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THEORETICAL ANALYSIS

Basic operation is the comparison marked as (1)

Analyze B(n)

Operation (1) is a compare operation, which executes n times in total because outer loop turns n times and it executes 1 time at each turn.
# of basic operations: n
B(n) ∈ Θ(n)

Analyze W(n)

Regardless of the values in the input, basic operation will execute n times because the outer loop runs n time. And each time, basic operation executes once.
# of basic operations: n
W(n) ∈ Θ(n)

Analyze A(n)

As stated in the best and worst cases, basic operation count doesn't depend on the input values. In each case, basic operation will execute n times.
# of basic operations: n
A(n) ∈ Θ(n)

Basic operations are the three assignments marked as (2)

If we analyze every element of the array, we can conclude that;
If for all i, arr[i] = 0 => only the first if block will be executed and number of basic operations in that block is (n*(n+1)/2)*log(n). Since the for loop executes from i to n-1, it executes (n*(n+1)/2) times in total. And the while loop executes log(n) times each time.

If for all i, arr[i] = 1 => only the second if block will be executed and number of basic operations in that block is (n^3)*log(n). Since 2 for loops executes n time each, it executes (n^2) times in total. And the outer loop executes n times. And the while loop executes log(n) times each time.

If for all i, arr[i] = 2 => only the third if block will be executed and number of basic operations in that block is (n*(n+1)*(2n+1)*n)/6. Since the first loop inside that if block executes n time and inner for loop executes up to square of the index of the outer for loop, it will be executed n*(n+1)*(2n+1)/6 times for every element of the array.

Analyze B(n)

In the case of all elements in the array are 0, the number of basic operations becomes minimum. So the best case occurs when all elements are 0. And in that case;
# of basic operations: (n*(n+1)/2)*log(n).
B(n) ∈ Θ(n^2 * log(n))

Analyze W(n)

In the case of all elements in the array are 2, the number of basic operations becomes maximum. So the worst case occurs when all elements are 2. And in that case;
# of basic operations: (n*(n+1)*(2n+1)*n)/6
W(n) ∈ Θ(n^4)

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Analyze A(n)

Average number of basic operations:

∑i=0 to n:

1/3*(n-i)*log(n) + 1/3*n^2*log(n) + 1/3*n*(n+1)*(2n+1)/6

= 1/3*(n*(n+1)/2)log(n) + 1/3*n^3*log(n) + 1/3*n^2*(n+1)*(2n+1)/6

A(n) ∈ Θ(n^4)

Basic operation is two assignments marked as (3)

Analyze B(n)

If arr[i] equals to 0, no basic operation is executed. So if all elements of the array equal to 0, it will be best case and no basic operation will be executed. In that case;

basic operations: 0

B(n) ∈ Θ(1)

Analyze W(n)

If for all i, arr[i] = 1 => only the second if block will be executed and number of basic operations in that block is (n^3)*log(n). Since 2 for loops executes n time each, it executes (n^2) times in total. And the outer loop executes n times. And the while loop executes log(n) times each time.

If for all i, arr[i] = 2 => only the third if block will be executed and number of basic operations in that block is (n*(n+1)*(2n+1)*n)/6. Since the first loop inside that if block executes n time and inner for loop executes up to square of the index of the outer for loop, it will be executed n*(n+1)*(2n+1)/6 times for every element of the array.

Number of basic operations is maximum in the case of all elements of the array are 2, so the worst case will be with an array that all elements are equal to 2.

basic operations: n*(n+1)*(2n+1)/6

W(n) ∈ Θ(n^4)

Analyze A(n)

Average number of basic operations:

∑i=0 to n:

1/3*0 + 1/3*n^2*log(n) + 1/3*n*(n+1)*(2n+1)/6

= 0 + 1/3*n^3*log(n) + 1/3*n^2*(n+1)*(2n+1)/6

A(n) ∈ Θ(n^4)

Basic operations are the two loop incrementations marked as (4)

If we analyze every element of the array, we can conclude that;

If for all i, arr[i] = 0 => basic operation is executed 0 times.

If for all i, arr[i] = 1 => First loop executes n times, and also the inner loop executes n times. And the outer main loop executes n times. So basic operation will execute n^3 times.

If for all i, arr[i] = 2 => Loop executes n times and outer main loop executes n times. So basic operation executes n^2 times.

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Analyze B(n)

Best case occurs when all the elements in the array are equal to 0 because no basic operation is executed in that scenario.
# basic operations: 0
B(n) ∈ Θ(1)

Analyze W(n)

Worst case occurs when all the elements in the array are equal to 1 because its growth rate is the larger one.
# basic operations: n^3
W(n) ∈ Θ(n^3)

Analyze A(n)

Average number of basic operations:
∑i=0 to n:
1/3*0 + 1/3*n^2 + 1/3*n
= 0 + 1/3*n^3* + 1/3*n^2
A(n) ∈ Θ(n^3)

Basic operation is the assignment marked as (5)

If we analyze every element of the array, we can conclude that;
If for all i, arr[i] = 0 => basic operation is executed n*(n+1)/2 times. Because in each loop, it executes n-i times, and if we sum them up from i = 0 to n, we conclude that it will be executed n*(n+1)/2 times.

If for all i, arr[i] = 1 => basic operation is executed 0 times
If for all i, arr[i] = 2 => basic operation is executed 0 times

Analyze B(n)

Best case occurs when input array contains only either 1 or 2. Because no basic operation is executed in that scenario.
# basic operations: 0
B(n) ∈ Θ(1)

Analyze W(n)

Worst case occurs when all the elements in the array are equal to 0. Because it is the only scenario when basic operations are executed.
# basic operations: n*(n+1)/2
W(n) ∈ Θ(n^2)

Analyze A(n)

Average number of basic operations:
∑i=0 to n:
1/3*(n-i) + 1/3*0 + 1/3*0
= 1/3*(n*(n+1)/2) + 0 + 0
A(n) ∈ Θ(n^2)

IDENTIFICATION OF BASIC OPERATION(S)

Operation 2 must be the basic operation. Because it executes in every if block. So in that way, execution of the basic operation represents characteristic of the input array.

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REAL EXECUTION

Best Case

N Size	Time Elapsed (seconds)
1	0,000001
5	0,000017
10	0,000064
25	0,000445
50	0,00208
75	0,005315
100	0,009386
150	0,025148
200	0,042173
250	0,066736

Worst Case

N Size	Time Elapsed (seconds)
1	0,000031
5	0,000103
10	0,001037
25	0,020487
50	0,243445
75	1,107594
100	3,524954
150	17,686174
200	56,409354
250	140,480636

Average Case

N Size	Time Elapsed (seconds)
1	0,0000016
5	0,000057
10	0,000236
25	0,01007
50	0,103765
75	0,424619
100	1,37562
150	6,369159
200	19,56555
250	43,94550

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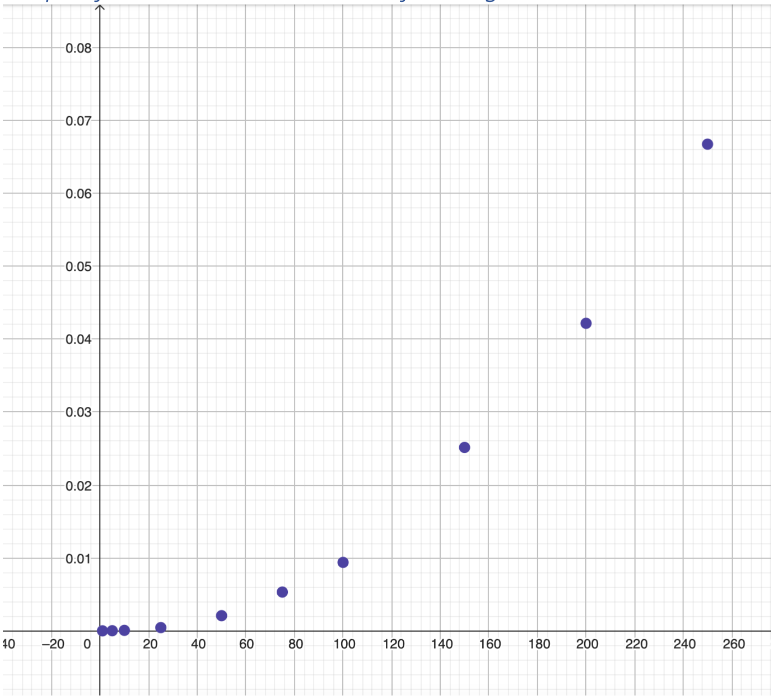
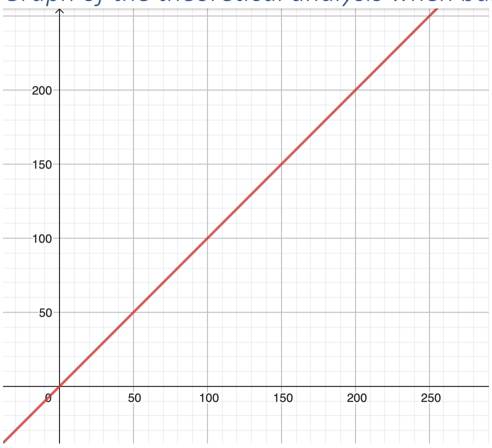
COMPARISON

Best Case

Graph of the real execution time of the algorithm



Graph of the theoretical analysis when basic operation is the operation marked as (1)



Compared to real best case execution graph, if we take 2 arbitrary points from the graphs, we can see that growth rates are not similar.

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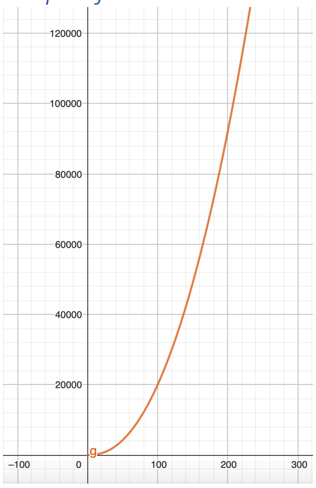
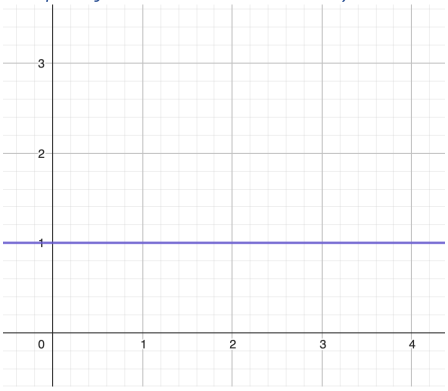
Graph of the theoretical analysis when basic operation is the operation marked as (2)



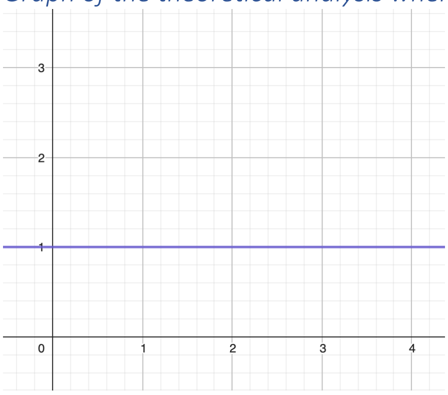
Compared to real best case execution graph, if we compare the values of graphs for n=100 and n=200, we can see that their growth rates are similar.

Graph of the theoretical analysis when basic operation is the operation marked as (3)



Compared to real best case execution graph, if we take 2 arbitrary points from the graphs, we can see that growth rates are not similar.

Graph of the theoretical analysis when basic operation is the operation marked as (4)



Compared to real best case execution graph, if we take 2 arbitrary points from the graphs, we can see that growth rates are not similar.

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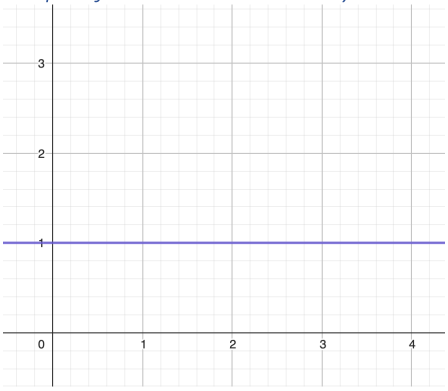
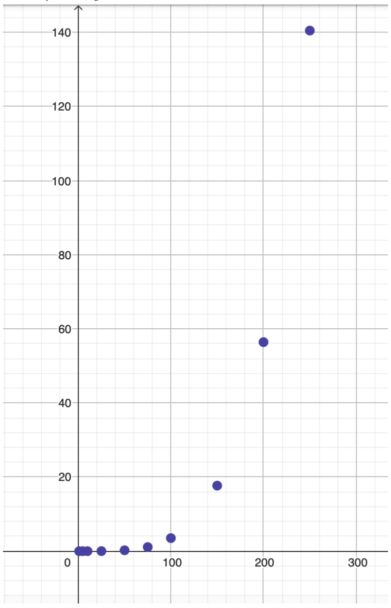
Graph of the theoretical analysis when basic operation is the operation marked as (5)



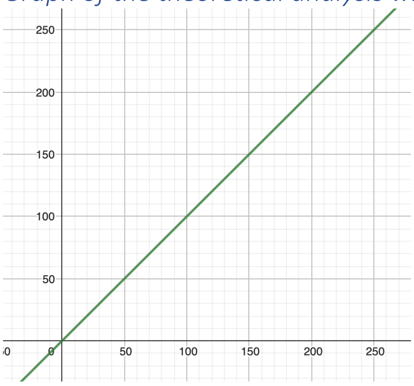
Compared to real best case execution graph, if we take 2 arbitrary points from the graphs, we can see that growth rates are not similar.

Worst Case

Graph of the real execution time of the algorithm



Graph of the theoretical analysis when basic operation is the operation marked as (1)



Compared to real best case execution graph, if we take 2 arbitrary points from the graphs, we can see that growth rates are not similar.

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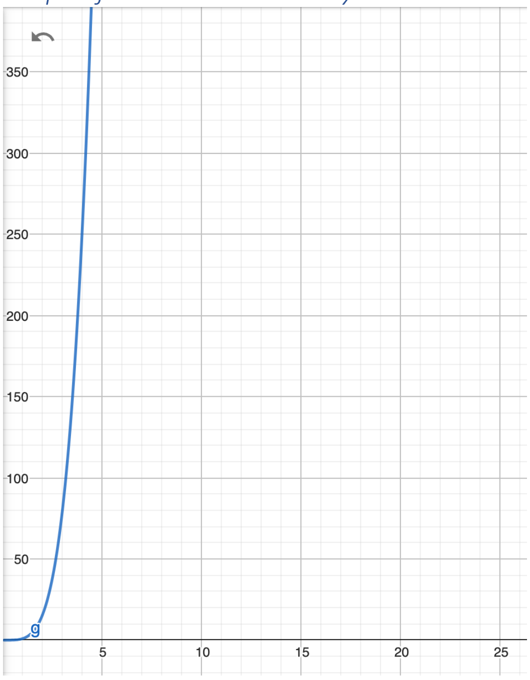
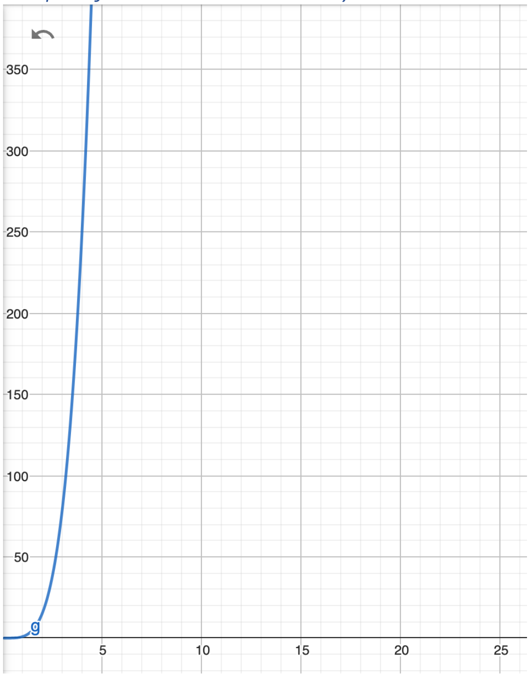
Graph of the theoretical analysis when basic operation is the operation marked as (2)



Compared to real best case execution graph, if we take 2 arbitrary points from the graphs, we can see that growth rates are not similar. From n=100 to 200, real execution graph grows 4 to 56. Which means 14x. And this graph is representing n^4, which should result in 16x growth. These numbers are similar, so we can say that these graphs are similar.

Graph of the theoretical analysis when basic operation is the operation marked as (3)



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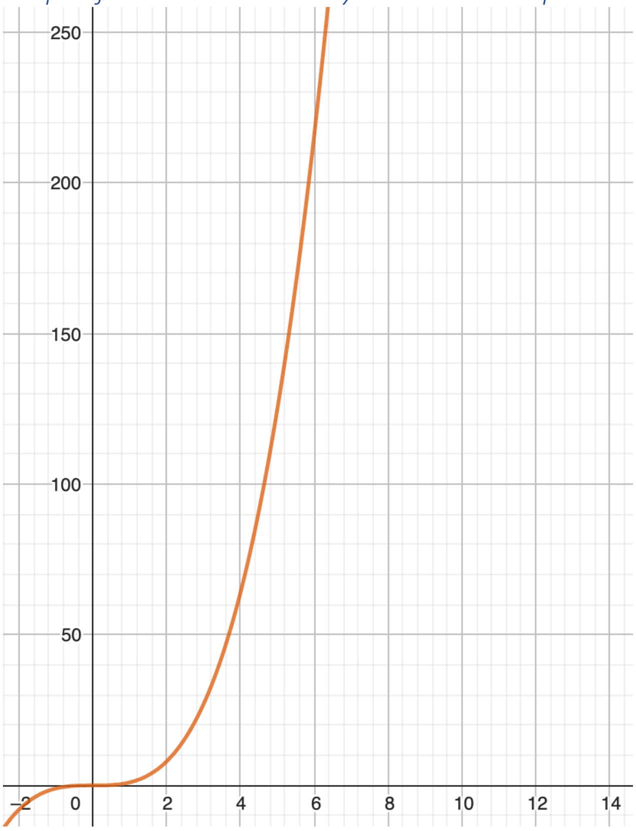
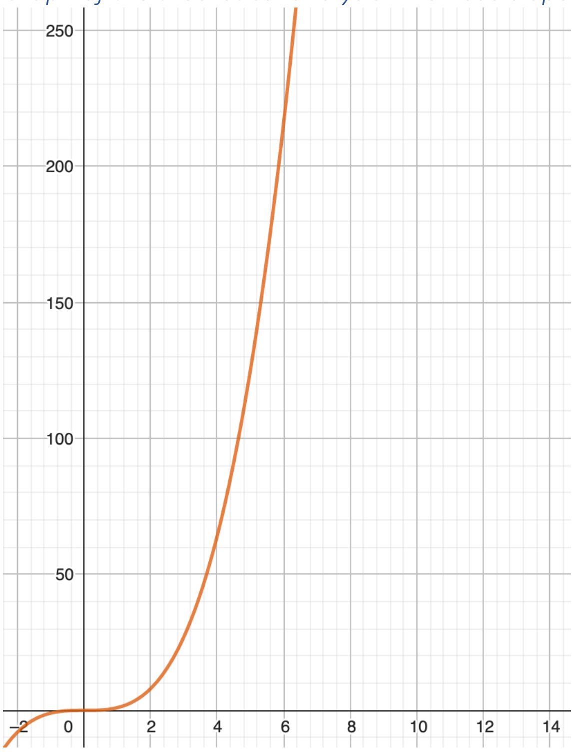
Graph of the theoretical analysis when basic operation is the operation marked as (4)



Compared to real best case execution graph, if we take 2 arbitrary points from the graphs, we can see that growth rates are not similar.

Graph of the theoretical analysis when basic operation is the operation marked as (5)



Compared to real best case execution graph, if we take 2 arbitrary points from the graphs, we can see that growth rates are not similar.

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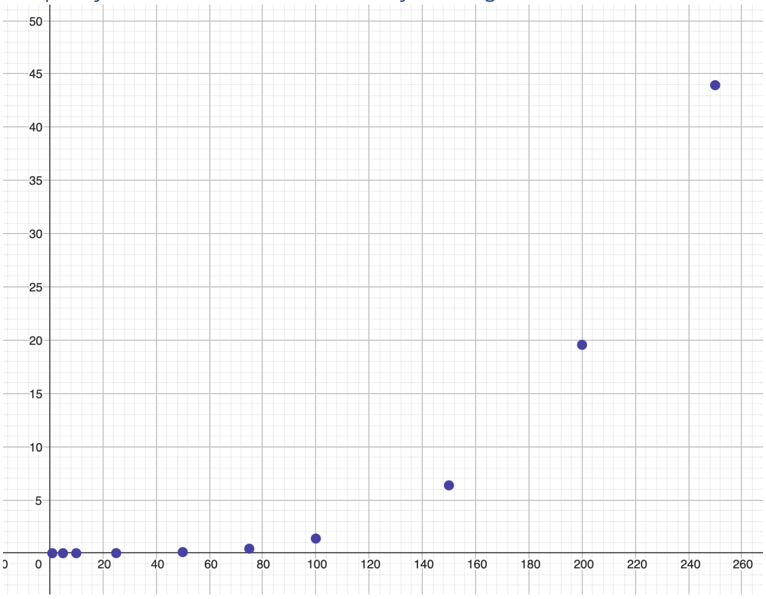
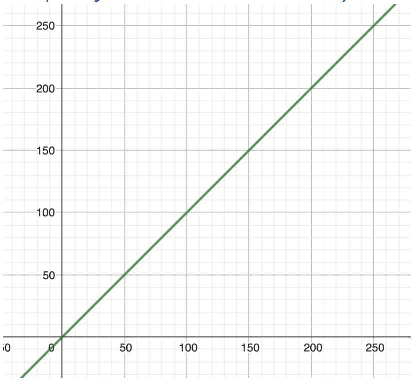
Comments

Average Case

Graph of the real execution time of the algorithm



Graph of the theoretical analysis when basic operation is the operation marked as (1)



Compared to real best case execution graph, if we take 2 arbitrary points from the graphs, we can see that growth rates are not similar.

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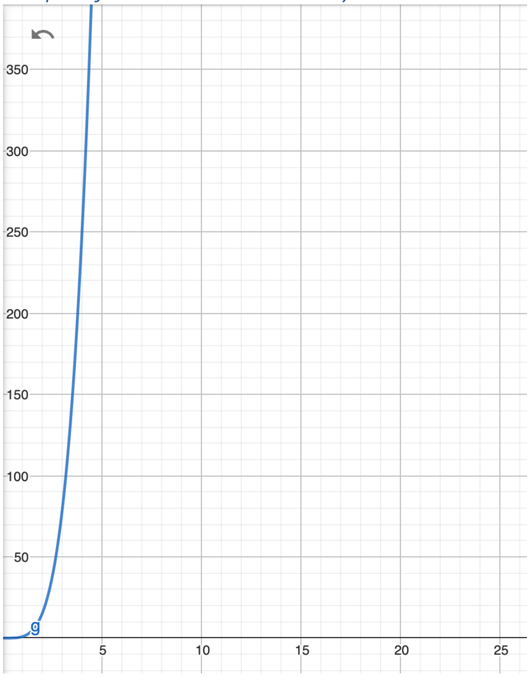
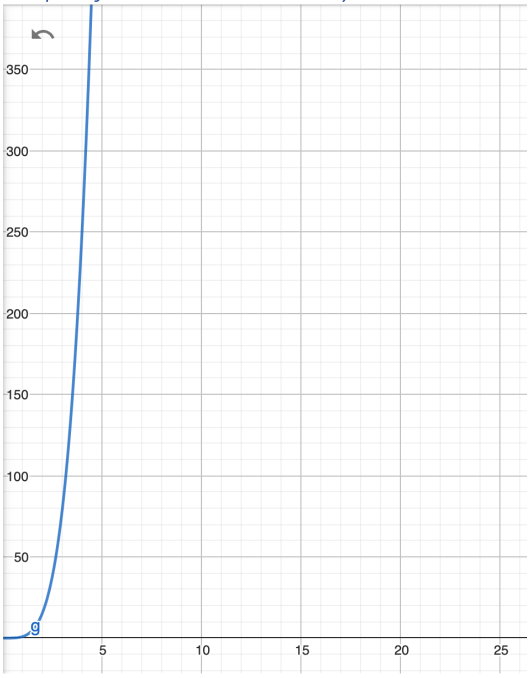
Graph of the theoretical analysis when basic operation is the operation marked as (2)



Compared to real best case execution graph, if we take 2 arbitrary points from the graphs, we can see that growth rates are not similar. From n=100 to 200, real execution graph grows 1,5 to 20. Which means 13.5x. And this graph is representing n^4, which should result in 16x growth. These numbers are similar, so we can say that these graphs are similar.

Graph of the theoretical analysis when basic operation is the operation marked as (3)



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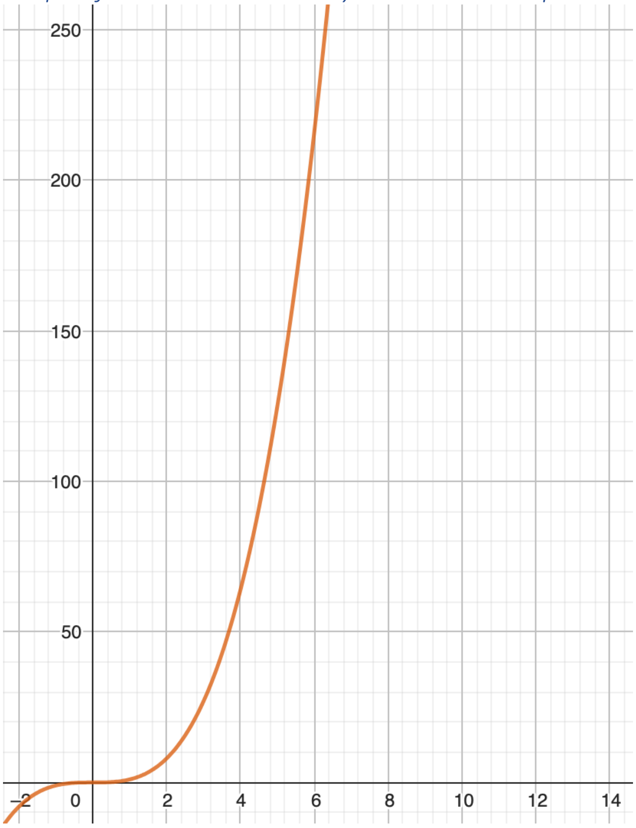
Graph of the theoretical analysis when basic operation is the operation marked as (4)



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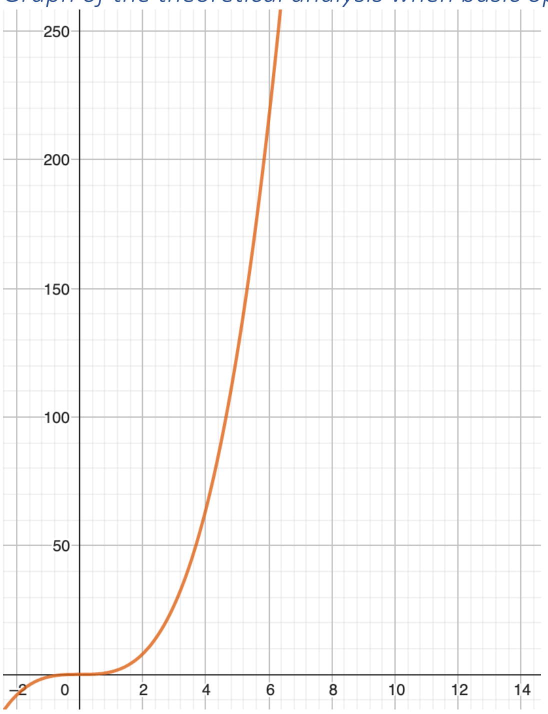
Graph of the theoretical analysis when basic operation is the operation marked as (5)



Compared to real best case execution graph, if we take 2 arbitrary points from the graphs, we can see that growth rates are not similar.

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Comments

After comparing every graph above, we concluded that when we choose operation(2) as the basic operation, theoretical data complies with real execution results the most.