Intergenerational Earnings Mobility Trends and Childhood Skill Formation

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Abstract

There is no clear trend in intergenerational mobility despite rising inequality. I focus on the role of child skill formation in the family and argue that functional form and distributional assumptions play a crucial role in their association. I estimate the child skill-formation function nonparametrically using an EM algorithm with nonlinear quantile regressions and compare it with the commonly used CES case. The nonparametric case has features, e.g., how returns differ across families, that CES cannot capture and can create potential mechanisms for the lack of trend in intergenerational mobility. I will use the estimated child skill formation function in a quantitative life-cycle model to evaluate such mechanisms.

1 Introduction

We live in a world with rising inequality, and understanding how inequality is transmitted between generations is more important than ever to design public policy to provide equal opportunities for future generations. One of the leading mechanisms for the transmission of economic well-being between generations is the childhood skill formation process in the family. Richer families invest more resources towards their children's skills than poorer families, and the gap is getting wider over time (Corak, 2013; Blanden, Doepke and Stuhler, 2022). Therefore, it is worrying that we might have less mobility across generations as the inequality rises and the ability of families to provide opportunities for their children diverges over time.

Contrary to the expectation, my estimation of the trend in intergenerational mobility in earnings is flat, and the US is not becoming less mobile over time. My results are based on PSID in line with the literature that uses other data or methodologies (Lee and Solon, 2009; Chetty et al., 2014*b*; Song et al., 2020). A flat mobility trend is a surprising finding given the negative association between inequality and intergenerational mobility across countries and commuting zones within the US, the so-called Great Gatsby curve (Corak, 2013; Chetty et al., 2014*a*).

The childhood skill formation function can explain the flat intergenerational earnings mobility trends given the rising inequality. To explore this channel, I estimate a function without any shape or distributional restriction. The skill formation function takes investment as input, e.g., parental time and expenditure, and tells how much skill a child would get. Imagine a functional form with very high skill returns for low levels of investments while returns are much lower for high levels of investment. In this case, even if richer families would invest much more than poorer families, the extra skill their children will get could be much lower than what would be for the children of low-income families. This story makes it possible to have better mobility even in a higher inequality environment. Motivated by this insight, I estimate a childhood skill formation function without making form or distributional assumptions to capture such mechanisms.

Using a simple theoretical model, I show that the functional form of childhood skill formation plays a crucial role in determining intergenerational mobility when inequality rises. There are two periods in the model, childhood, and adulthood. An altruistic parent decides on childhood skill investment and bequest in the first period. The child's skill is realized following a skill formation function in the second period. The child goes to the labor market and gets earnings depending on the skill level. I derive an expression for the intergenerational mobility of earnings and inequality. I check the comparative statics for an increase in returns to skill in the labor market. I show that mobility can go in any direction depending shape of the skill formation function while inequality rises unambiguously.

Becker et al. (2018) conduct a similar theoretical analysis with Cobb-Douglas functional form assumption for the skill formation. I generalize their setting by leaving functional form unspecified and show that results are sensitive to that. [1]

My estimation of intergenerational mobility in earnings shows that the trend is flat. I regress parents' earnings to children's earnings and allow the coefficient to be flexible across cohorts. I want to run this regression with lifetime earnings, which is impossible because of data restrictions. Instead, using PSID, I approximate the lifetime earnings of parents and children around age 40, i.e., I take three years around age 40 and take an average. The results suggest that there is no significant difference in mobility across generations.

I estimate a childhood skill production function without any shape and distributional assumptions using the PSID Child Development supplement dataset to capture potential features that can lead to flat mobility in an economy with rising inequality. Restrictive functional forms such as CES or Cobb-Douglas have a few parameters that restrict their flexibility in different dimensions.

In the data, I can observe child-related expenditure and the total time parents spend with their kids on a typical day. Parental time and expenditure are inputs in the childhood formation function. On the output side of the skill formation function, there are three cognitive tests as measures of childhood skills. I developed an empirical model where skill is an unobservable factor, and the skill level at a particular age depends on the lagged skill level, parent's skill level, and investment consists of time and expenditure. Also, the model includes a mapping between skill level at the end of childhood and years

[1] They also acknowledge the role of functional form for their results in a footnote and say that finding the correct functional form is an empirical question for future research.

of education.

I use a simulated EM algorithm similar to Arellano, Blundell and Bonhomme (2017) to estimate the empirical model. Skill is an abstract concept, and we can only observe some imperfect measures, e.g., test scores. Not observing a skill variable makes the estimation challenging. I approximate the unknown true skill formation function with orthogonal polynomials for a set of quantiles to be flexible in both functional form and uncertainty distribution. The algorithm starts with a guess of parameters, and I simulate skills conditional on guessed parameters and the data. In the second step, I update the parameter values with quantile regressions and maximum likelihood estimation and go back to the first step. A large number of iterations over these two steps gives a sequence of estimates converging to the true values. I also estimate a constant elasticity of substitution (CES) production with a normal disturbance function for comparison.

The results show that the skill investment and skill level are substitutes, i.e. the investment is more productive for disadvantaged children with low levels of skills. This also means that any missing investment at earlier ages can be substituted at later ages. This is against to the results on **?** where they estimate CES skill production function from NLSY dataset. Where as the findings of Agostinelli and Wiswall (2021) are in line with this paper since they use translog and include an interaction term between skill level and investment.

The parental skills and investment are complements hence more educated parents are more productive. However it is interesting to see that the returns to investment is decreasing for all parent but at faster rate for high educated parents. This means they hit the flat part of the function faster than low educated parents.

The uncertainty of childhood skill production function is more negatively skewed for more educated families, whereas the CES case cannot capture this feature because of the normal distribution assumption. Children of more educated parents, on average, acquire higher skills. However, because of negative skewness, they are more likely to end up at a lower level of skill distribution.

All of these features of the flexible skill production function can explain the flat mobility trend with rising inequality. Disadvantaged children can catch up because the investment is more productive for them. Although more educated parents are more productive given investment level, they hit the flat region of the skill production function more quickly. Lastly, more negative risk for children of high educated parents can create a mean reversion effect.

The paper's main contribution is a nonparametric estimation of childhood skill process function i.e. without shape and distributional restriction. Cunha, Heckman and Schennach (2010) provides a nonparametric identification but they assume CES functional form with log additive normal noise. Agostinelli and Wiswall (2021) estimates a translog function, in particular Cobb-Douglass and one interaction term between investment and skill level with a log additive noise. Log additive noise does not allow heterogeneous higher moments conditional on inputs. My specification includes all possible interactions in first and second orders and I allow all parameters to depend on the uncertainty.

In the future research, I will build a life cycle model with families and children where they decide how much to invest for childhood skills alongside other decisions like bequest, consumption, savings, and labor supply. The main exercise will be analyzing the implications of rising inequality driven by higher returns to skill. I will compare the exercise results in the flexible skill formation function and restrictive CES case. Based on the estimation results, I conjecture that the quantitative exercise can produce flat mobility even with rising inequality. There may not be a need for an intensive policy to improve the opportunity inequality of future generations.

Section 2 provides the details of the estimation of the trend in intergenerational elasticity of earnings and gives the results. Section 3 explains the theoretical motivation for an estimation a fully flexible production function. Section 4 briefly summarizes the plan for the estimation and Section 5 concludes.

2 Data and Estimation of Intergenerational Elasticity of Earnings Trend

I focus on trends in intergenerational elasticity in log household earnings. Arguably, the ideal way to measure this elasticity would be using life-time earnings of parents and children. However, this requires a panel data covers at least entire two generations. Typically, we can observe earnings at different ages for each individual such as in the survey datasets. Even if it is possible to observe all individuals at the same age, it differs across generations because of short time length of the dataset like administrative tax datasets.

There are several different approaches in the literature to address this problem. Lee and Solon (2009) uses income at different ages for each individual as an observation and controls for a polynomial of both children's and parents' age at the time of measurement to account for the life cycle bias. They use PSID and find no trend in intergenerational elasticity of income.

Justman and Krush (2013); Justman, Krush and Millo (2017) take a two step approach. First, predict the life-time income of fathers and sons and using predicted income they estimate international elasticity of income for different cohorts. They conclude that there is an upward trend in the intergenerational elasticity of income alongside the rise in the inequality.

The last approach uses average income of children and parents on certain ages and compare elasticity for different cohorts. Chetty et al. (2014*b*) measure parents' income average of ages when children were between 15-19 years old and their adult income is measured around age 30. They conclude that there is no trend for cohorts born years between 1970-1985. In a recent paper, Davis and Mazumder (2020) also follows a similar approach and compare earlier cohorts who were born in 1950 and 1960 using NLSY dataset and they find increase in the elasticity.

I follow the last approach and measure both parents' and children's household earnings around age 40 in PSID. In particular, I run the following regression,

$$y_{ic}^{child} = \alpha + \beta y_{ic}^{parent} + \gamma_c y_{ic}^{parent} + \epsilon_{ic}, \tag{2.1}$$

where y_{ic}^{child} is log household earnings of child i from cohort group c, and y_{ic}^{parent} is household earnings of parents of child i from cohort group c and γ_c is cohort group dummies. It is measured as average household earnings over three years around age 40 for children and the parent who is the head of the household. I repeat the estimation also for age 30.

I use only biological parents who lived with their children in their childhood (until age 18) at least one year. If the head of house changes during childhood, I assign whoever was the head longer as the head of household between father and mother. This only matters whose age is going to used for calculation of average household earnings.

This approach requires to observe both patents and children at ages around age 40 hence after selecting only those observations data gets really thin. I grouped cohorts in ten years groups and the cohort born in between 1950 and 1960 is omitted.

Table 1 provides results. In column (1), earnings measured around age 40, surprisingly the elasticity drops for 1961-1970 cohort almost by half and it is significant at 5% level. In the column (2), I drop the observations in top and bottom top 1% of parents' household earnings. The coefficient for the cohort 1961-1970 drops by half and loses its significance. It is hard to say if this surprising result is driven by a fundamental change on social mobility on tails or driven by a few outliers by chance. Columns (3) and (4) repeat the same exercise but uses average earnings around age 30. The cohort group dummies are not significant but the signs are different between two specification but with very large p-values.

I found no evidence for increasing trend in intergenerational elasticity of earnings, if anything there is decrease for cohort born after 1970 driven by the top and bottom tails of earnings distribution. This can be surprising given rising in equality and the positive association between intergenerational elasticity and inequality across countries.

In the next chapter, for child skill formation channel, the relationship between this elasticity and rising inequality depends on the shape of childhood skill production function.

Table 1: Intergenerational Elasticity of Earnings for Ten Tears Cohort Groups

	Dependent variable:			
	Children's Log Earnings			
	Age 40	(Drop %1)	Age 30	(Drop %1)
	(1)	(2)	(3)	(4)
Parents' Log Earnings	0.500***	0.507***	0.264**	0.612*
	(0.079)	(0.078)	(0.126)	(0.363)
	p = 0.000	p = 0.000	p = 0.037	p = 0.093
Parents' Log Earnings x (61-70)	−0.026	−0.010	0.037	−0.230
	(0.094)	(0.097)	(0.143)	(0.369)
	p = 0.784	p = 0.918	p = 0.798	p = 0.534
Parents' Log Earnings x (71-80)	−0.213**	−0.111	0.146	−0.169
	(0.101)	(0.104)	(0.137)	(0.367)
	p = 0.036	p = 0.286	p = 0.285	p = 0.645
Parents' Log Earnings x (81-90)			0.082	−0.222
			(0.136)	(0.367)
			p = 0.546	p = 0.545
Observations	1,416	1,388	1,707	1,672
Note:				*p<0.1; **p<0.05; ***p<0.01

3 Theoretical Motivation

This section provides a simple two period model to make the point that the shape of childhood skill function matters for the implications of higher inequality for intergenerational elasticity of earnings.

I use a version of simple two period model in Becker et al. (2018). I keep both childhood skill production function and return to skills function unspecified to highlight the effect of functional forms and I do not include any randomness in the model for simplicity.

There are two periods: childhood and parenthood. Parents maximize the following utility function,

$$V(I_p) = \max_{c,b_c,y} u(c)+\delta I_c, \tag{3.1}$$

$$\text{s.t.} \quad c + \frac{b_c}{R_k} + y = I_p, \quad b_c \geq 0, \tag{3.2}$$

where c consumption, b_c bequest for the child, R_k intergenerational return, y investment for child skills. I_p and I_c are income of the parent and the child which is sum of earnings and bequest, $I_j = E_j + b_j$, for $j \in \{p, c\}$.

Earnings are function of human capital, $E_j = g(H_j)$ where H_j is human capital or skills for $j \in \{p, c\}$. Lastly, the skills are produces by a production function depending on the investment and parent's skills,

$$H_c = F(y, H_p). \tag{3.3}$$

The solution for an unconstrained parent is where the two kind of investments, skill investment and bequest are equal,

$$\frac{\partial I_c}{\partial y} = R_k \quad \Longrightarrow \quad g'(H_c)F_y(y^*, H_p) = R_k, \tag{3.4}$$

and for the constrained parent the return of skill investment should be equal to the marginal utility,

$$\delta\frac{\partial I_c}{\partial y} = u'(c) \Longrightarrow g'(H_c)F_y(y^*, H_p) = \delta^{-1}u'(c). \tag{3.5}$$

By taking derivative of solution equations 3.4 and 3.5, we can derive expressions for intergenerational elasticity of earnings and skills. In the unconstrained case, these given by,

$$\frac{\log E_c}{\log E_p} = \frac{g'(H_c)/g(H_c)}{g'(H_p)/g(H_p)} \frac{\partial H_c}{\partial H_p}, \tag{3.6}$$

$$\frac{\partial H_c}{\partial H_p} = F_y \frac{\partial y^*}{\partial H_p} + F_H, \tag{3.7}$$

$$\frac{\partial y^*}{\partial H_p} = -\frac{\frac{g''(H_c)}{g'(H_c)} F_H + \frac{F_{yH}}{F_y}}{\frac{g''(H_c)}{g'(H_c)} F_y + \frac{F_{yH}}{F_y}}. \tag{3.8}$$

Therefore, the elasticities depend on curvatures and cross derivatives of functions, e.g. $\frac{g'}{g''}$, $\frac{F_{yy}}{F_y}$ and $\frac{F_{yH}}{F_y}$.

To make it easier to convey the idea, I use a graphical argument to show that it is even possible to have decreasing intergenerational elasticity of skills as a response to rising inequality through rising returns to skills. For the graphical argument in Figure , I assume the return to skills are linear, i.e $E = g(H) = rH$.

Figure 1 demonstrates an example in the case of unconstrained parent. The upper right plot gives the solution of optimal investment for the parents with low and high skills, i.e./ Equation 3.4. The rise in return to skills increases the return on investment on skills and shifts the curves upwards. The effect of higher investments on children's skill depends on the curvature and cross derivative in skill production function, as it is illustrated in the lower right plot. The lower left plot shows the final effect on the intergenerational association of skills after the rise of skill return. In this particular example, the slope gets smaller. It is possible to prove this mathematically too, for example choosing an high order polynomial for production function, this is the case for certain parameter values. Also, the function can be convex and constrained parents might invest in the convex region as it is illustrated in Figure 2. This can increase the response on the bottom of income distribution.

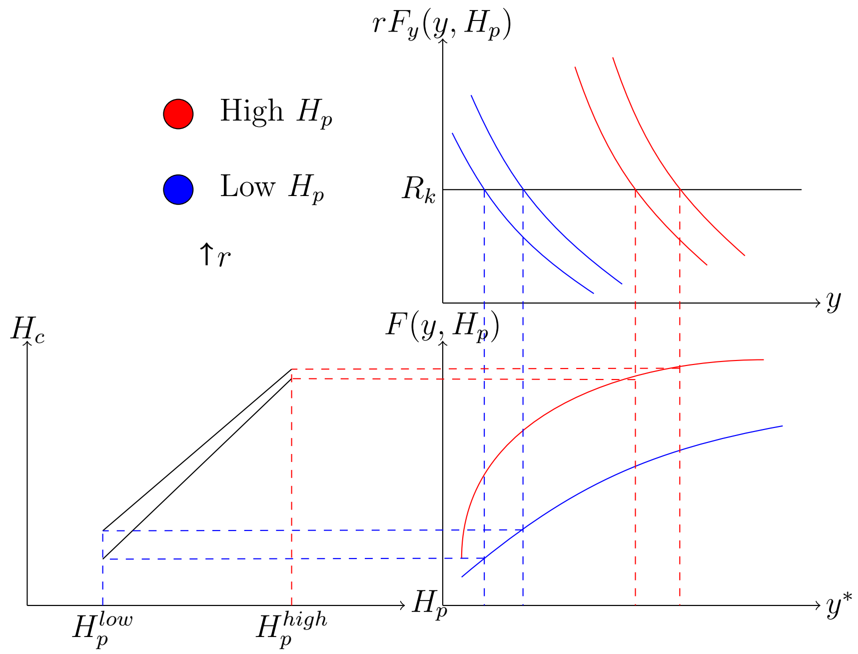


Figure 1: Effect of rising returns to skills on intergenerational skill transmission.

4 Estimation of Childhood Skill Production Function

4.1 Empirical Model

The childhood skill production depends on past skills, parents' skill and investment. Let θ_{it} be skill level of child i at time t,

$$\ln\theta_{t+1} = F(\ln\theta_t, \ln I_t, \ln\theta_P, u_t), \tag{4.1}$$

where I_t is investment, aggregate of parental time and money, θ_P is aggregated skills of mother and father and lastly u_{it} is random variable normalized to uniform distribution. Notice that $F(\cdot)$ is in fact the quantile function skills conditional on inputs and any distribution can

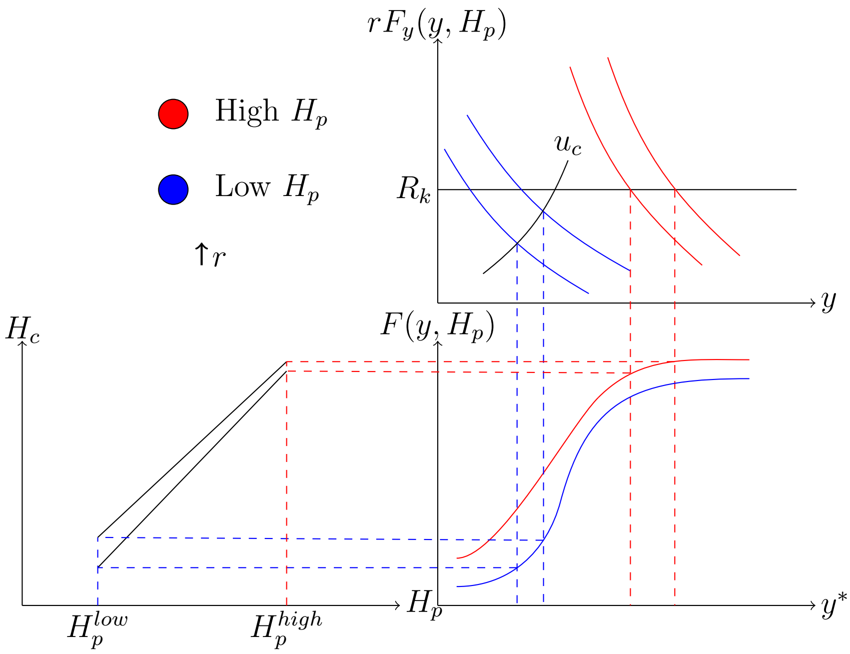


Figure 2: Effect of rising returns to skills on intergenerational skill transmission with convex production function.

be expressed in this way.[2]

The aggregate investment consists of parental time and monetary expenditure for child development. It is given by Cobb-Douglas aggregator,

$$\ln I_t = \ln Time_t^{mom} + \gamma^{tdad} \ln Time_t^{dad} + \gamma^{exp} \ln Exp_t, \qquad (4.2)$$

where $Time_t^{mom}$ and $Time_t^{dad}$ are active time mother and father spend together with the child and Exp_t is the total expenditure for

[2]For example if we had an additive normal noise, i.e.

$$\theta_{t+1} = \tilde{F}(\theta_t, I_t, \theta_P) + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0,1)$$

quantile function representation is given by,

$$\theta_{t+1} = F(\theta_t, I_t, \theta_P, u_t) = \tilde{F}(\theta_t, I_t, \theta_P) + \Phi^{-1}(u_t),$$

where $\Phi^{-1}(\cdot)$ is the inverse of standard normal c.d.f.

child development. The aggregate parents skill is given by,

$$\ln\theta_P = \ln\theta_{mom} + \gamma_\theta^{dad}\ln\theta_{dad} + \gamma_\theta^{int.}\ln\theta_{dad}\ln\theta_{mom}, \tag{4.3}$$

where the interaction coefficient γ_θ^{int} determines if there is super modularity between parents' skills and allows to analyze any effect of sorting between couples.

I need to specify the distribution of the skills in the initial period,

$$\ln\theta_0 = F_0(\ln\theta_P^0, age_0, u_0) \tag{4.4}$$

where θ_P^0 is aggregate parents skill with the same functional form in the equation 4.3 and age_0 is the age of the child in the first period of the data set.

Time and expenditure policy functions are given by another quantile function,

$$\begin{aligned} \ln Time_t^k &= F_k(\ln y_t, \ln\theta_t, \ln\theta_P^k, u_t^k), \quad k \in \{mom, dad\}, \\ \ln Exp_t &= F_k(\ln y_t, \ln\theta_t, \ln\theta_P^{Exp}, u_t^{Exp}), \\ \text{with} \quad & u^{mom}, u^{dad}, u^{Exp} \sim U(0,1), \end{aligned} \tag{4.5}$$

where y_t is total household income and θ_P^{tmom} is aggregate parents skill. Specifying policy function in this flexible reduced way allows them to be consistent with multiple models.

The skills are not directly observable and we have only measures of them. In the data, there are three different cognitive tests for children and question level information is available. Let $Prob_{mq}$ be the probability of answering question i correct in test m,

$$Prob_{mq} = \frac{exp\,(\alpha_m + \beta_m\theta - d_q)}{1 + exp\,(\alpha_m + \beta_m\theta - d_q)}, \tag{4.6}$$

where α_m and β_m are location and scale parameters of the test m and d_q is the difficulty level of the question q.

Lastly, I estimate a mapping between skill levels and years of education of the child when they become adults. I specify that the conditional distribution of years of education as binomial distribution with probability p,

$$p_{edu} = \frac{exp\left(f(\ln\theta_T, \ln\theta_P, age_T)\right)}{1 + exp\left(f(\ln\theta_T, \ln\theta_P, age_T)\right)}, \tag{4.7}$$

where θ_T is the skill level of child and age_T is the age of the child in the last period T.

The standard arguments in Cunha, Heckman and Schennach (2010) works for identification of the non-parametric production function with multiple measures available for the variables are not directly measured, e.g. test scores for skills. In a recent paper, **?** showed that there is a trade off between restrictions on the shape of function and measurement. I will put restriction on measurement as in Cunha, Heckman and Schennach (2010) to allow the identification of the fully flexible production function. A formal proof of the identification of 4.9 is the immediate next step of the project.

4.2 Estimation

The aim is to estimate the skill formation function in the equation 4.9 without restricting functional form and distribution of uncertainty. I approximate the function with hermite polynomials and allow the coefficients vary over quantiles,

$$F(\ln\theta_t, \ln I_t, \ln\theta_P, u_t) = \sum_{k=0}^{K_\theta} a_k(u_t)\varphi_k(\ln\theta_t, \ln I_t, \ln\theta_P), \tag{4.8}$$

where $\psi_k(\cdot)$'s are orthogonal polynomials and $a_k(u_t)$ are coefficients specific to the quantile $u_t \in [0, 1]$. I take a grid of quantiles $\{u^0, u^1, \ldots, u^L\}$ and estimate coefficients for them with quantile regression. For the off the grid quantiles, I use linear interpolation. I follow the same approach for the investment policy functions (eq. 4.5) and the initial skill distribution (eq. 4.4).

The skill measures and the mapping to the years of education are estimated with maximum likelihood estimator given their parametric specification.

Also in the CES specification, I replace the skill formation function with the CES production function with a normal noise.

$$\ln \theta_{t+1} = \ln \left[A \left(\alpha_\theta \theta_t^\phi + \alpha_I I_t^\phi + \alpha_{\theta_P} \theta_P^\phi \right)^{\frac{1}{\phi}} \right] + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0, \sigma_\theta), \tag{4.9}$$

As the estimation algorithm, I follow Arellano and Bonhomme (2016); Arellano, Blundell and Bonhomme (2017) and use an EM algortihm. The challenge is that skills are not observable and I cannot run quantile regressions and maximum likelihood estimations. In the EM algorithm, I start with a guess of parameters and simulate skills based on the implied distribution conditional on the data. The conditional distribution of skills depends on the parameters of the empirical model. This simulation step is called E step.

Using the sample of skills, I update the parameters by set of quantile regressions and maximum likelihood estimators. This is called M step. In this iteration the parameters converges to the true values and fluctuates around them. I do 500 EM iteration and take average of the last 250 parameter estimates as the final estimates.

4.3 Results

I compare the estimation results of flexible and CES cases by using several plots. Figure 3 plots the skill function at its mean with respect to log of aggregate investment. Notice that the estimation results of flexible case is concave in logs and really flat in the high levels investments while CES function is convex.

The strong concavity and flat region at the top of flexible can lead to a constant or higher mobility in a rising inequality environment. When the inequality rises because of higher skill return in the labor market because of technological change, all families will increase their investment for their children's skils. Because the skill function is really flat for high-income families, their children's skill will not increase too much but this is not the case low-income families.

In Figure 4, I plot the 1st derivative of the skill function with respect to log aggregate investment for different levels of past skill levels to see the patterns in the return of skill investment. Notice

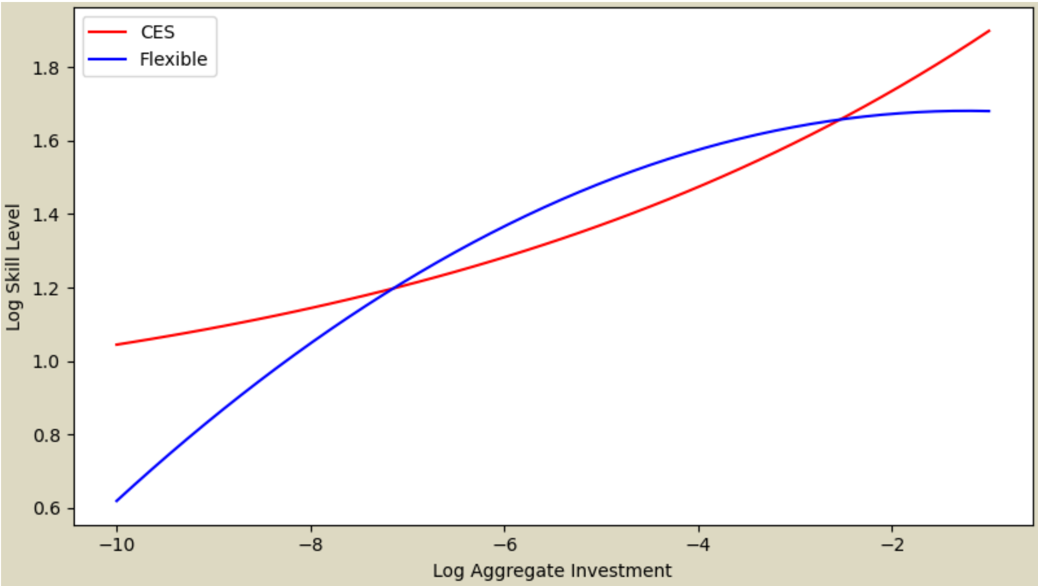
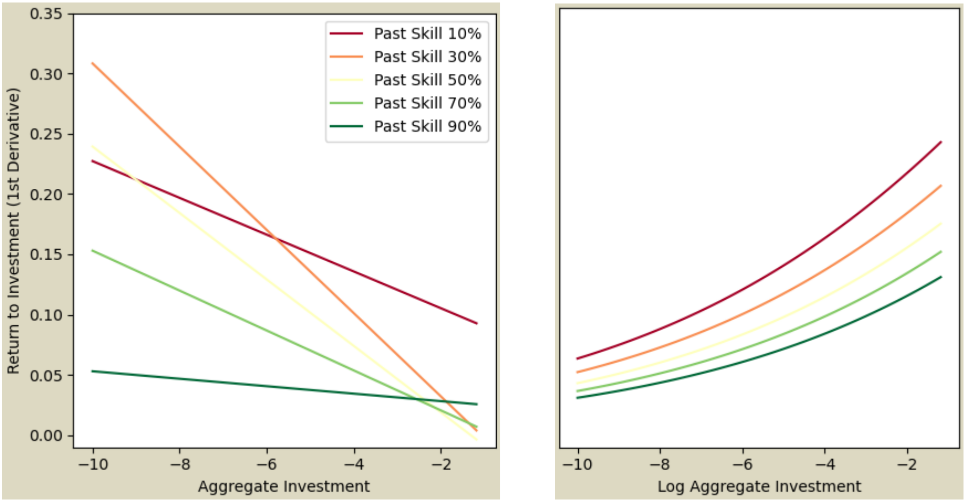


Figure 3: Childhood skill function at mean with respect to aggregate investment. The other two inputs, child's lag skill level and parents' skill level are fixed at their median value.

that this is also the elasticity of child skills with respect to investment because I use log scale. We see that in both cases the returns are higher for children with lower levels of past skills and this means that the skill level and investment are substitutes. The investment is more productive for disadvantaged children with low skill levels possible because of low investment in early ages. In other words, it is possible to substitute the missing early investment with later investment. When we look at the pattern of the return, we see that they are decreasing in the flexible case with different speeds for different levels of past skill while it is increasing in CES case.

In Figure 5, I plot a similar graph but this time for different levels of parents' skill. We see that in the flexible case more educated parents are more productive hence the parents' skill and investment are complements. However this is not the case in CES, because CES forces that all inputs are either complements or substitutes all together. It is interesting to see that in the flexible case, the returns are decreasing for all parents and much faster for more educated parents. This



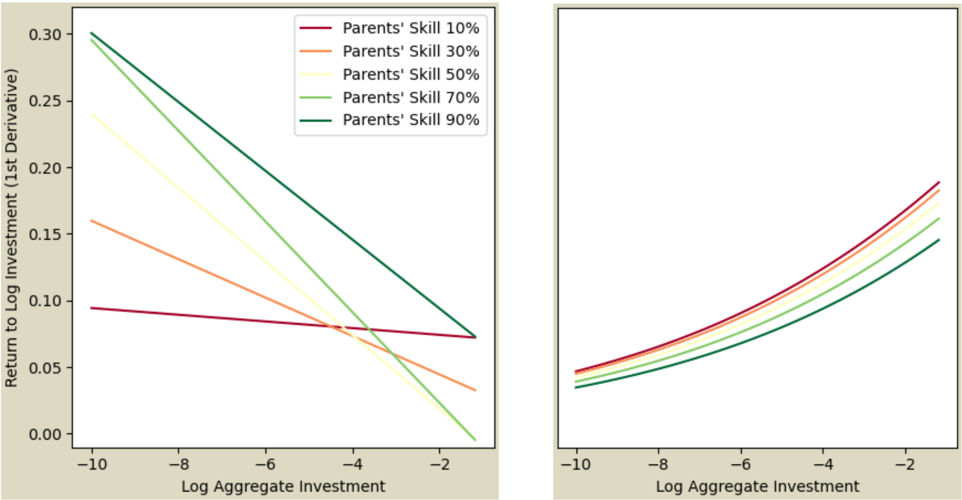
(a) Flexible (b) CES with Normal Noise

Figure 4: Return (Elasticity) of Childhood Skills with respect to log aggregate investment for different levels of past skill level. Parents' skill level is fixed to its median value.

means that the more educated parents hit the flat part of the skill function more quickly than low educated parents. This feature also can contribute to the fixed mobility in rising inequality story because children of low educated parents are able to catch up while children of high educated parents suffer from the flat skill function.

Figure 6 plots the skewness in the childhood skill formation with respect to parents' skill level for the flexible case because skewness is zero in CES case by assumption. We see that it is more negatively skewed for more educated parents, i.e. children of more educated parents are more subject to negative risk while the risk is more symmetric of children of low educated parents. In other words, the high educated parent are able to increase the mean skill for their children but cannot do much for the lower bound. This also can contribute to the catch up mechanism for children of low educated parents.

I would like to point out that CES is restrictive because only one substitution parameter governs substitution patterns of all inputs and the concavity/convexity in log scale. Figure 7 illustrates the shape of



(a) Flexible

(b) CES with Normal Noise

Figure 5: Return (Elasticity) of Childhood Skills with respect to log aggregate investment for different levels of parents' skill level. Past skill level is fixed to its median value.

A	ϕ	α_θ	α_I
3.0	0.17	0.44	0.17

Table 2: CES case estimation results.

function and returns with respect to parents skill level and past skill level. Figure 7a repeats the baseline case where the estimation results suggest that the inputs are substitutes. Figure 7b plots the Cobb-Douglas case where the returns in logs are constant for all past skill level or parents' skill level. Lastly, Figure 7c illustrates the complement case. We see that when it switched from substitutes to complements the shape of function, the patterns and order of returns changes all together. There is no reason to thing that these features should be connected and results of flexible case assures that they go to different directions than the ones CES forces them.

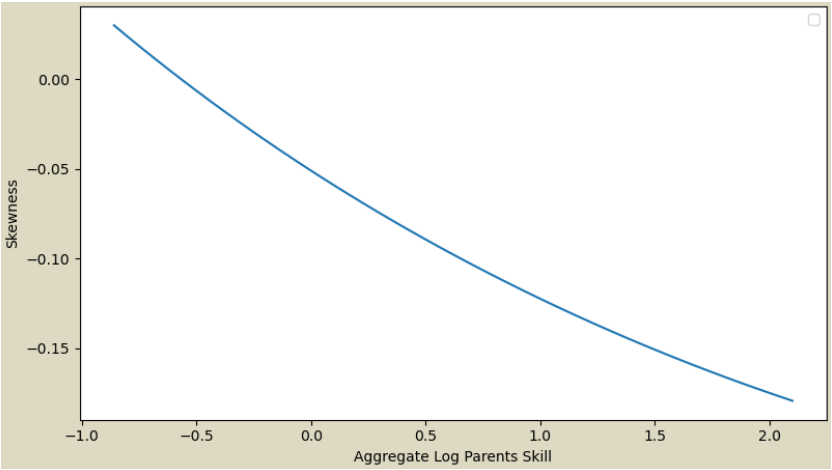


Figure 6: Skewness of childhood skill formation function conditional on parents' skill level.

5 Conclusion

The rising inequality can be worrying because it can lead much lower mobility. The intuition is that while high income families are able to provide better education opportunities for their children, low income families may lack enough resources and this gap will increase as the inequality increases. However there is good news in the data, the intergenerational earnings mobility seems constant despite substantially rising inequality.

I show that the childhood skill formation function play a key role in transmission of inequality in a theoretical model and I estimate a very flexible childhood skill formation function as opposed to restrictive CES function form.

My estimation result shows that some features of childhood skill formation function can be an explanation for the flat mobility trend given rising inequality. In particular, the investment is more productive for disadvantaged children and they might be able to catch up. Also high educated families reach the flat part of the skill function more quickly than low educated parents. Hence the children of low educated parents can enjoy higher returns. Lastly, the more negative skewness for high educated parent can create a mean reversion effect and improve mobility.

The estimation results for CES case shows that the CES functional form cannot capture these features. One single parameter governs substitution patterns for all inputs as well as concavity. There is no reason for these features of the skill function to be connected. My flexible estimation shows that this is indeed the case.

Lastly, my results have important policy implications. First, there is no need to panic for the possibility of fading American dream because of rising inequality at least through the skill formation channel. However, it is known that the mobility is already low in the US compared to other developed countries. To improve that the policy should focus on the disadvantaged children even if at later ages in order to maximize the effect in terms of more skills. Also, increasing productivity of low educated parents through parental education can be good strategy.

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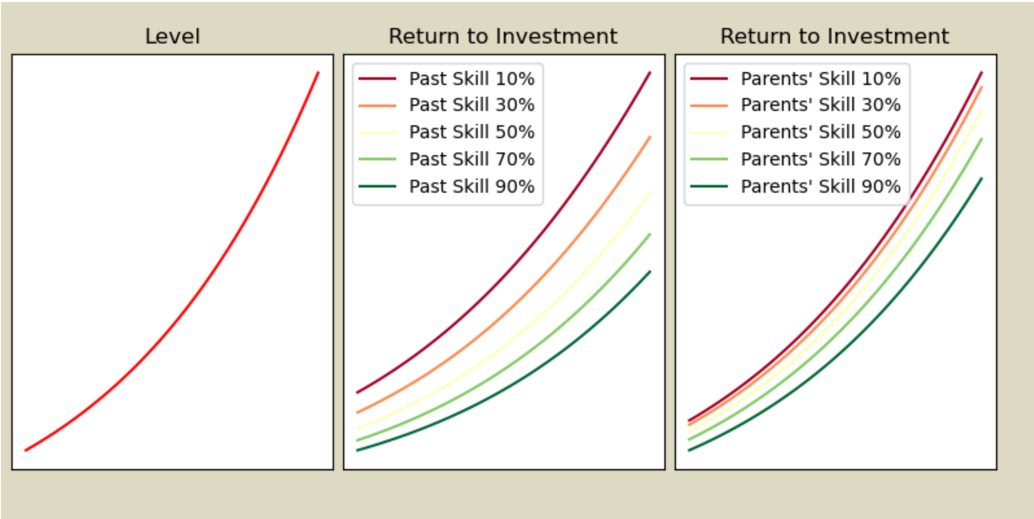
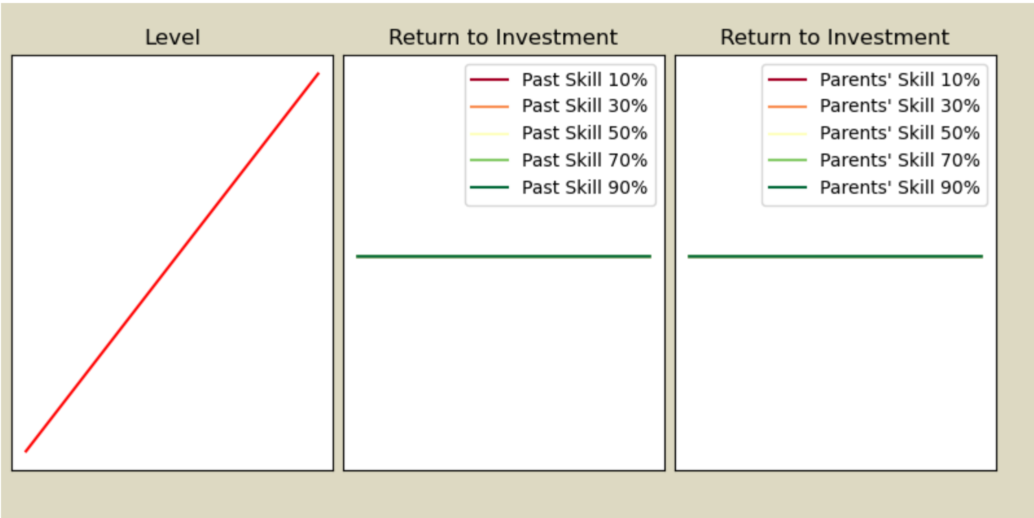
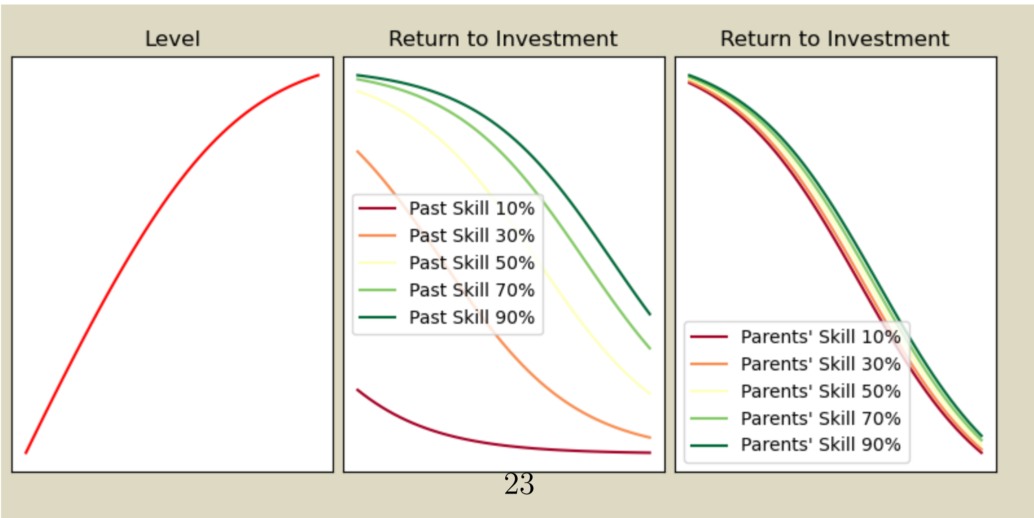


(a) Baseline: Substitutes ($\phi = 0.17$)



(b) Cobb-Douglas ($\phi = 0$)



(c) Complements ($\phi = -0.5$)

Figure 7: CES skill function with different substitution parameters.