

# Konvolüsyon

1D

Maske

1	2	1
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Görüntünün bir satırı

1	1	2	2	1	1	2	2	1	1
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Konvolüsyon Sonucu

	5								
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1D

1	2	1
---	---	---

1	1	2	2	1	1	2	2	1	1
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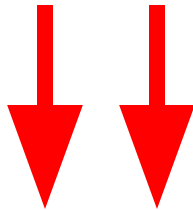
	$\frac{5}{4}$								
--	---------------	--	--	--	--	--	--	--	--

Maske toplamı:  
 $1+2+1 = 4$

1D

1	2	1
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1	1	2	2	1	1	2	2	1	1
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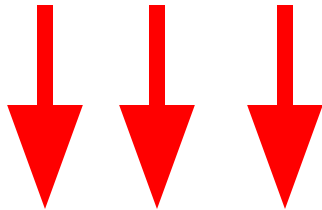


	$\frac{5}{4}$	$\frac{7}{4}$							
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1D

1	2	1
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1	1	2	2	1	1	2	2	1	1
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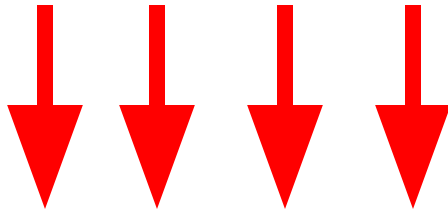


	$\frac{5}{4}$	$\frac{7}{4}$	$\frac{7}{4}$						
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1D

1	2	1
---	---	---

1	1	2	2	1	1	2	2	1	1
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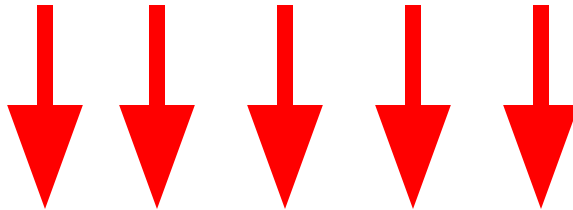


	$\frac{5}{4}$	$\frac{7}{4}$	$\frac{7}{4}$	$\frac{5}{4}$					
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1D

1	2	1
---	---	---

1	1	2	2	1	1	2	2	1	1
---	---	---	---	---	---	---	---	---	---

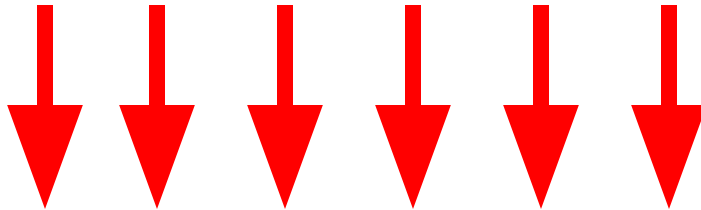


	$\frac{5}{4}$	$\frac{7}{4}$	$\frac{7}{4}$	$\frac{5}{4}$	$\frac{5}{4}$				
--	---------------	---------------	---------------	---------------	---------------	--	--	--	--

1D

1	2	1
---	---	---

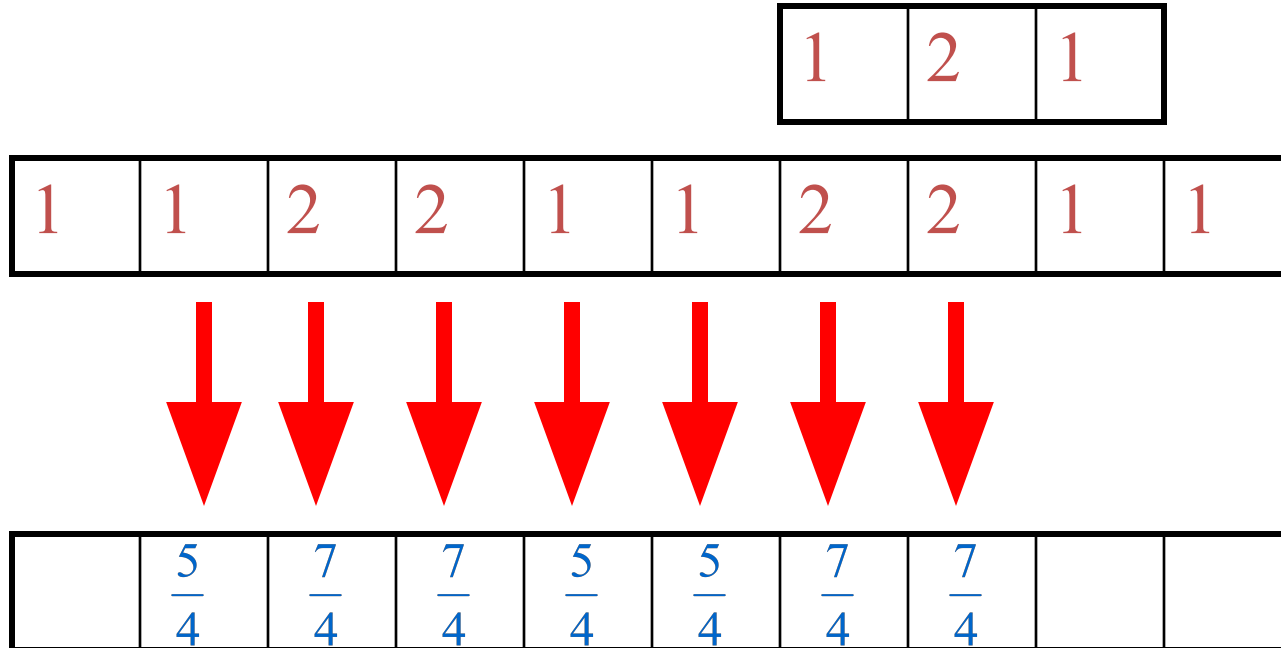
1	1	2	2	1	1	2	2	1	1
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	$\frac{5}{4}$	$\frac{7}{4}$	$\frac{7}{4}$	$\frac{5}{4}$	$\frac{5}{4}$	$\frac{7}{4}$			
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1D

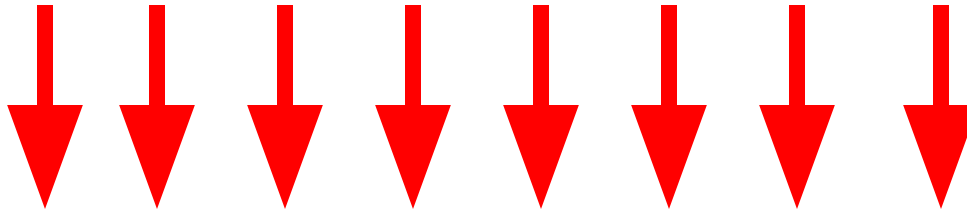


1D

İşlem  
tamam!!

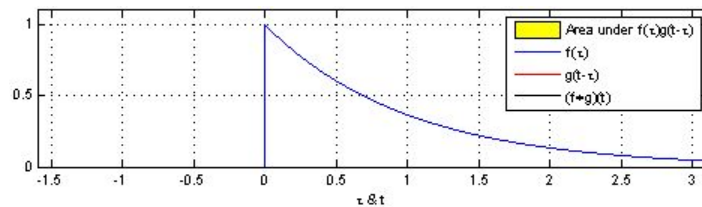
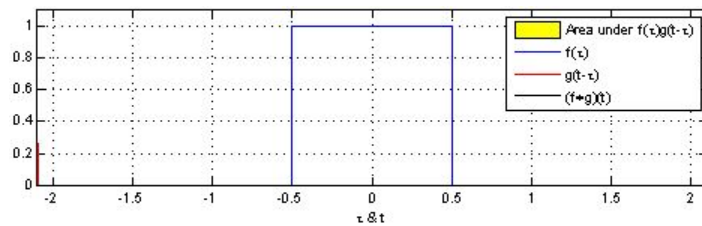
1	2	1
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1	1	2	2	1	1	2	2	1	1
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	$\frac{5}{4}$	$\frac{7}{4}$	$\frac{7}{4}$	$\frac{5}{4}$	$\frac{5}{4}$	$\frac{7}{4}$	$\frac{7}{4}$	$\frac{5}{4}$	
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# 1D



$$(f * g)(t) \stackrel{\text{def}}{=} \int_{-\infty}^{\infty} f(\tau) g(t - \tau) d\tau$$

$$(f * g)(t) = \int_0^t f(\tau) g(t - \tau) d\tau \quad \text{for } f, g : [0, \infty) \rightarrow \mathbb{R}$$

# 2D

- Maske iki boyutlu olmalı, Örneğin:  
**3x3**, 5x5, 7x7, ....

$$\frac{1}{9}$$

1	1	1
1	1	1
1	1	1

**Giriş**

1	2	0	1	3	
2	1	4	2	2	
1	0	1	0	1	
1	2	1	0	2	
2	5	3	1	2	

**Çıkış**


$$\frac{12}{9}$$

# 2D

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

**Giriş**

1	2	0	1	3	
2	1	4	2	2	
1	0	1	0	1	
1	2	1	0	2	
2	5	3	1	2	

**Çıkış**

	$\frac{12}{9}$	$\frac{11}{9}$			

# 2D

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

**Giriş**

1	2	0	1	3	
2	1	4	2	2	
1	0	1	0	1	
1	2	1	0	2	
2	5	3	1	2	

**Çıkış**

	$\frac{12}{9}$	$\frac{11}{9}$	$\frac{14}{9}$		

# 2D

**w**

 $\frac{1}{9}$ 

1	1	1
1	1	1
1	1	1

Matlab Komutu

```
f = imread('img1.bmp');  
w = [1 1 1;1 1 1;1 1 1]/9;  
g = conv2(f, w, 'same');
```

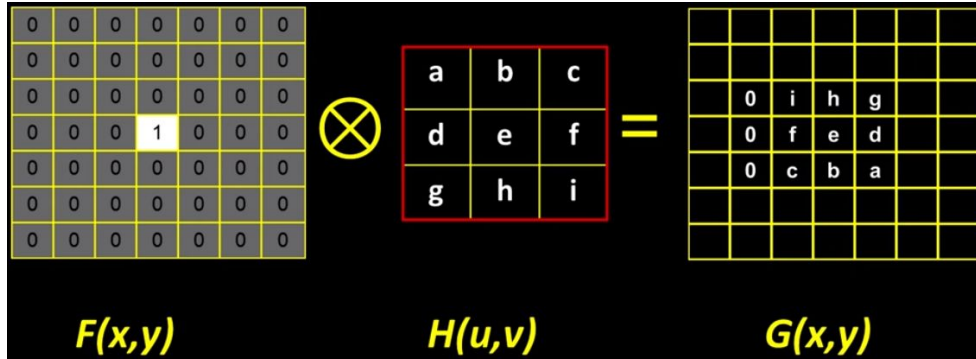
**f**

1	2	0	1	3	
2	1	4	2	2	
1	0	1	0	1	
1	2	1	0	2	
2	5	3	1	2	

**g**

	$\frac{12}{9}$	$\frac{11}{9}$	$\frac{14}{9}$		
	$\frac{13}{9}$	$\frac{11}{9}$	$\frac{13}{9}$		
	$\frac{16}{9}$	$\frac{12}{9}$	$\frac{11}{9}$		

# Konvolüsyonda Maske Neden Ters Çevrilir?



Impulse görüntüyle Korelasyon yapıldığında görüntü tersine döner.  
Ancak Konvolüsyon yapıldığında orjinal görüntünün kendisi elde edilir.

Impulse fonksiyonu sistem hakkında bilgi sahibi olmak için kullanılır.



# Uygulama alanı ve Matlab kullanımı

## UYGULAMA ALANLARI

- Nokta, Çizgi, Kenar Yakalama
- Nesne Yakalama
- Yumuşatma, Keskinleştirme
- Gürültü temizleme
- Morfolojik işlemler

## MATLAB KULLANIMI

**conv:** 1-D Convolution.

- $C = \text{conv}(A, B)$  convolves vectors A and B.

**conv2:** Two dimensional convolution.

- $C = \text{conv2}(A, B)$  performs the 2-D convolution of matrices A and B.


# Matematiksel olarak

## Cross-correlation

$$G[i, j] = \sum_{u=-k}^k \sum_{v=-k}^k H[u, v] F[i + u, j + v]$$

$$G = H \otimes F$$

Flip in both dimensions  
(bottom to top, right to left)



## Convolution

$$G[i, j] = \sum_{u=-k}^k \sum_{v=-k}^k H[u, v] F[i - u, j - v]$$

$$G = H \star F$$

# Konvolüsyonun özellikleri

- Linear & shift invariant

- Commutative:

$$f * g = g * f$$

- Associative

$$(f * g) * h = f * (g * h)$$

- Identity:

$$\text{unit impulse } e = [\dots, 0, 0, 1, 0, 0, \dots]. \quad f * e = f$$

- Differentiation:  $\frac{\partial}{\partial x}(f * g) = \frac{\partial f}{\partial x} * g$

## Hesaplama Karmaşıklığı

NxN görüntü

WxW filtre (kernel, maske)

$$NxNxWxW \Rightarrow N^2W^2$$

# Konvolüsyon maskesinin ayrılabilirliği

**c** **r** **H**

1	x	1 2 1	=	1 2 1
2				2 4 2
1				1 2 1

$$G = H * F = (C * R) * F = C * (R * F)$$

Böylece herbiri  $W \times N \times N$  maliyetinde iki konvolüsyon işlemi yapılır.  
Bu ise klasik konvolüsyon işlemini oldukça hızlandırır:

$$2 \cdot W \cdot N^2 \ll W^2 \cdot N^2$$

# Sınır işlemleri (padding)

