Pattern Recognition
F. Meriaudeau

HW1

Note: Though some of these exercises specifically require MATLAB, you may use MATLAB to help with plotting, computation, and/or verification of your results. This is in fact encouraged.

1. Bayesian methods for two-class, one-dimensional problem.

The conditional density of class1 (w1) in a 1D measurement space is normal (Gaussian) with mean 0 ($\mu 1 = 0$) and variance5 ($\sigma_1^2 = 5$). The class 2 conditional density is also normal with $\mu 2 = 2$ and $\sigma_2^2 = 1$.
(a) Give the mathematical representation of the two conditional densities.
(b) Sketch the two density functions on the same figure.
(c) Give the equation for the likelihood ratio.
(d) Assume that $P(\omega 1) = P(\omega 2) = 0.5$.

i. Using the MAP approach, which class should we choose if x = 3.
ii. Using the ML approach, which class should we choose if x = 3.
iii. How many decision regions do you see? Describe those regions based on your sketch.
iv. Write the integral equation that gives the overall probability of error based on the MAP method.
v. Find the decision boundary (or boundaries) using analytical methods (not the sketch).

(e) Now assume that $P(\omega 1) = 0.8$ and $P(\omega 2) = 0.2$ and a zero-one loss function.
Sketch the product of the conditional density and its corresponding prior for both w1and w1
i. Using the MAP approach, which class should we choose if x = 3.
ii. Using the ML approach, which class should we choose if x = 3.
iii. How many decision regions do you see? Describe those regions based on your sketch.
iv. Write the integral equation that gives the overall probability of error based on the MAP method.
v. Find the decision boundary (or boundaries) using analytical methods (not the sketch) and check to see that it matches your sketch.
vi. What kind of loss values (rather than zero-one) would alter the decisions?

2. DHS - Ch. 2, Exercise 2. Suppose two equally probable one-dimensional densities are of the form

$$p(x|\omega_i)\alpha \exp(-|x-a_i|/b_i) \ for\ i=1,2\ and\ b_i > 0$$

(a) Write an analytic expression for each density, that is, normalize each function for arbitrary ai and bi.
(b) Calculate the likelihood ratio as a function of your four variables.
(c) Plot the likelihood ratio $p(x|\omega 1)/p(x|\omega 2)$for the case a1 = 0, b1 = 1, a2 = 1, and b2 = 2.

3. Euclidean distance vs. Mahalanobis distance.

(a) Define these two distances. You can use equations.
(b) Comment in no more than three sentences on the differences between the two distances.
(c) Suppose we have two 2D Gaussian distributions defined by $\mu 1 = (1\ 1)^t$ and $\mu 2 = (4\ 4)^t$ with covariances given by

$$\sum_1 = \begin{pmatrix} 0{,}475 & -0{,}425 \\ -0{,}425 & 0{,}475 \end{pmatrix} \text{ and } \sum_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Given a vector $x = (2\ 2)^t$, to which class would we say x belongs using Euclidean distance? To which class would we assign x based on Mahalanobis distance? Show your work.