Student Information

Full Name : Emre Geçit
Id Number : 2521581

Q. 1

1. $\neg(p \wedge q) \leftrightarrow (\neg q \rightarrow p)$	
2. $(\neg(p \wedge q) \wedge (\neg q \rightarrow p)) \vee ((p \wedge q) \wedge \neg(\neg q \rightarrow p))$	Logical equivalency
3. $(\neg(p \wedge q) \wedge (q \vee p)) \vee ((p \wedge q) \wedge \neg(q \vee p))$	Lemma implication
4. $(\neg(p \wedge q) \wedge (q \vee p)) \vee ((p \wedge q) \wedge (\neg q \wedge \neg p))$	De Morgan's law
5. $((\neg p \vee \neg q) \wedge (q \vee p)) \vee (p \wedge q \wedge \neg q \wedge \neg p)$	Associative law
6. $((\neg p \vee \neg q) \wedge (q \vee p)) \vee F$	Negation law
7. $((\neg p \vee \neg q) \wedge (q \vee p))$	Negation law
8. $((\neg p \vee \neg q) \wedge (p \vee q)))$	Commutative law
9. $((p \vee q) \wedge (\neg p \vee \neg q))$	Commutative law

Q. 2

a. $\forall x \forall y \forall z \forall t(E(x, y) \wedge E(z, y) \wedge (x \neq y)) \rightarrow \neg(I(x, t) \wedge I(z, t))$
b. $\exists x \forall y \forall z(I(x, y) \wedge S(x, z) \rightarrow (x = z))$
c. Let m denote the Medicine Faculty.
$\forall j \forall x1 \forall x2 \forall x3(J(j, m) \wedge A(x1, j) \wedge A(x2, j) \wedge A(x3, j) \rightarrow ((x1 = x2) \vee (x2 = x3) \vee (x1 = x3)))$

Q. 3

a.

1. $p \vee \neg q$		Premise
2. $p \vee r$		Premise
| 3. $r \to q$		Assumption
| 4. p		Assumption
| 5. r		Assumption
| 6. q		$\to$ e 3, 5
| | 7. $\neg q$		Assumption
| | 8. $\bot$		$\neg e$ 6, 7
| | 9. p		$\bot$ e 8
| 10. p		Assumption
| 11. p		$\vee e$ 1, 7-9, 10
12. p		$\vee e$ 2, 4, 5-11
13. $(r \to q) \to p$		$\to$i 3-12

b.

| 1. $(q \to p) \to q$		Assumption
| 2. $q \vee \neg q$		LEM
| 3. q		Assumption
| 4. $\neg q$		Assumption
| | 5. q		Assumption
| | 6. $\bot$		$\neg e$ 4, 5
| | 7. p		$\neg e$ 6
| 8. $q \to p$		$\to e$ 5-7
| 9. q		$\to e$ 1, 8
10. q		$\vee e$ 2, 3-4, 5-10

Q. 4

a.

1.	$\neg(\forall x(P(x) \to Q(x)))$	Premise
2.	$\neg\exists x(P(x) \wedge \neg Q(x))$	Assumption
3.	$P(x0) \wedge \neg Q(x0)$	Assumption
4.	$\exists(P(x) \wedge \neg Q(x))$	$\exists$ i 3
5.	$\perp$	$\neg e$ 2, 4
6.	$\neg(P(x0) \wedge \neg Q(x0))$	$\neg$e 3-5
7.	$P(x0)$	Assumption
8.	$\neg Q(x0)$	Assumption
9.	$P(x0) \wedge \neg Q(x0)$	$\wedge$ i 7, 8
10.	$\perp$	$\neg$ e 6, 9
11.	$\neg\neg Q(x0)$	$\neg$ i 8-10
12.	$P(x0) \to Q(x0)$	$\to$ i 7-11
13.	$\forall x(P(x) \to Q(x))$	$\forall$ i 2-12
14.	$\perp$	$\neg$ i 1, 13
15.	$\exists x(P(x) \wedge \neg Q(x))$	$\neg$ e 2-14