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1. Probability Questions

Question 1.1

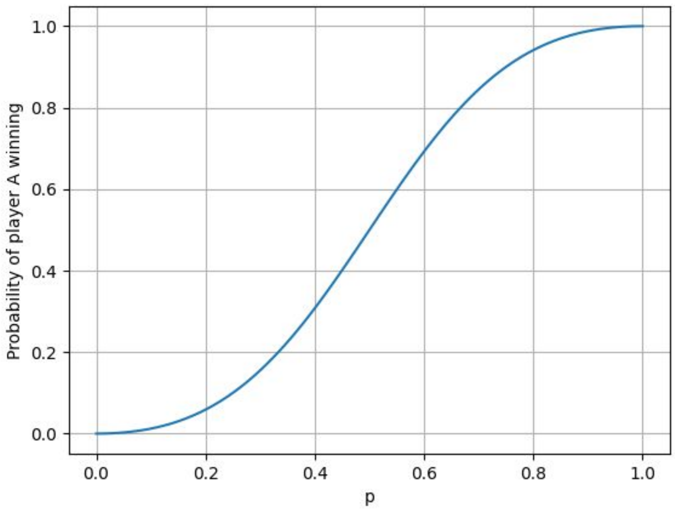
Probability = Probability of exactly one win + Probability of exactly one lose

Probability = $\binom{6}{1}(0.6)^1(0.4)^5 + \binom{6}{5}(0.6)^5(0.4)^1 = 0.223488$

Question 1.2

Assume that the probability of player A winning the game is K.

$K = p^2 + p(1-p)K + (1-p)pK = p^2 + 2Kp(1-p) \Rightarrow K = \frac{p^2}{2p^2-2p+1}$



4. Principal Component Analysis

Question 4.1

The eigenvector of the covariance matrix with the largest eigenvalue shows the direction of the largest variance of the data. When we maximize the $Var(z_1) = w_1^T Cov(X) w_1$ and keep the constraint $\|w_1\| = 1$, we try to find the w_1 that shows the direction of the largest variance. The covariance matrix is a positive definite symmetric matrix, so the eigenvectors are orthonormal. Therefore, w_1 will be the eigenvector of the covariance matrix with the largest eigenvalue because they show the same direction and their norms are 1.

Question 4.2

The covariance matrix is a positive definite symmetric matrix and the eigenvectors are different from each other, so the eigenvectors are orthonormal. When we try to find the second maximum $\|w_1$ at the $Var(z_1) = w_1^T Cov(X) w_1$ and keep the constraint $\|w_1\| = 1$, we try to find the w_1 that shows the direction of the second largest variance. The eigenvector of the covariance matrix with the second largest eigenvalue shows the direction of the second largest variance of the data. Therefore, w_1 will be the eigenvector of the covariance matrix with the second largest eigenvalue because they show the same direction and their norms are 1.