Student Information

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Answer 1

a) Write a context-free grammar for the language $L_1 = \{w | w \in \{a, b\}^* \wedge \text{ w has twise as many b's as a's}\}$.

Let G be the grammar (V, Σ, R, S) for the language L_1 where

$$
\begin{aligned}
V &= \{S, a, b\}, \\
\Sigma &= \{a, b\}, \\
R &= \{S \rightarrow aSbSb, \\
&\quad S \rightarrow bSaSb, \\
&\quad S \rightarrow bSbSa, \\
&\quad S \rightarrow e\}.
\end{aligned}
$$

b) Write a context-free grammar for the language $L_2 = \{a^n b^m | m, n \in \mathbb{N} \wedge m \leq n \leq 2m\}$.

Let G be the grammar (V, Σ, R, S) for the language L_2 where

$$
\begin{aligned}
V &= \{S, a, b\}, \\
\Sigma &= \{a, b\}, \\
R &= \{S \rightarrow aSbSa, \\
&\quad S \rightarrow aSaSb, \\
&\quad S \rightarrow bSaSa, \\
&\quad S \rightarrow bSa, \\
&\quad S \rightarrow aSb, \\
&\quad S \rightarrow e\}.
\end{aligned}
$$

c) Formally define and draw a PDA that accepts L_1.

Let $(K, \Sigma, V, \Delta, p, \{q\})$ be the sextuple for the language L_1 where K, Σ, V, R are defined on part a. Δ contains the following transitions:

1) $((p, e, e), (q, S))$

2) $((p, e, A), (q, x))$ for each rule $A \to x$ in R

3) $((q, a, a), (q, e))$ for each $a \in \Sigma$

Then,

$$\begin{aligned}\Delta = \{&((p, e, e), (q, S)),\\ &((p, e, S), (q, aSbSa)),\\ &((p, e, S), (q, bSaSa)),\\ &((p, e, S), (q, aSaSb)),\\ &((p, e, S), (q, e)),\\ &((p, a, a), (q, e)),\\ &((p, b, b), (q, e))\}.\end{aligned}$$

d) Write a context-free grammar for the language $L_3 = L_1 \cup L_2$.

Let $G_1 = (V_1, \Sigma_1, R_1, S_1)$ for the language L_1 and $G_2 = (V2, \Sigma_2, R_2, S_2)$ for the language L_2.

Then $G_3 = (V_1 \cup V2 \cup \{S\}, \Sigma_1 \cup \Sigma_2, R_1 \cup R_2 \cup \{S \to S_1, S \to S_2\})$ for the language L_3.

$$\begin{aligned}V_3 &= \{S, S_1, S_2, a, b\}\\ \Sigma &= \{a, b\}\\ R &= \{S \to S_1,\\ &\quad S \to S_2,\\ &\quad S_1 \to aS_1bS_1b,\\ &\quad S_1 \to bS_1aS_1b,\\ &\quad S_1 \to bS_1bS_1a,\\ &\quad S_1 \to e,\\ &\quad S_2 \to aS_2bS_2a,\\ &\quad S_2 \to aS_2aS_2b,\\ &\quad S_2 \to bS_2aS_2a,\\ &\quad S_2 \to bS_2a,\\ &\quad S_2 \to aS_2b,\\ &\quad S_2 \to e\}\end{aligned}$$

Answer 2

Given $G_1 = \{V, \Sigma, R, S\}$ where $V = \{0, 1, S, A\}$, $\Sigma = \{0, 1\}$, and $R = \{S \to AS|e, A \to A1|0A1|01\}$

a) Show that G_1 is ambiguous.

For the string 00111, there are more than one possible rightmost derivations.

1) $S \to AS \to A \to 0A1 \to 0A11 \to 00111$

2) $S \to AS \to A \to A1 \to 0A11 \to 00111$

b) Give an unambiguous grammar for $L(G_1)$. (i.e. disambiguate the given grammar.)

The reason why this grammar is ambiguous is that whenever the derivation includes the rules $A \to 0A1$ and $A \to A1$ these rules' application order can be swapped without affecting the final string. This issue can be solved in a way that restricts the application order of the rules. For this purpose, we will define a new nonterminal B, and define a transition from A to B. After some modifications, G_1 is as follows:

$$
\begin{aligned}
V &= \{S, A, B, 0, 1\} \\
\Sigma &= \{0, 1\} \\
R &= \{S \to AS, \\
&\quad\; S \to e, \\
&\quad\; A \to 0A1, \\
&\quad\; A \to B, \\
&\quad\; B \to B1, \\
&\quad\; B \to 01\}
\end{aligned}
$$