Student Information

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Answer 1

$\sum_{n=1}^{\infty} a_n x^n = \sum_{n=1}^{\infty} a_{n-1} x^{n-1} + \sum_{n=1}^{\infty} 2^n x^n$

$\sum_{n=1}^{\infty} a_n x^n = \sum_{n=1}^{\infty} a_{n-1} x^{n-1} + \sum_{n=1}^{\infty} u^n$ (substitute 2x with u)

$A(x) - 1 = A(x) * x + \sum_{n=1}^{\infty} u^n$

$A(x) - 1 = A(x) * x + \dfrac{1}{1-u}$

$A(x) - 1 = A(x) * x + \dfrac{1}{1-2x}$

$(x-1) * A(x) = \dfrac{1}{1-2x}$

$A(x) = \dfrac{1}{(1-2x)*(x-1)}$

$A(x) = \dfrac{2}{1-2x} + \dfrac{-1}{1-x}$ (partial fractions)

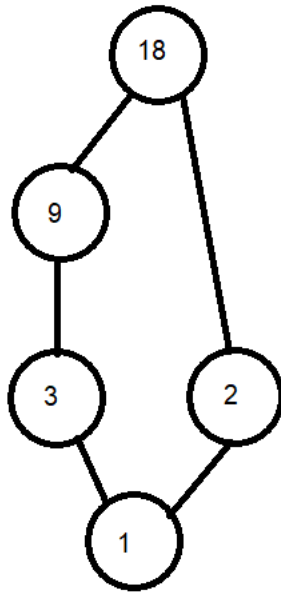$A(x) = -1 * (1, 1, 1, ..., 1, ...) + 2 * (1, 2, 4, ..., 2^n, ...)$

$a_n = -1 + 2 * 2^n$

$a_n = 2^{n+1} - 1$

Answer 2

a) Hasse diagram of R:

b) Matrix representation of R:
$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

c) None of the pairs have more than one Greatest Lower Bound or Lowest Upper Bound, therefore (A, R) is a lattice.

d) $R_s = R \cup R^{-1}$

$$R = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$R^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$R_s = R_s = R \cup R^{-1} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

e) 2 and 9 are not comparable since 2|9 and 9|2 are both false, 3 and 18 are comparable since 3|18 is true.

Answer 3

a) For the diagonal there can be 2^n possibilities. For the cells that are not in the diagonal, for each symmetric pair of cells, there can be 3 possible states ((0,0),(0,1),(1,0)). There are $(n^2 - n)/2$ symmetric cell pairs. Therefore, there are $2^n * 3^{(n^2-n)/2}$ possible binary anti-symmetric relations for a set with n elements.

b) For a reflexive relation, the diagonal should be filled with ones, so for the diagonal there is one possibility. For the cells that are not in the diagonal, for each symmetric pair of cells, there can be 3 possible states ((0,0),(0,1),(1,0)). There are $(n^2 - n)/2$ symmetric cell pairs. Therefore, there are $3^{(n^2-n)/2}$ possible binary, reflexive and anti-symmetric relations for a set with n elements.