MATLAB, Lab 6 – Group work

In 1963 an American metheorologist – Edward Lorentz was trying to develop a simplified mathematical model for atmospheric convection. He considered an isolated fluid volume that was heated from the bottom and cooled from above. He applied method known as the Galerkin approximation and obtained a simplified model of three differential equations:

$$\frac{dx}{dt} = \sigma(y - x)$$
$$\frac{dy}{dt} = rx - y - xz$$
$$\frac{dz}{dt} = xy - bz$$

where the variable *x* is proportional to the intensity of a convective motion, y is proportional to the temperature difference between ascending and descending currents and z accounts for the distortion of temperature profile from linearity. $\sigma > 1$ is referred to as Prandtl number and can be interpreted as the ratio of viscous diffusion over thermal diffusion. r describes the temperature difference on a heated layer, while b depends on the geometry of a fluid cell.

Lorentz observed that for $\sigma = 10$, $r = 28$, $b = \frac{8}{3}$ the system behaves in a quite unpredicted way and gives different results each time he ran the simulation. He finally concluded that this resulted from the fact that the accuracy of the computer was reaching only 6 decimal places while the system was sensitive to much smaller changes. He published a research paper, where this matter was described in the following way:

> *Two states differing by imperceptible amounts may eventually evolve into two considerably different states ... If, then, there is any error whatever in observing the present state — and in any real system such errors seem inevitable — an acceptable prediction of an instantaneous state in the distant future may well be impossible....In view of the inevitable inaccuracy and incompleteness of weather observations, precise very-long-range forecasting would seem to be nonexistent.*

In spite of the fact that the article was published in a meteorological journal, it is regarded as the beginning of a new discipline of mathematics known as the “chaos theory”. This discipline is devoted to considerations of the problem of sensitive dependence of some systems on minor changes in the initial conditions. In another paper Lorentz described this phenomenon in a very picturesque way:

> *One meteorologist remarked that if the theory were correct, one flap of a sea gull’s wings would be enough to alter the course of the weather forever. The controversy has not yet been settled, but the most recent evidence seems to favor the sea gulls.*

This observation is often known as the “butterfly effect”.

Task

Design a code that solves the Lorentz system. Apply the values of constants given above or play with them to find some other values for which the solution is chaotic. Analyze two cases with slightly different initial conditions. For each case present the results in two forms:

- Time evolution of each variable x(t), y(t), z(t),
- 3-dimensional curve in phase space (x,y,z)

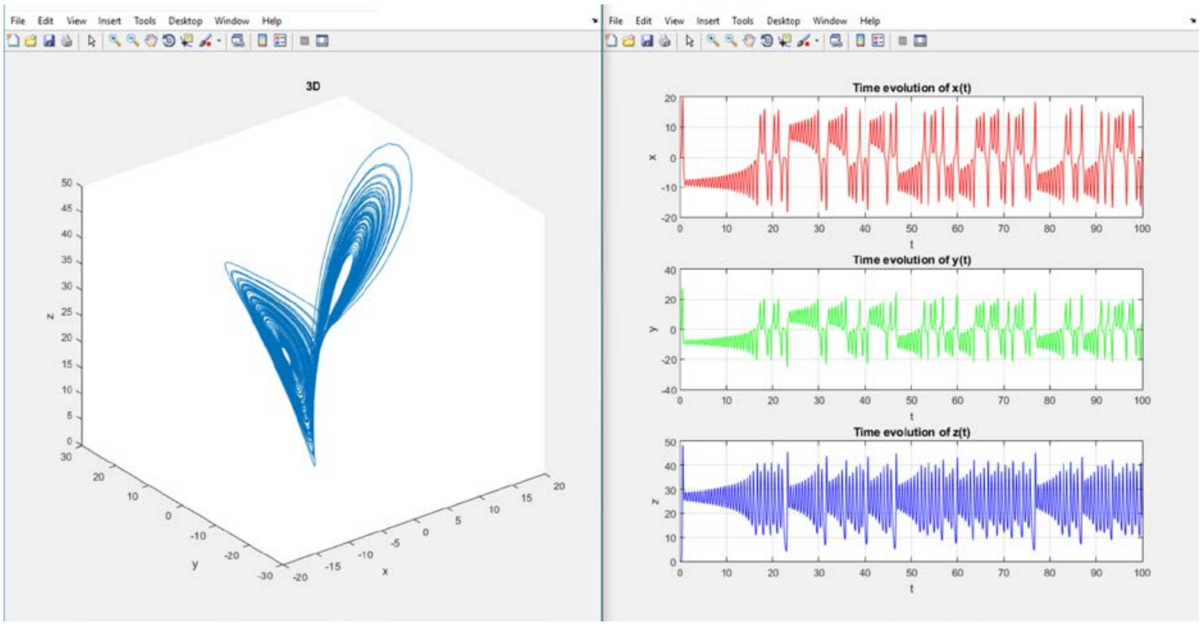
Code (main program):

```
t=[0 100]; % Time vector
xinit=[0.1;0.1;0.1]; % Initial conditions
[t,x]=ode45(@lorenz , t , xinit );

figure(1)
plot3(x(:,1),x(:,2),x(:,3))
title('3D')
xlabel('x');
ylabel('y');
zlabel('z');

figure(2)
subplot(3,1,1)
plot(t,x(:,1),'r')
title('Time evolution of x(t)');
grid on;
xlabel('t');
ylabel('x');


subplot(3,1,2)
plot(t,x(:,2),'g')
title('Time evolution of y(t)')
grid on;
xlabel('t');
ylabel('y');


subplot(3,1,3)
plot(t,x(:,3),'b')
title('Time evolution of z(t)')
grid on;
xlabel('t');
ylabel('z');
```

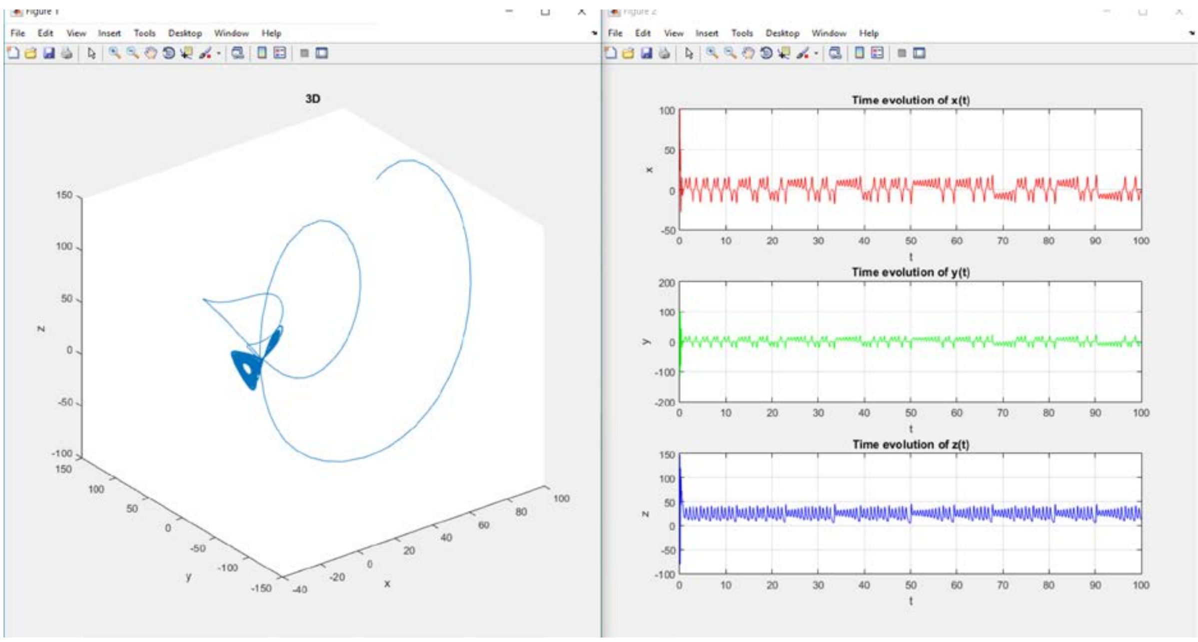
Code (function called by ODE procedure):

```
function dx=lorenz(t,x)

sigma=10; r=28; b=8/3; % Parameters

dx1=sigma*(x(2)-x(1));
dx2=r*x(1)-x(2)-x(1)*x(3);
dx3=x(1)*x(2)-b*x(3);

dx=[dx1;dx2;dx3]; %Put them together in a vector
```

Screenshot (case 1):



Screenshot (case 2):



Comments:

When we choose [0.1,0.1,0.1] point as initial point we obtain the figures shown in case 1.
If we select [100,100,100] point as initial point we will see the figures shown in case 2.