C4

William Bevington Callum O'Brien Alex Pace

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1 Parametric Curves

Parametric curves occur when x and y are defined in terms of a parameter, a parameter being a value that is the same in both the functions for x and for y. In a parametric curve C with parameter t;

$$C : x = f(t), y = g(t)$$

One differentiates a parametric eqution using the chain rule. If we were to differentiate y with respect to x by the chain rule, we would find

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t}\frac{\mathrm{d}t}{\mathrm{d}x} = \frac{f'(t)}{g'(t)}$$

Integration of parametric curves is done by integrating with respect to t. If one has to integrate with respect to something else, one can tranform it hence;

$$\int_{f(t)=a}^{b} g(t)\mathrm{d}f(t) = \int_{t=f^{-1}(a)}^{b} g(t)f'(t)\mathrm{d}t$$

This is then isomorphic to integration of non-parametric curves.

2 Implicit Differentiation

$$\frac{\mathrm{d}}{\mathrm{d}x}xy = \frac{\mathrm{d}y}{\mathrm{d}x}x + y$$

$$\frac{\mathrm{d}}{\mathrm{d}x}y^n = \frac{\mathrm{d}y}{\mathrm{d}x}ny^{n-1}$$

3 Forming Differential Equations

Differential equations describe scenarios in which;

$$\frac{\partial y}{\partial x} \propto y$$

Newton's law of cooling states that the rate of loss of temperature $-\frac{\partial\theta}{\partial t}$ is proportional to the the difference between the temperature θ of the body and the temperature θ_0 of its surroundings;

$$\frac{\partial\theta}{\partial t} = -k(\theta - \theta_0)$$