

1. $f(x) = \frac{2x+1}{(1+x)(1-x)^2}, \quad x \in \mathbb{R}, \quad |x| < 1$

Given that $f(x) = \frac{A}{1+x} + \frac{B}{1-x} + \frac{C}{(1-x)^2}$ where A , B and C are constants,

- (a) find the values of A , B and C . (4)
- (b) Hence, or otherwise, find the series expansion of $f(x)$ in ascending powers of x up to and including the term in x^2 . Simplify fully each term. (6)

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Question 1 continued



Question 1 continued

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Question 1 continued

Q1

(Total 10 marks)



$$x = \cos t, \quad y = \sin^2 2t, \quad 0 \leq t \leq \pi$$

- (a) State

- (i) the greatest and least value of x ,

- (ii) the greatest and least value of y .

(2)

- (b) Find the cartesian equation of the curve C .

(3)

- (c) Find the exact coordinates of the points where C meets the x -axis and the y -axis.

(3)



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Question 2 continued



Question 2 continued



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Question 2 continued

Q2

(Total 8 marks)



3. (a) Given that $y = 3^x$, find $\frac{dy}{dx}$

(1)

(b) Find an equation of the tangent to the curve

$$y = 3^x - 3^{-x} + 2$$

at the point $(0, 2)$.

(5)

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Question 3 continued

Q3

(Total 6 marks)



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Question 4 continued



Question 4 continued

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Question 4 continued

Q4

(Total 9 marks)



5. Integrate the following with respect to x .

(a) $x^2 \ln x$

(4)

(b) $\sec 2x \tan 2x + \sec^2 x$

(3)

Using the substitution $u = 2 + \cos\theta$, or otherwise,

(c) find the exact value of

$$\int_0^{\frac{\pi}{2}} \frac{\sin 2\theta}{2 + \cos \theta} d\theta$$

giving your answer in the form $a \ln b + c$, where a , b and c are constants to be found.

(8)



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Question 5 continued

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Question 5 continued

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Question 5 continued

Q5

(Total 15 marks)



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Question 6 continued



Question 6 continued



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Question 6 continued

(Total 14 marks)

Q6

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7.

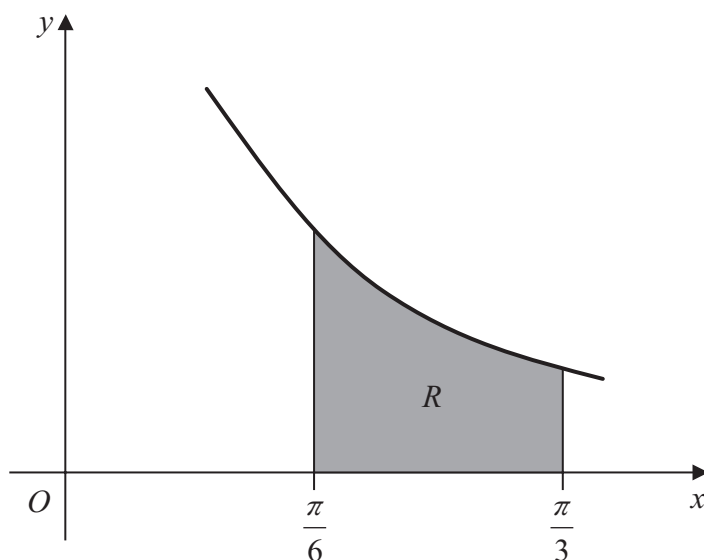


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \operatorname{cosec} x$.

The finite region R is bounded by the curve, the x -axis and the lines $x = \frac{\pi}{6}$ and $x = \frac{\pi}{3}$ as shown shaded in Figure 1.

- (a) Use calculus to find the value of the area of R to 3 decimal places. (3)
- (b) Use the trapezium rule, with 2 strips of equal width, to estimate the area of R .
Give your answer to 3 decimal places. (5)
- (c) Find the value of the error of your estimate in part (b). (1)
- (d) Find the exact value of the volume of the solid formed when the region R is rotated through 2π radians about the x -axis. Give your answer in the form $a\pi\sqrt{3}$ where a is a constant. (4)



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Question 7 continued



Question 7 continued



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Question 7 continued



