PYTHON FINAL PROJECT:
Attempting to Calculate the Mass of a Supermassive Black Hole
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For this project, I chose to analyze the relationship between the properties of stars orbiting a central supermassive black hole, or SMBH, and the ultimate mass of that black hole. A SMBH is a massive astronomical object where gravity is strong enough that even light cannot escape. The mass of a SMBH can range from hundreds of thousands to billions of times that of our Sun. They can typically be found at the center of large galaxies, including our Milky Way. I chose to look into the effects of SMBHs and their mass because there are such uncertainties around SMBH at the center of galaxies and how they affect the growth of galaxies over time. The mass of an SMBH can reveal clues about how it formed and grew, through the accretion of matter, mergers with other black holes, or the collapse of massive stars in the early universe. Knowing the mass of a SMBH can help to determine the ultimate energy output through quasars, or test the predictions of general relativity, for example. It is also important to determine the mass to see whether objects other than a SMBH might contribute to enclosed mass estimates of galaxies, like dark matter.

For a star orbiting a black hole in a nearly circular orbit, the relationship between the gravitational force and the centripetal force can be used to determine the predicted mass of the SMBH. Using Newton's version of Kepler's Third Law,

$$M = (rv^2)/G,$$

the mass can be calculated if the orbital velocities and orbital radii are known for multiple stars around a central SMBH. The orbital velocity of a star around a black hole is determined by tracking its position and the Doppler shift in the star's spectral lines, which results from orbiting a large mass, and then calculating the velocity.

The data I will be using came from the research paper “High-Precision Stellar Radial Velocities in the Galactic Center”, by the Astrophysical Journal. It reported on stellar radial velocities and their use in determining the mass and existence of a SMBH in our galactic center, Sagittarius A*. The paper included data from 85 cool stars in the central parsec of the Milky Way. Among many things, the presented data provided the average velocity, in kilometers per second, as well as the orbital radius, in arcseconds, for each observed star. The paper claimed that the velocity errors were much smaller than those previously gathered and some of the data had already been reduced to include only the stars with high-quality spectra as well as only the cool-star orbits. It was also said that there were “no large systematic errors in the absolute velocity calibration”. With this information, I did not feel it was necessary to reduce the data due to any inconsistencies or extremities.

Since the orbital radius data was collected in arcseconds, I started by converting those values into kilometers by first converting them to astronomical units and then to kilometers. This conversion was needed so that the units for distance would cancel out within the equation for Kepler’s Third Law. I included these values in a new column added to the existing data frame, labeled “r (km)”. Then the average orbital velocities of each star needed to be squared since that is the format for Kepler’s Third Law. So I created one more additional column, labeled “Vave Squared”, that took the values of average velocities and squared them.

After reworking the data to fit the needed format, I first thought of separately analyzing the relationship between the orbital radius and the orbital velocity squared, to the predicted mass of a SMBH. I started by examining the orbital radius as our x variable in determining the predicted mass. Kepler’s Third Law was used as the equation for the position of data points, with y being the predicted mass, x being the orbital radius of each star, and the slope being the orbital velocity of each star divided by the gravitational constant. After plotting this data there was a slight linear relationship so I used a NumPy polyfit function to graph the line of best fit, which resulted in a positive linear slope. The positive linear slope shows that, predictively, as the orbital radius of a star circling a SMBH increases so should the mass of that black hole. This seems realistic since a SMBH would need a larger mass to produce a greater gravitation inward force that would keep a distant star orbiting.

After looking at this relationship, I then repeated the process, using Kepler’s Third Law, but instead had x be the squared orbital velocity of each star. The plot of this data also revealed a linear relationship, so I once again chose a polyfit function to determine the line of best fit. The line of best fit for this graph had a positive linear slope but was slightly steeper. Again this seems to work well because when a star’s orbital velocity increases, the SMBH needs a stronger gravitational force, which increases with increasing mass, to balance the outward pointing centrifugal force of the star.

While both these graphs provide reasonable relationships that support Kepler’s Third Law, in order to determine the mass of the SMBH, that the stars within the data set are orbiting, the relationship between the orbital velocity and orbital radius of each star must be evaluated. So by rearranging Kepler’s Third Law into

$$v^2 = (MG)/r,$$

the velocity squared of each star now served as the y variable, while the inverse of the orbital radius of each star became the x variable. The line of best fit for this equation would then produce a slope that would be equivalent to the predicted mass of the SMBH we are studying, multiplied by the gravitational constant. After plotting the points for each star, it was uncertain to me if there was a linear relationship or not. However, for the equation to make sense, we needed a singular value for the slope, which only a linear relationship approximation would produce. So once again I used a Numpy polyfit function to determine the line of best fit, and it did indeed result in a positive linear slope. Then all I needed to do was single out the predicted mass by taking the slope and dividing it by the gravitational constant. This final answer was produced with some distance units not being canceled out, so I had to convert the answer so that we would be left with only the mass. Once this was done, calculated was a mass of 1.4386e26 kg for the SMBH at the Milky Way’s galactic center. This value did result to be smaller than what should be predicted, however, this could be due to possible calculation errors or simply not enough data values used.