

# C4

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## 1 Parametric Curves

Parametric curves occur when  $x$  and  $y$  are defined in terms of a parameter, a parameter being a value that is the same in both the functions for  $x$  and for  $y$ . In a parametric curve  $C$  with parameter  $t$ ;

$$C : x = f(t), y = g(t)$$

One differentiates a parametric equation using the chain rule. If we were to differentiate  $y$  with respect to  $x$  by the chain rule, we would find

$$\frac{dy}{dx} = \frac{dy}{dt} \frac{dt}{dx} = \frac{f'(t)}{g'(t)}$$

Integration of parametric curves is done by integrating with respect to  $t$ . If one has to integrate with respect to something else, one can transform it hence;

$$\int_{f(t)=a}^b g(t) df(t) = \int_{t=f^{-1}(a)}^b g(t) f'(t) dt$$

This is then isomorphic to integration of non-parametric curves.

## 2 Implicit Differentiation

$$\begin{aligned} \frac{d}{dx} xy &= \frac{dy}{dx} x + y \\ \frac{d}{dx} y^n &= \frac{dy}{dx} n y^{n-1} \end{aligned}$$

## 3 Forming Differential Equations

Differential equations describe scenarios in which;

$$\frac{\partial y}{\partial x} \propto y$$

Newton's law of cooling states that the rate of loss of temperature  $-\frac{\partial \theta}{\partial t}$  is proportional to the difference between the temperature  $\theta$  of the body and the temperature  $\theta_0$  of its surroundings;

$$\frac{\partial \theta}{\partial t} = -k(\theta - \theta_0)$$