Bilkent University

EEE321: Signals and Systems

Lab Assignment 6

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Section - 01

Part 1:

$$y[0]=\sum_{\ell=1}^{N} a[\ell].y[-\ell]+b[0].x[0]+\sum_{k=1}^{M} b[k].x[-k] = b[0].x[0] \quad \text{since } x[n], y[n]=0 \text{ for } n<0$$

$$y[1]=a[1].y[0]+\sum_{\ell=2}^{N} a[\ell].y[1-\ell]+b[0].x[1]+b[1].x[0]+\sum_{k=2}^{M} b[k].x[1-k]$$

$$y[1]=a[1].y[0]+b[0].x[1]+b[1].x[0]=a[1].b[0].x[0]+b[0].x[1]+b[1].x[0]$$

$$Y(z)=\sum_{n=0}^{\infty}\sum_{\ell=1}^{N} a[\ell]y[n-\ell].z^{-n}+\sum_{n=0}^{\infty}\sum_{k=0}^{M} b[k].x[n-k].z^{-n}=\sum_{\ell=1}^{N} a[\ell]z^{-\ell}.Y(z)+\sum_{k=0}^{M} b[k].z^{-k}.X(z)$$

$$Y(z)\left(1-\sum_{\ell=1}^{N} a[\ell]z^{-\ell}\right)=\sum_{k=0}^{M} b[k]z^{-k}X(z) \qquad H(z)=\frac{Y(z)}{X(z)}=\frac{\sum_{k=0}^{M} b[k]z^{-k}}{1-\sum_{\ell=1}^{N} a[\ell].z^{-\ell}}$$

$$P=M, \quad Q=N, \quad c_n[p]=b[k], \quad c_d[q]=-a[\ell]$$

Figure 1: Calculations for part 1

Part2:

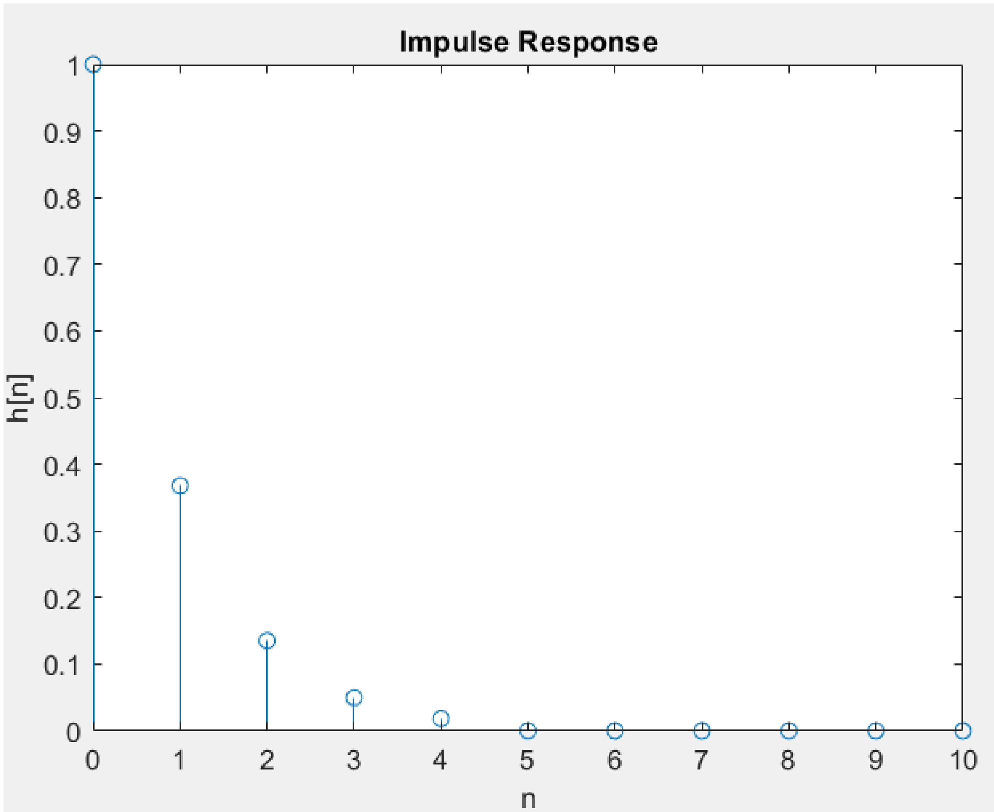


Figure 2: Impulse response

b) Non-zero values of h[n] are the same with the coefficients

c) h[n] is a FIR since it has a finite length, converges to zero. length=M=5

d) $H(z)=\frac{\sum_{k=0}^{M-1} b[k].z^{-k}}{1-\sum_{\ell=1}^{N} a[\ell].z^{-\ell}} = \sum_{k=0}^{M-1} e^{-k}.z^{-k} = \sum_{k=0}^{M-1} \left((ez)^{-1}\right)^{k} = \frac{1-\left((ez)^{-1}\right)^{M}}{1-(ez)^{-1}} = \frac{1-(ez)^{-M}}{1-(ez)^{-1}}$

$H(e^{jw}) = \sum_{k=0}^{M-1} (e.e^{jw})^{-k} = \sum_{k=0}^{M-1} e^{-k-kjw} = \frac{1-e^{-M-Mjw}}{1-e^{-1-jw}}$

e) It is a Low-pass filter. 3dB bandwidth is the range that contains freq values that are less than 3dB distance from the maximum dB value in the graph. That is equal to ≈ 2. Cut-off frequencies are values at both ends of the bandwidth range. $f_L \cong -1$, $f_H \cong +1$.

Figure 3: Answers for part 2

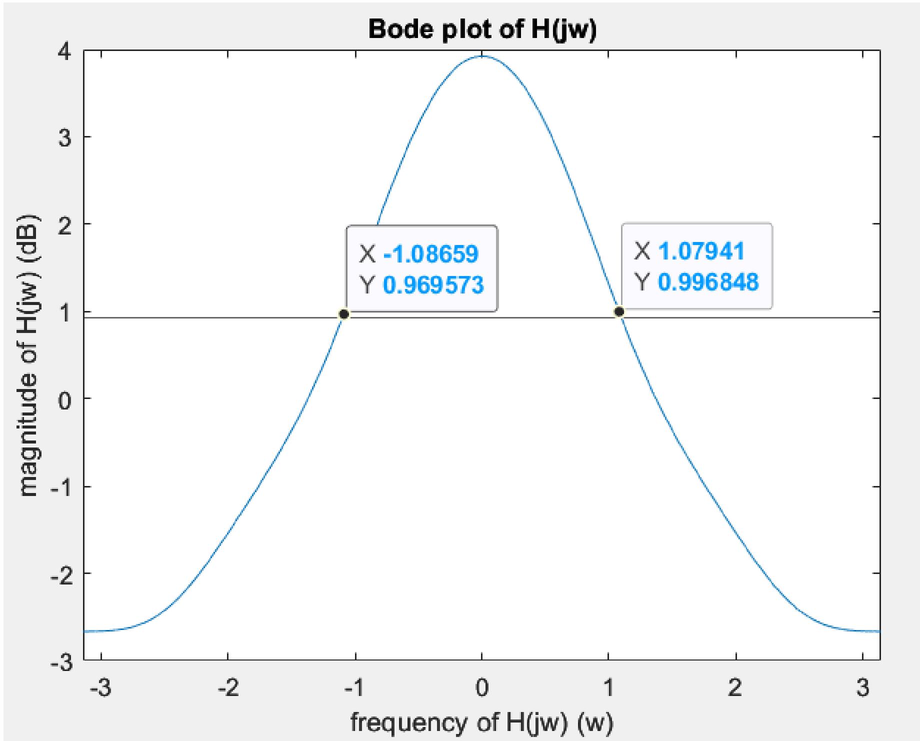


Figure 4: Bode plot of H(jw)

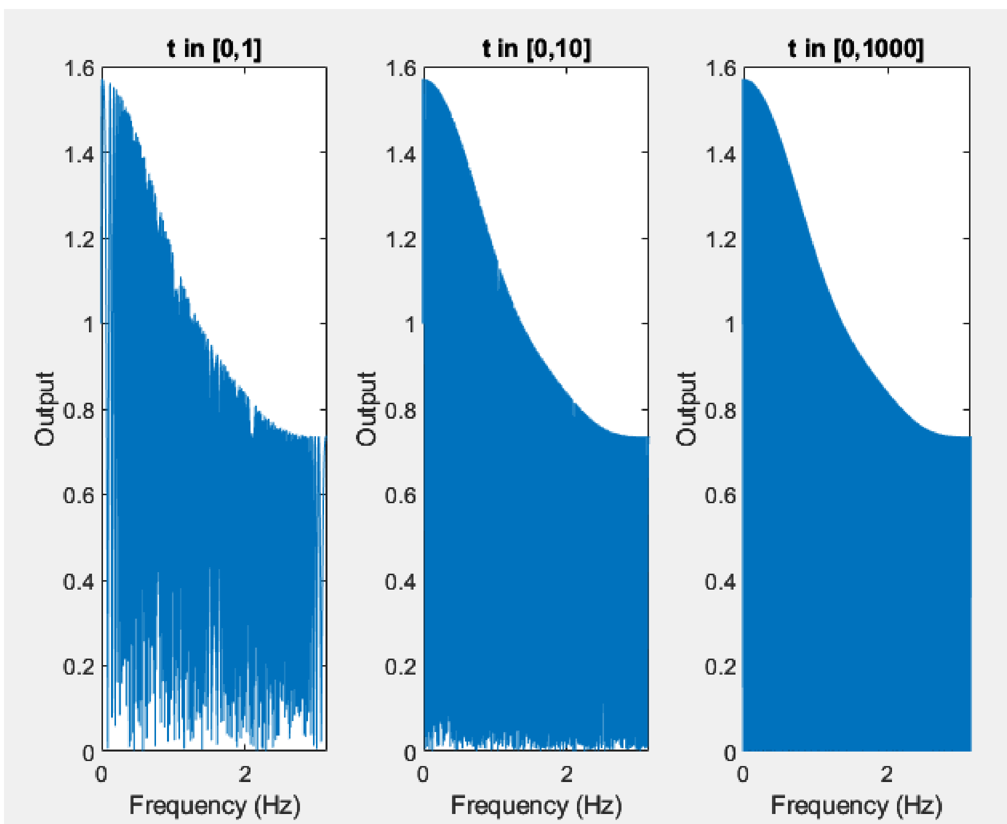


Figure 5: Plots for part 2

Part 3:

a) $$H(z) = \frac{(z-0{,}89-j0{,}45)}{(z-0{,}62-j0{,}77)(z-0{,}51-j\cdot 0{,}85)}$$

b) $$H(z) = \frac{Y(z)}{X(z)} = \frac{(1-z_1 z^{-1})}{(1-p_1 z^{-1})(1-p_2 z^{-1})}$$

$$Y(z) - Y(z)p_1 z^{-1} - Y(z)p_2 z^{-1} + Y(z)p_1 p_2 z^{-2} = X(z) - X(z) z_1 z^{-1}$$

$$y[n] = p_1 y[n-1] + p_2 y[n-1] - p_1 p_2 y[n-2] + x[n] - z_1 x[n-1]$$

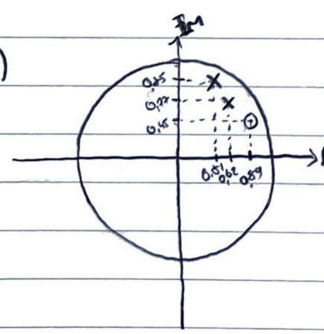
$$y[n] = \sum_{\ell=1}^{2} a[\ell]\, y[n-\ell] + \sum_{k=0}^{1} b[k] \cdot x[n-k]$$

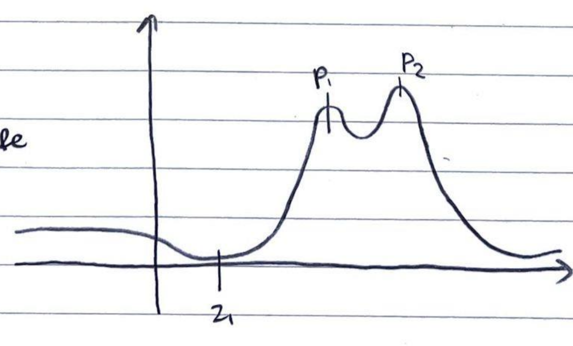
$$a[\ell] = [p_1 + p_2,\ -p_1 \cdot p_2] \qquad b[k] = [1,\ -z_1]$$

c) $$H(z) = \frac{A}{1-p_1 z^{-1}} + \frac{B}{1-p_2 z^{-1}}, \quad A = H(z)\cdot(z-p_1)\Big|_{z=p_1}, \quad B = H(z)\cdot(z-p_2)\Big|_{z=p_2}$$

$$\frac{1}{1-az^{-1}} \xrightarrow{\text{Inverse z trans}} a^n u[n] \quad \text{so} \quad \rightarrow H[n] = A\, p_1^n u[n] + B\, p_2^n u[n]$$

d)





e) $|p_1|$ and $|p_2| < 1$ $\rightarrow$ Stable

f) IIR since $y[n]$ depends on $y[n-\ell]$ for $1 \le \ell \le N$

Figure 6: Answers for part 3

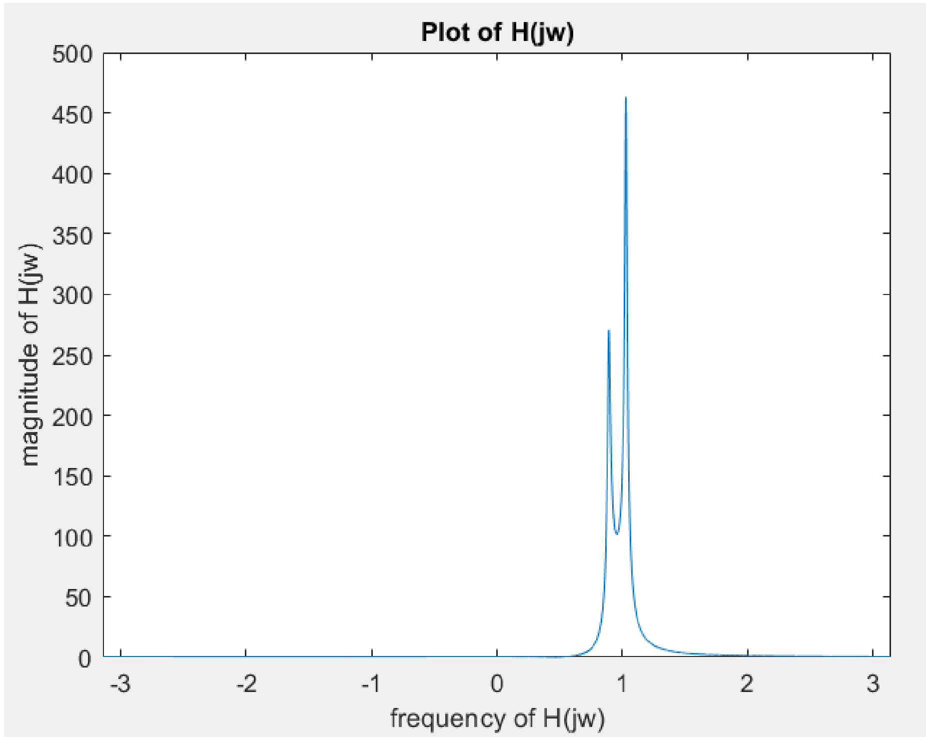


Figure 7: plot for part 3.g

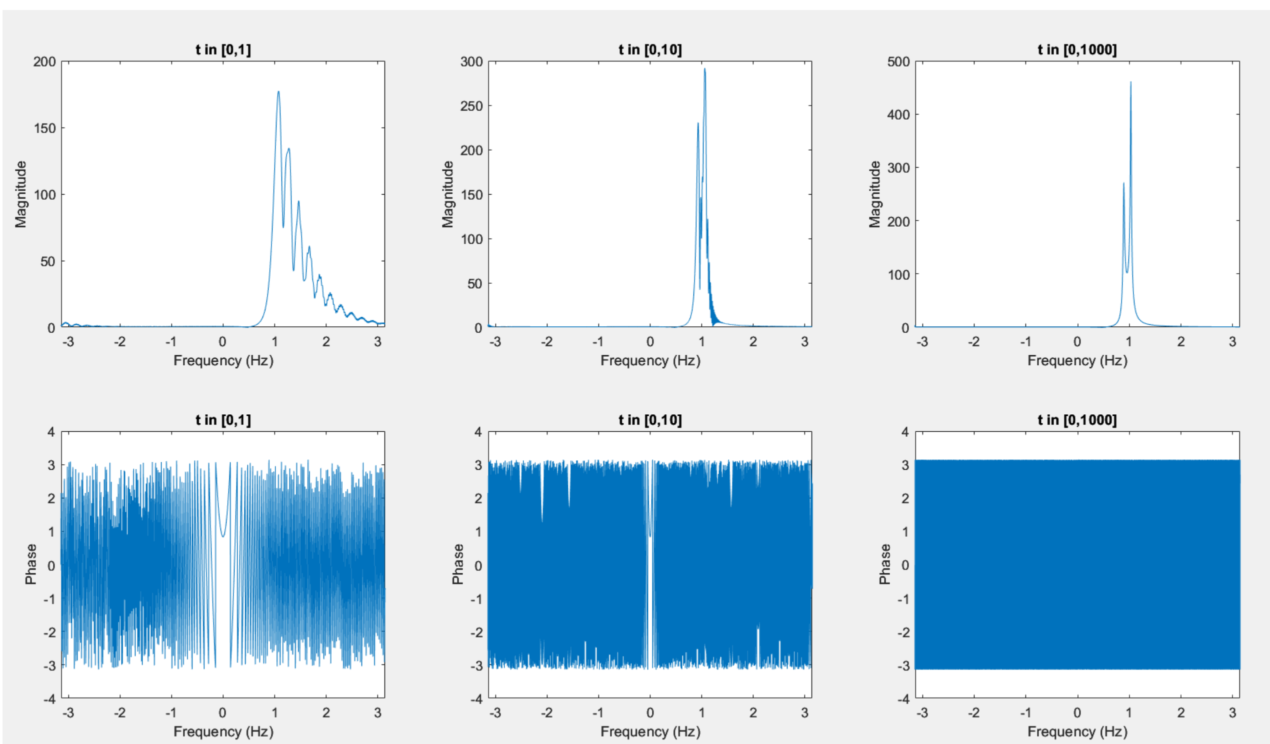


Figure 8: plots for part 3.h

Appendix

```
%% PART 2

clear
a = 0;
M = 5 + 0;
k = 0:M-1;
b = exp(-k);
x = [1 zeros(1, 10)];
h = DTLTI(a, b, x, 11);


%% PART 2.a

tiledlayout(1, 1);
nexttile;
stem(0:10, h);
title('Impulse Response');
xlabel('n');
ylabel('h[n]')


%% PART 2.e

w   = - pi:0.001:pi - 0.001;
H_w = (1 - exp(-M*(1i*w + 1)))./(1 - exp(-(1i*w + 1)));
H_dB = 20*log10(abs(H_w));
tiledlayout(1, 1);
nexttile;
plot(w, H_dB);                              %%%%% hw olarak mi olmali yoksa hdB mi
olmali ?????
title('Bode plot of H(jw)');
xlabel('frequency of H(jw) (w)');
ylabel('magnitude of H(jw) (dB)');
yline(max(H_dB) - 3);
xlim([- pi, pi - 0.001]);


%% PART 2.f

t = 0:1/1400:1-1/1400;                 %%%%label ne olmai burada frekans mi yoksa time
mi olacak
x = cos(2*pi*(((700-0)/1)*(t.^2)/2 + 0*t));
y = DTLTI(a, b, x, 1400);
tiledlayout(1, 3);
nexttile;
plot(0:pi/1400:pi-1/1400, abs(y));
title('t in [0,1]');
xlabel('Frequency (Hz)');
ylabel('Output');
```

```
xlim([0, pi]);
t = 0:1/1400:10-1/1400;
x = cos(2*pi*(((700-0)/10)*(t.^2)/2 + 0*t));
y = DTLTI(a, b, x, 14000);
nexttile;
plot(0:pi/14000:pi-1/14000, abs(y));
title('t in [0,10]');
xlabel('Frequency (Hz)');
ylabel('Output');
xlim([0, pi]);
t = 0:1/1400:1000-1/1400;
x = cos(2*pi*(((700-0)/1000)*(t.^2)/2 + 0*t));
y = DTLTI(a, b, x, 1400000);
nexttile;
plot(0:pi/1400000:pi-1/1400000, abs(y));
title('t in [0,1000]');
xlabel('Frequency (Hz)');
ylabel('Output');
xlim([0, pi]);
%%%%%%%%%%%%%%%%%% geenral trend

%% PART 3.g

w   = - pi:1/1400:pi - 1/1400;
H_w = (exp(1i*w)).*(exp(1i*w) - (0.89 + 1i*0.45))./((exp(1i*w) - (0.62 +
1i*0.77)).*(exp(1i*w) - (0.51 + 1i*0.85))); %%%%% c ile carpmak neden etkilemiyor
%H_w = (1 - exp(-1i*w)*(0.89 + 1i*0.45))./(1 - (exp(-1i*w))*(0.62 + 1i*0.77)).*(1 -
(exp(-1i*w)*(0.51 + 1i*0.85)));
H_dB = 20*log10(abs(H_w));
tiledlayout(1, 1);
nexttile;
plot(w, abs(H_w));                            %%%%% hw olarak mi olmali yoksa hdB mi
olmali ?????
title('Plot of H(jw)');
xlabel('frequency of H(jw)');
ylabel('magnitude of H(jw)');
xlim([- pi, pi]);


%% PART 3.h

a = [(0.62 + 1i*0.77) + (0.51 + 1i*0.85), - (0.62 + 1i*0.77)*(0.51 + 1i*0.85)];
b = [1, - (0.89 + 1i*0.45)];
t = 0:1/1400:1-1/1400;
x = exp(2i*pi*(((700 - (-700))/1)*(t.^2)/2 + (-700)*t));
y = DTLTI(a, b, x, 1400);
subplot(2, 3, 1);
plot(-pi:2*pi/1400:pi-1/1400, abs(y));
title('t in [0,1]');
xlabel('Frequency (Hz)');
ylabel('Magnitude');
xlim([-pi, pi]);
subplot(2, 3, 4);
plot(-pi:2*pi/1400:pi-1/1400, angle(y));
title('t in [0,1]');
```

```
xlabel('Frequency (Hz)');
ylabel('Phase');
xlim([-pi, pi]);
t = 0:1/1400:10-1/1400;
x = exp(2i*pi*(((700 - (-700))/10)*(t.^2)/2 + (-700)*t));
y = DTLTI(a, b, x, 14000);
subplot(2, 3, 2);
plot(-pi:2*pi/14000:pi-1/14000, abs(y));
title('t in [0,10]');
xlabel('Frequency (Hz)');
ylabel('Magnitude');
xlim([-pi, pi]);
subplot(2, 3, 5);
plot(-pi:2*pi/14000:pi-1/14000, angle(y));
title('t in [0,10]');
xlabel('Frequency (Hz)');
ylabel('Phase');
xlim([-pi, pi]);
t = 0:1/1400:1000-1/1400;
x = exp(2i*pi*(((700 - (-700))/1000)*(t.^2)/2 + (-700)*t));
y = DTLTI(a, b, x, 1400000);
subplot(2, 3, 3);
plot(-pi:2*pi/1400000:pi-1/1400000, abs(y));
title('t in [0,1000]');
xlabel('Frequency (Hz)');
ylabel('Magnitude');
xlim([-pi, pi]);
subplot(2, 3, 6);
plot(-pi:2*pi/1400000:pi-1/1400000, angle(y));
title('t in [0,1000]');
xlabel('Frequency (Hz)');
ylabel('Phase');
xlim([-pi, pi]);


%% FUNCTIONS

function [y] = DTLTI(a, b, x, N_y)
    y = zeros(1,N_y);
    for n = 0:N_y - 1
        for l = 1:length(a)
            if (n - l) < 0
                y(n + 1) = y(n + 1);
            else
                y(n + 1) = a(l)*y(n + 1 - l) + y(n + 1);
            end
        end
        for k = 0:length(b) - 1
            if (n - k) < 0
                 y(n + 1) = y(n + 1);
            else
                y(n + 1) = b(k + 1)*x(n + 1 - k) + y(n + 1);
            end
        end
    end
```

end