

Chapter 6 Exercises

<Your Name Goes Here>

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Exercise 1 on page 216

Solve the following system by Gaussian Elimination:

$$\begin{array}{rcrcrcrcrcl} x_1 & + & x_2 & + & x_3 & & = & 2 \\ 2x_1 & + & x_2 & + & x_3 & & = & 3 \\ x_1 & - & x_2 & + & 3x_3 & & = & 8 \end{array}$$

Up to 10 points are possible for a correct answer with a good analysis.

Solution

$$\begin{array}{cccccc} 1 & 1 & 1 & = & 2 \\ 2 & 1 & 1 & = & 3 \\ 1 & -1 & 3 & = & 8 \end{array}$$

$$\begin{array}{cccccc} 1 & 1 & 1 & = & 2 \\ 1 & 0 & 0 & = & 1 \\ 2 & 0 & 4 & = & 10 \end{array}$$

$$\begin{array}{cccccc} 1 & 0 & 0 & = & 1 \\ 1 & 1 & 1 & = & 2 \\ 0 & 0 & 4 & = & 8 \end{array}$$

$$\begin{array}{cccccc} 1 & 0 & 0 & = & 1 \\ 0 & 1 & 1 & = & 1 \\ 0 & 0 & 1 & = & 2 \end{array}$$

$$\begin{array}{cccccc} 1 & 0 & 0 & = & 1 \\ 0 & 1 & 0 & = & -1 \\ 0 & 0 & 1 & = & 2 \end{array}$$

$$x_1 = 1 \quad x_2 = -1 \quad x_3 = 2$$

Exercise 2 on page 216

a. Solve the system of the previous question by the LU decomposition method.

b. From the standpoint of general algorithm design techniques, how would you classify the LU decomposition method?

*Up to 10 points are possible for a correct answer with all the steps shown for part *a*.*

*Up to 2 points are possible for a correct answer, well explained, for part *b*.*

Solution

a

b

Transform and Conquer - We are not reducing

Exercise 3 on page 216

Solve the system of Problem 1 by computing the inverse of its coefficient matrix and then multiplying it by the right-hand side vector.

Up to 10 points are possible for a correct answer with all the steps shown.

$$\begin{array}{ccccccc} 1 & 1 & 1 & : & 1 & 0 & 0 \\ 2 & 1 & 1 & : & 0 & 1 & 0 \\ 1 & -1 & 3 & : & 0 & 0 & 1 \end{array}$$

$$\begin{array}{ccccccc} -1 & 0 & 0 & : & 1 & -1 & 0 \\ 2 & 1 & 1 & : & 0 & 1 & 0 \\ 1 & -1 & 3 & : & 0 & 0 & 1 \end{array}$$

$$\begin{array}{ccccccc} -1 & 0 & 0 & : & 1 & -1 & 0 \\ 0 & 1 & 1 & : & 2 & -1 & 0 \\ 1 & -1 & 3 & : & 0 & 0 & 1 \end{array}$$

$$\begin{array}{ccccccc} -1 & 0 & 0 & : & 1 & -1 & 0 \\ 0 & 1 & 1 & : & 2 & -1 & 0 \\ 0 & -1 & 3 & : & 1 & -1 & 1 \end{array}$$

$$\begin{array}{ccccccc} 1 & 0 & 0 & : & -1 & 1 & 0 \\ 0 & 1 & 1 & : & 2 & -1 & 0 \\ 0 & -1 & 3 & : & 1 & -1 & 1 \end{array}$$

$$\begin{array}{ccccccc} 1 & 0 & 0 & : & -1 & 1 & 0 \\ 0 & 1 & 1 & : & 2 & -1 & 0 \\ 0 & 0 & 4 & : & 3 & -2 & 1 \end{array}$$

$$\begin{array}{ccccccc} 1 & 0 & 0 & : & -1 & 1 & 0 \\ 0 & 1 & 1 & : & 2 & -1 & 0 \\ 0 & 0 & 1 & : & \frac{3}{4} & -\frac{2}{4} & \frac{1}{4} \end{array}$$

$$\begin{array}{ccccccc} 1 & 0 & 0 & : & -1 & 1 & 0 \\ 0 & 1 & 0 & : & \frac{5}{4} & -\frac{2}{4} & -\frac{1}{4} \\ 0 & 0 & 1 & : & \frac{3}{4} & -\frac{2}{4} & \frac{1}{4} \end{array}$$

Bonus Exercises

Up to 30 bonus points are possible for good answers, well explained, for Exercises 4, 5 and 6 on page 206.