3D CSEM modelling in the frequency and Laplace domains

One story of success and one of failure

Dieter Werthmüller

5th December 2019

Thursday Talk – Applied Geophysics & Petrophysics

TU Delft

Faculty of Civil Engineering and Geosciences > Department of Geoscience & Engineering > Section of Applied Geophysics & Petrophysics

The Project

GitaroJIM.eu

MarTERA.eu

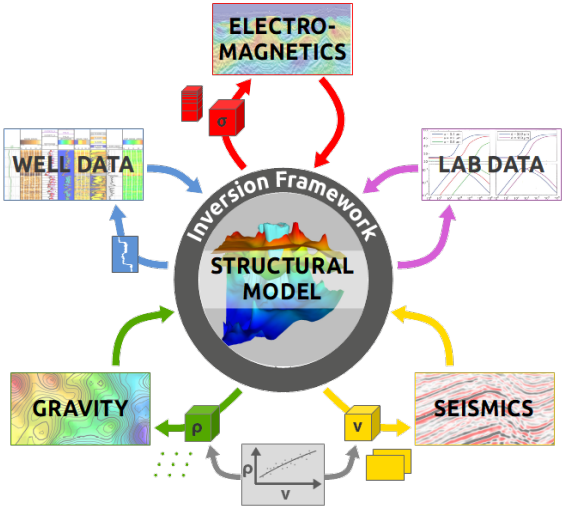
Project duration: 3 years (June 2018 – May 2021)

Gitaro.JIM – Partners

Supported by University of Southampton and Equinor

Member of the Delphi Consortium

Gitaro.JIM – Objectives

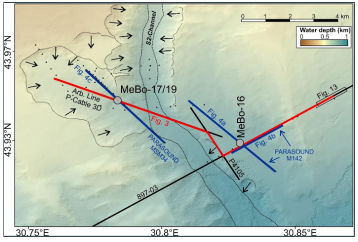


Gitaro.JIM – Data

Methane Hydrates

Danube Paleo Delta

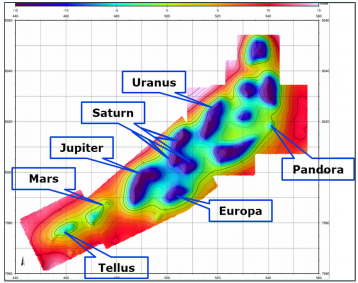
Black Sea



Oil & Gas

Nordkapp Basin

Barents Sea



Seafloor Massive Sulfides

Palinuro

Tyrrhenian Sea

- 3D CSEM modelling; frequency domain
- Comparison to existing code; frequency domain and time domain
- Inversion
- Anisotropy
- Topopgraphy

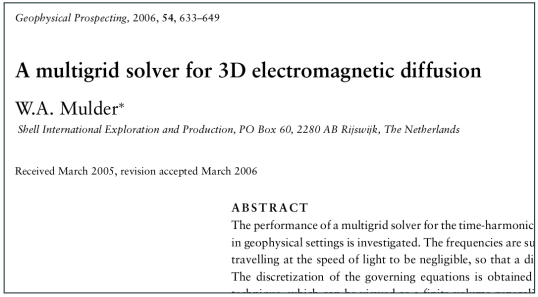
The Code

Multigrid CSEM code by Mulder et al.

W.A. Mulder, T.B. Jönsthövel, M. Wirianto, E.C. Slob, R.-E. Plessix, ...

$$-s\sigma\hat{\mathbf{E}} - \nabla \times \mu^{-1}\nabla \times \hat{\mathbf{E}} = s\hat{\mathbf{J}}$$

- Matlab/C
- Multigrid
- Finite Volumes
- VTI resistivity



Geophysical Prospecting, 2006, **54**, 633–649

A multigrid solver for 3D electromagnetic diffusion

W.A. Mulder*
Shell International Exploration and Production, PO Box 60, 2280 AB Rijswijk, The Netherlands

Received March 2005, revision accepted March 2006

ABSTRACT
The performance of a multigrid solver for the time-harmonic
in geophysical settings is investigated. The frequencies are su
travelling at the speed of light to be negligible, so that a di
The discretization of the governing equations is obtained

emg3d – Open-source multigrid solver for 3D EM diffusion

- Python/Numba
- Apache v2.0
- empymod.github.io

Installation

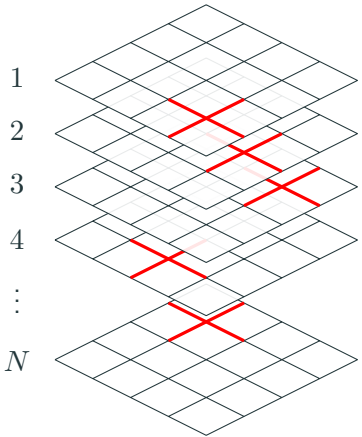
conda: `conda install -c conda-forge emg3d`

pip: `pip install emg3d`

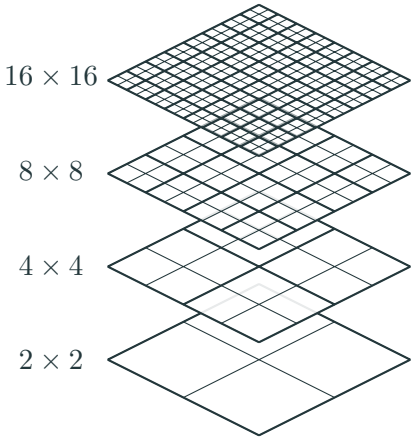
Requirements: `NumPy`, `SciPy`, `Numba`, `empymod`; `Python 3.7+`

Multigrid method

Matrix-free solver

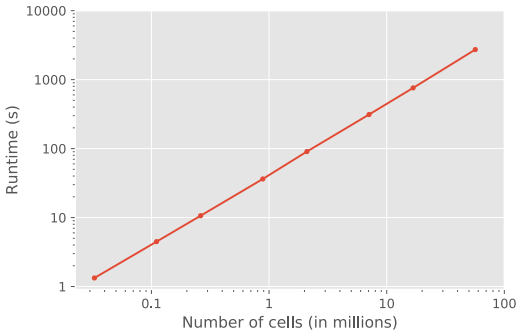


Restriction and prolongation

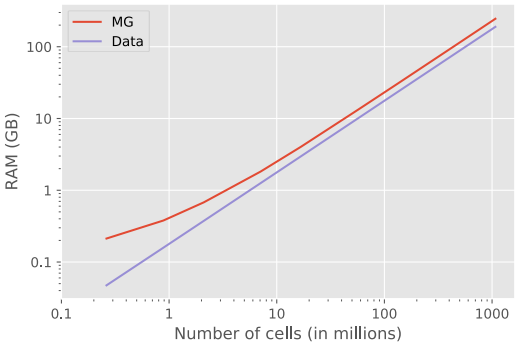


Multigrid – CPU & RAM

CPU



RAM



Measured on `texel`, single thread

Frequency Domain

emg3d – Minimal Example

```
import emg3d
import discretize

grid = discretize.TensorMesh(
        [[(25, 48)], [(50, 32)], [(30, 32)]],
        x0='CCC')

model = emg3d.utils.Model(
        grid, res_x=1.5, res_y=1.8, res_z=3.3)

sfield = emg3d.utils.get_source_field(
        grid, src=[0, 0, 0, 0, 0], freq=10)

efield = emg3d.solver.solver(
        grid, model, sfield, verb=3)
```

emg3d – Minimal Example

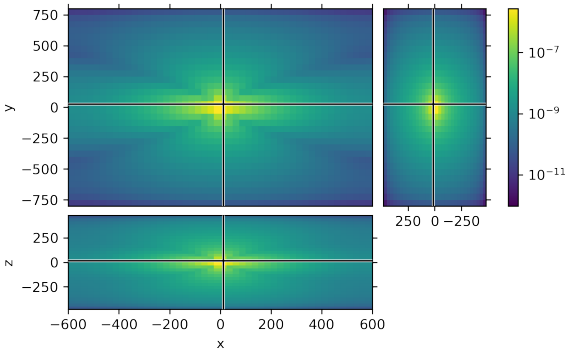
```python
import emg3d
import discretize

grid = discretize.TensorMesh(
        [[(25, 48)], [(50, 32)], [(30, 32)]],
        x0='CCC')

model = emg3d.utils.Model(
        grid, res_x=1.5, res_y=1.8, res_z=3.3)

sfield = emg3d.utils.get_source_field(
        grid, src=[0, 0, 0, 0, 0], freq=10)

efield = emg3d.solver.solver(
        grid, model, sfield, verb=3)
```

```
:: emg3d START :: 10:28:27 :: v0.9.1

   MG-cycle       : 'F'                 sslsolver : False
   semicoarsening : False [0]           tol       : 1e-06
   linerelaxation : False [0]           maxit     : 50
   nu_{i,1,c,2}   : 0, 2, 1, 2          verb      : 3
   Original grid  :  48 x  32 x  32     => 49,152 cells
   Coarsest grid  :   3 x   2 x   2     => 12 cells
   Coarsest level :   4 ;   4 ;   4

   [hh:mm:ss]  rel. error                  [abs. error, last/prev]   l s

      h_
     2h_ \                  /
     4h_  \          /\    /
     8h_   \    /\  /  \  /
    16h_    \/\/  \/    \/

   [10:28:27]   2.623e-02  after   1 F-cycles   [1.464e-06, 0.026]   0 0
   [10:28:28]   2.253e-03  after   2 F-cycles   [1.258e-07, 0.086]   0 0
   [10:28:28]   3.051e-04  after   3 F-cycles   [1.704e-08, 0.135]   0 0
   [10:28:29]   5.501e-05  after   4 F-cycles   [3.071e-09, 0.180]   0 0
   [10:28:29]   1.170e-05  after   5 F-cycles   [6.532e-10, 0.213]   0 0
   [10:28:30]   2.745e-06  after   6 F-cycles   [1.533e-10, 0.235]   0 0
   [10:28:30]   6.874e-07  after   7 F-cycles   [3.838e-11, 0.250]   0 0

   > CONVERGED
   > MG cycles        : 7
   > Final rel. error : 6.874e-07

:: emg3d END   :: 10:28:30 :: runtime = 0:00:03
```

emg3d – Minimal Example

```
import emg3d
import discretize

grid = discretize.TensorMesh(
        [[(25, 48)], [(50, 32)], [(30, 32)]],
        x0='CCC')

model = emg3d.utils.Model(
        grid, res_x=1.5, res_y=1.8, res_z=3.3)

sfield = emg3d.utils.get_source_field(
        grid, src=[0, 0, 0, 0, 0], freq=10)

efield = emg3d.solver.solver(
        grid, model, sfield, verb=3)
```

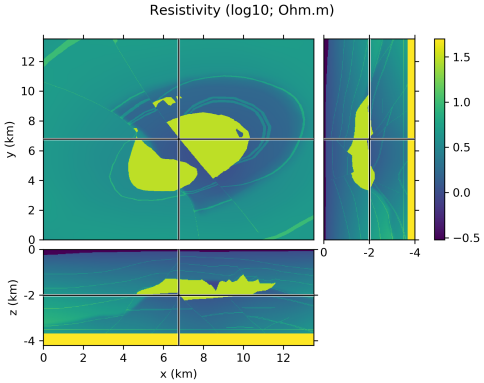
```
grid.plot_3d_slicer(
      efield.fx.ravel('F'), view='abs', vType='Ex',
      pcolorOpts={'norm': LogNorm()})
```



emg3d – SEG-EAGE salt model †

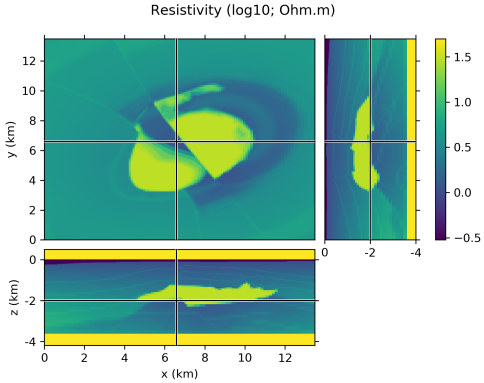
≈ 96 million cells (only model)

$(20\text{m} \times 20\text{m} \times 20\text{m})$



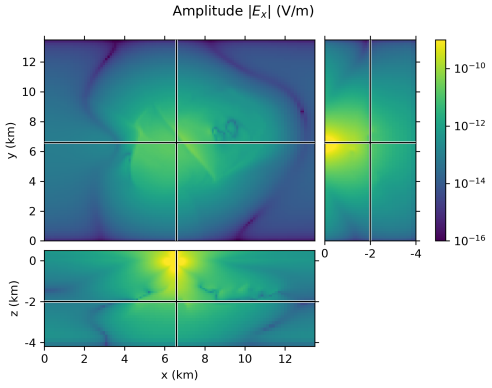
≈ 2.1 million cells (+ air & boundary)

(x/y: $55 - 182\,\text{m}$; z: $20 - 517\,\text{m}$)



† SEG-EAGE salt model: Aminzadeh et al. (1997); Velocity-to-resistivity conversion: Mulder (2007).

emg3d – SEG-EAGE salt model †




```
:: emg3d START :: 10:36:50 :: v0.9.1

   MG-cycle       : 'F'                 sslsolver : 'bicgstab'
   semicoarsening : False [0]           tol       : 1e-06
   linerelaxation : False [0]           maxit     : 50 (1)
   nu_{i,1,c,2}   : 0, 2, 1, 2          verb      : 3
   Original grid  : 128 x 128 x 128     => 2,097,152 cells
   Coarsest grid  :   2 x   2 x   2     => 8 cells
   Coarsest level :   6 ;   6 ;   6

   [hh:mm:ss]  rel. error            solver              MG          l s

   [10:37:03]   1.591e-01  after                        1 F-cycles    0 0
   [10:37:15]   3.299e-02  after                        2 F-cycles    0 0
   [10:37:16]   2.291e-02  after   1 bicgstab-cycles
   [10:37:26]   6.756e-03  after                        3 F-cycles    0 0
   [10:37:37]   1.035e-03  after                        4 F-cycles    0 0
   [10:37:38]   8.113e-04  after   2 bicgstab-cycles
   [10:37:51]   3.055e-04  after                        5 F-cycles    0 0
   [10:38:01]   6.347e-05  after                        6 F-cycles    0 0
   [10:38:02]   4.581e-05  after   3 bicgstab-cycles
   [10:38:14]   1.753e-05  after                        7 F-cycles    0 0
   [10:38:27]   4.368e-06  after                        8 F-cycles    0 0
   [10:38:28]   3.969e-06  after   4 bicgstab-cycles
   [10:38:43]   2.619e-06  after                        9 F-cycles    0 0
   [10:38:55]   1.626e-06  after                       10 F-cycles    0 0
   [10:38:56]   1.127e-06  after   5 bicgstab-cycles
   [10:39:07]   8.366e-07  after                       11 F-cycles    0 0
   [10:39:08]   7.448e-07  after   6 bicgstab-cycles

   > CONVERGED
   > Solver steps      : 6
   > MG prec. steps    : 11
   > Final rel. error : 7.448e-07

:: emg3d END   :: 10:39:08 :: runtime = 0:02:18
```

Time Domain

Plessix et al., 2007 & Mulder et al., 2008

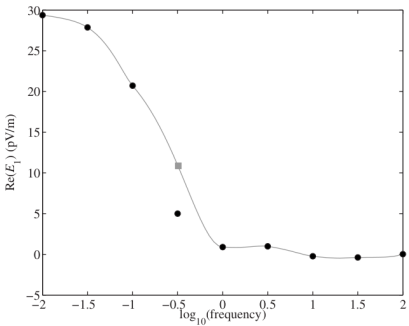
Automatic gridding

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} \approx \quad 503.3/\sqrt{f\sigma}\,,$$

- Minimum cell width $\mathrm{func}(\delta_\mathrm{s}, L_\mathrm{s}, \mathrm{pps})$
- Domains: $\mathrm{pps} \cdot \delta_\mathrm{ave}$
  - Core domain: $\mathrm{pps} \approx 4 - 8\ (3 - 8)$
  - Boundary domain: $\mathrm{pps} \approx 4\ (5)$
  - Air (upwards z): 50 km
- Mapping by volume averaging
- **Find optimal α for a fixed #cells**

Adaptive frequency selection

- Start: $f_\mathrm{min} - f_\mathrm{max}$, 1/dec
- Add more frequencies left and right of f_i if $|E_\mathrm{int}(f_i) - E_\mathrm{calc}(f_i)| > \mathrm{tol}$



Changes to existing schemes

(a) code

- Flexible #cell, in each direction
- Mult. with any low prime, $\{2, 3, 5\} \cdot 2^n$
  e.g., $\mathbf{64}, 80, 96, \mathbf{128}, 160, 192, \mathbf{256}$

(b) automatic gridding

- Define core domain
- **Search for minimal number of cells**
  - α-range for core domain
  - α-range for boundary domain
- Careful regarding the air layer

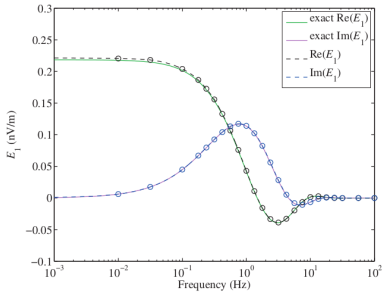
(c) f-selection and Fourier transform

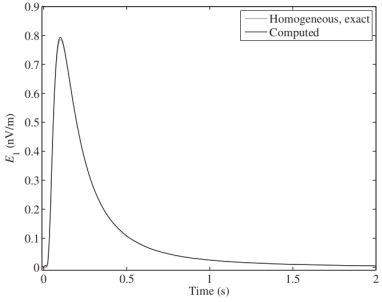
- Regular spacing (log-scale)
- A logarithmic Fourier transform
  - Digital Linear Filters (DLF)[†]
  - FFTLog [‡]

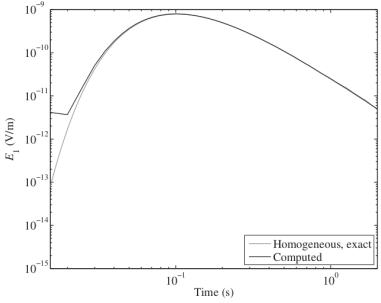
[†] Ghosh (1970); [‡] Hamilton (2000)

Mulder et al., 2008; Figures 2-4; homogeneous fullspace

- 26 frequencies, 0.01 Hz−100 Hz
- Runtime: 3h 47 min 12 s
- The peak value has an error of about 1 %

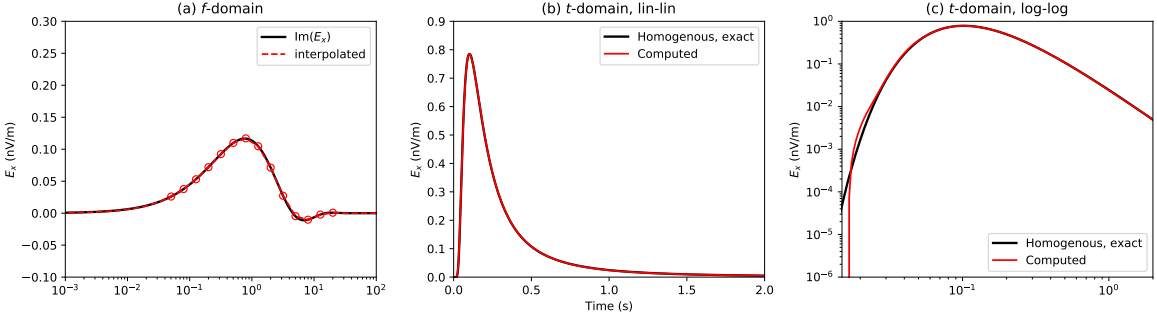






Our result, 2019

- 14 frequencies, 0.05 Hz−20 Hz
- Runtime: 1 min 40 s
- The peak value has an error of about 0.01 %



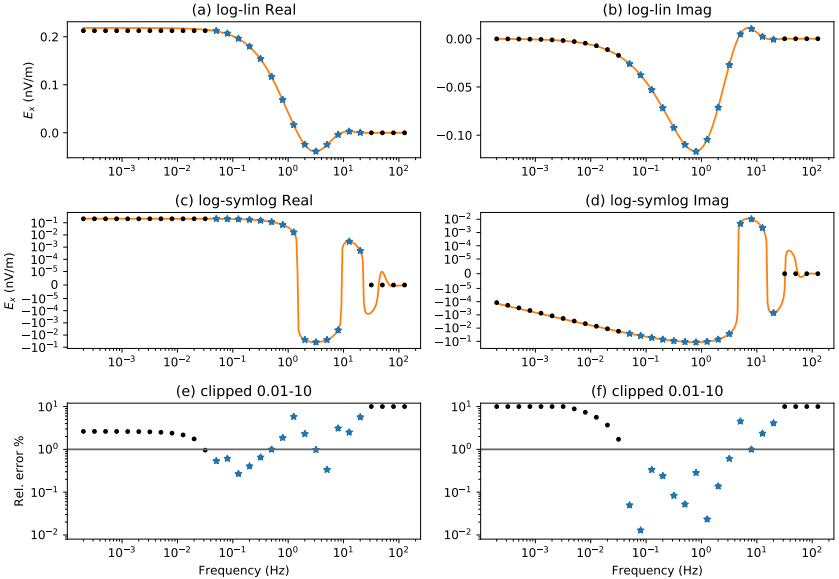
Analysis – Overview and gridding

Main reasons for speed-up of 136x:

- ++ Automatic gridding; transform; cell sizes
- + Computers faster in the last 11 years
- ≈ Different implementation (Matlab/C vs Python/Numba)

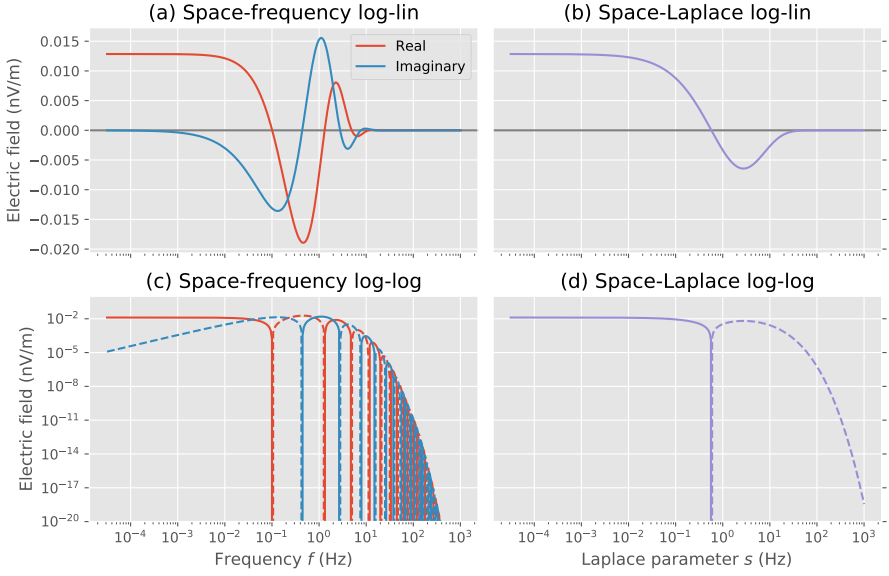
```
**** TOTAL RUNTIME :: 0:01:40 ****

20.036 Hz:  4/1 it;   4 s;  a: 1.00 / 1.18;  n: 80 x 24 x 24;  d: 20 /  138
12.642 Hz:  7/1 it;   6 s;  a: 1.00 / 1.22;  n: 80 x 24 x 24;  d: 20 /  192
 7.977 Hz:  7/1 it;   6 s;  a: 1.00 / 1.25;  n: 80 x 24 x 24;  d: 20 /  265
 5.033 Hz:  7/1 it;   6 s;  a: 1.00 / 1.29;  n: 80 x 24 x 24;  d: 20 /  364
 3.176 Hz:  7/1 it;   5 s;  a: 1.00 / 1.30;  n: 80 x 24 x 24;  d: 24 /  422
 2.004 Hz:  7/1 it;   4 s;  a: 1.00 / 1.30;  n: 64 x 24 x 24;  d: 30 /  531
 1.264 Hz:  7/1 it;   4 s;  a: 1.00 / 1.30;  n: 64 x 24 x 24;  d: 37 /  669
 0.798 Hz:  7/1 it;   7 s;  a: 1.00 / 1.20;  n: 64 x 24 x 24;  d: 40 /  616
 0.503 Hz:  7/1 it;   7 s;  a: 1.00 / 1.23;  n: 64 x 24 x 24;  d: 40 /  893
 0.318 Hz:  7/1 it;   9 s;  a: 1.00 / 1.25;  n: 64 x 24 x 24;  d: 40 / 1137
 0.200 Hz:  7/1 it;   7 s;  a: 1.00 / 1.28;  n: 64 x 24 x 24;  d: 40 / 1623
 0.126 Hz:  7/1 it;   7 s;  a: 1.00 / 1.30;  n: 64 x 24 x 24;  d: 40 / 2047
 0.080 Hz:  7/1 it;  13 s;  a: 1.00 / 1.25;  n: 64 x 32 x 32;  d: 40 / 2220
 0.050 Hz:  7/1 it;  12 s;  a: 1.00 / 1.27;  n: 64 x 32 x 32;  d: 40 / 2955
```

Analysis – Frequency Calculation



Laplace Domain

Laplace s: $-s\sigma\hat{\mathbf{E}} - \nabla \times \mu^{-1}\nabla \times \hat{\mathbf{E}} = s\hat{\mathbf{J}} \quad \Rightarrow \quad s = \mathrm{i}\omega = 2\mathrm{i}\pi f$ **or** s



Laplace – faster calculation, faster convergence

Frequency domain

- 9–10 s per F-cycle
- 5 BiCGSTAB cycles; 9 MG cycles
- 1 min 32 s

```
   [22:03:37]   1.565e-01  after                        1 F-cycles    0 0
   [22:03:47]   2.998e-02  after                        2 F-cycles    0 0
   [22:03:47]   2.047e-02  after   1 bicgstab-cycles
   [22:03:57]   5.506e-03  after                        3 F-cycles    0 0
   [22:04:07]   6.357e-04  after                        4 F-cycles    0 0
   [22:04:08]   5.097e-04  after   2 bicgstab-cycles
   [22:04:18]   1.757e-04  after                        5 F-cycles    0 0
   [22:04:28]   2.832e-05  after                        6 F-cycles    0 0
   [22:04:28]   1.956e-05  after   3 bicgstab-cycles
   [22:04:38]   6.460e-06  after                        7 F-cycles    0 0
   [22:04:48]   1.568e-06  after                        8 F-cycles    0 0
   [22:04:49]   1.375e-06  after   4 bicgstab-cycles
   [22:04:58]   8.652e-07  after                        9 F-cycles    0 0
   [22:04:59]   7.268e-07  after   5 bicgstab-cycles

   > CONVERGED
   > Solver steps     : 5
   > MG prec. steps   : 9
   > Final rel. error : 7.268e-07

:: emg3d END   :: 22:04:59 :: runtime = 0:01:32
```

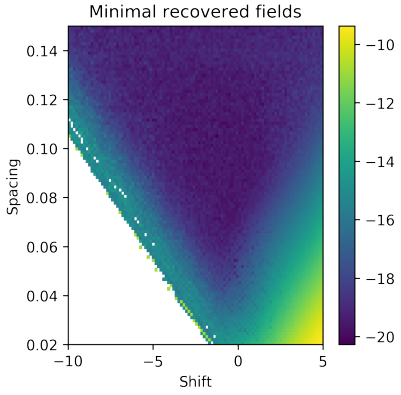
Laplace domain

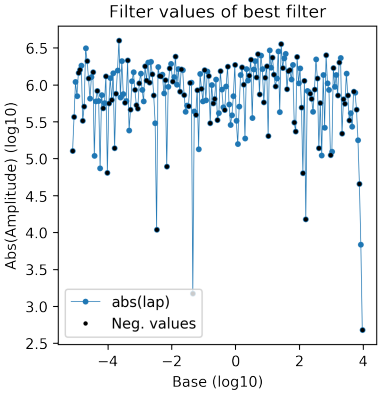
- 7–8 s per F-cycle
- 4 BiCGSTAB cycles; 8 MG cycles
- 1 min 2 s $\Rightarrow$ 1/3 faster!

```
   [22:00:48]   1.508e-01  after                        1 F-cycles    0 0
   [22:00:56]   2.805e-02  after                        2 F-cycles    0 0
   [22:00:56]   1.900e-02  after   1 bicgstab-cycles
   [22:01:04]   4.924e-03  after                        3 F-cycles    0 0
   [22:01:11]   5.519e-04  after                        4 F-cycles    0 0
   [22:01:12]   4.401e-04  after   2 bicgstab-cycles
   [22:01:19]   1.468e-04  after                        5 F-cycles    0 0
   [22:01:27]   2.277e-05  after                        6 F-cycles    0 0
   [22:01:27]   1.551e-05  after   3 bicgstab-cycles
   [22:01:35]   4.829e-06  after                        7 F-cycles    0 0
   [22:01:42]   8.981e-07  after                        8 F-cycles    0 0
   [22:01:43]   7.426e-07  after   4 bicgstab-cycles

   > CONVERGED
   > Solver steps     : 4
   > MG prec. steps   : 8
   > Final rel. error : 7.426e-07

:: emg3d END   :: 22:01:43 :: runtime = 0:01:02
```

DLF Laplace-to-time

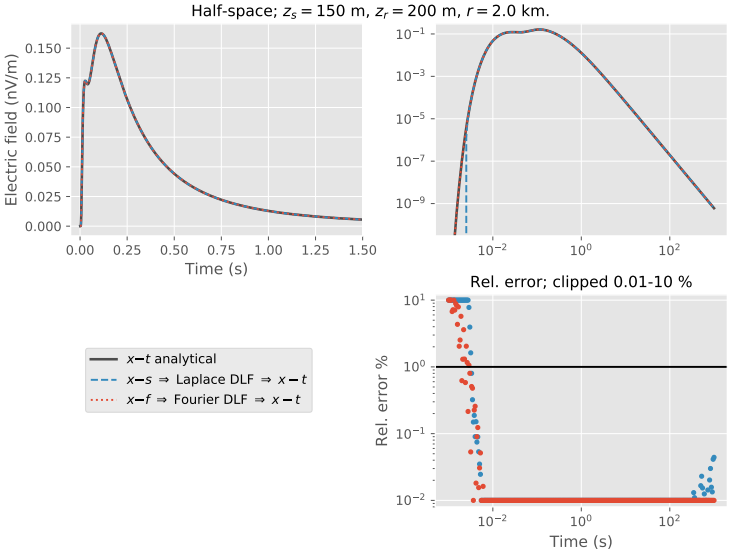
$$F(r) = \int_0^\infty f(l)K(lr)\mathrm{d}l \qquad \xrightarrow{r = e^x, l = e^{-y}} \qquad F(r) \approx \sum_{n=1}^{N} \frac{f(b_n/r)h_n}{r}$$



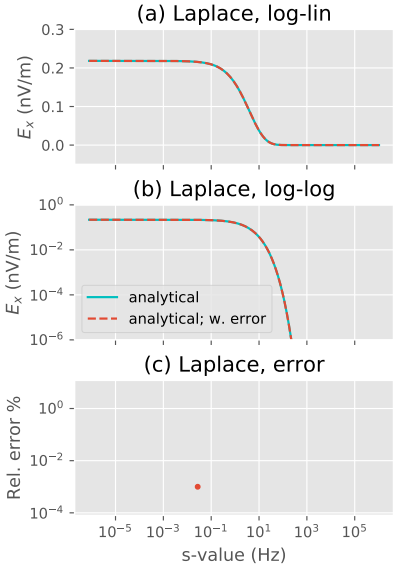


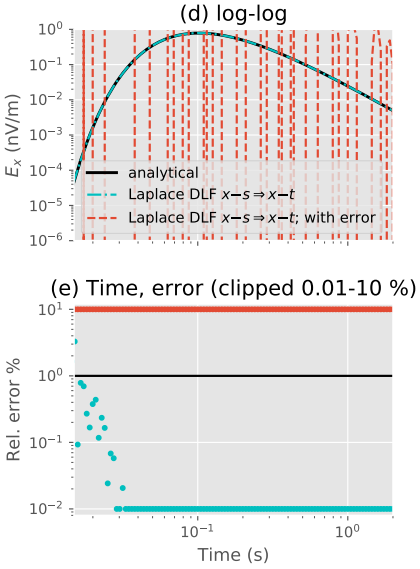
† DLF-method: Ghosh (1970); Used tool: Werthmüller et al. (2019)

Test using empymod



Laplace – It doesn't work





Summary

Summary

- **emg3d**
- **Frequency-domain** calculation
- **Laplace-domain** calculation
- **Time-domain** calculation
  - Significant speed-up using the f-domain
  - Not working using the s-domain

Slides: **github.com/empymod** $\Rightarrow$ presentations $\Rightarrow$ ThursdayTalk2019

References

Aminzadeh, F., Brac, J., and Kunz, T., 1997, **SEG/EAGE 3-D Salt and Overthrust Models:** SEG, Tulsa, Oklahoma; wiki.seg.org/wiki/SEG/EAGE_Salt_and_Overthrust_Models.

Ghosh, D. P., 1970, **The application of linear filter theory to the direct interpretation of geoelectrical resistivity measurements:** Ph.D. Thesis, TU Delft; uuid: 88a568bb-ebee-4d7b-92df-6639b42da2b2.

Hamilton, A. J. S., 2000, **Uncorrelated modes of the non-linear power spectrum:** Monthly Notices of the Royal Astronomical Society, 312, pages 257–284; doi: 10.1046/j.1365-8711.2000.03071.x.

Jönsthövel, T. B., C. W. Oosterlee, and W. A. Mulder, 2006, **Improving multigrid for 3-D electro-magnetic diffusion on stretched grids:** European Conference on Computational Fluid Dynamics; UUID: df65da5c-e43f-47ab-b80d-2f8ee7f35464.

Mulder, W. A., 2006, **A multigrid solver for 3D electromagnetic diffusion:** Geophysical Prospecting, 54, 633–649; doi: 10.1111/j.1365-2478.2006.00558.x.

Mulder, W. A., 2007, **A robust solver for CSEM modelling on stretched grids:** EAGE Technical Program Expanded Abstracts, D036; doi: 10.3997/2214-4609.201401567.

Mulder, W. A., M. Wirianto, and E. Slob, 2008, **Time-domain modeling of electromagnetic diffusion with a frequency-domain code:** Geophysics, 73, F1–F8; doi: 10.1190/1.2799093.

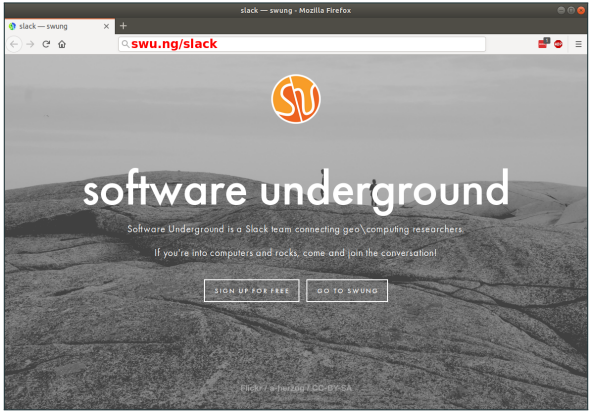
Plessix, R.-E., M. Darnet, and W. A. Mulder, 2007, **An approach for 3D multisource, multifrequency CSEM modeling:** Geophysics, 72, SM177–SM184; doi: 10.1190/1.2744234.

Werthmüller, D., 2017, **An open-source full 3D electromagnetic modeler for 1D VTI media in Python: empymod:** Geophysics, 82(6), WB9–WB19; doi: 10.1190/geo2016-0626.1.

Werthmüller, D., K. Key, and E. C. Slob, 2019, **A tool for designing digital filters for the Hankel and Fourier transforms in potential, diffusive, and wavefield modeling:** Geophysics, 84(2), F47–F56; doi: 10.1190/geo2018-0069.1.

Werthmüller, D., W. A. Mulder, and E. C. Slob, 2019, **emg3d: A multigrid solver for 3D electromagnetic diffusion:** Journal of Open Source Software, 4(39), 1463; doi: 10.21105/joss.01463.

Swung (software underground)



Hackathon EAGE 2020: 06/07 June 2020 (#amstel-hack-2020)

3D CSEM modelling in the frequency and Laplace domains

One story of success and one of failure

Dieter Werthmüller

5th December 2019

Thursday Talk – Applied Geophysics & Petrophysics

TU Delft

Faculty of Civil Engineering and Geosciences > Department of Geoscience & Engineering > Section of Applied Geophysics & Petrophysics

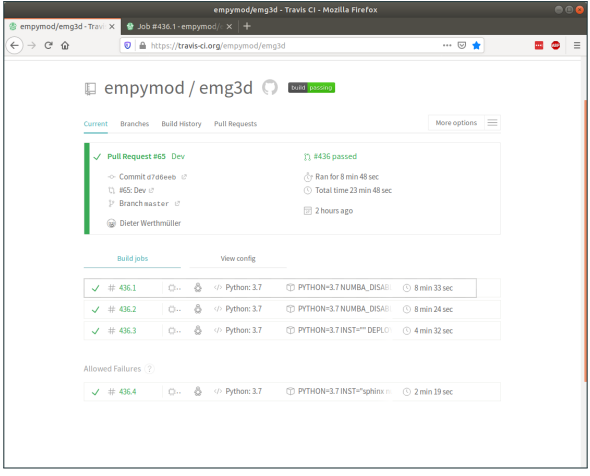
Appendix

emg3d – CI workflow

- Commit to GitHub
- Travis CI - run all tests
  - Report to coveralls
  - Check all hyperlinks
  - Deploy to PyPi (*if tag*)
  - Deploy to conda-forge (*if tag*)
- Codacy
- ReadTheDocs and Gallery
- Zenodo (*if tag*)
- Benchmarks (`asv`, *manual*)

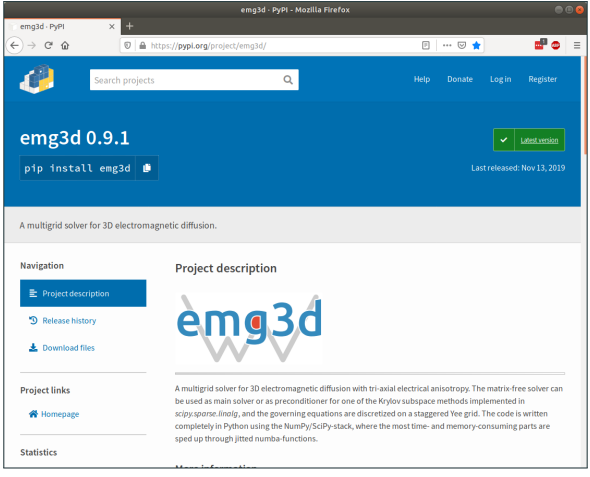
emg3d – CI workflow

- Commit to GitHub
- Travis CI - run all tests
  - Report to coveralls
  - Check all hyperlinks
  - Deploy to PyPi (*if tag*)
  - Deploy to conda-forge (*if tag*)
- Codacy
- ReadTheDocs and Gallery
- Zenodo (*if tag*)
- Benchmarks (`asv`, *manual*)



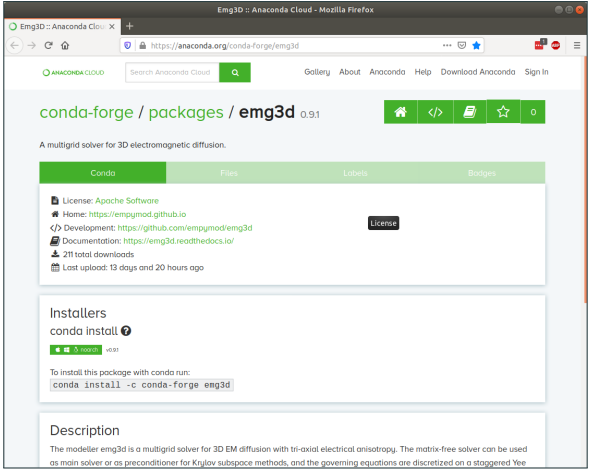
emg3d – CI workflow

- Commit to GitHub
- Travis CI - run all tests
  - Report to coveralls
  - Check all hyperlinks
  - Deploy to PyPi (*if tag*)
  - Deploy to conda-forge (*if tag*)
- Codacy
- ReadTheDocs and Gallery
- Zenodo (*if tag*)
- Benchmarks (`asv`, *manual*)



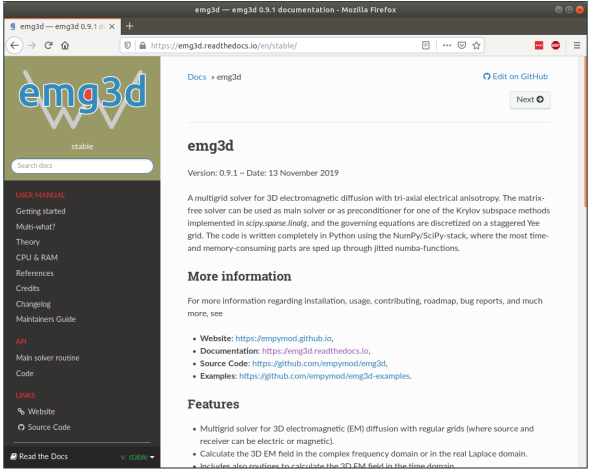
emg3d – CI workflow

- Commit to GitHub
- Travis CI - run all tests
  - Report to coveralls
  - Check all hyperlinks
  - Deploy to PyPi (*if tag*)
  - Deploy to conda-forge (*if tag*)
- Codacy
- ReadTheDocs and Gallery
- Zenodo (*if tag*)
- Benchmarks (`asv`, *manual*)



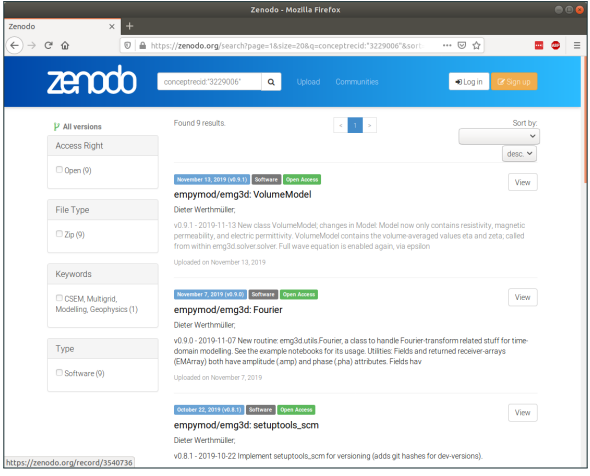
emg3d – CI workflow

- Commit to GitHub
- Travis CI - run all tests
  - Report to coveralls
  - Check all hyperlinks
  - Deploy to PyPi (*if tag*)
  - Deploy to conda-forge (*if tag*)
- Codacy
- ReadTheDocs and Gallery
- Zenodo (*if tag*)
- Benchmarks (`asv`, *manual*)



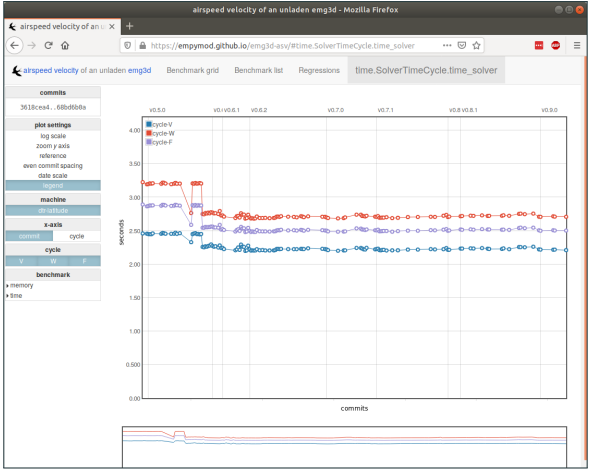
emg3d – CI workflow

- Commit to GitHub
- Travis CI - run all tests
  - Report to coveralls
  - Check all hyperlinks
  - Deploy to PyPi (*if tag*)
  - Deploy to conda-forge (*if tag*)
- Codacy
- ReadTheDocs and Gallery
- Zenodo (*if tag*)
- Benchmarks (`asv`, *manual*)



emg3d – CI workflow

- Commit to GitHub
- Travis CI - run all tests
  - Report to coveralls
  - Check all hyperlinks
  - Deploy to PyPi (*if tag*)
  - Deploy to conda-forge (*if tag*)
- Codacy
- ReadTheDocs and Gallery
- Zenodo (*if tag*)
- Benchmarks (`asv`, *manual*)

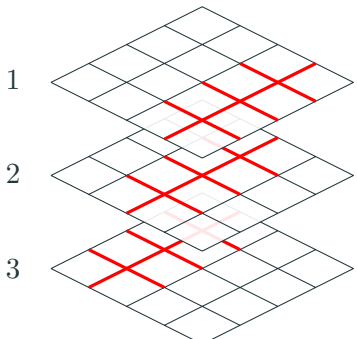


emg3d – Links

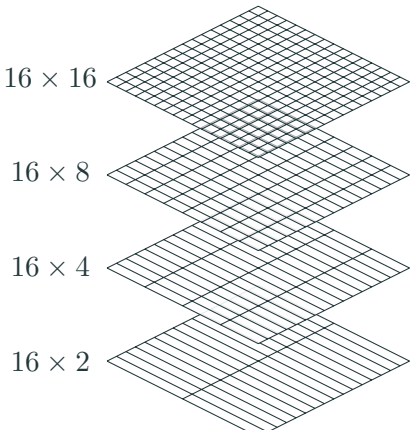
Website:	empymod.github.io
Code:	github.com/empymod/emg3d
Examples:	github.com/empymod/emg3d-examples
Docs:	emg3d.rtfd.io
Travis:	travis-ci.org/empymod/emg3d
Coveralls:	coveralls.io/github/empymod/emg3d
Codacy:	app.codacy.com/project/prisae/emg3d/dashboard
Benchmarks:	empymod.github.io/emg3d-asv
Zenodo:	doi.org/10.5281/zenodo.3229006

Multigrid – Line relaxation & Semicoarsening

Line relaxation



Semicoarsening



Analysis – Result