**BİLKENT UNIVERSITY - ENGINEERING FACULTY**

**DEPARTMENT OF COMPUTER ENGINEERING**

# IE 400

# Principles of Engineering Management

# 2020-2021 Fall

## Term Project

## Team Members

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28 December 2020

**a)**

Parameters:

N: Total number of villages (=30)

$D_{ij}$ : Distance between village i and j $\forall$ i,j = 1, … , N

Decision Variables:

$X_j$: 1, if Santa chooses village j as a center
0, otherwise $\forall$ j = 1, … , N

$Y_{ij}$ : 1, if parents at village i walk to the village j which is a center
0, otherwise $\forall$ i,j = 1, … , N

dMax: Longest distance that a parent should walk

**MODEL**

Objective Function:

Minimizing the the longest distance that a parent should walk:

min dMax

Constraints:

This constraint ensures that all villages are visited once:

$$\sum_{j=1}^{N} Y_{ij} = 1 \qquad \forall\ i = 1, \dots, N$$

This constraint ensures that there are 4 villages selected as centers:

$$\sum_{j=1}^{N} X_j = 4$$

This constraint ensures that the parents only walk to the villages which are centers:

$$Y_{ij} \leq X_j \qquad \forall\ i = 1, \dots, N \text{ and } j = 1, \dots, N$$

This constraint ensures that the walk that a parent takes cannot exceed the longest distance that a parent should walk:

$$dMax \geq \sum_{i=1}^{N} \sum_{j=1}^{N} Y_{ij} \times D_{ij}$$

This constraint ensures that $X_j$ is a binary variable:

$X_j \; \varepsilon \; \{0, 1\}$ $\qquad\qquad \forall \; j = 1, \dots, N$

This constraint ensures that $Y_{ij}$ is a binary variable:

$Y_{ij} \; \varepsilon \; \{0, 1\}$ $\qquad\qquad \forall \; i = 1, \dots, N$ and $j = 1, \dots, N$

**RESULT**

By solving this model with given data, we found that the centers are at the villages with numbers 15, 19, 24, 30. In addition, the minimum longest distance is 50.5.

**b)**

Parameters:

- N: Total number of villages (=30)
- $D_{ij}$: Distance between village i and j $\quad \forall \; i,j = 1, \dots, N$
- $P_{ij}$: Probability that the road between village i and village j is out of use due to snow $\quad \forall \; i,j = 1, \dots, N$
- pMax: The maximum probability that a road can have to be able to use it (=0.60)

Decision Variables:

$X_j$: 1, if Santa chooses village j as a center
0, otherwise $\quad \forall \; j = 1, \dots, N$

$Y_{ij}$: 1, if $P_{ij}$ < pMax so that parents at village i can walk to the village j which is a center
0, otherwise $\quad \forall \; i,j = 1, \dots, N$

dMax: Longest distance that a parent should walk

**MODEL**

Objective Function:

Minimizing the longest distance that a parent walks:

min dMax

Constraints:

This constraint ensures that each village is visited once:

$$\sum_{j=1}^{N} Y_{ij} = 1 \qquad \forall \; i = 1, \dots, N$$

This constraint ensures that there are 4 villages selected as centers:

$$\sum_{j=1}^{N} X_j = 4$$

This constraint ensures that the walk that a parent takes cannot exceed the longest distance that a parent should walk:

$$dMax \geq \sum_{i=1}^{N} \sum_{j=1}^{N} Y_{ij} \times D_{ij}$$

This constraint ensures that the parents only walk to the villages which are centers:

$$Y_{ij} \leq X_j \qquad \forall\ i = 1, \dots, N \text{ and } j = 1, \dots, N$$

This constraint ensures that $X_j$ is a binary variable:

$$X_j \; \varepsilon \; \{0, 1\} \qquad \forall\ j = 1, \dots, N$$

This constraint ensures that $Y_{ij}$ is a binary variable:

$$Y_{ij} \; \varepsilon \; \{0, 1\} \qquad \forall\ i = 1, \dots, N \text{ and } j = 1, \dots, N$$

**<u>RESULT</u>**

By solving this model with given data, we found that the centers are at the villages with numbers 9, 13, 20, 24. In addition, the minimum longest distance is 54.5.

**c)**

<u>Parameters:</u>

N: Total number of villages (= 30)

$D_{ij}$: Distance between village i and j $\forall$ i,j = 1, … , N

$P_{ij}$: Probability that the road between village i and village j is out of use due to snow $\forall$ i,j = 1, … , N

pMax: The maximum probability that a road can have to be able to use it (= 0.60)

speed: The speed of the snowplows (= 40)

<u>Decision Variables:</u>

$X_{ij}$: 1, if $P_{ij}$ is ≤ pMax so that Santa can use the road between village i and j

0, otherwise $\forall$ i,j = 1, … , N

$U_i$ : Subtour elimination variables $\forall\ i = 1, \dots, N$

## MODEL

Objective Function:

Minimizing the time it takes for Santa to visit all the villages and turn back to his initial position:

$$\min \sum_{i=1}^{N} \sum_{j=1}^{N} [(X_{ij} \times D_{ij}) \div \text{speed}]$$

Constraints:

This constraint ensures that Santa arrives each village once:

$$\sum_{i=1}^{N} X_{ij} = 1 \qquad \forall\ j = 1, \dots, N$$

This constraint ensures that Santa leaves each village once:

$$\sum_{j=1}^{N} X_{ij} = 1 \qquad \forall\ i = 1, \dots, N$$

These two constraints ensure that there is no subtour:

$$U_i - U_j + N \times X_{ij} \leq N - 1 \qquad i \neq j \text{ and } \forall\ i,j = 2, \dots, N$$

$$U_1 = 1$$

This constraint ensures that Santa does not leave the current village to go there:

$$X_{ii} = 0 \qquad \forall\ i = 1, \dots, N$$

This constraint ensures that $X_{ij}$ is a binary variable:

$$X_{ij} \in \{0, 1\} \qquad \forall\ i = 1, \dots, N \text{ and } j = 1, \dots, N$$

This constraint ensures that $U_i$ is integer:

$$U_i \text{ is integer} \qquad \forall\ i = 1, \dots, N$$

## RESULT

By solving this model with given data, we found that the path Santa takes is:

$1 \rightarrow 21 \rightarrow 11 \rightarrow 19 \rightarrow 26 \rightarrow 10 \rightarrow 2 \rightarrow 24 \rightarrow 29 \rightarrow 8 \rightarrow 7 \rightarrow 15 \rightarrow 3 \rightarrow 4 \rightarrow 23 \rightarrow 20 \rightarrow 5 \rightarrow 6 \rightarrow 28$
$\rightarrow 12 \rightarrow 16 \rightarrow 22 \rightarrow 18 \rightarrow 13 \rightarrow 9 \rightarrow 27 \rightarrow 25 \rightarrow 14 \rightarrow 17 \rightarrow 30 \rightarrow 1$

In addition, the minimum time it takes for Santa to visit all the villages and turn back to his initial position is 27.875.

**d)**

Parameters:

N: Total number of villages (= 30)

$D_{ij}$ : Distance between village i and j $\forall$ i, j = 1, … , N

speed: The speed of the snowplows (= 40)

time_threshold: The time limit for the volunteers to turn back to their initial positions (= 10)

D: Maximum distance that a volunteer can travel (speed $\times$ time_threshold = 400)

Decision Variables:

$X_{ij}$ : 1, if volunteer goes from village i to village j directly
0, otherwise i ≠ j and $\forall$ i, j = 1, … , N

V: Volunteer count

$Y_{ij}$ : Spare distance capacity a volunteer has after covering the node j.
i ≠ j and $\forall$ j = 1, … , N

**MODEL**

Objective Function:

Minimizing the number of volunteers needed:

min V

Constraints:

This constraint ensures that exactly V number of volunteers leave the village 1:

$$\sum_{i=2}^{N} X_{1i} = V$$

This constraint ensures that exactly V number of volunteers return back to the village 1:

$$\sum_{i=2}^{N} X_{i1} = V$$

This constraint ensures that each village is visited exactly once, and volunteers does not leave and arrive to the same village:

$$\sum_{i=1,\ i \neq j}^{N} X_{ij} = 1 \qquad \forall\ j = 2, \ldots, N$$

This constraint ensures that each village is visited exactly once, and volunteers does not leave and arrive to the same village:

$$\sum_{j=1,\ i \neq j}^{N} X_{ij} = 1 \qquad \forall\ i = 2, \ldots, N$$

This constraint ensures the subtour elimination and balances inflow and outflow at each village:

$$\sum_{j=1,\, i \neq j}^{N} Y_{ij} - \sum_{j=1,\, i \neq j}^{N} Y_{ji} - \sum_{j=1,\ i \neq j}^{N} (X_{ij} \times D_{ij}) = 0 \qquad \forall\ i = 2, \ldots, N$$

This constraint ensures that the distance that the volunteer will take to go to a village does not exceed the spare distance capacity of the volunteer (distance traveled up to the village does not exceed D):

$$Y_{ij} \leq D \times X_{ij} \qquad i \neq j\ \ \forall\ i = 1, \ldots, N,\ j = 1, \ldots, N$$

This constraint ensures that $Y_{ij}$ does not have a negative value:

$$Y_{ij} \geq 0 \qquad i \neq j\ \ \forall\ i = 1, \ldots, N,\ j = 1, \ldots, N$$

This constraint ensures that the volunteer will take to go to a village from village 1 does not exceed the spare distance capacity of the volunteer:

$$Y_{1i} = D_{1i} \times X_{1i} \qquad \forall\ i = 2, \ldots, N$$

This constraint ensures that required number of volunteers can be at most N:

$$V \leq 30$$

This constraint ensures that $X_{ij}$ is a binary variable:

$$X_{ij}\ \varepsilon\ \{0, 1\} \qquad \forall\ i = 1, \ldots, N,\ j = 1, \ldots, N$$

**RESULT**

By solving this model with given data, we found that:

A path is 1 → 13 → 9 → 27 → 19 → 25 → 12 → 28 → 11 → 1

Time is 9.825.

A path is $1 \rightarrow 15 \rightarrow 14 \rightarrow 8 \rightarrow 29 \rightarrow 10 \rightarrow 4 \rightarrow 3 \rightarrow 1$

Time is 9.85.

A path is $1 \rightarrow 2 \rightarrow 30 \rightarrow 21 \rightarrow 26 \rightarrow 16 \rightarrow 24 \rightarrow 1$

Time is 9.45.

A path is $1 \rightarrow 5 \rightarrow 20 \rightarrow 6 \rightarrow 17 \rightarrow 23 \rightarrow 7 \rightarrow 18 \rightarrow 22 \rightarrow 1$

Time is 9.9.

Therefore, the minimum number of volunteers is 4.