C4

Callum O'Brien

September 22, 2015

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1 Parametric Equations

A parametric equation is one in the form;

$$x = f(t), y = g(t)$$

To convert them to cartesian form ($y = h(x)$), one can rearrange to the form;

$$t = f^{-1}(x), y = g(t) \Rightarrow y = g(f^{-1}(x))$$

If that doesn't work, try squaring both functions and approach the problem that way.

1.1 Integration

$$x = f(t), y = g(t)$$

$$\int_{x=a}^{b} y \, \mathrm{d}x = \int_{t=f^{-1}(a)}^{f^{-1}(b)} y \, \frac{\mathrm{d}x}{\mathrm{d}t} \mathrm{d}t$$

1.1.1 Example

Given $x = t^2$ and $y = 2t(3 - t)$, evalulate:

$$\int_{x=0}^{9} y \, \mathrm{d}x$$

$$\int_{x=0}^{9} y \, \mathrm{d}x = \int_{t=0}^{3} y \, \frac{\mathrm{d}x}{\mathrm{d}t} \mathrm{d}t \tag{1}$$

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t} t^2 = 2t \tag{2}$$

Hence

$$\int_{x=0}^{9} y \, \mathrm{d}x = \int_{t=0}^{3} 4t^2(3 - t)\mathrm{d}t = t^3(4 - t)|_{t=0}^{3} = 108 - 81 = 27 \tag{3}$$

TL;DR Multiply by $\frac{\mathrm{d}x}{\mathrm{d}t}$ then integrate with respect to t

2 Implicit Differentiation

$$\frac{\mathrm{d}}{\mathrm{d}x} xy = \frac{\mathrm{d}y}{\mathrm{d}x} x + y$$

$$\frac{\mathrm{d}}{\mathrm{d}x} y^n = \frac{\mathrm{d}y}{\mathrm{d}x} n y^{n-1}$$

3 Forming Differential Equations

Differential equations describe scenarios in which;

$$\frac{\partial y}{\partial x} \propto y$$

Newton's law of cooling states that the rate of loss of temperature $-\frac{\partial \theta}{\partial t}$ is proportional to the the difference between the temperature θ of the body and the temperature θ_0 of its surroundings;

$$\frac{\partial \theta}{\partial t} = -k(\theta - \theta_0)$$