C3

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1 Partial Fractions

1.1 Splitting a Fraction with Two or More Linear Factors in the Denominator

$$\frac{x+3}{(x+2)(x+1)} = \frac{a}{x+2} + \frac{b}{x+1} = \frac{a(x+1)+b(x+2)}{(x+2)(x+1)}$$

$$\therefore x+3 = a(x+1)+b(x+2)$$

Here, two methods can be used; equating coefficients and substitution. Equating coefficients is rather self explanatory, and involves creating simultaneous equations from the fact that coefficients of different powers of x will be equal on both the left- and right-hand-side of the above equation, then solving for A and B;

$$\textit{coefficients of } x\text{:}\ 1 = a+b$$

$$\textit{constants:}\ 3 = a+2b$$

The former rearranges to

$$b = 1-a$$

Substituting this into the latter shows

$$3 = a+2(1-a) = 2-a \therefore a = -1 \Rightarrow b = 2$$

Hence,

$$\frac{x+3}{(x+2)(x+1)} = \frac{2}{x+1} - \frac{1}{x+2}$$

Substitution involves substituting values for x which neglect one of the unknowns in our equation. In the above example, one would substitute the values -1 and -2 to neglect the terms containting a and b repectively;

$$x \to -1,\ b = 2$$

$$x \to -2,\ -a = 1 \therefore a = -1$$

And once again, we arrive at the same partial fractions,

$$\frac{x+3}{(x+2)(x+1)} = \frac{2}{x+1} - \frac{1}{x+2}$$

1.2 Splitting a Fraction with a Squared Linear Factor in the Denominator

$$\frac{}{()^2} = \frac{a}{} + \frac{b}{()^2} = \frac{a() + b}{()^2}$$
$$= a() + b$$