Detection of a Transient Signal with Unknown Arrival Time and Amplitude

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1 Introduction

Detecting weak, short-lived signals in noise is a basic task in statistical signal processing. In many practical settings the signal is only partially known: it may occur at an unknown time within the observation window and with an unknown amplitude. This leads to a composite hypothesis testing problem, where the "signal-present" hypothesis corresponds to a family of distributions indexed by unknown parameters [1, Ch. 4–5].

In this project we study transient detection in a discrete-time observation vector $y \in \mathbb{R}^N$ under the model $H_0 : y = v$ (noise only) versus $H_1 : y = As_\tau + v$ (signal plus noise), where s_τ is a known template embedded at an unknown arrival time τ and scaled by an unknown (two-sided) amplitude $A \in \mathbb{R}$, and where the noise is modeled as white Gaussian $v \sim \mathcal{N}(0, \sigma^2 I)$.

Because no prior distribution is assumed for (A, τ), we adopt a frequentist approach and construct a generalized likelihood ratio test (GLRT). The resulting detector has a simple and interpretable form: it computes a matched-filter score for each candidate shift and then scans (maximizes) over τ to account for the unknown arrival time. Since the maximization over shifts induces dependence across the scan statistics, the threshold required to achieve a desired false-alarm probability is selected by Monte Carlo calibration under H_0.

Numerical experiments for a rectangular pulse illustrate the key behaviors: the null distribution of the scan statistic is right-skewed/heavy-tailed due to the maximization step; ROC curves improve monotonically as the amplitude increases; and at a fixed false-alarm level (e.g. $\alpha = 0.05$) the detection probability transitions from near chance at $A = 0$ (where H_1 reduces to H_0) to near-certain detection for sufficiently large amplitudes.

2 Theory

2.1 Problem setup

We observe a finite-length discrete-time measurement vector

$$y = \big(y[0], y[1], \dots, y[N-1]\big)^T \in \mathbb{R}^N, \tag{1}$$

and we want to decide whether it contains a transient signal buried in additive noise. This type of detection problem fits the standard framework of signal detection and parameter estimation, in particular, signals to be detected often contain unknown parameters that must be accounted for in the detector design [1, p. 169].

2.1.1 Signal model

Let

$$s = \big(s[0], s[1], \dots, s[L-1]\big)^T \in \mathbb{R}^L \tag{2}$$

be a known template describing the transient shape, with length $L \le N$. The transient occurs at an unknown time τ, where we restrict

$$\tau \in \mathcal{T} := \{0, 1, \dots, N-L\}, \tag{3}$$

so that the pulse fits entirely inside the observation window. To embed the length-L template into the N-dimensional observation space, define the shifted template $s_\tau \in \mathbb{R}^N$ by

$$(s_\tau)[n] = \begin{cases} s[n-\tau], & n = \tau, \tau+1, \dots, \tau+L-1, \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

This is simply the template starting at position τ with zeros elsewhere. The transient is assumed to enter the data with an unknown amplitude

$$A \in \mathbb{R}. \tag{5}$$

In this project I allow A to be positive or negative (a two-sided amplitude model), meaning the pulse polarity is not assumed to be known.

2.1.2 Noise model

The measurement is corrupted by additive noise

$$v \in \mathbb{R}^N. \tag{6}$$

As a baseline model, I assume

$$v \sim \mathcal{N}(0, \sigma^2 I_N), \tag{7}$$

i.e. zero-mean white Gaussian noise with known variance $\sigma^2 > 0$.

2.1.3 Hypothesis testing formulation

The detection problem is formulated as a binary hypothesis test:

$$\mathcal{H}_0 : \quad y = v \qquad \text{(noise only)}, \tag{8}$$

$$\mathcal{H}_1 : \quad y = A\, s_\tau + v \qquad \text{(signal + noise)}. \tag{9}$$

Under $\mathcal{H}_1$, both the arrival time $\tau \in \mathcal{T}$ and the amplitude $A \in \mathbb{R}$ are unknown. Hence $\mathcal{H}_1$ is a composite hypothesis: rather than a single distribution, it represents a family of distributions indexed by the parameter pair (A, τ) [1, p. 169]. The goal is to construct a decision rule of the form

$$\text{decide } \mathcal{H}_1 \iff T(y) > \gamma, \tag{10}$$

where $T(y)$ is a test statistic and γ is a threshold. Hence the problem is to decide a suitable test statistic and threshold.

2.2 Deriving the test statistic and threshold

Before deriving a detector, it is useful to emphasize that there are (at least) two broad design philosophies for handling unknown parameters in detection problems [1, ch. 4–5]. One is a Bayesian approach, where unknown parameters are modeled as random and one specifies (i) an observation model (likelihood), (ii) a prior distribution for the unknown parameters, and (iii) a loss/cost function that quantifies the penalty of making an error [1, p. 114]. This is conceptually attractive, but it requires assuming priors and costs, which we will try to avoid. The second approach is frequentist: parameters are treated as unknown but fixed, and one typically designs tests by controlling the false-alarm probability (Neyman–Pearson) and then maximizing detection performance subject to this constraint. In composite hypothesis settings, a common frequentist strategy is to combine estimation and detection via the generalized likelihood ratio test (GLRT), which replaces unknown parameters by their maximum-likelihood (ML) estimates under each hypothesis [1, p. 170]. A practical challenge in the frequentist route is that threshold selection may not admit a clean closed form once the test statistic involves maximization/search over unknown parameters; in such cases, thresholds can be calibrated numerically, which we will discuss 2.2.4 as well as in chapter 3.

In this report I adopt the frequentist approach. In what follows, I derive a GLRT-based test statistic for the transient detection problem with unknown amplitude and arrival time, and then describe how the detection threshold is chosen to satisfy a prescribed false-alarm probability.

2.2.1 GLRT-based test statistic

Under the white Gaussian noise assumption $v \sim \mathcal{N}(0, \sigma^2 I_N)$, the conditional density of Y given (A, τ) under $\mathcal{H}_1$ is

$$f_1(y; A, \tau) = \frac{1}{(2\pi\sigma^2)^{N/2}} \exp\left(-\frac{1}{2\sigma^2} \|y - As_\tau\|^2 \right), \qquad (A, \tau) \in \mathbb{R} \times \mathcal{T}, \tag{11}$$

and under $\mathcal{H}_0$ it is

$$f_0(y) = \frac{1}{(2\pi\sigma^2)^{N/2}} \exp\left(-\frac{1}{2\sigma^2} \|y\|^2 \right). \tag{12}$$

Note, that Since (A, τ) are unknown under $\mathcal{H}_1$, the GLRT compares

$$\Lambda_{\mathrm{GLRT}}(y) := \frac{\max_{A \in \mathbb{R},\ \tau \in \mathcal{T}} f_1(y; A, \tau)}{f_0(y)} \ \gtrless_{\mathcal{H}_0}^{\mathcal{H}_1} \ \eta, \tag{13}$$

for some threshold $\eta > 0$ chosen to satisfy a false-alarm constraint.

2.2.2 Maximization over the amplitude (two-sided case)

Fix an arrival time $\tau \in \mathcal{T}$ and consider the likelihood under $\mathcal{H}_1$,

$$f_1(y; A, \tau) = \frac{1}{(2\pi\sigma^2)^{N/2}} \exp\left(-\frac{1}{2\sigma^2} \|y - As_\tau\|^2 \right).$$

For fixed τ, the only term that depends on A is the exponential factor. Since the exponential function is strictly increasing, maximizing $f_1(y; A, \tau)$ over A is equivalent to maximizing its exponent, i.e.

$$\arg\max_{A \in \mathbb{R}} f_1(y; A, \tau) \ = \ \arg\max_{A \in \mathbb{R}} \left(-\frac{1}{2\sigma^2} \|y - As_\tau\|^2 \right) \ = \ \arg\min_{A \in \mathbb{R}} \|y - As_\tau\|^2.$$

Next, expand the quadratic function of A:

$$\begin{aligned} \|y - As_\tau\|^2 &= (y - As_\tau)^T (y - As_\tau) \\ &= y^T y - 2A\, s_\tau^T y + A^2\, s_\tau^T s_\tau \\ &= \|y\|^2 - 2A\, s_\tau^T y + A^2 \|s_\tau\|^2. \end{aligned} \tag{14}$$

This is a parabola in A with positive coefficient $\|s_\tau\|^2$, so its minimum is found by setting the derivative to zero:

$$\frac{d}{dA} \|y - As_\tau\|^2 = -2\, s_\tau^T y + 2A\|s_\tau\|^2 = 0.$$

Solving for A gives the ML estimate

$$\widehat{A}(\tau) = \frac{s_\tau^T y}{\|s_\tau\|^2}. \tag{15}$$

To compute the maximized likelihood value, substitute $A = \widehat{A}(\tau)$ back into (14). Using the optimality condition $A\|s_\tau\|^2 = s_\tau^T y$ at $A = \widehat{A}(\tau)$, we obtain

$$\begin{aligned} \min_{A \in \mathbb{R}} \|y - As_\tau\|^2 &= \|y\|^2 - 2\widehat{A}(\tau)\, s_\tau^T y + \widehat{A}(\tau)^2 \|s_\tau\|^2 \\ &= \|y\|^2 - 2\frac{(s_\tau^T y)^2}{\|s_\tau\|^2} + \frac{(s_\tau^T y)^2}{\|s_\tau\|^2} \\ &= \|y\|^2 - \frac{(s_\tau^T y)^2}{\|s_\tau\|^2}. \end{aligned} \tag{16}$$

Therefore the likelihood maximized over A is

$$\max_{A \in \mathbb{R}} f_1(y; A, \tau) = \frac{1}{(2\pi\sigma^2)^{N/2}} \exp\left(-\frac{1}{2\sigma^2} \left[\|y\|^2 - \frac{(s_\tau^T y)^2}{\|s_\tau\|^2} \right] \right). \tag{17}$$

2.2.3 Maximization over arrival time

The remaining GLRT step is to maximize the likelihood over the (unknown) arrival time $\tau \in \mathcal{T}$:

$$\max_{\tau\in\mathcal{T}} \max_{A\in\mathbb{R}} f_1(y; A, \tau).$$

Using (17), and noting that the prefactor $(2\pi\sigma^2)^{-N/2}$ does not depend on τ, we have

$$\begin{aligned} \max_{\tau\in\mathcal{T}} \max_{A\in\mathbb{R}} f_1(y; A, \tau) &\propto \max_{\tau\in\mathcal{T}} \exp\left(-\frac{1}{2\sigma^2} \left[\|y\|^2 - \frac{(s_\tau^T y)^2}{\|s_\tau\|^2} \right] \right) \\ &= \exp\left(-\frac{1}{2\sigma^2} \|y\|^2 \right) \max_{\tau\in\mathcal{T}} \exp\left(\frac{1}{2\sigma^2} \frac{(s_\tau^T y)^2}{\|s_\tau\|^2} \right). \end{aligned} \tag{18}$$

Since the exponential function is strictly increasing, maximizing the exponential over τ is equivalent to maximizing its argument. Therefore the GLRT is equivalent to a threshold test on

$$T(y) := \max_{\tau\in\mathcal{T}} \frac{(s_\tau^T y)^2}{\|s_\tau\|^2}, \qquad \text{decide } \mathcal{H}_1 \iff T(y) > \gamma, \tag{19}$$

for some threshold γ.

2.2.4 Neyman-Pearson with Monte Carlo calibration

Although the statistic $T(y)$ has a simple closed form, choosing γ to satisfy a Neyman–Pearson false-alarm constraint

$$\mathbb{P}\big(T(Y) > \gamma \mid \mathcal{H}_0\big) = \alpha \tag{20}$$

requires the distribution of $T(Y)$ under $\mathcal{H}_0$ [1, p. 29]. Note that α refers to the global false-alarm probability of the entire detector, i.e. after taking the maximum over all candidate arrival times $\tau \in \mathcal{T}$. In other words, α controls the probability that the scan-based test declares $\mathcal{H}_1$ at any shift when in fact $\mathcal{H}_0$ is true, rather than a per-shift false-alarm probability. Under $\mathcal{H}_0$, for each fixed τ,

$$\frac{s_\tau^T Y}{\sigma\|s_\tau\|} \sim \mathcal{N}(0,1) \quad \Rightarrow \quad \frac{(s_\tau^T Y)^2}{\sigma^2\|s_\tau\|^2} \sim \chi_1^2,$$

but $T(Y)$ is the maximum of these quantities. Moreover, the random variables $\{s_\tau^T Y\}_{\tau\in\mathcal{T}}$ are generally correlated because neighboring templates overlap, with covariance

$$\mathrm{Cov}\big(s_\tau^T Y,\ s_{\tau'}^T Y \mid \mathcal{H}_0\big) = \sigma^2\, s_\tau^T s_{\tau'}.$$

As a result, $T(Y)$ is the maximum of a correlated collection of χ^2-type variables. Hence threshold selection in our case is analytically difficult to deduce. A practical approach is to calibrate γ by Monte Carlo simulation [2, p. 325] [3, p. 7] [4, p. 4].

3 Experimental Setup

3.1 Signal, noise, and search space

We use the signal and noise model of Section 2:

$$H_0: \ y = v, \qquad H_1: \ y = As_\tau + v, \qquad v \sim \mathcal{N}(0, \sigma^2 I_N),$$

where the arrival time τ is constrained to $\mathcal{T} = \{0, 1, \ldots, N-L\}$ so that the length-L pulse fits inside the length-N observation window. In all experiments, the template is a rectangular pulse of length L:

$$s[n] = 1, \qquad n = 0, 1, \ldots, L-1,$$

embedded into $\mathbb{R}^N$ via the zero-padded shift definition in (4). Since $\|s_\tau\|^2 = \|s\|^2 = L$ for all τ, the denominator in $T(y)$ is constant across shifts in the rectangular-pulse setting. For the paper-quality runs we used:

$$N = 400, \quad L = 60, \quad \sigma^2 = 1, \quad |\mathcal{T}| = N - L + 1 = 341.$$

Thus the scan maximization is performed over 341 candidate arrival times.

3.2 Monte Carlo calibration of γ under H_0

Because $T(Y)$ is the maximum of a correlated collection of χ^2-type quantities, its null distribution does not admit a simple closed form in general, and we therefore calibrate γ by Monte Carlo simulation under H_0. Concretely, for each target α we:

1. Generate M amount of i.i.d. noise-only samples $Y^{(i)} \sim \mathcal{N}(0, \sigma^2 I_N)$.
2. Compute $T(Y^{(i)})$ for each trial.
3. Set $\gamma(\alpha)$ to the empirical $(1-\alpha)$-quantile of the values $\{T(Y^{(i)})\}_{i=1}^{M}$, so that approximately an α fraction exceed the threshold.

In the reported runs we used $M = 200{,}000$. We consider the following false-alarm levels:

$$\alpha \in \{0.001,\ 0.005,\ 0.01,\ 0.02,\ 0.05,\ 0.1\}.$$

A histogram of the simulated null values $\{T(Y^{(i)})\}_{i=1}^{M}$ together with the calibrated $\gamma(0.05)$ is included as a diagnostic figure (Fig. 1), illustrating that γ lies in the upper tail of the empirical H_0 distribution.

3.3 Performance metrics and Monte Carlo estimation

After calibrating $\gamma(\alpha)$, we estimate false-alarm and detection probabilities by Monte Carlo with an independent set of trials. For each chosen threshold $\gamma(\alpha)$ we estimate

$$P_F(\alpha) := \mathbb{P}(T(Y) > \gamma(\alpha) \mid H_0), \qquad P_D(\alpha; A) := \mathbb{P}(T(Y) > \gamma(\alpha) \mid H_1, A).$$

For a fixed amplitude A, the ROC curve is obtained by sweeping α (which sweeps $\gamma(\alpha)$), and plotting the estimated pairs $\big(\widehat{P}_F(\alpha), \widehat{P}_D(\alpha; A)\big)$. In the reported runs we used $M = 50{,}000$ trials per operating point, and we report Monte Carlo uncertainty via an approximate 95% confidence interval

$$\widehat{p} \pm 1.96\sqrt{\widehat{p}(1-\widehat{p})/M_{\text{eval}}},$$

where $\widehat{p}$ denotes either $\widehat{P}_F$ or $\widehat{P}_D$.

3.4 Amplitude sweeps and figure set

To visualize performance across signal strength, we evaluate multiple amplitudes

$$A \in \{0.0, 0.1, 0.2, \ldots, 1.0\}.$$

This grid focuses on the "transition" regime where detection performance moves from near-chance ($P_D \approx P_F$) to near-certain detection ($P_D \approx 1$), yielding informative ROC curves (Fig. 4) and an informative P_D-versus-A curve at a fixed operating point $\alpha = 0.05$ (Fig. 5). In addition to aggregate Monte Carlo performance curves, we include single-run illustrations to connect the statistic to the data:

- one realization under H_0 (noise only) and under H_1 (signal+noise) (Fig. 2);
- the scan curve $\tau \mapsto \frac{(s_\tau^T y)^2}{\|s_\tau\|^2}$ with the threshold γ, as well as the true and estimated arrival times (Fig. 3);
- a small table row reporting $(A_{\text{true}}, \tau_{\text{true}})$ and $(\widehat{A}, \widehat{\tau})$ together with $T(y)$, γ, and the final decision at $\alpha = 0.05$.

3.5 Implementation and reproducibility

All experiments are implemented in Python and executed from the repository scripts. Calibration, ROC evaluation, the P_D-vs-A sweep, and sanity checks are produced by the experiment scripts, and figures are saved to the run directory. Random number generator seeds are fixed per script to make the outputs reproducible.

The implementation used to generate all numerical results and figures is available in a public repository: `https://github.com/emrekaplaner123/transient-signal-detection`. For exact reproducibility, the experiments in this report correspond to the configuration file `experiments/configs/paper.yaml`. Running `scripts/05_make_all_figures.py`, `scripts/06_single_run_example.py` and `scripts/07_null_distribution.py` produces the full set of figures used in the report.

4 Numerical Results

This section reports the empirical behavior of the GLRT scan detector using the Monte Carlo calibration and evaluation procedure described in Section 3. We organize the results into (i) threshold calibration diagnostics under H_0, (ii) a single-run illustration showing how the scan behaves on concrete data, and (iii) aggregate performance curves (ROC and P_D versus amplitude).

4.1 Null distribution under H_0 and calibrated threshold

Figure 1 shows a histogram estimate of the null distribution of the scan statistic $T(Y)$ under H_0. The distribution is right-skewed, which is expected since T is the maximum over τ of a collection of squared (normalized) correlations. The dashed vertical line marks the calibrated threshold γ for $\alpha = 0.05$, chosen as the empirical $(1-\alpha)$-quantile of the Monte Carlo sample $\{T(Y^{(i)})\}$. In this run, the calibrated threshold was $\gamma(0.05) = 10.767$.

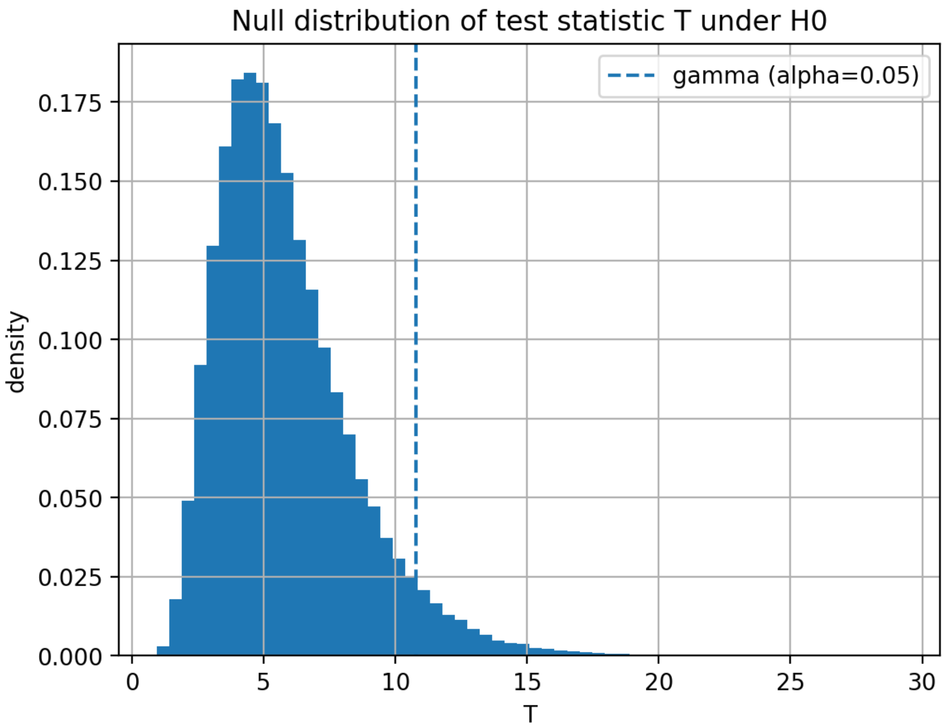

Figure 1: Empirical null distribution of the scan statistic $T(Y)$ under H_0 estimated via Monte Carlo. The dashed line indicates the calibrated threshold γ for $\alpha = 0.05$, selected so that $\mathbb{P}_{H_0}(T > \gamma) \approx \alpha$.

4.2 Single-run illustration: data, scan curve, and decision

To connect the hypothesis test to the observed data, we include one realization under H_0 and one under H_1 together with the scan curve. Figure 2 plots (i) a noise-only sample y under H_0 and (ii) a signal-plus-noise sample under H_1 with amplitude A_{true} and arrival time τ_{true}. Figure 3 plots the per-shift scores

$$T_\tau(y) \;=\; \frac{(s_\tau^T y)^2}{\|s_\tau\|^2}, \qquad \tau \in \mathcal{T},$$

together with the calibrated threshold γ (horizontal dashed line), the true arrival time τ_{true} (vertical line), and the estimated arrival time $\widehat{\tau} = \arg\max_\tau T_\tau(y)$ (vertical line). The test decides H_1 if $\max_\tau T_\tau(y) > \gamma$. In the displayed example, the maximum exceeds γ, so the test rejects H_0 at level $\alpha = 0.05$. The corresponding amplitude estimate is $\widehat{A} = \frac{s_{\widehat{\tau}}^T y}{\|s_{\widehat{\tau}}\|^2}$.

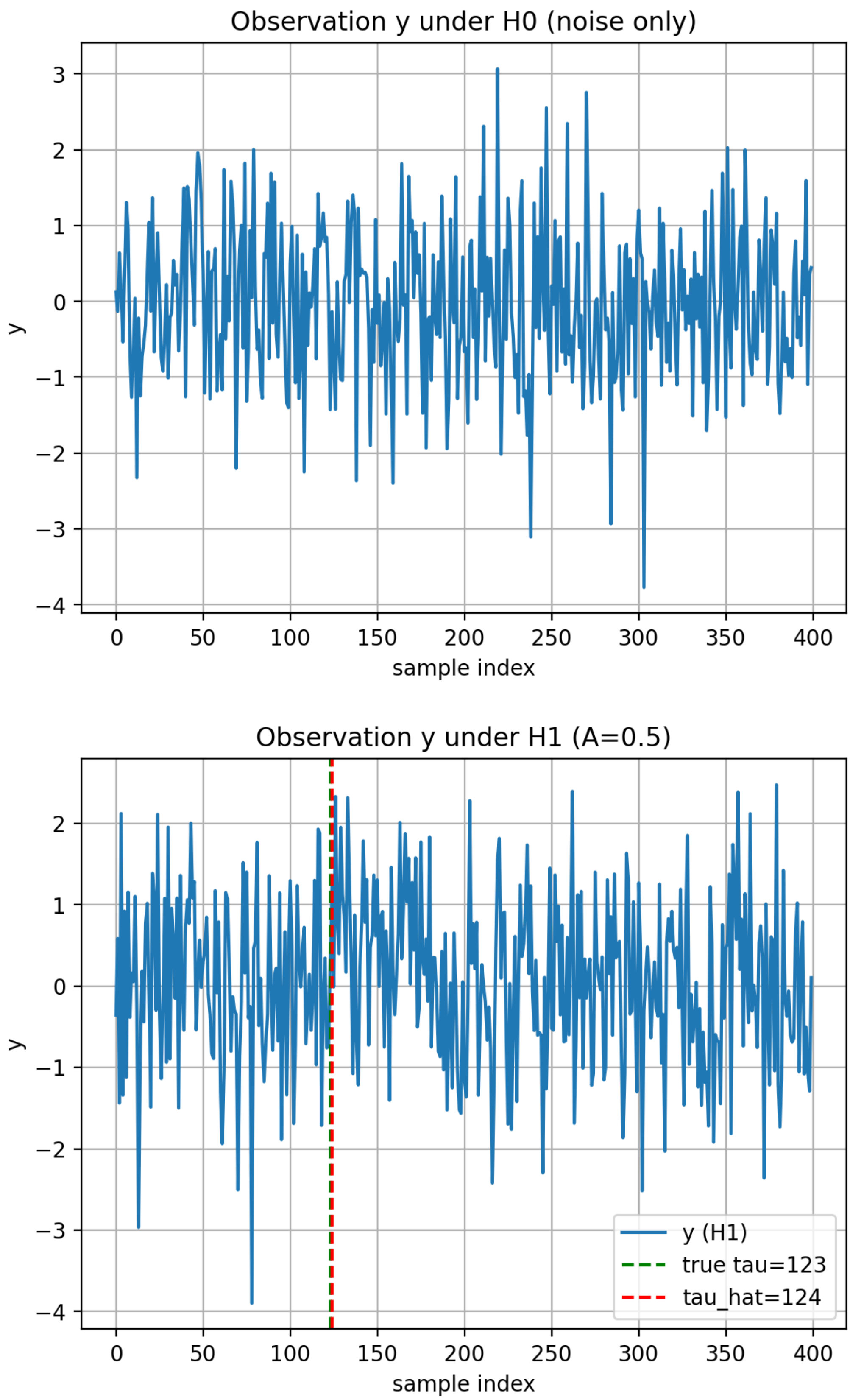

Figure 2: Top: one realization under H_0 (noise only). Bottom: one realization under H_1 (signal + noise) with dashed vertical lines marking τ_{true} and $\widehat{\tau}$.

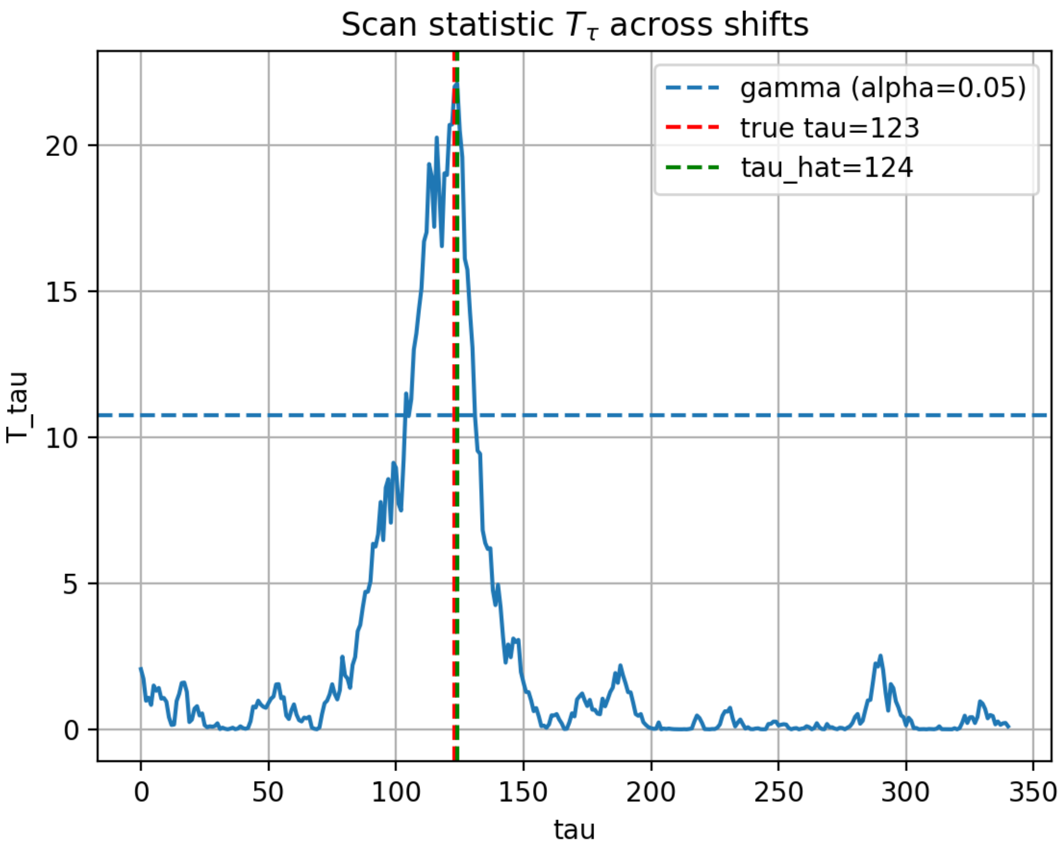

Figure 3: Scan curve $\tau \mapsto T_\tau(y)$ for a single H_1 realization. The horizontal dashed line is the calibrated threshold γ for $\alpha = 0.05$. The vertical lines mark τ_{true} and the estimate $\widehat{\tau} = \arg\max_\tau T_\tau(y)$. The detector declares H_1 when the maximum exceeds γ.

Reported single-run values. For transparency, Table 1 summarizes the true parameters and the estimates from the displayed H_1 run, together with the realized test statistic value $T(y)$, the calibrated threshold γ at $\alpha = 0.05$, and the resulting decision.

A_{true}	τ_{true}	$\widehat{A}$	$\widehat{\tau}$	$T(y)$	γ	Decision
0.5	123	0.607	124	22.106	10.767	Reject H_0 (declare signal)

Table 1: Single-run example summary at $\alpha = 0.05$. The detector declares H_1 when $T(y) > \gamma$.

4.3 ROC curves across amplitude

We next evaluate the receiver operating characteristic (ROC) curves by sweeping the operating point α, using the corresponding calibrated thresholds $\gamma(\alpha)$. For each amplitude A, we estimate the pair $(P_F(\alpha), P_D(\alpha; A))$ by Monte Carlo and plot P_D versus P_F.

Figure 4 shows representative ROC curves for several amplitudes. As expected, increasing A shifts the ROC curve upward: for a fixed false-alarm rate P_F, the detection probability P_D increases with signal strength. As a consistency check, when $A = 0$ the hypotheses coincide (H_1 reduces to H_0), and the ROC curve lies close to the diagonal $P_D \approx P_F$.

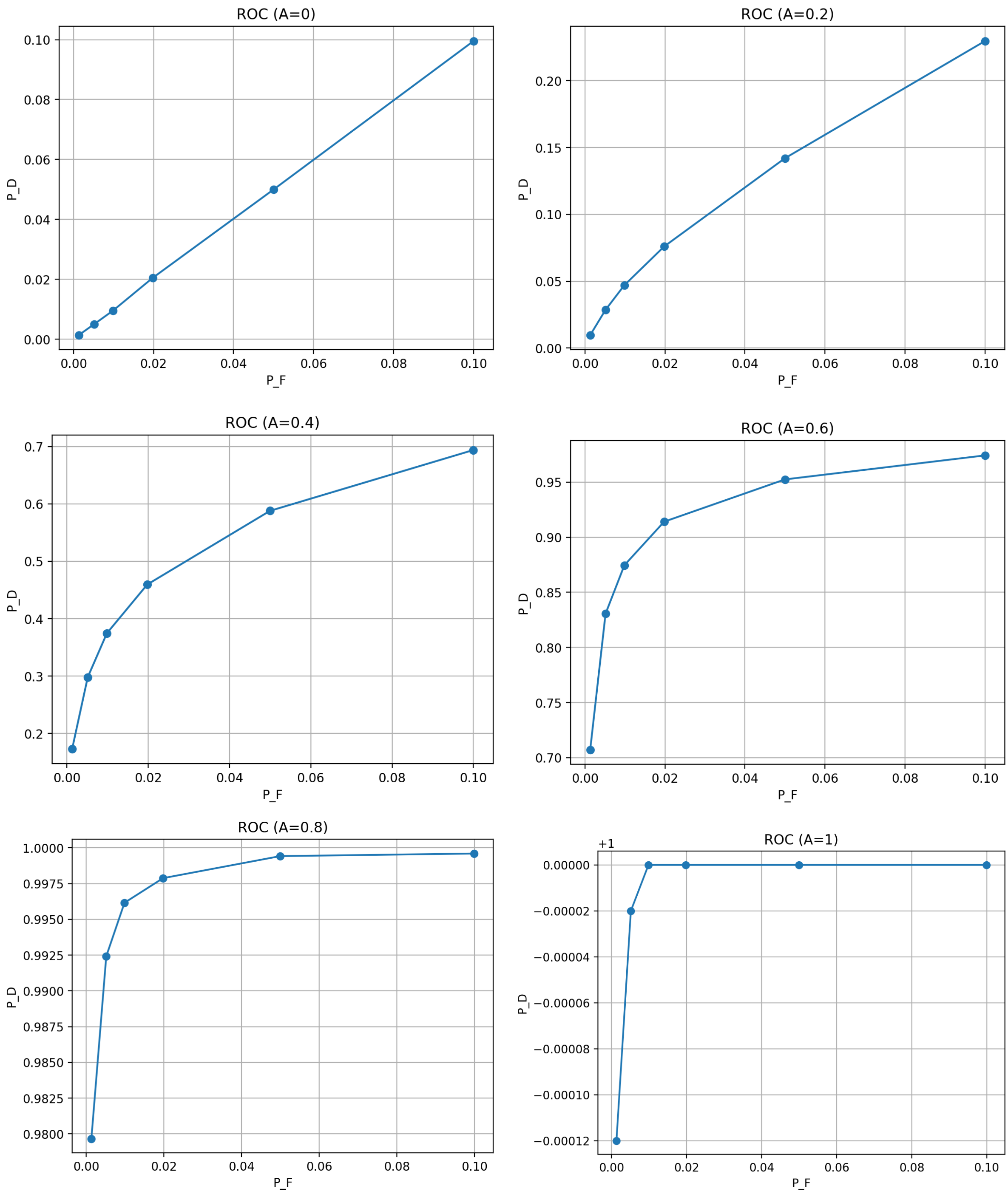

Figure 4: Representative ROC curves for different amplitudes A. Each curve is obtained by sweeping α (and hence $\gamma(\alpha)$) and plotting the estimated (P_F, P_D) pairs. Larger amplitudes yield uniformly better detection performance.

4.4 P_D versus amplitude at fixed α

Finally, we fix the false-alarm level at $\alpha = 0.05$, use the corresponding calibrated threshold $\gamma(0.05)$, and estimate the detection probability as a function of amplitude:

$$A \;\mapsto\; P_D(A) := \mathbb{P}(T(Y) > \gamma(0.05) \mid H_1, A).$$

Figure 5 shows that $P_D(A)$ increases rapidly from near-chance performance at $A = 0$ (where $P_D \approx P_F \approx 0.05$) to near-certain detection for sufficiently large A.

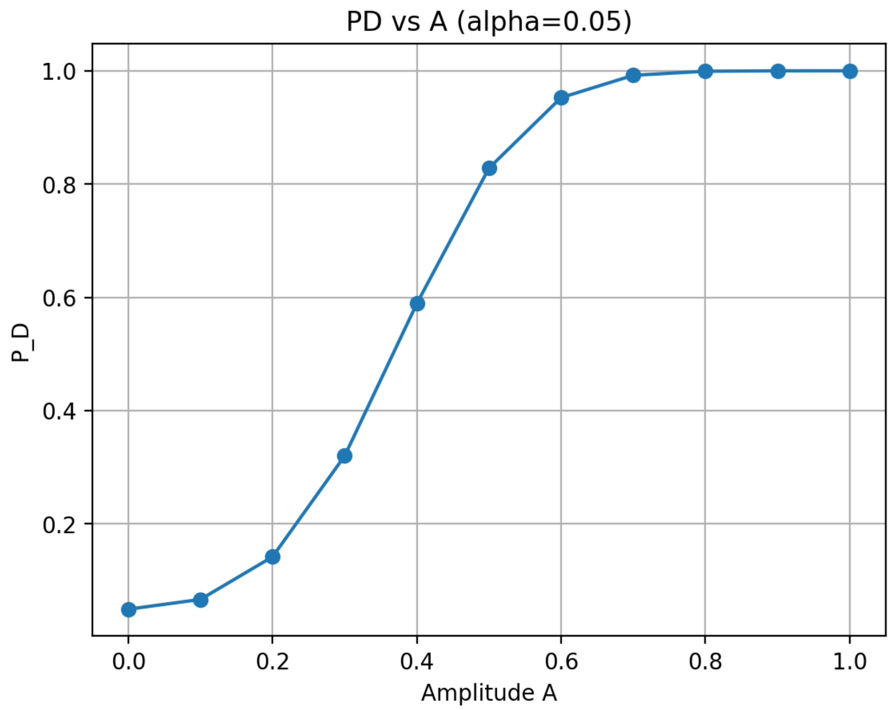

Figure 5: Estimated detection probability P_D as a function of amplitude A at a fixed false-alarm level $\alpha = 0.05$. The point at $A = 0$ satisfies $P_D \approx P_F \approx 0.05$ since H_1 reduces to H_0.

5 Discussion and Conclusion

5.1 Main findings

This project derived and implemented a GLRT scan detector for a rectangular transient with unknown arrival time τ and unknown amplitude A in white Gaussian noise. The resulting statistic

$$T(y) = \max_{\tau \in \mathcal{T}} \frac{(s_\tau^T y)^2}{\|s_\tau\|^2}$$

can be interpreted as a matched-filter scan over candidate arrival times, followed by a maximization step that accounts for the unknown timing. The numerical results confirm three expected qualitative behaviors:

- **Right-skewed null distribution.** Under H_0, $T(Y)$ is the maximum of a correlated family of squared correlations, producing a heavy right tail (Fig. 1).
- **Signal strength improves detection.** As the amplitude A increases, ROC curves move upward, i.e. for fixed P_F the detection probability P_D increases (Fig. 4). At a fixed operating point $\alpha = 0.05$, $P_D(A)$ increases sharply from near-chance performance at $A = 0$ to near-certain detection for larger A (Fig. 5).
- **Sanity check at $A = 0$.** When $A = 0$ the hypotheses coincide (H_1 reduces to H_0), and the ROC curve lies close to the diagonal relationship $P_D \approx P_F$, as expected.

5.2 Threshold calibration and interpretation

Because the maximization over τ induces dependence across shifts, a closed-form threshold γ is generally not available. Calibrating $\gamma(\alpha)$ by Monte Carlo under H_0 directly targets the global false-alarm constraint

$$P\big(T(Y) > \gamma(\alpha) \mid H_0\big) = \alpha,$$

and automatically accounts for the correlation created by overlapping shifted templates. The diagnostic histogram in Fig. 1 provides a visual check that the chosen $\gamma(0.05)$ lies in the appropriate upper tail.

5.3 Conclusion

Overall, the experiments demonstrate that a GLRT scan detector provides a simple and effective frequentist solution to transient detection with unknown amplitude and arrival time. With Monte Carlo calibration, the method achieves controlled false-alarm probability while exhibiting the expected monotone improvement in detection power with increasing signal amplitude.

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Portions of the code were drafted with assistance from an AI language model and were subsequently reviewed, edited, and validated by the author.

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