Master Computer Vision

Probabilistic Robotics

Solved Exercises

Author:
Muhammad Usman

Course Co ordinator:
Pere Ridao Smith

March 24, 2014

1 Question E1:

When an Extended Kalman Filter supposed to work better?

a when the uncertainty of the estimate is big and the process model is highly non Linear nearby the current estimate.

b When the uncertainty of the estimate is big and the process model is quite linear nearby the current estimate.

c When the uncertainty of the estimate is small and the process model is highly non linear nearby the current estimate.

d When the uncertainty of the estimate is small and the process model is quite linear nearby the current estimate.

1.1 Solution

Whenever we use Kalman Filter we make an assumption that uncertainty in robot pose can be represented by a Gaussian random vector. This is very strong assumption and it make our life much easier as far as computation is concerned. We know that Gaussian when passed through linear function remains Gaussian and this allows us to use Kalman filter for localization of robot. But when a Gaussian is passed through non linear function it is destroyed. So we try to make system work by using Extended Kalman filter in which we approximate non linear function with linear one if non linearity is small enough to be approximated. This will work as far as uncertainty is small enough so linear approximation is good or the non linearity is so small that the linear approximation is good. As less certain the robot higher will be its Gaussian belief, and more it will be affected by the non linearities in state transition and measurement.[1] So d is the correct option.

2 Question E2:

Assume a 1 DOF mobile robot moving in a hallway while using odometery to measure the displacement and a camera for detecting the door, but not being able to identify which one is observed. Mark the false sentence.

a The solution can be implemented using histogram filter.

b The solution cannot be implemented using Bayes filter.

c The solution can be implemented using the Particle filter.

d The solution can be implemented using the EKF.

2.1 Solution:

Lets comment on each of the candidate solution independently.

a Histogram filter is just the discreet implementation of Bayes Filter. It is very close to original Bayes filter and it can be used to localize our robot. So this statement is true.

b Bayes filter uses continuous Pdf and that's why it cannot be used by itself as computer works on discreet data and we used probabilistic algorithms that use tractable approximations to the Bayes filter. If we implement Bayes filter by considering Pdf's as Histograms it becomes Histogram filter and corresponding algorithm for localization is called Grid Localisation and similarly if we represent Pdf using Particles it become Particle filters. So this statement is true.

c Our model is Linear and particle filter can deal with both linear and non linear models so ye this statement is true, we can use particle filter to localize this robot.

d Our motion model of system is linear and EKF is extension of Kalman filter to deal with non linearity. SO this statement is false we don't need to use EKF when we can use Kalman filter for this system.

3 Question E3:

Which simplification shave been applied to compute the increment of heading in the following odometery equations($\triangle\theta_k = \frac{dr-dl}{w}$)?

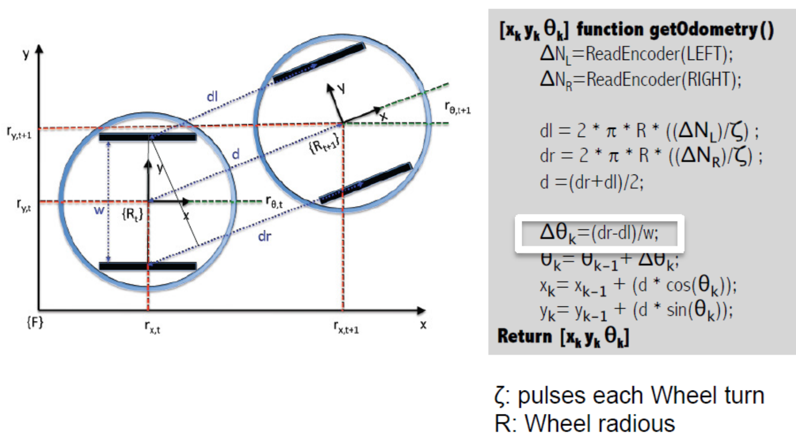

Figure 1: Get Odometery Function

3.1 Solution:

The given equation is actually calculation of change in robot pose. We bring differential motion into our calculation and we consider very small and tiny motions so that it becomes linear as normally it would be a curve. This is our first basic assumption. After this we will calculate the robot displacement and we will arrive at the calculation of robot pose change. Below is the illustration how we can get out triangle from which we can get change in pose simply by using trigonometry. In the figure shown below we have shown how we get our change in robot pose and we also show our right angle triangle. As in case of right angle triangle we know that,

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

But we are in differential motion in which θ is very very small and we know that for very very small θ,

$$\sin\theta \approx \theta$$

$$\cos\theta \approx 1$$

$$\tan\theta \approx \theta$$

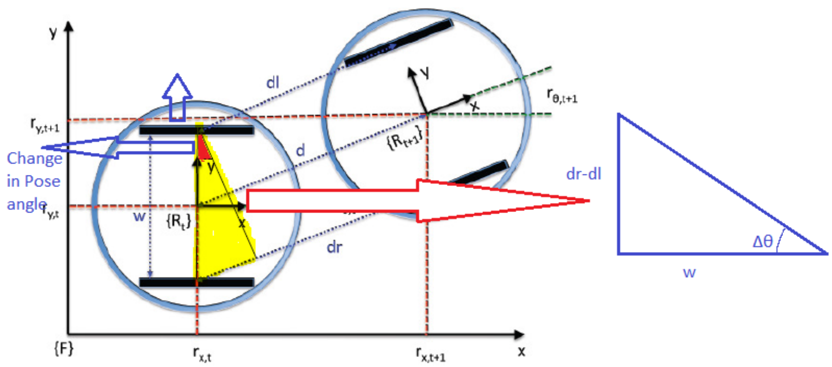

Figure 2: Illustration

Now as in our case by considering the right angle triangle we shown,

$$\sin \triangle\theta = dr - dl$$

$$\cos \triangle\theta = w$$

Hence,

$$\triangle\theta = \frac{dr - dl}{w}$$

4 Question E4:

A 1DOF Mobile robot moving in a non circular hallway is able to measure the square of its distance with respect to the origin of the corridor. The range sensor noise is also quadratic with the distance. The motion model is based on the use of odometery. Answer the following questions assuming that the robot position belief can be modelled under the Gaussian assumption:

1 Propose a well suited filter to estimate the robot position? Justify your choice.

2 Provide the formulation of the corresponding filter:

  a The motion model equation (filter prediction equation) showing the noise.

  b The measurement model (filter observation equation) showing the noise.

4.1 Solution:

As given in question robot is able to measure square of its distance with respect to the origin of corridor and the noise of range sensor is also quadratic so they are non linear. The motion model of system it will depend on previous position plus odometery and noise. But when we will come up with its measurement model it will involve measurement from sensor and its noise so it will be non linear model. Although robot position belief can be modelled under the Gaussian assumption so the best option is to implement Extended Kalman filter. We can also use Monte Carlo(particle filter) to localize but Extended Kalman Filter is more efficient. The state is predicted using non linear model.

5 Question E5:

A Mobile Robot uses a docking station for battery charging. When the robot is commanded to dock (u_t = DOCK INTO STATION) , if its not already docked it succeeds with 0.8 probabilities but fails with 0.2 probability. Otherwise, this is if it is already docked, it remains docked with 0.9 probability but can fail with 0.9 probability. Whenever the robot is docked, a docking detection sensor detects the situation (z_t = ROBOT IS DOCKED) with a 0.7 probability but fails in detection with 0.3 probability. If the docking station is free, the docking detection sensor detects the situation with 0.6 probability but fails in detection with 0.4 probability. The following sequence of actions/sensor readings have been executed: (1) te action u_t = DOCK INTO STATION is executed. (2) The sensor detects z_t = ROBOT IS DOCKED. Fill the Table of the Robot Belief of being DOCKED.

Executed Action	Probability of ROBOT IS DOCKED	Probability of ROBOT IS NOT DOCKED
Initial Guess	0.5	0.5
u_t = DOCK INTO STATION	0.85	0.15
z_t = ROBOT IS DOCKED	0.9084	0.0916

5.1 Solution:

Now i will explain the calculations and for the ease of visualization i have used D to denote Docket position and $\overline{D}$ to denote position where robot is not docked. First part of calculation is also know as prediction.
Given Data:

$$P(X = D|U = D, X = \overline{D}) = 0.8$$
$$P(X = \overline{D}|U = D, X = \overline{D}) = 0.2$$
$$P(X = D|U = D, X = D) = 0.9$$
$$P(X = \overline{D}|U = D, X = D) = 0.1$$

Data Related to Sensor accuracy:

$$P(Z_t = D|X = D) = 0.7$$
$$P(Z_t = \overline{D}|X = D) = 0.3$$
$$P(Z_t = \overline{D}|X = \overline{D}) = 0.6$$
$$P(Z_t = D|X = \overline{D}) = 0.4$$

Initial Guess:

$$P(X_0 = D) = 0.5$$
$$P(X_0 = \overline{D}) = 0.5$$

$\boldsymbol{U_t}$ = Dock into Station:

$$P(X_1 = D) = P(X = D|U = D, X = D)P(X_0 = D) + P(X = D|U = D, X = \overline{D})P(X_0 = \overline{D})$$
$$P(X_1 = D) = (0.9)(0.5) + (0.8)(0.5) = 0.85$$
$$P(X_1 = \overline{D}) = P(X = \overline{D}|U = D, X = D)P(X_0 = D) + P(X = \overline{D}|U = D, X = \overline{D})P(X_0 = \overline{D})$$
$$P(X_1 = \overline{D}) = (0.1)(0.5) + (0.2)(0.5) = 0.85$$

Update after sensor data $\boldsymbol{z_t}$ = ROBOT IS DOCKED :

$$P(X_2 = D) = \eta P(Z_t = D|X = D)P(X_1 = D) = \eta(0.7)(0.85) = 0.595\eta \tag{1}$$

$$P(X_2 = \overline{D}) = \eta P(Z_t = D|X = \overline{D})P(X_1 = \overline{D}) = \eta(0.4)(0.15) = 0.06\eta \tag{2}$$

From (1) and (2) Lets find value of η the normalizer

$$0.595\eta + 0.06\eta = 1$$

So

$$\eta = 1.52672$$

And Our Updated belief will be,

$$P(X_2 = D) = 0.595\eta = 0.9084$$

$$P(X_2 = \overline{D}) = 0.06\eta = 0.0916$$

It can be seen that how measurement from sensor decreases the uncertainty.

6 Question E6:

A mobile Robot is being localized within a soccer field. For this purpose, 6 Landmarks are located in the corners as well as in the ends of the line in the middle of the field. The robot is able to detect only the range to the landmarks. It is also equipped with the dead reckonig system providing $[\triangle x \triangle y \triangle \theta]$. For the sequence of events shown in the next figure, plot the evolution of particles. Explain the process briefly.

6.1 Solution

On the attached figure i have showed the evolution of particles according to the sequence of events. At start the position of robot is unknown so particles are present everywhere as robot can be anywhere in the soccer field. In the next step robot moves and as all the particles but still they are present everywhere as position of robot is unknown. In the third step land mark 2 is detected and measurement of range from the sensor arrived, now particles gathered around the measured range from the landmark 2 but its a semicircle as robot can be anywhere around the measured range from landmark 2. In the next step robot remains stationary so particles will also remain stationary as no movement no prediction and no update. Now landmark 3 is detected and range is measured and we have two ranges so two semi circles (regions) specified by the measured range from landmarks and our particles will be inside the region where both these semi circles intersect as shown in figure. Again robot remain static so no prediction and no update. Land mark four is detected in the next step and now our particles will spread in the region of intersection of the ranges measured from all three landmarks and now we can somehow confidently localize our robot in soccer field. In the next step robot moved and our particles will also moved and as no update from sensors so uncertainty will increase and particles will spread a little bit as seen in figure.

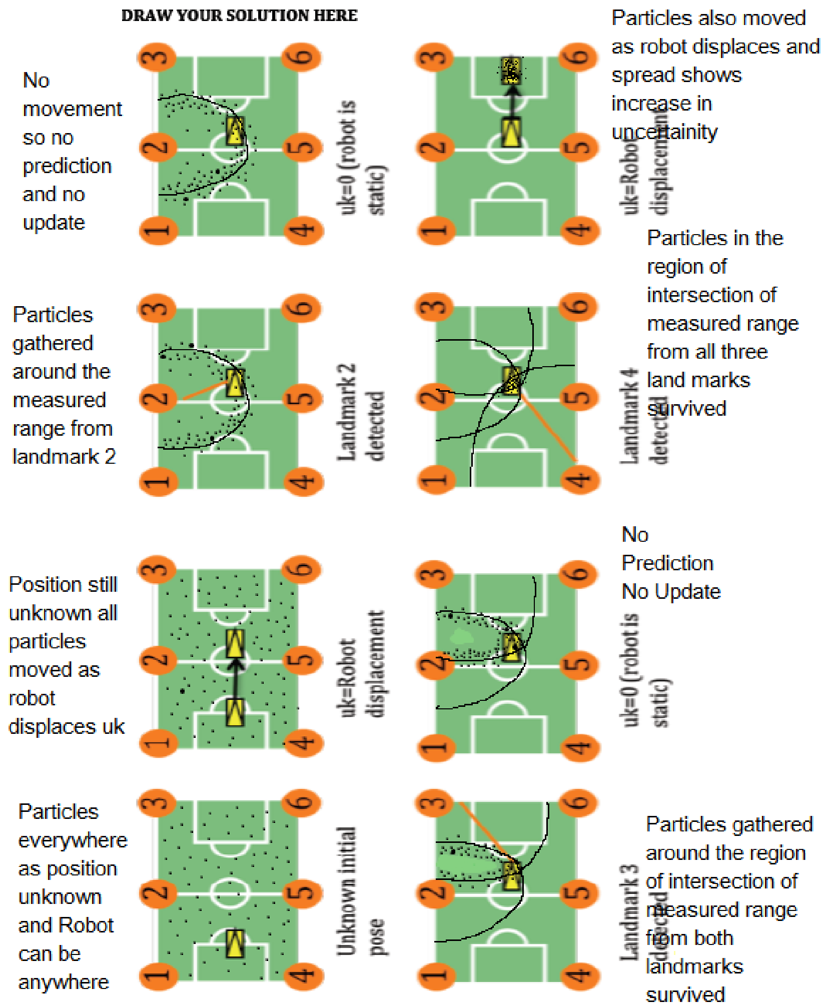

Figure 3: Evolution of Particles

References

[1] Lecture Notes, PERE Ridao Smith