Intergenerational Mobility Trends and Childhood Skill Formation

Oct 20st, 2022

Emre Enes Yavuz

Northwestern
Department of Economics

Rising Inequality → Intergenerational Mobility in Earnings

Through Childhood Skill Formation

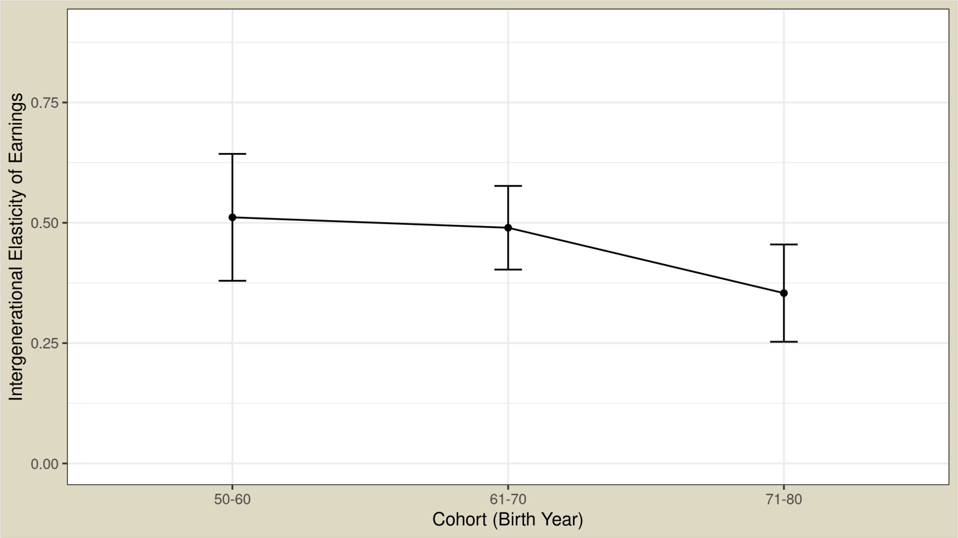
- Substantial rise inequality in lifetime earnings.

  % 23 increase in var of log lifetime earnings for men from 1957 to 1983. (Guvenen et. al. 2021)

- Gap in childhood skill investment getting wider (Corak 2013, Blanden et. el. 2022)
  - High-income parents spends more money and more time for children.

- The worry: Intergenerational mobility in earnings can go down.

Mobility trends seems flat over time!



Childhood Skill Formation Function can explain trends.

Childhood Skill Formation Function

It transforms investment (parental time and expenditures) to skills.

It is possible to get flat mobility even with rising inequality if...

- returns are really low for children of high-income families,
- while they are high for low-income families.

I estimate this function without any functional or distributional form assumptions.

Results

Childhood Skill Formation Function

Inputs are investment, current skill level and parents education.

- Investments are more productive for
  - low-skilled children and
  - high-educated parents but returns decreases faster.
- Uncertainty is more negatively skewed for high-educated parents.
- CES or Cobb-Douglas cannot capture these features.

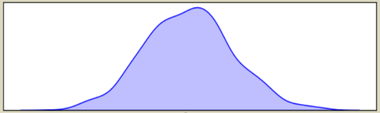
Literature

Estimation of Childhood Skill Formation Function

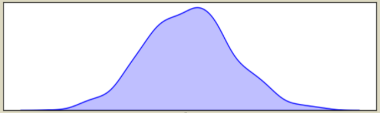
- Cunha, Heckman and Schennach (2010) ECTA
- Attanasio et. el. (2020) AER
- Agostinelli and Wiswall (2021) (Working Paper)

Inequality and Intergenerational Mobility

- Becker et. al. 2018 JPE

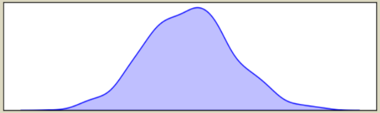


Parents' Earnings

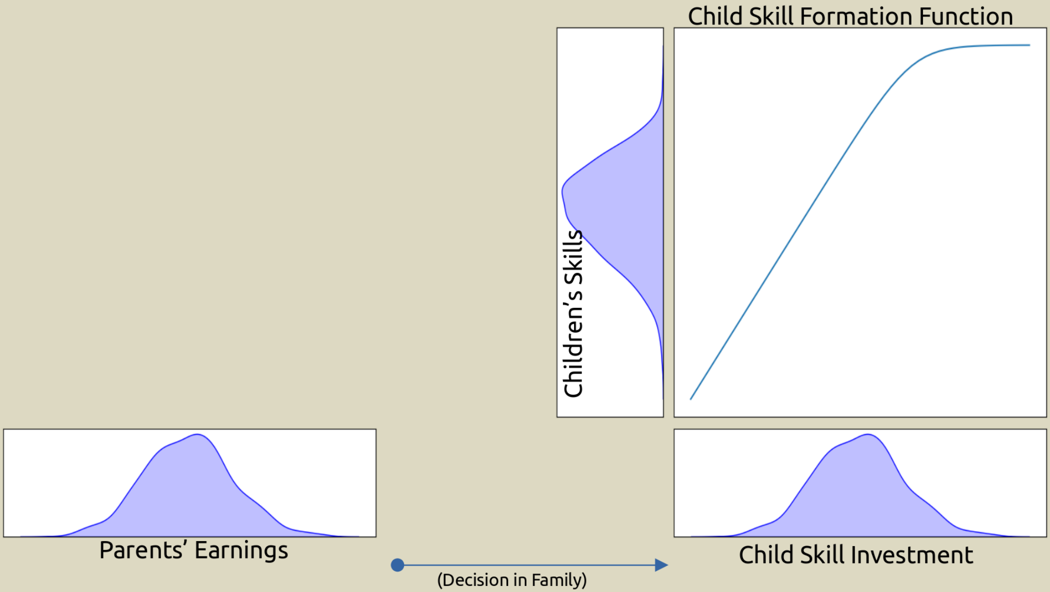


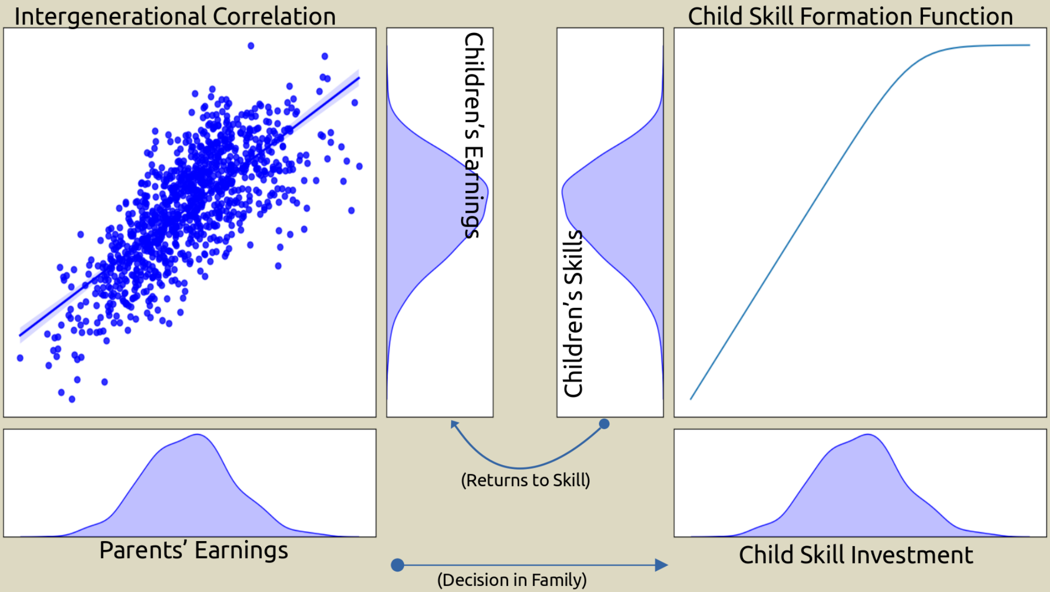
Parents’ Earnings

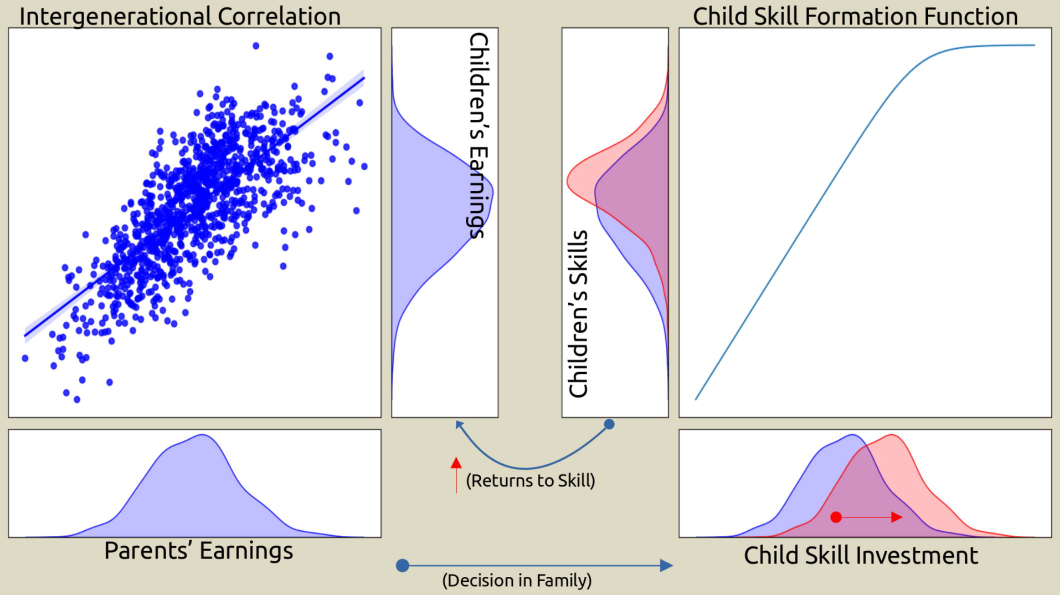
(Decision in Family)

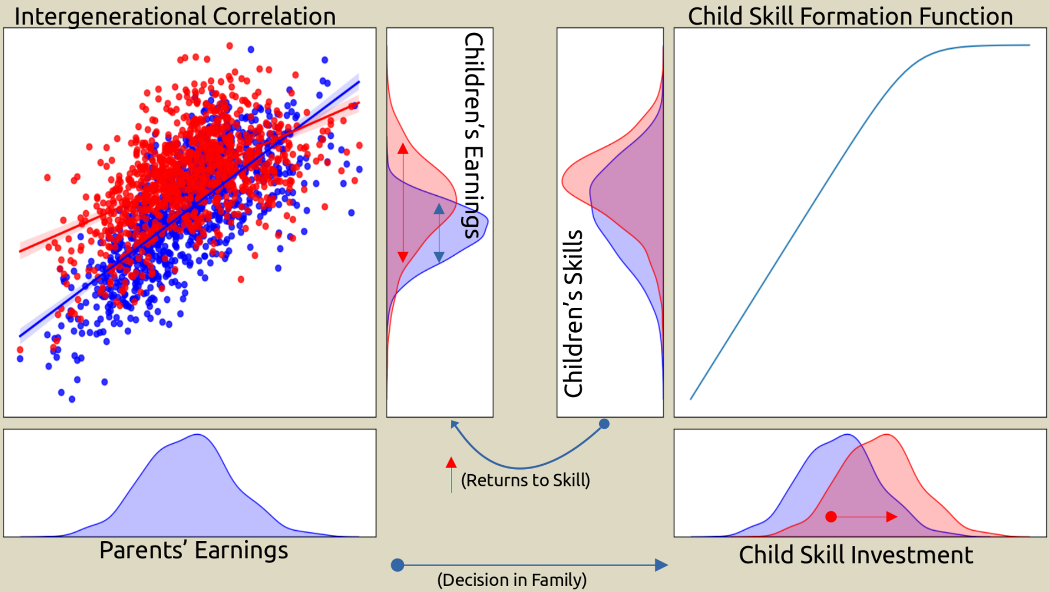


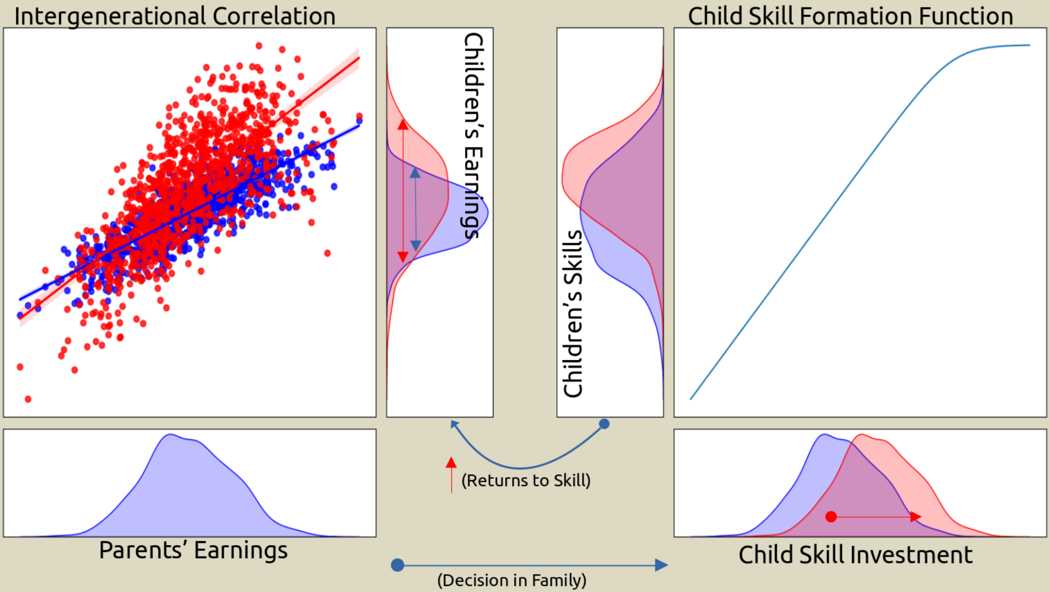
Child Skill Investment

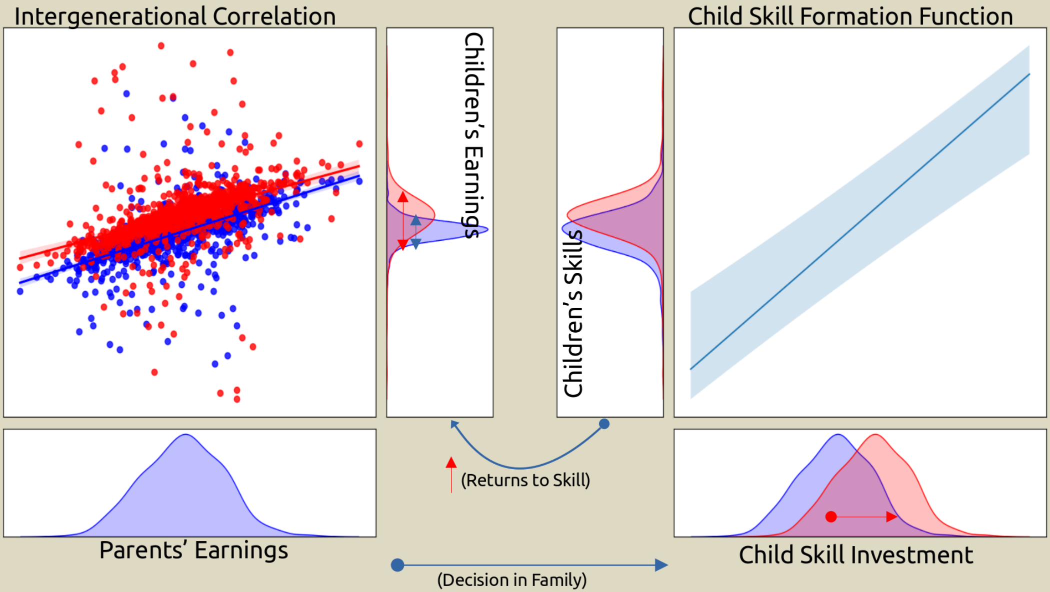

Child Skill Formation Function
Children's Skills
Parents' Earnings
Child Skill Investment
(Decision in Family)


Intergenerational Correlation
Children's Earnings
Children's Skills
Child Skill Formation Function
(Returns to Skill)
Parents' Earnings
Child Skill Investment
(Decision in Family)


Intergenerational Correlation
Children's Earnings
Children's Skills
Child Skill Formation Function
(Returns to Skill)
Parents' Earnings
Child Skill Investment
(Decision in Family)


Intergenerational Correlation
Children's Earnings
Children's Skills
Child Skill Formation Function
Parents' Earnings
(Returns to Skill)
Child Skill Investment
(Decision in Family)


Intergenerational Correlation
Child Skill Formation Function
Children's Earnings
Children's Skills
(Returns to Skill)
Parents' Earnings
Child Skill Investment
(Decision in Family)


Intergenerational Correlation
Children's Earnings
Child Skill Formation Function
Children's Skills
(Returns to Skill)
Parents' Earnings
Child Skill Investment
(Decision in Family)

Roadmap

- Estimate a flexible child skill formation function.
  - Approximate unknown functional form.
  - No distributional form assumption.
- Estimate a CES case with normal disturbance.
  - Typically used in the literature (Cunha et. al. 2010).
- Compare two case and comment on potential implications.
  - How returns differs across families.
  - Non-normal features of conditional skill distribution.

PSID Child Development Supplement

- Three waves: 1997, 2002, 2007.
  - Followed children up to age 12 at 1997 until age 18.
- Inputs:
  - Time diaries of children.
  - Child related expenditures.
- Output: Standardized tests to measure skills.
  - Three cognitive tests.
- Parents' Skill: Years of Education.

Child Skill Formation Function

Let θ_j be child skill level at age j,

$$\theta_{j+1} = F(\theta_j, I_j, \theta_P, u_j), \quad u_j \sim U[0,1].$$

Inputs: Past Skill Level (θ_j), Investment (I_j), Parents' Skill (θ_P)

CES with Normal Disturbance:

$$\theta_{j+1} = \left[\gamma_\theta \theta_j^\phi + \gamma_I I_j^\phi + (1 - \gamma_\theta - \gamma_I)\theta_P^\phi\right]^{\frac{1}{\phi}} exp(\varepsilon_j), \quad \varepsilon_j \sim \mathcal{N}(0, \sigma_\varepsilon).$$

Child Skill Formation Function

Investment, I_j:

$$I_j = h(m_j, t_j^{father}, t_j^{mother}).$$

Parents' Skill Level, θ_P:

$$\theta_P = g(\theta_{father}, \theta_{mother}),$$

Initial Skill Distribution

Let initial level of skills depend on parents' skill,

$$\theta_0 = F_0(age_0, \theta_P, u_0),$$

with $u_0 \sim U[0,1]$.

Last Period Skill to Years Of Education

- Binomial distribution for years of education.
- Its probability parameter given by,

$$p = \frac{\exp\left[f\left(\theta_T, \theta_P, age_T\right)\right]}{1 + \exp\left(f\left(\theta_T, \theta_P, age_T\right)\right)}.$$

Investment Policy Function

$$m_j = M(\theta_j, \theta_P, y_j, u_j^M),$$
$$t_j^k = T^k(\theta_j, \theta_P, y_j, u_{kj}^T) \quad \text{for} \quad k = father, mother,$$

where y_j is total household income and $u_j^M, u_{kj}^T \sim U[0,1]$.

- Policy function is not explicitly derived from a model.
- But, it can approximate one from a model.

Skill Measures

Probability of answering a question i in test t correct is given by,

$$Prob_{ti} = \frac{\exp\left(\alpha_t + \beta_t\theta - d_i\right)}{1 + \exp\left(\alpha_t + \beta_t\theta - d_i\right)}.$$

- Normalize one test as $\alpha_t = 0, \beta_t = 1$.

Parametrize Functions

Approximate skill formation function,

$$F(\theta_j, I_j, \theta_P, u_j) = \sum_{k=0}^{K} a_k(u_j)\varphi_k(\theta_j, I_j, \theta_P).$$

where,

- φ_k order k orthogonal polynomial,
- $a^k(u_j)$ is polynomial coefficient.
- Estimate for a grid $\{u^0, u^1, \ldots, u^L\}$ with quantile regression.

EM Algorithm

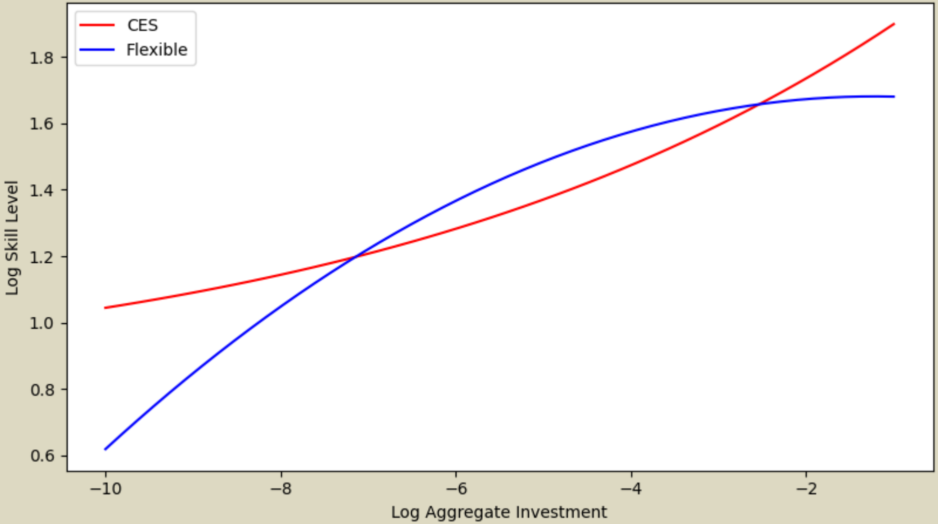
Start with a guess of parameter values;

- **E step:** Simulate skills using guessed parameters.
- **M step:** Update parameters using simulated skills.

Keep iterating EM Steps:

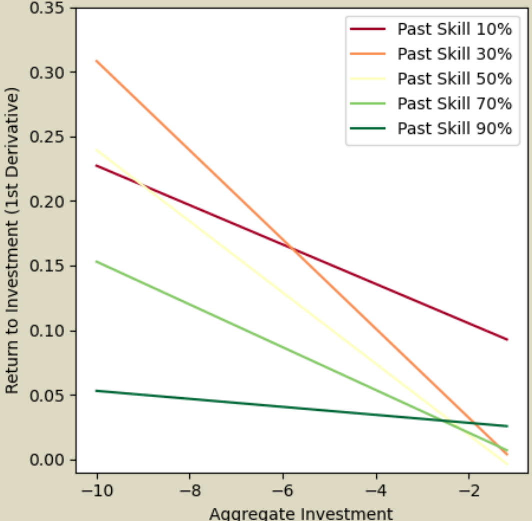
- Parameter estimates converge and fluctuate around true value.

Flexible vs. CES at Mean



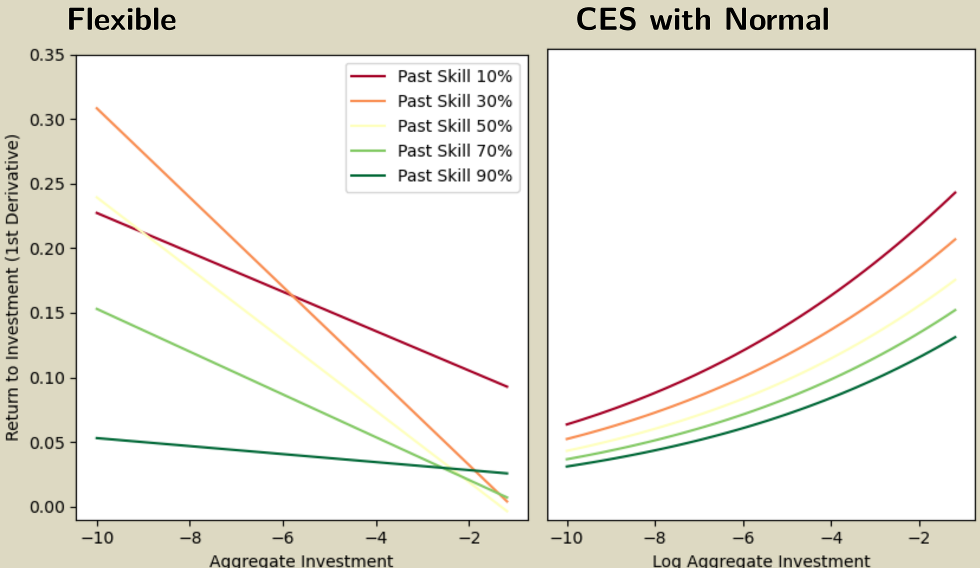
Current Skill Level and Investment are Substitutes

Flexible



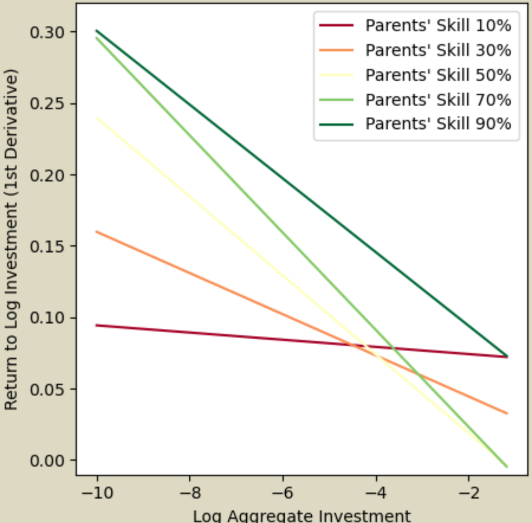
CES with Normal

Current Skill Level and Investment are Substitutes



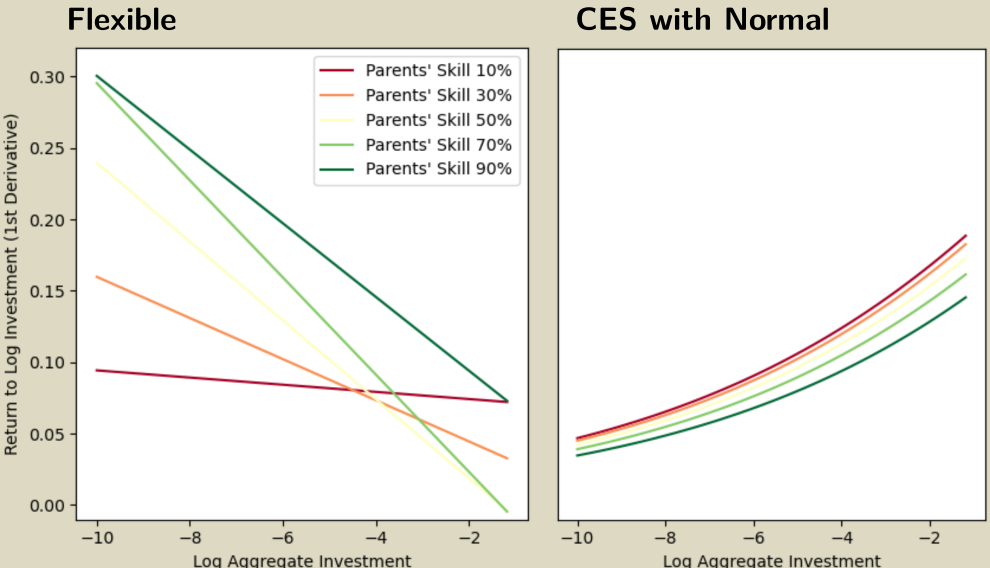
Parents' Education and Investment are Complements

Flexible

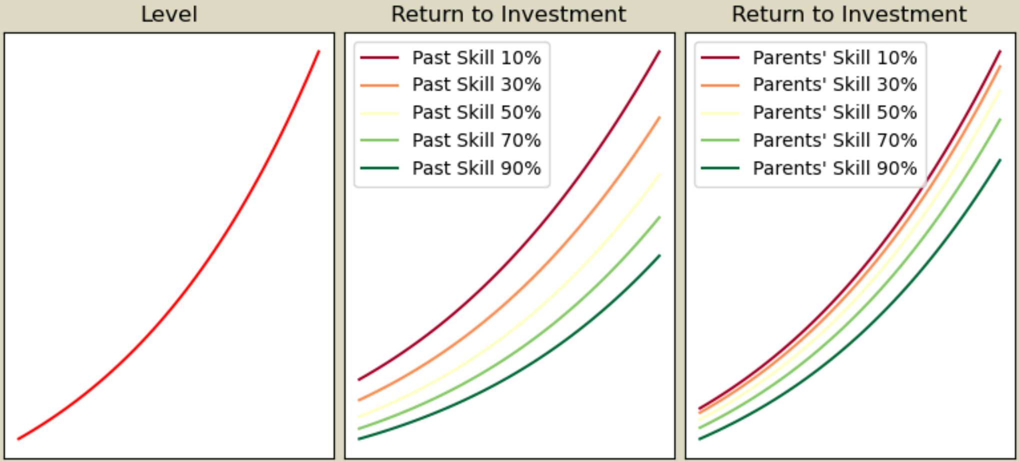


CES with Normal

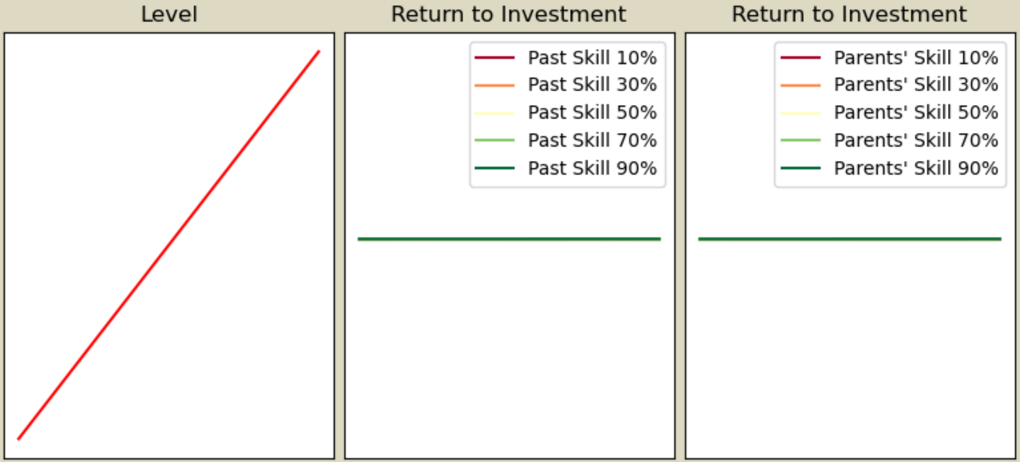
Parents' Education and Investment are Complements



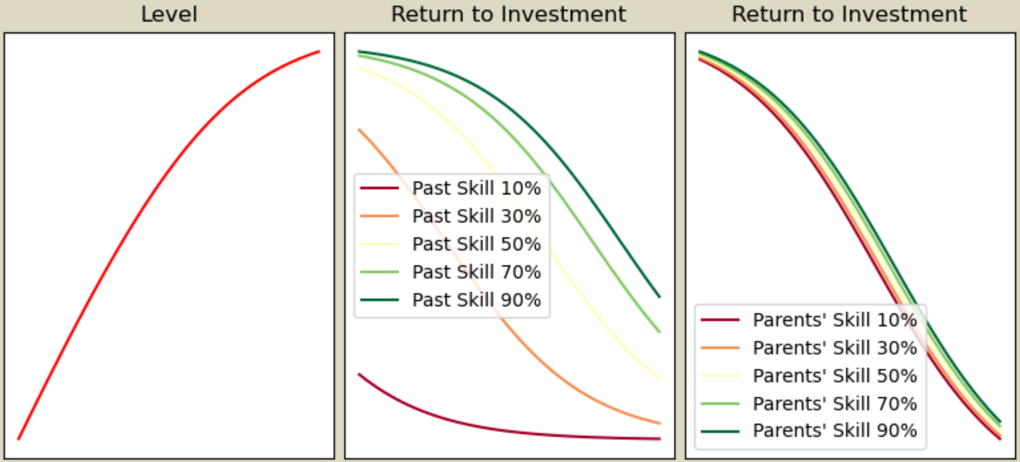
Baseline CES: Substitutes, $\phi = 0.18$



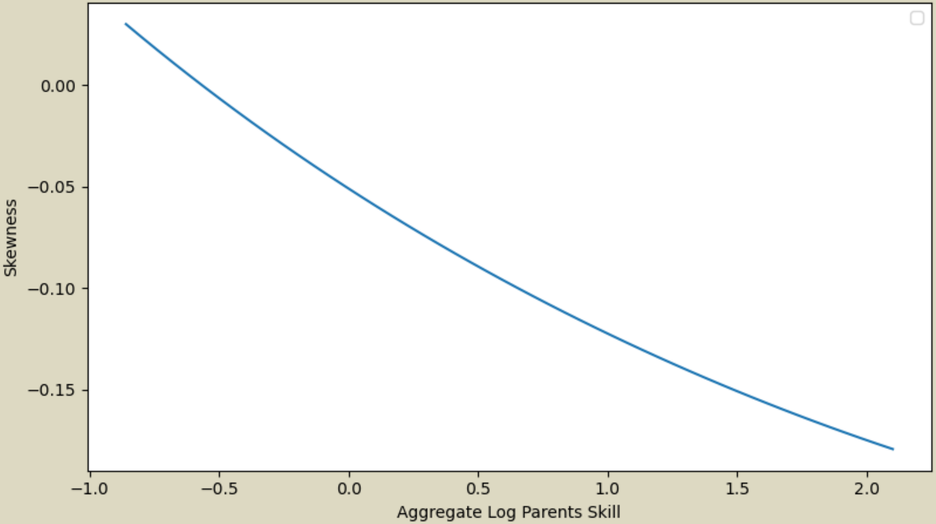
Cobb-Douglas: $\phi = 0$



Alternative CES: Complements, $\phi = -0.5$



More Negatively Skewed for More Educated Parents



Conclusion and Next Steps

- Estimated a flexible childhood skill formation function.
- Different features from standard CES with normal disturbance,
  - e.g. returns and skewness.
- These features can lead mechanisms for flat mobility with rising inequality.

Conclusion and Next Steps

Use a quantitative model to assess role of childhood skill formation.

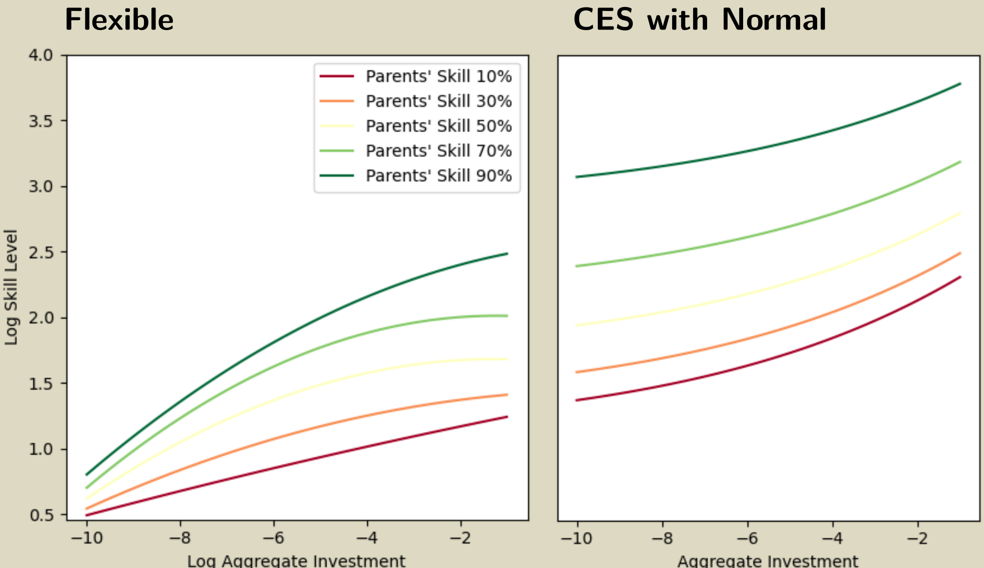
Ingredients:

- Lifecycle model with families consist of two parents and children.
- Decisons on saving, labor supply, skill investment, intervivos transfer.

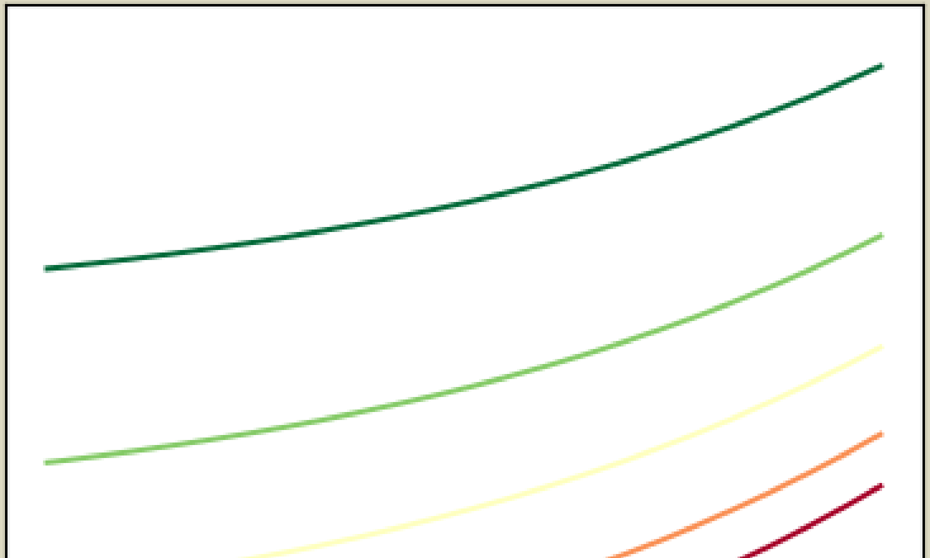
Main Exercise:

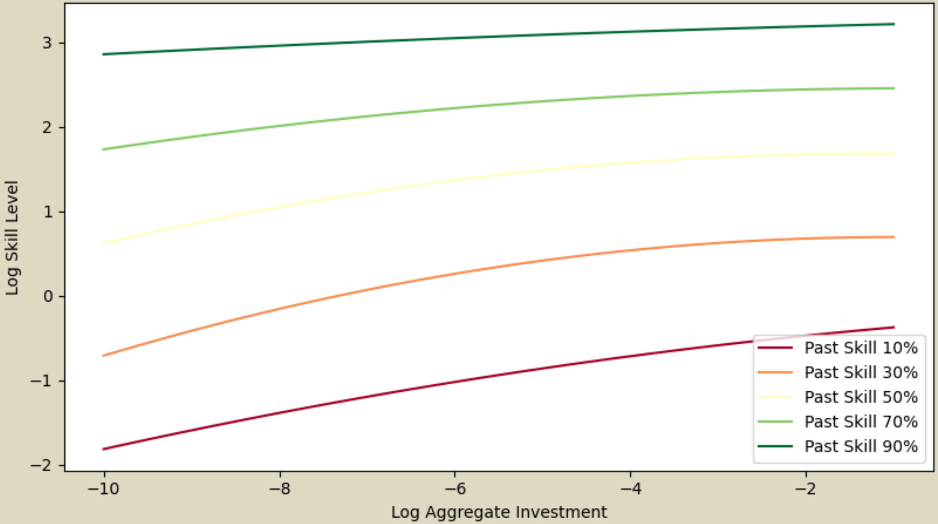
- Let inequality rise through exogenous rise in skill premium.
- Families will react and increase childhood skill investment.
- Look at implications for intergenerational mobility.

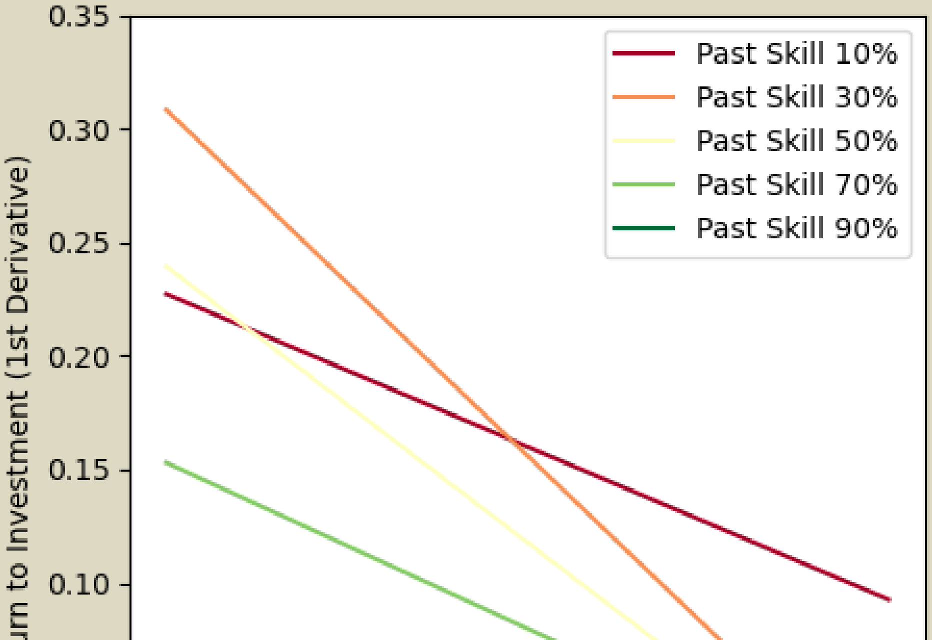
Returns Decrease Faster for More Educated Parents

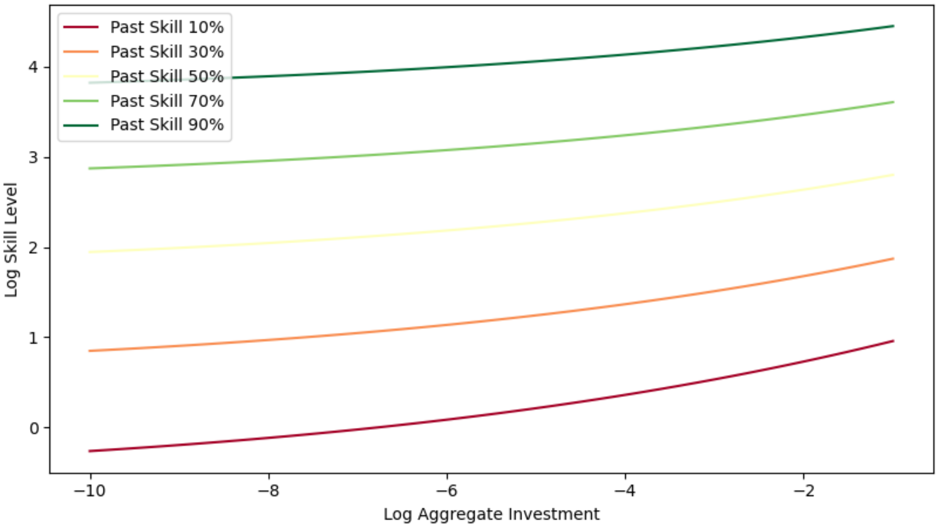


Returns are more Similar in CES

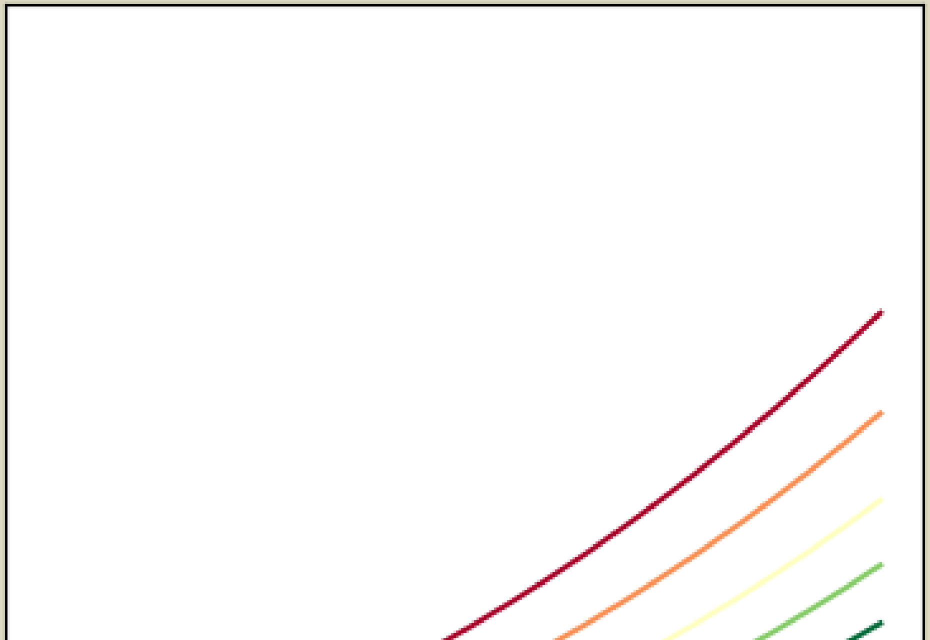



Log Skill Level
Log Aggregate Investment
3
2
1
0
−1
−2
−10
−8
−6
−4
−2
Past Skill 10%
Past Skill 30%
Past Skill 50%
Past Skill 70%
Past Skill 90%


0.35
0.30
0.25
0.20
0.15
0.10
urn to Investment (1st Derivative)
Past Skill 10%
Past Skill 30%
Past Skill 50%
Past Skill 70%
Past Skill 90%



Past Skill 10%
Past Skill 30%
Past Skill 50%
Past Skill 70%
Past Skill 90%
Log Skill Level
Log Aggregate Investment
4
3
2
1
0
−10
−8
−6
−4
−2



Moment Condition

- Coefficients can be estimated with quantile regression.
- Moment condition for quantile u^l,

$$\{a_0(u^l),\ldots,a_K(u^l)\} = \underset{\tilde{a}_0,\ldots,\tilde{a}_K}{\arg\min}\,\mathbb{E}\left[\rho_{u^l}\left(\theta_{j+1} - \sum_{k=0}^{K}\tilde{a}_k\varphi_k(\theta_j, I_j, \theta_P)\right)\right].$$

Moment Condition

- Coefficients can be estimated with quantile regression.
- Moment condition for quantile u^l,

$$a(u^l) = \arg\min_{\tilde{a}} \mathbb{E}\left[R_{u^l}(\theta, \tilde{a})\right].$$

- Can't minimize sample counterparts because θ is unobserved.

EM Algorithm

Apply law of total probability,

$$a(u^l) = \arg\min_{\tilde{a}} \mathbb{E}_Z \left[\mathbb{E}_{\theta|Z;\tilde{a}} \left[R_{u_l}(\theta, \tilde{a}) \right] \right].$$

- Inner $\mathbb{E}$ taken with densinty of skills conditional on observables.
- $f(\theta|Z;\tilde{a})$ can be derived with Bayes rule.
- Dual role of parameters $\tilde{a}$.

EM Algorithm

Solve with iteration, start with a_s,

$$a_{s+1} = \arg\min_{\tilde{a}} \mathbb{E}_Z \left[\mathbb{E}_{\theta|Z;a_s} \left[R_{u_l}(\theta, \tilde{a}) \right] \right].$$

- We can get sample of θ using $f(\theta|Z;a_s)$.
- Get M draws of θ for each observation.

EM Algorithm

Solve with iteration, start with $\hat{a}_s$,

$$\hat{a}_{s+1} = \arg\min_{\tilde{a}} \sum_{i=1}^{N} \sum_{m=1}^{M} R_{u_l}(\theta_{im}, \tilde{a}).$$

with $\theta_{im} \sim f(\theta | Z_i, \hat{a}_s)$

- Gives a Markovian sequence of parameter estimates, $\hat{a}_s$.
- It converges to be fluctuating around the true value. (Nielsen 2000, Arellano and Bonhomme 2016)
- Repeat iteration for a large number, S.
- Set final estimates as average of last $S/2$ iteration.

Example 1 – CES Case

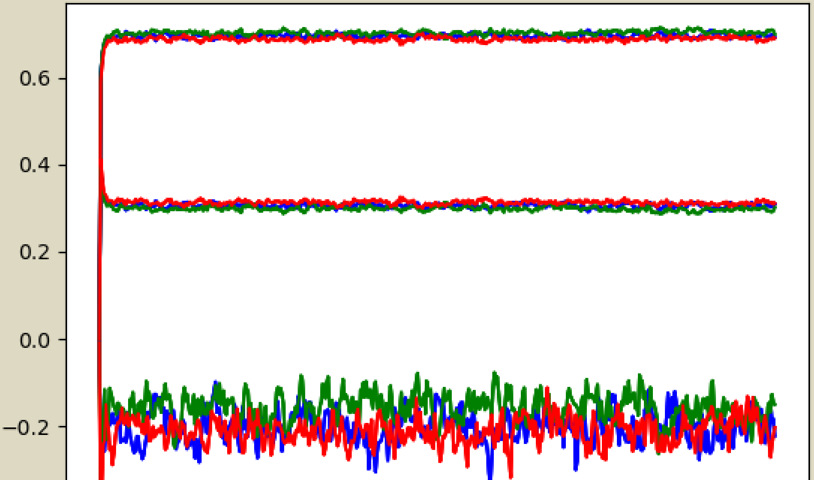
$$\theta_{j+1} = \left[\gamma_1 \theta_j^{\phi} + \gamma_2 I_j^{\phi}\right]^{\frac{1}{\phi}} \exp(\eta_{j+1}),$$

with $\theta_0 \sim \mathcal{N}(\sigma_0)$, $I_j \sim \mathcal{N}(\sigma_I)$, $\eta_{j+1} \sim \mathcal{N}(\sigma_\eta)$.

Let parameter values be;

$$\gamma_1 = 0.7, \quad \gamma_2 = 0.3, \quad \phi = -0.2.$$

Example 1 – CES Case



Conclusion and Next Steps

- Great Gatsby seems to not hold over time.
- Functional form of skill formation is key.
- I will estimate a flexible one using a new algorithm.

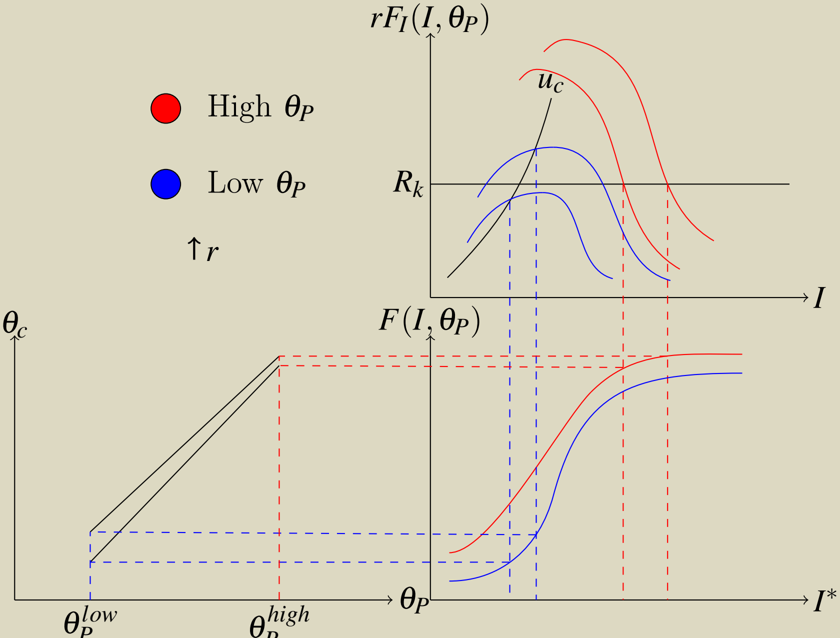
Next Steps

- Build a quantitative model with families with children.
- Use the estimated skill formation function.
- Let returns to skill increase so that inequality rises.
- Check implications for intergenerational mobility.

Solution

- For a constrained parent:

$$\delta \frac{\partial Y_C}{\partial I} = u'(c) \Longrightarrow rF_I(I^*, \theta_P) = \delta^{-1} u'(c).$$


High θ_P
Low θ_P
$\uparrow r$
$rF_I(I,\theta_P)$
u_c
R_k
I
$F(I,\theta_P)$
θ_c
θ_P
I^*
θ_P^{low}
θ_P^{high}

Some Results from Becker et al 2018

Version 1: Linear Returns and Quadratic Production Function

$$
\begin{aligned}
E_C =& r(\theta_C) = r\theta_C \\
\theta_C =& F\left(I, \theta_P\right) = \mu + \kappa I + \varphi I^2 + \theta I \theta_P + \delta \theta_P + \gamma \theta_P^2
\end{aligned}
$$

- $\frac{\partial \theta_C}{\partial \theta_P}$ is independent of r.
- Adding $I^2 \theta_P$ overturns the result.

Some Results from Becker et al 2018

Version 2: General Returns and Cobb-Douglas Prod. Function

$$E_C = g(\theta_C) = r\theta_C^{\sigma}$$
$$\theta_C = F(I, \theta_P) = I^{\alpha}\theta_P^{\beta}$$

- $\frac{\partial \log H_c}{\partial \log H_p}$ is independent of r, increasing in σ.

Conclusion

- Functional forms matters for implication of rising inequality.
- Nature of rising inequality matters.

Measurement Scale Parameters

$$Z_{jk} = \mu_{jk} + \alpha_{jk}\theta_{jk} + \varepsilon_{jk} \qquad Z_{j1} = \theta_{jk} + \varepsilon_{jk}.$$

- Assume we observe at least two measurements for two ages, $J \geq 2$ and $K \geq 2$.
- Scale parameters are identified by covariances, e.g for m,

$$\frac{cov\left(Z_{jk}, Z_{j+1,1}\right)}{cov\left(Z_{j1}, Z_{j+1,1}\right)} = \alpha_{jk}\frac{cov\left(\theta_j, \theta_{j+1,1}\right)}{cov\left(\theta_j, \theta_{j+1,1}\right)} = \alpha_{jk}.$$

- We can modify measurements,

$$\tilde{Z}_{jk} = \frac{Z_jk}{\alpha_{jk}} = \frac{\mu_{jk}}{\alpha_{jk}} + \theta_j + \frac{\varepsilon_{jk}}{\alpha_{jk}}.$$

Identification of Joint Distribution

$$\Lambda \equiv \left(\{\theta_j, m_j, t_{1j}, t_{2j}, y_j\}_{j=0}^{J}, \theta_{P1}, \theta_{P2} \right).$$

Joint distribution of Λ is identified,

- If we have two measures with error for each variable.

$$Z_1 = \Lambda + \varepsilon_1$$
$$Z_2 = M + \Lambda + \varepsilon_2$$

- If measured without error set both to that measurement.

Identification of Joint Distribution

- If $\mathbb{E}[\varepsilon_1|\Lambda,\varepsilon_2] = 0$ and $\varepsilon_2 \perp\!\!\!\perp \Lambda$, p.d.f. of Λ is given by,

$$p_\Lambda(\Lambda) = (2\pi)^{-L} \int e^{-i\chi\cdot\Lambda} \exp\left(\int_0^{\chi} \frac{E\left[iZ_1 e^{i\zeta\cdot Z_2}\right]}{E\left[e^{i\zeta\cdot Z_2}\right]} \cdot d\zeta \right) d\chi.$$

(Result follows from Cunha, Heckman and Schennach (2010))

Proof:

- Plug measurements and apply assumptions.
- This gives Fourier transformation of characteristic function.
- That is equal to density.

Child Skill Formation Function

Child skill formation function (and others) can be described by joint distribution.

$$u_j \equiv \Pr\left[\theta_{j+1} \leq \bar{\theta} \mid \theta_j, I_j, \theta_P\right] \equiv G\left(\bar{\theta} \mid \theta_j, I_j, \theta_P\right).$$

where G is conditional c.d.f. can be obtained from joint p.d.f.

$$\theta_{j+1} = F(\theta_j, I_j, \theta_P, u_j) = G^{-1}\left(u_j \mid \theta_j, I_j, \theta_P\right).$$

Conclusion

$F(\cdot)$ is a conditional quantile function.

A Model of Intergenerational Mobility

- Based on Becker et al 2018.
- Two periods: Childhood, Adulthood.
- Parent decides on child skill investment and transfers.

Parent's Problem

$$V(Y_P) = \max_{c,b_C,I} u(c) + \delta Y_C$$

$$\text{s.t.} \quad c + \frac{b_C}{R_k} + I = Y_p$$

- Income sonsists of earnings and transfer,

$$Y_C = E_C + b_C.$$

- Earning is function of skill (θ_C),

$$E_C = r\theta_C.$$

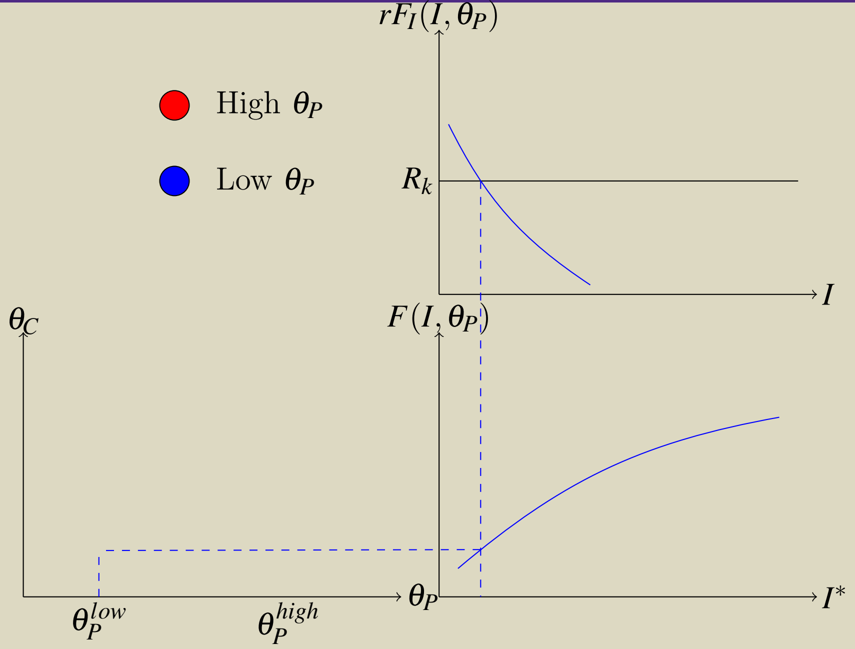
- Skill is function of investment and parent's skill,

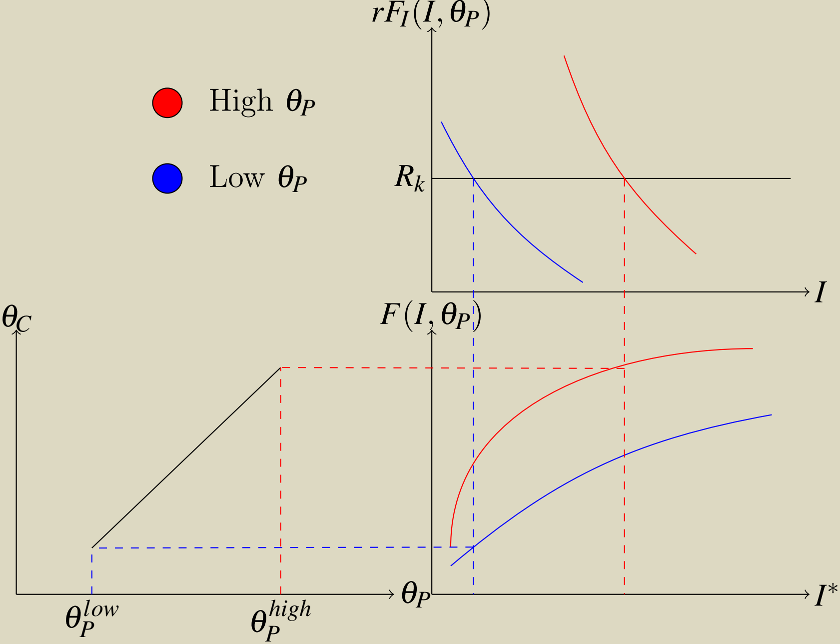
$$\theta_C = F(I, \theta_P).$$

Solution

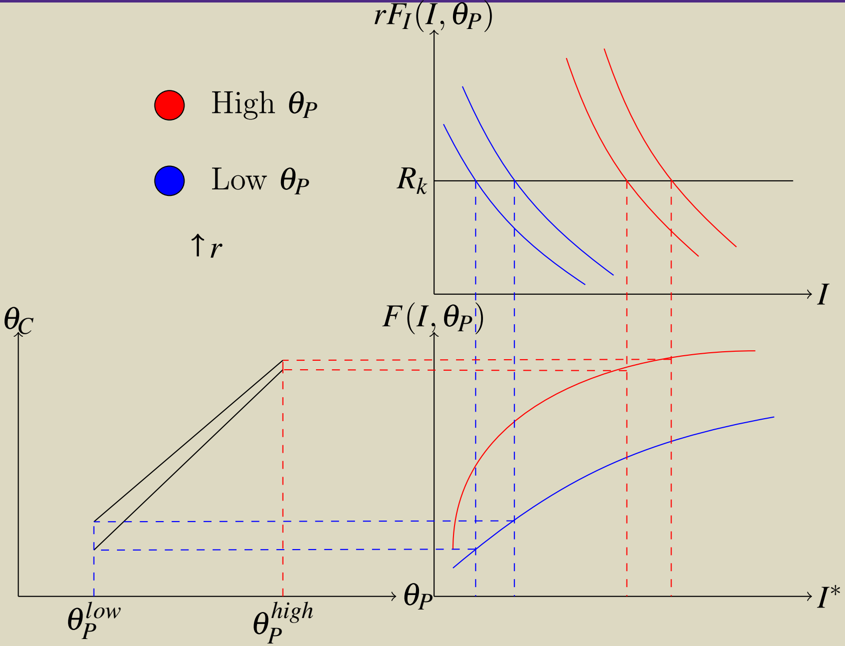
- For an unconstrained parent:

$$\frac{\partial Y_C}{\partial I} = R_k \quad \Longrightarrow \quad rF_I(I^*, \theta_P) = R_k$$

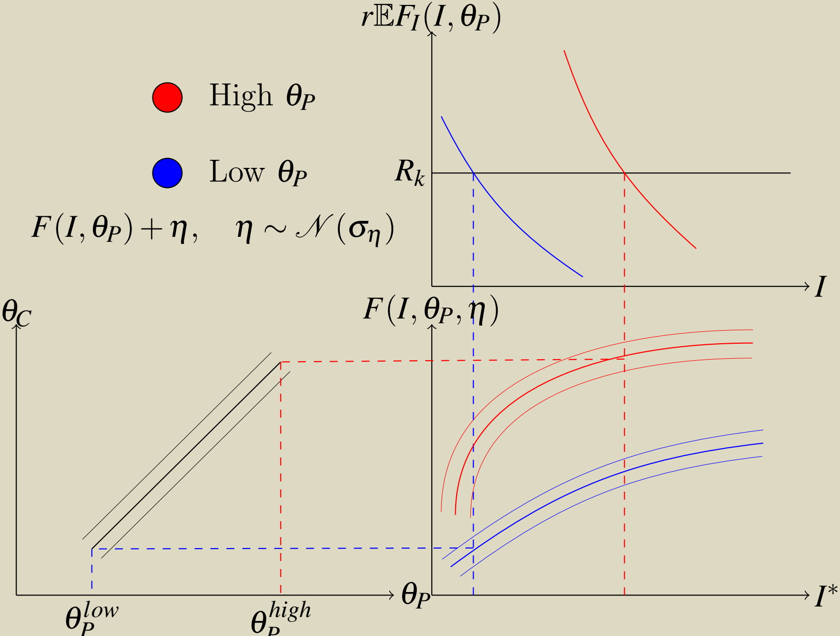

High θ_P
Low θ_P
$rF_I(I, \theta_P)$
R_k
I
$F(I, \theta_P)$
θ_C
θ_P
I^*
θ_P^{low}
θ_P^{high}

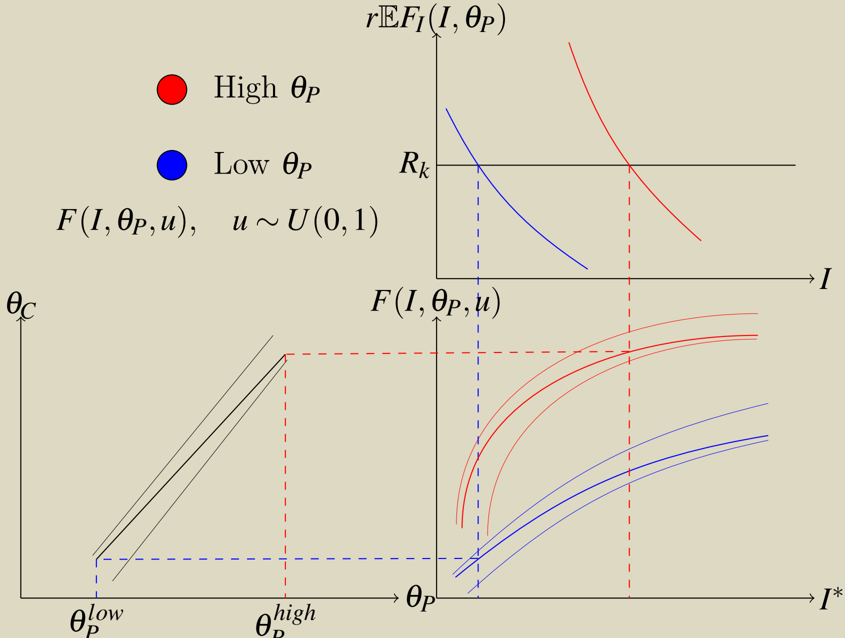


High θ_P
Low θ_P
$rF_I(I, \theta_P)$
R_k
I
$F(I, \theta_P)$
I^*
θ_C
θ_P
θ_P^{low}
θ_P^{high}



$rF_I(I, \theta_P)$
High θ_P
Low θ_P
$\uparrow r$
R_k
I
$F(I, \theta_P)$
θ_C
θ_P
θ_P^{low}
θ_P^{high}
I^*


$r\mathbb{E}F_I(I,\theta_P)$
High θ_P
Low θ_P
R_k
$F(I,\theta_P)+\eta, \quad \eta \sim \mathcal{N}(\sigma_\eta)$
I
θ_C
$F(I,\theta_P,\eta)$
θ_P
I^*
θ_P^{low}
θ_P^{high}



$r\mathbb{E}F_I(I,\theta_P)$
High θ_P
Low θ_P
R_k
$F(I,\theta_P,u), \quad u \sim U(0,1)$
I
θ_C
$F(I,\theta_P,u)$
θ_P
I^*
θ_P^{low}
θ_P^{high}

Identification of Joint Distribution

$$\Lambda \equiv \left(\left\{ \theta_j, m_j, t_j^{father}, t_j^{mother}, y_j \right\}_{j=0}^{J}, \theta_{father}, \theta_{mother} \right).$$

- If we have two measurements for two ages for each variable.
- The joint distribution of Λ is identified.

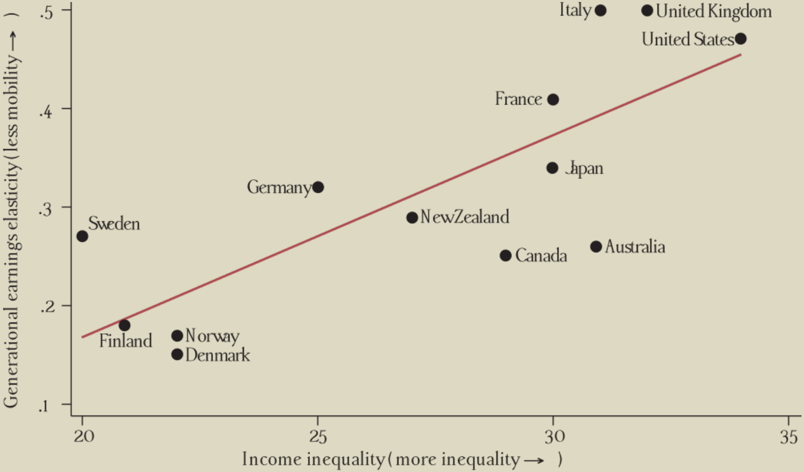
Measurements

- Normalize one of the measurements, say $k = 1$, for all ages j,

$$\mu_{jk} = 0 \quad \text{and} \quad \alpha_{jk} = 1$$
$$Z_{j1} = \theta_j + \varepsilon_{j1}$$

- This normalizes measurement unit of skill.
- Also, assumes that this measurement is age invariant.
  - Two children of two different ages with same skill level have same expected measurement.

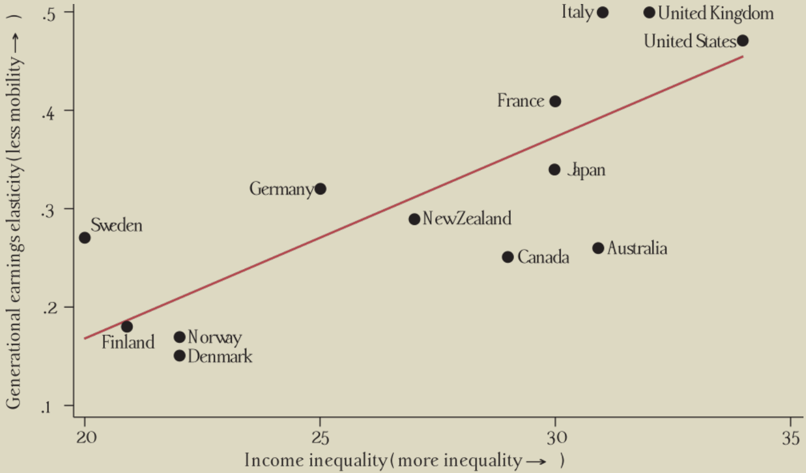
Great Gatsby Curve



Source: Corak (2013) and OECD.

Great Gatsby Curve

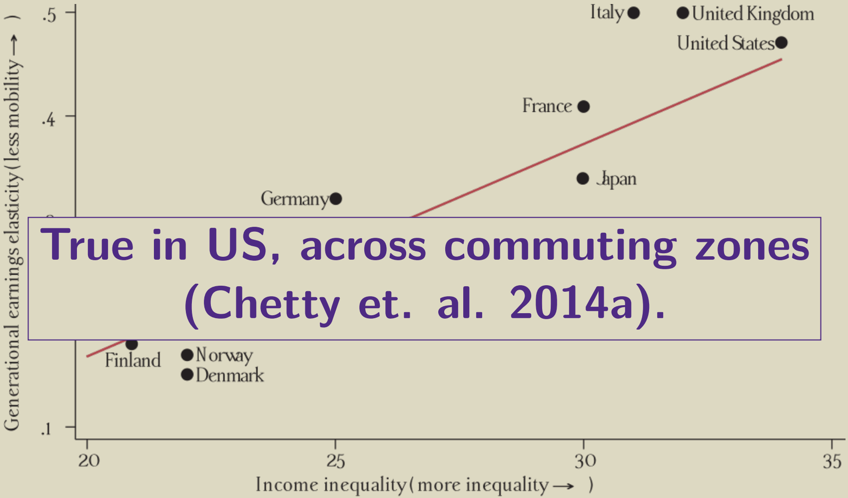
More inequality means more correlation between children's and parents' earnings.



Source: Corak (2013) and OECD.

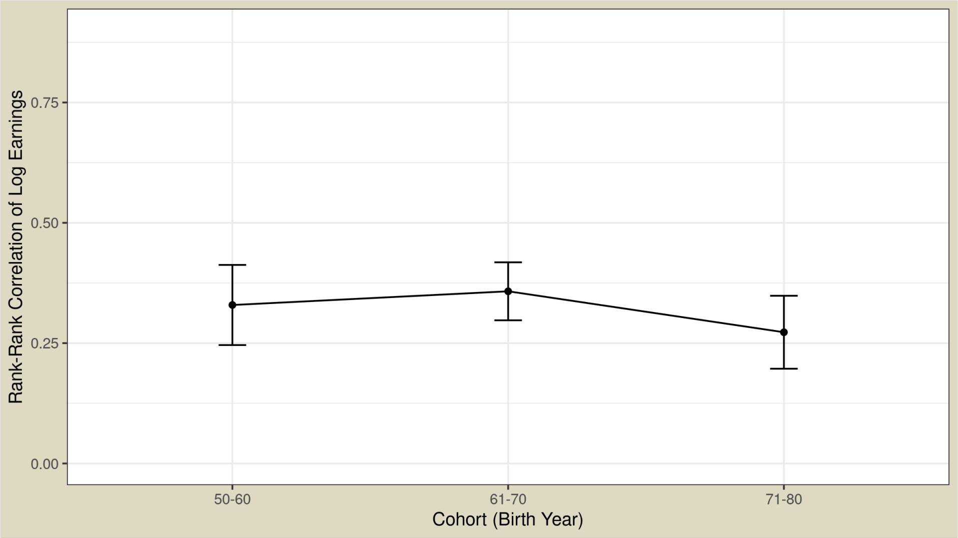
Great Gatsby Curve

More inequality means more correlation between children's and parents' earnings.



True in US, across commuting zones (Chetty et. al. 2014a).

No Upward Trend in Intergenerational Rank-Rank Correlation



Estimate Trend

in Intergenerational Elasticity of Earnings

$$\ln y_{ic}^{child} = \alpha_c + \beta_c \ln y_{ic}^{parent}$$

- y_{ic} is approximated by average over ages around 40.
  - As in Mazumder (2016).
- Group cohorts in 10 years in PSID.