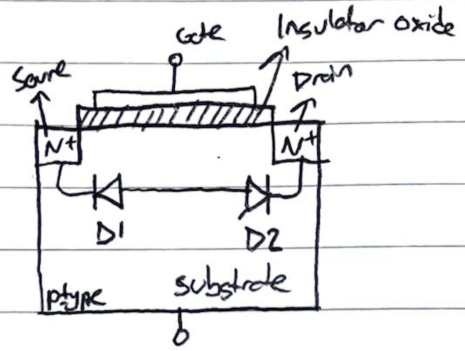
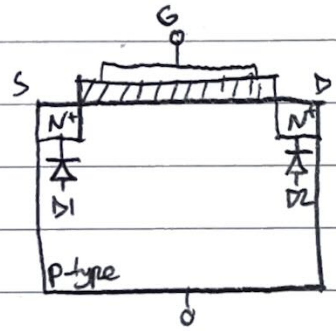
EMRE UNCU - 22003884 HW-1 / /

Q1



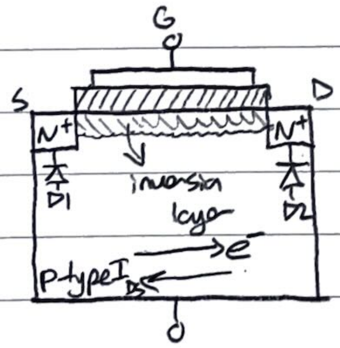
There is a insulator (→ isolator) oxide between the gate and the substrate, it prevents the current from the gate to the substrate.
Substrate is tied to the most negative voltage in circuit so the D1 (source-subs) and D2 (drain-subs) diodes are off, thus there is no current to substrate.

a)

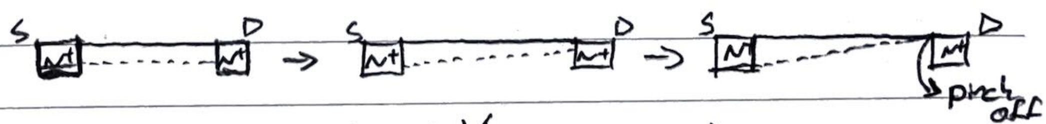


An inversion layer is needed for the current conduction D to S. When $V_{GS} < V_{th}$ the inversion layer cannot be created. So $I_{DS}=0$ and transistor is Off

b)

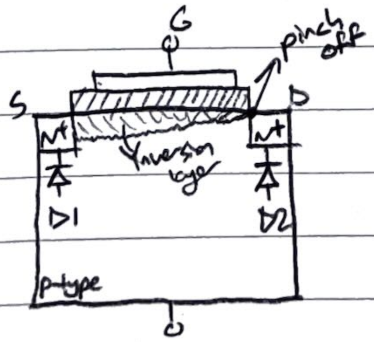


When $V_{GS} > V_{th}$ and V_{DS} is small, the inversion layer can be created, the I_{DS} current occured from drain to source because of the $\bar{e}$ flow from source to drain. I_{DS} increases as V_{DS} increases.
So $I_{DS} > 0$ and transistor is ON in TRIODE state



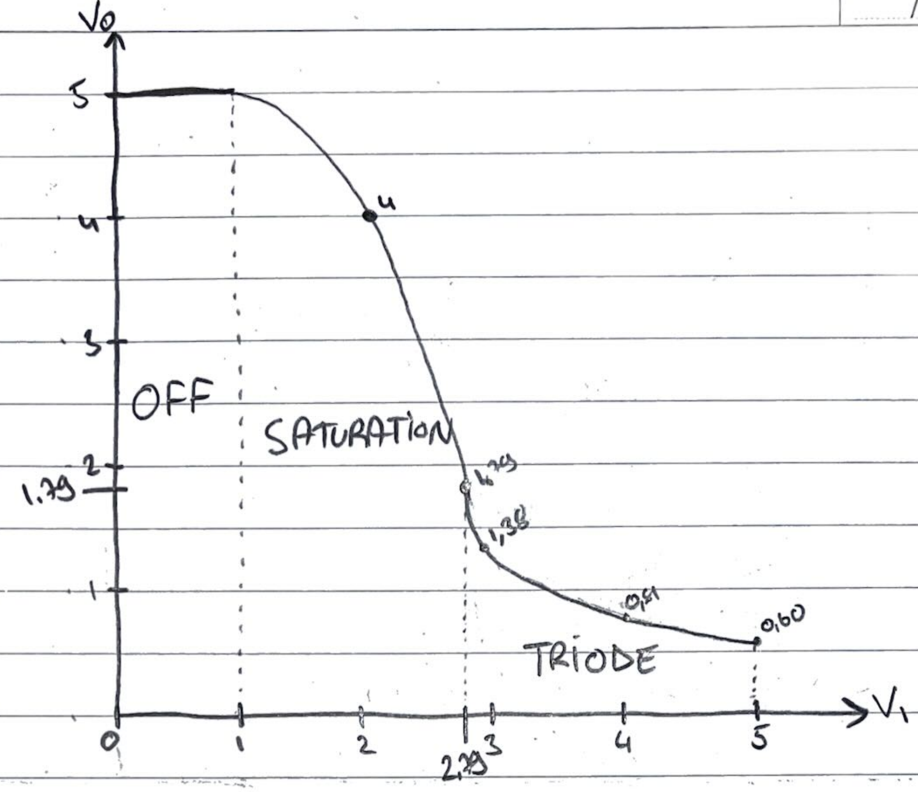
⟶ as V_{DS} increases ⟶

c)



When $V_{GS} > V_{th}$ and V_{DS} is large, As V_{DS} increases the $\bar{e}$ distribution shifts towards the source. After some points, ie. $V_{DS} = V_{GS} - V_{Th}$ the pinch off points occurs, so $\bar{e}$ density at drain becomes zero. After the pinch off point I_{DS} remains constant even if V_{DS} increases. because $\bar{e}$'s get swept by E field. Transistor is ON and in Saturation state

Q2



OFF

$V_I < V_{TH} = 1$ for $0 < V_I < 1$

So transistor is OFF

$I_D = 0$

$V_O = 5 - 5I_D$

$V_O = 5$

SATURATION

for $V_I > V_{TH} = 1$

transistor is ON

Let assume transistor is in SAT

until transition point

On transition point:

$V_{DS} = V_{GS} - V_{TH} \Rightarrow V_O = V_I - 1$ *

$I_D = k_n (V_{GS} - V_{TH})^2 = 0.2 (V_I - 1)^2$

$I_D = 0.2 . V_O^2$ and $V_O = 5 - 5I_D$

so

$V_O^2 + V_O - 5 = 0$

$V_O = 1.79V$ and $\cancel{V_O = -2.79V}$

$V_I = V_O + 1 = 2.79V$

so M1 is in SAT for $1 \leq V_I \leq 2.79$

$I_D = 0.2.(V_I - 1)^2$ and $V_O = 5 - 5I_D$

$V_O = 5 - (V_I - 1)^2$ for $1 \leq V_I \leq 2.79V$

for $V_I = 2 \Rightarrow V_O > V_{GS} - V_{TH} \Rightarrow SAT$ ✓

$4 > 2 - 1$

TRIODE

Assumption ⇒ triode for $2.79 < V_I < 5V$

$I_D = k_n (2.(V_{GS} - V_{TH})^2 . V_{DS} - V_{DS}^2$

$I_D = 0.2 (2(V_I - 1)^2 . V_O - V_O^2)$

$V_O = 5 - R_D I_D = 5 - 5I_D$

so

$5 - V_O = 2(V_I - 1) . V_O - V_O^2$

$V_O^2 + (1 - 2V_I) V_O + 5 = 0$

$$V_O = \frac{(2V_I - 1) - \sqrt{(1 - 2V_I)^2 - 20}}{2}$$

for $2.79 < V_I < 5$

for $V_I = 4 \Rightarrow V_O < V_{GS} - V_{TH} \Rightarrow$ triode ✓

$0.81 < 4 - 1$

Q3

for M3 $V_S = 0V,\ V_D = 2V,\ V_{DS} = 2V,\ V_G = 2V,\ V_{GS} = 2V$

Assume M3 is in saturation $\Rightarrow I_D = k_n (V_{GS} - V_{TH})^2 = \frac{k'_n}{2} \cdot \frac{W}{L} \cdot (V_{GS} - V_{TH})^2 = 100 . 5 . (2-1)^2 = 500\,\mu A$

$I_D = 500\,\mu A$

$V_{GS} > V_{TH} \Rightarrow 2 > 1$
$V_{DS} \geq V_{GS} - V_{TH} \Rightarrow 2 > 1$
$\Rightarrow$ M3 is in SATURATION

for M2 $V_S = 2V,\ V_D = V_x,\ V_{DS} = V_x - 2,\ V_G = V_x,\ V_{GS} = V_x - 2$

Assume M2 is in saturation $\Rightarrow I_D = 500\,\mu A = 100 . 10 . (V_x - 2 - 1)^2 = 1000 . (V_x - 3)^2\,\mu A$

$V_x = 3 + \frac{1}{\sqrt{2}}$

$V_{GS} > V_{TH} \Rightarrow 1 + \frac{1}{\sqrt{2}} > 1$
$V_{DS} \geq V_{GS} - V_{TH} \Rightarrow 1 + \frac{1}{\sqrt{2}} \geq \frac{1}{\sqrt{2}}$
$\Rightarrow$ M2 is in SATURATION

for M1 $V_S = 3 + \frac{1}{\sqrt{2}},\ V_D = 5V,\ V_{DS} = 2 - \frac{1}{\sqrt{2}},\ V_G = 5V,\ V_{GS} = 2 - \frac{1}{\sqrt{2}}$

Assume M1 is in saturation $\Rightarrow I_D = 500\,\mu A = 100 . \left(\frac{W}{L}\right)_3 . \left(2 - \frac{1}{\sqrt{2}} - 1\right)^2 = 100 \left(\frac{W}{L}\right)_3 . \left(1 - \frac{1}{\sqrt{2}}\right)^2 \mu A$

$(W/L)_3 = 58.28$

$V_{GS} > V_{TH} \Rightarrow 2 - \frac{1}{\sqrt{2}} > 1$
$V_{DS} \geq V_{GS} - V_{TH} \Rightarrow 2 - \frac{1}{\sqrt{2}} > 1 - \frac{1}{\sqrt{2}}$
$\Rightarrow$ M1 is in SATURATION

Q4

a) $V_G = 10.\frac{30}{30+70} = 3V$, $V_D - R_D.I_D = 0 \Rightarrow V_D = 3I_D$ $\quad V_{SD} = 10 - 8I_D$

$V_S + R_S I_D = 10 \Rightarrow V_S = 10 - 5I_D$ $\quad V_{SG} = 7 - 5I_D$

Assume M1 is in Saturation $\Rightarrow I_D = K_p(V_{SG} - |V_{tp}|)^2 = 0{,}25(7 - 5I_D - 1)^2 = 0{,}25(6 - 5I_D)^2$

$\underline{\underline{I_D = 0{,}83\,mA}}$

$I_D \neq 1.73$ because $V_{SG} > |V_{tp}|$

So $V_{SD} = 10 - 8(0{,}83) = \underline{3{,}36V}$

$V_{SG} = 7 - 5(0{,}83) = 2.85V$

$V_{SG} > |V_{tp}| \Rightarrow 2.85 > 1$

$V_{SD} \geq V_{SG} - |V_{tp}| \Rightarrow 3{,}36 \geq 1{,}85$

$\Rightarrow$ M1 is in $\underline{\underline{\text{SATURATION}}}$

b) $V_G = 10.\frac{30}{30+70} = 3V$, $V_D - R_D I_D = 0 \Rightarrow V_D = 3I_D$ $\quad V_{SD} = 10 - 4I_D$

$V_S + R_S I_D = 10 \Rightarrow V_S = 10 - I_D$ $\quad V_{SG} = 7 - I_D$

Assume M1 is in saturation $\Rightarrow I_D = 0{,}25(7 - I_D - 1)^2 = 0{,}25(6 - I_D)^2$ $\quad I_{D_1} = 13.29\,mA$, $I_{D_2} = 2.71\,mA$

Both I_{D_1} and I_{D_2} are not satisfy $V_{SG} > |V_{tp}|$ and $V_{SD} \geq V_{SG} - |V_{tp}|$

so M1 is not in saturation

$I_D = K_p(2(V_{SG} - |V_{tp}|)V_{SD} - V_{SD}^2) = 0{,}25(2(6 - I_D)(10 - 4I_D) - (10 - 4I_D)^2)$ $\quad I_{D_1} = 2.16\,mA$, $I_{D_2} = -1{,}16\,mA$

Let choose $\underline{\underline{I_{D_1} = 2.16\,mA}}$

$\underline{\underline{V_{SD} = 10 - 4(2.16) = 1.36V}}$

$V_{SG} = 7 - (2{,}16) = 4{,}84V$

$V_{SG} > |V_{tp}| \Rightarrow 4.84 > 1$

$V_{SD} < V_{SG} - |V_{tp}| \Rightarrow 1.36 < 3.84$

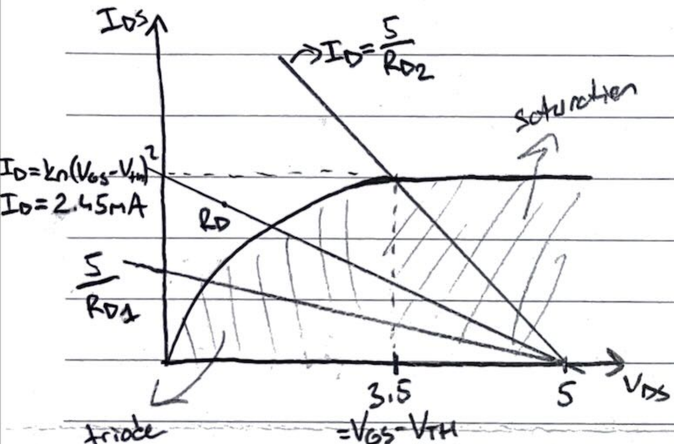
$\Rightarrow$ M1 is in $\underline{\underline{\text{TRIODE}}}$

Q5

a) $V_S = 0V$, $V_D = 5 - R_D I_D$, $V_{DS} = 5 - R_D I_D$, $V_G = 5V$, $V_{GS} = 5V$

$V_{GS} > V_{TH} \Rightarrow 5 > 1.5$

$V_{DS} \geq V_{GS} - V_{TH} \Rightarrow 5 - R_D I_D \geq 5 - V_{TH} \Rightarrow \boxed{V_{TH} \geq R_D I_D}$ for satisfy saturation



When R_D increases the intersection of the curve with the R_D line shifts to left. As R_D increases the intersection slides to triode area from sat area. Also satisfy the $V_{DS} \geq V_{GS} - V_{TH}$ becomes harder because $5 - R_D I_D \geq 5 - 1.5$. R_D should be choosen small to keep M1 in saturation.

b)

Assume M1 is in saturation $\Rightarrow I_D = K_n(V_{GS} - V_{TH})^2 = 0.2(5-1.5)^2 = 2.45mA$

$V_{GS} > V_{TH} \Rightarrow 5 > 1.5$

$V_{DS} \geq V_{GS} - V_{TH} \Rightarrow 5 - R_D I_D \geq 5 - V_{TH} \Rightarrow V_{TH} \geq R_D I_D \Rightarrow 1.5 \geq R_D(2.45mA)$

so $\boxed{0 \leq R_D \leq 612\,\Omega}$ to keep M1 in saturation state.

Q6

a) for M1: $V_S = 0V$, $V_D = V_o$, $V_{DS} = V_o$, $V_G = 2V$, $V_{GS} = 2V$

M1 in saturation $\Rightarrow I_D = K_n (V_{GS} - V_{tn})^2 = K_n \cdot (2-1)^2 \Rightarrow I_D = K_n$

$V_{GS} > V_{tn} \Rightarrow 2 > 1$

$V_{DS} \geq V_{GS} - V_{tn} \Rightarrow V_o \geq 1$

for M2: $V_S = 5V$, $V_D = V_o$, $V_{SD} = 5 - V_o$, $V_G = V_b$, $V_{SG} = 5 - V_b$

M2 in triod $\Rightarrow I_D = K_n = K_p(2(V_{SG} - |V_{tp}|) \cdot V_{SD} - V_{SD}^2) = K_p(2(5 - V_b - 1) \cdot (5 - V_o) - (5 - V_o)^2)$

$= \cancel{K_n} = \cancel{K_p}(-V_o^2 + (2V_b + 2)V_o + 15 - 10V_b) \Rightarrow V_o^2 - (2V_b + 2)V_o + 10V_b - 14 = 0$

$V_{o_1} = \frac{2(V_b+1) + \sqrt{4V_b^2 - 32V_b + 60}}{2}$, $V_{o_2} = \frac{2(V_b+1) - \sqrt{4V_b^2 - 32V_b + 60}}{2}$ $\Rightarrow V_o = (V_b + 1) + \sqrt{(V_b - 5)(V_b - 3)}$

We choose V_{o_1} because of (*)

M2 in triod: $V_{SG} > |V_{tp}| \Rightarrow 5 - V_b > 1 \Rightarrow V_b < 4$

: $V_{SD} < V_{SG} - |V_{tp}| \Rightarrow 5 - V_o < 4 - V_b \Rightarrow V_o > V_b + 1$ (*)

M2 is ON: $V_{SD} > 0 \Rightarrow 5 - V_o > 0 \Rightarrow V_o < 5 \Rightarrow (V_b + 1) + \sqrt{(V_b - 5)(V_b - 3)} < 5$

to satisfy this eq V_b should be smaller than 3 so range of V_b is $\boxed{(-\infty, 3)}$

b) for M1: $V_S = 0V$, $V_D = V_o$, $V_{DS} = V_o$, $V_G = 3V$, $V_{GS} = 3V$

M1 in saturation $\Rightarrow I_D = K_n (V_{GS} - V_{tn})^2 = K_n \cdot (3-1)^2 \Rightarrow I_D = 4K_n$

for M2: $V_S = 5V$, $V_D = V_o$, $V_{SD} = 5 - V_o$, $V_G = V_b$, $V_{SG} = 5 - V_b$

M2 in triod $\Rightarrow I_D = 4K_n = K_p(2(V_{SG} - |V_{tp}|) \cdot V_{SD} - V_{SD}^2) \Rightarrow 4 = 2(5 - V_b - 1)(5 - V_o) - (5 - V_o)^2$

$V_o^2 - (2V_b + 2)V_o + 10V_b - 11 = 0$

$V_{o_1} = V_b + 1 + \sqrt{V_b^2 - 8V_b + 9}$, $V_{o_2} = V_b + 1 - \sqrt{V_b^2 - 8V_b + 9}$ $\Rightarrow V_o = (V_b + 1) + \sqrt{V_b^2 - 8V_b + 9}$

again V_{o_1} is choosen since $V_o > V_b + 1$

M2 in triod: $V_{SG} > |V_{tp}| \Rightarrow 5 - V_b > 1 \Rightarrow V_b < 4$

: $V_{SD} < V_{SG} - |V_{tp}| \Rightarrow 5 - V_o < 4 - V_b \Rightarrow V_o > V_b + 1$ (*)

$(V_b - (4 - \sqrt{7}))(V_b - (4 + \sqrt{7}))$

M2 is ON: $V_{SD} > 0 \Rightarrow 5 - V_o > 0 \Rightarrow V_o < 5 \Rightarrow (V_b + 1) + \sqrt{V_b^2 - 8V_b + 9} < 5$

to satisfy this eq V_b should be smaller than $4 - \sqrt{7} = 1.35$

range of V_b is $\boxed{(-\infty, 1.35)}$