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1 Input:  $\epsilon_0$ , initial learning rate
2 Input:  $\alpha$ , decay rate of learning rate
3 Input:  $\beta_1$ , 1st order moment for plain gradients
4 Input:  $\beta_2$ , 2nd order moment for squared gradients
5 Input:  $\zeta$ , small constant to avoid zero division
6 Input:  $m$ , minibatch size
7 Input:  $k$ , epoch size
8 Input:  $\theta$ , initial weights
9 Input:  $\mathbf{X}$ , training dataset inputs
10 Input:  $\mathbf{y}$ , training dataset targets
11 Initialize:  $s \leftarrow 0$ , accumulation variable for historical gradients
12 Initialize:  $r \leftarrow 0$ , accumulation variable for historical squared gradients
13 Initialize:  $t \leftarrow 0$ , counter of each gradient update
14 Initialize:  $j \leftarrow 1$ , current epoch
15 while  $j \leq k$  do
16     update learning rate  $\epsilon_j \leftarrow \epsilon_0 + \alpha(\epsilon_{j-1} - \epsilon_0)$ 
17     while stopping criteria is not satisfied do
18          $\{\mathbf{x}^1 \dots \mathbf{x}^m\}, \{\mathbf{y}^1 \dots \mathbf{y}^m\} \leftarrow$  get a sample from  $\mathbf{X}$  and  $\mathbf{y}$  randomly
19         calculate estimation of gradient  $\hat{g} \leftarrow \frac{1}{m} \sum_{i=1}^m L(f(\mathbf{x}^i; \theta), \mathbf{y}^i)$ 
20         accumulate historical gradients  $s \leftarrow \beta_1 s + (1 - \beta_1) \hat{g}$ 
21         accumulate historical squared gradients  $r \leftarrow \beta_2 r + (1 - \beta_2) \hat{g} \odot \hat{g}$ 
22          $t \leftarrow t + 1$ 
23         apply bias correction to 1st order momentum  $\hat{s} = \frac{s}{1 - \beta_1^t}$ 
24         apply bias correction to 2nd order momentum  $\hat{r} = \frac{r}{1 - \beta_2^t}$ 
25         calculate step size  $\Delta\theta \leftarrow -\epsilon_j \frac{\hat{s}}{\sqrt{\zeta + \hat{r}}}$ 
26         update weights  $\theta \leftarrow \theta + \Delta\theta$ 
27      $j \leftarrow j + 1$  go to next epoch

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