

Student Information

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Answer 1

a) $N \geq 0.25 \left(\frac{z_{\alpha/2}}{\varepsilon} \right)^2$
 $1 - \alpha = 0.99 \implies \alpha = 0.01$
 $z_{\alpha/2} = z_{0.005} \approx 2.5758$
 $N \geq 0.25 \left(\frac{2.5758}{0.02} \right)^2$
 $N \geq 4146.716$

The size of the Monte Carlo study must be at least $N = 4147$.

b)

- $E(X) = \frac{\alpha}{\lambda}$ for a Gamma distributed random variable X . Substituting $\alpha = 190$ and $\lambda = 0.15$ we get:

$$E(X)_{gamma} = \frac{190}{0.15} = 1266.\bar{6}$$

The expected value for the weight of an automobile is $1266.\bar{6}$

- $E(X)_{gamma} = \frac{\alpha}{\lambda} = \frac{110}{0.01} = 11000$
- Expected value for the total weights of all automobiles that pass over the bridge on a day is equal to the number of automobiles that pass over the bridge on a day multiplied by the expected value for the weight of an automobile.

We know the expected value for the weight of an automobile is $E(X) = 1266.\bar{6}$.

The number of automobiles that pass over the bridge on a day is a Poisson random variable with $\lambda = 50$.

$$E(X)_{poisson} = \lambda = 50$$

Expected value for the total weights of all automobiles that pass over the bridge on a day is:

$$50 \times 1266.\bar{6} = 63333.\bar{3}$$

- The number of trucks that pass over the bridge on a day is a Poisson random variable with $\lambda = 10$. Expected value for the total weights of all trucks that pass over the bridge on a day is

$$10 \times 11000 = 110000$$

Answer 2

- Probability that the total weight of all the vehicles that pass over the bridge on a day is more than 200 tons = 0.225464
- Estimated total weight = 173324.013013
- Standard deviation = 36312.385849

For testing the accuracy of the estimator of the total weight of all the vehicles, we will construct a %99 confidence interval.

$$\begin{aligned} & \hat{\theta} \pm z_{\alpha/2} \cdot s(\hat{\theta}) \\ & 173324.013013 \pm 2.5758 * \frac{36312.385849}{\sqrt{4147}} \\ & 173324.013013 \pm 1452.4 \\ & [171871.56731, 174776.45872] \end{aligned}$$

A confidence interval of %99 only has a margin error of 1.4 tons, that is very small with compared to 173.3 tons, only a %0.84 margin error. This margin also includes the expected value we have calculated. We can conclude that our estimator is pretty accurate.