Bilkent University
EEE321: Signals and Systems
Lab Assignment 2

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Section - 01

Part 1:

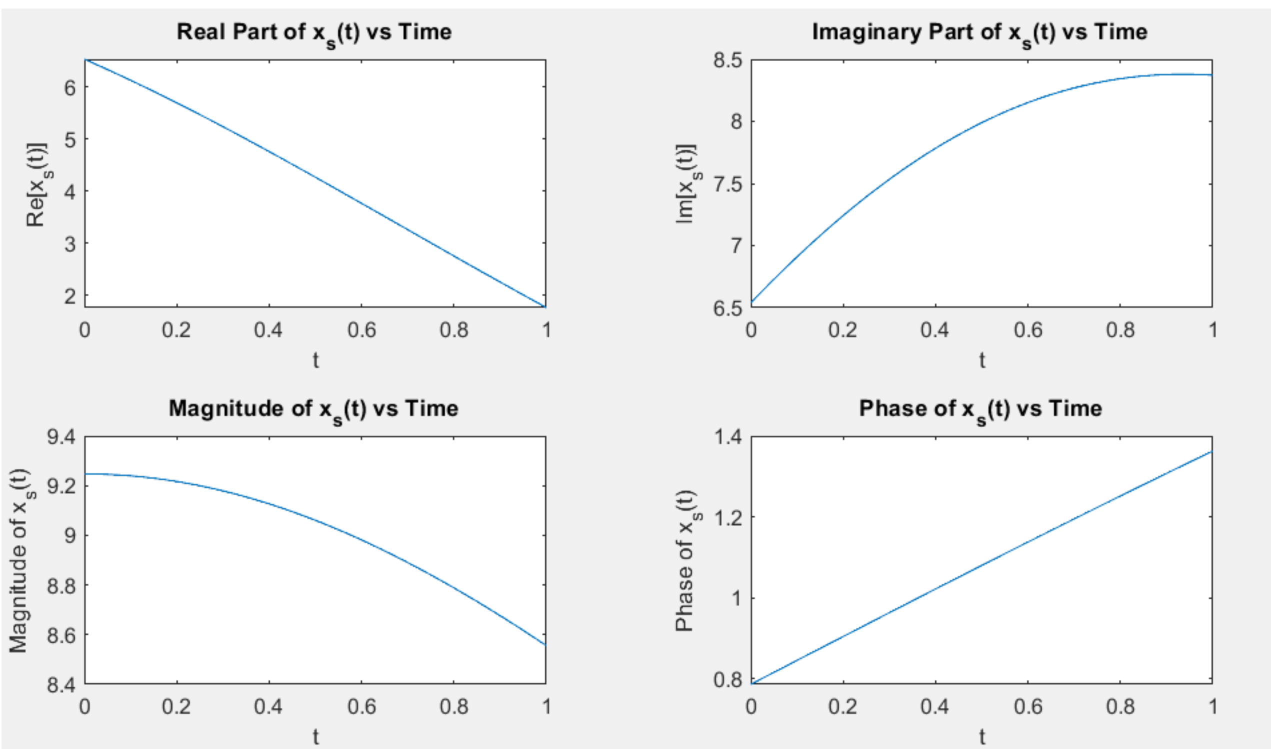

Figure 1: Plots for Part 1

Part 2:

$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{j\frac{2\pi kt}{T}}$$

$$X_k = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) e^{-j\frac{2\pi kt}{T}} dt$$

$$\tilde{x}(t) = \sum_{k=-K}^{K} X_k e^{j\frac{2\pi kt}{T}} \quad where\ K \in Z^+$$

$$x(t) = \begin{cases} 1 - 2t^2, & if\ -\frac{W}{2} < t < \frac{W}{2} \\ 0, & otherwise \end{cases} \quad for\ t \in \left[-\frac{T}{2}, \frac{T}{2}\right)\ and\ W < T$$

When $T = 2\ and\ W = 1.0$, the graph of x(t) in the range $-1.5T < t < 1.5T$ is sketched as follows:

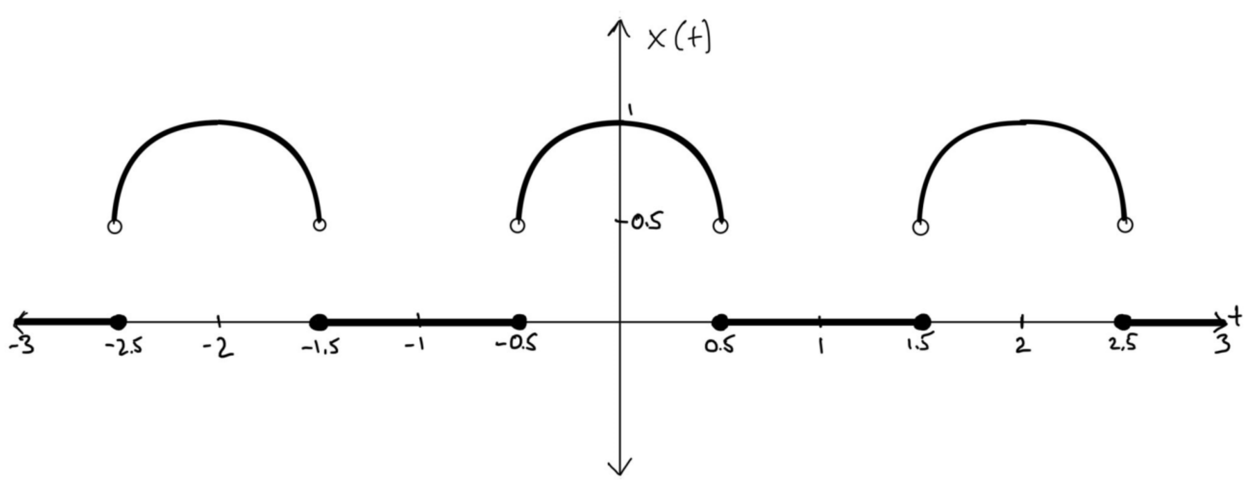

Figure 2: Sketch of the x(t) vs time

$$X_k = \frac{1}{T}\int_{-\frac{W}{2}}^{\frac{W}{2}} (1-2t^2)\, e^{-j\omega k t}\, dt = \frac{1}{T}\left[\int_{-\frac{W}{2}}^{\frac{W}{2}} e^{-j\omega k t}\, dt - 2\int_{-\frac{W}{2}}^{\frac{W}{2}} t^2 e^{-j\omega k t}\, dt\right]$$

$$\boxed{\omega = \frac{2\pi}{T}}$$

$$\int u\,dv = uv - \int v\,du$$

$$\int_{-\frac{W}{2}}^{\frac{W}{2}} t^2 e^{-j\omega t}\, dt = t^2\frac{e^{-j\omega k t}}{-j\omega k} - \frac{2}{-j\omega k}\int_{-\frac{W}{2}}^{\frac{W}{2}} t\, e^{-j\omega k t}\, dt$$

when $u = t^2$, $u' = 2t$, $v' = e^{-j\omega k t}$, $v = \frac{e^{-j\omega k t}}{-j\omega k t}$

$$\int_{-\frac{W}{2}}^{\frac{W}{2}} t\, e^{-j\omega k t}\, dt = t\frac{e^{-j\omega k t}}{-j\omega k} - \int_{-\frac{W}{2}}^{\frac{W}{2}} \frac{e^{-j\omega k t}}{-j\omega k t}\, dt$$

when $u = t$, $u' = 1$, $v' = e^{-j\omega k t}$, $v = \frac{e^{-j\omega k t}}{-j\omega k t}$

$$= t\frac{e^{-j\omega k t}}{-j\omega k} - \frac{e^{-j\omega k t}}{-\omega^2 k^2}$$

So $$\int_{-\frac{W}{2}}^{\frac{W}{2}} t^2 e^{-jk\omega t}\, dt = e^{-j\omega k t}\left(\frac{t^2}{-j\omega k} + \frac{2t}{\omega^2 k^2} + \frac{2}{j\omega^3 k^3}\right) + C$$

And $$X_k = \frac{1}{T}\left[\frac{e^{-j\omega k t}}{-j\omega k} - 2e^{-j\omega k t}\left(\frac{t^2}{-j\omega k} + \frac{2t}{\omega^2 k^2} + \frac{2}{j\omega^3 k^3}\right)\right]\Bigg|_{-\frac{W}{2}}^{\frac{W}{2}} = \frac{1}{T}\left[e^{-j\omega k t}\left(\frac{1-2t^2}{-j\omega k} - \frac{4t}{\omega^2 k^2} - \frac{4}{j\omega^3 k^3}\right)\right]\Bigg|_{-\frac{W}{2}}^{\frac{W}{2}}$$

$$X_k = \frac{1}{T}\left[\left(\frac{2-W^2}{\omega k} + \frac{8}{\omega^3 k^3}\right)\overbrace{\left(\frac{e^{j\omega k\frac{W}{2}} - e^{-j\omega k\frac{W}{2}}}{2j}\right)}^{\sin\left(\omega k\frac{W}{2}\right)} - \frac{4W}{\omega^2 k^2}\overbrace{\left(\frac{e^{j\omega k\frac{W}{2}} + e^{-j\omega k\frac{W}{2}}}{2}\right)}^{\cos\left(\omega k\frac{W}{2}\right)}\right]$$

So $$X_k = \left(\frac{2-W^2}{2\pi k} + \frac{T^2}{\pi^3 k^3}\right)\sin\left(\frac{\pi W k}{T}\right) - \frac{WT}{\pi^2 k^2}\cos\left(\frac{\pi W k}{T}\right) \;//$$

Figure 3: Calculation for Xk of x(t)

Part 3:

When $T = 2, W = 1.0, K = 20 + D_{11}, and\ t = [-5: 0.001: 5]$, the time plots of the real and imaginary parts of x(t) are as follows:

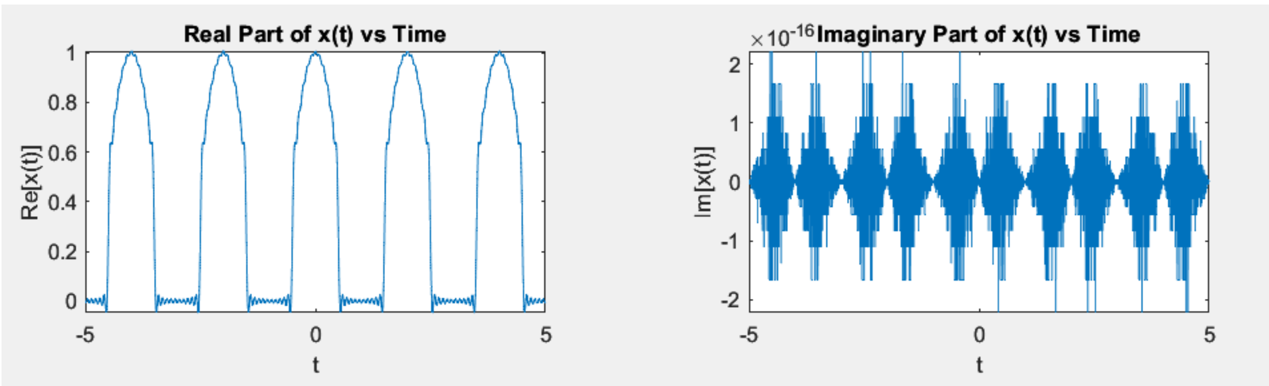

Figure 4: Plots for Part 3

When the maximum and minimum values of the real and imaginary parts of x(t) are examined, the following results are obtained:

real_max = 1.0068, real_min = -0.0465

imaginary_max = 2.2204e-16, imaginary_min = -2.2204e-16

When the above values are compared, it can be seen that the values of the imaginary part are minimal compared to those of the real part. Therefore, we can ignore these numbers and assume that the imaginary part equals zero. The rounding error is why MATLAB does not calculate the imaginary part as equal to zero even though x is a real-valued signal. As another example, if $\sin(\pi/6 - 0.5)$ is given, MATLAB will show the result as -5.5511e-17 while the expected value is 0.

The plots created by changing the K value without changing the T, W, and t values are as follows:

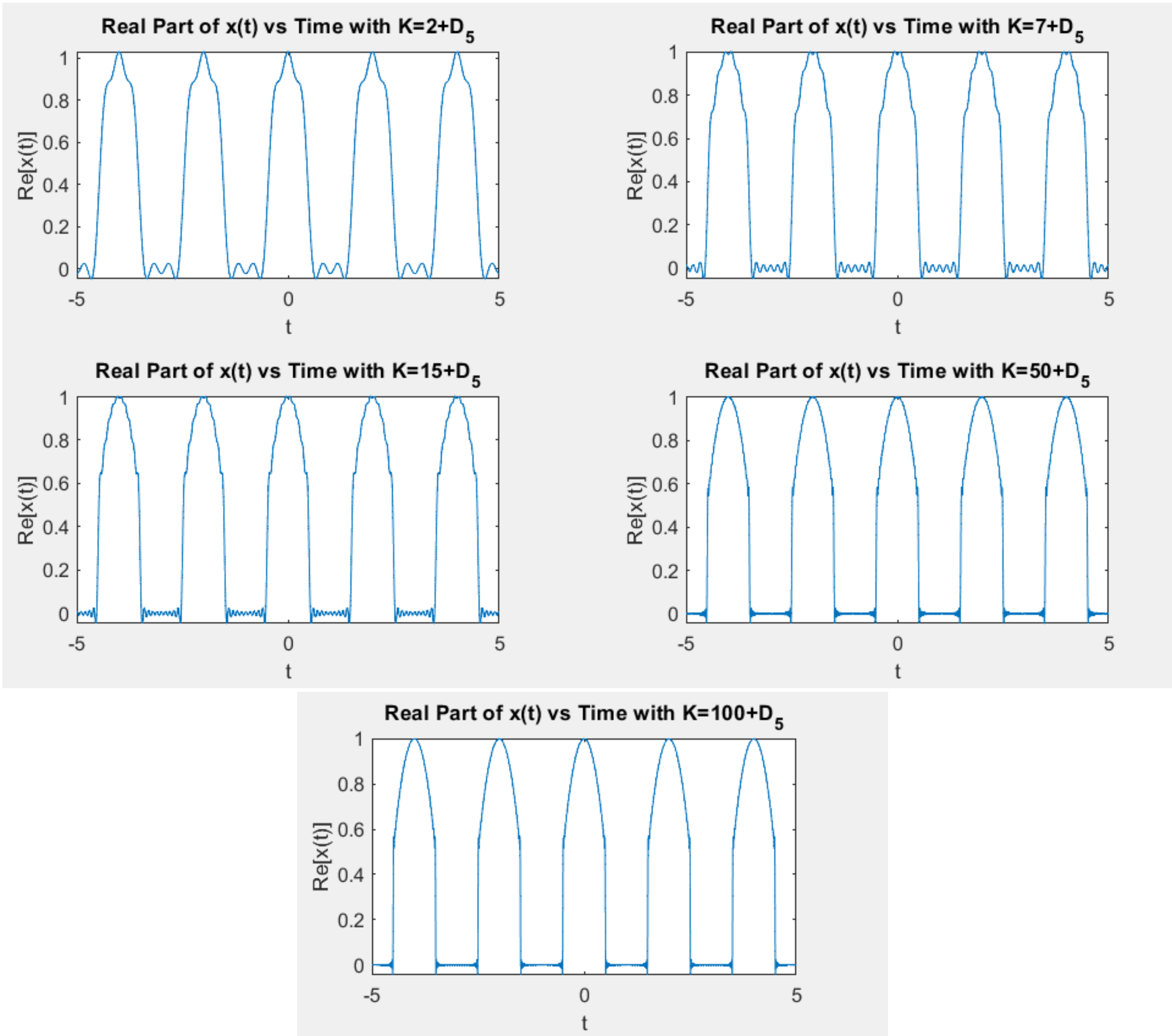

Figure 5: Plots of Part 3 with five different K values

Looking at the graphs above, it can be seen that as the K value increases, $\tilde{x}(t)$ becomes more and more similar to $x(t)$, that is, as the K value increases, a more successful approximation to $x(t)$ can be made. Also, when K goes to infinity, $\tilde{x}(t)$ can be expected to be equal to $x(t)$. It can be said that as K increases, the oscillations next to the discontinuities decrease but do not disappear completely.

Part 4:

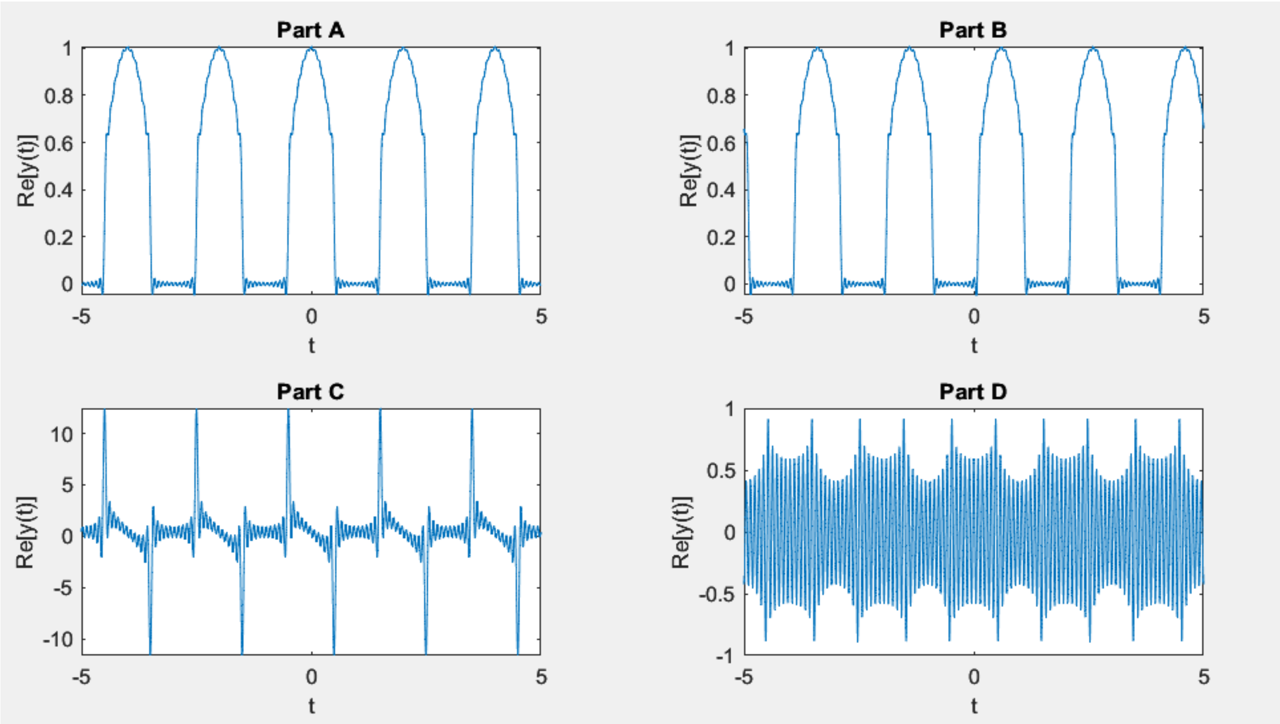

Figure 6: Plots for Part 4

Part A:

Adding the k=-k expression into the for loop will be sufficient to make the desired change. This operation makes the function time reversed, but since the function is even, there is no change in the plot.

Part B:

The effect of multiplying the FSE coefficients by $e^{-j\frac{2\pi k t_0}{T}}$ is to shift the x(t) signal to the right. To make this change, the “`X_k(k+K+1) = (X_k(k+K+1))*(exp((-1j*2*pi*k*t_0)/T));`” was added. As a result, it can be seen that the plot is shifted to the right by 0.6

Part C:

This operation takes the derivative of x(t). To make this change, the “`X_k(k+K+1) = (X_k(k+K+1))*((1j*k*2*pi)/T);`” was added. As a result, it can be seen that the plot was differentiated.

Part D:

In this section, the order of the coefficients was changed in the sections where k<0 and k>0, and k=0 remained constant. For this, the following part has been added to the code:

```
X_k = [flip(X_k(1:K)),zeros(1,K+1)] + [zeros(1,K+1),flip(X_k(K+2:2*K+1))];
```

MATLAB Code:

```
%% PART 1
    t       = 0:0.001:1;
    n       = mod(22003884,41);
    A       = rand(1, n)*3*(1+1j);
    omega   = rand(1, n)*pi;

    xs      = SUMCS(t, A, omega);
    real_p  = real(xs);
    imag_p  = imag(xs);
    abs_p   = abs(xs);
    angle_p = angle(xs);

    tiledlayout(2,2)

% Real Part
    nexttile
    plot(t, real_p);
    xlabel('t');
    ylabel('Re[x_s(t)]');
    title('Real Part of x_s(t) vs Time');

% Imaginary Part
    nexttile
    plot(t, imag_p);
    xlabel('t');
    ylabel('Im[x_s(t)]');
    title('Imaginary Part of x_s(t) vs Time');

% Magnitude
    nexttile
    plot(t, abs_p);
    xlabel('t');
    ylabel('Magnitude of x_s(t)');
    title('Magnitude of x_s(t) vs Time');

% Phase
    nexttile
    plot(t, angle_p);
    xlabel('t');
    ylabel('Phase of x_s(t)');
    title('Phase of x_s(t) vs Time');


%% PART 3
    D_11 = mod(22003884,11);
    D_5  = mod(22003884,5);
    t    = -5:0.001:5;
    T    = 2;
    W    = 1;
    K    = 20+D_11;
```

```
    %K     = 2+D_5;
    %K     = 7+D_5;
    %K     = 15+D_5;
    %K     = 50+D_5;
    %K     = 100+D_5;
    %K     = 1000+D_5;

    xt = FSWave(t,K,T,W);
    real_p = real(xt);
    imag_p = imag(xt);
    real_max = max(real_p);
    real_min = min(real_p);
    imaginary_max = max(imag_p);
    imaginary_min = min(imag_p);

    tiledlayout(2,2)

% Real Part
    nexttile
    plot(t, real_p);
    xlabel('t');
    ylabel('Re[x(t)]');
    title('Real Part of x(t) vs Time');

% Imaginary Part
    nexttile
    plot(t,imag_p);
    xlabel('t');
    ylabel('Im[x(t)]');
    title('Imaginary Part of x(t) vs Time');


%% PART 4
    tiledlayout(2,2)

% Part 4_A
    xt_A = FSWave_A(t,K,T,W);
    real_p_A = real(xt_A);
    nexttile
    plot(t, real_p_A);
    xlabel('t');
    ylabel('Re[y(t)]');
    title('Part A');

% Part 4_B
    xt_B = FSWave_B(t,K,T,W);
    real_p_B = real(xt_B);
    nexttile
    plot(t, real_p_B);
    xlabel('t');
    ylabel('Re[y(t)]');
    title('Part B');

% Part 4_C
    xt_C = FSWave_C(t,K,T,W);
```

```
    real_p_C = real(xt_C);
    nexttile
    plot(t, real_p_C);
    xlabel('t');
    ylabel('Re[y(t)]');
    title('Part C');

% Part 4_D
    xt_D = FSWave_D(t,K,T,W);
    real_p_D = real(xt_D);
    nexttile
    plot(t, real_p_D);
    xlabel('t');
    ylabel('Re[y(t)]');
    title('Part D');


%% FUNCTIONS
    function [xs] = SUMCS(t, A, omega)
        xs = zeros(1, length(t));
        M  = length(A);
        for i = 1:M
            xs = xs + A(i)*exp(1j*omega(i)*t);
        end
    end


    function [xt] = FSWave(t, K, T, W)
        X_k  = (-K:K);
        a    = (-K:K)*2*pi/T; %?????????????????????????????????????
        for k = -K:K
            if k == 0
                X_k(K+1)    = (W-W^3/6)/T;
            else
                X_k(K+1+k) = ((2-(W^2))/(2*pi*k) + T^2/((pi^3)*(k^3)))*sin((pi*W*k)/T)
- ((W*T)/((pi^2)*(k^2)))*cos((pi*W*k)/T);
            end
        end
        X_N = SUMCS(t,X_k(1:K),a(1:K));
        X_0 = X_k(K+1);
        X_P = SUMCS(t,X_k(K+2:2*K+1),a(K+2:2*K+1));
        xt  = X_N + X_0 + X_P;
    end

    function [xt] = FSWave_A(t, K, T, W)
        X_k  = (-K:K);
        a    = (-K:K)*2*pi/T;
        for k = -K:K
            k = -k; %#ok<FXSET>
            if k == 0
                X_k(K+1)    = (W-W^3/6)/T;
            else
                X_k(K+1+k) = ((2-(W^2))/(2*pi*k) + T^2/((pi^3)*(k^3)))*sin((pi*W*k)/T)
- ((W*T)/((pi^2)*(k^2)))*cos((pi*W*k)/T);
            end
```

```
        end
        X_N = SUMCS(t,X_k(1:K),a(1:K));
        X_0 = X_k(K+1);
        X_P = SUMCS(t,X_k(K+2:2*K+1),a(K+2:2*K+1));
        xt  = X_N + X_0 + X_P;
    end

    function [xt] = FSWave_B(t, K, T, W)
        X_k  = (-K:K);
        a    = (-K:K)*2*pi/T;
        t_0  = 0.6;
        for k = -K:K
            if k == 0
                X_k(K+1)   = (W-W^3/6)/T;
            else
                X_k(K+1+k) = ((2-(W^2))/(2*pi*k) + T^2/((pi^3)*(k^3)))*sin((pi*W*k)/T)
- ((W*T)/((pi^2)*(k^2)))*cos((pi*W*k)/T);
                X_k(k+K+1) = (X_k(k+K+1))*(exp((-1j*2*pi*k*t_0)/T));
            end
        end
        X_N = SUMCS(t,X_k(1:K),a(1:K));
        X_0 = X_k(K+1);
        X_P = SUMCS(t,X_k(K+2:2*K+1),a(K+2:2*K+1));
        xt  = X_N + X_0 + X_P;
    end

    function [xt] = FSWave_C(t, K, T, W)
        X_k  = (-K:K);
        a    = (-K:K)*2*pi/T;
        for k = -K:K
            if k == 0
                X_k(K+1)   = (W-W^3/6)/T;
            else
                X_k(K+1+k) = ((2-(W^2))/(2*pi*k) + T^2/((pi^3)*(k^3)))*sin((pi*W*k)/T)
- ((W*T)/((pi^2)*(k^2)))*cos((pi*W*k)/T);
                X_k(k+K+1) = (X_k(k+K+1))*((1j*k*2*pi)/T);
            end
        end
        X_N = SUMCS(t,X_k(1:K),a(1:K));
        X_0 = X_k(K+1);
        X_P = SUMCS(t,X_k(K+2:2*K+1),a(K+2:2*K+1));
        xt  = X_N + X_0 + X_P;
    end

    function [xt] = FSWave_D(t, K, T, W)
        X_k  = (-K:K);
        a    = (-K:K)*2*pi/T;
        for k = -K:K
            if k == 0
                X_k(K+1)   = (W-W^3/6)/T;
            else
                X_k(K+1+k) = ((2-(W^2))/(2*pi*k) + T^2/((pi^3)*(k^3)))*sin((pi*W*k)/T)
- ((W*T)/((pi^2)*(k^2)))*cos((pi*W*k)/T);
            end
        end
```

```
    X_k = [flip(X_k(1:K)),zeros(1,K+1)] + [zeros(1,K+1),flip(X_k(K+2:2*K+1))];
    X_N = SUMCS(t,X_k(1:K),a(1:K));
    X_0 = X_k(K+1);
    X_P = SUMCS(t,X_k(K+2:2*K+1),a(K+2:2*K+1));
    xt  = X_N + X_0 + X_P;
end
```