Engel's Treadmill

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Abstract

Modern economic growth is characterized by constant growth in income per capita along with secular changes in the composition of the economy. We develop an endogenous growth model with directed technical change across sectors where these two facts emerge endogenously in equilibrium. The direction of innovation is determined by the market size, which evolves endogenously due to demand nonhomotheticities across sectors. Along the aggregate balanced growth path, there is perpetual unbalanced growth across sectors due to the two-way interaction between increasing income and directed technical change. As income grows, demand shifts to more income elastic sectors, triggering more innovation in those sectors, which further increases income, and so on. We refer to this perpetual process as "Engel's Treadmill." The model predicts that, along the balanced growth path, the relative price of more income-elastic sectors declines at a faster rate. Using US PCE price data for 138 sectors from 1959 through 2020, we provide evidence consistent with this prediction. We also show that, consistent with our theory, high income-elastic sectors experience higher innovation rate and job creation from entrants.

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1 Introduction

Modern economic growth has brought unprecedented sustained increases in the standard of living to the Western world (DeLong, 2000; Crafts and O'Rourke, 2014). As pointed out in Kuznets' Nobel address (Kuznets, 1973) and later on formalized by endogenous growth theory, technological progress has played a critical role in generating this sustained growth in income per capita. Remarkably, this sustained aggregate growth has gone hand-in-hand with uneven growth across sectors of the economy. Figure 14 illustrates these points for the US over the 1899-2007 period. Figure 11a shows that aggregate growth per capita is well approximated by a constant growth rate. By contrast, Figure 11b shows that the relative contribution of different sectors to aggregate output is considerably heterogeneous over time. Some sectors have been shrinking, e.g., transportation services and textiles, while the contribution has been hump-shaped for machinery and communication services, and increasing for financial and insurance services.[1]

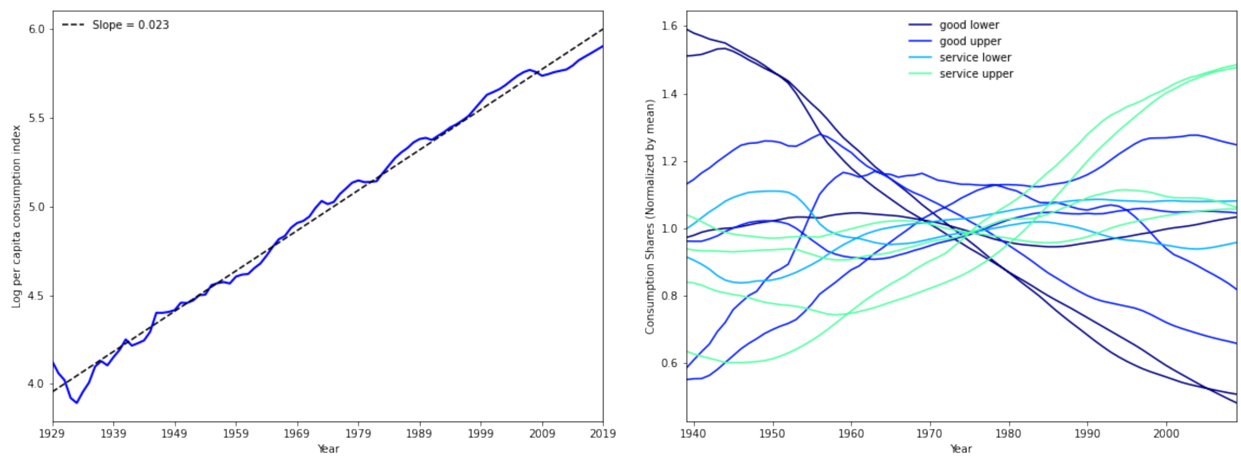
In this paper, we present a theory that jointly rationalizes the steady increase in aggregate income per capita and the uneven growth patterns experienced by different sectors over time. We develop a model of endogenous growth with directed technical change across a continuum of sectors. Our theory emphasizes the role of demand nonhomotheticities in determining the effective market size of different sectors. Due to these differences in market size, innovation shifts towards more income elastic sectors as income grows. We show that, in equilibrium, the two-way interactions between *(i)* income growth brought by technological progress and *(ii)* changes in the direction of innovation brought by income growth are consistent with an aggregate balanced growth path and the rise and fall in the relative importance of different sectors over time.

The core of our mechanism builds on embedding Engel's Law (i.e., the idea that the composition of households' consumption baskets moves towards more income-elastic goods as they become richer) into a directed technical change framework (Acemoglu, 2002; Gancia and Zilibotti, 2005). Taken together, these two elements enable demand to "pull" innovation across sectors (Schmookler, 1966).[2] Much like in Sisyphus' Greek parable in which he was punished to eternally roll a boulder up a steep hill only to start over again before reaching the top, households pursue the consumption of a luxury good only to eventually deem it a necessity and shift their expenditures to a pursuit of other more luxurious goods. That is, consumers in our economy change

[1]Figure 12 in appendix F shows that these patterns extend to a finer disaggregation of the US economy to twenty sectors.

[2]Pasinetti (1981) is, to the best of our knowledge, the first to argue that nonhomotheticities in demand should interact with the demand-pull effect of innovation. In this sense, our paper provides a neoclassical rendition of his insight.

Figure 1:
US Aggregate Consumption Growth and Nomalized Consumption Shares, 1929-2019



their concept of what necessities are as they become richer, shifting their demand away from what was once considered a luxury, but now a necessity, towards other more luxurious goods. The income-growth induced conversion of luxury goods into necessities implies that innovation is continuously redirected toward other luxury goods spawning yet further income growth in a perpetual process that we coin "Engel's treadmill." As a result, the growth process, despite appearing stable in the aggregate, is intrinsically heterogeneous over time.

We provide a sharp characterization of the sectoral evolution of innovation, price and market size along the balanced growth path. Our model implies that prices should fall relatively faster in more income-elastic sectors and that innovation inputs and outputs should grow faster in more income-elastic sectors. We use disaggregated price data for 150 product categories of the US Personal Consumption Expenditures (PCE) from 1958 through 2017 and patent data from the US Patent Office from 1974 through 2017 that corroborate these predictions of the model. Preliminary results for R&D expenditures are also supportive of the model prediction. We estimate sectoral income elasticities from the Consumer Expenditure Survey (CE) using the method proposed by Aguiar and Bils (2015), which leverages cross-sectional household variation in income to identify income elasticities. Then, armed with these income elasticities, we regress sectoral price and patent growth on income elasticities and find a statistically significant relationship of the expected sign.

Before proceeding, it is worth reiterating that the model presented here emphasizes the role of domestic demand in driving endogenous technological changes. We abstract from other potentially important forces driving sectoral growth and innovation, such as trade or technology shocks. We do not deem these other forces inconsequential and, in particular, we believe that extending our framework to an open-economy world would yield interesting results. However, we want to point out that an explanation based on technologies linked to different sectors exogenously appearing and being adopted is at odds with the following observation. Despite the fact that different sets of countries have gone through their development process at very different points in time, the ordering in the rise and fall of the various sectors across countries is highly correlated. For example consider on the one hand, early birds in modern growth, such as the United Kingdom, France and the US, and the late bloomers, Japan, South Korea and Taiwan, on the other hand who entered modern growth later on. We divide these economies into 12 sectors and compute the order in which these sectors have peaked in each country. Figure 2 shows the ranking for the US on the horizntal axis and for other countries on the vertical axis: the ordering of the peaks is highly correlated (0.82) across countries.[3] This suggests a strong role for domestic demand in determining the evolution of sectoral shares.

Detailed Outline of the Paper Section 2 presents our baseline model. The model is similar to a standard expanding variety endogenous growth model (e.g., as in Acemoglu, 2009). Its main departure is that it features a two-dimensional nested product space, representing goods across and within sectors. Nonhomothetic CES preferences are used to model preferences across sectors. Consistent with the vast literature in structural change, we assume that sectors are gross complements.[4] Within sectors, preferences are homothetic and goods are gross substitutes. Innovation is of the expanding variety type (horizontal innovation) à la Romer (1990). In our baseline model, labor is constant over time and it can be used to innovate or produce goods. Long-run growth is ensured thanks to knowledge spillovers.[5]

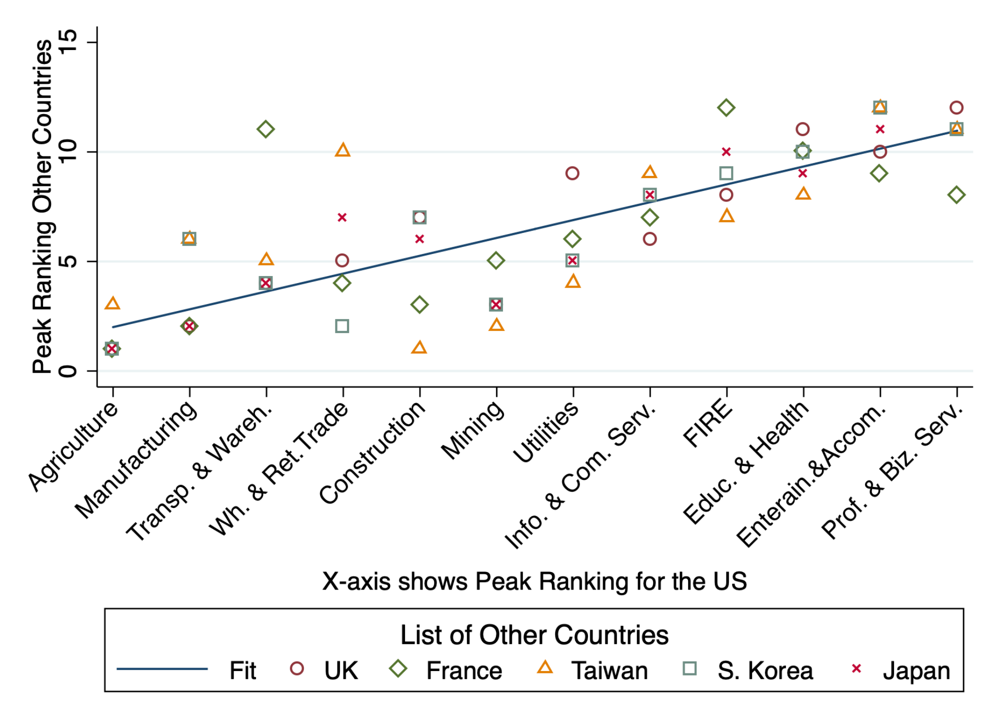
The model features an aggregate balanced growth path (BGP) nearly identical to an off-the-shelf one-dimensional product space version without nonhomotheticities. Moreover, the BGP is stable in the sense that an economy with an arbitrary initial distribution of sectoral products converges to the distribution that generates a BGP. Despite the parsimony of the model in the aggre-

[3] If we collapse the twelve sectors of this analysis to nine, we can extend our analysis to all OECD countries (joining prior to year 2000). In this case the correlation in rankings is 0.64. Matsuyama (2002) makes a similar argument on the importance of the domestic market in driving sectoral take-offs along the development path.

[4] See among others Buera and Kaboski (2009); Herrendorf et al. (2014); Boppart (2014); Comin et al. (2021).

[5] We show in different extensions that our main results go through in a semi-endogenous rendition of the model based in Jones (1995) and a also in Schumpeterian (vertical innovation) framework.

Figure 2: Ranking of Sectoral Peaks, US and other Advanced Economies



Notes: Sectoral shares computed using World KLEMS data for all countries.

gate, we show that sectors feature unbalanced growth at any point in time *along* the BGP. Sectors take-off sequentially as measured by their share in aggregate output according to their income elasticity ranking, featuring the "flying geese" pattern discussed in Matsuyama (2002). Sectors eventually decline in significance as they become necessities. We show that the expenditure share in each sector features a hump-shaped pattern with a rise and a subsequent fall.

We characterize the sectoral evolution of prices and innovation along the BGP. Our model predicts that innovation growth, both measured as number of ideas/new products (output) or R&D spending (input), is increasing in the income elasticity of the sector. We show that this also implies that sectoral prices decline faster in more income elastic sectors. We provide evidence consistent with these predictions in Section 3, which is to be discussed next. We also show in the Schumpeterian growth rendition of the model that product turnover is faster in more income elastic sectors (we plan to test for this too).

We compare the allocations in the decentralized equilibrium to the Social Planner's. An externality arises from the nonhomothetic demand when the decentralized equilibrium BGP is compared to the Pareto optimal growth path. An increase in a household's expenditures will, due to the directed technological change, also change the composition of the realized basket of goods in a favorable manner to them. Since this is unaccounted for by the households, equilibrium aggregate growth is lower than what would be optimal. A simple R&D subsidy can restore optimality–as in the simple one-sector expanding variety model.

Section 3 presents evidence from the US consistent with the key predictions of the model. First, we show that the US sectoral peak ordering of the twelve sectors presented in Figure 2 correlate very strongly (0.79) with the income elasticity of these sectors–suggesting an important role of the US market size. Then, we present evidence consistent with the mechanism of the model. First, we use disaggregated price data over 150 product categories from the PCE. We use the finest matching between PCE and the consumer expenditure survey categories and estimate the income elasticity of each industry following the Aguiar and Bils (2015) methodology. We show that there is a negative correlation between the price growth between 1958 and 2017 and income elasticity. The result is also robust to adding broad product category fixed effects (e.g., goods vs. services or durables vs. nondurables). Second, we show that innovation output across sectors as measured by patents has grown faster in more income elastic sectors. Using patent classes matched to different 3-digit NAICS industries, we show that there is positive correlation between industry patent growth between 1974 and 2015 and our estimated sectoral income elasticity. This correlation persists when we add broad industry (1-digit) fixed effects.

Section **??** presents two extensions of the baseline model. First, we present a Schumpeterian growth rendition of our model. We first present a version in which entrants and incumbents have access to different innovation technology. We then show that our baseline model's predictions are also obtained in a modified version of the model in which patents are only instantaneous. This formulation also goes through in the Schumpeterian model. Finally, in the context of instantaneous patents, Section 2.8 extends the baseline model to the case of gross substitutes across sectors. Having goods be substitutes across sectors changes the behavior of the nonhomothetic CES preferences, and it also implies that sectors will eventually be substituted away from and decline to zero. Unsurprisingly, the sectoral dynamics differ greatly when sectors are gross substitutes. Intermediate products within sectors face shrinking demand and are gradually retired from production. Thus, in this version of the model ideas endogenously become obsolete.

Related Literature Our paper relates to several vast and rich literatures. First, the core result of our paper on endogenously-generated, demand-induced take-offs of different sectors along the growth path relates to a classical tradition in growth and development, notably Nurkse (1963). As in our model, this literature emphasized the role of market size and demand complementarities across sectors as key to understand the development process. Our model generates a sequential take-off of sectors, also known as the "flying-geese" pattern. This result is similar to Matsuyama (2002) (see also Foellmi and Zweimuller, 2006). However, the economic mechanism in these papers necessarily relies on a trickle-down effect from rich to poor consumers (combined with learning-by-doing or an innovation decision). By contrast, we assume a representative consumer and obtain the flying-geese result solely from an endogenous change in the direction of innovation over time.

The paper also relates to theories that have studied structural change with balanced growth. Following the seminal work of Kongsamut et al. (2001), most studies in this literature have taken sectoral productivity growth as exogenous.[6] A notable exception is Foellmi and Zweimüller (2008). In the last section of their paper, they combine nonhomothetic, hierarchical preferences with an expanding variety model to generate a BGP. In their setup, however, there is no margin for endogenous changes in the direction of innovation. Innovation only happens at the extensive margin, i.e., by adding more products to a one-dimensional product hierarchy. By contrast, we have two margins for innovation (within and across sectors). This allows us to study the endogenous evolution of the direction of innovation across sectors, which cannot be done in their model. Another important technical difference is that we have a unique BGP.[7] In two preceding working papers, Weiss and Boppart (2013) and Comin et al. (2016) also develop models of directed technological change under nonhomothetic preferences.[8] However, they restrict their attention to models where nonhomotheticities apply to two or three sectors, respectively. As a result, the perpetual two-way interaction between aggregate growth and demand-directed innovation generating "Engel's treadmill" emphasized in this paper is absent in these papers.[9] Both papers provide evidence that TFP grows faster in more income-elastic sectors in the context of broad sectors of the economy. Comin et al. also provide evidence on patenting and R&D growing faster for the service

[6]See Ngai and Pissarides (2007) and Boppart (2014) for other theories consistent with BGP and exogenous sectoral productivity growth.

[7]Herrendorf and Valentinyi (2015) propose a theory to rationalize reallocation from goods to services along a BGP based on the assumption that the returns to variety are larger in the goods sector. Their theory does not make use of nonhomothetic preferences.

[8]See also Acemoglu et al. (2012) and Aghion et al. (2016) for a different application of demand affecting the direction of innovation in the context of clean and dirty technologies.

[9]Weiss and Boppart (2013) study the role of backward linkages through the input-output table in generating demand for and innovation toward different inputs demanded by two final sectors over which demand is nonhomothetic. Comin et al. (2016) document structural change over innovation and provide a model of directed technological change with knowledge spillovers across sectors to quantitatively account for structural change in the US since 1850. Their model does not generate a BGP on the aggregate.

sector. The more disaggregated level evidence on prices and innovation provided in this paper complements their findings.

At a more technical level, this paper relates to the extant work that has used nonhomothetic CES preferences (Hanoch, 1975) for the study of structural change in open and closed economies (e.g., Matsuyama, 2019; Comin et al., 2016, 2021; Duernecker et al., 2017; Sposi, 2019). Relative to these papers, we show how to incorporate these preferences into an endogenous growth model that delivers aggregate implications identical to an off-the-shelf one-sector growth model. In so doing, we extend the home-market effect insight in Matsuyama (2019) to a dynamic setting, and we offer a tractable framework that complements the work in Comin et al. (2016). This enables us to study the two-way interaction between rising income and sectoral price and innovation dynamics. Moreover, to the best of our knowledge, this paper is also the first to provide a closed-form representation of the implicitly defined nonhomothetic CES aggregator and show that the price distribution needed for the closed-form result arises as an equilibrium outcome–which we anticipate to be useful in other contexts.

As we have discussed, the idea of nonhomotheticities in demand affecting the direction of innovation goes back, at least, to Pasinetti (1981).[10] There is recent empirical evidence supporting this mechanism. Beerli et al. (2020) document a sizable causal effect of changes in market size on the direction of innovation in the context of the Chinese durable good industry driven by heterogeneity in the slopes of Engel curves. Weiss and Boppart (2013) document that changes in firms' market size due to structural change lead to increases in firms' TFP growth rate. In a more granular setting, Jaravel (2018) also finds substantial evidence of directed innovation towards higher income elastic sectors. He documents that more income elastic products have lower inflation because they have increasing demand, which leads to more entry and larger variety in these product categories.[11]

[10] See also Katona (1964) for a discussion of the age of mass consumption as redefining the concept of necessities and luxuries.

[11] There is additional evidence on the role of demand-pull effects driving innovation. For example, Acemoglu and Linn (2004) and Costinot et al. (2019) provide evidence on the effect of market size on innovation and product entry in the pharmaceutical industry.

2 Baseline Model

2.1 Environment

Household Preferences, Endowments, and Demographics The economy is populated by a mass L of homogeneous households. Each household is endowed with one unit of labor that is inelastically supplied. Households have preferences over an infinite stream of consumption bundles $\{\boldsymbol{C}_t\}_{t=0}^{\infty}$ according to

$$\int_0^{\infty} e^{-\delta t} \ln C(\boldsymbol{C_t}) dt \tag{1}$$

where $\delta > 0$ is the discount factor and $C(\cdot)$ is the intra-period utility aggregator over the consumption bundle $\boldsymbol{C}_t$.

Households can smooth consumption over time through investments in an asset A_t which represents shares in the portfolio of all firms in the economy. The household budget constraint is thus given by

$$\dot{A}_t = r_t A_t + W_t - E_t,$$

where W_t, Π_t, and E_t denotes the wage rate, aggregate profits, and household expenditures in the economy.

At time t, the goods available to households to construct their consumption bundle belong to the product space $(\varepsilon, i) \in [0, \infty) \times [0, N_{\varepsilon,t}]$. The households' preferences over these goods are given by a nested CES structure. The outer nest is indexed by ε and defined through nonhomothetic CES preferences, while the inner nest is indexed by i and is a homothetic CES. Formally, within-period household preferences are defined by

$$1 = \left(\int_0^{\infty} \left(\varepsilon^{-\beta} g(U_t)^{-\varepsilon} C_{\varepsilon,t} \right)^{\frac{\rho-1}{\rho}} d\varepsilon \right)^{\frac{\rho}{\rho-1}} \qquad \text{and} \qquad C_{\varepsilon,t} = \left(\int_0^{N_{\varepsilon,t}} C_{\varepsilon i,t}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \tag{2}$$

where $\rho, \sigma > 0$ and $g(\cdot) : \mathbb{R} \to [0, 1)$ is a monotonically increasing, continuously differentiable concave function that corresponds to the within-period aggregator in Equation (1), $C_t \equiv g(U_t)$.[12] For our baseline model we assume that goods are complements across the outer nest and substitutes within, $0 < \rho < 1 < \sigma < \infty$. A natural interpretation of the outer nest is as different sectors in the economy, and the inner nest of goods within the sector. Section 2.8 discusses the case with

[12]For example, the functional form $g(U_t) = 1 - \frac{1}{1+U}$ satisfies these conditions. An elaboration on the effect of the utility aggregator is given in appendix **??**.

$\rho > 1$, that is when goods across different sectors are substitutes rather than compliments. The parameter $\beta \geq 0$ enables the existence of sector-dependent taste-parameters which vary monotonically in terms of a sector's expenditure elasticity rank ordering. The inclusion of this is not important for the main mechanism of our model. Therefore, for clarity, we will derive our model for $\beta = 0$, while providing the relevant results for $\beta > 0$ in tandem when necessary.

Innovation and Production Technologies Production of each intermediate in the production set is linear in labor

$$Y_{\varepsilon i,t} = L_{\varepsilon i,t}. \tag{3}$$

New products can be created in any ε sector through an innovation technology which is identical across ε sectors. The innovation flow of new products in sector ε is given by

$$\dot{N}_{\varepsilon,t} = \eta N_t L_{R\varepsilon,t} \tag{4}$$

where $L_{R\varepsilon,t}$ is total amount of labor hired for research in sector ε, $N_{\varepsilon,t}$ is the total number of product varieties in sector ε, and $N_t = \int_0^\infty N_{\varepsilon,t} d\varepsilon$ is the total number of product varieties in the economy at time t.

Markets and Patents Labor markets are competitive while firms selling to households engage in monopolistic competition. There is free entry in the innovation sector where Firms are awarded a perpetual patent upon successful innovation of a new product.

2.2 Equilibrium Characterization

We begin our analysis with the competitive equilibrium of the economy, that is, households maximize utility given their budget constraint taking prices as given, firms maximize profits, and goods and labor markets clear.

Household Optimality First, we derive household demand and expenditure along the lines of Comin et al. (2021). Given total household expenditure, E_t, and the price vector $\{P_{\varepsilon i,t}\}$, cost minimization implies

$$C_{\varepsilon i,t} = P_{\varepsilon i,t}^{-\sigma} P_{\varepsilon,t}^{\sigma-\rho} E_t^{\rho} C_t^{\varepsilon(1-\rho)} \tag{5}$$

$$\text{with} \quad P_{\varepsilon,t} = \left(\int_0^{N_{\varepsilon,t}} P_{\varepsilon i,t}^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} \quad \text{and} \quad E_t = \left(\int_0^{\infty} (C_t^{\varepsilon} P_{\varepsilon,t})^{1-\rho} d\varepsilon \right)^{\frac{1}{1-\rho}}. \tag{6}$$

In order to concisely characterize the household's optimal inter-temporal allocations, we need first to characterize a closed-form mapping between expenditures and aggregate utility which is implicitly defined in (6). Foreshadowing the properties of the equilibrium, we know that prices across sectors are characterized by to an exponential function.

$$P_{\varepsilon,t} = \zeta_t \exp(\chi_t \varepsilon) \tag{7}$$

Note that the parameters ζ_t and χ_t are only viewed as parameters from the perspective of the household, and in general will be determined by equilibrium forces. Moreover, as far as sectoral prices are nominal terms, ζ_t can be scaled up or down as suited and we thus refer to it as the overall price-level, while χ_t fully characterizes the relative prices across sectors. With $\rho < 1$, the integral in (6) will be well defined when $\ln C_t + \chi_t < 0$. This will be verified later when equilibrium prices are determined. The closed-form mapping between aggregate utility and expenditures for the household is given by

$$C_t = \exp\left(-\chi_t - \frac{\zeta_t^{1-\rho}}{1-\rho} E_t^{-(1-\rho)} \right). \tag{8}$$

Using this mapping the household euler equation is given by

$$\frac{\dot{E}_t}{E_t} = \frac{1}{2-\rho} \left((r_t - \delta) + (1-\rho) \frac{\dot{\zeta}_t}{\zeta_t} \right), \tag{9}$$

We refer to ζ_t as the overall price level. The household transversality condition is given by

$$\lim_{t \to \infty} \exp\left(- \int_0^t r_s ds \right) N_t V_t = 0, \tag{10}$$

where $N_t V_t = A_t$ is the combined present value of all firms.

Firm Optimality Equation (5) shows that the demand for good εi is isoelastic in its own price. Under monopolistic competition, the firm producing good εi finds optimal to set a constant markup over the marginal cost, W, determined by the within sector elasticity of substitution

$$P_{\varepsilon i,t} = \frac{\sigma}{\sigma - 1} W_t. \tag{11}$$

Note that all which is needed to characterize the distribution of prices across sectors is the distribution of products within sectors. The corresponding firm profits are

$$\Pi_{\varepsilon i,t} = \frac{1}{\sigma - 1} W_t Y_{\varepsilon i,t}. \tag{12}$$

Since all firms in a ε-sector are identical, we have that the sectoral index $P_{\varepsilon i}$ in Equation (5) is

$$P_{\varepsilon,t} = \frac{\sigma}{\sigma - 1} W_t N_{\varepsilon,t}^{-\frac{1}{\sigma-1}}. \tag{13}$$

Combining this result, with market clearing for good εi

$$Y_{\varepsilon i,t} = LC_{\varepsilon i,t} \tag{14}$$

and the demand Equation (5), a firm's profits is

$$\Pi_{\varepsilon i,t} = \frac{L}{\sigma} \left(\frac{\sigma}{\sigma - 1} \right)^{1-\rho} W_t^{1-\rho} N_{\varepsilon,t}^{-\frac{\sigma-\rho}{\sigma-1}} E_t^{\rho} C_t^{\varepsilon(1-\rho)}. \tag{15}$$

Free Entry and the Distribution of Products and Prices With a perpetual patent, the value of a product at any given time, t, is equal to the sum of all its future discounted profits,

$$V_{\varepsilon i,t} = \int_t^{\infty} \exp\left(- \int_t^s r(\tau) d\tau \right) \Pi_{\varepsilon i}(s) ds. \tag{16}$$

The mass of firms N_ε occupying each ε sector is endogenously determined by the free entry. Firms can select any ε sector in which to innovate. Aggregating the research technology (4) over all sectors yields the aggregate research technology

$$\dot{N}_t = \eta N_t L_{R,t} \tag{17}$$

where $\dot{N}_t = \int_0^\infty \dot{N}_{\varepsilon,t} d\varepsilon$ and $L_{R,t} = \int_0^\infty L_{R\varepsilon,t} d\varepsilon$ denote the aggregate flow of new products and employment in research. Firms innovate new product varieties in each ε sector until the flow value of doing research is equal to the labor cost of doing so

$$\eta N_t V_{\varepsilon i,t} = W_t. \tag{18}$$

Since the cost of research across ε is the same, firms will enter until the value of products are identical across them too. The only difference across ε sectors is the number of products within each sector, i.e. N_ε. Using the definition of the net present value of an innovation across different products εi, it follows that $\Pi_{\varepsilon i,t} = \Pi_t$ almost everywhere. The symmetry in profits across products implies that each product is produced in the same amount and employs the same amount of labor. For this reason the labor market clearing condition is given by

$$L = L_{Y,t} + L_{R,t} = N_t L_{\epsilon i,t} + L_{R,t} \tag{19}$$

where $L_{Y,t}$ is the total amount of labor used for production. Furthermore, the definition of aggregate expenditures as the sum of all individual purchases reduces to

$$LE_t = \frac{\sigma}{\sigma - 1} W_t L_{Y,t} \tag{20}$$

The symmetry in profits across sectors also lets us obtain an expression for the number of products in each sector, by rearranging (15) and using (20) to get

$$N_{\varepsilon,t} = \left(\frac{LE_t}{\sigma \Pi_t} \left(\frac{L_{Y,t}}{L} \right)^{\rho - 1} C_t^{\varepsilon(1-\rho)} \right)^{\frac{\sigma-1}{\sigma-\rho}}. \tag{21}$$

This expression still explicitly relies on the aggregate utility level C, so it remains to fully characterize the closed-form mapping between household aggregate utility and expenditures. Rather than use an exogenous assumed price distribution as we did for the household problem, we can now write it using (13) and (21). With the price distribution in hand, we can solve the integral in expenditure function implied by the nonhomothetic CES outer nest in (6), this yields the following mapping between utility and expenditures

$$\ln C_t = -\frac{\sigma - \rho}{(\sigma - 1)(1 - \rho)} \left(\frac{LE_t}{\sigma \Pi_t} \left(\frac{L_{Y,t}}{L} \right)^{\sigma - 1} \right)^{-\frac{1-\rho}{\sigma-\rho}}, \tag{22}$$

which features strong monotonicity between utility and expenditures. This also verifies that that the same mapping in the partial equilibrium case for the household is indeed well defined.[13]

We can now concisely characterize the distribution of products across sectors, ε, without needing to allude to utility levels. Substituting (22) into (21) and reorganizing results in the following expression

$$N_{\varepsilon,t} = N_t \Psi_t(N_t) \exp\left(-\Psi_t(N_t) \cdot \varepsilon\right) \tag{24}$$

$$\text{where} \quad \Psi_t(N_t) = \left(\left(\frac{L_{Y,t}}{L} \right)^{\sigma-1} N_t \right)^{-\frac{1-\rho}{\sigma-\rho}} \quad \text{and} \quad N_t = \frac{LE_t}{\sigma \Pi_t}. \tag{25}$$

Note how the shape of the distribution changes as expenditures increase. The mass point at zero becomes higher and the decay becomes slower indicating that there is always positive growth across all sectors as expenditures increase albeit faster growth in higher ε sectors.[14][15]

With the product distribution in hand, the price distribution follows using (13),

$$P_{\varepsilon,t} = \underbrace{E_t \left(\frac{L_{Y,t}}{L} \right)^{-1} \left(N_t \Psi_t(N_t) \right)^{-\frac{1}{\sigma-1}}}_{\zeta_t} \exp\left(\underbrace{\frac{1}{\sigma-1} \Psi_t(N_t)}_{\chi_t} \cdot \varepsilon \right), \tag{26}$$

which in turn provides us with equilibrium characterizations of the price-level, ζ_t, and the shape of the exponential price distribution, χ_t.

2.3 Balanced Aggregate Growth

Our balanced growth path (BGP) will be defined by having a constant interest rate, r, a constant share of workers in research production, L_R, and the number of products growing at a constant rate g_N. We solve for this BGP by normalizing the price-level $\zeta_t = 1$, which in our baseline model is equivalent to normalizing the sector zero good, $P_{0,t} = 1$. Common alternative normalizations

[13] When $\beta > 0$, this closed-form mapping between utility and expenditures becomes

$$\ln C_t = -\frac{1}{\alpha(\sigma-1)} \left(\Gamma\left(1 + \beta\alpha(\sigma-1)\right) \left(\frac{LE_t}{\sigma \Pi_t} \left(\frac{L_Y}{L} \right)^{\sigma-1} \right)^{-\alpha} \right)^{\frac{1}{1+\beta\alpha(\sigma-1)}}, \tag{23}$$

where $\Gamma(\cdot)$ is the standard gamma function and only enters as a normalization constant.

[14] This condition is not satisfied when $\rho > 1$. An alternatively framework in which patents are momentary is provided in Section 2.8 for the case of substitutes.

[15] This relation between aggregate expenditures and total number of products also holds in a standard expanding varieties model as in Acemoglu (2009). This can be seen from the fact that one can also derive it by combining (12), (19), and (20), rather than by integrating over the product distribution.

are provided in appendix E where it is shown how they map into variations in the price-level. The resulting BGP will exist when $\frac{\eta L}{\sigma-1} > \delta$, which ensures that there is positive growth. The transversality condition is always satisfied within our specified parametric bounds.

From (18), (20), and (25), we have proportional growth rates in expenditures, wages, and the profits and present value of products,[16]

$$g_E = g_W = \frac{1}{\sigma - \rho} g_N, \qquad g_\Pi = g_V = \left(\frac{1}{\sigma - \rho} - 1\right) g_N. \tag{27}$$

The growth rate of the overall price-level, g_ζ, is zero by normalization. The interest rate is thus determined by (9) to be

$$r = \delta + \left(\frac{1}{\sigma - \rho} + \alpha\right) g_N. \tag{28}$$

Lastly, the growth rate in the number of products and the research labor share is solved for using (12), (17), and (19) yielding

$$g_N = \eta L_R = \frac{1}{1 + \alpha + \frac{1}{\sigma-1}} \left(\frac{\eta L}{\sigma - 1} - \delta\right). \tag{29}$$

2.4 Nonbalanced Sectoral Growth

The sectoral growth dynamics can be concisely described in terms of the total number of products. Since the balanced growth path features constant growth in total number of products, we will characterize how the mass of products across sectors N_ε grows with the total mass of products N. As mentioned earlier, there is always positive growth in products across all sectors due to the fact that goods are complements. From equations (24) and (25), we can fully describe the sectoral dynamics along the BGP. Recall that $\alpha = \frac{1-\rho}{\sigma-\rho} \in (0, 1)$. The sectoral dynamics are fully captured

[16] Note that the growth rate of profits and present value of a product can be either positive or negative depending on the relative elasticity of substitution across and within sectors. This property of nested CES preferences and monopolostic competition is discussed in Matsuyama, 1995.

by the following two equations:[17]

$$N_{\varepsilon,t} = \left(\left(\frac{L_Y}{L} \right)^{\sigma-1} N_t \right)^{-\alpha} \exp\left(- \left(\left(\frac{L_Y}{L} \right)^{\sigma-1} N_t \right)^{-\alpha} \cdot \varepsilon \right) N_t \tag{32}$$

$$\frac{\dot{N}_{\varepsilon,t}}{N_{\varepsilon,t}} = \left((1-\alpha) + \alpha \left(\left(\frac{L_Y}{L} \right)^{\sigma-1} N_t \right)^{-\alpha} \cdot \varepsilon \right) \frac{\dot{N}_t}{N_t} \tag{33}$$

Since the total mass of products features exponential growth, we can plot the sectoral dynamics against the logarithm of it to understand how they vary as a function of time. Figure 3 depicts these dynamics. There is a sequential relationship in growth where sectors featuring smaller ε increase in mass initially, after which higher ε sectors begin to take off. This is shown in the top panels of each column. Here we see that sectors take off sequentially with sectors defined by larger ε growing slightly faster than their predecessors. Moreover, as expenditures increase in the limit, all ε sectors grow at the rate $(1-\alpha)g_N$.[18]

So far, we have characterized the sectoral dynamics along the BGP where the distribution of products always follows a gamma distribution. When the initial distribution is not the one induced by the BGP, one can show that the sectoral distribution converges to the gamma distribution featured by the BGP. Appendix C provides the proof.

2.5 Expenditure Elasticities and Expenditure Share Peaks

One can derive the expenditure elasticities by combining the household utility to expenditure mapping in (8) with the sectoral demand function,

$$C_{\varepsilon,t} = \left(\frac{P_{\varepsilon,t}}{E_t} \right)^{-\rho} C_t^{(1-\rho)\varepsilon}. \tag{34}$$

[17] When $\beta > 0$ the sectoral product distribution and its dynamics are characterized instead by

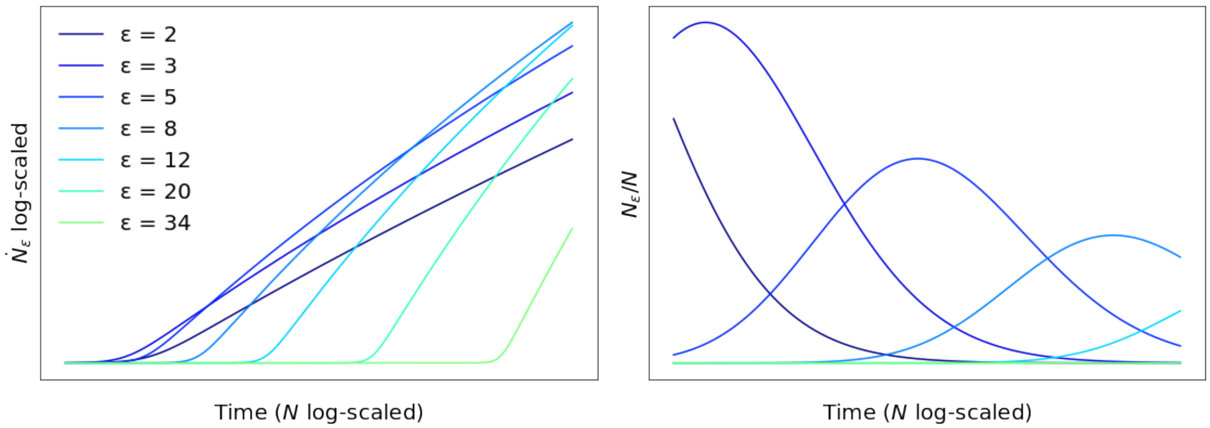
$$N_{\varepsilon,t} = \Psi(N_t)\varepsilon^{\beta\alpha(\sigma-1)} \exp\left(- \left(\Gamma\left(1+\beta\alpha(\sigma-1)\right)\Psi(N_t)\right)^{\frac{1}{1+\beta\alpha(\sigma-1)}} \cdot \varepsilon \right) N_t, \tag{30}$$

$$\frac{\dot{N}_{\varepsilon,t}}{N_{\varepsilon,t}} = \left((1-\alpha) + \frac{\alpha}{1+\beta\alpha(\sigma-1)} \left(\Gamma\left(1+\beta\alpha(\sigma-1)\right)\Psi(N_t)\right)^{\frac{1}{1+\beta\alpha(\sigma-1)}} \cdot \varepsilon \right) \frac{\dot{N}_t}{N_t}. \tag{31}$$

The key difference is that when $\beta > 0$ the product distribution follows a gamma distribution rather an exponential distribution. This creates a non-monotonic relationship between the expenditure-elasticity rank ordered sectors and their absolute size, which in turn allows for the overlapping sectoral dynamics in value-added shares that we see in figure 3. AS shown, this overlap is made more extreme by using a higher value of β. However, the peaks will always feature an exponential decay simply due to the total mass of products becoming ever larger, and therefore the peak value-added shares ε sectors will always be smaller for higher ε.

[18] Note, however, that technically there are always new sectors with larger ε which take off for any level of expenditures and so the proportion that each sector covers goes to zero.

Figure 3:
Sectoral dynamics over time.



The expenditure elasticity of demand is defined by $\eta_{\varepsilon,t} = \frac{\partial \ln C_{\varepsilon,t}}{\partial \ln E_t}$. The chosen nominal normalization in this matters since an increase in expenditures may be associated with a large increase in well-being or non at all depending on how the price-level changes over time as discussed in appendix E. For this analysis we will maintain the price-level normalization that we have used so far. The expenditure elasticity of demand is then given by

$$\eta_{\varepsilon,t} = \rho + (1-\rho)E_t^{\rho-2}\varepsilon. \tag{35}$$

Given the strongly monotonic and time-invariant mapping between ε and η_ε, it is clear that defining sectors by ε is equivalent to defining and ordering sectors by their associated expenditure elasticity. Moreover, the dependence on the aggregate expenditure level (hence utility level) implies that the expenditure elasticity of a given product or sector is diminishing as households get wealthier. At low income level's, nearly all products are luxuries goods, and as incomes grow, goods slowly turn from being luxuries into necessities in the spirit of Georgo Katona's *Mass Consumption Societies*.

Since sectors feature hump-shaped value-added shares over time, as depicted in the right panel of figure 3, it is of interest to explore how the sectoral peaks are associated with their expenditure elasticities. In other words, what is the expenditure elasticity of demand for a given sector at the moment it peaks in terms of it's value-added share? For a given ε sector, the household expenditure level at the time of it's peak can be derived to be

$$E_{\varepsilon}^{\text{peak}} = \frac{\sigma - 1}{\sigma} \varepsilon^{\frac{1}{1-\rho}}, \tag{36}$$

which captures what the right panel of figure 3 qualitatively does: that ε sectors peak sequentially at increasing expenditure levels. We can combine (36) and (35) to characterize at what expenditure elasticity different sectors peak. This is given by

$$\eta_{\varepsilon}^{\text{peak}} = \rho + (1 - \rho) \left(\frac{\sigma - 1}{\sigma} \right)^{2-\rho} \varepsilon^{-\frac{1}{1-\rho}}. \tag{37}$$

Thus we find that the expenditure elasticity of a given sector at the time of it's value-added share peak is smaller for larger ε sectors. One notable feature is that it implies some sectors will peak with an expenditure elasticity above 1 denoting them as luxury goods while "advanced" sectors will peak with expenditure an elasticity below one. Recall, again, however, that this result depends on which type of nominal normalization one uses to derive the BGP.

2.6 A NhCES Growth Externality

In this sort of growth model, the growth in 'well-being' is due to R&D which expands the variety of products. Because people have a love-for-variety, this increase in the mass of product types makes them better off. With nonhomothetic CES preferences, a new externality arises due to the change in the preferred composition of the consumption basket as people become better off. In the social planner problem, more labor may be allocated to research raising growth because the social planner internalizes the fact that there is an additional utility gain from the compositional change in the variety of products. In the competitive equilibrium, people do not fully incorporate the fact that a higher expenditure level will also change the available basket of goods in a favorable direction for them. The derivation of the social planner's problem is provided in appendix D.

The euler equations along the balanced growth paths for the social planner (SP) and the competitive equilibrium (CE) are displayed below. Recall that there is a bijective mapping between the number of products in the economy and the utility level of a household. This mapping holds identically in the social planner's problem, and therefore, in order to directly compare them, we compare the growth in total number of products.

$$\text{SP}: \qquad g_N = \frac{1}{\alpha + \frac{1}{\sigma-1}} \left(\frac{\eta L}{\sigma - 1} - \delta \right) \tag{38}$$

$$\text{CE}: \qquad g_N = \frac{1}{1 + \alpha + \frac{1}{\sigma-1}} \left(\frac{\eta L}{\sigma - 1} - \delta \right) \tag{39}$$

All else equal, the nonhomothetic CES externality makes growth rate in the competitive equilibrium lower than what would be optimal. When households are considering their optimal intertemporal allocation of expenditures, i.e. their expenditure growth rate, they do not take into account that a higher growth rate would change the composition of goods in the market in a favorable manner. They therefore end up choosing a growth rate that is lower than the pareto optimal outcome.[19]

We can compare the sectoral dynamics over time of the social planner's allocations with the competitive equilibrium ones characterized above. In both cases the equilibrium relationship between utility and the number of products is given by equation (89) and the resulting distribution of products given by (32). The higher growth rate in the social planner's solution is associated with a higher share of labor allocated to research, and hence a lower labor share in production, which appears in this equilibrium mapping. Thus both a higher growth rate, but a lower labor-share in production will impact the realized value-added shares at any given moment.

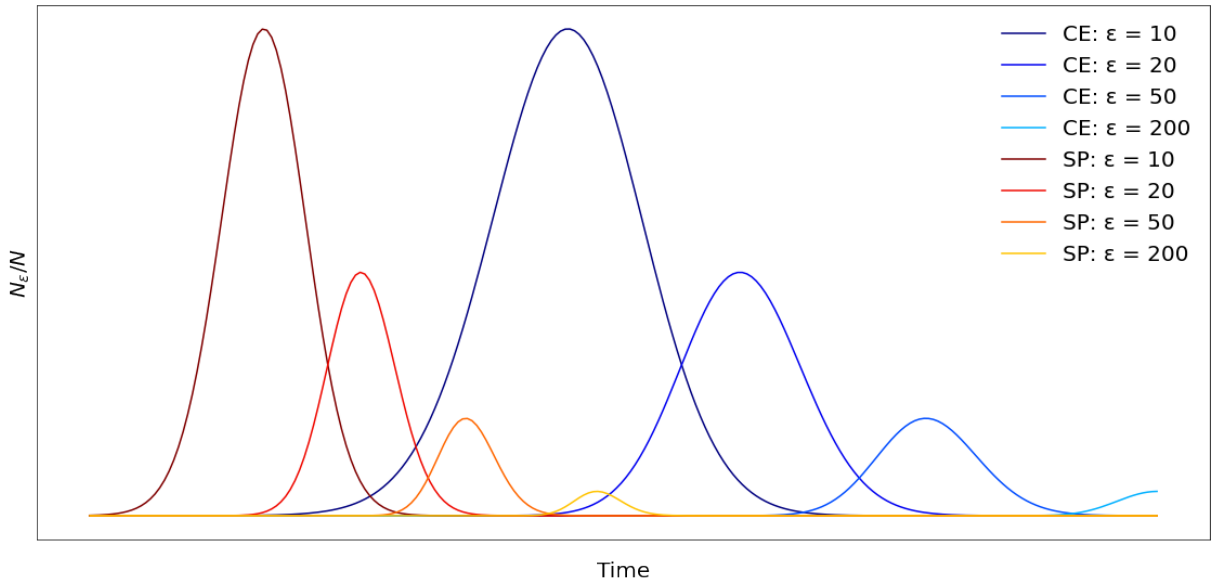
Figure 4 compares the evolution over time of sectoral shares for two economies that begin with the same level for products, but where one follows the allocations of the competitive equilibrium and the other the allocations of the social planner. Since the social planner labor allocations induce a faster growth rate in products, the sectors grow and peak earlier in time than under the competitive equilibrium. The size of each sector's peak is identical under both allocations as well, despite

[19] We have an exponential sectoral distribution because the innovation technology and free entry make the profit equal across sectors which depend on both the number of varieties in each sector and nonhomothetic utility term depending exponentially on ε. The latter gives the exponential distribution whereas the former governs the speed of the sectoral dynamics through α. The sectoral profits depends on number of varieties in the sector through the sectoral price index P_ε in the demand (see eq. (5)) hence it not surprising that α depends on elasticities of substitution, σ and ρ. If we allow for a more general love-of-variety effect by setting inner nest

$$C_{\varepsilon,t} = \left(\int_0^{N_{\varepsilon,t}} C_{\varepsilon i,t}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} N_{\varepsilon,t}^{\gamma - \frac{1}{\sigma-1}},$$

so that γ governs this channel instead of the elasticity of substitution parameter σ. In this case, σ is canceled out from the sectoral dynamics equation (32) and we have $\alpha = \frac{\gamma(1-\rho)}{\gamma(1-\rho)+1}$. This shows that σ is playing a role through the love-of-variety channel. Also, notice that if we interpret equation (5) as a production function for a sectoral final good, γ would govern how the labor productivity growth depends on the number of varieties.

Figure 4: Comparison of CE and SP Dynamics



the peaks occurring at different times with different underlying labor shares. The reason for this is that the product distribution in both cases always matches the sectoral weights in household preferences which in turn are a function of the aggregate utility level. In the competitive equilibrium with a higher amount of labor in production, a sector will peak when there are less total number products, but more being produced of each such that the same utility level is reached. While sectors peak in the competitive equilibrium for lower values of total products, the higher growth rate of total products in the social planner's allocation means that they will still peak earlier with respect to time.

The inefficiency in the competitive equilibrium is fully captured by a labor share in production that is too high leading to too low a number of total products at any given point in time. However, conditional on the sub-optimally high labor share of production, the direction of innovation across sectors in the competitive equilibrium is optimal resulting in peaks of the same magnitude as under the social planner's allocation. Thus a R&D subsidy that is non-targeted would suffice to achieve a pareto optimal equilibrium.

2.7 Growth Under Weak Scale Effects

Jones (1995) proposed a variation on the expanding varieties model, which features weaker scale effects in innovation. He referred to this version as a semi-endogenous growth model because perpetual growth requires and is determined by the growth in the underlying population. Nonetheless, it is in many ways a better description of endogenous growth as it does not rely on strong external scale effects in the innovation technology. Moreover, it does not bear the same counterfactual predictions that the balanced growth rate is proportional to the number of researchers. An increasing population or share of labor which goes to research, both true in the United State, would lead larger and larger growth rates every year in sharp contrast to the observed declining rate of growth.

The semi-endogenous growth model features two changes to our baseline model of section 2. First, the population grows over time at a constant rate, $g_L = l > 0$ with the new households receiving their equal share of wealth. Second, the research technology is given by

$$\dot{N}_t = \eta N_t^{\phi} L_{Rt}. \tag{40}$$

The parameter $\phi < 1$ parameterizes the strength of scale effects in innovation. When negative, innovation exhausts the effectiveness of innovating in the future, and when positive, innovation begets more innovation. Zero may thus be viewed as a natural point of reference where there are no external effects to innovation. Note that our baseline model may be interpreted as setting $l = 0$ and $\phi = 1$ which dynamics and steady state are quite different.

The implications to our baseline model are the aggregate relations characterizing the growth rate in the number of products and free entry.

$$\frac{\dot{N}_t}{N_t} = \eta N_t^{\phi-1} L_{Rt} \quad \text{and} \quad W_t = \eta N_t^{\phi} V_t. \tag{41}$$

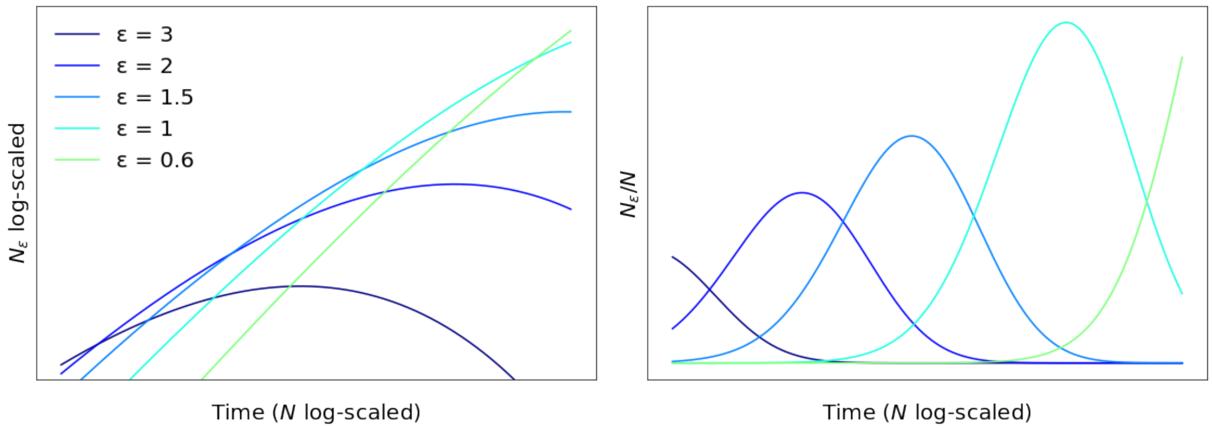
Performing a normalization and defining a BGP equivalent to the one in section 2, we can again characterize the features of the BGP in terms of the growth rate in the number of products, which in this case is $g_N = \frac{1}{1-\phi} l$. The associated growth rates along the BGP are given by

$$g_E = g_W = \frac{1}{\sigma - \rho} g_N, \qquad g_\Pi = g_V = (\frac{1}{\sigma - \rho} - \phi) g_N, \tag{42}$$

and the interest rate and labor shares by

$$r = \delta + \left(\frac{1}{\sigma - \rho} + \alpha \right) g_N \quad \text{and} \quad \frac{L_R}{L} = \frac{\frac{1}{\sigma-1} g_N}{\delta + \left(\frac{1}{\sigma-1} + \phi + \alpha \right) g_N}. \tag{43}$$

Figure 5:
Sectoral Dynamics under Substitutable Sectors.



The transversality condition necessitates that $\delta > (1 - \frac{\alpha}{1-\phi})l$. Combined with ensuring positive growth, the parametric constraints can be characterized as the following bounds on population growth

$$l \in \left(0, \frac{\delta}{1 - \frac{\alpha}{1-\phi}}\right). \tag{44}$$

Interestingly, the model admits a scenario where the population grows faster than the by which amount they discount the future as long as α is significantly less than $1 - \phi$. There is no solution when $\alpha > 1 - \phi$, as it would require a negative discount rate.

Since the only alterations to our model are at the aggregate level, it still features the same closed-form solutions between utility and expenditures, and therefore the sectoral dynamics are identical to those in the model with strong scale effects in innovation in section 2.

2.8 The Model with Substitute Sectors

The dynamics across sectors along the balanced growth path are depicted in 5. The difference is that with $\rho > 1$, the value of $\alpha < 0$ which fundamentally changes the sectoral dynamics. Figure 5 show the dynamics with $\alpha = -0.3$. Under the case of substitutes lower ε sectors have higher income elasticities opposite to what was true under complements. The underlying reasons for these preference differences are described in appendix **??**.

The biggest difference when compared to case of complements is that now sectors feature both a rise and decline. This can be seen in the top two panels of figure 5, which show the total mass and the change over time of various ε sectors. Any sector will eventually decline to zero. The second and third panels show that the growth of each sector eventually goes negative until it is not consumed at all.

Under complements, the low income elastic sectors strictly dominated in mass the high income elastic sectors despite the high income elastic sector eventually taking off and catching up. Here, we now have that the higher income-elastic sectors always strictly dominate the lower income elastic sectors. As described in section **??**, when utility is defined on $[1, \infty)$, the nonhomothetic CES preferences feature a strong link between relative preferences and income elasticities. The more you prefer a certain type of good now, the even more (in relative terms) you will prefer it in the future. This results in a nested sequential hump-shaped pattern where high-income elastic goods peak later than lower ones, but were still preferred to the lower income elastic ones at the earlier time before the peak. This is shown in the bottom panel of the same figure. This appears to be a rather counterfactual dynamic, and it was one of the reasons we chose not to have the model with substitute sectors as our baseline model.[20]

3 An Empirical Exploration of the Model

In this section, we explore the key predictions of the model. As a prologue to this, we begin by documenting a foundational assumption of our model – namely that there is a mapping between the peak ordering of US sectors and the rank order of the sectoral expenditure elasticities.[21] We then show that price and innovation growth across sectors correlate with sectoral expenditure elasticities as predicted by our model. The first step is to compute the income elasticities across sectors. We turn to this exercise first.

3.1 Estimating Expenditure Elasticities

We use cross-sectional data from the Consumer Expenditure survey to estimate the expenditure elasticity of different sectors. The finest level of expenditures (UCC categories) cover over 600 distinct categories. We map them to the PCE disaggregated categories (around 150) using the UCC-PCE bridge supplemented by a manual match for the missing correspondences. We also map UCC categories to the 3-digit NAICS sectors required for our patent data analysis and to the

[20]For instance, the nested hump-shaped pattern would mean that the United States at the end of the 19th century would have spent a larger proportion of GDP on space travel, than they did on agriculture.

[21]This has previously been documented for the case of three sectors (agriculture, manufacturing, and services) in Comin et al. (2021). Here we further disaggregate to 12 sectors.

12 sector categories used for the peak ordering in the introduction. For the elasticities that we use for the patent data exercise and analysis of the 12 categories in the introduction, we proceed in two steps. We match the UCC categories to the US Input-Output categories first and then compute the value added demand along the categories appearing in the IO table (this is similar to Buera et al., 2020 and Comin et al., 2020)

We construct our household sample as in Comin et al. (2020). This corresponds to the standard way of cleaning the Consumer Expenditure survey data, except that we allow for households whose household head is older than 65 to stay in the sample to allow for potentially important health expenditures. We use household data for the period 2000-2005. We estimate the Engel curve for household n, sector s, quarter t as in Aguiar and Bils (2015)

$$\ln\left(\frac{x_{st}^{n}}{\bar{x}_{st}}\right) = \alpha_{str} + \eta_s \ln E_t^n + \Gamma_s Z^n + u_{st}^n,$$

where x_{st}^n denotes quarterly expenditure in s, E_t^n total household expenditure, $\bar{x}_{st}$ is the average expenditure across households, α_{str} are time-region FE, and Z^n demographic controls (dummies for family size, number of earners and age of the household head). We instrument E_t^n with household income quintile and income. The object of interest is η_s which captures the expenditure elasticity.

3.2 Sequential Sectoral Take-off

Table 1 reports the estimated expenditure elasticities for the twelve sectors reported in the introduction. We find that they rank in a very similar way as in Aguiar and Bils (2015) even though these are specified over value-added consumption rather than expenditure. For example, Agriculutre has the lowest expenditure elasticity while Education and Health has the highest. Armed with these elasticity estimates, we can investigate whether the sectorsal peak ordering for the US coincides with the rank ordering of estimated expenditure elasticities. We find that the correlation between the two is 0.79 (reported at the bottom of Table 1). Figure 6 plots the peak order and expenditure elasticity ranking, where again the positive correlation is apparent.

3.3 Correlation between Price Growth and Expenditure Elasticities

Next, we investigate whether the sectoral price growth over long horizons is correlated with sectoral expenditure elasticities. To this end, we correlate the most finely disaggregated 150 PCE categories growth over the 1959-2020 period with the estimated expenditure elasticities. Table 2

Table 1: Estimated Expenditure Elasticities for Twelve Broad US Sectors

Estimated Expenditure Elasticity η_s Parameters	
Prof. & Biz. Serv	1.12 (0.01)
Entertainment, Restaurants	1.31 (0.02)
Education and Health	1.74 (0.05)
FIRE	1.07 (0.01)
Information and Communication	1.05 (0.01)
Utilities	0.90 (0.01)
Mining	0.91 (0.01)
Construction	0.85 (0.01)
Wholesale and Retail Trade	0.88 (0.01)
Transportation	1.04 (0.04)
Manufacturing	0.98 (0.01)
Agriculture	0.68 (0.02)
Correlation with Sectoral Peak Order:	0.79

Figure 6: Correlation Expenditure Elasticities and US Sectoral Peaks Ranking

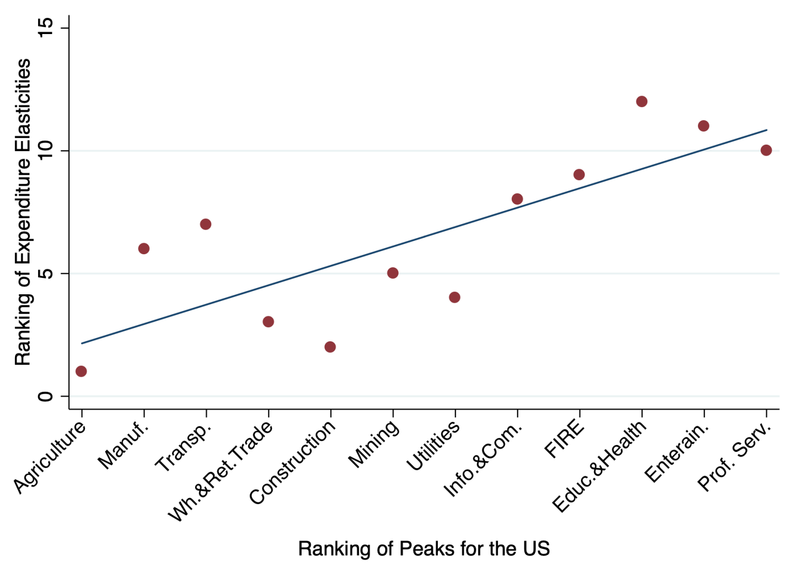


Table 2: Correlation of Price Growth and Expenditure Elasticity

	Dep. Var.: Ave. Annual Price Growth					
	Ave. 1959-2020			Ave. 1980-2020		
	(1)	(2)	(3)	(4)	(5)	(6)
Top Quintile Share η_ε	-2.8	-3.8	-3.2	-4.3	-4.6	-3.4
ε indexes PCE categories	(1.3)	(1.2)	(1.5)	(1.5)	(1.2)	(1.2)
Goods vs. Servs FE	N	Y	Y	N	Y	Y
2-digit Prod. Cat. FE	N	N	Y	N	N	Y
(partial) R^2	0.05	0.11	0.09	0.06	0.11	0.06

Notes: Numb. Obs. is 148. Robust std. errors. Obs. weighted by initial expenditure.

reports the results. In this current version we are using the share of expenditures going to the top quintile rather than the expenditure elasticity, since both are highly correlated (0.80) and it is perhaps easier to interpret. The next version of this paper will include the results with the estimated expenditure elasticities.

We find that there is a negative, statistically significant correlation between price growth and expenditure elasticity. Column (1) reports the result of a simple linear regression. The magnitude is also economically significant. We find that moving from a sector in the 10th percentile to the 90th implies a differential of yearly price growth of $-2.8 \cdot (0.50 - 0.23) = -0.76\%$. Figure 7 depicts this regression. Columns (2) and (3) introduce 1- and 2-digits fixed effects. We see that in this case the coefficient increases in magnitude – suggesting that the within variation in broad categories is more pronounced. Columns (4) to (6) report the same exercise for the last 40 years, with similar findings.

3.4 Correlation between Innovation Growth and Expenditure Elasticities

Finally, we investigate whether growth in patenting activity across sectors is correlated with sectoral expenditure elasticities. That is, we are assuming that patents are a proxy of the flow of ideas $\dot{N}_{\varepsilon,t}$ implied by our model. It is immediate to check that $\partial^2 \ln \dot{N}_{\varepsilon,t}/\partial\varepsilon\partial t > 0$, implying that the growth rate of patenting is increasing in ε.

In this exercise we use variation at 3-digit NAICS starting in 1974 (we plan to extend it backwards in time). We follow Comin et al. (2016) and use the correspondence from USPTO codes to NAICS implied by Compustat firms. That is, we compute the share of patent classes that belong to a sector by computing the share of patents of a class going into a sector in the Compustat firm

Figure 7: Correlation Price Growth and Expenditure Elasticity

sample. This matching has the substantial advantage that we have patents going to service sectors too.[22]

As in the previous section, we regress the average yearly growth (of patents in this case) on the expenditure elasticity of the sector. Table 3 reports the results. Column (1) reports the coefficient on the expenditure elasticity of a simple linear regression. As predicted by our theory, this coefficient is positive. The magnitude is also sizable, moving from the the 10th to the 90th percentile of the sector implies a differential in annual growth of around 0.5%. We find that the result is robust to the inclusion of broad industry fixed effects in column (2). This implies that the variation within broad sectors is also important. Finally, weighting patents by the citations received yields similar results as shown in columns (3) and (4).

4 Schumpeterian Growth Featuring Engel's Treadmill

Our growth mechanism may also be incorporated into a model of vertical innovation where incumbent and entering firms innovate in direct competition with each other. Rather than the development of new products, innovation leads to quality improvements in existing products. Maintaining a parallel to our baseline model, research and production requires a scarce resource as

[22] We impute the NAICS code to non-listed firms assuming that there is the same mapping between firm main industry code and patent classes.

Table 3: Estimated Correlation between Patent Growth and Expenditure Elasticity

	Equally Weighted		Citations Weighted	
	(1)	(2)	(3)	(4)
Exp. Elasticity η_ε	.024	.016	.025	.016
	(.007)	(.007)	(.006)	(.008)
Broad Ind. FE	N	Y	N	Y

Std. Errors: robust and clustered at NAICS 1, respectively. Weighted regression by number of obs. by industry.

input, namely labor, and growth is ensured thanks to aggregate knowledge spillovers.

Preferences and Production Products are still defined over a two-dimensional space with the alteration that there now exists a normalized unit mass of products within each sector. The product space is given by $(\varepsilon, i) \in [0, \infty) \times [0, 1]$. Sectors are still complementary while product varieties within a sector are subsititutes, with respective elasticities of substitution $0 < \rho < 1 < \sigma$. Each differentiated product within in a sector is associated with the quality level of the leading firm that is producing it. Only the highest quality version of a product is produced at any given time since different qualities are perfect substitutes. The quality level affects the household preferences for the product in the CES aggregator as follows

$$C_{\varepsilon,t} = \left(\int_0^1 q_{\varepsilon i,t} C_{\varepsilon i|q,t}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}, \tag{45}$$

where $C_{\varepsilon i|q,t}$ is amount consumption of the variety i in sector ε with quality level q, and similarly $q_{\varepsilon i,t}$ is the quality level of variety i in sector ε. The preferences across sectors are defined according to the NhCES aggregator as in the baseline model and the sectoral demand in equation (34).

Quality is incorporated into the production and research technologies by requiring one unit of labor for quality unit of a product produced, scaled appropriately. That is, higher quality goods take more labor to produce. The production technology is thus given by

$$Y_{\varepsilon i,t} = \frac{L_{\varepsilon i,t}}{q_{\varepsilon i,t}}. \tag{46}$$

Firms are awarded a perpetual patent for their quality innovation and compete monopolistically. This implies that the price of a given product, which the firm optimally set as a constant markup over marginal cost, is

$$P_{\varepsilon i,t} = \frac{\sigma}{\sigma - 1} W_t q_{\varepsilon i,t}. \tag{47}$$

Let the average quality within a sector be defined by $Q_{\varepsilon,t} = \int_0^1 q_{\varepsilon i,t} di$. From the within sector expenditure minimization problem, the sector-specific price index is fully characterized by a sector's average quality level,

$$P_{\varepsilon,t} = \frac{\sigma}{\sigma - 1} W_t Q_{\varepsilon,t}^{-\frac{1}{\sigma-1}} \tag{48}$$

which is analogous to the role the number of varieties played for the sectoral price-index in (13) in the baseline model. Note that in this setup, the equilibrium demand for any product within a sector is also identical, despite heterogeneous quality levels, and only depends on the sector's average quality level

$$C_{\varepsilon i|q,t} = C_{\varepsilon i,t} = Q_{\varepsilon,t}^{-\frac{\sigma}{\sigma-1}} C_{\varepsilon,t} \tag{49}$$

and thus also equal to average product demand in that sector. Profits for a specific product are dependent on its quality, however, and given by

$$\Pi_{\varepsilon i|q,t} = \frac{1}{\sigma - 1} L W_t q_{\varepsilon i,t} C_{\varepsilon i,t}. \tag{50}$$

Incumbent and Entrant Innovation There are two types of innovation processes, which we denote the incumbent technology and the entrant technology.[23] The incumbent technology enables a firm to improve on its current quality level by a factor $\lambda^I > 0$. The flow rate of incumbent innovations depends on how much incumbent R&D the firm performs, which requires labor, and is given by

$$z_{\varepsilon i,t}^I = \eta^I Q_t \frac{L_{R\varepsilon i,t}^I}{q_{\varepsilon i,t}}. \tag{51}$$

As in the production technology, more labor is necessary to innovate for higher quality products. Q_t captures the same strong scale effects as in our baseline model. Rather than being a function of the total number of products, the scale effects are now determined by the aggregate quality level $Q_t = \int_0^\infty Q_{\varepsilon,t} d\varepsilon$.[24] The leading incumbent firm will perform incumbent R&D until the labor cost equals the gains from the expected quality innovations of its product, that is, $W_t q_{\varepsilon i,t} = \eta^I Q_t \left(V_{\varepsilon i|\lambda^I q,t} - V_{\varepsilon i|q,t} \right)$, where $V_{\varepsilon i|q,t}$ is the cumulative sum of the expected future dis-

[23] The names are given based on which type of firm ends up performing the type of innovation technology. It is not based upon who has access to a given technology.

[24] These scale effects may similarly be weakened while including population growth to get a semi-endogenous growth version of the model.

counted profits for a product of quality $q_{\varepsilon i,t}$. We will see below that $V_{\varepsilon i|q,t}$ is linear in $q_{\varepsilon i,t}$, and thus the condition above becomes

$$W_t q_{\varepsilon i,t} = \eta^I(\lambda^I - 1)Q_t V_{\varepsilon i|q,t}. \tag{52}$$

The entrant research technology enables a firm to innovate for a product quality level that it does not own. An entrant innovation improves on the existing quality level by a factor of $\lambda^E > \lambda^I$. The flow rate of entrant innovations depends on how much entrant R&D firms are performing, which also requires labor, and is given by

$$z^E_{\varepsilon i,t} = \left(\eta^E Q_t \frac{L^E_{R\varepsilon i,t}}{q_{\varepsilon i,t}}\right)^{\varphi}, \tag{53}$$

where $L^E_{R\varepsilon i,t}$ is the total amount of entrant innovation for this product by all firms combined, and $\phi \in (0,1)$ implies that there are decreasing returns in total entrant innovation.[25] There is free entry in entrant research and, while taking other firm's research efforts as given, competing firms will perform research until marginal cost of research equals the expected marginal return from innovation, given by

$$W_t q_{\varepsilon i,t} = \varphi \lambda^E \eta^E Q_t V_{\varepsilon i|q,t} \left(\eta^E Q_t \frac{L^E_{R\varepsilon i,t}}{q_{\varepsilon i,t}}\right)^{\varphi - 1}. \tag{54}$$

Combining this with the incumbent innovation condition in (52) implies that the flow rate of innovation by entrants is constant across time and all sectors[26]

$$z^E_{\varepsilon i,t} = z^E = \left(\frac{\varphi \eta^E \lambda^E}{\eta^I(\lambda^I - 1)}\right)^{\frac{\varphi}{1-\varphi}}. \tag{55}$$

The growth rate of the aggregate quality level, Q_t, can then be backed out from the intensities of innovation by incumbent and entering firms. The expected amount of quality growth of a single product is $\dot{q}_{\varepsilon i,t} = (\lambda^I - 1)z^I_{\varepsilon i,t} q_{\varepsilon i,t} + (\lambda^E - 1)z^E q_{\varepsilon i,t}$. Substituting in for (51) and noting that the entrant innovation intensity is constant, integrating across the entire product space yields the following law of motion of aggregate quality

$$\frac{\dot{Q}_t}{Q_t} = (\lambda^I - 1)\eta^I L^I_{R,t} + (\lambda^E - 1)z^E \tag{56}$$

[25] This may be interpreted as many different potential entrants partially performing the same research and crowding out each other's efforts.

[26] While the intensity of innovation by entrants is constant across sectors and time, the amount of R&D labor dedicated is not since sectors with higher quality level require more labor for the same flow rate of innovation to take place.

where $L^I_{R,t}$ is economy-wide amount of labor that is allocated to incumbents' research efforts.

Closed Form Utility Mapping As in the baseline model, we can similarly derive a closed form mapping between household utility and expenditures. Here, we can derive the distribution of average quality across sectors as a function of utility by inverting the firms Hamiltonian-Jacobi-Bellman (HJB) equation. Combining this with the expenditure level derived from a household's expenditure minimization problem, yields the closed-form mapping.

The HJB associated with a specific product is

$$r_t V_{\varepsilon i|q,t} = \max_{L_{R\varepsilon i,t}} \Pi_{\varepsilon i|q,t} + \underbrace{(\lambda^I - 1) z^I_{\varepsilon i,t} V_{\varepsilon i|q,t} - w_t L_{R\varepsilon i,t}}_{=0} - z^E V_{\varepsilon i|q,t} + \dot{V}_{\varepsilon i|q,t} \tag{57}$$

Note that in equilibrium the two middle terms cancel for any level of incumbent research due to the constant returns to scale in the incumbent research technology. Note also for (52) that the the growth rate of in the value of a product conditional on it's current quality level only depends on the aggregate growth rates of quality and wages. Any level of incumbent research is optimal from a cost-versus-benefit perspective, and the equilibrium amount will be pinned down by the demand-induced market-size of each sector. The HJB can thus be significantly simplified to[27]

$$r_t + z^E - \frac{\dot{W}_t}{W_t} + \frac{\dot{Q}_t}{Q_t} = \frac{\Pi_{\varepsilon i|q,t}}{V_{\varepsilon i|q,t}}. \tag{58}$$

Substituting in for the profit condition (50), the product and sector demand conditions, (49) and (34), and the incumbent innovation condition (52), yields a formulation for the average quality levels across sectors as a function of household utility,

$$Q_{\varepsilon,t} = \left(\frac{\eta^I \left(\lambda^I - 1\right)}{\sigma - 1} \frac{L Q_t}{z^E + r_t - \frac{\dot{W}_t}{W_t} + \frac{\dot{Q}_t}{Q_t}} \left(\frac{L_{Y,t}}{L}\right)^{\rho} \right)^{\frac{\sigma-1}{\sigma-\rho}} C_t^{\frac{(1-\rho)(\sigma-1)}{\sigma-\rho}\varepsilon} \tag{59}$$

Analagous to derivation in the baseline model, we can take the household expenditure level in (6) and substitute in for the sectoral price index in (48) and the sectoral quality level just above. This

[27]Given the linearity in $q_{\varepsilon i,t}$ of profits in (50), the linearity of $V_{\varepsilon i|q,t}$ in $q_{\varepsilon i,t}$ follows immediately from this simplified HJB.

yields a well-defined integral which has the following closed-form solution

$$\ln C_t = -\frac{\sigma-\rho}{(\sigma-1)(1-\rho)}\left(\frac{\eta^I\left(\lambda^I-1\right)}{\sigma-1}\frac{LQ_t}{z^E+r_t-\frac{\dot{W}_t}{W_t}+\frac{\dot{Q}_t}{Q_t}}\left(\frac{L_{Y,t}}{L}\right)^{\sigma}\right)^{-\frac{1-\rho}{\sigma-\rho}} \tag{60}$$

Plugging this back into the sectoral quality level above and integrating over sectors leads to simplified expressions in equilibrium for the HJB average quality distribution, and aggregate utility in (58), (59), and (60)

$$r_t + z^E - \frac{\dot{V}_t}{V_t} = \frac{\eta^I(\lambda^I-1)}{\sigma-1}L_{Y,t}, \tag{61}$$

$$Q_{\varepsilon,t} = Q_t\Psi_t(Q_t)\exp\left(-\Psi_t(Q_t)\varepsilon\right) \tag{62}$$

$$\ln C_t = -\frac{\sigma-\rho}{(\sigma-1)(1-\rho)}\Psi_t(Q_t) \tag{63}$$

where $\Psi_t(Q_t) = \left(Q_t\left(\frac{L_{Y,t}}{L}\right)^{\sigma-1}\right)^{-\alpha}$ and $\alpha = \frac{1-\rho}{\sigma-\rho}$ as before. Note the complete parallel between the equilibrium quality distribution and closed-form utility mapping with that in the baseline model in (24) and (22). The price distribution across sectors follows immediately from plugging (62) into (48),

$$P_{\varepsilon,t} = \underbrace{\left(\frac{L_{Y,t}}{L}\right)^{-\frac{\sigma-1}{\sigma-\rho}} E_t Q_t^{-\frac{1}{\sigma-\rho}}}_{\zeta_t}\exp\left(\underbrace{\frac{1}{\sigma-1}\Psi_t(Q_t)}_{\chi_t}\cdot\varepsilon\right). \tag{64}$$

The last piece of the model is the household euler equation which is unchanged from the baseline and given by (9).

Steady State Growth Path We again find a growth path characterized by a constant interest rate and labor shares, balanced growth in aggregate variables, and unbalanced growth across sectors. In parallel to the baseline model we normalize the price-level $\zeta_t = 1$. The relations between the aggregate growth rates are

$$g_E = g_W = \frac{1}{\sigma-\rho}g_Q, \qquad g_\Pi = g_V = \left(\frac{1}{\sigma-\rho}-1\right)g_Q, \tag{65}$$

and the unbalanced sectoral growth rates are given by

$$\frac{\dot{Q}_\varepsilon}{Q_\varepsilon} = \left((1-\alpha)+\alpha\Psi(Q)\cdot\varepsilon\right)g_Q. \tag{66}$$

Table 4: Correlation of Incumbent Job Creation Share and Expenditure Elasticity

	Job Creation Share		
	(1)	(2)	(3)
Expenditure Elasticity η_ε	-0.15	-0.13	-0.08
ε indexes 3-digit NAICS industries	(0.05)	(0.05)	(0.05)
Time FE	Y	Y	Y
Goods vs. Servs FE	N	Y	Y
1-digit NAICS FE	N	N	Y
(partial) R^2	0.06	0.04	0.01

Notes: 616 observations. Industry clustered robust standard errors.

The growth rate of the aggregate average quality level, the interest rate, and labor shares to production, incumbent innovation, and entrant innovation are pinned down by

$$r = \frac{\eta^I(\lambda^I - 1)}{\sigma - 1} L_Y - z^E + \left(\frac{1}{\sigma - \rho} - 1\right) g_Q, \tag{67}$$

$$g_Q = \left(\frac{1}{\sigma - \rho} + \alpha\right)^{-1} (r - \delta), \tag{68}$$

$$L_R^I = \frac{1}{(\lambda^I - 1)\eta^I} \left(g_Q - (\lambda^E - 1) z^E\right), \tag{69}$$

$$L_R^E = \frac{1}{\eta^E} (z^E)^{\frac{1}{\varphi}}, \tag{70}$$

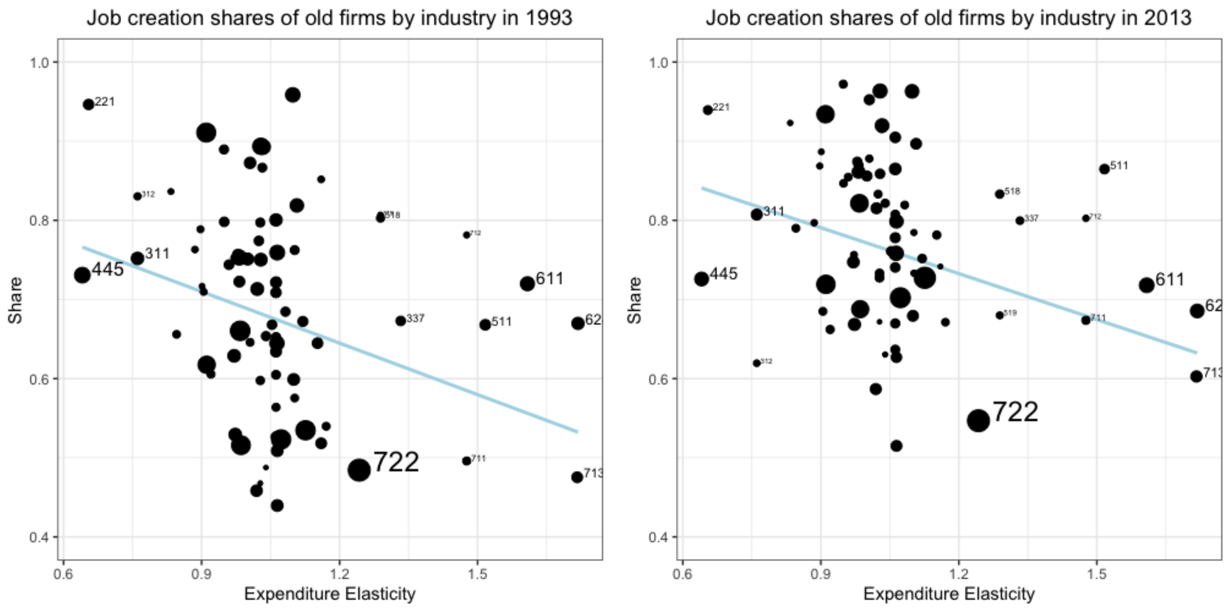
$$L = L_Y + L_R^I + L_R^E. \tag{71}$$

Empirical Implications from Nonhomothetic Schumpetarian Growth The steady state dynamics of the schumpetarian growth model are identical to that of the expanding varieties model, with aggregate average quality level taking the role of the total number of product varieties. The dynamics are driven by alternative forces, however, which we again can test in the data. Here, the driver of growth is a mix of innovation by incumbent firms and their competitors who seek to replace said incumbents.

5 Conclusion

This paper provides an endogenous growth model that features a unique and stable aggregate balanced growth path while also capturing the perpetual, non-linear dynamics across sectors resulting from directed innovation. The growth mechanism results from the two-way interaction

Figure 8: Incumbent Job Creation Shares



between non-homotheticities in demand with directed technical change in what we have called "Engel's Treadmill." In search of profits, innovation tends toward where markets are expanding and drives overall growth. Due to nonhomotheticities in demand, the resulting income growth induces households to change their demand patterns creating yet newer markets into which innovation will flow.

Our theory relies on and implies several testable predictions. First, at its core the theory deems that a sector's defining characteristic is its rank in the ordering of expenditure elasticities, which determines when the sector will take-off, peak, and decline. Second, our theory predicts that prices should decline more in sectors with higher expenditure elasticities. Third, the intensity of innovation is greater in sectors with higher expenditure elasticities. We find all of these correlations to be true in the data.

Notably, our theory is mute regarding the effects of trade and the potentially heterogenous technologies across sectors on structural change. We by no means dismiss these as being important contributors. However, the ability of our model to explain the consistent sectoral patterns across modern economies as well as the empirical predictions regarding price and innovation growth underlines the importance of the home-market sizes across sectors in structural change and growth. Nonetheless, the expansion of this theory into an open-economy setting is an obvious next step in this line of research.

References

Acemoglu, D. (2002). Directed Technical Change. *Review of Economic Studies*, 69(4):781–809.

Acemoglu, D. (2009). *Introduction to Modern Economic Growth*. Princeton University Press.

Acemoglu, D., Aghion, P., Bursztyn, L., and Hemous, D. (2012). The Environment and Directed Technical Change. *American Economic Review*, 102(1):131–166.

Acemoglu, D. and Linn, J. (2004). Market Size in Innovation: Theory and Evidence from the Pharmaceutical Industry. *The Quarterly Journal of Economics*, 119(3):1049–1090.

Aghion, P., Dechezlepretre, A., Hemous, D., Martin, R., and van Reenen, J. (2016). Carbon taxes, path dependency, and directed technical change: Evidence from the auto industry. *Journal of Political Economy*, 124(1):1 – 51.

Aguiar, M. and Bils, M. (2015). Has consumption inequality mirrored income inequality? *American Economic Review*, 105(9):2725–56.

Bakker, G., Crafts, N., and Woltjer, P. (2019). The Sources of Growth in a Technologically Progressive Economy: The United States, 1899–1941. *Economic Journal*, 129(622):2267–2294.

Beerli, A., Weiss, F. J., Zilibotti, F., and Zweim uller, J. (2020). Demand forces of technical change evidence from the chinese manufacturing industry. *China Economic Review*, 60:101157.

Boppart, T. (2014). Structural Change and the Kaldor Facts in a Growth Model With Relative Price Effects and Non-Gorman Preferences. *Econometrica*, 82:2167–2196.

Buera, F. J. and Kaboski, J. P. (2009). Can Traditional Theories of Structural Change Fit The Data? *Journal of the European Economic Association*, 7(2-3):469–477.

Buera, F. J., Kaboski, J. P., Rogerson, R., and Vizcaino, J. I. (2020). Skill-biased structural change. Discussion Papers 2020/09, University of Nottingham, Centre for Finance, Credit and Macroeconomics (CFCM).

Comin, D., Danieli, A., and Mestieri, M. (2020). Income-Driven Labor-Market Polarization. Working Papers 2020-050, Human Capital and Economic Opportunity Working Group.

Comin, D., Lashkari, D., and Mestieri, M. (2016). Demand-Pull, Technology-Push, and the Sectoral Direction of Innovation. 2016 Meeting Papers 1287, Society for Economic Dynamics.

Comin, D., Lashkari, D., and Mestieri, M. (2021). Structural Change With Long-Run Income and Price Effects. *Econometrica*, 89(1):311–374.

Costinot, A., Donaldson, D., Kyle, M., and Williams, H. (2019). The More We Die, The More We Sell? A Simple Test of the Home-Market Effect. *The Quarterly Journal of Economics*, 134(2):843–894.

Crafts, N. and O'Rourke, K. (2014). Twentieth century growth. In *Handbook of Economic Growth*, volume 2, chapter 06, pages 263–346. Elsevier, 1 edition.

DeLong, J. B. (2000). Cornucopia: The pace of economic growth in the twentieth century. Working Paper 7602, National Bureau of Economic Research.

Duernecker, G., Herrendorf, B., and Valentinyi, A. (2017). Structural Change within the Service Sector and the Future of Baumol's Disease. CEPR Discussion Papers 12467, C.E.P.R. Discussion Papers.

Foellmi, R. and Zweimuller, J. (2006). Income Distribution and Demand-Induced Innovations. *Review of Economic Studies*, 73(4):941–960.

Foellmi, R. and Zweimüller, J. (2008). Structural change, Engel's consumption cycles and Kaldor's facts of economic growth. *Journal of Monetary Economics*, 55(7):1317–1328.

Gancia, G. and Zilibotti, F. (2005). Horizontal Innovation in the Theory of Growth and Development. In Aghion, P. and Durlauf, S., editors, *Handbook of Economic Growth*, volume 1 of *Handbook of Economic Growth*, chapter 3, pages 111–170. Elsevier.

Grossman, G. M. and Helpman, E. (1991). *Innovation and growth in the global economy*. MIT press.

Hanoch, G. (1975). Production and Demand Models with Direct or Indirect Implicit Additivity. *Econometrica*, 43(3):395–419.

Herrendorf, B., Rogerson, R., and Valentinyi, A. (2014). Growth and Structural Transformation. In Aghion, P. and Durlauf, S., editors, *Handbook of Economic Growth*, volume 2 of *Handbook of Economic Growth*, chapter 6, pages 855–941. Elsevier.

Herrendorf, B. and Valentinyi, A. (2015). Endogenous Sector-Biased Technological Change and Industrial Policy. CEPR Discussion Papers 10869, C.E.P.R. Discussion Papers.

Jaravel, X. (2018). The Unequal Gains from Product Innovations: Evidence from the U.S. Retail Sector*. *The Quarterly Journal of Economics*, 134(2):715–783.

Jones, C. I. (1995). R&D-Based Models of Economic Growth. *Journal of Political Economy*, 103(4):759–784.

Katona, G. (1964). *The Mass Consumption Society*. McGraw-Hill.

Kongsamut, P., Rebelo, S., and Xie, D. (2001). Beyond balanced growth. *The Review of Economic Studies*, 68(4):869–882.

Kuznets, S. (1973). Modern Economic Growth: Findings and Reflections. *American Economic Review*, 63(3):247–258.

Matsuyama, K. (1995). Complementarities and Cumulative Processes in Models of Monopolistic Competition. *Journal of Economic Literature*, 33(2):701–729.

Matsuyama, K. (1999). Growing through cycles. *Econometrica*, 67(2):335–347.

Matsuyama, K. (2002). The Rise of Mass Consumption Societies. *Journal of Political Economy*, 110(5):1035–1070.

Matsuyama, K. (2019). Engel's Law in the Global Economy: Demand-Induced Patterns of Structural Change, Innovation, and Trade. *Econometrica*, 87(2):497–528.

Ngai, L. R. and Pissarides, C. A. (2007). Structural change in a multisector model of growth. *American Economic Review*, 97(1):429–443.

Nurkse, R. (1963). *Problems of Capital Formation in Underdeveloped Countries*. Oxford University Press.

Pasinetti, L. L. (1981). *Structural Change and Economic Growth: A Theoretical Essay on the Dynamics of the Wealth of Nations*. Cambridge University Press.

Romer, P. (1990). Endogenous technological change. *Journal of Political Economy*, 98(5):S71–102.

Schmookler, J. (1966). *Invention and Economic Growth*. Harvard University Press.

Sposi, M. (2019). Evolving comparative advantage, sectoral linkages, and structural change. *Journal of Monetary Economics*, 103(C):75–87.

Weiss, F. and Boppart, T. (2013). Non-homothetic preferences and industry directed technical change. 2013 Meeting Papers 916, Society for Economic Dynamics.

Appendix

A Alternative Normalizations of the Baseline Model

The model provides a clear exposition regarding the implications of the various possible normalizations in understanding the balanced growth path. Though the normalization of nominal variables, such as expenditures, have long been performed for the attractively simple solutions they provide, the intuition which goes along with them is often lacking. This section provides interpretations for some these normalizations in terms of their implications on the overall price-level.

We will first consider normalizing expenditures to one along the BGP, widely popularized by Grossman and Helpman (1991). Then we will promote the normalization of profits instead which provides a particular straightforward solution in our model. Lastly, we will contrast these to our baseline normalization of setting the price-level, $\zeta_t = 1$. In order of expenditures, profits, and price-level, these normalizations can be interpreted as imposing a deflationary, inflationary, or constant price-level along the balanced growth path. These normalization, of course, only affect nominal variables and therefore admit identical growth rates in products N and constant labor shares L_R and L_Y given by

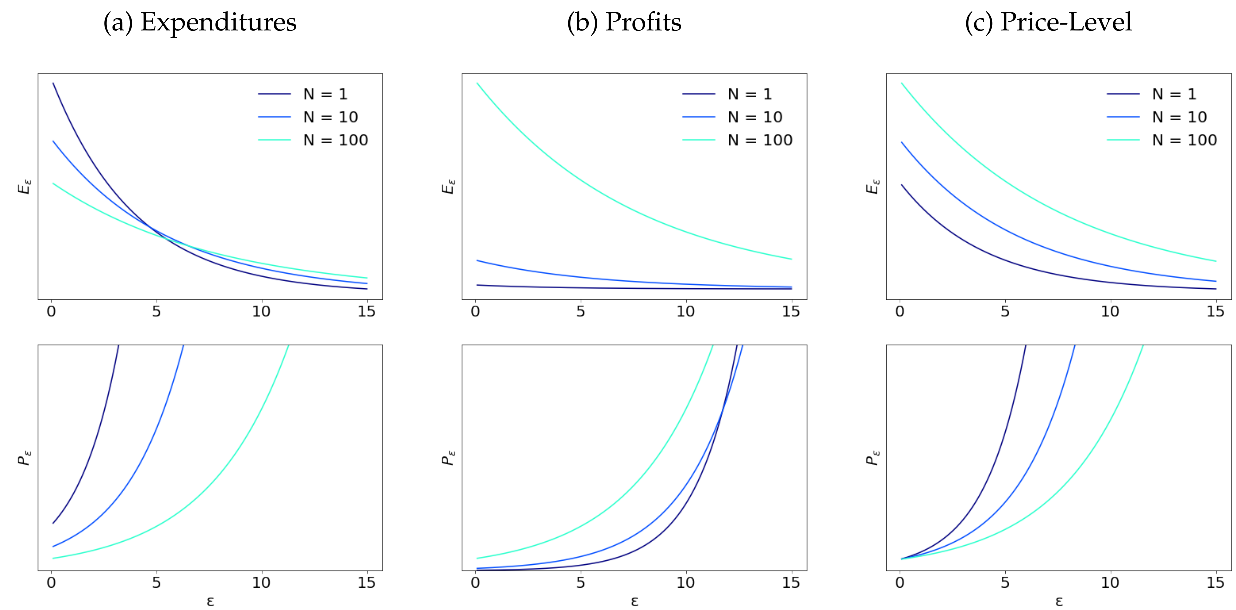
$$g_N = \eta L_R = \frac{1}{1+\alpha+\frac{1}{\sigma-1}}\left(\frac{\eta L}{\sigma-1}-\delta\right). \tag{72}$$

The effect of various normalizations on the expenditure and price distributions, $E_\varepsilon = E^\rho(P_\varepsilon C^\varepsilon)^{1-\rho}$ and $P_\varepsilon = \zeta_t \exp(\chi_t \varepsilon)$, are depicted in figures 9a, 9b, and 9c.

Under the expenditure normalization both expenditures and wages do not grow while a product's profits and present value grow at the negative growth rate of products, $g_\Pi = g_V = -g_N$. However, to keep expenditures fixed, the price-level deflates at a rate $g_\zeta = -\frac{1}{\sigma-\rho} g_N$. This gives rise to the expenditure and price dynamics depicted in 9a. As the economy grows, the expenditure distribution flattens out as household consume relatively more income elastic goods from higher ε sectors. At the same time the decreasing price-level means that the nominal expenditures on lower ε sectors is diminishing.

Under the profit normalization, both a product's profits and present value do not grow while expenditures and wages grow at the same rate as the number of products, $g_E = g_W = g_N$, making

Figure 9:
Alternative Normalizations



this normalized BGP particularly straightforward. In this case, the price-level may be inflating or deflating at a constant rate $g_\zeta = \left(1 - \frac{1}{\sigma-\rho}\right) g_N$ depending on the whether $\sigma - \rho \lesseqgtr 1$. Figure 9b depicts it for the more likely case with inflation where expenditures grow across all sectors while more so, relatively, for higher ε ones.

The BGP under the price-level normalization was provided in the section 2 with the relative growth rates provided in (27). Expenditures and wages grow a positive rate which may be smaller or larger then the growth rate products, and a product's profits and present value may be increasing or declining, depending on relative elasticities across and within sectors. There is no inflation or deflation by decree, only changes in the relative prices. Expenditures grow for all sectors as shown in figure 9c regardless of the relative elasticities intratemporal substitution, albeit at a slower pace than under the profit normalization. Here, the expenditure growth across all sectors is solely due to decreases in the relative prices of higher ε sectors, while under the profit normalization there is the additional nominal inflationary push.

B Uniqueness of the Equilibrium

We can show that the balanced growth path is indeed the unique equilibrium of the model. It is possible to show that the labor share in research has to be constant and this immediately implies that the growth rate of total number of varieties are constant. It is clear that from the equations 27 and 28, other growth rates and interest rate are constant because they can be written in terms of growth rate of total number varieties. Without imposing anything, using equations 9, 12, 16, 17, 18, 20 and price normalization, we can write the change of labor in the research in terms of its level.

$$\dot{L}_{R,t} = \frac{(L - L_{R,t})(L_{R,t} - L_R^*)}{\frac{1}{\sigma-\rho}\left[(2-\rho)\sigma - 1\right]} \tag{73}$$

where L_R^* is the balanced growth path level, an expression in terms of model parameters provided in the equation 29. Notice that the denominator is positive given $\sigma > 1 > \rho$. We plot the differential equation in Figure 10 to illustrate its dynamics.

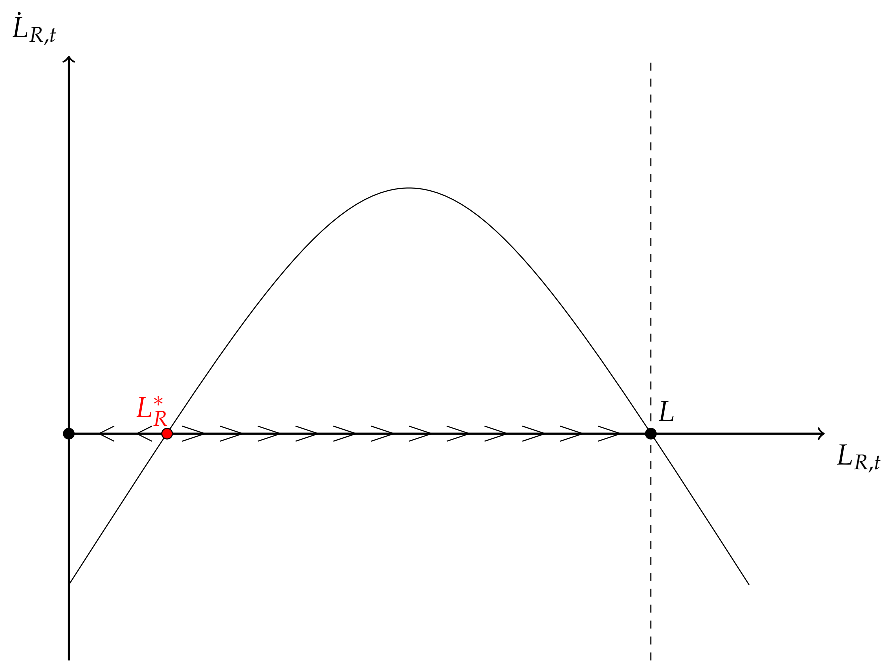


Figure 10: Change of labor employed in research sector in terms of its level.

First note that research labor cannot be higher than total labor, i.e. $L_{R,t} \in [0, L]$. If we have $L_{R,t} < L_R^*$ then $\dot{L}_{R,t} < 0$ hence the labor in research will always decrease and eventually we will have negative labor which is not possible. Similarly if $L_{R,t} > L_R^*$, it will increase and eventually will be higher than the total labor, also not possible. Also we cannot have $L_{R,t} = L$, because that will leave no labor for production. Therefore the unique solution is $L_{R,t} = L_R^*$ and $\dot{L}_{R,t} = 0$, i.e. labor share in research sector has to be constant and the unique solution is the balanced growth path.

C Stability of the BGP Sectoral Distribution

The model implies an exponential distribution of varieties in the balanced growth path. What about if the economy has a different distribution in the initial state? It is possible to show that an arbitrary distribution converges to the exponential distribution implied by the BGP as the economy grows. Using equation (21), the ratio of number of varieties in BGP for sector an arbitrary sector $\varepsilon = \tilde{\varepsilon}$ to the sector with $\varepsilon = 0$ can be written by,

$$\frac{N_{\tilde{\varepsilon},t}}{N_{0,t}} = C_t^{\tilde{\varepsilon}(\sigma-1)\alpha}. \tag{74}$$

Assume $\frac{N_{\tilde{\varepsilon},t_0}}{N_{0,t_0}} > C_{t_0}^{\tilde{\varepsilon}(\sigma-1)\alpha}$ and suppose towards contradiction that $\frac{N_{\tilde{\varepsilon},t}}{N_{0,t}} \frac{1}{C_t^{\tilde{\varepsilon}(\sigma-1)\alpha}}$ is increasing for $t \in [t_0, t_0 + \epsilon)$. Equation (15) gives the ratio of profits,

$$\frac{\Pi_{\tilde{\varepsilon},t}}{\Pi_{0,t}} = \left[\frac{N_{\tilde{\varepsilon},t}}{N_{0,t}} \frac{1}{C_t^{\tilde{\varepsilon}(\sigma-1)\alpha}} \right]^{-\frac{\sigma-\rho}{\sigma-1}}. \tag{75}$$

Therefore we have $\frac{\Pi_{\tilde{\varepsilon},t}}{\Pi_{0,t}} < \frac{\Pi_{\tilde{\varepsilon},t_0}}{\Pi_{0,t_0}}$ for $t \in [t_0, t_0 + \epsilon)$ and it can be generalized to for all $t \geq t_0$.[28] The assumption implies that there is innovation in sector $\varepsilon = \bar{\tilde{\varepsilon}}$, but there might or might not be innovation in sector $\varepsilon = 0$, hence firm entry conditions are given by,

$$\eta N_t^{\phi} V_{\tilde{\varepsilon}i,t} = W_t \quad \text{and} \quad \eta N_t^{\phi} V_{\tilde{0}i,t} \leq W_t. \tag{76}$$

This implies $V_{\tilde{\varepsilon}i,t} \geq V_{0i,t}$, combining with the definition of value of a firm and $\frac{\Pi_{\tilde{\varepsilon},t}}{\Pi_{0,t}} < \frac{\Pi_{\tilde{\varepsilon},t_0}}{\Pi_{0,t_0}}$ for $t \geq t_0$, we can obtain the result, $\frac{\Pi_{\tilde{\varepsilon},t_0}}{\Pi_{0,t_0}} > 1$. This contradicts with the initial assumption of $\frac{N_{\tilde{\varepsilon},t_0}}{N_{0,t_0}} > C_{t_0}^{\tilde{\varepsilon}(\sigma-1)\alpha}$ because it implies $\frac{\Pi_{\tilde{\varepsilon},t_0}}{\Pi_{0,t_0}} < 1$.

D The Social Planner Problem

The social planner problem attempts to optimize the discounted flow of utility for households subject to their nonhomothetic CES preferences across goods, the production technology, and the

[28] Suppose it is violated for $\bar{t} > t_0$, $\frac{\Pi_{\tilde{\varepsilon},\bar{t}}}{\Pi_{0,\bar{t}}} \geq \frac{\Pi_{\tilde{\varepsilon},t_0}}{\Pi_{0,t_0}}$. The profit is continuous in time, hence there exist $\tilde{t}$ such that $\frac{\Pi_{\tilde{\varepsilon},\tilde{t}}}{\Pi_{0,\tilde{t}}} = \frac{\Pi_{\tilde{\varepsilon},t_0}}{\Pi_{0,t_0}}$ and profit ratio is increasing in the neighborhood of $\tilde{t}$. Following from the equation (75), we have $\frac{N_{\tilde{\varepsilon},t}}{N_{0,\tilde{t}}} \frac{1}{C_{\tilde{t}}^{\tilde{\varepsilon}(\sigma-1)\alpha}} = \frac{N_{\tilde{\varepsilon},t_0}}{N_{0,t_0}} \frac{1}{C_{t_0}^{\tilde{\varepsilon}(\sigma-1)\alpha}}$. By the initial assumption $\frac{N_{\tilde{\varepsilon},t}}{N_{0,\tilde{t}}} \frac{1}{C_{\tilde{t}}^{\tilde{\varepsilon}(\sigma-1)\alpha}}$ is increasing in the neighborhood of $\tilde{t}$ and, hence, profit ratio is decreasing. This contradicts with the choice of $\tilde{t}$.

innovation technology. It does so by determining how much labor to allocate each type of good for production and how much labor to allocate to each sector for innovation subject to the aggregate supply of labor.

$$\max_{\{L_{\varepsilon i,t}\},\{L_{R\varepsilon,t}\}} L\int_0^\infty \exp\left(-\delta t\right)\ln(C_t)dt \tag{77}$$

$$\text{s.t.} \qquad 1 = \left(\int_0^\infty \left(C_t^{-\varepsilon}C_{\varepsilon,t}\right)^{\frac{\rho-1}{\rho}} d\varepsilon\right)^{\frac{\rho}{\rho-1}} \tag{78}$$

$$C_{\varepsilon,t} = \left(\int_0^{N_{\varepsilon,t}} C_{\varepsilon i,t}^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}} \tag{79}$$

$$Y_{\varepsilon i,t} = L_{\varepsilon i,t} \tag{80}$$

$$\dot{N}_{\varepsilon,t} = \eta N_t L_{R\varepsilon,t} \tag{81}$$

$$Y_{\varepsilon i,t} = LC_{\varepsilon i,t} \tag{82}$$

$$L = \int_0^\infty \int_0^{N_{\varepsilon,t}} L_{\varepsilon i,t} di d\varepsilon + \int_0^\infty L_{R\varepsilon,t} d\varepsilon \tag{83}$$

Similar to the household problem in the competitive equilibrium, the social planner problem can also be broken down into an intra- and an inter-temporal problem. The intra-temporal problem sets out to determine what the optimal distribution of goods across sectors is while the inter-temporal one determines how many new goods to create over time.[29] The intra-temporal problem can be written as follows. Given the labor allocation, $L_{Y,t}$ and total number of products, N_t, the social planner seeks to

$$\max_{\{N_{\varepsilon,t}\}} \quad C_t \quad \text{s.t.} \quad 1 = \int_0^\infty \left(C_t^{-\varepsilon}\frac{L_{Y,t}}{L}\frac{N_{\varepsilon,t}^{\frac{\sigma}{\sigma-1}}}{N_t}\right)^{\frac{\rho-1}{\rho}} d\varepsilon \tag{84}$$

$$N_t = \int_0^\infty N_{\varepsilon,t} d\varepsilon \tag{85}$$

Here, we have already imposed that the amount of labor for production of each product is identical and given by

$$L_{Y\varepsilon i,t} = \frac{L_{Y,t}}{N_t} \tag{86}$$

[29]The keen-eyed observer will note that the intra- versus inter-temporal breakdown of the problem is not entirely accurate since the intra-temporal problem of determining the optimal distribution of goods is indeed one that is enacted through the inter-temporal mechanism of research. This is innocuous, however, since the cost of innovation in any sector is the same. The problem is identical to one where the social planner determines the total number of new goods to innovate which are to be allocated to each sector in the next period. The problem of allocating new products across sectors is equivalent to allocation labor input to research across sectors.

This is optimal due to the households strict preference for variety. Any variation in labor input for production across sectors is absorbed by the variation in mass of products across sectors. Solving this problem yields the optimal distribution of products across sectors

$$\frac{N_{\varepsilon,t}}{N_t} = \left(C_t^{-\varepsilon(\sigma-1)} \left(\frac{L_{Y,t}}{L} \right)^{\sigma-1} N_t \right)^{-\frac{1-\rho}{\sigma-\rho}} \tag{87}$$

Plugging this back into the implicit preference constraint in (84), the constraint reduces to the optimal distribution of products across sectors

$$1 = \int_0^\infty \left(C_t^{-\varepsilon(\sigma-1)} \left(\frac{L_{Y,t}}{L} \right)^{\sigma-1} N_t \right)^{-\frac{1-\rho}{\sigma-\rho}} d\varepsilon = \int_0^\infty \frac{N_{\varepsilon,t}}{N_t}\, d\varepsilon \tag{88}$$

In hindsight, it is an obvious result that the optimal supplied basket of goods should perfectly match the shape of the preferences indicating the preferred basket of goods that is demanded. It also lets us define a mapping between utility C_t and the total mass of products N_t. Conditional on always having the optimal distribution of products, the total number of products is the indicator for the amount of development and wealth of the economy. Solving the integral above in (88) yields

$$\ln C_t = -\frac{\sigma-\rho}{(1-\rho)(\sigma-1)} \left(\left(\frac{L_{Y,t}}{L} \right)^{\sigma-1} N_t \right)^{-\frac{1-\rho}{\sigma-\rho}} \tag{89}$$

This appears identical to the equilibrium relationship we found in the competitive equilibrium in (22) which determined the distribution of product, but it will differ slightly from it due to the optimal labor share to L_Y/L is lower the optimal allocation. Moreover, it is not equivalent to the mapping in partial equilibrium in (8) which households use when making their decisions. This is the source of the nonhomothetic CES externality. Households do not fully incorporate their spending's effect on the realized composition of the basket of products. Nor do they realize that when they allocate more labor to research that they are modifying their future basket of goods.

We next turn to analyzing the inter-temporal allocations. The social planner seeks to maximize discounted utility over time by deciding how much labor to allocate to production, which we know is evenly dispersed among products, and how much to allocate to aggregate product innovation when the new products are allocated optimally among sectors.

$$\max_{L_{Y,t},L_{R,t},\dot{N}_t} \; L\int_0^\infty \exp\left(-\delta t\right)\ln(C_t)dt \tag{90}$$

$$\text{s.t.} \qquad \dot{N}_t = \eta N_t L_{R,t} \tag{91}$$

$$\ln(C_t) = -\frac{\sigma-\rho}{(1-\rho)(\sigma-1)}\left(\left(\frac{L_{Y,t}}{L}\right)^{\sigma-1} N_t\right)^{-\frac{1-\rho}{\sigma-\rho}} \tag{92}$$

$$L = L_{Y,t} + L_{R,t} \tag{93}$$

The amount of labor to allocate for research in each sector is accounted for in the optimal distribution of firms, and the amount of labor to allocate for production for each good is identical. The euler equation for social planner is provided in terms of the number of products which as an alternative to expenditure growth in the competitive equilibrium.

$$\frac{\dot{N}_t}{N_t} = \frac{1}{\alpha + \frac{1}{\sigma-1}}\left(\frac{\eta L}{\sigma-1} - \delta\right) \tag{94}$$

where $\alpha = \frac{1-\rho}{\sigma-\rho}$ as before.

E The Model with Substitutable Sectors

The model featuring substitute goods across sectors mirrors that with complimentary goods with a few alterations. The preferences and technology put forth in section 2.1 are the same. However, the elasticities of substitutions across goods and sectors are now ordered $1 < \rho < \sigma < \infty$. The transformation of utility $C = g(U)$ is set to ensure that transformed utility is always greater than one.[30] This is in order to get a well-defined solution as described in appendix **??**. A comparison of the substitute and complement models' preferences is done in the second and third rows of figure **??**. Under substitutes, lower ε goods are still preferred in absolute terms to higher ε goods, but the income elasticity is now larger for lower ε as well whereas under complementary sectors higher ε goods had higher income elasticities.

The main alteration in the substitute model is in regard to patents and the knowledge they encompass. First, rather than having a perpetual patent for the newly innovated product, the firm which innovates it now only has a momentary monopoly on selling the good after which any firm

[30]For example, the utility transformation $g(U) = 1 + U$ will do.

has the rights to produce it.[31] Second, in order to produce a good, a firm must have knowledge of how to produce the good using the patent which requires a flow cost κ in terms of labor to learn and maintain. The innovating firm has the knowledge of how to produce the good at the moment of innovation since they just invented it.[32]

Model Derivation The household demand is still characterized by (5). The production and innovation technologies are also as they were in (3) and (4). A newly innovated product thus has the same profits and pricing conditions as all products did in the model with complements given in (12). After the product is invented, any firm may enter and pay the fixed cost κ, which is defined in terms of labor, to learn how to produce it. Profits therefore become zero as firms will enter until no surplus remains. The price of the product can be recovered in order to satisfy the zero profit condition

$$\Pi_{\varepsilon i,t} = P_{\varepsilon i,t} Y_{\varepsilon i,t} - W_t L_{\varepsilon i,t} - W_t \kappa = 0. \tag{95}$$

For tractability, we set κ equal to inverse of research labor productivity $\frac{1}{\eta N_t}$. In other words, as one unit of labor become more productive in innovation, it is also more productive in learning and maintaining the production. The zero profit condition then makes the price of old products the same as that for new products.[33] In this case, however, the profits from existing products will be zero after the fixed cost is paid.

In contrast to the model with complementary sectors which always featured positive product growth across all sectors, the model with substitutes features sectors that will eventually decline as they are substituted out for more preferred "newer" sectors. Since there will no longer be innovation in every sector, the sectoral research technology in (4) now implies the following complementary slackness condition

$$W_t \geq \eta N_t V_{\varepsilon,t}, \quad L_{R\varepsilon,t} \geq 0, \quad \text{and} \quad (W_t - \eta N_t V_{\varepsilon,t}) L_{R_{\varepsilon,t}} = 0. \tag{96}$$

[31] The model of Foellmi and Zweimüller (2008), which may also be interpreted as a a model with substitute sectors, also necessitates a momentary patent rather than a perpetual one. This is in order to maintain a constant value of innovation despite the value of the product which is non-constant over time.

[32] It is worth mentioning that our baseline model with complementary sectors is also compatible with this innovation and production framework. We omitted it from there because it only adds unnecessary complexity to an expanding variety model which otherwise functions nearly identically to a "baseline" expanding varieties model as that in Acemoglu, 2009.

[33] It can be argued that the learning and maintenance cost could be lower than the innovation cost. However, in that case, the price set by competitive firms will be lower than monopoly price and the potential demand for newly innovated variety will be lower. This can create innovation cycles as pointed out in Matsuyama (1999). However we would like abstract the model from this kind of mechanisms since it is not a central point of the paper.

The momentary patent induces the value of creating a new product to be equal to the instantaneous profits from producing it. Since we have free entry in innovation across sectors, the value of innovating a product will be equalized across the sectors that innovate. Denote this value of innovation and its associated profits by

$$V_t = \Pi_t \tag{97}$$

In sectors that do not innovate we have by the complementary slackness condition that the value of innovating, and hence the profits, is less than in sectors that do innovate, $\Pi_{\varepsilon i,t} < \Pi_t$. In words, innovation is no longer profitable in the sufficiently "old" sectors as the demand per product in these sectors is declining and therefore less than the demand per product which the free entry condition imposes in "newer" sectors. Note that the fixed cost κ is constant across all sectors even where demand is declining. Firms will still enter and produce with the existing products that are in the declining sectors, and these firms will set price equal to that of sectors which are not in decline. However, these sectors' declining demand implies that only some of the existing products will be produced because there is no longer sufficient demand to earn weakly positive profits net the fixed cost if all products were produced. This model thus features endogenous retirement of old products as they eventually become obsolete.

The derivations of the utility to expenditure mapping, the distribution of products and prices, and the Euler equation are nearly identical to that in the complementary case above with the notable differences that now $\rho > 1$. The labor market clearing is now given by

$$L = L_{Y,t} + N_t \kappa + L_{R,t} \tag{98}$$

This is the only substantial difference in the results of the model and it affects the growth rate in the equilibrium. The sectoral distribution is given by the exactly same as equation (32) in the benchmark model. There is, however, a notably qualitative difference in the distribution of products and dynamics across sectors as the α parameter is now of opposite sign.

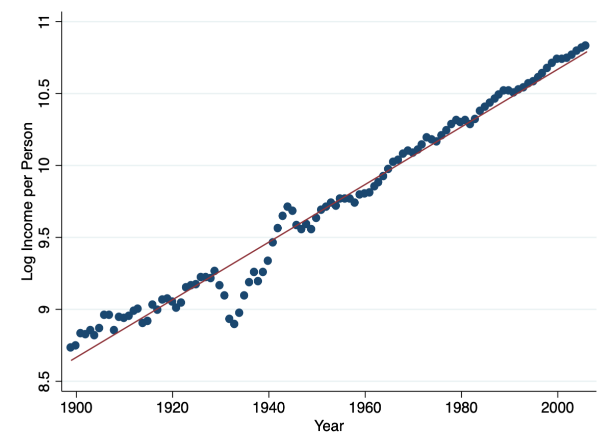
Balanced aggregate growth and unbalanced sectoral growth under substitute sectors The balanced growth path under substitute sectors is identical on face to that with complements with the same relations between growth rates. Since the patent is momentary, the supply side of the economy is completely atemporal. The growth rates of expenditures, wages, profits and the interest rate in terms of growth rate of total number of varieties are same as the benchmark model as in the equation (27). The growth rate of total number of firms and labor allocation are given by

$$g_N = \eta L - \sigma, \quad L_Y^* = \frac{\sigma - 1}{\eta}, \quad L_R^* = L - \frac{\sigma}{\eta}. \tag{99}$$

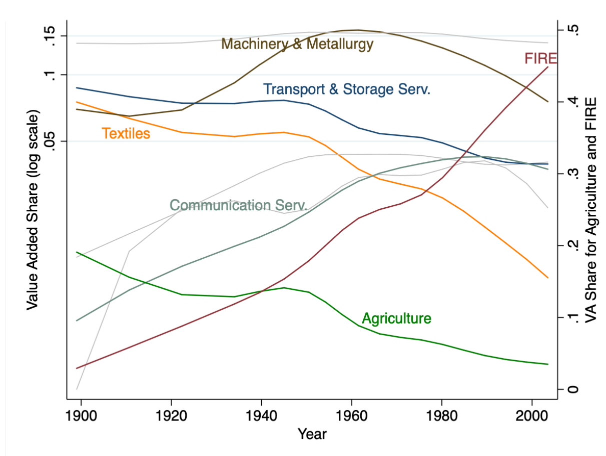
...

Figure 11: US Aggregate and Sectoral Growth, 1899-2007

(a) GDP per capita



(b) Sectoral Value Added



Notes: Data for income per capita come from the Maddison project database. Sectoral value added shares correspond to the private domestic economy. They are computed merging World Klems 2013 with historical national accounts from Bakker et al. (2019). The lines reported correspond to quadratic splines. The highlighted sector Agriculture includes food processing (AtB and 15t16 in ISIC Rev. 3 notation), Textiles correspond to 17t20, Machinery to 27t35 (incl. Metals and Transport Equipment), Transport to 60t63, Communication to 64, and FIRE to JtK. In gray without labels we have Chemicals (23t25), Utilities (E) and Wholesale and Retail Trade (G).

F Additional Figures and Tables

Figure 12: US Sectoral VA, 20-Sector Split, 1899-2005

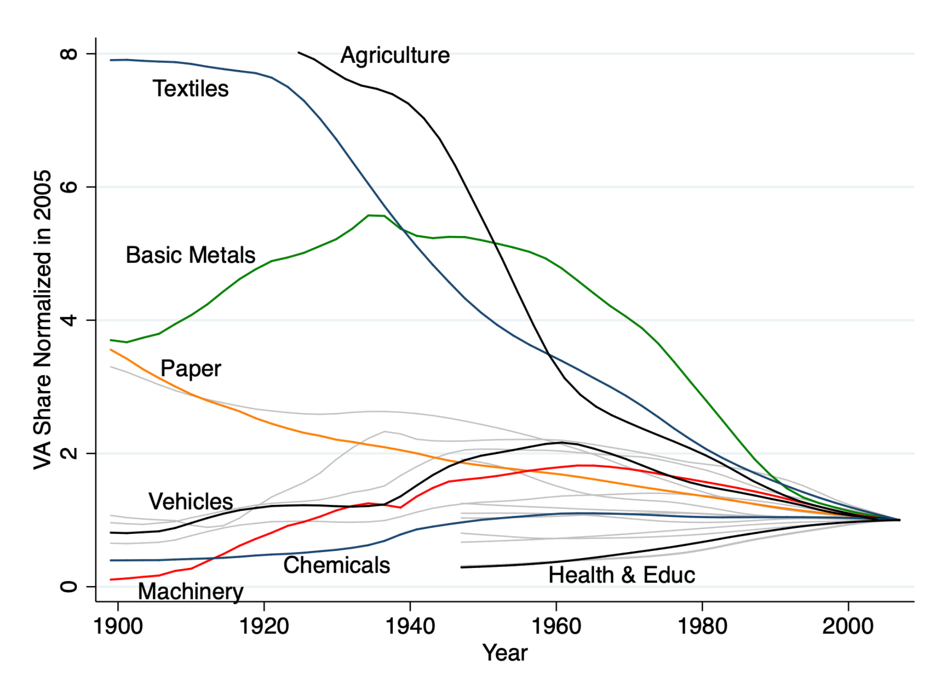


Table 5: Correlation of the Incumbent Employment Share and Expenditure Elasticity

	Employment Share		
	(1)	(2)	(3)
Expenditure Elasticity η_ε	-0.08	-0.05	-0.03
ε indexes 3-digit NAICS industries	(0.03)	(0.03)	(0.04)
Time FE	Y	Y	Y
Goods vs. Servs FE	N	Y	Y
1-digit NAICS FE	N	N	Y
(partial) R^2	0.04	0.02	0.01

Notes: 616 observations. Industry clustered robust standard errors.

Figure 13: Incumbent Employment Shares

Figure 14: Employment Growth

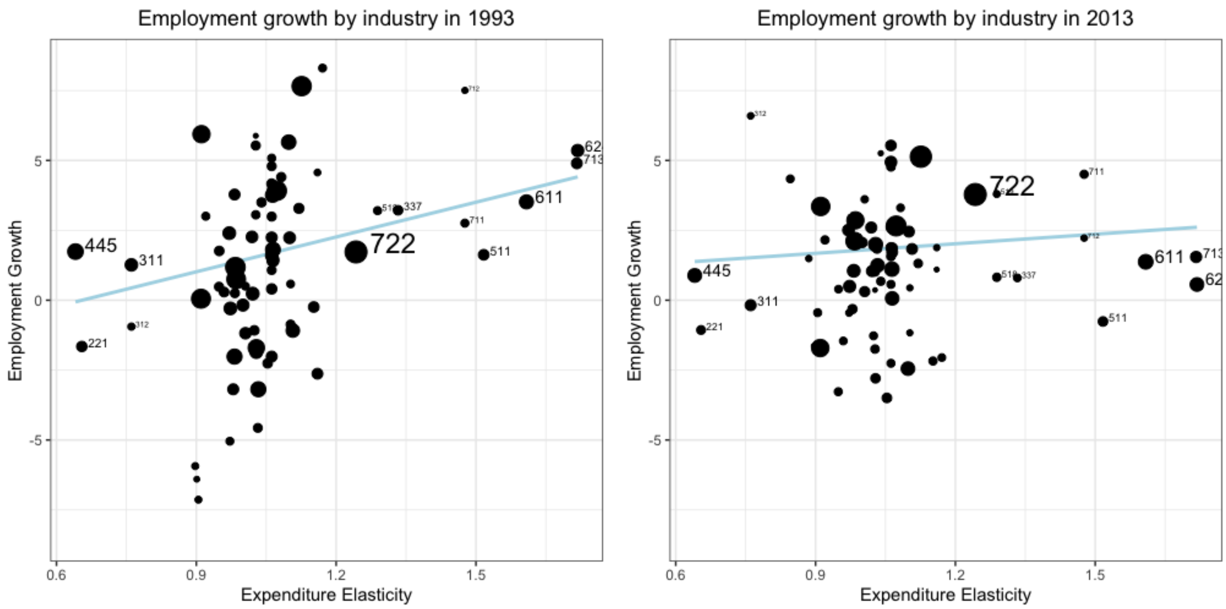


Table 6: Correlation of Employment growth and Expenditure Elasticity

	Employment Growth		
	(1)	(2)	(3)
Expenditure Elasticity η_ε	2.29	1.33	0.69
ε indexes 3-digit NAICS industries	(0.76)	(0.65)	(0.68)
Time FE	Y	Y	Y
Goods vs. Servs FE	N	Y	Y
1-digit NAICS FE	N	N	Y
(partial) R^2	0.01	0.00	0.00

Notes: 3076 observations. Industry clustered robust standard errors. Industry Observations weighted by employment.