

Student Information

Name : Emre Geçit

ID : 2521581

Answer 1

a) Write a context-free grammar for the language $L_1 = \{w | w \in \{a, b\}^* \wedge w \text{ has twice as many b's as a's}\}$.

Let G be the grammar (V, Σ, R, S) for the language L_1 where

$$\begin{aligned} V &= \{S, a, b\}, \\ \Sigma &= \{a, b\}, \\ R &= \{S \rightarrow aSbSb, \\ &\quad S \rightarrow bSaSb, \\ &\quad S \rightarrow bSbSa, \\ &\quad S \rightarrow e\}. \end{aligned}$$

b) Write a context-free grammar for the language $L_2 = \{a^n b^m | m, n \in \mathbb{N} \wedge m \leq n \leq 2m\}$.

Let G be the grammar (V, Σ, R, S) for the language L_2 where

$$\begin{aligned} V &= \{S, a, b\}, \\ \Sigma &= \{a, b\}, \\ R &= \{S \rightarrow aSbSa, \\ &\quad S \rightarrow aSaSb, \\ &\quad S \rightarrow bSaSa, \\ &\quad S \rightarrow bSa, \\ &\quad S \rightarrow aSb, \\ &\quad S \rightarrow e\}. \end{aligned}$$

c) Formally define and draw a PDA that accepts L_1 .

Let $(K, \Sigma, V, \Delta, p, \{q\})$ be the sextuple for the language L_1 where K, Σ, V, R are defined on part a. Δ contains the following transitions:

- 1) $((p, e, e), (q, S))$
- 2) $((p, e, A), (q, x))$ for each rule $A \rightarrow x$ in R
- 3) $((q, a, a), (q, e))$ for each $a \in \Sigma$

Then,

$$\begin{aligned} \Delta = \{ & ((p, e, e), (q, S)), \\ & ((p, e, S), (q, aSbSa)), \\ & ((p, e, S), (q, bSaSa)), \\ & ((p, e, S), (q, aSaSb)), \\ & ((p, e, S), (q, e)), \\ & ((p, a, a), (q, e)), \\ & ((p, b, b), (q, e)) \}. \end{aligned}$$

d) Write a context-free grammar for the language $L_3 = L_1 \cup L_2$.

Let $G_1 = (V_1, \Sigma_1, R_1, S_1)$ for the language L_1 and $G_2 = (V_2, \Sigma_2, R_2, S_2)$ for the language L_2 .

Then $G_3 = (V_1 \cup V_2 \cup \{S\}, \Sigma_1 \cup \Sigma_2, R_1 \cup R_2 \cup \{S \rightarrow S_1, S \rightarrow S_2\})$ for the language L_3 .

$$\begin{aligned} V_3 &= \{S, S_1, S_2, a, b\} \\ \Sigma &= \{a, b\} \\ R &= \{S \rightarrow S_1, \\ & \quad S \rightarrow S_2, \\ & \quad S_1 \rightarrow aS_1bS_1b, \\ & \quad S_1 \rightarrow bS_1aS_1b, \\ & \quad S_1 \rightarrow bS_1bS_1a, \\ & \quad S_1 \rightarrow e, \\ & \quad S_2 \rightarrow aS_2bS_2a, \\ & \quad S_2 \rightarrow aS_2aS_2b, \\ & \quad S_2 \rightarrow bS_2aS_2a, \\ & \quad S_2 \rightarrow bS_2a, \\ & \quad S_2 \rightarrow aS_2b, \\ & \quad S_2 \rightarrow e\} \end{aligned}$$

Answer 2

Given $G_1 = \{V, \Sigma, R, S\}$ where $V = \{0, 1, S, A\}$, $\Sigma = \{0, 1\}$, and $R = \{S \rightarrow AS|e, A \rightarrow A1|0A1|01\}$

a) Show that G_1 is ambiguous.

For the string 00111, there are more than one possible rightmost derivations.

1) $S \rightarrow AS \rightarrow A \rightarrow 0A1 \rightarrow 0A11 \rightarrow 00111$

2) $S \rightarrow AS \rightarrow A \rightarrow A1 \rightarrow 0A11 \rightarrow 00111$

b) Give an unambiguous grammar for $L(G_1)$. (i.e. disambiguate the given grammar.)

The reason why this grammar is ambiguous is that whenever the derivation includes the rules $A \rightarrow 0A1$ and $A \rightarrow A1$ these rules' application order can be swapped without affecting the final string. This issue can be solved in a way that restricts the application order of the rules. For this purpose, we will define a new nonterminal B , and define a transition from A to B . After some modifications, G_1 is as follows:

$$V = \{S, A, B, 0, 1\}$$

$$\Sigma = \{0, 1\}$$

$$R = \{S \rightarrow AS,$$

$$S \rightarrow e,$$

$$A \rightarrow 0A1,$$

$$A \rightarrow B,$$

$$B \rightarrow B1,$$

$$B \rightarrow 01\}.$$