Reverse Engineering the Correct Drag Reduction Model Coefficients from a Publication

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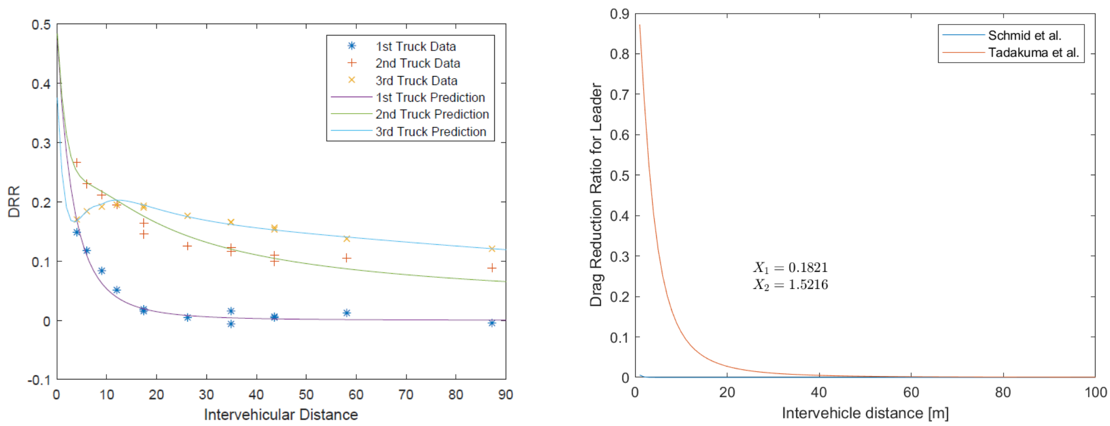
In the paper, "Prediction Model for Energy Consumption in Heavy-Duty Vehicle Formations" by Schmid et al. [1], the authors propose a physics-based drag reduction model for heavy-duty vehicle platoons. A table with the fitted model coefficients is provided at the end of the paper, but the results are not reproducible. Followup papers to [1] give the same coefficients, so the lack of reproducibility extends there as well [2, 3].

As an example, the drag reduction model for a lead vehicle from [1] is given in Equation 1.

$$DRR_1 = 1 - \left[1 - \left[\frac{X_1}{d_{i(i+1)} + X_1 X_2}\right]^3\right]^2 \tag{1}$$

DRR_1 stands for Drag Reduction Ratio of the first vehicle, and $d_{i(i+1)}$ is the intervehicular distance from the first truck to the next. X_1 and X_2 are coefficients that are given in Table 1 of [1] as 0.1821 and 1.5216. The curve that Equation 1 creates should give the Truck 1 trend in Figure 1a.



(a) Published DRR Prediction / Experimental Results

(b) DRR for truck 1 as calculated

Figure 1: Expected versus actual output from the DRR_1 model of [1]

However, this is not the case with the provided X_1 and X_2 values. As Figure 1b displays the resulting DRR curve. The provided coefficients lead to trivially small drag reduction values. The proper shape of the Tadakuma model also shown [4] provides confidence that the equations are mechanized correctly.

To investigate, the model values were extracted from Figure 1a using WebPlotDigitizer [5]. A screenshot of the digitization process is shown in Figure 2.

Without the explicit details of how the data was processed, it is difficult to know how the values were fit. Rather than refitting the uncertain original data, I opted to fit the provided curves, since the goal of this report is to identify the proper coefficients for [1]. The MATLAB function `fminsearch` was used to find the coefficients $X_1, ..., X_6$ that minimize the error between the digitized curves and the DRR functions. The loss functions to be minimized are set up as the Root Mean Square Error (RMSE) between the digitized fit and the DRR functions. To allow reproduction of the results herein, commented MATLAB code is provided at the end of this report.

Table 1 shows a comparison of the original values and the recalculated values. I am confident that only the X_1 value is incorrectly listed. The rest may be used as they are, but I recommend using the recalculated values in Table 1 since each pair of coefficients depends on the last pair (e.g. X_3, X_4 depends on X_1, X_2). The small differences in the X_2 through X_6 coefficients are not concerning.

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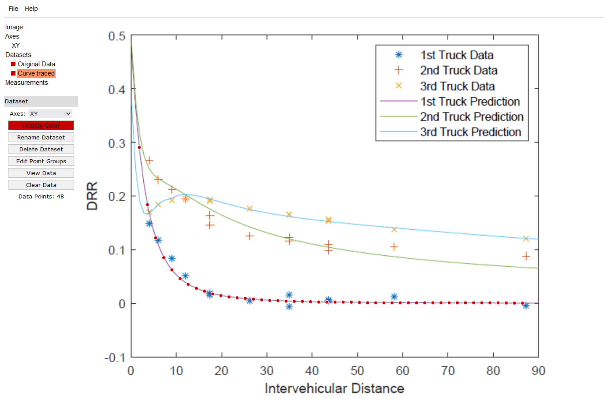

Figure 2: Using WebPlotDigitizer [5] to extract the DRR curves

	X_1	X_2	X_3	X_4	X_5	X_6
Published	0.1821	1.5216	0.6643	8.9340	0.3371	-0.0419
Recalculated	5.4543	1.5197	0.6610	8.9289	0.3374	-0.0422

Table 1: Original published values compared to the recalculated values

Figure 3 shows the validation that the refit worked. All curves are tracking line-on-line, and I have confidence that the coefficients are correct.

In summary, the drag reduction model provided in [1] was not reproducible. It was suspected that the model parameters provided were not all correct, so the coefficients were reverse engineered to see which of the six coefficients were offenders. The code is attached at the end of this document. As it turns out, only X_1 was misstated; it is something like 5.4543 instead of the listed 0.1821.

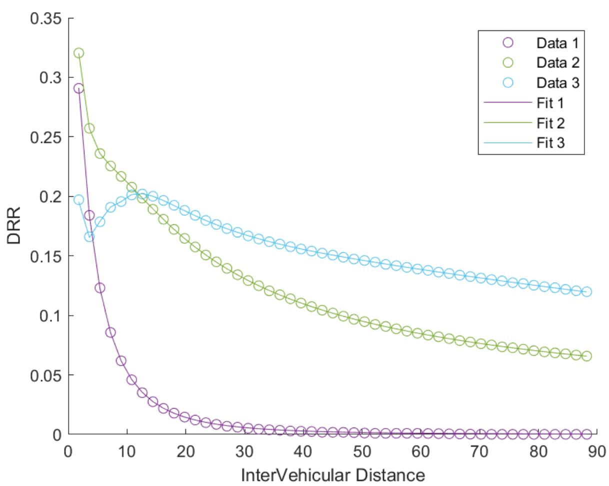

Figure 3: Validation of the recalculated coefficients

References

[1] M. Schmid, D. Liu, B. Eksioglu, N. Huynh, G. Comert, and B. College, "Prediction Model for Energy Consumption in Heavy-Duty Vehicle Formations," p. 7, 2020.

[2] D. Liu, B. Eksioglu, M. J. Schmid, N. Huynh, and G. Comert, "Optimizing Energy Savings for a Fleet of Commercial Autonomous Trucks," *IEEE Transactions on Intelligent Transportation Systems*, vol. 23, pp. 7570–7586, July 2022.

[3] B. Eksioglu, M. J. A. Schmid, N. Huynh, G. Comert, D. Liu, and others, "Framework for Accommodating Emerging Autonomous Vehicles," 2022. Publisher: Center for Connected Multimodal Mobility, Clemson University.

[4] K. Tadakuma, T. Doi, M. Shida, and K. Maeda, "Prediction formula of Aerodynamic Drag Reduction in Multiple-Vehicle Platooning Based on Wake Analysis and On-Road Experiments," *SAE International Journal of Passenger Cars - Mechanical Systems*, vol. 9, pp. 645–656, Apr. 2016.

[5] A. Rohatgi, "Webplotdigitizer: Version 4.6," 2022.

CODE

```
% Evan Stegner
% October 31st, 2023
% The purpose of this code is to reverse engineering the Drag Reduction
% Ratios from [1] M. Schmid, D. Liu, B. Eksioglu, N. Huynh, G. Comert, and
% B. College, 'Prediction Model for Energy Consumption in Heavy-Duty
% Vehicle Formations,' 2020.

%% Data Input
% Truck 1 DRR model digitized from Schmid et al. Figure 1
Xy1=[1.8000000000000007, 0.290760232400188; 3.599999999999998,
    0.18405548147132433; 5.4, 0.1230717506578698; 7.199999999999999,
    0.08568278614662289; 9, 0.061841406501878804; 10.8, 0.045890750354190 77;
     12.600000000000001, 0.03506992047084434; 14.399999999999999,
    0.02748900249145525; 16.200000000000003, 0.021867138764904093;
    18.000000000000004, 0.017891174304907387; 19.800000000000004,
    0.014589725838019207; 21.600000000000005, 0.01211059295221828;
    23.400000000000006, 0.009891678607513055; 25.200000000000003,
    0.008466445276975 82; 27.000000000000007, 0.006875560811393067;
    28.800000000000004, 0.006065071708101999; 30.60000000000001,
    0.005208900970720354; 32.4, 0.004377607292086472; 34.2,
    0.003921769490228155; 36, 0.0034984990140787264; 37.79999999999999,
    0.0028950306248736 18; 39.599999999999994, 0.002642756157895665;
    41.399999999999984, 0.002457798250561 9156; 43.19999999999999,
    0.0018618994842326542; 44.999999999999986, 0.001787399715995 73;
    46.799999999999976, 0.0017759584761007918; 48.59999999999998,
    0.001525996109400407; 50.39999999999998, 0.0011601372433605 04;
    52.19999999999997, 0.0009337707733712008; 53.99999999999998,
    0.0009320434167060521; 55.79999999999996, 0.0009320434167060521;
    57.59999999999997, 0.0009320433702950659; 59.399999999999956,
    0.0009007608481674811; 61.19999999999995, 0.0007913610195653398;
    62.99999999999995, 0.0006577030386119009; 64.79999999999995,
    0.0005116262423675577; 66.59999999999994, 0.00036496996789292524;
    68.39999999999995, 0.0002295735522485 0762; 70.19999999999993,
    0.00011727633249469793; 71.99999999999994, 0.00007668764626722613;
    73.79999999999993, 0.0000766871165645; 75.59999999999994,
    0.0000766871165645; 77.39999999999992, 0.0000766871165645;
    79.19999999999993, 0.0000766871165645; 80.99999999999991,
    0.0000766871165645; 82.79999999999993, 0.0000766871165645;
    84.59999999999991, 0.0000766871165645; 86.39999999999992,
    0.0000766871165645; 88.1999999999999, 0.0000766871165645];

X_data1 = Xy1(:,1);
y_data1 = Xy1(:,2);

% Truck 2 DRR model digitized from Schmid et al. Figure 1
Xy2 = [1.8000000000000007, 0.3204417851482591; 3.599999999999998,
    0.2571348142388094; 5.4, 0.23596748913585952; 7.199999999999999,
    0.22550343338731; 9, 0.21670608252097656; 10.8, 0.20773023064135032;
    12.600000000000001, 0.19844858922779718; 14.399999999999999,
    0.18914308805347657; 16.200000000000003, 0.18072335214606328;
    18.000000000000004, 0.17226489505164877; 19.800000000000004,
    0.1646739477635381; 21.600000000000005, 0.1575098315172549;
    23.400000000000006, 0.15070315868436707; 25.200000000000003,
    0.14499240971235094; 27.000000000000007, 0.13948435316740826;
    28.800000000000004, 0.1341440593506561; 30.60000000000001,
```

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    0.12928616200024828; 32.4, 0.12492483914540398; 34.2,
    0.12065264050576063; 36, 0.11730135142168019; 37.79999999999999,
    0.11395785779800971; 39.599999999999994, 0.11037500050045329;
    41.399999999999984, 0.10739146298040969; 43.19999999999999,
    0.10436384390381148; 44.999999999999986, 0.1019705719738262;
    46.799999999999976, 0.09936108838094215; 48.59999999999998,
    0.09673540789242663; 50.39999999999998, 0.09474829982341182;
    52.19999999999997, 0.09237289030099272; 53.99999999999998,
    0.090722410064633; 55.79999999999996, 0.0887203381505609;
    57.59999999999997, 0.08678239573928237; 59.399999999999956,
    0.08500395281090628; 61.19999999999995, 0.08345032373463956;
    62.99999999999995, 0.08206742063356309; 64.79999999999995,
    0.08045065180111444; 66.59999999999994, 0.07882541127769388;
    68.39999999999995, 0.07768175873662209; 70.19999999999993,
    0.07617778662579289; 71.99999999999994, 0.07518369892292698;
    73.79999999999993, 0.07354624846345714; 75.59999999999994,
    0.07279334637944385; 77.39999999999992, 0.0716473731012115;
    79.19999999999993, 0.07034134393183472; 80.99999999999991,
    0.0693545119783131; 82.79999999999993, 0.06850401071805623;
    84.59999999999991, 0.06772982721272491; 86.39999999999992,
    0.06669341974897497; 88.1999999999999, 0.06577681139640407];
X_data2 = Xy2(:,1);
y_data2 = Xy2(:,2);

% Truck 3 DRR model digitized from Schmid et al. Figure 1
Xy3 = [1.8000000000000007, 0.19711657453806553; 3.599999999999998,
    0.16600406922807842; 5.4, 0.1786403653052795; 7.199999999999999,
    0.19080792698917126; 9, 0.19560292288775782; 10.8, 0.2011412939184538;
    12.600000000000001, 0.20194093582768047; 14.399999999999999,
    0.20007366924529602; 16.200000000000003, 0.19655937643265586;
    18.000000000000004, 0.19248509627106236; 19.800000000000004,
    0.18821966472771384; 21.600000000000005, 0.18401304026242665;
    23.400000000000006, 0.17977524024721836; 25.200000000000003,
    0.17637710495976422; 27.000000000000007, 0.17288698241016298;
    28.800000000000004, 0.1697538425322842; 30.60000000000001,
    0.166772020552802; 32.4, 0.16420133324504815; 34.2, 0.16191382933181286;
     36, 0.15991574755720145; 37.79999999999999, 0.15768532506202432;
    39.599999999999994, 0.15577845009554153; 41.399999999999984,
    0.1540424622258918; 43.19999999999999, 0.1523048158386141;
    44.999999999999986, 0.1506197013423557; 46.799999999999976,
    0.1488313036124656; 48.59999999999998, 0.14731525126663503;
    50.39999999999998, 0.14612611404204623; 52.19999999999997,
    0.14471412275148166; 53.99999999999998, 0.14284891769579267;
    55.79999999999996, 0.1418146450481379; 57.59999999999997,
    0.14035727668499248; 59.399999999999956, 0.13894109893020612;
    61.19999999999995, 0.13781679988761697; 62.99999999999995,
    0.1363096780133326; 64.79999999999995, 0.1351995877435287;
    66.59999999999994, 0.13381514400585826; 68.39999999999995,
    0.13265283946405854; 70.19999999999993, 0.1313232001885113;
    71.99999999999994, 0.13008373402988138; 73.79999999999993,
    0.12901332831749368; 75.59999999999994, 0.12749111112717065;
    77.39999999999992, 0.12674109302799352; 79.19999999999993,
    0.12531701653252747; 80.99999999999991, 0.12412588642918199;
    82.79999999999993, 0.12324569766604282; 84.59999999999991,
    0.12186822991854518; 86.39999999999992, 0.12070515430572515;
    88.1999999999999, 0.11987228700129982];
X_data3 = Xy3(:,1);
y_data3 = Xy3(:,2);
```

```matlab

%% DRR Functions
% DRR for truck 1
DRR1 = @(X_12v) 1-(1-(X_12v(1)./(X_data1+X_12v(1)*X_12v(2))).^3).^2 ;
%loss function for DRR1 error, Root Mean Square Error
f1 = @(X_12v) sqrt(mean(( DRR1(X_12v) -y_data1).^2));

% Centerline deficit for truck 2
zeta = @(X_12v,X_34v) X_34v(1).*(1-DRR1(X_12v)).^X_34v(2).*X_data2.^(-2/3)
% DRR for truck 2
DRR2 = @(X_12v,X_34v) 1-(1-zeta(X_12v,X_34v)).^2.*(1-(DRR1(X_12v)))
%loss function for DRR2 error, Root Mean Square Error
f2 = @(X_12v,X_34v) sqrt(mean(( DRR2(X_12v,X_34v) -y_data2).^2));

% Damping term for truck 3+
del_Cdc = @(X_56v) 1 + min(repmat(0.23391,size(X_data3)), X_56v(1).*exp(X_56v(2).*X_data3));
% Centerline deficit for truck 3
zeta2 = @(X_12v,X_34v) X_34v(1).*(1-DRR1(X_12v)).^X_34v(2).*X_data3.^(-2/3)
% DRR for truck 3
DRR3 = @(X_12v,X_34v,X_56v) 1-(1-DRR2(X_12v,X_34v)).*...
    (1-zeta2(X_12v,X_34v)).^2.*...
    del_Cdc(X_56v)
%loss function for DRR3 error, Root Mean Square Error
f3 = @(X_12v,X_34v,X_56v) sqrt(mean(( DRR3(X_12v,X_34v,X_56v) -y_data3).^2));

%% Coefficient Search

options = optimset('Display','iter','PlotFcns',@optimplotfval,'TolFun',1e-10,'TolX',1e-10);
% finding X1 and X2
X_ =fminsearch(f1,[10,10],options)

% Use the new X1 and X2 in loss function for X3 and X4
fun2= @(X_34v) f2(X_,X_34v);
% finding X3 and X4
X__ = fminsearch(fun2,[10,10],options)

% Use the new X3 and X4 in loss function for X5 and X6
fun3= @(X_56v) f3(X_,X__,X_56v);
% finding X5 and X6
X___ = fminsearch(fun3,[0.3,-0.1],options)

%% Validation
% Plot the validation
figure()

% Use original colors from the paper
colororder([0.49,0.18,0.56; ...
    0.47,0.67,0.19; ...
    0.30,0.75,0.93; ...
    0.49,0.18,0.56;...
    0.47,0.67,0.19; ...
            0.30,0.75,0.93])
scatter(X_data1,y_data1);hold on
scatter(X_data2,y_data2);hold on
scatter(X_data3,y_data3);hold on
```

```
plot(X_data1,DRR1(X_))
plot(X_data2,DRR2(X_,X__))
plot(X_data3,DRR3(X_,X__,X___))
legend('Data 1','Data 2','Data 3','Fit 1','Fit 2','Fit 3')
xlabel('Intervehicular Distance')
ylabel('DRR')

% Output Table
fprintf('|\tX1 \t\t X2\t\t X3 \t X4 \t X5 \t X6\t\t|\n')
fprintf('|\t%5.4f\t%5.4f\t%5.4f\t%5.4f\t%5.4f\t%5.4f\t|\n',X_(1),X_(2),...
    X__(1),X__(2),X___(1),X___(2))
```