Paper Reference

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Surname

Initial(s)

Signature

Paper Reference(s)

6668/01

Edexcel GCE

Further Pure Mathematics FP2
Advanced/Advanced Subsidiary

Friday 21 June 2013 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 9 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

Turn over

1.
$$\frac{d^2 y}{dx^2} + x \frac{dy}{dx} = 2\cos x$$

(a) Find $\dfrac{d^3 y}{dx^3}$ in terms of x, $\dfrac{dy}{dx}$ and $\dfrac{d^2 y}{dx^2}$.

(3)

At $x = 0$, $y = 1$ and $\dfrac{dy}{dx} = 3$

(b) Find the value of $\dfrac{d^3 y}{dx^3}$ at $x = 0$

(1)

(c) Express y as a series in ascending powers of x, up to and including the term in x^3.

(3)

Question 1 continued

(Total 7 marks)

Q1

P 4 1 8 0 4 A 0 3 2 8

2. (a) Sketch, on the same axes,

(i) $y = |2x - 3|$

(ii) $y = 4 - x^2$

(3)

(b) Find the set of values of x for which

$$4 - x^2 > |2x - 3|$$

(6)

Question 2 continued

P 4 1 8 0 4 A 0 5 2 8

3.
$$f(x) = \ln(1 + \sin kx)$$

where k is a constant, $x \in \mathbb{R}$ and $-\dfrac{\pi}{2} < kx < \dfrac{3\pi}{2}$

(a) Find $f'(x)$

(2)

(b) Show that $f''(x) = \dfrac{-k^2}{1 + \sin kx}$

(3)

(c) Find the Maclaurin series of $f(x)$, in ascending powers of x, up to and including the term in x^3.

(4)

Question 3 continued

Turn over

Question 3 continued

Question 3 continued

Q3

(Total 9 marks)

P 4 1 8 0 4 A 0 9 2 8

4. Find the general solution of the differential equation

$$x\frac{dy}{dx} + (1 + x\cot x)y = \sin x, \qquad 0 < x < \pi$$

giving your answer in the form $y = f(x)$.

(9)

Question 4 continued

P 4 1 8 0 4 A 0 1 1 2 8

Question 4 continued

Question 4 continued

Q4

(Total 9 marks)

P 4 1 8 0 4 A 0 1 3 2 8

5. (a) Express $\dfrac{2}{r(r+1)(r+2)}$ in partial fractions.

(3)

(b) Using your answer to part (a) and the method of differences, show that

$$\sum_{r=1}^{n} \frac{2}{r(r+1)(r+2)} = \frac{n(n+3)}{2(n+1)(n+2)}$$

(4)

P 4 1 8 0 4 A 0 1 4 2 8

Question 5 continued

Question 5 continued

P 4 1 8 0 4 A 0 1 6 2 8

Question 5 continued

P 4 1 8 0 4 A 0 1 7 2 8

6. Solve the equation

$$z^5 = -16\sqrt{3} + 16i$$

giving your answers in the form $r(\cos\theta + i\sin\theta)$, where $r > 0$ and $-\pi < \theta < \pi$.

(8)

Question 6 continued

(Total 8 marks)

Q6

7. (a) Find the value of the constant λ for which $y = \lambda x e^{2x}$ is a particular integral of the differential equation

$$\frac{d^2 y}{dx^2} - 4y = 6e^{2x}$$

(4)

(b) Hence, or otherwise, find the general solution of the differential equation

$$\frac{d^2 y}{dx^2} - 4y = 6e^{2x}$$

(3)

20

Question 7 continued

8. A complex number z is represented by the point P on an Argand diagram.

(a) Given that $|z| = 1$, sketch the locus of P.

(1)

The transformation T from the z-plane to the w-plane is given by

$$w = \frac{z + 7i}{z - 2i}$$

(b) Show that T maps $|z| = 1$ onto a circle in the w-plane.

(5)

(c) Show that this circle has its centre at $w = -5$ and find its radius.

(2)

Question 8 continued

P 4 1 8 0 4 A 0 2 3 2 8

Question 8 continued
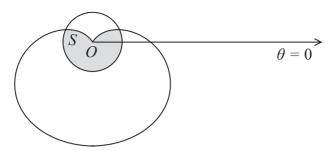
Question 8 continued

9.

Figure 1

Figure 1 shows a sketch of the curves given by the polar equations

$$r = 1 \quad \text{and} \quad r = 2 - 2\sin\theta$$

(a) Find the coordinates of the points where the curves intersect.

(3)

The region S, between the curves, for which $r < 1$ and for which $r < 2 - 2\sin\theta$, is shown shaded in Figure 1.

(b) Find, by integration, the area of the shaded region S, giving your answer in the form $a\pi + b\sqrt{3}$, where a and b are rational numbers.

(8)

Question 9 continued

Turn over

Question 9 continued

Q9

(Total 11 marks)

TOTAL FOR PAPER: 75 MARKS

END