

FP3

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1 Hyperbolic Functions

The hyperbolic functions are analogs of the ordinary trigonometric, or circular functions. The basic hyperbolic functions are, as one might expect, analogous to sine and cosine; they are hyperbolic sine and hyperbolic cosine.

$$\sinh : \mathbb{R} \rightarrow \mathbb{R} : x \mapsto \frac{e^x - e^{-x}}{2}$$

$$\cosh : \mathbb{R} \rightarrow \{x \mid x \in \mathbb{R}, x \geq 1\} : x \mapsto \frac{e^x + e^{-x}}{2}$$

From these one can derive the hyperbolic tangent, hyperbolic secant, hyperbolic cosecant and hyperbolic cotangent functions in much the same way as their circular counterparts.

$$\tanh : \mathbb{R} \rightarrow \{x \mid x \in \mathbb{R}, x \in [-1, 1]\} : x \mapsto \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\operatorname{sech} : \mathbb{R} \rightarrow \{y \mid y \in \mathbb{R}, y \in [0, 1]\} : x \mapsto \frac{2}{e^x + e^{-x}}$$

$$\operatorname{csch} : \{x \mid x \in \mathbb{R}, x \neq 0\} \rightarrow \{y \mid y \in \mathbb{R}, y \neq 0\} : x \mapsto \frac{2}{e^x - e^{-x}}$$

$$\coth : \{x \mid x \in \mathbb{R}, x \neq 0\} \rightarrow \{y \mid y \in \mathbb{R}, y \notin [-1, 1]\} : x \mapsto \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

The inverse functions of hyperbolic sine, hyperbolic cosine and hyperbolic tangent are area hyperbolic sine, area hyperbolic cosine and area hyperbolic tangent;

$$\operatorname{arsinh} : \mathbb{R} \rightarrow \mathbb{R} : x \mapsto \ln \left(x + \sqrt{x^2 + 1} \right)$$

$$\operatorname{arcosh} : \{x \mid x \in \mathbb{R}, x \geq 1\} \rightarrow \mathbb{R} : x \mapsto \ln \left(x + \sqrt{x^2 - 1} \right)$$

$$\operatorname{artanh} : \{x \mid x \in \mathbb{R}, x \in [-1, 1]\} \rightarrow \mathbb{R} : x \mapsto \frac{1}{2} \ln \left(\frac{x+1}{1-x} \right)$$