

Practical works – n°2

*Systems*

• **Exercise 1 – Causality**

**1.1** Considering the system defined by the equation  $y_k = (x_k + x_{k+1})/2$ , check its causality property by examining the response to the signal  $H(k - 4)$  or `step(4,N)`. When plotting, include the abscissa range  $[1 : N]$ .

**1.2** Propose a modification to obtain a causal version

• **Exercise 2 – Stability**

**2.1** Program the primitive (accumulator) operator `prim(f)` applied on the signal **f** of length **N**. The value of the vector returned by `prim` at the index **k** will correspond to  $F_k$  with  $k \leq N$ . Note  $F_k = \sum_{q=-\infty}^k f_q$ . Discuss on the result of the primitive operator applied to the signal  $H(k - 4)$ . Is the primitive operator stable ?

**2.2** What is the impulse response of the primitive operator (in the discrete domain) ?

**2.3** Test the stability of the system defined by the equation:  $y_k = x_k + 2y_{k-1}$ . Plot the impulse response.

**2.4** Test the stability of the system defined by the equation:  $y_k = x_k + y_{k-1}/3$ . Plot the impulse response. Write the response  $y$  as a convolution operation (truncate the impulse response).

• **Exercise 3 – Invariance and linearity**

**3.1** Define the following signals:  $\mathbf{x}_a = [0\ 0\ 0\ 1\ 2\ 3\ 4\ 5\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]$ ;  $\mathbf{x}_b = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 4\ 3\ 2\ 1\ 0\ 0\ 0\ 0\ 0]$ . Compute the responses  $y_a, y_b$  according to the equation  $y = 3x_{k-1} - 2x_k + x_{k+1}$

**3.2** Prove the system defined by the previous equation is linear (and invariant). Write the equation as a convolution equation.

**3.3** Propose a nonlinear/noninvariant system.

• **Exercise 4 – Convolution**

**4.1** Try to write a simple version of the convolution function (do not process the limits of the signal).

**4.2** Generate (matlab function `randn`) an observation  $x_n$  (length 1000 points or more) of the normal/gaussian random process  $\mathcal{N}$ . Plot the distribution of the values of this observation. Compute the convolution product ( $y$ ) of  $x_n$  with the values  $h = [18\ 8\ 5\ 2\ 1]$  (`conv(x, h, 'same')`). Compute the cross-correlation of  $x_n$  with  $y$  and observe the result. Conclusion ?