Week 14: *Class Review*

EMSE 6035: Marketing Analytics for Design Decisions

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December 01, 2021

Week 14: *Class Review*

1. Final Report & Presentation

2. Sensitivity Analysis

BREAK

3. Exam Review

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Analysis

1. Clean data
2. Modeling
   - Simple logit
   - Mixed logit
   - One sub-group model
3. Analysis
   - WTP for key features
   - Market simulation
   - Sensitivity analysis

Report

1. Introduction
2. Survey Design
3. Data Analysis
4. Results (plots / text)
5. Recommendations

Final Presentation

- In class, 12/15
- 10 minutes (strict)
- External Panel of Reviewers
- Slides due on Blackboard by midnight on 12/14

How to design good slides

Hitchcock's rule

Hitchcock's rule

The size of any object in your frame should be proportional to its importance to the story at that moment

Watch this example

Hitchcock's rule

The size of any object in your ~~frame~~ **slide** should be proportional to its importance to the story at that moment

...and finally you will read this

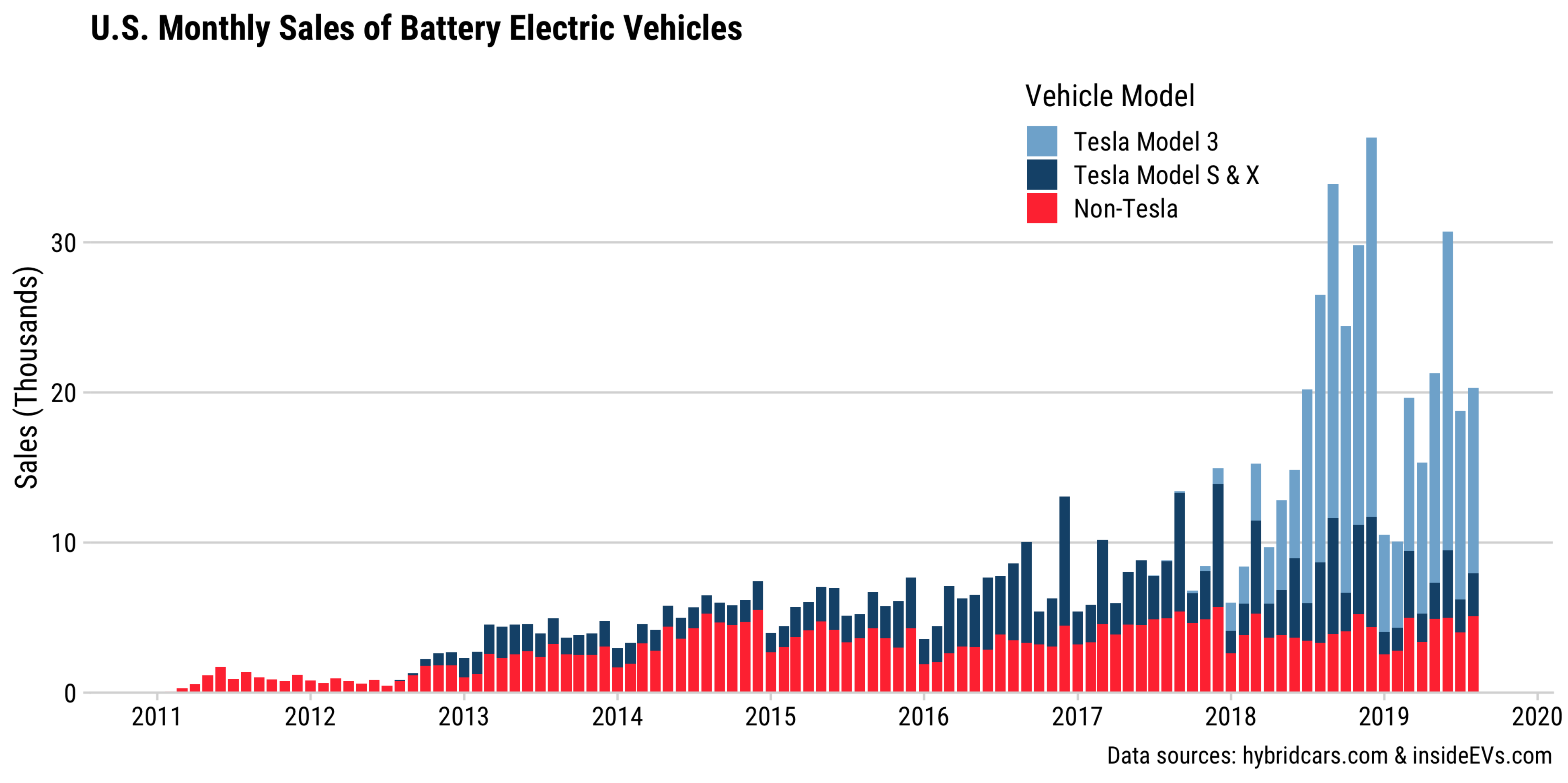
You will read this first

and then you will read this

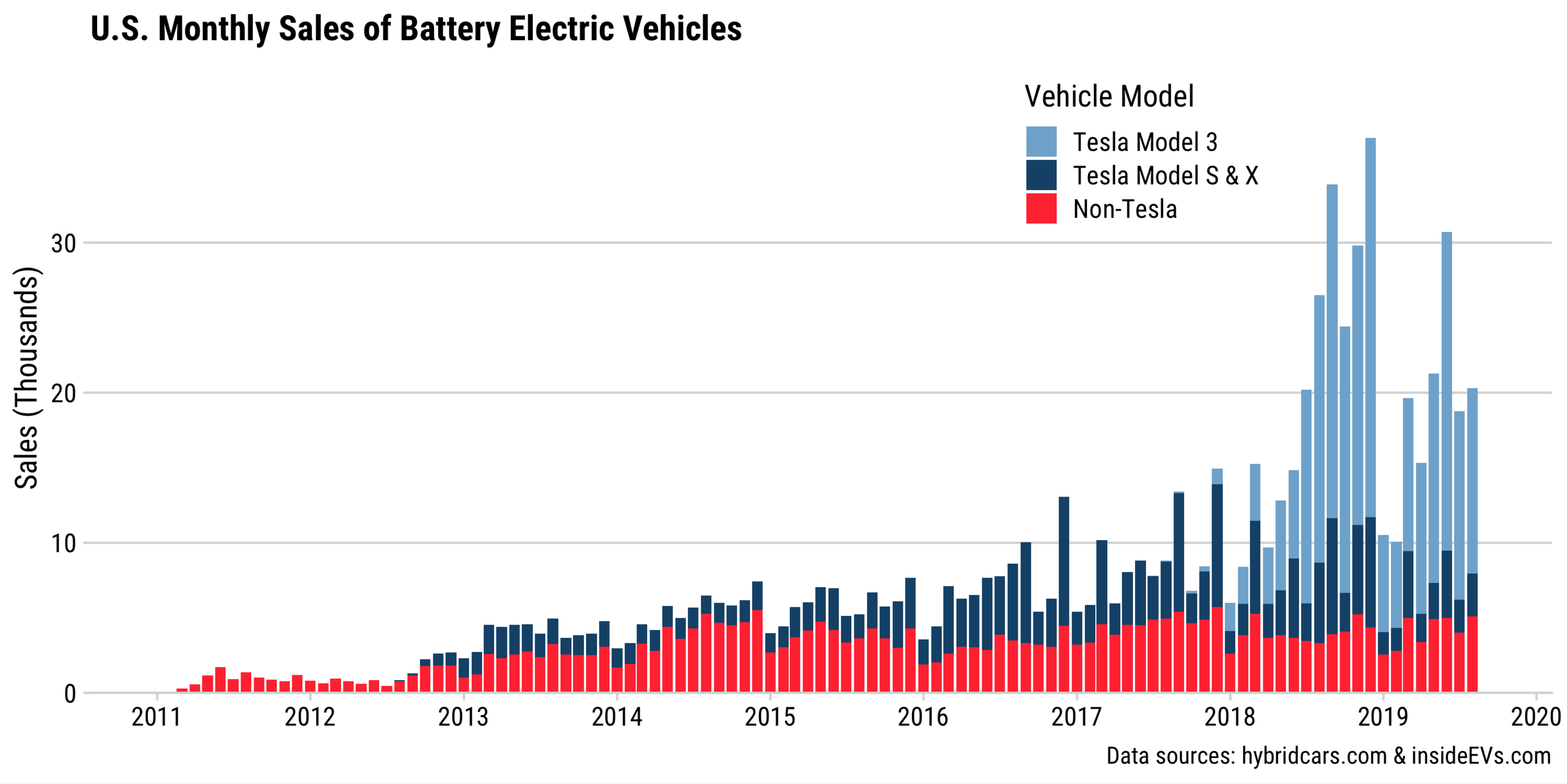
Main point at top and use a big font!

(see Stephanie Evergreen's blog post "So What?")

Except for Tesla, EV adoption in the U.S. is **flat**



Tesla's Model 3 is a Game Changer for EVs



> 40pt font for titles

> 24pt font for all other text

(Exception: footer text can be small)

Avoid fonts like

Comic Sans

Papyrus

They make your work look amateurish

Consider using a light-colored background
(tan / gray)

Use high contrast between font and background color

Dark text on a light background works well

Light text on a dark background also works well

Use high contrast between font and background color

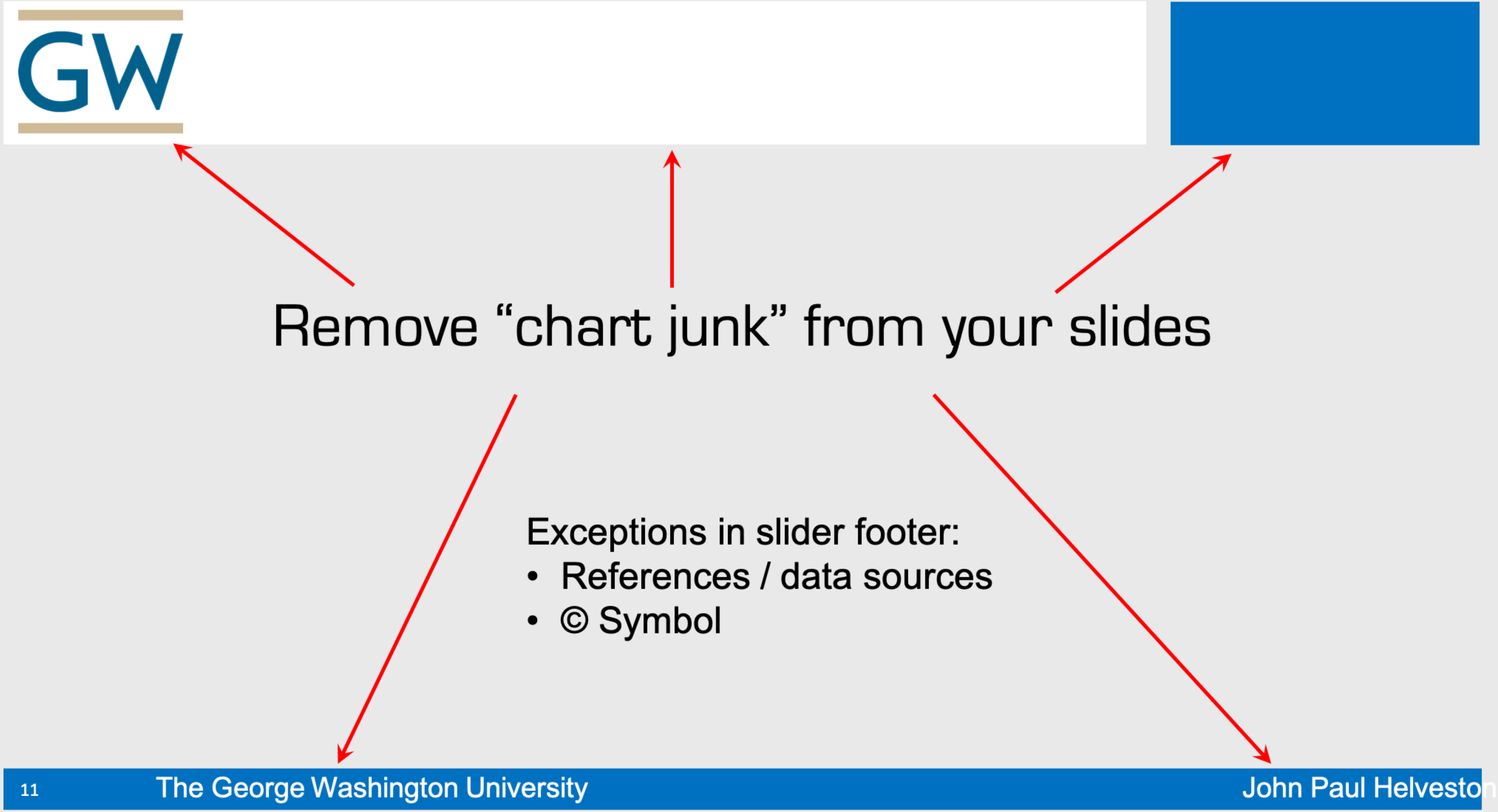
Yellow text on a white background is horrible

Blue text on a black background is horrible

1 slide, 1 idea

Break up main points into multiple slides

Number your slides!


GW
Remove “chart junk” from your slides
Exceptions in slider footer:
• References / data sources
• © Symbol
11
The George Washington University
John Paul Helveston

Example of an acceptable slide footer

↓

If you are in person, consider using handouts (1-2 pages)

How to design good slides

- **Hitchcock's rule**: The size of any object on your slide should be proportional to its importance to the story at that moment
- **Slide titles**: A single statement about what slide means (in big font!)
- **Use large font sizes** (>40 titles, >24 text)
- Consider using a **light-colored background** (tan / gray)
- Use **high contrast** between font and background color
- **Don't use silly fonts** like Comic Sans, Papyrus, etc.
- **1 slide, 1 idea**: Break up main points into multiple slides
- **Slide numbers**: bottom-left or bottom-right
- **Remove "chart junk"**: logos, etc. (exception: small footers)

- **Consider using handouts**

- **Don't pack the slide with bullet lists** (see what I did there?)

Week 14: *Class Review*

1. Final Report & Presentation
2. Sensitivity Analysis

BREAK

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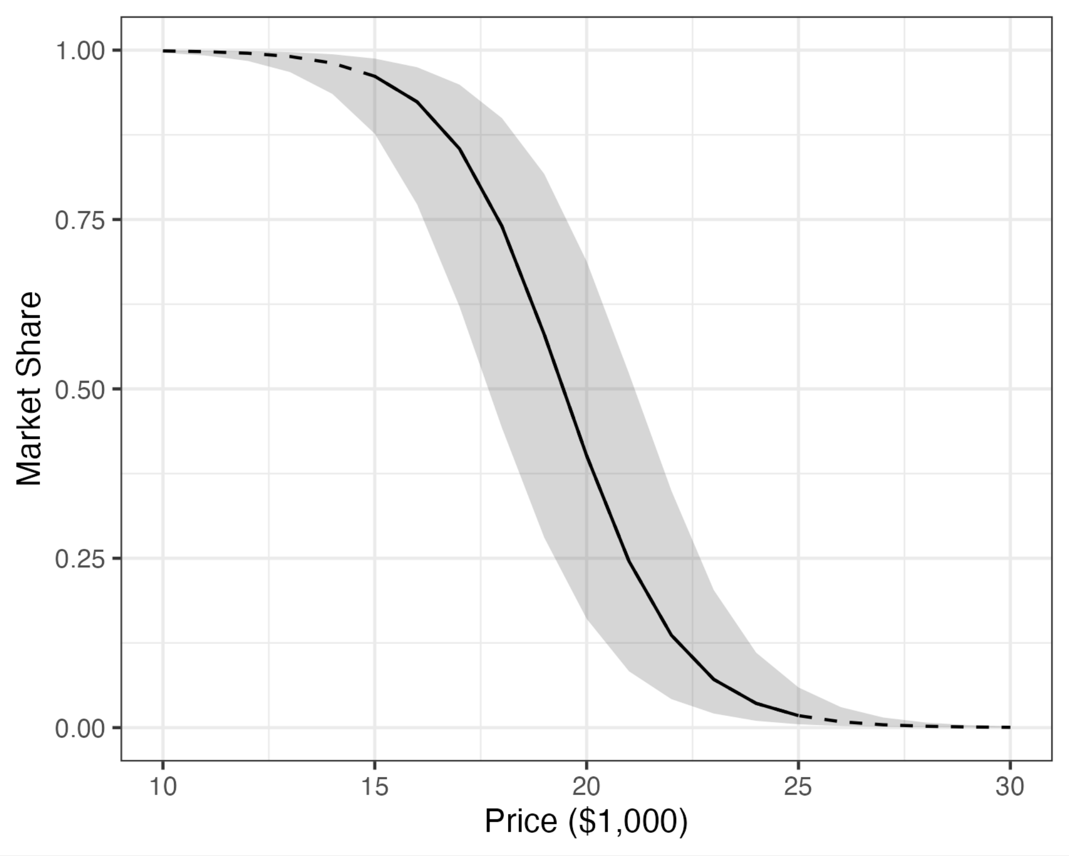
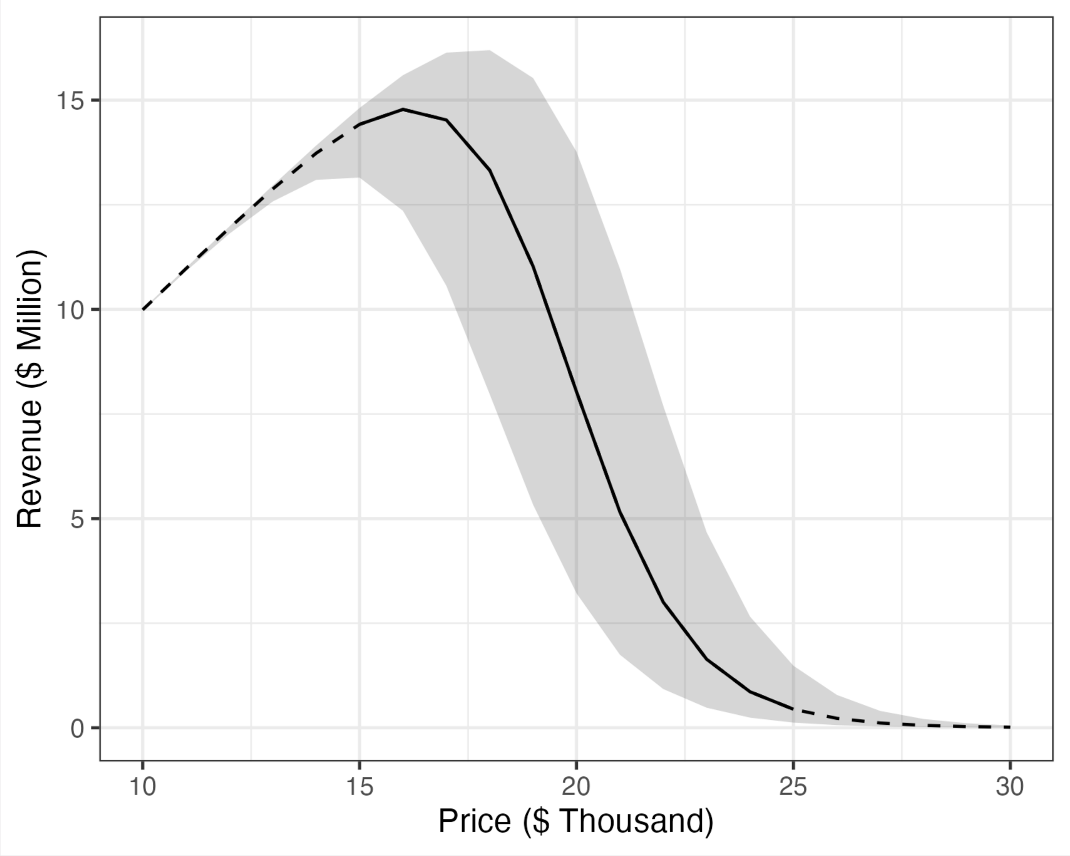
Market share sensitivity to price

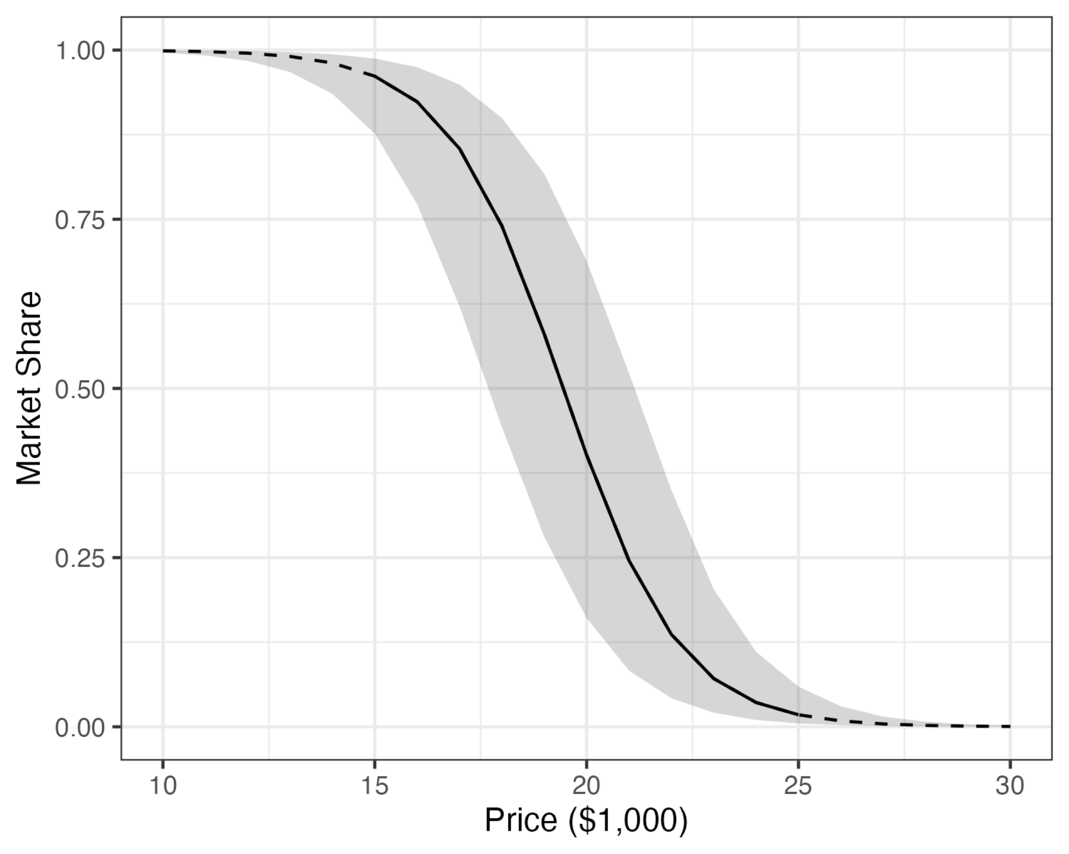


Revenue sensitivity to price



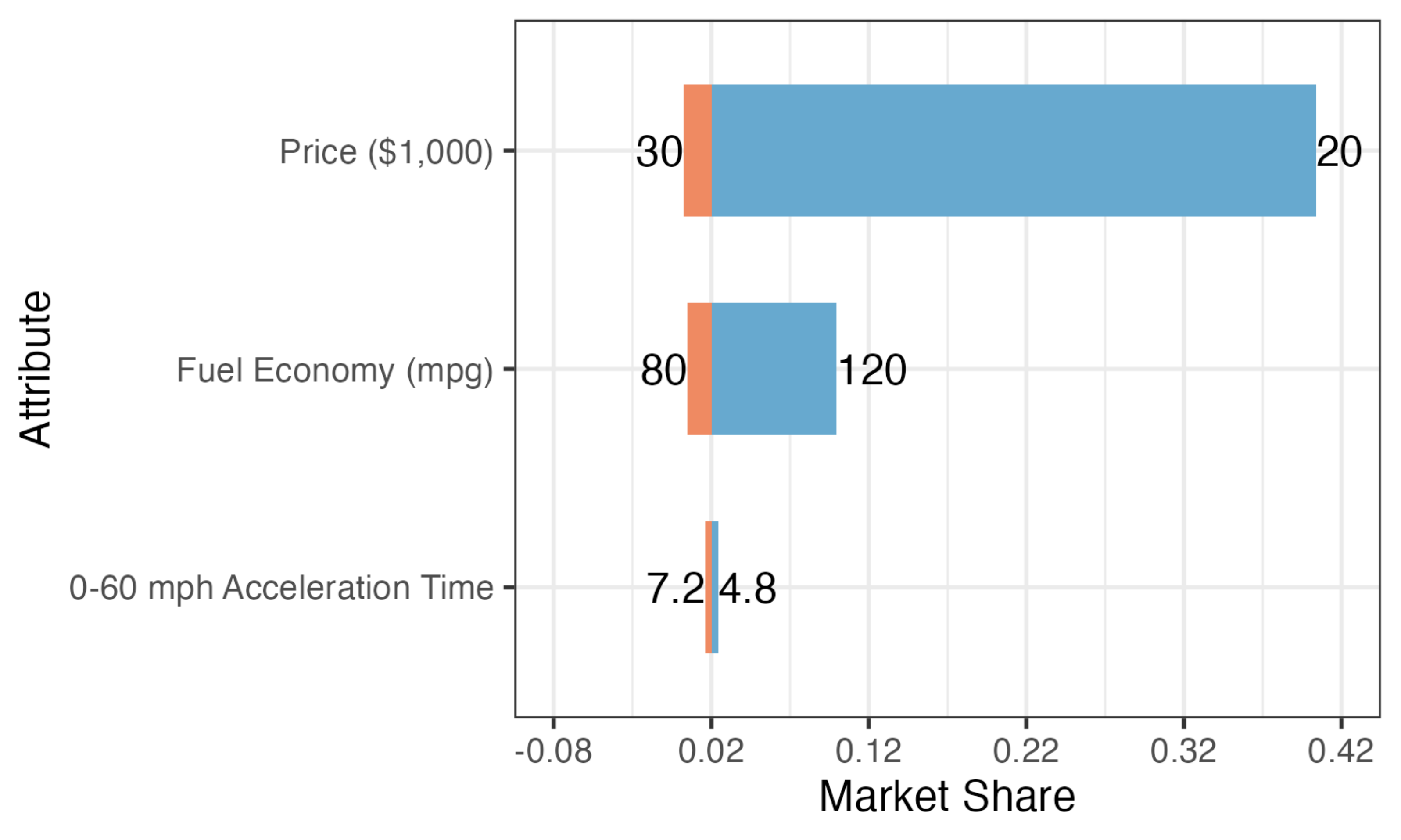
$$R = Q * P$$

Market share sensitivity to price

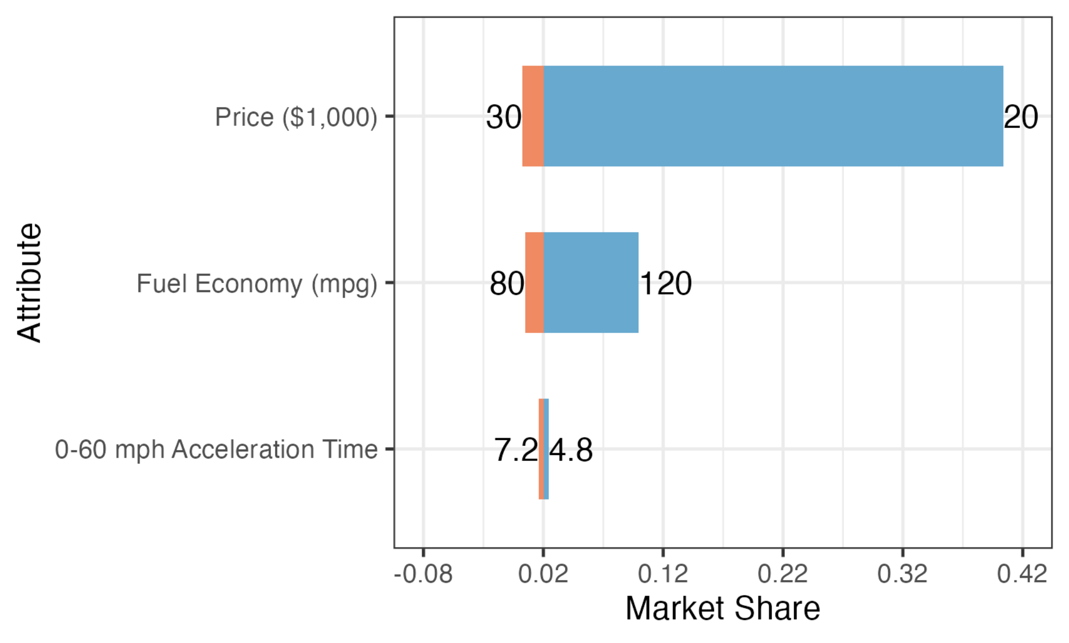


Observations

- Solid line reflects *interpolation* (attribute range in survey)
- Dashed line reflects *extrapolation* (beyond attribute range in survey)
- Ribbon reflects *parameter uncertainty*


Attribute
Price ($1,000)
Fuel Economy (mpg)
0-60 mph Acceleration Time
30
20
80
120
7.2
4.8
-0.08
0.02
0.12
0.22
0.32
0.42
Market Share

Market share sensitivity to all attributes



Observations

- Middle point reflects baseline market share:
  - **Price**: $25,000
  - **Fuel Economy**: 100 mpg
  - **0-60 mph Accel. time**: 6 sec
- Boundaries on each attribute should reflect max feasible attribute bounds

Sensitivity analyses

1. Open `logitr-cars`
2. Open `code/9.1-compute-sensitivity.R`
3. Open `code/9.2-plot-sensitivity.R`

Break

05:00

Week 14: *Class Review*

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BREAK

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Things I'm covering

- Data wrangling in R
- Utility models
- Maximum likelihood estimation
- Optimization
- Uncertainty
- Design of experiment
- WTP
- Market simulations
- Using R for all of the above
  (e.g., estimating models wiht `logitr`)

Things I'm **not** covering

- formr.org
- Heterogeneity
  (mixed logit, sub-groups)

Data wrangling in R

Steps to importing external data files

1. Create a path to the data

```r
library(here)
path_to_data <- here('data', 'data.csv')
path_to_data
```

```
#> [1] "/Users/jhelvy/gh/0gw/MADD/2021-Fall/class/14-review/data/data.csv"
```

2. Import the data

```r
library(tidyverse)
data <- read_csv(path_to_data)
```

Steps to importing external data files

```r
library(tidyverse)

data <- read_csv(here::here('data', 'data.csv'))
```

The main `dplyr` "verbs"

"Verb"	What it does
`select()`	Select columns by name
`filter()`	Keep rows that match criteria
`arrange()`	Sort rows based on column(s)
`mutate()`	Create new columns

Example data frame

```r
beatles <- tibble(
    firstName   = c("John", "Paul", "Ringo", "George"),
    lastName    = c("Lennon", "McCartney", "Starr", "Harrison"),
    instrument  = c("guitar", "bass", "drums", "guitar"),
    yearOfBirth = c(1940, 1942, 1940, 1943),
    deceased    = c(TRUE, FALSE, FALSE, TRUE)
)

beatles
```

```
#> # A tibble: 4 × 5
#>   firstName lastName  instrument yearOfBirth deceased
#>   <chr>     <chr>     <chr>            <dbl> <lgl>
#> 1 John      Lennon    guitar            1940 TRUE
#> 2 Paul      McCartney bass              1942 FALSE
#> 3 Ringo     Starr     drums             1940 FALSE
#> 4 George    Harrison  guitar            1943 TRUE
```

`filter()` and `select()`:

Get the **first & last name** of members born after 1941 & are still living

```
beatles %>%
  filter(yearOfBirth > 1941, deceased == FALSE) %>%
  select(firstName, lastName)
```

```
#> # A tibble: 1 × 2
#>   firstName lastName
#>   <chr>     <chr>
#> 1 Paul      McCartney
```

Create new variables with `mutate()`

Use the `yearOfBirth` variable to compute the age of each band member

```
beatles %>%
    mutate(age = 2021 - yearOfBirth) %>%
    arrange(age)
```

```
#> # A tibble: 4 × 6
#>   firstName lastName  instrument yearOfBirth deceased   age
#>   <chr>     <chr>     <chr>            <dbl> <lgl>    <dbl>
#> 1 George    Harrison  guitar            1943 TRUE        78
#> 2 Paul      McCartney bass              1942 FALSE       79
#> 3 John      Lennon    guitar            1940 TRUE        81
#> 4 Ringo     Starr     drums             1940 FALSE       81
```

Handling if/else conditions

`ifelse(<condition>, <if TRUE>, <else>)`

```
beatles %>%
    mutate(playsGuitar = ifelse(instrument == "guitar", TRUE, FALSE))
```

```
#> # A tibble: 4 × 6
#>   firstName lastName  instrument yearOfBirth deceased playsGuitar
#>   <chr>     <chr>     <chr>            <dbl> <lgl>    <lgl>
#> 1 John      Lennon    guitar            1940 TRUE     TRUE
#> 2 Paul      McCartney bass              1942 FALSE    FALSE
#> 3 Ringo     Starr     drums             1940 FALSE    FALSE
#> 4 George    Harrison  guitar            1943 TRUE     TRUE
```

Utility models

Random utility model

The utility for alternative j is

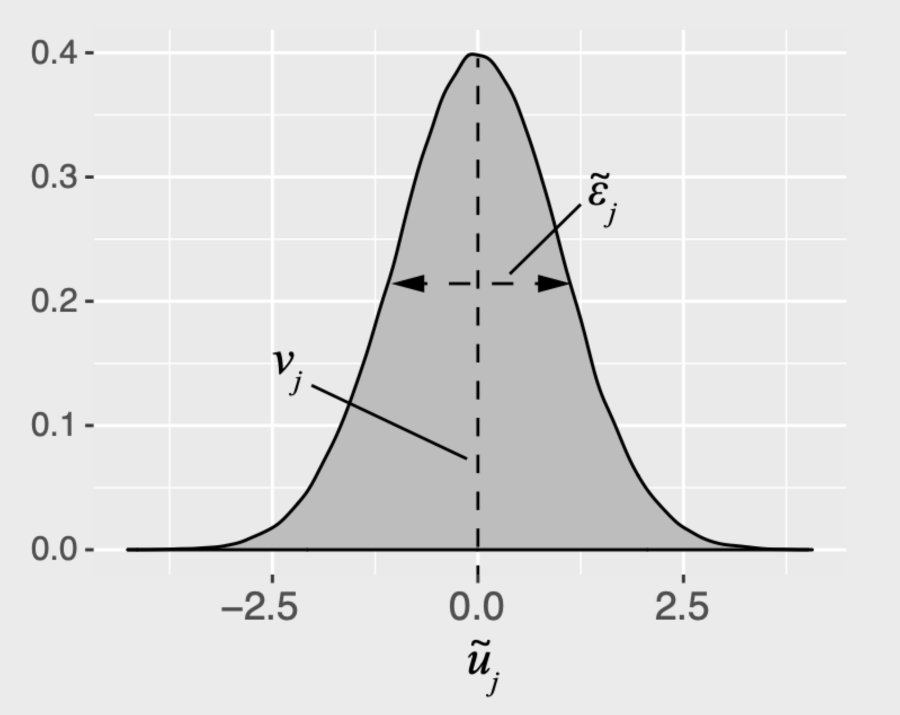
$$\tilde{u}_j = v_j + \tilde{\varepsilon}_j$$

v_j = Things we observe (non-random variables)

$\tilde{\varepsilon}_j$ = Things we *don't* observe (random variable)

Logit model: Assume that $\tilde{\varepsilon}_j$ ~ Gumbel Distribution

$$\tilde{u}_j = v_j + \tilde{\varepsilon}_j$$



Probability of choosing alternative j:

$$P_j = \frac{e^{v_j}}{\sum_k e^{v_k}}$$

Notation Convention

Continuous: x_j

$$u_j = \beta_1 x_j^{\text{price}} + \dots$$

```
#>   price
#> 1     1
#> 2     2
#> 3     3
```

Discrete: δ_j

$$u_j = \beta_1 \delta_j^{\text{ford}} + \beta_2 \delta_j^{\text{gm}} \dots$$

```
#>   brand brand_BMW brand_Ford brand_GM
#> 1  Ford         0          1        0
#> 2    GM         0          0        1
#> 3   BMW         1          0        0
```

Dummy-coded variables

Dummy coding: 1 = "Yes", 0 = "No"

Data frame with one variable: *brand*

```r
data <- data.frame(
    brand = c("Ford", "GM", "BMW"))

data
```

```
#>   brand
#> 1  Ford
#> 2    GM
#> 3   BMW
```

Add dummy columns for each brand

```r
library(fastDummies)

dummy_cols(data, "brand")
```

```
#>   brand brand_BMW brand_Ford brand_GM
#> 1  Ford         0          1        0
#> 2    GM         0          0        1
#> 3   BMW         1          0        0
```

Modeling *continuous* variable

$$v_j = \beta_1 x^{\text{price}}$$

```r
model <- logitr(
    data   = data,
    choice = "choice",
    obsID  = "obsID",
    pars   = "price"
)
```

Coef.	Interpretation
β1	how utility changes with increasing *price*

Modeling *discrete* variable

$$v_j = \beta_1 \delta_j^{\text{ford}} + \beta_2 \delta_j^{\text{gm}}$$

```r
model <- logitr(
    data   = data,
    choice = "choice",
    obsID  = "obsID",
    pars   = c("brand_Ford", "brand_GM")
)
```

Reference level: *price=10*

Coef.	Interpretation
β1	utility for *Ford* relative to *BMW*
β2	utility for *GM* relative to *BMW*

Estimating utility models

1. Open `logitr-cars.Rproj`
2. Open `code/3.1-model-mnl.R`

mnl_dummy

All dummy-code variables

```
pars = c(
  "price_20", "price_25",
  "fuelEconomy_25", "fuelEconomy_30",
  "accelTime_7", "accelTime_8",
  "powertrain_Electric")
```

Reference Levels:

- Price: 15
- Fuel Economy: 20
- Accel. Time: 6
- Powertrain: "Gasoline"

mnl_linear

All continuous (linear), except for `powertrain_Electric`

```
pars = c(
  'price', 'fuelEconomy', 'accelTime',
  'powertrain_Electric')
```

Reference Levels:

- Powertrain: "Gasoline"

Practice Question 1

20:00

Let's say our utility function is:

$$v_j = \beta_1 x_j^{\text{price}} + \beta_2 x_j^{\text{cacao}} + \beta_3 \delta_j^{\text{hershey}} + \beta_4 \delta_j^{\text{lindt}}$$

And we estimate the following coefficients:

Parameter	Coefficient
β_1	-0.1
β_2	0.1
β_3	-2.0
β_4	-0.1

What are the expected probabilities of choosing each of these bars using a logit model?

Attribute	Bar 1	Bar 2	Bar 3
Price	\$1.20	\$1.50	\$3.00
% Cacao	10%	60%	80%
Brand	Hershey	Lindt	Ghirardelli

Maximum likelihood estimation

Maximum likelihood estimation

$$\tilde{u}_j = \boldsymbol{\beta}'\mathbf{x}_j + \tilde{\varepsilon}_j$$

$$= \boxed{\beta_1} x_{j1} + \boxed{\beta_2} x_{j2} + \ldots + \tilde{\varepsilon}_j$$

Weights that denote the *relative* value of attributes $x_{j1}, x_{j2}, \ldots$

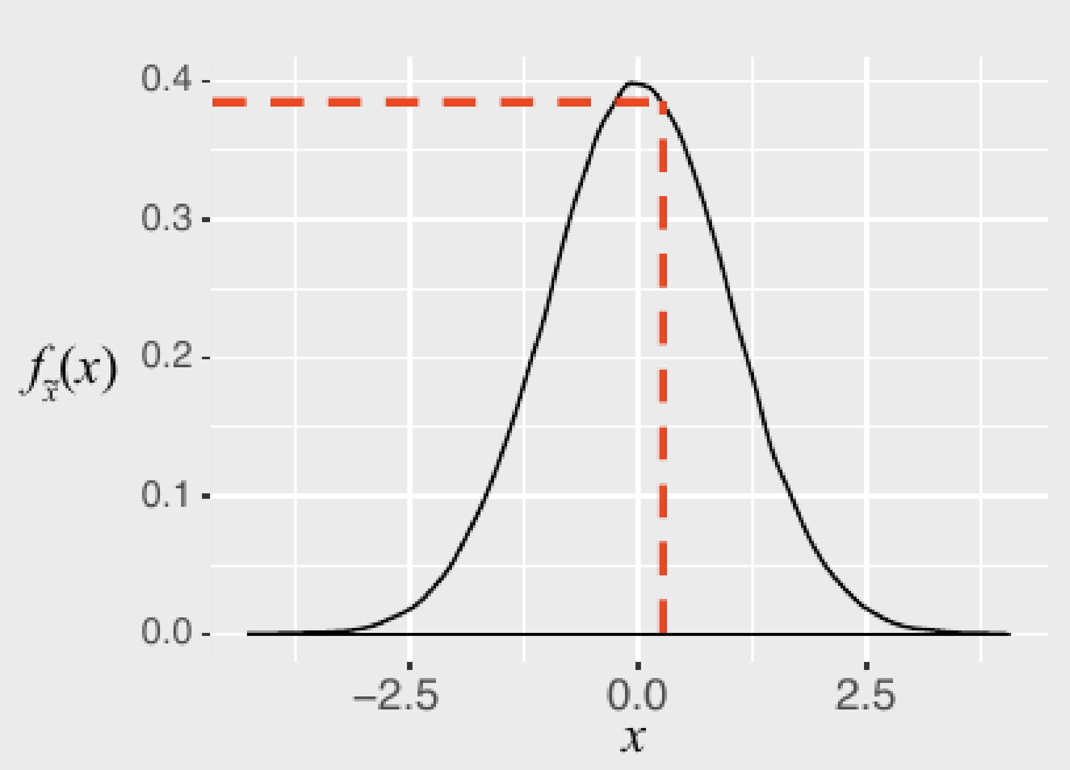
Estimate β_1 , β_2 , ... , by minimizing the negative log-likelihood function:

$$\text{minimize} - \ln(\mathcal{L}) = -\sum_{j=1}^{J} y_j \ln[P_j(\boldsymbol{\beta}|\mathbf{x})]$$

with respect to $\boldsymbol{\beta}$

$y_j = 1$ if alternative j was chosen
$y_j = 0$ if alternative j was not chosen

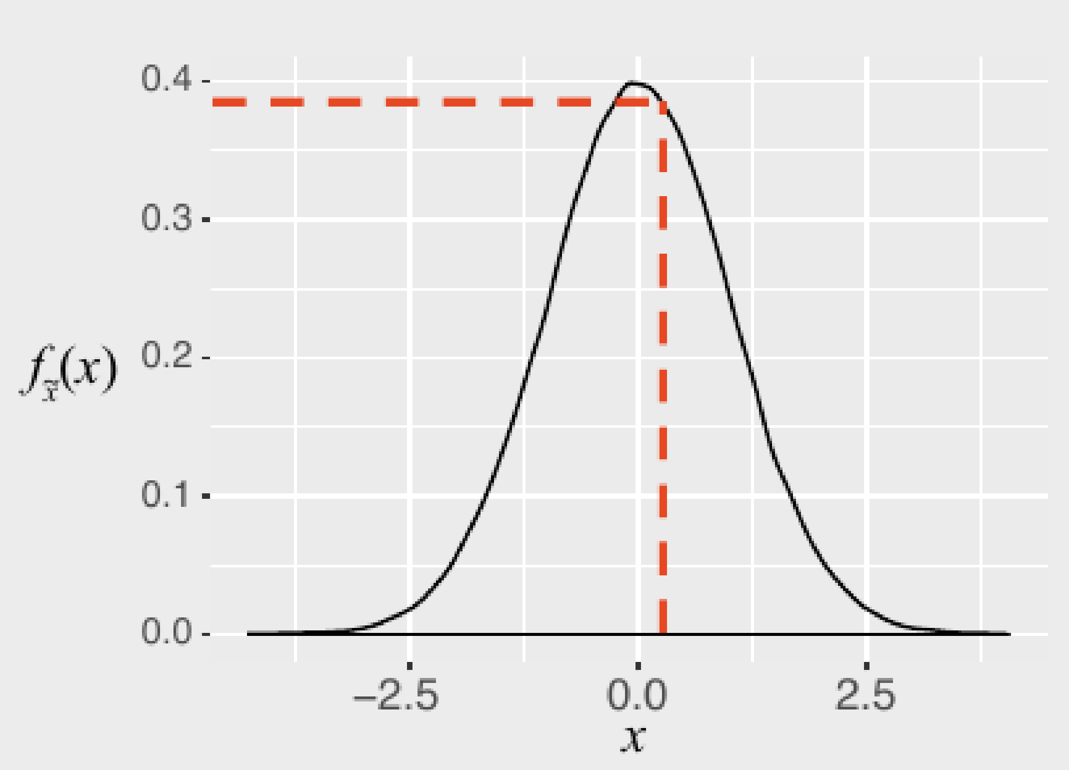
Computing the likelihood



x: an observation

$f(x)$: probability of observing x

Computing the likelihood



x: an observation

$f(x)$: probability of observing x

$\mathcal{L}(\theta|x)$: probability that θ are the true parameters, given that observed x

$$\mathcal{L}(\theta|x) = f(x_1)f(x_2)\dots f(x_n)$$

Log-likelihood converts multiplication to summation:

$$\ln \mathcal{L}(\theta|x) = \ln f(x_1) + \ln f(x_2)\dots \ln f(x_n)$$

Practice Question 2

Observations - Height of students (inches):

```
#>  [1] 65 69 66 67 68 72 68 69 63 70
```

a) Let's say we know that the height of students, $\tilde{x}$, in a classroom follows a normal distribution. A professor obtains the above height measurements students in her classroom. What is the log-likelihood that $\tilde{x} \sim \mathcal{N}(68, 4)$? In other words, compute $\ln \mathcal{L}(\mu = 68, \sigma = 4)$.

b) Compute the log-likelihood function using the same standard deviation $(\sigma = 4)$ but with the following different values for the mean, $\mu : 66, 67, 68, 69, 70$. How do the results compare? Which value for μ produces the highest log-likelihood?

Optimization

Optimality conditions

<u>First order necessary condition</u>

x^* is a "stationary point" when

$$\frac{df(x^*)}{dx} = 0$$

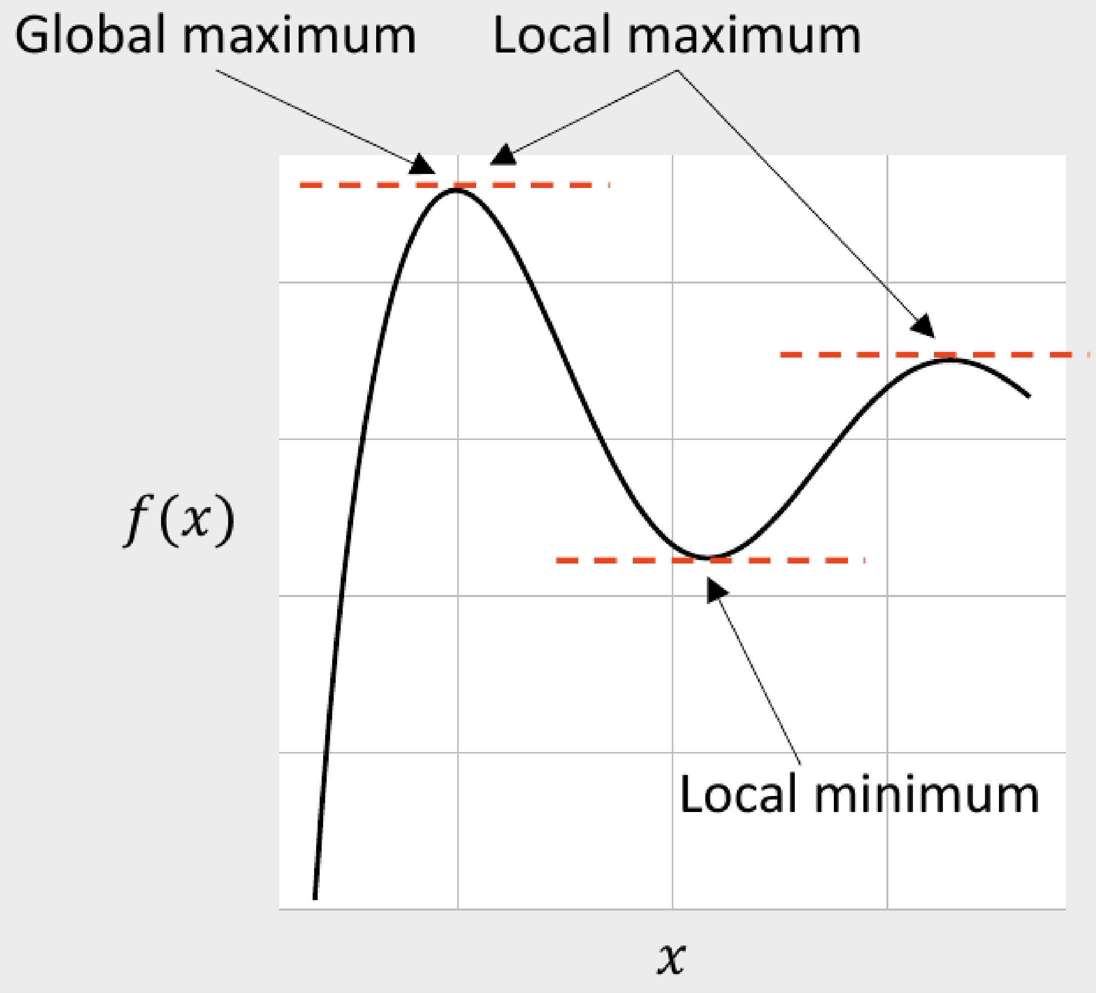
<u>Second order sufficiency condition</u>

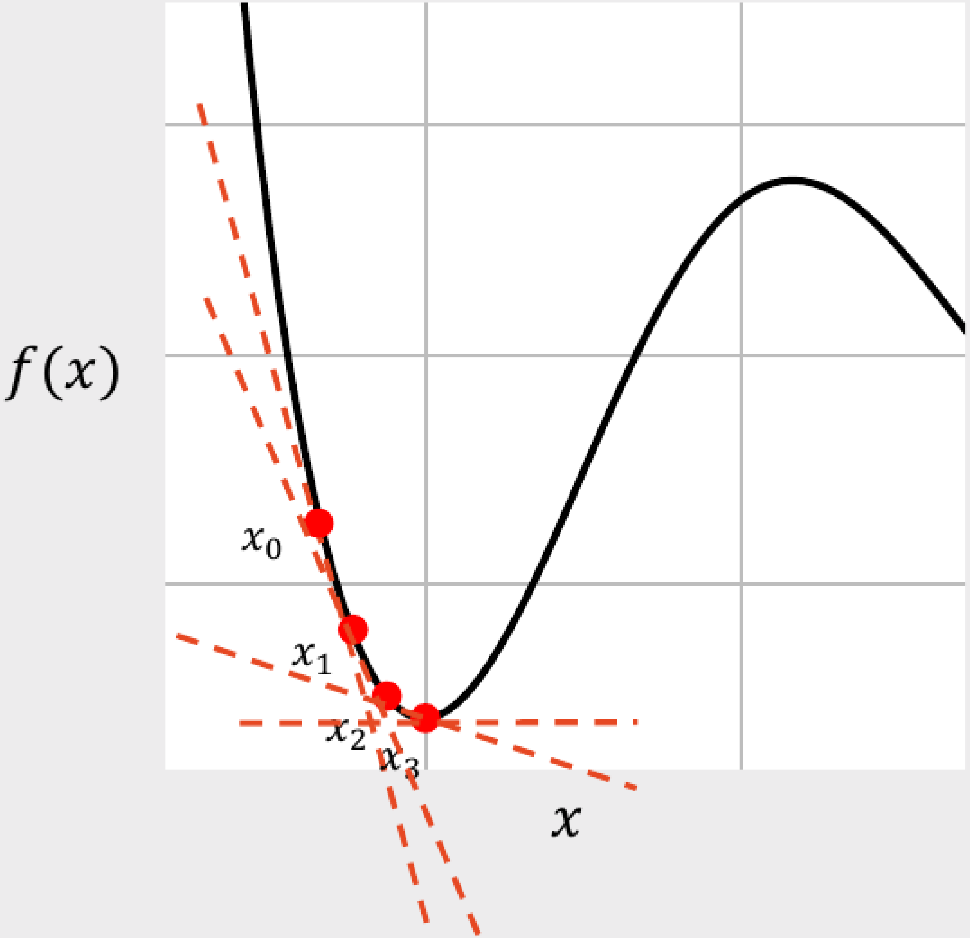
x^* is a local *maximum* when

$$\frac{d^2f(x^*)}{dx^2} < 0$$

x^* is a local *minimum* when

$$\frac{d^2f(x^*)}{dx^2} > 0$$





<u>Gradient Descent Method</u>:

1. Choose a starting point, x_0
2. At that point, compute the gradient, $\nabla f(x_0)$
3. Compute the next point, with a step size γ :

$$x_{n+1} = x_n - \gamma \nabla f(x_n)$$

Very small number

*Stop when $\nabla f(x_n) < \delta$

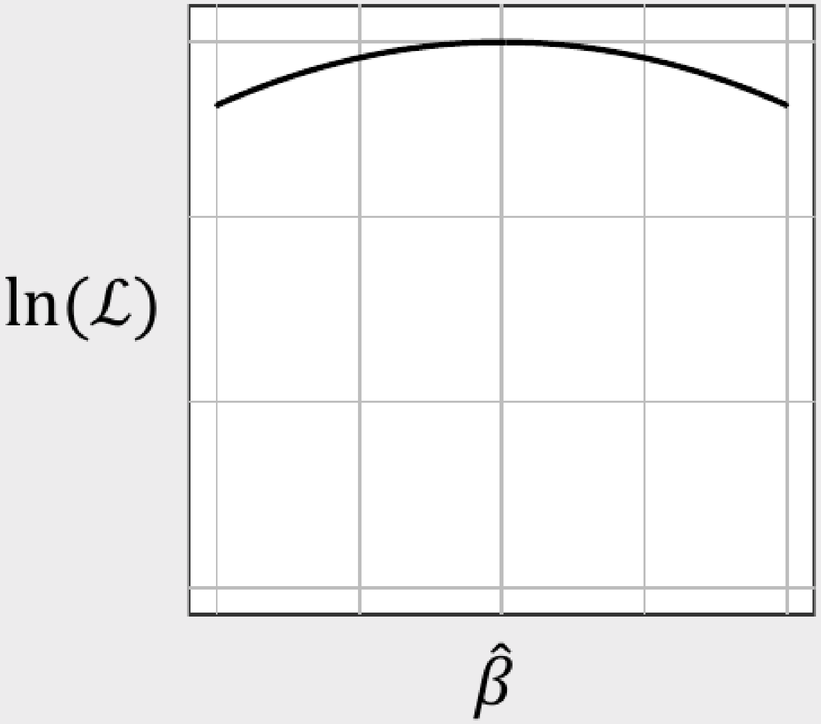
or

*Stop when $(x_{n+1} - x_n) < \delta$

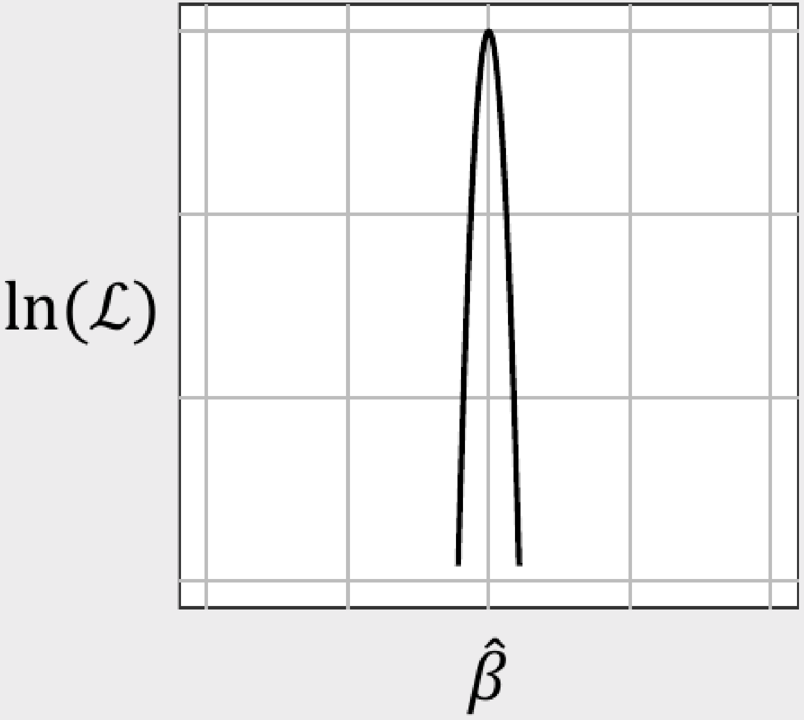
Uncertainty

The certainty of $\widehat{\boldsymbol{\beta}}$ is inversely related to the curvature of the log-likelihood function

Greater variance in $\ln(\mathcal{L})$,
Less certainty in $\hat{\beta}$



Less variance in $\ln(\mathcal{L})$,
Greater certainty in $\hat{\beta}$



The *curvature* of the log-likelihood function is inversely related to the hessian

$$\sum_{\boldsymbol{\beta}} = -\left[\underbrace{\nabla^2_{\boldsymbol{\beta}} \ln(\mathcal{L})}_{\text{Hessian}}\right]^{-1}$$

Covariance of $\widehat{\boldsymbol{\beta}}$

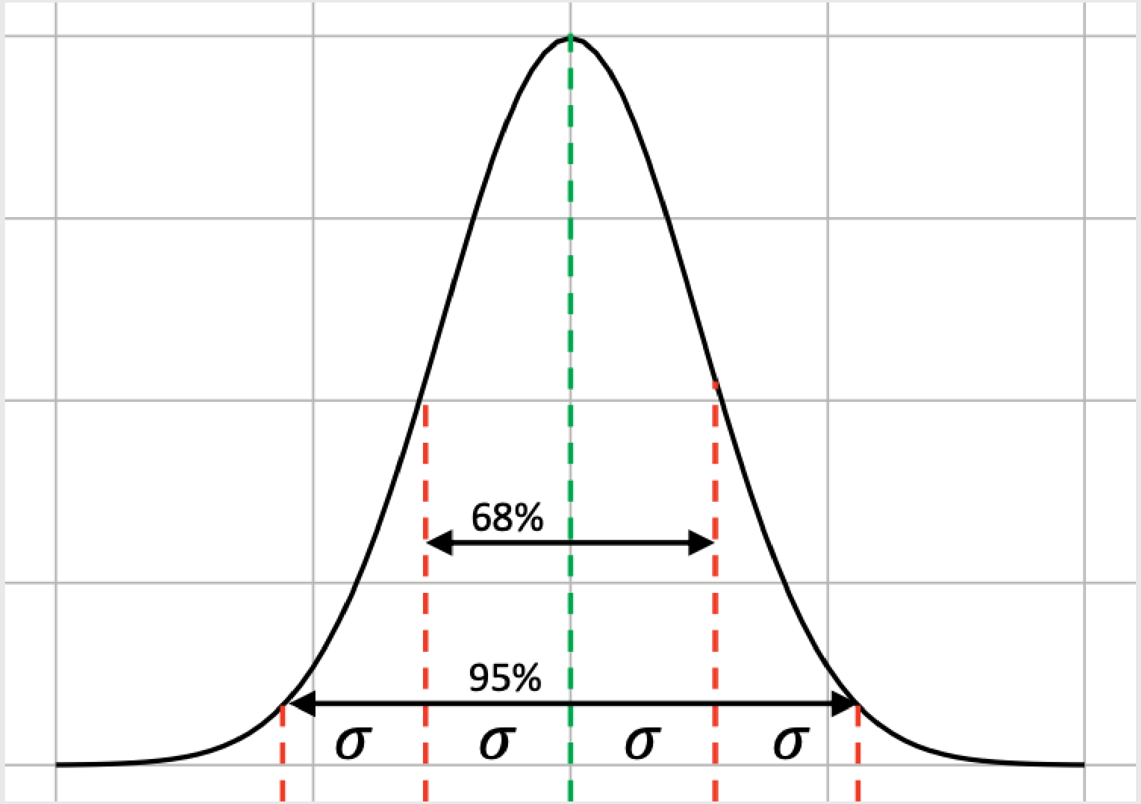
The *curvature* of the log-likelihood function is inversely related to the hessian

$$\sum\nolimits_{\boldsymbol{\beta}} = -\overbrace{\left[\nabla^2_{\boldsymbol{\beta}} \ln(\mathcal{L})\right]}^{\text{Hessian}}{}^{-1} = \begin{bmatrix} \sigma^2_{11} & \cdots & \sigma^2_{m1} \\ \vdots & \ddots & \vdots \\ \sigma^2_{1n} & \cdots & \sigma^2_{mn} \end{bmatrix}$$

Covariance of $\widehat{\boldsymbol{\beta}}$

Usually report parameter uncertainty ("standard errors") with σ values

Est.	Std. Err.
$\hat{\beta}_1$	σ_1
$\hat{\beta}_2$	σ_2
$\vdots$	$\vdots$
$\hat{\beta}_m$	σ_m



$\hat{\beta}$

A 95% confidence interval is approximately $[\hat{\beta} - 2\sigma, \hat{\beta} + 2\sigma]$

Two approaches for obtaining confidence interval

Using Standard Errors

1. Get coefficients, `beta`
2. Get covariance matrix, `covariance`
3. `se = sqrt(diag(covariance))`
4. `coef_ci = c(beta - 2*se, beta + 2*se)`

Using Simulated Draws

1. Get coefficients, `beta`
2. Get covariance matrix, `covariance`
3. `draws <- as.data.frame(MASS::mvrnorm(10^5, beta, covariance))`
4. `coef_ci <- ci(draws, ci = 0.95)`

In-class example

```r
# 1. Get coefficients
beta <- c(
    price = -0.7, mpg = 0.1, elec = -4.0)

# 2. Get covariance matrix
hessian <- matrix(c(
    -6000,   50,   60,
       50, -700,   50,
       60,   50, -300),
    ncol = 3, byrow = TRUE)

covariance <- -1*solve(hessian)
```

Model from `logitr`

```r
beta <- coef(model)
covariance <- vcov(model)
```

Practice Question 3

Suppose we estimate the following utility model describing preferences for cars:

$$u_j = \alpha p_j + \beta_1 x_j^{mpg} + \beta_2 x_j^{elec} + \varepsilon_j$$

Compute a 95% confidence interval around the coefficients using:

a) Standard errors b) Simulated draws

The estimated model produces the following results:

Parameter	Coefficient
α	-0.7
β_1	0.1
β_2	-0.4

Hessian:

$$\begin{bmatrix} -6000 & 50 & 60 \\ 50 & -700 & 50 \\ 60 & 50 & -300 \end{bmatrix}$$

Design of experiment

Wine Pairings Example

meat	wine
fish	white
fish	red
steak	white
steak	red

Main Effects

1. **Fish** or **Steak**?
2. **Red** or **White** wine?

Interaction Effects

1. **Red** or **White** wine *with **Steak***?
2. **Red** or **White** wine *with **Fish***?

"D-optimal" designs maximize **main** effect information but confound **interaction** effect information

$$D = \left(\frac{|\boldsymbol{I}(\boldsymbol{\beta})|}{n^p} \right)^{1/p}$$

where p is the number of coefficients in the model and n is the total sample size

WTP

Willingness to Pay (WTP)

$$\tilde{u}_j = \alpha p_j + \boldsymbol{\beta} x_j + \tilde{\varepsilon}_j$$

$$\boldsymbol{\omega} = \frac{\boldsymbol{\beta}}{-\alpha}$$

Computing WTP with draws

$$\hat{\omega} = \frac{\hat{\beta}}{-\hat{\alpha}}$$

```
draws_other <- draws[,2:ncol(draws)]
draws_price <- draws[,1]
draws_wtp <- draws_other / (-1*draws_price)
head(draws_wtp)
```

```
#>              [,1]      [,2]
#> [1,] 0.19349651 -5.759953
#> [2,] 0.18011082 -5.582842
#> [3,] 0.10050502 -5.478047
#> [4,] 0.04873818 -5.918177
#> [5,] 0.16091506 -5.743675
#> [6,] 0.14522956 -5.723287
```

Mean WTP with confidence interval

```
maddTools::ci(draws_wtp)
```

```
#>         mean       lower      upper
#> 1  0.1434948  0.03485731  0.2519324
#> 2 -5.7178574 -5.98148271 -5.4708053
```

Willingness to Pay (WTP)

"Preference Space"

$$\tilde{u}_j = \alpha p_j + \boldsymbol{\beta} x_j + \tilde{\varepsilon}_j$$

"WTP Space"

$$\boldsymbol{\omega} = \frac{\boldsymbol{\beta}}{-\alpha}$$

$$\boldsymbol{\lambda} = -\alpha$$

$$\tilde{u}_j = \boldsymbol{\lambda}(\boldsymbol{\omega} x_j - p_j) + \tilde{\varepsilon}_j$$

WTP space models have non-convex log-likelihood functions!

Use multi-start loop with random starting points

Market simulations

Simulate Market Shares

1. Define a market, X
2. Compute shares:

$$\hat{P}_j = \frac{e^{\hat{\boldsymbol{\beta}}' \boldsymbol{X}_j}}{\sum_{k=1}^{J} e^{\hat{\boldsymbol{\beta}}' \boldsymbol{X}_k}}$$

Simulate Market Shares

$$\hat{v} = \widehat{\boldsymbol{\beta}}'\mathbf{x}$$

$$= \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \vdots & \vdots & \dots & \vdots \\ x_{J1} & x_{J2} & \dots & x_{Jn} \end{bmatrix} \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \vdots \\ \hat{\beta}_n \end{bmatrix}$$

$$= \begin{bmatrix} \hat{\beta}_1 x_{11} + \hat{\beta}_2 x_{12} + \dots + \hat{\beta}_n x_{1n} \\ \hat{\beta}_1 x_{21} + \hat{\beta}_2 x_{22} + \dots + \hat{\beta}_n x_{2n} \\ \vdots \\ \hat{\beta}_1 x_{J1} + \hat{\beta}_2 x_{J2} + \dots + \hat{\beta}_n x_{Jn} \end{bmatrix} = \begin{bmatrix} \hat{v}_1 \\ \hat{v}_2 \\ \vdots \\ \hat{v}_J \end{bmatrix}$$

Simulate Market Shares

$$\hat{v} = \widehat{\boldsymbol{\beta}}'\mathbf{x}$$

$$= \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ x_{J1} & x_{J2} & \cdots & x_{Jn} \end{bmatrix} \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \vdots \\ \hat{\beta}_n \end{bmatrix}$$

$$= \begin{bmatrix} \hat{\beta}_1 x_{11} + \hat{\beta}_2 x_{12} + \ldots + \hat{\beta}_n x_{1n} \\ \hat{\beta}_1 x_{21} + \hat{\beta}_2 x_{22} + \ldots + \hat{\beta}_n x_{2n} \\ \vdots \\ \hat{\beta}_1 x_{J1} + \hat{\beta}_2 x_{J2} + \ldots + \hat{\beta}_n x_{Jn} \end{bmatrix} = \begin{bmatrix} \hat{v}_1 \\ \hat{v}_2 \\ \vdots \\ \hat{v}_J \end{bmatrix}$$

In R:

```
X %*% beta
```

Simulating Market Shares with Uncertainty

Rely on the `predict()` function to compute shares with uncertainty.

Internally, it:

1. Takes draws of $\boldsymbol{\beta}$
2. Computes P_j for each draw
3. Returns mean and confidence interval computed from draws

Review the `logitr-cars` examples

Your Turn

15:00

As a team:

- Read in and clean your final data.
- Estimate a baseline model.
- Set your baseline market simulation case.
- Compute sensitivities to price and other attributes.