Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	9	/	0	1	Signature	

Paper Reference(s)

6669/01

Edexcel GCE

Further Pure Mathematics FP3

Advanced/Advanced Subsidiary

Monday 24 June 2013 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 32 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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PEARSON

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1. A hyperbola H has equation

$$\frac{x^2}{4k^2} - \frac{y^2}{k^2} = 1$$

where k is a positive constant.

(a) Find the eccentricity of H.

(2)

Given that the distance between the foci of H is $6\sqrt{5}$,

(b) find the value of k.

(2)

Leave blank

Question 1 continued

Q1

(Total 4 marks)

Leave blank

2. The curve C has parametric equations

$$x = 2t + \ln \sec 2t, \quad y = 2t - \ln \sec 2t, \quad 0 \leqslant t \leqslant \frac{\pi}{6}$$

Show that the length of C is $\sqrt{2} \ln (2 + \sqrt{3})$.

(7)

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Question 2 continued

Leave blank

Question 2 continued

Leave blank

Question 2 continued

Q2

(Total 7 marks)

Leave blank

3. The line l_1 has vector equation $\mathbf{r} = \begin{pmatrix} 0 \\ -3 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$, where λ is a parameter.

The line l_2 has vector equation $\left(\mathbf{r} - \begin{pmatrix} 4 \\ -7 \\ 7 \end{pmatrix} \right) \times \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} = \mathbf{0}$.

(a) Verify that the point with coordinates (2, –1, 3) lies on both l_1 and l_2.

(3)

(b) Find a vector perpendicular to both l_1 and l_2.

(3)

(c) Find a cartesian equation of the plane containing both l_1 and l_2.

(3)

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Question 3 continued

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Q3

(Total 9 marks)

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4. (a) Starting from the definition of sinh x in terms of exponentials, show that

$$\text{arsinh } x = \ln(x + \sqrt{(x^2 + 1)})$$

(5)

(b) Find the exact values of x for which

$$4x - 2 = \sinh \ln 4x$$

(5)

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Q4

(Total 10 marks)

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5. The ellipse E has equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where a and b are constants.

(a) Show that an equation of the normal to E at the point $P(a\cos\theta, b\sin\theta)$ is

$$ax\sin\theta - by\cos\theta = (a^2 - b^2)\cos\theta\sin\theta$$

(5)

The normal to E at P cuts the x-axis at R and the y-axis at S. The point M is the mid-point of RS.

(b) Find a cartesian equation of the locus of M as θ varies.

(4)

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Q5

(Total 9 marks)

Leave blank

6. (a) Differentiate, with respect to x,

$$\arctan\left(\frac{3}{x}\right),\ x > 0$$

simplifying your answer.

(3)

(b) Use calculus to find the exact value of

$$\int_{\sqrt{3}}^{3} x \arctan\left(\frac{3}{x}\right) \mathrm{d}x$$

giving your answer in the form $a + b\sqrt{3} + c\pi$, where a, b and c are rational numbers.

(8)

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Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 11 marks)

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7. $$\mathbf{A} = \begin{pmatrix} 2 & 4 & -6 \\ 0 & 2 & 0 \\ 1 & 0 & -5 \end{pmatrix}$$

(a) Show that 2 is an eigenvalue of **A** and find the other two eigenvalues of **A**.

(4)

(b) Find an eigenvector of **A** corresponding to the eigenvalue 2.

(3)

A transformation T: $\mathbb{R}^3 \rightarrow \mathbb{R}^3$ is represented by the matrix **A**. The transformation T maps the plane Π_1 with equation $\mathbf{r}.\begin{pmatrix} 0 \\ 4 \\ -5 \end{pmatrix} = 20$ onto the plane Π_2.

(c) Find a vector equation of Π_2, giving your answer in the form $\mathbf{r}.\mathbf{n} = p$.

(6)

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Question 7 continued

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Question 7 continued

Leave blank

Question 7 continued

Q7

(Total 13 marks)

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8. (a) Differentiate, with respect to x,

$$x^{n-1}\sqrt{(x^2+1)}, \qquad n \neq 1$$

(2)

(b) Given that

$$I_n = \int \frac{x^n}{\sqrt{(x^2+1)}}\,\mathrm{d}x,$$

using your answer to (a) or otherwise, show that

$$nI_n + (n-1)I_{n-2} = x^{n-1}\sqrt{(x^2+1)}, \qquad n \geqslant 2$$

(5)

(c) Hence evaluate $\int_0^1 \frac{x^2}{\sqrt{(x^2+1)}}\,\mathrm{d}x$, giving your answer in terms of a natural logarithm.

(5)

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Question 8 continued

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Question 8 continued

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Question 8 continued

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Question 8 continued

Q8

(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

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