Student Information

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Answer 1

a) Write a context-free grammar for the language $L_1 = \{w|w \in \{a,b\}^* \wedge$ w has twise as many b's as a's$\}$.

Let G be the grammar (V, Σ, R, S) for the language L_1 where

$$V = \{S, a, b\},$$
$$\Sigma = \{a, b\},$$
$$R = \{S \rightarrow aSbSb,$$
$$S \rightarrow bSaSb,$$
$$S \rightarrow bSbSa,$$
$$S \rightarrow e\}.$$

b) Write a context-free grammar for the language $L_2 = \{a^n b^m | m, n \in \mathbb{N} \wedge m \leq n \leq 2m\}$.

Let G be the grammar (V, Σ, R, S) for the language L_2 where

$$V = \{S, a, b\},$$
$$\Sigma = \{a, b\},$$
$$R = \{S \rightarrow aSbSa,$$
$$S \rightarrow aSaSb,$$
$$S \rightarrow bSaSa,$$
$$S \rightarrow bSa,$$
$$S \rightarrow aSb,$$
$$S \rightarrow e\}.$$

c) Formally define and draw a PDA that accepts L_1.

Let $(K, \Sigma, V, \Delta, p, \{q\})$ be the sextuple for the language L_1 where K, Σ, V, R are defined on part a.

Δ contains the following transitions:

1) $((p, e, e), (q, S))$

2) $((p, e, A), (q, x))$ for each rule $A \to x$ in R

3) $((q, a, a), (q, e))$ for each $a \in \Sigma$

Then,

$$\Delta = \{((p, e, e), (q, S)),$$
$$((p, e, S), (q, aSbSa)),$$
$$((p, e, S), (q, bSaSa)),$$
$$((p, e, S), (q, aSaSb)),$$
$$((p, e, S), (q, e)),$$
$$((p, a, a), (q, e)),$$
$$((p, b, b), (q, e))\}.$$

d) Write a context-free grammar for the language $L3 = L_1 \cup L_2$. Let $G_1 = (V1, \Sigma1, R1, S1)$ for the language L_1 and $G_2 = (V2, \Sigma2, R2, S2)$ for the language L_2.

Then $G_3 = (V1 \cup V2 \cup \{S\}, \Sigma1 \cup \Sigma2, R1 \cup R2 \cup \{S \to S1, S \to S2\})$ for the language $L3$.

$$V_3 = \{S, S_1, S_2, a, b\}$$
$$\Sigma = \{a, b\}$$
$$R = \{S \to S_1,$$
$$S \to S_2,$$
$$S_1 \to aS_1bS_1b$$
$$S_1 \to bS_1aS_1b$$
$$S_1 \to bS_1bS_1a$$
$$S_1 \to e$$
$$S_2 \to aS_2bS_2a$$
$$S_2 \to aS_2aS_2b$$
$$S_2 \to bS_2aS_2a$$
$$S_2 \to bS_2a$$
$$S_2 \to aS_2b$$
$$S_2 \to e$$
$$\}.$$

Answer 2

a)

b)

Answer 3

a)

b)