University of Burgundy
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Applied Mathematics

Optimization

Chapter 1: Function minimization 2D

1. Powell's method

Powell's method start with one initial point (x0) and two vectors (e1 and e2, they are linearly independent). The following iterative process describes the algorithm:

1. Define searching step distance D.
2. Start at x0, follow the direction e1 or -e1, find the minimum point x1.
3. Start at x1, follow the direction e2 or -e2, find the minimum point x2.
4. e1 = e2, e2 = x2 - x0, x2 = x0, go to step 2.

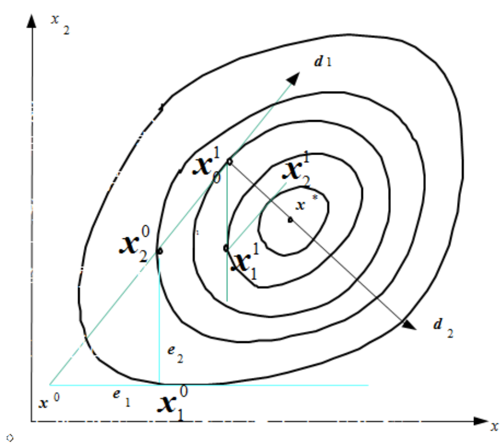

Figure 1:

Our test function is $f(x, y) = -e^{-x^2-y^2} + e^{-(x+1)^2-(y+1)^2}$. And the result we got is shown as figure below. We found the minimum value is $z = -0.88911$ at point $x = 0.13893andy = 0.11597$.

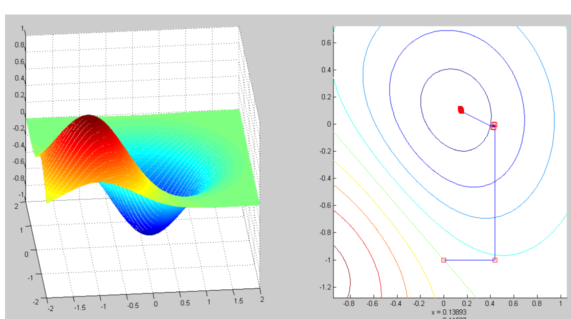

Figure 2:

The accuracy is not enough because the included angle between two searching vectors become smaller and smaller which leads it goes straight.

2. Quasi-Newton method: DFP

In essence, Broyden, DFP and BFGS is the same process with different Hesse matrix. Hesse matrix is used for calculate the direction of searching process. The algorithm can be implemented as following process:

1. Initialize start point x^0, Hesse matrix H_0, error ε, iteration counter $k = 0$ and maximum iterative time n.
2. If $f'(x^k) < \varepsilon$, go to step 6.
3. Get searching direction $d^k = -H_k f'(x^k)$.
4. Find t^k which leads $f(x^k + t^k d^k) = minf(x^k + td^k)$. Then s_k is given by $s_k = t^k d^k$, $y_k = f'(x^k + s_k) - f'(x^k)$.
5. Get $H_{k+1} = H_k - \frac{H_k y_k y_k^T H_k}{y_k^T H_k y_k} + \frac{s_k s_k^T}{y_k^T s_k}$, $k = k + 1$. If $k < n$, go to step 2.
6. Minimum point is x^k.

For step 4, we employ golden section method and interval is obtain by advance and retreat strategy. The flowchart of advance and retreat strategy is shown as figure below:

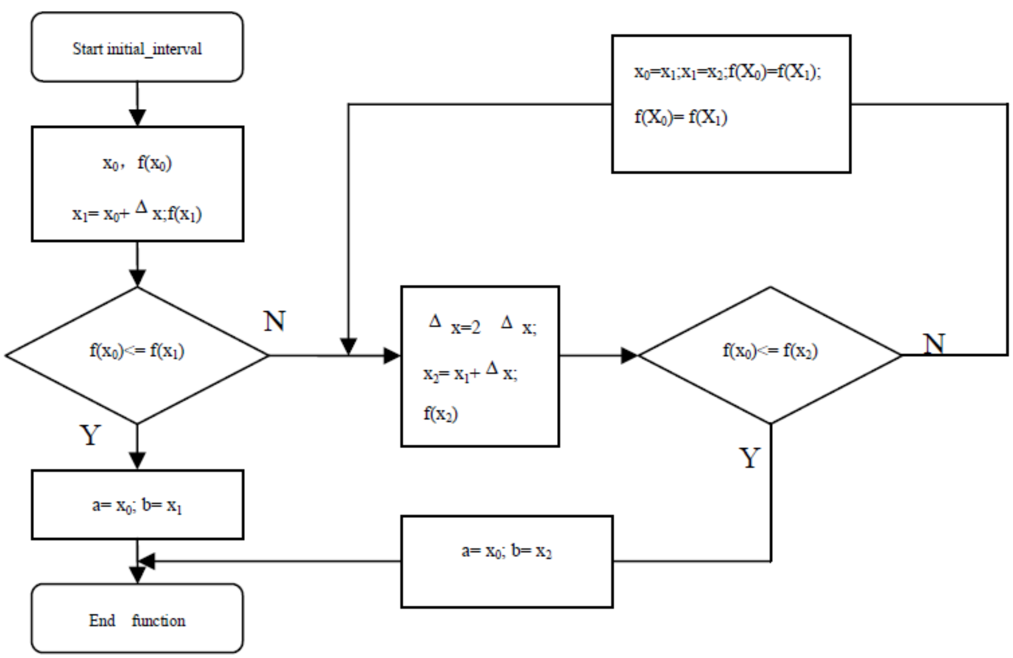

Figure 3:

Ideally, we can find the minimum by two iterations. Due to the accuracy of calculation, the first two iterations is not enough to get a good result. consequently, we still need many iterations in our program. The following figure shows the result we got.

After 30 iterations we found the minimum value is 3 at point (0,-1).

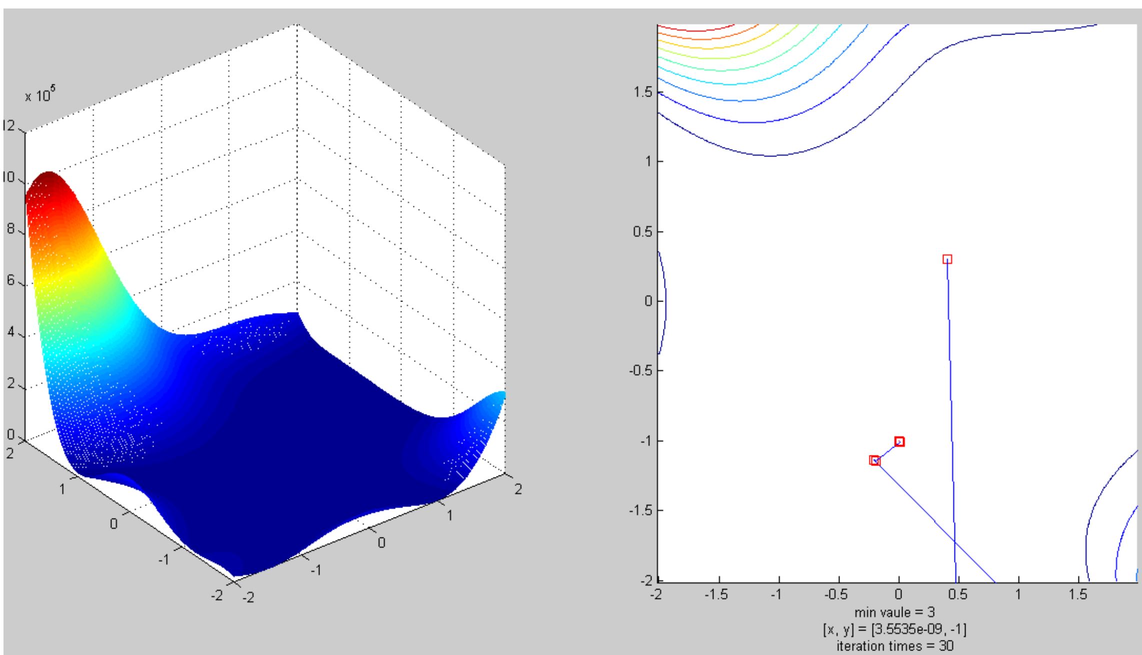

Figure 4: