Projective Geometry deals with properties that are invariant under projections. Hence angles and distances are not preserved, but collinearity is. In many ways, it is more fundamental than Euclidean Geometry, and also simpler in terms of its axiomatic presentation. Projective Geometry is also "global" in a sense that Euclidean Geometry is not: In Euclidean Geometry lines may or may not meet; if not, this is an indication that something is "missing". In Projective Geometry two lines always meet, and thus there is perfect duality between the concepts of points and lines.

Synthetic Projective Geometry was a time-honored subject in Secondary Schools in the past, and its ancestry goes back to the Ancient Greeks (Pappus and Appolonius), with a renewed interest during the Renaissance. It was not however systematically developed until the 17th and 18th centuries (Desargues, Poncelet and Monge) and reached its apogee during the last century. Although we will only peripherally touch upon the axiomatic and synthetic aspects, the general notions of projective spaces constitute the basic setting for Algebraic Geometry.

As for the axiomatic and synthetic aspects of Projective Geometry, there exists a host of classical references. The most elegant and least involved is probably Hartshorne's [**?**], while works by *e.g.* Coxeter (with predictable titles) go into more detail. However, we will not be overly concerned with those aspects.

As is well known, two lines may or may not meet. If not, they are said to be parallel. The notion of *parallel lines* is easily seen to be an equivalence relation among lines. We may then "force" two lines always to meet by "postulating" a missing point at "infinity". Infinity will consists formally of the equivalence classes of lines (with respect to being parallel) and each line will be augmented by its equivalence class. In addition, we will consider infinity as a line. In this way, we have formally forced every pair of lines to meet, and still through two points there will be a unique line. The ensuing construct will be called the *projective plane*.

Such a construction is of course quite unsatisfactory. It appears forced and unnatural and very contrived; although, from a formal point of view, it may be impeccable. However, it is not very geometric, and it assigns to the missing points ("the line at infinity") a special status that they do not deserve.

A more geometric presentation is to consider the task, of say, a Renaissance artist using perspective to represent three dimensional reality onto a flat canvas. We may consider him standing on a floor F, ideally extending indefinitely, and painting on an equally vast canvas C. If his eye is represented by the point ω (for simplicity we may think of him as a cyclops) then to every point p on the floor we may associate the point p' on the canvas which is given by the intersection of the line (ωp) with C. This sets up a correspondence between points on F and their pictorial representations on C.

Remarque. Of course, the image on the canvas is not limited to what contains the plane floor; to every point in space p different from ω, we may associate a point $p' \in C$ as above. In this case to each point on the canvas corresponds an entire ray in space.

However, a moment of reflection reveals that not each point on the canvas corresponds to a point on the floor. If we consider the plane through ω parallel to F, its intersection H with C is a line corresponding to the "missing" points of F. The line H is, of course, the horizon, and the images of parallel lines on F will all be lines meeting at H. So H is the natural "compactification" of the plane F, the line at infinity. Conversely, not all points on F will be represented on C; the line M on F given by the intersection with a plane through ω parallel to C, will have no image on C. And lines on F meeting on M will be mapped onto parallel lines on C.