MPHY0030
Programming Foundations for Medical Image Analysis
Assessed Coursework 2020-21

Part 2 Report

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Report Q1: Is the polynomial part needed and why? [1]

In the paper by Fornefett et al, 2001, it is mentioned that the polynomial part is not required for the calculation of both the inverse multiquadric and the Gaussian due to them being positive definite.

Report Q2: Write down the linear algebra used to represent the spline fitting problem and the solution. Cite the key formula in the report in the relevant part of your code. [6]

$$\begin{pmatrix} \mathbf{K}+\lambda\mathbf{W}^{-1} & \mathbf{P} \\ \mathbf{P}^{\mathrm{T}} & 0 \end{pmatrix}\begin{pmatrix} \boldsymbol{\alpha} \\ \boldsymbol{\beta} \end{pmatrix} = \begin{pmatrix} \mathbf{q}_k \\ 0 \end{pmatrix}$$

The figure above is a system of linear equations, where K is an n x n matrix given by,

$$K_{ij} = R(\|\mathbf{p}_i - \mathbf{p}_j\|)$$

and P is an n x M matrix given by.

$$P_{ij} = \phi_j(\mathbf{p}_i)$$

For the equation to cover our approximation, the system can be extended into;

$$\begin{pmatrix} \mathbf{K}+\lambda\mathbf{W}^{-1} & \mathbf{P} \\ \mathbf{P}^{\mathrm{T}} & 0 \end{pmatrix}\begin{pmatrix} \boldsymbol{\alpha} \\ \boldsymbol{\beta} \end{pmatrix} = \begin{pmatrix} \mathbf{q}_k \\ 0 \end{pmatrix}$$

with

$$\mathbf{W} = \mathrm{diag}\left\{1/\sigma_1^2, \ldots, 1/\sigma_n^2\right\},$$

For our Gaussian approximation, since we are not interested in the polynomial part, the linear equation below can be constructed, which allows us to find the coefficients, alpha.

$$(\mathbf{K} + \lambda\mathbf{W}^{-1})\boldsymbol{\alpha} = \mathbf{q}_k$$

Report Q3: What is the best linear algebra algorithm should be implemented to solve this spline fitting problem and why? [1]

Principal component analysis can be implemented to solve this spline fitting problem as it is a popular dimensionality reduction method, which can be used on 3D datasets with numerous columns. It is generally used to create visual projections of high-dimensional data.

Report Q4: What are the control points here? Can we choose any points as control points at evaluation stage, and why? [2]

We cannot choose any points in the evaluation stage. The evaluation stage uses the control points which will be coming from and previously inputted into the fitting stage. Furthermore, this data will allow us to depict whether we have successfully completed the interpolation or not.

Report Q5: Do we need the weighting parameter lambda at evaluation stage, and why? [2]

I have used lambda for the fitting stage and not for the evaluation stage. I do have to input it into the evaluation stage since I am calling the function 'fit(~)' in my evaluation stage. It should only be used for the datasets that have localisation errors, but evaluation stage does not work with randomly interpolated points, therefore, theoretically, is should be used in the evaluation stage.

Report Q6: Describe the details of your vectorisation strategies for kernel computing for large point sets. [2]

For large data sets, matrix vectorisation must be used cut down on runtime. The strategy of looping of rows and columns is effective but extremely time consuming since it is required to iterative over every single component of a matrix. On the other hand, a matrix could be easily used for the computation of the kernel.

Report Q7: Discussion of the utility of the Gaussian kernel parameter, sigma. [1]

According to Fornefett et al, sigmas are the individual weights of the landmarks which. represent landmark localization errors.

Report Q8: What is a reasonable approach to randomly displace the control points, (e.g. what distribution the random transform need to draw from and is there any constraint needed), such that the resulting transformation represented by the moved control points are considered biophysically reasonable. [2]

For the control points to be only allowed to move halfway between the next point. If we imagine a point Y, in between X and Z, equally spaced apart. Y should be allowed to move, at max, halfway to either X or Z. This means that when points are transformed, they will not overlap and cause tearing on the image.