Intro2Astro 2025
Week 5

Emre Tuncer

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Section I: Life & Habitability

1.a. Habitable Zone Calculation

To find the bounds of the habitable zone, we rearrange the equilibrium temperature equation to solve for the radial distance, a.

Given the equation:

$$T_{eq} = \left(\frac{(1-A)L_*}{16\pi\sigma a^2} \right)^{\frac{1}{4}}$$

We solve for a:

$$T_{eq}^4 = \frac{(1-A)L_*}{16\pi\sigma a^2}$$

$$a^2 = \frac{(1-A)L_*}{16\pi\sigma T_{eq}^4}$$

$$a = \sqrt{\frac{(1-A)L_*}{16\pi\sigma T_{eq}^4}}$$

The given constants are:

- Bond Albedo, $A = 0.3$
- Stellar Luminosity, $L_* = 3.827 \times 10^{26}$ W
- Stefan-Boltzmann constant, $\sigma = 5.670 \times 10^{-8}$ W m^{-2} K^{-4}
- 1 au $= 1.496 \times 10^{11}$ m

The habitable zone is defined by the temperatures at which water is liquid:

- Inner bound (boiling point of water): $T_{eq} = 373$ K
- Outer bound (freezing point of water): $T_{eq} = 273$ K

Inner Bound Calculation (a_{inner} at 373 K):

$$a_{inner} = \sqrt{\frac{(1-0.3)(3.827 \times 10^{26})}{16\pi(5.670 \times 10^{-8})(373)^4}}$$

$$a_{inner} = \sqrt{\frac{0.7 \times 3.827 \times 10^{26}}{16\pi(5.670 \times 10^{-8})(1.933 \times 10^{10})}}$$

$$a_{inner} = \sqrt{\frac{2.6789 \times 10^{26}}{5.517 \times 10^{4}}} \approx \sqrt{4.855 \times 10^{21}} \approx 6.968 \times 10^{10} \text{ m}$$

In astronomical units (au):

$$a_{inner} \approx \frac{6.968 \times 10^{10} \text{ m}}{1.496 \times 10^{11} \text{ m/au}} \approx 0.466 \text{ au}$$

Outer Bound Calculation (a_{outer} at 273 K):

$$a_{outer} = \sqrt{\frac{(1-0.3)(3.827 \times 10^{26})}{16\pi(5.670 \times 10^{-8})(273)^4}}$$

$$a_{outer} = \sqrt{\frac{0.7 \times 3.827 \times 10^{26}}{16\pi(5.670 \times 10^{-8})(5.554 \times 10^{9})}}$$

$$a_{outer} = \sqrt{\frac{2.6789 \times 10^{26}}{1.583 \times 10^{4}}} \approx \sqrt{1.692 \times 10^{22}} \approx 1.301 \times 10^{11} \text{ m}$$

In astronomical units (au):

$$a_{outer} \approx \frac{1.301 \times 10^{11} \text{ m}}{1.496 \times 10^{11} \text{ m/au}} \approx 0.870 \text{ au}$$

Result: The calculated radial extent of the habitable zone is from approximately **0.47 au** to **0.87 au**.

1.b. Model Underestimate or Overestimate

To determine if the model is an underestimate or overestimate, we calculate the equilibrium temperature of Earth at its orbit of 1 au using this model.

$$T_{eq,Earth} = \left(\frac{(1-0.3)(3.827 \times 10^{26})}{16\pi(5.670 \times 10^{-8})(1.496 \times 10^{11})^2}\right)^{\frac{1}{4}}$$

$$T_{eq,Earth} = \left(\frac{2.6789 \times 10^{26}}{16\pi(5.670 \times 10^{-8})(2.238 \times 10^{22})}\right)^{\frac{1}{4}}$$

$$T_{eq,Earth} = \left(\frac{2.6789 \times 10^{26}}{6.379 \times 10^{16}}\right)^{\frac{1}{4}} \approx (4.199 \times 10^{9})^{\frac{1}{4}} \approx 254.5 \text{ K}$$

The calculated equilibrium temperature for Earth is approximately 255 K (-18°C). Earth's actual average surface temperature is about 288 K (15°C). Since Earth's actual temperature is warmer than the temperature predicted by the model for a planet at 1 au, and our calculated habitable zone only extends to 0.87 au, this model **underestimates** the habitable zone.

Reason: The primary reason for this discrepancy is that the "bare-rocky" model neglects the **greenhouse effect**. Earth's atmosphere contains greenhouse gases (like HO and CO) which trap outgoing infrared radiation, warming the surface to temperatures above the equilibrium temperature. This additional warming effect allows liquid water to exist at orbital distances where this simple model would predict freezing temperatures, thus extending the outer boundary of the true habitable zone.

Section II: Interpreting Atmospheric Absorption Spectra

2. Labeling Absorption Features

The provided figure shows the percentage of solar radiation absorbed or scattered by Earth's atmosphere. The major absorption features can be attributed to the following molecules at specific wavelengths:

- **O and O (Oxygen and Ozone):** Responsible for the strong absorption of ultraviolet (UV) radiation at wavelengths shorter than approximately 0.3 µm. This is seen as the sharp cutoff on the far left of the graph where absorption is 100

- **HO (Water Vapor):** Water has numerous, significant absorption bands throughout the infrared portion of the spectrum. Prominent absorption features are visible around 1.1 µm, 1.4 µm, 1.9 µm, and 2.7 µm. There is also a very broad and strong absorption region starting around 5-7 µm and extending to longer wavelengths.

- **O (Molecular Oxygen):** A distinct, sharp, and narrow absorption band is visible at approximately 0.76 µm.

- **CO (Carbon Dioxide):** Carbon dioxide has several notable absorption bands. There are features around 1.6 µm, 2.0 µm, and a very strong, distinct peak around 4.3 µm. Another significant absorption band for CO occurs around 15 µm.
- **CH (Methane):** Methane contributes smaller absorption features, with noticeable bands around 2.3 µm, 3.3 µm and near 8 µm.

Section III: Characterizing Atmospheric Loss

3.a. Escape Velocity of Terra II

The escape velocity, v_e, is calculated using the formula:

$$v_e = \sqrt{\frac{2GM}{r}}$$

Given parameters for Terra II:

- Mass, $M_{TerraII} = 4 \times M_\oplus = 4 \times (5.972 \times 10^{24} \text{ kg}) = 2.3888 \times 10^{25}$ kg
- Radius, $r_{TerraII} = 2 \times R_\oplus = 2 \times (6.378 \times 10^6 \text{ m}) = 1.2756 \times 10^7$ m
- Gravitational Constant, $G = 6.6743 \times 10^{-11}$ $\mathbf{m^3}$ $\mathbf{kg^1}$ $\mathbf{s^2}$

Now, we calculate the escape velocity:

$$v_e = \sqrt{\frac{2(6.6743 \times 10^{-11})(2.3888 \times 10^{25})}{1.2756 \times 10^7}}$$

$$v_e = \sqrt{\frac{3.1887 \times 10^{15}}{1.2756 \times 10^7}} = \sqrt{2.4998 \times 10^8} \approx 15810.7 \text{ m/s}$$

To match the units on the diagram, we convert from m/s to km/s:

$$v_e = \frac{15810.7 \text{ m/s}}{1000} \approx 15.81 \text{ km/s}$$

The escape velocity on Terra II is approximately **15.81 km/s**.

3.b. Atmosphere Retention Prediction

To predict if Terra II can retain an atmosphere, we plot it on the Cosmic Shoreline diagram using the following coordinates:

- **Escape Velocity (x-axis):** 15.81 km/s
- **Historic XUV fluence (y-axis):** 100 times that of Earth (10^2 on the log scale).

Locating the point (15.81, 10^2) on the diagram places Terra II in the upper-left, reddish-colored region labeled "Atmosphere lost". This point is clearly above the "Cosmic Shoreline" which separates planets that retain their atmospheres from those that lose them.

Prediction: Based on the Cosmic Shoreline model, Terra II is **likely to have lost its atmosphere**. Its position on the diagram indicates that the high XUV radiation from its star combined with an insufficient escape velocity would lead to atmospheric stripping over time.