Edexcel GCE

Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Tuesday 22 June 2010 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 9 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

Turn over

1.
$$z = 2 - 3i$$

(a) Show that $z^2 = -5 - 12i$.

(2)

Find, showing your working,

(b) the value of $|z^2|$,

(2)

(c) the value of $\arg(z^2)$, giving your answer in radians to 2 decimal places.

(2)

(d) Show z and z^2 on a single Argand diagram.

(1)

Question 1 continued

(Total 7 marks)

2. $\mathbf{M} = \begin{pmatrix} 2a & 3 \\ 6 & a \end{pmatrix}$, where a is a real constant.

(a) Given that $a = 2$, find $\mathbf{M}^{-1}$.

(3)

(b) Find the values of a for which $\mathbf{M}$ is singular.

(2)

Question 2 continued

3. $$f(x) = x^3 - \frac{7}{x} + 2, \quad x > 0$$

(a) Show that $f(x) = 0$ has a root α between 1.4 and 1.5

(2)

(b) Starting with the interval $[1.4, 1.5]$, use interval bisection twice to find an interval of width 0.025 that contains α.

(3)

(c) Taking 1.45 as a first approximation to α, apply the Newton-Raphson procedure once to $f(x) = x^3 - \frac{7}{x} + 2$ to obtain a second approximation to α, giving your answer to 3 decimal places.

(5)

Question 3 continued

N 3 5 3 8 7 A 0 7 2 8

Question 3 continued

Question 3 continued

(Total 10 marks)

Q3

N 3 5 3 8 7 A 0 9 2 8

4.
$$f(x) = x^3 + x^2 + 44x + 150$$

Given that $f(x) = (x+3)(x^2 + ax + b)$, where a and b are real constants,

(a) find the value of a and the value of b.

(2)

(b) Find the three roots of $f(x) = 0$.

(4)

(c) Find the sum of the three roots of $f(x) = 0$.

(1)

Question 4 continued

N 3 5 3 8 7 A 0 1 1 2 8

5. The parabola C has equation $y^2 = 20x$.

(a) Verify that the point $P(5t^2, 10t)$ is a general point on C.

(1)

The point A on C has parameter $t = 4$.
The line l passes through A and also passes through the focus of C.

(b) Find the gradient of l.

(4)

Question 5 continued

Q5

N 3 5 3 8 7 A 0 1 3 2 8

6. Write down the 2×2 matrix that represents

    (a) an enlargement with centre $(0, 0)$ and scale factor 8,

<div align="right">(1)</div>

    (b) a reflection in the x-axis.

<div align="right">(1)</div>

    Hence, or otherwise,

    (c) find the matrix **T** that represents an enlargement with centre $(0, 0)$ and scale factor 8, followed by a reflection in the x-axis.

<div align="right">(2)</div>

$$\mathbf{A} = \begin{pmatrix} 6 & 1 \\ 4 & 2 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} k & 1 \\ c & -6 \end{pmatrix}, \text{ where } k \text{ and } c \text{ are constants.}$$

    (d) Find **AB**.

<div align="right">(3)</div>

    Given that **AB** represents the same transformation as **T**,

    (e) find the value of k and the value of c.

<div align="right">(2)</div>

Question 6 continued

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Question 6 continued

N 3 5 3 8 7 A 0 1 6 2 8

Question 6 continued

(Total 9 marks)

Q6

7.
$$f(n) = 2^n + 6^n$$

(a) Show that $f(k+1) = 6f(k) - 4(2^k)$.

(3)

(b) Hence, or otherwise, prove by induction that, for $n \in \mathbb{Z}^+$, $f(n)$ is divisible by 8.

(4)

Question 7 continued

(Total 7 marks)

Q7

N35387A01928

8. The rectangular hyperbola H has equation $xy = c^2$, where c is a positive constant.

The point A on H has x-coordinate $3c$.

(a) Write down the y-coordinate of A.

(1)

(b) Show that an equation of the normal to H at A is

$$3y = 27x - 80c$$

(5)

The normal to H at A meets H again at the point B.

(c) Find, in terms of c, the coordinates of B.

(5)

Question 8 continued

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Question 8 continued

N35387A02228

Question 8 continued

(Total 11 marks)

Q8

9. (a) Prove by induction that

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$$

(6)

Using the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$,

(b) show that

$$\sum_{r=1}^{n}(r+2)(r+3) = \frac{1}{3}n(n^2 + an + b),$$

where a and b are integers to be found.

(5)

(c) Hence show that

$$\sum_{r=n+1}^{2n}(r+2)(r+3) = \frac{1}{3}n(7n^2 + 27n + 26)$$

(3)

Question 9 continued

Question 9 continued

Question 9 continued

Question 9 continued

(Total 14 marks)

TOTAL FOR PAPER: 75 MARKS

END

N 3 5 3 8 7 A 0 2 8 2 8