Chaotic Oscillator Lab

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Abstract: The purpose of this lab report is to document the observation of the chaotic circuit with its niche behaviour and the calculation of its corresponding Feigenbaum Number in the X14 lab. This involves finding the resonant frequency by varying frequency at a constant input amplitude and finding the specific frequency for maximal output amplitude, and it involves taking samples of maximum voltage values in points of bifurcation by varying input amplitude using a frequency close to but not at resonance. This process includes apparatuses like an oscilloscope, a signal generator, and other electrical components.

1. Introduction

The objective of this experiment is to observe chaotic behaviour in an LD circuit with varying amplitude, leading to concrete conclusions in bifurcations at different driving amplitudes while presenting an understanding of how chaotic behaviour operates with the calculation of the Feigenbaum Number.

Chaos Theory helps us discover unpredictable behaviour of chaotic circuits which were thought to be random and have no correlation in the past. Predicting such behaviour has assertive applications in real world including encryption, signal processing, and power systems.

This experiment is exploring the period-doubling bifurcations and route to chaos in a driven LD circuit.

2. Background/theory

2.1 Chaos Theory

Chaos is defined as "the non-periodic behaviour of deterministic nonlinear dynamic systems that is highly sensitive to initial conditions" [1]. In electronics, behaviour such that the output of a system demonstrates high vulnerability under slight input changes can be observed in many circuits, one of which is the series LD circuit with an AC source [1,5,6].

Chaos Theory is a comprehensive field where it is aimed to predict chaotic behaviour of systems inclined to chaotic behaviour under various subfields, including economics, philosophy, and physics, which includes electronics. [4] Chaotic behaviour of electrical circuits is noted to be predictable, as well as other fields [1,3,5].

2.2 Non-linear behaviour of inductor-diode circuit

Non-linearity is called when a desired result for output of a system is not directly proportional to the input element of this system, resulting in the system relationship not being able to be shown as a straight-line on a graph, unlike a linear system.

In an inductor-diode circuit with an AC source, non-linearity is elementarily present in the diode. Diodes have a non-linear relationship as an electrical component not obeying Ohm's Law $V = RI$ (1) where relationship is clearly linear between Voltage V and Current I with a constant coefficient R, whilst a real diodes V-I relationship is demonstrated with an exponential behaviour due to its one-direction conductivity property.

"The circuit is driven by a sinusoidal input voltage and the diode provides for the system's nonlinearity while its state-of-bias-related capacitance, combined with the inductance, gives the system the necessary degrees of freedom in order to produce chaos." [1]. Where the diode with its semiconductor material property with a p-n junction stores charge and electric field, where the junction acting as a capacitor, and therefore having the mentioned capacitance. In the circuit, "the diode appears to act like a charging or discharging capacitor as its space-charge width varies accordingly" [1]. due to the position change of holes-electrons within the junction.

2.3 Period-Doubling Bifurcation

Period-doubling bifurcation is a periodic system's slight change in input causing the emergence of a new period, double of the previous one. The frequency of occurrence of this type of bifurcation increases as this stated input is increased more, where at some point it is so frequent that period doubles even with the smallest increases in the parameter, leading to chaotic behaviour as seen in Figure 1.

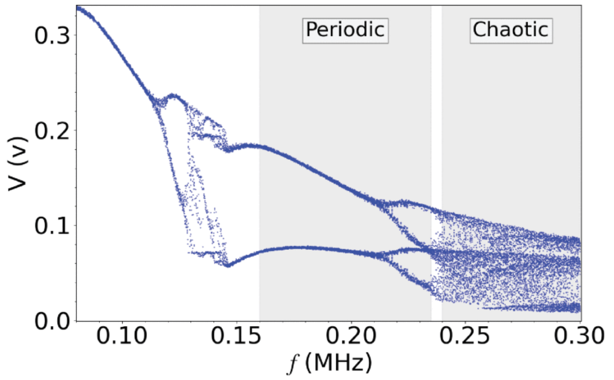

Figure 1: Example Period-Doubling Bifurcation Diagram of an RLD Circuit [5]

In an LD circuit, the driving AC source exposes in a forward current, where "the larger the forward current, the greater the amount of charges that cross the junction and the longer the system needs to return to its reverse bias equilibrium. If the reverse current is unable to reach equilibrium before the forward bias, then the next cycle will depend upon the previous cycle, and, this will be equivalent to different parameters at each cycle's initial conditions" [1] leading to this period-doubling behaviour of LD circuit.

2.4 Feigenbaum Constant

Named after its founder, Mitchell Jay Feigenbaum, Feigenbaum constant δ describes "the rate of transition to chaos" [3] with the occurring period-doubling bifurcations in nonlinear dynamical systems in their route to chaos. It is defined as the ratio of the intervals between the points where period-doubling bifurcations occur as a system parameter is varied, shown in Equation 2 [1,3]:

$$\delta = \frac{a_{n-1} - a_{n-2}}{a_n - a_{n-1}} \tag{2}$$

Where δ is the Feigenbaum number and a_n is the amplitude at the point of nth successive bifurcation. Its percentage uncertainty $\%\Delta a$ can be calculated by Equation 3:

$$\%\Delta\delta = \left(\frac{\Delta a_{n-1} + \Delta a_{n-2}}{a_{n-1} - a_{n-2}} + \frac{\Delta a_n + \Delta a_{n-1}}{a_n - a_{n-1}}\right) \times 100 \tag{3}$$

Δa is the absolute uncertainty of the amplitude measurement which is

2.5 Resonance Effects in LD Circuits

Resonance is the phenomena in which maximum amplitude of a periodic-behaving system is achieved by the system response is in phase with a periodic driving-force, whether this would be a mass-spring system or an RLC circuit.

In an RLC circuit, resonance specifically demonstrates "the signal frequency that corresponds to the maximum amplitude of the output voltage" [1] under a driving AC voltage input. Considering the junction of the diode acting as a capacitor, driving frequency of the LD circuit of this experiments interest can be defined with (4):

$$f_r = \frac{1}{2\pi\sqrt{LC_j}} \tag{4}$$

While resonance allows a maximum amplitude for the output, f_r also provides stability to the circuit due to in-phase oscillation, making it harder to reach chaos at f_r. Therefore, a frequency value that both gets the advantage of high driving amplitude and being out-of-phase is optimal for observing chaotic behaviour and route-to-chaos.

3. Experimental methods

3.1 Setup & Apparatus

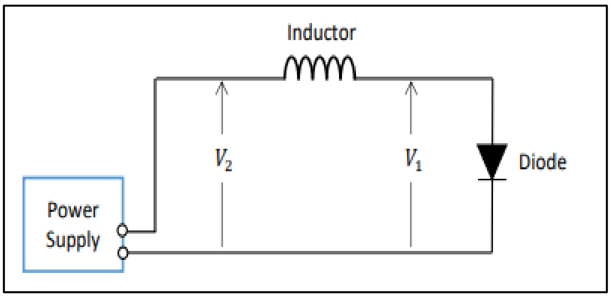

Figure 1: Circuit diagram for the chaotic oscillator [6]

Table I: Apparatus

1	Rigol DG1000Z Series Function/Arbitrary Waveform Generator
2	Tektronix MDO4000B Series Mixed Domain Oscilloscope
3	Inductor
4	Diode
5	Breadboard
6	Cables

3.2 Setting up the Circuit

Create the circuit above with the mentioned components (Table I) by firstly check the Rigol DG1000Z Series Arbitrary Waveform Generator output is off. Afterwards, you can start building the actual circuit by connecting the positive terminal of the waveform generator to a chosen power rail on the breadboard. Connect any of the two sides of the inductor to this power rail, then connect the other side of the inductor to a chosen terminal strip of the breadboard. Then connect the anode of the diode to this terminal strip, and its other side, i.e. cathode, to another chosen terminal strip, which a new power rail is also going to be connected. Then connect this power rail to the ground, then to the negative side of the signal generator which is in black. After setting up the probe, connect the channel 1 of the oscilloscope to measure V_1, between two power rails by the help of two cables. Do the same for channel 2 of the oscilloscope to measure V_2, between the anode and the cathode of the diode. Remember to do all of this while being cautious about only using insulated parts of the cables for your safety.

3.3 Methodology

Set signal generator to frequency 180kHz and V_{p-p} 200mV [6] (these values may vary across different inductors and diodes) and turn the output on. Reduce the frequency and note the variation in output voltage V_2. Take the specific frequency at which V_2 amplitude is at the maximum value, i.e. resonance frequency, and write down its value with correct unit. Now choose a frequency close to but not at resonance and explore the effect of increasing the amplitude of the input signal from the signal generator. Record your observations, including which values of amplitude they occurred. Identify and note down the input V_{p-p} at which the output signal bifurcates. Use these amplitudes to estimate Feigenbaum Number (2). Calculate the uncertainty of the Feigenbaum Number (3). Remember to use the measurement tool while gathering data from the oscilloscope in order to collect more accurate results.

4. Results

When frequency is lowered from 180kHz using the signal generator, V_{max} was reached at 120kHz, with a peak-to-peak voltage V_{p-p} of 122mV, meaning that 120kHz is equal to resonant frequency f_r, and the fixed frequency value of choice for investigating period-doubling should be close to 120kHz, e.g. 180kHz as used in this experiment. While a single-period waveform was observed at lower V_{p-p} values, increasing V_{p-p} lead the single line to double at 1560mV_{p-p}, forming two waveforms, which means the first successful period-doubling bifurcation has occurred. At 2080mV_{p-p}, it doubled again to 4 periods, and doubled again at 2210mV_{p-p} to form 8 waveforms in total as seen in Figure 3. The doubling pattern continued to repeat with much smaller V_{p-p} increases and an exponential sensitivity was observed to these increases, especially after 3rd bifurcation point. This sensitivity disallows the measurement of high-amplitude bifurcation points on route-to-chaos; therefore, only the first three bifurcation points were used in the Feigenbaum Number calculation.

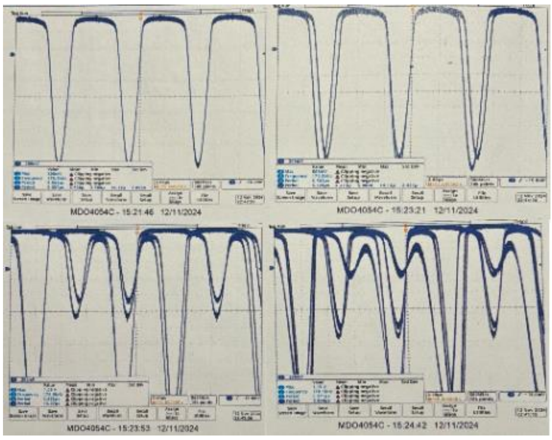

Figure 3: Period doubling in LD Circuit (Respectively: no bifurcation, 1st, 2nd, and 3rd bifurcation)

With the values found the Feigenbaum number δ can be calculated by (2):

$$\delta = \frac{2080 - 1560}{2210 - 2080} = 4$$

Also, we can calculate the uncertainty of the Feigenbaum Number in order to debate if our results are accurate and precise in the Discussion section. The percentage uncertainty of the Feigenbaum Number can be calculated by (3):

$$\%\Delta\delta = \left(\frac{10+10}{2080-1560} + \frac{10+10}{2210-2080}\right) \times 100 = 19.230769\%$$

As the smallest division of the oscilloscope, which is a digital instrument is 10mV for each amplitude measurement so their absolute uncertainty is ±10mV.

5. Discussion

In this experiment, observations demonstrate how the nonlinear properties of an LD circuit with a driving AC source results in period-doubling bifurcation, leading to chaotic behaviour. One of these observations is that as the driving amplitude is increased, an exponential increase of the rate at which the bifurcations occur, leading to abnormal periodic waveforms. Another observation was the additive effect of high driving amplitude and out-of-phase driving force on route-to-chaos due to the close-to-resonance driving-frequency. Overall, these observations fulfil the theoretical expectations of the experiment by being consistent with other studies on similar topics [1,5].

The calculation of Feigenbaum Number from V_{p-p} values at successive bifurcation points has resulted in a Feigenbaum Number of 4 with an uncertainty of +-19.23%, compared to the theoretical value of the Feigenbaum number of 4.6692... Considering the uncertainty value calculated, we can determine our range values by $4 \pm 4 \times 0.19230769$, giving an uncertainty range in between 3.230769 and 4.769230, meaning that the theoretical value is inside the margin of uncertainty, meaning that the calculated Feigenbaum Number has a fair value.

The usage of the informal method of visual measurement rather a software-based gathering may have caused huge accuracy errors due to human error. Also, with just varying the input signal variables by hand rather than using software leads to imprecise measurements of bifurcation points.

Additionally, noise and environmental factors may have caused measurement errors in oscilloscope tools where for instance maximum $V_{p\text{-}p}$ values were being gathered by the maximum readings of the oscilloscope, which can be influenced by noise. Also, the circuit components were not ideal, as a nature of the practice.

6. Conclusions

This experiment investigated the period-doubling bifurcation in an LD circuit by increasing the driving amplitude and observing the points at which period-doubling bifurcation occurs. The nonlinear circuit initially exhibited single-period waveforms from the input, while at 1560m$V_{p\text{-}p}$ the period doubled to form two waveforms, then at 2080m$V_{p\text{-}p}$ the period doubled again to form 4-period waveform, then again period-doubling has occurred and the resulting number of waveforms became 8 at 2210m$V_{p\text{-}p}$. With its doubling pattern due to the nonlinear property of the circuit, after the 3rd bifurcation with 8 waveforms, the circuit output V_2 exhibited chaotic behaviour, proving the point of expected route-to-chaos via bifurcations, which is predictable by the use of Feigenbaum Number.

These results back the presence of chaos in LD circuits that indicate bifurcation points with correlated spaces in between, determined by a coefficient defining the rate at which bifurcation occur, which results in a complex waveform in even slightly higher amplitudes. In the experiment, the coefficient, i.e. Feigenbaum Number has been calculated by using the points at which the period-doubling bifurcations occur, and a reasonable value of $\delta = 4$ has been found such that the real value of 4.66920… is within the range of uncertainty of our value, meaning that the experiment was successive.

These findings highlight the significant impact of nonlinear circuits with chaotic behaviour on different fields such as encryption and communications. For better results, software tools like MATLAB can be used while gathering data like amplitude and frequency from the oscilloscope, possibly using the moving average tool. For further study, even higher driving amplitudes can be carried out in order to investigate the limits of the nonlinear behaviour of diodes in AC driven LD circuits. Also, the effect of various types of diodes on the chaotic behaviour could've been investigated. Finally, for the constant frequency that should informally be close to resonant frequency, an optimal value could've been found as some function of the resonance frequency that would allow a more routinely experimental method.

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