Module 2D - Optics Module

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Contents

Objectives: Understanding basics of optics with regard to raytracing explained in later modules.
Prerequisite Knowledge: Highschool Physics
Prerequisite Modules: 1A - Calculus, 1B - Linear Algebra
Difficulty: Intermediate

Summary: Theory pertaining to electromagnetic energy propagation and reflection/absorption calculations are provided.

1 Theory

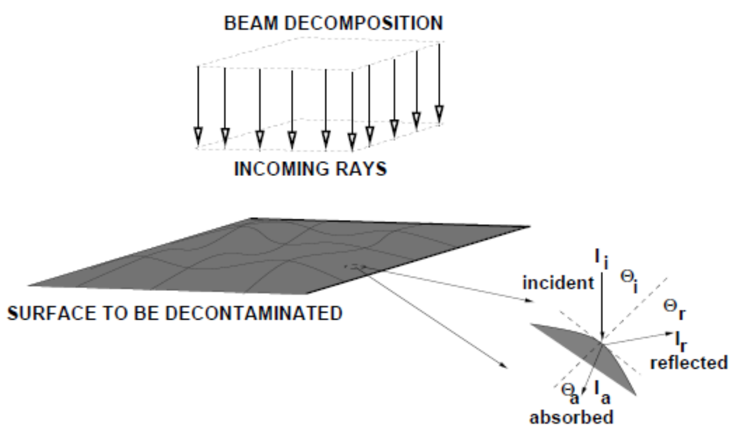

Figure 1.1: An electromagnetic pulse applied to a surface.

1.1 Electromagnetic energy propagation

1.1.1 Beam-ray decomposition

In order to connect the concept of a ray with a pulse/beam, since $\bar{I}$ is the energy per unit area per unit time, we obtain the energy associated with an entire pulse/beam by multiplying the irradiance by the cross-sectional area of an initially coherent beam, $\bar{I}A^b$, where A^b is the cross-sectional area of the beam (comprising all of the rays). The energy per unit time (power) for a ray in the pulse/beam is then given by

$$I = \bar{I}A^r = \bar{I}A^b/N_r, \tag{1.1}$$

where N_r is the number of rays in the beam (Figure 1.1) and A_r can be considered the area associated with a ray. Essentially, rays are a mathematical construction/discretization of a pulse/beam We refer the reader to Gross [1], Zohdi [3-9] for details.

1.1.2 Reflection and absorption of rays

Following a framework found in Zohdi [3-9] for details, we consider a ray of light incident upon a material interface which produces a reflected ray and a transmitted/absorbed (refracted) ray (Figure 1.1), the amount of incident electromagnetic energy per unit time, power (I_i), that is reflected (I_r) is given by the total reflectance $\boldsymbol{I\!R} \stackrel{\text{def}}{=} \frac{I_r}{I_i}$, where $0 \leq \boldsymbol{I\!R} \leq 1$. $\boldsymbol{I\!R}$ is given by Equation 1.3, for unpolarized electromagnetic radiation. We have the following observations:

- The angle between the point of contact of a ray (Figure 1.1) and the outward normal to the surface at that point is the angle of incidence, θ_i. The classical reflection law states that the angle at which a ray is reflected is the same as the angle of incidence and that the incoming (incident, θ_i) and outgoing (reflected, θ_r) ray lays in the same plane, and $\theta_i = \theta_r$

- The classical refraction law states that, if the ray passes from one medium into a second one (with a different index of refraction), and, if the index of refraction of the second medium is less than that of the first, the angle the ray makes with the normal to the interface is always less than the angle of incidence, where $\hat{n} \overset{\text{def}}{=} \frac{v_i}{v_a} = \sqrt{\frac{\epsilon_a \mu_a}{\epsilon_1 \mu_i}} = \frac{\sin\theta_i}{\sin\theta_a}, \theta_a$ being the angle of the absorbed ray (Figure 1.1), v_a is the propagation speed in the absorbing medium, v_i is the propagation speed in the incident medium, ϵ_a is the electric permittivity of the absorbing medium, μ_a magnetic permeability of the absorbing medium, ϵ_i is the electric permittivity in the incident medium and μ_i magnetic permeability in the incident medium.

- Recall, all electromagnetic radiation travels, in a vacuum, at the speed $c \approx 2.99792458 \times 10^8 \pm 1.1 m/s$. In any another medium $v = \frac{1}{\sqrt{\epsilon\mu}}$ for electromagnetic waves.[1]

- We define $\hat{n}$ as the ratio of the refractive indices of the ambient (incident) medium (n_i) and absorbing medium (n_a), $\hat{n} = n_a/n_i$, where $\hat{\mu}$ is the ratio of the magnetic permeabilities of the surrounding incident medium (μ_i) and scattering/absorbing medium (μ_a), $\hat{\mu} = \mu_a/\mu_i$. Thus, we have

$$\hat{n} = \frac{n_a}{n_i} = \sqrt{\frac{\epsilon_a \mu_a}{\epsilon_i \mu_i}} \Rightarrow \epsilon_a \mu_a = (\hat{n})^2 \epsilon_i \mu_i. \tag{1.2}$$

- For a pulse of light, the reflectivity $\boldsymbol{I\!R}$ can be shown to be (see [1] for example)

$$\boldsymbol{I\!R} = \frac{I_r}{I_i} = \frac{1}{2}\left(\left(\frac{\hat{n}^2\cos\theta_i - \left(\hat{n}^2 - \sin^2\theta_i\right)^{\frac{1}{2}}}{\frac{\hat{n}^2}{\mu}\cos\theta_i + \left(\hat{n}^2 - \sin^2\theta_i\right)^{\frac{1}{2}}}\right)^2 + \left(\frac{\cos\theta_i - \frac{1}{\hat{\mu}}\left(\hat{n}^2 - \sin^2\theta_i\right)^{\frac{1}{2}}}{\cos\theta_i + \frac{1}{\hat{\mu}}\left(\hat{n}^2 - \sin^2\theta_i\right)^{\frac{1}{2}}}\right)^2\right) \tag{1.3}$$

  where I_i is the incoming irradiance, I_r the reflected irradiance, $\hat{n}$ is the ratio of the refractive indices of the of absorbing (n_a) and incident media (n_i), where the refractive index is defined as the ratio of the speed of light in a vacuum (c) to that of the medium (v), where the speed of electromagnetic waves is $c = \frac{1}{\sqrt{\epsilon_o \mu_o}}$, where ϵ is the electric permittivity and μ is the magnetic permeability.

- We consider applications with non-magnetic media and frequencies where the magnetic permeability is virtually the same for both the incident medium (usually the atmosphere) and the scattering/absorbing medium. Thus, for the remainder of the work, we shall take $\hat{\mu} = 1 (\mu_o = \mu_i = \mu_a)$, thus

$$\hat{n} = \frac{n_a}{n_i} = \sqrt{\frac{\epsilon_a \mu_a}{\epsilon_i \mu_i}} \Rightarrow \epsilon_a \mu_a = (\hat{n})^2 \epsilon_i \mu_i \Rightarrow \epsilon_a = (\hat{n})^2 \epsilon_i \tag{1.4}$$

  This yields

$$\boldsymbol{I\!R} = \frac{I_r}{I_i} = \frac{1}{2}\left(\left(\frac{\hat{n}^2\cos\theta_i - \left(\hat{n}^2 - \sin^2\theta_i\right)^{\frac{1}{2}}}{\hat{n}^2\cos\theta_i + \left(\hat{n}^2 - \sin^2\theta_i\right)^{\frac{1}{2}}}\right)^2 + \left(\frac{\cos\theta_i - \left(\hat{n}^2 - \sin^2\theta_i\right)^{\frac{1}{2}}}{\cos\theta_i + \left(\hat{n}^2 - \sin^2\theta_i\right)^{\frac{1}{2}}}\right)^2\right) \tag{1.5}$$

- Notice that as $\hat{n} \to 1$ we have complete absorption, while as $\hat{n} \to \infty$ we have complete reflection. The total amount of absorbed power by the material is $(1 - \boldsymbol{I\!R})I_i$.

The next section supplies the theory underpinning electromagnetic wave propagation and rays.

1.2 Electromagnetic wave propagation and rays

Following a framework found in Zohdi [3-9], the propagation of electromagnetic waves in free space can be described by a simplified form of Maxwell's equations (see Jackson [2], Zohdi [6])

[1] The free space electric permittivity is $\epsilon_o = \frac{1}{c^2\mu_o} = 8.8542 \times 10^{-12} CN^{-1}m^{-1}$ and the free space magnetic permeability is $\mu_o = 4\pi \times 10^{-7} WbA^{-1}m^{-1} = 1.2566 \times 10^{-6} WbA^{-1}m^{-1}$.

$$\nabla \times \boldsymbol{E} = -\mu_o \frac{\partial \boldsymbol{H}}{\partial t}, \quad \text{and} \quad \nabla \times \boldsymbol{H} = \epsilon_o \frac{\partial \boldsymbol{E}}{\partial t} \tag{1.6}$$

where $\nabla \cdot \boldsymbol{H} = 0, \nabla \cdot \boldsymbol{E} = 0, \boldsymbol{E}$ is the electric field, $\boldsymbol{H}$ is the magnetic field, ϵ_o is the free space permittivity and μ_o is the free space permeability. Using standard vector identities, one can show that

$$\nabla \times (\nabla \times \boldsymbol{E}) = -\mu_o \epsilon_o \frac{\partial^2 \boldsymbol{E}}{\partial t^2}, \quad \text{and} \quad \nabla \times (\nabla \times \boldsymbol{H}) = -\mu_o \epsilon_o \frac{\partial^2 \boldsymbol{H}}{\partial t^2} \tag{1.7}$$

and that

$$\nabla^2 \boldsymbol{E} = \frac{1}{c^2} \frac{\partial^2 \boldsymbol{E}}{\partial t^2}, \quad \text{and} \quad \nabla^2 \boldsymbol{H} = \frac{1}{c^2} \frac{\partial^2 \boldsymbol{H}}{\partial t^2} \tag{1.8}$$

where the speed of electromagnetic waves is $c = \frac{1}{\sqrt{\epsilon_o \mu_o}}$. All electromagnetic radiation travels, in a vacuum, at the speed $c \approx 2.99792458 \times 10^8 \pm 1.1 m/s$. In any another medium, for electromagnetic waves, the propagation speed is $v = \frac{1}{\sqrt{\epsilon \mu}}$, where ϵ and μ are the electric permittivity and magnetic permeability of that medium, respectively.[2]

1.2.1 Plane harmonic wave fronts

Now consider the special case of plane harmonic waves, for example of the form

$$\boldsymbol{E} = \boldsymbol{E}_o \cos(\boldsymbol{k} \cdot \boldsymbol{x} - \omega t) \quad \text{and} \quad \boldsymbol{H} = \boldsymbol{H}_o \cos(\boldsymbol{k} \cdot \boldsymbol{x} - \omega t) \tag{1.9}$$

where $\boldsymbol{x}$ is an initial position vector to the wave front, where k is the direction of propagation. We refer to the phase as $\phi = \boldsymbol{k} \cdot \boldsymbol{x} - \omega t$, and $\omega = \frac{2\pi}{\tau}$ as the angular frequency, where τ is the period. For plane waves, the wave front is a plane on which ϕ is constant, which is orthogonal to the direction of propagation, characterized by $\boldsymbol{k}$. In the case of harmonic waves, we have

$$\boldsymbol{k} \times \boldsymbol{E} = \mu_o \omega \boldsymbol{H} \quad \text{and} \quad \boldsymbol{k} \times \boldsymbol{H} = -\epsilon_o \omega \boldsymbol{E}, \tag{1.10}$$

and $\boldsymbol{k} \cdot \boldsymbol{E} = 0$ and $\boldsymbol{k} \cdot \boldsymbol{H} = 0$. The three vectors, $\boldsymbol{k}, \boldsymbol{E}$ and $\boldsymbol{H}$ constitute a mutually orthogonal triad.[3] The direction of wave propagation is given by $\frac{\boldsymbol{E} \times \boldsymbol{H}}{\|\boldsymbol{E} \times \boldsymbol{H}\|}$. Electromagnetic waves traveling through space carry electromagnetic energy which flows in the direction of wave propagation. The energy per unit area per unit time flowing perpendicularly into a surface in free space is given by the Poynting vector $\boldsymbol{S} = \boldsymbol{E} \times \boldsymbol{H}$.

1.2.2 Natural (random) electromagnetic energy propagation

Since at high-frequencies $\boldsymbol{E}$, $\boldsymbol{H}$ and $\boldsymbol{S}$ oscillate rapidly, it is impractical to measure instantaneous values of $\boldsymbol{S}$ directly. Consider the harmonic representations in Equation 1.9 which leads to $\boldsymbol{S} = \boldsymbol{E}_o \times \boldsymbol{H}_o \cos^2(\boldsymbol{k} \cdot \boldsymbol{x} - \omega t)$, and consequently the average value over a longer time interval ($\mathcal{T}$) than the time scale of rapid random oscillation,

$$\langle S \rangle_{\mathcal{T}} = \boldsymbol{E}_o \times \boldsymbol{H}_o \left\langle \cos^2(\boldsymbol{k} \cdot \boldsymbol{x} - \omega t) \right\rangle_{\mathcal{T}} = \frac{1}{2} \boldsymbol{E}_o \times \boldsymbol{H}_o, \tag{1.11}$$

leading to the definition of the irradiance

$$I \stackrel{\text{def}}{=} \langle \|\boldsymbol{S}\| \rangle_{\mathcal{T}} = \frac{1}{2} \|\boldsymbol{E}_o \times \boldsymbol{H}_o\| = \frac{1}{2} \sqrt{\frac{\epsilon_o}{\mu_o}} \|\boldsymbol{E}_o\|^2. \tag{1.12}$$

Thus, the rate of flow of energy is proportional to the square of the amplitude of the electric field.

[2] The free space electric permittivity is $\epsilon_o = \frac{1}{c^2 \mu_o} = 8.8542 \times 10^{-12} C N^{-1} m^{-1}$ and the free space magnetic permeability is $\mu_o = 4\pi \times 10^{-7} Wb A^{-1} m^{-1} = 1.2566 \times 10^{-6} Wb A^{-1} m^{-1}$.

[3] By combining the relations in Equation 1.10, one obtains $\|\boldsymbol{k}\| = \frac{\omega}{c}$.

1.2.3 Reflection and absorption of energy-Fresnel relations

We consider a plane harmonic wave incident upon a plane boundary separating two different materials, specifically vacuum and surface, which produces a reflected wave and an absorbed (refracted) wave (Figure 1.1). Two cases for the electric field vector are considered:

- (1) electric field vectors that are parallel ($\parallel$) to the plane of incidence and
- (2) electric field vectors that are perpendicular ($\perp$) to the plane of incidence.

In either case, the tangential components of the electric and magnetic fields are required to be continuous across the interface. Consider case (1). We have the following general vectorial representations

$$\boldsymbol{E}_{\parallel} = E_{\parallel} \cos(\boldsymbol{k} \cdot \boldsymbol{x} - \omega t) e_1 \quad \text{and} \quad \boldsymbol{H}_{\parallel} = H_{\parallel} \cos(\boldsymbol{k} \cdot \boldsymbol{x} - \omega t) \boldsymbol{e}_2, \tag{1.13}$$

where e_1 and e_2 are orthogonal to the propagation direction $\boldsymbol{k}$. By employing the law of refraction ($n_i \sin\theta_i = n_a \sin\theta_a$) we obtain the following conditions relating the incident, reflected and absorbed components of the electric field quantities

$$E_{\parallel i} \cos\theta_i - E_{\parallel r} \cos\theta_r = E_{\parallel a} \cos\theta_a \quad \text{and} \quad H_{\perp i} + H_{\perp r} = H_{\perp a}. \tag{1.14}$$

Since, for plane harmonic waves, the magnetic and electric field amplitudes are related by $H = \frac{E}{v\mu}$. we have

$$E_{\parallel i} + E_{\parallel r} = \frac{\mu_i}{\mu_a} \frac{v_i}{v_a} E_{\parallel a} = \frac{\mu_i}{\mu_a} \frac{n_a}{n_i} E_{\parallel a} \stackrel{\text{def}}{=} \frac{\hat{n}}{\hat{\mu}} E_{\parallel a}, \tag{1.15}$$

where $\hat{\mu} \stackrel{\text{def}}{=} \frac{\mu_a}{\mu_i}, \hat{n} \stackrel{\text{def}}{=} \frac{n_a}{n_i}$ and where v_i, v_r and v_a are the values of the velocity in the incident, reflected and absorbed directions.[4] By again employing the law of refraction, we obtain the Fresnel reflection and transmission/absorption coefficients, generalized for the case of unequal magnetic permeabilities

$$r_{\parallel} = \frac{E_{\parallel r}}{E_{\parallel i}} = \frac{\frac{\hat{n}}{\hat{\mu}} \cos\theta_i - \cos\theta_a}{\frac{\hat{n}}{\hat{\mu}} \cos\theta_i + \cos\theta_a} \quad \text{and} \quad a_{\parallel} = \frac{E_{\parallel a}}{E_{\parallel i}} = \frac{2\cos\theta_i}{\cos\theta_a + \frac{\hat{n}}{\hat{\mu}} \cos\theta_i}. \tag{1.16}$$

Following the same procedure for case (2), where the components of $\boldsymbol{E}$ are perpendicular to the plane of incidence, we have

$$r_{\perp} = \frac{E_{\perp r}}{E_{\perp i}} = \frac{\cos\theta_i - \frac{\hat{n}}{\hat{\mu}} \cos\theta_a}{\cos\theta_s + \frac{\hat{n}}{\hat{\mu}} \cos\theta_a} \quad \text{and} \quad a_{\perp} = \frac{E_1 a}{E_{\perp i}} = \frac{2\cos\theta_i}{\cos\theta_i + \frac{\hat{n}}{\hat{\mu}} \cos\theta_a}. \tag{1.17}$$

Our primary interest is in the reflections. We define the reflectances as

$$\boldsymbol{I\!R}_{\parallel} \stackrel{\text{def}}{=} r_{\parallel}^2 \quad \text{and} \quad \boldsymbol{I\!R}_{\perp} \stackrel{\text{def}}{=} r_{\perp}^2 \tag{1.18}$$

Particularly convenient forms for the reflections are

$$r_{\parallel} = \frac{\frac{\hat{n}^2}{\hat{\mu}} \cos\theta_i - \left(\hat{n}^2 - \sin^2\theta_i\right)^{\frac{1}{2}}}{\frac{\hat{n}^2}{\hat{\mu}} \cos\theta_i + \left(\hat{n}^2 - \sin^2\theta_i\right)^{\frac{1}{2}}} \quad \text{and} \quad r_{\perp} = \frac{\cos\theta_i - \frac{1}{\hat{\mu}} \left(\hat{n}^2 - \sin^2\theta_i\right)^{\frac{1}{2}}}{\cos\theta_i + \frac{1}{\hat{\mu}} \left(\hat{n}^2 - \sin^2\theta_i\right)^{\frac{1}{2}}} \tag{1.19}$$

Thus, the total energy reflected can be characterized by

$$\boldsymbol{I\!R} \stackrel{\text{def}}{=} \left(\frac{E_r}{E_i}\right)^2 = \frac{E_{\perp r}^2 + E_{\parallel r}^2}{E_i^2} = \frac{I_{\parallel r} + I_{\perp r}}{I_i} \tag{1.20}$$

If the resultant plane of oscillation of the (polarized) wave makes an angle of γ_i with the plane of incidence, then

$$E_{\parallel i} = E_i \cos\gamma_i \quad \text{and} \quad E_{\perp i} = E_i \sin\gamma_i, \tag{1.21}$$

[4] Throughout the analysis we assume that $\hat{n} \geq 1$

and it follows from the previous definition of I that

$$I_{\|i} = I_i \cos^2 \gamma_i \quad \text{and} \quad I_{\perp i} = I_i \sin^2 \gamma_i. \tag{1.22}$$

Substituting these expression back into the expressions for the reflectances yields

$$\boldsymbol{I\!R} = \frac{I_{\|r}}{I_i} \cos^2 \gamma_i + \frac{I_{\perp r}}{I_i} \sin^2 \gamma_i = \boldsymbol{I\!R}_{\|} \cos^2 \gamma_i + \boldsymbol{I\!R}_{\perp} \sin^2 \gamma_i. \tag{1.23}$$

For natural or unpolarized electromagnetic radiation, the angle γ_i varies rapidly in a random manner, as does the field amplitude. Thus, since

$$\left\langle \cos^2 \gamma_i(t) \right\rangle_{\mathcal{T}} = \frac{1}{2} \quad \text{and} \quad \left\langle \sin^2 \gamma_i(t) \right\rangle_{\mathcal{T}} = \frac{1}{2}, \tag{1.24}$$

and therefore for natural electromagnetic radiation

$$I_{\|i} = \frac{I_i}{2} \quad \text{and} \quad I_{\perp i} = \frac{I_i}{2}. \tag{1.25}$$

and therefore

$$r_{\|}^2 = \left(\frac{E_{|r}^2}{E_{||i}^2} \right)^2 = \frac{I_{\|r}}{I_{||i}} \quad \text{and} \quad r_{\perp}^2 = \left(\frac{E_{1r}^2}{E_{\perp i}^2} \right)^2 = \frac{I_{\perp r}}{I_{\perp i}}. \tag{1.26}$$

Thus, the total reflectance becomes

$$\boldsymbol{I\!R} = \frac{1}{2} \left(\boldsymbol{I\!R}_{\|} + \boldsymbol{I\!R}_{\perp} \right) = \frac{1}{2} \left(r_{\|}^2 + r_{\perp}^2 \right), \tag{1.27}$$

where $0 \leq \boldsymbol{I\!R} \leq 1$. For the cases where $\sin \theta_a = \frac{\sin \theta_i}{\hat{n}} > 1$, one may rewrite reflection relations as

$$r_{\|} = \frac{\frac{\hat{n}^2}{\hat{\mu}} \cos \theta_i - j \left(\sin^2 \theta_i - \hat{n}^2 \right)^{\frac{1}{2}}}{\frac{\hat{n}^2}{\hat{\mu}} \cos \theta_i + j \left(\sin^2 \theta_i - \hat{n}^2 \right)^{\frac{1}{2}}} \quad \text{and} \quad r_{\perp} = \frac{\cos \theta_i - \frac{1}{\hat{\mu}} j \left(\sin^2 \theta_i - \hat{n}^2 \right)^{\frac{1}{2}}}{\cos \theta_i + \frac{1}{\hat{\mu}} j \left(\sin^2 \theta_i - \hat{n}^2 \right)^{\frac{1}{2}}}, \tag{1.28}$$

where, $j = \sqrt{-1}$, and in this complex case[5]

$$\boldsymbol{I\!R}_{\|} \stackrel{\text{def}}{=} r_{\|} \bar{r}_{\|} = 1, \quad \text{and} \quad \boldsymbol{I\!R}_{\perp} \stackrel{\text{def}}{=} r_{\perp} \bar{r}_{\perp} = 1, \tag{1.29}$$

where $\bar{r}_{\|}$ and $\bar{r}_{\perp}$ are complex conjugates. Thus, for angles above the critical angle θ_i^*, all of the energy is reflected. Notice that as $\hat{n} \to 1$ we have complete absorption, while as $\hat{n} \to \infty$ we have complete reflection. The amount of absorbed irradiance by the surface is $I_a = (1 - \boldsymbol{I\!R}) I_i$.

1.2.4 Reflectivity

To observe the dependency of $\boldsymbol{I\!R}$ on $\hat{n}$ and θ_i we can explicitly write

$$\boldsymbol{I\!R} = \frac{1}{2} \left(\left(\frac{\frac{\hat{n}^2}{\hat{\mu}} \cos \theta_i - \left(\hat{n}^2 - \sin^2 \theta_i \right)^{\frac{1}{2}}}{\frac{\hat{n}^2}{\hat{\mu}} \cos \theta_i + \left(\hat{n}^2 - \sin^2 \theta_i \right)^{\frac{1}{2}}} \right)^2 + \left(\frac{\cos \theta_i - \frac{1}{\hat{\mu}} \left(\hat{n}^2 - \sin^2 \theta_i \right)^{\frac{1}{2}}}{\cos \theta_i + \frac{1}{\hat{\mu}} \left(\hat{n}^2 - \sin^2 \theta_i \right)^{\frac{1}{2}}} \right)^2 \right). \tag{1.30}$$

We observe:

- As $\hat{n} \to \infty$, $\boldsymbol{I\!R} \to 1$, no matter what the angle of incidence's value. We note that as $\hat{n} \to 1$, provided that $\hat{\mu} = 1$, $\boldsymbol{I\!R} \to 0$, i.e. all incident energy is absorbed (it is transparent).

[5] The limiting case $\frac{\sin \theta_i^*}{\hat{n}} = 1$, is the critical angle $\left(\theta_i^* \right)$ case.

- With increasing $\hat{n}$, the angle for minimum reflectance grows larger. As mentioned previously, for the remainder of the work, we shall take $\hat{\mu} = 1\,(\mu_o = \mu_i = \mu_a)$, thus

$$\hat{n} = \frac{n_a}{n_i} = \sqrt{\frac{\epsilon_a \mu_a}{\epsilon_i \mu_i}} \Rightarrow \epsilon_a \mu_a = (\hat{n})^2 \epsilon_i \mu_i \Rightarrow \epsilon_a = (\hat{n})^2 \epsilon_i. \tag{1.31}$$

- The previous assumption yields

$$\mathbb{R} = \frac{I_r}{I_i} = \frac{1}{2}\left(\left(\frac{\hat{n}^2 \cos\theta_i - \left(\hat{n}^2 - \sin^2\theta_i\right)^{\frac{1}{2}}}{\hat{n}^2 \cos\theta_i + \left(\hat{n}^2 - \sin^2\theta_i\right)^{\frac{1}{2}}}\right)^2 + \left(\frac{\cos\theta_i - \left(\hat{n}^2 - \sin^2\theta_i\right)^{\frac{1}{2}}}{\cos\theta_i + \left(\hat{n}^2 - \sin^2\theta_i\right)^{\frac{1}{2}}}\right)^2\right) \tag{1.32}$$

Remark: We now recall Equation 1.1 connects the concept of a ray with a pulse/beam and observe:

- Since $\bar{I}$ is the energy per unit area per unit time, we obtain the energy associated with an entire pulse/beam by multiplying the irradiance by the cross-sectional area of an initially coherent beam, $\bar{I}A^b$, where A^b is the cross-sectional area of the beam (comprising all of the rays).
- The energy per unit time (power) for a ray in the pulse/beam is then given by $I = \bar{I}A^r = \bar{I}A^b/N_r$, where N_r is the number of rays in the beam (Figure 1.1) and A^r can be considered the area associated with a ray.
- The reflection relation, Equation 1.30 can then be used to compute changes in the magnitude of the reflected rays (and the amount absorbed), with directional changes given by the laws of reflection.

We refer the reader to Gross [1] and Zohdi [3-9] for details.

2 Example

Consider a UV ray hitting an operating table in a hospital surgery room. The light is placed above the table with an offset in y-direction. The resulting ray right before it hits the surface has the velocity $v = [0, \frac{c}{\sqrt{2}}, -\frac{c}{\sqrt{2}}]$, where $c = 3\times10^8 m/s$. The steps to calculate the absorptivity of the surface is shown below.

We can then compute the angle of incidence (θ_i) via the cosine formula between the ray velocity vector (v) and the outward unit normal vector of the table surface (n). The table is assumed flat, therefore has the surface normal $n = [0, 0, 1]$. Then, the angle of incidence is:

$$\cos\theta_i = \frac{v_j \cdot n_j}{\|v_j\|\|n_j\|} \Rightarrow \theta_i = \cos^{-1}\left(\frac{v_j \cdot n_j}{\|v_j\|\|n_j\|}\right) = \cos^{-1}\left(\frac{-c/\sqrt{2}}{c}\right) = 45^\circ \tag{2.1}$$

The component of ray velocity normal to the surface of the table is given by

$$v_{j,\perp} = \|v_j\| \cos\theta_i n_j = -\frac{c}{\sqrt{2}} \tag{2.2}$$

We can calculate the outgoing reflected velocity (v_j^{ref}) by turning the inbound normal velocity outward by subtracting it twice:

$$v_j^{\text{ref}} = v_j - 2v_{j,\perp} = [0, \frac{c}{\sqrt{2}}, -\frac{c}{\sqrt{2}} - (-\frac{2c}{\sqrt{2}})] = [0, \frac{c}{\sqrt{2}}, \frac{c}{\sqrt{2}}] \tag{2.3}$$

Next, we consider the material properties of the operating table. We define $\hat{n}$ as the ratio of the refractive indices of the ambient (incident) medium (n_i) and absorbing medium (n_a) such that

$$\hat{n} = n_a/n_i \tag{2.4}$$

The absorbing medium refractive index, n_a, can be assumed to be of aluminum, $n_a = 1.20$[1]. We assume the incident refractive index to be that of a vacuum as $n_i = 1$.

For this model, we will consider applications with non-magnetic media and frequencies where the magnetic permeability is virtually the same for both the incident and absorbing medium. Thus, we can assume $\mu_i \approx \mu_a$. With this assumption, we define the reflectivity as follows:

$$\boldsymbol{I\!R}(\hat{n}, \theta_i) = \frac{I_r}{I_i} = \frac{1}{2}\left(\left(\frac{\hat{n}^2\cos\theta_i - (\hat{n}^2 - \sin^2\theta_i)^{\frac{1}{2}}}{\hat{n}^2\cos\theta_i + (\hat{n}^2 - \sin^2\theta_i)^{\frac{1}{2}}}\right)^2 + \left(\frac{\cos\theta_i - (\hat{n}^2 - \sin^2\theta_i)^{\frac{1}{2}}}{\cos\theta_i + (\hat{n}^2 - \sin^2\theta_i)^{\frac{1}{2}}}\right)^2\right) = 0.0125 \tag{2.5}$$

where all parameters are defined above. Recall this is a dimensionless parameter and exists in the range of $\in [0, 1]$.

We can use the reflectance to obtain the total amount of absorbed power by a material as follows:

$$P_{abs} = (1 - \boldsymbol{I\!R})P_r = 0.9875P_r \tag{2.6}$$

This shows that the current setup for the disinfecting lamp would result in the operating table absorbing 98.75% of the energy. It is important to determine the energy absorbed by the surface to correctly position the lamp, as well as choose the correct amount of light intensity for decontamination of surfaces.

[1] Refractive index of Al. https://refractiveindex.info/?shelf=mainbook=Alpage=Rakic

3 Assignment

In this assignment, you will be introduced to the basics of reflectance calculations. If you use Python or another language for simplifying answers or evaluating specific numbers, clearly set up the equations before showing your numerical results.

The template code to complete the project is given here on `GitHub`.

Problem 1

Write a Python function that would take in refractive index, incident velocity vector, and normal vector and calculate the incident angle, reflected velocity, and absorbance. Use the given template code to fill in the blanks in the function `myOptics`.

Problem 2

For the given surface normal, ray velocity, and surface refractive index combinations, calculate: (a) incidence angle θ_i, (b) ray velocity vector v_j after reflection, (c) percentage of energy absorbed P_{abs} at the surface point. Assume ray velocity magnitude to be the speed of light, $c = 3 \times 10^8 m/s$ and ambient refractive index to be $n_i = 1$.

Problem 2.1

$n = [0.47, -0.58, 0.67]$, $v_f = [0.44, -0.58, 0.69]$, $n_a = 15$

Problem 2.2

$n = [0.48, -0.58, 0.66]$, $v_f = [0.39, -0.58, 0.72]$, $n_a = 18.99$

Problem 2.3

$n = [0.30, 0.76, 0.58]$, $v_f = [0.58, 0.50, 0.65]$, $n_a = 15.38$

Problem 2.4

$n = [0.58, 0.23, 0.78]$, $v_f = [0.09, -0.58, 0.81]$, $n_a = 17.69$

Problem 2.5

$n = [0.31, -0.58, 0.76]$, $v_f = [0.46, 0.68, 0.58]$, $n_a = 14.97$

Problem 2.6

$n = [0.29, 0.76, 0.58]$, $v_f = [0.58, 0.08, 0.81]$, $n_a = 1.80$

Problem 2.7

$n = [0.58, 0.49, 0.65]$, $v_f = [0.09, -0.58, 0.81]$, $n_a = 3.79$

Problem 2.8

$n = [0.47, -0.58, 0.67]$, $v_f = [0.40, 0.71, 0.58]$, $n_a = 11.21$

Problem 2.9

$n = [0.35, 0.74, 0.58]$, $v_f = [0, 0.82, 0.58]$, $n_a = 17.58$

Problem 2.10

$n = [0.13, 0.81, 0.58]$, $v_f = [0.28, -0.58, 0.77]$, $n_a = 19.80$

4 Solution

Problem 1

Write a Python function that would take in refractive index, incident velocity vector, and normal vector and calculate the incident angle, reflected velocity, and absorbance. Use the given template code to fill in the blanks in the function `myOptics`.
Solution Cell Block:

```

############################################### Main Function ################################################

def myOptics(n,v_f,n_hat): #function that will solve for incidence angle,reflected ray velocity, and P_abs

    # define speed of light
    c = 3e8

    #RELATION: n . v_f. = ||n||.||v_f||.cos(theta_i)

    # calculate angle of incidence (theta_i)
    theta_i = np.arccos(np.dot(n,v_f) / (np.linalg.norm(v_f)*np.linalg.norm(n))) #FILL IN HERE

    # calculate angle of incidence (theta_i) in degrees
    theta_deg = theta_i * 180 / np.pi #FILL IN HERE

    # calculate reflected ray velocity (v_ref)
    v_ref = v_f - 2 * np.dot(n,v_f) * n #FILL IN HERE

    # calculate reflectivity (R)
    IR = (1/2)*(((n_hat**2*np.cos(theta_i)-np.sqrt(n_hat**2-np.sin(theta_i)**2))/\
                           (n_hat**2*np.cos(theta_i)+np.sqrt(n_hat**2-np.sin(theta_i)**2)))**2+\
                           ((np.cos(theta_i)-np.sqrt(n_hat**2-np.sin(theta_i)**2))/\
                           (np.cos(theta_i)+np.sqrt(n_hat**2-np.sin(theta_i)**2)))**2) #FILL IN HERE

    # calculate absorbed power (P_abs) as a percentage
    P_abs = (1-IR)*100  #FILL IN HERE

    return theta_deg, list(v_ref), P_abs
```

Problem 2

For the given surface normal, ray velocity, and surface refractive index combinations, calculate: (a) incidence angle θ_i, (b) ray velocity vector v_j after reflection, (c) percentage of energy absorbed P_{abs} at the surface point. Assume ray velocity magnitude to be the speed of light, $c = 3 \times 10^8 m/s$ and ambient refractive index to be $n_i = 1$.

Problem 2.1

$n = [0.47, -0.58, 0.67]$, $v_f = [0.44, -0.58, 0.69]$, $n_a = 15$
Solution Cell Block:

```

############################################### Problem 2.1 ################################################

# define normal vector
n = np.array([0.47, -0.58, 0.67]) #FILL IN HERE

# define velocity vector
```

```
v_f = np.array([0.44, -0.58, 0.69]) #FILL IN HERE

# define refractive index
n_a = 15 #FILL IN HERE

# call myOptics function
theta_deg, v_ref, P_abs = myOptics(n,v_f,n_a)

print('Incidence angle (deg):',theta_deg) # print angle of incidence
print('Reflected ray velocity:',v_ref) # print reflected ray velocity
print('Absorbed energy (\%):',P_abs) # print absorbed power

# call plotInteractive function
plotInteractive(n,v_f,v_ref)
```

Problem 2.2

$n = [0.48, -0.58, 0.66]$, $v_f = [0.39, -0.58, 0.72]$, $n_a = 18.99$
Solution Cell Block:

```

############################################## Problem 2.2 ###############################################

# define normal vector
n = np.array([0.48,-0.58,0.66]) #FILL IN HERE

# define velocity vector
v_f = np.array([0.39,-0.58,0.72]) #FILL IN HERE

# define refractive index
n_a = 18.99 #FILL IN HERE

# call myOptics function
theta_deg, v_ref, P_abs = myOptics(n,v_f,n_a)

print('Incidence angle (deg):',theta_deg) # print angle of incidence
print('Reflected ray velocity:',v_ref) # print reflected ray velocity
print('Absorbed energy (\%):',P_abs) # print absorbed power

# call plotInteractive function
plotInteractive(n,v_f,v_ref)
```

Problem 2.3

$n = [0.30, 0.76, 0.58]$, $v_f = [0.58, 0.50, 0.65]$, $n_a = 15.38$
Solution Cell Block:

```

############################################## Problem 2.3 ###############################################

# define normal vector
n = np.array([0.30,0.76,0.58]) #FILL IN HERE

# define velocity vector
v_f = np.array([0.58,0.50,0.65]) #FILL IN HERE

# define refractive index
n_a = 15.38 #FILL IN HERE

# call myOptics function
theta_deg, v_ref, P_abs = myOptics(n,v_f,n_a)

print('Incidence angle (deg):',theta_deg) # print angle of incidence
```

```
print('Reflected ray velocity:',v_ref) # print reflected ray velocity
print('Absorbed energy (\%):',P_abs) # print absorbed power

# call plotInteractive function
plotInteractive(n,v_f,v_ref)
```

Problem 2.4

$n = [0.58, 0.23, 0.78]$, $v_f = [0.09, -0.58, 0.81]$, $n_a = 17.69$
Solution Cell Block:

```

############################################## Problem 2.4
    ###############################################

# define normal vector
n = np.array([0.58,0.23,0.78]) #FILL IN HERE

# define velocity vector
v_f = np.array([0.09,-0.58,0.81]) #FILL IN HERE

# define refractive index
n_a = 17.69 #FILL IN HERE

# call myOptics function
theta_deg, v_ref, P_abs = myOptics(n,v_f,n_a)

print('Incidence angle (deg):',theta_deg) # print angle of incidence
print('Reflected ray velocity:',v_ref) # print reflected ray velocity
print('Absorbed energy (\%):',P_abs) # print absorbed power

# call plotInteractive function
plotInteractive(n,v_f,v_ref)
```

Problem 2.5

$n = [0.31, -0.58, 0.76]$, $v_f = [0.46, 0.68, 0.58]$, $n_a = 14.97$
Solution Cell Block:

```

############################################## Problem 2.5
    ###############################################

# define normal vector
n = np.array([0.31,-0.58,0.76]) #FILL IN HERE

# define velocity vector
v_f = np.array([0.46,0.68,0.58]) #FILL IN HERE

# define refractive index
n_a = 14.97 #FILL IN HERE

# call myOptics function
theta_deg, v_ref, P_abs = myOptics(n,v_f,n_a)

print('Incidence angle (deg):',theta_deg) # print angle of incidence
print('Reflected ray velocity:',v_ref) # print reflected ray velocity
print('Absorbed energy (\%):',P_abs) # print absorbed power

# call plotInteractive function
plotInteractive(n,v_f,v_ref)
```

Problem 2.6

$n = [0.29, 0.76, 0.58]$, $v_f = [0.58, 0.08, 0.81]$, $n_a = 1.80$
Solution Cell Block:

```

################################################ Problem 2.6 ################################################

# define normal vector
n = np.array([0.29,0.76,0.58]) #FILL IN HERE

# define velocity vector
v_f = np.array([0.58,0.08,0.81]) #FILL IN HERE

# define refractive index
n_a = 1.80 #FILL IN HERE

# call myOptics function
theta_deg, v_ref, P_abs = myOptics(n,v_f,n_a)

print('Incidence angle (deg):',theta_deg) # print angle of incidence
print('Reflected ray velocity:',v_ref) # print reflected ray velocity
print('Absorbed energy (\%):',P_abs) # print absorbed power

# call plotInteractive function
plotInteractive(n,v_f,v_ref)
```

Problem 2.7

$n = [0.58, 0.49, 0.65]$, $v_f = [0.09, -0.58, 0.81]$, $n_a = 3.79$
Solution Cell Block:

```

################################################ Problem 2.7 ################################################

# define normal vector
n = np.array([0.58,0.49,0.65]) #FILL IN HERE

# define velocity vector
v_f = np.array([0.09,-0.58,0.81]) #FILL IN HERE

# define refractive index
n_a = 3.79 #FILL IN HERE

theta_deg, v_ref, P_abs = myOptics(n,v_f,n_a) # call myOptics function

print('Incidence angle (deg):',theta_deg) # print angle of incidence
print('Reflected ray velocity:',v_ref) # print reflected ray velocity
print('Absorbed energy (\%):',P_abs) # print absorbed power

# call plotInteractive function
plotInteractive(n,v_f,v_ref)
```

Problem 2.8

$n = [0.47, -0.58, 0.67]$, $v_f = [0.40, 0.71, 0.58]$, $n_a = 11.21$
Solution Cell Block:

```

################################################ Problem 2.8 ################################################

# define normal vector
n = np.array([0.47,-0.58,0.67]) #FILL IN HERE
```

```

# define velocity vector
v_f = np.array([0.40,0.71,0.58]) #FILL IN HERE

# define refractive index
n_a = 11.21 #FILL IN HERE

# call myOptics function
theta_deg, v_ref, P_abs = myOptics(n,v_f,n_a)

print('Incidence angle (deg):',theta_deg) # print angle of incidence
print('Reflected ray velocity:',v_ref) # print reflected ray velocity
print('Absorbed energy (\%):',P_abs) # print absorbed power

# call plotInteractive function
plotInteractive(n,v_f,v_ref)
```

Problem 2.9

$n = [0.35, 0.74, 0.58]$, $v_f = [0, 0.82, 0.58]$, $n_a = 17.58$
Solution Cell Block:

```

############################################## Problem 2.9
    ###############################################

# define normal vector
n = np.array([0.35,0.74,0.58]) #FILL IN HERE

# define velocity vector
v_f = np.array([0,0.82,0.58]) #FILL IN HERE

# define refractive index
n_a = 17.58 #FILL IN HERE

# call myOptics function
theta_deg, v_ref, P_abs = myOptics(n,v_f,n_a)

print('Incidence angle (deg):',theta_deg) # print angle of incidence
print('Reflected ray velocity:',v_ref) # print reflected ray velocity
print('Absorbed energy (\%):',P_abs) # print absorbed power

# call plotInteractive function
plotInteractive(n,v_f,v_ref)
```

Problem 2.10

$n = [0.13, 0.81, 0.58]$, $v_f = [0.28, -0.58, 0.77]$, $n_a = 19.80$
Solution Cell Block:

```

############################################## Problem 2.10
    ###############################################

# define normal vector
n = np.array([0.13,0.81,0.58]) #FILL IN HERE

# define velocity vector
v_f = np.array([0.28,-0.58,0.77]) #FILL IN HERE

# define refractive index
n_a = 19.80 #FILL IN HERE

# call myOptics function
theta_deg, v_ref, P_abs = myOptics(n,v_f,n_a)
```

```

print('Incidence angle (deg):',theta_deg) # print angle of incidence
print('Reflected ray velocity:',v_ref) # print reflected ray velocity
print('Absorbed energy (\%):',P_abs) # print absorbed power

# call plotInteractive function
plotInteractive(n,v_f,v_ref)
```

5 Ethical Considerations for this Project

A goal of this project is to enable advancements in science and engineering through to address critical national challenges associated with next generation food systems. There are deep ethical considerations associated with any technology, in particular for food systems. While technology has tremendous potential to identify greater efficiencies, when it is created without appropriate consideration for who will have access to and control over new resources, or how the new technologies will impact those who work in the system, the efficiencies identified may come at the cost of greater societal inequity. It is important to pursue harnessing technology to disrupt existing inequities, rather than further entrench existing power structures. The following areas should be considered:

- Labor: 1) occupational health, 2) food manufacturing, and 3) outdoor agriculture labor;
- Producers: 1) Small- to mid-size farms, 2) urban agriculture, and 3) research in farm transitions; Technology: 1) research in technology and democracy;
- Health Human Rights: 1) land rights, 2) social justice, and 3) decolonization in agriculture;

Please consider the following questions:

- What are the societal implications of the technology that you are developing?
- Can this technology be distributed fairly and equitably to a wide variety of entities in agricultural industry?
- Are there any potential unintended consequences of this technology becoming available?
- Are there any harmful "spinoffs" of this technology?
- Are there any useful "spinoffs" of this technology?

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