Time-domain CSEM modelling

using frequency- and Laplace-domain computations

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Part I: frequency to time

Fast Fourier transform of EM data for computationally expensive kernels †

Solver

- Fast
- Robust over wide range of frequencies

Gridding

- Adaptive
- f-dependent

Transform

As few frequencies as possible

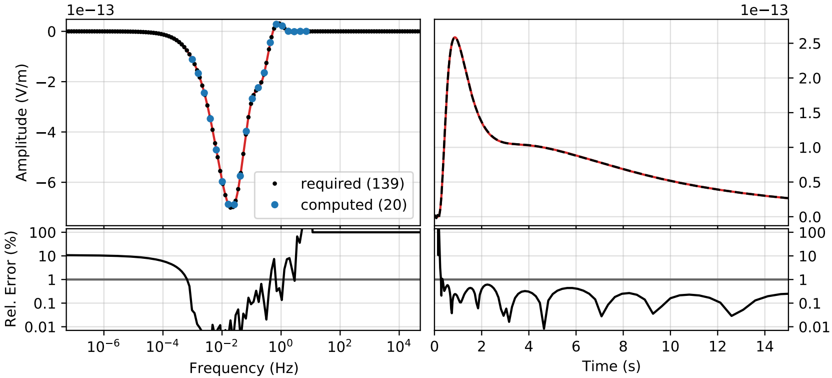
- Log-scale (DLF; FFTLog)
- Interpolation
- Zero-padding

Conclusions

- 15–25 frequencies are usually enough
- Equally applies for $t \rightarrow f$

† Werthmüller, Mulder, and Slob, 2021, GJI, 10.1093/gji/ggab171.

Example using FFTLog and Digital Linear Filters



- $f < f_{\min}$ PCHIP
- $f_{\min} < f < f_{\max}$ Cubic spline
- $f > f_{\max}$ 0

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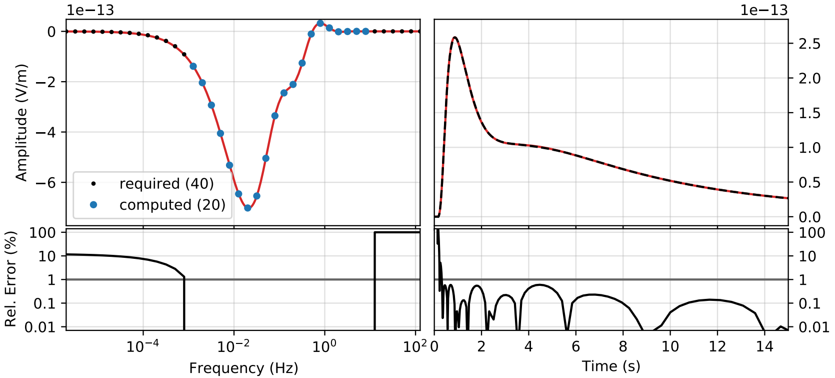

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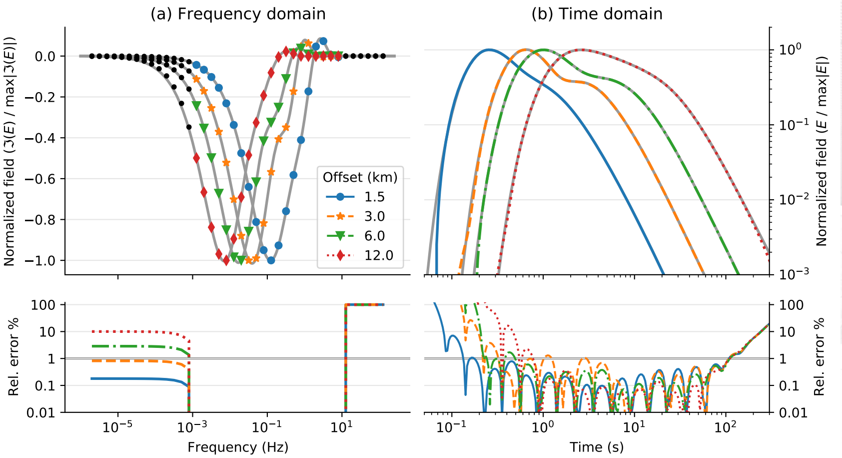

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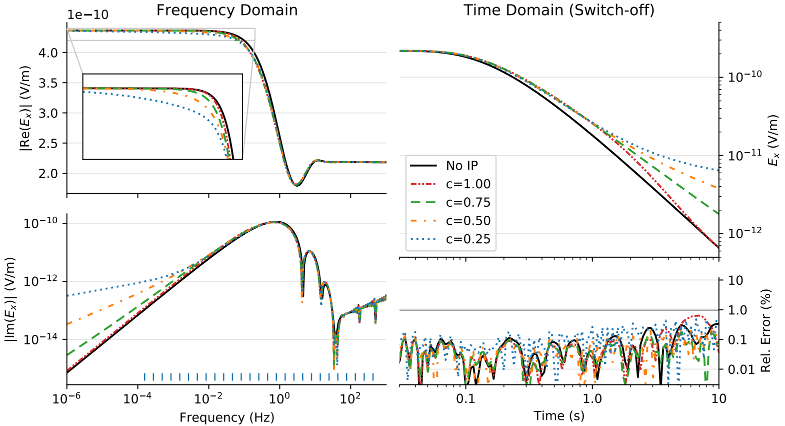
Example: Induced Polarization

Also works for 3D or any model, as the transform is unaware of the complexity.



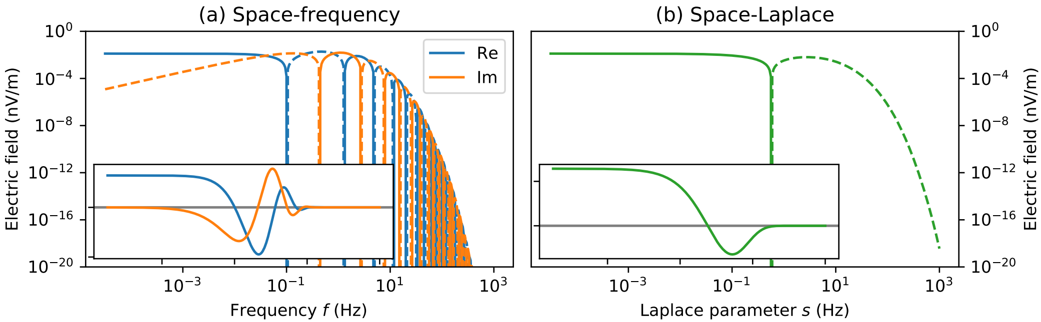
$$\sigma(\omega) = \sigma_\infty + \frac{\sigma_0 - \sigma_\infty}{1 + (\mathrm{i}\omega\tau)^c}$$

27 frequencies instead of 747 frequencies (201 pt filter)

Part II: Laplace to time

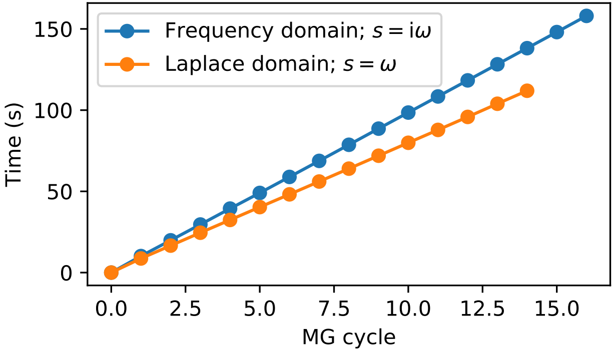
Laplace-domain computation: Motivation

$$\mathrm{i}\omega \rightarrow s: \qquad s\mu\tilde{\sigma}\mathbf{E} + \nabla^2\mathbf{E} = -s\mu\mathbf{J}_\mathrm{s}$$



⇒ Faster (1) Computation (2) Convergence ⇐

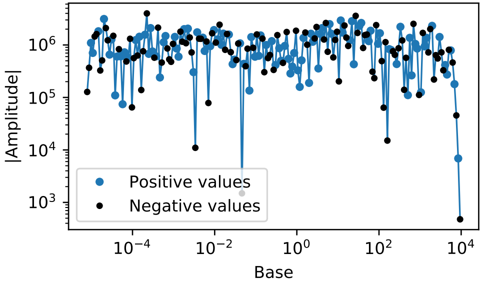
Computation speed (layered; 3D) & Convergence (3D)

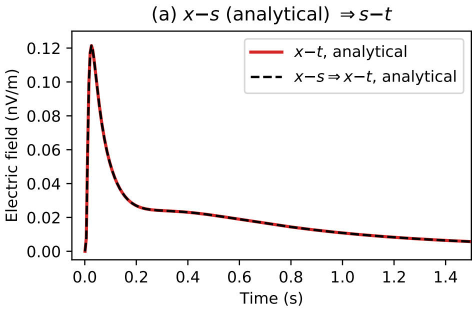


- Overall Laplace comp. roughly 2/3 of frequency comp.

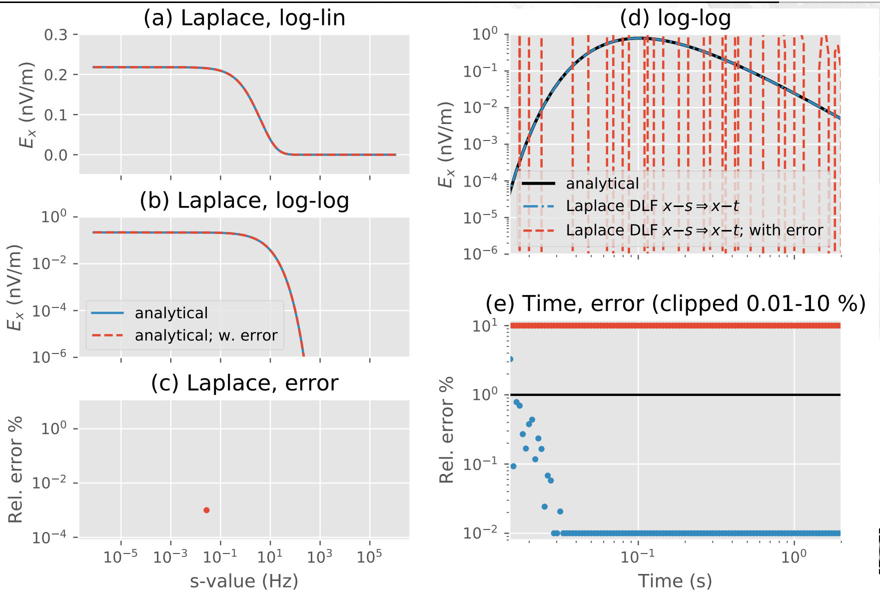
Digital linear filter for Laplace

$$F(r) = \int_0^\infty f(l) K(lr) \mathrm{d}l \quad \Rightarrow \quad F(r) \approx \sum_{n=1}^{N} \frac{f(b_n/r)h_n}{r}$$





Problem: robustness of the approach



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Part III: Laplace to frequency

Laplace-to-frequency domain

- Same motivation as for $s \to t$: computation speed and convergence
- Possible to design a linear digital filter
- Same limitation as $s \to t$
- Additionally: DLF seems to be offset dependent!

Wrap-up

Conclusions

- $f \rightarrow t$: Solver; Method; Gridding; **15–25 frequencies are generally enough**
- **Laplace: roughly 2/3 computation time** (computation; convergence)
- $s \rightarrow t$; $s \rightarrow f$: **DLF works; but only for very precise results**

References

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Used open-source codes: `empymod` (layered models) & `emg3d` (3D models), emsig.xyz.

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