Mark Scheme (Results)

June 2013

GCE Further Pure Mathematics FP3 (6669/01)
Original Paper

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Summer 2013
Publications Code N/A
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

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June 2013
6669 Further Pure Mathematics 3
Mark Scheme

Question Number	Scheme	Marks
1.	(a) $b^2 = a^2(e^2-1) \Rightarrow k^2 = 4k^2(e^2-1)$	M1
	Leading to $e = \frac{\sqrt{5}}{2}$ awrt 1.1	A1 (2)
	(b) $2ae = 6\sqrt{5} \Rightarrow 4k \times \frac{\sqrt{5}}{2} = 6\sqrt{5}$	M1
	$k = 3$	A1 (2) **[4]**
2.	$\frac{dx}{dt} = 2 + 2\tan 2t$, $\frac{dy}{dt} = 2 - 2\tan 2t$	B1
	$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = (2+2\tan 2t)^2 + (2-2\tan 2t)^2$	M1
	$= 8 + 8\tan^2 2t = 8\sec^2 2t$	M1
	$\int\left(\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right)^{\frac{1}{2}} dt = 2\sqrt{2}\int \sec 2t\, dt$	
	$= \sqrt{2}\ln(\sec 2t + \tan 2t) \quad (+C)$	M1 A1
	$s = \left[\sqrt{2}\ln(\sec 2t + \tan 2t)\right]_0^{\frac{\pi}{6}} = \sqrt{2}\ln(2+\sqrt{3})$ ✱ cso	M1 A1 (7) **[7]**

Question Number	Scheme	Marks
3.	(a) $\lambda = 1 \Rightarrow \mathbf{r} = \begin{pmatrix} 0 \\ -3 \\ 4 \end{pmatrix} + \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} \Rightarrow (2, -1, 3) \in l_1$	B1
	$\left[\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ -7 \\ 7 \end{pmatrix}\right] \times \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 6 \\ -4 \end{pmatrix} \times \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} = \begin{pmatrix} 12-12 \\ 4-4 \\ 6-6 \end{pmatrix}$	M1
	$= \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \mathbf{0} \Rightarrow (2, -1, 3) \in l_2$	A1 (3)
	(b) $\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} \times \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ -5 \\ -8 \end{pmatrix}$ or any multiple	M1 A2(1, 0)
		(3)
	(c) $\mathbf{r}.\mathbf{n} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} . \begin{pmatrix} 1 \\ -5 \\ -8 \end{pmatrix} = 2 + 5 - 24 = -17$	M1
	Leading to $x - 5y - 8z = -17$ or any multiple	M1 A1 (3)
		[9]
	Alternative to (c)	
	Three points on the plane are $(2, -1, 3), (0, -3, 4), (4, -7, 7)$	
	$x + by + cz = d$	
	$-2 - b + 3c = d$	
	$-3b + 4c = d$	
	$4 - 7b + 7c = d$	M1
	Solving to $b = -5, c = -8, d = -17$	M1 A1 (3)

Question Number	Scheme		Marks	
4.	(a) $y = \text{arsinh}\, x \Rightarrow \sinh y = x$			
	$\dfrac{e^y - e^{-y}}{2} = x$		M1	
	$e^{2y} - 2x e^y - 1 = 0$		M1	
	$e^y = \dfrac{2x + \sqrt{(4x^2 + 4)}}{2}$	condone $\pm$	M1 A1	
	$= x + \sqrt{(x^2 + 1)}$			
	$y = \ln\left(x + \sqrt{(1 + x^2)}\right)$ ✱	cso	A1	(5)
	(b) $\text{arsinh}(4x - 2) = \ln 4x$			
	$\ln\left(4x - 2 + \sqrt{\left[(4x-2)^2 + 1\right]}\right) = \ln 4x$		M1	
	$4x - 2 + \sqrt{\left[(4x-2)^2 + 1\right]} = 4x$		M1	
	$(4x-2)^2 + 1 = 4$			
	$(4x-2)^2 = 3$ *or* $16x^2 - 16x + 1 = 0$		M1 A1	
	$x = \dfrac{2 \pm \sqrt{3}}{4}$	accept exact equivalents	A1	(5)
				[10]
	Alternative to (b)			
	$4x - 2 = \dfrac{e^{\ln 4x} - e^{-\ln 4x}}{2}$		M1	
	$= 2x - \dfrac{1}{8x}$		M1	
	Leading to $16x^2 - 16x + 1 = 0$		M1 A1	
	$x = \dfrac{2 \pm \sqrt{3}}{4}$		A1	(5)

Question Number	Scheme		Marks	
5.	(a) $\dfrac{\mathrm{d}x}{\mathrm{d}\theta} = -a\sin\theta,\quad \dfrac{\mathrm{d}y}{\mathrm{d}\theta} = b\cos\theta$		B1	
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{b\cos\theta}{a\sin\theta} \Rightarrow m' = \dfrac{a\sin\theta}{b\cos\theta}$		M1 A1	
	$y - b\sin\theta = \dfrac{a\sin\theta}{b\cos\theta}\left(x - a\cos\theta\right)$		M1	
	Leading to $ax\sin\theta - by\cos\theta = \left(a^2 - b^2\right)\cos\theta\sin\theta$ ✱	cso	A1	(5)
	(b) x-coordinate of R: $\dfrac{a^2 - b^2}{a}\cos\theta$			
	y-coordinate of S: $-\dfrac{a^2 - b^2}{b}\sin\theta$	both	B1	
	$M : \left(\dfrac{a^2 - b^2}{2a}\cos\theta, -\dfrac{a^2 - b^2}{2b}\sin\theta\right)$		M1	
	$\dfrac{x^2}{\left(\dfrac{a^2 - b^2}{2a}\right)^2} + \dfrac{y^2}{\left(\dfrac{a^2 - b^2}{2b}\right)^2} = 1$	or equivalent	M1 A1	(4)
				[9]

Question Number	Scheme	Marks
6.	(a) $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\arctan\left(\dfrac{3}{x}\right)\right)=\dfrac{1}{1+\left(\dfrac{3}{x}\right)^2}\times-\dfrac{3}{x^2}$	M1 A1
	$=-\dfrac{3}{x^2+9}$	A1 (3)
	Note: $\arctan\left(\dfrac{3}{x}\right)$ can be written as $\operatorname{arccot}\left(\dfrac{x}{3}\right)$ or as $\dfrac{\pi}{2}-\arctan\left(\dfrac{x}{3}\right)$. The mark scheme above essentially still applies.	
	(b) $\displaystyle\int x\arctan\left(\frac{3}{x}\right)\mathrm{d}x=\frac{x^2}{2}\arctan\left(\frac{3}{x}\right)+\frac{3}{2}\int\frac{x^2}{x^2+9}\mathrm{d}x$	M1 A1
	$\displaystyle= \quad ... \quad +\frac{3}{2}\int\frac{x^2+9-9}{x^2+9}\mathrm{d}x$	
	$\displaystyle= \quad ... \quad +\frac{3}{2}\int\left(1-\frac{9}{x^2+9}\right)\mathrm{d}x$	M1
	$\displaystyle= \quad ... \quad +\frac{3}{2}x-\frac{9}{2}\arctan\left(\frac{x}{3}\right) \quad (+C)$	M1 A1
	Note: $\dots \; +\dfrac{3}{2}x+\dfrac{9}{2}\arctan\left(\dfrac{3}{x}\right) \quad (+C)$ is also a correct form.	
	$\displaystyle\Big[\ ...\ \Big]_{\sqrt{3}}^{3}=\frac{9}{2}\arctan 1+\frac{9}{2}-\frac{9}{2}\arctan 1-\left(\frac{3}{2}\arctan\sqrt{3}+\frac{3\sqrt{3}}{2}-\frac{9}{2}\arctan\frac{1}{\sqrt{3}}\right)$	M1
	$\displaystyle=\frac{9}{2}-\frac{3\sqrt{3}}{2}+\frac{\pi}{4}$	M1 A1 (8)
		[11]

Question Number	Scheme	Marks
7.	(a) $\begin{vmatrix} 2-\lambda & 4 & -6 \\ 0 & 2-\lambda & 0 \\ 1 & 0 & -5-\lambda \end{vmatrix} = 0$	
	$(2-\lambda)(2-\lambda)(-5-\lambda)+6(2-\lambda)=0$	M1
	Leading to $(2-\lambda)(\lambda-1)(\lambda+4)=0$	M1
	$\lambda = 2; 1, -4$	B1; A1 (4)
	The B for $\lambda = 2$ can be awarded at any stage and for direct verification	
	(b) $\begin{pmatrix} 2 & 4 & -6 \\ 0 & 2 & 0 \\ 1 & 0 & -5 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 2\begin{pmatrix} x \\ y \\ z \end{pmatrix}$	
	$2x+4y-6z=2x \qquad (2y=3z)$	
	$(2y=2y)$	
	$x-5z=2z \qquad (x=7z)$	M1
	Let $z=2$, then $x=14$, $y=3$	M1
	An eigenvector is $\begin{pmatrix} 14 \\ 3 \\ 2 \end{pmatrix}$ or any multiple	A1 (3)
	(c) A cartesian equation of Π_1 is $4y-5z=20$ and a parametric form is	
	$\left(s, t, \tfrac{4}{5}t-4\right)$	M1
	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 & 4 & -6 \\ 0 & 2 & 0 \\ 1 & 0 & -5 \end{pmatrix}\begin{pmatrix} s \\ t \\ \tfrac{4}{5}t-4 \end{pmatrix} = \begin{pmatrix} 2s-\tfrac{4}{5}t+24 \\ 2t \\ s-4t+20 \end{pmatrix}$	M1 A1
	$x = 2s-\tfrac{4}{5}t+24,\ y=2t,\ z=s-4t+20$	M1
	Eliminating s and t	
	$5x-10z=36t-80$	
	$5x-10z=18y-80$	
	$5x-18y-10z=-80$	M1
	A vector equation of Π_2 is $\mathbf{r}.\begin{pmatrix} 5 \\ -18 \\ -10 \end{pmatrix} = -80$	A1 (6)
		[13]

Question Number	Scheme	Marks
8.	(a) $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(x^{n-1}\left(x^2+1\right)^{\frac{1}{2}}\right)=(n-1)x^{n-2}\left(x^2+1\right)^{\frac{1}{2}}+x^n\left(x^2+1\right)^{-\frac{1}{2}}$	M1 A1 (2)
	Accept unsimplified forms	
	(b) $\dfrac{nx^n}{\left(x^2+1\right)^{\frac{1}{2}}}+\dfrac{(n-1)x^{n-2}}{\left(x^2+1\right)^{\frac{1}{2}}}=\dfrac{(n-1)x^n+(n-1)x^{n-2}}{\left(x^2+1\right)^{\frac{1}{2}}}+\dfrac{x^n}{\left(x^2+1\right)^{\frac{1}{2}}}$	M1
	$=\dfrac{(n-1)x^{n-2}\left(x^2+1\right)}{\left(x^2+1\right)^{\frac{1}{2}}}+\dfrac{x^n}{\left(x^2+1\right)^{\frac{1}{2}}}$	
	$=(n-1)x^{n-2}\left(x^2+1\right)^{\frac{1}{2}}+x^n\left(x^2+1\right)^{-\frac{1}{2}}$ (the answer to (a))	M1 A1
	The argument above can be reversed	
	Hence, using (a) and integrating	
	$nI_n+(n-1)I_{n-2}=x^{n-1}\sqrt{\left(x^2+1\right)}$ ✱ cso	M1 A1 (5)
	(c) $I_0=\displaystyle\int_0^1\frac{1}{\sqrt{\left(x^2+1\right)}}\mathrm{d}x=\left[\operatorname{arsinh}x\right]_0^1=\operatorname{arsinh}1\quad\left(=\ln\left(1+\sqrt{2}\right)\right)$	B1
	Using (b) $2I_2+I_0=\left[x\sqrt{\left(x^2+1\right)}\right]_0^1=\sqrt{2}$	M1 A1
	$I_2=\dfrac{\sqrt{2}-I_0}{2}=\dfrac{\sqrt{2}-\ln\left(1+\sqrt{2}\right)}{2}$	M1 A1 (5)
		[12]
	Alternative to (b)	
	$I_n=\displaystyle\int x^{n-1}\frac{x}{\sqrt{\left(x^2+1\right)}}\mathrm{d}x=x^{n-1}\left(x^2+1\right)^{\frac{1}{2}}-\int(n-1)x^{n-2}\left(x^2+1\right)^{\frac{1}{2}}\mathrm{d}x$	M1 A1
	$=x^{n-1}\left(x^2+1\right)^{\frac{1}{2}}-\displaystyle\int\frac{(n-1)x^{n-2}\left(x^2+1\right)}{\left(x^2+1\right)^{\frac{1}{2}}}\mathrm{d}x$	M1
	$=x^{n-1}\left(x^2+1\right)^{\frac{1}{2}}-(n-1)\displaystyle\int\frac{x^n}{\left(x^2+1\right)^{\frac{1}{2}}}\mathrm{d}x-(n-1)\int\frac{x^{n-2}}{\left(x^2+1\right)^{\frac{1}{2}}}\mathrm{d}x$	
	$=x^{n-1}\left(x^2+1\right)^{\frac{1}{2}}-(n-1)I_n-(n-1)I_{n-2}$	M1
	Hence $nI_n+(n-1)I_{n-2}=x^{n-1}\sqrt{\left(x^2+1\right)}$ ✱ cso	A1 (5)

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Order Code N/A Summer 2013

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