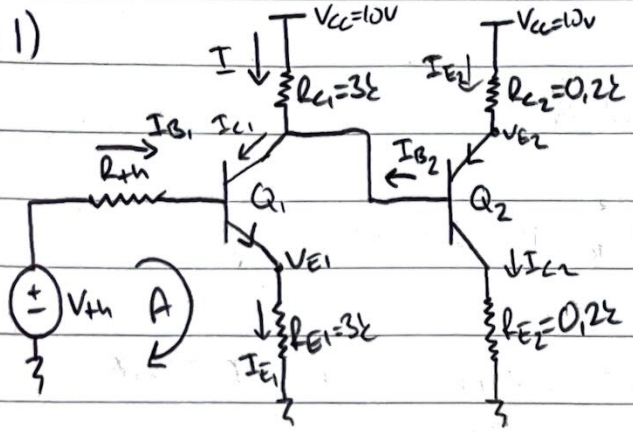
Q1)



$R_{th} = R_1 // R_2 = 33,3\Omega$

$V_{th} = \frac{V_{cc} \cdot R_2}{R_1 + R_2} = 3,33\,V$

Assume Q_1 is forward Active

KVL @ A

$V_{th} = I_{B_1} \cdot R_{th} + V_{BE(ON)} + (\beta+1) I_{B_1} \cdot R_{E_1}$

$3,3 = I_{B_1} \cdot 33,3 + 0,7 + 101 \cdot I_{B_1} \cdot 3$

$I_{B_1} = 7,73\,\mu A$

$I_{C_1} = 0.77\,mA$

$I_{E_1} = 0,78\,mA$

$I_{C_1} = I + I_{B_2} = \frac{V_{cc} - V_{C_1}}{R_{C_1}} + I_{B_2} = \frac{10 - V_{C_1}}{3} + I_{B_2}$ (1)

(1) and (2)

$I_{C_1} = \frac{10 - V_{C_1}}{3} + \frac{9,3 - V_{C_1}}{202} = 0,77\,mA$

$V_{C_1} = 7,90V \quad I = \frac{10 - V_{C_1}}{3} = 0,7\,mA$

$V_{E_1} = I_{E_1} \cdot R_{E_1} = 0,78 \cdot 3 = 2,34V$

$V_{B_1} = V_{E_1} + V_{BE(ON)} = 2,34 + 0,7 = 3,04$

$V_{C_1} = 7,90V \quad V_{E_1} = 2,34V \quad V_{B_1} = 3,04V$

$V_{CE_1} = 5.56V$

Check for state Q_1

$V_{CE_1} > V_{CE(SAT)} \Rightarrow 5.56 > 0,2$ ✓

Assume Q_2 is forward Active

$V_{E_2} = V_{B_2} + V_{EB} = V_{C_1} + V_{EB} = V_{C_1} + 0,7$

$I_{E_2} = \frac{V_{cc} - V_{E_2}}{R_{C_2}} = \frac{10 - V_{C_1} + 0,7}{0,2} = (\beta+1) I_{B_2} = 101\, I_{B_2}$

$I_{B_2} = \frac{9,3 - V_{C_1}}{101 \cdot 0,2}$ (2)

$I_{C_1} = I + I_{B_2} \Rightarrow 0,77 = 0,7 + I_{B_2}$

$I_{B_2} = 70.0\,\mu A$

$I_{C_2} = 7.00\,mA$

$I_{E_2} = 7,07\,mA$

$V_{C_2} = I_{C_2} \cdot R_{E_2} = 7 \cdot 0,2 = 1,4V$

$V_{B_2} = V_{C_1} = 7,90V$

$V_{E_2} = V_{B_2} + V_{EB(ON)} = 7,9 + 0,7 = 8,60V$

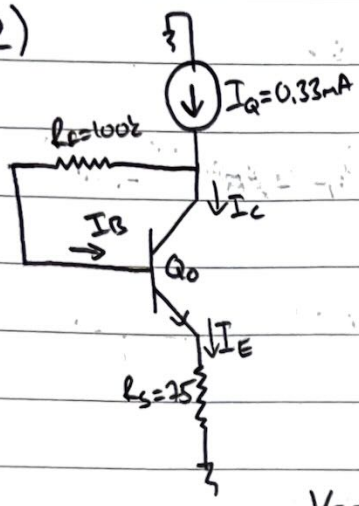
$V_{C_2} = 1,40V \quad V_{E_2} = 8,60V \quad V_{B_2} = 7,90V$

$V_{EC_2} = 7,20V$

Check for Q_2

$V_{EC_2} > V_{EC(SAT)} \Rightarrow 7.20 > 0,2$ ✓

Q2)



a)

Assure Q_0 is forward Active

$I_Q = I_B + I_C = (\beta+1) I_B = 101 I_B = 0{,}33\,mA$

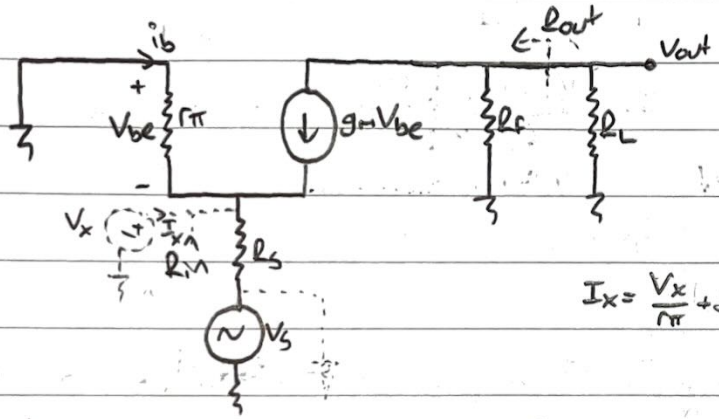
$I_B = 3{,}27\,\mu A \quad I_C = 0{,}327\,mA \quad I_E = 033\,mA$

$V_E = I_E . R_S = 0{,}025\,V \quad V_B = V_E + V_{BE(on)} = 0{,}725\,V \quad V_C = V_B + I_B R_f = 1{,}052\,V$

Check for $Q_0 \Rightarrow V_{CE} > V_{CE(SAT)} \Rightarrow 1{,}027 > 0{,}2$ ✓

$$g_m = \frac{I_C}{V_T} = \frac{0{,}327}{0{,}026} = 12{,}58\ \frac{MA}{V}$$

b)



for R_{in}, by putting test source

$V_b = 0 \quad V_e = V_x \quad I_x = -\frac{V_b - V_e}{r_\pi} - g_m V_{be}$

$$I_x = \frac{V_x}{r_\pi} + g_m V_x = V_x\left(\frac{1}{r_\pi} + g_m\right) = V_x\left(\frac{1+\beta}{r_\pi}\right) \Rightarrow \boxed{R_{in} = \frac{V_x}{I_x} = \frac{r_\pi}{1+\beta}}$$

$R_{in} = 78{,}7\,\Omega$

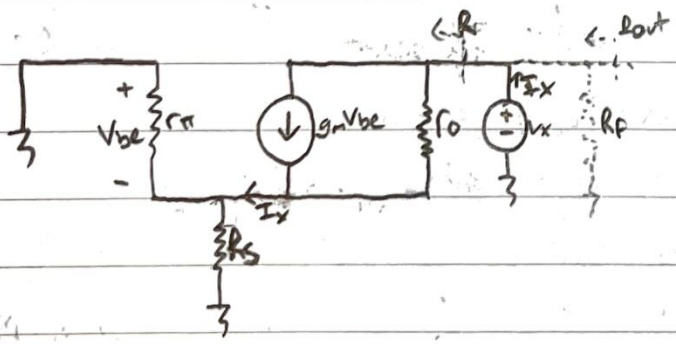
for R_{out} $\quad V_b = 0 \quad V_{be} = -V_e$

KCL @ E

$$g_m V_{be} = \frac{V_e}{R_S} + \frac{V_e}{r_\pi} \Rightarrow -g_m V_e = V_e\left(\frac{1}{R_S} + \frac{1}{r_\pi}\right) \Rightarrow V_e \text{ must be } 0V$$

So $V_{be} = 0$ and $\boxed{R_{out} = R_f = 100\,k\Omega}$

c)



$R_{out} = R_f // R$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{12{,}58} = 7{,}95\,k\Omega$$

$V_b = 0 \quad V_{be} = -V_e$

KCL @ E

$$I_x = \frac{V_e}{R_S} + \frac{V_e}{r_\pi} \Rightarrow V_e = I_x (R_S // r_\pi)$$

$$V_x = I_x\left(1 + (R_S // r_\pi)(1 + g_m r_o)\right)$$

$$R = \frac{V_x}{I_x} = 1 + (R_S // r_\pi)(1 + g_m r_o) = 95.05\,k\Omega$$

$R_{out} = R // R_f = \left(1 + (R_S // r_\pi)(1 + g_m r_o)\right) // R_f$

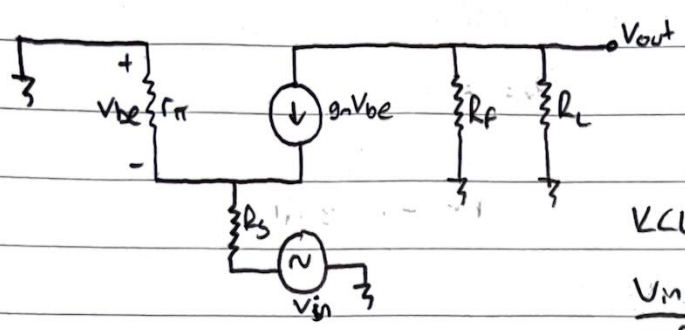
KCL @ X

$$I_x = \frac{V_x - V_e}{r_o} + g_m V_{be} \Rightarrow V_x = I_x r_o + g_m V_e r_o + V_e$$

$$V_x = I_x r_o + V_e(1 + g_m r_o) = I_x r_o + I_x (R_S // r_\pi)(1 + g_m r_o)$$

$$\boxed{R_{out} = 48{,}73\,k\Omega}$$

d)



$V_b = 0V \quad V_{be} = -V_e$

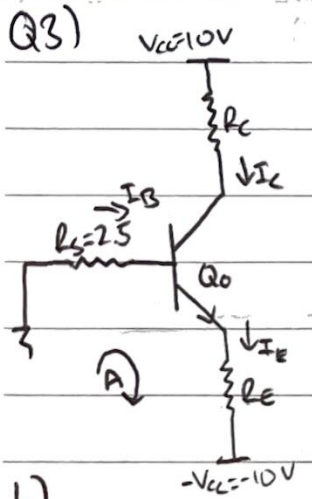
$V_{out} = -g_m V_{be} (R_F // R_L) = g_m V_e (R_F // R_L)$

KCL @ E

$$\frac{V_{in} - V_e}{R_S} - \frac{V_e}{r_\pi} + g_m V_{be} = \frac{V_{in}}{R_S} - V_e\left(\frac{1}{R_S} + \frac{1}{r_\pi} + \frac{\beta}{r_\pi}\right) = 0$$

$$A_V = \frac{V_{out}}{V_{in}} = \frac{\frac{\beta}{r_\pi} \cdot V_e \cdot (R_F // R_L)}{R_S V_e \left(\frac{1}{R_S} + \frac{1}{r_\pi} + \frac{\beta}{r_\pi}\right)} = \frac{\beta (R_F // R_L)}{r_\pi \cdot R_S \cdot \left(\frac{1}{R_S} + \frac{1}{r_\pi} + \frac{\beta}{r_\pi}\right)} \Rightarrow \boxed{A_V = 9.52}$$

Q3)



a) $I_E = 0.5mA \quad I_B = 4.95\mu A \quad I_C = 0.495mA$

Assume Q_0 is FA KVL @ A

$$I_B R_S + V_{BE(ON)} + I_E R_E = -V_{CC} \Rightarrow \frac{4.95}{1000} \cdot 2.5 + 0.7 + 0.5 \cdot R_E = 10$$

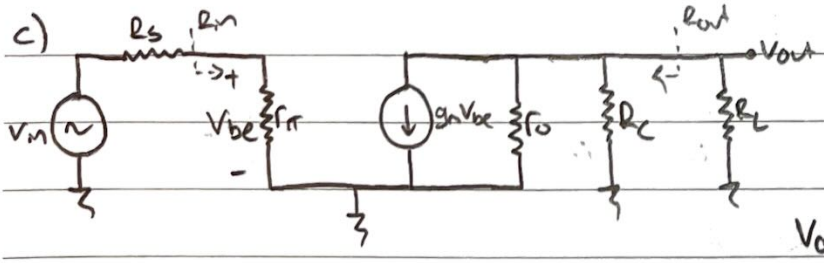
$$\Rightarrow \boxed{R_E = 18.58 k\Omega}$$

b)

$V_C = 5V \quad I_C = \frac{V_{CC} - V_C}{R_C} = \frac{10-5}{R_C} = 0.495mA \Rightarrow \boxed{R_C = 10.1 k\Omega}$

Check $Q_0 \Rightarrow V_E = -10 + I_E R_E = -0.71V \quad V_{CE} = 5.71V > V_{CE(SAT)}$ ✓

c)



$V_e = 0 \quad V_{be} = V_b = V_{in} \cdot \frac{r_\pi}{r_\pi + R_S}$

$V_{out} = -g_m V_{be} (r_o // R_C // R_L) = -g_m V_b (r_o // R_C // R_L)$

$$g_m = \frac{I_C}{V_T} = \frac{0.495}{0.026} = 19.04 \frac{mA}{V}$$

$$V_{out} = -g_m \frac{V_{in} \cdot r_\pi}{r_\pi + R_S} (r_o // R_C // R_L)$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{19.04} = 5.25 k\Omega$$

$$A_V = \frac{V_{out}}{V_{in}} = \frac{-g_m r_\pi}{r_\pi + R_S} (r_o // R_C // R_L) \Rightarrow \boxed{A_V = -63.22}$$

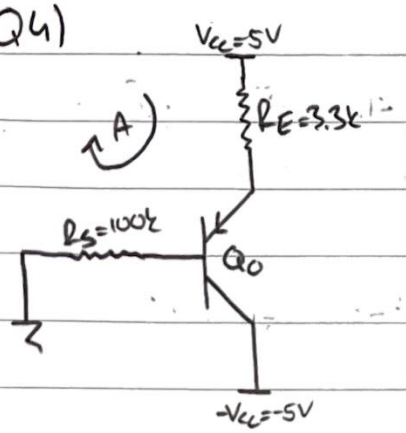
d) $\boxed{R_{in} = r_\pi = 5.25\text{k}\Omega}$ $\quad V_b = 0V,\ V_e = 0V \Rightarrow g_m V_{be} = 0A \Rightarrow \boxed{R_{out} = r_o // R_C = 9.61\text{k}\Omega}$

e) $A_V = \frac{V_{out}}{V_S} = \frac{-g_m r_\pi}{r_\pi + R_S} \cdot (r_o // R_C // R_L) = \frac{-g_m R_{in}}{R_{in}+R_S}(r_o // R_C // R_L) \Rightarrow \frac{V_S \cdot R_{in}}{R_{in}+R_S} = \frac{-V_{out}}{g_m (r_o // R_C // R_L)}$

$V_{in} = V_S \cdot \frac{R_{in}}{R_{in}+R_S}$ $\qquad V_{out} = A_{OC} V_{in} \cdot \frac{R_L}{R_L + R_{out}} = A_{OC} \cdot \frac{V_S R_{in}}{R_{in}+R_S} \cdot \frac{R_L}{R_L + R_{out}}$

$A_{OC} = V_{out} \cdot \frac{-g_m (r_o // R_C // R_L)}{V_{out}} \cdot \frac{R_L + R_{out}}{R_L} \Rightarrow \boxed{A_{OC} = -183.02}$

Q4)



a)

Assume Q_0 is FA $\qquad$ KVL @ A

$V_{CC} = I_E R_E + V_{EB(ON)} + I_B R_S = (\beta+1) I_B R_E + V_{EB(ON)} + I_B R_S$

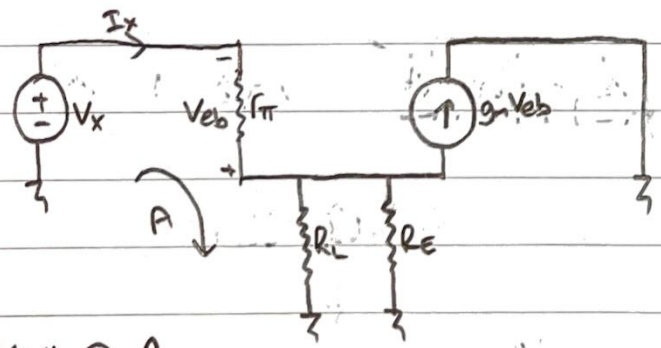
$5 = 101 \cdot 3.3 \cdot I_B + 0.7 + 100 I_B$

$I_B = 9.92\mu A, \quad I_C = 0.992 mA \quad \boxed{I_E = 1.00 mA}$

Check for $Q_0 \Rightarrow V_E = V_{CC} - I_E R_E = 1.70V \quad \therefore V_{EC} = 1.70 > V_{EC(SAT)}$ ✓

b)

R_{in}



KVL @ A

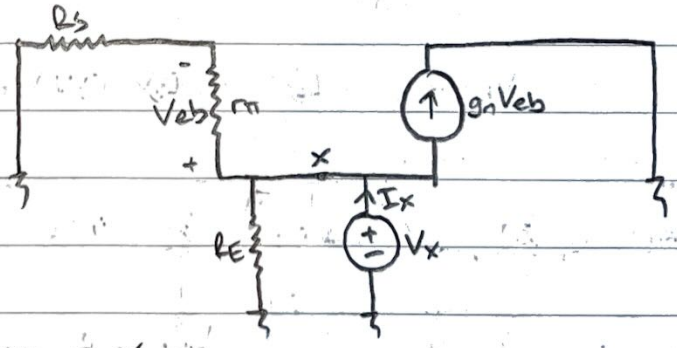
$V_x = I_x r_\pi + (\beta+1) I_x (R_E // R_L)$

$R_{in} = \frac{V_x}{I_x} = r_\pi + (\beta+1)(R_E // R_L) \Rightarrow \boxed{R_{in} = 80.13\text{k}\Omega}$

$g_m = \frac{I_C}{V_T} = \frac{0.992}{0.026} = 38.15 \frac{mA}{V}$

$r_\pi = \frac{\beta}{g_m} = 2.62\ k\Omega$

R_{out}

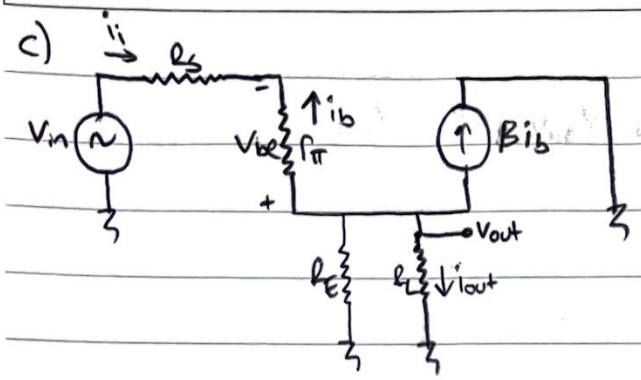


KCL @ X

$I_x = \frac{V_x}{r_\pi + R_S} + \frac{V_x}{R_E} + g_m \overset{V_e = V_x}{V_{eb}} = V_x \left(\frac{1}{r_\pi + R_S} + \frac{1}{R_E} + \frac{\beta}{r_\pi} \right)$

$R_{out} = \frac{V_x}{I_x} = \frac{1}{\left(\frac{1}{r_\pi + R_S} + \frac{1}{R_E} + \frac{\beta}{r_\pi} \right)} \Rightarrow \boxed{R_{out} = 25.99\ \Omega}$

c)



KCL @ V_{out}

$$i_b = \frac{V_{out} - V_{in}}{r_\pi + R_S} \qquad \frac{V_{out}}{R_E // R_L} = -(\beta+1) i_b = \frac{(\beta+1)(V_{in} - V_{out})}{r_\pi + R_S}$$

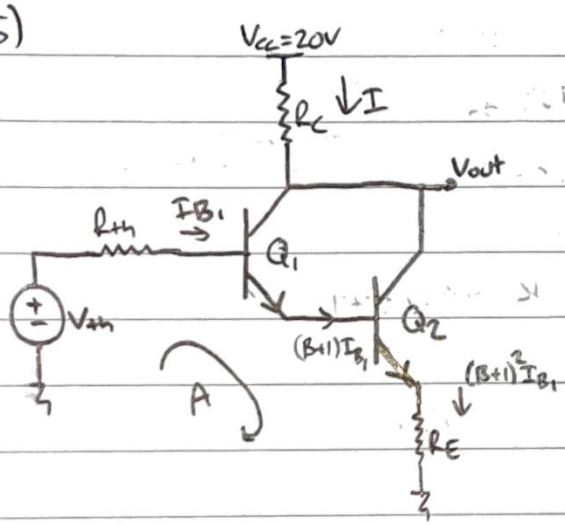
$$\frac{V_{out}}{R_E // R_L} = \frac{V_{in}(\beta+1)}{r_\pi + R_S} - \frac{V_{out}(\beta+1)}{r_\pi + R_S} \Rightarrow V_{out}\left(\frac{1}{R_E // R_L} + \frac{\beta+1}{r_\pi + R_S}\right) = \frac{V_{in}(\beta+1)}{r_\pi + R_S}$$

$$A_V = \frac{V_{out}}{V_{in}} = \frac{\frac{\beta+1}{r_\pi + R_S}}{\frac{1}{R_E // R_L} + \frac{\beta+1}{r_\pi + R_S}} = \frac{(\beta+1)(R_E // R_L)}{r_\pi + R_S + (\beta+1)(R_E // R_L)} \Rightarrow \boxed{A_V = 0{,}43}$$

d)

$$i_{in} = -i_b \qquad i_{out} = i_e \cdot \frac{R_E}{R_E + R_L} = (\beta+1) i_b \frac{R_E}{R_E + R_L} \qquad A_i = \frac{i_{out}}{i_{in}} = \frac{(\beta+1) \cdot R_E}{R_E + R_L} \Rightarrow \boxed{A_i = 77{,}51}$$

5)



Assume Q_1 and Q_2 is FA

$R_{th} = R_1 // R_2 = 14{,}11\ k\Omega$ $\qquad V_{th} = V_{cc} \cdot \frac{R_2}{R_1 + R_2} = 3{,}4\ V$

KVL @ A

$$V_{th} = I_{B_1} R_{th} + V_{BE(ON)} + V_{BE(ON)} + (\beta+1)^2 I_{B_1} \cdot R_E$$

$$3{,}4 = I_{B_1} \cdot 14{,}11 + 1{,}4 + (101)^2 \cdot I_{B_1} \cdot 0{,}2$$

$I_{B_1} = 0{,}924\ \mu A$ $\qquad \boxed{I_{C_1} = 0{,}097\ mA}$ $\qquad I_{E_1} = 0{,}098\ mA$

$I_{B_2} = 0{,}098\ mA$ $\qquad \boxed{I_{C_2} = 9{,}83\ mA}$ $\qquad I_{E_2} = 9{,}93\ mA$

$I = I_{C_1} + I_{C_2} = 9{,}93\ mA$

$V_{C_1} = V_{C_2} = V_{CC} - I \cdot R_C = 5{,}10V$ $\qquad V_{B_1} = V_{th} - I_{B_1} R_{th} = 3{,}39V$ $\qquad V_{E_1} = V_{B_1} - V_{BE(ON)} = 2{,}69V$

$V_{B_2} = V_{E_1} = 2{,}69V$ $\qquad V_{E_2} = V_{B_2} - V_{BE(ON)} = 1{,}99V$

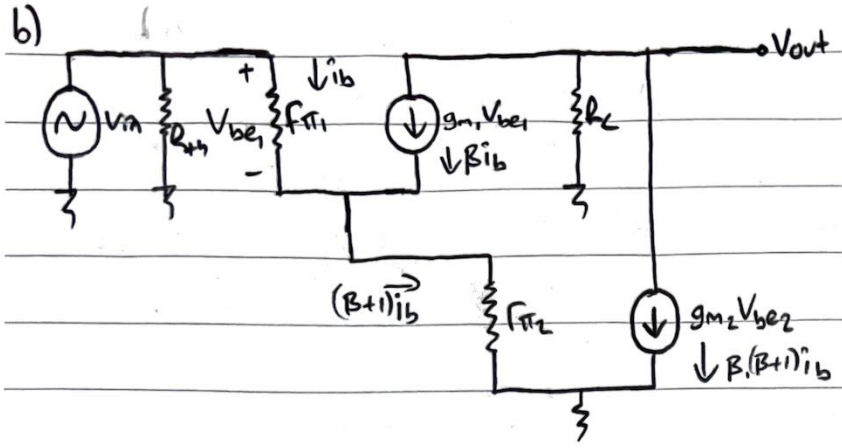
$\boxed{V_{CE_1} = 2{,}41V}$ $\qquad$ Check $Q_1 \Rightarrow V_{CE_1} > V_{CE(SAT)}$ ✓

$\boxed{V_{CE_2} = 3{,}11V}$ $\qquad$ Check $Q_2 \Rightarrow V_{CE_2} > V_{CE(SAT)}$ ✓

$$g_{m_1} = \frac{I_{C_1}}{V_T} = \frac{0{,}097}{0{,}026} = 3{,}73\ \frac{mA}{V} \qquad r_{\pi_1} = \frac{\beta}{g_{m_1}} = 26{,}81\ k\Omega$$

$$g_{m_2} = \frac{I_{C_2}}{V_T} = \frac{9{,}83}{0{,}026} = 378{,}01\ \frac{mA}{V} \qquad r_{\pi_2} = \frac{\beta}{g_{m_2}} = 0{,}26\ k\Omega$$

b)



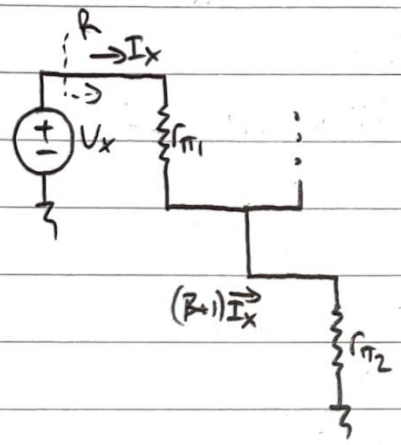
$V_{e_2} = 0V \quad V_{be_2} = V_{b_2} = V_{e_1}$

$V_{out} = -R_C(\beta i_b + \beta.(\beta+1) i_b)$

$$A_V = \frac{V_{out}}{V_{in}} = \frac{-i_b.R_C(\beta+\beta.(\beta+1))}{i_b(r_{\pi_1}+(\beta+1)r_{\pi_2})} \Rightarrow \boxed{A_V = -288{,}30}$$

$V_{in} = i_b r_{\pi_1} + (\beta+1) i_b r_{\pi_2}$

c)

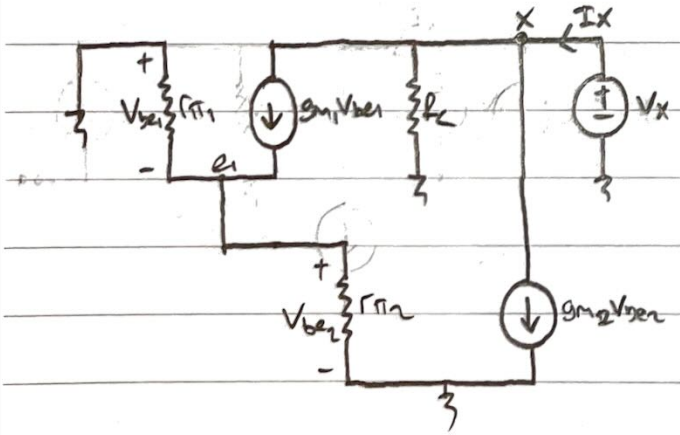


$V_x = I_x.r_{\pi_1} + (\beta+1) I_x . r_{\pi_2}$

$$R = \frac{V_x}{I_x} = r_{\pi_1} + (\beta+1) r_{\pi_2}$$

$R_{in} = R_{th} // R = R_{th} // (r_{\pi_1} + (\beta+1) r_{\pi_2})$

$\boxed{R_{in} = 11{,}14\ k\Omega}$



KCL @ e_1

$$\frac{V_{e_1}}{r_{\pi_1}} + \frac{V_{e_1}}{r_{\pi_2}} = g_{m_1} V_{be_1} \Rightarrow \frac{V_{e_1}}{r_{\pi_1}+r_{\pi_2}} = -g_{m_1} V_{e_1}$$

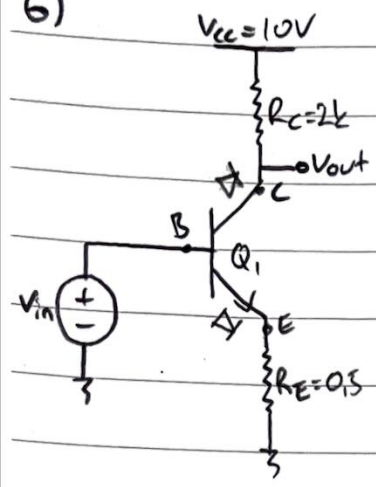
V_{e_1} must be $0V$ So, $V_{e_1} = V_{b_2} = 0V$

$V_{be_1} = V_{be_2} = 0V$

KCL @ X

$$I_x = \frac{V_x}{R_C} + \overset{(0V)}{g_{m_1} V_{be_1}} + \overset{(0V)}{g_{m_2} V_{be_2}} = \frac{V_x}{R_C} \Rightarrow R_{out} = \frac{V_x}{I_x} \Rightarrow \boxed{R_{out} = R_C = 1{,}5 k\Omega}$$

6)



for $0 \leq V_{in} < 0,7 = V_{BE(on)} \Rightarrow Q_1$ is OFF

$\Rightarrow V_{out} = V_{cc} - I_c R_c = V_{cc} = 10V$

for $V_{CE} \geq 0,2 = V_{CE(SAT)}$ = Q_1 is Forward Active

If $V_{CE} = 0,2$ and Q_1 is FA

$V_C = V_{cc} - I_c R_c = 10 - 2I_C = 10 - 2\beta I_B$

$V_E = I_E . R_E = I_E . 0,5 = (\beta+1) I_B 0,5 \simeq \beta I_B . 0,5$ as β is large

$V_{CE} = 0,2 = 10 - 2\beta I_B - \beta I_B . 0,5 \Rightarrow \beta I_B = I_C = 3,92 mA$

when $V_{out} = V_{cc} - I_c R_c = 10 - 3,92 . 2 = 2,16V$ Q_1 is SAT

If $I_C \simeq I_E = 3,92 mA \rightarrow V_E = I_E . R_E = 1,96V \rightarrow V_B = V_{in} = V_E + V_{BE(on)} = 2,66V$

for $0,7 \leq V_{in} \leq 2,66 \Rightarrow Q_1$ is FA $\Rightarrow I_E = \frac{V_{in} - V_{BE(on)}}{R_E} = \frac{V_{in} - 0,7}{0,5} \simeq I_C$

$V_{out} = V_{cc} - I_c R_c = 10 - 2I_C \simeq 10 - 4V_{in} + 2,8 = 12,8 - 4V_{in} = V_{out} \Rightarrow$ linear

for $V_{in} > 2,66 V \rightarrow Q_1$ is SAT $\Rightarrow V_{CE} = V_{CE(SAT)} = 0,2V$ and $I_C < \beta I_B$

If $V_{CE} = 0,2V$ and $V_{BE} = 0,7V \Rightarrow V_{BC} = 0,5V \Rightarrow V_B - V_C = V_{in} - V_{out} = 0,5V$

$$V_{out} = \begin{cases} 10V, & \text{if } 0 \leq V_{in} < 0,7 \\ 12,8 - 4V_{in}, & \text{if } 0,7 \leq V_{in} < 2,66 \\ V_{in} - 0,5V, & \text{if } 2,66 \leq V_{in} \leq 10 \end{cases}$$