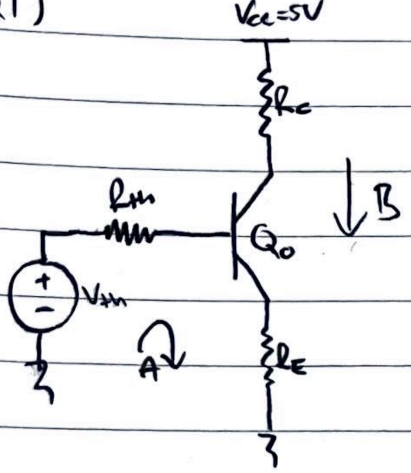
EMRE UNCU 22003884 HW4

Q1)



a)

$R_{th} = R_1 // R_2 = 13.2\,k\Omega \qquad V_{th} = \dfrac{V_{cc} \cdot R_2}{R_1 + R_2} = 2V$

Assume Q_0 is FA

KVL @ A

$V_{th} = I_B \cdot R_{th} + V_{BE(on)} + (\beta+1) R_E$

$I_B = \dfrac{V_{th} - V_{BE(on)}}{R_{th} + (\beta+1) R_E} = \dfrac{2 - 0{,}7}{13{,}2 + 101{,}3} \Rightarrow I_B = 4{,}11\,\mu A$

$\boxed{I_C = 0{,}41\,mA}$

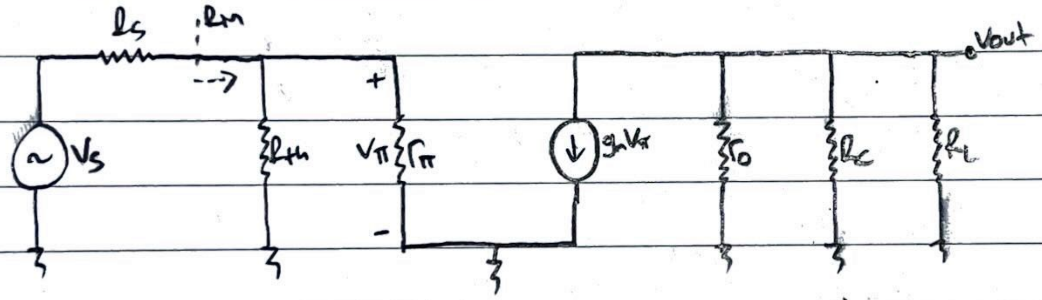
$I_E = 0{,}42\,mA$

KVL @ B

$V_{cc} - I_C R_C - V_{CE} - I_E \cdot R_E = 0 \Rightarrow V_{CE} = 5 - 0{,}41 \cdot 4 - 0{,}42 \cdot 3 \Rightarrow \boxed{V_{CE} = 2{,}1V} > 0{,}2 = V_{CE(SAT)}$

State is FA ✓

b) @ midband → external capacitors ⇒ short internal capacitors ⇒ open



$g_m = \dfrac{I_C}{V_T} = \dfrac{0{,}41}{0{,}026} = 15{,}77\,\dfrac{mA}{V}$

$r_\pi = \dfrac{\beta}{g_m} = \dfrac{100}{15{,}77} = 6{,}34\,k\Omega$

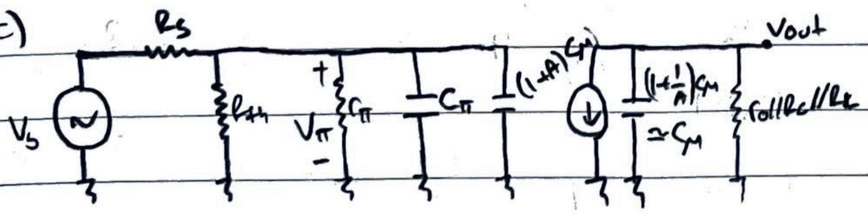
$\boxed{R_{in} = R_{th} // r_\pi = 4.28\,k\Omega}$

$V_{out} = -g_m V_\pi (R_L // R_C // r_o)$

$V_\pi = \dfrac{V_S \cdot (R_{th} // r_\pi)}{R_S + (R_{th} // r_\pi)}$

$A = \dfrac{V_{out}}{V_S} = \dfrac{-g_m (R_L // R_C // r_o)(R_{th} // r_o)}{R_S + (R_{th} // r_\pi)} \Rightarrow A = -17.92$

c)



$f_T = \dfrac{g_m}{2\pi (C_\pi + C_\mu)} = 700 \cdot 10^6 = \dfrac{15{,}77 \cdot 10^{-3}}{2\pi (10^{-12} + C_\pi)}$

$C_\pi = 2.6\,pF$

$A = \dfrac{V_{out}}{V_S} = \dfrac{g_m V_\pi (r_o // R_C // R_L)}{V_\pi} = 34.66$

$\tau_1 = C_1 \cdot (R_S // R_{th} // r_\pi) \Rightarrow f_1 = \dfrac{1}{2\pi \tau_1} = 461\,GHz$

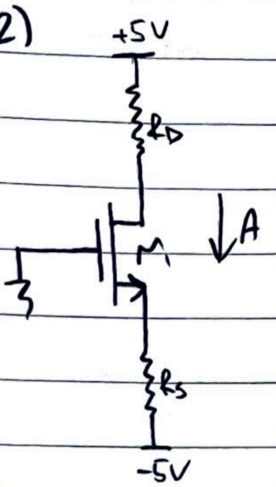
$C_1 = C_\pi + (1+A) C_\mu = 38{,}26\,pF$

$C_2 = C_\mu = 1\,pF$

$\tau_2 = C_2 (r_o // R_C // R_L) \Rightarrow \boxed{f_2 = \dfrac{1}{2\pi \tau_2} = 72{,}41\,GHz}$

lower one for upper cutoff

Q2)



a) $V_G = 0$ $\quad V_{GS} = -V_S$

Assume M_1 in SAT

$I_D = K_n(V_{GS}-V_{tN})^2 = 2(V_S+1)^2 = \frac{V_S-(-5)}{10}$ $\quad$ ~~$V_S = -0.53V$~~ ✗ $\quad V_S = -1.42V$ ✓

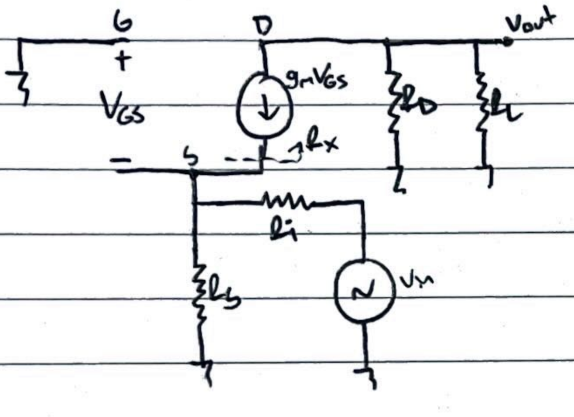
KVL @ A $\quad \boxed{I_D = 0.35\,mA}$

$V_{DD} - I_D R_D - I_D R_S - (-V_{DD}) = \boxed{V_{DS} = 4.75V} > V_{GS}-V_{tN} = 0.42$

M_1 in SAT ✓

$g_m = 2\sqrt{K_n I_D} = 1.67\,\frac{mA}{V}$

b)



$V_{out} = -g_m V_{GS}(R_D // R_L) = g_m V_S (R_D // R_L)$

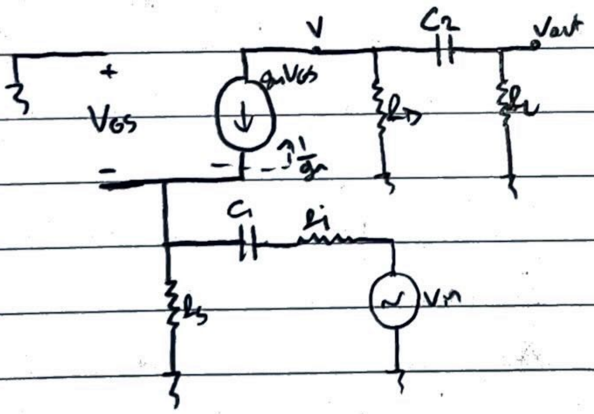
KCL @ S with test source

$I_x = -g_m(-V_S) = g_m V_x \Rightarrow R_x = \frac{1}{g_m}$

$V_S = \frac{V_{in} \cdot (R_S // R_x)}{R_i + (R_S // R_x)} \Rightarrow V_{in} = \frac{V_S\left(R_{in} + (R_S // \frac{1}{g_m})\right)}{R_S // \frac{1}{g_m}}$

$A_0 = \frac{V_{out}}{V_{in}} = \frac{g_m \cdot (R_D // R_L)(R_S // \frac{1}{g_m})}{R_i + (R_S // \frac{1}{g_m})} \Rightarrow A_0 = 0.82$

c)

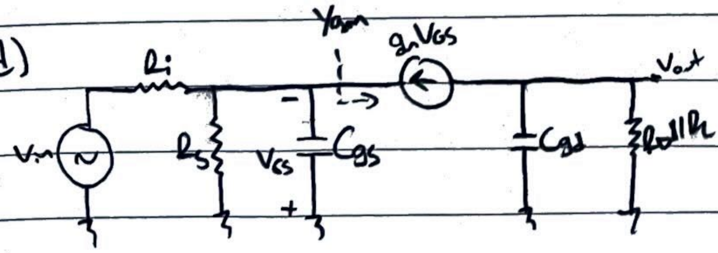


tree circuit

$\tau_2 = C_2(R_D + R_L) \Rightarrow f_2 = \frac{1}{2\pi\tau_2} = 8.84\,Hz$ ✗ larger one

$\tau_1 = C_1(R_i + R_S // \frac{1}{g_m}) = \boxed{f_1 = \frac{1}{2\pi\tau_1} = 62.05\,Hz}$

d)



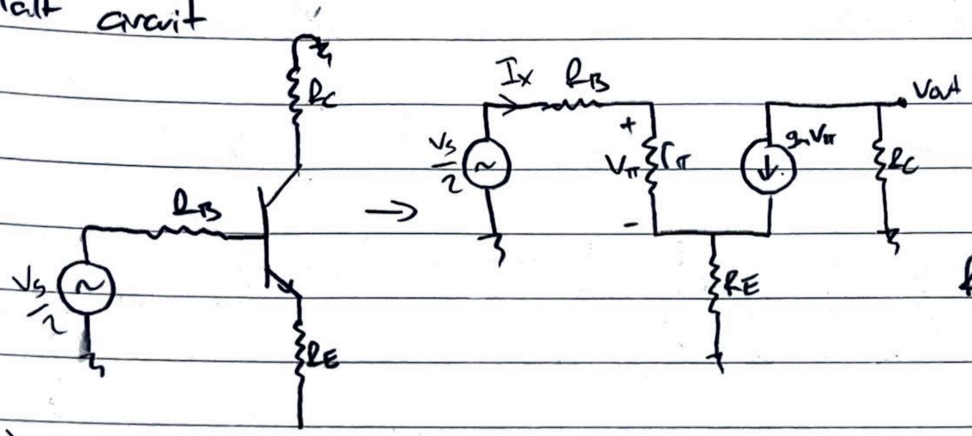
$\tau_{gd} = C_{gd}(R_D // R_L) \Rightarrow \boxed{f_{gd} = \frac{1}{2\pi\tau_{gd}} = 23.87\,MHz}$ smaller one

$\tau_{gs} = C_{gs}(R_i // R_S // \frac{1}{g_m}) \Rightarrow f_{gs} = \frac{1}{2\pi\tau_{gs}} = 30.22\,MHz$

Q3) Assume R_{EE} and β large enough $\Rightarrow I_C = I_E = \frac{I}{2} = 0{,}5$, $g_m = \frac{I_C}{V_T} = 19{,}2\,\frac{mA}{V}$, $r_\pi = \frac{\beta}{g_m} = 5{,}22k\Omega$

a)

Half circuit



$R_x = \frac{\frac{V_s}{2}}{I_x} = R_B + r_\pi + (\beta+1)R_E$

$R_x = 25{,}35k\Omega$

$R_{id} = 2R_x$

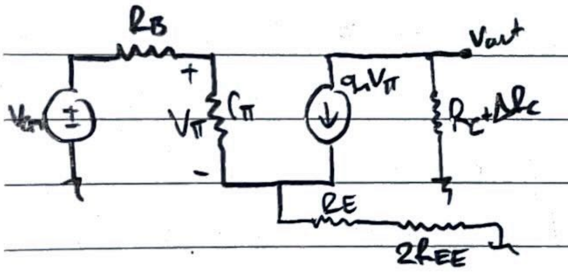
$\boxed{R_{id} = 50.7k\Omega}$

b) for half circuit $V_{out_1} = -g_m V_\pi . R_C$ $\quad V_\pi = \frac{V_s}{2} \cdot \frac{r_\pi}{R_B + r_\pi + (\beta+1)R_E}$ $\quad V_{out} = V_{out_2} - V_{out_1}$

$V_{out_2} = -V_{out_1} \Rightarrow V_{out} = -2V_{out_1}$

$A_{dm} = \frac{V_{out}}{V_s} = \frac{-2V_{out_1}}{V_s} = \frac{g_m . R_C . r_\pi}{R_B + r_\pi + (\beta+1)R_E} \Rightarrow \boxed{A_{dm} = 39.4}$

c) Half circuit



$V_{out_1} = -g_m(R_C + \Delta R_C)V_\pi$ $\quad V_\pi = V_{cm}\frac{r_\pi}{R_B + r_\pi + (\beta+1)(R_E + 2R_{EE})}$

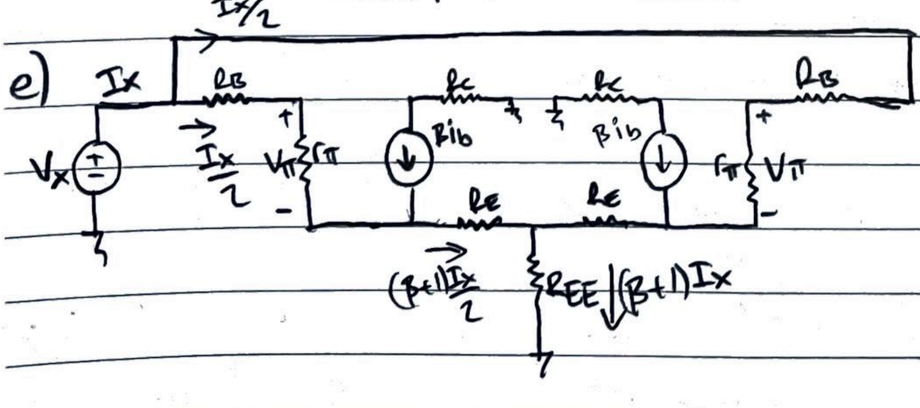
$V_{out_2} = -g_m(R_C - \Delta R_C)V_\pi$

$V_{out} = V_{out_2} - V_{out_1} = (2\Delta R_C).g_m.V_\pi$

$A_{cm} = \frac{V_{out}}{V_{cm}} = \frac{(2\Delta R_C).g_m.r_\pi}{R_B + r_\pi + (\beta+1)(R_E + 2R_{EE})} \Rightarrow$ $\quad \Delta R_C = 0{,}1k\Omega$ $\quad \boxed{A_{cm} = 0{,}000495}$

d) $CMRR = 20\log\left|\frac{A_{dm}}{A_{cm}}\right| \Rightarrow \boxed{CMRR = 98dB}$

e)



$V_x = \frac{I_x}{2}R_B + \frac{I_x}{2}r_\pi + (\beta+1)\frac{I_x}{2}R_E + (\beta+1)I_x.R_{EE}$

$R_{icm} = \frac{R_B}{2} + \frac{r_\pi}{2} + \frac{(\beta+1)R_E}{2} + (\beta+1)R_{EE} \Rightarrow \boxed{R_{icm} = 20{,}212\,k\Omega}$

Q4)

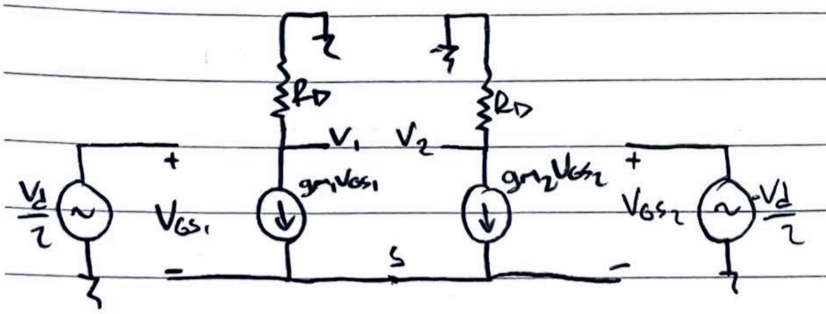
$I_{D_1} = k_1 (V_{GS}-V_{TN})^2$

$I_{D_2} = k_2 (V_{GS}-V_{TN})^2$

$k_2 = 2k_1 \Rightarrow I_{D_2} = 2I_{D_1}$

$g_{m_1} = 2\sqrt{k_1 I_{D_1}}$

$g_{m_2} = 2\sqrt{2k_1 \cdot 2I_{D_1}} \rightarrow g_{m_2} = 2g_{m_1}$



$g_{m_1}V_{GS_1} = -\overset{(2g_{m_1})}{g_{m_2}}V_{GS_2} \Rightarrow V_{GS_1} = -2V_{GS_2}$

$V_{GS_1} = \frac{V_d}{2} - V_S$, $V_{GS_2} = \frac{-V_d}{2} - V_S \Rightarrow V_S = \frac{-V_d}{6}$

$V_1 = -g_{m_1}V_{GS_1}R_D = -g_{m_1}\left(\frac{V_d}{2}+\frac{V_d}{6}\right)R_D$ $\quad V_2 = -g_{m_2}V_{GS_2}R_D = -2g_{m_1}\left(\frac{-V_d}{2}+\frac{V_d}{6}\right)R_D$

$V_2 - V_1 = \frac{4}{3} g_{m_1} \cdot V_d \cdot R_D$ $\quad \boxed{A_{dm} = \frac{V_2-V_1}{V_d} = \frac{4g_{m_1}R_D}{3}}$

Q5)

a) Drain of M1 $\Rightarrow V_D = V_{CM} - V_{BE(on)} = V_{CM} - 0{,}7 \Rightarrow V_S = 0 \Rightarrow V_{DS} = V_{CM} - 0{,}7$

$V_G = 3V$, $V_{GS} = 3V$ for M_1 in sat $V_{DS} = V_{CM} - 0{,}7 > V_{GS} - V_{TN} = 2 \Rightarrow V_{CM} > 2{,}7V$ for M_1 in sat

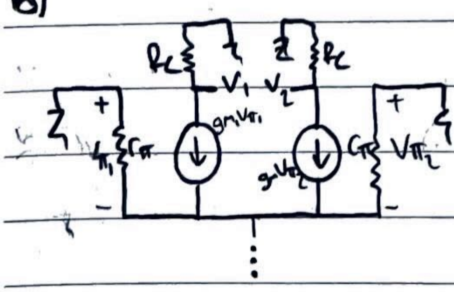
$I_0 = K_N(V_{GS}-V_{TN})^2 = 2mA \Rightarrow I_E = I_0/2 = 1mA$, $I_C = \frac{100}{101} mA$

$V_C = 10 - I_C R_C = 6{,}04V$, $V_{CE} = 6{,}04 - V_{CM} + 0{,}7 > V_{CE(SAT)} = 0{,}2$

$V_{CM} < 6{,}54$ for FA

$\boxed{2{,}7 < V_{CM} < 6{,}54}$

b)

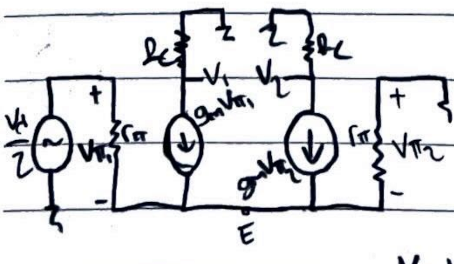


Superposition for w_1 $\quad V_{\pi_1} = V_{\pi_2}$

$V_{out} = V_2 - V_1 = -g_m R_C V_{\pi_2} + g_m R_C V_{\pi_1} = 0V$

Superposition for w_2 $\quad$ KCL @ E

$\frac{\frac{V_d}{2}-V_e}{r_\pi} + g_m\left(\frac{V_d}{2}-V_e\right) + g_m\left(-\frac{V_d}{2}-V_e\right) + \frac{-\frac{V_d}{2}-V_e}{r_\pi} = 0$

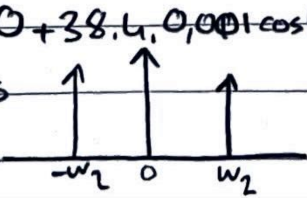


$\frac{-2V_e}{r_\pi} - g_m V_e = 0$ → this can only happen when $V_e = 0V$

$V_{out} = V_2 - V_1 = +g_m R_C \cdot \frac{V_d}{2} + g_m R_C \cdot \frac{V_d}{2} = g_m R_C V_d$ $\quad g_m = \frac{I_C}{V_T} = 38\frac{mA}{V}$

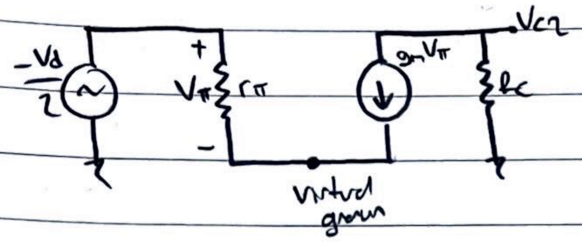
$V_{out} = 0 + 38 \cdot 4 \cdot 0{,}001 \cos(w_2 t) \Rightarrow \boxed{V_{out} = 0{,}15\cos(w_2 t)}$

freq spectrum of cos at w_2 is 2 impuls at w_2 and $-w_2$



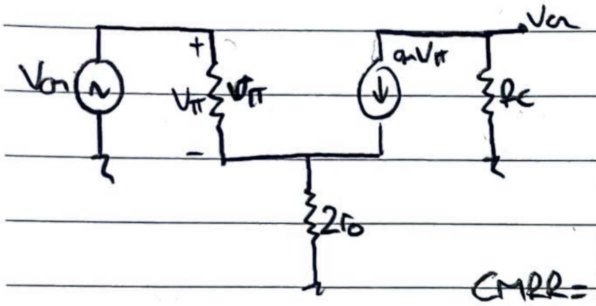
c) M_1 seen as resistor from above circuit $\Rightarrow r_o = \frac{1}{\lambda I_D} = 25k\Omega$

diff mode half circuit



$$V_{c2} = -g_m R_C V_\pi = g_m R_C \frac{V_d}{2}$$

$$A_{dm_2} = \frac{V_{c2}}{V_d} = \frac{g_m R_C}{2}$$



$$V_{c2} = -g_m R_C V_\pi = -g_m R_C \frac{r_\pi}{r_\pi + (\beta+1)2r_o} \cdot V_{cm}$$

$$A_{cm_2} = \frac{V_{c2}}{V_{cm}} = -g_m R_C \frac{r_\pi}{r_\pi + (\beta+1)2r_o}$$

$$CMRR = \left| \frac{A_{dm_2}}{A_{cm_2}} \right| = \frac{r_\pi + (\beta+1)2r_o}{2r_\pi}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{38} = 2{,}63k\Omega$$

$$\boxed{CMRR = 960{,}58}$$