

Name: **SOLUTION**

[10p] Consider the graph G whose adjacency matrix representation is given below: $G_{ij} = c(e_{i \rightarrow j})$

Let node 0 be the source and node 9 be the sink. Find the maximum flow.

Show your work. (No point if the solution is obtained by brute force or guessing.)

G	0	1	2	3	4	5	6	7	8	9
0	0	6	0	0	0	6	0	0	2	0
1	0	0	3	0	0	3	0	0	0	0
2	0	0	0	2	0	6	0	3	0	0
3	0	0	0	0	4	0	0	0	0	0
4	0	0	0	0	0	0	0	4	0	7
5	0	0	0	0	0	0	3	0	2	0
6	0	0	0	0	0	0	0	2	1	0
7	0	0	0	0	0	0	0	0	0	3
8	0	0	0	0	0	0	0	0	0	6
9	0	0	0	0	0	0	0	0	0	0

Use Ford-Fulkerson algorithm to find the max flow or alternatively consider the s-t cut (A,B) with $A=\{0,1,5,6\}$ and $B=\{2,3,4,7,8,9\}$.

$$\begin{aligned}
 \text{cap}(A,B) &= \sum_{e_{A \rightarrow B}} c(e) = c(e_{0 \rightarrow 2}) + c(e_{1 \rightarrow 2}) + c(e_{5 \rightarrow 8}) + c(e_{6 \rightarrow 7}) + c(e_{6 \rightarrow 8}) \\
 &= 2 + 3 + 2 + 2 + 1 \\
 &= 10
 \end{aligned}$$

It can be shown that this is a min cut, so the max flow is **10**.