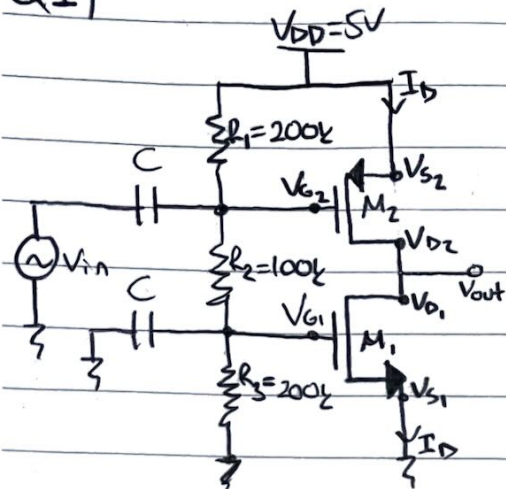


Q1)



$$a) V_{G_2} = \frac{(200k + 100k) \cdot 5V}{200k + 100k + 200k} = 3V, V_{G_1} = \frac{200k \cdot 5V}{200k + 100k + 200k} = 2V$$

$$V_{S_1} = 0V \quad V_{G_1} = 2V \quad V_{D_1} = V_{out}$$

$$V_{GS_1} = 2V \quad V_{DS_1} = V_{out}$$

$$V_{S_2} = 5V \quad V_{G_2} = 3V \quad V_{D_2} = V_{out}$$

$$V_{SG_2} = 2V_{in} \quad V_{SD_2} = 5 - V_{out}$$

$$0.25(2-1)^2(1+0.02V_{out}) = 0.25(3-1)^2(1+0.01(5-V_{out}))$$

Assume SAT for M_1 Assume SAT for M_2

$$I_D = K_n (V_{GS_1} - V_{th})^2 (1 + \lambda_1 V_{DS_1})$$

$$I_D = K_p (V_{SG_2} - |V_{thp}|)^2 (1 + \lambda_2 V_{SD_2})$$

$$I_D = 0.25(2-1)^2(1+0.02V_{out}) \quad (1)$$

$$I_D = 0.25(3-1)^2(1+0.01(5-V_{out})) \quad (2)$$

$$0.25(1+0.02V_{out}) = 0.25(1+0.01(5-V_{out})) \quad \text{by } (1)=(2)$$

$$2V_{out} = 5 - V_{out} \Rightarrow V_{out} = 5/3$$

$$V_{DS_1} = V_{out} \Rightarrow V_{DS_1} = 1.67V$$

$$V_{SD_2} = 5 - V_{out} \Rightarrow V_{SD_2} = 3.33V$$

$$\text{SAT} \rightarrow V_{GS_1} > V_{th} \Rightarrow 2V > 1V \quad \checkmark$$

$$\text{SAT} \rightarrow V_{SG_2} > |V_{thp}| \Rightarrow 2V > 1V \quad \checkmark$$

$$V_{DS_1} \geq V_{GS_1} - V_{th} \Rightarrow 1.67V > 1V \quad \checkmark$$

$$V_{SD_2} \geq V_{SG_2} - |V_{thp}| \Rightarrow 3.33V > 1V \quad \checkmark$$

$$I_D = 0.25(1+0.02 \cdot 1.67) \Rightarrow I_D = 0.26mA$$

$$g_{m_1} = 2\sqrt{K_n \cdot I_D} = 2\sqrt{0.25 \cdot 0.26}$$

$$g_{m_2} = 2\sqrt{K_p \cdot I_D} = 2\sqrt{0.25 \cdot 0.26}$$

$$g_{m_1} = 0.51 \text{ mA/V}$$

$$g_{m_2} = 0.51 \text{ mA/V}$$

$$r_{o_1} = \frac{1}{\lambda_1 I_D} = \frac{1}{0.02 \cdot 0.26}$$

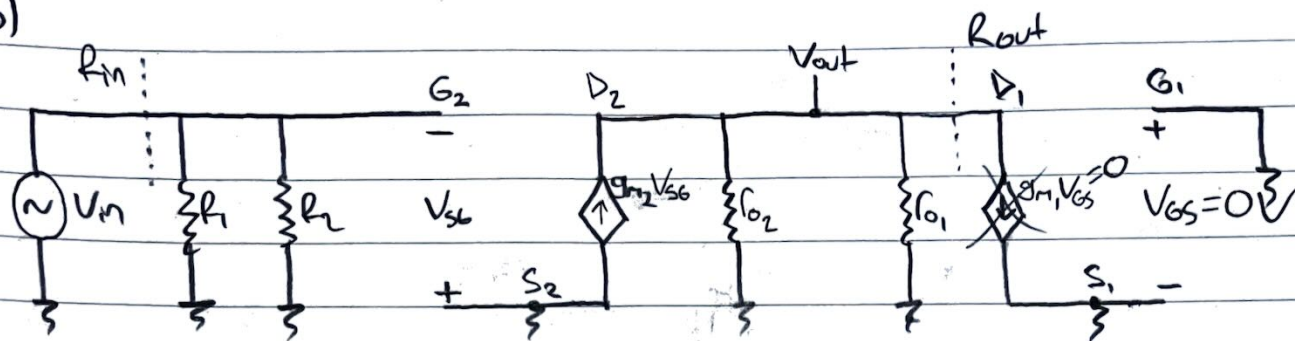
$$r_{o_2} = \frac{1}{\lambda_2 I_D} = \frac{1}{0.01 \cdot 0.26}$$

$$r_{o_1} = 193.57 \text{ k}\Omega$$

$$r_{o_2} = 387.15 \text{ k}\Omega$$

 $\lambda_1 \text{ and } \lambda_2 \neq 0 \text{ for } I_D \neq 0$

b)



$$V_{in} = -V_{S6}$$

$$V_{out} = g_{m2} V_{S6} (r_{o1} \parallel r_{o2}) =$$

$$A_v = \frac{V_{out}}{V_{in}} = \frac{g_{m2} V_{S6} (r_{o1} \parallel r_{o2})}{-V_{S6}}$$

$$A_v = -g_{m2} \left(\frac{r_{o1} \cdot r_{o2}}{r_{o1} + r_{o2}} \right) = -0.51 \left(\frac{193.57 \cdot 387.15}{193.57 + 387.15} \right)$$

$$A_v = \frac{V_{out}}{V_{in}} = -65.82$$

c) $R_{in} = R_1 \parallel R_2$

$$R_{in} = \frac{200 \cdot 100}{200 + 100} \Rightarrow R_{in} = 66.67 \text{ k}\Omega$$

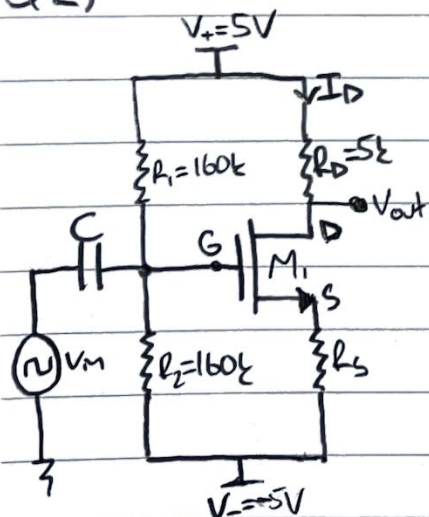
$$R_{out} \quad V_{in} \rightarrow \text{short} \rightarrow V_{S6} = 0V$$

$$g_{m2} V_{S6} = 0 \rightarrow \text{open} \quad I_D = 0$$

$$R_{out} = r_{o2} \parallel r_{o1}$$

$$R_{out} = \frac{193.57 \cdot 387.15}{193.57 + 387.15} \Rightarrow R_{out} = 129.05 \text{ k}\Omega$$

Q2)



$$a) V_G = (5 + (-5)) \cdot \frac{160}{160 + 160} = 0V \quad V_D = V_{out} = 5 - I_D R_D$$

$$V_S = -5 + I_D R_S \quad V_{GS} = 5 - I_D R_S \quad V_{DS} = 10 - I_D R_D - I_D R_S$$

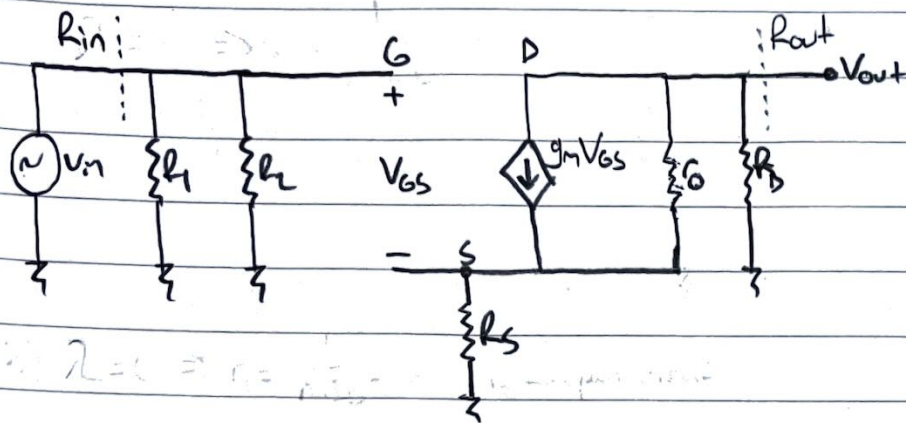
$$\text{Assume } M_1 \text{ in SAT} \quad I_D = K_n (V_{GS} - V_{TN})^2$$

$$\text{If } V_{out} = 0 \Rightarrow V_D = 0 = 5 - I_D \cdot 5 \Rightarrow I_D = 1 \text{ mA}$$

$$I_D = 0.25 (5 - 1 \cdot R_S - 1)^2 = 1 \Rightarrow R_S = 2 \text{ k}\Omega$$

$$\text{SAT} \rightarrow V_{GS} > V_{TN} \Rightarrow 5 - 1 \cdot 2 = 3V > 1V \quad \checkmark$$

$$V_{DS} \geq V_{GS} - V_{TN} \Rightarrow 10 - 1 \cdot 5 - 1 \cdot 2 = 3V \geq 5 - 1 \cdot 2 - 1 = 2V \quad \checkmark$$



$$g_m = 2\sqrt{k_n I_D} = 2\sqrt{0.25 \cdot 1}$$

$$g_m = 1 \text{ mA/V}$$

b) $\lambda = 0 \Rightarrow r_o = \frac{1}{\lambda I_D} = \infty \Rightarrow r_o$ is open circuit

$$V_{in} = V_{GS} + (g_m V_{GS}) R_S, \quad V_{out} = -(g_m V_{GS}) R_D$$

$$A_v = \frac{V_{out}}{V_{in}} = \frac{-V_{GS} \cdot g_m \cdot R_D}{V_{GS} (1 + g_m R_S)} = \frac{-g_m R_D}{1 + g_m R_S} = \frac{-5}{3} \Rightarrow \boxed{A_v = -1.67}$$

c) $r_o = \frac{1}{\lambda I_D} = \frac{1}{0.01 \cdot 1} = 100 \text{ k}\Omega$

$$V_G = V_{in}, \quad V_{GS} = V_{in} - V_S$$

KCL @ S: $0 = \frac{V_S}{R_S} - g_m V_{GS} + \frac{V_S - V_{out}}{r_o} \Rightarrow V_S \left(\frac{1}{R_S} + g_m + \frac{1}{r_o} \right) = g_m V_{in} + \frac{V_{out}}{r_o} \quad (1)$

KCL @ D: $g_m V_{GS} + \frac{V_{out} - V_S}{r_o} + \frac{V_{out}}{R_D} = 0 \Rightarrow V_S \left(\frac{1}{r_o} + g_m \right) = g_m V_{in} + V_{out} \left(\frac{1}{r_o} + \frac{1}{R_D} \right) \quad (2)$

Using (1) and (2):

$$\frac{(1)}{(2)} = \frac{\frac{1}{R_S} + g_m + \frac{1}{r_o}}{\frac{1}{r_o} + g_m} = \frac{g_m V_{in} + \frac{V_{out}}{r_o}}{g_m V_{in} + V_{out} \left(\frac{1}{r_o} + \frac{1}{R_D} \right)}$$

$$g_m V_{in} \left(\frac{1}{r_o} + g_m \right) + \frac{V_{out}}{r_o} \left(\frac{1}{r_o} + g_m \right) = g_m V_{in} \left(\frac{1}{R_S} + g_m + \frac{1}{r_o} \right) + V_{out} \left(\frac{1}{R_D} + \frac{1}{r_o} \right) \left(\frac{1}{R_S} + g_m + \frac{1}{r_o} \right)$$

$$V_{out} \left(\frac{1}{r_o} \left(\frac{1}{r_o} + g_m \right) - \left(\frac{1}{r_o} + \frac{1}{R_D} \right) \left(\frac{1}{R_S} + g_m + \frac{1}{r_o} \right) \right) = -V_{in} \left(g_m \left(\frac{1}{R_S} + g_m + \frac{1}{r_o} \right) - g_m \left(\frac{1}{r_o} + g_m \right) \right)$$

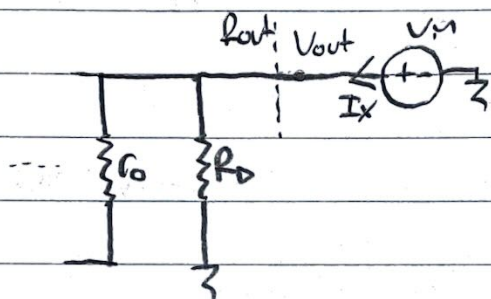
$$\frac{V_{out}}{V_{in}} = \frac{g_m \left(\frac{1}{R_S} + g_m + \frac{1}{r_o} - \frac{1}{r_o} - g_m \right)}{\frac{1}{r_o} \left(\frac{1}{r_o} + g_m \right) - \left(\frac{1}{r_o} + \frac{1}{R_D} \right) \left(\frac{1}{R_S} + g_m + \frac{1}{r_o} \right)} \xrightarrow[r \rightarrow 0]{\substack{\lim_{r_o \rightarrow \infty} \\ \frac{1}{r_o} = 0}} \frac{V_{out}}{V_{in}} = \frac{-g_m \left(\frac{1}{R_S} \right)}{\frac{1}{R_D} \left(\frac{1}{R_S} + g_m \right)} = \frac{-g_m R_D}{1 + g_m R_S}$$

Same as part b

d) $R_{in} = \frac{R_1 \cdot R_2}{R_1 + R_2} \Rightarrow \boxed{R_{in} = 80 \text{ k}\Omega}$

for R_{out}

$V_G = 0V$



$$R_{out} = \frac{V_{in}}{I_x}$$

KCL @ S

$$\frac{V_S}{R_S} + g_m V_{GS} + \frac{V_S - V_{out}}{r_o} = 0 \Rightarrow V_S = V_x \left(\frac{1}{r_o} \right) \left(\frac{1}{R_S} + g_m + \frac{1}{r_o} \right)^{-1}$$

Substitute

KCL @ D

$$I_x = \frac{V_{out}}{R_D} + g_m V_{GS} + \frac{V_{out} - V_S}{r_o} \Rightarrow I_x = V_x \left(\frac{1}{R_D} + \frac{1}{r_o} \right) - V_S \left(g_m + \frac{1}{r_o} \right)$$

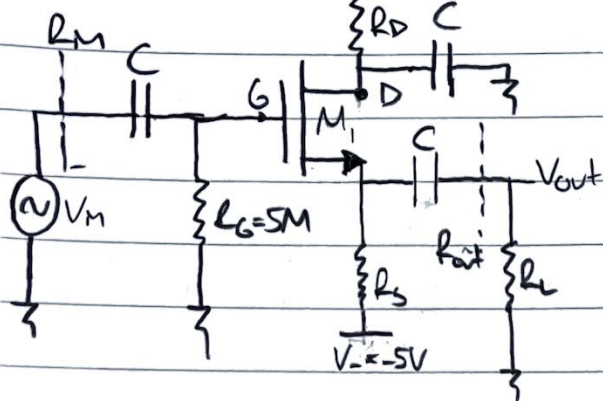
$$I_x = V_x \left(\frac{1}{R_D} + \frac{1}{r_o} \right) - V_x \left(\frac{1}{r_o} \right) \left(\frac{1}{R_S} + g_m + \frac{1}{r_o} \right) \left(g_m + \frac{1}{r_o} \right) = V_x \left(\frac{1}{R_D} + \frac{1}{r_o} - \left(\frac{1}{r_o} \right) \left(\frac{1}{R_S} + g_m + \frac{1}{r_o} \right) \left(g_m + \frac{1}{r_o} \right) \right)$$

$$I_x = V_x \left(\frac{1}{5} + \frac{1}{100} - \frac{1}{100} \cdot \left(\frac{1}{2} + 1 + \frac{1}{100} \right) \left(1 + \frac{1}{100} \right) \right) \Rightarrow \frac{V_x}{I_x} = \boxed{R_{out} \approx 5 \text{ k}\Omega}$$

Q3)

$$V_t = 5V$$

a)

Assume M_1 in SAT

$$V_G = 0V$$

$$V_S = -5 + I_D R_S$$

$$V_D = 5 - I_D R_D$$

$$V_{GS} = 5 - I_D R_S$$

$$V_{DS} = 10 - I_D R_S - I_D R_D$$

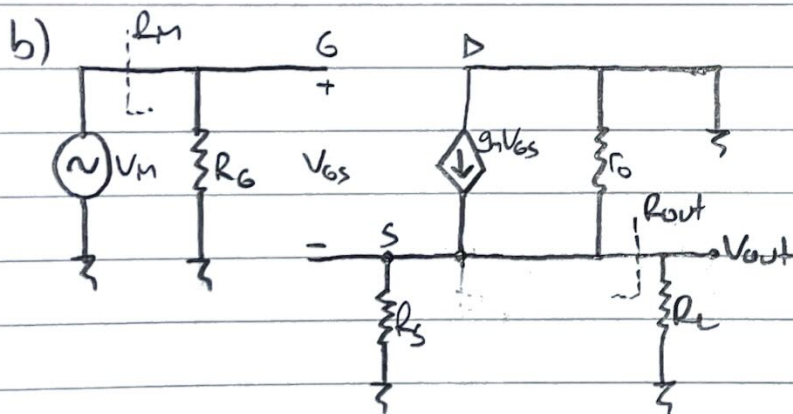
$$I_D = K_n (V_{GS} - V_{TN})^2 \quad \text{If } I_D = 0.2 \text{ and } V_D = 1V$$

$$V_D = 5 - I_D R_D = 5 - 0.2 R_D = 1 \Rightarrow R_D = 20k\Omega$$

$$I_D = K_n (V_{GS} - V_{TN})^2 = 0.5 (V_{GS} - 1)^2 = 0.2 \Rightarrow V_{GS} = 1.63 = 5 - 0.2 R_S \Rightarrow R_S = 16.85k\Omega$$

$$SAT \rightarrow V_{GS} > V_{TN} \Rightarrow 1.63 > 1 \quad \checkmark$$

$$V_{DS} \geq V_{GS} - V_{TN} \Rightarrow 2.63 \geq 1.63 - 1 \quad \checkmark$$



$$r_o = \frac{1}{\lambda I_D} = \frac{1}{0.01 \cdot 0.2} \Rightarrow r_o = 500k\Omega$$

$$V_{out} = V_S \quad V_D = 0V$$

$$g_m = 2\sqrt{K_n I_D} = 2\sqrt{0.5 \cdot 0.2} \Rightarrow g_m = 0.63 \frac{mA}{V}$$

KCL @ S

$$g_m V_{GS} = \frac{V_S}{R_S} + \frac{V_S}{r_o} \Rightarrow V_{out} = \frac{g_m V_{GS}}{\frac{1}{r_o} + \frac{1}{R_S}}$$

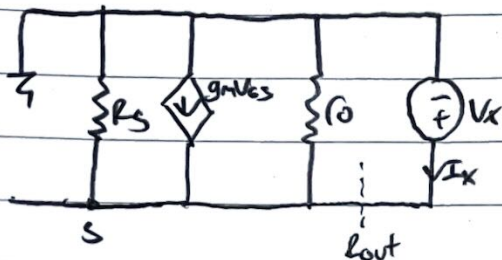
$$V_{in} = V_{GS} + V_S = V_{GS} + V_{out} \Rightarrow V_{in} = V_{GS} + \frac{g_m V_{GS}}{\frac{1}{r_o} + \frac{1}{R_S}}$$

$$A_v = \frac{V_{out}}{V_{in}} = \frac{\frac{V_{GS} g_m}{\frac{1}{r_o} + \frac{1}{R_S}}}{V_{GS} \left(1 + \frac{g_m}{\left(\frac{1}{r_o} + \frac{1}{R_S} \right)} \right)} = \frac{\frac{0.63}{\frac{1}{500} + \frac{1}{16.85}}}{1 + \frac{0.63}{\frac{1}{500} + \frac{1}{16.85}}} \Rightarrow A_{voc} = 0.81$$

c)

$$R_{in} = R_G \Rightarrow \boxed{R_{in} = 5M\Omega}$$

for R_{out} $V_G = 0V$ $V_S = V_X$ $V_{GS} = -V_S = -V_X$

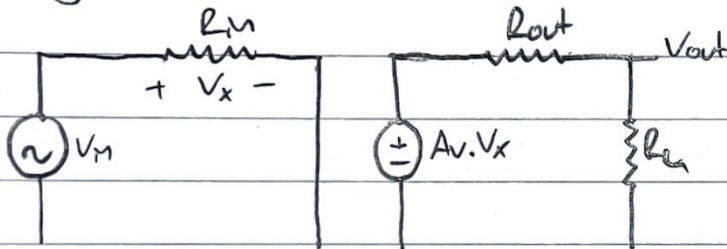


$$I_X = \frac{V_X}{R_G} + \frac{V_X}{R_S} - g_m V_{GS} = V_X \left(\frac{1}{R_G} + \frac{1}{R_S} + g_m \right)$$

$$R_{out} = \frac{V_X}{I_X} = \frac{1}{\frac{1}{R_G} + \frac{1}{R_S} + g_m} = \frac{1}{\frac{1}{500} + \frac{1}{16.85} + 0.63}$$

$$\boxed{R_{out} = 1.45k\Omega}$$

d) Using Black Box Model

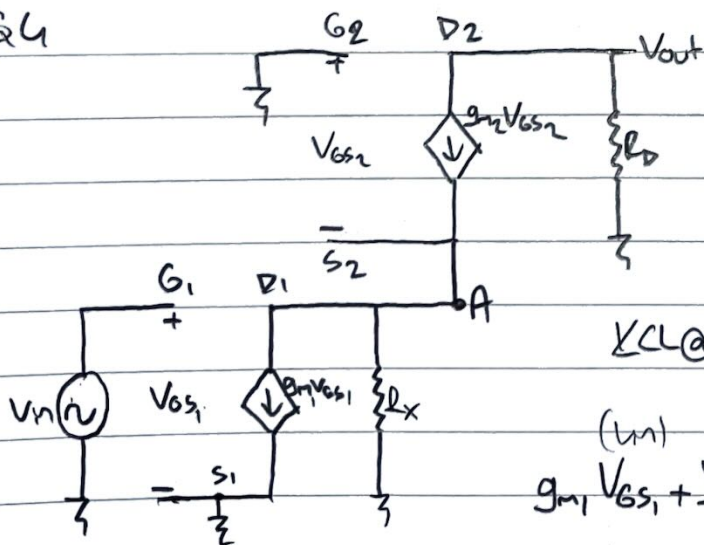


$V_X = V_{in}$

$$V_{out} = A_v V_X \cdot \frac{R_L}{R_{out} + R_L}$$

$$A_v = \frac{V_{out}}{V_{in}} = 0.91 \cdot \frac{10}{1.45 + 10} \Rightarrow \boxed{A_v = 0.79}$$

Q4



$V_{G2} = 0V$ $V_{S2} = -V_{D2} = -V_A$

$V_{G1} = V_{in}$ $V_{S1} = V_{G1} = V_{in}$

$V_{out} = -g_{m2} V_{GS2} R_D = g_{m2} V_A R_D$

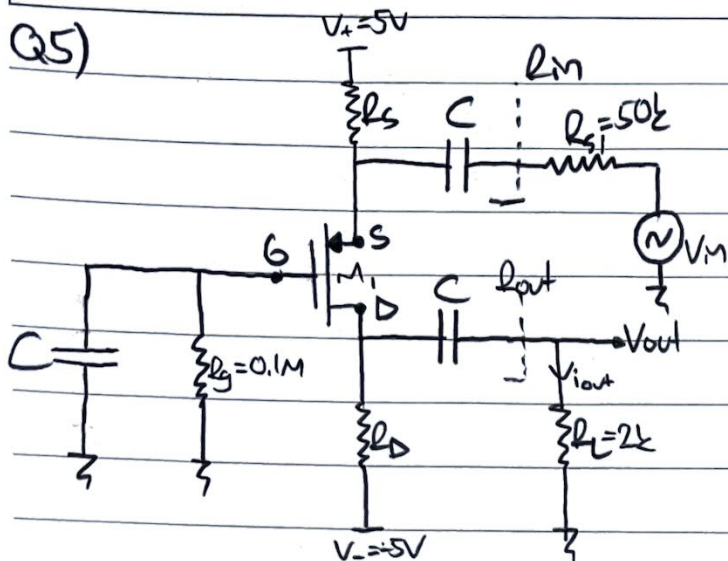
KCL @ A

$$g_{m1} V_{GS1} + \frac{V_A}{R_X} - g_{m2} V_{GS2} = g_{m1} V_{in} + \frac{V_A}{R_X} + g_{m2} V_A = 0$$

$$V_{in} = \frac{-V_A}{g_{m1}} \left(\frac{1}{R_X} + g_{m2} \right)$$

$$A_v = \frac{V_{out}}{V_{in}} = \frac{V_A \cdot g_{m2} \cdot R_D}{\frac{-V_A}{g_{m1}} \left(\frac{1}{R_X} + g_{m2} \right)} \Rightarrow \boxed{A_v = \frac{-g_{m1} \cdot g_{m2} \cdot R_D}{\frac{1}{R_X} + g_{m2}}}$$

Q5)



a)

$$V_G = 0V \quad V_S = 5 - I_D R_S \quad V_D = -5 + I_D R_D$$

$$V_{SG} = 5 - I_D R_S \quad V_{SD} = 10 - I_D R_S - I_D R_D$$

Assume Min SAT $I_D = 0.5$ $V_{SD} = 5V$

$$I_D = k_p (V_{SG} - |V_{TP}|)^2 = 0.5 (V_{SG} - 1)^2 = 0.5$$

$$V_{SG} = 5 - I_D R_S = 5 - 0.5 \cdot 6 = 2$$

$$R_S = 6k \Omega$$

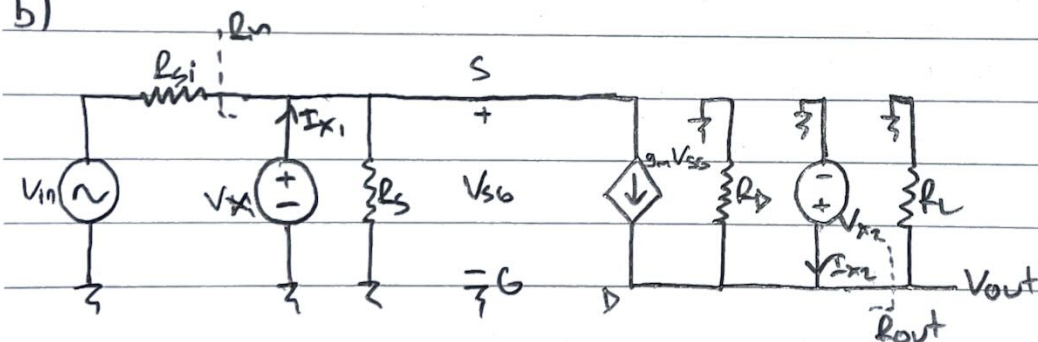
$$R_D = 4k \Omega$$

$$V_{SD} = 10 - I_D R_S - I_D R_D = 10 - 0.5 \cdot 6 - 0.5 \cdot 4 = 5$$

$$SAT \rightarrow V_{SG} > |V_{TP}| \Rightarrow 2 > 1 \quad \checkmark$$

$$V_{SD} \geq V_{SG} - |V_{TP}| \Rightarrow 5 \geq 2 - 1 \quad \checkmark$$

b)



$$g_m = 2\sqrt{0.5 \cdot 0.5} = 1 \text{ mA/V}$$

$$V_G = 0V \quad V_{SG} = V_S$$

for R_{in} , by using $V_{X1} \Rightarrow R_{in} = V_{X1} / I_{X1}$ ignore left side of R_{in} kill V_{in}
ignore V_{X2} source

$$KCL @ S \quad I_{X1} = \frac{V_{X1}}{R_S} + g_m V_{GS} = V_{X1} \left(\frac{1}{R_S} + g_m \right) \quad R_{in} = \frac{V_{X1}}{I_{X1}} = \frac{1}{\frac{1}{R_S} + g_m} = \frac{1}{\frac{1}{6} + 1} \Rightarrow R_{in} = 0.86k \Omega$$

for R_{out} , by using $V_{X2} \Rightarrow R_{out} = V_{X2} / I_{X2}$ ignore right side of R_{out} kill V_{in}
ignore V_{X1} source $V_{in} = 0$

$$KCL @ D \quad I_{X2} = \frac{V_{X2}}{R_D} - g_m V_{SG} = \frac{V_{X2}}{R_D} - g_m V_S$$

$$KCL @ S \quad g_m V_{SG} + \frac{V_S}{R_S} + \frac{V_S}{R_{Si}} = 0 \Rightarrow V_S = 0V$$

$$\Rightarrow I_{X2} = \frac{V_{X2}}{R_D} \Rightarrow R_{out} = \frac{V_{X2}}{I_{X2}} = R_D \Rightarrow R_{out} = 4k \Omega$$

c) KCL @ S

$$g_m V_{SG} + \frac{V_S}{R_S} + \frac{V_S - V_{in}}{R_{Si}} = V_S \left(g_m + \frac{1}{R_S} + \frac{1}{R_{Si}} \right) - \frac{V_{in}}{R_{Si}} = V_S \left(1 + \frac{1}{6} + \frac{1}{50} \right) - \frac{0.5 \cdot 10^{-3}}{50} = 0$$

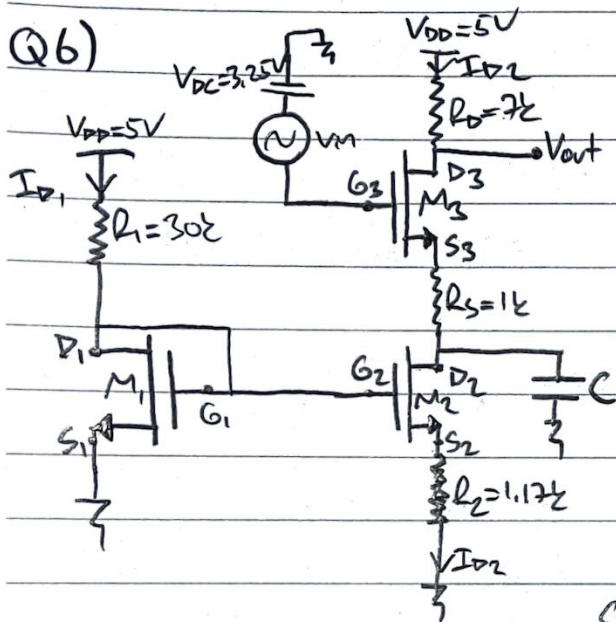
KCL @ D

$$V_S = 8.43 \cos(\omega t) \mu V$$

$$-g_m V_{SG} + \frac{V_D}{R_D} + i_{out} = -g_m V_S + \frac{i_{out} R_L}{R_D} + i_{out} = -1.843 \cdot 10^{-6} + \frac{i_{out} \cdot 3}{2} = 0$$

$$i_{out} = 5.62 \cos(\omega t) \mu A$$

Q6)



Assume M_1 in SAT

$$V_{G1} = V_{D1} = 5 - I_{D1} R_1 \quad V_{GS1} = V_{DS1} = V_{G1}$$

$$V_{S1} = 0V$$

$$I_{D1} = K_{N1} (V_{GS1} - V_{TN})^2 = K_{N1} (5 - I_{D1} R_1 - V_{TN})^2$$

$$I_{D1} = 0.1 (5 - 30 I_{D1} - 1)^2 = 0.1 (16 - 240 I_{D1} + 900 I_{D1}^2)$$

$$0.18 \text{ mA} \Rightarrow V_{GS1} = 5 - 0.18 \cdot 30 < V_{TN} \quad \times$$

$$I_{D1} = 0.1 \text{ mA} \Rightarrow V_{GS1} = 5 - 0.1 \cdot 30 = 2V$$

$$\text{SAT} \Rightarrow V_{GS1} > V_{TN} \Rightarrow 2 > 1 \quad \checkmark$$

$$M_1 \quad V_{DS1} > V_{GS1} - V_{TN} \Rightarrow 2 > 1 \quad \checkmark$$

$$V_{G1} = 2V$$

$$V_{D1} = 2V$$

$$V_{S1} = 0V$$

Assume M_2 in SAT

$$V_{G2} = V_{G1} = 2V \quad V_{S2} = I_{D2} R_2 \quad V_{GS2} = 2 - I_{D2} R_2$$

$$I_{D2} = K_{N2} (V_{GS2} - V_{TN})^2 = K_{N2} (2 - I_{D2} R_2 - V_{TN})^2 = 0.5 (2 - I_{D2} (1.17) - 1)^2 = 0.68 I_{D2}^2 - 1.17 I_{D2} + \frac{1}{2}$$

$$2.94 \text{ mA} \Rightarrow V_{GS2} = 2 - 2.94 \cdot 1.17 < V_{TN} \quad \times$$

$$I_{D2} = 0.25 \text{ mA} \Rightarrow V_{GS2} = 2 - 0.25 \cdot 1.17 = 1.71V$$

$$I_{D2} = 0.25 \text{ mA}$$

$$V_{S2} = 0.29V$$

$$V_{G2} = 2V$$

$$V_{D2} = 1.5V$$

$$I_{D1} = 0.1 \text{ mA}$$

$$I_{D2} = 0.25 \text{ mA}$$

Assume M_3 in SAT

$$V_{G3} = 3.25 \quad V_{S3} = V_{D2} + I_{D2} R_3 \quad V_{D3} = 5 - I_{D3} R_D$$

$$V_{G3} = 3.25V$$

$$V_{D3} = 3.25V$$

$$V_{S3} = 1.75V$$

$$V_{GS3} = 3.25 - V_{D2} - I_{D2} R_3$$

$$V_{DS3} = 5 - I_{D2} R_D - V_{D2} - I_{D2} R_3$$

$$I_{D2} = K_{N3} (V_{GS3} - V_{TN})^2 = K_{N3} (3.25 - V_{D2} - I_{D2} R_S - 1)^2 = 1 (3.25 - V_{D2} - 0.25 \cdot 1 - 1)^2 = 0.25$$

$$\rightarrow 2.5V \Rightarrow V_{GS3} = 3.25 - 2.5 - 0.25 \cdot 1 < 1 \quad \times$$

$$V_{D2} \rightarrow 1.5V \Rightarrow V_{GS3} = 3.25 - 1.5 - 0.25 \cdot 1 = 1.5V \quad V_{S3} = V_{D2} + I_{D2} R_S = 1.75V$$

$$SAT \rightarrow V_{GS2} > V_{TN} \Rightarrow 1.71 > 1 \quad \checkmark$$

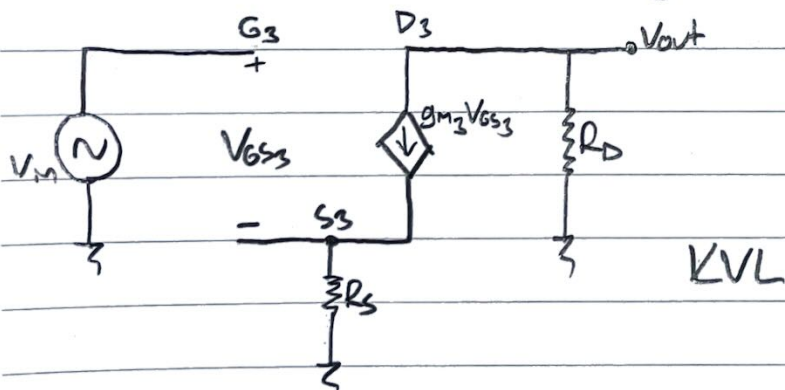
$$M2 \quad V_{DS2} \geq V_{GS2} - V_{TN} \Rightarrow 1.5 - 0.25 > 1.71 - 1 \quad \checkmark$$

$$SAT \rightarrow V_{GS3} > V_{TN} \Rightarrow 1.5 > 1 \quad \checkmark$$

$$M3 \quad V_{DS3} \geq V_{GS3} - V_{TN} \Rightarrow 3.25 - 1.75 > 1.5 - 1 \quad \checkmark$$

So, M_1 , M_2 , and M_3 are in SATURATION

b) In AC, drain of M_2 is ground, due to the short circuit on capacitor.
So, M_2 and M_3 can be ignored in small signal analysis equivalent circuit



$$g_{m3} = 2\sqrt{K_{N3} I_{D2}} = 2\sqrt{1 \cdot 0.25} = 1 \text{ mA/V}$$

$$V_{in} = V_{GS3} + g_{m3} V_{GS3} R_S$$

KCL @ D_3

$$-\frac{V_{out}}{R_D} = g_{m3} V_{GS3} \Rightarrow V_{out} = -g_{m3} V_{GS3} R_D$$

$$A_v = \frac{V_{out}}{V_{in}} = \frac{-g_{m3} V_{GS3} R_D}{V_{GS3} + g_{m3} V_{GS3} R_S} = \frac{-g_{m3} R_D}{1 + g_{m3} R_S} = \frac{-1.7}{1 + 1.1} \Rightarrow \boxed{A_v = -3.5}$$