Student Information

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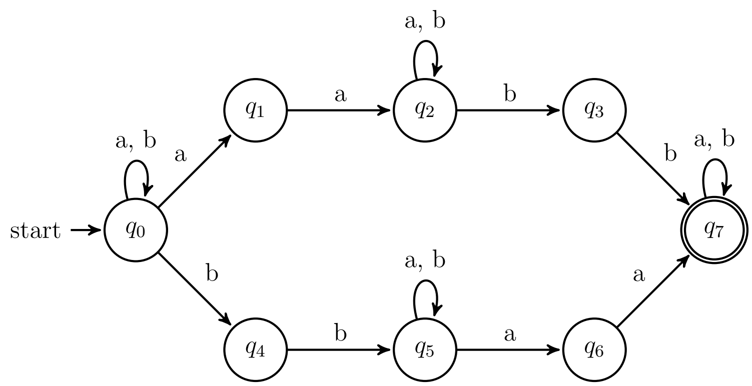
Answer 1

a)

$$((a \cup b)^*aa(a \cup b)^*bb(a \cup b)^*) \cup ((a \cup b)^*bb(a \cup b)^*aa(a \cup b)^*)$$

b)



Q $= \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}$
$\Sigma = \{a, b\}$
$s = q_0$
$F = \{q_7\}$

$\Delta)$

states	a	b
q_0	$\{q_0, q_1\}$	$\{q_0, q_4\}$
q_1	$\{q_2\}$	$\{\}$
q_2	$\{q_2\}$	$\{q_2, q_3\}$
q_3	$\{\}$	$\{q7\}$
q_4	$\{\}$	$\{q_5\}$
q_5	$\{q_5, q_6\}$	$\{q_5\}$
q_6	$\{q_7\}$	$\{\}$
q_7	$\{q_7\}$	$\{q_7\}$

c)

states	a	b	
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0, q_4\}$	$\{q_0, q_1\}$, $\{q_0, q_4\}$ added
$\{q_0, q_1\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_4\}$	$\{q_0, q_1, q_2\}$ added
$\{q_0, q_4\}$	$\{q_0, q_1\}$	$\{q_0, q_4, q_5\}$	$\{q_0, q_4, q_5\}$ added
$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_2, q_3, q_4\}$	$\{q_0, q_2, q_3, q_4\}$ added
$\{q_0, q_4, q_5\}$	$\{q_0, q_1, q_5, q_6\}$	$\{q_0, q_4, q_5\}$	$\{q_0, q_1, q_5, q_6\}$ added
$\{q_0, q_2, q_3, q_4\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_2, q_3, q_4, q_5, q_7\}$	$\{q_0, q_2, q_3, q_4, q_5, q_7\}$ added
$\{q_0, q_1, q_5, q_6\}$	$\{q_0, q_1, q_2, q_5, q_6, q_7\}$	$\{q_0, q_4, q_5\}$	$\{q_0, q_1, q_2, q_5, q_6, q_7\}$ added
$\{q_0, q_2, q_3, q_4, q_5, q_7\}$	$\{q_0, q_1, q_2, q_5, q_6, q_7\}$	$\{q_0, q_2, q_3, q_4, q_5, q_7\}$	no new state
$\{q_0, q_1, q_2, q_5, q_6, q_7\}$	$\{q_0, q_1, q_2, q_5, q_6, q_7\}$	$\{q_0, q_2, q_3, q_4, q_5, q_7\}$	no new state

Q $= \{\{q_0\}, \{q_0, q_1\}, \{q_0, q_4\}, \{q_0, q_1, q_2\}, \{q_0, q_4, q_5\}, \{q_0, q_2, q_3, q_4\}, \{q_0, q_1, q_5, q_6\},$
$\{q_0, q_2, q_3, q_4, q_5, q_7\}, \{q_0, q_1, q_2, q_5, q_6, q_7\}\}$

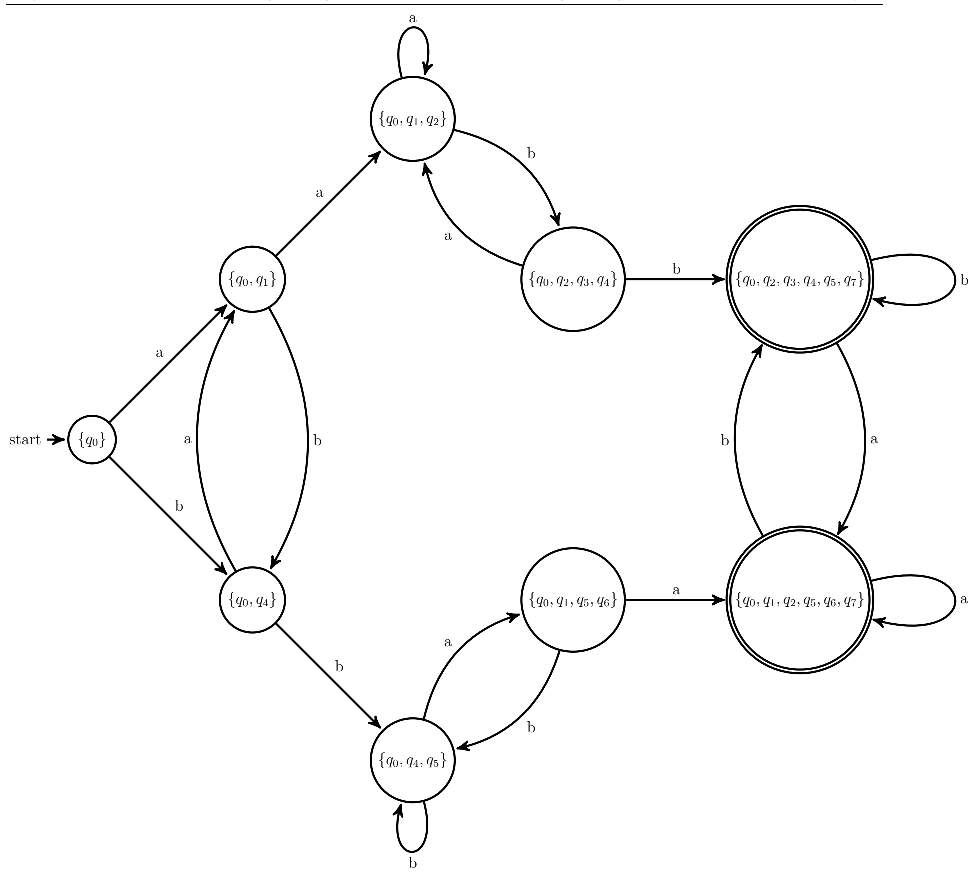
$\Sigma = \{a, b\}$

$s = \{q_0\}$

$F = \{\{q_0, q_2, q_3, q_4, q_5, q_7\}, \{q_0, q_1, q_2, q_5, q_6, q_7\}\}$

Δ)

states	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0, q_4\}$
$\{q_0, q_1\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_4\}$
$\{q_0, q_4\}$	$\{q_0, q_1\}$	$\{q_0, q_4, q_5\}$
$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_2, q_3, q_4\}$
$\{q_0, q_4, q_5\}$	$\{q_0, q_1, q_5, q_6\}$	$\{q_0, q_4, q_5\}$
$\{q_0, q_2, q_3, q_4\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_2, q_3, q_4, q_5, q_7\}$
$\{q_0, q_1, q_5, q_6\}$	$\{q_0, q_1, q_2, q_5, q_6, q_7\}$	$\{q_0, q_4, q_5\}$
$\{q_0, q_2, q_3, q_4, q_5, q_7\}$	$\{q_0, q_1, q_2, q_5, q_6, q_7\}$	$\{q_0, q_2, q_3, q_4, q_5, q_7\}$
$\{q_0, q_1, q_2, q_5, q_6, q_7\}$	$\{q_0, q_1, q_2, q_5, q_6, q_7\}$	$\{q_0, q_2, q_3, q_4, q_5, q_7\}$



d)

For NFA:

- $(q_0, bbabb)$
  - $(q_0, babb)$
    - (q_0, abb)
      - (q_0, bb)
        - (q_0, b)
          - (q_0, e)
          - (q_4, e)
        - (q_4, b)
          - (q_5, e)
      - (q_1, bb)
    - (q_4, abb)
  - $(q_4, babb)$
    - (q_5, abb)
      - (q_5, bb)
        - (q_5, b)
      - (q_6, bb)

For each possible path through the NFA, we either reach empty string without otherside the accepting state or we reach a state that does not accept our current character. Therefore, ω' is not accepted by our NFA.

For DFA:
$(q_0, bbabb) \vdash (q_0q_4, babb) \vdash (q_0q_4q_5, abb) \vdash (q_0q_1q_5q_6, bb) \vdash (q_0q_4q_5, b) \vdash (q_0q_4q_5, e)$

Since we reached the empty string otherside the accepting state, we conclude that ω' is not accepted by our DFA.

Answer 2

a)

Complement of a regular language is another regular language by the closure properties

Theorem: Complement of an irregular language is an irregular language.

Proof: Let A be an irregular language that does not obey the theorem. Then $\overline{A}$ is a regular language. Then, by the closure properties, $\overline{\overline{A}}$ is again, a regular language. But this contradicts with the first definition. Then we conclude, there is no irregular language of which its complement is a regular language.

By combining and using the properties above, we can conclude that there is a biconditionality between the regularity of language L_1 and L_2. That is, L_2 is regular if and only if L_1 is regular and L_2 is irregular if and only if L_1 is irregular.

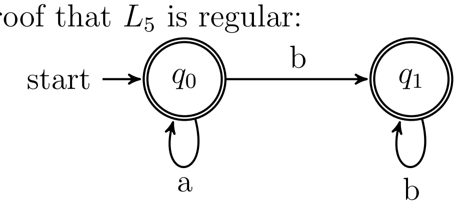
Assume that L_1 is regular.
Let l be the pumping length.
There is a split $w = xyz$ for all $|w| \geq l$, $|xy| \leq l$, $y \neq e$ and $xy^i z \in L_1$.
For even l: p = l
For odd l: p = $l + 1$

$w = a^{\mathrm{p}/2+1} b^{\mathrm{p}/2}$
First possible split: x=$a^{p/2+1}$, y=b^s, z=$b^{p/2-s}$ (s $\leq$ p/2 - 1)
For i $\geq$ 3, $xy^i z$ is not in the language. This split is not valid.
Second possible split: x=$a^{p/2+1-t}$, y=$a^t b^s$, z=$b^{p/2-s}$ (s $\leq$ p/2 - 1)
For i $>$ 1, $xy^i z$ is not in the language. This split is not valid.
Third possible split: x=$a^{p/2+1-t}$, y=a^t, z=$b^{p/2}$
For i = 0, $xy^i z$ is not in the language. This split is not valid.

Since no split can satisfy the pumping lemma conditions, L_1 is not regular. By the theorems above, neither L_2 is regular.

b)

Proof that L_5 is regular:



Since a NFA can be drawn for L_5, L_5 is regular.

Proof that L_6 is regular:
Since L_6 can be expressed by a regular expression L_6 is regular.

$L_4 \cup L_5 \cup L_6 = L_5 \cup L_6$ since L_4 is a subset of L_5. (In L_5's definition, if we choose m = n $\neq$ 0 we reach L_4). Thus, $L_4 \cup L_5 = L_5$

$L_5 \cup L_6$ is regular since L_5 and L_6 are both regular. (by the closure properties)