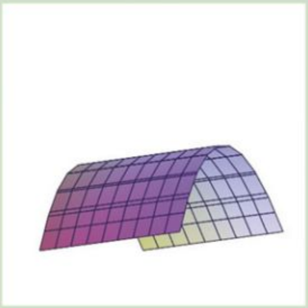
$k_1= 0\ \ k_2<0$



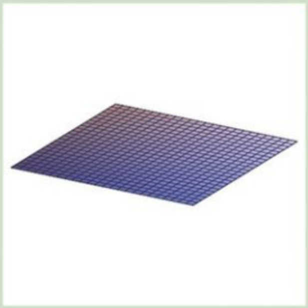
$k_1>0\ \ k_2<0$



$k_1>0,\ k_2>0$



$k_1>0\ \ k_2= 0$



$k_1= 0\ \ k_2= 0$