Problem

This problem investigates the wavelet transform and its application to image denoising.

- *Develop a Matlab function for computing the J-level wavelet transform of an NxN image (assume N is a power of 2).*

Your function can repeatedly use the instruction that I wrote and discussed in class (downsampling, filtering, computation of the high pass-filter), or you can start from scratch. Your function should take three arguments: an input image, the number of levels J you wish to compute and the analysis low-pass filter.

Your function should output an array of NxN wavelet coefficients (in the arrangement discussed in class).

- *Develop a Matlab function for computing the inverse J-level wavelet transform of an NxN array of wavelet coefficients.*

Your function could repeatedly use the instruction that I wrote and discussed in class (upsampling, filtering, …). Your function should take three arguments: an input array of wavelet coefficients, the number of levels J in that array and the **analysis** low-pass filter. Your function should output an image reconstructed from the input coefficients.

- *Test your forward and inverse transform code by applying it to the Lena image with the Daubechies D4 filter:*

```
h0=[0.48296      0.83652      0.22414      -0.12941];
```

You can test it with other (grayscale) images if you want, too.

- *Image Denoising: analyse of the denoising performance of the wavelet transform.*

Add a small amount of Gaussian white noise (with variance varying from 2 to 20), compute the transforms, set all coefficients to zero except those whose magnitude is larger than 3σ (you can also try factors other than 3), and reconstruct an estimate of the original image by applying the corresponding inverse transform. Note that we have used the Hard threshold

$$T^{hard}\left(d_{l,k}\right) = \begin{cases} d_{j,k} & \text{if } \left|d_{l,k}\right| \geq \lambda \\ 0 & \text{if } \left|d_{l,k}\right| < \lambda \end{cases}$$

Compare the denoising performance of the Soft Threshold of the wavelet transform with the Hard Threshold of the wavelet transform using the test image(s).

$$T^{soft}(d_{j,k}) = \begin{cases} sign(d_{j,k})\left(|d_{j,k}| - \lambda\right) \text{if } |d_{j,k}| \geq \lambda \\ 0 \text{ if } |d_{j,k}| < \lambda \end{cases}$$