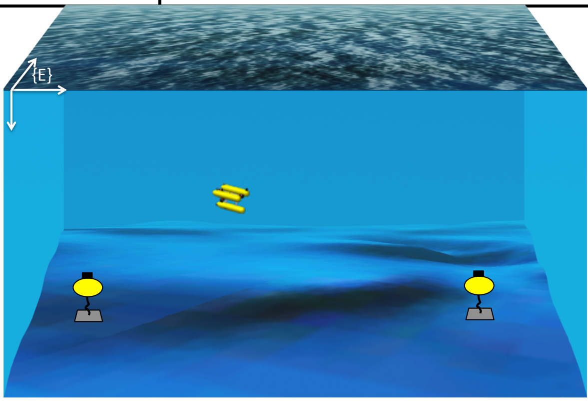
PARTICLE FILTER EXERCICES

PF1 An AUV moves in the **3D volume** of coverture of 2 underwater transpoders τ_1 and τ_2 located at known positions with respect to the earth fixed frame {E}. When the robot interrogates transponder τ_i, it measures the distance between them (the robot and the transponder). The vehicle is also equipped with a dead reckoning system which provides displacements ${}^B\Delta\eta$ represented in the body fixed frame. Assuming a planar seafloor at a known depth, answer the following questions:

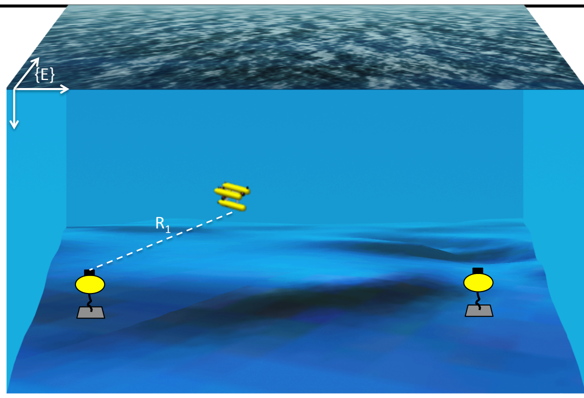
a) Draw, over the figures shown in the answer's section, the particle evolution from t_1 to t_4.
b) If the robot is equipped with a depth sensor, draw again the evolution of particles from t_1 to t_4, assuming that depth and range measurements take place almost simultaneously (no need to predict in the middle).
c) If no prediction is carried out between the range and depth measurement, how would you compute the P(Z|X) for both measurements together?

EXERCICE 7	STUDENT NAME:

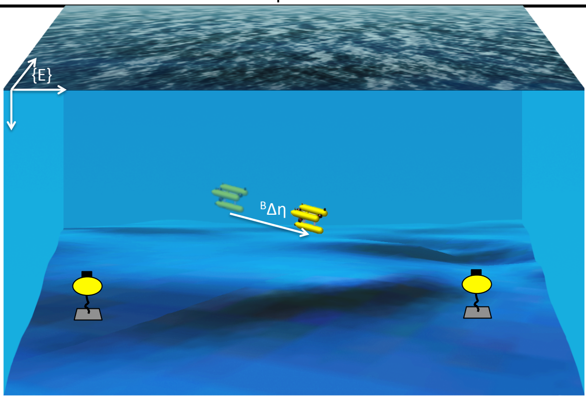
Without Depth measurement



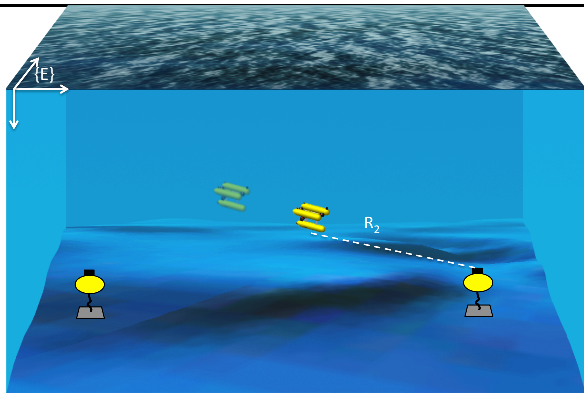
t_1: The AUV position is unknown



t_2: The range R1 to transponder 1 is measured.

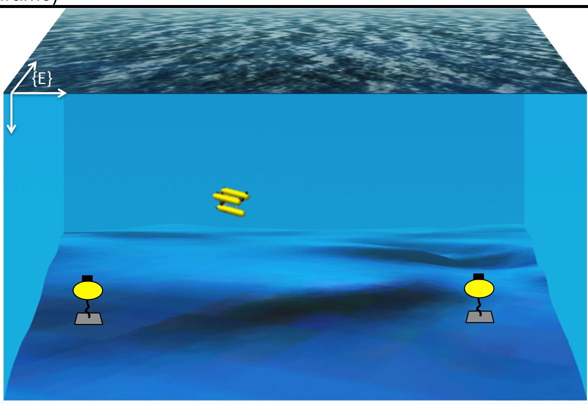


t_3: The AUV displaces a quantity $^B\Delta\eta$ (known in the robot body frame)

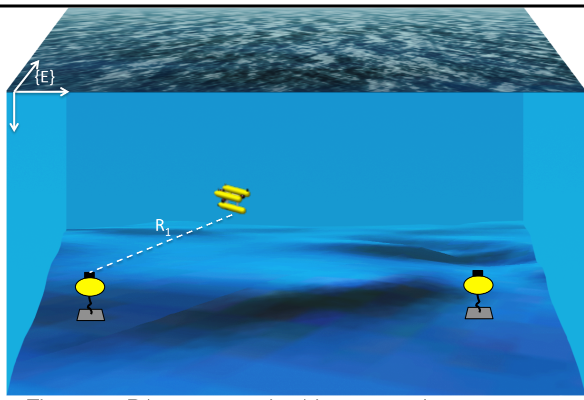


t_4: The range R2 to transponder 2 is measured.

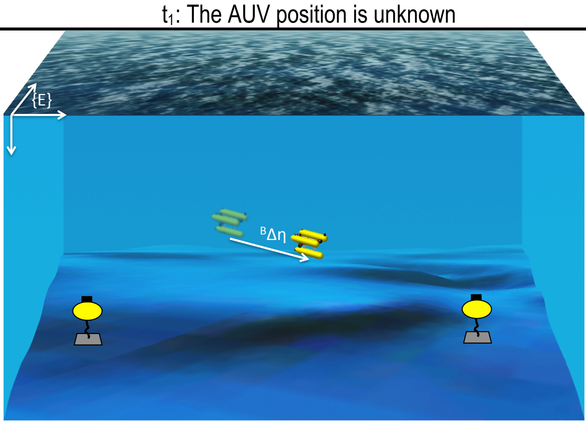
With Depth measurement



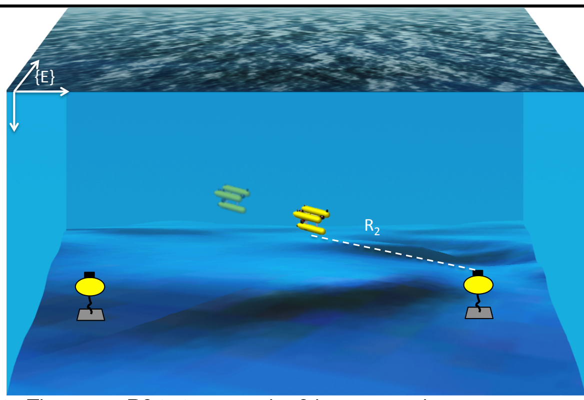
t_1: The AUV position is unknown



t_2: The range R1 to transponder 1 is measured.



t_3: The AUV displaces a quantity $^B\Delta\eta$ (known in the robot body frame)



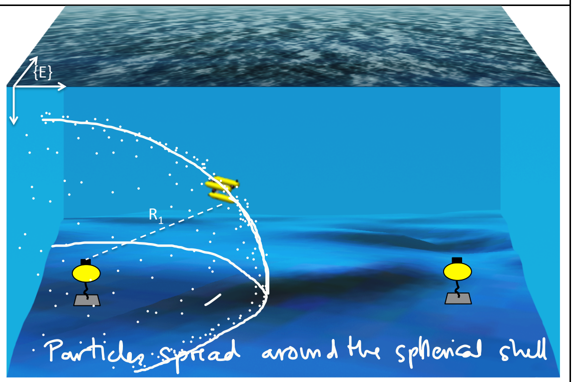
t_4: The range R2 to transponder 2 is measured.

SOLUTION TO PF1

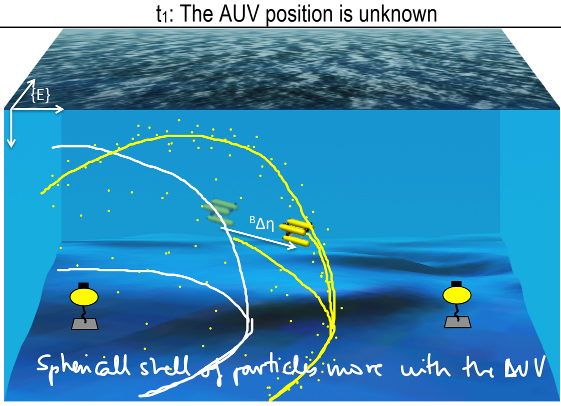
EXERCICE 7 | **STUDENT NAME:** SOLUTION

Without Depth measurement

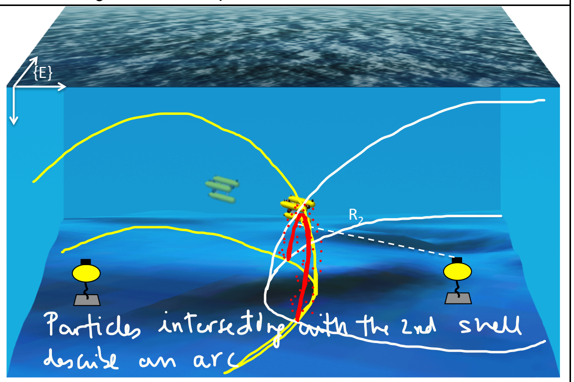
t_1: The AUV position is unknown



t_2: The range R1 to transponder 1 is measured.

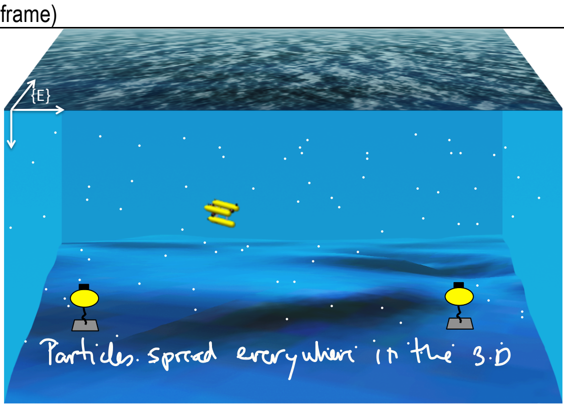


t_3: The AUV displaces a quantity ${}^{B}\Delta\eta$ (known in the robot body frame)

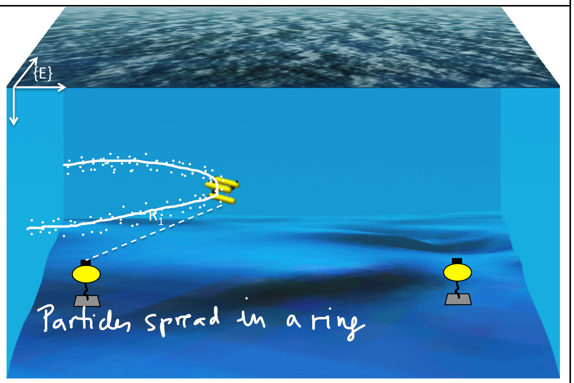


t_4: The range R2 to transponder 2 is measured.

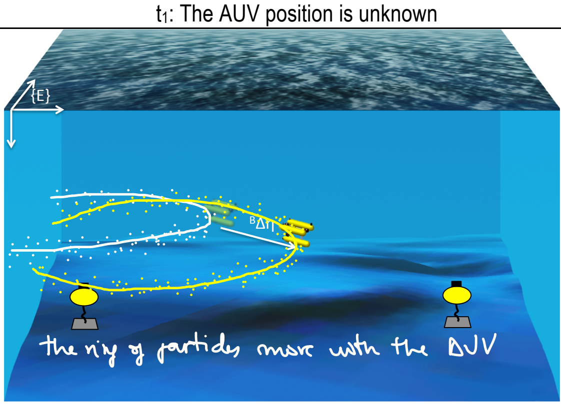
With Depth measurement



t_1: The AUV position is unknown



t_2: The range R1 to transponder 1 is measured.

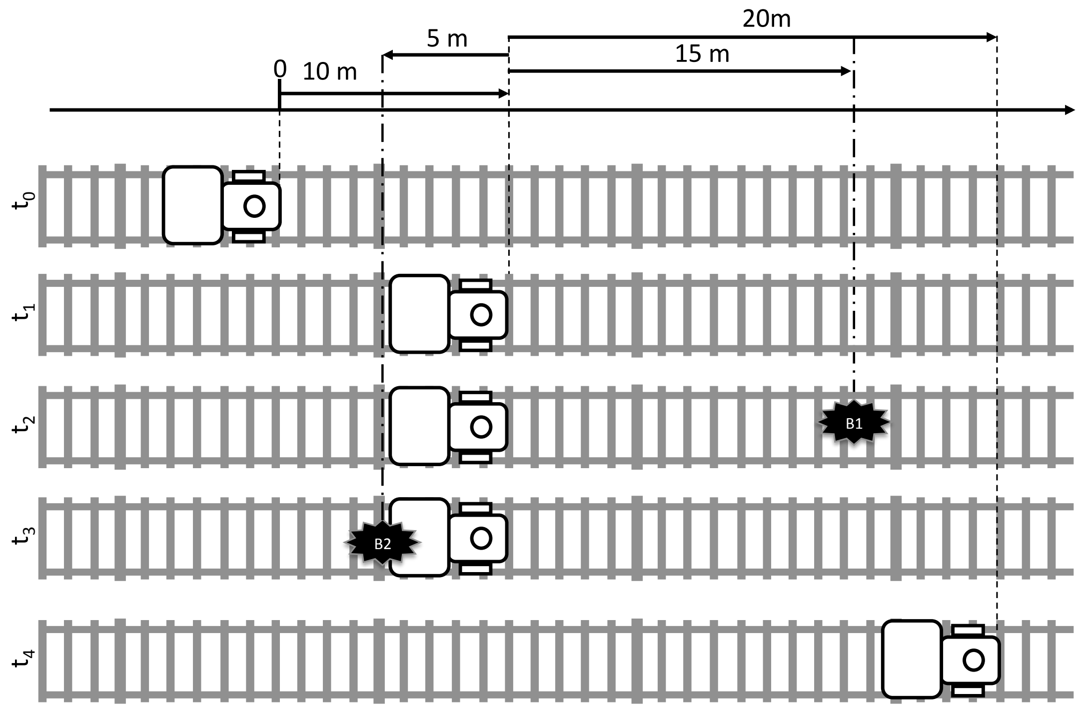


t_3: The AUV displaces a quantity ${}^{B}\Delta\eta$ (known in the robot body frame)

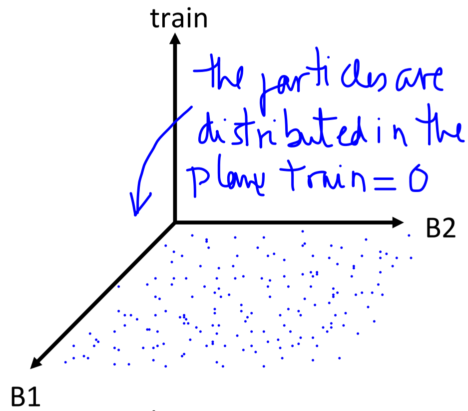
t_4: The range R2 to transponder 2 is measured.

PF2. A train is moving on a rail using odometry to measure its motion. There are to fixed beacons at unknown positions and a train sensor is able to measure the distance to those beacons. We assume the initial position of the train to be zero (the selected reference frame). We are interested in joinly estimating the robot and beacons position (SLAM). To this aim a particle filter containing the robot an both beacon positions (each one 1 DOF) is used. You are requested to draw the particles distribution after the following situations:

a) t_0**:** Initial distribution assuming the train is at zero position with zero uncertainty. Both beacons positions are unknown.
b) t_1: The train moves 10 meters to right.
c) t_2: Beacon 1 is detect at 15 meters at the right of the train.
d) t_3: Beacon 2 is detected a 5 meters at the left of the train.
e) t_4: The train moves 20 meters to right.

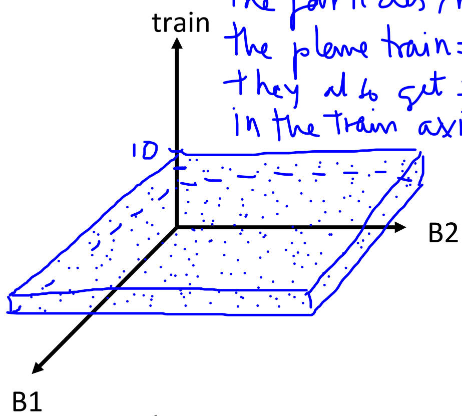


SOLUTION TO EXERCICE 2



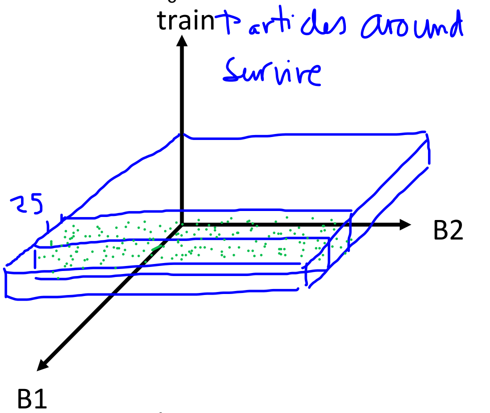
the particles are distributed in the plane train=0

t_0



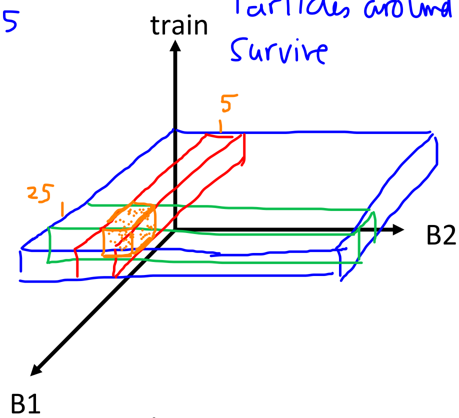
the particles move to the plane train=10. they also get spread in the train axis

t_1



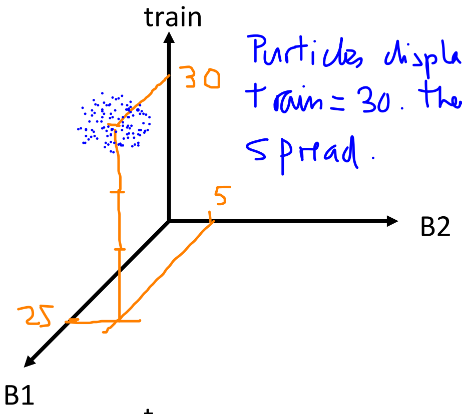
Particles around B1=25 survive

t_2



Particles around B2=5 survive

t_4



Particles displace to train=30. they also spread.

t_5

BAYES FILTER EXERCICES

BF1 A Robot manipulator is equipped with a tactile sensor in its gripper. When the robot is commanded to grasp an object, if the object was not already grasped it succeeds with 0.7 probabilities but fails with a 0.3 probability, otherwise it continues grasping the object with 0.9 probability but can fail with 0.1 probability. Whenever an object is hold within the gripper, the tactile sensor detects the situation with a 0.6 probability but fails in the detection with a 0.4 probability. If the gripper is free, the tactile sensor detects the situation with 0.8 probability but fails with 0.2 probability. The following sequence of actions/sensor readings have been executed: 1) the action GRASP is executed 2) the tactile sensor detects NO OBJECT GRASPED. Fill the table with the evolution of the robot belief of having grasped (or not) the object.

Executed action	**Probability of OBJECT GRASPED**	**Probability of NO OBJECT GRASPED**
	0.5 (Initial guess)	0.5 (Initial guess)
u_t=GRASP THE OBJECT		
z_t=NO OBJECT GRASPED		

SOLUTION

$$P(X=G \mid U=G, X=G) = 0.9$$
$$P(X=\bar{G} \mid U=G, X=G) = 0.1$$
$$P(X=G \mid U=G, X=\bar{G}) = 0.7$$
$$P(X=\bar{G} \mid U=G, X=\bar{G}) = 0.3$$
$$P(Z=G \mid X=G) = 0.6$$
$$P(Z=\bar{G} \mid X=G) = 0.4$$
$$P(Z=G \mid X=\bar{G}) = 0.2$$
$$P(Z=\bar{G} \mid X=\bar{G}) = 0.8$$
$$P(X_0=G) = P(X_0=\bar{G}) = 0.5$$

After executing the command to grasp the object we get:

$$\begin{cases} \mathrm{P}(\mathrm{X}_1=G) = P(X=G \mid U=G, X=G)P(X_0=G) + P(X=G \mid U=G, X=\bar{G})P(X_0=\bar{G}) \\ \mathrm{P}(\mathrm{X}_1=G) = 0.9\times 0.5 + 0.7\times 0.5 = 0.8 \\ \mathrm{P}(\mathrm{X}_1=\bar{G}) = P(X=\bar{G} \mid U=G, X=G)P(X_0=G) + P(X=\bar{G} \mid U=G, X=\bar{G})P(X_0=\bar{G}) \\ \mathrm{P}(\mathrm{X}_1=\bar{G}) = 0.1\times 0.5 + 0.3\times 0.5 = 0.2 \end{cases}$$

After sensing the object as grasped by means of the tactile sensor we get:

$$\begin{cases} \mathrm{P}(\mathrm{X}_2=G) = \eta P(Z=\bar{G} \mid \mathrm{X}_1=G)\mathrm{P}(\mathrm{X}_1=G) = \eta 0.4\times 0.8 = 0.32\eta = 0.6667 \\ \mathrm{P}(\mathrm{X}_2=\bar{G}) = \eta P(Z=\bar{G} \mid \mathrm{X}_1=\bar{G})\mathrm{P}(\mathrm{X}_1=\bar{G}) = \eta 0.8\times 0.2 = 0.16\eta = 0.3333 \\ 0.32\eta + 0.16\eta = 1 \Rightarrow \eta = 1/(0.32+0.16) = 2.0833 \end{cases}$$

EKF1. Let $\mathbf{x}_k = \left[\eta_k^T \; \nu_k^T\right]^T = [x \; y \; z \; \psi \; u \; v \; w \; r]^T$ be the 4DOF (Roll and Pitch are assumed to be zero) state vector containing the pose and velocity vector of an underwater robot, where x, y, z and ψ correspond to the 3D position and heading of the vehicle and u, v, w are the corresponding linear velocities being r the angular velocity (yaw derivative). The robot pose is referenced to a boat fixed frame **{B}** and the velocity is referenced to the robot fixed frame **{R}**. Let $\mathbf{n_k}$ =[$n_x \; n_y \; n_z \; n_\psi \; n_u \; n_v \; n_w \; n_r$]^T be a zero mean Gaussian vector. The close form of the motion model is given by:

$$\bar{\mathbf{x}}_\mathbf{k} = f(\mathbf{x_{k-1}}, \mathbf{n_{k-1}}), \tag{4}$$

$$\begin{bmatrix} x \\ y \\ z \\ \psi \\ u \\ v \\ w \\ r \end{bmatrix}_k = \begin{bmatrix} x + (uT + n_u\frac{T^2}{2})cos(\psi) - (vT + n_v\frac{T^2}{2})sin(\psi) \\ y + (uT + n_u\frac{T^2}{2})sin(\psi) + (vT + n_v\frac{T^2}{2})cos(\psi) \\ z + wT + n_w\frac{T^2}{2} \\ \psi + rT + n_r\frac{T^2}{2} \\ u + n_uT \\ v + n_vT \\ w + n_wT \\ r + n_rT \end{bmatrix}_{k-1}$$

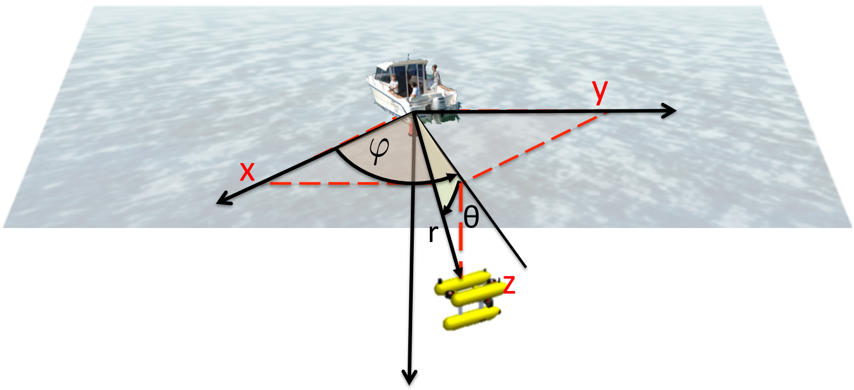
1. Let $\mathbf{z_k}$=[$u_D \; v_D \; w_D$]^T be the vector of measurements of the linear velocity (in {R} frame) from the velocity sensor (D subindex stands for DVL, the velocity sensor). Assuming zero mean Gaussian noise vector $\mathbf{v_k}$ with $\mathbf{R_k}$ covariance write the observation equation for the case where only velocity measurements are used. Is it linear or non-linear?

$$\boldsymbol{z_k} = \boldsymbol{H_k x_k} + \boldsymbol{v_k}$$

$$\begin{bmatrix} \boldsymbol{u_D} \\ \boldsymbol{v_D} \\ \boldsymbol{w_D} \end{bmatrix}_{\boldsymbol{k}} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \end{bmatrix}_{\boldsymbol{K}} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{z} \\ \boldsymbol{\psi} \\ \boldsymbol{u} \\ \boldsymbol{v} \\ \boldsymbol{w} \\ \boldsymbol{r} \end{bmatrix}_{\boldsymbol{k}} + \begin{bmatrix} \boldsymbol{v_u} \\ \boldsymbol{v_v} \\ \boldsymbol{v_w} \end{bmatrix}_{\boldsymbol{k}}$$

It is linear since only linear operations appear (products and additions).

2. Now an acoustic localization sensor is used which provides the robot pose with respect to the boat {B} in cylindrical coordinates z=[r φ θ]^T



You are requested to compute the robot position in the Cartesian coordinates fixed to the boat {B} based on the cylindrical coordinates provided by the sensor and use it to write the observation equation for the case when only the localization sensor is used. Is it linear or non-linear?

The conversion from

$$\boldsymbol{z_k = h(x_k, v_k)}$$

$$\begin{bmatrix} \boldsymbol{r} \\ \boldsymbol{\varphi} \\ \boldsymbol{\theta} \end{bmatrix} = \begin{bmatrix} \sqrt{\boldsymbol{x^2 + y^2 + z^2}} \\ \boldsymbol{atan2(y, x)} \\ \boldsymbol{atan2(z, \sqrt{x^2 + y^2})} \end{bmatrix} + \begin{bmatrix} \boldsymbol{v_u} \\ \boldsymbol{v_v} \\ \boldsymbol{v_w} \end{bmatrix}_{\boldsymbol{k}}$$

It is a non linear equation since it involves non linear operations like the root square of the atan2.

3. For the previous case 2, you are requested to provide the H_k Jacobian matrix to be used within the corresponding EKF.

$$\boldsymbol{H_k} = \left[\frac{\boldsymbol{\partial h(x_k, v_k)}}{\boldsymbol{\partial x_k}}\right]$$

$$= \begin{bmatrix} \frac{\boldsymbol{x}}{\sqrt{\boldsymbol{x^2+y^2+z^2}}} & \frac{\boldsymbol{y}}{\sqrt{\boldsymbol{x^2+y^2+z^2}}} & \frac{\boldsymbol{z}}{\sqrt{\boldsymbol{x^2+y^2+z^2}}} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \frac{\boldsymbol{-2y}}{\boldsymbol{(x^2+y^2)x}} & \frac{\boldsymbol{1/x}}{\boldsymbol{1+\left(\frac{y}{x}\right)^2}} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \frac{\boldsymbol{-zy}}{\boldsymbol{(x^2+y^2+z^2)}\sqrt{\boldsymbol{x^2+y^2}}} & \frac{\boldsymbol{-zx}}{\boldsymbol{(x^2+y^2+z^2)}\sqrt{\boldsymbol{x^2+y^2}}} & \frac{\boldsymbol{2z}\sqrt{\boldsymbol{x^2+y^2}}}{\boldsymbol{x^2+y^2+z^2}} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}$$

EXERCICES TO BE SOLVED

E1 When an EKF is supposed to work better?:

a. when the uncertainty of the estimate is big and the process model is highly non-linear nearby the current estimate.

b. when the uncertainty of the estimate is big and the process model is quite linear nearby the current estimate.

c. when the uncertainty of the estimate is small and the process model is highly non-linear nearby the current estimate.

d. when the uncertainty of the estimate is small and the process model is quite linear nearby the current estimate.

E2 Assume a 1 DOF mobile robot moving in a hallway while using odometry to measure the displacement and a camera for detecting the door, but not being able to identify which one is observed. Mark the FALSE sentence:

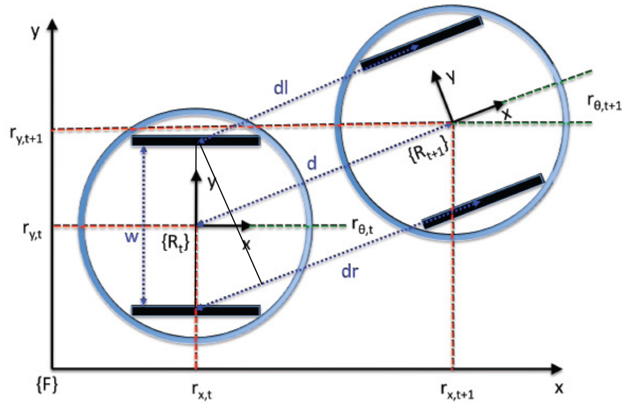
e. The solution can be implemented using the histogram filter

f. The solution cannot be implemented using the bayes filter

g. The solution can be implemented using the particle filter

h. The solution can be implemented using the EKF

E3 Which simplifications have been applied to compute the increment of heading in the following odometry equations ($\Delta\theta_k=(dr-dl)/w;$)?




```
[xk yk θk] function getOdometry()
    ΔNL=ReadEncoder(LEFT);
    ΔNR=ReadEncoder(RIGHT);

    dl = 2 * π * R * ((ΔNL)/ζ) ;
    dr = 2 * π * R * ((ΔNR)/ζ) ;
    d =(dr+dl)/2;

    Δθk=(dr-dl)/w;
    θk= θk-1+ Δθk;
    xk= xk-1 + (d * cos(θk));
    yk= yk-1 + (d * sin(θk));
Return [xk yk θk]
```

ζ: pulses each Wheel turn
R: Wheel radious

E4 A 1DOF mobile robot moving in a non-circular hallway is able to measure the square of its distance with respect to the origin of the corridor. The range sensor noise is also quadratic with the distance. The motion model is based on the use of odometry. Answer the following questions assuming that the robot position belief can be modeled under the Gaussian assumption:

1. Propose a well-suited filter to estimate the robot position? Justify your choice.
2. Provide a formulation of the corresponding filter:
   a. the motion model equation (filter prediction equation) showing the noise.
   b. the measurement model (filter observation equation) showing the noise.

E5 A Mobile robot uses a docking station for battery charging. When the robot is commanded to dock (u_t=DOCK-INTO-STATION), if it is not already docked it succeeds with 0.8 probabilities but fails with a 0.2 probability. Otherwise, this is if it is already docked, it remains docked with 0.9 probability but can fail with 0.1 probability. Whenever the robot is docked, a docking-detection-sensor detects the situation (z_t=ROBOT-IS-DOCKED) with a 0.7 probability but fails in the detection with a 0.3 probability. If the docking station is free, the docking-detection-sensor detects the situation (z_t=ROBOT-IS-NOT-DOCKED) with 0.6 probability but fails with 0.4 probability. The following sequence of actions/sensor readings have been executed: 1) the action u_t=DOCK-INTO-STATION is executed 2) the sensor detects z_t=ROBOT-IS-DOCKED. Fill the table with the evolution of the robot belief of being docked (or not).

Executed action	**Probability of ROBOT-IS-DOCKED**	**Probability of ROBOT-IS-NOT-DOCKED**
	0.5 (Initial guess)	0.5 (Initial guess)
u_t=DOCK-INTO-STATION		
z_t=ROBOT-IS-DOCKED		

E6 A mobile robot is being localized within a soccer field. For this purpose, 6 landmarks are located in the corners as well as in the ends of the line in the middle of the field. The robot is able to detect only the range to the landmarks. It is also equipped with a dead reckoning system providing [Δx Δy $\Delta\theta$]. For the sequence of events shown in the next figure, plot the evolution of the particles. Explain the process briefly.

DRAW YOUR SOLUTION HERE