FP3

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1 Hyperbolic Functions

The hyperbolic functions are analogs of the ordinary trigonometric, or circular functions. The basic hyperbolic functions are, as one might expect, analygous to sine and cosine; they are hyperbolic sine and hyperbolic cosine.

$$\sinh : \mathbb{R} \to \mathbb{R} : x \mapsto \frac{e^x - e^{-x}}{2}$$

$$\cosh : \mathbb{R} \to \{x \mid x \in \mathbb{R},\, x \geq 1\} : x \mapsto \frac{e^x + e^{-x}}{2}$$

From these one can derive the hyperbolic tangent, hyperbolic secant, hyperbolic cosecant and hyperbolic cotangent functions in much the same way as their circular counterparts.

$$\tanh : \mathbb{R} \to \{x \mid x \in \mathbb{R},\, x \in [-1, 1]\} : x \mapsto \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\operatorname{sech} : \mathbb{R} \to \{y \mid y \in \mathbb{R},\, y \in [0, 1)\} : x \mapsto \frac{2}{e^x + e^{-x}}$$

$$\operatorname{csch} : \{x \mid x \in \mathbb{R},\, x \neq 0\} \to \{y \mid y \in \mathbb{R},\, y \neq 0\} : x \mapsto \frac{2}{e^x - e^{-x}}$$

$$\coth : \{x \mid x \in \mathbb{R},\, x \neq 0\} \to \{y \mid y \in \mathbb{R},\, y \notin [-1, 1]\} : x \mapsto \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

The inverse functions of hyperbolic sine, hyperbolic cosine and hyperbolic tangent are area hyperbolic sine, area hyperbolic cosine and area hyperbolic tangent;

$$\operatorname{arsinh} : \mathbb{R} \to \mathbb{R} : x \mapsto \ln\left(x + \sqrt{x^2 + 1}\right)$$

$$\operatorname{arcosh} : \{x \mid x \in \mathbb{R},\, x \geq 1\} \to \mathbb{R} : x \mapsto \ln\left(x + \sqrt{x^2 - 1}\right)$$

$$\operatorname{artanh} : \{x \mid x \in \mathbb{R},\, x \in [-1, 1]\} \to \mathbb{R} : x \mapsto \frac{1}{2} \ln\left(\frac{x + 1}{1 - x}\right)$$