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Ecole Nationale Supérieure des Mines SAINT-ETIENNE

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General Adaptive Neighborhood Image Processing and Analysis

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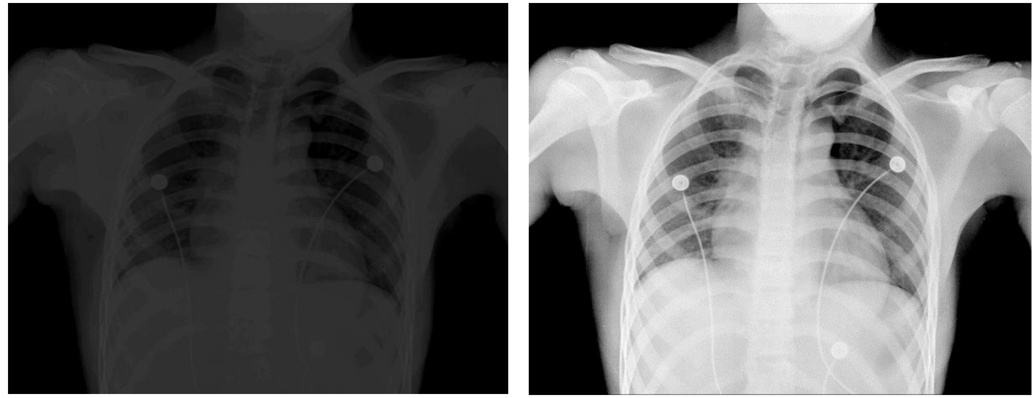
MINISTÈRE DE L'ÉCONOMIE DE L'INDUSTRIE ET DE L'EMPLOI

Context

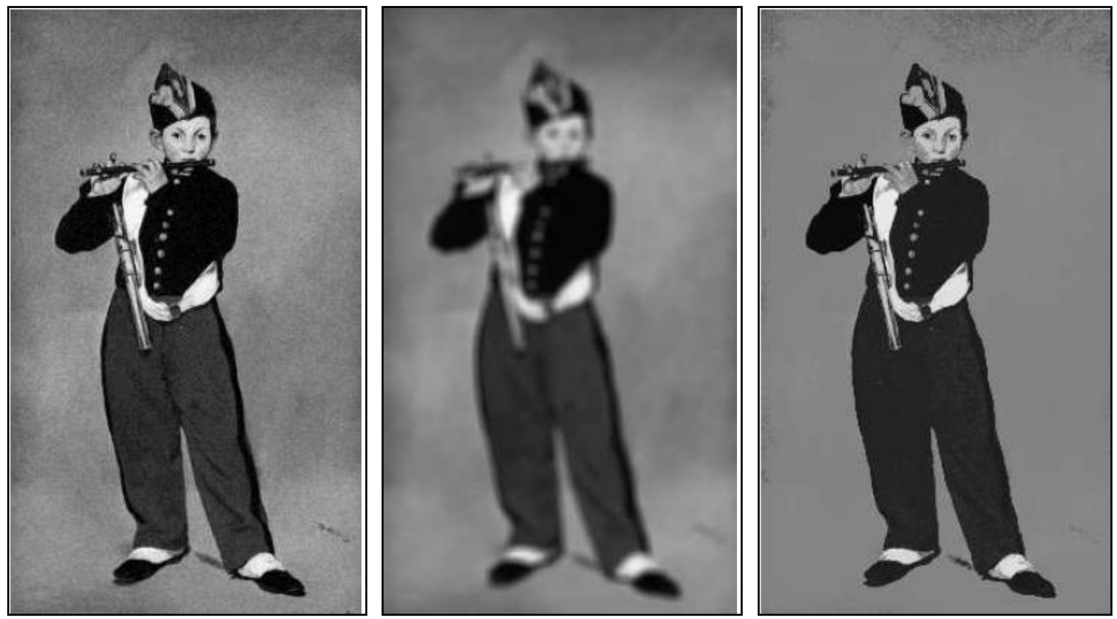
- **Image Filtering**
  - Restoration
  - Enhancement
  - Motion estimation



X-ray chest image enhancement

- **Drawbacks of Classical Image Filtering**
  - Blurring transitions
  - Creating artificial patterns
  - Moving edges

- **Need of Adaptive Image Filtering**



Original image *Classical filtering* *Adaptive filtering*

Context

- **Literature about Adaptive Image Filtering**
  - PDEs
    - Diffusion controled by the image gradient, …
  - Local neighborhood
    - Adaptive weights
    - Variable size
  - Geodesy
    - Reconstruction
  - …

GANIP
(General Adaptive Neighborhood Image Processing)

Context

- **PhD (2005) - Habilitation Degree (2012) / Johan Debayle**
  - GANIP: General Adaptive Neighborhood Image Processing
  - Adaptive Image Representation
  - Adaptive Image Processing and Analysis

- **Applications**
  - Image restoration
  - Image enhancement
  - Image segmentation
  - Image characterization
  - Image registration

- **Main Publications about GANIP**
  - 3 book chapters
  - 10 international journal papers

Motivation

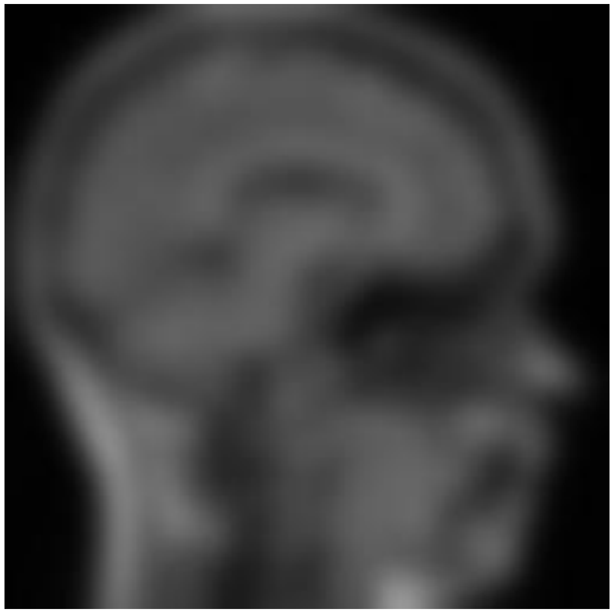
- **Adaptivity with Spatial Structures**

Spatially invariant
(fixed-shape, fixed-size operational window)

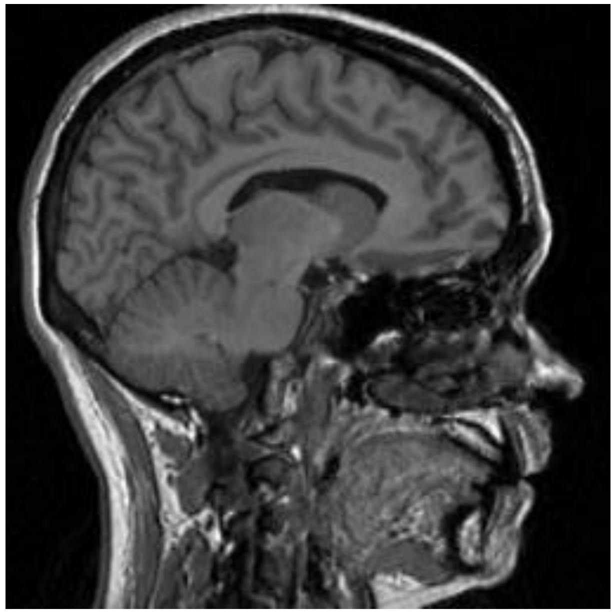
Spatially adaptive
(variable-shape, variable-size operational window)

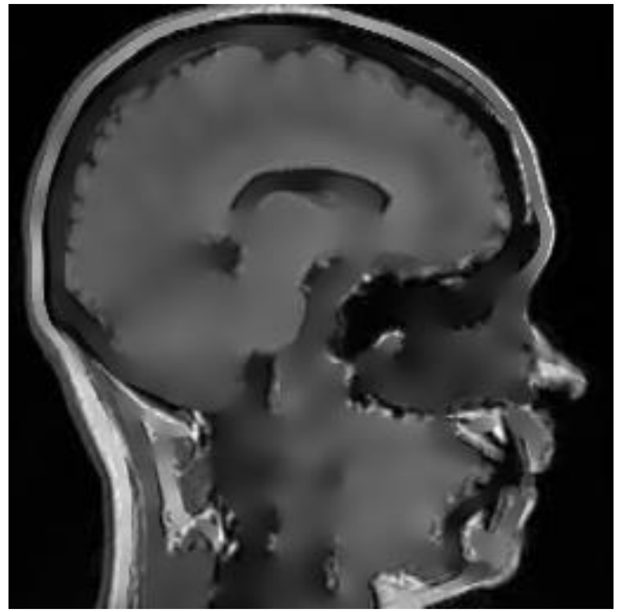
Motivation

- **Adaptivity with Scales**



Extrinsic analyzing scales (isotropic diffusion)





Intrinsic analyzing scales (anisotropic diffusion)

Motivation

- **Adaptivity with Intensities**

- **Generalized Linear Image Processing (GLIP) Framework / $\oplus$, $\ominus$**
  - Physically relevant
  - Mathematically rigorous
  - Computationally effective
  - Practically fruitful

- **Classical Linear Image Processing (CLIP)**
  - (+, -)
  - not a good candidate!

- **Other GLIP Structures**
  - MHIP: Multiplicative Homomorphic Image Processing
  - LRIP: Log-Ratio Image Processing
  - LIP: Logarithmic Image Processing

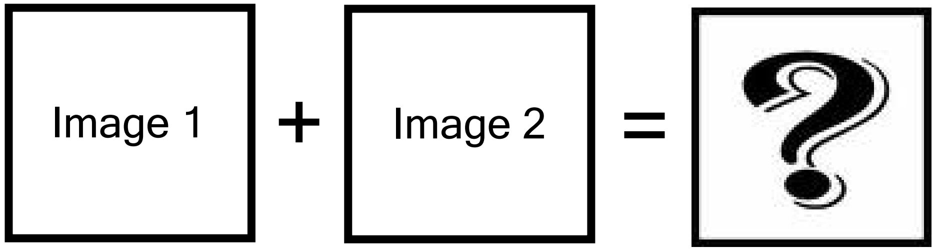
Example of GLIP Structures and Operations

CLIP	MHIP	LRIP	LIP
initial intensity value range			
$(0, +\infty)$	$(0, +\infty)$	$(0, M)$	$(0, M)$
extended intensity value range (defining the vector space)			
$(-\infty, +\infty)$	$(0, +\infty)$	$(0, M)$	$(-\infty, M)$
homomorphism related to the CLIP vector space			
$f \mapsto f$	$f \mapsto \ln(f)$	$f \mapsto \ln\left(\frac{f}{M-f}\right)$	$f \mapsto -M \times \ln\left(\frac{M-f}{M}\right)$
vector addition			
usual $+$	$f \boxplus g = fg$	$f \mathbin{\Diamond\!\!\!\!+} g = \frac{M}{\left(\frac{M-f}{f}\right)\left(\frac{M-g}{g}\right)+1}$	$f \mathbin{\triangle\!\!\!\!+} g = f + g - \frac{fg}{M}$
scalar multiplication			
usual $\times$	$\alpha \boxtimes f = \exp(\alpha \times \ln(f))$	$\alpha \mathbin{\Diamond\!\!\!\!\times} f = \frac{M}{\left(\frac{M-f}{f}\right)^{\alpha}+1}$	$\alpha \mathbin{\triangle\!\!\!\!\times} f = M - M\left(1 - \frac{f}{M}\right)^{\alpha}$
opposite			
usual $-$	$\boxminus f = \frac{1}{f}$	$\mathbin{\Diamond\!\!\!\!-} f = M - f$	$\mathbin{\triangle\!\!\!\!-} f = \frac{-Mf}{M-f}$
vector subtraction			
usual $-$	$f \boxminus g = \frac{f}{g}$	$f \mathbin{\Diamond\!\!\!\!-} g = \frac{M}{\left(\frac{M-f}{f}\right)\left(\frac{g}{M-g}\right)+1}$	$f \mathbin{\triangle\!\!\!\!-} g = M\left(\frac{f-g}{M-g}\right)$

Example: Logarithmic Image Processing (LIP)

[JM 1988, AIEP 2001]

- **Motivation**

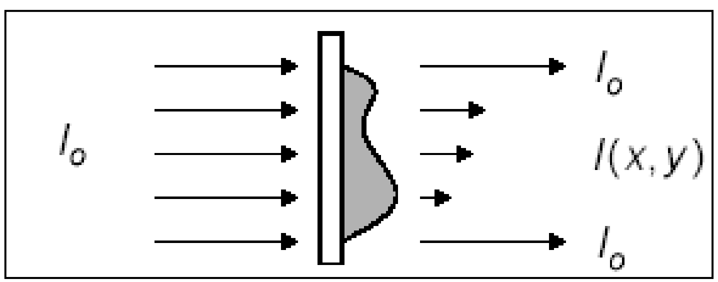


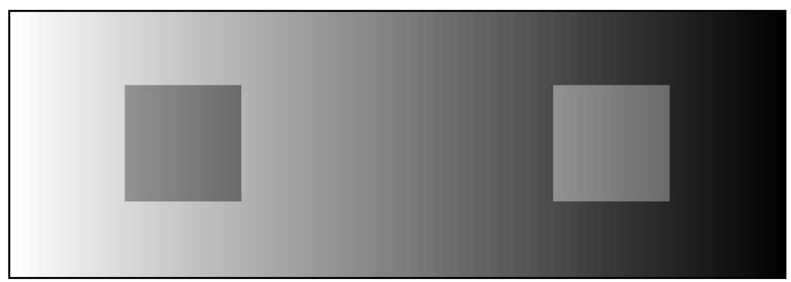
- **Elementary Operations**

$$f \,\triangle\!\!\!\!\!\!+\; g = f + g - \frac{fg}{M}$$

$$\alpha \,\triangle\!\!\!\!\!\!\times\; f = M - M\left(1 - \frac{f}{M}\right)^{\alpha}, \quad \alpha \in \mathbb{R}$$

- **Adapted to Image Formation Laws, Human Visual Perception Laws…**

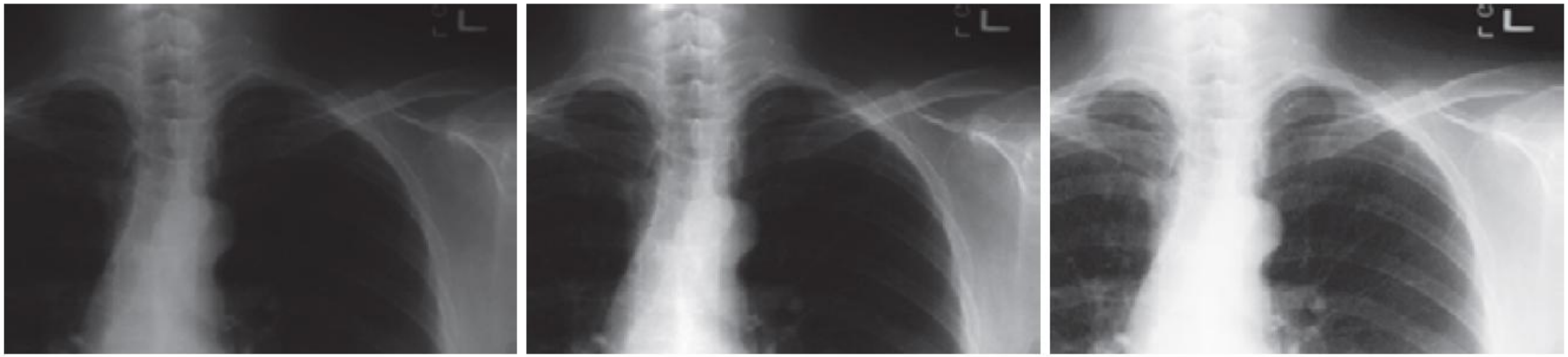




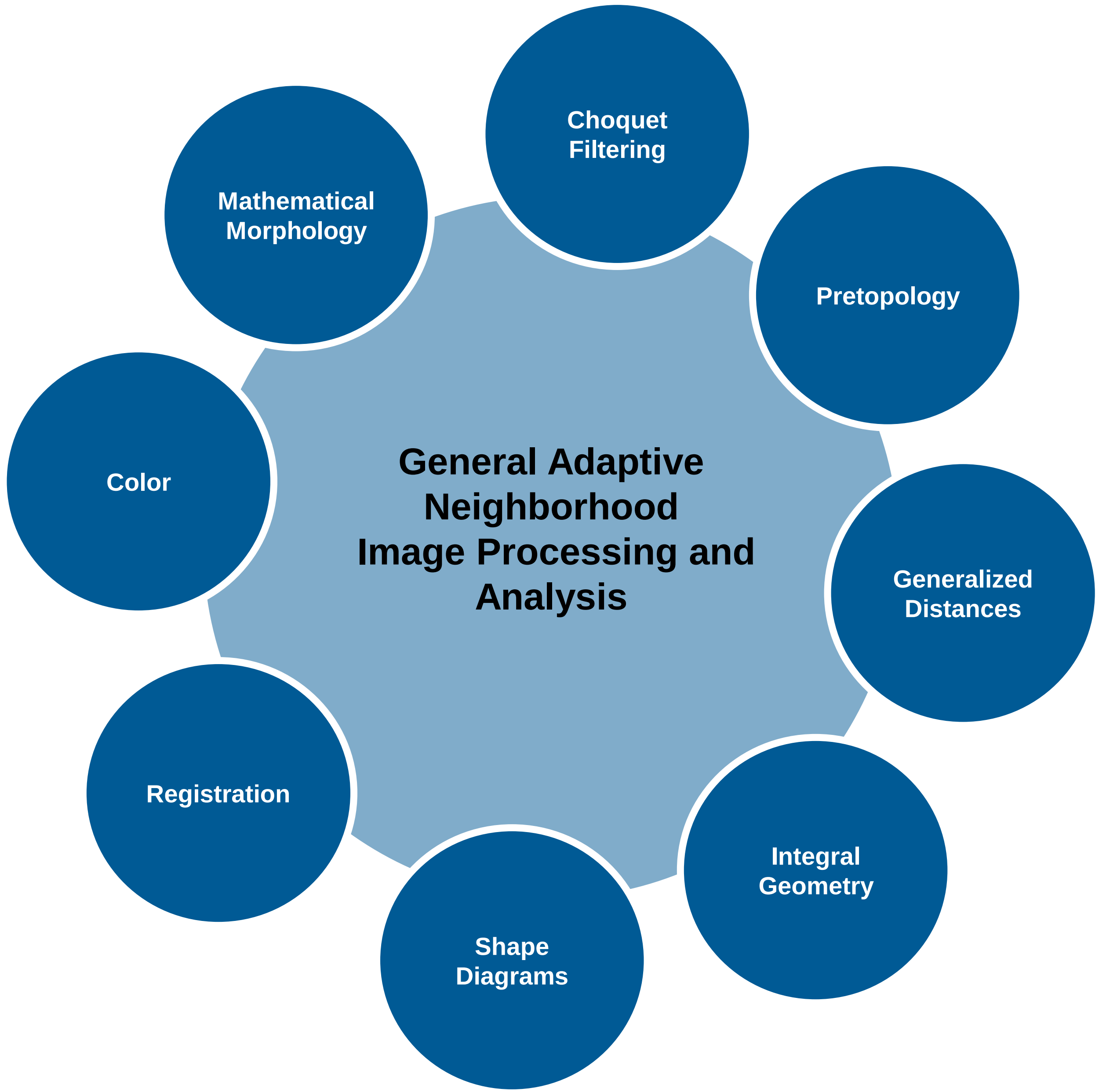
Example: Logarithmic Image Processing (LIP)

[JM 1988, AIEP 2001]

- **Application Example: Thorax X-Ray Image Enhancement**



(a) original image (b) CLIP enhancement (c) LIP enhancement


General Adaptive Neighborhood Image Processing and Analysis
Choquet Filtering
Pretopology
Generalized Distances
Integral Geometry
Shape Diagrams
Registration
Color
Mathematical Morphology

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Main Publications

[IAS 2005, JMIV 2006]

Image Anal Stereol 2005;24:145-158
Original Research Paper

SPATIALLY ADAPTIVE MORPHOLOGICAL IMAGE FILTERING USING INTRINSIC STRUCTURING ELEMENTS

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(Accepted October 21, 2005)

ABSTRACT

This paper deals with spatially adaptive morphological filtering, extending the theory of mathematical morphology to the paradigm of adaptive neighborhood. The basic idea in this approach is to substitute the extrinsically-defined, fixed-shape, fixed-size structuring elements generally used by morphological operators, by intrinsically-defined, variable-shape, variable-size structuring elements. These last so-called intrinsic structuring elements fit to the local features of the image, with respect to a selected analyzing criterion such as luminance, contrast, thickness, curvature or orientation. The resulting spatially-variant morphological operators perform efficient image processing, without any a priori knowledge of the studied image and some of which satisfy multiscale properties. Moreover, in a lot of practical cases, the elementary adaptive morphological operators are connected, which is topologically relevant. The proposed approach is practically illustrated in several application examples, such as morphological multiscale decomposition, morphological hierarchical segmentation and boundary detection.

Keywords: adaptive neighborhood, connected operators, intrinsic spatial analysis, mathematical morphology, multiscale representation.

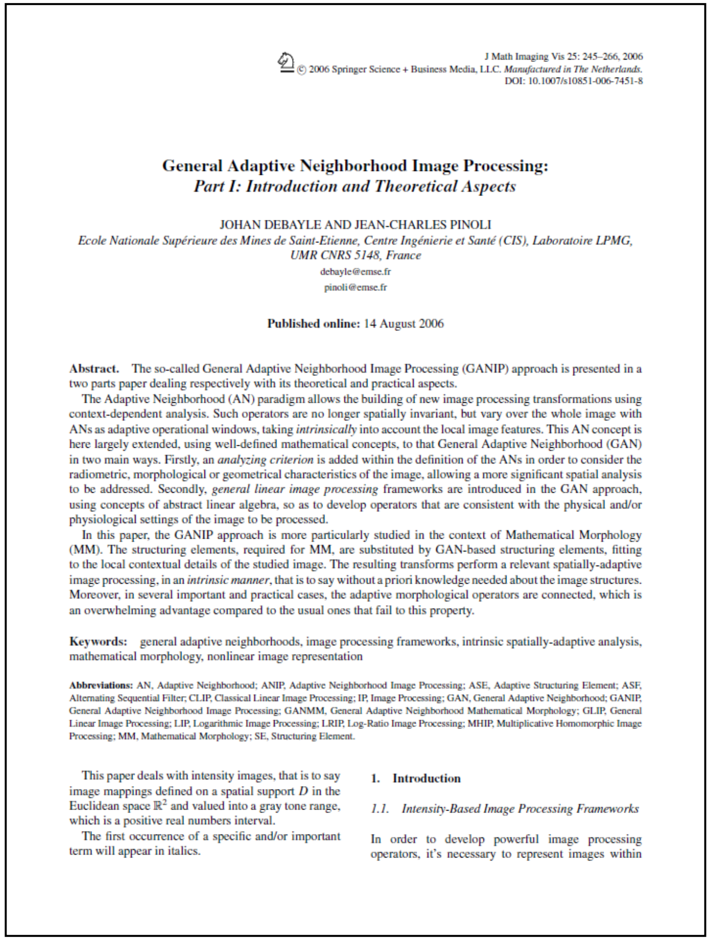
INTRODUCTION

Firstly, a lot of image processing techniques use *spatially-invariant* transformations, with fixed operational windows. This kind of operators, such as morphological operators or convolution filters, give efficient and compact computing structures, in the sense where data and operators are independent. However, due to their fixed operational windows, they consequently have several strong drawbacks such as creating artificial patterns, changing the detailed parts of large objects, damaging transitions or removing significant details (Arce and Foster, 1989).

Alternative approaches towards *spatially-variant* image processing have been proposed (Gordon and Rangayyan, 1984; Perona and Malik, 1990; Salembier, 1992; Alvarez *et al.*, 1993; Charif-Chefchaouni and Schonfeld, 1994; Vogt, 1994; Braga Neto, 1996; Cuisenaire, 2005; Lerallut *et al.*, 2005) with the introduction of adaptive operators, where the adaptive concept results from the spatial adjustment of the operational window. A spatially adaptive operator will no longer be spatially-invariant, but must vary over the whole image with adaptive windows, taking locally into account the image context. Such transforms perform efficient image processing.

Secondly, usual image processing operators have some limitations concerning their operational windows (adaptive or not). In fact, these last ones are usually *extrinsically* defined with regard to the local features of the image. A priori constraints are imposed upon the size and/or the shape of the operational windows, which is not the most appropriate. For instance, spatially-invariant approaches such as wavelets (Mallat, 1989), morphological pyramids (Sun and Maragos, 1989; Laporterie *et al.*, 2002), and isotropic scale-spaces (Lindeberg, 1994; Heijmans and Boomgaard, 2000) use sliding windows extrinsically defined with regard to the analyzing scales. Indeed, their size and shape are fixed on the whole image for each scale, *i.e.*, a priori determined, independently of the image context. In an other example (Vogt, 1994), spatially-variant morphological operators are used, where the shape of morphological structuring elements that automatically adjust the gray tones in a range image is rectangular or ellipsoidal, involving a priori knowledge about the image context.

Therefore, *intrinsic* approaches, using self-defined operational windows that fit to the local content of the image, without any a priori spatial constraints, are more appropriate. Following this idea, image processing based on the Adaptive Neighborhood (AN) paradigm (Paranjape *et al.*, 1994) has been proposed. A set of adaptive neighborhoods (ANs set) is defined around each point within the image, whose extent depends on the local features of the image in which the given point is situated. Thus, for each point to be processed, its associated ANs set is used as (intrinsic) operational windows of the considered transformation. The resulting operators



J Math Imaging Vis 25: 245–266, 2006

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General Adaptive Neighborhood Image Processing: *Part I: Introduction and Theoretical Aspects*

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Abstract. The so-called General Adaptive Neighborhood Image Processing (GANIP) approach is presented in a two parts paper dealing respectively with its theoretical and practical aspects.

The Adaptive Neighborhood (AN) paradigm allows the building of new image processing transformations using context-dependent analysis. Such operators are no longer spatially invariant, but vary over the whole image with ANs as adaptive operational windows, taking *intrinsically* into account the local image features. This AN concept is here largely extended, using well-defined mathematical concepts, to that General Adaptive Neighborhood (GAN) in two main ways. Firstly, an *analyzing criterion* is added within the definition of the ANs in order to consider the radiometric, morphological or geometrical characteristics of the image, allowing a more significant spatial analysis to be addressed. Secondly, *general linear image processing* frameworks are introduced in the GAN approach, using concepts of abstract linear algebra, so as to develop operators that are consistent with the physical and/or physiological settings of the image to be processed.

In this paper, the GANIP approach is more particularly studied in the context of Mathematical Morphology (MM). The structuring elements, required for MM, are substituted by GAN-based structuring elements, fitting to the local contextual details of the studied image. The resulting transforms perform a relevant spatially-adaptive image processing, in an *intrinsic manner*, that is to say without a priori knowledge needed about the image structures. Moreover, in several important and practical cases, the adaptive morphological operators are connected, which is an overwhelming advantage compared to the usual ones that fail to this property.

Keywords: general adaptive neighborhoods, image processing frameworks, intrinsic spatially-adaptive analysis, mathematical morphology, nonlinear image representation

Abbreviations: AN, Adaptive Neighborhood; ANIP, Adaptive Neighborhood Image Processing; ASE, Adaptive Structuring Element; ASF, Alternating Sequential Filter; CLIP, Classical Linear Image Processing; IP, Image Processing; GAN, General Adaptive Neighborhood; GANIP, General Adaptive Neighborhood Image Processing; GANMM, General Adaptive Neighborhood Mathematical Morphology; GLIP, General Linear Image Processing; LIP, Logarithmic Image Processing; LRIP, Log-Ratio Image Processing; MHIP, Multiplicative Homomorphic Image Processing; MM, Mathematical Morphology; SE, Structuring Element.

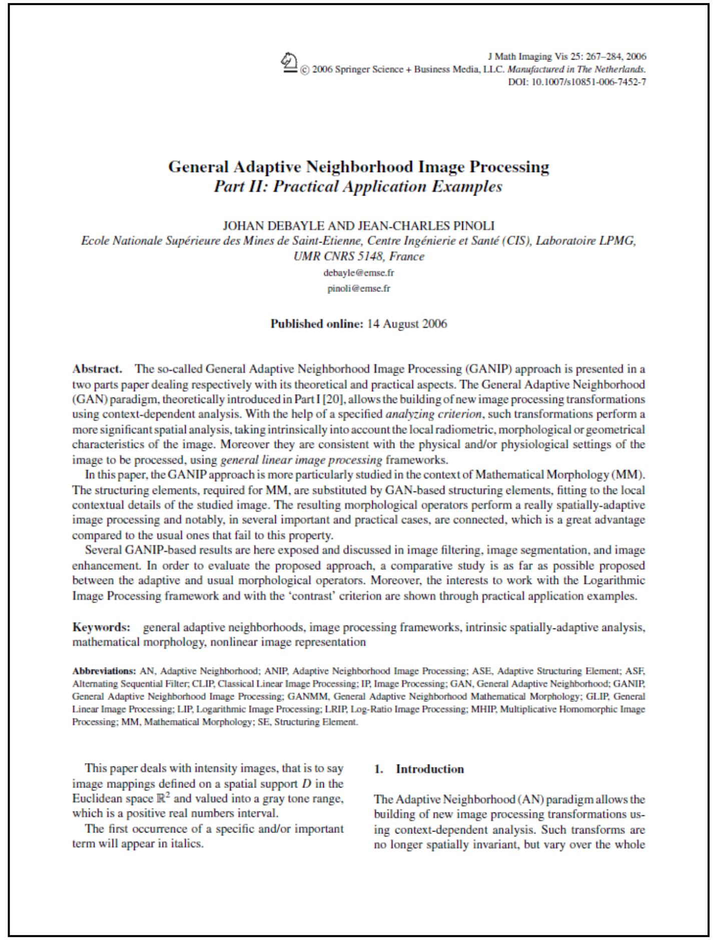
This paper deals with intensity images, that is to say image mappings defined on a spatial support D in the Euclidean space $\mathbb{R}^2$ and valued into a gray tone range, which is a positive real numbers interval.

The first occurrence of a specific and/or important term will appear in italics.

1. Introduction

1.1. Intensity-Based Image Processing Frameworks

In order to develop powerful image processing operators, it's necessary to represent images within



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General Adaptive Neighborhood Image Processing *Part II: Practical Application Examples*

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Published online: 14 August 2006

Abstract. The so-called General Adaptive Neighborhood Image Processing (GANIP) approach is presented in a two parts paper dealing respectively with its theoretical and practical aspects. The General Adaptive Neighborhood (GAN) paradigm, theoretically introduced in Part I [20], allows the building of new image processing transformations using context-dependent analysis. With the help of a specified *analyzing criterion*, such transformations perform a more significant spatial analysis, taking intrinsically into account the local radiometric, morphological or geometrical characteristics of the image. Moreover they are consistent with the physical and/or physiological settings of the image to be processed, using *general linear image processing* frameworks.

In this paper, the GANIP approach is more particularly studied in the context of Mathematical Morphology (MM). The structuring elements, required for MM, are substituted by GAN-based structuring elements, fitting to the local contextual details of the studied image. The resulting morphological operators perform a really spatially-adaptive image processing and notably, in several important and practical cases, are connected, which is a great advantage compared to the usual ones that fail to this property.

Several GANIP-based results are here exposed and discussed in image filtering, image segmentation, and image enhancement. In order to evaluate the proposed approach, a comparative study is as far as possible proposed between the adaptive and usual morphological operators. Moreover, the interests to work with the Logarithmic Image Processing framework and with the 'contrast' criterion are shown through practical application examples.

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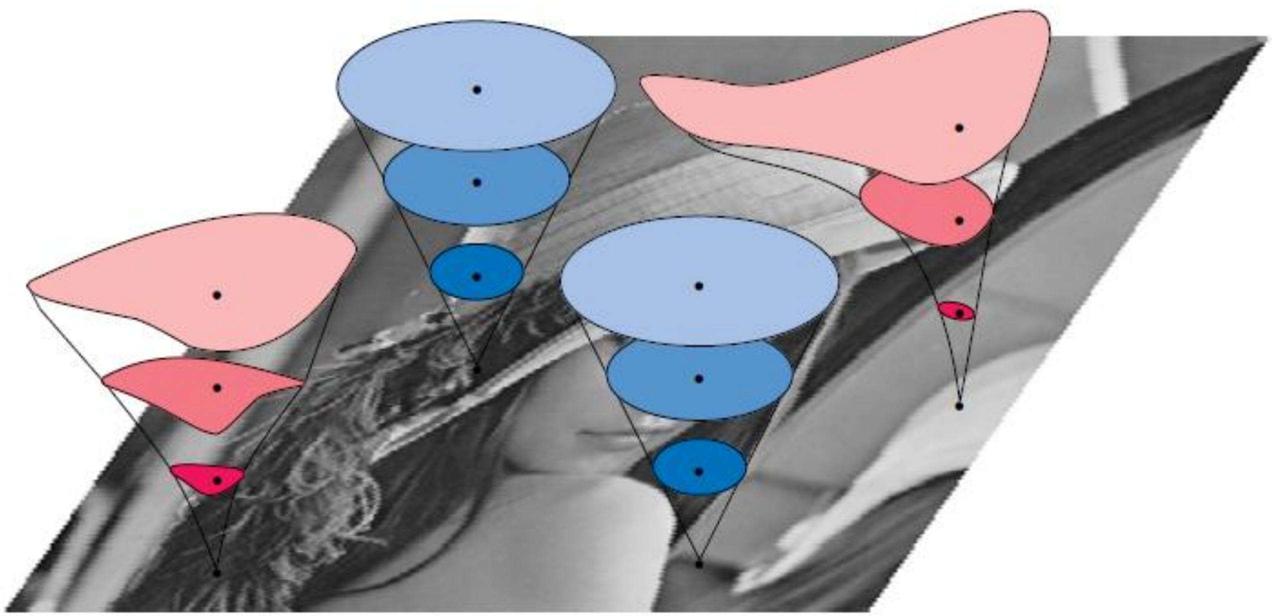
1. Introduction

The Adaptive Neighborhood (AN) paradigm allows the building of new image processing transformations using context-dependent analysis. Such transforms are no longer spatially invariant, but vary over the whole

Motivation

- **New Image Representation**

- **Local Neighborhoods**
  - Adaptivity with scales (intrinsic scales)
  - Adaptivity with spatial structures (variable operational windows)
  - Adaptivity with intensities (psychophysical relevance)

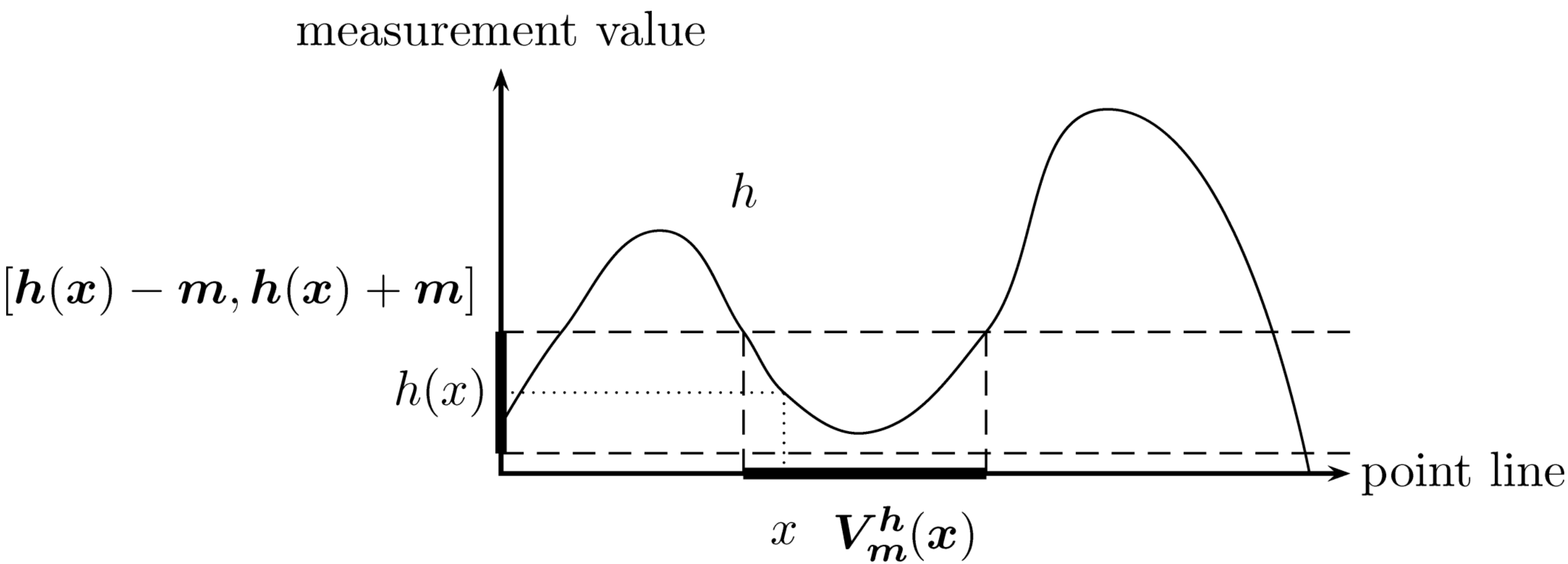


Weak GANs

- **Definition**
  - h: criterion mapping
  - m: homogeneity tolerance
  - ⬡: Generalized Linear Image Processing (GLIP) framework

$$V^{h}_{m_{⬡}}(x) = C_{h^{-1}([h(x) \ominus_{⬡} m_{⬡}, h(x) \oplus_{⬡} m_{⬡}])}(x)$$

- **Computation**



Illustration

- **Retina Image**

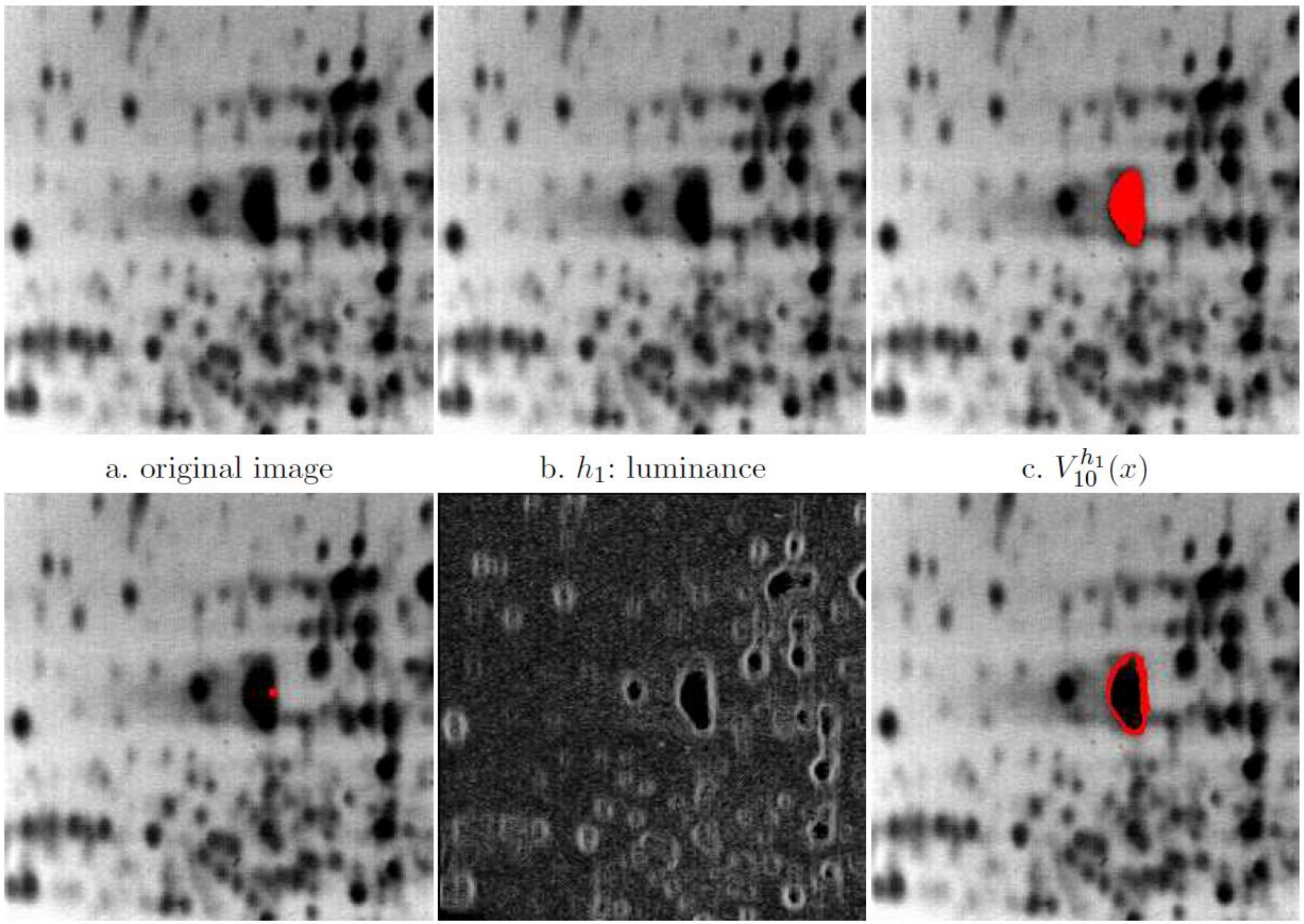
(a) x and y

(b) $V_{20}^{h}(x)$ and $V_{20}^{h}(y)$

Illustration

- **Impact of the Criterion Mapping**

Electrophoresis gel image



a. original image   b. h_1: luminance   c. $V_{10}^{h_1}(x)$

d. seed point x e. h_2: contrast   f. $V_{30_\triangle}^{h_2}(x)$

Illustration

- **Impact of the Homogeneity Tolerance**

Original image

4 seed points

m=25

Properties

- **Reflexivity** $x \in V^h_{m_\hexagon}(x)$

- **Increasing with Respect to *m***

$$\begin{pmatrix} (m^1_\hexagon, m^2_\hexagon) \in E^{\oplus} \times E^{\oplus} \\ m^1_\hexagon \leq m^2_\hexagon \end{pmatrix} \Rightarrow V^h_{m^1_\hexagon}(x) \subseteq V^h_{m^2_\hexagon}(x)$$

- **Equality between Iso-Valued Points**

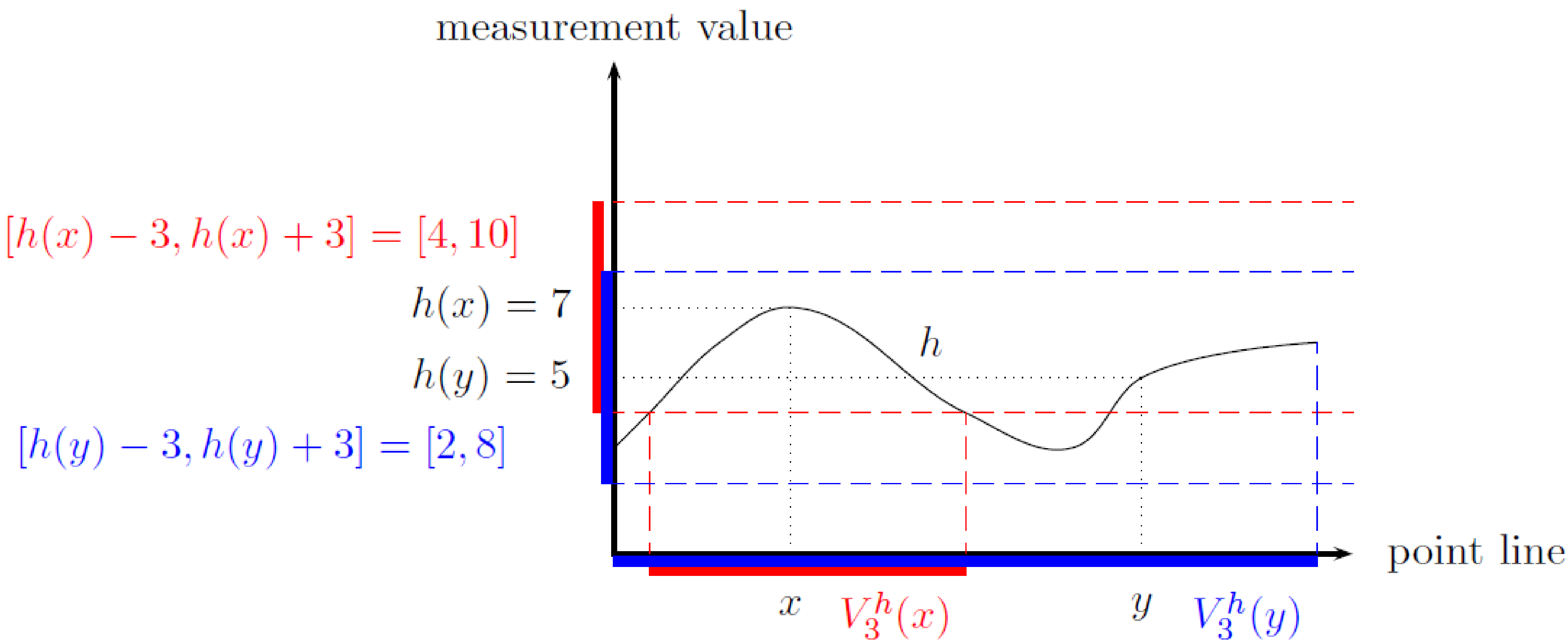
$$\begin{pmatrix} (x,y) \in D^2 \\ x \in V^h_{m_\hexagon}(y) \\ h(x) = h(y) \end{pmatrix} \Rightarrow V^h_{m_\hexagon}(x) = V^h_{m_\hexagon}(y)$$

- **Translation Invariance** $c \in E \Rightarrow V^{h \oplus c}_{m_\hexagon}(x) = V^h_{m_\hexagon}(x)$

- **Multiplication Compatibility** $\alpha \in \mathbb{R}^+ \setminus \{0\} \Rightarrow V^{\alpha \otimes h}_{m_\hexagon}(x) = V^h_{\frac{1}{\alpha} \otimes m_\hexagon}(x)$

Properties

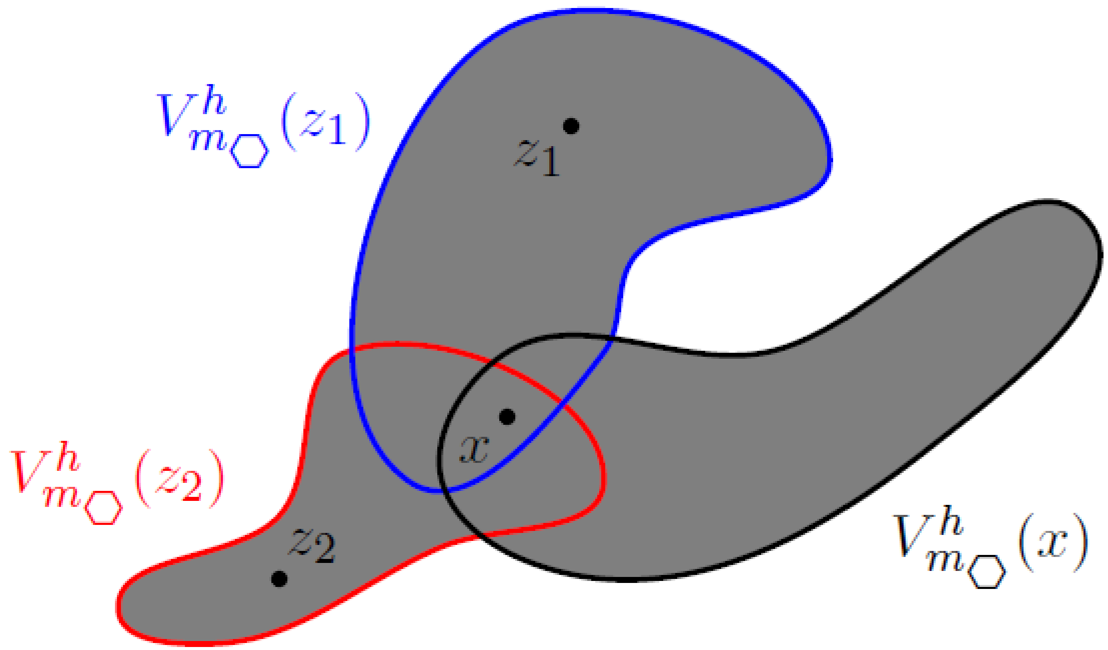
- **No Symmetry**



Strong GANs

- **Definition**
  - h: criterion mapping
  - m: homogeneity tolerance
  - $\hexagon$: Generalized Linear Image Processing (GLIP) framework

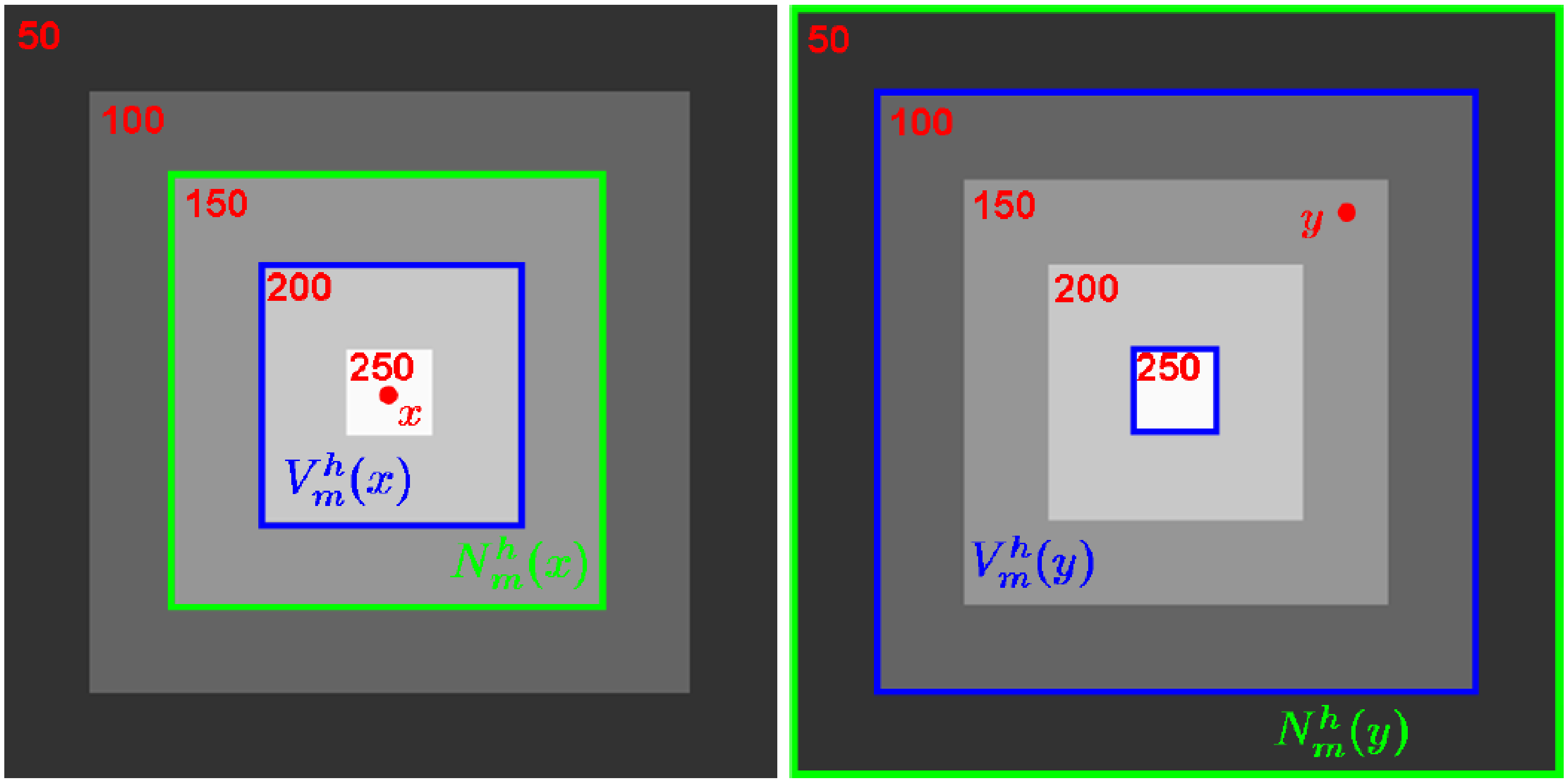
$$N_{m_{\hexagon}}^{h}(x) = \bigcup_{z \in D} \{ V_{m_{\hexagon}}^{h}(z) | x \in V_{m_{\hexagon}}^{h}(z) \}$$



Strong GANs

- **Illustration**
  - Using m=50

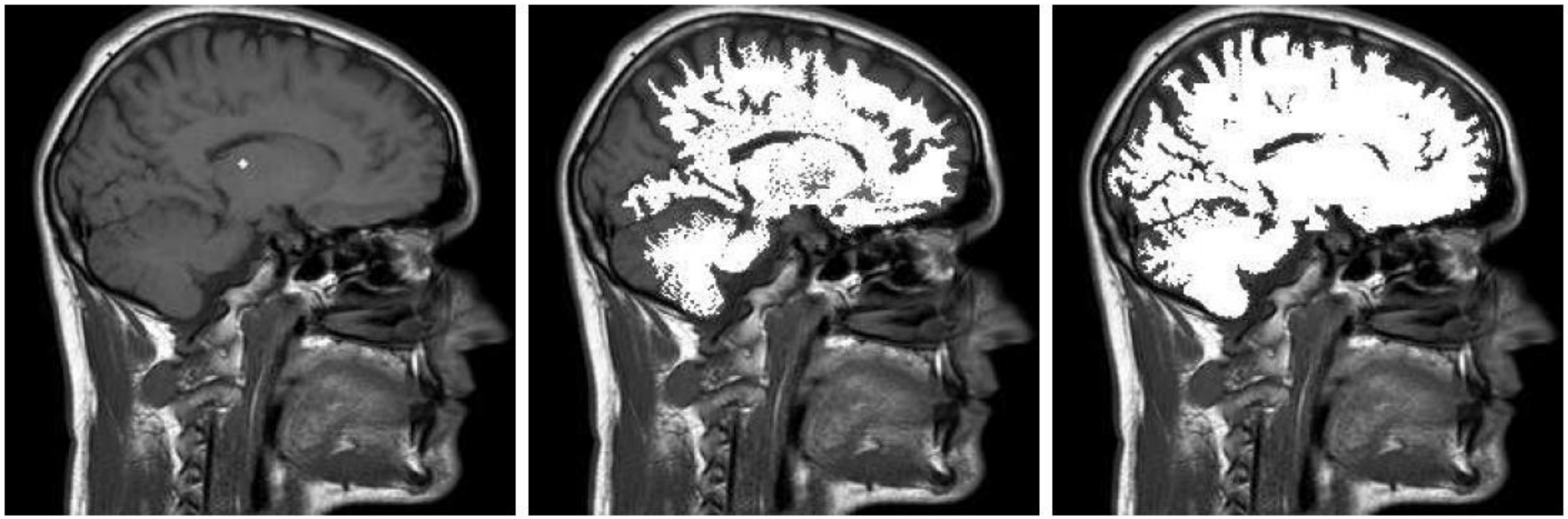
$$N^h_{m_{\hexagon}}(x) = \bigcup_{z \in D} \{V^h_{m_{\hexagon}}(z) | x \in V^h_{m_{\hexagon}}(z)\}$$



(a) GANs of x (b) GANs of y

Illustration

- **MR Head Image**



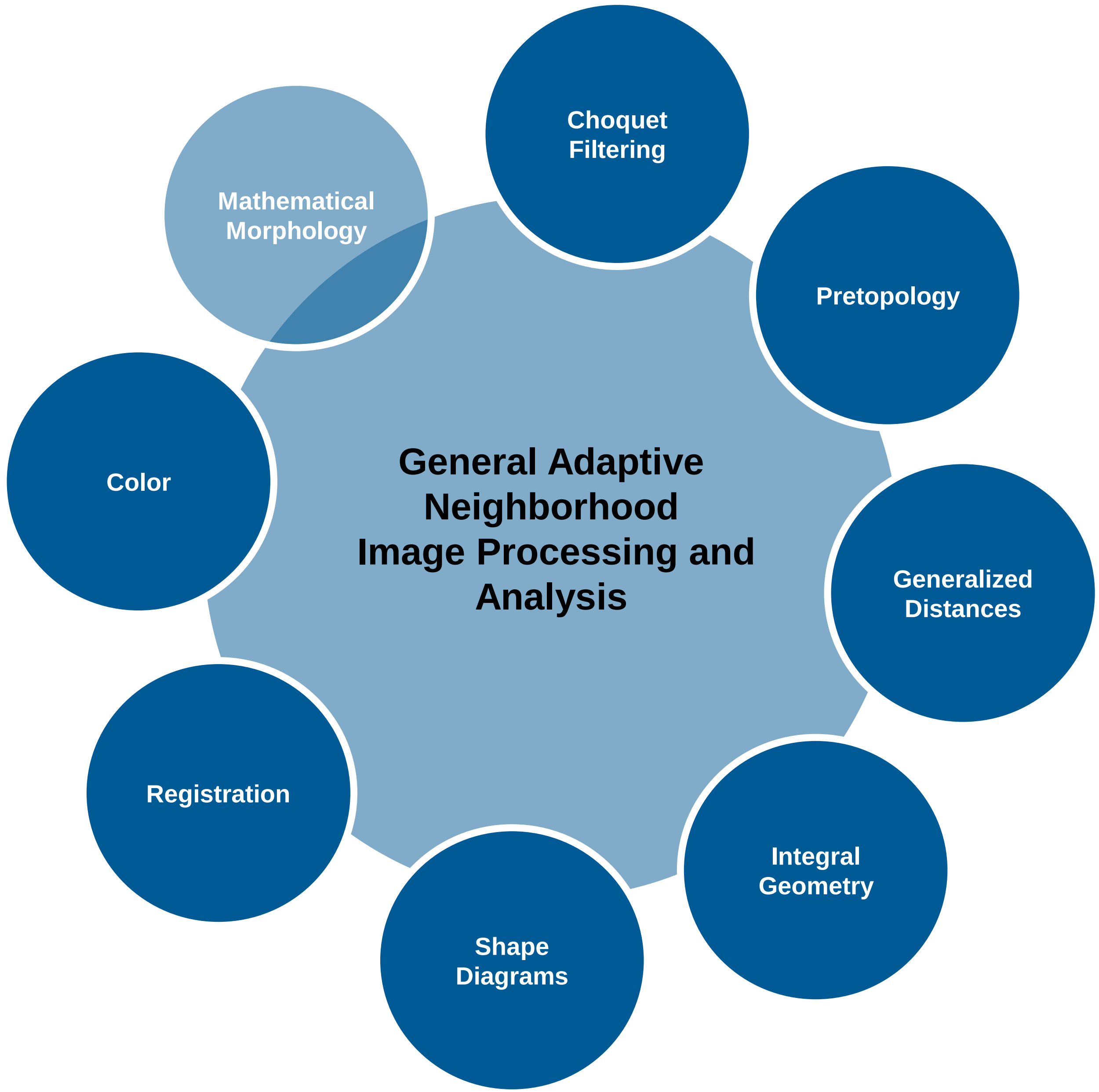
(a) original image f with a seed point x (b) Weak GAN $V_{10}^{f}(x)$ (c) Strong GAN $N_{10}^{f}(x)$

Properties

- **Reflexivity** $x \in N^h_{m_{\hexagon}}(x)$
- **Geometric Nesting** $V^h_{m_{\hexagon}}(x) \subseteq N^h_{m_{\hexagon}}(x) \subseteq V^h_{2 \otimes m_{\hexagon}}(x)$
- **Symmetry** $x \in N^h_{m_{\hexagon}}(y) \Leftrightarrow y \in N^h_{m_{\hexagon}}(x)$
- **Increasing with Respect to *m***

$$\begin{pmatrix} (m^1_{\hexagon}, m^2_{\hexagon}) \in E^{\oplus} \times E^{\oplus} \\ m^1_{\hexagon} \leq m^2_{\hexagon} \end{pmatrix} \Rightarrow N^h_{m^1_{\hexagon}}(x) \subseteq N^h_{m^2_{\hexagon}}(x)$$

- **Translation Invariance** $c \in E \Rightarrow N^{h \oplus c}_{m_{\hexagon}}(x) = N^h_{m_{\hexagon}}(x)$
- **Multiplication Compatibility** $\alpha \in \mathbb{R}^+ \backslash \{0\} \Rightarrow N^{\alpha \otimes h}_{m_{\hexagon}}(x) = N^h_{\frac{1}{\alpha} \otimes m_{\hexagon}}(x)$


Choquet Filtering
Mathematical Morphology
Pretopology
Color
General Adaptive Neighborhood Image Processing and Analysis
Generalized Distances
Registration
Integral Geometry
Shape Diagrams

Main Publications

[**JMIV 2006**, ICIP 2009]

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General Adaptive Neighborhood Image Processing: *Part I: Introduction and Theoretical Aspects*

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Abstract. The so-called General Adaptive Neighborhood Image Processing (GANIP) approach is presented in a two parts paper dealing respectively with its theoretical and practical aspects. The General Adaptive Neighborhood (GAN) paradigm, theoretically introduced in Part I [20], allows the building of new image processing transformations using context-dependent analysis. With the help of a specified *analyzing criterion*, such transformations perform a more significant spatial analysis, taking intrinsically into account the local radiometric, morphological or geometrical characteristics of the image. Moreover they are consistent with the physical and/or physiological settings of the image to be processed, using *general linear image processing* frameworks.

In this paper, the GANIP approach is more particularly studied in the context of Mathematical Morphology (MM). The structuring elements, required for MM, are substituted by GAN-based structuring elements, fitting to the local contextual details of the studied image. The resulting morphological operators perform a really spatially-adaptive image processing and notably, in several important and practical cases, are connected, which is a great advantage compared to the usual ones that fail to this property.

Several GANIP-based results are here exposed and discussed in image filtering, image segmentation, and image enhancement. In order to evaluate the proposed approach, a comparative study is as far as possible proposed between the adaptive and usual morphological operators. Moreover, the interests to work with the Logarithmic Image Processing framework and with the 'contrast' criterion are shown through practical application examples.

Keywords: general adaptive neighborhoods, image processing frameworks, intrinsic spatially-adaptive analysis, mathematical morphology, nonlinear image representation

Abbreviations: AN, Adaptive Neighborhood; ANIP, Adaptive Neighborhood Image Processing; ASE, Adaptive Structuring Element; ASF, Alternating Sequential Filter; CLIP, Classical Linear Image Processing; IP, Image Processing; GAN, General Adaptive Neighborhood; GANIP, General Adaptive Neighborhood Image Processing; GANMM, General Adaptive Neighborhood Mathematical Morphology; GLIP, General Linear Image Processing; LIP, Logarithmic Image Processing; LRIP, Log-Ratio Image Processing; MHIP, Multiplicative Homomorphic Image Processing; MM, Mathematical Morphology; SE, Structuring Element.

This paper deals with intensity images, that is to say image mappings defined on a spatial support D in the Euclidean space $\mathbb{R}^2$ and valued into a gray tone range, which is a positive real numbers interval.

The first occurrence of a specific and/or important term will appear in italics.

1. Introduction

The Adaptive Neighborhood (AN) paradigm allows the building of new image processing transformations using context-dependent analysis. Such transforms are no longer spatially invariant, but vary over the whole

GENERAL ADAPTIVE NEIGHBORHOOD MATHEMATICAL MORPHOLOGY

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ABSTRACT

This paper aims to present a novel framework, entitled General Adaptive Neighborhood Image Processing (GANIP), focusing on the area of adaptive morphology. The usual fixed-shape structuring elements required in Mathematical Morphology (MM) are substituted by adaptive (GAN-based) spatial structuring elements. GANIP and MM results to the so-called General Adaptive Neighborhood Mathematical Morphology (GANMM). Several GANMM-based image filters are defined. They satisfy strong morphological and topological properties such as connectedness. The practical results in the fields of image restoration and image enhancement confirm and highlight the theoretical advantages of the GANMM approach.

Index Terms— Adaptive filters , Image representations, Image restoration, Image enhancement, Morphological operations

1. INTRODUCTION

A novel framework entitled General Adaptive Neighborhood Image Processing (GANIP) [1] has been recently introduced in order to propose an original image representation and mathematical structure for adaptive processing and analysis of gray-tone images. In this paper, the GANIP framework is first presented and particularly studied in the context of Mathematical Morphology (MM). The central idea is based on the key notion of adaptivity which is simultaneously associated with the analyzing scales, the spatial structures and the intensity values of the image to be addressed. GANIP enables to define new adaptive operators. Such operators are no longer spatially invariant, but vary over the whole image with General Adaptive Neighborhoods (GANs) as adaptive operational windows, taking intrinsically into account the local image features. The paper deals both with GANIP and MM resulting to the so-called General Adaptive Neighborhood Mathematical Morphology (GANMM) which provides a novel framework in the area of adaptive morphology [2, 3, 4]. The structuring elements, required for MM, are substituted by GAN-based structuring elements, fitting to the local contextual details of the studied image. GANMM leads to a relevant spatially-adaptive morphological image processing and analysis, without a priori knowledge needed about the image structures. Several GANMM-based image processing and analysis issues are then addressed in the field of image restoration and image enhancement. They are illustrated in several biomedical areas. In order to evaluate the GANMM approach, a comparative study is proposed between the adaptive and usual morphological operators (e.g. filters by reconstruction). The practical results confirm and highlight the theoretical advantages of the GANMM approach.

2. GANIP FRAMEWORK

In the so-called General Adaptive Neighborhood Image Processing (GANIP) framework [1], a set of General Adaptive Neighborhoods (GANs set) is identified about each point in the image to be analyzed. These GANs are subsets of the image spatial support and are used as adaptive spatial windows for image transformations (such as adaptive morpholgical operations) or quantitative image analysis.

2.1. Image space

In order to define powerful image processing operators, it is necessary to represent intensity images within mathematical frameworks (most of the time of a vectorial nature) based on a *physically and/or psychophysically relevant image formation process*. In addition, their mathematical structures and operations (the vector addition and then the scalar multiplication) have to be consistent with the physical nature of the images and/or the human visual system, and computationally effective. Thus, *General Linear Image Processing* (GLIP) frameworks [5] have been explored, in order to include situations in which signals or images are combined by processes other than the usual vector addition. Consequently, operators based on such *intensity-based image processing frameworks* should be consistent with the physical and/or physiological settings of the images to be processed.

The space of *image* (resp. *criterion*) *mappings*, defined on the spatial support D and valued in a real numbers interval $\bar{E}$ (resp. E), is represented in a GLIP framework, denoted $\mathcal{I}$ (resp. $\mathcal{C}$). The GLIP framework $\mathcal{I}$ (resp. $\mathcal{C}$) is then supplied

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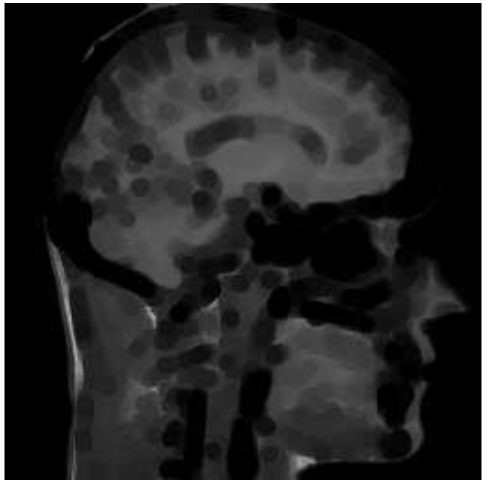
GAN Dilation and Erosion

- **Structuring Elements: Strong GANs**

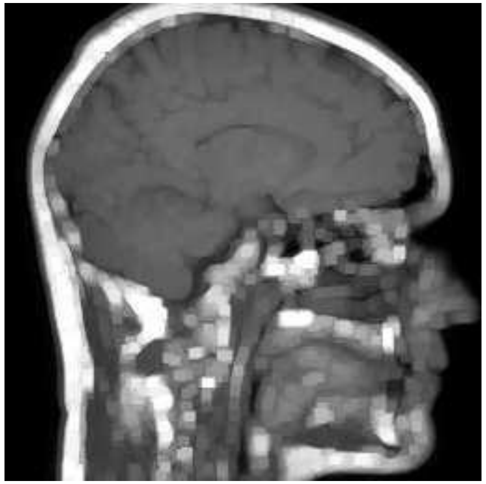
- **Definition**

$$D^{h}_{m_{\hexagon}}(f)(x) = \sup_{w \in N^{h}_{m_{\hexagon}}(x)} f(w)$$

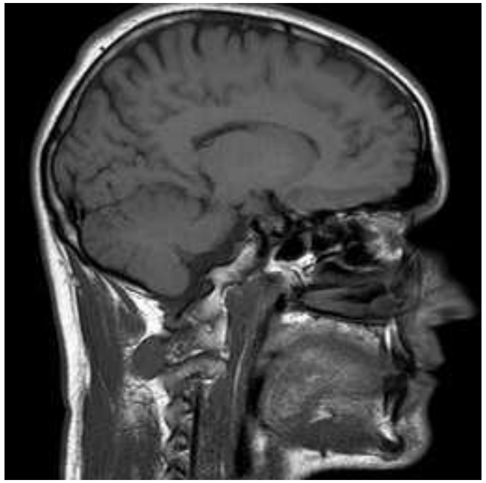
$$E^{h}_{m_{\hexagon}}(f)(x) = \inf_{w \in N^{h}_{m_{\hexagon}}(x)} f(w)$$



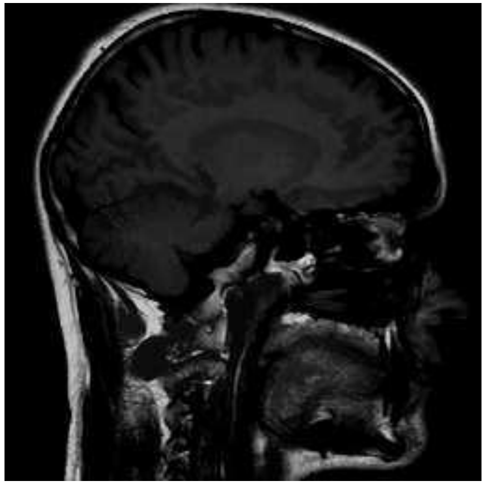
Classical erosion



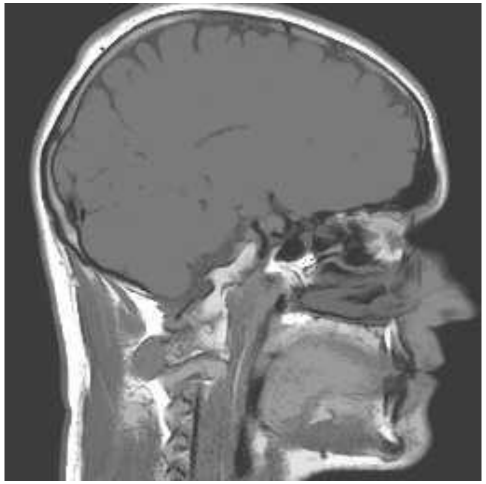
Classical dilation



Original image



GAN erosion

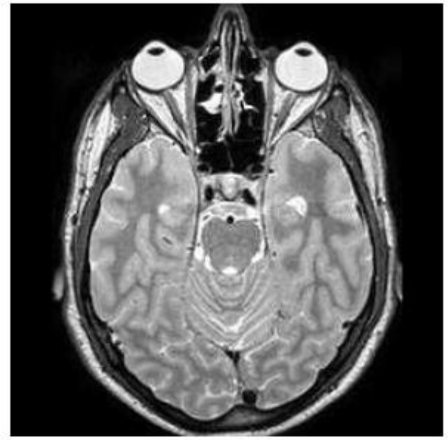


GAN dilation

GAN Closing and Opening

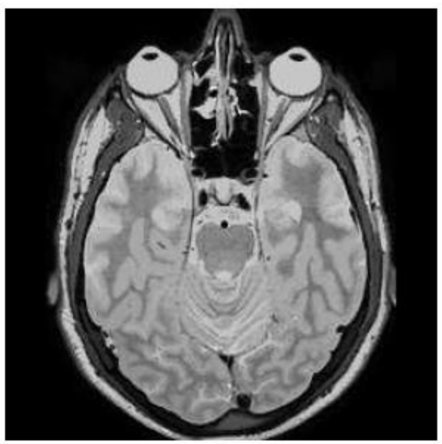
- **Definition**

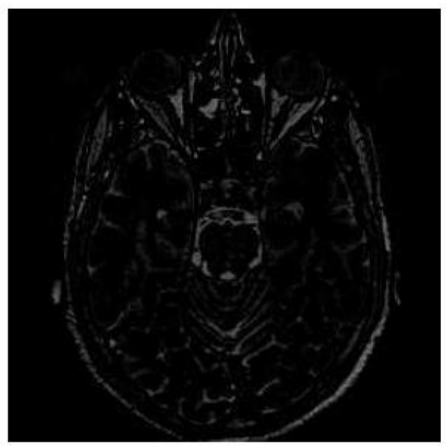
$$\mathrm{C}^h_{m_{\hexagon}}(f) = \mathrm{E}^h_{m_{\hexagon}} \circ \mathrm{D}^h_{m_{\hexagon}}(f)$$
$$\mathrm{O}^h_{m_{\hexagon}}(f) = \mathrm{D}^h_{m_{\hexagon}} \circ \mathrm{E}^h_{m_{\hexagon}}(f)$$

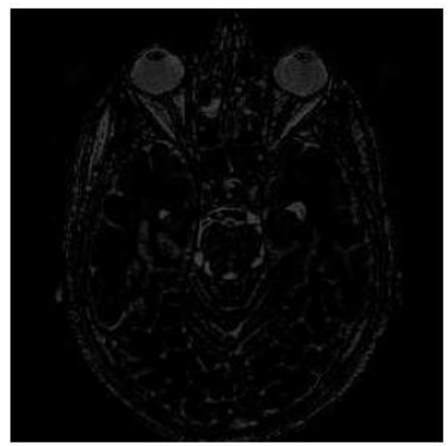

(a) Original f

- **Reconstruction vs. Adaptive Morphology**


(b) Opening by reconstruction


(c) GAN opening


(d) residue: (a)-(b)


(e) residue: (a)-(c)

Advanced GAN Morphological Operators

- **Adaptive Alternating Filters**

$$
\begin{aligned}
\mathrm{CO}^h_{m_{\hexagon}}(f) &= \mathrm{C}^h_{m_{\hexagon}} \circ \mathrm{O}^h_{m_{\hexagon}}(f) \\
\mathrm{OC}^h_{m_{\hexagon}}(f) &= \mathrm{O}^h_{m_{\hexagon}} \circ \mathrm{C}^h_{m_{\hexagon}}(f)
\end{aligned}
$$

- **Adaptive Sequential Filters**

$$
\begin{aligned}
\mathrm{D}^h_{m,p}(f) &= \underbrace{\mathrm{D}^h_m \circ \cdots \circ \mathrm{D}^h_m}_{p \text{ times}}(f) \\
\mathrm{E}^h_{m,p}(f) &= \underbrace{\mathrm{E}^h_m \circ \cdots \circ \mathrm{E}^h_m}_{p \text{ times}}(f)
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{C}^h_{m,p}(f) &= \mathrm{E}^h_{m,p} \circ \mathrm{D}^h_{m,p}(f) \\
\mathrm{O}^h_{m,p}(f) &= \mathrm{D}^h_{m,p} \circ \mathrm{E}^h_{m,p}(f)
\end{aligned}
$$

- **Adaptive Alternating Sequential Filters**

$$
\begin{aligned}
\mathrm{ASFOC}^h_{m,n}(f) &= \mathrm{OC}^h_{m,p_n} \circ \cdots \circ \mathrm{OC}^h_{m,p_1}(f) \\
\mathrm{ASFCO}^h_{m,n}(f) &= \mathrm{CO}^h_{m,p_n} \circ \cdots \circ \mathrm{CO}^h_{m,p_1}(f)
\end{aligned}
$$

Properties

- **Increasing**

$$f_1 \leq f_2 \Rightarrow \begin{cases} \mathrm{D}^h_{m_\hexagon}(f_1) \leq \mathrm{D}^h_{m_\hexagon}(f_2) \\ \mathrm{E}^h_{m_\hexagon}(f_1) \leq \mathrm{E}^h_{m_\hexagon}(f_2) \\ \mathrm{C}^h_{m_\hexagon}(f_1) \leq \mathrm{C}^h_{m_\hexagon}(f_2) \\ \mathrm{O}^h_{m_\hexagon}(f_1) \leq \mathrm{O}^h_{m_\hexagon}(f_2) \end{cases}$$

- **Adjunction (Morphological Duality)**

$$\mathrm{D}^h_{m_\hexagon}(f_1) \leq f_2 \Leftrightarrow f_1 \leq \mathrm{E}^h_{m_\hexagon}(f_2)$$

- **Extensiveness, Anti-Extensiveness**

$$\mathrm{O}^h_{m_\hexagon}(f) \leq f \leq \mathrm{C}^h_{m_\hexagon}(f)$$

- **Distributivity**

$$\begin{cases} \bigvee_{i \in I} [\mathrm{D}^h_{m_\hexagon}(f_i)] = \mathrm{D}^h_{m_\hexagon}(\bigvee_{i \in I} [f_i]) \\ \bigwedge_{i \in I} [\mathrm{E}^h_{m_\hexagon}(f_i)] = \mathrm{E}^h_{m_\hexagon}(\bigwedge_{i \in I} [f_i]) \end{cases}$$

Properties

- **Duality with Respect to Opposite**

$$\begin{cases} \tilde{\ominus}\,\mathrm{D}^{h}_{m_{\hexagon}}(f) = \mathrm{E}^{h}_{m_{\hexagon}}(\,\tilde{\ominus}\,f) \\ \tilde{\ominus}\,\mathrm{C}^{h}_{m_{\hexagon}}(f) = \mathrm{O}^{h}_{m_{\hexagon}}(\,\tilde{\ominus}\,f) \end{cases}$$

- **Increasing, Decreasing with Respect to the Homegeneity Tolerance**

$$\begin{pmatrix} (m^1_{\hexagon}, m^2_{\hexagon}) \in E^{\oplus} \times E^{\oplus} \\ m^1_{\hexagon} \leq m^2_{\hexagon} \end{pmatrix} \Rightarrow \begin{cases} \mathrm{D}^{h}_{m^1_{\hexagon}}(f) \leq \mathrm{D}^{h}_{m^2_{\hexagon}}(f) \\ \mathrm{E}^{h}_{m^1_{\hexagon}}(f) \geq \mathrm{E}^{h}_{m^2_{\hexagon}}(f) \end{cases}$$

- **Idempotence**

$$\begin{cases} \mathrm{C}^{h}_{m_{\hexagon}}(\mathrm{C}^{h}_{m_{\hexagon}}(f)) = \mathrm{C}^{h}_{m_{\hexagon}}(f) \\ \mathrm{O}^{h}_{m_{\hexagon}}(\mathrm{O}^{h}_{m_{\hexagon}}(f)) = \mathrm{O}^{h}_{m_{\hexagon}}(f) \end{cases}$$

- **Translation Invariance**

$$c \in \tilde{E} \Rightarrow \begin{cases} \mathrm{D}^{h\,\oplus c}_{m_{\hexagon}}(f) = \mathrm{D}^{h}_{m_{\hexagon}}(f) \\ \mathrm{E}^{h\,\oplus c}_{m_{\hexagon}}(f) = \mathrm{E}^{h}_{m_{\hexagon}}(f) \\ \mathrm{C}^{h\,\oplus c}_{m_{\hexagon}}(f) = \mathrm{C}^{h}_{m_{\hexagon}}(f) \\ \mathrm{O}^{h\,\oplus c}_{m_{\hexagon}}(f) = \mathrm{O}^{h}_{m_{\hexagon}}(f) \end{cases}$$

Properties

- **Multiplication Compatibility**

$$\alpha \in \mathbb{R}^{+}\backslash\{0\} \Rightarrow \begin{cases} \mathrm{D}_{m_{\hexagon}}^{\alpha \otimes h}(f) = \mathrm{D}_{\frac{1}{\alpha}\otimes m_{\hexagon}}^{h}(f) \\ \mathrm{E}_{m_{\hexagon}}^{\alpha \otimes h}(f) = \mathrm{E}_{\frac{1}{\alpha}\otimes m_{\hexagon}}^{h}(f) \\ \mathrm{C}_{m_{\hexagon}}^{\alpha \otimes h}(f) = \mathrm{C}_{\frac{1}{\alpha}\otimes m_{\hexagon}}^{h}(f) \\ \mathrm{O}_{m_{\hexagon}}^{\alpha \otimes h}(f) = \mathrm{O}_{\frac{1}{\alpha}\otimes m_{\hexagon}}^{h}(f) \end{cases}$$

- **Translation Commutativity**

$$c \in E \Rightarrow \begin{cases} \mathrm{D}_{m_{\hexagon}}^{h}(f \,\tilde{\oplus}\, c) = \mathrm{D}_{m_{\hexagon}}^{h}(f) \,\tilde{\oplus}\, c \\ \mathrm{E}_{m_{\hexagon}}^{h}(f \,\tilde{\oplus}\, c) = \mathrm{E}_{m_{\hexagon}}^{h}(f) \,\tilde{\oplus}\, c \\ \mathrm{C}_{m_{\hexagon}}^{h}(f \,\tilde{\oplus}\, c) = \mathrm{C}_{m_{\hexagon}}^{h}(f) \,\tilde{\oplus}\, c \\ \mathrm{O}_{m_{\hexagon}}^{h}(f \,\tilde{\oplus}\, c) = \mathrm{O}_{m_{\hexagon}}^{h}(f) \,\tilde{\oplus}\, c \end{cases}$$

- **Multiplication Commutativity**

$$\alpha \in \mathbb{R} \Rightarrow \begin{cases} \mathrm{D}_{m_{\hexagon}}^{h}(\alpha \,\tilde{\otimes}\, f) = \alpha \,\tilde{\otimes}\, \mathrm{D}_{m_{\hexagon}}^{h}(f) \\ \mathrm{E}_{m_{\hexagon}}^{h}(\alpha \,\tilde{\otimes}\, f) = \alpha \,\tilde{\otimes}\, \mathrm{E}_{m_{\hexagon}}^{h}(f) \\ \mathrm{C}_{m_{\hexagon}}^{h}(\alpha \,\tilde{\otimes}\, f) = \alpha \,\tilde{\otimes}\, \mathrm{C}_{m_{\hexagon}}^{h}(f) \\ \mathrm{O}_{m_{\hexagon}}^{h}(\alpha \,\tilde{\otimes}\, f) = \alpha \,\tilde{\otimes}\, \mathrm{O}_{m_{\hexagon}}^{h}(f) \end{cases}$$

Properties

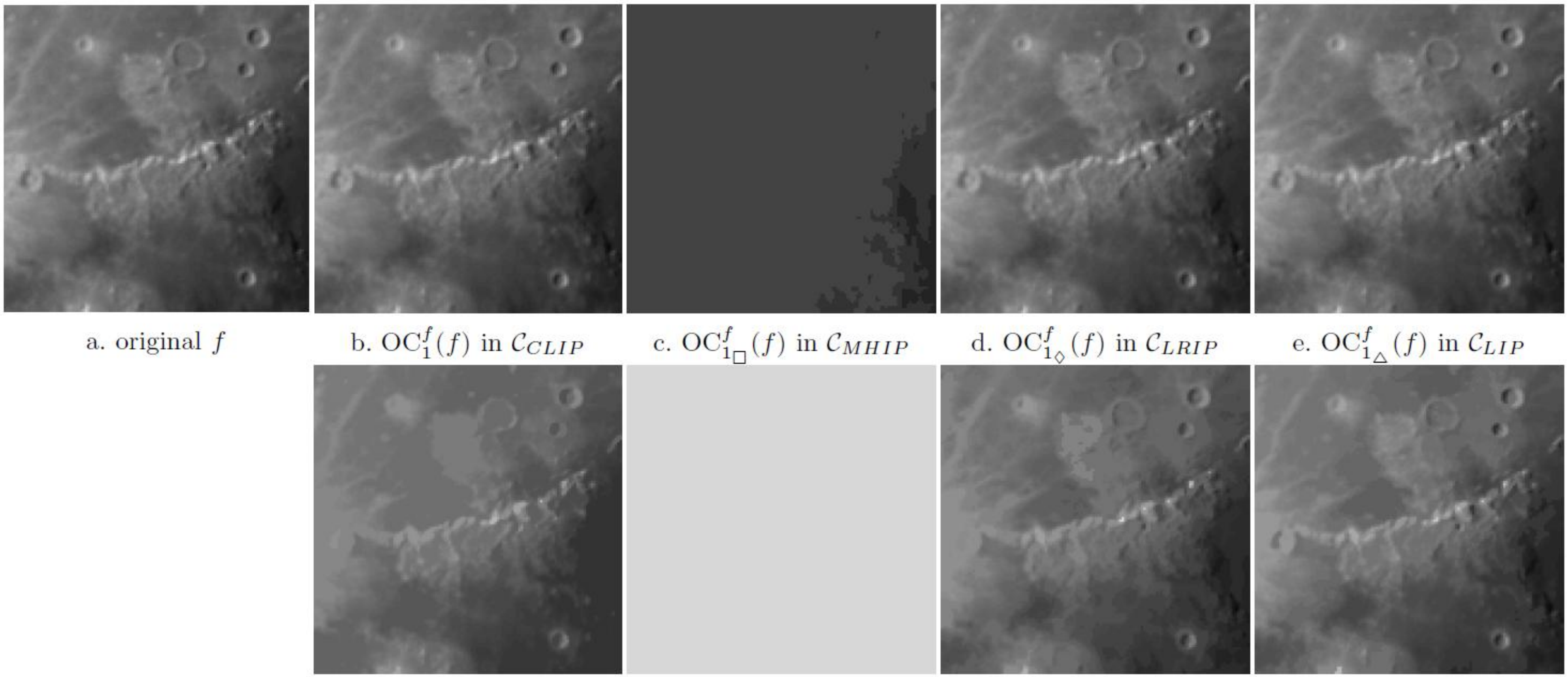
- **Connectivity**

$$\begin{pmatrix} \mathcal{I} = \mathcal{C} \\ f \in \mathcal{I} \end{pmatrix} \Rightarrow \left\{ \begin{array}{l} f \mapsto \mathrm{D}^{f}_{m_{\hexagon}}(f) \\ f \mapsto \mathrm{E}^{f}_{m_{\hexagon}}(f) \\ f \mapsto \mathrm{C}^{f}_{m_{\hexagon}}(f) \\ f \mapsto \mathrm{O}^{f}_{m_{\hexagon}}(f) \end{array} \right. \quad \textit{are connected operators}$$

Application Example

Image Restoration of 'Moon'

- Impact of the GLIP framework

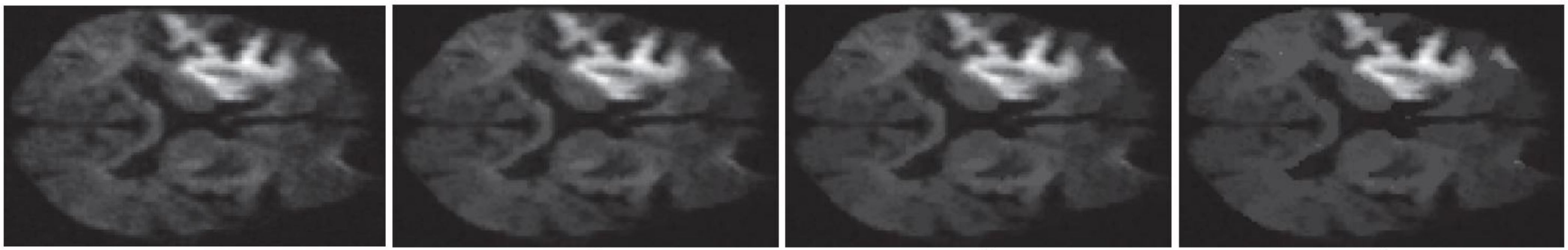


a. original f b. $OC_1^f(f)$ in $\mathcal{C}_{CLIP}$ c. $OC_{1_\square}^f(f)$ in $\mathcal{C}_{MHIP}$ d. $OC_{1_\diamond}^f(f)$ in $\mathcal{C}_{LRIP}$ e. $OC_{1_\triangle}^f(f)$ in $\mathcal{C}_{LIP}$

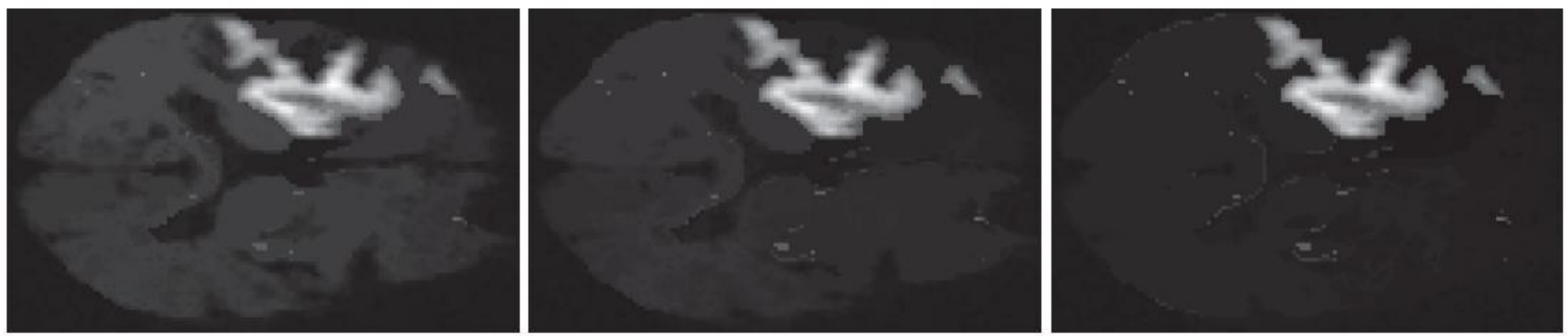
f. $OC_{20}^f(f)$ in $\mathcal{C}_{CLIP}$ g. $OC_{20_\square}^f(f)$ in $\mathcal{C}_{MHIP}$ h. $OC_{20_\diamond}^f(f)$ in $\mathcal{C}_{LRIP}$ i. $OC_{20_\triangle}^f(f)$ in $\mathcal{C}_{LIP}$

Application Example

- **Image Restoration of Cerebro Vascular Accidents (CVA)**
  - GAN sequential filtering



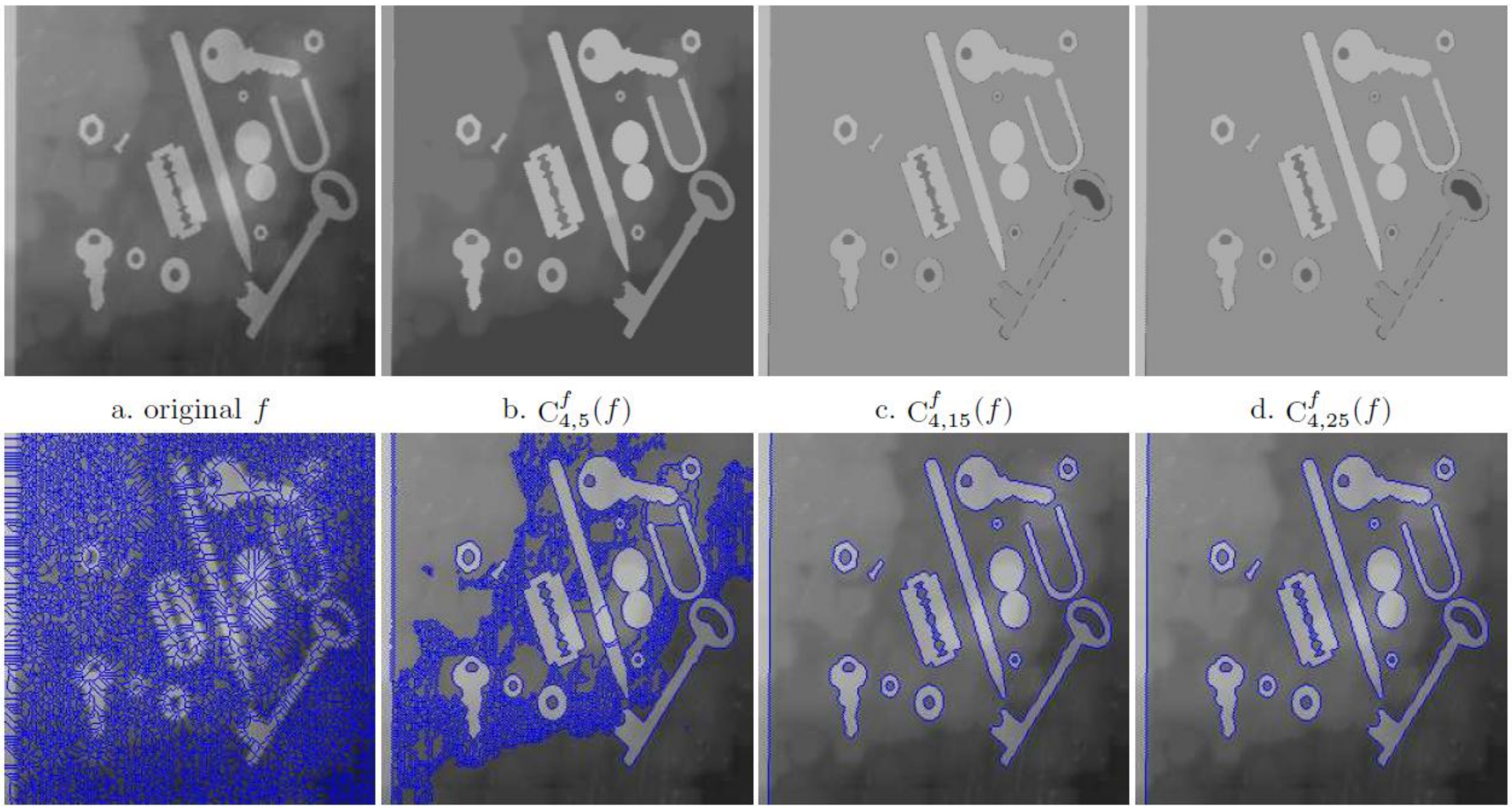
(a) original f (b) $O^f_{7,1}(f)$ (c) $O^f_{7,2}(f)$ (d) $O^f_{7,4}(f)$



(e) $O^f_{7,6}(f)$ (f) $O^f_{7,8}(f)$ (g) $O^f_{7,10}(f)$

Application Example

- **Multiscale Image Segmentation**
  - GAN sequential filtering and watershed transformation of the gradient



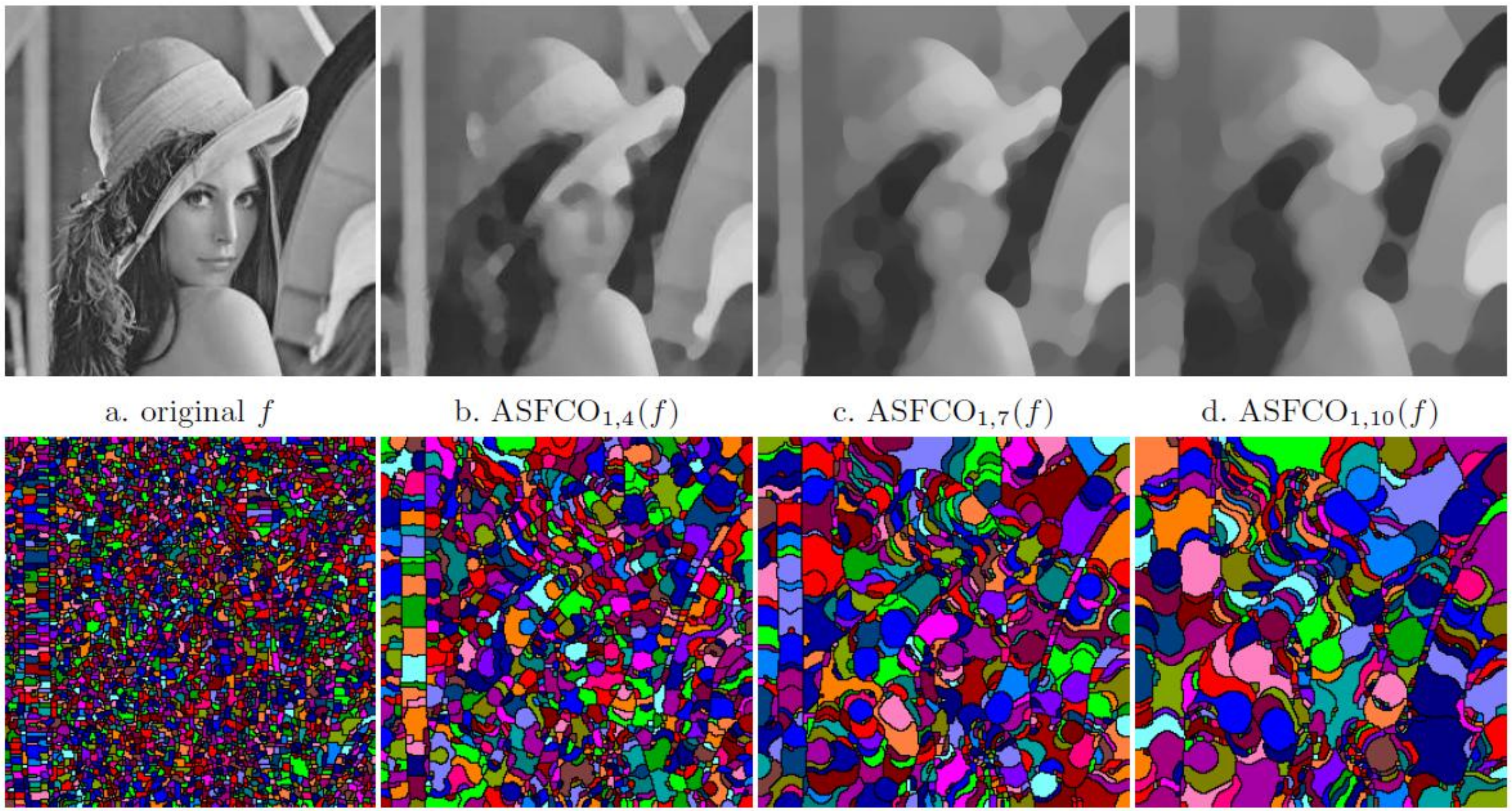
a. original f b. $C^{f}_{4,5}(f)$ c. $C^{f}_{4,15}(f)$ d. $C^{f}_{4,25}(f)$

e. $Seg(f)$ f. $Seg(C^{f}_{4,5}(f))$ g. $Seg(C^{f}_{4,15}(f))$ h. $Seg(C^{f}_{4,25}(f))$

Application Example

Multiscale Image Segmentation

- Classical alternating sequential filtering and watershed transformation of the gradient

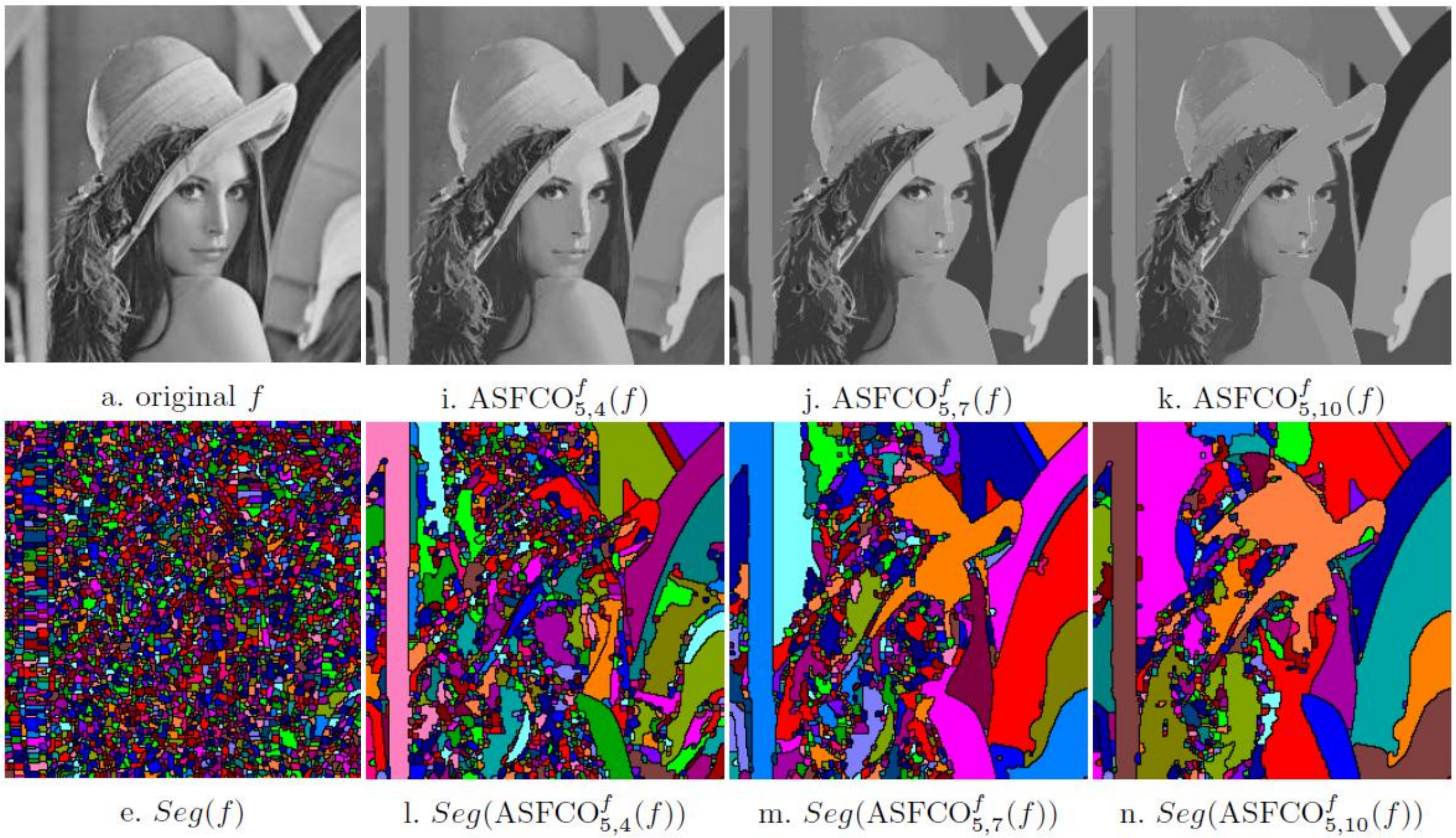


a. original f b. $\mathrm{ASFCO}_{1,4}(f)$ c. $\mathrm{ASFCO}_{1,7}(f)$ d. $\mathrm{ASFCO}_{1,10}(f)$

e. $Seg(f)$ f. $Seg(\mathrm{ASFCO}_{1,4}(f))$ g. $Seg(\mathrm{ASFCO}_{1,7}(f))$ h. $Seg(\mathrm{ASFCO}_{1,10}(f,0))$

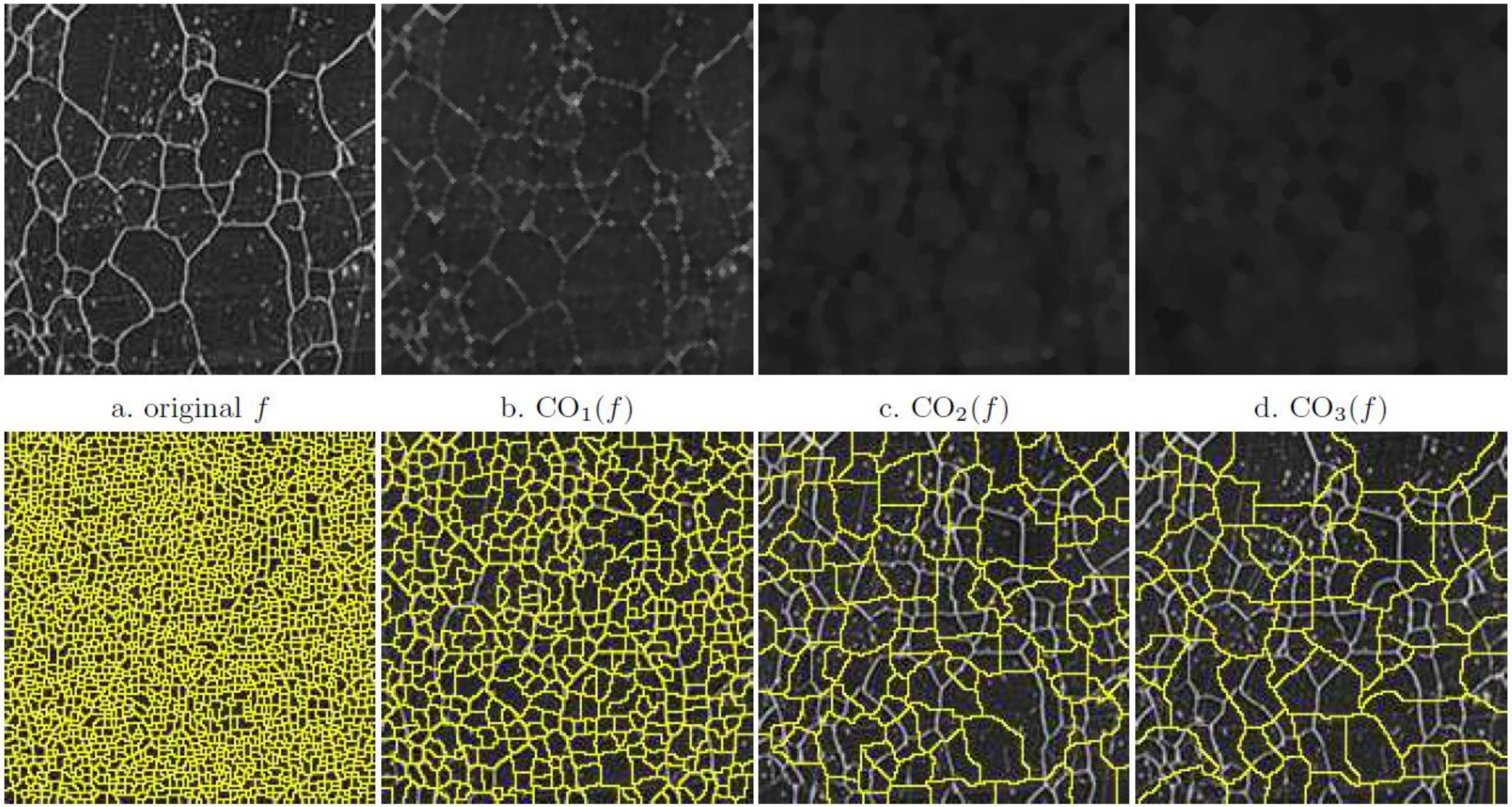
Application Example

- **Multiscale Image Segmentation**
  - GAN alternating sequential filtering and watershed transformation of the gradient



Application Example

- **Image Segmentation of Metallurgic Grain Boundaries**
  - Classical alternating filtering and watershed transformation

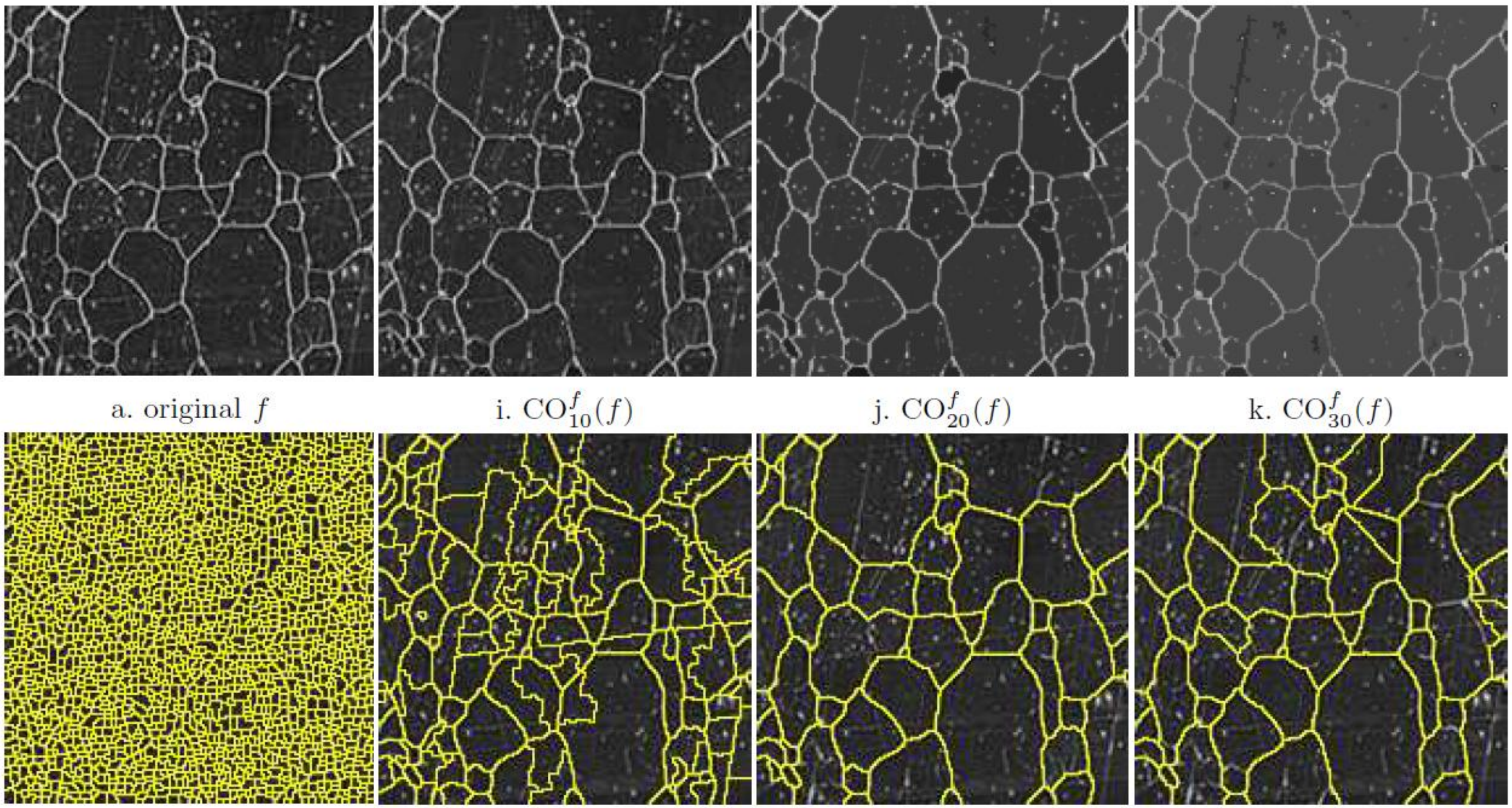


a. original f b. $CO_1(f)$ c. $CO_2(f)$ d. $CO_3(f)$

e. $Wat(f)$ f. $Wat(CO_1(f))$ g. $Wat(CO_2(f))$ h. $Wat(CO_3(f))$

Application Example

- **Image Segmentation of Metallurgic Grain Boundaries**
  - GAN alternating filtering and watershed transformation

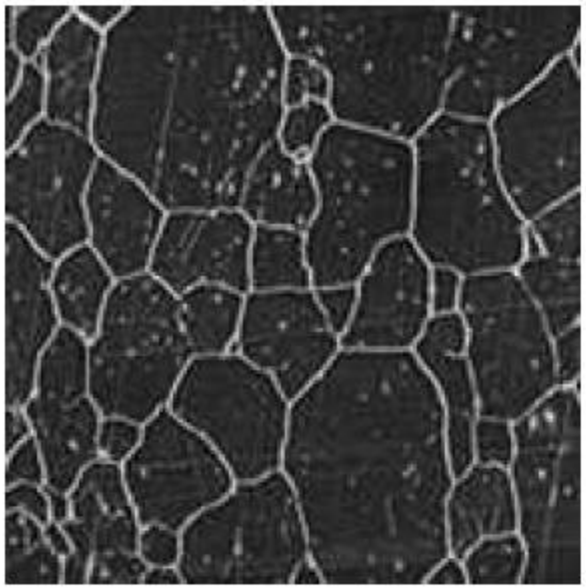


a. original f | i. $\mathrm{CO}^{f}_{10}(f)$ | j. $\mathrm{CO}^{f}_{20}(f)$ | k. $\mathrm{CO}^{f}_{30}(f)$

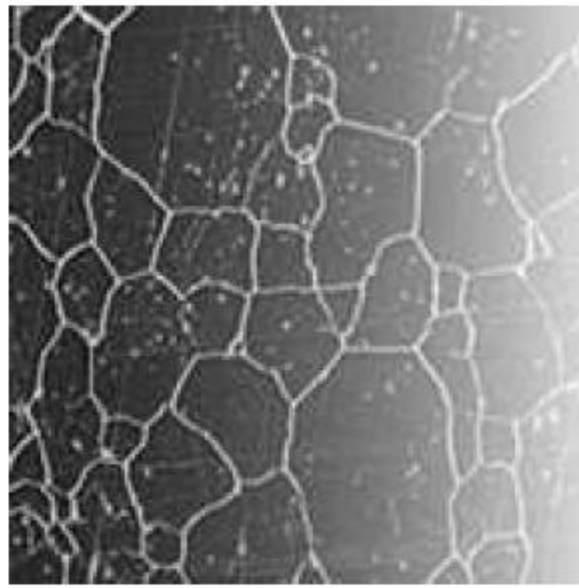
e. $Wat(f)$ | l. $Wat(\mathrm{CO}^{f}_{10}(f))$ | m. $Wat(\mathrm{CO}^{f}_{20}(f))$ | n. $Wat(\mathrm{CO}^{f}_{30}(f))$

Application Example

- **Image Segmentation of Metallurgic Grain Boundaries**
  - In uneven illumination conditions



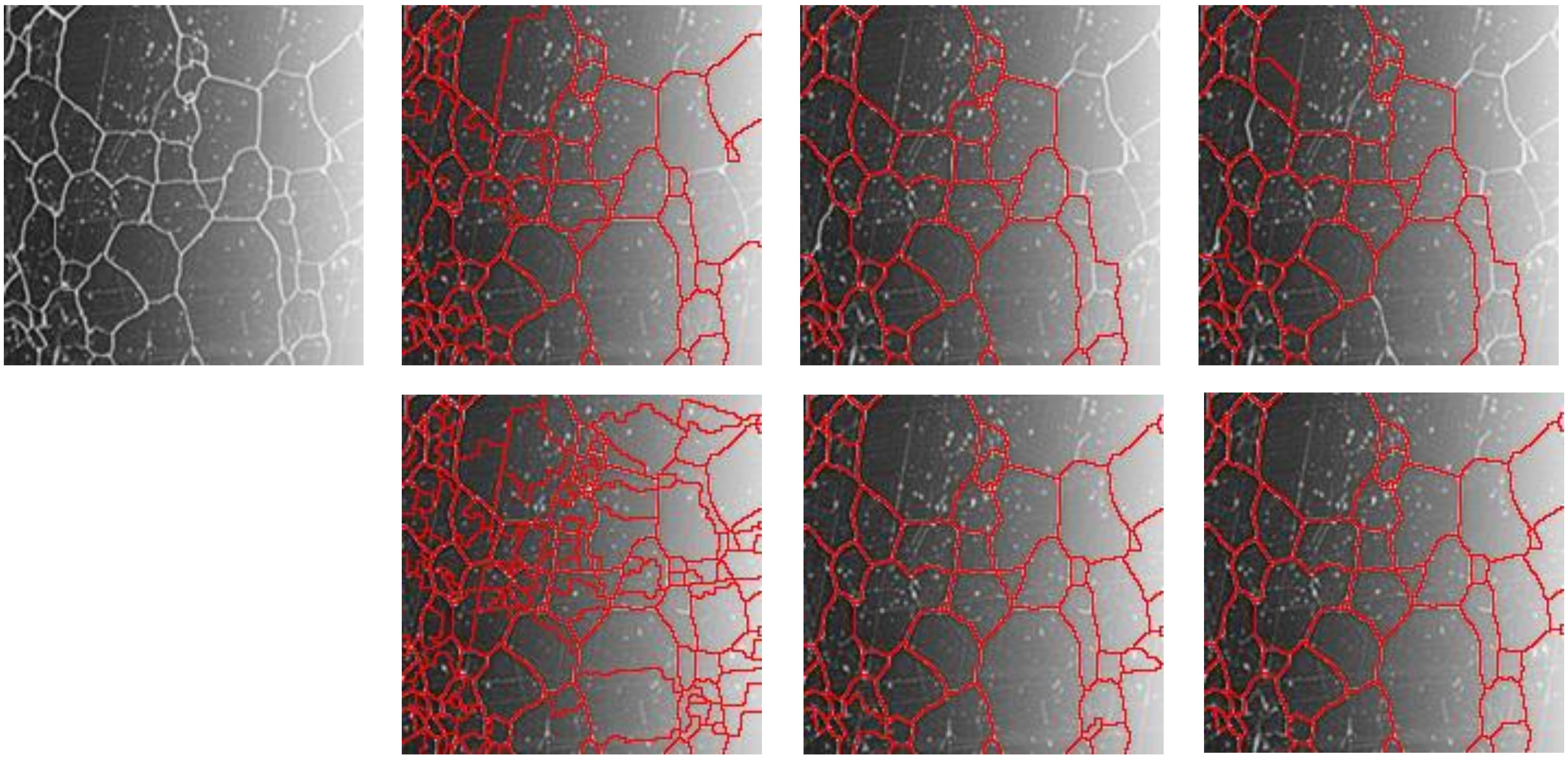
a. original f



b. non-uniformly lightened $\check{f}$

Application Example

- **Image Segmentation of Metallurgic Grain Boundaries**
  - In uneven illumination conditions
  - GAN filtering: CLIP vs. LIP



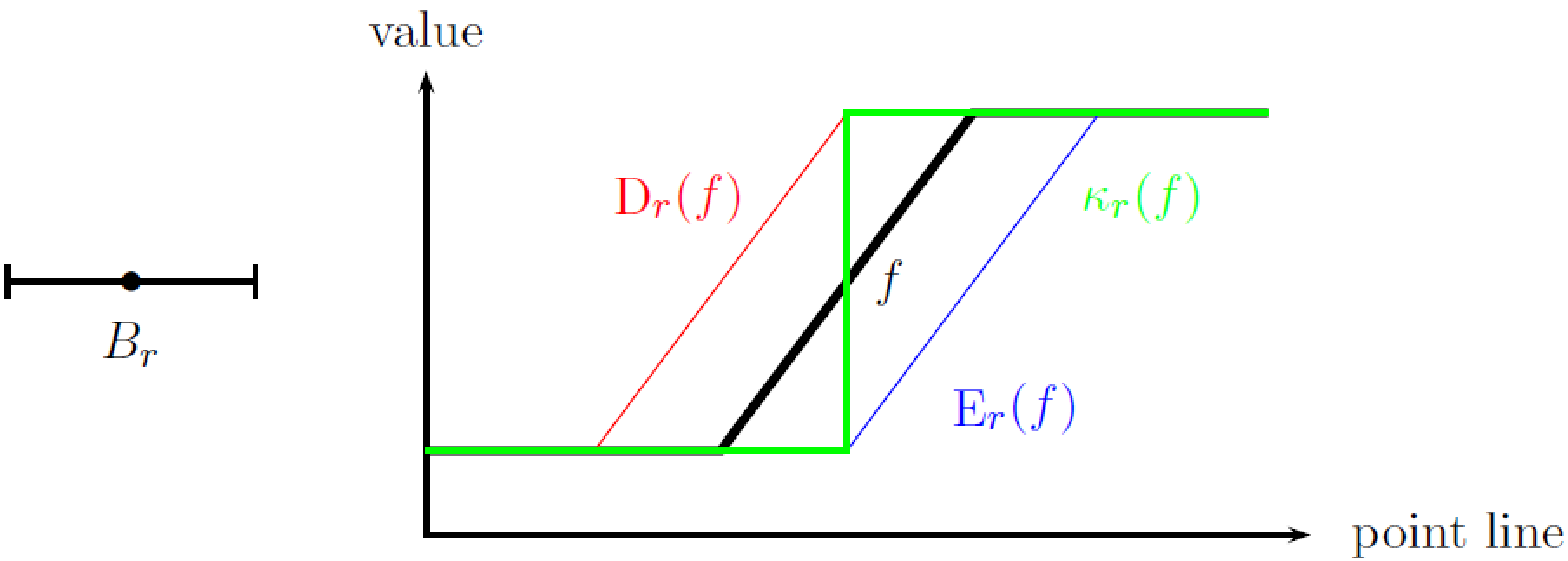
CLIP

LIP

Application Example

- **Image Enhancement of Retinal Vessels**
  - Toggle contrast operator

$$\kappa_r(f)(x) = \begin{cases} \mathrm{D}_r(f)(x) & \textit{if } \mathrm{D}_r(f)(x) - f(x) < f(x) - \mathrm{E}_r(f)(x) \\ \mathrm{E}_r(f)(x) & \textit{otherwise} \end{cases}$$



Application Example

- **Image Enhancement of Retinal Vessels**
  - GAN toggle contrast operator

$$\kappa_{m_\triangle}^{C(f)}(f)(x) = \begin{cases} \mathrm{D}_{m_\triangle}^{C(f)}(f)(x) & if \ \mathrm{D}_{m_\triangle}^{C(f)}(f)(x) - f(x) < f(x) - \mathrm{E}_{m_\triangle}^{C(f)}(f)(x) \\ \mathrm{E}_{m_\triangle}^{C(f)}(f)(x) & otherwise \end{cases}$$

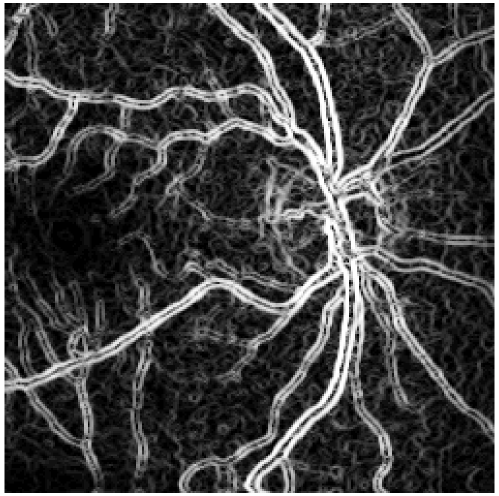
  - LIP contrast

$$C(f)(x) = \frac{1}{\#V(x)} \triangle\!\!\!\!\times \sum_{y \in V(x)}^{\triangle\!\!\!\!+} (\max(f(x), f(y)) \triangle \min(f(x), f(y)))$$

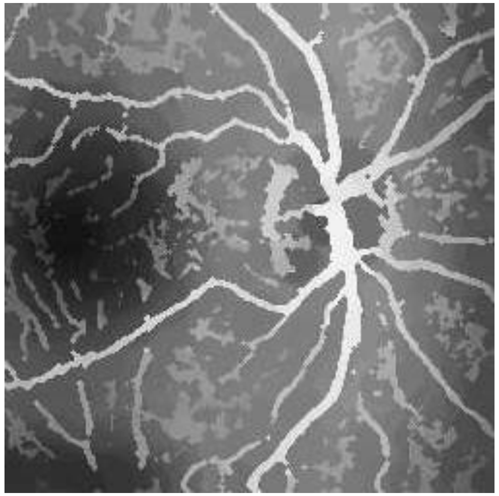
Application Example

- **Image Enhancement of Retinal Vessels**
  - Classical vs. GAN toggle contrast operator

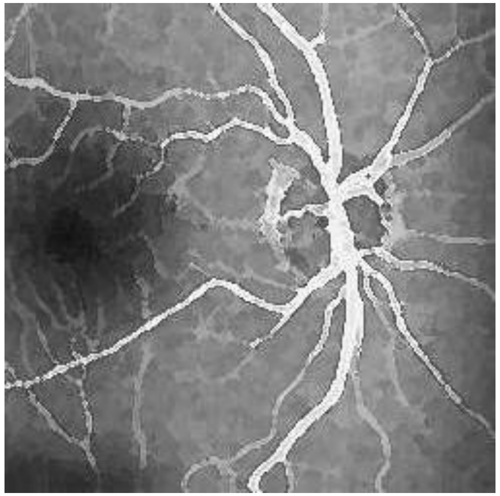
Original image



LIP contrast



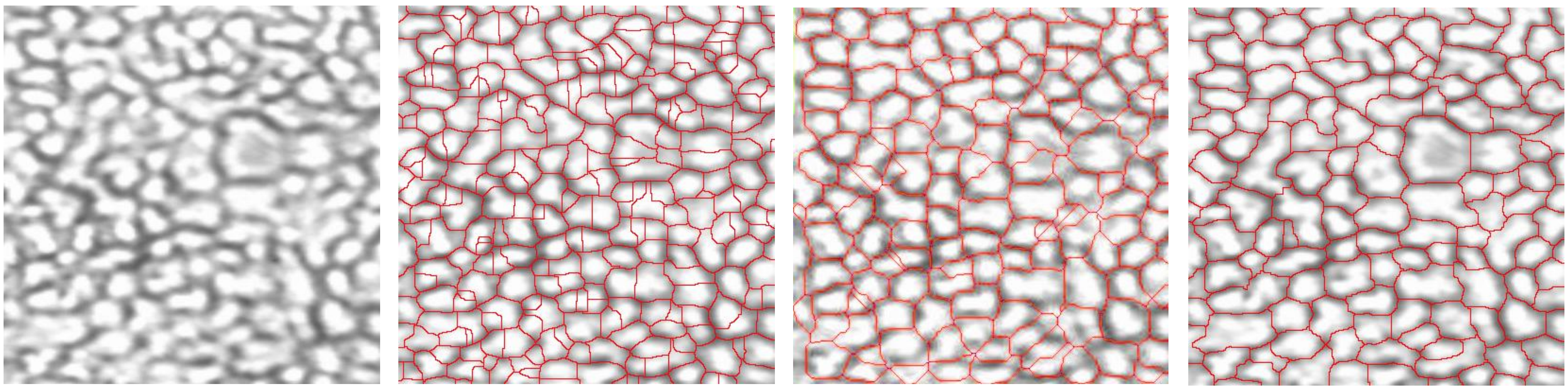
Classical Filtering



GAN Filtering

Application Example

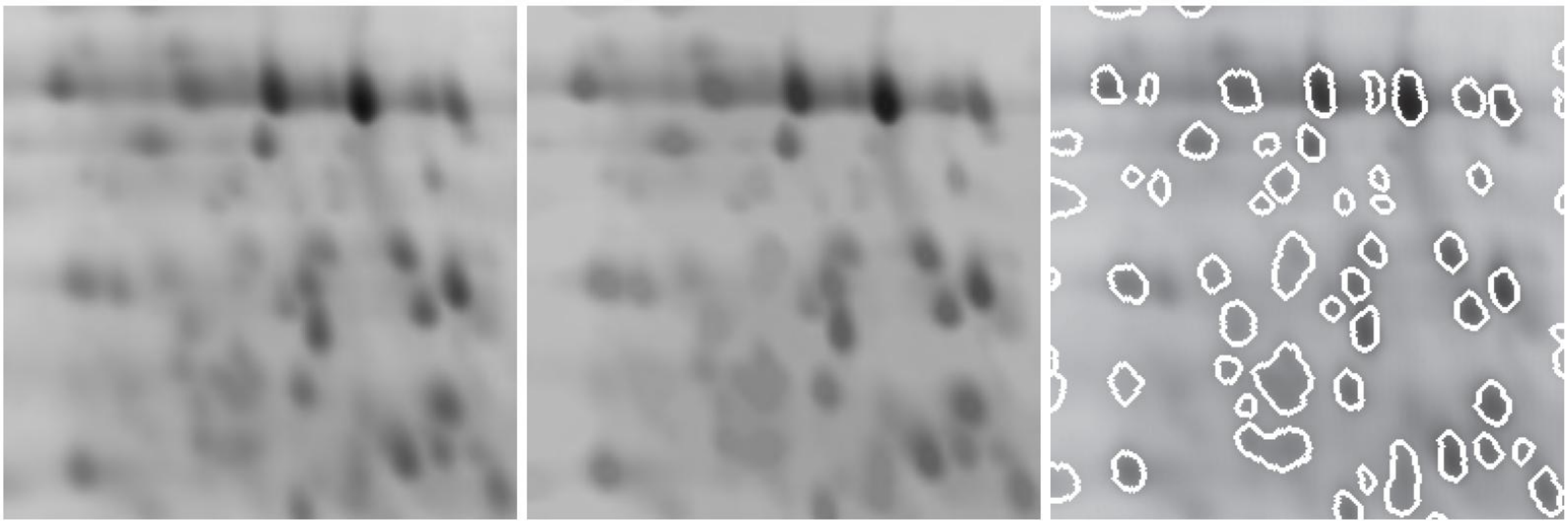
- **Image Segmentation of Cornea Cells**
  - Closing-opening and watershed tranformation



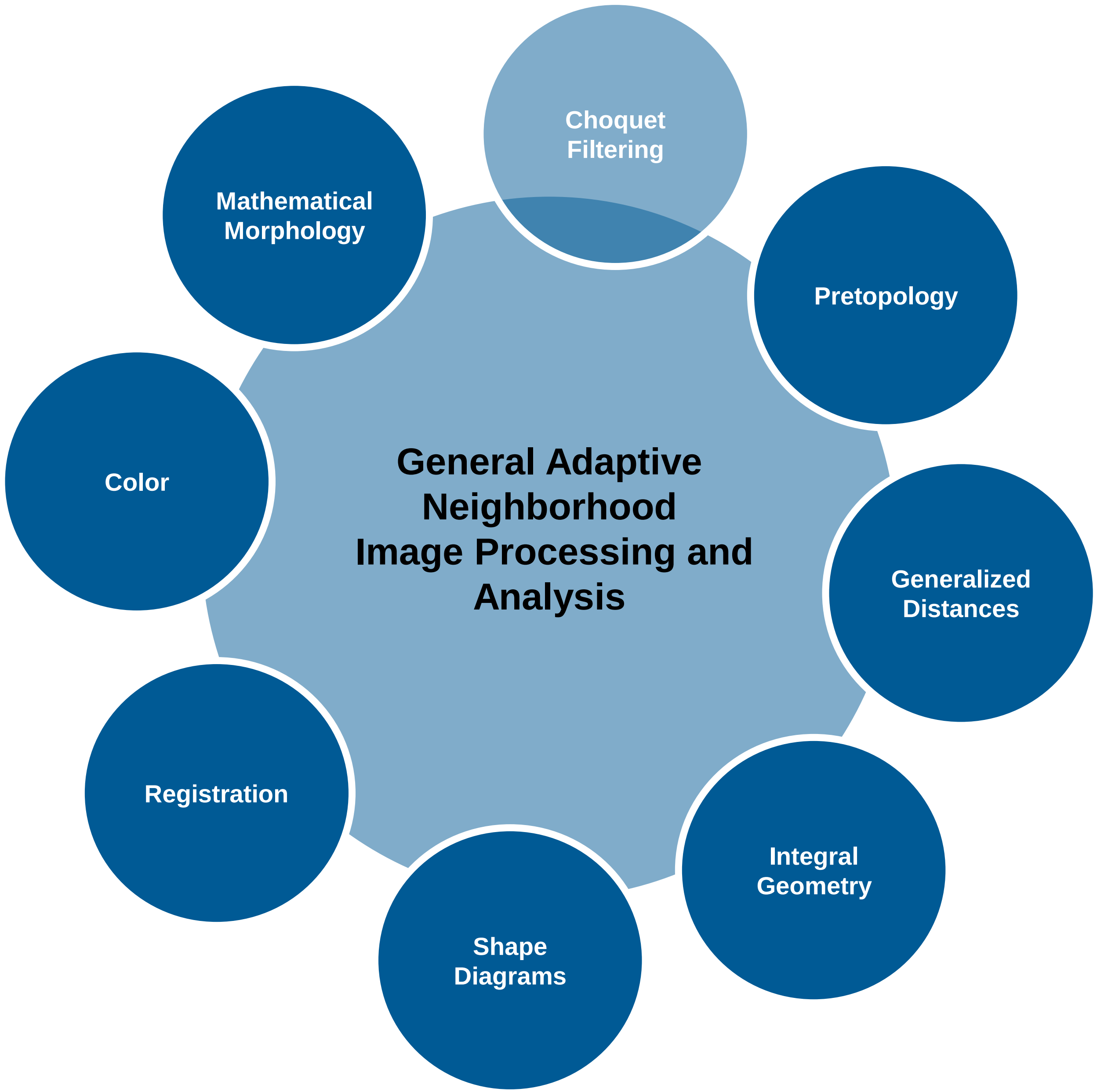
Original image *Classical processing* *Samba processing* *GAN processing*

Application Example

- **Image Segmentation of Protein Electrophoresis Gel**
  - Opening-closing and constrained watershed tranformation



(a) original image (b) adaptive image filtering (c) adaptive image segmentation


Choquet Filtering
Mathematical Morphology
Pretopology
Color
General Adaptive Neighborhood Image Processing and Analysis
Generalized Distances
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[**JMIV 2009**, ECSIA 2009]

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DOI 10.1007/s10851-009-0163-0

General Adaptive Neighborhood Choquet Image Filtering

Johan Debayle · Jean-Charles Pinoli

Published online: 3 July 2009

Abstract A novel framework entitled General Adaptive Neighborhood Image Processing (GANIP) has been recently introduced in order to propose an original image representation and mathematical structure for adaptive image processing and analysis. The central idea is based on the key notion of adaptivity which is simultaneously associated with the analyzing scales, the spatial structures and the intensity values of the image to be addressed. In this paper, the GANIP framework is briefly exposed and particularly studied in the context of Choquet filtering (using fuzzy measures), which generalizes a large class of image filters. The resulting spatially-adaptive operators are studied with respect to the general GANIP framework and illustrated in both the biomedical and materials application areas. In addition, the proposed GAN-based filters are practically applied and compared to several other denoising methods through experiments on image restoration, showing a high performance of the GAN-based Choquet filters.

Keywords Choquet filtering · Fuzzy measure · General adaptive neighborhoods · Image restoration

1 Introduction

In the General Adaptive Neighborhood Image Processing (GANIP) approach [12, 13], an intensity image is represented with a class of local neighborhoods defined for each point of the image to be studied. These so-called General Adaptive Neighborhoods (GANs) are simultaneously adaptive with the spatial structures, the analyzing scales and the intensity values.

This paper first outlines the Logarithmic Image Processing (LIP) [26, 41] framework (Sect. 2), a specific General Linear Image Processing (GLIP) one [35, 40], involved in the GANIP approach. The GLIP operations, acting on image intensities, are consistent with the physical and/or psychophysical laws and characteristics associated to the image class (transmitted light images, human visual images …). After a brief theoretical introductory survey of the GANIP framework (Sect. 3), Choquet filtering [34] (generalizing several image transforms) is briefly exposed (Sect. 4) and extended with the use of the GAN-based image representation (Sect. 5). Different illustration examples are then proposed in Sect. 6 so as to show the impact of the parameters of these GAN-based Choquet filters. Thereafter (Sect. 7), some evaluation criteria for image restoration are presented. The proposed approach is then practically tested and compared to specific denoising methods through experiments on image restoration (Sect. 8), highlighting the effectiveness of the GAN-based Choquet filters. Finally, the conclusion and perspectives of the proposed approach are discussed in Sects. 9 and 10, respectively.

2 LIP Framework

In order to develop powerful image processing operators, it is necessary to represent intensity images within mathematical frameworks (most of the time of a vectorial nature) based on a *physically and/or psychophysically relevant image formation process* [48]. In addition, their mathematical struc-

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GENERAL ADAPTIVE NEIGHBORHOOD REPRESENTATION FOR ADAPTIVE CHOQUET IMAGE FILTERING

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ABSTRACT

This paper aims to first outline the General Adaptive Neighborhood Image Processing (GANIP) approach. An intensity image is represented with a set of local neighborhoods defined for each point of the image to be studied. These so-called General Adaptive Neighborhoods (GANs) are simultaneously adaptive with the spatial structures, the analyzing scales and the physical settings of the image to be addressed and/or the human visual system. Thereafter, the GAN representation is used in the context of Choquet filtering. The Choquet filters are defined with the help of fuzzy integrals which are based on fuzzy measures (i.e., extension of probability measures). Those image operators generalize a large class of image filters (linear filters, rank filters, order filters) used for image restoration and enhancement. The GAN-based Choquet filters are applied on real images and compared with the corresponding classical ones through different illustrations in the biomedical and materials application areas. The results highlight the performance of the proposed operators and open interesting pathways for adaptive image processing.

Keywords: Adaptive image processing, Choquet filtering, General adaptive neighborhood, Image representation, Multiscale analysis.

INTRODUCTION

This paper first outline the General Adaptive Neighborhood Image Processing (GANIP) approach (Debayle and Pinoli, 2006) which is an extension of the approach proposed in (Debayle and Pinoli, 2005). An intensity image is then represented with a set of local neighborhoods defined for each point of the image to be studied. These so-called General Adaptive Neighborhoods (GANs) are simultaneously adaptive with the spatial structures, the analyzing scales and the intensity values. After a brief theoretical introductory survey of the GAN paradigm (Section 2), Choquet filtering (generalizing several image transforms) is briefly exposed and extended to this GAN-based image representation (Section 3). Different illustration examples are proposed in this section so as to compare the GAN-based and the classical Choquet filters, highlighting the efficiency of the adaptive operators.

THE GENERAL ADAPTIVE NEIGHBORHOOD (GAN) PARADIGM

This paper deals with 2D intensity images, that is to say image mappings defined on a spatial support D in the Euclidean space $\mathbb{R}^2$ and valued into a gray tone range, which is a real number interval. The General Adaptive Neighborhood paradigm has been introduced (Debayle and Pinoli, 2006) in order to propose an original image representation for adaptive processing and analysis. The central idea is the notion of adaptivity which is simultaneously associated to the analyzing scales, the spatial structures and the intensity values of the image to be addressed.

ADAPTIVITY WITH ANALYZING SCALES

A multiscale image representation such as wavelet decomposition (Mallat, 1989) or isotropic scale-space (Lindeberg, 1994), generally takes into account analyzing scales which are global and a priori defined, that is to say *extrinsic* scales. This kind of multiscale analysis presents a main drawback since a priori knowledge, relating to the features of the studied image, is consequently required. On the contrary, an *intrinsic* multiscale representation such as anisotropic scale-space (Perona and Malik, 1990), takes advantage of scales which are determined by the image itself. Such a decomposition does not need any a priori information.

ADAPTIVITY WITH SPATIAL STRUCTURES

The image processing techniques using *spatially invariant* transformations, with fixed operational windows, give efficient computing structures, in the sense where data and operators are independant. Nevertheless, they consequently have several drawbacks such as creating artificial patterns, changing the detailed parts of large objects, damaging transitions

Fuzzy Measure

- **Mathematical Definition**

$$\mu : 2^X \to [0,1], \text{ such that:}$$

- $\mu(\emptyset) = 0; \mu(X) = 1$
- $\mu(A) \leq \mu(B)$ if $A \subseteq B$

- **Generalization of Probabiliy Measures**
  - Additional constraint: additivity for disjoint sets

Choquet Integral

- **Mathematical Definition**

isotropic neighborhood

Image *f*

$$X = \{x_0, \ldots, x_{K-1}\}$$

$$0 = f(x_{(-1)}) \leq f(x_{(0)}) \leq f(x_{(1)}) \leq \ldots, \leq f(x_{(K-1)}),$$
$$A_{(i)} := \{x_{(i)}, \ldots, x_{(K-1)}\} \text{ and } A_{(K)} = \emptyset.$$

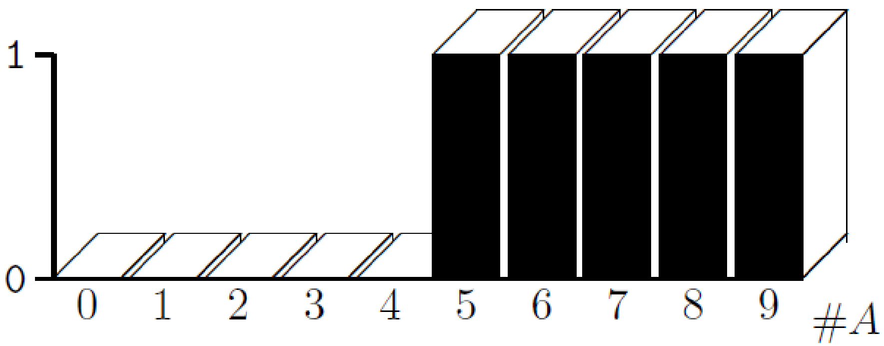
$$\boxed{\begin{aligned} C_\mu(f) &= \sum_{i=0}^{K-1} (f(x_{(i)}) - f(x_{(i-1)}))\mu(A_{(i)}) \\ &= \sum_{i=0}^{K-1} (\mu(A_{(i)}) - \mu(A_{(i+1)}))f(x_{(i)}) \end{aligned}}$$

Choquet Filtering

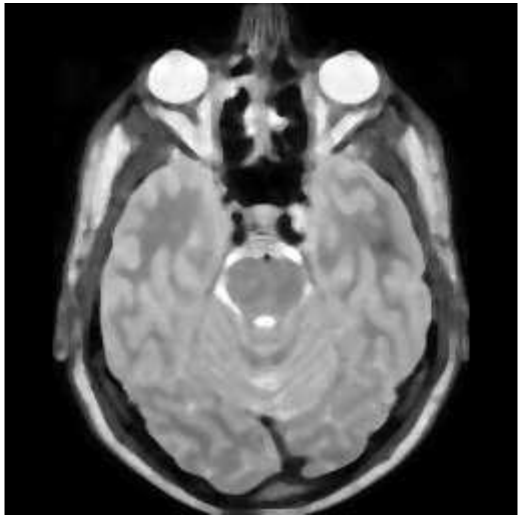
- **Large Class of Filters**
  - Linear filters (mean, Gaussian, …)
  - Rank filters (min, max, median, …)
  - Order filters (n-power, quasimidrange, …)

- **Example: 3x3 Median Filter**

$$\forall A \subseteq W \quad \mu(A) = \begin{cases} 0 \text{ if } \#A \leq \frac{\#W}{2} \\ 1 \text{ otherwise} \end{cases}$$

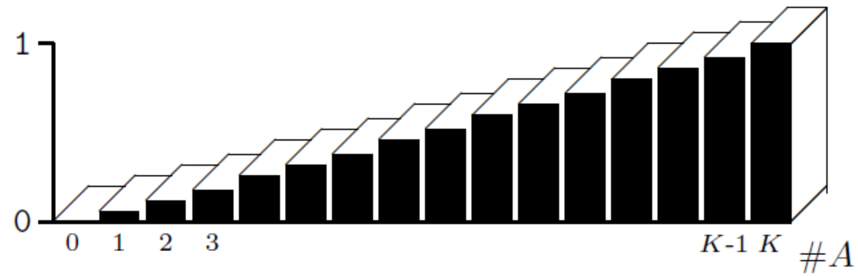


Original image

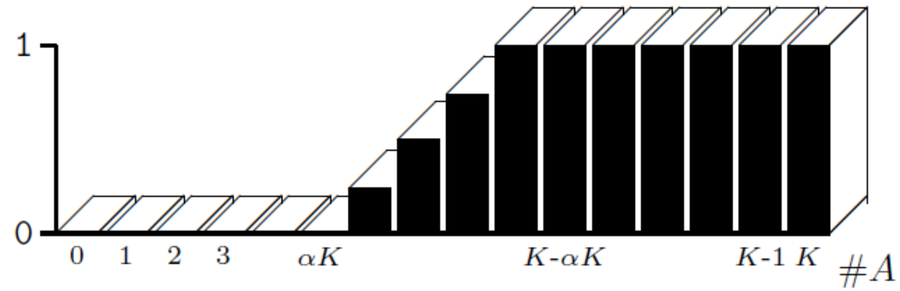


Filtered image

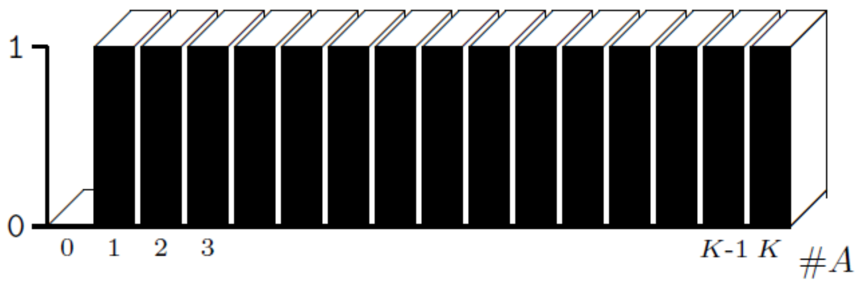
Example of Fuzzy Measures



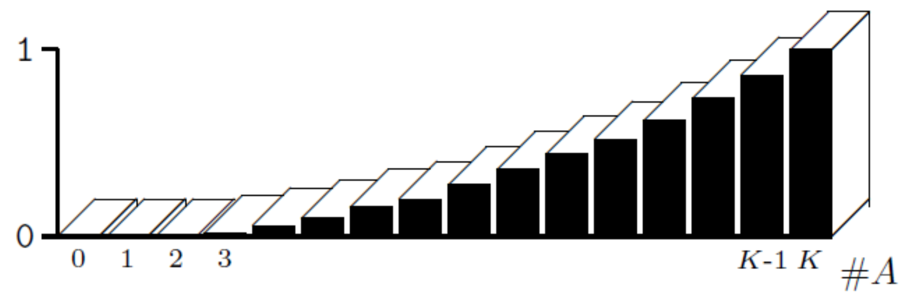
Mean filtering



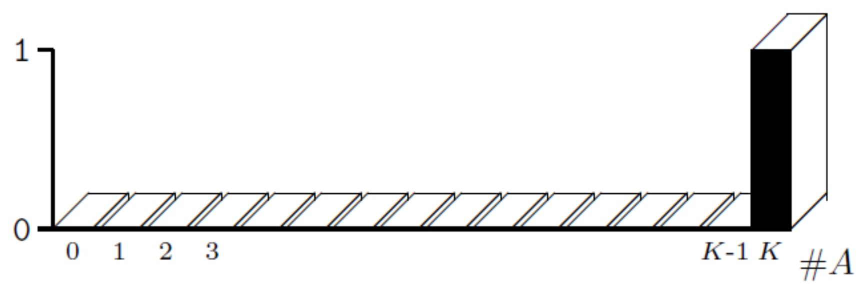
α-trimmed mean filtering



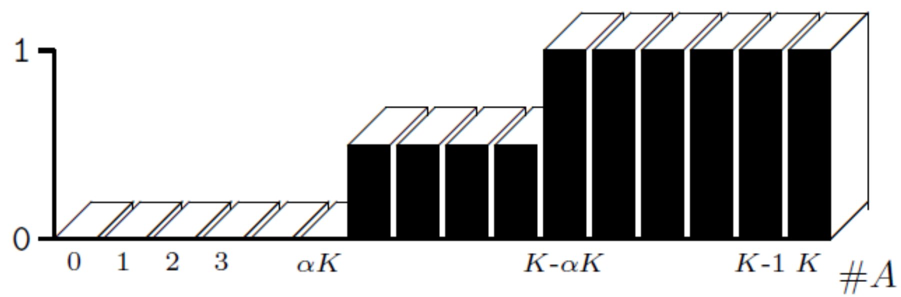
Max filtering



n-power filtering



Min filtering

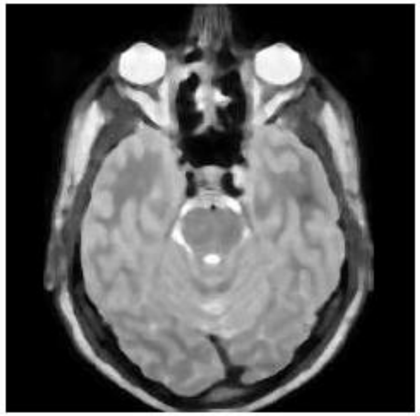


α-quasi-midrange filtering

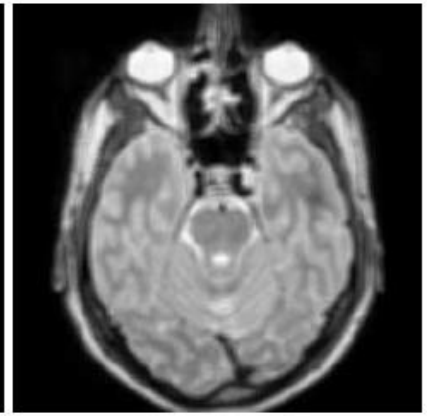
Illustration

- **Classical Choquet Filters**

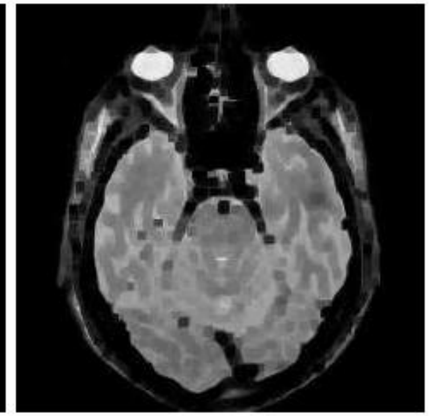
(a) original image



(b) median filtering

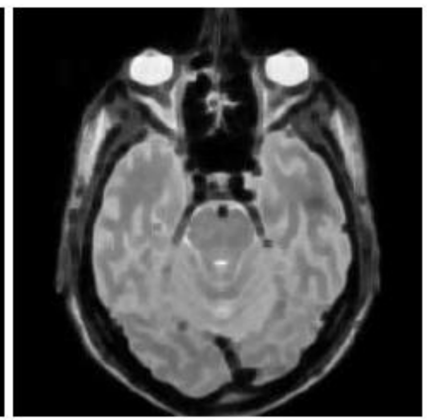


(c) mean filtering

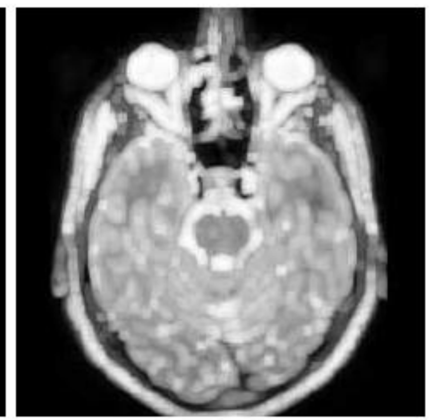


(d) min filtering

(e) max filtering



(f) 5−power filtering



(g) $\frac{1}{5}$−power filtering

GAN Choquet Filtering

- **Mathematical Definition**

adaptive neighborhood

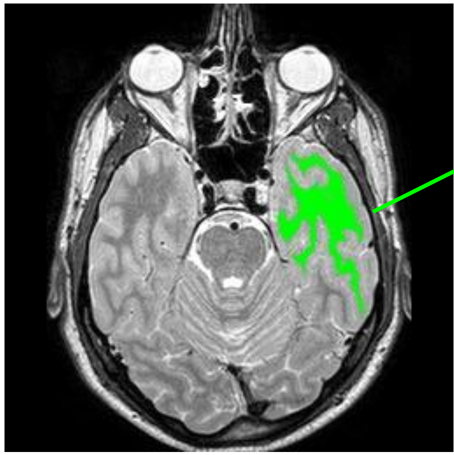


Image *f*

$$V^h_{m_{\hexagon}}(y) = \{x_0, \ldots, x_{K-1}\}$$

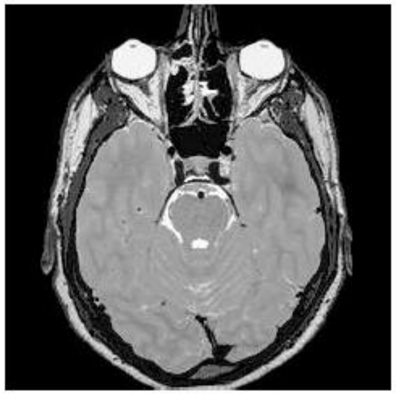
$$0 = f(x_{(-1)}) \leq f(x_{(0)}) \leq f(x_{(1)}) \leq \ldots, \leq f(x_{(K-1)}),$$
$$A_{(i)} := \{x_{(i)}, \ldots, x_{(K-1)}\} \text{ and } A_{(K)} = \emptyset.$$

$$\boxed{\mathrm{CF}^h_{m_{\hexagon}}(f)(y) = \sum_{x_i \in V^h_{m_{\hexagon}}(y)} (\mu_y(A_{(i)}) - \mu_y(A_{(i+1)}))f(x_{(i)})}$$

Illustration

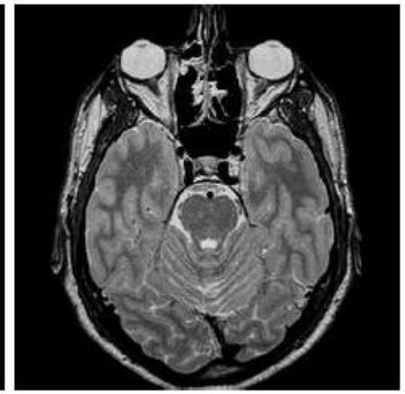
- **GAN Choquet Filters**

(a) original image

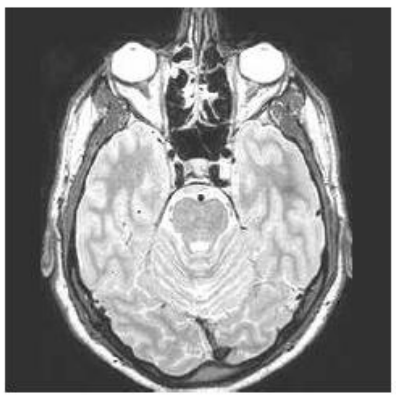


(b) GAN median filtering

(c) GAN mean filtering

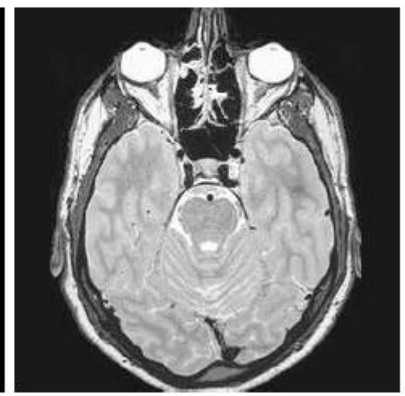


(d) GAN min filtering



(e) GAN max filtering

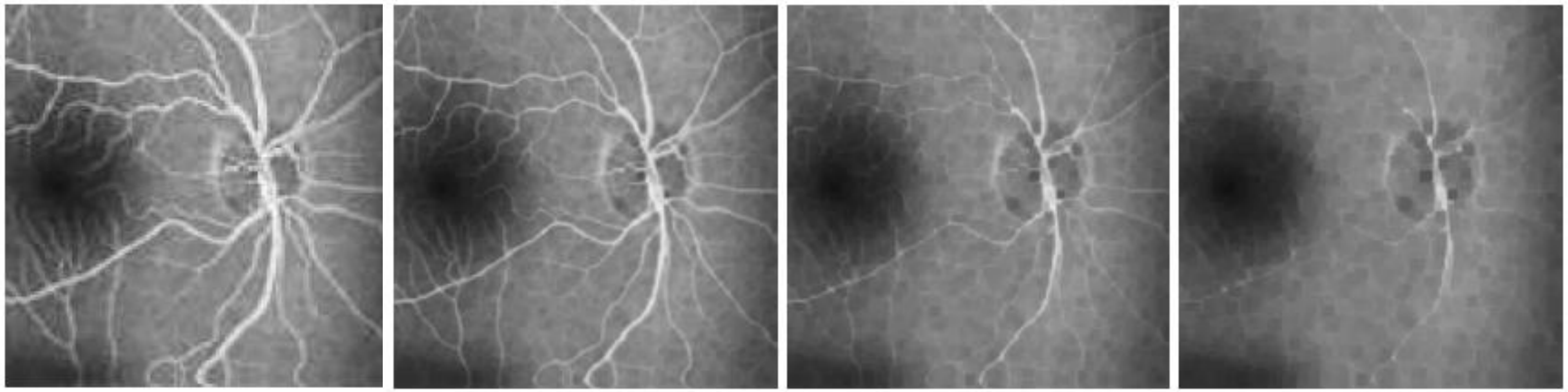
(f) GAN $5-$power filtering



(g) GAN $\frac{1}{5}-$power filtering

Illustration

- **Impact of the Homogeneity Tolerance**

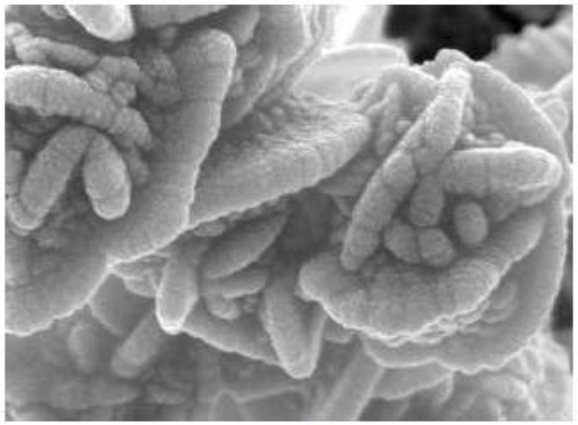


(a) original image (b) 3x3 min filtering (c) 5x5 min filtering (d) 7x7 min filtering

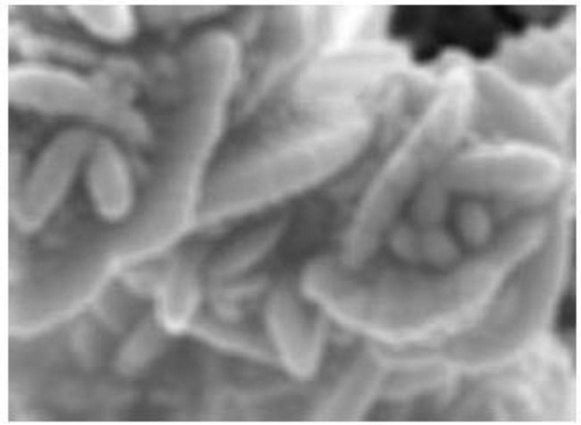
(e) GAN-based min filtering, m=20, CLIP (f) GAN-based min filtering, m=40, CLIP (g) GAN-based min filtering, m=60, CLIP

Illustration

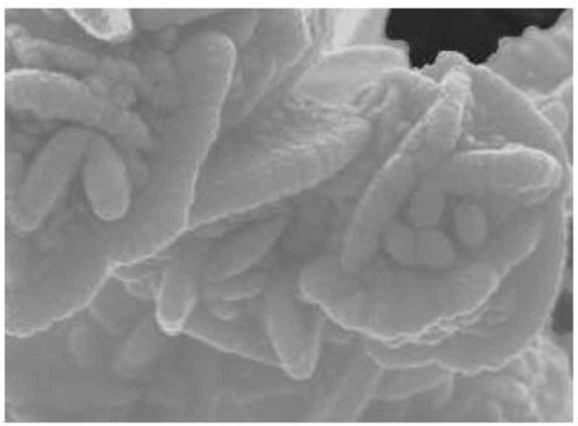
- Impact of the GLIP Model



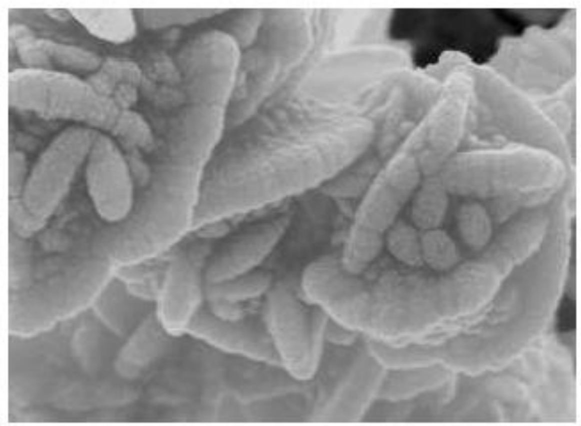
(a) original image



(b) 7x7 mean filtering



(c) GAN-based mean filtering, m=70, CLIP



(d) GAN-based mean filtering, $m_{\triangle}$=70, LIP

Illustration

- **Impact of the Criterion Mapping**

(a) original image

(b) GAN max filtering using the luminance criterion with $m_\triangle = 40$

(c) criterion mapping: LIP contrast

(d) GAN max filtering using the contrast criterion with $m_\triangle = 2$

Application Example

- **Image Restoration of the Painting 'Le Fifre' (E. Manet)**
  - Classical vs. GAN mean filtering

Original image

Classical mean filtering (r=1)

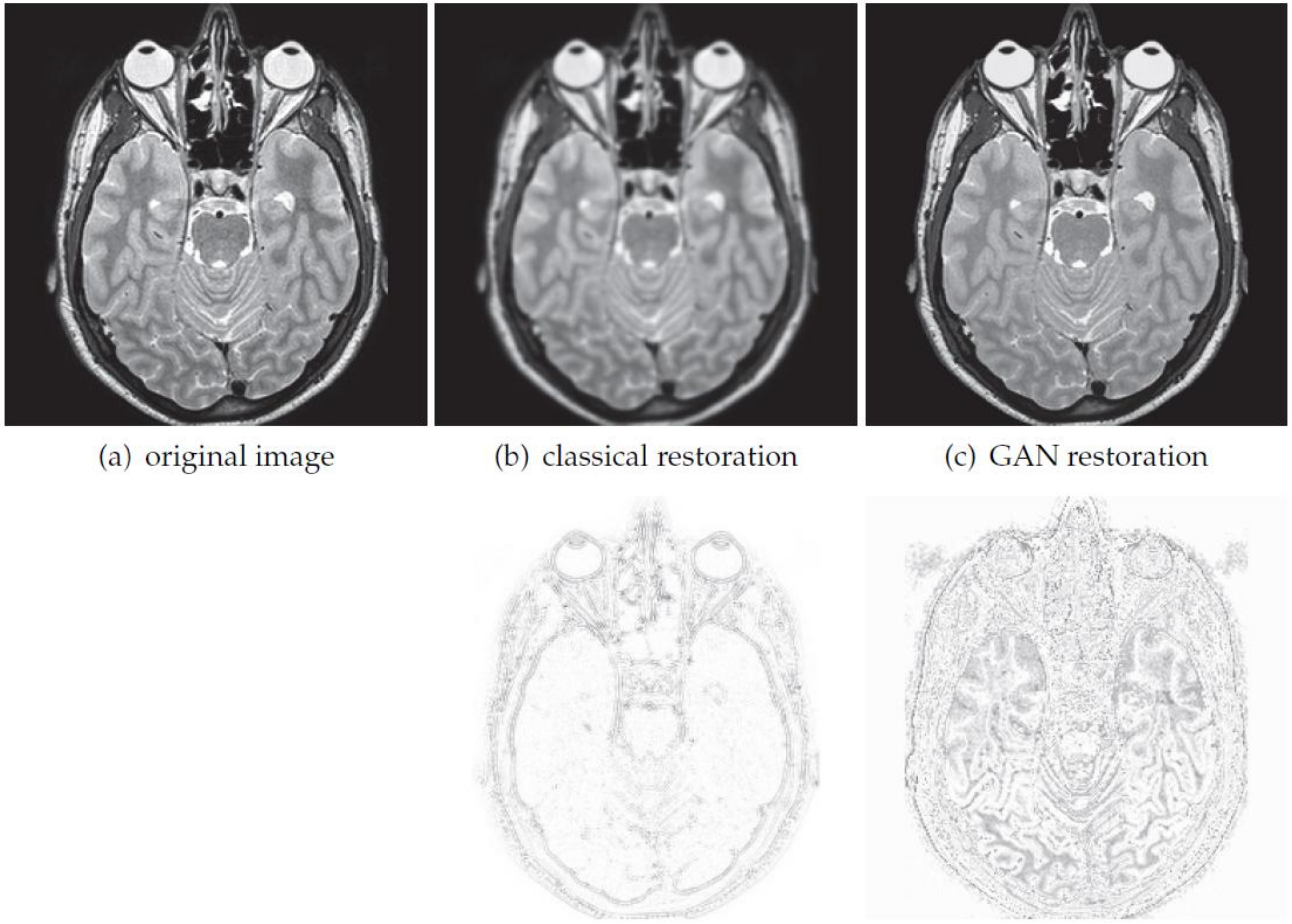
Classical mean filtering (r=3)

Adaptive mean filtering (m=10)

Adaptive mean filtering (m=50)

Application Example

- **Image Restoration of MR Brain**
  - Classical vs. GAN mean filtering



(a) original image (b) classical restoration (c) GAN restoration

(d) residue of the classical filtering (e) residue of the GAN filtering

Application Example

- **Image Restoration / Visual Quality**


(a) original image


(b) noisy image


(c) Bilateral filtering


(d) Wiener's filtering


(e) Total variation filtering

Gaussian noise

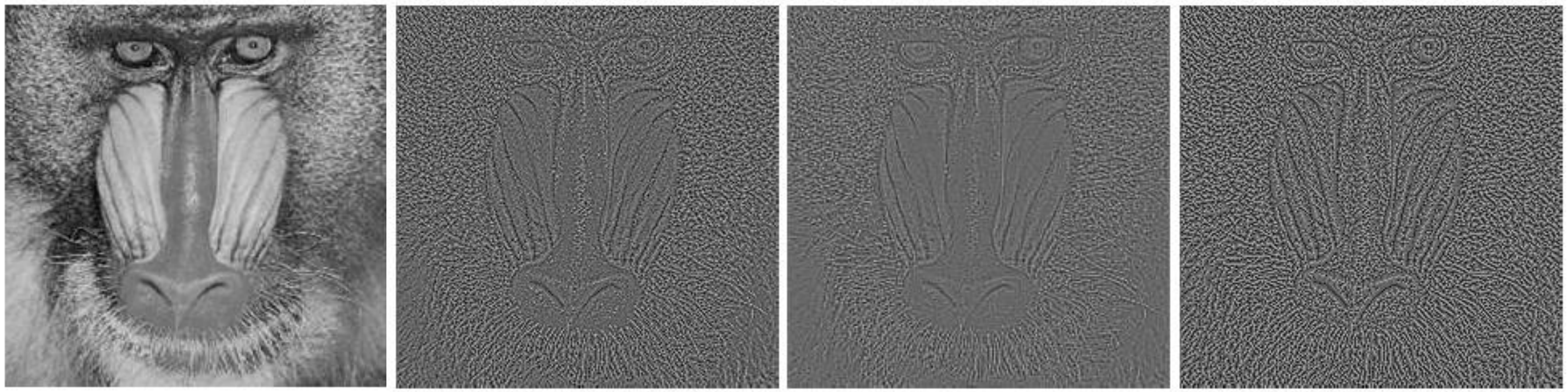

(f) Anisotropic diffusion filtering


(g) Mean filtering


(h) GAN-based mean filtering

Application Example

- Image Restoration / Residue Comparison



(a) original image (b) Bilateral filtering (c) Wiener's filtering (d) Total variation filtering

(e) Anisotropic diffusion filtering (f) Mean filtering (g) GAN-based mean filtering

Application Example

- **Image Restoration / Quantitative Comparison**

$$\mathrm{LNRMSE}_W = \sqrt{\frac{\sum_x (g(x) - f(x))^2}{\sum_x (h(x) - f(x))^2}}$$

- f: noise-free image
- g: restored image
- h: noisy image (Gaussian)

(a) original image (b) noisy image

Lena W_1/σ	10	20	30	40	50
Anisotropic diffusion	0.64	0.44	0.41	0.37	0.34
Bilateral	0.62	0.43	0.39	0.35	0.33
Classical mean	0.75	0.46	0.41	0.36	0.32
GAN-based mean	$\boxed{0.60}$	$\boxed{0.41}$	$\boxed{0.35}$	$\boxed{0.31}$	$\boxed{0.29}$
Total variation	0.61	0.43	0.41	0.38	0.39
Wiener	0.81	0.52	0.48	0.46	0.41
Lena W_2/σ	10	20	30	40	50
Anisotropic diffusion	0.50	0.41	0.39	0.37	0.35
Bilateral	0.50	0.39	0.42	0.37	0.35
Classical mean	0.69	0.43	0.44	0.38	0.33
GAN-based mean	0.49	$\boxed{0.33}$	$\boxed{0.34}$	$\boxed{0.31}$	$\boxed{0.28}$
Total variation	$\boxed{0.48}$	0.37	0.42	0.39	0.40
Wiener	0.70	0.52	0.50	0.47	0.41
Cameraman W_3/σ	10	20	30	40	50
Anisotropic diffusion	$\boxed{0.75}$	0.75	0.66	0.65	0.58
Bilateral	0.78	0.74	0.65	0.64	0.57
Classical mean	2.24	1.23	0.87	0.74	0.63
GAN-based mean	0.78	$\boxed{0.73}$	$\boxed{0.64}$	$\boxed{0.63}$	$\boxed{0.55}$
Total variation	0.77	0.75	0.65	0.64	0.57
Wiener	1.37	0.90	0.71	0.68	0.61
Cameraman W_4/σ	10	20	30	40	50
Anisotropic diffusion	0.38	0.32	0.34	0.30	0.33
Bilateral	0.44	0.36	0.39	0.40	0.41
Classical mean	1.05	0.61	0.50	0.44	0.44
GAN-based mean	$\boxed{0.37}$	$\boxed{0.28}$	$\boxed{0.26}$	$\boxed{0.25}$	$\boxed{0.31}$
Total variation	0.43	0.39	0.39	0.39	0.42
Wiener	0.63	0.55	0.49	0.48	0.51

Application Example

- **Image Restoration**

a. noise-free image f b. noisy image g c. $Med_1(g)$ d. $V\text{-}Med^{g}_{20}(g)$

Salt & pepper noise

- **The weak GANs are not adapted!**

Application Example

- **Image Restoration**
  - Definition of new GANs

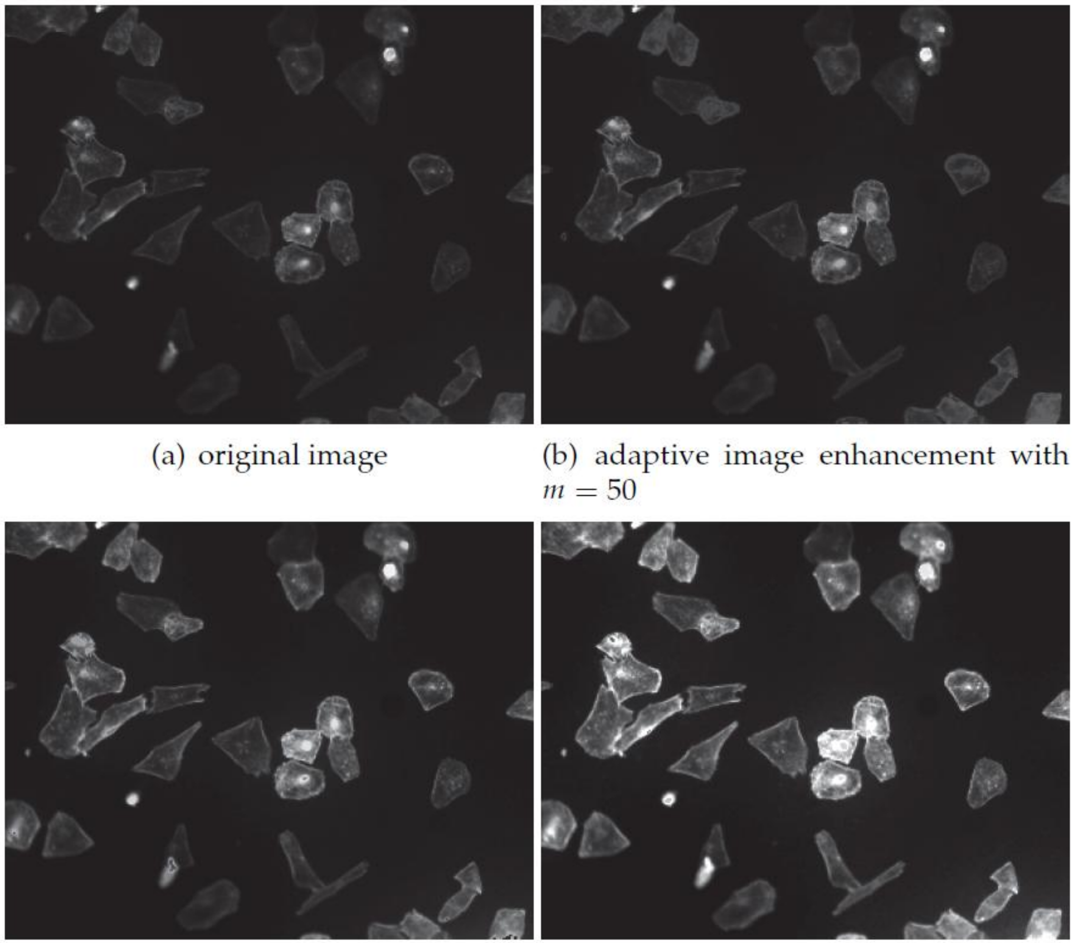
$$Z^f_{m_{\hexagon}}(x) = \begin{cases} \bigcup_{y \in B_1(x)} \{V^f_{m_{\hexagon}}(y)\} & \textit{if } A(V^f_{m_{\hexagon}}(x)) \leq \pi * q^2 \\ V^f_{m_{\hexagon}}(x) & \textit{otherwise} \end{cases}$$

b. noisy image g c. $Med_1(g)$

a. noise-free image f d. $V\text{-}Med^g_{20}(g)$ e. $Z\text{-}Med^g_{20}(g)$

Application Example

- **Image Enhancement of Osteoblast Cells**
  - Using the GAN max filtering

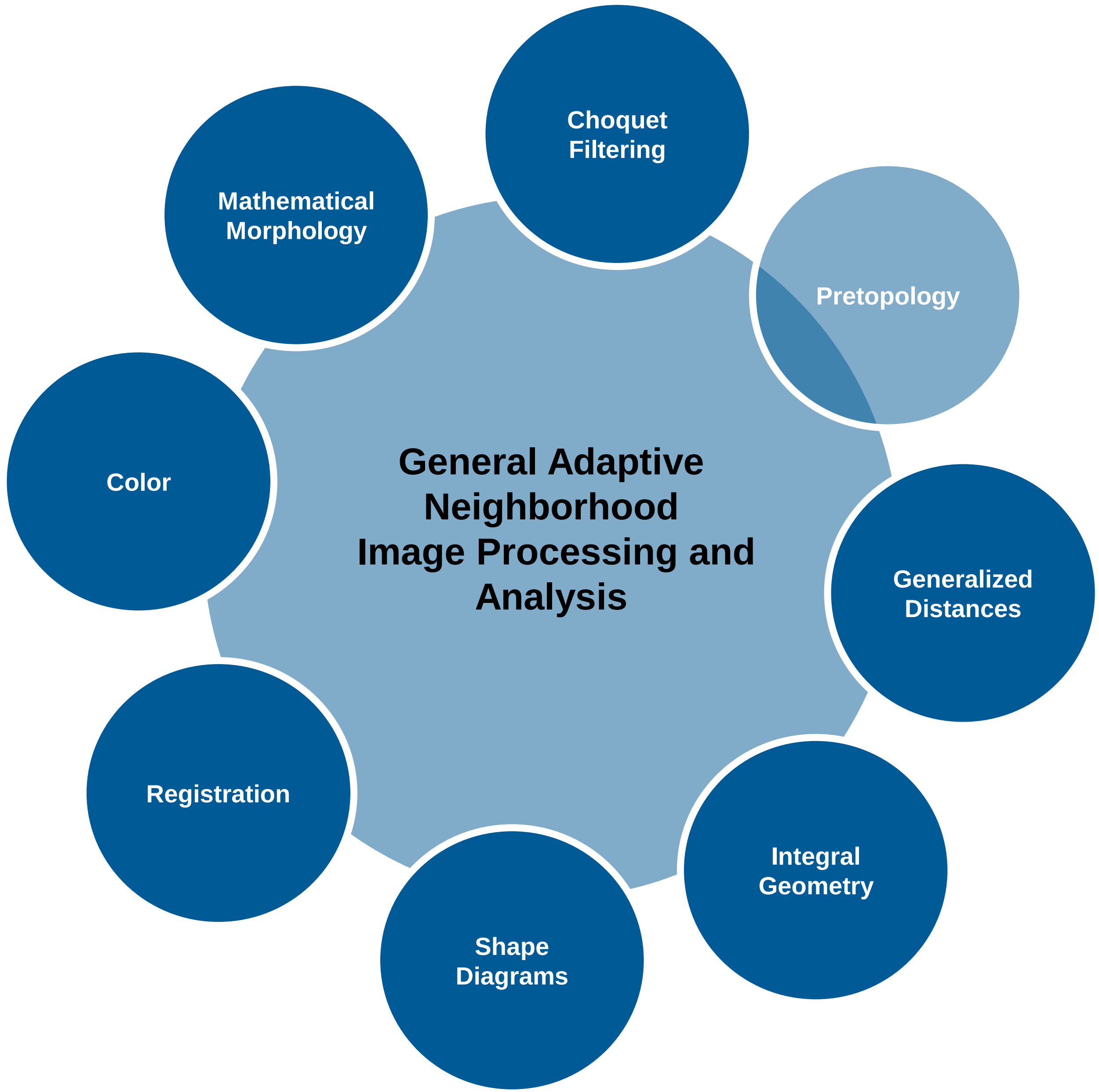


(a) original image

(b) adaptive image enhancement with $m = 50$

(c) adaptive image enhancement with $m = 100$

(d) adaptive image enhancement with $m = 150$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[JMIV 2011, JSTSP 2012]

J Math Imaging Vis (2011) 41:210–221
DOI 10.1007/s10851-011-0271-5

General Adaptive Neighborhood-Based Pretopological Image Filtering

Johan Debayle · Jean-Charles Pinoli

Published online: 10 March 2011

Abstract This paper introduces pretopological image filtering in the context of the General Adaptive Neighborhood Image Processing (GANIP) approach. Pretopological filters act on gray level image while satisfying some topological properties. The GANIP approach enables to get an image representation and mathematical structure for adaptive image processing and analysis. Then, the combination of pretopology and GANIP leads to efficient image operators. They enable to process images while preserving region structures without damaging image transitions. More precisely, GAN-based pretopological filters and GAN-based viscous pretopological filters are proposed in this paper. The viscous notion enables to adjust the filtering activity to the image gray levels. These adaptive filters are evaluated through several experiments highlighting their efficiency with respect to the classical operators. They are practically applied in both the biomedical and material application areas for image restoration, image background subtraction and image enhancement.

Keywords Adaptive image processing · General adaptive neighborhoods · Pretopology · Viscous filtering

1 Introduction

The framework entitled General Adaptive Neighborhood Image Processing (GANIP) has been introduced [11, 12] in order to propose an original image representation and mathematical structure for adaptive image processing and analysis. For each point of the image spatial support, a set of adaptive local neighborhoods (GANs) is introduced. They can be used as operational windows for setting up adaptive image filters such as proposed in [8, 9, 13, 32].

These specific neighborhoods are here studied from a topological (more precisely pretopological) point of view [5]. Indeed, they enable to define generalized topologies [5, 37, 40], i.e. based on less axioms than classical topologies [5, 6, 35]. For example, several pretopological structures (i.e. satisfying specific axioms) have been studied [23, 37, 40] and particularly applied for image processing [1, 2, 4, 24, 28, 29, 36]. Nevertheless, the resulting image transformations are generally not adaptive with respect to the spatial structures. Indeed, the pretopological adherency of a point is restricted to points within a fixed-shape and fixed-size analyzing (operational) window.

In this paper, pretopological structures are proposed in accordance with the GANIP framework. In this way, efficient adaptive image processing filters are defined, studied and compared to classical operators. More precisely, GAN-based pretopological filters and GAN-based viscous pretopological filters are proposed in this paper. The viscous notion [25, 44] enables to adjust the filtering activity to the image gray levels. The theoretical advantages of the proposed GAN-based pretopological (viscous) filters are practically highlighted on real application examples for image restoration, image background subtraction and image enhancement.

The paper first outlines the concept and definitions of generalized topologies (Sect. 2). Then, the GANIP approach is briefly exposed in Sect. 3. The definition and properties of the GAN-based pretopologies are proposed and studied in Sect. 4. Thus, GAN-based pretopological filters (Sect. 5)

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Spatially and Intensity Adaptive Morphology

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Abstract—**In this paper, spatially and intensity adaptive morphology is introduced and studied in the context of the General Adaptive Neighborhood Image Processing (GANIP) approach. The combination of GAN (General Adaptive Neighborhood)-based filtering and semi-flat morphology is particularly efficient in the sense that the filtering is adaptive to the image spatial structures (structuring elements are spatially variant) and its activity is controlled according to the image intensities (level sets are processed at different scales). The resulting morphological filters show a high image processing performance while preserving the image regions and details without damaging its transitions. The effectiveness of these adaptive operators are practically highlighted on real application examples for image background removing, image restoration and image enhancement.**

Index Terms—**Adaptive morphology, connected operators, general adaptive neighborhood image processing, generalized linear image processing, image filtering, semi-flat morphology, stack filtering.**

I. Introduction

In mathematical morphology [1], [2], several adaptive approaches have been proposed in the literature for taking into account the data information. Indeed, the usefulness and necessity of adaptive algorithms are evident if one considers the variability of images [3]: the intrinsic variability of the data from one image to another, the *a priori* knowledge (e.g. context or noise) that needs to be incorporated in the processing, and the requirements of the processing in terms of resulting image properties (for instance the preservation of certain image structures). Adaptive morphology can be classified in two main approaches: spatially adaptive morphology and intensity adaptive morphology. The first class defines morphological operators [3]–[13] where the size and/or shape of the structuring element depends on the local spatial structures present in the image. The second class proposes operators [14]–[18] that process the image level sets differently (i.e. according to their intensities).

The aim of this paper is to combine both the spatially adaptive and the intensity adaptive approaches. In this way, the General Adaptive Neighborhood Image Processing (GANIP) framework, introduced a few years ago [8], first enables to define spatially adaptive morphological operators. Indeed, in the GANIP approach, a set of General Adaptive Neighborhoods (GANs set) is identified around each point in the image to be analyzed. A GAN is a subset of the spatial support constituted by connected points whose measurement values, in relation to a selected criterion (such as luminance, contrast, thickness...), fit within a specified homogeneity tolerance. These GANs are used as adaptive structuring elements for defining spatially adaptive morphological operators. Thereafter, these operators can be extended by varying the homogeneity tolerance according to the image point intensities, resulting in spatially and intensity adaptive operators.

The paper is organized in the following way after this first introductory section. The second section callbacks the General Adaptive Neighborhood Image Processing (GANIP) framework. More particularly, the Section III is devoted to spatially adaptive morphology using the GANIP framework. Section IV introduces intensity adaptive morphology. Section V combines spatially and intensity adaptive morphology by defining and studying several GAN semi-flat morphological filters. In the last section, the proposed image transforms are successfully applied on real application examples for image background removing, image restoration and image enhancement.

II. General Adaptive Neighborhood Image Processing (GANIP)

A. General Adaptive Neighborhood (GAN) Paradigm

The General Adaptive Neighborhood paradigm has been introduced [19] in order to propose an original image representation for adaptive processing and analysis. The central idea is the notion of adaptivity which is simultaneously associated to the analyzing scales, the spatial structures and the intensity values of the image to be addressed.

In the so-called General Adaptive Neighborhood Image Processing (GANIP) approach [8], [20], a set of General Adaptive Neighborhoods (GANs set) is identified around each point in the image to be analyzed.

These GANs are used as adaptive windows for further image transformations or quantitative image analysis.

B. Generalized Linear Image Processing (GLIP)

The space of *image mappings*, defined on the spatial support $D \subseteq \mathbb{R}^2$ and valued in a real number interval $\tilde{E} \subseteq \mathbb{R}$, is represented in a Generalized Linear Image Processing (GLIP) framework [21], denoted $\mathcal{I}$. The GLIP framework $\mathcal{I}$ is then supplied with an ordered vectorial structure, using the formal vector addition $\oplus$, the formal scalar multiplication $\otimes$ and the classical partial order relation $\geq$ defined as:

$$\forall (f,g) \in \mathcal{I}^2 \quad f \geq g \Leftrightarrow (\forall x \in D, \quad f(x) \geq g(x)) \qquad (1)$$

The most frequently used framework is the *Classical Linear Image Processing* (CLIP) framework where the vector addition and the vector multiplication are the usual $+$ and $\times$ op-

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Generalized Topologies

- **Context: Image Topological Analysis**

- **Pseudoclosure Function**

$$\mathrm{clo}: \begin{cases} \mathcal{P}(D) & \rightarrow & \mathcal{P}(D) \\ A & \mapsto & \mathrm{clo}(A) \end{cases}$$

  - Properties

    - $\mathrm{clo}(\emptyset) = \emptyset$
    - $\forall A \in \mathcal{P}(D) \quad A \subseteq \mathrm{clo}(A)$

- **Pseudointerior Function**

$$\mathrm{int}: \begin{cases} \mathcal{P}(D) \rightarrow \mathcal{P}(D) \\ A \quad\quad \mapsto \mathrm{int}(A) = {}^c(\mathrm{clo}({}^cA)) \end{cases}$$

- **Axioms**

  - (K0) $\mathrm{clo}(\emptyset) = \emptyset$
  - (K1) $A \subseteq B \Rightarrow \mathrm{clo}(A) \subseteq \mathrm{clo}(B)$ (isotonic)
  - (K2) $A \subseteq \mathrm{clo}(A)$ (expanding)
  - (K3) $\mathrm{clo}(A \cup B) \subseteq \mathrm{clo}(A) \cup \mathrm{clo}(B)$ (sub-additive)
  - (K4) $\mathrm{clo}(\mathrm{clo}(A)) = \mathrm{clo}(A)$ (idempotent)

  - (K'0) $\exists A | x \notin \mathrm{clo}(A)$
  - (KA) $\mathrm{clo}(D) = D$
  - (KB) $A \cup B = D \Rightarrow \mathrm{clo}(A) \cup \mathrm{clo}(B) = D$

Generalized Topologies

- **Generalized Topological Axioms**

Generalized topologies	(K0): $\mathrm{clo}(\emptyset) = \emptyset$	(K1): $A \subseteq B \Rightarrow \mathrm{clo}(A) \subseteq \mathrm{clo}(B)$	(K2): $A \subseteq \mathrm{clo}(A)$	(K3): $\mathrm{clo}(A \cup B) \subseteq \mathrm{clo}(A) \cup \mathrm{clo}(B)$	(K4): $\mathrm{clo}(\mathrm{clo}(A)) = \mathrm{clo}(A)$
Extended topology	•	•			
Brissaud space	•		•		
Neighborhood space	•	•	•		
Closure space	•	•	•		•
Smyth space	•	•		•	
Pretopology	•	•	•	•	
Topology	•	•	•	•	•

GAN-based Pretopologies

- **GAN Pseudoclosure**

$$\mathrm{V}^h_{m_{\hexagon}} : \begin{cases} \mathcal{P}(D) \rightarrow \mathcal{P}(D) \\ A \qquad \mapsto \bigcup\limits_{z \in A} \{V^h_{m_{\hexagon}}(z)\} \end{cases}$$

$$\mathrm{N}^h_{m_{\hexagon}} : \begin{cases} \mathcal{P}(D) \rightarrow \mathcal{P}(D) \\ A \qquad \mapsto \bigcup\limits_{z \in A} \{N^h_{m_{\hexagon}}(z)\} \end{cases}$$

1. $\mathrm{V}^h_{m_{\hexagon}}(\emptyset) = \emptyset, \mathrm{N}^h_{m_{\hexagon}}(\emptyset) = \emptyset$
2. $\forall A \in \mathcal{P}(D)\ A \subseteq \mathrm{V}^h_{m_{\hexagon}}(A), A \subseteq \mathrm{N}^h_{m_{\hexagon}}(A)$

- **GAN Pseudointerior**

$$\breve{\mathrm{V}}^h_{m_{\hexagon}} : \begin{cases} \mathcal{P}(D) \rightarrow \mathcal{P}(D) \\ A \qquad \mapsto {}^c(\mathrm{V}^h_{m_{\hexagon}}({}^cA)) \end{cases}$$

$$\breve{\mathrm{N}}^h_{m_{\hexagon}} : \begin{cases} \mathcal{P}(D) \rightarrow \mathcal{P}(D) \\ A \qquad \mapsto {}^c(\mathrm{N}^h_{m_{\hexagon}}({}^cA)) \end{cases}$$

Properties

Axioms Systems

1. (K'0) : $\begin{cases} \exists A \mid x \notin \mathrm{V}^h_{m_{\hexagon}}(A) \\ \exists A \mid x \notin \mathrm{N}^h_{m_{\hexagon}}(A) \end{cases}$
2. (K0) : $\begin{cases} \mathrm{V}^h_{m_{\hexagon}}(\emptyset) = \emptyset \\ \mathrm{N}^h_{m_{\hexagon}}(\emptyset) = \emptyset \end{cases}$
3. (K1) : $A \subseteq B \Rightarrow \begin{cases} \mathrm{V}^h_{m_{\hexagon}}(A) \subseteq \mathrm{V}^h_{m_{\hexagon}}(B) \\ \mathrm{N}^h_{m_{\hexagon}}(A) \subseteq \mathrm{N}^h_{m_{\hexagon}}(B) \end{cases}$
4. (KA) : $\begin{cases} \mathrm{V}^h_{m_{\hexagon}}(D) = D \\ \mathrm{N}^h_{m_{\hexagon}}(D) = D \end{cases}$
5. (KB) : $A \cup B = D \Rightarrow \begin{cases} \mathrm{V}^h_{m_{\hexagon}}(A) \cup \mathrm{V}^h_{m_{\hexagon}}(B) = D \\ \mathrm{N}^h_{m_{\hexagon}}(A) \cup \mathrm{N}^h_{m_{\hexagon}}(B) = D \end{cases}$
6. (K2) : $\begin{cases} A \subseteq \mathrm{V}^h_{m_{\hexagon}}(A) \\ A \subseteq \mathrm{N}^h_{m_{\hexagon}}(A) \end{cases}$
7. (K3) : $\begin{cases} \mathrm{V}^h_{m_{\hexagon}}(A \cup B) \subseteq \mathrm{V}^h_{m_{\hexagon}}(A) \cup \mathrm{V}^h_{m_{\hexagon}}(B) \\ \mathrm{N}^h_{m_{\hexagon}}(A \cup B) \subseteq \mathrm{N}^h_{m_{\hexagon}}(A) \cup \mathrm{N}^h_{m_{\hexagon}}(B) \end{cases}$

Translation Invariance

$$c \in E^{\oplus} \Rightarrow \begin{cases} \mathrm{V}^{h \oplus c}_{m_{\hexagon}}(A) = \mathrm{V}^h_{m_{\hexagon}}(A) \\ \mathrm{N}^{h \oplus c}_{m_{\hexagon}}(A) = \mathrm{N}^h_{m_{\hexagon}}(A) \end{cases}$$

Multiplication Compatibility

$$\alpha \in \mathbb{R}^{+} \backslash \{0\} \Rightarrow \begin{cases} \mathrm{V}^{\alpha \otimes h}_{m_{\hexagon}}(A) = \mathrm{V}^h_{\frac{1}{\alpha} \otimes m_{\hexagon}}(A) \\ \mathrm{N}^{\alpha \otimes h}_{m_{\hexagon}}(A) = \mathrm{N}^h_{\frac{1}{\alpha} \otimes m_{\hexagon}}(A) \end{cases}$$

Space Comparison

$$\forall ({m_{\hexagon}}^1, {m_{\hexagon}}^2) \in E^{\oplus} \times E^{\oplus}$$

$${m_{\hexagon}}^1 \leq {m_{\hexagon}}^2 \Rightarrow \begin{cases} \mathrm{V}^h_{{m_{\hexagon}}^1}(A) \subseteq \mathrm{V}^h_{{m_{\hexagon}}^2}(A) \\ \mathrm{N}^h_{{m_{\hexagon}}^1}(A) \subseteq \mathrm{N}^h_{{m_{\hexagon}}^2}(A) \end{cases}$$

GAN-based Pretopologies

- **Properties**

$$\mathrm{V}^h_{m_{\hexagon}} : \begin{cases} \mathcal{P}(D) \to \mathcal{P}(D) \\ A \quad \mapsto \bigcup_{z \in A} \{V^h_{m_{\hexagon}}(z)\} \end{cases}$$

$$\mathrm{N}^h_{m_{\hexagon}} : \begin{cases} \mathcal{P}(D) \to \mathcal{P}(D) \\ A \quad \mapsto \bigcup_{z \in A} \{N^h_{m_{\hexagon}}(z)\} \end{cases}$$

Generalized topologies	(K0): $\mathrm{clo}(\emptyset) = \emptyset$	(K1): $A \subseteq B \Rightarrow \mathrm{clo}(A) \subseteq \mathrm{clo}(B)$	(K2): $A \subseteq \mathrm{clo}(A)$	(K3): $\mathrm{clo}(A \cup B) \subseteq \mathrm{clo}(A) \cup \mathrm{clo}(B)$	(K4): $\mathrm{clo}(\mathrm{clo}(A)) = \mathrm{clo}(A)$
Extended topology	•	•			
Brissaud space	•		•		
Neighborhood space	•	•	•		
Closure space	•	•	•		•
Smyth space	•	•		•	
Pretopology	•	•	•	•	
Topology	•	•	•	•	•

Stack Filtering

- **Level Sets: Image Decomposition**

$$X_t(f) = \{x \in D; f(x) \geq t\}$$

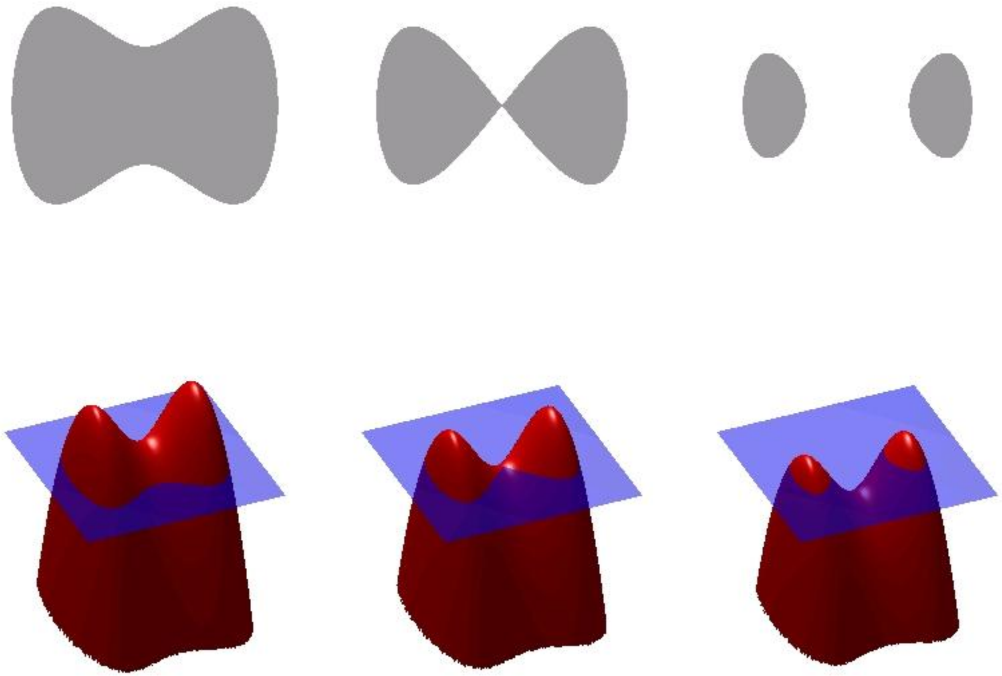
- **Image Reconstruction**

$$f = \bigvee_t t.\chi_t(f)$$

$$\chi_t(x) = \begin{cases} 1 \text{ if } x \in X_t(f) \\ 0 \text{ otherwise} \end{cases}$$

- **Stack Filter**
  - Operator acting on level sets

$$\Psi(f) = \bigvee_t t.\psi(\chi_t(f))$$



Example

- **Morphological Pseudoclosure and Pseudointerior**

$$\mathrm{Clo}_r(f) = \bigvee_t t\,.\,\mathrm{clo}_{B_r}(\chi_t(f))$$ (Classical dilation)

$$\mathrm{Int}_r(f) = \bigvee_t t\,.\,\mathrm{int}_{B_r}(\chi_t(f))$$ (Classical erosion)

(a) original image (b) classical pseudointerior filtering (c) classical pseudo-closure filtering

GAN Pretopological Filtering

- **GAN-based Stack Filtering**

$$\mathrm{CloV}^h_{m_{\hexagon}}(f) = \bigvee_t t.\,\mathrm{V}^h_{m_{\hexagon}}(\chi_t(f))$$

$$\mathrm{IntV}^h_{m_{\hexagon}}(f) = \bigvee_t t.\,\breve{\mathrm{V}}^h_{m_{\hexagon}}(\chi_t(f))$$

$$\boxed{\begin{aligned}\mathrm{CloN}^h_{m_{\hexagon}}(f) &= \bigvee_t t.\,\mathrm{N}^h_{m_{\hexagon}}(\chi_t(f)) \\ \mathrm{IntN}^h_{m_{\hexagon}}(f) &= \bigvee_t t.\,\breve{\mathrm{N}}^h_{m_{\hexagon}}(\chi_t(f))\end{aligned}}$$

GAN morphology

- **Properties**

$$f \leq \mathrm{CloV}^h_{m_{\hexagon}}(f) \leq \mathrm{CloN}^h_{m_{\hexagon}}(f)$$

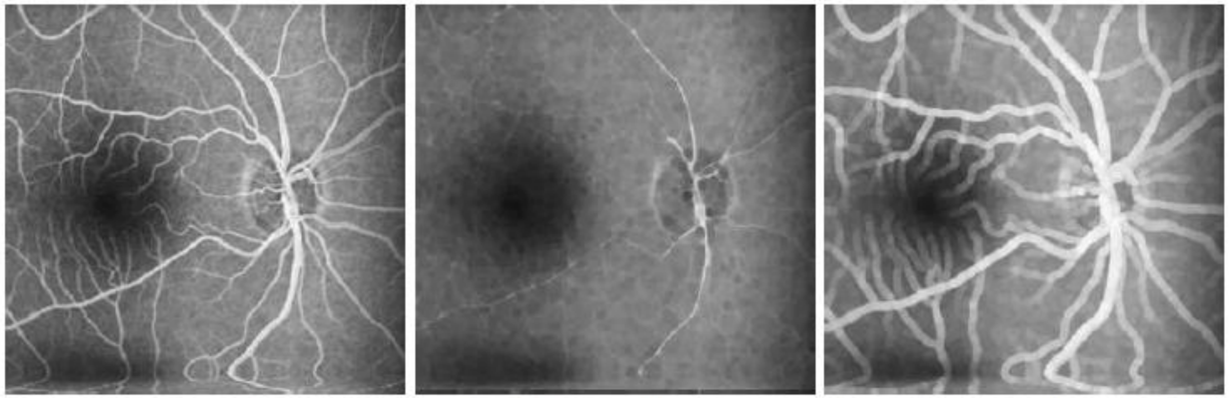
$$f \geq \mathrm{IntV}^h_{m_{\hexagon}}(f) \geq \mathrm{IntN}^h_{m_{\hexagon}}(f)$$

(a) original image (b) Adaptive weak pseudointerior filtering (c) Adaptive weak pseudoclosure filtering

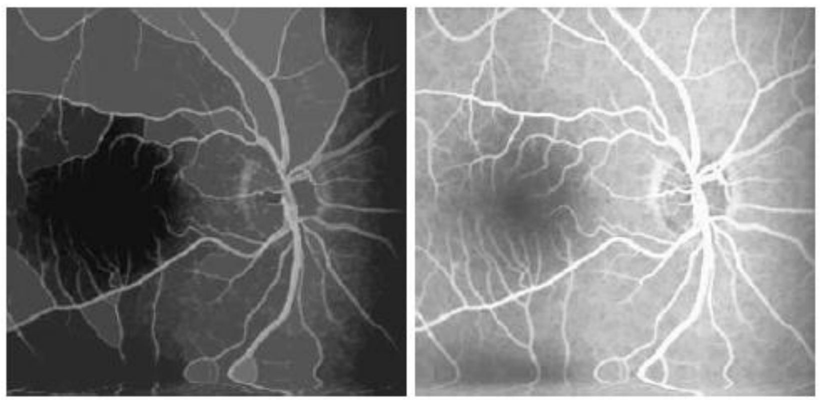
(d) Adaptive strong pseudointerior filtering (e) Adaptive strong pseudoclosure filterin

Illustration

- Classical vs. Adaptive Pretopological Filtering



(a) original image (b) Classical pseudointerior filtering (c) Classical pseudo-closure filtering



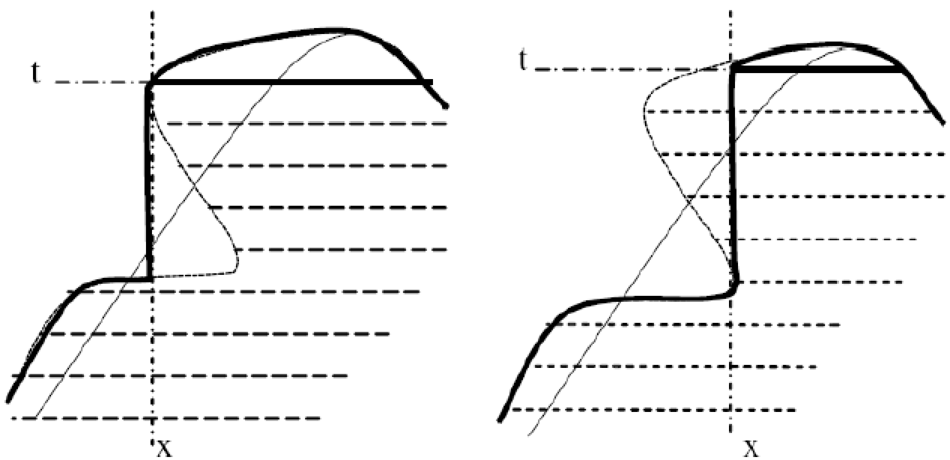
(d) Adaptive strong pseudointerior filtering (e) Adaptive strong pseudoclosure filtering

Viscous Stack Filtering

- **Definition**
  - Family of set operators

$$\Phi(f) = \bigvee_t t.\phi_t(\chi_t(f))$$

- **Image Reconstruction**
  - Decreasing family: OK
  - General family: upper or lower envelope



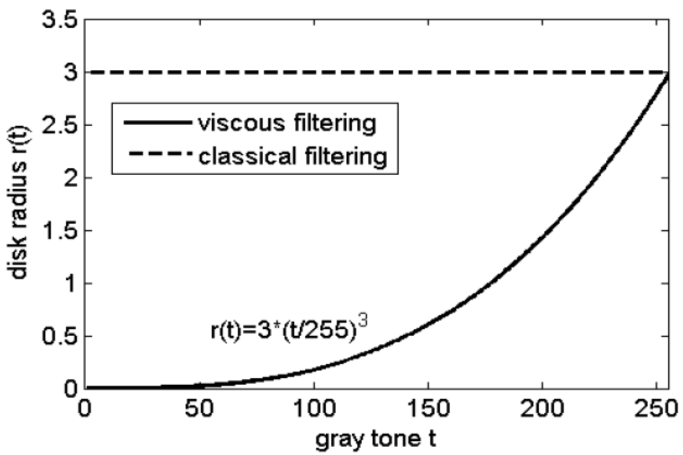
(a) upper envelope (b) lower envelope

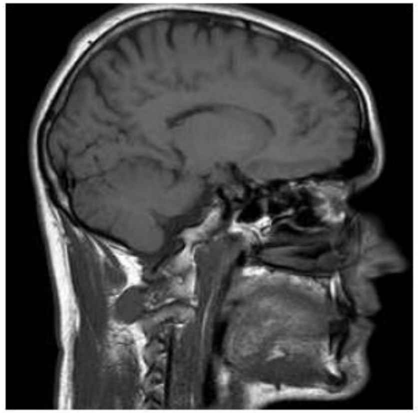
Viscous Pretopological Filtering

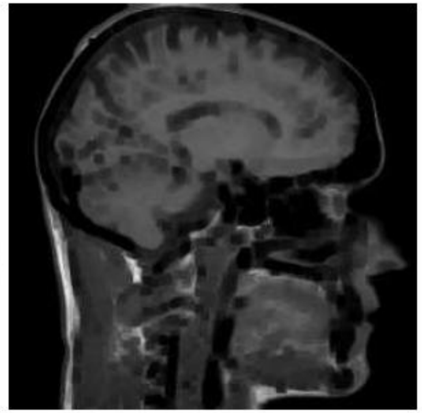
- **Illustration**

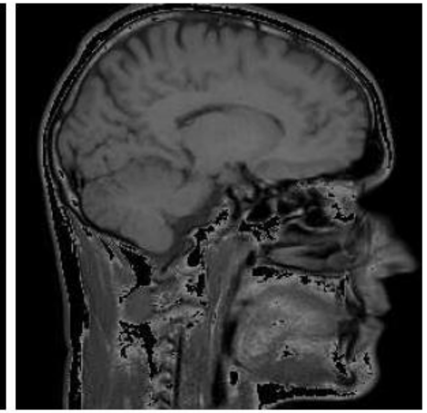
$$\mathrm{VisClo}_{\{r\}}(f) = \bigvee_t t.\,\mathrm{clo}_{B_{r(t)}}(\chi_t(f))$$

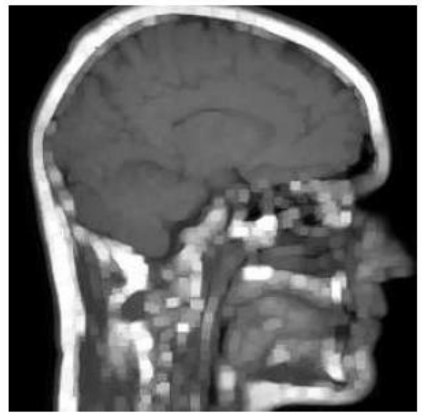
$$\mathrm{ViscInt}_{\{r\}}(f) = \bigvee_t t.\,\mathrm{int}_{B_{r(t)}}\,\chi_t(f))$$

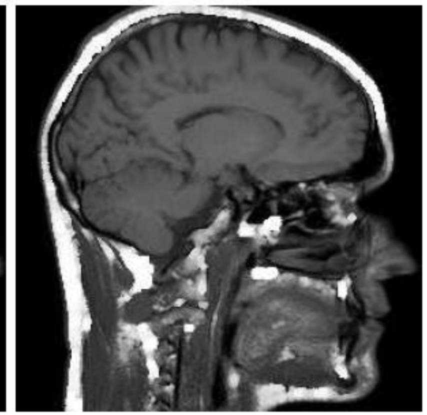



(a) original image


(b) classical pseudointerior


(c) viscous pseudointerior


(d) classical pseudo-closure


(e) viscous pseudoclosure

GAN Viscous Pretopological Filtering

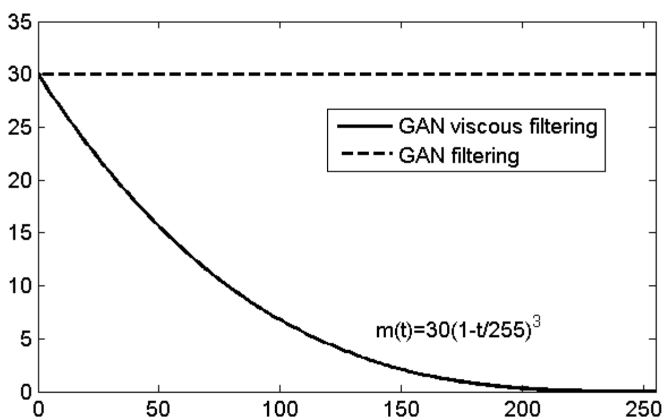
- **GANs: Varying homogeneity tolerance**

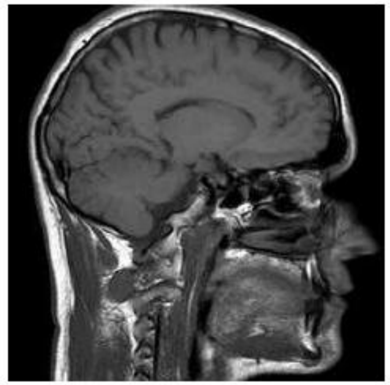
$$\mathrm{ViscCloV}^{h}_{\{m_{\hexagon}\}}(f) = \bigvee_{t} t.\, \mathrm{V}^{h}_{m_{\hexagon}(t)}(\chi_t(f))$$

$$\mathrm{ViscIntV}^{h}_{\{m_{\hexagon}\}}(f) = \bigvee_{t} t.\, \breve{\mathrm{V}}^{h}_{m_{\hexagon}(t)}(\chi_t(f))$$

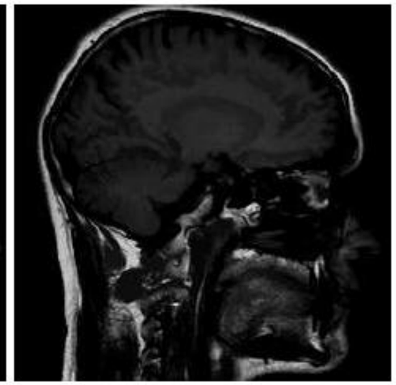
$$\mathrm{ViscCloN}^{h}_{\{m_{\hexagon}\}}(f) = \bigvee_{t} t.\, \mathrm{N}^{h}_{m_{\hexagon}(t)}(\chi_t(f))$$

$$\mathrm{ViscIntN}^{h}_{\{m_{\hexagon}\})}(f) = \bigvee_{t} t.\, \breve{\mathrm{N}}^{h}_{m_{\hexagon}(t)}(\chi_t(f))$$

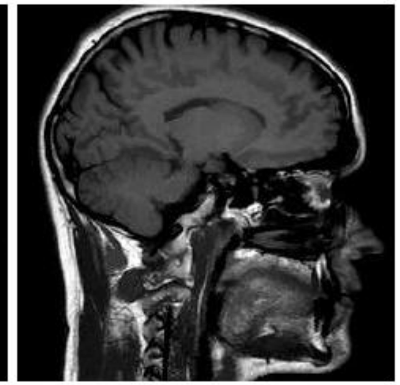




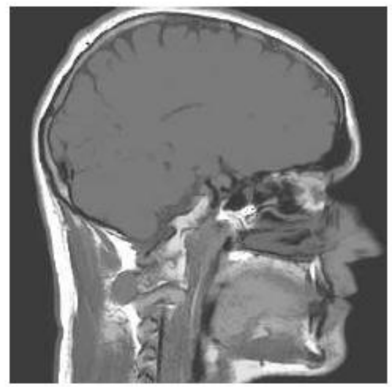
(a) original image



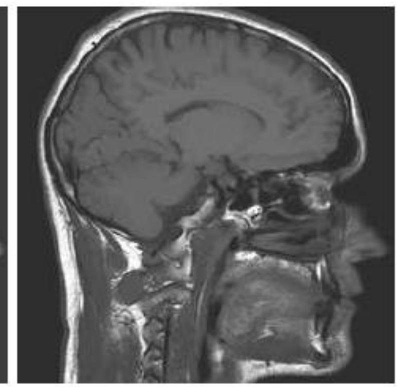
(b) GAN pseudointerior



(c) GAN viscous pseudointerior



(d) GAN pseudointerior



(e) GAN viscous pseudoclosure

GAN Viscous Morphological Filtering

- **GAN Viscous Dilation and Erosion**

$$\mathrm{D}_{\{m_{\hexagon}\}}^{h(f_0)}(f) = \bigvee_t t\mathrm{D}_{m_{\hexagon}(t)}^{h(f_0)}(\chi_t(f))$$

Upper enveloppe

$$\mathrm{E}_{\{m_{\hexagon}\}}^{h(f_0)}(f) = \bigvee_t t.\bigwedge_{s \leq t} \mathrm{E}_{m_{\hexagon}(s)}^{h(f_0)}(\chi_s(f))$$

Lower enveloppe

- **Morphological Duality!**

- **GAN Viscous Opening and Closing**

$$\begin{aligned} \mathrm{O}_{\{m_{\hexagon}\}}^{h(f_0)}(f) &= \mathrm{D}_{\{m_{\hexagon}\}}^{h(f_0)} \circ \mathrm{E}_{\{m_{\hexagon}\}}^{h(f_0)}(f) \\ \mathrm{C}_{\{m_{\hexagon}\}}^{h(f_0)}(f) &= \mathrm{E}_{\{m_{\hexagon}\}}^{h(f_0)} \circ \mathrm{D}_{\{m_{\hexagon}\}}^{h(f_0)}(f) \end{aligned}$$

Properties

- **Adjunction (Morphological Duality)**

$$\mathrm{D}_{\{m_{\hexagon}\}}^{h(f_0)}(f_1) \leq f_2 \Leftrightarrow f_1 \leq \mathrm{E}_{\{m_{\hexagon}\}}^{h(f_0)}(f_2)$$

- **Extensiveness, Anti-Extensiveness**

$$\mathrm{O}_{\{m_{\hexagon}\}}^{h(f_0)}(f) \leq f \leq \mathrm{C}_{\{m_{\hexagon}\}}^{h(f_0)}(f)$$

- **Increasing**

$$f_1 \leq f_2 \Rightarrow \begin{cases} \mathrm{D}_{\{m_{\hexagon}\}}^{h(f_0)}(f_1) \leq \mathrm{D}_{\{m_{\hexagon}\}}^{h(f_0)}(f_2) \\ \mathrm{E}_{\{m_{\hexagon}\}}^{h(f_0)}(f_1) \leq \mathrm{E}_{\{m_{\hexagon}\}}^{h(f_0)}(f_2) \\ \mathrm{C}_{\{m_{\hexagon}\}}^{h(f_0)}(f_1) \leq \mathrm{C}_{\{m_{\hexagon}\}}^{h(f_0)}(f_2) \\ \mathrm{O}_{\{m_{\hexagon}\}}^{h(f_0)}(f_1) \leq \mathrm{O}_{\{m_{\hexagon}\}}^{h(f_0)}(f_2) \end{cases}$$

- **Distributivity**

$$\begin{cases} \bigvee_{i \in I}[\mathrm{D}_{\{m_{\hexagon}\}}^{h(f_0)}(f_i)] = \mathrm{D}_{\{m_{\hexagon}\}}^{h(f_0)}(\bigvee_{i \in I}[f_i]) \\ \bigwedge_{i \in I}[\mathrm{E}_{\{m_{\hexagon}\}}^{h(f_0)}(f_i)] = \mathrm{E}_{\{m_{\hexagon}\}}^{h(f_0)}(\bigwedge_{i \in I}[f_i]) \end{cases}$$

- **Duality with Respect to Opposite**

$$\begin{cases} \tilde{\ominus}\, \mathrm{D}_{\{m_{\hexagon}\}}^{h(f_0)}(f) = \mathrm{E}_{\{m_{\hexagon}\}}^{h(f_0)}(\tilde{\ominus}\, f) \\ \tilde{\ominus}\, \mathrm{C}_{\{m_{\hexagon}\}}^{h(f_0)}(f) = \mathrm{O}_{\{m_{\hexagon}\}}^{h(f_0)}(\tilde{\ominus}\, f) \end{cases}$$

- **Idempotence**

$$\begin{cases} \mathrm{C}_{\{m_{\hexagon}\}}^{h(f_0)}(\mathrm{C}_{\{m_{\hexagon}\}}^{h(f_0)}(f)) = \mathrm{C}_{\{m_{\hexagon}\}}^{h(f_0)}(f) \\ \mathrm{O}_{\{m_{\hexagon}\}}^{h(f_0)}(\mathrm{O}_{\{m_{\hexagon}\}}^{h(f_0)}(f)) = \mathrm{O}_{\{m_{\hexagon}\}}^{h(f_0)}(f) \end{cases}$$

Properties

- **Increasing, Decreasing**

$$\forall t, \quad m^1_{\hexagon}(t) \leq m^2_{\hexagon}(t) \Rightarrow$$

$$\begin{cases} \mathrm{D}^{h(f_0)}_{\{m^1_{\hexagon}\}}(f) \leq \mathrm{D}^{h(f_0)}_{\{m^2_{\hexagon}\}}(f) \\ \mathrm{E}^{h(f_0)}_{\{m^1_{\hexagon}\}}(f) \geq \mathrm{E}^{h(f_0)}_{\{m^2_{\hexagon}\}}(f) \end{cases}$$

- **Translation Invariance**

$$c \in \tilde{E} \Rightarrow \begin{cases} \mathrm{D}^{h(f_0) \oplus c}_{\{m_{\hexagon}\}}(f) = \mathrm{D}^{h(f_0)}_{\{m_{\hexagon}\}}(f) \\ \mathrm{E}^{h(f_0) \oplus c}_{\{m_{\hexagon}\}}(f) = \mathrm{E}^{h(f_0)}_{\{m_{\hexagon}\}}(f) \\ \mathrm{C}^{h(f_0) \oplus c}_{\{m_{\hexagon}\}}(f) = \mathrm{C}^{h(f_0)}_{\{m_{\hexagon}\}}(f) \\ \mathrm{O}^{h(f_0) \oplus c}_{\{m_{\hexagon}\}}(f) = \mathrm{O}^{h(f_0)}_{\{m_{\hexagon}\}}(f) \end{cases}$$

- **Multiplication Compatibility**

$$\alpha \in \mathbb{R}^+ \backslash \{0\} \Rightarrow$$

$$\begin{cases} \mathrm{D}^{\alpha \otimes h(f_0)}_{\{m_{\hexagon}\}}(f) = \mathrm{D}^{h(f_0)}_{\{\frac{1}{\alpha} \otimes m_{\hexagon}\}}(f) \\ \mathrm{E}^{\alpha \otimes h(f_0)}_{\{m_{\hexagon}\}}(f) = \mathrm{E}^{h(f_0)}_{\{\frac{1}{\alpha} \otimes m_{\hexagon}\}}(f) \\ \mathrm{C}^{\alpha \otimes h(f_0)}_{\{m_{\hexagon}\}}(f) = \mathrm{C}^{h(f_0)}_{\{\frac{1}{\alpha} \otimes m_{\hexagon}\}}(f) \\ \mathrm{O}^{\alpha \otimes h(f_0)}_{\{m_{\hexagon}\}}(f) = \mathrm{O}^{h(f_0)}_{\{\frac{1}{\alpha} \otimes m_{\hexagon}\}}(f) \end{cases}$$

Properties

- **Translation Commutativity**

$$c \in E \Rightarrow$$

$$\begin{cases} \mathrm{D}_{\{m_{\hexagon}\}}^{h(f_0)}(f \,\tilde{\oplus}\, c) = \mathrm{D}_{\{m_{\hexagon}\}}^{h(f_0)}(f) \,\tilde{\oplus}\, c \\ \mathrm{E}_{\{m_{\hexagon}\}}^{h(f_0)}(f \,\tilde{\oplus}\, c) = \mathrm{E}_{\{m_{\hexagon}\}}^{h(f_0)}(f) \,\tilde{\oplus}\, c \\ \mathrm{C}_{\{m_{\hexagon}\}}^{h(f_0)}(f \,\tilde{\oplus}\, c) = \mathrm{C}_{\{m_{\hexagon}\}}^{h(f_0)}(f) \,\tilde{\oplus}\, c \\ \mathrm{O}_{\{m_{\hexagon}\}}^{h(f_0)}(f \,\tilde{\oplus}\, c) = \mathrm{O}_{\{m_{\hexagon}\}}^{h(f_0)}(f) \,\tilde{\oplus}\, c \end{cases}$$

- **Multiplication Commutativity**

$$\alpha \in \mathbb{R} \Rightarrow$$

$$\begin{cases} \mathrm{D}_{\{m_{\hexagon}\}}^{h(f_0)}(\alpha \,\tilde{\otimes}\, f) = \alpha \,\tilde{\otimes}\, \mathrm{D}_{\{m_{\hexagon}\}}^{h(f_0)}(f) \\ \mathrm{E}_{\{m_{\hexagon}\}}^{h(f_0)}(\alpha \,\tilde{\otimes}\, f) = \alpha \,\tilde{\otimes}\, \mathrm{E}_{\{m_{\hexagon}\}}^{h(f_0)}(f) \\ \mathrm{C}_{\{m_{\hexagon}\}}^{h(f_0)}(\alpha \,\tilde{\otimes}\, f) = \alpha \,\tilde{\otimes}\, \mathrm{C}_{\{m_{\hexagon}\}}^{h(f_0)}(f) \\ \mathrm{O}_{\{m_{\hexagon}\}}^{h(f_0)}(\alpha \,\tilde{\otimes}\, f) = \alpha \,\tilde{\otimes}\, \mathrm{O}_{\{m_{\hexagon}\}}^{h(f_0)}(f) \end{cases}$$

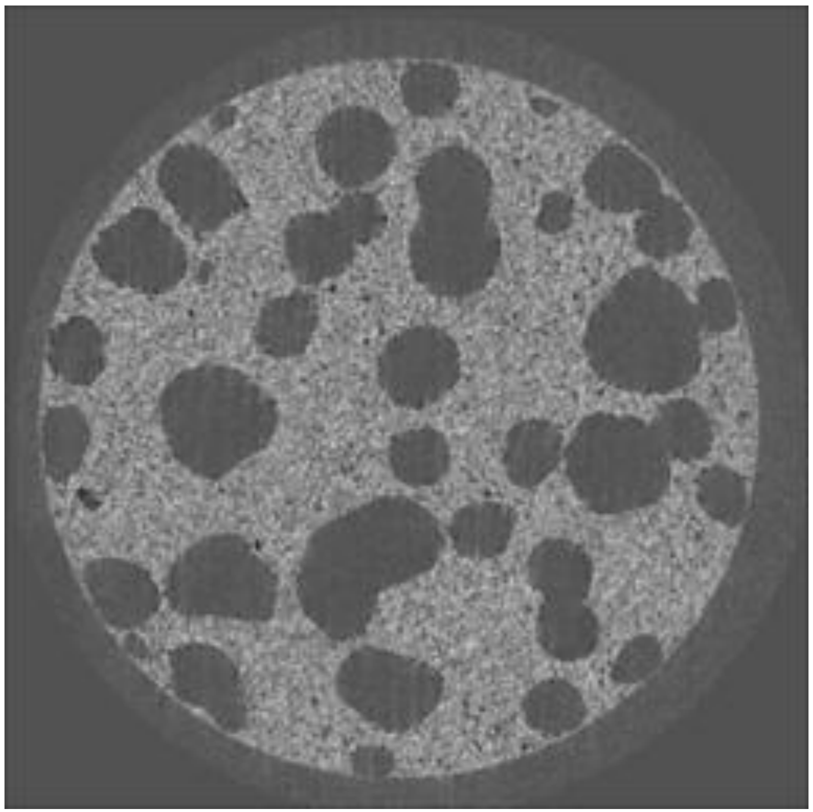
- **Connectivity**

$$\begin{pmatrix} \mathcal{I} = \mathcal{C} \\ f \in \mathcal{I} \end{pmatrix} \Rightarrow$$

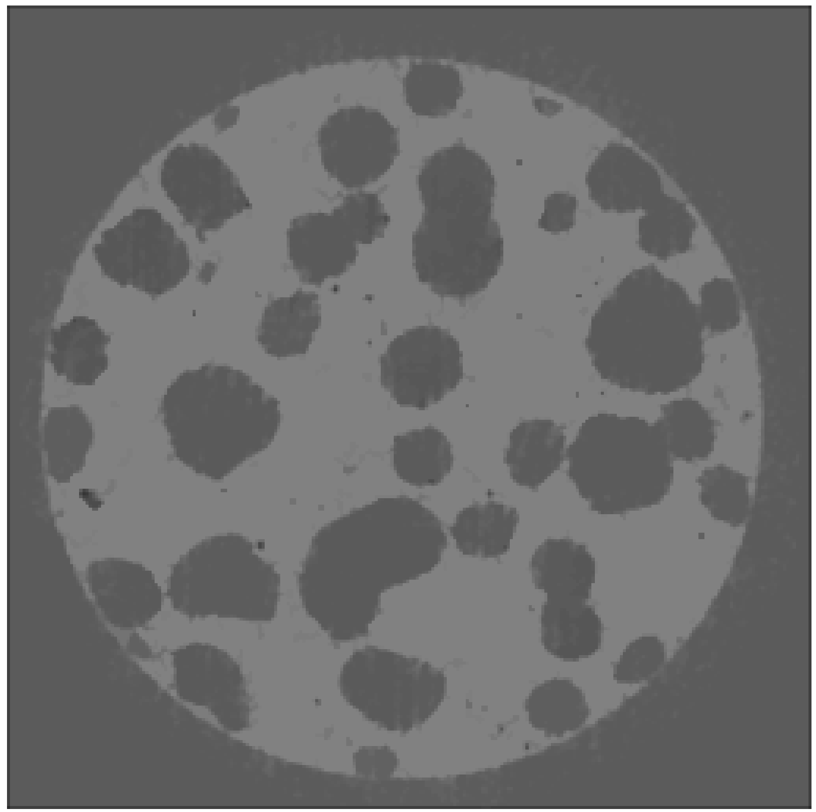
$$\begin{cases} f \mapsto \mathrm{D}_{\{m_{\hexagon}\}}^{f}(f) \\ f \mapsto \mathrm{E}_{\{m_{\hexagon}\}}^{f}(f) \\ f \mapsto \mathrm{C}_{\{m_{\hexagon}\}}^{f}(f) \\ f \mapsto \mathrm{O}_{\{m_{\hexagon}\}}^{f}(f) \end{cases} \text{are connected operators}$$

Application Example

- **Image Restoration of Cement-based Materials**
  - GAN viscous closing-opening: $m(t) = 120 * (t/256)^3$



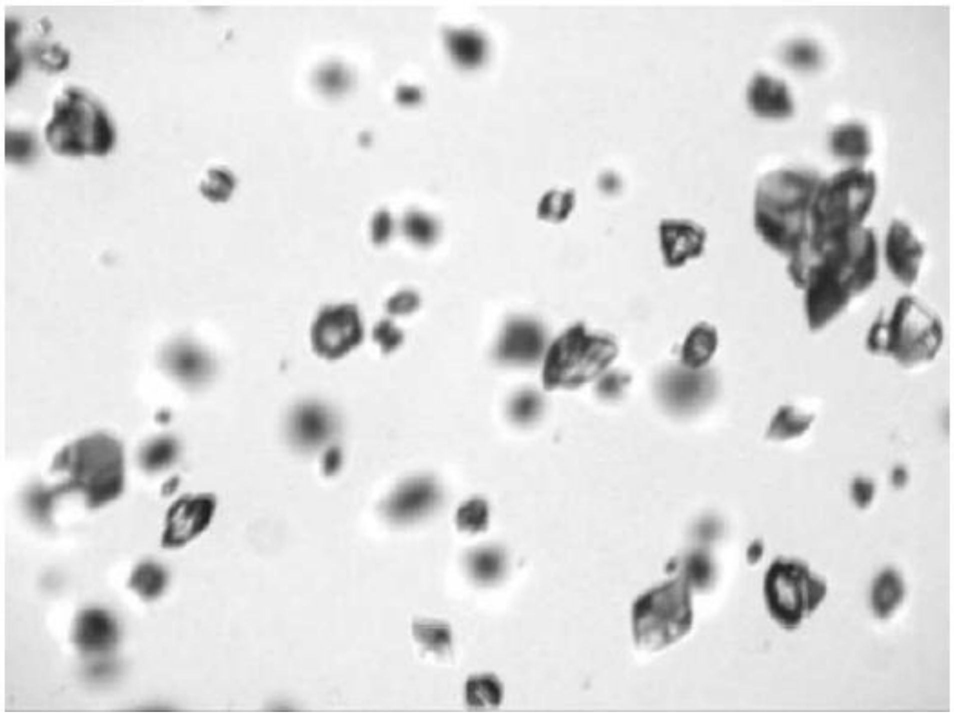
Original image



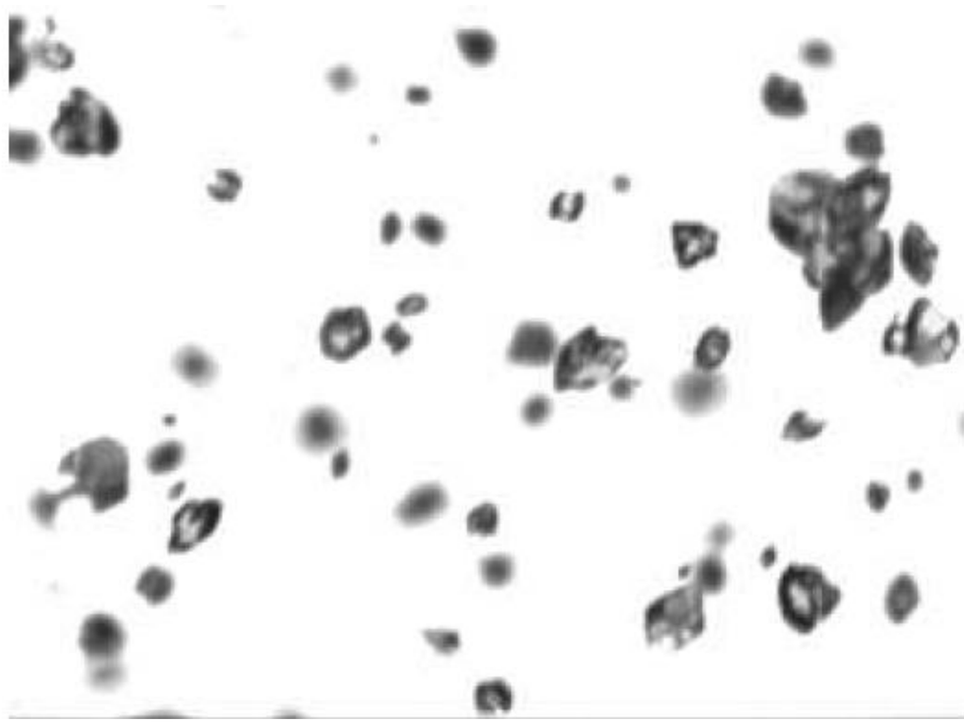
Enhanced image

Application Example

- **Image Background Subtraction of Crystals**
  - GAN viscous dilation: $t \mapsto m(t) = 30 * (t/255)^3$



Original image

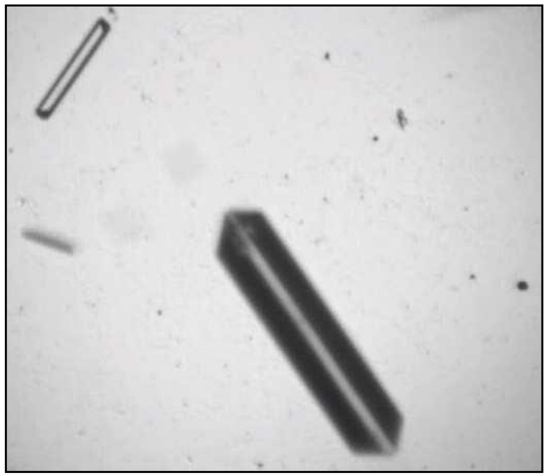


Enhanced image

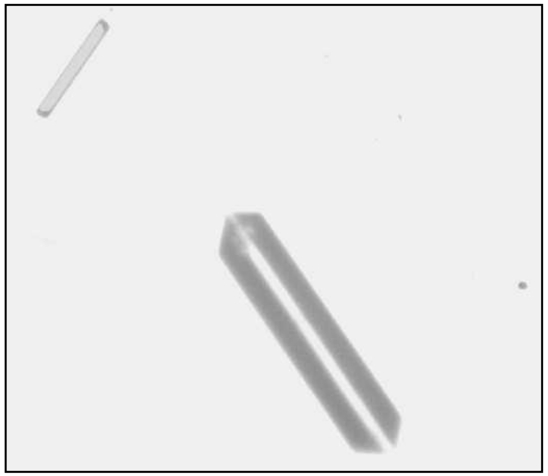
Application Example

○ Image Background Subtraction of Crystals

- GAN viscous dilation

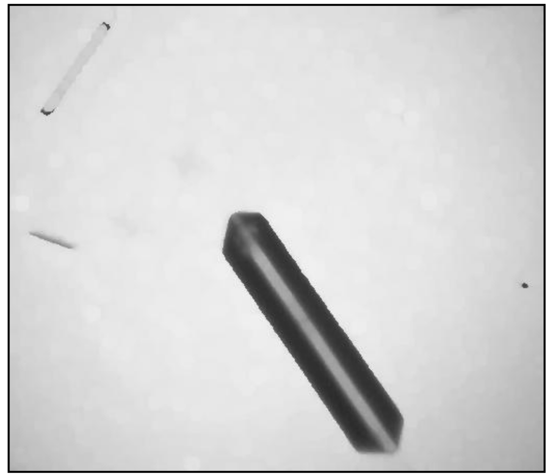


Original image



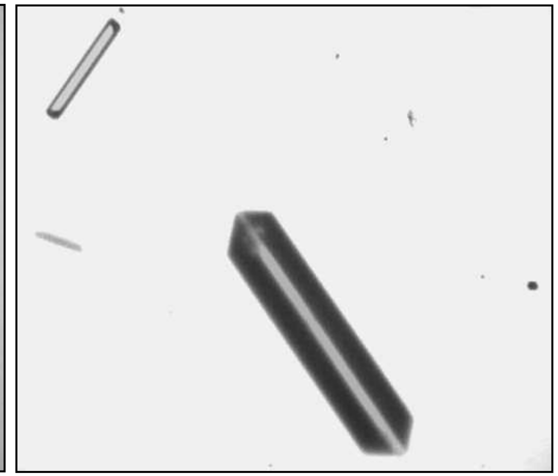
Classical viscous filtering

$r(t) = 10 * (t/256)^3$



GAN filtering

$m = 50$



GAN viscous filtering

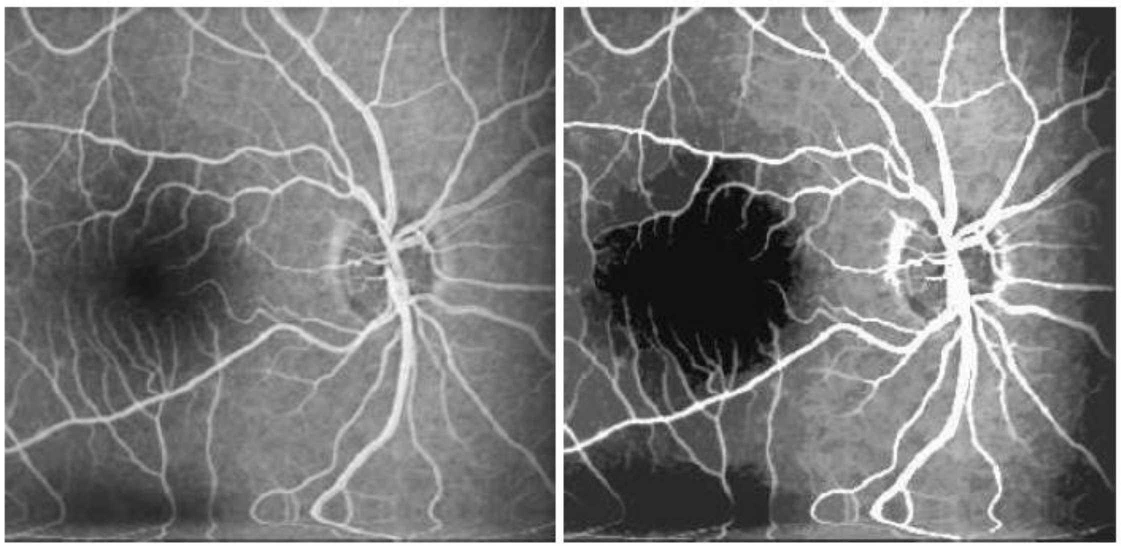
$m(t) = 50 * (t/256)^3$

Application Example

- **Image Enhancement of Retinal Vessels**
  - GAN viscous toggle filtering: $t \mapsto m(t) = 90 * (t/255)^5$

$$\kappa^h_{\{m_\hexagon\}}(f)(x) = \begin{cases} \mathrm{VisCloV}^h_{\{m_\hexagon\}}(f)(x) \text{ if} \\ \quad \mathrm{VisCloV}^h_{\{m_\hexagon\}}(f)(x) - f(x) \\ \quad < f(x) - \mathrm{VisIntV}^h_{\{\breve{m}_\hexagon\}}(f)(x) \\ \mathrm{VisIntV}^h_{\{\breve{m}_\hexagon\}}(f)(x) \text{ otherwise} \end{cases}$$

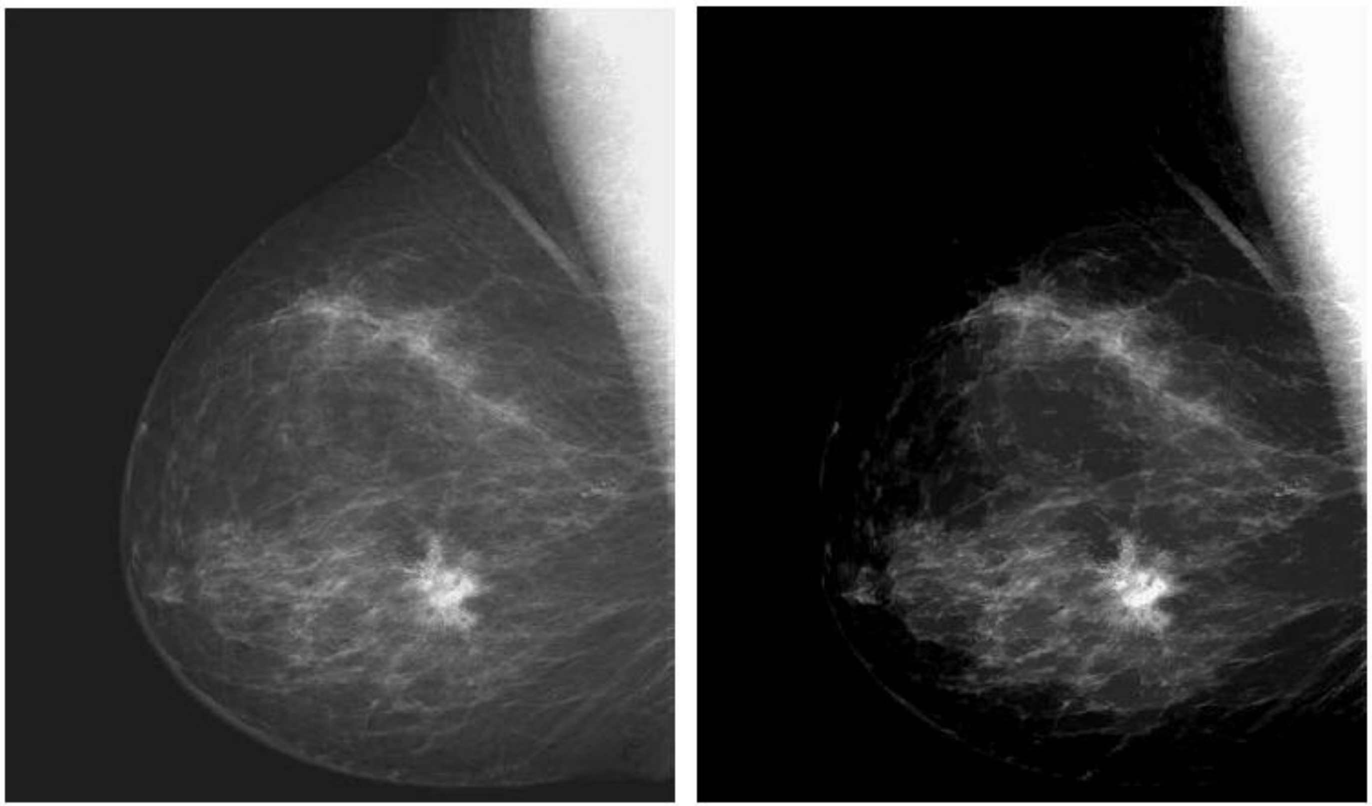
where $\breve{m}_\hexagon(t) = m_\hexagon(M - t)$.



(a) original image (b) GAN viscous pseudoclosure

Application Example

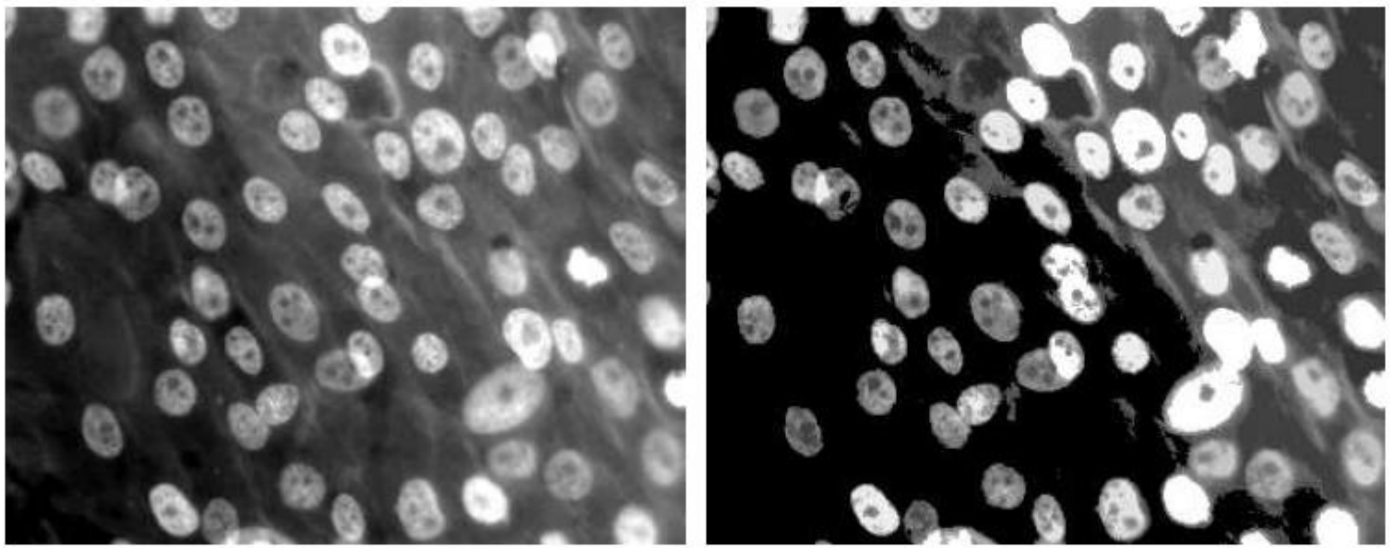
- **Image Enhancement of Breast**
  - GAN viscous toggle filtering: $t \mapsto m(t) = 90 * (t/255)^5$



(c) original image (breast) (d) enhanced image

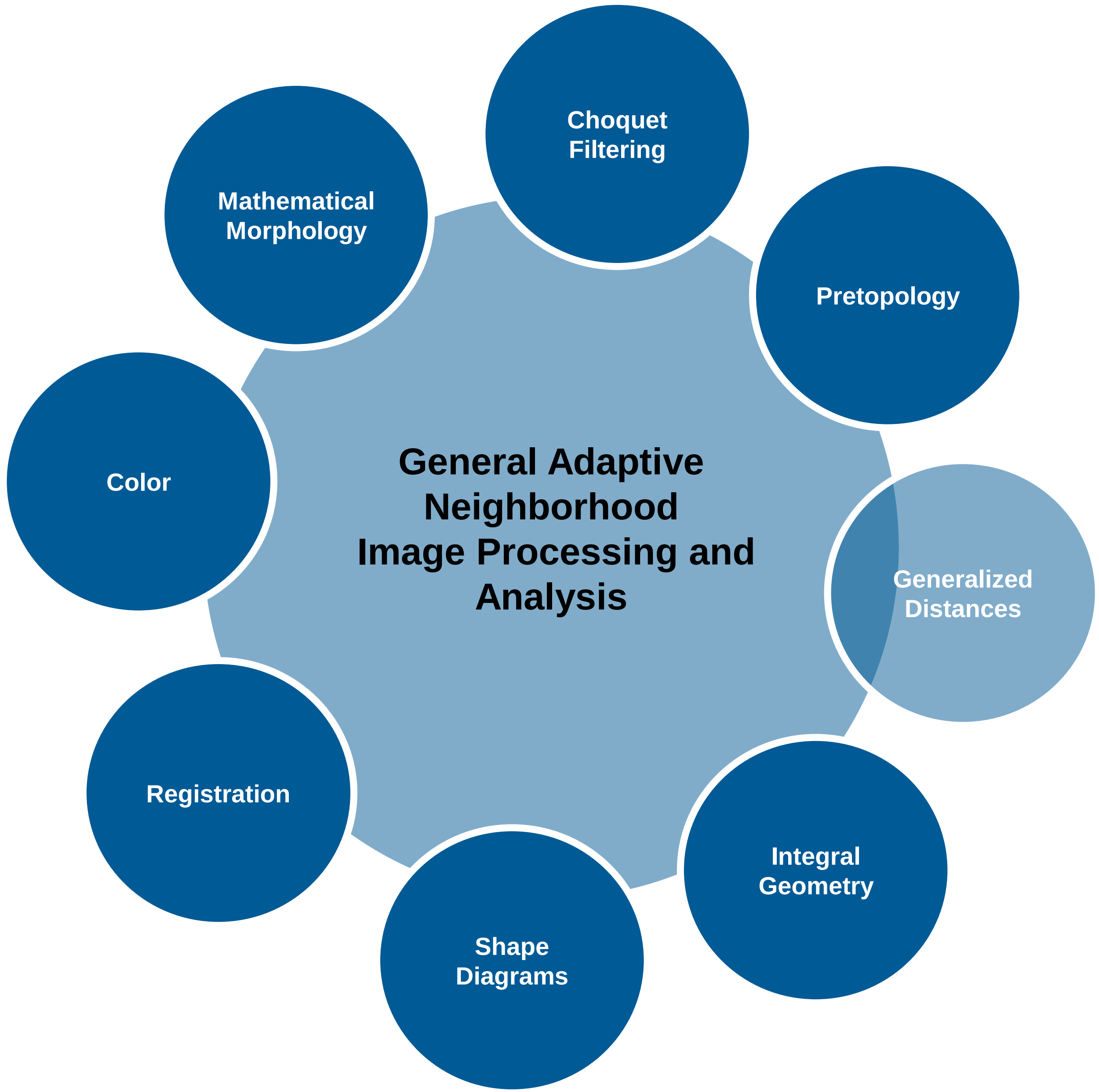
Application Example

- **Image Enhancement of Osteoblast Cells**
  - GAN viscous toggle filtering: $t \mapsto m(t) = 90 * (t/255)^5$



(e) original image (osteoblast cells)

(f) enhanced image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[ICIP 2011, **PR 2012**]

2011 18th IEEE International Conference on Image Processing

GENERAL ADAPTIVE DISTANCE TRANSFORMS ON GRAY TONE IMAGES: APPLICATION TO IMAGE SEGMENTATION.

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ABSTRACT

This paper aims to introduce and study two novel generalized metrics on gray tone images. These generalized metrics are based on the General Adaptive Neighborhood Image Processing (GANIP) framework that enables to represent an image by spatial neighborhoods, named General Adaptive Neighborhoods (GAN) that fit to their local context. These metrics are generalized in the sense that they do not satisfy all the axioms of a standard mathematical metric. This notion of adaptive generalized metrics leads to the definition of relevant GAN distance maps and GAN nearest neighbor transforms successfully applied for image segmentation.

Index Terms— Adaptive generalized metric, Distance transform, General adaptive neighborhood, Generalized distance map, Image segmentation

1. INTRODUCTION

Distance transformations have been among the earliest image processing algorithms. They were initially implemented only for binary images [1] and used for set skeletonization, image segmentation, pattern recognition… A distance transform of a binary image specifies the distance from each background point to the nearest foreground point (i.e., belonging to the structure of interest). Thereafter, geodesic distance transforms on gray tone images have been investigated such as [2, 3, 4] where the gray tone surface (the foreground) acts as the geodesic mask. A gray tone distance transform calculates distances as minimal cost paths on the image surface. Nevertheless, these gray tone transforms propagate local distances without considering the topological characteristics of the image spatial structures. Indeed, each local distance is computed through a fixed mask operation according both to a spatial distance and a gray tone distance between neighbor points (within the mask region). This approach can lead to unrealistic minimal paths between image points, because a large distance value can occur between points within the same spatial structure.

The aim of this paper is to propose two novel gray tone distance transforms taking into account adaptatively the image spatial structures. To overcome the limitation described above, the fixed mask is replaced by the real surrounding region (i.e. the local region determined by the image itself) for each point to be processed. The proposed distances are based on specific adaptive neighborhoods, namely General Adaptive Neighborhoods (GANs) [5], which locally represent the image spatial structures.

The paper is organized as follows. Section 2 briefly exposes the GAN paradigm and section 3 introduces different GAN distance transforms. Section 4 show real examples of image segmentation using the proposed approach before concluding (Section 5).

2. GENERAL ADAPTIVE NEIGHBORHOODS (GANS)

In the so-called General Adaptive Neighborhood Image Processing (GANIP) framework [5], a set of General Adaptive Neighborhoods (GANs set) is identified about each point in the image to be analyzed. These GANs (Fig. 1) are subsets of the image spatial support and are used as adaptive spatial (operational) windows for image transformations or quantitative image analysis.

2.1. Weak GANs

The space of *image* (resp. *criterion*) *mappings*, defined on the spatial support D and valued in a real numbers interval $\bar{E}$ (resp. E), is represented in a General Linear Image Processing (GLIP) [6] framework, denoted $\mathcal{I}$ (resp. $\mathcal{C}$). The GLIP framework $\mathcal{I}$ (resp. $\mathcal{C}$) is then supplied with an ordered vectorial structure, using the formal vector addition $\bar{\oplus}$ (resp. $\oplus$), the formal scalar multiplication $\bar{\otimes}$ (resp. $\otimes$) and the classical partial order relation $\geq$ defined directly from those of real numbers.
For each point $x \in D$ and for an image $f \in \mathcal{I}$, its associated GAN denoted $V^h_{m_\bigcirc}(x)$ is included as subset in D (Fig.

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PATTERN RECOGNITION

Adaptive generalized metrics, distance maps and nearest neighbor transforms on gray tone images

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Generalized distance map
Image segmentation
Nearest neighbor transform

A B S T R A C T

This paper aims to introduce and study two novel metrics on gray tone images. These metrics are based on the General Adaptive Neighborhood Image Processing (GANIP) framework that enables to represent an image by spatial neighborhoods, named General Adaptive Neighborhoods (GAN) that fit to their local context. These metrics are generalized in the sense that they do not satisfy all the axioms of a standard mathematical metric. This notion of adaptive generalized metrics leads to the definition of relevant GAN distance maps and GAN nearest neighbor transforms used for image segmentation.

1. Introduction

Distance transformations have been among the earliest image processing algorithms. They were initially implemented only for binary images [1,2] and used for set skeletonization, image segmentation, pattern recognition, etc. A distance transform of a binary image specifies the distance from each point of the background to the nearest foreground point (i.e. belonging to the object). The distance calculations are based on propagating local distance values across this binary image with a mask operation.

Thereafter, geodesic distance transforms on gray tone images have been investigated [3–9] where the gray tone surface (the foreground) acts as the geodesic mask. A gray tone distance transform calculates distances as minimal cost paths on the image surface. Hence, the shortest path becomes the geographical minimal geodesic on the gray tone height map. These gray tone transforms have been further used for calculating nearest neighbor transforms which divides the image spatial support into adjacent regions, so that each point belongs to the region surrounding its nearest seed point. The image is then segmented according to seed points or regions (feature points) and a given gray tone distance transform. The well-known watershed algorithm [10] is an example of nearest neighbor transform using seed points as image minima and a specific distance transform.

Nevertheless, these gray tone transforms propagate local distances without considering the topological characteristics of the image spatial structures. Indeed, each local distance is computed through a fixed mask operation according to both a spatial distance and a gray tone distance between neighbor points (within the mask region). This approach can lead to unrealistic minimal paths between image points, because a large distance value can occur between points within the same spatial structure.

The aim of this paper is to propose novel gray tone distance transforms taking into account adaptively the image spatial structures. To overcome the limitation described above, the fixed mask is replaced by the real surrounding region (i.e. the local region determined by the image itself) for each point to be processed. The proposed distances are based on specific adaptive neighborhoods, namely General Adaptive Neighborhoods [11], which locally represent the image spatial structures. In particular, this framework ensures that the distance between two points inside a same local image structure will be smaller than the distance between two points belonging to two different local image structures.

The paper is organized in the following way. First (Section 2), the General Adaptive Neighborhood Image Processing (GANIP) framework [11] is presented and illustrated. Thereafter, two GAN-based generalized metrics (Section 3), between points or sets are introduced leading to the definition of new gray tone distance

Abbreviations: CLIP, Classical Linear Image Processing; DT, distance transform; DTOCS, distance transform on curved space; GAN, General Adaptive Neighborhood; GANIP, General Adaptive Neighborhood Image Processing; GLIP, General Linear Image Processing; GTT, geodesic time transform; LIP, logarithmic image processing; NNT, nearest neighbor transform; SKIZ, skeleton by influence zones; WDTOCS, weighted distance transform on curved space

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Generalized Distances

- **Context: Gray-Tone Distance**
- **Generalized Distance:** $d : D \times D \to \mathbb{R}$
- **Several Axioms**

(M0) positivity: $\forall (x,y) \in D^2 \quad d(x,y) \geq 0$

(M1) reflexivity: $\forall x \in D \quad d(x,x) = 0$

(M2) symmetry: $\forall (x,y) \in D^2 \quad d(x,y) = d(y,x)$

(M3) triangle inequality: $\forall (x,y,z) \in D^3 \quad d(x,y) \leq d(x,z) + d(z,y)$

(M4) separation: $\forall (x,y) \in D^2 \quad d(x,y) = 0 \Rightarrow x = y$

Axioms	(M0) positivity	(M1) reflexivity	(M2) symmetry	(M3) triangle inequality	(M4) separation
Semi-quasi-pseudometric	•	•			
Semi-pseudometric	•	•	•		
Semi-quasimetric	•	•		•	
Quasi-pseudometric	•	•			•
Quasimetric	•	•		•	•
Semimetric	•	•	•	•	
Pseudometric	•	•	•		•
Metric	•	•	•	•	•

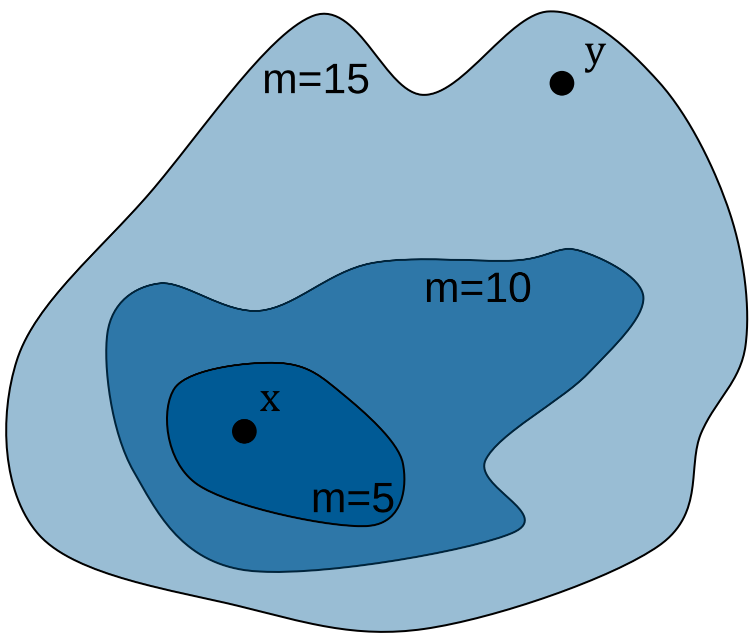
GAN Generalized Distances

- **Definition**

$$dV_{\hexagon}^{h}(x,y) = \inf_{m_{\hexagon} \in E^{\oplus}} \{m_{\hexagon} ; x \in V_{m_{\hexagon}}^{h}(y)\}$$

$$dN_{\hexagon}^{h}(x,y) = \inf_{m_{\hexagon} \in E^{\oplus}} \{m_{\hexagon} ; x \in N_{m_{\hexagon}}^{h}(y)\}$$

- **Illustration**



GAN Generalized Distances

- **Properties**

(M0) positivity:

$$\forall (x,y) \in D^2 \begin{cases} dV^h_{\hexagon}(x,y) \geq 0 \; ^{\oplus} \\ dN^h_{\hexagon}(x,y) \geq 0 \; ^{\oplus} \end{cases}$$

(M1) reflexivity:

$$\forall x \in D \begin{cases} dV^h_{\hexagon}(x,x) = 0 \; ^{\oplus} \\ dN^h_{\hexagon}(x,x) = 0 \; ^{\oplus} \end{cases}$$

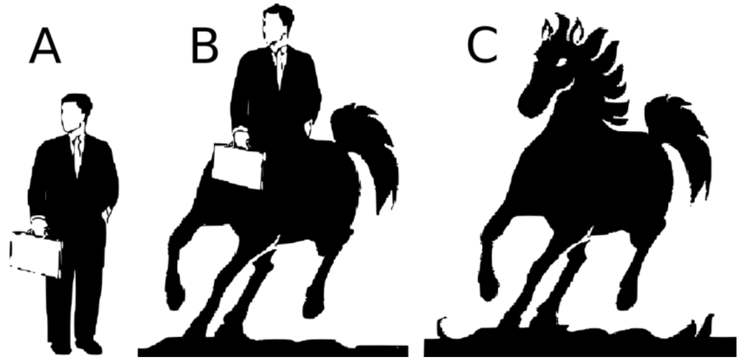
(M2) symmetry:

$$\forall (x,y) \in D^2 \quad dN^h_{\hexagon}(x,y) = dN^h_{\hexagon}(y,x)$$

Axioms	(M0) positivity	(M1) reflexivity	(M2) symmetry	(M3) triangle inequality	(M4) separation
Semi-quasi-pseudometric	•	•			
Semi-pseudometric	•	•	•		
Semi-quasimetric	•	•		•	
Quasi-pseudometric	•	•			•
Quasimetric	•	•		•	•
Semimetric	•	•	•	•	
Pseudometric	•	•	•		•
Metric	•	•	•	•	•

GAN Generalized Distances

- **Properties**
  - No triangle inequality: not a problem for the human visual perception!



$$d(A,C) \geq d(A,B) + d(B,C)$$

  - No symmetry for the weak GANs

GAN Generalized Distances

- **Extension to Subsets**

$$
\begin{aligned}
dV_{\hexagon}^{h}(x, A) &= \inf_{y \in A} dV_{\hexagon}^{h}(x, y) \\
&= \inf_{m_{\hexagon} \in E^{\oplus}} \{ m_{\hexagon}, x \in \mathrm{V}_{m_{\hexagon}}^{h}(A) \} \\
dN_{\hexagon}^{h}(x, A) &= \inf_{y \in A} dN_{\hexagon}^{h}(x, y) \\
&= \inf_{m_{\hexagon} \in E^{\oplus}} \{ m_{\hexagon}, x \in \mathrm{N}_{m_{\hexagon}}^{h}(A) \}
\end{aligned}
$$

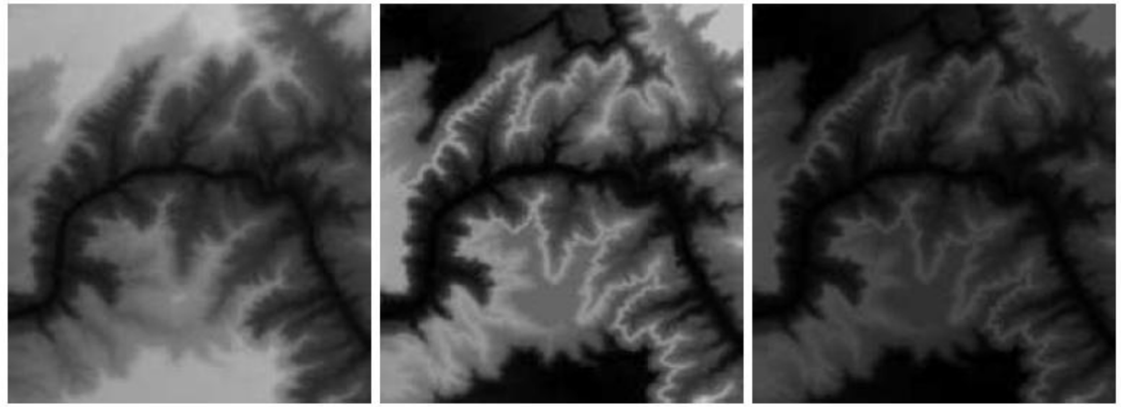
$$
\begin{aligned}
dV_{\hexagon}^{h}(A, B) &= \inf_{y \in B} dV_{\hexagon}^{h}(x, A) \\
dN_{\hexagon}^{h}(A, B) &= \inf_{y \in B} dN_{\hexagon}^{h}(x, A)
\end{aligned}
$$

GAN Distance Maps

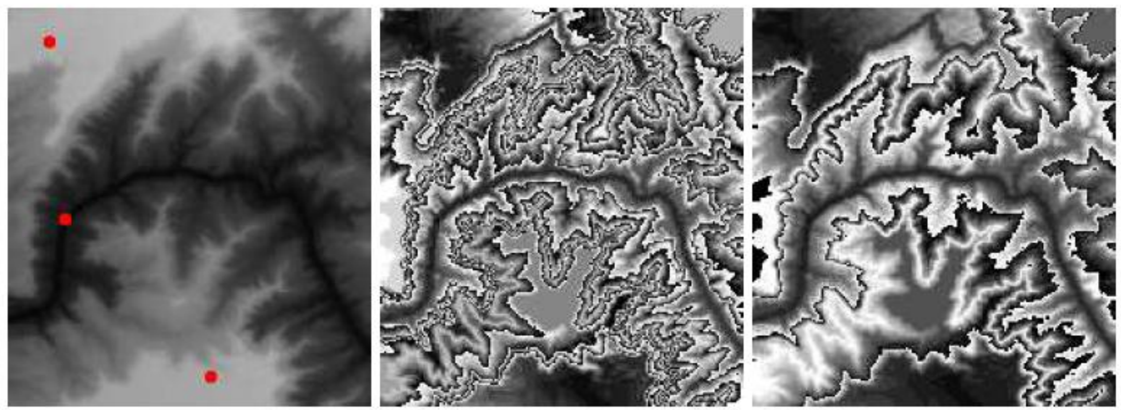
- **GAN Distance Maps**
  - From a selection of markers S

$$dmV^h_{\hexagon}(S) \quad : \quad \begin{cases} D & \rightarrow & E^{\oplus} \\ x & \mapsto & dV^h_{\hexagon}(x,S) \end{cases}$$

$$dmN^h_{\hexagon}(S) \quad : \quad \begin{cases} D & \rightarrow & E^{\oplus} \\ x & \mapsto & dN^h_{\hexagon}(x,S) \end{cases}$$



(a) original 'Grand Canyon' image f (b) weak GAN distance map (c) strong GAN distance map



(d) seed points of the original image (e) map (b) modulo 20 (f) map (c) modulo 20

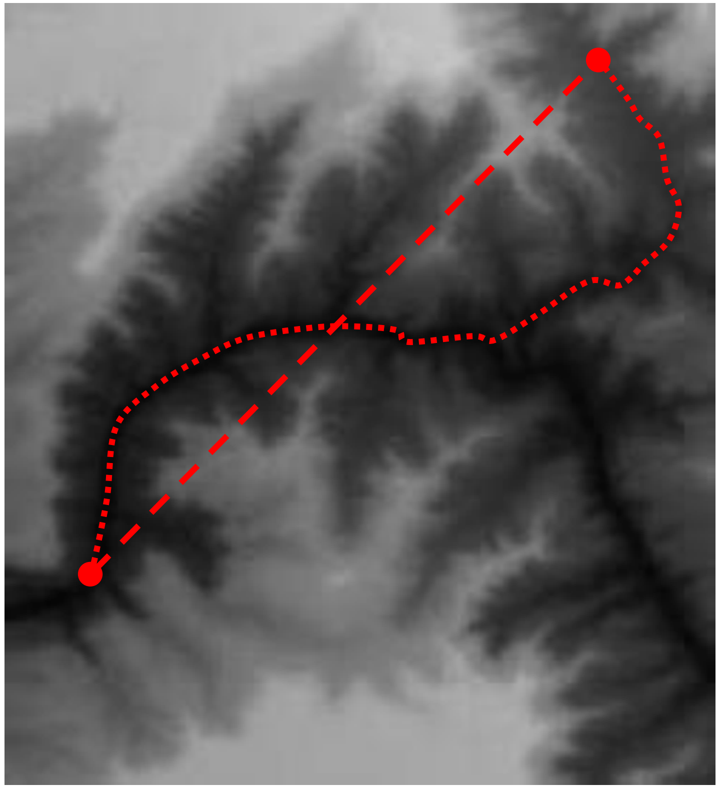
GAN Distance Maps

- **Comparaison with other Gray-Tone Distance Maps**
  - Minimal geodesic path between points
  - Geodesic time transform (GTT)

$$t_f(P) = \sum_{i=1}^{l} \frac{f(p_{i+1}) + f(p_i)}{2} t(p_i, p_{i+1})$$

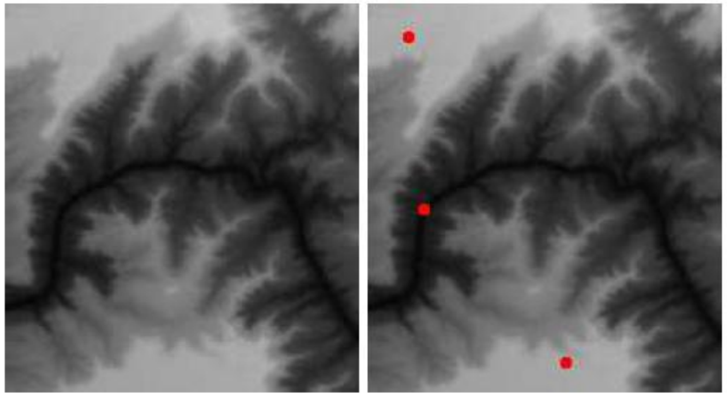
  - Distance transform on curved space (DTOCS)

$$d(p_i, p_{i+1}) = |f(p_i) - f(p_{i+1})| + 1$$

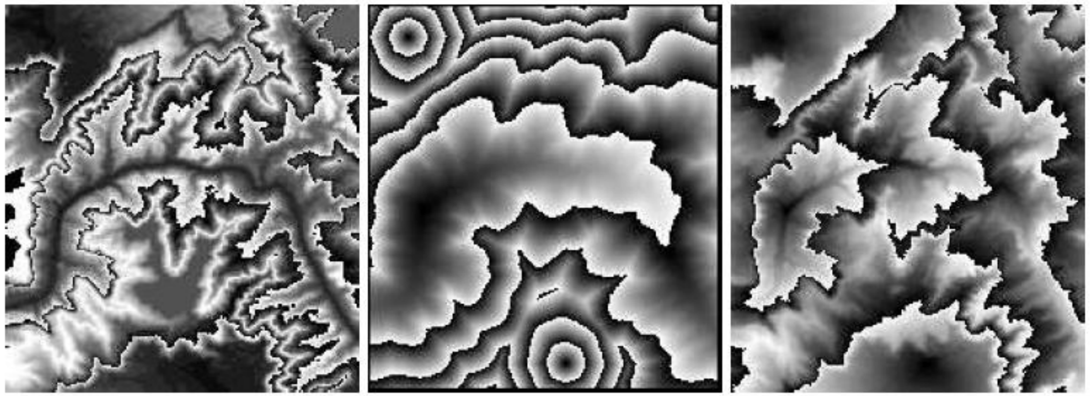


GAN Distance Maps

- **Comparaison with other Gray-Tone Distance Maps**



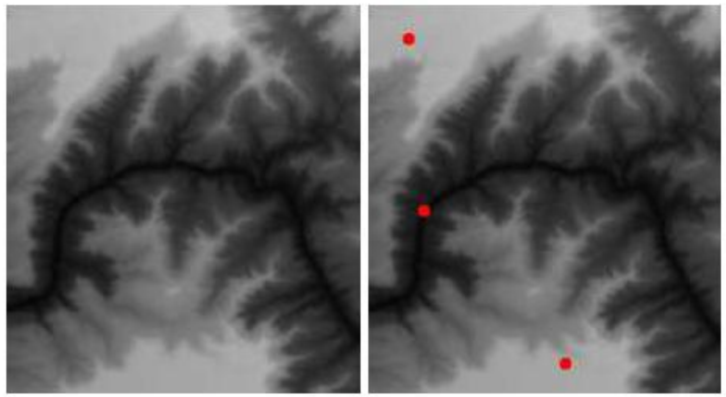
(a) original image f (b) three seed points



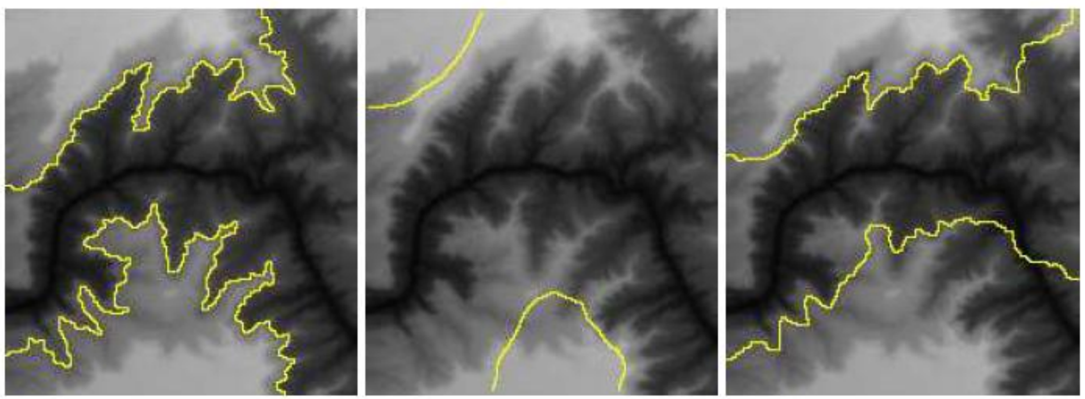
(c) strong GAN distance map (d) GTT map (e) DTOCS map

GAN Nearest Neighbor Transform

- **Watershed of the GAN Distance Map**



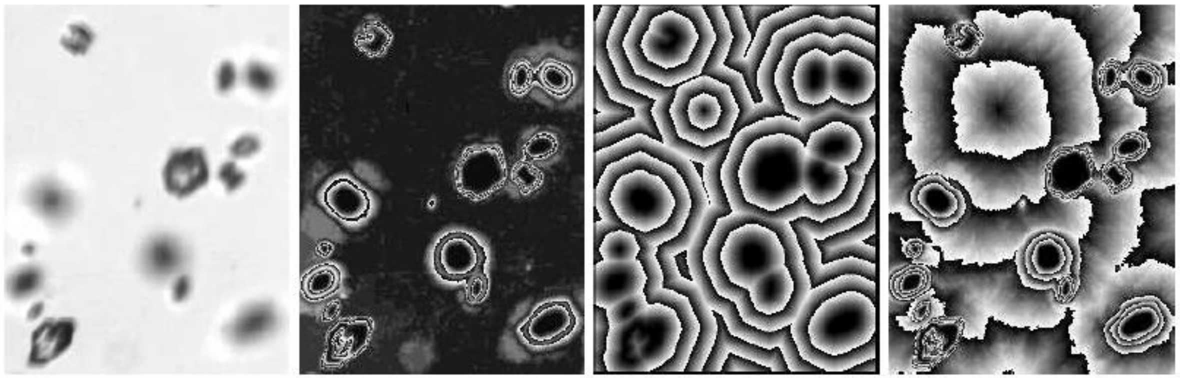
(a) original image f (b) three seed points



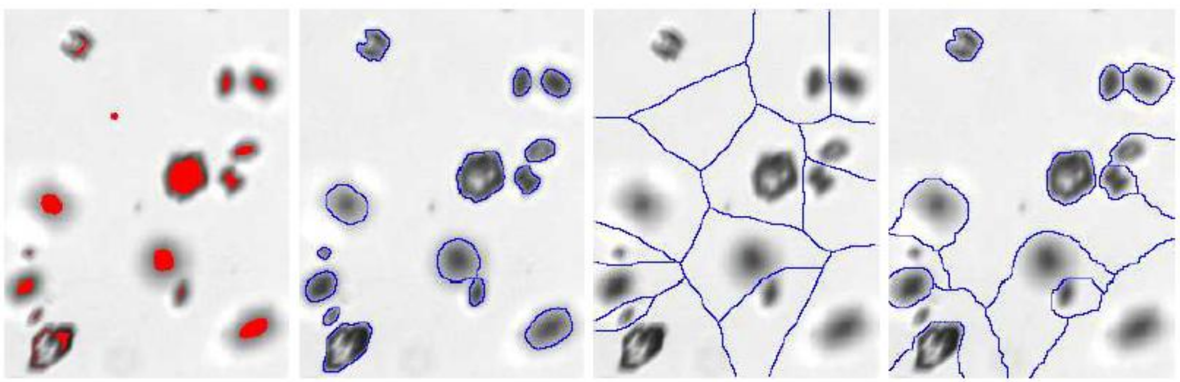
(c) borders marked on strong GAN-DT image (d) borders marked on GTT image (e) borders marked on DTOCS image

Application Example

- Image Segmentation of Crystals



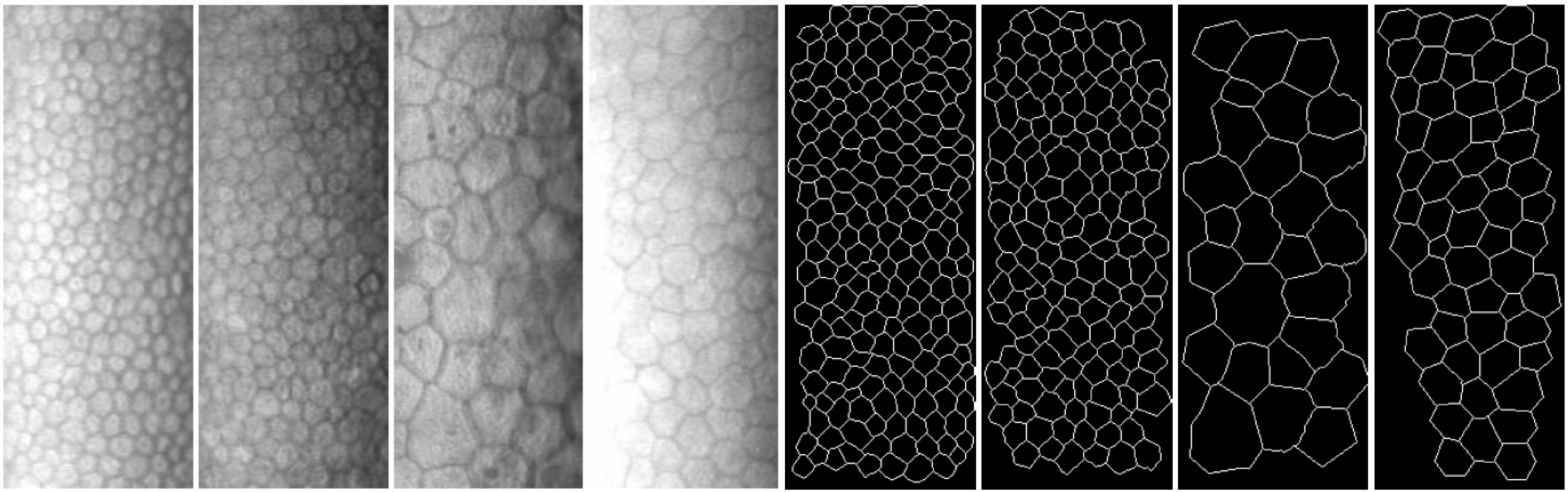
(a) original image f (b) strong GAN distance map (c) GTT distance map (d) DTOCS distance map



(e) seed points (f) GAN nearest neighbor transform (g) GTT nearest neighbor transform (h) DTOCS nearest neighbor transform

Application Example

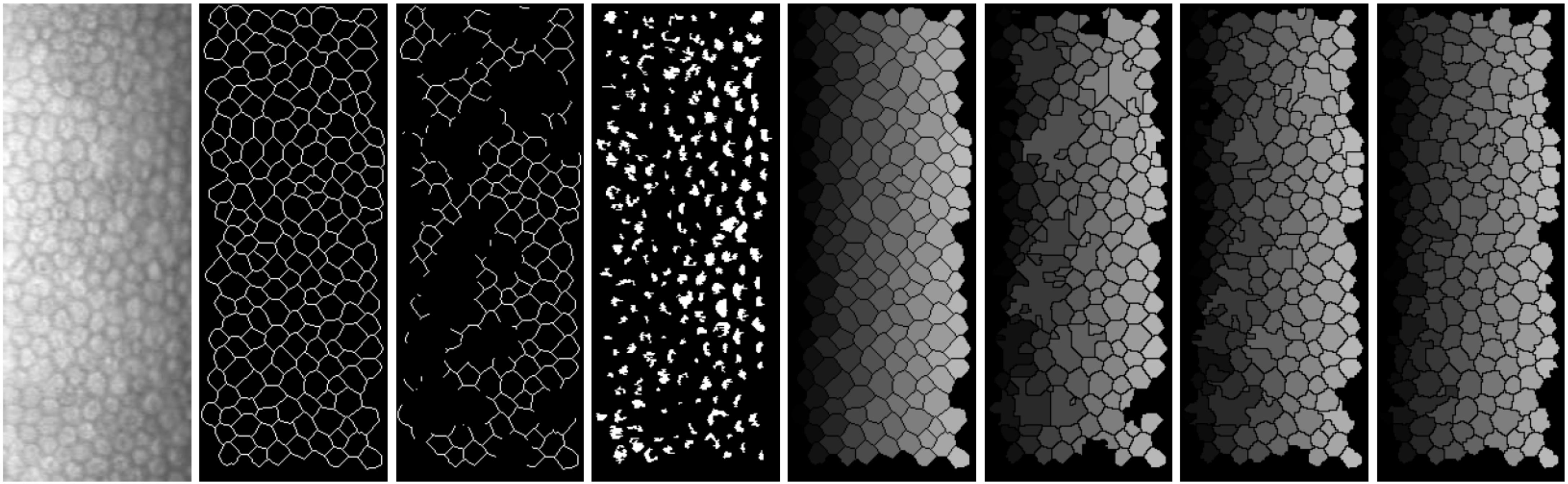
- **Image Segmentation of Cornea Cells**
  - Reference segmentation



(a) image 2 (b) image 4 (c) image 11 (d) image 15 (e) segmented image 2 by an expert (f) segmented image 4 by an expert (g) segmented image 11 by an expert (h) segmented image 15 by an expert

Application Example

- **Image Segmentation of Cornea Cells**
  - Segmentation methodology

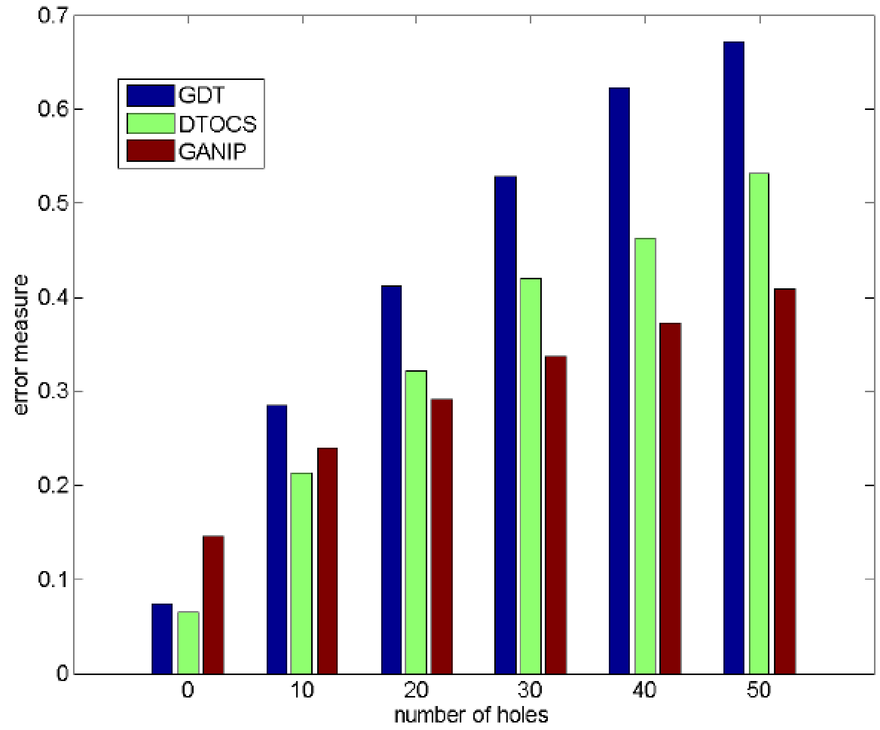


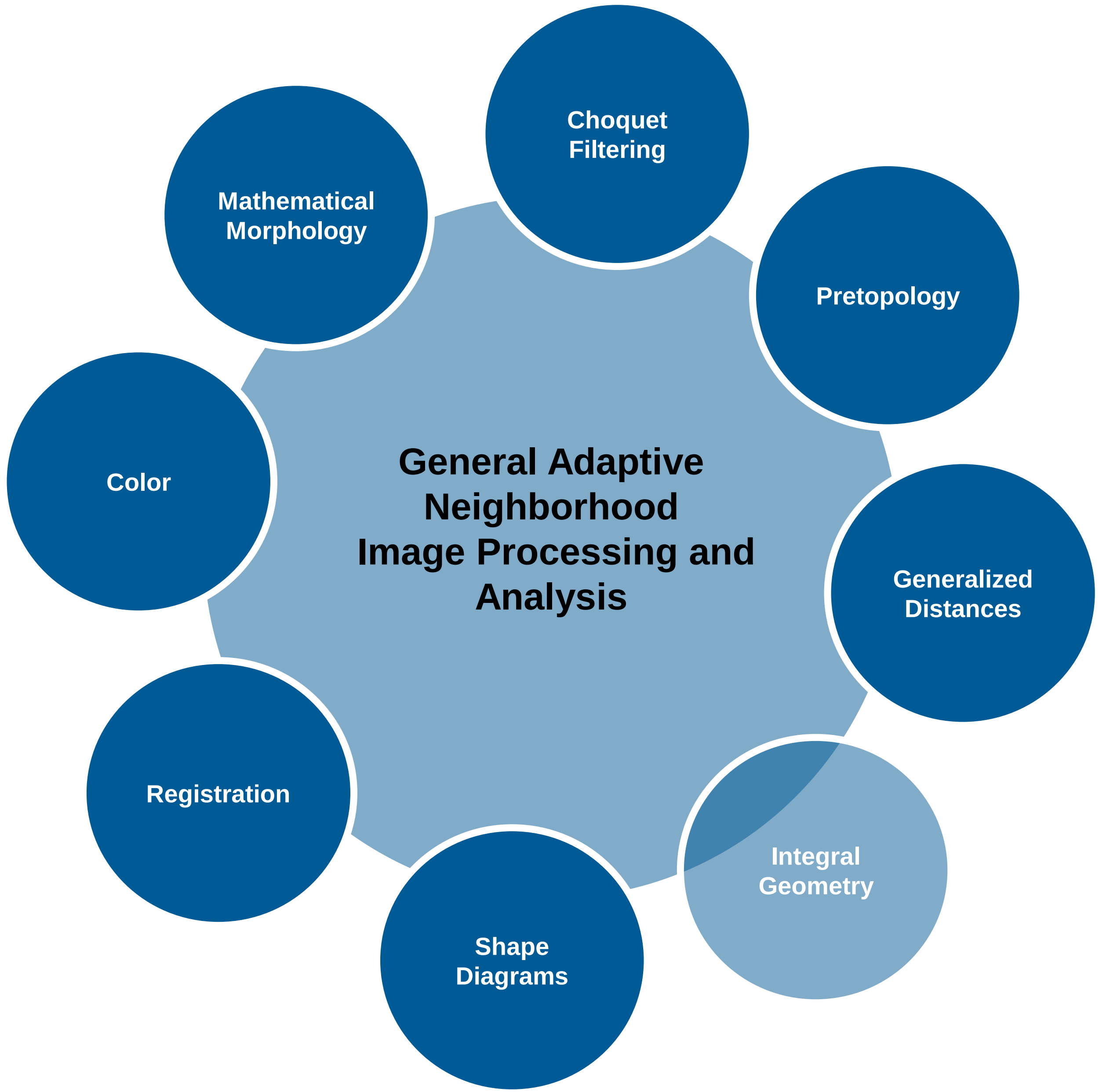
(a) image 5 (b) squeleton of the expert (c) squeleton with 50 holes (d) cell markers (e) reference segmentation (f) GTT segmentation (g) DTOCS segmentation (h) GANIP segmentation

Application Example

- **Image Segmentation of Cornea Cells**
  - Quantitative evaluation

$$d(S,R) = \frac{1}{n}\sum_{i=1}^{n}\frac{\mu(S_i \cup R_i) - \mu(S_i \cap R_i)}{\mu(R_i)}$$




Choquet Filtering
Mathematical Morphology
Pretopology
Color
General Adaptive Neighborhood Image Processing and Analysis
Generalized Distances
Registration
Integral Geometry
Shape Diagrams

Ecole Nationale Supérieure des Mines SAINT-ETIENNE

GAN-based Integral Geometry

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INSPIRING INNOVATION | INNOVANTE PAR TRADITION

Main Publications

[ECSIA 2009, **IJSIP 2010**]

GENERAL ADAPTIVE NEIGHBORHOOD-BASED MINKOWSKI MAPS FOR GRAY-TONE IMAGE ANALYSIS

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ABSTRACT

In quantitative image analysis, Minkowski functionals are standard parameters for topological and geometrical measurements. Nevertheless, they are often limited to binary images and achieved in a global and monoscale way. The use of General Adaptive Neighborhoods (GANs) enables to overcome these limitations. The GANs are spatial neighborhoods defined around each point of the spatial support of a gray-tone image, according to three (GAN) axiomatic criteria: a criterion function (luminance, contrast, …), an homogeneity tolerance with respect to this criterion, and an algebraic model for the image space. Thus, the GANs are simultaneously adaptive with the analyzing scales, the spatial structures and the image intensities.
The aim of this paper is to introduce the GAN-based Minkowski functionals, which allow a gray-tone image analysis to be realized in a local, adaptive and multiscale way. The Minkowski functionals are computed on the GAN of each point of the image, enabling to define the so-called Minkowski maps which assign the geometrical or the topological functional to each point. The impact of the GAN characteristics, as well as the impact of multiscale morphological transformations, is analyzed in a qualitative way through these maps. The GAN-based Minkowski maps are illustrated on the test image 'Lena' and also applied in the biomedical and materials areas.

Keywords: General adaptive neighborhood, GLIP Mathematical morphology, Minkowski functionals, Minkowski maps, Multiscale image representation, Pattern analysis.

INTRODUCTION

This paper aims to introduce a novel approach for analyzing a gray-tone image in a local, adaptive and multiscale way. A segmentation process, generally used before quantitative image analysis, is not here required. The quantitative description is directly applied on the raw gray-tone images. Geometrical and topological measurements, through Minkowski functionals, are performed on spatial neighborhoods associated to each point of the image. These specific neighborhoods, named General Adaptive Neighborhoods (GANs) (Debayle and Pinoli, 2006), are simultaneously adaptive with the analyzing scales, the spatial structures and the image intensities. It enables to define the so-called GAN-based Minkowski maps which assign a measurement (based on the local Minkowski functionals) to each point of the image to be studied.

First, this paper recalls the notions of general adaptive neighborhood and of Minkowski functionals. Then, the next section introduces the GAN-based Minkowski maps. Thereafter, the impact of the GAN axiomatic criteria (analyzing criterion, homogeneity tolerance, algebraic model), as well as the impact of a multiscale morphological transformation is analyzed in a qualitative way through these maps. The GAN-based Minkowski maps are illustrated on the test image 'Lena' and also in both the biomedical and materials areas.

GENERAL ADAPTIVE NEIGHBORHOODS

The GANIP (General Adaptive Neighborhood Image Processing) approach (Debayle and Pinoli, 2006) provides a general and operational framework for adaptive processing and analysis of gray-tone images. It is based on an image representation by means of spatial neighborhoods, named General Adaptive Neighborhoods (GANs). Indeed, GANs are simultaneously adaptive with:

- the spatial structures: the size and the shape of the neighborhoods are adapted to the local context of the image,
- the analyzing scales: the scales are given by the image itself, and not *a priori* fixed,
- the intensity values: the neighborhoods are defined according to the GLIP (Generalized Linear Image Processing) mathematical framework, enabling to consider the physical and/or psychophysical settings of the image class.

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Integral Geometry and General Adaptive Neighborhoods for Multiscale Image Analysis

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Appeared: 25/05/2010

Abstract: In quantitative image analysis, Minkowski functionals are becoming standard parameters for topological and geometrical measurements. Nevertheless, they are limited to binary images or to sections of gray-tone images and are achieved in a global and monoscale way. The use of General Adaptive Neighborhoods (GANs) enables to overcome these limitations. The GANs are spatial neighborhoods defined around each point of the spatial support of a gray-tone image, according to three (GAN) axiomatic criteria: a criterion function (luminance, contrast, …), an homogeneity tolerance with respect to this criterion, and an algebraic model for the image space. Thus, the GANs are simultaneously adaptive with the analyzing scales, the spatial structures and the image intensities.
This paper aims to introduce the GAN-based Minkowski functionals, which allow a gray-tone image analysis to be realized in a local, adaptive and multiscale way. The Minkowski functionals are computed on the GAN of each point of the spatial support of a gray-tone image, enabling to define the so-called Minkowski maps by assigning the Minkowski functional value to each point. The histograms of these maps provide a statistical distribution of the topology and geometry of the gray-tone image structures, and not only of the image intensities. The impact of the GAN characteristics, as well as the impact of multiscale transformations, are analyzed in a qualitative global and local way through these GAN-based Minkowski maps and histograms. This multiscale image analysis is illustrated on the test image 'Lena' and also applied in both the biomedical and materials areas.

Keywords: GAN-based Minkowski maps and histograms, General Adative Neighborhoods, Integral Geometry, Minkowski functionals, Multiscale image analysis

NOMENCLATURE

This section presents a list of all the used symboles and their meaning.

A	area	$\times$	scalar multiplication of the CLIP framework
P	perimeter	LIP	Logarithmic Image Processing
χ	Euler number	$\triangle$	LIP framework
A_A	specific area	$\triangle\!\!\!\!+$	vector addition of the LIP framework
P_A	specific perimeter	$\triangle\!\!\!\!-$	vector subtraction of the LIP framework
χ_A	specific Euler number	$\triangle\!\!\!\!\times$	scalar multiplication of the LIP framework
μ	specific Minkowski functional	$\mathcal{I}$	space of gray-tone images
GLIP	General Linear Image Processing	$\mathcal{C}$	space of analyzing criteria
$\bigcirc$	GLIP framework	$\mathbb{E}_{\bigcirc}$	intensity value range
$\oplus$	vector addition of the GLIP framework	D	image spatial support
$\ominus$	vector subtraction of the GLIP framework	f	gray-tone image
$\otimes$	scalar multiplication of the GLIP framework	h	analyzing criterion
CLIP	Classical Linear Image Processing	$m_{\bigcirc}$	homogeneity tolerance
$+$	vector addition of the CLIP framework	$V^h_{m_{\bigcirc}}(x)$	GAN of $x \in D$
$-$	vector subtraction of the CLIP framework	$\mu^h_{m_{\bigcirc}}$	GAN-based Minkowski map

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Integral Geometry

- **Context: Gray-Tone Local Measurements**

- **2-D Minkowski Functionals**
  - Valid for union of convex sets
  - Image: union of pixels (convex sets)

  - A: Area
  - P: Perimeter
  - χ: Euler-Poincaré Number

$$w : \begin{cases} \mathcal{P}(D) & \rightarrow & \mathbb{R} \\ Z & \mapsto & w(Z) \end{cases}$$

- **Properties**
  - Increasing (for convex sets)
  - Invariance under rigid motions (rotation, translation, reflectance, …)
  - Homogeneity
  - Additivity
  - Continuity (for convex sets)

GAN-based Minkowski Functionals

- **Adaptive Minkowski Maps**

$$\mu^h_{m_{\hexagon}} : \begin{cases} D & \rightarrow \quad \mathbb{R} \\ x & \mapsto \mu\left(V^h_{m_{\hexagon}}(x)\right) \end{cases}$$

$$\mu = \{A, P, \chi\}$$

- **Density Functional Values**
  - Normalization by the image area

(a) original image f

(b) $(\chi_A)^f_{40}$ (c) $(P_A)^f_{40}$ (d) $(A_A)^f_{40}$

Gray-scale lower and upper bound values

(a)	0	255
(b)	-812.10^{-5}	1.10^{-5}
(c)	4.10^{-5}	9686.10^{-5}
(d)	1.10^{-5}	42926.10^{-5}

Properties

- **Equality between Iso-Valued Points**

$$\begin{pmatrix} x_1 \in V^h_{m_{\hexagon}}(x_2) \\ h(x_1) = h(x_2) \end{pmatrix} \Rightarrow \gamma^h_{m_{\hexagon}}(x_1) = \gamma^h_{m_{\hexagon}}(x_2)$$

- **Translation Invariance**

$$\forall c \in \mathbb{E} : \gamma^{h \oplus c}_{m_{\hexagon}}(x) = \gamma^h_{m_{\hexagon}}(x)$$

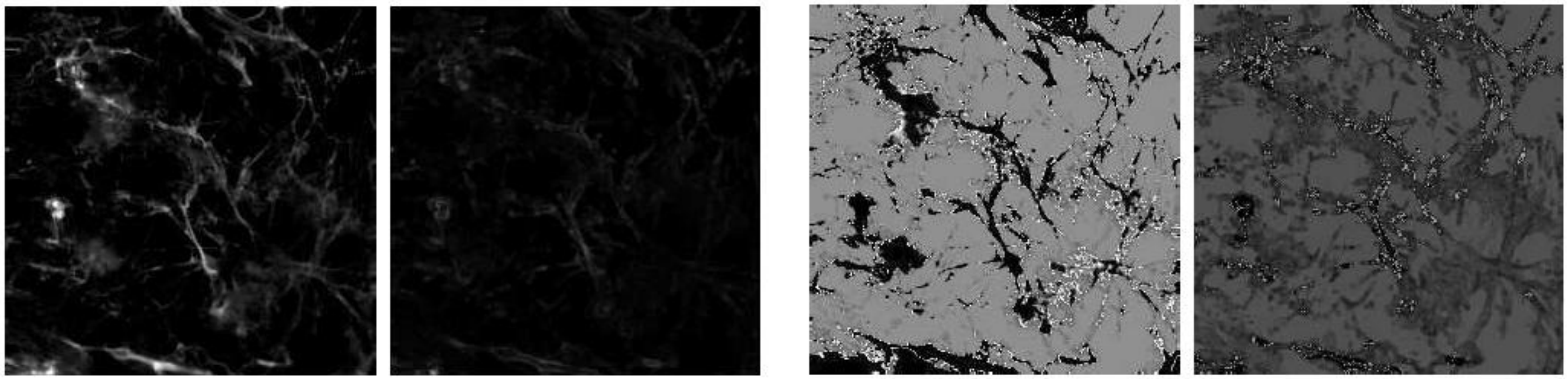
- **Multiplication Compatibility**

$$\forall \alpha \in \mathbb{R}^{+*} : \gamma^{\alpha \otimes h}_{m_{\hexagon}}(x) = \gamma^h_{\frac{1}{\alpha} \otimes m_{\hexagon}}(x)$$

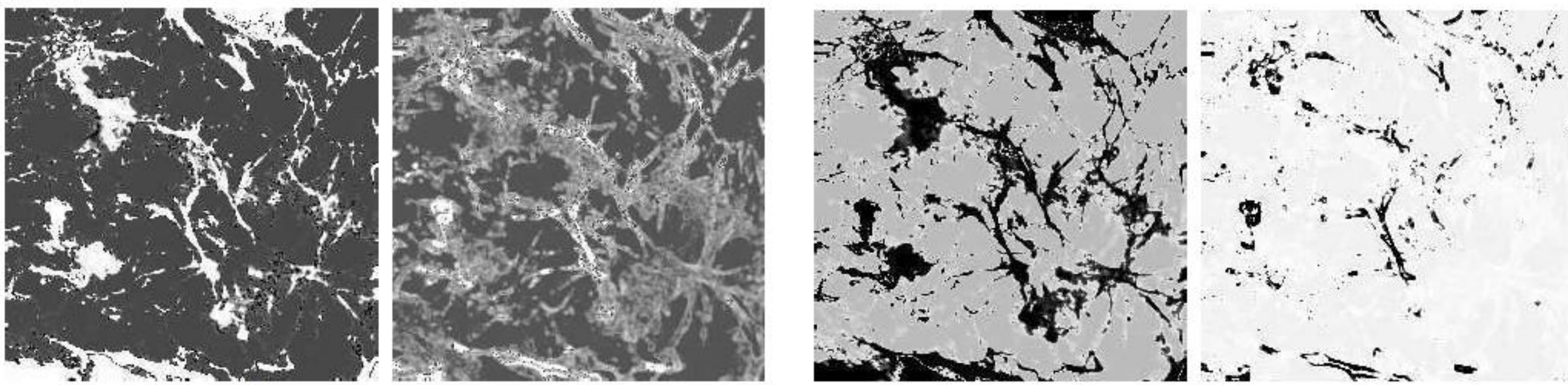
- **Increasing Property for the Area Functional**

Illustration

- **Impact of the Criterion Mapping**
  - Analysis of a fibronection image



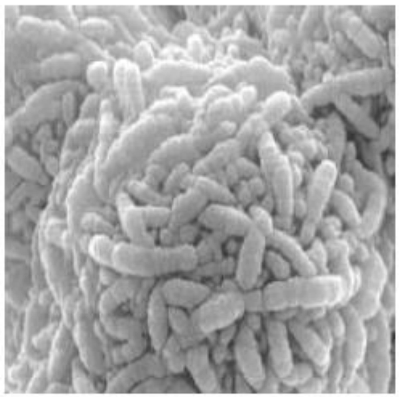
(a) original image f (b) criterion map c (e) $(P_A)^f_{20}$ (f) $(P_A)^c_{20}$



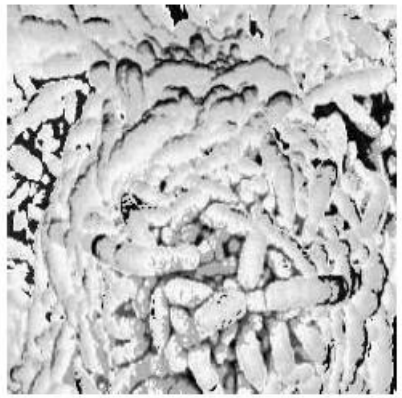
(c) $(\chi_A)^f_{20}$ (d) $(\chi_A)^c_{20}$ (g) $(A_A)^f_{20}$ (h) $(A_A)^c_{20}$

Illustration

- **Impact of the GLIP Model**
  - Analysis of Zinc Sulfide (ZnS) image (Scanning Electron Microscopy)

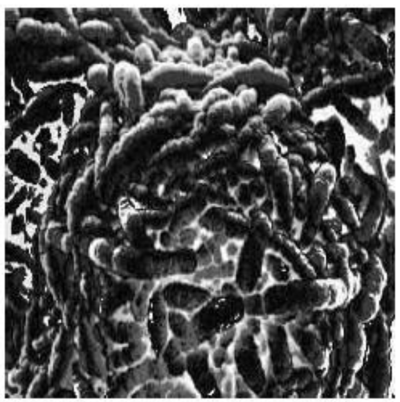


(a) original image f



(d) $(P_A)^f_{50}$

(e) $(P_A)^f_{50_\triangle}$



(b) $(\chi_A)^f_{50}$

(c) $(\chi_A)^f_{50_\triangle}$

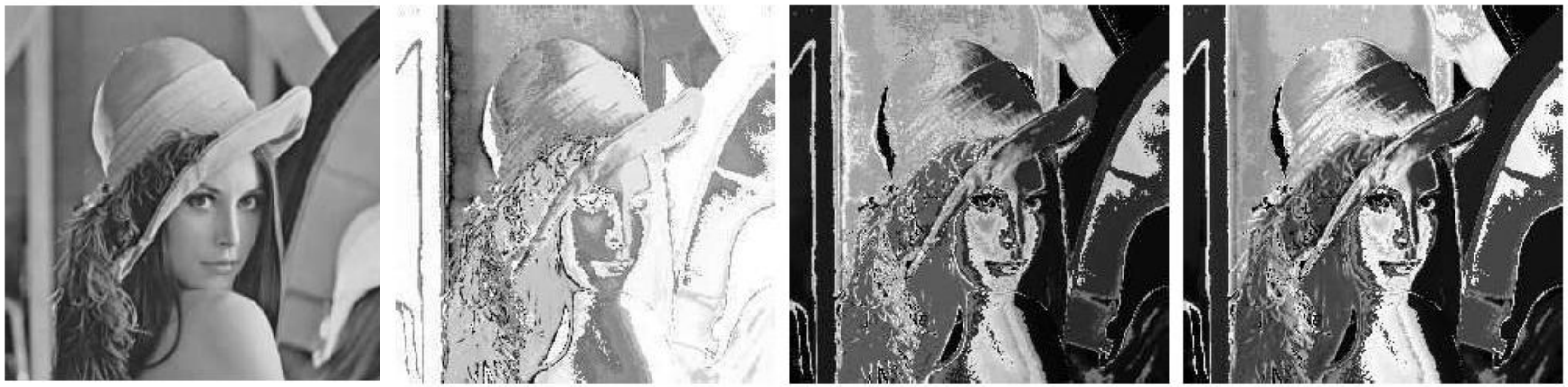
(f) $(A_A)^f_{50}$

(g) $(A_A)^f_{50_\triangle}$

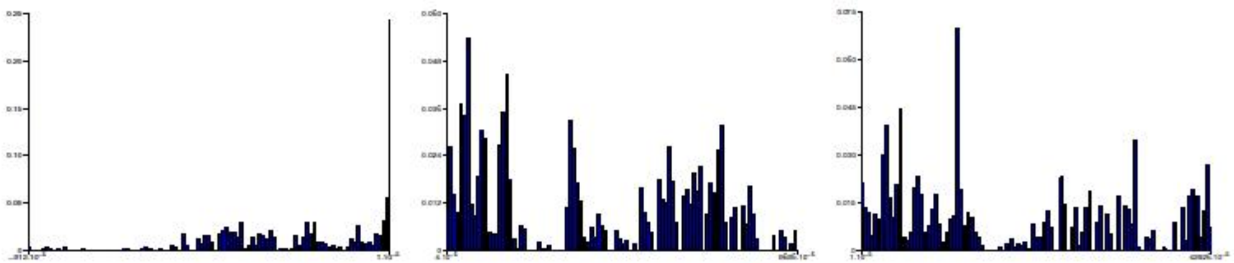
GAN-based Minkowski Histograms

- **Distribution of GAN-based Minkowski Density Functional Values**

$$\mathrm{H}\left(\mu^{h}_{m_{\hexagon}}\right)(t) = \frac{d\mathcal{L}^2\left(\left\{x \in D : \mu^{h}_{m_{\hexagon}}(x) < t\right\}\right)}{\mathcal{L}^2(D)dt}$$



(a) original image f (b) $(\chi_A)^f_{40}$ (c) $(P_A)^f_{40}$ (d) $(A_A)^f_{40}$



(e) $\mathrm{H}\left((\chi_A)^f_{40}\right)$ (f) $\mathrm{H}\left((P_A)^f_{40}\right)$ (g) $\mathrm{H}\left((A_A)^f_{40}\right)$

GAN-based Minkowski Functions

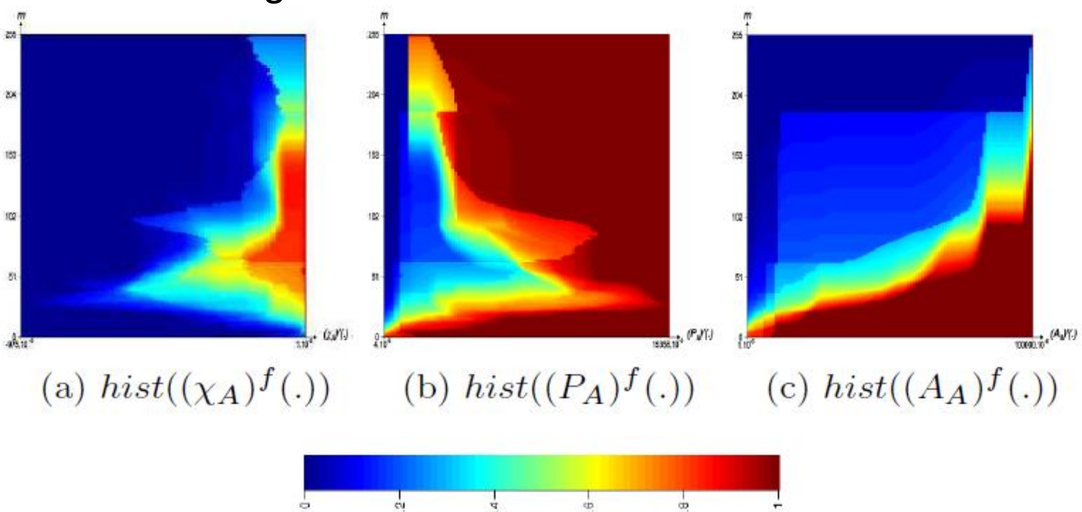
- **Minkowski Functions**

$$W : \begin{cases} \mathcal{P}(D) \times \mathbb{R} & \rightarrow & \mathbb{R} \\ (Z, p) & \mapsto & w_p(Z) \end{cases}$$

- **Varying Parameter**
  - Homogeneity tolerance

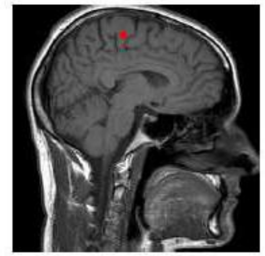
$$\mu^h_{\diamond}(.) : m \mapsto \mu^h_{m\hexagon}(.)$$

- **Representation**
  - Joint histogram

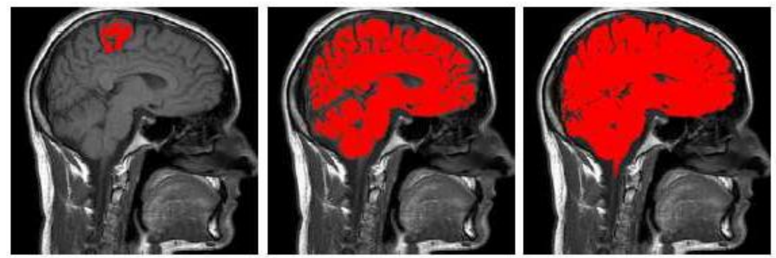


(a) $hist((\chi_A)^f(.))$ (b) $hist((P_A)^f(.))$ (c) $hist((A_A)^f(.))$

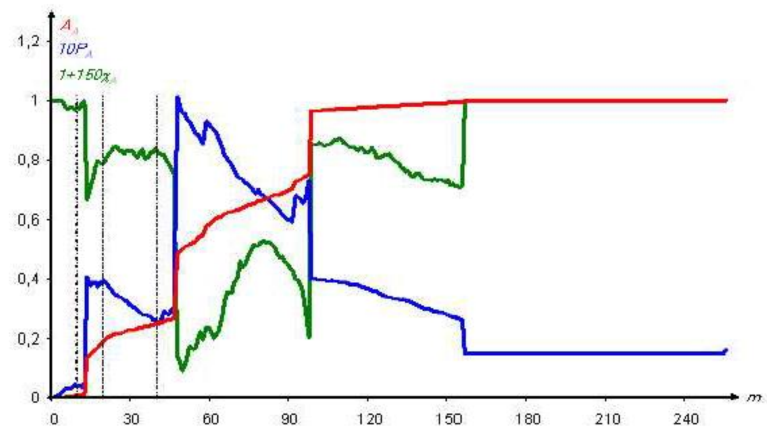
(d) color scale



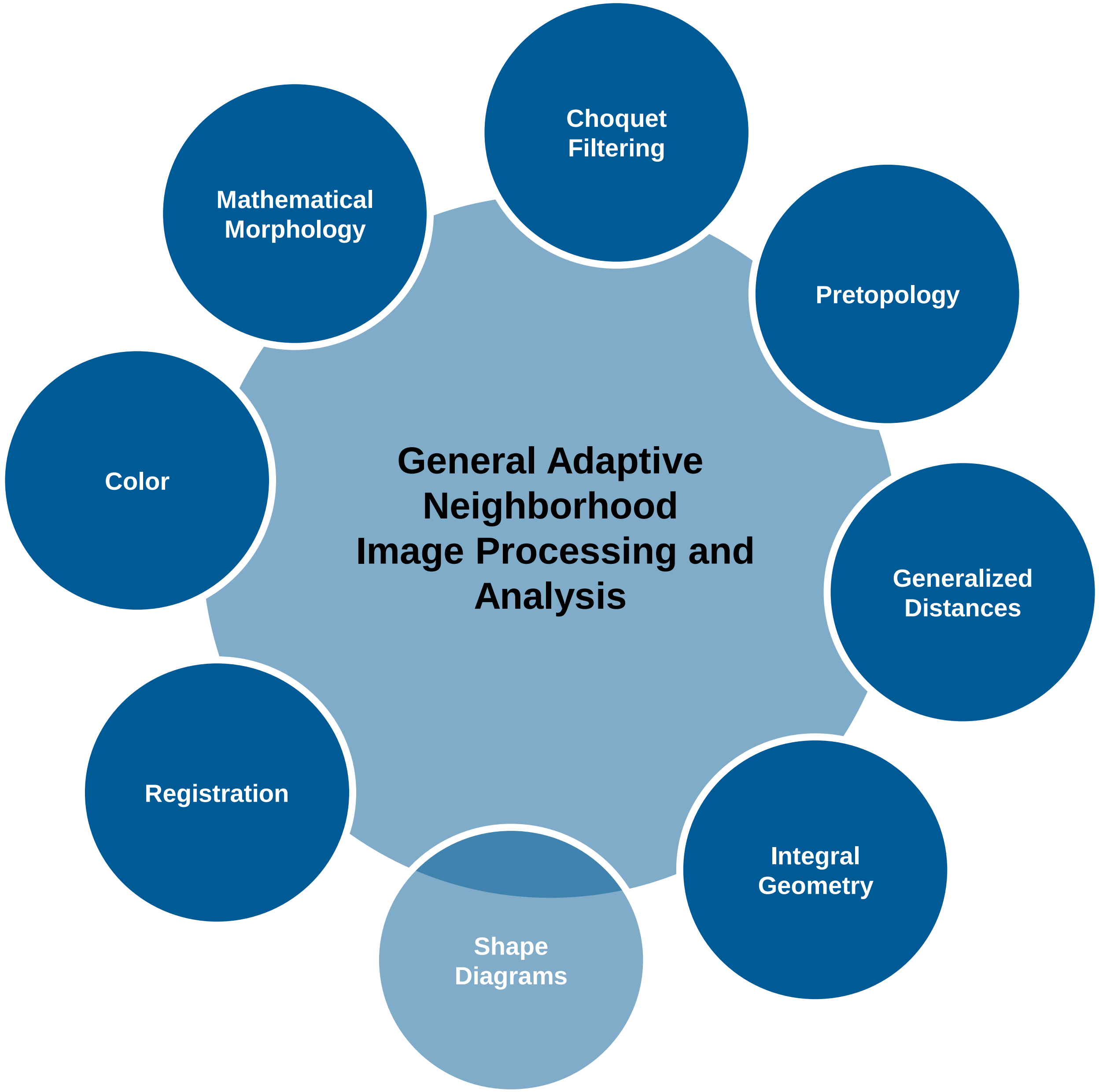
(a) x



(b) $V^f_{10}(x)$ (c) $V^f_{20}(x)$ (d) $V^f_{40}(x)$



(e) $(\chi_A)^f(x)$, $(P_A)^f(x)$ et $(A_A)^f(x)$


Choquet Filtering
Mathematical Morphology
Pretopology
Color
General Adaptive Neighborhood Image Processing and Analysis
Generalized Distances
Registration
Integral Geometry
Shape Diagrams

Main Publications

[JMIV 2013]

J Math Imaging Vis
DOI 10.1007/s10851-013-0439-2

Adaptive Shape Diagrams for Multiscale Morphometrical Image Analysis

Séverine Rivollier · Johan Debayle · Jean-Charles Pinoli

Abstract Shape diagrams are integral geometric representations in the Euclidean plane introduced to study 2D connected compact sets. Such a set is represented by a point within a shape diagram whose coordinates are morphometrical functionals defined as normalized ratios of geometrical functionals. In addition, the General Adaptive Neighborhoods (GANs) are spatial neighborhoods defined around each point of the spatial support of a gray-tone image, that fit with the image local structures. The aim of this paper is to introduce and study the GAN-based shape diagrams, which allow a gray-tone image morphometrical analysis to be realized in a local, adaptive and multiscale way. The GAN-based shape diagrams will be illustrated on standard images and also applied in the biomedical and materials areas.

Keywords General adaptive neighborhood · Integral geometric functionals · Morphometrical functionals · Multiscale image analysis · Pattern analysis · Shape diagrams

1 Introduction

This paper deals with intensity images, that is to say image mappings defined on a spatial support within the Euclidean space $\mathbb{R}^2$ and valued into a gray-tone range, which is a positive real numbers interval. It aims at introducing and studying a novel approach for morphometrically analyzing a gray-tone image in a local, adaptive and multiscale way. A segmentation process, generally used before quantitative image analysis, is not here required. The quantitative description is directly applied on the raw gray-tone images. Both shape analysis and intrinsic spatial analysis are generally not taken into account for gray-tone image characterization, such as granulometric analysis. The novel approach developed in this paper combines two existing concepts, the shape diagrams and the general adaptive neighborhoods, which provide both shape analysis and intrinsic spatial analysis, respectively. The shape diagrams [5, 56–58, 61] are based on morphometrical functionals, and allow to analyze the shape of sets. The General Adaptive Neighborhoods (GANs) [10, 11] are spatial neighborhoods enabling an adaptive intrinsic spatial analysis of gray-tone images to be realized. They are associated to each point belonging to the image spatial support, and are simultaneously adaptive with the analyzing scales, the spatial structures and the image intensities.

The purpose of this article is to morphometrically characterize gray-tone images (without any segmentation step) by combining the GANs and the shape diagrams. Morphometrical functionals and functions are performed on the GANs associated to each image point. They enable to define the so-called GAN-based (adaptive) shape diagrams which provide morphometrical distributions of the gray-tone image local structures.

The two first sections of this paper recall the notions of general adaptive neighborhoods and of shape diagrams, respectively. By combining these two notions, the third section introduces the GAN-based shape diagrams. Thereafter, the impact of the GAN axiomatic criteria, as well as the impact of a morphological transformation is analyzed through these

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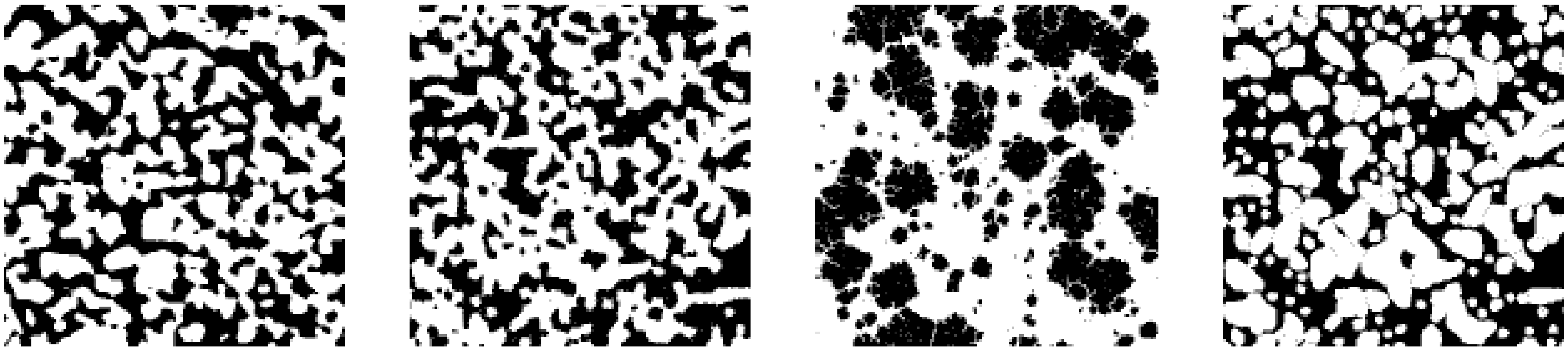
Springer

Context

- **Quantification of Complex Structures**
  - Integral geometry is not enough!

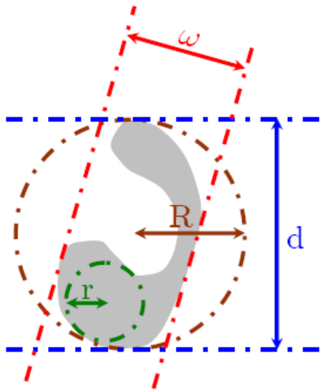


Images (borosilicate glasses) having the same Minkowski functionals (Euler number, area, perimeter)

- **Need of Shape Descriptors**

Shape Diagrams

- **Context: Morphometrical Analysis**
- **Geometric Functionals**
  - ω and d: minimum and maximum Feret diameters
  - r and R: radii of the inscribed and circumscribed circles
  - P: perimeter
  - A: area



- **Geometric Inequalities**
  - Convex and non-convex sets
- **Morphometrical Functionals**

Geometrical functionals	Geometric inequalities	Morphometrical functionals
r, R	$r \leq R$	r / R
ω, R	$\omega \leq 2R$	$\omega / 2R$
A, R	$A \leq \pi R^2$	$A / \pi R^2$
P, R	$P \leq 2\pi R$	$P / 2\pi R$
d, R	$d \leq 2R$	$d / 2R$
r, d	$2r \leq d$	$2r / d$
ω, d	$\omega \leq d$	ω / d
A, d	$4A \leq \pi d^2$	$4A / \pi d^2$
P, d	$P \leq \pi d$	$P / \pi d$
R, d	$\sqrt{3} R \leq d$	$\sqrt{3} R / d$
r, P	$2\pi r \leq P$	$2\pi r / P$
ω, P	$\pi \omega \leq P$	$\pi \omega / P$
A, P	$4\pi A \leq P^2$	$4\pi A / P^2$
d, P	$2d \leq P$	$2d / P$
R, P	$4R \leq P$	$4R / P$
r, A	$\pi r^2 \leq A$	$\pi r^2 / A$
ω, A	$\omega^2 \leq \sqrt{3} A$	$\omega^2 / \sqrt{3} A$
r, ω	$2r \leq \omega$	$2r / \omega$
ω, r	$\omega \leq 3r$	$\omega / 3r$

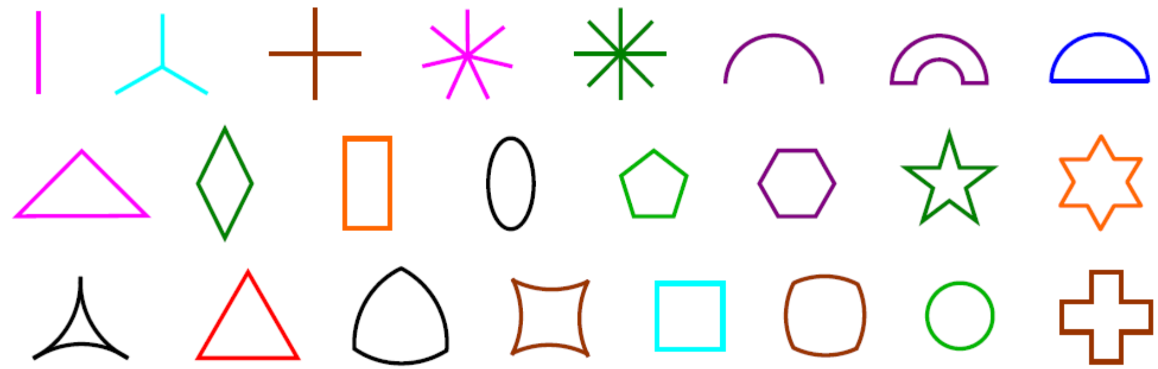
Definition & Properties

- **Definition**

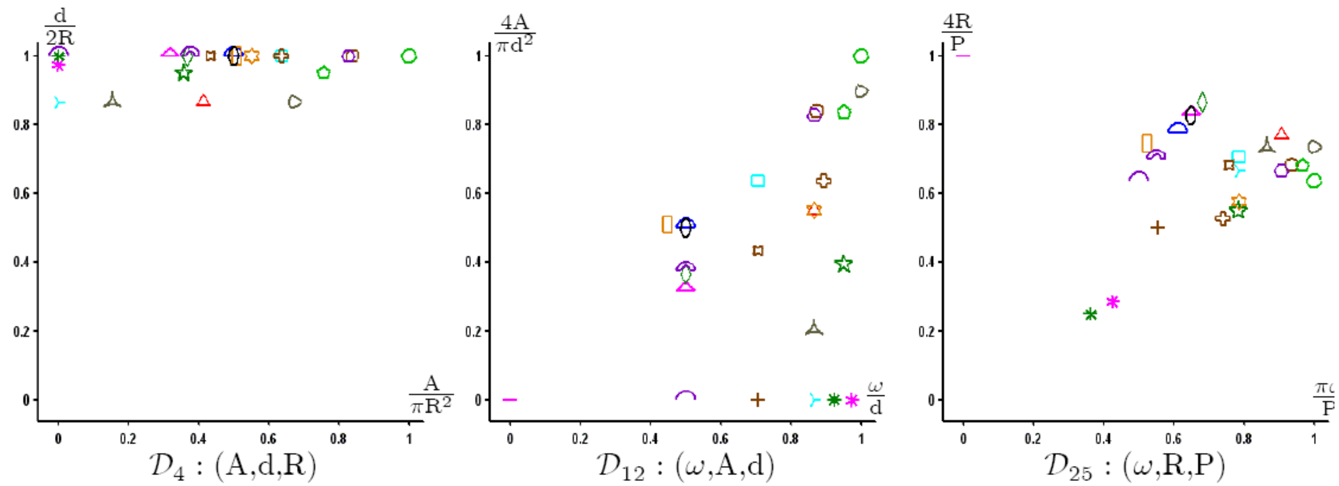
$$\mathcal{D} : \left\{ \begin{array}{ccc} \mathcal{K}(\mathbb{E}^2) & \rightarrow & [0,1]^2 \\ S & \mapsto & (x,y) \end{array} \right.$$

- **Study on different sets**
  - Analytic convex sets
  - Simply connected sets
  - Discretized sets

- **Properties**
  - Dispersion
  - Continuity
  - Overlapping
  - Convexity Discrimination

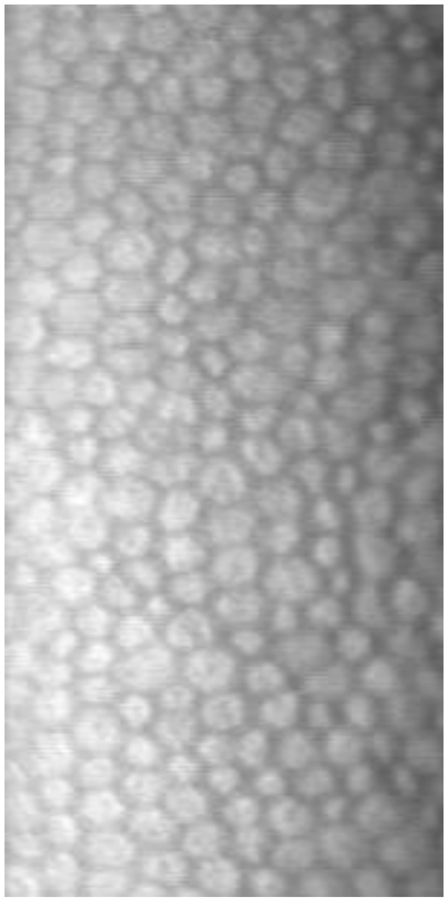


24 analytic sets



Example: Cornea Cell Analysis

- **Shape Homogeneity**



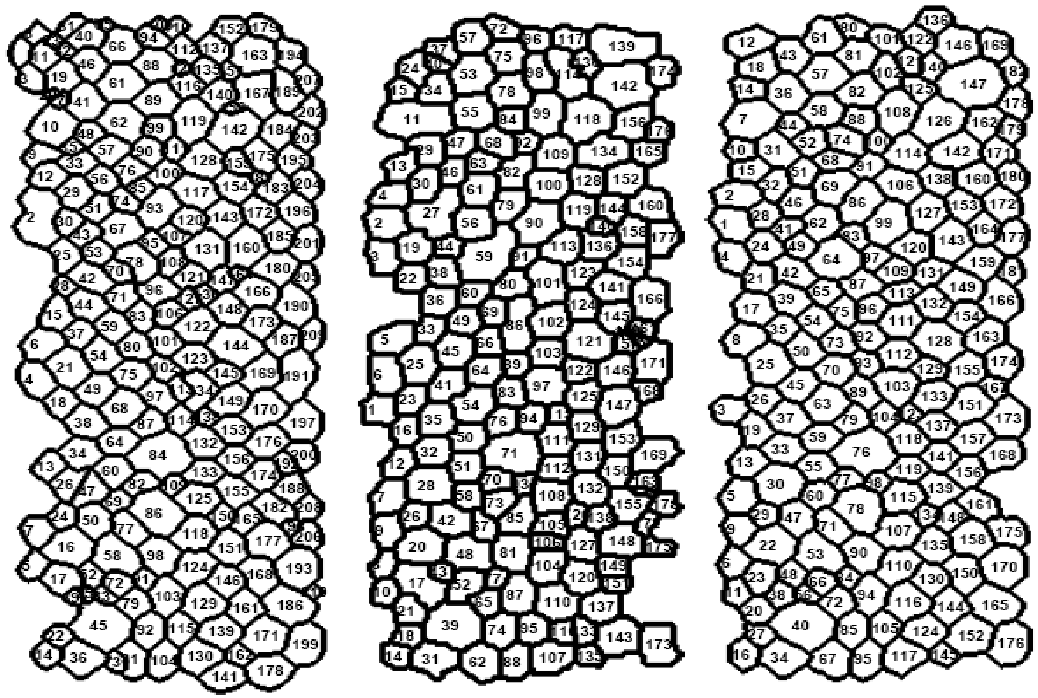


Image A Image B Image C

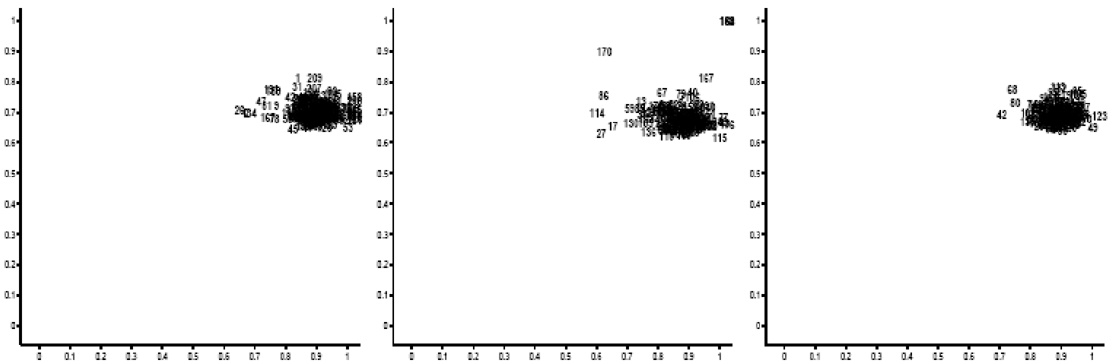


Image A - $\mathcal{D}_{24}$: (A, R, P) Image B - $\mathcal{D}_{24}$: (A, R, P) Image C - $\mathcal{D}_{24}$: (A, R, P)

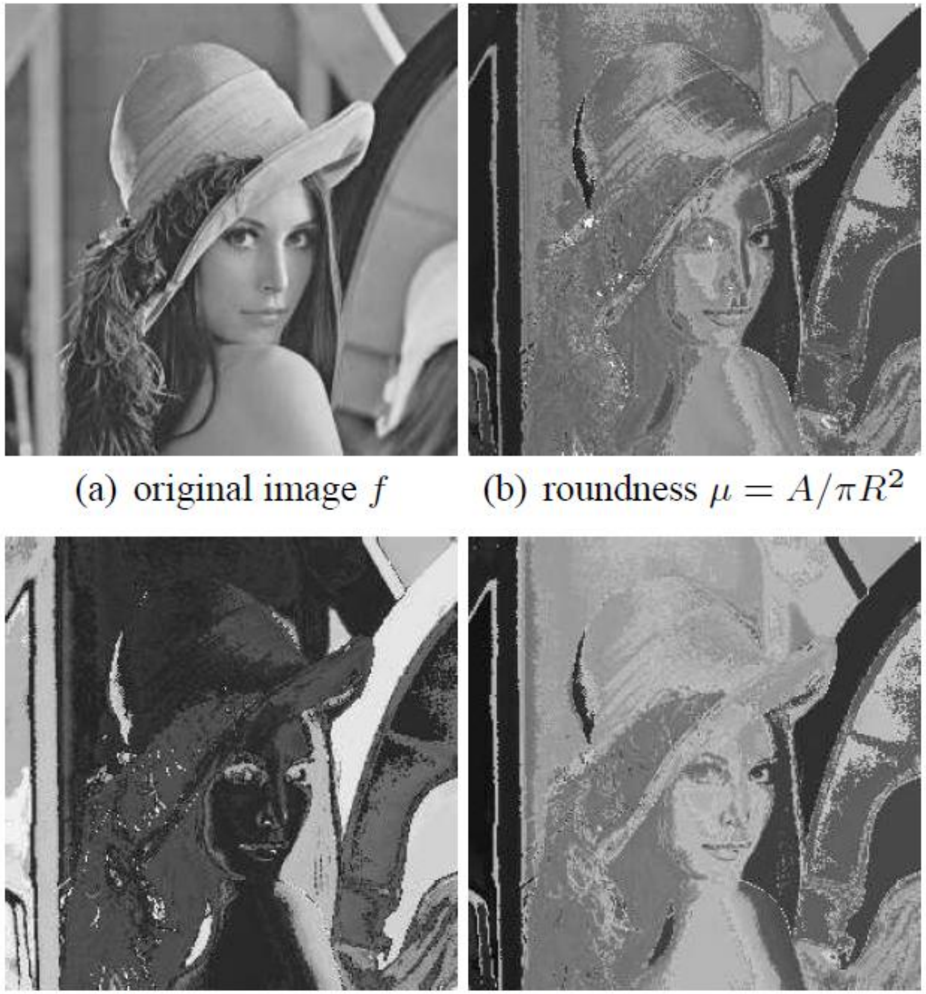
GAN Morphometrical Functionals

- **Gray-Tone Image Morphometry?**

- **Combination**
  - GANs
  - Morphometrical Functionals

- **GAN Morphometrical Functionals**
  - Defined from GAN geometrical functionals

$$\mu^h_{m_{\hexagon}}(x) = A\frac{\gamma^h_{m_{\hexagon}}(x)}{\delta^h_{m_{\hexagon}}(x)} = A\frac{\gamma(V^h_{m_{\hexagon}}(x))}{\delta(V^h_{m_{\hexagon}}(x))}$$



(a) original image f (b) roundness $\mu = A/\pi R^2$

(c) thinness $\mu = 2d/P$ (d) diameter constancy $\mu = \omega/d$

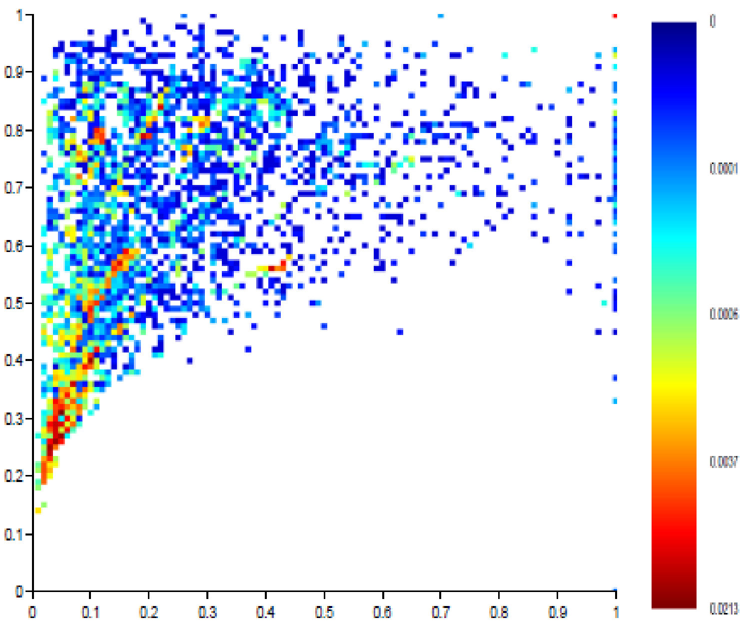
GAN-based Shape Diagrams

- **Definition**

$$\int_{[0,t_1]\times[0,t_2]} \mathcal{D}_k\left(V^h_{m_{\hexagon}}\right)(u)du = \frac{\mathcal{L}^2\left(\left\{x \in \mathrm{D} : \mu^h_{m_{\hexagon}}(x) \leq t_1, \nu^h_{m_{\hexagon}}(x) \leq t_2\right\}\right)}{\mathcal{L}^2(\mathrm{D})}$$

- **Illustration**

(a) Lena f



(b) $\mathcal{D}_{24}\left(V^f_{20}\right)$

Properties

- **Equality between Iso-Valued Points**

$$\begin{pmatrix} x_1 \in V^h_{m_{\hexagon}}(x_2) \\ h(x_1) = h(x_2) \end{pmatrix} \Rightarrow \mu^h_{m_{\hexagon}}(x_1) = \mu^h_{m_{\hexagon}}(x_2) \quad (1)$$

- **Translation Invariance**

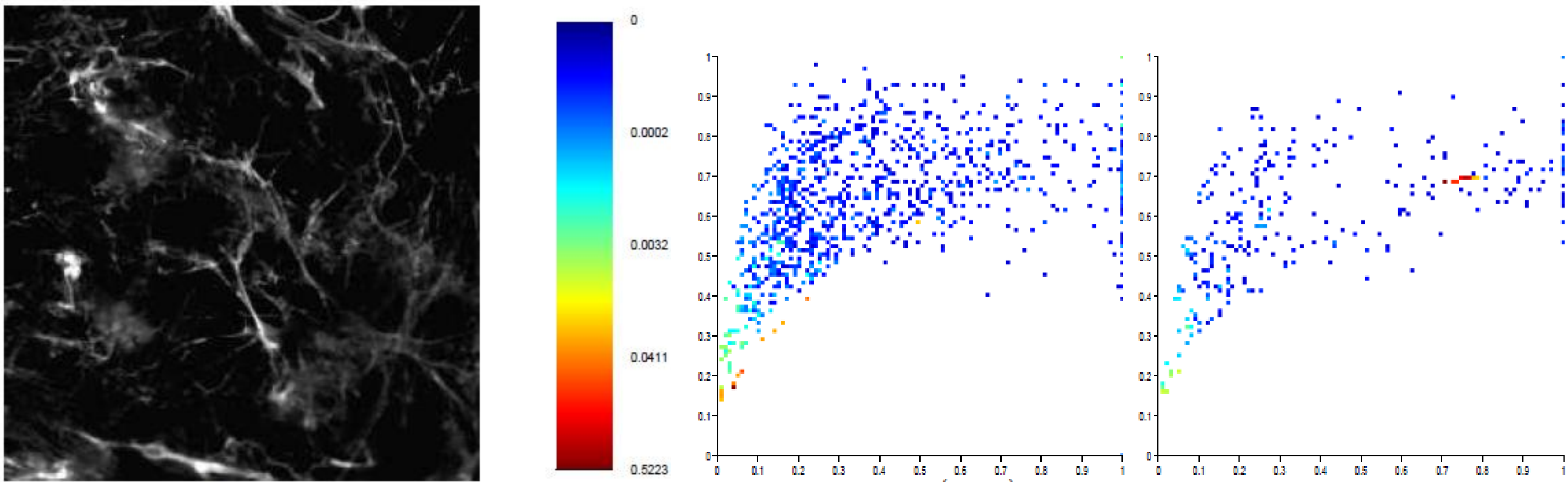
$$\forall c \in \mathbb{E} : \mu^{h \oplus c}_{m_{\hexagon}}(x) = \mu^h_{m_{\hexagon}}(x)$$

- **Multiplication Compatibility**

$$\forall \alpha \in \mathbb{R}^{+*} : \mu^{\alpha \otimes h}_{m_{\hexagon}}(x) = \mu^h_{\frac{1}{\alpha} \otimes m_{\hexagon}}(x)$$

Illustration

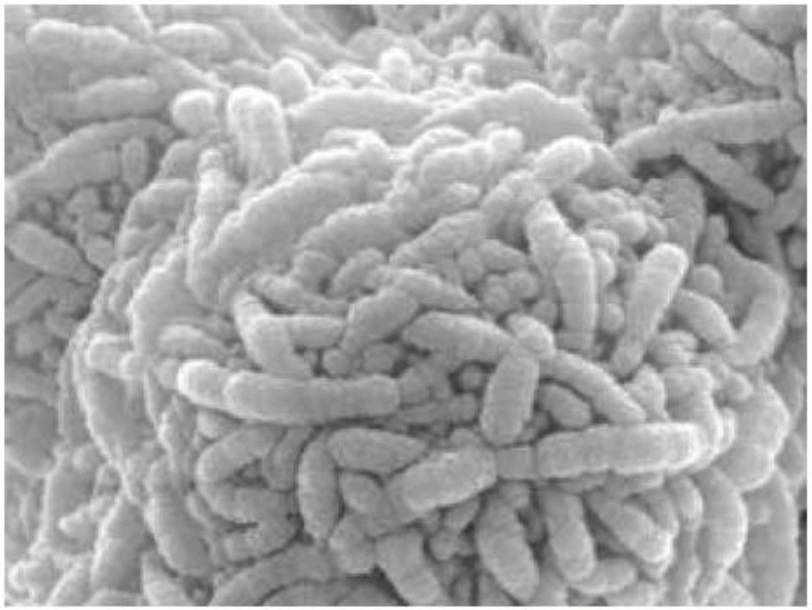
- **Impact of the Criterion Mapping**
  - Fibronectin image analysis



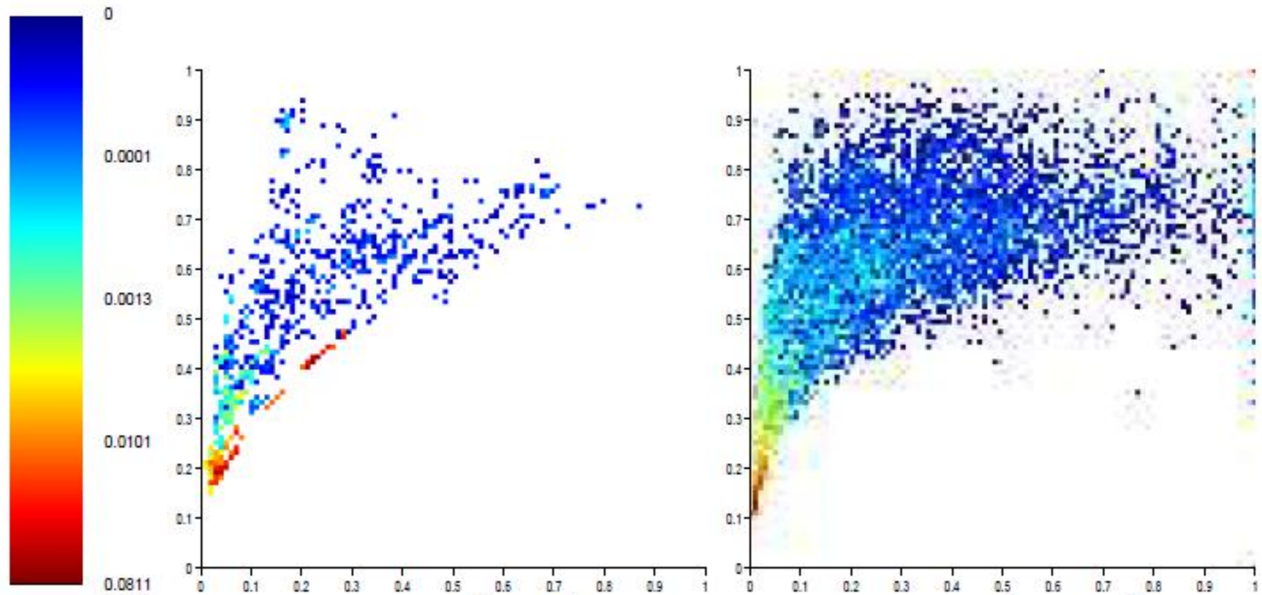
(a) Fibronectin f (b) $\mathcal{D}_{24}\left(V_{20}^{f}\right)$ (c) $\mathcal{D}_{24}\left(V_{20}^{c}\right)$

Illustration

- **Impact of the GLIP Framework**
  - Zinc Sulfide (ZnS) image analysis



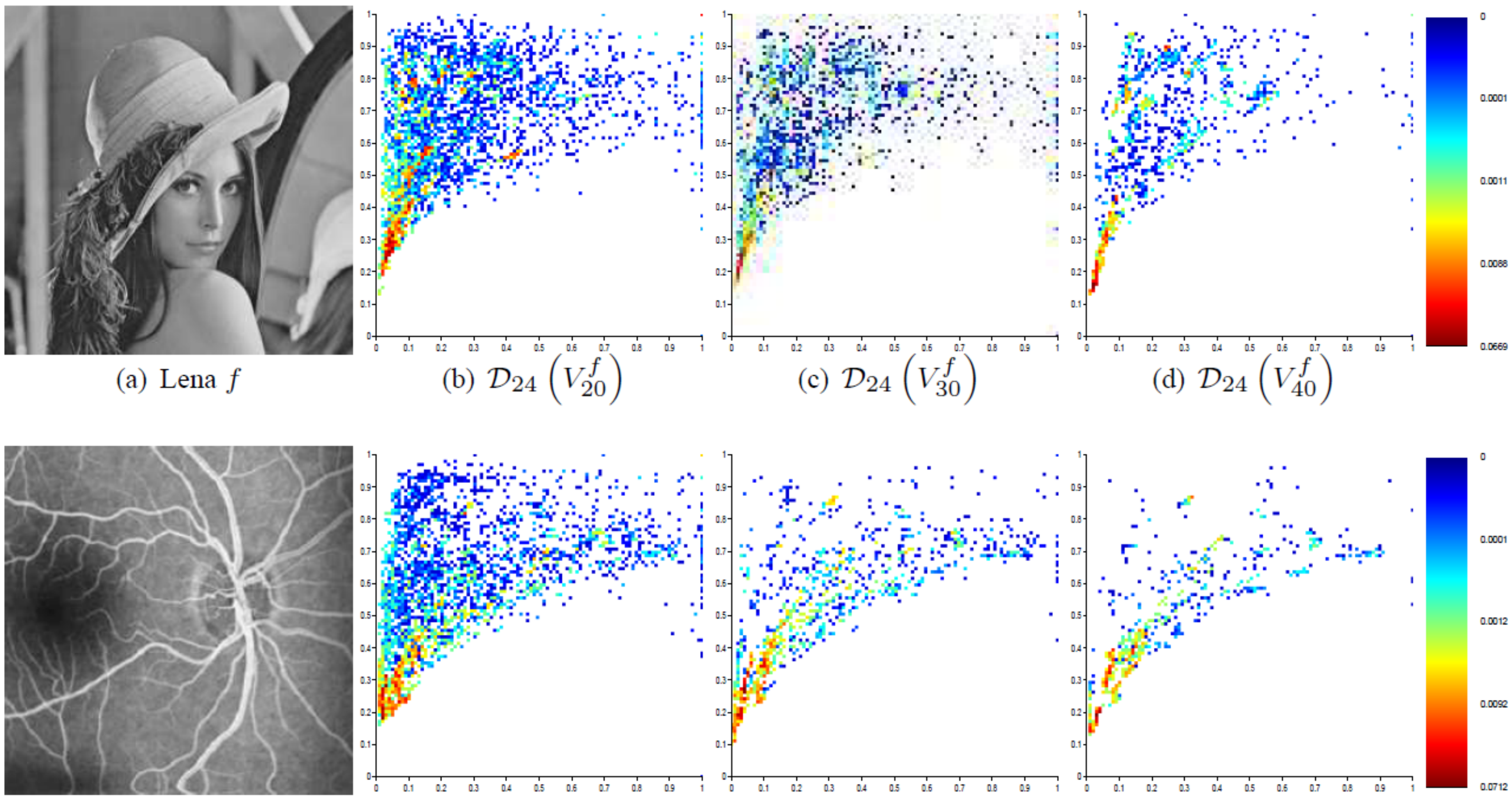
(a) Zinc sulfide f



(b) $\mathcal{D}_{24}\left(V^{f}_{50}\right)$ (c) $\mathcal{D}_{24}\left(V^{f}_{50_{\triangle}}\right)$

Illustration

- **Impact of the Homogeneity Tolerance**

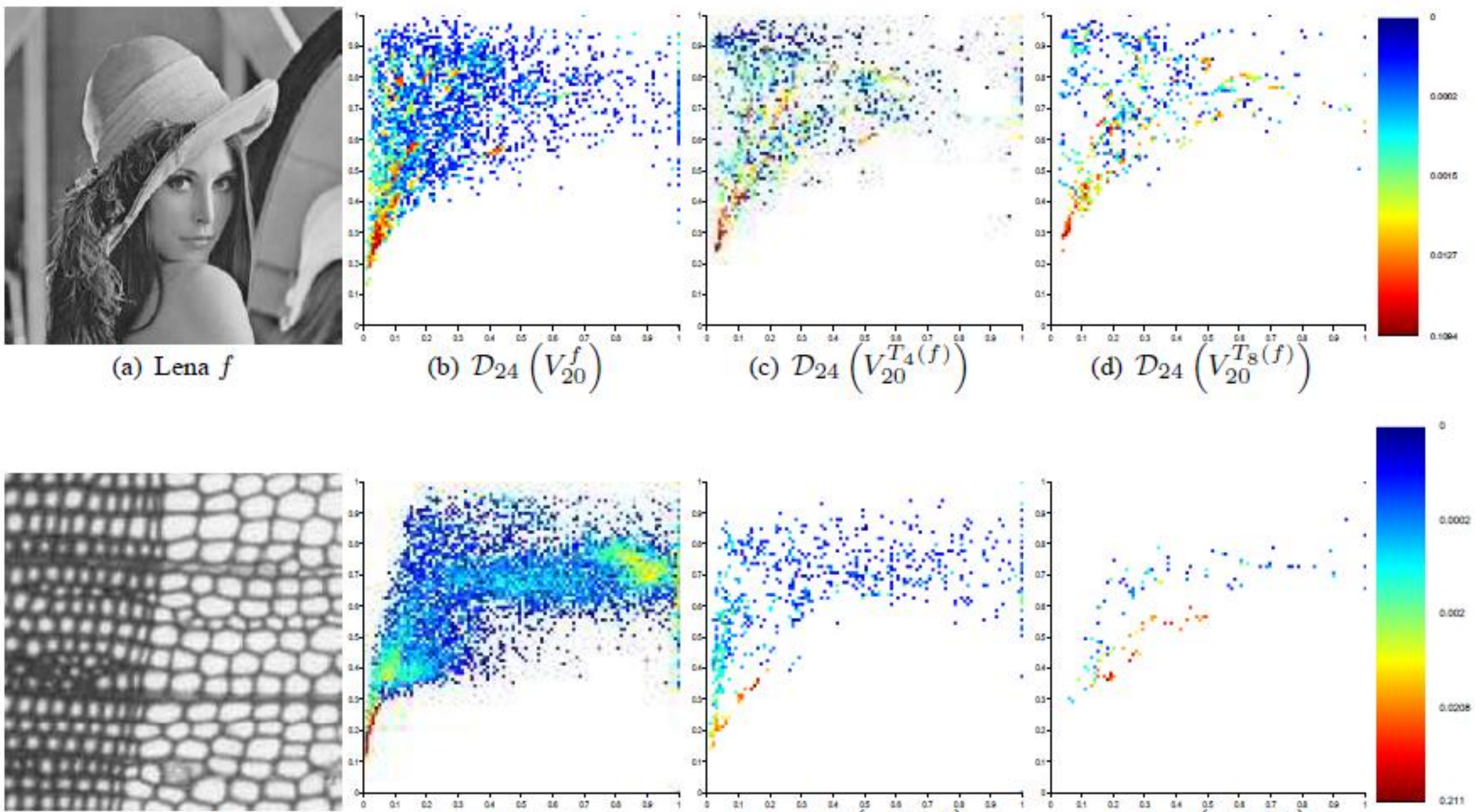


(a) Lena f (b) $\mathcal{D}_{24}\left(V_{20}^{f}\right)$ (c) $\mathcal{D}_{24}\left(V_{30}^{f}\right)$ (d) $\mathcal{D}_{24}\left(V_{40}^{f}\right)$

(e) retina g (f) $\mathcal{D}_{24}\left(V_{20}^{g}\right)$ (g) $\mathcal{D}_{24}\left(V_{30}^{g}\right)$ (h) $\mathcal{D}_{24}\left(V_{40}^{g}\right)$

Illustration

- **Impact of a Morphological Transformation (Dilation)**

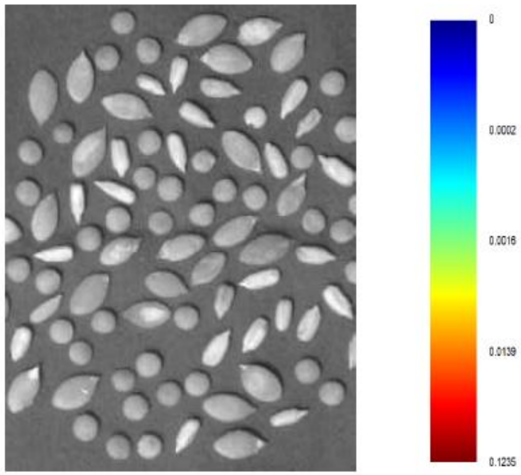


(a) Lena f (b) $\mathcal{D}_{24}\left(V_{20}^{f}\right)$ (c) $\mathcal{D}_{24}\left(V_{20}^{T_4(f)}\right)$ (d) $\mathcal{D}_{24}\left(V_{20}^{T_8(f)}\right)$

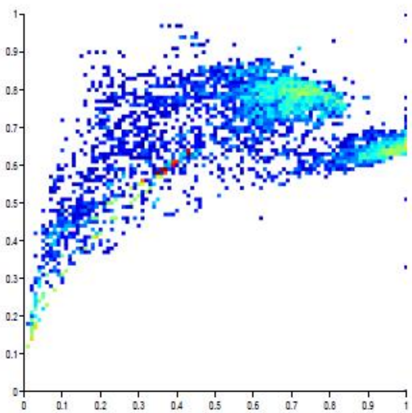
(e) wood g (f) $\mathcal{D}_{24}\left(V_{20}^{g}\right)$ (g) $\mathcal{D}_{24}\left(V_{20}^{T_4(g)}\right)$ (h) $\mathcal{D}_{24}\left(V_{20}^{T_8(g)}\right)$

Application Example

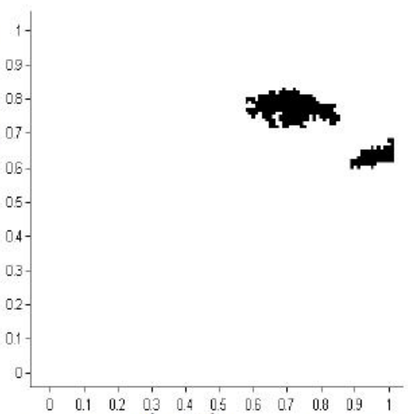
- **Classification of Grain Shapes**



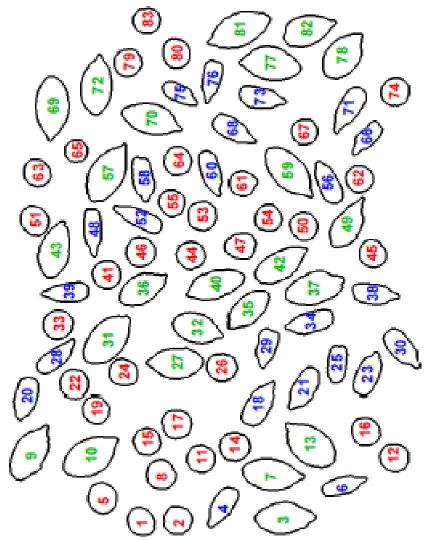
(a) Grains f



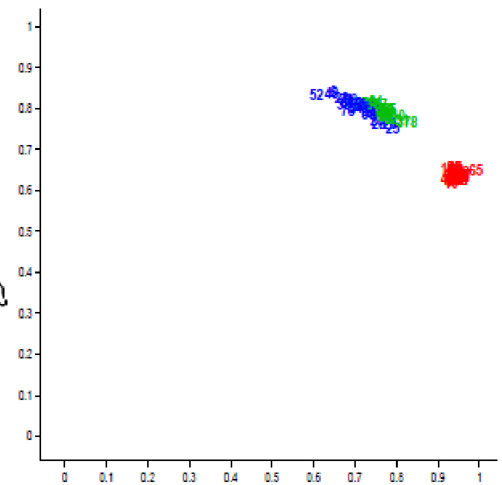
(b) $\mathcal{D}_{24}\left(V_{40}^{f}\right)$



(c) $\mathcal{D}_{24}\left(V_{40}^{f}\right)$ thresholded



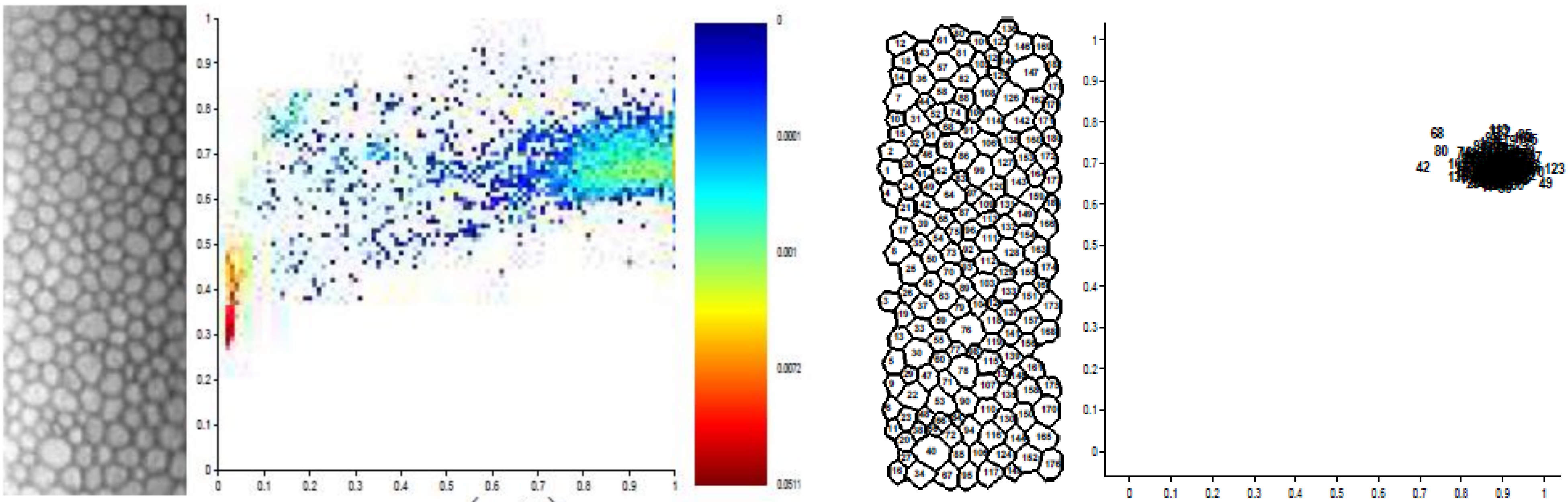
(d) segmented image



(e) $\mathcal{D}_{24}$

Application Example

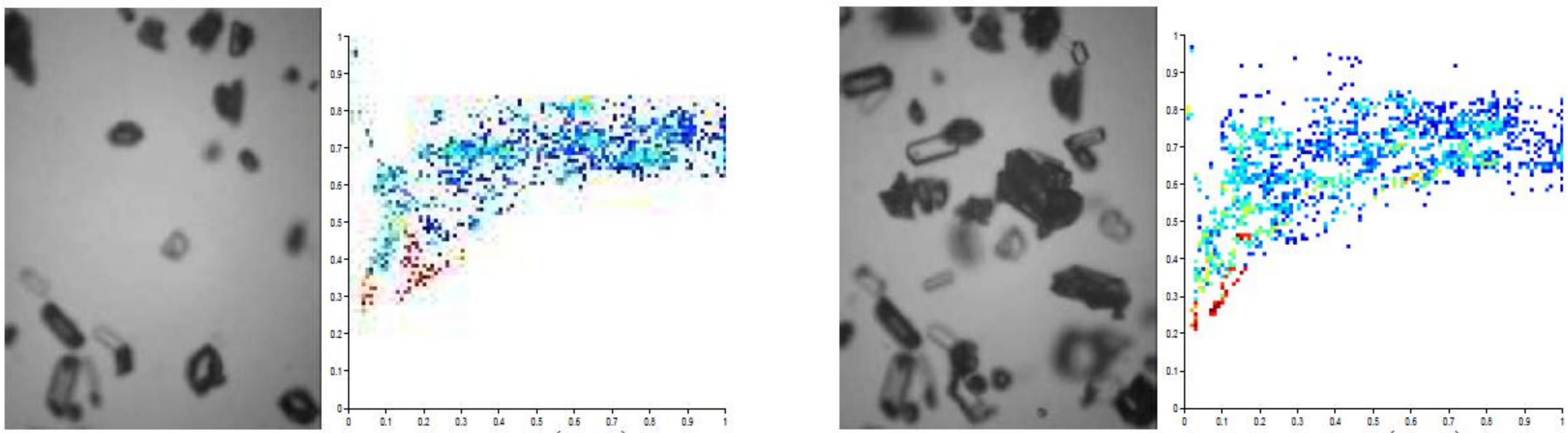
- **Homogeneity of Corneal Cell Shapes**



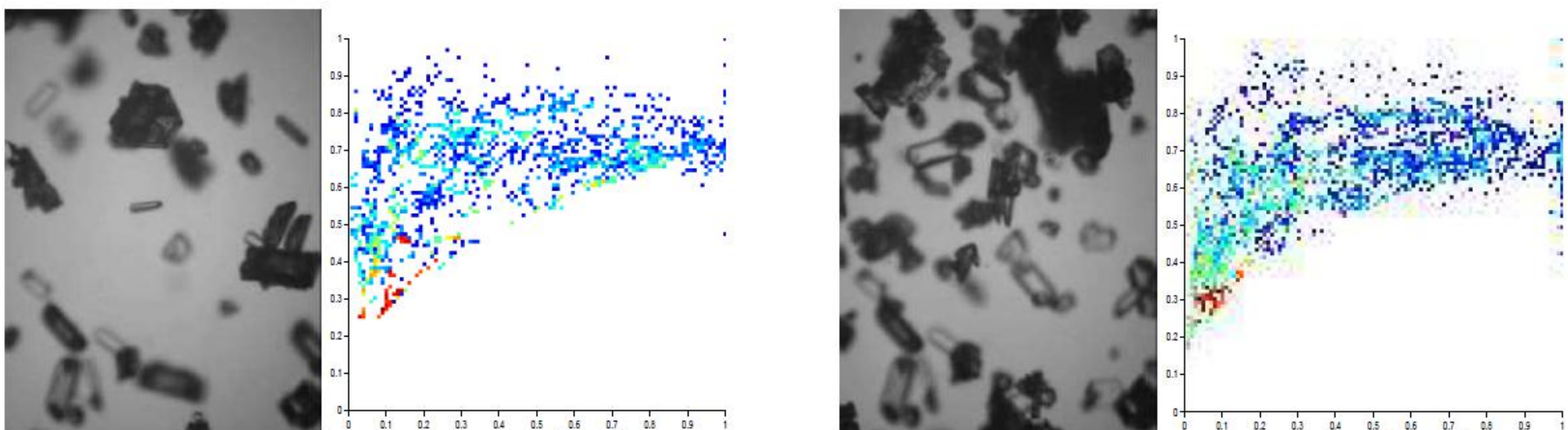
(a) Cornea f (b) $\mathcal{D}_{24}\left(V^{f}_{20}\right)$ (c) segmented image (d) $\mathcal{D}_{24}$

Application Example

- **Crystal Shape Evolution**



(a) f_0: time t_0 (b) $\mathcal{D}_{24}\left(V_{20}^{f_0}\right)$ (e) f_4: time $\sim t_0 + 4s$ (f) $\mathcal{D}_{24}\left(V_{20}^{f_4}\right)$



(c) f_2: time $\sim t_0 + 2s$ (d) $\mathcal{D}_{24}\left(V_{20}^{f_2}\right)$ (g) f_6: time $\sim t_0 + 6s$ (h) $\mathcal{D}_{24}\left(V_{20}^{f_6}\right)$

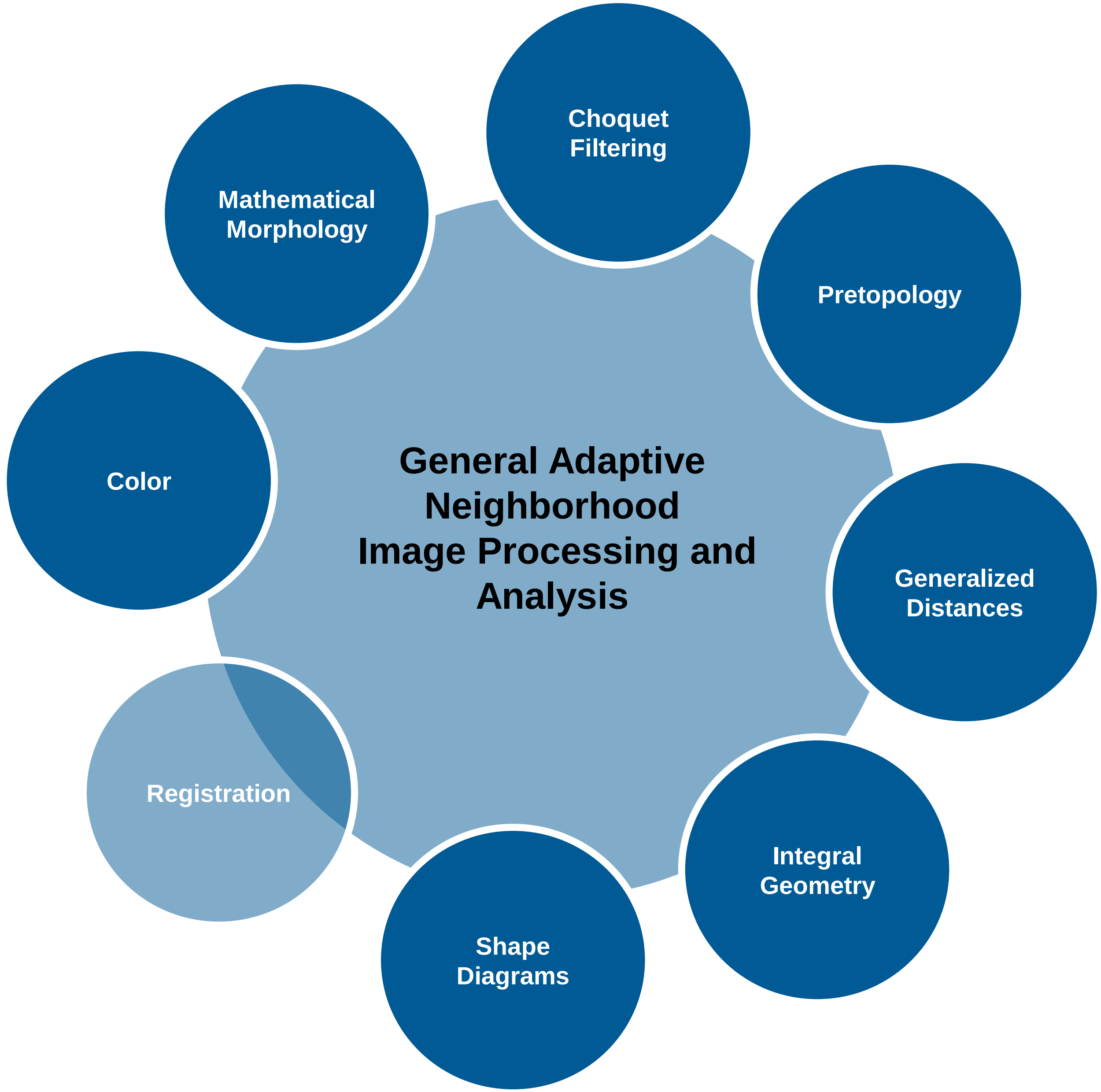
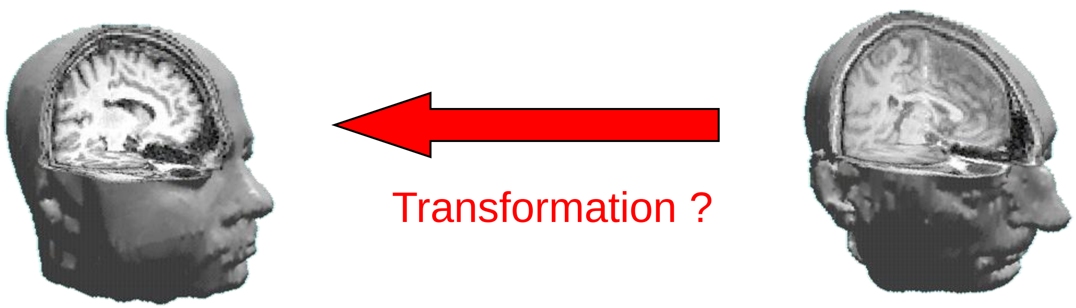
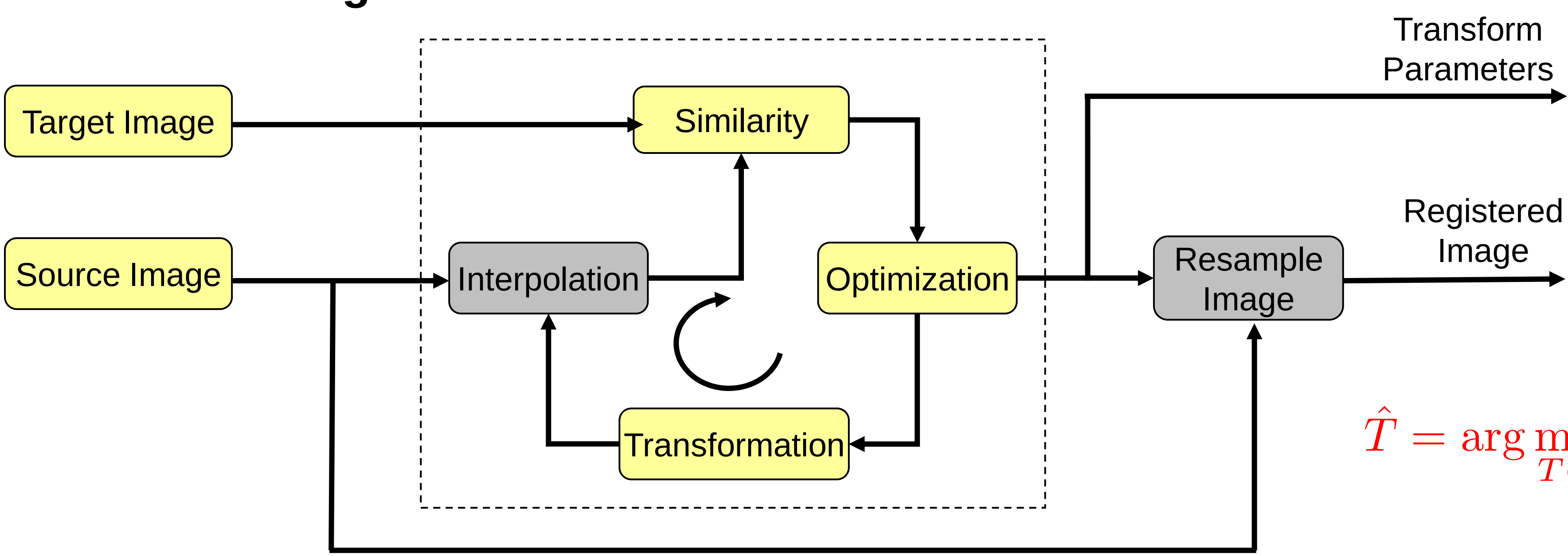

General Adaptive Neighborhood Image Processing and Analysis
Choquet Filtering
Pretopology
Generalized Distances
Integral Geometry
Shape Diagrams
Registration
Color
Mathematical Morphology

Image Registration

- **Process of Putting Two Images into Spatial Correspondence**



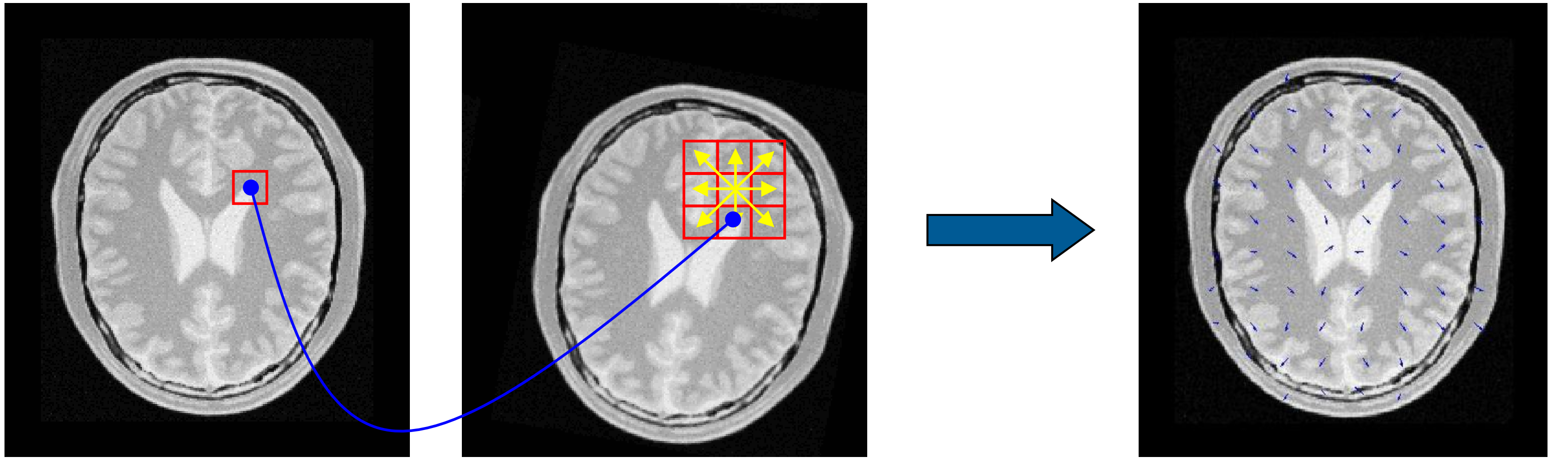
- **General Registration Scheme**



$$\hat{T} = \arg\max_{T \in E} C(I, J \circ T)$$

Block Matching

- **Displacement Field Computation**

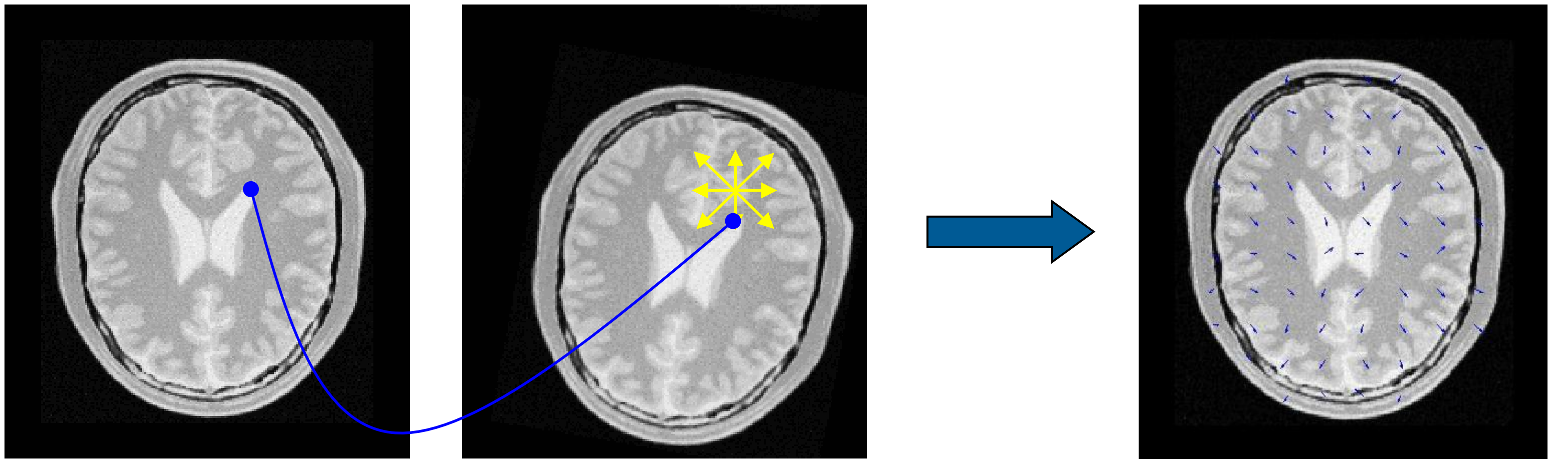


- **Similarity Measures between Blocks**
  - Sum of (absolute) squared differences
  - Correlation coefficient
  - Mutual Information

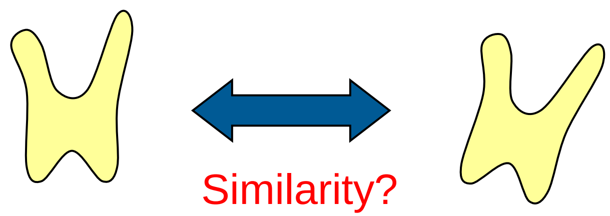
- **Transformation Estimation:** $\boxed{\hat{T} = \arg\min_{T} \sum_{i=1}^{n} d\left(T\left(m_i\right) - m'_i\right)}$

GAN Block Matching

- Displacement Field Computation

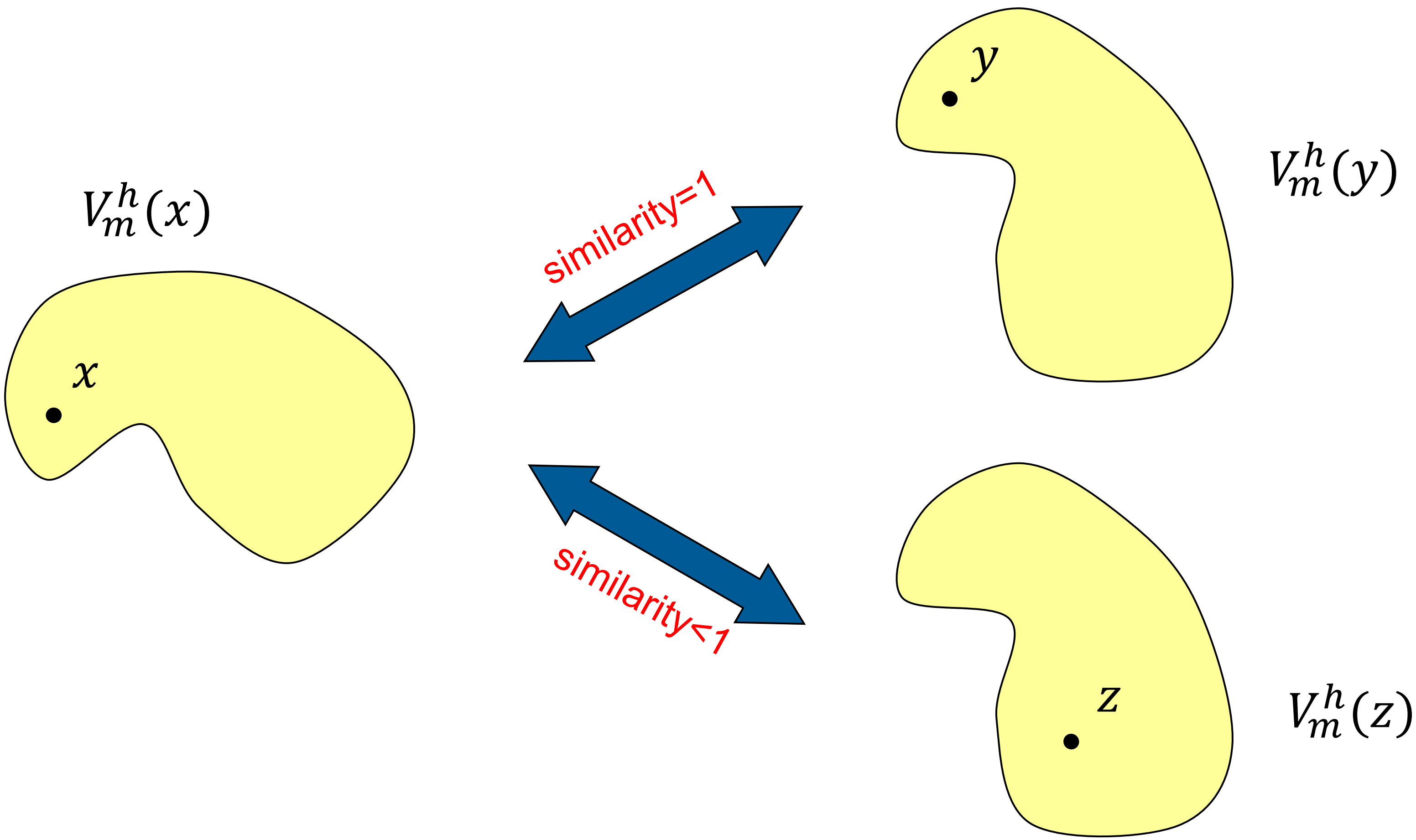


- Similarity Measures between GANs
  - Hausdorff (not robust, not invariant)
  - Moments: Hu, Flusser, Zernike, … (invariant, not robust)
  - Fourier descriptors (invariant, only star-shaped metric)
  - Shape matrices (invariant, time consuming)



GAN Block Matching

- **Not Only Shape Similarity Measures!**

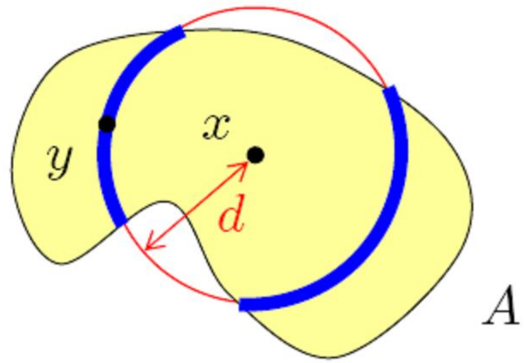


GAN Block Matching

- **Proposed Measure**
  - Normalized histogram of distances from points in A to its barycenter

$$h_A^*(d) = \frac{1}{\mathcal{H}^2(A)} \mathcal{H}^1(A \cap \partial B(x_A, d))$$

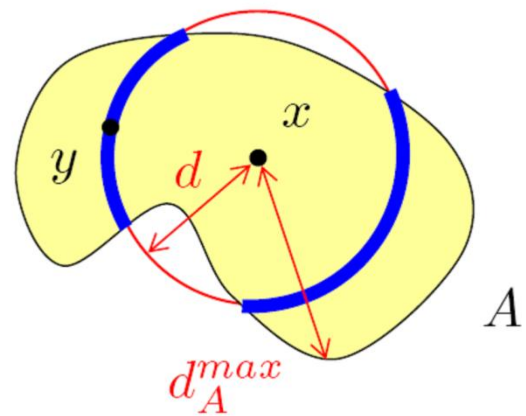
  - Invariance to rigid transformation



- **Extension for Scaling Invariance**

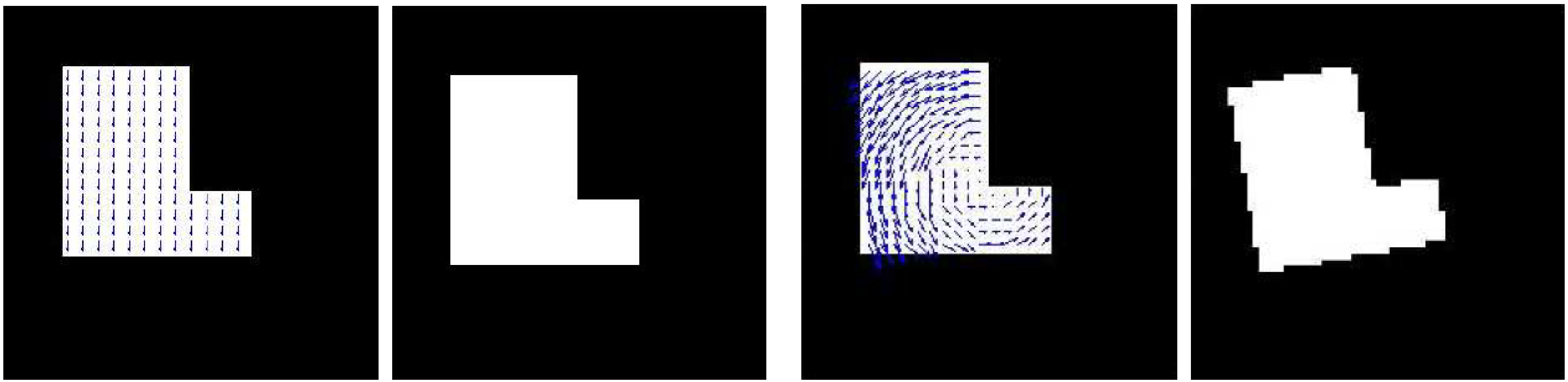
$$h_A^*(d) = \frac{1}{\mathcal{H}^2(A)} \mathcal{H}^1\left(\frac{1}{d_A^{max}} A \cap \partial B(x_A, d)\right)$$

$$d_A^{max} = \max_{y \in A} d(x_A, y)$$



Illustration

- **Model Image**



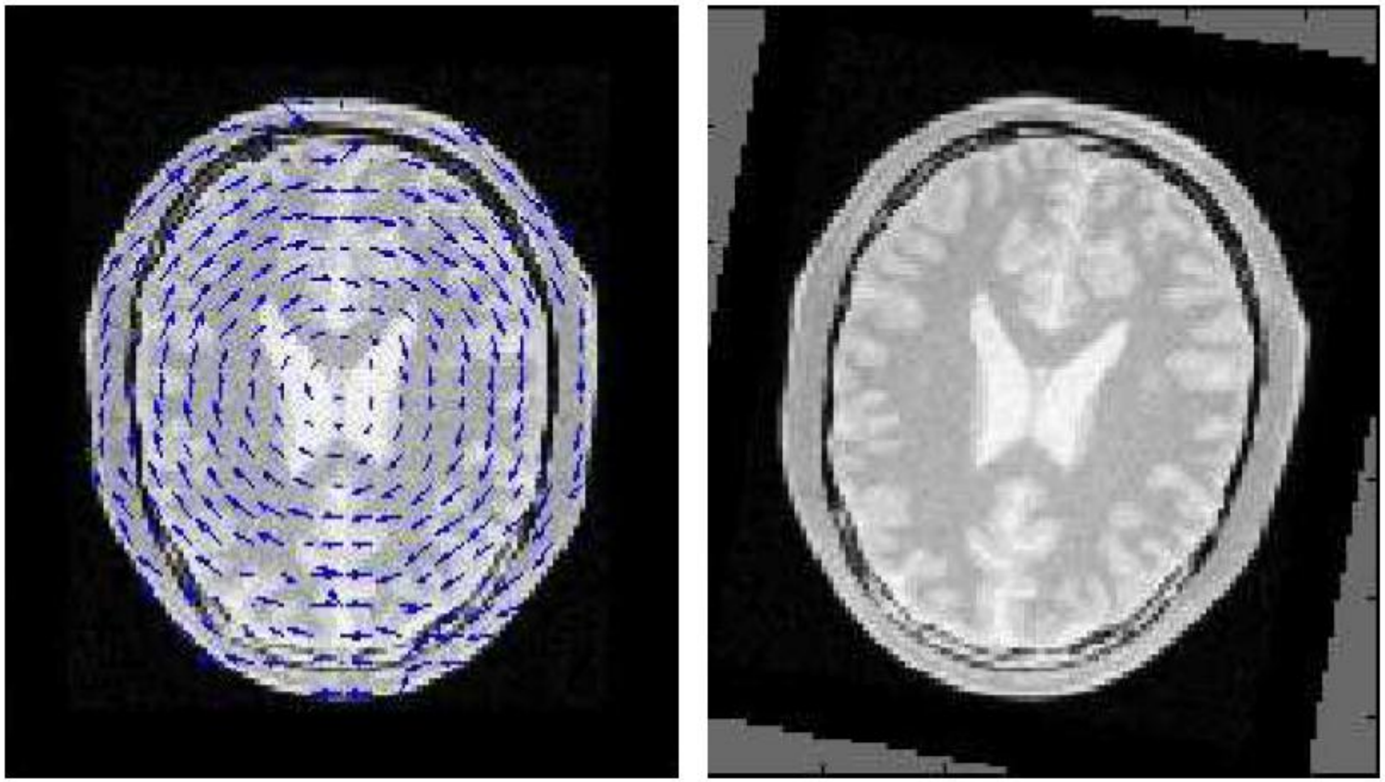
(a) floating image J_1 (b) reference image I_1 (c) floating image J_2 (d) reference image I_2

Translation

Rotation

Illustration

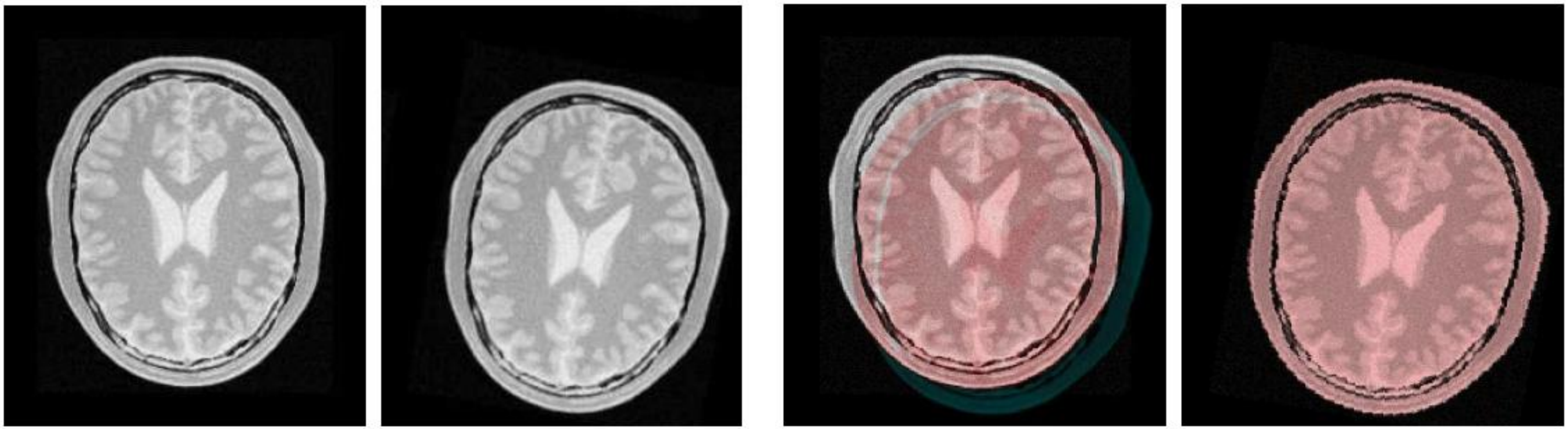
- MR Brain Image



(a) floating image J (b) reference image I

Application Example

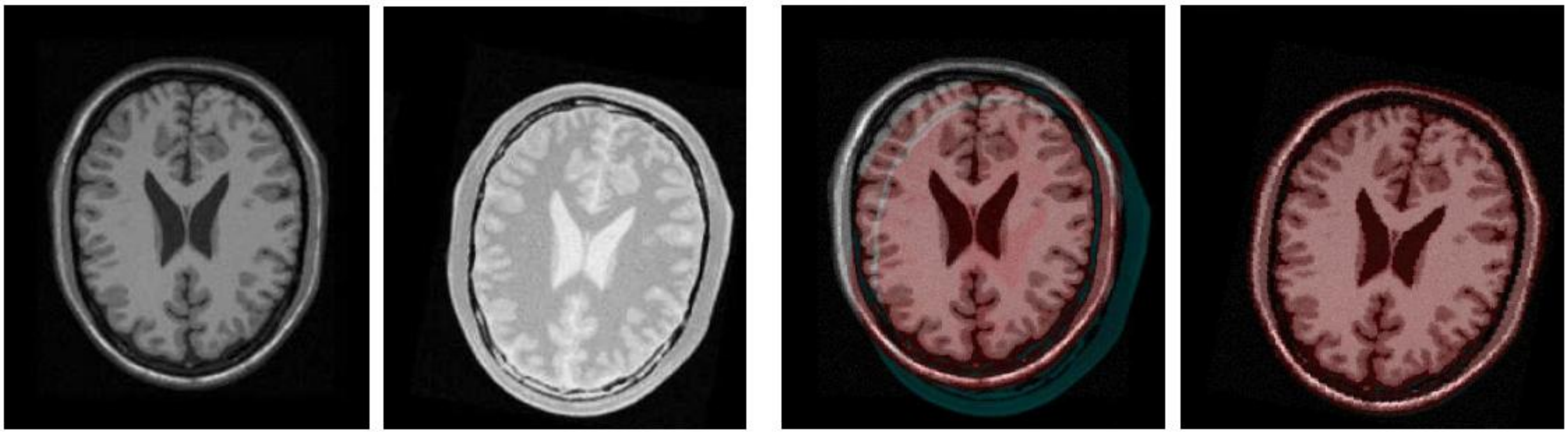
- Monomodal Brain Image Registration



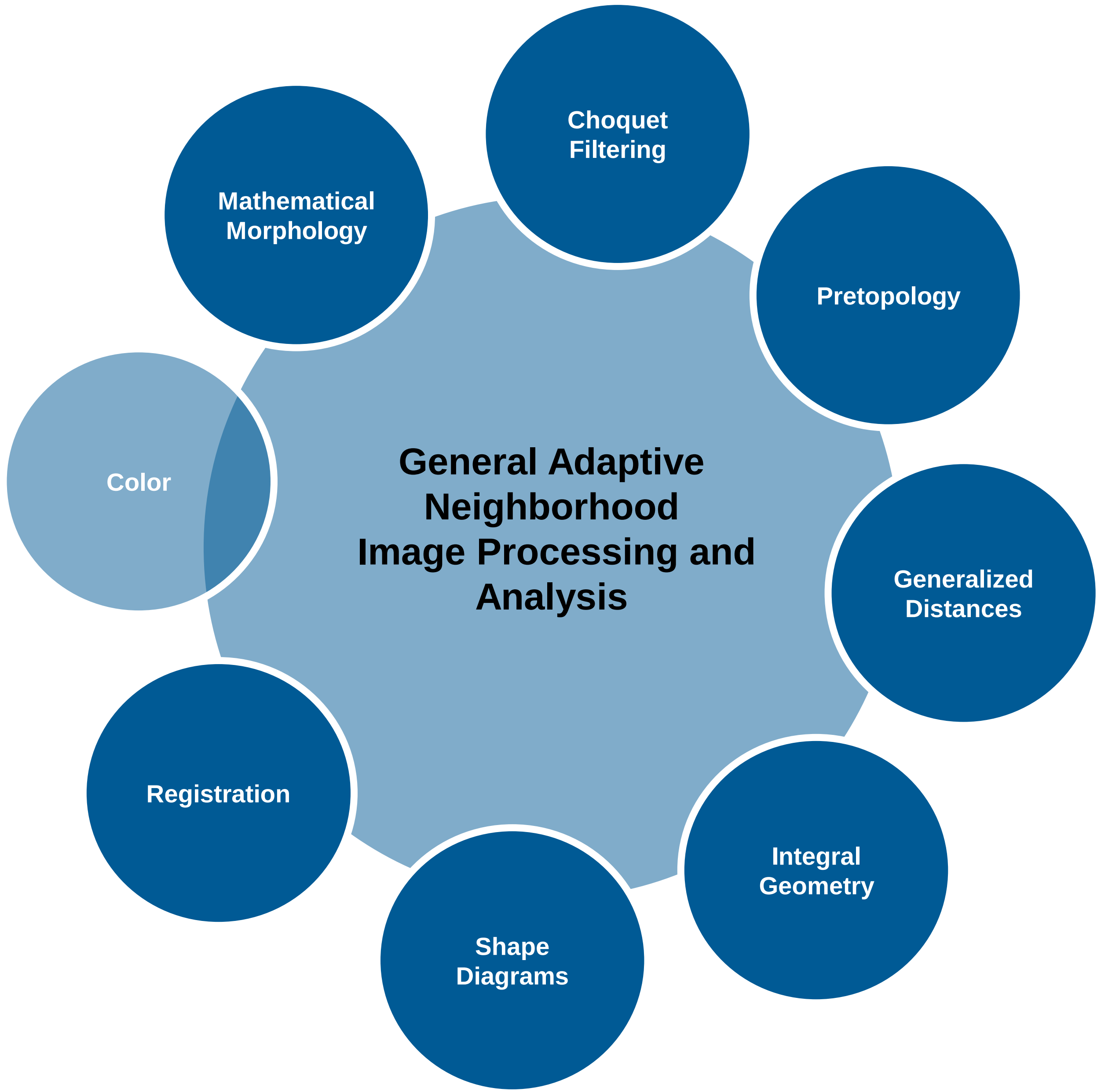
(a) PD MR brain image (b) PD MR brain image (c) before registration (d) after registration

Application Example

- **Multimodal MR Brain Image Registration**



(a) T1 MR brain image (b) PD MR brain image (c) before registration (d) after registration


Choquet Filtering
Mathematical Morphology
Pretopology
Color
General Adaptive Neighborhood Image Processing and Analysis
Generalized Distances
Registration
Integral Geometry
Shape Diagrams

Main Publications

[ALCIP 2013]

Spatially Adaptive Color Image Processing

Johan Debayle and Jean-Charles Pinoli

Abstract This chapter is focused on spatially adaptive image processing for color images in the context of the General Adaptive Neighborhood Image Processing (GANIP) approach. The GANIP was first defined for gray-tone images and is here extended to color images. A set of local adaptive neighborhoods is defined for each image point, depending on the color intensity function of the image. These adaptive neighborhoods are then used as spatially adaptive operational windows for defining adaptive Choquet filters and adaptive morphological filters. The resulting adaptive operators are successfully applied and compared with the classical operators for image restoration, enhancement and segmentation of color images.

1 Introduction

1.1 The General Adaptive Neighborhood Image Processing (GANIP) approach

The General Adaptive Neighborhood paradigm has been introduced [11] in order to propose an original image representation for adaptive processing and analysis of gray-tone images. The central idea is the notion of adaptivity which is simultaneously associated to the analyzing scales, the spatial structures and the intensity values of the image to be addressed.

In the so-called General Adaptive Neighborhood Image Processing (GANIP) approach [11, 12], a set of General Adaptive Neighborhoods (GANs set) is identified around each point in the image to be analyzed. A GAN is a subset of the spatial

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1

Color GANs

Definition

$$V^h_{m_\hexagon}(x) = C_{h^{-1}([h(x)\ominus m_\hexagon, h(x)\oplus m_\hexagon])}(x)$$

$$\begin{aligned} & h^{-1}([h(x)\ominus m_\hexagon, h(x)\oplus m_\hexagon]) \\ = \quad & \{y; h(y) \in [h(x)\ominus m_\hexagon, h(x)\oplus m_\hexagon]\} \\ = \quad & \{y; \boxed{|h(y)\ominus h(x)|_\hexagon \leq m_\hexagon}\} \end{aligned}$$

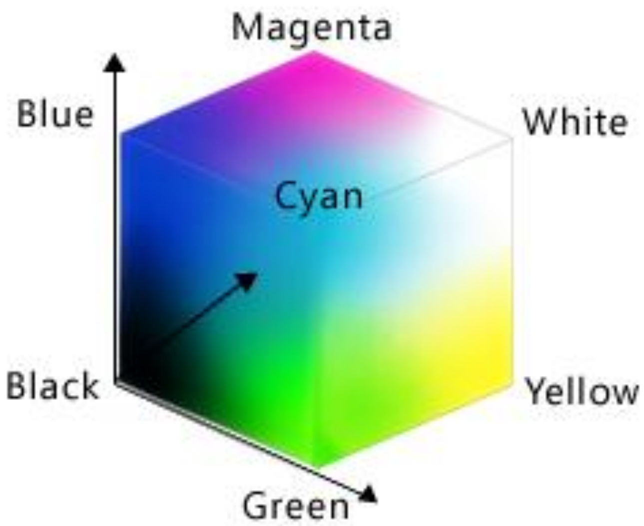
-> distance

Extension to Color Images

- Color Space?
- Which Distance?
- Color GLIP framework?

RGB Color Space

- **Illustration**



- **Usual Distance**

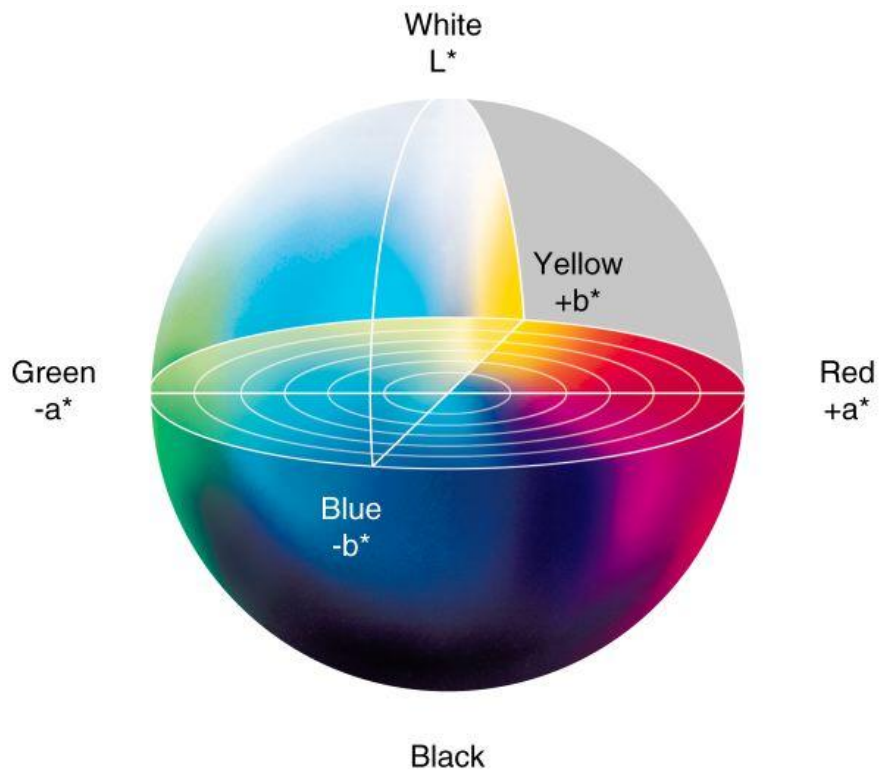
$$||c_1 - c_2||_{RGB} = \sqrt{(c_1^R - c_2^R)^2 + (c_1^G - c_2^G)^2 + (c_1^B - c_2^B)^2}$$

L*a*b* Color Space

- **Definition**

$$\begin{cases} L^* = 116 f(Y/Y_n) - 16, \\ a^* = 500(f(X/X_n) - f(Y/Y_n)), \\ b^* = 200(f(Y/Y_n) - f(Z/Z_n)). \end{cases}$$

$$f(t) = \begin{cases} t^{\frac{1}{3}} & \text{if } t > (\frac{6}{29})^3, \\ \frac{1}{3}(\frac{29}{6})^2 t + \frac{4}{29} & \text{otherwise.} \end{cases}$$



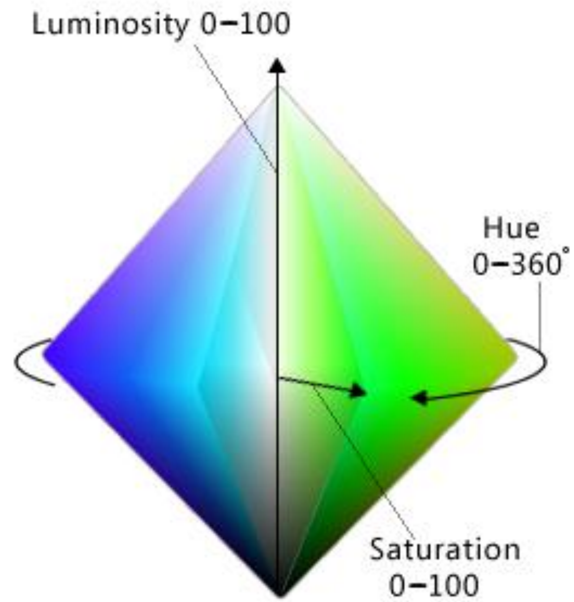
- **Usual Distance**

$$||c_1 - c_2||_{L^*a^*b^*} = \sqrt{(c_1^{L^*} - c_2^{L^*})^2 + (c_1^{a^*} - c_2^{a^*})^2 + (c_1^{b^*} - c_2^{b^*})^2}$$

HSL Color Space

- **Definition**

$$
\begin{cases}
H = 60^\circ \times \begin{cases} \dfrac{G-B}{\max(R,G,B)-\min(R,G,B)} & \text{if } R = \max(R,G,B), \\ \dfrac{B-R}{\max(R,G,B)-\min(R,G,B)} + 2 & \text{if } G = \max(R,G,B), \\ \dfrac{R-G}{\max(R,G,B)-\min(R,G,B)} + 4 & \text{if } B = \max(R,G,B). \end{cases} \\
S = \begin{cases} \dfrac{\max(R,G,B)-\min(R,G,B)}{\max(R,G,B)+\min(R,G,B)} & \text{if } L \leq 0.5, \\ \dfrac{\max(R,G,B)-\min(R,G,B)}{2-\max(R,G,B)-\min(R,G,B)} & \text{if } L > 0.5. \end{cases} \\
L = \dfrac{\max(R,G,B)+\min(R,G,B)}{2}
\end{cases}
$$



- **Usual Distance**

$$||c_1 - c_2||_{HSL} = \sqrt{(c_1^L - c_2^L)^2 + (c_1^S)^2 + (c_2^S)^2 - 2c_1^S c_2^S \cos(c_1^H \div c_2^H)}$$

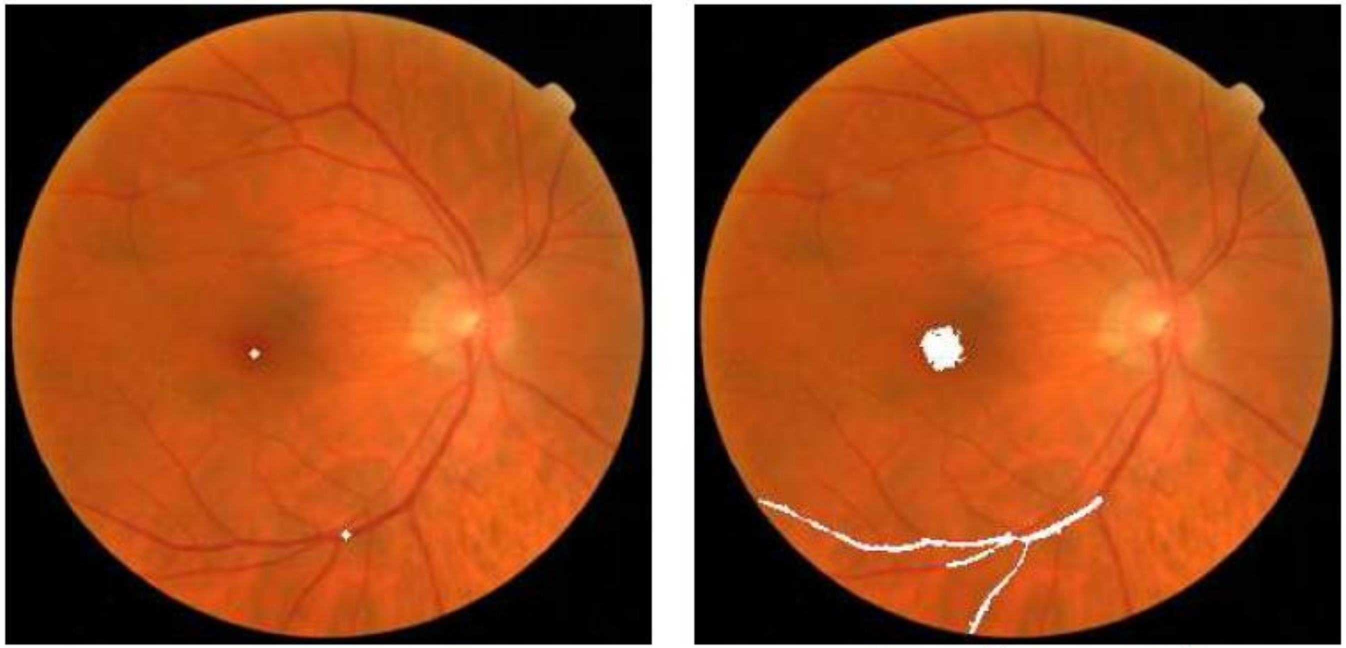
$$h_1 \div h_2 = \begin{cases} |h_1 - h_2| & \text{if } |h_1 - h_2| \leq 180^\circ, \\ 360^\circ - |h_1 - h_2| & \text{if } |h_1 - h_2| > 180^\circ. \end{cases}$$

Color GANs within the CLIP Framework

- **Definition**

$$V_m^{f_0}(x) = C_{\{y \in D; ||f_0(y) - f_0(x)||_E \leq m\}}(x)$$

- **Illustration**



(a) original image f_0 with two seed points x and y (white dots)

(b) CANs $V_{25}^{f_0}(x)$ and $V_{25}^{f_0}(y)$

Properties

- **Reflexivity**

$$x \in V_m^{f_0}(x)$$

- **Increasing with Respect to Homogeneity Tolerance**

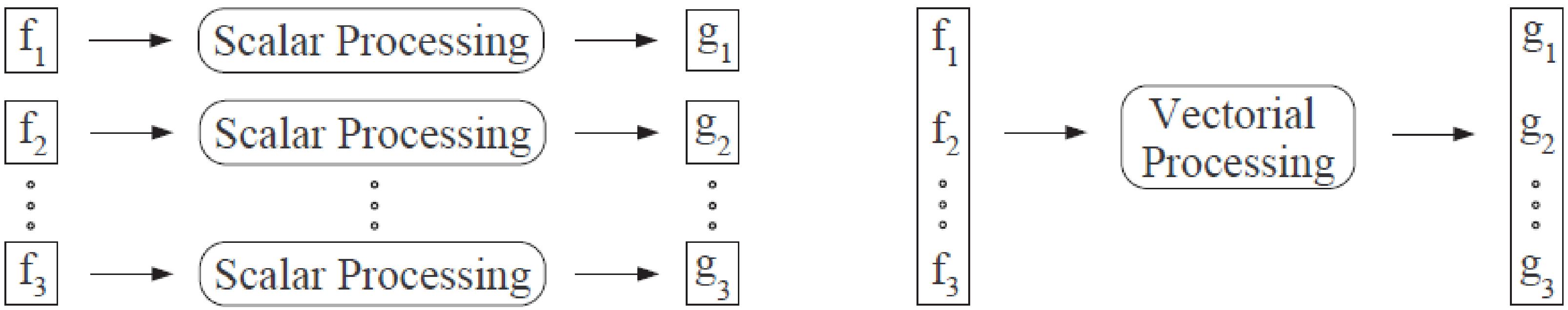
$$\begin{pmatrix} (m_1, m_2) \in \mathbb{R}^+ \times \mathbb{R}^+ \\ m_1 \leq m_2 \end{pmatrix} \Rightarrow V_{m_1}^{f_0}(x) \subseteq V_{m_2}^{f_0}(x)$$

- **Equality between Iso-Valued Points**

$$\begin{pmatrix} (x, y) \in D^2 \\ x \in V_m^{f_0}(y) \\ f_0(x) = f_0(y) \end{pmatrix} \Rightarrow V_m^{f_0}(x) = V_m^{f_0}(y)$$

GAN Color Image Processing

- **Marginal or Vectorial Processing?**



  - Marginal processing leads to false colors!

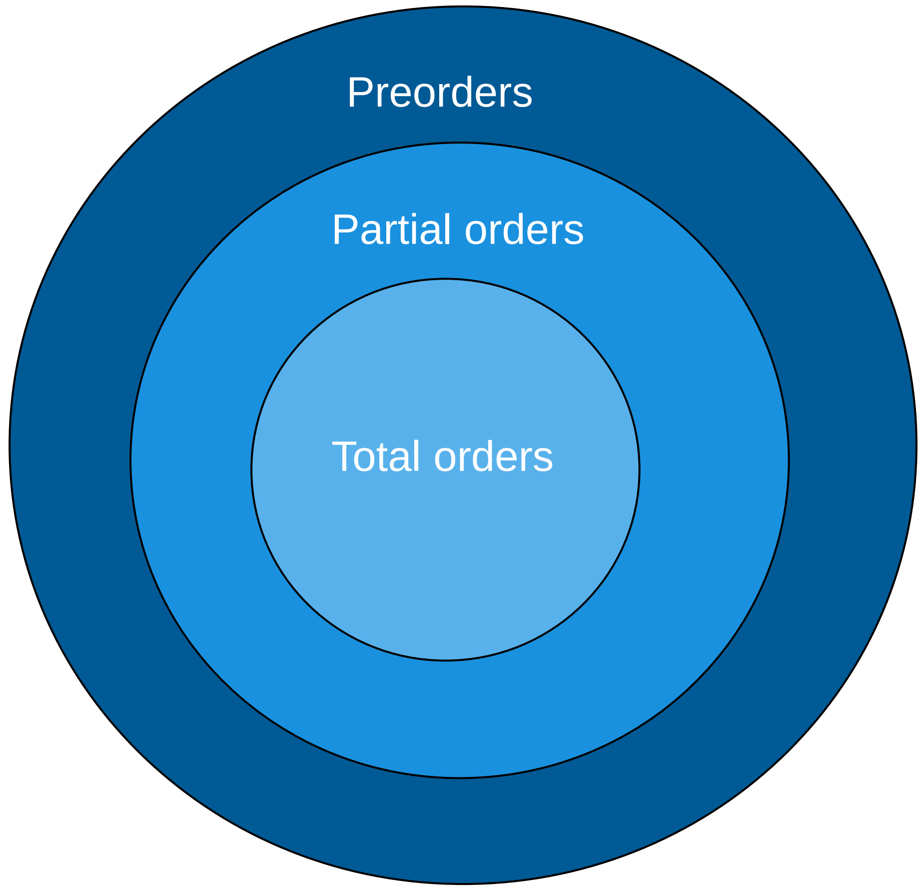
- **Vector Ordering Relation**
  - Needed for Choquet or morphological filtering

Ordering Relations

- **Preorder**
  - $a \leq a$ (reflexivity)
  - if $a \leq b$ and $b \leq c$ then $a \leq c$ (transitivity)

- **Partial Order**
  - $a \leq a$ (reflexivity)
  - if $a \leq b$ and $b \leq c$ then $a \leq c$ (transitivity)
  - if $a \leq b$ and $b \leq a$ then $a = b$ (antisymmetry)

- **Total Order**
  - $a \leq a$ (reflexivity)
  - if $a \leq b$ and $b \leq c$ then $a \leq c$ (transitivity)
  - if $a \leq b$ and $b \leq a$ then $a = b$ (antisymmetry)
  - $a \leq b$ or $b \leq a$ (totality)



Vector Ordering Relations

- **(Partial) Marginal Ordering**

$$c_1 \prec_E c_2 \Leftrightarrow c_1^A < c_2^A, c_1^B < c_2^B, c_1^C < c_2^C$$

Vectors may not be comparable but extrema exist

- **(Total) Lexicographical Order:** $A \to B \to C$

$$c_1 \prec_E c_2 \Leftrightarrow \begin{cases} c_1^A < c_2^A \text{ or,} \\ c_1^A = c_2^A \text{ and } c_1^B < c_2^B \text{ or,} \\ c_1^A = c_2^A \text{ and } c_1^B = c_2^B \text{ and } c_1^C < c_2^C. \end{cases}$$

Extrema are unique

- **Reduced (Total Preorder)**

$$c_1 \prec_E c_2 \Leftrightarrow \left\| c_1 - c_{ref} \right\|_E < \left\| c_2 - c_{ref} \right\|_E$$

Multiple extrema may exist

GAN Color Choquet Filtering

- **Definition**

$$CF_m^{f_0}(f)(y) = \sum_{x_i \in V_m^{f_0}(y)} \left(\mu_y(A_{(i)}) - \mu_y(A_{(i+1)})\right) f(x_{(i)})$$

- **Example on a Painting Image from the Artist Gamze Aktan**
  - Within the RGB color space using the lexicographical order: $R \to G \to B$

(a) original image (b) adaptive mean filtering (c) adaptive median filtering (d) adaptive min filtering (e) adaptive max filtering

Illustration

- **GAN Mean Filtering within Different Color Spaces**

(a) original image (b) RGB, classical mean (c) RGB, adaptive mean (d) L*a*b*, classical mean (e) L*a*b*, adaptive mean

(f) HSL, classical mean (g) HSL, adaptive mean

Painting image from the artist Gamze Aktan

GAN Color Mathematical Morphology

$$R_m^{f_0}(x) = \bigcup_{z \in D} \{V_m^{f_0}(z) | x \in V_m^{f_0}(z)\}$$

- **Definition**

$$\mathrm{D}_m^{f_0}(f)(x) = \sup_E \{f(w); w \in R_m^{f_0}(x)\} \qquad \mathrm{O}_m^{f_0}(f) = \mathrm{D}_m^{f_0} \circ \mathrm{E}_m^{f_0}(f)$$

$$\mathrm{E}_m^{f_0}(f)(x) = \inf_E \{f(w); w \in R_m^{f_0}(x)\} \qquad \mathrm{C}_m^{f_0}(f) = \mathrm{E}_m^{f_0} \circ \mathrm{D}_m^{f_0}(f)$$

- **Example on a Painting Image from the Artist Gamze Aktan**
  - Within the L*a*b* color space using the lexicographical order: $L^* \to a^* \to b^*$

(a) original image (b) adaptive erosion (c) adaptive dilation (d) adaptive opening (e) adaptive closing

Properties

- **Increasing**

$$f_1 \prec_E f_2 \Rightarrow \begin{cases} \mathrm{D}_m^{f_0}(f_1) \prec_E \mathrm{D}_m^{f_0}(f_2) \\ \mathrm{E}_m^{f_0}(f_1) \prec_E \mathrm{E}_m^{f_0}(f_2) \\ \mathrm{C}_m^{f_0}(f_1) \prec_E \mathrm{C}_m^{f_0}(f_2) \\ \mathrm{O}_m^{f_0}(f_1) \prec_E \mathrm{O}_m^{f_0}(f_2) \end{cases}$$

- **Adjunction (Morphological Duality)**

$$\mathrm{D}_m^{f_0}(f_1) \prec_E f_2 \Leftrightarrow f_1 \prec_E \mathrm{E}_m^{f_0}(f_2)$$

- **Increasing / Decreasing**

$$\begin{pmatrix} (m_1, m_2) \in \mathbb{R}^+ \times \mathbb{R}^+ \\ m_1 \leq m_2 \end{pmatrix} \Rightarrow \begin{cases} \mathrm{D}_{m_1}^{f_0}(f) \prec_E \mathrm{D}_{m_2}^{f_0}(f) \\ \mathrm{E}_{m_2}^{f_0}(f) \prec_E \mathrm{E}_{m_1}^{f_0}(f) \end{cases}$$

- **Distributivity**

$$\begin{cases} \sup_{E,\, i \in I}[\mathrm{D}_m^{f_0}(f_i)] = \mathrm{D}_m^{f_0}(\sup_{E,\, i \in I}[f_i]) \\ \inf_{E,\, i \in I}[\mathrm{E}_m^{f_0}(f_i)] = \mathrm{E}_m^{f_0}(\inf_{E,\, i \in I}[f_i]) \end{cases}$$

- **Idempotence**

$$\begin{cases} \mathrm{C}_m^{f_0} \circ \mathrm{C}_m^{f_0}(f) = \mathrm{C}_m^{f_0}(f) \\ \mathrm{O}_m^{f_0} \circ \mathrm{O}_m^{f_0}(f) = \mathrm{O}_m^{f_0}(f) \end{cases}$$

- **Extensiveness / Anti-Extensiveness**

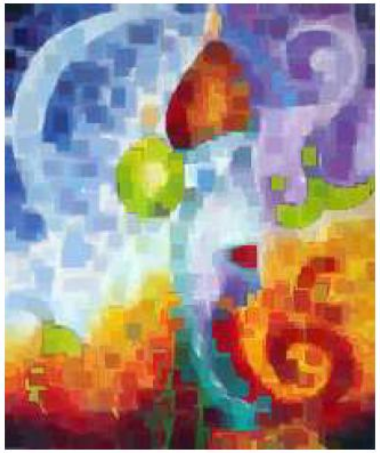
$$\mathrm{O}_m^{f_0}(f) \prec_E f \prec_E \mathrm{C}_m^{f_0}(f)$$

Illustration

- **Classical vs. Geodesic vs. GAN Filtering**
  - Within the HSL color space using the lexicographical order: $L \rightarrow S \rightarrow H$

(a) original image

(b) classical opening



(c) classical closing

(d) opening by reconstruction

(e) closing by reconstruction

(f) adaptive opening

(g) adaptive closing

Painting image from the artist Gamze Aktan

Application Example

- **Image Restoration of Cells (Fluorescence Microscopy)**

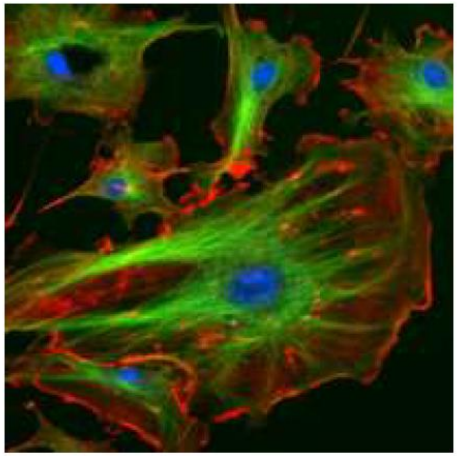
- **Morphological Center**
  - Morphological median filter using alternating operators

$$\zeta_m^{f_0}(f) = \inf_E(\sup_E(f, \inf_E(\psi_m^{f_0}(f), \phi_m^{f_0}(f))), \sup_E(\psi_m^{f_0}(f), \phi_m^{f_0}(f)))$$

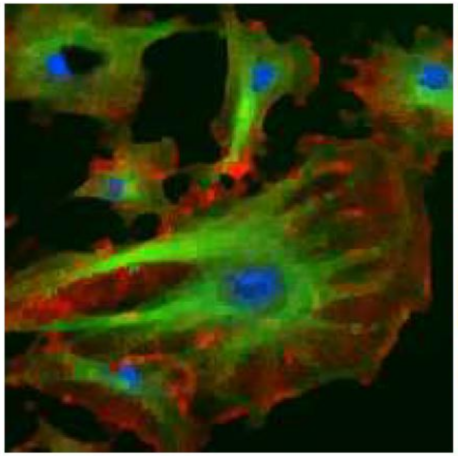
$$\text{where: } \psi_m^{f_0}(f) = \mathrm{O}_m^{f_0} \circ \mathrm{C}_m^{f_0} \circ \mathrm{O}_m^{f_0}(f) \text{ and } \phi_m^{f_0}(f) = \mathrm{C}_m^{f_0} \circ \mathrm{O}_m^{f_0} \circ \mathrm{C}_m^{f_0}(f)$$

Application Example

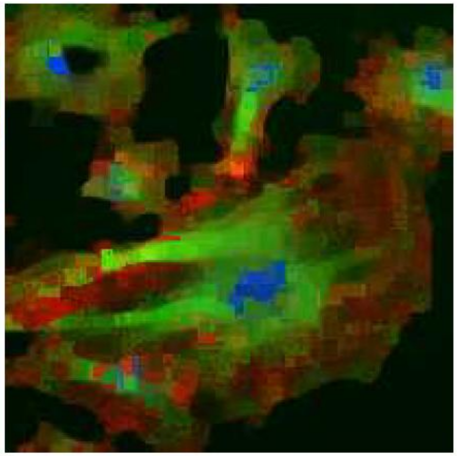
- **Image Restoration of Cells (Fluorescence Microscopy)**



(a) original image

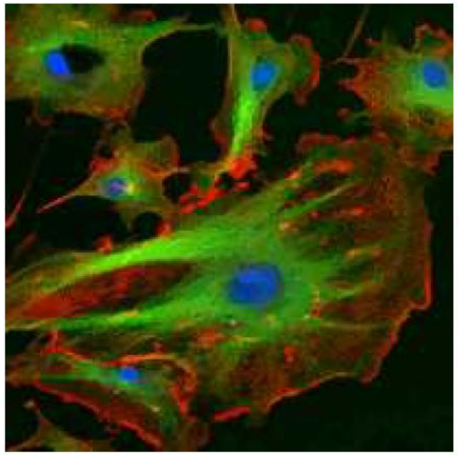


(b) classical morphological centre, $r = 1$

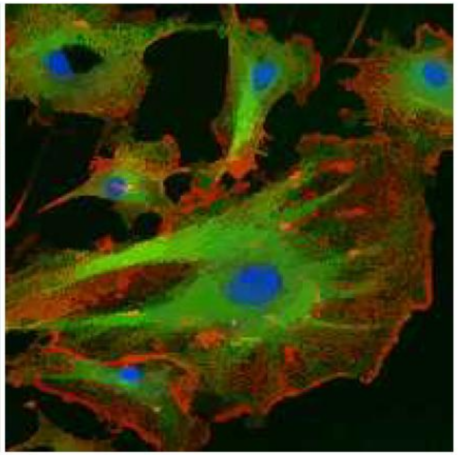


(c) classical morphological centre, $r = 2$

HSL color space using the lexicographical order: $L \to S \to H$



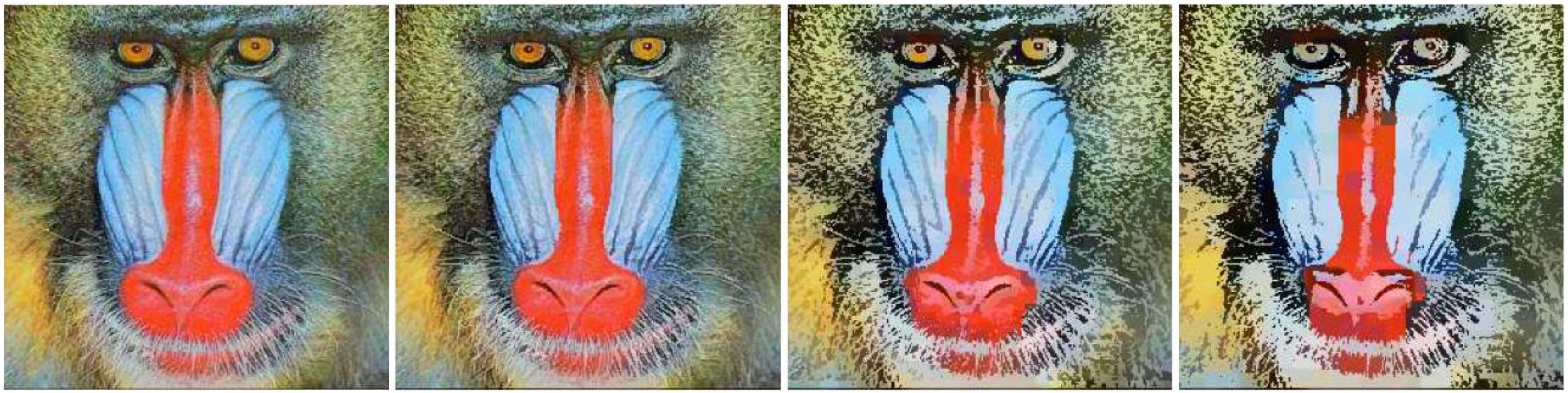
(d) adaptive morphological centre, $m = 20$



(e) adaptive morphological centre, $m = 30$

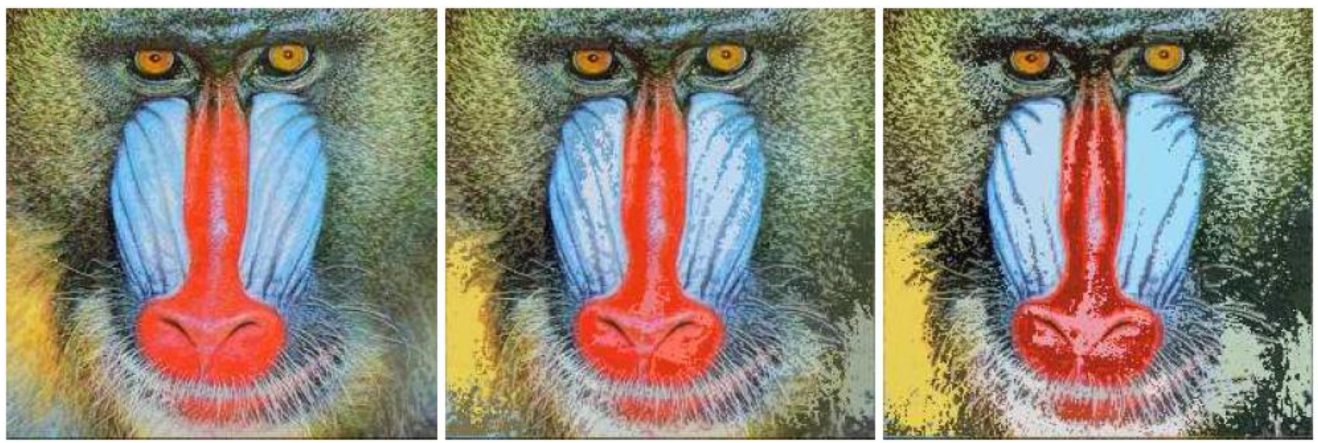
Application Example

- Image Enhancement of the ‘Baboon’ Image



(a) original image (b) classical toggle contrast, $r = 1$ (c) classical toggle contrast, $r = 5$ (d) classical toggle contrast, $r = 10$

RGB color space using the lexicographical order: $G \rightarrow R \rightarrow B$

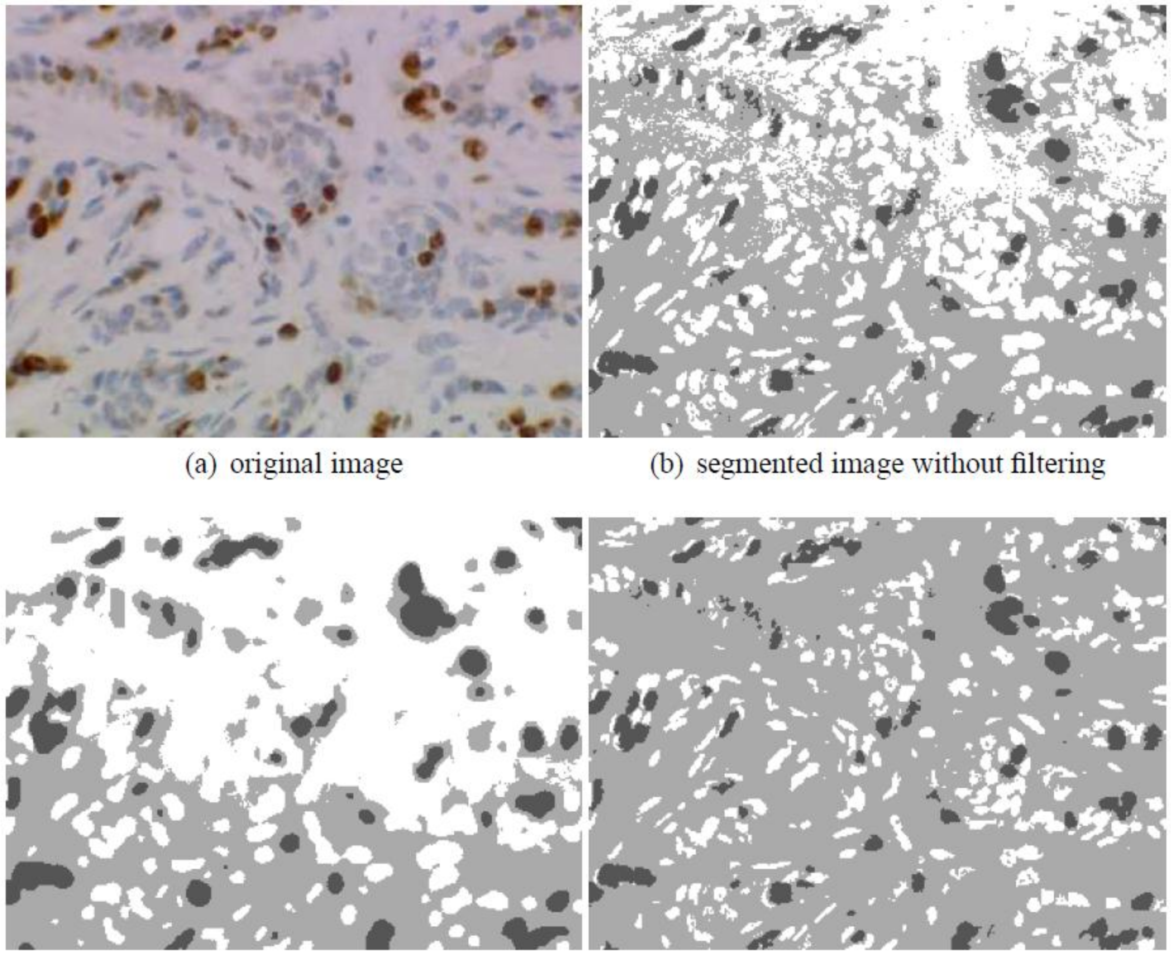


(e) adaptive toggle contrast, $m = 25$ (f) adaptive toggle contrast, $m = 50$ (g) adaptive toggle contrast, $m = 75$

Application Example

- **Image Segmentation of Cells (ADCIS Aphelion)**
  - By kmeans clustering

L*a*b* color space



(a) original image

(b) segmented image without filtering

(c) segmented image with classical mean filtering

(d) segmented image with adaptive mean filtering

References

http://www.emse.fr/~debayle/bib/Keyword/GENERAL-ADAPTIVE-NEIGHBORHOOD.html