Bilkent University

EEE321: Signals and Systems

Lab Assignment 3

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Section - 01

Part 1.1:

In this part, it is asked to write a function that prepares the analog signal to be transmitted when a phone number containing only numerical digits is dialed.

	1209 Hz	1336 Hz	1477 Hz	1633 Hz
697 Hz	1	2	3	A
770 Hz	4	5	6	B
852 Hz	7	8	9	C
941 Hz	*	0	#	D

Table 1: DTMF Frequencies

For this, first an array containing the following frequency pairs was created using the table above.

```
frequencies = [941, 1336; 697,1209; 697,1336; 697,1477;   % 0 1 2 3
                          770,1209; 770,1336; 770,1477;   %   4 5 6
                          852,1209; 852,1336; 852,1477];  %   7 8 9
```

Afterwards, the desired DTMFTRA(number) function was created using this array and after the phone number input was given, it was listened to with the soundsc(x,8192) command. The sound heard was the same as the keystroke sounds made when entering a number on old phones.

Part 1.2:

a) $x(t) = e^{j2\pi f_0 t} = e^{j\omega_0 t} \Rightarrow \boxed{X(\omega) = 2\pi\delta(\omega-\omega_0)}$ $\quad \omega_0 = 2\pi f_0 t$

b) $x(t) = \cos(2\pi f_0 t) = \cos(\omega_0 t) \Rightarrow X(\omega) = \int_{-\infty}^{\infty} \cos(\omega_0 t) e^{-j\omega t} dt$

$$X(\omega) = \int_{-\infty}^{\infty} \left(\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}\right) e^{-j\omega t} dt = \frac{1}{2}\left(\underbrace{\int_{-\infty}^{\infty} e^{j\omega_0 t} \cdot e^{-j\omega t} dt}_{2\pi\delta(\omega-\omega_0)} + \underbrace{\int_{-\infty}^{\infty} e^{-j\omega_0 t} \cdot e^{-j\omega t} dt}_{2\pi\delta(\omega+\omega_0)}\right) \rightarrow \text{by linearity}$$

$\boxed{X(\omega) = \pi\delta(\omega-\omega_0) + \pi\delta(\omega+\omega_0)}$ $\quad \omega_0 = 2\pi f_0 t$

c) $x(t) = \sin(2\pi f_0 t) = \sin(\omega_0 t) \Rightarrow X(\omega) = \int_{-\infty}^{\infty} \sin(\omega_0 t) e^{-j\omega t} dt$

$$X(\omega) = \int_{-\infty}^{\infty} \left(\frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}\right) e^{-j\omega t} dt = \frac{1}{2j}\left(\underbrace{\int_{-\infty}^{\infty} e^{j\omega_0 t} e^{-j\omega t} dt}_{2\pi\delta(\omega-\omega_0)} - \underbrace{\int_{-\infty}^{\infty} e^{-j\omega_0 t} e^{-j\omega t} dt}_{2\pi\delta(\omega+\omega_0)}\right) \rightarrow \text{by linearity}$$

$\boxed{X(\omega) = -\pi j\delta(\omega-\omega_0) + \pi j\delta(\omega+\omega_0)}$ $\quad \omega_0 = 2\pi f_0 t$

d) $x(t) = \text{rect}\left(\frac{t}{T_0}\right) = \begin{cases} 1, & -\frac{T_0}{2} < t < \frac{T_0}{2} \\ 0, & \text{otherwise} \end{cases}$ $\quad X(\omega) = \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} 1 \cdot e^{-j\omega t} dt = \frac{1}{-j\omega}\left(e^{-j\omega\frac{T_0}{2}} - e^{j\omega\frac{T_0}{2}}\right)$

$$= \frac{2}{\omega}\left(\frac{e^{j\omega\frac{T_0}{2}} - e^{-j\omega\frac{T_0}{2}}}{2j}\right) \boxed{= \frac{2}{\omega}\sin\left(\frac{\omega T_0}{2}\right)}$$

e) $x(t) = e^{j2\pi f_0 t}\,\text{rect}\left(\frac{t}{T_0}\right) = e^{j\omega_0 t}\,\text{rect}\left(\frac{t}{T_0}\right)$

multiplying with $e^{j\omega_0 t}$ in time domain means shifting ω_0 in frequency domain

$\boxed{X(\omega) = \frac{2}{\omega-\omega_0}\sin\left(\frac{(\omega-\omega_0)T_0}{2}\right)}$ $\quad \omega_0 = 2\pi f_0 t$

f) $x(t) = \cos(2\pi f_0 t) \cdot \text{rect}\left(\frac{t}{T_0}\right) = \cos(\omega_0 t) \cdot \text{rect}\left(\frac{t}{T_0}\right) = \left(\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}\right)\text{rect}\left(\frac{t}{T_0}\right)$

$= \frac{1}{2}\left(e^{j\omega_0 t}\,\text{rect}\left(\frac{t}{T_0}\right) + e^{-j\omega_0 t}\,\text{rect}\left(\frac{t}{T_0}\right)\right)$ by linearity and answer e

$\boxed{X(\omega) = \left(\frac{1}{\omega-\omega_0}\right)\sin\left(\frac{(\omega-\omega_0)T_0}{2}\right) + \left(\frac{1}{\omega+\omega_0}\right)\sin\left(\frac{(\omega+\omega_0)T_0}{2}\right)}$ $\quad \omega_0 = 2\pi f_0 t$

Figure 1: Calculations for PART 1.2

g) $x(t) = rect\left(\frac{t-t_0}{T_0}\right)$

t_0 shifting in time domain means multiplying with e^{-jwt_0} in freq domain

$\boxed{X(w) = e^{-jwt_0} \cdot \frac{2}{w} \sin\left(\frac{wT_0}{2}\right)}$ using answer from d

h) $x(t) = e^{j2\pi f_0 t} rect\left(\frac{t-t_0}{T_0}\right) = e^{jw_0 t} \cdot rect\left(\frac{t-t_0}{T_0}\right)$

multiply $e^{jw_0 t}$ + t_0 shifting in time domain $\longleftrightarrow$ w_0 shifting + multiplying $e^{-jw_0 t}$ in freq domain

i.e both freq and time shifted

$\boxed{X(w) = e^{-j(w-w_0)t_0} \cdot \left(\frac{2}{w-w_0}\right) \sin\left(\frac{(w-w_0)T_0}{2}\right)}$ $w_0 = 2\pi f_0 t$

i) $x(t) = \cos(2\pi f_0 t)\, rect\left(\frac{t-t_0}{T_0}\right) = \cos(w_0 t) \cdot rect\left(\frac{t-t_0}{T_0}\right) = \left(\frac{e^{jw_0 t} + e^{-jw_0 t}}{2}\right) rect\left(\frac{t-t_0}{T_0}\right)$

$= \frac{1}{2}\left(e^{jw_0 t} rect\left(\frac{t-t_0}{T_0}\right) + e^{-jw_0 t} rect\left(\frac{t-t_0}{T_0}\right)\right)$ by linearity and answer from h

$\boxed{= \frac{1}{2}\left(e^{-j(w-w_0)t_0}\left(\frac{2}{w-w_0}\right)\sin\left(\frac{(w-w_0)T_0}{2}\right)\right) + \frac{1}{2}\left(e^{-j(w+w_0)t_0}\left(\frac{2}{w+w_0}\right)\sin\left(\frac{(w+w_0)T_0}{2}\right)\right)}$

$w_0 = 2\pi f_0 t$

Figure 2: Calculations for PART 1.2

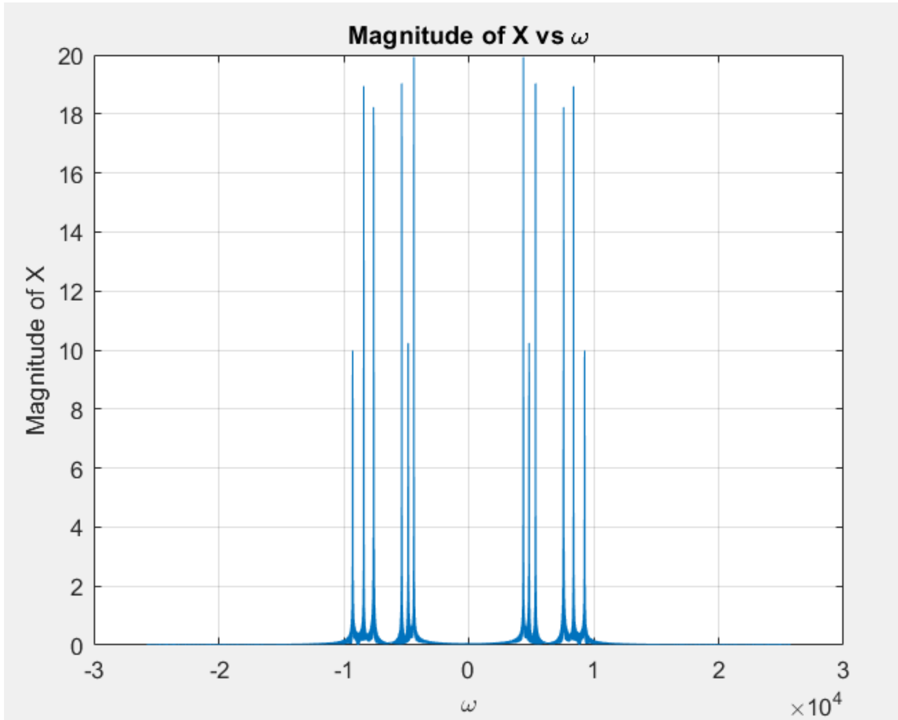

Figure 3: Magnitude of X vs omega

The dialed number is [4 8 8 3 1] and peak values are:

$$4378\, rad/sec \leftrightarrow \quad 696.7\, Hz \sim \ 697\, Hz$$

$$4840\, rad/sec \leftrightarrow \quad 770.3\, Hz \sim \ 770\, Hz$$

$$5343\, rad/sec \leftrightarrow \quad 850.3\, Hz \sim \ 852\, Hz$$

$$7575\, rad/sec \leftrightarrow \quad 1205.6\, Hz \sim 1209\, Hz$$

$$8399\, rad/sec \leftrightarrow \quad 1336.7\, Hz \sim 1336\, Hz$$

$$9269\, rad/sec \leftrightarrow \quad 1475.2\, Hz \sim 1477\, Hz$$

These values match the frequency values of the numbers in Table 1, but they are not sufficient to determine each number individually and their order in dialed number. Therefore, the x(t) signal must be multiplied by five different rectangular signals and each digit in the dialed number must be obtained separately.

```
x_1 = x.*[ones(1,2048), zeros(1,8192)];
x_2 = x.*[zeros(1,2048), ones(1,2048), zeros(1,6144)];
x_3 = x.*[zeros(1,4096), ones(1,2048), zeros(1,4096)];
x_4 = x.*[zeros(1,6144), ones(1,2048), zeros(1,2048)];
x_5 = x.*[zeros(1,8192), ones(1,2048)];
```

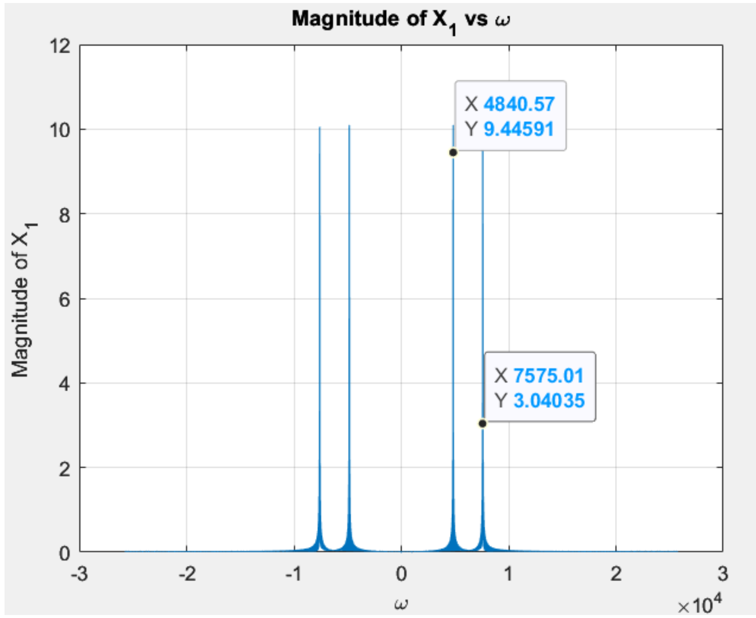

Figure 4: Magnitude of X_1 vs omega

$$4840, 7575\, rad/\sec \quad \sim \quad 770, 1209\, Hz$$

The first digit dialed is 4 from Table 1

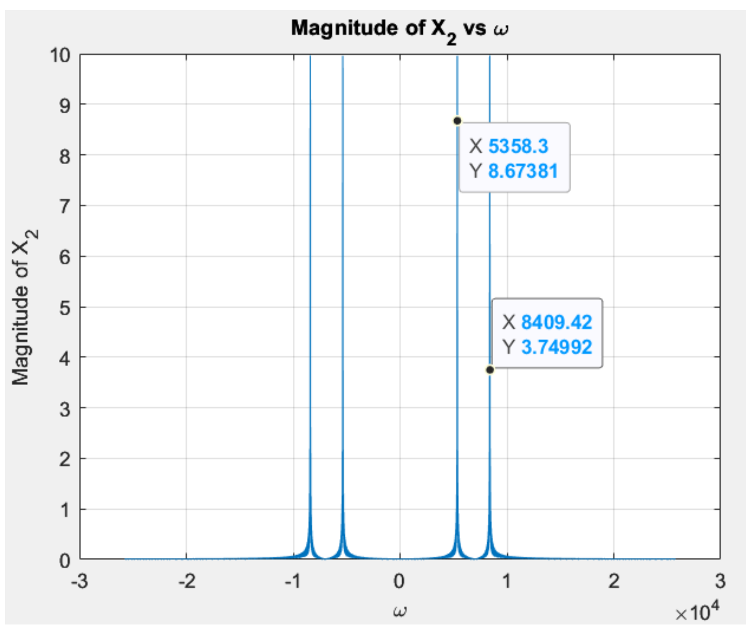

Figure 5: Magnitude of X_2 vs omega

$$5358, 8409\, rad/\sec \quad \sim \quad 852, 1336\, Hz$$

The second digit dialed is 8 from Table 1

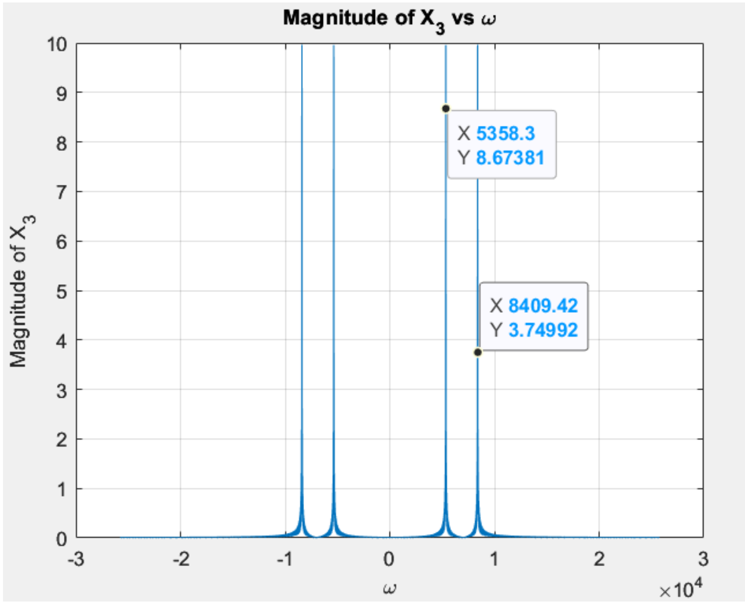

Figure 6: Magnitude of X_3 vs omega

$$5358, 8409\ rad/\sec \quad \sim \quad 852, 1336\ Hz$$

The third digit dialed is 8 from Table 1

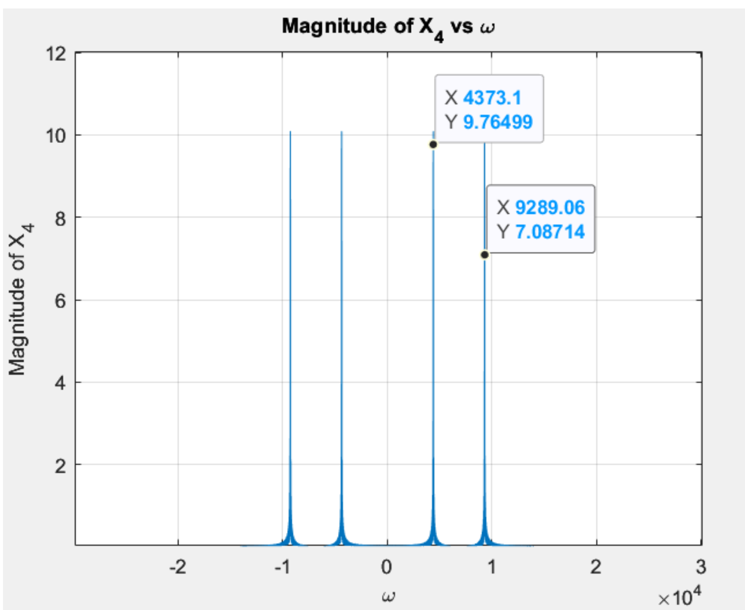

Figure 7: Magnitude of X_4 vs omega

$$4373, 9289\ rad/\sec \quad \sim \quad 697, 1477\ Hz$$

The fourth digit dialed is 3 from Table 1

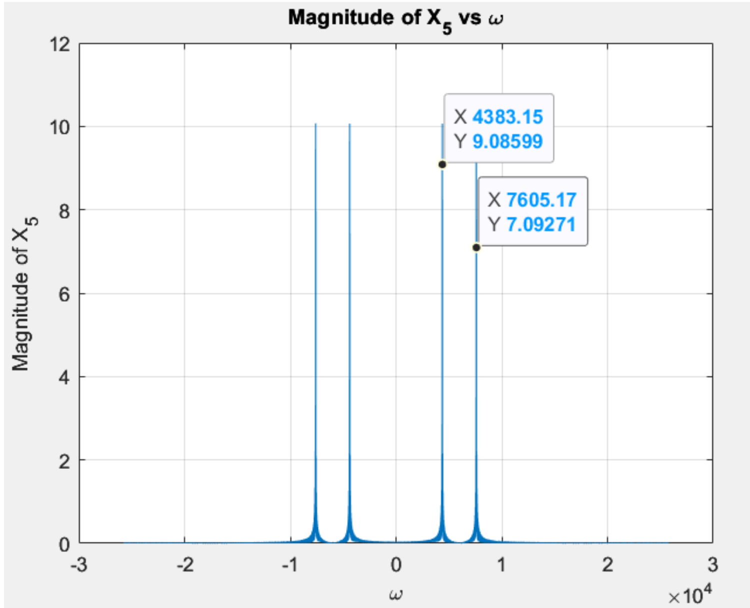

Figure 8: Magnitude of X_5 vs omega

$$4383, 7605\, rad/\sec \quad \sim \quad 697, 1209\, Hz$$

The fifth digit dialed is 1 from Table 1

Thus, all digits are obtained separately and the dialed number is [4 8 8 3 1]. Although the correct frequency values were obtained in the first method, since the frequencies of all digits appeared together, it was not possible to identify which frequency value belonged to which digit and therefore the digits and their orders. However, thanks to the second method, the frequency values of each digit were determined separately and thus each digit was obtained in its order.

Part 2:

The recording for this part is, “one two three Emre Bilkent four five six signal seven”.

```
audio = audiorecorder(8192,8,1,2);
recordblocking(audio,12);
x = transpose(getaudiodata(audio));
play(audio)
```

a) Since it is a LTI system if we assume that $x(t)=\delta(t)$ the system response, $y(t)$ becomes $h(t)$. So,

$$y(t)=x(t)+\sum_{i=1}^{M}A_1 x(t-t_i) \rightarrow h(t)=\delta(t)+\sum_{i=1}^{M}A_i\delta(t-t_i)$$

b)

$$h(t)\rightarrow H(w)=1+\sum_{i=1}^{M}A_i e^{-jwt_i} \quad \text{because } F\{\delta(t)\}=1 \ ; F\{\delta(t-t_i)\}=e^{-jwt_i}$$

c) from part a and convolution property:

$$y(t)=x(t)*h(t) \longleftrightarrow Y(w)=X(w).H(w)$$

d)

$$Y(w)=X(w).H(w) \Rightarrow X(w)=\frac{Y(w)}{H(w)}$$

Figure 9: Calculations for PART 2

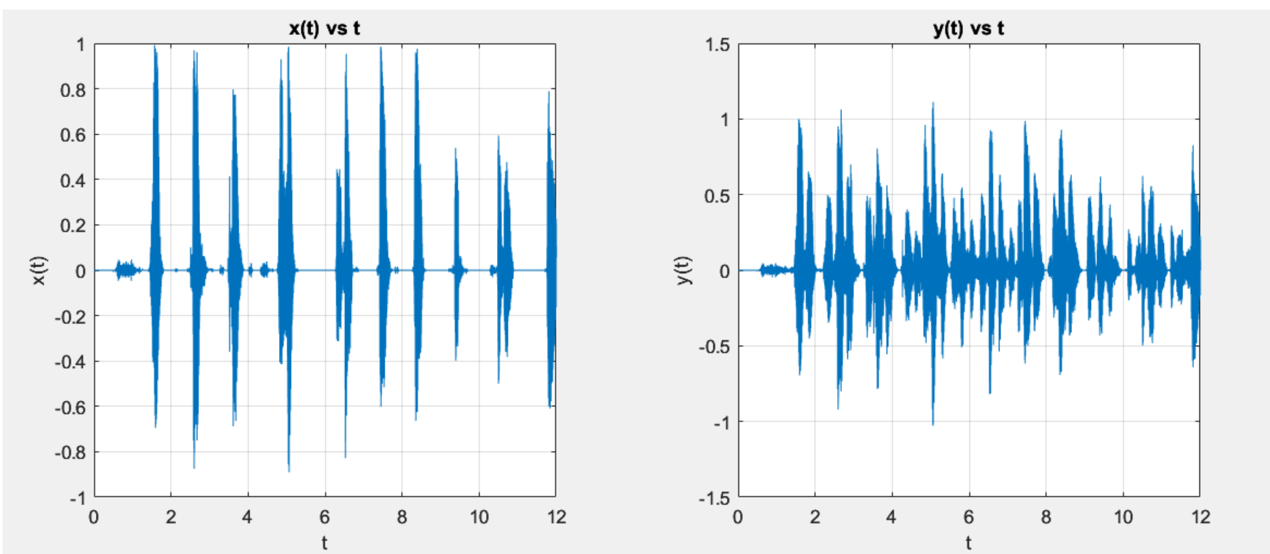

Figure 10: plots of x(t) vs t and y(t) vs t

Although the words spoken were slightly understood when the y(t) sound was listened to, a disturbing sound emerged because there were 6 different echoes next to each word.

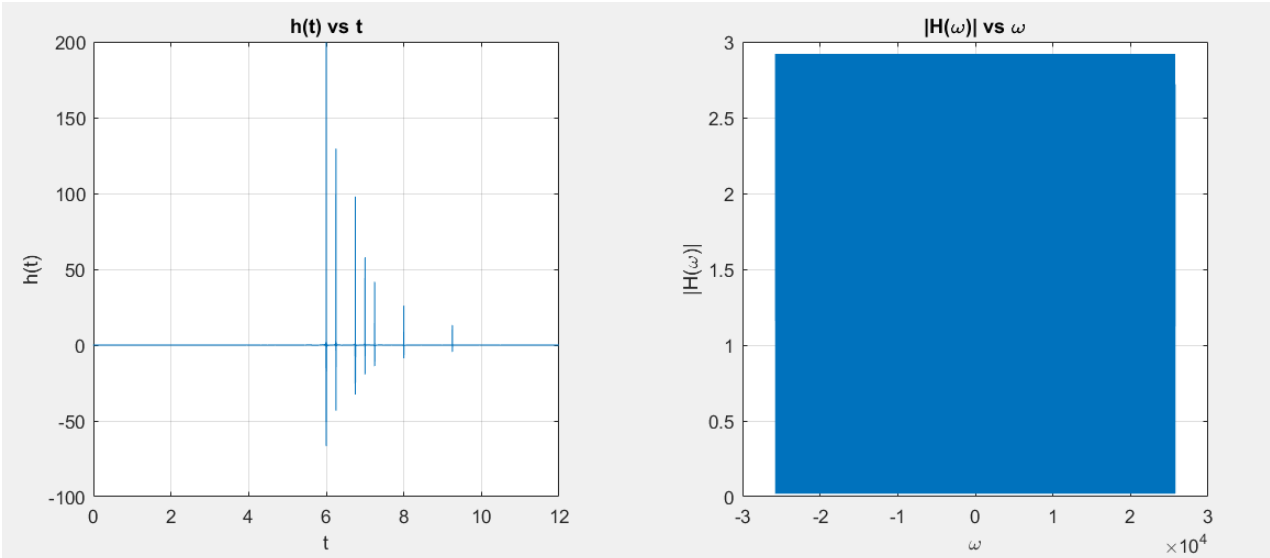

Figure 11: plots of h(t) vs t and |H(w)| vs w

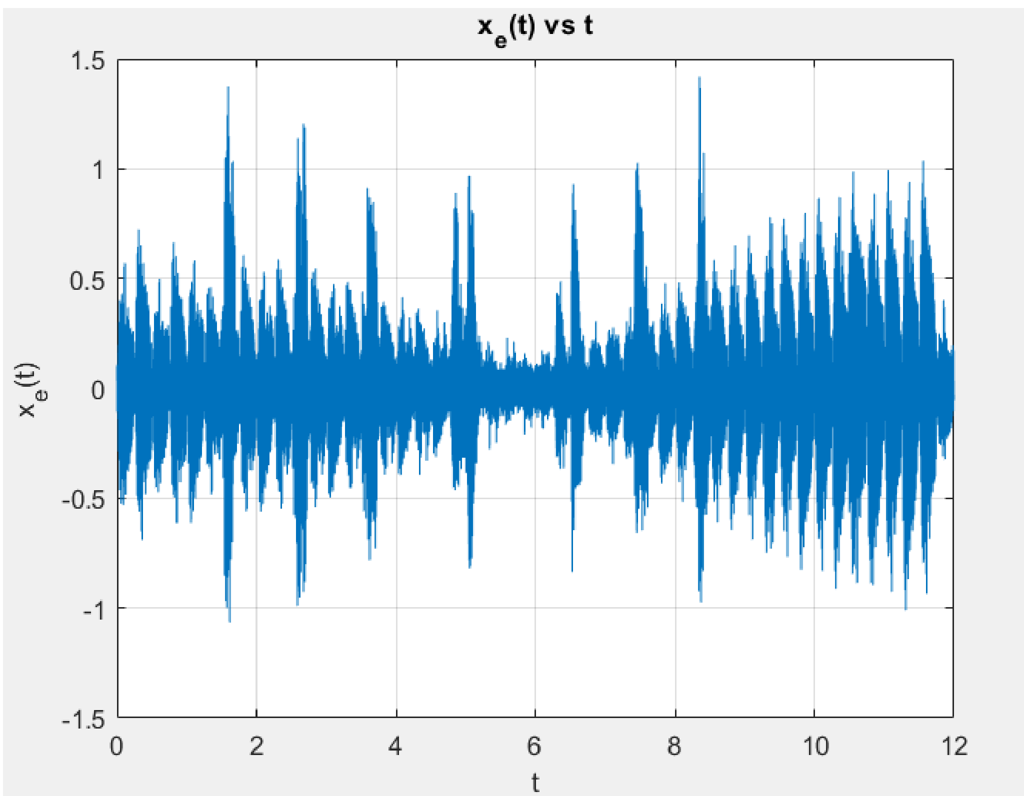

Figure 12: plot of x_e(t) vs t

When the sound $x_e(t)$ was listened to, it was noticed that the echoes in the sound y(t) disappeared, that is, a sound more similar to the original sound was obtained and the words were understood clearly. But the last word spoken, "seven", was heard repeatedly and in a low voice. If the signal of the word "seven" is ignored, the resulting graph is almost identical to the original graph.

MATLAB Code:

```
%% PART 1.1 DTMF Transmitter

number = [0 5 5 1 4 3 2 8 4 3 2];
x = DTMFTRA(number);
soundsc(x, 8192)


%% PART 1.2 DTMF Receiver

number = [4 8 8 3 1];
x = DTMFTRA(number);
soundsc(x, 8192)
X = FT(x);
omega = linspace(-8192*pi, 8192*pi, 10241);
omega = omega(1:10240);
plot(omega, abs(X));
xlabel('\omega');
ylabel('Magnitude of X');
title('Magnitude of X vs \omega');
grid ON;


%% PART 1.2 DIGIT 1

x_1 = x.*[ones(1,2048),zeros(1,8192)];
soundsc(x_1, 8192)
X = FT(x_1);
plot(omega, abs(X));
xlabel('\omega');
ylabel('Magnitude of X_1');
title('Magnitude of X_1 vs \omega');
grid ON;


%% PART 1.2 DIGIT 2

x_2 = x.*[zeros(1,2048),ones(1,2048),zeros(1,6144)];
soundsc(x_2, 8192)
X = FT(x_2);
plot(omega, abs(X));
xlabel('\omega');
ylabel('Magnitude of X_2');
title('Magnitude of X_2 vs \omega');
grid ON;
```

```matlab
%% PART 1.2 DIGIT 3

x_3 = x.*[zeros(1,4096),ones(1,2048), zeros(1,4096)];
soundsc(x_3, 8192)
X = FT(x_3);
plot(omega, abs(X));
xlabel('\omega');
ylabel('Magnitude of X_3');
title('Magnitude of X_3 vs \omega');
grid ON;


%% PART 1.2 DIGIT 4

x_4 = x.*[zeros(1,6144),ones(1,2048),zeros(1,2048)];
soundsc(x_4, 8192)
X = FT(x_4);
plot(omega, abs(X));
xlabel('\omega');
ylabel('Magnitude of X_4');
title('Magnitude of X_4 vs \omega');
grid ON;


%% PART 1.2 DIGIT 5

x_5 = x.*[zeros(1,8192),ones(1,2048)];
soundsc(x_5, 8192)
X = FT(x_5);
plot(omega, abs(X));
xlabel('\omega');
ylabel('Magnitude of X_5');
title('Magnitude of X_5 vs \omega');
grid ON;


%% PART 2 RECORD

audio = audiorecorder(8192,8,1,2);
recordblocking(audio,12);
x = transpose(getaudiodata(audio));


%% PART 2 PLAY

play(audio)
```

```matlab
%% FUNCTIONS

function [x] = DTMFTRA (number)
    frequencies = [941, 1336; 697,1209; 697,1336; 697,1477;   % 0 1 2 3
                              770,1209; 770,1336; 770,1477;   %   4 5 6
                              852,1209; 852,1336; 852,1477];  %   7 8 9
    x = [];
    for i = 1:length(number)
        t = (i-1)*0.25:1/8192:i*0.25;
        freq  = frequencies(number(i) + 1, :);
        signal = cos(2 * pi * freq(1) * t) + cos(2 * pi * freq(2) * t);
        x = cat(2, x, signal(2:length(signal)));
    end
end

function output=FT(input)
    M=size(input,2);
    t=exp(1j*pi*(M-1)/M*(0:1:M-1));
    output=exp(-1j*pi*(M-1)^2/(2*M))*t.*1/(M)^0.5.*fft(input.*t);
end

function output=IFT(input)
    M=size(input,2);
    t=exp(-1j*pi*(M-1)/M*(0:1:M-1));
    output=real(exp(1j*pi*(M-1)^2/(2*M))*t.*(M)^0.5.*ifft(input.*t));
end
```