Student Information

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Answer 1

a)

b)

Answer 2

a) These are not enough in order to make meaningful comments about the distribution of the data. We also need the standard deviation of the data.

b) We test the null hypothesis $H_0 : \mu = 7.5$ against a one-sided left-tail alternative $H_A : \mu < 7.5$, because we are only interested to know if the mean of rating μ is less than 7.5.

Step 1: Test statistic. We are given $\sigma = 0.8$, $n = 256$, $\alpha = 0.05$, $\mu_0 = 7.5$, and from the sample $\bar{X} = 7.4$. The test statistic is

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{7.4 - 7.5}{0.8/\sqrt{256}} = -2.$$

Step 2: Acceptance and rejection regions. The critical value is

$$z_\alpha = z_{0.05} = 1.645.$$

With the left-tail alternative, we

$$\begin{gathered}\text{reject } H_0 \text{ if } Z < -1.645. \\ \text{accept } H_0 \text{ if } Z \geq -1.645.\end{gathered}$$

Our test statistic $Z = -2$ belongs to the rejection region; therefore, we reject the null hypothesis.

Restaurant A would not be in my list of candidate restaurants to order food from.

c) $\sigma = 1.0$, $n = 256$, $\alpha = 0.05$, $\mu_0 = 7.5$, $\bar{X} = 7.4$

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{7.4 - 7.5}{1.0/\sqrt{256}} = -1.6$$

Our test statistic $Z = -1.6$ belongs to the acceptance region; therefore, we accept the null hypothesis.

In this case, restaurant A would be in my list of candidate restaurants.

d) I will consider placing an order from a restaurant if and only if the rating of that restaurant is not significantly lower than 7.5. There is no need for resorting to a statistical test for values restaurants with a rating greater than or equal to 7.5, a greater rating will always be desired. We should resort statistical tests if the rating is lower than 7.5.

Answer 3

a) We test the null hypothesis $H_0 : \mu_A \geq \mu_B + 90$ against one sided left-tail alternative $H_A : \mu_A < \mu_B + 90$.

Since the variances are equal, we can use the pooled sample variance formula.

$$s_p^2 = \frac{(n-1)s_X^2 + (m-1)s_Y^2}{n+m-2} = \frac{19 \cdot 27.04 + 31 \cdot 519.84}{50} = 332.576$$

$$s_p \simeq 18.237$$

Test statistic t

$$t = \frac{\bar{X} - \bar{Y} - D}{s_p\sqrt{\frac{1}{n} + \frac{1}{m}}} = \frac{211 - 133 - 90}{18.237\sqrt{\frac{1}{20} + \frac{1}{32}}} \simeq -2.31$$

$t_\alpha = t_{0.01} = 2.403$ ($n + m - 2 = 50$ degrees of freedom)
Since the alternative hypothesis is left-tailed

$$\text{reject } H_0 \text{ if } t < -2.403.$$
$$\text{accept } H_0 \text{ if } t \geq -2.403.$$

Our test statistic $t = -2.31$ is greater than the critical value $-t_\alpha = -2.403$; therefore, we accept the null hypothesis.

The researcher can claim that the computer B provides a 90-minute or better improvement.

b) Use the same hypotheses from part a.
Test statistic t

$$t = \frac{\bar{X} - \bar{Y} - D}{\sqrt{\frac{s_X^2}{n} + \frac{s_Y^2}{m}}} = \frac{211 - 133 - 90}{\sqrt{\frac{27.04}{20} + \frac{519.84}{32}}} \simeq -2.86$$

Degrees of freedom v (Satterthwaite's approximation)

$$v = \frac{(\frac{s_X^2}{n} + \frac{s_Y^2}{m})^2}{\frac{s_X^4}{n^2(n-1)} + \frac{s_Y^4}{m^2(m-1)}} = \frac{(\frac{27.04}{20} + \frac{519.84}{32})^2}{\frac{731.1616}{400 \cdot 19} + \frac{270233.6256}{1024 \cdot 31}} \simeq 36$$

$t_{0.01} = 2.434$ (36 degrees of freedom)

Since the hypothesis is left-tailed

$$\text{reject } H_0 \text{ if } t < -2.434.$$
$$\text{accept } H_0 \text{ if } t \geq -2.434.$$

Our test statistic $t = -2.86$ is less than the critical value $-t_{0.01} = -2.434$; therefore, we reject the null hypothesis.

In this case, the researcher cannot claim that the computer B provides a 90-minute or better improvement.