# TR29124 C++ Special Math Functions 2.0

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### **Mathematical Special Functions**

#### 1.1 Introduction and History

The first significant library upgrade on the road to C++2011, TR1, included a set of 23 mathematical functions that significantly extended the standard transcendental functions inherited from C and declared in <cmath>.

Although most components from TR1 were eventually adopted for C++11 these math functions were left behind out of concern for implementability. The math functions were published as a separate international standard IS 29124 - Extensions to the C++ Library to Support Mathematical Special Functions.

For C++17 these functions were incorporated into the main standard.

#### 1.2 Contents

The following functions are implemented in namespace std:

- assoc\_laguerre Associated Laguerre functions
- · assoc legendre Associated Legendre functions
- beta Beta functions
- comp\_ellint\_1 Complete elliptic functions of the first kind
- · comp ellint 2 Complete elliptic functions of the second kind
- comp\_ellint\_3 Complete elliptic functions of the third kind
- · cyl bessel i Regular modified cylindrical Bessel functions
- cyl\_bessel\_j Cylindrical Bessel functions of the first kind
- cyl\_bessel\_k Irregular modified cylindrical Bessel functions
- · cyl neumann Cylindrical Neumann functions or Cylindrical Bessel functions of the second kind
- ellint\_1 Incomplete elliptic functions of the first kind
- ellint\_2 Incomplete elliptic functions of the second kind
- · ellint 3 Incomplete elliptic functions of the third kind

- · expint The exponential integral
- · hermite Hermite polynomials
- · laguerre Laguerre functions
- · legendre Legendre polynomials
- · riemann zeta The Riemann zeta function
- · sph\_bessel Spherical Bessel functions
- sph\_legendre Spherical Legendre functions
- · sph neumann Spherical Neumann functions

The hypergeometric functions were stricken from the TR29124 and C++17 versions of this math library because of implementation concerns. However, since they were in the TR1 version and since they are popular we kept them as an extension in namespace \_\_gnu\_cxx:

- · conf\_hyperg Confluent hypergeometric functions
- hyperg Hypergeometric functions

In addition a large number of new functions are added as extensions:

- · airy\_ai Airy functions of the first kind
- · airy bi Airy functions of the second kind
- · bincoef Binomial coefficients
- chebyshev\_t Chebyshev polynomials of the first kind
- · chebyshev\_u Chebyshev polynomials of the second kind
- · chebyshev\_v Chebyshev polynomials of the third kind
- · chebyshev w Chebyshev polynomials of the fourth kind
- clausen Clausen integrals
- · clausen c Clausen cosine integrals
- clausen\_s Clausen sine integrals
- · comp\_ellint\_d Incomplete Legendre D elliptic integral
- conf\_hyperg\_lim Confluent hypergeometric limit functions
- · coshint Hyperbolic cosine integral
- · cosint Cosine integral
- cyl\_hankel\_1 Cylindrical Hankel functions of the first kind
- · cyl\_hankel\_2 Cylindrical Hankel functions of the second kind
- · dawson Dawson integrals
- · dilog Dilogarithm functions
- · dirichlet beta Dirichlet beta function

1.2 Contents 3

- · dirichlet\_eta Dirichlet beta function
- · dirichlet lambda Dirichlet lambda function
- · double factorial -
- ellint\_d Legendre D elliptic integrals
- · ellint rc Carlson elliptic functions R C
- · ellint\_rd Carlson elliptic functions R\_D
- ellint\_rf Carlson elliptic functions R\_F
- ellint\_rg Carlson elliptic functions R\_G
- · ellint\_rj Carlson elliptic functions R\_J
- expint\_e1 -
- · factorial Factorials
- · fresnel\_c Fresnel cosine integrals
- fresnel\_s Fresnel sine integrals
- gamma\_I Lower incomplete gamma functions
- gamma\_p Regularized lower incomplete gamma functions
- gamma\_q Regularized upper incomplete gamma functions
- gamma\_u upper incomplete gamma functions
- gegenbauer Gegenbauer polynomials
- heuman\_lambda Heuman lambda functions
- · hurwitz zeta Hurwitz zeta functions
- · ibeta Regularized incomplete beta functions
- jacobi Jacobi polynomials
- · jacobi sn Jacobi sine amplitude functions
- jacobi cn Jacobi cosine amplitude functions
- jacobi\_dn Jacobi delta amplitude functions
- · jacobi\_zeta Jacobi zeta functions
- Ibincoef Log binomial coefficients
- · Idouble\_factorial Log double factorials
- legendre\_q Legendre functions of the second kind
- Ifactorial Log factorials
- Ipochhammer I Log lower Pochhammer functions
- lpochhammer\_u Log upper Pochhammer functions
- · owens t Owens T functions

- pochhammer\_I Lower Pochhammer functions
- pochhammer\_u Upper Pochhammer functions
- · psi Psi of digamma function
- · radpoly Radial polynomials
- sinhc Hyperbolic sinus cardinal function
- sinhc\_pi -
- · sinc Sinus cardinal function
- sinc pi -
- · sinhint Hyperbolic sine integral
- sinint Sine integral
- sph\_bessel\_i Spherical regular modified Bessel functions
- sph bessel k Spherical iregular modified Bessel functions
- · sph\_hankel\_1 Spherical Hankel functions of the first kind
- · sph\_hankel\_2 Spherical Hankel functions of the first kind
- · sph harmonic Spherical
- · zernike Zernike polynomials

#### 1.3 General Features

#### 1.3.1 Argument Promotion

The arguments suppled to the non-suffixed functions will be promoted according to the following rules:

- 1. If any argument intended to be floating point is given an integral value That integral value is promoted to double.
- 2. All floating point arguments are promoted up to the largest floating point precision among them.

#### 1.3.2 NaN Arguments

If any of the floating point arguments supplied to these functions is invalid or NaN (std::numeric\_limits<Tp>::quiet\_← NaN), the value NaN is returned.

#### 1.4 Implementation

We strive to implement the underlying math with type generic algorithms to the greatest extent possible. In practice, the functions are thin wrappers that dispatch to function templates. Type dependence is controlled with std::numeric\_limits and functions thereof.

We don't promote float to double or double to long double reflexively. The goal is for float functions to operate more quickly, at the cost of float accuracy and possibly a smaller domain of validity. Similarly, long double should give you more dynamic range and slightly more pecision than double on many systems.

1.5 Testing 5

#### 1.5 Testing

These functions have been tested against equivalent implementations from the Gnu Scientific Library, GSL and <a href="http://www.boost.org/doc/libs/1\_60\_0/libs/math/doc/html/index. $\leftarrow$ html>Boost and the ratio

 $\frac{|f - f_{test}|}{|f_{test}|}$ 

is generally found to be within 10^-15 for 64-bit double on linux-x86\_64 systems over most of the ranges of validity.

Todo Provide accuracy comparisons on a per-function basis for a small number of targets.

#### 1.6 General Bibliography

#### See also

Abramowitz and Stegun: Handbook of Mathematical Functions, with Formulas, Graphs, and Mathematical Tables Edited by Milton Abramowitz and Irene A. Stegun, National Bureau of Standards Applied Mathematics Series - 55 Issued June 1964, Tenth Printing, December 1972, with corrections Electronic versions of A&S abound including both pdf and navigable html.

for example http://people.math.sfu.ca/~cbm/aands/

The old A&S has been redone as the NIST Digital Library of Mathematical Functions: http://dlmf.nist. ← gov/ This version is far more navigable and includes more recent work.

An Atlas of Functions: with Equator, the Atlas Function Calculator 2nd Edition, by Oldham, Keith B., Myland, Jan, Spanier, Jerome

Asymptotics and Special Functions by Frank W. J. Olver, Academic Press, 1974

Numerical Recipes in C, The Art of Scientific Computing, by William H. Press, Second Ed., Saul A. Teukolsky, William T. Vetterling, and Brian P. Flannery, Cambridge University Press, 1992

The Special Functions and Their Approximations: Volumes 1 and 2, by Yudell L. Luke, Academic Press, 1969

Mathematical Special Functions	

6

### **Todo List**

```
page Mathematical Special Functions
    Provide accuracy comparisons on a per-function basis for a small number of targets.

Member std::__detail::__dawson_cont_frac (_Tp __x)
    this needs some compile-time construction!

Member std::__detail::__expint_E1 (_Tp __x)
    Find a good asymptotic switch point in E_1(x).

Member std::__detail::__expint_En_recursion (unsigned int __n, _Tp __x)
    Find a principled starting number for the E_n(x) downward recursion.

Member std::__detail::__hurwitz_zeta (_Tp __s, std::complex < _Tp > __a)
    This __hurwitz_zeta prefactor is prone to overflow. positive integer orders s?
```

8	Todo List

# **Module Index**

#### 3.1 Modules

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C++ Mathematical Special Functions			 										17
C++17/IS29124 Mathematical Special Functions													18
GNU Extended Mathematical Special Functions													40

10	Module Index

# Namespace Index

4.1	<b>Namespace</b>	List
-----	------------------	------

Here is a list of all namespaces with brief descriptions:

gnu_cxx	 	
std	 	
std:: detail	 	

12	Namespace Index

# **Chapter 5**

# **Class Index**

E 4	$\sim$ 1	10.00
5.1	Class	i let

Here are the classes, structs, unions and interfaces with brief descriptions:		
std::detail::_Factorial_table< _Tp >	235	

14	Class Index

# **Chapter 6**

# File Index

# 6.1 File List

Here is a list of all files with brief descriptions:

bits/sf_airy.tcc
bits/sf_bessel.tcc
bits/sf_beta.tcc
bits/sf_cardinal.tcc
bits/sf_chebyshev.tcc
bits/sf_dawson.tcc
bits/sf_ellint.tcc
bits/sf_expint.tcc
bits/sf_fresnel.tcc
bits/sf_gamma.tcc
bits/sf_gegenbauer.tcc
bits/sf_hankel.tcc
bits/sf_hankel_new.tcc
bits/sf_hermite.tcc
bits/sf_hydrogen.tcc
bits/sf_hyperg.tcc
bits/sf_hypint.tcc
bits/sf_jacobi.tcc
bits/sf_laguerre.tcc
bits/sf_legendre.tcc
bits/sf_mod_bessel.tcc
bits/sf_owens_t.tcc
bits/sf_polylog.tcc
bits/sf_theta.tcc
bits/sf_trigint.tcc
bits/sf_zeta.tcc
hits/specfup h

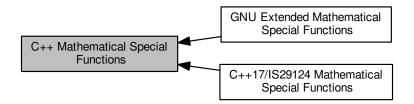
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# **Chapter 7**

# **Module Documentation**

# 7.1 C++ Mathematical Special Functions

Collaboration diagram for C++ Mathematical Special Functions:



# **Modules**

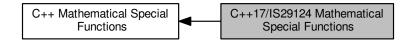
- C++17/IS29124 Mathematical Special Functions
- GNU Extended Mathematical Special Functions

# 7.1.1 Detailed Description

A collection of advanced mathematical special functions.

# 7.2 C++17/IS29124 Mathematical Special Functions

Collaboration diagram for C++17/IS29124 Mathematical Special Functions:



# **Functions**

```
template<typename</li>Tp >
   __gnu_cxx::__promote< _Tp >::__type std::assoc_laguerre (unsigned int __n, unsigned int __m, _Tp __x)
• float std::assoc_laguerref (unsigned int __n, unsigned int __m, float __x)

    long double std::assoc laguerrel (unsigned int n, unsigned int m, long double x)

template<typename _Tp >
   _gnu_cxx::__promote< _Tp >::__type std::assoc_legendre (unsigned int __l, unsigned int __m, _Tp __x)

    float std::assoc legendref (unsigned int I, unsigned int m, float x)

    long double std::assoc_legendrel (unsigned int __l, unsigned int __m, long double __x)

• template<typename Tpa, typename Tpb>
    gnu cxx:: promote 2< Tpa, Tpb >:: type std::beta ( Tpa a, Tpb b)

    float std::betaf (float a, float b)

    long double std::betal (long double __a, long double __b)

template<typename _Tp >
   __gnu_cxx::__promote< _Tp >::__type std::comp_ellint_1 (_Tp __k)

    float std::comp_ellint_1f (float __k)

    long double std::comp ellint 1l (long double k)

• template<typename _{\rm Tp}>
    _gnu_cxx::__promote< _Tp >::__type std::comp_ellint_2 (_Tp __k)

    float std::comp ellint 2f (float k)

    long double std::comp ellint 2l (long double k)

• template<typename Tp, typename Tpn >
    _gnu_cxx::__promote_2< _Tp, _Tpn >::__type std::comp_ellint_3 (_Tp __k, _Tpn __nu)

    float std::comp_ellint_3f (float __k, float __nu)

      Return the complete elliptic integral of the third kind \Pi(k,\nu) for float modulus k.

    long double std::comp_ellint_3l (long double __k, long double __nu)

      Return the complete elliptic integral of the third kind \Pi(k,\nu) for long double modulus k.
• template<typename _Tpnu , typename _Tp >
    _gnu_cxx::__promote_2< _Tpnu, _Tp >::__type std::cyl_bessel_i (_Tpnu __nu, _Tp __x)

    float std::cyl_bessel_if (float __nu, float __x)

    long double std::cyl bessel il (long double nu, long double x)

    template<typename _Tpnu , typename _Tp >

   __gnu_cxx::__promote_2< _Tpnu, _Tp >::__type std::cyl_bessel_j (_Tpnu __nu, _Tp __x)

    float std::cyl_bessel_if (float __nu, float __x)

• long double std::cyl_bessel_jl (long double __nu, long double __x)
```

```
• template<typename _Tpnu , typename _Tp >
    _gnu_cxx::__promote_2< _Tpnu, _Tp >::__type std::cyl_bessel_k (_Tpnu __nu, _Tp __x)

    float std::cyl bessel kf (float nu, float x)

    long double std::cyl_bessel_kl (long double __nu, long double __x)

• template<typename Tpnu, typename Tp >
    _gnu_cxx::__promote_2< _Tpnu, _Tp >::__type std::cyl_neumann (_Tpnu __nu, _Tp __x)

    float std::cyl neumannf (float nu, float x)

    long double std::cyl_neumannl (long double __nu, long double __x)

    template<typename</li>
    Tp , typename
    Tpp >

   _gnu_cxx::__promote_2< _Tp, _Tpp >::__type std::ellint_1 (_Tp __k, _Tpp __phi)

    float std::ellint_1f (float __k, float __phi)

    long double std::ellint 11 (long double k, long double phi)

    template<typename _Tp , typename _Tpp >

    _gnu_cxx::__promote_2< _Tp, _Tpp >::__type std::ellint_2 (_Tp __k, _Tpp __phi)

    float std::ellint 2f (float k, float phi)

      Return the incomplete elliptic integral of the second kind E(k,\phi) for float argument.

    long double std::ellint_2l (long double __k, long double __phi)

      Return the incomplete elliptic integral of the second kind E(k, \phi).
template<typename _Tp , typename _Tpn , typename _Tpp >
   _gnu_cxx::_promote_3< _Tp, _Tpn, _Tpp >::_type std::ellint_3 (_Tp __k, _Tpn __nu, _Tpp __phi)
      Return the incomplete elliptic integral of the third kind \Pi(k, \nu, \phi).

    float std::ellint_3f (float __k, float __nu, float __phi)

      Return the incomplete elliptic integral of the third kind \Pi(k, \nu, \phi) for float argument.
• long double std::ellint 3l (long double k, long double nu, long double phi)
      Return the incomplete elliptic integral of the third kind \Pi(k, \nu, \phi).

    template<typename</li>
    Tp >

   _gnu_cxx::__promote< _Tp >::__type std::expint (_Tp __x)

    float std::expintf (float __x)

    long double std::expintl (long double x)

template<typename</li>Tp >
   _gnu_cxx::__promote< _Tp >::__type std::hermite (unsigned int __n, _Tp __x)

    float std::hermitef (unsigned int __n, float __x)

    long double std::hermitel (unsigned int n, long double x)

template<typename _Tp >
    _gnu_cxx::__promote< _Tp >::__type std::laguerre (unsigned int __n, _Tp __x)

    float std::laguerref (unsigned int n, float x)

    long double std::laguerrel (unsigned int __n, long double __x)

• template<typename _Tp >
    _gnu_cxx::__promote< _Tp >::__type std::legendre (unsigned int __l, _Tp __x)

    float std::legendref (unsigned int I, float x)

    long double std::legendrel (unsigned int __I, long double __x)

template<typename _Tp >
    gnu cxx:: promote < Tp >:: type std::riemann zeta ( Tp s)

    float std::riemann_zetaf (float __s)

    long double std::riemann zetal (long double s)

template<typename _Tp >
    gnu cxx:: promote < Tp >:: type std::sph bessel (unsigned int n, Tp x)

    float std::sph besself (unsigned int n, float x)

    long double std::sph_bessell (unsigned int __n, long double __x)

template<typename _Tp >
   gnu cxx:: promote < Tp >:: type std::sph legendre (unsigned int I, unsigned int m, Tp theta)
```

- float std::sph\_legendref (unsigned int \_\_l, unsigned int \_\_m, float \_\_theta)
- long double std::sph\_legendrel (unsigned int \_\_l, unsigned int \_\_m, long double \_\_theta)
- template<typename \_Tp >
   \_\_gnu\_cxx::\_\_promote< \_Tp >::\_\_type std::sph\_neumann (unsigned int \_\_n, \_Tp \_\_x)
- float std::sph neumannf (unsigned int n, float x)
- long double std::sph\_neumannl (unsigned int \_\_n, long double \_\_x)

# 7.2.1 Detailed Description

A collection of advanced mathematical special functions for C++17 and IS29124.

#### 7.2.2 Function Documentation

7.2.2.1 template<typename \_Tp > \_\_gnu\_cxx::\_\_promote<\_Tp>::\_\_type std::assoc\_laguerre ( unsigned int \_\_n, unsigned int \_\_n, \_Tp \_\_x ) [inline]

Return the associated Laguerre polynomial  $L_n^m(x)$  of nonnegative order n, nonnegative degree m and real argument x.

The associated Laguerre function of real degree  $\alpha$ ,  $L_n^{\alpha}(x)$ , is defined by

$$L_n^{\alpha}(x) = \frac{(\alpha+1)_n}{n!} {}_1F_1(-n;\alpha+1;x)$$

where  $(\alpha)_n$  is the Pochhammer symbol and  ${}_1F_1(a;c;x)$  is the confluent hypergeometric function.

The associated Laguerre polynomial is defined for integral degree  $\alpha=m$  by:

$$L_n^m(x) = (-1)^m \frac{d^m}{dx^m} L_{n+m}(x)$$

where the Laguerre polynomial is defined by:

$$L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n} (x^n e^{-x})$$

and x >= 0.

See also

laguerre for details of the Laguerre function of degree n

# **Template Parameters**

_Tp	The floating-point type of the argumentx.

#### **Parameters**

n	The order of the Laguerre function, $\underline{\hspace{0.2cm}}$ n >= 0.
m	The degree of the Laguerre function, $\underline{\hspace{0.1cm}}$ $m >= 0$ .
X	The argument of the Laguerre function, $\underline{}x >= 0$ .

#### **Exceptions**

Definition at line 371 of file specfun.h.

7.2.2.2 float std::assoc\_laguerref ( unsigned int \_\_n, unsigned int \_\_m, float \_\_x ) [inline]

Return the associated Laguerre polynomial  $L_n^m(x)$  of order n, degree m, and float argument x.

#### See also

assoc\_laguerre for more details.

Definition at line 323 of file specfun.h.

7.2.2.3 long double std::assoc\_laguerrel ( unsigned int \_\_n, unsigned int \_\_m, long double \_\_x ) [inline]

Return the associated Laguerre polynomial  $L_n^m(x)$  of order n, degree m and long double argument x.

# See also

assoc\_laguerre for more details.

Definition at line 334 of file specfun.h.

7.2.2.4 template<typename \_Tp > \_\_gnu\_cxx::\_\_promote<\_Tp>::\_\_type std::assoc\_legendre ( unsigned int \_\_I, unsigned int \_

Return the associated Legendre function  $P_l^m(x)$  of degree l, order m, and real argument x.

The associated Legendre function is derived from the Legendre function  $P_l(x)$  by the Rodrigues formula:

$$P_l^m(x) = (1 - x^2)^{m/2} \frac{d^m}{dx^m} P_l(x)$$

# See also

legendre for details of the Legendre function of degree 1

# **Template Parameters**

_ <i>Tp</i>   The f	oating-point type of the argumentx.
---------------------	-------------------------------------

#### **Parameters**

		The degree $_{1} >= 0$ .
	m	The order $_{m} <= 1$ .
Ì	X	The argument, abs (x) <= 1.

#### **Exceptions**

std::domain_error	if $abs(\underline{x}) > 1$ .

Definition at line 419 of file specfun.h.

7.2.2.5 float std::assoc\_legendref ( unsigned int \_\_I, unsigned int \_\_m, float \_\_x ) [inline]

Return the associated Legendre function  $P_l^m(x)$  of degree l, order m, and float argument x.

See also

assoc\_legendre for more details.

Definition at line 386 of file specfun.h.

7.2.2.6 long double std::assoc\_legendrel( unsigned int \_\_I, unsigned int \_\_I, long double \_\_x) [inline]

Return the associated Legendre function  $P_l^m(x)$  of degree l, order m, and long double argument x.

See also

assoc\_legendre for more details.

Definition at line 397 of file specfun.h.

7.2.2.7 template<typename \_Tpa , typename \_Tpb > \_\_gnu\_cxx::\_\_promote\_2<\_Tpa, \_Tpb>::\_\_type std::beta ( \_Tpa \_\_a, \_Tpb \_\_b ) [inline]

Return the beta function, B(a, b), for real parameters a, b.

The beta function is defined by

$$B(a,b) = \int_0^1 t^{a-1} (1-t)^{b-1} dt = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

where a > 0 and b > 0

**Template Parameters** 

_Тра	The floating-point type of the parametera.
_Tpb	The floating-point type of the parameterb.

#### **Parameters**

ſ	a	The first argument of the beta function, $\underline{} = 0$ .
	b	The second argument of the beta function, $\underline{\hspace{0.2cm}}$ b $> 0$ .

# **Exceptions**

Definition at line 464 of file specfun.h.

**7.2.2.8** float std::betaf (float \_\_a, float \_\_b) [inline]

Return the beta function, B(a,b), for float parameters a, b.

See also

beta for more details.

Definition at line 433 of file specfun.h.

**7.2.2.9** long double std::betal (long double \_a, long double \_b) [inline]

Return the beta function, B(a, b), for long double parameters a, b.

See also

beta for more details.

Definition at line 443 of file specfun.h.

7.2.2.10 template<typename\_Tp>\_\_gnu\_cxx:\_\_promote<\_Tp>::\_type std::comp\_ellint\_1(\_Tp \_k) [inline]

Return the complete elliptic integral of the first kind K(k) for real modulus  ${\bf k}$ .

The complete elliptic integral of the first kind is defined as

$$K(k) = F(k, \pi/2) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - k^2 sin^2 \theta}}$$

where  $F(k,\phi)$  is the incomplete elliptic integral of the first kind and the modulus |k|<=1.

See also

ellint\_1 for details of the incomplete elliptic function of the first kind.

# **Template Parameters**

_ <i>Tp</i>	The floating-point type of the modulusk.

#### **Parameters**

# **Exceptions**

$$std::domain\_error \mid if abs(\__k) > 1$$
.

Definition at line 512 of file specfun.h.

**7.2.2.11** float std::comp\_ellint\_1f(float \_\_k) [inline]

Return the complete elliptic integral of the first kind E(k) for float modulus k.

See also

comp\_ellint\_1 for details.

Definition at line 479 of file specfun.h.

**7.2.2.12** long double std::comp\_ellint\_1I( long double \_\_k ) [inline]

Return the complete elliptic integral of the first kind E(k) for long double modulus k.

See also

comp ellint 1 for details.

Definition at line 489 of file specfun.h.

Return the complete elliptic integral of the second kind E(k) for real modulus k.

The complete elliptic integral of the second kind is defined as

$$E(k) = E(k, \pi/2) = \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 \theta}$$

where  $E(k,\phi)$  is the incomplete elliptic integral of the second kind and the modulus |k| <= 1.

See also

ellint\_2 for details of the incomplete elliptic function of the second kind.

# **Template Parameters**

_Tp	The floating-point type of the modulusk.
Parameters	

 $\underline{\hspace{0.1cm}}$  **The modulus**, abs ( $\underline{\hspace{0.1cm}}$  k) <= 1

**Exceptions** 

std::domain\_error | if abs (\_\_k) > 1.

Definition at line 559 of file specfun.h.

7.2.2.14 float std::comp\_ellint\_2f (float \_\_k ) [inline]

Return the complete elliptic integral of the second kind E(k) for float modulus k.

See also

comp\_ellint\_2 for details.

Definition at line 527 of file specfun.h.

**7.2.2.15** long double std::comp\_ellint\_2l ( long double \_\_k ) [inline]

Return the complete elliptic integral of the second kind E(k) for long double modulus k.

See also

comp ellint 2 for details.

Definition at line 537 of file specfun.h.

Return the complete elliptic integral of the third kind  $\Pi(k,\nu)=\Pi(k,\nu,\pi/2)$  for real modulus k.

The complete elliptic integral of the third kind is defined as

$$\Pi(k,\nu) = \Pi(k,\nu,\pi/2) = \int_0^{\pi/2} \frac{d\theta}{(1-\nu\sin^2\theta)\sqrt{1-k^2\sin^2\theta}}$$

where  $\Pi(k,\nu,\phi)$  is the incomplete elliptic integral of the second kind and the modulus |k|<=1.

#### See also

ellint\_3 for details of the incomplete elliptic function of the third kind.

#### **Template Parameters**

_Tp	The floating-point type of the modulusk.
_Tpn	The floating-point type of the argumentnu.

#### **Parameters**

k	The modulus, abs $(\underline{}k) \le 1$
nu	The argument

#### **Exceptions**

Definition at line 610 of file specfun.h.

7.2.2.17 float std::comp\_ellint\_3f (float \_\_k, float \_\_nu ) [inline]

Return the complete elliptic integral of the third kind  $\Pi(k,\nu)$  for float modulus k.

#### See also

comp\_ellint\_3 for details.

Definition at line 574 of file specfun.h.

7.2.2.18 long double std::comp\_ellint\_3l(long double \_\_k, long double \_\_nu) [inline]

Return the complete elliptic integral of the third kind  $\Pi(k,\nu)$  for long double modulus k.

#### See also

comp\_ellint\_3 for details.

Definition at line 584 of file specfun.h.

7.2.2.19 template<typename \_Tpnu , typename \_Tp > \_\_gnu\_cxx::\_\_promote\_2<\_Tpnu, \_Tp>::\_\_type std::cyl\_bessel\_i ( \_Tpnu \_\_nu, \_Tp \_\_x ) [inline]

Return the regular modified Bessel function  $I_{\nu}(x)$  for real order  $\nu$  and argument x >= 0.

The regular modified cylindrical Bessel function is:

$$I_{\nu}(x) = i^{-\nu} J_{\nu}(ix) = \sum_{k=0}^{\infty} \frac{(x/2)^{\nu+2k}}{k! \Gamma(\nu+k+1)}$$

#### **Template Parameters**

_Tpnu	The floating-point type of the ordernu.
_ <i>Tp</i>	The floating-point type of the argumentx.

#### **Parameters**

nu	The order
x	The argument, $\underline{}x >= 0$

#### **Exceptions**

-t-ll	4 . 0
sta::domain_error	Ifx < 0 .

Definition at line 656 of file specfun.h.

7.2.2.20 float std::cyl\_bessel\_if ( float \_\_nu, float \_\_x ) [inline]

Return the regular modified Bessel function  $I_{\nu}(x)$  for float order  $\nu$  and argument x >= 0.

# See also

cyl\_bessel\_i for setails.

Definition at line 625 of file specfun.h.

**7.2.2.21** long double std::cyl\_bessel\_il ( long double \_\_nu, long double \_\_x ) [inline]

Return the regular modified Bessel function  $I_{\nu}(x)$  for long double order  $\nu$  and argument x >= 0.

# See also

cyl\_bessel\_i for setails.

Definition at line 635 of file specfun.h.

Return the Bessel function  $J_{\nu}(x)$  of real order  $\nu$  and argument x >= 0.

The cylindrical Bessel function is:

$$J_{\nu}(x) = \sum_{k=0}^{\infty} \frac{(-1)^k (x/2)^{\nu+2k}}{k! \Gamma(\nu+k+1)}$$

# **Template Parameters**

_Tpnu	The floating-point type of the ordernu.
_ <i>Tp</i>	The floating-point type of the argumentx.

#### **Parameters**

nu	The order
X	The argument, $\underline{}x >= 0$

# **Exceptions**

std::domain_error	if v < 0
stadomaii_enti	" <u></u>

Definition at line 702 of file specfun.h.

**7.2.2.23** float std::cyl\_bessel\_jf ( float \_\_nu, float \_\_x ) [inline]

Return the Bessel function of the first kind  $J_{\nu}(x)$  for float order  $\nu$  and argument x>=0.

See also

cyl\_bessel\_j for setails.

Definition at line 671 of file specfun.h.

7.2.2.24 long double std::cyl\_bessel\_il ( long double \_\_nu, long double \_\_x ) [inline]

Return the Bessel function of the first kind  $J_{\nu}(x)$  for long double order  $\nu$  and argument x>=0.

See also

cyl\_bessel\_j for setails.

Definition at line 681 of file specfun.h.

7.2.2.25 template<typename \_Tpnu , typename \_Tp > \_\_gnu\_cxx::\_\_promote\_2<\_Tpnu, \_Tp>::\_\_type std::cyl\_bessel\_k ( \_Tpnu \_\_nu, \_Tp \_\_x ) [inline]

Return the irregular modified Bessel function  $K_{\nu}(x)$  of real order  $\nu$  and argument x.

The irregular modified Bessel function is defined by:

$$K_{\nu}(x) = \frac{\pi}{2} \frac{I_{-\nu}(x) - I_{\nu}(x)}{\sin \nu \pi}$$

where for integral  $\nu=n$  a limit is taken:  $lim_{\nu\to n}$ . For negative argument we have simply:

$$K_{-\nu}(x) = K_{\nu}(x)$$

# **Template Parameters**

_Tpnu	The floating-point type of the ordernu.
_ <i>Tp</i>	The floating-point type of the argumentx.

#### **Parameters**

nu	The order
X	The argument, $\underline{}x >= 0$

# **Exceptions**

std::domain_error	$if_{x} < 0$ .
_	

Definition at line 754 of file specfun.h.

**7.2.2.26** float std::cyl\_bessel\_kf ( float \_\_nu, float \_\_x ) [inline]

Return the irregular modified Bessel function  $K_{\nu}(x)$  for float order  $\nu$  and argument x>=0.

See also

cyl\_bessel\_k for setails.

Definition at line 717 of file specfun.h.

7.2.2.27 long double std::cyl\_bessel\_kl ( long double \_\_nu, long double \_\_x ) [inline]

Return the irregular modified Bessel function  $K_{\nu}(x)$  for long double order  $\nu$  and argument x>=0.

See also

cyl\_bessel\_k for setails.

Definition at line 727 of file specfun.h.

Return the Neumann function  $N_{\nu}(x)$  of real order  $\nu$  and argument x >= 0.

The Neumann function is defined by:

$$N_{\nu}(x) = \frac{J_{\nu}(x)\cos\nu\pi - J_{-\nu}(x)}{\sin\nu\pi}$$

where x >= 0 and for integral order  $\nu = n$  a limit is taken:  $\lim_{\nu \to n} d\nu$ 

# **Template Parameters**

_Tpnu	The floating-point type of the ordernu.
_ <i>Tp</i>	The floating-point type of the argumentx.

#### **Parameters**

nu	The order
$\underline{}$ The argument, $\underline{}$ $x >= 0$	

# **Exceptions**

```
std::domain_error \mid if \__x < 0.
```

Definition at line 802 of file specfun.h.

**7.2.2.29** float std::cyl\_neumannf (float \_\_nu, float \_\_x ) [inline]

Return the Neumann function  $N_{\nu}(x)$  of float order  $\nu$  and argument x.

See also

cyl\_neumann for setails.

Definition at line 769 of file specfun.h.

7.2.2.30 long double std::cyl\_neumannl( long double \_\_nu, long double \_\_x ) [inline]

Return the Neumann function  $N_{\nu}(x)$  of long double order  $\nu$  and argument x.

See also

cyl\_neumann for setails.

Definition at line 779 of file specfun.h.

7.2.2.31 template<typename\_Tp , typename\_Tpp > \_\_gnu\_cxx::\_\_promote\_2<\_Tp, \_Tpp>::\_\_type std::ellint\_1 ( \_Tp \_\_k, \_Tpp \_\_phi ) [inline]

Return the incomplete elliptic integral of the first kind  $F(k,\phi)$  for real modulus k and angle  $\phi$ .

The incomplete elliptic integral of the first kind is defined as

$$F(k,\phi) = \int_0^{\phi} \frac{d\theta}{\sqrt{1 - k^2 sin^2 \theta}}$$

For  $\phi = \pi/2$  this becomes the complete elliptic integral of the first kind, K(k).

See also

comp ellint 1.

**Template Parameters** 

_ <i>Tp</i>   The floating-point type of the modulusk.
--

_ <i>Tpp</i>	The floating-point type of the anglephi.

#### **Parameters**

k	The modulus, abs (k) <= 1
phi	The integral limit argument in radians

#### **Exceptions**

```
std::domain_error | if abs (__k) > 1 .
```

Definition at line 850 of file specfun.h.

Return the incomplete elliptic integral of the first kind  $E(k,\phi)$  for float modulus k and angle  $\phi$ .

See also

ellint\_1 for details.

Definition at line 817 of file specfun.h.

Return the incomplete elliptic integral of the first kind  $E(k,\phi)$  for long double modulus k and angle  $\phi$ .

See also

ellint 1 for details.

Definition at line 827 of file specfun.h.

Return the incomplete elliptic integral of the second kind  $E(k, \phi)$ .

The incomplete elliptic integral of the second kind is defined as

$$E(k,\phi) = \int_0^\phi \sqrt{1 - k^2 sin^2 \theta}$$

For  $\phi = \pi/2$  this becomes the complete elliptic integral of the second kind, E(k).

See also

comp\_ellint\_2.

# **Template Parameters**

_ <i>Tp</i>	The floating-point type of the modulusk.
_Трр	The floating-point type of the anglephi.

#### **Parameters**

k	The modulus, abs (k) <= 1
phi	The integral limit argument in radians

#### Returns

The elliptic function of the second kind.

#### **Exceptions**

$$std::domain\_error \mid if abs(\__k) > 1$$
.

Definition at line 898 of file specfun.h.

Return the incomplete elliptic integral of the second kind  $E(k,\phi)$  for float argument.

#### See also

ellint\_2 for details.

Definition at line 865 of file specfun.h.

Return the incomplete elliptic integral of the second kind  $E(k, \phi)$ .

#### See also

ellint\_2 for details.

Definition at line 875 of file specfun.h.

Return the incomplete elliptic integral of the third kind  $\Pi(k, \nu, \phi)$ .

The incomplete elliptic integral of the third kind is defined by:

$$\Pi(k,\nu,\phi) = \int_0^\phi \frac{d\theta}{(1-\nu\sin^2\theta)\sqrt{1-k^2\sin^2\theta}}$$

For  $\phi=\pi/2$  this becomes the complete elliptic integral of the third kind,  $\Pi(k,\nu)$ .

#### See also

comp\_ellint\_3.

# **Template Parameters**

_Tp	The floating-point type of the modulusk.
_Tpn	The floating-point type of the argumentnu.
_Трр	The floating-point type of the anglephi.

#### **Parameters**

k	The modulus, abs (k) <= 1
nu	The second argument
phi	The integral limit argument in radians

# Returns

The elliptic function of the third kind.

# **Exceptions**

std::domain_erro	if abs (k) > 1 .
------------------	------------------

Definition at line 951 of file specfun.h.

Return the incomplete elliptic integral of the third kind  $\Pi(k,\nu,\phi)$  for float argument.

#### See also

ellint\_3 for details.

Definition at line 913 of file specfun.h.

7.2.2.39 long double std::ellint\_3I ( long double \_\_k, long double \_\_nu, long double \_\_phi ) [inline]

Return the incomplete elliptic integral of the third kind  $\Pi(k, \nu, \phi)$ .

# See also

ellint\_3 for details.

Definition at line 923 of file specfun.h.

7.2.2.40 template<typename\_Tp > \_\_gnu\_cxx::\_\_promote<\_Tp>::\_\_type std::expint(\_Tp\_\_x) [inline]

Return the exponential integral Ei(x) for real argument x.

The exponential integral is given by

$$Ei(x) = -\int_{-x}^{\infty} \frac{e^t}{t} dt$$

# **Template Parameters**

_ <i>Tp</i>	The floating-point type of the argumentx.

#### **Parameters**

\_\_x The argument of the exponential integral function.

Definition at line 991 of file specfun.h.

7.2.2.41 float std::expintf (float \_x ) [inline]

Return the exponential integral Ei(x) for float argument x.

See also

expint for details.

Definition at line 965 of file specfun.h.

**7.2.2.42** long double std::expintl (long double \_x ) [inline]

Return the exponential integral Ei(x) for long double argument x.

See also

expint for details.

Definition at line 975 of file specfun.h.

Return the Hermite polynomial  $H_n(x)$  of order n and real argument x.

The Hermite polynomial is defined by:

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2}$$

The Hermite polynomial obeys a reflection formula:

$$H_n(-x) = (-1)^n H_n(x)$$

# **Template Parameters**

_Tp	The floating-point type of the argumentx.

# **Parameters**

n The order
-------------

\_\_x The argument

Definition at line 1039 of file specfun.h.

**7.2.2.44** float std::hermitef (unsigned int \_\_n, float \_\_x ) [inline]

Return the Hermite polynomial  $H_n(x)$  of nonnegative order n and float argument x.

See also

hermite for details.

Definition at line 1006 of file specfun.h.

**7.2.2.45** long double std::hermitel ( unsigned int \_\_n, long double \_\_x ) [inline]

Return the Hermite polynomial  $H_n(x)$  of nonnegative order n and long double argument x.

See also

hermite for details.

Definition at line 1016 of file specfun.h.

Returns the Laguerre polynomial  $L_n(x)$  of nonnegative degree n and real argument x >= 0.

The Laguerre polynomial is defined by:

$$L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n} (x^n e^{-x})$$

**Template Parameters** 

$\_\mathit{Tp}$ The floating-point type of the argument $\_\_x$ .	
---	--

#### **Parameters**

	n	The nonnegative order
_		The argument $_x >= 0$

# **Exceptions**

Definition at line 1083 of file specfun.h.

7.2.2.47 float std::laguerref (unsigned int \_\_n, float \_\_x ) [inline]

Returns the Laguerre polynomial  $L_n(x)$  of nonnegative degree n and float argument x>=0.

See also

laguerre for more details.

Definition at line 1054 of file specfun.h.

**7.2.2.48** long double std::laguerrel ( unsigned int \_\_n, long double \_\_x ) [inline]

Returns the Laguerre polynomial  $L_n(x)$  of nonnegative degree n and long double argument x >= 0.

See also

laguerre for more details.

Definition at line 1064 of file specfun.h.

Return the Legendre polynomial  $P_l(x)$  of nonnegative degree l and real argument |x| <= 0.

The Legendre function of order l and argument x,  $P_l(x)$ , is defined by:

$$P_l(x) = \frac{1}{2^l l!} \frac{d^l}{dx^l} (x^2 - 1)^l$$

#### **Template Parameters**

_Tp	The floating-point type of the argumentx.

# **Parameters**

	The degree $l>=0$
X	The argument abs (x) <= 1

# **Exceptions**

Definition at line 1128 of file specfun.h.

7.2.2.50 float std::legendref ( unsigned int \_\_I, float \_\_x ) [inline]

Return the Legendre polynomial  $P_l(x)$  of nonnegative degree l and float argument |x| <= 0.

See also

legendre for more details.

Definition at line 1098 of file specfun.h.

**7.2.2.51** long double std::legendrel ( unsigned int \_\_I, long double \_\_x ) [inline]

Return the Legendre polynomial  $P_l(x)$  of nonnegative degree l and long double argument |x| <= 0.

See also

legendre for more details.

Definition at line 1108 of file specfun.h.

 $\textbf{7.2.2.52} \quad template < typename \_Tp > \underline{\quad} gnu\_cxx::\underline{\quad} promote < \underline{\quad} Tp > ::\underline{\quad} type \ std::riemann\_zeta \ (\ \underline{\quad} Tp \underline{\quad} s \ ) \quad \texttt{[inline]}$ 

Return the Riemann zeta function  $\zeta(s)$  for real argument s.

The Riemann zeta function is defined by:

$$\zeta(s) = \sum_{k=1}^{\infty} k^{-s} \text{ for } s > 1$$

and

$$\zeta(s) = \frac{1}{1 - 2^{1 - s}} \sum_{k = 1}^{\infty} (-1)^{k - 1} k^{-s} \text{ for } 0 <= s <= 1$$

For s < 1 use the reflection formula:

$$\zeta(s) = 2^s \pi^{s-1} \sin(\frac{\pi s}{2}) \Gamma(1-s) \zeta(1-s)$$

**Template Parameters** 

_Tp	The floating-point type of the arguments.
-----	---

# **Parameters**

```
__s | The argument s != 1
```

Definition at line 1179 of file specfun.h.

7.2.2.53 float std::riemann\_zetaf (float \_\_s ) [inline]

Return the Riemann zeta function  $\zeta(s)$  for float argument s.

See also

riemann\_zeta for more details.

Definition at line 1143 of file specfun.h.

**7.2.2.54** long double std::riemann\_zetal ( long double \_\_s ) [inline]

Return the Riemann zeta function  $\zeta(s)$  for long double argument s.

See also

riemann zeta for more details.

Definition at line 1153 of file specfun.h.

Return the spherical Bessel function  $j_n(x)$  of nonnegative order n and real argument x >= 0.

The spherical Bessel function is defined by:

$$j_n(x) = \left(\frac{\pi}{2x}\right)^{1/2} J_{n+1/2}(x)$$

#### **Template Parameters**

To	The fleeting point type of the argument
_1p	The floating-point type of the argumentx.

#### **Parameters**

n	The integral order $n >= 0$
X	The real argument $x >= 0$

#### **Exceptions**

Definition at line 1223 of file specfun.h.

**7.2.2.56** float std::sph\_besself ( unsigned int \_\_n, float \_\_x ) [inline]

Return the spherical Bessel function  $j_n(x)$  of nonnegative order n and float argument x >= 0.

See also

sph\_bessel for more details.

Definition at line 1194 of file specfun.h.

7.2.2.57 long double std::sph\_bessell ( unsigned int \_n, long double \_x ) [inline]

Return the spherical Bessel function  $j_n(x)$  of nonnegative order n and long double argument x>=0.

See also

sph bessel for more details.

Definition at line 1204 of file specfun.h.

7.2.2.58 template<typename\_Tp > \_\_gnu\_cxx::\_\_promote<\_Tp>::\_\_type std::sph\_legendre( unsigned int \_\_I, unsigned int \_\_m, \_\_Tp \_\_theta ) [inline]

Return the spherical Legendre function of nonnegative integral degree 1 and order m and real angle  $\theta$  in radians.

The spherical Legendre function is defined by

$$Y_l^m(\theta,\phi) = (-1)^m \left[ \frac{(2l+1)}{4\pi} \frac{(l-m)!}{(l+m)!} \right] P_l^m(\cos\theta) \exp^{im\phi}$$

# **Template Parameters**

_ <i>Tp</i>	The floating-point type of the angle _	_theta.

#### **Parameters**

1	The order $_{1} >= 0$	
m	The degreem >= 0 andm <=1	
theta	The radian polar angle argument	

Definition at line 1270 of file specfun.h.

7.2.2.59 float std::sph\_legendref ( unsigned int \_\_*l*, unsigned int \_\_*m*, float \_\_*theta* ) [inline]

Return the spherical Legendre function of nonnegative integral degree 1 and order m and float angle  $\theta$  in radians.

#### See also

sph legendre for details.

Definition at line 1238 of file specfun.h.

7.2.2.60 long double std::sph\_legendrel ( unsigned int \_\_l, unsigned int \_\_m, long double \_\_theta ) [inline]

Return the spherical Legendre function of nonnegative integral degree 1 and order m and long double angle  $\theta$  in radians.

# See also

sph\_legendre for details.

Definition at line 1249 of file specfun.h.

Return the spherical Neumann function of integral order  $n \ge 0$  and real argument  $x \ge 0$ .

The spherical Neumann function is defined by

$$n_n(x) = \left(\frac{\pi}{2x}\right)^{1/2} N_{n+1/2}(x)$$

#### **Template Parameters**

_ <i>Tp</i>	The floating-point type of the argumentx.

# **Parameters**

n	The integral order $n >= 0$

```
\underline{\phantom{a}} The real argument \underline{\phantom{a}} x >= 0
```

# **Exceptions**

```
std::domain_error | if __x < 0 .
```

Definition at line 1314 of file specfun.h.

```
7.2.2.62 float std::sph_neumannf ( unsigned int __n, float __x ) [inline]
```

Return the spherical Neumann function of integral order n >= 0 and float argument x >= 0.

# See also

sph\_neumann for details.

Definition at line 1285 of file specfun.h.

```
7.2.2.63 long double std::sph_neumannl ( unsigned int _n, long double _x ) [inline]
```

Return the spherical Neumann function of integral order n >= 0 and long double x >= 0.

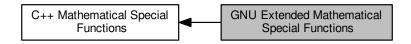
#### See also

sph\_neumann for details.

Definition at line 1295 of file specfun.h.

# 7.3 GNU Extended Mathematical Special Functions

Collaboration diagram for GNU Extended Mathematical Special Functions:



# **Enumerations**

enum { \_\_gnu\_cxx::\_GLIBCXX\_JACOBI\_SN, \_\_gnu\_cxx::\_GLIBCXX\_JACOBI\_CN, \_\_gnu\_cxx::\_GLIBCXX\_J
 ACOBI\_DN }

#### **Functions**

```
template<typename</li>Tp >
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::airy_ai (_Tp __x)

    float __gnu_cxx::airy_aif (float __x)

    long double gnu cxx::airy ail (long double x)

template<typename _Tp >
   __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::airy_bi (_Tp __x)

    float gnu cxx::airy bif (float x)

    long double gnu cxx::airy bil (long double x)

template<typename_Tp>
    \_gnu\_cxx::\_promote\_num\_t < \_Tp > \_\_gnu\_cxx::bernoulli \ (unsigned \ int \ \_\_n)

    float gnu cxx::bernoullif (unsigned int n)

    long double gnu cxx::bernoullil (unsigned int n)

template<typename_Tp>
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::bincoef (unsigned int __n, unsigned int __k)

    float gnu cxx::bincoeff (unsigned int n, unsigned int k)

• long double gnu cxx::bincoefl (unsigned int n, unsigned int k)
template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::chebyshev_t (unsigned int __n, Tp x)

    float gnu cxx::chebyshev tf (unsigned int n, float x)

    long double __gnu_cxx::chebyshev_tl (unsigned int __n, long double __x)

template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::chebyshev_u (unsigned int __n, _Tp __x)

    float gnu cxx::chebyshev uf (unsigned int n, float x)

    long double __gnu_cxx::chebyshev_ul (unsigned int __n, long double __x)

template<typename</li>Tp >
   gnu cxx:: promote num t< Tp > gnu cxx::chebyshev v (unsigned int n, Tp x)

    float gnu cxx::chebyshev vf (unsigned int n, float x)

    long double __gnu_cxx::chebyshev_vl (unsigned int __n, long double __x)

template<typename _Tp >
  __gnu_cxx::_promote_num_t< _Tp > __gnu_cxx::chebyshev_w (unsigned int __n, _Tp __x)
```

```
    float __gnu_cxx::chebyshev_wf (unsigned int __n, float __x)

    long double __gnu_cxx::chebyshev_wl (unsigned int __n, long double __x)

template<typename _Tp >
   __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::clausen (unsigned int __m, _Tp __w)
template<typename _Tp >
  std::complex< __gnu_cxx::__promote_num_t< _Tp >> __gnu_cxx::clausen (unsigned int __m, std::complex<
  _{\mathsf{Tp}} > _{\mathsf{w}}

 template<typename _Tp >

   __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::clausen_c (unsigned int __m, _Tp __w)

    float gnu cxx::clausen cf (unsigned int m, float w)

    long double gnu cxx::clausen cl (unsigned int m, long double w)

template<typename_Tp>
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::clausen_s (unsigned int __m, _Tp __w)

    float gnu cxx::clausen sf (unsigned int m, float w)

    long double <u>__gnu_cxx::clausen_sl</u> (unsigned int __m, long double __w)

    float gnu cxx::clausenf (unsigned int m, float w)

• std::complex< float > __gnu_cxx::clausenf (unsigned int __m, std::complex< float > __w)
• long double gnu cxx::clausenl (unsigned int m, long double w)

    std::complex < long double > __gnu_cxx::clausenl (unsigned int __m, std::complex < long double > __w)

    template<typename Tk >

   _gnu_cxx::__promote_num_t< _Tk > __gnu_cxx::comp_ellint_d (_Tk __k)

    float __gnu_cxx::comp_ellint_df (float __k)

    long double gnu cxx::comp ellint dl (long double k)

float __gnu_cxx::comp_ellint_rf (float __x, float __y)

    long double gnu cxx::comp ellint rf (long double x, long double y)

    template<typename _Tx , typename _Ty >

   _gnu_cxx::__promote_num_t< _Tx, _Ty > __gnu_cxx::comp_ellint_rf (_Tx __x, _Ty __y)

    float gnu cxx::comp ellint rg (float x, float y)

    long double __gnu_cxx::comp_ellint_rg (long double __x, long double __y)

    template<typename _Tx , typename _Ty >

   _gnu_cxx::__promote_num_t< _Tx, _Ty > __gnu_cxx::comp_ellint_rg (_Tx __x, _Ty __y)
• template<typename _Tpa , typename _Tpc , typename _Tp >
   _gnu_cxx::__promote_3< _Tpa, _Tpc, _Tp >::__type <u>__gnu_cxx::conf_hyperg</u> (_Tpa __a, _Tpc __c, _Tp __x)
• template<typename _Tpc , typename _Tp >
  __gnu_cxx::__promote_2< _Tpc, _Tp >::__type __gnu_cxx::conf_hyperg_lim (_Tpc __c, _Tp __x)

    float gnu cxx::conf hyperg limf (float c, float x)

    long double gnu cxx::conf hyperg liml (long double c, long double x)

    float gnu cxx::conf hypergf (float a, float c, float x)

    long double gnu cxx::conf hypergl (long double a, long double c, long double x)

template<typename _Tp >
    gnu cxx:: promote num t< Tp> gnu cxx::coshint (Tpx)

    float gnu cxx::coshintf (float x)

    long double gnu cxx::coshintl (long double x)

    template<typename</li>
    Tp >

   __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::cosint (_Tp __x)

    float gnu cxx::cosintf (float x)

    long double gnu cxx::cosintl (long double x)

    template<typename _Tpnu , typename _Tp >

 __z)
```

```
• template<typename _Tpnu , typename _Tp >
     std::complex< gnu cxx:: promote num t< Tpnu, Tp>> gnu cxx::cyl hankel 1 (std::complex< ←
     Tpnu > nu, std::complex < Tp > x

    std::complex< float > __gnu_cxx::cyl_hankel_1f (float __nu, float __z)

    std::complex < float > __gnu_cxx::cyl_hankel_1f (std::complex < float > __nu, std::complex < float > __x)

• std::complex < long double > gnu cxx::cyl hankel 1l (long double nu, long double z)
• std::complex < long double > gnu cxx::cyl hankel 1l (std::complex < long double > nu, std::complex < long
     double > __x)
template<typename _Tpnu , typename _Tp >
     std::complex< \underline{\quad} gnu\_cxx::\underline{\quad} promote\_num\_t< \underline{\quad} Tpnu, \underline{\quad} Tp>> \underline{\quad} gnu\_cxx::cyl\_hankel\_2 \ (\underline{\quad} Tpnu \ \underline{\quad} nu, \underline{\quad} Tp>> \underline{\quad} gnu\_cxx::cyl\_hankel\_2 \ (\underline{\quad} Tpnu \ \underline{\quad} nu, \underline{\quad} Tp>> \underline{\quad} gnu\_cxx::cyl\_hankel\_2 \ (\underline{\quad} Tpnu \ \underline{\quad} nu, \underline{\quad} Tp>> \underline{\quad} gnu\_cxx::cyl\_hankel\_2 \ (\underline{\quad} Tpnu \ \underline{\quad} nu, \underline{\quad} Tp>> \underline{\quad} gnu\_cxx::cyl\_hankel\_2 \ (\underline{\quad} Tpnu \ \underline{\quad} nu, \underline{\quad} Tp>> \underline{\quad} gnu\_cxx::cyl\_hankel\_2 \ (\underline{\quad} Tpnu \ \underline{\quad} nu, \underline{\quad} Tp>> \underline{\quad} gnu\_cxx::cyl\_hankel\_2 \ (\underline{\quad} Tpnu \ \underline{\quad} nu, \underline{\quad} Tp>> \underline{\quad} gnu\_cxx::cyl\_hankel\_2 \ (\underline{\quad} Tpnu \ \underline{\quad} nu, \underline{\quad} Tp>> \underline{\quad} gnu\_cxx::cyl\_hankel\_2 \ (\underline{\quad} Tpnu \ \underline{\quad} nu, \underline{\quad} Tp>> \underline{\quad} gnu\_cxx::cyl\_hankel\_2 \ (\underline{\quad} Tpnu \ \underline{\quad} nu, \underline{\quad} Tp>> \underline{\quad} gnu\_cxx::cyl\_hankel\_2 \ (\underline{\quad} Tpnu \ \underline{\quad} nu, \underline{\quad} Tp>> \underline{\quad} gnu\_cxx::cyl\_hankel\_2 \ (\underline{\quad} Tpnu \ \underline{\quad} nu, \underline{\quad} Tp>> \underline{\quad} gnu\_cxx::cyl\_hankel\_2 \ (\underline{\quad} Tpnu \ \underline{\quad} nu, \underline{\quad} \underline
• template<typename _Tpnu , typename _Tp >
     std::complex< gnu cxx:: promote num t< Tpnu, Tp>> gnu cxx::cyl hankel 2 (std::complex< ←
     Tpnu > __nu, std::complex< _Tp > __x)

    std::complex< float > __gnu_cxx::cyl_hankel_2f (float __nu, float __z)

    std::complex < float > __gnu_cxx::cyl_hankel_2f (std::complex < float > __nu, std::complex < float > __x)

    std::complex < long double > gnu cxx::cyl hankel 2l (long double nu, long double z)

• std::complex < long double > gnu cxx::cyl hankel 2l (std::complex < long double > nu, std::complex < long
     double > \underline{\hspace{1cm}} x)
template<typename</li>Tp >
          _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::dawson (_Tp __x)

    float __gnu_cxx::dawsonf (float __x)

    long double <u>gnu_cxx::dawsonl</u> (long double <u>x</u>)

template<typename</li>Tp >
          gnu cxx:: promote num t < Tp > gnu cxx::digamma (Tp z)

    float gnu cxx::digammaf (float z)

    long double gnu cxx::digammal (long double z)

template<typename _Tp >
       __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::dilog (_Tp __x)

    float gnu cxx::dilogf (float x)

    long double gnu cxx::dilogl (long double x)

template<typename</li>Tp >
     _Tp __gnu_cxx::dirichlet_beta (_Tp __s)
• float __gnu_cxx::dirichlet betaf (float s)

    long double gnu cxx::dirichlet betal (long double s)

template<typename _Tp >
     _Tp __gnu_cxx::dirichlet_eta (_Tp __s)

    float gnu cxx::dirichlet etaf (float s)

    long double gnu cxx::dirichlet etal (long double s)

template<typename _Tp >
          gnu cxx:: promote num t< Tp > gnu cxx::double factorial (int n)

    float gnu cxx::double factorialf (int n)

    long double __gnu_cxx::double_factoriall (int __n)

• template<typename Tk, typename Tp, typename Ta, typename Tb>
          gnu_cxx::_promote_num_t<_Tk,_Tp,_Ta,_Tb>__gnu_cxx::ellint_cel(_Tk__k_c,_Tp__p,_Ta__a,_Tb
          b)

    float __gnu_cxx::ellint_celf (float __k_c, float __p, float __a, float __b)

    long double gnu cxx::ellint cell (long double k c, long double p, long double a, long double b)

• template<typename _Tk , typename _Tphi >
       __gnu_cxx::__promote_num_t< _Tk, _Tphi > __gnu_cxx::ellint_d (_Tk __k, _Tphi __phi)

    float gnu cxx::ellint df (float k, float phi)

• long double gnu cxx::ellint dl (long double k, long double phi)
```

```
• template<typename _{\rm Tp}, typename _{\rm Tk} >
   _gnu_cxx::__promote_num_t< _Tp, _Tk > __gnu_cxx::ellint_el1 (_Tp __x, _Tk __k_c)
• float gnu cxx::ellint el1f (float x, float k c)

    long double __gnu_cxx::ellint_el1l (long double __x, long double __k_c)

• template<typename Tp, typename Tk, typename Ta, typename Tb>
    gnu_cxx::_promote_num_t< _Tp, _Tk, _Ta, _Tb > __gnu_cxx::ellint_el2 (_Tp __x, _Tk __k_c, _Ta __a, _Tb
   b)

    float __gnu_cxx::ellint_el2f (float __x, float __k_c, float __a, float __b)

    long double __gnu_cxx::ellint_el2l (long double __x, long double __k_c, long double __a, long double __b)

• template<typename Tx, typename Tk, typename Tp>
  __gnu_cxx::__promote_num_t<_Tx,_Tk,_Tp > __gnu_cxx::ellint_el3 (_Tx __x, _Tk __k_c, _Tp __p)

    float __gnu_cxx::ellint_el3f (float __x, float __k_c, float __p)

    long double __gnu_cxx::ellint_el3l (long double __x, long double __k_c, long double __p)

• template<typename _Tp , typename _Up >
    _gnu_cxx::__promote_num_t< _Tp, _Up > __gnu_cxx::ellint_rc (_Tp __x, _Up __y)
• float gnu cxx::ellint rcf (float x, float y)

    long double gnu cxx::ellint rcl (long double x, long double y)

- template<typename _Tp , typename _Up , typename _Vp >
    gnu cxx:: promote num t< Tp, Up, Vp > gnu cxx::ellint rd (Tp x, Up y, Vp z)

    float gnu cxx::ellint rdf (float x, float y, float z)

    long double gnu cxx::ellint rdl (long double x, long double y, long double z)

- template<typename _Tp , typename _Up , typename _Vp >
    _gnu_cxx::__promote_num_t< _Tp, _Up, _Vp > __gnu_cxx::ellint_rf (_Tp __x, _Up __y, _Vp __z)

    float gnu cxx::ellint rff (float x, float y, float z)

• long double __gnu_cxx::ellint_rfl (long double __x, long double __y, long double __z)
template<typename _Tp , typename _Up , typename _Vp >
    _gnu_cxx::__promote_num_t< _Tp, _Up, _Vp > __gnu_cxx::ellint_rg (_Tp __x, _Up __y, _Vp __z)

    float gnu cxx::ellint rgf (float x, float y, float z)

    long double __gnu_cxx::ellint_rgl (long double __x, long double __y, long double __z)

ullet template<typename _Tp , typename _Up , typename _Vp , typename _Wp >
  __gnu_cxx::_promote_num_t< _Tp, _Up, _Vp, _Wp > __gnu_cxx::ellint_rj (_Tp __x, _Up __y, _Vp __z, _Wp
  __p)

    float __gnu_cxx::ellint_rjf (float __x, float __y, float __z, float __p)

    long double __gnu_cxx::ellint_rjl (long double __x, long double __y, long double __z, long double __p)

• template<typename Tp >
  _Tp __gnu_cxx::ellnome (_Tp __k)

    float gnu cxx::ellnomef (float k)

• long double __gnu_cxx::ellnomel (long double __k)
• template<typename_Tp>
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::expint_e1 (_Tp __x)

    float gnu cxx::expint e1f (float x)

    long double __gnu_cxx::expint_e1l (long double __x)

template<typename _Tp >
   __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::expint_en (unsigned int __n, _Tp __x)

    float gnu cxx::expint enf (unsigned int n, float x)

    long double __gnu_cxx::expint_enl (unsigned int __n, long double __x)

template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::factorial (unsigned int __n)

    float gnu cxx::factorialf (unsigned int

    long double <u>__gnu_cxx::factoriall</u> (unsigned int __n)

template<typename _Tp >
   __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::fresnel_c (_Tp __x)
```

```
    float __gnu_cxx::fresnel_cf (float __x)

• long double __gnu_cxx::fresnel_cl (long double __x)
template<typename _Tp >
   __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::fresnel_s (_Tp __x)

    float __gnu_cxx::fresnel_sf (float __x)

    long double <u>__gnu_cxx::fresnel_sl</u> (long double <u>__x)</u>

ullet template<typename _Tn , typename _Tp >
   _gnu_cxx::__promote_num_t< _Tn, _Tp > __gnu_cxx::gamma_l (_Tn __n, _Tp __x)
• float __gnu_cxx::gamma_lf (float __n, float __x)

    long double __gnu_cxx::gamma_ll (long double __n, long double __x)

• template<typename Ta, typename Tp>
    _gnu_cxx::__promote_num_t< _Ta, _Tp > __gnu_cxx::gamma_p (_Ta __a, _Tp __x)

    float gnu cxx::gamma pf (float a, float x)

    long double __gnu_cxx::gamma_pl (long double __a, long double __x)

template<typename _Ta , typename _Tp >
    _gnu_cxx::__promote_num_t< _Ta, _Tp > __gnu_cxx::gamma_q (_Ta __a, _Tp __x)

    float gnu cxx::gamma gf (float a, float x)

    long double gnu cxx::gamma ql (long double a, long double x)

• template<typename _{\rm Tn}, typename _{\rm Tp} >
    gnu cxx:: promote num t< Tn, Tp > gnu cxx::gamma u (Tn n, Tp x)

    float gnu cxx::gamma uf (float n, float x)

    long double gnu cxx::gamma ul (long double n, long double x)

    template<typename _Talpha , typename _Tp >

    _gnu_cxx::__promote_num_t< _Talpha, _Tp > __gnu_cxx::gegenbauer (unsigned int __n, _Talpha __alpha,
  _Tp __x)
• float gnu cxx::gegenbauerf (unsigned int n, float alpha, float x)

    long double __gnu_cxx::gegenbauerl (unsigned int __n, long double __alpha, long double __x)

    template<typename _Tk , typename _Tphi >

   __gnu_cxx::__promote_num_t< _Tk, _Tphi > __gnu_cxx::heuman_lambda (_Tk __k, _Tphi __phi)

    float gnu cxx::heuman lambdaf (float k, float phi)

    long double gnu cxx::heuman lambdal (long double k, long double phi)

• template<typename _Tp , typename _Up >
    _gnu_cxx::__promote_num_t< _Tp, _Up > __gnu_cxx::hurwitz_zeta (_Tp __s, _Up __a)

    template<typename _Tp , typename _Up >

  std::complex< _Tp > __gnu_cxx::hurwitz_zeta (_Tp __s, std::complex< _Up > __a)

    float gnu cxx::hurwitz zetaf (float s, float a)

    long double gnu cxx::hurwitz zetal (long double s, long double a)

    template<typename _Tpa , typename _Tpb , typename _Tpc , typename _Tp >

    _gnu_cxx::__promote_4< _Tpa, _Tpb, _Tpc, _Tp >::__type __gnu_cxx::hyperg (_Tpa __a, _Tpb __b, _Tpc
   __c, _Tp ___x)

    float __gnu_cxx::hypergf (float __a, float __b, float __c, float __x)

    long double __gnu_cxx::hypergl (long double __a, long double __b, long double __c, long double __x)

- template<typename _Ta , typename _Tb , typename _Tp >
    _gnu_cxx::__promote_num_t< _Ta, _Tb, _Tp > __gnu_cxx::ibeta (_Ta __a, _Tb __b, _Tp __x)

    template<typename _Ta , typename _Tb , typename _Tp >

    _gnu_cxx::__promote_num_t< _Ta, _Tb, _Tp > __gnu_cxx::ibetac (_Ta __a, _Tb __b, _Tp __x)

    float gnu cxx::ibetacf (float a, float b, float x)

    long double gnu cxx::ibetacl (long double a, long double b, long double x)

    float gnu cxx::ibetaf (float a, float b, float x)

    long double gnu cxx::ibetal (long double a, long double b, long double x)
```

```
ullet template<typename _Talpha , typename _Tbeta , typename _Tp >
   _gnu_cxx::__promote_num_t< _Talpha, _Tbeta, _Tp > __gnu_cxx::jacobi (unsigned __n, _Talpha __alpha,
  Tbeta beta, Tp x)
• template<typename _Kp , typename _Up >
   _gnu_cxx::__promote_num_t< _Kp, _Up > __gnu_cxx::jacobi_cn (_Kp __k, _Up __u)
• float gnu cxx::jacobi cnf (float k, float u)
• long double gnu cxx::jacobi cnl (long double k, long double u)
• template<typename _Kp , typename _Up >
    _gnu_cxx::__promote_num_t< _Kp, _Up > __gnu_cxx::jacobi_dn (_Kp __k, _Up __u)
• float gnu cxx::jacobi dnf (float k, float u)

    long double __gnu_cxx::jacobi_dnl (long double __k, long double __u)

• template<typename _Kp , typename _Up >
    _gnu_cxx::__promote_num_t< _Kp, _Up > __gnu_cxx::jacobi_sn (_Kp __k, _Up __u)

    float gnu cxx::jacobi snf (float k, float u)

    long double __gnu_cxx::jacobi_snl (long double __k, long double __u)

• template<typename Tk, typename Tphi >
    gnu_cxx::__promote_num_t< _Tk, _Tphi > __gnu_cxx::jacobi_zeta ( Tk k, Tphi phi)

    float gnu cxx::jacobi zetaf (float k, float phi)

    long double __gnu_cxx::jacobi_zetal (long double __k, long double __phi)

    float gnu cxx::jacobif (unsigned n, float alpha, float beta, float x)

    long double __gnu_cxx::jacobil (unsigned __n, long double __alpha, long double __beta, long double __x)

template<typename _Tp >
   _gnu_cxx::_promote_num_t< _Tp > __gnu_cxx::lbincoef (unsigned int __n, unsigned int __k)

    float gnu cxx::lbincoeff (unsigned int n, unsigned int k)

• long double __gnu_cxx::lbincoefl (unsigned int __n, unsigned int __k)
template<typename _Tp >
   gnu cxx:: promote num t < Tp > gnu cxx::ldouble factorial (int n)

    float gnu cxx::ldouble factorialf (int n)

    long double __gnu_cxx::ldouble_factoriall (int __n)

template<typename</li>Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::legendre_q (unsigned int __n, _Tp __x)

    float __gnu_cxx::legendre_qf (unsigned int __n, float __x)

    long double __gnu_cxx::legendre_ql (unsigned int __n, long double __x)

template<typename _Tp >
    gnu cxx:: promote num t < Tp > gnu cxx::lfactorial (unsigned int n)

    float __gnu_cxx::lfactorialf (unsigned int __n)

    long double gnu cxx::lfactoriall (unsigned int n)

template<typename _Tp >
   __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::logint (_Tp __x)

    float gnu cxx::logintf (float x)

    long double gnu cxx::logintl (long double x)

• template<typename _Tp , typename _Tn >
    _gnu_cxx::__promote_num_t< _Tp, _Tn > __gnu_cxx::lpochhammer_l (_Tp __a, _Tn __n)

    float gnu cxx::lpochhammer lf (float a, float n)

    long double gnu cxx::lpochhammer II (long double a, long double n)

• template<typename _Tp , typename _Tn >
   _gnu_cxx::__promote_num_t< _Tp, _Tn > __gnu_cxx::lpochhammer_u (_Tp __a, _Tn __n)

    float gnu cxx::lpochhammer uf (float a, float n)

    long double gnu cxx::lpochhammer ul (long double a, long double n)

    template<typename _Tph , typename _Tpa >

    gnu cxx:: promote num t< Tph, Tpa > gnu cxx::owens t (Tph h, Tpa a)

    float gnu cxx::owens tf (float h, float a)
```

```
    long double __gnu_cxx::owens_tl (long double __h, long double __a)

• template<typename _Tp , typename _Tn >
    _gnu_cxx::__promote_num_t< _Tp, _Tn > __gnu_cxx::pochhammer_l (_Tp __a, _Tn __n)

    float __gnu_cxx::pochhammer_lf (float __a, float __n)

    long double gnu cxx::pochhammer II (long double a, long double n)

• template<typename _Tp , typename _Tn >
    _gnu_cxx::__promote_num_t< _Tp, _Tn > __gnu_cxx::pochhammer_u (_Tp __a, _Tn __n)

    float __gnu_cxx::pochhammer_uf (float __a, float __n)

• long double __gnu_cxx::pochhammer_ul (long double __a, long double __n)
• template<typename _Tp , typename _Wp >
   _gnu_cxx::__promote_num_t< _Tp, _Wp > __gnu_cxx::polylog (_Tp __s, _Wp __w)

    template<typename</li>
    Tp , typename
    Wp >

  std::complex< __gnu_cxx::__promote_num_t< _Tp, _Wp >> __gnu_cxx::polylog (_Tp __s, std::complex< _Tp
  > w)

    float __gnu_cxx::polylogf (float __s, float __w)

    std::complex < float > gnu cxx::polylogf (float s, std::complex < float > w)

    long double __gnu_cxx::polylogl (long double __s, long double __w)

    std::complex < long double > __gnu_cxx::polylogl (long double __s, std::complex < long double > __w)

template<typename</li>Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::psi (_Tp __x)

    float __gnu_cxx::psif (float __x)

    long double __gnu_cxx::psil (long double __x)

template<typename</li>Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::radpoly (unsigned int __n, unsigned int __m, Tp rho)
• float __gnu_cxx::radpolyf (unsigned int __n, unsigned int __m, float __rho)

    long double __gnu_cxx::radpolyl (unsigned int __n, unsigned int __m, long double __rho)

template<typename _Tp >
  __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::sinc (_Tp __x)
template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::sinc_pi (_Tp __x)

    float gnu cxx::sinc pif (float x)

    long double __gnu_cxx::sinc_pil (long double __x)

 float __gnu_cxx::sincf (float __x)

    long double gnu cxx::sincl (long double x)

template<typename _Tp >
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::sinhc (_Tp __x)
template<typename_Tp>
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::sinhc_pi (_Tp __x)

    float gnu cxx::sinhc pif (float x)

    long double gnu cxx::sinhc pil (long double x)

    float gnu cxx::sinhcf (float x)

    long double gnu cxx::sinhcl (long double x)

template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::sinhint (_Tp __x)

    float gnu cxx::sinhintf (float x)

    long double <u>__gnu_cxx::sinhintl</u> (long double <u>__x)</u>

template<typename_Tp>
   gnu cxx:: promote num t< Tp> gnu cxx::sinint (Tpx)

    float gnu cxx::sinintf (float x)

    long double <u>__gnu_cxx::sinintl</u> (long double <u>__x)</u>

template<typename_Tp>
  __gnu_cxx::_promote_num_t< _Tp > __gnu_cxx::sph_bessel_i (unsigned int __n, _Tp __x)
```

```
    float gnu cxx::sph_bessel_if (unsigned int __n, float __x)

    long double gnu cxx::sph bessel il (unsigned int n, long double x)

template<typename Tp >
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::sph_bessel_k (unsigned int __n, _Tp __x)
• float gnu cxx::sph bessel kf (unsigned int n, float x)

    long double __gnu_cxx::sph_bessel_kl (unsigned int __n, long double __x)

template<typename</li>Tp >
  std::complex< __gnu_cxx::__promote_num_t< _Tp >> __gnu_cxx::sph_hankel_1 (unsigned int __n, _Tp __z)

    template<typename</li>
    Tp >

  std::complex< __gnu_cxx::_promote_num_t< _Tp >> __gnu_cxx::sph_hankel_1 (unsigned int __n, std↔
  ::complex < _Tp > __x)
• std::complex< float > gnu cxx::sph hankel 1f (unsigned int n, float z)

    std::complex < float > gnu cxx::sph hankel 1f (unsigned int n, std::complex < float > x)

• std::complex< long double > __gnu_cxx::sph_hankel_1l (unsigned int __n, long double __z)
• std::complex < long double > gnu cxx::sph hankel 1l (unsigned int n, std::complex < long double > x)
template<typename</li>Tp >
  std::complex < __gnu_cxx::__promote_num_t < _Tp > > __gnu_cxx::sph_hankel_2 (unsigned int __n, _Tp __z)
template<typename _Tp >
  std::complex< gnu cxx:: promote num t< Tp >> gnu cxx::sph hankel 2 (unsigned int n, std←
  ::complex < Tp > x)

    std::complex< float > __gnu_cxx::sph_hankel_2f (unsigned int __n, float __z)

    std::complex < float > __gnu_cxx::sph_hankel_2f (unsigned int __n, std::complex < float > __x)

• std::complex < long double > __gnu_cxx::sph_hankel_2l (unsigned int __n, long double z)
• std::complex < long double > gnu cxx::sph hankel 2l (unsigned int n, std::complex < long double > x)
• template<typename _Ttheta , typename _Tphi >
 std::complex< __gnu_cxx::_promote_num_t< _Ttheta, _Tphi >> __gnu_cxx::sph_harmonic (unsigned int ←
   I, int m, Ttheta theta, Tphi phi)
• std::complex < float > gnu cxx::sph harmonicf (unsigned int I, int m, float theta, float phi)
• std::complex < long double > __gnu_cxx::sph_harmonicl (unsigned int __l, int __m, long double __theta, long
  double phi)
• template<typename Tpnu, typename Tp >
  gnu_cxx::_promote_num_t< _Tpnu, _Tp > __gnu_cxx::theta_1 (_Tpnu __nu, _Tp __x)
float __gnu_cxx::theta_1f (float __nu, float __x)

    long double gnu cxx::theta 1l (long double nu, long double x)

• template<typename _Tpnu , typename _Tp >
   _gnu_cxx::__promote_num_t< _Tpnu, _Tp > __gnu_cxx::theta_2 (_Tpnu __nu, _Tp __x)
• float gnu cxx::theta 2f (float nu, float x)

    long double gnu cxx::theta 2l (long double nu, long double x)

• template<typename _Tpnu , typename _Tp >
   _gnu_cxx::__promote_num_t< _Tpnu, _Tp > __gnu_cxx::theta_3 (_Tpnu __nu, _Tp __x)
• float gnu cxx::theta 3f (float nu, float x)

    long double __gnu_cxx::theta_3l (long double __nu, long double __x)

• template<typename _Tpnu , typename _Tp >
   _gnu_cxx::__promote_num_t< _Tpnu, _Tp > __gnu_cxx::theta_4 (_Tpnu __nu, _Tp __x)

    float __gnu_cxx::theta_4f (float __nu, float __x)

    long double __gnu_cxx::theta_4l (long double __nu, long double __x)

template<typename _Tpk , typename _Tp >
   gnu cxx:: promote num t < Tpk, Tp > gnu cxx::theta c ( Tpk k, Tp x)

    float gnu cxx::theta cf (float k, float x)

    long double __gnu_cxx::theta_cl (long double __k, long double __x)

template<typename _Tpk , typename _Tp >
  __gnu_cxx::__promote_num_t< _Tpk, _Tp > __gnu_cxx::theta_d (_Tpk __k, _Tp __x)
```

```
• float __gnu_cxx::theta_df (float __k, float __x)
```

long double \_\_gnu\_cxx::theta\_dl (long double \_\_k, long double \_\_x)

• template<typename \_Tpk , typename \_Tp >

```
__gnu_cxx::__promote_num_t< _Tpk, _Tp > __gnu_cxx::theta_n (_Tpk __k, _Tp __x)
```

- float gnu cxx::theta nf (float k, float x)
- long double \_\_gnu\_cxx::theta\_nl (long double \_\_k, long double \_\_x)
- template<typename \_Tpk , typename \_Tp >

- float gnu cxx::theta sf (float k, float x)
- long double gnu cxx::theta sl (long double k, long double x)
- $\bullet \;\; {\rm template}{<} {\rm typename} \; {\rm \_Trho} \; , \\ {\rm typename} \; {\rm \_Tphi} > \\$

```
__gnu_cxx::__promote_num_t< _Trho, _Tphi > __gnu_cxx::zernike (unsigned int __n, int __m, _Trho __rho, Tphi phi)
```

- float \_\_gnu\_cxx::zernikef (unsigned int \_\_n, int \_\_m, float \_\_rho, float \_\_phi)
- long double \_\_gnu\_cxx::zernikel (unsigned int \_\_n, int \_\_m, long double \_\_rho, long double \_\_phi)

# 7.3.1 Detailed Description

An extended collection of advanced mathematical special functions for GNU.

# 7.3.2 Enumeration Type Documentation

#### 7.3.2.1 anonymous enum

Enumerator

\_GLIBCXX\_JACOBI\_SN \_GLIBCXX\_JACOBI\_CN \_GLIBCXX\_JACOBI\_DN

Definition at line 1734 of file specfun.h.

# 7.3.3 Function Documentation

Return the Airy function Ai(x) of real argument x.

The Airy function is defined by:

$$Ai(x) = \frac{1}{\pi} \int_0^\infty \cos\left(\frac{t^3}{3} + xt\right) dt$$

**Template Parameters** 

\_Tp | The real type of the argument

### **Parameters**

x	The argument

Definition at line 2676 of file specfun.h.

7.3.3.2 float \_\_gnu\_cxx::airy\_aif(float \_\_x) [inline]

Return the Airy function Ai(x) for float argument x.

See also

airy\_ai for details.

Definition at line 2641 of file specfun.h.

7.3.3.3 long double \_\_gnu\_cxx::airy\_ail ( long double \_\_x ) [inline]

Return the Airy function Ai(x) for long double argument x.

See also

airy\_ai for details.

Definition at line 2655 of file specfun.h.

7.3.3.4 template<typename \_Tp > \_\_gnu\_cxx::\_\_promote\_num\_t<\_Tp> \_\_gnu\_cxx::airy\_bi( \_Tp \_\_x ) [inline]

Return the Airy function Bi(x) of real argument x.

The Airy function is defined by:

$$Bi(x) = \frac{1}{\pi} \int_0^\infty \left[ \exp\left(-\frac{t^3}{3} + xt\right) + \sin\left(\frac{t^3}{3} + xt\right) \right] dt$$

## **Template Parameters**

_ <i>Tp</i>	The real type of the argument

## **Parameters**

Definition at line 2728 of file specfun.h.

7.3.3.5 float \_\_gnu\_cxx::airy\_bif( float \_\_x ) [inline]

Return the Airy function Bi(x) for float argument x.

See also

airy\_bi for details.

Definition at line 2692 of file specfun.h.

```
7.3.3.6 long double __gnu_cxx::airy_bil( long double __x ) [inline]
```

Return the Airy function Bi(x) for long double argument x.

See also

airy\_bi for details.

Definition at line 2706 of file specfun.h.

Return the Bernoulli number of integer order n.

The Bernoulli numbers are defined by

### **Parameters**

```
__n The order.
```

Definition at line 3704 of file specfun.h.

```
7.3.3.8 float __gnu_cxx::bernoullif ( unsigned int __n ) [inline]
```

Return the Bernoulli number of integer order n as a float.

See also

bernoulli for details.

Definition at line 3679 of file specfun.h.

```
7.3.3.9 long double __gnu_cxx::bernoullil( unsigned int __n ) [inline]
```

Return the Bernoulli number of integer order n as a long double.

See also

bernoulli for details.

Definition at line 3689 of file specfun.h.

```
7.3.3.10 template<typename_Tp > __gnu_cxx::__promote_num_t<_Tp> __gnu_cxx::bincoef ( unsigned int __n, unsigned int __n,
```

Definition at line 3644 of file specfun.h.

**7.3.3.11** float \_\_gnu\_cxx::bincoeff ( unsigned int \_\_n, unsigned int \_\_k ) [inline]

Definition at line 3632 of file specfun.h.

7.3.3.12 long double \_\_gnu\_cxx::bincoefl ( unsigned int \_\_n, unsigned int \_\_k ) [inline]

Definition at line 3636 of file specfun.h.

Return the Chebyshev polynomial of the first kind  $T_n(x)$  of non-negative order n and real argument x.

The Chebyshev polynomial of the first kind is defined by:

$$T_n(x) = \cos(n\theta)$$

where  $\theta = \arccos(x)$ ,  $-1 \le x \le +1$ .

### **Template Parameters**

_ <i>Tp</i>	The real type of the argument

### **Parameters**

 n The non-negative integral order
 $x$ The real argument $-1 \le x \le +1$

Definition at line 1935 of file specfun.h.

7.3.3.14 float \_\_gnu\_cxx::chebyshev\_tf ( unsigned int \_\_n, float \_\_x ) [inline]

Return the Chebyshev polynomials of the first kind  $T_n(x)$  of non-negative order n and float argument x.

See also

chebyshev\_t for details.

Definition at line 1906 of file specfun.h.

7.3.3.15 long double \_\_gnu\_cxx::chebyshev\_tl( unsigned int \_\_n, long double \_\_x ) [inline]

Return the Chebyshev polynomials of the first kind  $T_n(x)$  of non-negative order n and real argument x.

See also

chebyshev t for details.

Definition at line 1916 of file specfun.h.

Return the Chebyshev polynomial of the second kind  $U_n(x)$  of non-negative order n and real argument x.

The Chebyshev polynomial of the second kind is defined by:

$$U_n(x) = \frac{\sin[(n+1)\theta]}{\sin(\theta)}$$

where  $\theta = \arccos(x)$ ,  $-1 \le x \le +1$ .

## **Template Parameters**

_Tp	The real type of the argument

#### **Parameters**

n	The non-negative integral order
X	The real argument $-1 \le x \le +1$

Definition at line 1979 of file specfun.h.

7.3.3.17 float \_\_gnu\_cxx::chebyshev\_uf ( unsigned int \_\_n, float \_\_x ) [inline]

Return the Chebyshev polynomials of the second kind  $U_n(x)$  of non-negative order n and float argument x.

## See also

chebyshev\_u for details.

Definition at line 1950 of file specfun.h.

7.3.3.18 long double \_\_gnu\_cxx::chebyshev\_ul ( unsigned int \_\_n, long double \_\_x ) [inline]

Return the Chebyshev polynomials of the second kind  $U_n(x)$  of non-negative order n and real argument x.

## See also

chebyshev\_u for details.

Definition at line 1960 of file specfun.h.

Return the Chebyshev polynomial of the third kind  $V_n(x)$  of non-negative order n and real argument x.

The Chebyshev polynomial of the third kind is defined by:

$$V_n(x) = \frac{\cos\left[\left(n + \frac{1}{2}\right)\theta\right]}{\cos\left(\frac{\theta}{2}\right)}$$

where  $\theta = \arccos(x)$ ,  $-1 \le x \le +1$ .

## **Template Parameters**

_Tp   The real type of the argument	
-------------------------------------	--

## **Parameters**

n	The non-negative integral order
x	The real argument $-1 \le x \le +1$

Definition at line 2024 of file specfun.h.

7.3.3.20 float \_\_gnu\_cxx::chebyshev\_vf( unsigned int \_\_n, float \_\_x ) [inline]

Return the Chebyshev polynomials of the third kind  $V_n(x)$  of non-negative order n and float argument x.

### See also

chebyshev\_v for details.

Definition at line 1994 of file specfun.h.

7.3.3.21 long double \_\_gnu\_cxx::chebyshev\_vI ( unsigned int \_\_n, long double \_\_x ) [inline]

Return the Chebyshev polynomials of the third kind  $V_n(x)$  of non-negative order n and real argument x.

#### See also

chebyshev\_v for details.

Definition at line 2004 of file specfun.h.

Return the Chebyshev polynomial of the fourth kind  $W_n(x)$  of non-negative order n and real argument x.

The Chebyshev polynomial of the fourth kind is defined by:

$$W_n(x) = \frac{\sin\left[\left(n + \frac{1}{2}\right)\theta\right]}{\sin\left(\frac{\theta}{2}\right)}$$

where  $\theta = \arccos(x)$ ,  $-1 \le x \le +1$ .

### **Template Parameters**

= 1	_ <i>Tp</i>	The real type of the argument
-----	-------------	-------------------------------

## **Parameters**

n	The non-negative integral order
X	The real argument $-1 \le x \le +1$

Definition at line 2069 of file specfun.h.

7.3.3.23 float \_\_gnu\_cxx::chebyshev\_wf( unsigned int \_\_n, float \_\_x) [inline]

Return the Chebyshev polynomials of the fourth kind  $W_n(x)$  of non-negative order n and float argument x.

## See also

chebyshev\_w for details.

Definition at line 2039 of file specfun.h.

```
7.3.3.24 long double __gnu_cxx::chebyshev_wl ( unsigned int __n, long double __x ) [inline]
```

Return the Chebyshev polynomials of the fourth kind  $W_n(x)$  of non-negative order n and real argument x.

See also

chebyshev\_w for details.

Definition at line 2049 of file specfun.h.

Return the Clausen function of integer order m and complex argument w.

The Clausen function is defined by

#### **Parameters**

m	
w	The complex argument

Definition at line 4647 of file specfun.h.

Definition at line 4668 of file specfun.h.

7.3.3.27 templateTp > \underline{gnu\\_cxx::\\_promote\\_num\\_t} < Tp > \underline{gnu\\_cxx::clausen\\_c} ( unsigned int 
$$\underline{m}$$
,  $Tp \underline{w}$ ) [inline]

Return the Clausen cosine function of order m and real argument x.

The Clausen cosine function is defined by

#### **Parameters**

m	
W	

Definition at line 4608 of file specfun.h.

```
7.3.3.28 float __gnu_cxx::clausen_cf ( unsigned int __m, float __w ) [inline]
```

Return the Clausen cosine function of order m and real argument x.

See also

clausen c for details.

Definition at line 4583 of file specfun.h.

```
7.3.3.29 long double __gnu_cxx::clausen_cl ( unsigned int __m, long double __w ) [inline]
```

Return the Clausen cosine function of order m and real argument x.

See also

clausen c for details.

Definition at line 4592 of file specfun.h.

Return the Clausen sine function of order m and real argument x.

The Clausen sine function is defined by

#### **Parameters**

m	
W	

Definition at line 4569 of file specfun.h.

```
7.3.3.31 float __gnu_cxx::clausen_sf ( unsigned int __m, float __w ) [inline]
```

Return the Clausen sine function of order m and real argument x.

See also

clausen s for details.

Definition at line 4544 of file specfun.h.

```
7.3.3.32 long double __gnu_cxx::clausen_sl ( unsigned int __m, long double __w ) [inline]
```

Return the Clausen sine function of order m and real argument x.

See also

clausen\_s for details.

Definition at line 4553 of file specfun.h.

```
7.3.3.33 float __gnu_cxx::clausenf ( unsigned int __m, float __w ) [inline]
```

Return the Clausen function of integer order m and complex argument w.

See also

clausen for details.

Definition at line 4622 of file specfun.h.

7.3.3.34 std::complex < float > \_\_gnu\_cxx::clausenf ( unsigned int \_\_m, std::complex < float > \_\_w ) [inline]

Definition at line 4656 of file specfun.h.

7.3.3.35 long double \_\_gnu\_cxx::clausenl ( unsigned int \_\_m, long double \_\_w ) [inline]

Return the Clausen function of integer order m and complex argument w.

#### See also

clausen for details.

Definition at line 4631 of file specfun.h.

7.3.3.36 std::complex < long double >  $\_$ gnu\_cxx::clausenl ( unsigned int  $\_$ m, std::complex < long double >  $\_$ w ) [inline]

Definition at line 4660 of file specfun.h.

 $7.3.3.37 \quad template < typename \_Tk > \_gnu\_cxx::\_promote\_num\_t < \_Tk > \_gnu\_cxx::comp\_ellint\_d ( \_Tk \_k ) \quad [inline]$ 

Return the complete Legendre elliptic integral D(k) of real modulus k.

The complete Legendre elliptic integral D is defined by

$$D(k) = \int_0^{\pi/2} \frac{\sin^2 \theta d\theta}{\sqrt{1 - k^2 \sin 2\theta}}$$

## **Template Parameters**

_ <i>Tk</i>	The type of the modulus k

#### **Parameters**

$$\underline{\hspace{0.5cm}}$$
 The modulus  $-1$   $<=$   $\underline{\hspace{0.5cm}}$   $k$   $<=$   $+1$ 

Definition at line 3890 of file specfun.h.

7.3.3.38 float \_\_gnu\_cxx::comp\_ellint\_df(float \_\_k) [inline]

Return the complete Legendre elliptic integral D(k) of float modulus k.

See also

comp\_ellint\_d for details.

Definition at line 3863 of file specfun.h.

7.3.3.39 long double \_\_gnu\_cxx::comp\_ellint\_dl( long double \_\_k) [inline]

Return the complete Legendre elliptic integral D(k) of long double modulus k.

See also

comp\_ellint\_d for details.

Definition at line 3873 of file specfun.h.

Return the complete Carlson elliptic function  $R_F(x,y,z)$  for float arguments.

See also

comp ellint rf for details.

Definition at line 2849 of file specfun.h.

Return the complete Carlson elliptic function  $R_F(x,y)$  for long double arguments.

See also

comp\_ellint\_rf for details.

Definition at line 2859 of file specfun.h.

Return the complete Carlson elliptic function  $R_F(x,y)$  for real arguments.

The complete Carlson elliptic function of the first kind is defined by:

$$R_F(x,y) = R_F(x,y,y) = \frac{1}{2} \int_0^\infty \frac{dt}{(t+x)^{1/2}(t+y)}$$

### **Parameters**

X	The first argument.
у	The second argument.

Definition at line 2877 of file specfun.h.

Return the Carlson complementary elliptic function  $R_G(x,y)$ .

See also

comp\_ellint\_rg for details.

Definition at line 3082 of file specfun.h.

7.3.3.44 long double \_\_gnu\_cxx::comp\_ellint\_rg ( long double \_\_x, long double \_\_y ) [inline]

Return the Carlson complementary elliptic function  $R_G(x, y)$ .

See also

comp\_ellint\_rg for details.

Definition at line 3091 of file specfun.h.

Return the complete Carlson elliptic function  $R_G(x,y)$  for real arguments.

The complete Carlson elliptic function is defined by:

$$R_G(x,y) = R_G(x,y,y) = \frac{1}{4} \int_0^\infty dt t(t+x)^{-1/2} (t+y)^{-1} (\frac{x}{t+x} + \frac{2y}{t+y})$$

### **Parameters**

X	The first argument.
y	The second argument.

Definition at line 3110 of file specfun.h.

Return the confluent hypergeometric function  ${}_1F_1(a;c;x)$  of real numeratorial parameter a, denominatorial parameter c, and argument x.

The confluent hypergeometric function is defined by

$$_{1}F_{1}(a;c;x) = \sum_{n=0}^{\infty} \frac{(a)_{n}x^{n}}{(c)_{n}n!}$$

where the Pochhammer symbol is  $(x)_k = (x)(x+1)...(x+k-1), (x)_0 = 1$ 

### **Parameters**

a	The numeratorial parameter
c	The denominatorial parameter
x	The argument

Definition at line 1377 of file specfun.h.

Return the confluent hypergeometric limit function  ${}_{0}F_{1}(z;x)$  of real numeratorial parameter c and argument x.

The confluent hypergeometric limit function is defined by

$$_{0}F_{1}(;c;x) = \sum_{n=0}^{\infty} \frac{x^{n}}{(c)_{n}n!}$$

where the Pochhammer symbol is  $(x)_k=(x)(x+1)...(x+k-1),$   $(x)_0=1$ 

### **Parameters**

c	The denominatorial parameter
x	The argument

Definition at line 1473 of file specfun.h.

```
7.3.3.48 float __gnu_cxx::conf_hyperg_limf(float __c, float __x) [inline]
```

Return the confluent hypergeometric limit function  ${}_0F_1(;c;x)$  of float numeratorial parameter c and argument x.

See also

conf hyperg lim for details.

Definition at line 1444 of file specfun.h.

```
7.3.3.49 long double __gnu_cxx::conf_hyperg_liml( long double __c, long double __x) [inline]
```

Return the confluent hypergeometric limit function  ${}_0F_1(;c;x)$  of long double numeratorial parameter c and argument x.

See also

conf\_hyperg\_lim for details.

Definition at line 1454 of file specfun.h.

```
7.3.3.50 float __gnu_cxx::conf_hypergf ( float __a, float __c, float __x ) [inline]
```

Return the confluent hypergeometric function  ${}_1F_1(a;c;x)$  of float numeratorial parameter a, denominatorial parameter c, and argument x.

See also

conf\_hyperg for details.

Definition at line 1345 of file specfun.h.

```
7.3.3.51 long double __gnu_cxx::conf_hypergl( long double __a, long double __c, long double __x) [inline]
```

Return the confluent hypergeometric function  ${}_1F_1(a;c;x)$  of long double numeratorial parameter a, denominatorial parameter c, and argument x.

See also

conf\_hyperg for details.

Definition at line 1356 of file specfun.h.

Return the hyperbolic cosine integral Chi(x) of real argument x.

The hyperbolic cosine integral is defined by

$$Chi(x) = -\int_{x}^{\infty} \frac{\cosh(t)}{t} dt = \gamma_E + \ln(x) + \int_{0}^{x} \frac{\cosh(t) - 1}{t} dt$$

## **Template Parameters**

_ <i>Tp</i>	The type of the real argument

#### **Parameters**

\_\_x The real argument

Definition at line 1727 of file specfun.h.

7.3.3.53 float \_\_gnu\_cxx::coshintf(float \_\_x) [inline]

Return the hyperbolic cosine integral of float argument x.

See also

coshint for details.

Definition at line 1699 of file specfun.h.

7.3.3.54 long double \_\_gnu\_cxx::coshintl( long double \_\_x ) [inline]

Return the hyperbolic cosine integral Chi(x) of long double argument x.

See also

coshint for details.

Definition at line 1709 of file specfun.h.

Return the cosine integral Ci(x) of real argument x.

The cosine integral is defined by

$$Ci(x) = -\int_{x}^{\infty} \frac{\cos(t)}{t} dt = \gamma_E + \ln(x) + \int_{0}^{x} \frac{\cos(t) - 1}{t} dt$$

### **Parameters**

\_\_x The real upper integration limit

Definition at line 1644 of file specfun.h.

7.3.3.56 float \_\_gnu\_cxx::cosintf(float \_\_x) [inline]

Return the cosine integral Ci(x) of float argument x.

See also

cosint for details.

Definition at line 1618 of file specfun.h.

7.3.3.57 long double \_\_gnu\_cxx::cosintl( long double \_\_x ) [inline]

Return the cosine integral Ci(x) of long double argument x.

See also

cosint for details.

Definition at line 1628 of file specfun.h.

Return the cylindrical Hankel function of the first kind  $H_n^{(1)}(x)$  of real order  $\nu$  and argument x >= 0.

The cylindrical Hankel function of the first kind is defined by:

$$H_{\nu}^{(1)}(x) = \left(\frac{\pi}{2x}\right)^{1/2} \left[J_{n+1/2}(x) + iN_{n+1/2}(x)\right]$$

where  $J_{\nu}(x)$  and  $N_{\nu}(x)$  are the cylindrical Bessel and Neumann functions respectively (

See also

cyl bessel and cyl neumann).

**Template Parameters** 

_ <i>Tp</i>   The real type of the argument
---

## **Parameters**

nu	The real order
Z	The real argument

Definition at line 2378 of file specfun.h.

Return the complex cylindrical Hankel function of the first kind  $H_{\nu}^{(1)}(x)$  of complex order  $\nu$  and argument x.

The cylindrical Hankel function of the first kind is defined by

$$H_{\nu}^{(1)}(x) = J_{\nu}(x) + iN_{\nu}(x)$$

#### **Template Parameters**

_Tpnu	The complex type of the order
_ <i>Tp</i>	The complex type of the argument

#### **Parameters**

nu	The complex order
X	The complex argument

Definition at line 4167 of file specfun.h.

Return the cylindrical Hankel function of the first kind  $H_{\nu}^{(1)}(x)$  of float order  $\nu$  and argument x >= 0.

See also

cyl\_hankel\_1 for details.

Definition at line 2345 of file specfun.h.

7.3.3.61 std::complex 
$$\_$$
gnu\_cxx::cyl\_hankel\_1f ( std::complex< float >  $\_$ nu, std::complex< float >  $\_$ x ) [inline]

Return the complex cylindrical Hankel function of the first kind  $H_{\nu}^{(1)}(x)$  of std::complex<float> order  $\nu$  and argument x.

See also

cyl\_hankel\_1 for more details.

Definition at line 4136 of file specfun.h.

Return the cylindrical Hankel function of the first kind  $H_{\nu}^{(1)}(x)$  of long double order  $\nu$  and argument x >= 0.

See also

cyl\_hankel\_1 for details.

Definition at line 2356 of file specfun.h.

7.3.3.63 std::complex < long double >  $\_$ gnu\_cxx::cyl\_hankel\_1I ( std::complex < long double >  $\_$ nu, std::complex < long double >  $\_$ nu, std::complex < long double >  $\_$ nu, std::complex < long double

Return the complex cylindrical Hankel function of the first kind  $H_{\nu}^{(1)}(x)$  of std::complex<long double> order  $\nu$  and argument x.

See also

cyl\_hankel\_1 for more details.

Definition at line 4147 of file specfun.h.

Return the cylindrical Hankel function of the second kind  $H_n^{(2)}(x)$  of real order  $\nu$  and argument x >= 0.

The cylindrical Hankel function of the second kind is defined by:

$$H_{\nu}^{(2)}(x) = \left(\frac{\pi}{2x}\right)^{1/2} \left[J_{n+1/2}(x) - iN_{n+1/2}(x)\right]$$

where  $J_{
u}(x)$  and  $N_{
u}(x)$  are the cylindrical Bessel and Neumann functions respectively (

See also

cyl\_bessel and cyl\_neumann).

## **Template Parameters**

_ <i>Tp</i>	The real type of the argument

### **Parameters**

nu	The real order
Z	The real argument

Definition at line 2427 of file specfun.h.

Return the complex cylindrical Hankel function of the second kind  $H^{(2)}_{\nu}(x)$  of complex order  $\nu$  and argument x.

The cylindrical Hankel function of the second kind is defined by

$$H_{\nu}^{(2)}(x) = J_{\nu}(x) - iN_{\nu}(x)$$

**Template Parameters** 

_Tpnu	The complex type of the order

_ <i>Tp</i>	The complex type of the argument

### **Parameters**

nu	The complex order
x	The complex argument

Definition at line 4214 of file specfun.h.

```
7.3.3.66 std::complex<float> __gnu_cxx::cyl_hankel_2f(float __nu, float __z) [inline]
```

Return the cylindrical Hankel function of the second kind  $H_{\nu}^{(2)}(x)$  of float order  $\nu$  and argument x >= 0.

See also

```
cyl_hankel_2 for details.
```

Definition at line 2394 of file specfun.h.

```
7.3.3.67 std::complex < float > __nu, std::complex < float > __nu, std::complex < float > __x )  [ \texttt{inline} ]
```

Return the complex cylindrical Hankel function of the second kind  $H^{(2)}_{\nu}(x)$  of std::complex<float> order  $\nu$  and argument x.

See also

```
cyl hankel 2 for more details.
```

Definition at line 4183 of file specfun.h.

```
7.3.3.68 std::complex < long double > _gnu_cxx::cyl_hankel_2l( long double __nu, long double __z) [inline]
```

Return the cylindrical Hankel function of the second kind  $H_{\nu}^{(2)}(x)$  of long double order  $\nu$  and argument x >= 0.

See also

```
cyl_hankel_2 for details.
```

Definition at line 2405 of file specfun.h.

```
7.3.3.69 std::complex < long double > \_gnu_cxx::cyl_hankel_2l ( std::complex < long double > \_nu, std::complex < long double > \_x ) [inline]
```

Return the complex cylindrical Hankel function of the second kind  $H_{\nu}^{(2)}(x)$  of std::complex<long double> order  $\nu$  and argument x.

See also

```
cyl hankel 2 for more details.
```

Definition at line 4194 of file specfun.h.

7.3.3.70 template<typename\_Tp > \_\_gnu\_cxx::\_\_promote\_num\_t<\_Tp> \_\_gnu\_cxx::dawson( \_Tp \_\_x ) [inline]

Return the Dawson integral, F(x), for real argument x.

The Dawson integral is defined by:

$$F(x) = e^{-x^2} \int_0^x e^{y^2} dy$$

and it's derivative is:

$$F'(x) = 1 - 2xF(x)$$

### **Parameters**

```
\underline{\hspace{0.5cm}} The argument -inf < x < inf.
```

Definition at line 3420 of file specfun.h.

7.3.3.71 float \_\_gnu\_cxx::dawsonf(float \_\_x) [inline]

Return the Dawson integral, F(x), for float argument x.

### See also

dawson for details.

Definition at line 3392 of file specfun.h.

7.3.3.72 long double \_\_gnu\_cxx::dawsonl(long double \_\_x) [inline]

Return the Dawson integral, F(x), for long double argument x.

#### See also

dawson for details.

Definition at line 3401 of file specfun.h.

Definition at line 2793 of file specfun.h.

7.3.3.74 float \_\_gnu\_cxx::digammaf(float \_\_z) [inline]

Definition at line 2781 of file specfun.h.

7.3.3.75 long double \_\_gnu\_cxx::digammal( long double \_\_z ) [inline]

Definition at line 2785 of file specfun.h.

7.3.3.76 template<typename\_ $Tp > \underline{gnu\_cxx::\_promote\_num\_t < \underline{Tp} > \underline{gnu\_cxx::dilog(\_Tp\_x)}$  [inline]

Return the dilogarithm function  $\psi(z)$  for real argument.

The dilogarithm is defined by:

$$Li_2(x) = \sum_{k=1}^{\infty} \frac{x^k}{k^2}$$

#### **Parameters**

\_\_x The argument.

Definition at line 2834 of file specfun.h.

7.3.3.77 float \_\_gnu\_cxx::dilogf(float \_\_x) [inline]

Return the dilogarithm function  $\psi(z)$  for float argument.

See also

dilog for details.

Definition at line 2808 of file specfun.h.

7.3.3.78 long double \_\_gnu\_cxx::dilogl(long double \_\_x) [inline]

Return the dilogarithm function  $\psi(z)$  for long double argument.

See also

dilog for details.

Definition at line 2818 of file specfun.h.

7.3.3.79 template<typename\_Tp > \_Tp \_\_gnu\_cxx::dirichlet\_beta(\_Tp \_\_s) [inline]

Return the Dirichlet beta function of real argument s.

The Dirichlet beta function is defined by:

$$\beta(s) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)^s}$$

An important reflection formula is:

$$\beta(1-s) = \left(\frac{2}{\pi}\right)^s \sin(\frac{\pi s}{2})\Gamma(s)\beta(s)$$

### **Parameters**

s

Definition at line 4530 of file specfun.h.

7.3.3.80 float \_\_gnu\_cxx::dirichlet\_betaf ( float \_\_s ) [inline]

Return the Dirichlet beta function of real argument s.

See also

dirichlet beta for details.

Definition at line 4501 of file specfun.h.

**7.3.3.81** long double \_\_gnu\_cxx::dirichlet\_betal ( long double \_\_s ) [inline]

Return the Dirichlet beta function of real argument s.

See also

dirichlet\_beta for details.

Definition at line 4510 of file specfun.h.

Return the Dirichlet eta function of real argument s.

The Dirichlet eta function is defined by

$$\eta(s) = \sum_{k=1}^{\infty} \frac{(-1)^k}{k^s} = (1 - 2^{1-s}) \zeta(s)$$

An important reflection formula is:

$$\eta(-s) = 2\frac{1 - 2^{-s-1}}{1 - 2^{-s}} \pi^{-s-1} s \sin(\frac{\pi s}{2}) \Gamma(s) \eta(s+1)$$

**Parameters** 

\_\_s

Definition at line 4487 of file specfun.h.

7.3.3.83 float \_\_gnu\_cxx::dirichlet\_etaf( float \_\_s ) [inline]

Return the Dirichlet eta function of real argument s.

See also

dirichlet\_eta for details.

Definition at line 4457 of file specfun.h.

**7.3.3.84** long double \_\_gnu\_cxx::dirichlet\_etal( long double \_\_s ) [inline]

Return the Dirichlet eta function of real argument s.

See also

dirichlet\_eta for details.

Definition at line 4466 of file specfun.h.

7.3.3.85 template<typename\_Tp > \_\_gnu\_cxx::\_\_promote\_num\_t<\_Tp> \_\_gnu\_cxx::double\_factorial(int \_\_n) [inline]

Definition at line 3581 of file specfun.h.

7.3.3.86 float \_\_gnu\_cxx::double\_factorialf(int \_\_n) [inline]

Definition at line 3569 of file specfun.h.

7.3.3.87 long double \_\_gnu\_cxx::double\_factorial( int \_\_n ) [inline]

Definition at line 3573 of file specfun.h.

Return the Bulirsch complete elliptic integral  $cel(k_c, p, a, b)$  of real complementary modulus  $k_c$ , and parameters p, a, and b.

The Bulirsch complete elliptic integral is defined by

$$cel(k_c, p, a, b) = \int_0^{\pi/2} \frac{a\cos^2\theta + b\sin^2\theta}{\cos^2\theta + p\sin^2\theta} \frac{d\theta}{\sqrt{\cos^2\theta + k_c^2\sin^2\theta}}$$

## **Parameters**

k_c	The complementary modulus $k_c=\sqrt{1-k^2}$
p	The parameter
a	The parameter
b	The parameter

Definition at line 4120 of file specfun.h.

Return the Bulirsch complete elliptic integral  $cel(k_c, p, a, b)$  of real complementary modulus  $k_c$ , and parameters p, a, and b.

See also

ellint\_cel for details.

Definition at line 4088 of file specfun.h.

7.3.3.90 long double  $\_gnu\_cxx::ellint\_cell$  ( long double  $\_k\_c$ , long double  $\_p$ , long double  $\_a$ , long double  $\_b$  ) [inline]

Return the Bulirsch complete elliptic integral  $cel(k_c, p, a, b)$ .

See also

ellint cel for details.

Definition at line 4097 of file specfun.h.

Return the incomplete Legendre elliptic integral  $D(k,\phi)$  of real modulus k and angular limit  $\phi$ .

The Legendre elliptic integral D is defined by

$$D(k,\phi) = \int_0^\phi \frac{\sin^2 \theta d\theta}{\sqrt{1 - k^2 \sin^2 \theta}}$$

#### **Parameters**

k	The modulus $-1 \le \underline{} k \le +1$
phi	The angle

Definition at line 3933 of file specfun.h.

Return the incomplete Legendre elliptic integral  $D(k,\phi)$  of float modulus k and angular limit  $\phi$ .

See also

ellint\_d for details.

Definition at line 3905 of file specfun.h.

Return the incomplete Legendre elliptic integral  $D(k,\phi)$  of long double modulus k and angular limit  $\phi$ .

See also

ellint\_d for details.

Definition at line 3915 of file specfun.h.

Return the Bulirsch elliptic integral  $el1(x, k_c)$  of the first kind of real tangent limit x and complementary modulus  $k_c$ .

The Bulirsch elliptic integral of the first kind is defined by

$$el1(x, k_c) = el2(x, k_c, 1, 1) = \int_0^{\arctan x} \frac{1 + 1 \tan^2 \theta}{\sqrt{(1 + \tan^2 \theta)(1 + k_c^2 \tan^2 \theta)}} d\theta$$

#### **Parameters**

X	The tangent of the angular integration limit
k_c	The complementary modulus $k_c=\sqrt{1-k^2}$

Definition at line 3979 of file specfun.h.

Return the Bulirsch elliptic integral  $el1(x,k_c)$  of the first kind of float tangent limit x and complementary modulus  $k_c$ .

### See also

ellint\_el1 for details.

Definition at line 3949 of file specfun.h.

Return the Bulirsch elliptic integral  $el1(x, k_c)$  of the first kind of real tangent limit x and complementary modulus  $k_c$ .

### See also

ellint el1 for details.

Definition at line 3960 of file specfun.h.

7.3.3.97 template < typename \_Tp , typename \_Tk , typename \_Ta , typename \_Tb > \_\_gnu\_cxx::\_\_promote\_num\_t < \_Tp, \_Tk, \_Ta, \_Tb > \_\_gnu\_cxx::ellint\_el2 ( \_Tp \_x, \_Tk \_
$$k_c$$
, \_Ta \_a, \_Tb \_b ) [inline]

Return the Bulirsch elliptic integral of the second kind  $el2(x, k_c, a, b)$ .

The Bulirsch elliptic integral of the second kind is defined by

$$el2(x, k_c, a, b) = \int_0^{\arctan x} \frac{a + b \tan^2 \theta}{\sqrt{(1 + \tan^2 \theta)(1 + k_c^2 \tan^2 \theta)}} d\theta$$

## **Parameters**

X	The tangent of the angular integration limit
k_c	The complementary modulus $k_c=\sqrt{1-k^2}$
a	The parameter
b	The parameter

Definition at line 4025 of file specfun.h.

```
7.3.3.98 float _gnu_cxx::ellint_el2f (float _x, float _k_c, float _a, float _b) [inline]
```

Return the Bulirsch elliptic integral of the second kind  $el2(x, k_c, a, b)$ .

See also

ellint\_el2 for details.

Definition at line 3994 of file specfun.h.

7.3.3.99 long double 
$$\_$$
gnu\_cxx::ellint\_el2l ( long double  $\_$ x, long double  $\_$ k\_c, long double  $\_$ a, long double  $\_$ b ) [inline]

Return the Bulirsch elliptic integral of the second kind  $el2(x, k_c, a, b)$ .

See also

ellint\_el2 for details.

Definition at line 4004 of file specfun.h.

7.3.3.100 template \_\_gnu\_cxx::\_\_promote\_num\_t<\_Tx, \_Tk, \_Tp> \_\_gnu\_cxx::ellint\_el3 ( \_Tx \_ x, \_Tk \_ 
$$k_c$$
, \_Tp \_  $p$  ) [inline]

Return the Bulirsch elliptic integral of the third kind  $el3(x, k_c, p)$  of real tangent limit x, complementary modulus  $k_c$ , and parameter p.

The Bulirsch elliptic integral of the third kind is defined by

$$el3(x, k_c, p) = \int_0^{\arctan x} \frac{d\theta}{(\cos^2 \theta + p \sin^2 \theta) \sqrt{\cos^2 \theta + k_c^2 \sin^2 \theta}}$$

## **Parameters**

Γ	x	The tangent of the angular integration limit
	k_c	The complementary modulus $k_c=\sqrt{1-k^2}$
	p	The paramenter

Definition at line 4072 of file specfun.h.

Return the Bulirsch elliptic integral of the third kind  $el3(x, k_c, p)$  of float tangent limit x, complementary modulus  $k_c$ , and parameter p.

See also

ellint\_el3 for details.

Definition at line 4041 of file specfun.h.

7.3.3.102 long double \_gnu\_cxx::ellint\_el3l ( long double \_x, long double \_k\_c, long double \_p) [inline]

Return the Bulirsch elliptic integral of the third kind  $el3(x, k_c, p)$  of long double tangent limit x, complementary modulus  $k_c$ , and parameter p.

### See also

ellint el3 for details.

Definition at line 4052 of file specfun.h.

Return the Carlson elliptic function  $R_C(x,y) = R_F(x,y,y)$  where  $R_F(x,y,z)$  is the Carlson elliptic function of the first kind.

The Carlson elliptic function is defined by:

$$R_C(x,y) = \frac{1}{2} \int_0^\infty \frac{dt}{(t+x)^{1/2}(t+y)}$$

Based on Carlson's algorithms:

- B. C. Carlson Numer. Math. 33, 1 (1979)
- B. C. Carlson, Special Functions of Applied Mathematics (1977)
- Numerical Recipes in C, 2nd ed, pp. 261-269, by Press, Teukolsky, Vetterling, Flannery (1992)

## **Parameters**

Γ	x	The first argument.
	у	The second argument.

Definition at line 2969 of file specfun.h.

Return the Carlson elliptic function  $R_C(x, y)$ .

#### See also

ellint rc for details.

Definition at line 2935 of file specfun.h.

Return the Carlson elliptic function  $R_C(x, y)$ .

### See also

ellint\_rc for details.

Definition at line 2944 of file specfun.h.

Return the Carlson elliptic function of the second kind  $R_D(x,y,z) = R_J(x,y,z,z)$  where  $R_J(x,y,z,p)$  is the Carlson elliptic function of the third kind.

The Carlson elliptic function of the second kind is defined by:

$$R_D(x,y,z) = \frac{3}{2} \int_0^\infty \frac{dt}{(t+x)^{1/2}(t+y)^{1/2}(t+z)^{3/2}}$$

Based on Carlson's algorithms:

- B. C. Carlson Numer. Math. 33, 1 (1979)
- B. C. Carlson, Special Functions of Applied Mathematics (1977)
- · Numerical Recipes in C, 2nd ed, pp. 261-269, by Press, Teukolsky, Vetterling, Flannery (1992)

#### **Parameters**

X	The first of two symmetric arguments.
у	The second of two symmetric arguments.
Z	The third argument.

Definition at line 3068 of file specfun.h.

Return the Carlson elliptic function  $R_D(x, y, z)$ .

See also

ellint rd for details.

Definition at line 3032 of file specfun.h.

7.3.3.108 long double \_\_gnu\_cxx::ellint\_rdl ( long double \_\_x, long double \_\_y, long double \_\_z ) [inline]

Return the Carlson elliptic function  $R_D(x, y, z)$ .

See also

ellint\_rd for details.

Definition at line 3041 of file specfun.h.

Return the Carlson elliptic function  $R_F(x,y,z)$  of the first kind for real arguments.

The Carlson elliptic function of the first kind is defined by:

$$R_F(x,y,z) = \frac{1}{2} \int_0^\infty \frac{dt}{(t+x)^{1/2}(t+y)^{1/2}(t+z)^{1/2}}$$

### **Parameters**

X	The first of three symmetric arguments.
у	The second of three symmetric arguments.
Z	The third of three symmetric arguments.

Definition at line 2921 of file specfun.h.

Return the Carlson elliptic function  $R_F(x,y,z)$  of the first kind for float arguments.

### See also

ellint\_rf for details.

Definition at line 2892 of file specfun.h.

Return the Carlson elliptic function  $R_F(x,y,z)$  of the first kind for long double arguments.

#### See also

ellint rf for details.

Definition at line 2902 of file specfun.h.

Return the symmetric Carlson elliptic function of the second kind  $R_G(x, y, z)$ .

The Carlson symmetric elliptic function of the second kind is defined by:

$$R_G(x,y,z) = \frac{1}{4} \int_0^\infty dt t [(t+x)(t+y)(t+z)]^{-1/2} \left(\frac{x}{t+x} + \frac{y}{t+y} + \frac{z}{t+z}\right)$$

Based on Carlson's algorithms:

- B. C. Carlson Numer. Math. 33, 1 (1979)
- B. C. Carlson, Special Functions of Applied Mathematics (1977)
- Numerical Recipes in C, 2nd ed, pp. 261-269, by Press, Teukolsky, Vetterling, Flannery (1992)

## **Parameters**

X	The first of three symmetric arguments.

у	The second of three symmetric arguments.
Z	The third of three symmetric arguments.

Definition at line 3159 of file specfun.h.

7.3.3.113 float \_\_gnu\_cxx::ellint\_rgf(float \_\_x, float \_\_y, float \_\_z) [inline]

Return the Carlson elliptic function  $R_G(x, y)$ .

See also

ellint\_rg for details.

Definition at line 3124 of file specfun.h.

7.3.3.114 long double \_\_gnu\_cxx::ellint\_rgl ( long double \_\_x, long double \_\_y, long double \_\_z ) [inline]

Return the Carlson elliptic function  $R_G(x, y)$ .

See also

ellint\_rg for details.

Definition at line 3133 of file specfun.h.

Return the Carlson elliptic function  $R_J(x,y,z,p)$  of the third kind.

The Carlson elliptic function of the third kind is defined by:

$$R_J(x,y,z,p) = \frac{3}{2} \int_0^\infty \frac{dt}{(t+x)^{1/2}(t+y)^{1/2}(t+z)^{1/2}(t+p)}$$

Based on Carlson's algorithms:

- B. C. Carlson Numer. Math. 33, 1 (1979)
- B. C. Carlson, Special Functions of Applied Mathematics (1977)
- Numerical Recipes in C, 2nd ed, pp. 261-269, by Press, Teukolsky, Vetterling, Flannery (1992)

#### **Parameters**

X	The first of three symmetric arguments.
у	The second of three symmetric arguments.
Z	The third of three symmetric arguments.

\_\_p The fourth argument.

Definition at line 3018 of file specfun.h.

7.3.3.116 float \_\_gnu\_cxx::ellint\_rjf ( float \_\_x, float \_\_y, float \_\_z, float \_\_p ) [inline]

Return the Carlson elliptic function  $R_J(x, y, z, p)$ .

See also

ellint rj for details.

Definition at line 2983 of file specfun.h.

7.3.3.117 long double \_\_gnu\_cxx::ellint\_rjl( long double \_\_x, long double \_\_y, long double \_\_z, long double \_\_p) [inline]

Return the Carlson elliptic function  $R_J(x, y, z, p)$ .

See also

ellint\_rj for details.

Definition at line 2992 of file specfun.h.

7.3.3.118 template<typename\_Tp > \_Tp \_\_gnu\_cxx::ellnome( \_Tp \_\_k ) [inline]

Return the elliptic nome function q(k) of modulus k.

The elliptic nome function is defined by

$$q(k) =$$

Parameters

\_\_k | The modulus 
$$-1 <= k <= +1$$

Definition at line 4880 of file specfun.h.

7.3.3.119 float \_\_gnu\_cxx::ellnomef(float \_\_k) [inline]

Return the elliptic nome function q(k) of modulus k.

See also

ellnome for details.

Definition at line 4855 of file specfun.h.

7.3.3.120 long double  $\_gnu\_cxx$ ::ellnomel ( long double  $\_k$  ) [inline]

Return the elliptic nome function q(k) of long double modulus k.

```
See also
```

ellnome for details.

Definition at line 4865 of file specfun.h.

```
7.3.3.121 template<typename_Tp > __gnu_cxx::_promote_num_t<_Tp>__gnu_cxx::expint_e1( _Tp __x ) [inline]
```

Definition at line 3436 of file specfun.h.

```
7.3.3.122 float __gnu_cxx::expint_e1f( float __x ) [inline]
```

Definition at line 3427 of file specfun.h.

```
7.3.3.123 long double __gnu_cxx::expint_e1l( long double __x ) [inline]
```

Definition at line 3431 of file specfun.h.

Definition at line 3455 of file specfun.h.

```
7.3.3.125 float __gnu_cxx::expint_enf( unsigned int __n, float __x ) [inline]
```

Definition at line 3443 of file specfun.h.

```
7.3.3.126 long double __gnu_cxx::expint_enl ( unsigned int __n, long double __x ) [inline]
```

Definition at line 3447 of file specfun.h.

Definition at line 3560 of file specfun.h.

```
7.3.3.128 float __gnu_cxx::factorialf ( unsigned int __n ) [inline]
```

Definition at line 3548 of file specfun.h.

```
7.3.3.129 long double __gnu_cxx::factoriall ( unsigned int __n ) [inline]
```

Definition at line 3552 of file specfun.h.

7.3.3.130 template<typename\_Tp>\_\_gnu\_cxx::\_\_promote\_num\_t<\_Tp>\_\_gnu\_cxx::fresnel\_c(\_Tp\_\_x) [inline]

Return the Fresnel cosine integral of argument x.

The Fresnel cosine integral is defined by

$$C(x) = \int_0^x \cos(\frac{\pi}{2}t^2)dt$$

### **Parameters**

X	The argument

Definition at line 3378 of file specfun.h.

7.3.3.131 float \_\_gnu\_cxx::fresnel\_cf(float \_\_x) [inline]

Definition at line 3359 of file specfun.h.

7.3.3.132 long double \_\_gnu\_cxx::fresnel\_cl( long double \_\_x ) [inline]

Definition at line 3363 of file specfun.h.

Return the Fresnel sine integral of argument x.

The Fresnel sine integral is defined by

$$S(x) = \int_0^x \sin(\frac{\pi}{2}t^2)dt$$

## **Parameters**

X	The argument

Definition at line 3350 of file specfun.h.

7.3.3.134 float \_\_gnu\_cxx::fresnel\_sf( float \_\_x ) [inline]

Definition at line 3331 of file specfun.h.

**7.3.3.135** long double \_\_gnu\_cxx::fresnel\_sl( long double \_\_x ) [inline]

Definition at line 3335 of file specfun.h.

7.3.3.136 template<typename \_Tn , typename \_Tp > \_\_gnu\_cxx::\_\_promote\_num\_t<\_Tn, \_Tp> \_\_gnu\_cxx::gamma\_I ( \_Tn \_\_n, \_Tp \_\_x ) [inline]

Definition at line 2772 of file specfun.h.

7.3.3.137 float \_\_gnu\_cxx::gamma\_lf ( float \_\_n, float \_\_x ) [inline]

Definition at line 2760 of file specfun.h.

```
7.3.3.138 long double __gnu_cxx::gamma_ll( long double __n, long double __x) [inline]
```

Definition at line 2764 of file specfun.h.

```
7.3.3.139 template<typename_Ta, typename_Tp > __gnu_cxx::__promote_num_t<_Ta, _Tp> __gnu_cxx::gamma_p ( _Ta __a, __Tp __x ) [inline]
```

Definition at line 3758 of file specfun.h.

```
7.3.3.140 float __gnu_cxx::gamma_pf(float __a, float __x) [inline]
```

Definition at line 3746 of file specfun.h.

```
7.3.3.141 long double __gnu_cxx::gamma_pl ( long double __a, long double __x ) [inline]
```

Definition at line 3750 of file specfun.h.

```
7.3.3.142 template<typename _Ta , typename _Tp > __gnu_cxx::__promote_num_t<_Ta, _Tp > __gnu_cxx::gamma_q ( _Ta __a, __Tp __x ) [inline]
```

Definition at line 3779 of file specfun.h.

```
7.3.3.143 float __gnu_cxx::gamma_qf(float __a, float __x) [inline]
```

Definition at line 3767 of file specfun.h.

```
7.3.3.144 long double __gnu_cxx::gamma_ql ( long double __a, long double __x ) [inline]
```

Definition at line 3771 of file specfun.h.

```
7.3.3.145 template<typename_Tn , typename_Tp > __gnu_cxx::__promote_num_t<_Tn, _Tp> __gnu_cxx::gamma_u ( _Tn __n, __Tp __x ) [inline]
```

Definition at line 2751 of file specfun.h.

```
7.3.3.146 float __gnu_cxx::gamma_uf(float __n, float __x) [inline]
```

Definition at line 2739 of file specfun.h.

```
7.3.3.147 long double __gnu_cxx::gamma_ul ( long double __n, long double __x ) [inline]
```

Definition at line 2743 of file specfun.h.

7.3.3.148 template<typename \_Talpha , typename \_Tp > \_\_gnu\_cxx::\_\_promote\_num\_t<\_Talpha, \_Tp > \_\_gnu\_cxx::gegenbauer ( unsigned int \_\_n, \_Talpha \_\_alpha, \_Tp \_\_x ) [inline]

Return the Gegenbauer polynomial  $C_n^{\alpha}(x)$  of degree n and real order  $\alpha > -1/2, \alpha \neq 0$  and argument x.

The Gegenbauer polynomials are generated by a three-term recursion relation:

$$C_n^{\alpha}(x) = \frac{1}{n} \left[ 2x(n+\alpha-1)C_{n-1}^{\alpha}(x) - (n+2\alpha-2)C_{n-2}^{\alpha}(x) \right]$$

and  $C_0^{\alpha}(x) = 1$ ,  $C_1^{\alpha}(x) = 2\alpha x$ .

### **Template Parameters**

_Talpha	The real type of the order
_Tp	The real type of the argument

#### **Parameters**

	The non-negative integral degree
alph	The real order
	The real argument

Definition at line 2177 of file specfun.h.

7.3.3.149 float \_\_gnu\_cxx::gegenbauerf ( unsigned int \_\_n, float \_\_alpha, float \_\_x ) [inline]

Return the Gegenbauer polynomial  $C_n^{\alpha}(x)$  of degree n and float order  $\alpha > -1/2, \alpha \neq 0$  and argument x.

See also

gegenbauer for details.

Definition at line 2144 of file specfun.h.

7.3.3.150 long double \_\_gnu\_cxx::gegenbauerl( unsigned int \_\_n, long double \_\_alpha, long double \_\_x) [inline]

Return the Gegenbauer polynomial  $C_n^{\alpha}(x)$  of degree n and long double order  $\alpha > -1/2, \alpha \neq 0$  and argument x.

See also

gegenbauer for details.

Definition at line 2155 of file specfun.h.

7.3.3.151 template<typename \_Tk , typename \_Tphi > \_\_gnu\_cxx::\_\_promote\_num\_t<\_Tk, \_Tphi> \_\_gnu\_cxx::heuman\_lambda ( \_\_Tk \_k, \_Tphi \_\_phi ) [inline]

Return the Heuman lambda function  $\Lambda(k,\phi)$  of modulus k and angular limit  $\phi$ .

The complete Heuman lambda function is defined by

$$\Lambda(k,\phi) = \frac{F(1-m,\phi)}{K(1-m)} + \frac{2}{\pi}K(m)Z(1-m,\phi)$$

where  $m=k^2$ , K(k) is the complete elliptic function of the first kind, and Z(k,phi) is the Jacobi zeta function.

## **Template Parameters**

_ <i>Tk</i>	the floating-point type of the modulus
_Tphi	the floating-point type of the angular limit argument

### **Parameters**

k	The modulus
phi	The angle

Definition at line 3848 of file specfun.h.

7.3.3.152 float \_\_gnu\_cxx::heuman\_lambdaf ( float \_\_k, float \_\_phi ) [inline]

Definition at line 3822 of file specfun.h.

7.3.3.153 long double \_\_gnu\_cxx::heuman\_lambdal( long double \_\_k, long double \_\_phi ) [inline]

Definition at line 3826 of file specfun.h.

Return the Hurwitz zeta function of real argument s, and parameter a.

The the Hurwitz zeta function is defined by

$$\zeta(s,a) = \sum_{n=0}^{\infty} \frac{1}{(a+n)^s}$$

# Parameters

s	The argument
a	The parameter

Definition at line 3200 of file specfun.h.

7.3.3.155 template<typename \_Tp , typename \_Up > std::complex<\_Tp> \_\_gnu\_cxx::hurwitz\_zeta ( \_Tp \_\_s, std::complex< \_Up > \_\_a )

Return the Hurwitz zeta function of real argument s, and complex parameter a.

See also

hurwitz\_zeta for details.

Definition at line 3214 of file specfun.h.

7.3.3.156 float \_\_gnu\_cxx::hurwitz\_zetaf ( float \_\_s, float \_\_a ) [inline]

Return the Hurwitz zeta function of float argument s, and parameter a.

See also

hurwitz zeta for details.

Definition at line 3174 of file specfun.h.

7.3.3.157 long double \_\_gnu\_cxx::hurwitz\_zetal ( long double \_\_s, long double \_\_a ) [inline]

Return the Hurwitz zeta function of long double argument s, and parameter a.

See also

hurwitz zeta for details.

Definition at line 3184 of file specfun.h.

Return the hypergeometric function  ${}_2F_1(a,b;c;x)$  of real numeratorial parameters a and b, denominatorial parameter c, and argument x.

The hypergeometric function is defined by

$$_{2}F_{1}(a,b;c;x) = \sum_{n=0}^{\infty} \frac{(a)_{n}(b)_{n}x^{n}}{(c)_{n}n!}$$

where the Pochhammer symbol is  $(x)_k = (x)(x+1)...(x+k-1), (x)_0 = 1$ 

#### **Parameters**

a	The first numeratorial parameter
b	The second numeratorial parameter
c	The denominatorial parameter
X	The argument

Definition at line 1426 of file specfun.h.

Return the hypergeometric function  ${}_2F_1(a,b;c;x)$  of @ float numeratorial parameters a and b, denominatorial parameter c, and argument x.

See also

hyperg for details.

Definition at line 1393 of file specfun.h.

Return the hypergeometric function  ${}_2F_1(a,b;c;x)$  of long double numeratorial parameters a and b, denominatorial parameter c, and argument x.

See also

hyperg for details.

Definition at line 1404 of file specfun.h.

Return the regularized incomplete beta function of parameters a, b, and argument x.

The regularized incomplete beta function is defined by

$$I_x(a,b) = \frac{B_x(a,b)}{B(a,b)}$$

where

$$B_x(a,b) = \int_0^x t^{a-1} (1-t)^{b-1} dt$$

is the non-regularized beta function and B(a,b) is the usual beta function.

#### **Parameters**

a	The first parameter
b	The second parameter
X	The argument

Definition at line 3291 of file specfun.h.

Return the regularized complementary incomplete beta function of parameters a, b, and argument x.

The regularized complementary incomplete beta function is defined by

$$I_x(a,b) = I_x(a,b)$$

## **Parameters**

a	The parameter
b	The parameter
X	The argument

Definition at line 3322 of file specfun.h.

Definition at line 3300 of file specfun.h.

References \_\_gnu\_cxx::ibetaf().

Definition at line 3304 of file specfun.h.

References \_\_gnu\_cxx::ibetal().

7.3.3.165 float \_\_gnu\_cxx::ibetaf (float \_\_a, float \_\_b, float \_\_x ) [inline]

Return the regularized incomplete beta function of parameters a, b, and argument x.

See ibeta for details.

Definition at line 3257 of file specfun.h.

Referenced by gnu cxx::ibetacf().

Return the regularized incomplete beta function of parameters a, b, and argument x.

See ibeta for details.

Definition at line 3267 of file specfun.h.

Referenced by gnu cxx::ibetacl().

Return the Jacobi polynomial  $P_n^{(\alpha,\beta)}(x)$  of degree n and float orders  $\alpha,\beta>-1$  and argument x.

The Jacobi polynomials are generated by a three-term recursion relation:

$$2n(\alpha+\beta+n)(\alpha+\beta+2n-2)P_{n}^{(\alpha,\beta)}(x) = (\alpha+\beta+2n-1)((\alpha^{2}-\beta^{2})+x(\alpha+\beta+2n-2)(\alpha+\beta+2n))P_{n-1}^{(\alpha,\beta)}(x) - 2(\alpha+n-1)(\beta+n-1)(\alpha+\beta+2n-2)(\alpha+2n-2)(\alpha+2n-2)(\alpha+2n-2)(\alpha+2n-2)(\alpha+2n-2)(\alpha+2n-2)(\alpha+2n-2$$

# **Template Parameters**

_Talpha	The real type of the order $\alpha$
_Tbeta	The real type of the order $\beta$
_ <i>Tp</i>	The real type of the argument

## **Parameters**

n	The non-negative integral degree
alpha	The real order
beta	The real order
X	The real argument

Definition at line 2129 of file specfun.h.

References std::\_\_detail::\_\_beta().

Return the Jacobi elliptic cn(k, u) integral of real modulus k and argument u.

The Jacobi elliptic cn integral is defined by

$$cos(\phi) = cn(k, F(k, \phi))$$

where  $F(k, \phi)$  is the elliptic integral of the first kind.

# **Template Parameters**

_ <i>Kp</i>	The type of the real modulus
_Up	The type of the real argument

## **Parameters**

k	The real modulus
u	The real argument

Definition at line 1838 of file specfun.h.

Return the Jacobi elliptic cn(k,u) integral of float modulus k and argument u.

## See also

jacobi\_cn for details.

Definition at line 1802 of file specfun.h.

Return the Jacobi elliptic cn(k,u) integral of long double modulus k and argument u.

# See also

jacobi\_cn for details.

Definition at line 1815 of file specfun.h.

Return the Jacobi elliptic dn(k,u) integral of real modulus k and argument u.

The Jacobi elliptic dn integral is defined by

$$\sqrt{1 - k^2 \sin(\phi)} = dn(k, F(k, \phi))$$

where  $F(k,\phi)$  is the elliptic integral of the first kind.

# **Template Parameters**

_ <i>Kp</i>	The type of the real modulus
_Up	The type of the real argument

## **Parameters**

k	The real modulus

и	The real argument

Definition at line 1890 of file specfun.h.

Return the Jacobi elliptic dn(k, u) integral of float modulus k and argument u.

See also

jacobi dn for details.

Definition at line 1854 of file specfun.h.

Return the Jacobi elliptic dn(k, u) integral of long double modulus k and argument u.

See also

jacobi\_dn for details.

Definition at line 1867 of file specfun.h.

Return the Jacobi elliptic sn(k,u) integral of real modulus k and argument u.

The Jacobi elliptic sn integral is defined by

$$\sin(\phi) = sn(k, F(k, \phi))$$

where  $F(k,\phi)$  is the elliptic integral of the first kind.

**Template Parameters** 

_ <i>Kp</i>	The type of the real modulus
_Up	The type of the real argument

#### **Parameters**

k	The real modulus
u	The real argument

Definition at line 1786 of file specfun.h.

Return the Jacobi elliptic sn(k, u) integral of float modulus k and argument u.

See also

jacobi\_sn for details.

Definition at line 1750 of file specfun.h.

7.3.3.176 long double \_\_gnu\_cxx::jacobi\_snl( long double \_\_k, long double \_\_u) [inline]

Return the Jacobi elliptic sn(k, u) integral of long double modulus k and argument u.

## See also

jacobi\_sn for details.

Definition at line 1763 of file specfun.h.

Return the Jacobi zeta function of k and  $@c\phi$ .

The Jacobi zeta function is defined by

$$Z(m,\phi) = E(m,\phi) - \frac{E(m)F(m,\phi)}{K(m)}$$

where  $E(m,\phi)$  is the elliptic function of the second kind, E(m) is the complete ellitic function of the second kind, and  $F(m,\phi)$  is the elliptic function of the first kind.

# **Template Parameters**

_Tk	the real type of the modulus
_Tphi	the real type of the angle limit

# **Parameters**

k	The modulus
phi	The angle

Definition at line 3813 of file specfun.h.

7.3.3.178 float \_gnu\_cxx::jacobi\_zetaf(float \_k, float \_phi) [inline]

Definition at line 3788 of file specfun.h.

7.3.3.179 long double \_\_gnu\_cxx::jacobi\_zetal ( long double \_\_k, long double \_\_phi ) [inline]

Definition at line 3792 of file specfun.h.

7.3.3.180 float \_gnu\_cxx::jacobif ( unsigned \_n, float \_alpha, float \_beta, float \_x ) [inline]

Return the Jacobi polynomial  $P_n^{(\alpha,\beta)}(x)$  of degree n and float orders  $\alpha,\beta>-1$  and argument x.

# See also

jacobi for details.

Definition at line 2085 of file specfun.h.

References std:: detail:: beta().

```
7.3.3.181 long double __gnu_cxx::jacobil ( unsigned __n, long double __alpha, long double __beta, long double __x ) [inline]
```

Return the Jacobi polynomial  $P_n^{(\alpha,\beta)}(x)$  of degree n and long double orders  $\alpha,\beta>-1$  and argument x.

See also

jacobi for details.

Definition at line 2096 of file specfun.h.

References std:: detail:: beta().

Definition at line 3665 of file specfun.h.

```
7.3.3.183 float __gnu_cxx::lbincoeff ( unsigned int __n, unsigned int __k ) [inline]
```

Definition at line 3653 of file specfun.h.

```
7.3.3.184 long double __gnu_cxx::lbincoefl ( unsigned int __n, unsigned int __k ) [inline]
```

Definition at line 3657 of file specfun.h.

Definition at line 3623 of file specfun.h.

```
7.3.3.186 float __gnu_cxx::ldouble_factorialf(int __n) [inline]
```

Definition at line 3611 of file specfun.h.

```
7.3.3.187 long double __gnu_cxx::ldouble_factoriall(int__n) [inline]
```

Definition at line 3615 of file specfun.h.

Definition at line 3737 of file specfun.h.

```
7.3.3.189 float __gnu_cxx::legendre_qf( unsigned int __n, float __x ) [inline]
```

Return the Legendre function of the second kind  $Q_l(x)$  for float argument.

See also

legendre\_q for details.

Definition at line 3719 of file specfun.h.

7.3.3.190 long double \_\_gnu\_cxx::legendre\_ql( unsigned int \_\_n, long double \_\_x ) [inline]

Return the Legendre function of the second kind  $Q_l(x)$  for long double argument.

See also

legendre\_q for details.

Definition at line 3729 of file specfun.h.

Definition at line 3602 of file specfun.h.

7.3.3.192 float \_\_gnu\_cxx::lfactorialf ( unsigned int \_\_n ) [inline]

Definition at line 3590 of file specfun.h.

**7.3.3.193** long double \_\_gnu\_cxx::lfactoriall ( unsigned int \_\_n ) [inline]

Definition at line 3594 of file specfun.h.

7.3.3.194 template<typename\_Tp > \_\_gnu\_cxx::\_\_promote\_num\_t<\_Tp> \_\_gnu\_cxx::logint( \_Tp \_\_x ) [inline]

Return the logarithmic integral of argument x.

The logarithmic integral is defined by

$$li(x) = \int_0^x \frac{dt}{ln(t)}$$

**Parameters** 

\_\_x The real upper integration limit

Definition at line 1565 of file specfun.h.

7.3.3.195 float \_\_gnu\_cxx::logintf(float \_\_x) [inline]

Return the logarithmic integral of argument x.

See also

logint for details.

Definition at line 1541 of file specfun.h.

7.3.3.196 long double \_\_gnu\_cxx::logintl( long double \_\_x ) [inline]

Return the logarithmic integral of argument x.

See also

logint for details.

Definition at line 1550 of file specfun.h.

Definition at line 3497 of file specfun.h.

Definition at line 3485 of file specfun.h.

Definition at line 3489 of file specfun.h.

Definition at line 3476 of file specfun.h.

Definition at line 3464 of file specfun.h.

Definition at line 3468 of file specfun.h.

Return the Owens T function T(h, a) of shape factor h and integration limit a.

The Owens T function is defined by

$$T(h,a) = \frac{1}{2\pi} \int_0^a \frac{\exp\left[-\frac{1}{2}h^2(1+x^2)\right]}{1+x^2} dx$$

## **Parameters**

h	The shape factor
a	The integration limit

Definition at line 5091 of file specfun.h.

```
7.3.3.204 float __gnu_cxx::owens_tf(float __h, float __a) [inline]
```

Return the Owens T function T(h, a) of shape factor h and integration limit a.

See also

owens t for details.

Definition at line 5063 of file specfun.h.

```
7.3.3.205 long double __gnu_cxx::owens_tl( long double __h, long double __a) [inline]
```

Return the Owens T function T(h,a) of long double shape factor h and integration limit a.

See also

owens t for details.

Definition at line 5073 of file specfun.h.

```
7.3.3.206 template<typename _Tp , typename _Tn > __gnu_cxx::__promote_num_t<_Tp, _Tn> __gnu_cxx::pochhammer_l ( _Tp __a, _Tn __n ) [inline]
```

Definition at line 3539 of file specfun.h.

```
7.3.3.207 float __gnu_cxx::pochhammer_lf(float __a, float __n) [inline]
```

Definition at line 3527 of file specfun.h.

```
7.3.3.208 long double __gnu_cxx::pochhammer_ll(long double __a, long double __n) [inline]
```

Definition at line 3531 of file specfun.h.

```
7.3.3.209 template<typename_Tp , typename_Tn > __gnu_cxx::__promote_num_t<_Tp, _Tn> __gnu_cxx::pochhammer_u ( _Tp __a, _Tn __n ) [inline]
```

Definition at line 3518 of file specfun.h.

```
7.3.3.210 float __gnu_cxx::pochhammer_uf(float __a, float __n) [inline]
```

Definition at line 3506 of file specfun.h.

7.3.3.211 long double \_\_gnu\_cxx::pochhammer\_ul ( long double \_\_a, long double \_\_n ) [inline]

Definition at line 3510 of file specfun.h.

Return the complex polylogarithm function of real thing s and complex argument w.

The polylogarithm function is defined by

## **Parameters**

s	
W	

Definition at line 4403 of file specfun.h.

Return the complex polylogarithm function of real thing  ${\tt s}$  and complex argument  ${\tt w}$ .

The polylogarithm function is defined by

## **Parameters**

	c	
L	3	
Г	W	

Definition at line 4443 of file specfun.h.

```
7.3.3.214 float __gnu_cxx::polylogf ( float __s, float __w ) [inline]
```

Return the real polylogarithm function of real thing s and real argument w.

# See also

polylog for details.

Definition at line 4376 of file specfun.h.

```
7.3.3.215 std::complex<float> __gnu_cxx::polylogf(float __s, std::complex< float > __w) [inline]
```

Return the complex polylogarithm function of real thing s and complex argument w.

# See also

polylog for details.

Definition at line 4416 of file specfun.h.

7.3.3.216 long double \_\_gnu\_cxx::polylogl( long double \_\_s, long double \_\_w ) [inline]

Return the complex polylogarithm function of real thing s and complex argument w.

See also

polylog for details.

Definition at line 4386 of file specfun.h.

7.3.3.217 std::complex<long double> \_\_gnu\_cxx::polylogl( long double \_\_s, std::complex< long double> \_\_w ) [inline]

Return the complex polylogarithm function of real thing s and complex argument w.

See also

polylog for details.

Definition at line 4426 of file specfun.h.

 $\textbf{7.3.3.218} \quad template < typename \_Tp > \_gnu\_cxx::\_promote\_num\_t < \_Tp > \_gnu\_cxx::psi(\_Tp \_x \ ) \quad \texttt{[inline]}$ 

Return the psi or digamma function of argument x.

The the psi or digamma function is defined by

$$\psi(x) =$$

#### **Parameters**

x The parameter

Definition at line 3242 of file specfun.h.

7.3.3.219 float \_\_gnu\_cxx::psif(float \_\_x) [inline]

Definition at line 3223 of file specfun.h.

**7.3.3.220** long double \_\_gnu\_cxx::psil( long double \_\_x ) [inline]

Definition at line 3227 of file specfun.h.

7.3.3.221 template<typename \_Tp > \_\_gnu\_cxx::\_\_promote\_num\_t<\_Tp> \_\_gnu\_cxx::radpoly ( unsigned int \_\_n, unsigned int \_\_

Return the radial polynomial  $R_n^m(\rho)$  for non-negative degree n, order m <= n, and real radial argument  $\rho$ .

The radial polynomials are defined by

$$R_n^m(\rho) = \sum_{k=0}^{\frac{n-m}{2}} \frac{(-1)^k (n-k)!}{k! (\frac{n+m}{2} - k)! (\frac{n-m}{2} - k)!} \rho^{n-2k}$$

for n-m even and identically 0 for n-m odd. The radial polynomials can be related to the Jacobi polynomials:

$$R_n^m(\rho) =$$

#### See also

jacobi for details on the Jacobi polynomials.

# **Template Parameters**

_ <i>Tp</i>	The real type of the radial coordinate
-------------	--

#### **Parameters**

n	The non-negative degree.
m	The non-negative azimuthal order
rho	The radial argument

Definition at line 2287 of file specfun.h.

```
7.3.3.222 float __gnu_cxx::radpolyf( unsigned int __n, unsigned int __n, float __rho ) [inline]
```

Return the radial polynomial  $R_n^m(\rho)$  for non-negative degree n, order m <= n, and float radial argument  $\rho$ .

## See also

radpoly for details.

Definition at line 2248 of file specfun.h.

References std::\_\_detail::\_\_poly\_radial\_jacobi().

```
7.3.3.223 long double __gnu_cxx::radpolyl ( unsigned int __n, unsigned int __m, long double __rho ) [inline]
```

Return the radial polynomial  $R_n^m(\rho)$  for non-negative degree n, order m <= n, and long double radial argument  $\rho$ .

# See also

radpoly for details.

Definition at line 2259 of file specfun.h.

References std::\_\_detail::\_\_poly\_radial\_jacobi().

```
7.3.3.224 template<typename_Tp > __gnu_cxx::_promote_num_t<_Tp> __gnu_cxx::sinc(_Tp__x) [inline]
```

Definition at line 1527 of file specfun.h.

```
7.3.3.225 template<typename_Tp > __gnu_cxx::_promote_num_t<_Tp > __gnu_cxx::sinc_pi( _Tp __x ) [inline]
```

Definition at line 1500 of file specfun.h.

```
7.3.3.226 float __gnu_cxx::sinc_pif( float __x ) [inline]
```

Definition at line 1485 of file specfun.h.

**7.3.3.227** long double \_\_gnu\_cxx::sinc\_pil ( long double \_\_x ) [inline]

Definition at line 1492 of file specfun.h.

7.3.3.228 float \_\_gnu\_cxx::sincf(float \_\_x) [inline]

Definition at line 1512 of file specfun.h.

7.3.3.229 long double \_\_gnu\_cxx::sincl( long double \_\_x ) [inline]

Definition at line 1519 of file specfun.h.

7.3.3.230 template<typename\_Tp > \_\_gnu\_cxx::\_\_promote\_num\_t<\_Tp> \_\_gnu\_cxx::sinhc(\_Tp \_\_x) [inline]

Definition at line 2329 of file specfun.h.

7.3.3.231 template<typename\_Tp > \_\_gnu\_cxx::\_promote\_num\_t<\_Tp> \_\_gnu\_cxx::sinhc\_pi(\_Tp \_\_x) [inline]

Definition at line 2308 of file specfun.h.

7.3.3.232 float \_\_gnu\_cxx::sinhc\_pif(float \_\_x) [inline]

Definition at line 2296 of file specfun.h.

**7.3.3.233** long double \_\_gnu\_cxx::sinhc\_pil( long double \_\_x ) [inline]

Definition at line 2300 of file specfun.h.

7.3.3.234 float \_\_gnu\_cxx::sinhcf(float \_\_x) [inline]

Definition at line 2317 of file specfun.h.

7.3.3.235 long double \_\_gnu\_cxx::sinhcl( long double \_\_x ) [inline]

Definition at line 2321 of file specfun.h.

 $\textbf{7.3.3.236} \quad template < typename \_Tp > \underline{\quad} gnu\_cxx::\underline{\quad} promote\_num\_t < \underline{\quad} Tp > \underline{\quad} gnu\_cxx::sinhint (\ \underline{\quad} Tp \underline{\quad} x\ ) \quad [\ inline]$ 

Return the hyperbolic sine integral Shi(x) of real argument x.

The hyperbolic sine integral is defined by

$$Shi(x) = \int_0^x \frac{sinh(t)}{t} dt$$

# **Template Parameters**

_ <i>Tp</i>	The type of the real argument

#### **Parameters**

```
__x The argument
```

Definition at line 1685 of file specfun.h.

Return the hyperbolic sine integral of float argument x.

## See also

sinhint for details.

Definition at line 1658 of file specfun.h.

Return the hyperbolic sine integral Shi(x) of long double argument x.

# See also

sinhint for details.

Definition at line 1668 of file specfun.h.

Return the sine integral Si(x) of real argument x.

The sine integral is defined by

$$Si(x) = \int_0^x \frac{\sin(t)}{t} dt$$

# **Parameters**

X	The real upper integration limit

Definition at line 1604 of file specfun.h.

Return the sine integral Si(x) of float argument x.

# See also

sinint for details.

Definition at line 1579 of file specfun.h.

7.3.3.241 long double \_\_gnu\_cxx::sinintl( long double \_\_x ) [inline]

Return the sine integral Si(x) of long double argument x.

See also

sinint for details.

Definition at line 1589 of file specfun.h.

Return the regular modified spherical Bessel function  $i_n(x)$  of nonnegative order n and real argument x>=0.

The spherical Bessel function is defined by:

$$i_n(x) = \left(\frac{\pi}{2x}\right)^{1/2} I_{n+1/2}(x)$$

#### **Template Parameters**

_ <i>Tp</i>	The floating-point type of the argumentx.

## **Parameters**

n	The integral order $n >= 0$
X	The real argument $x >= 0$

# **Exceptions**

std::domain\_error | if 
$$_{x} < 0$$
.

Definition at line 2567 of file specfun.h.

7.3.3.243 float \_\_gnu\_cxx::sph\_bessel\_if ( unsigned int \_\_n, float \_\_x ) [inline]

Return the regular modified spherical Bessel function  $i_n(x)$  of nonnegative order n and float argument x >= 0.

See also

sph bessel i for details.

Definition at line 2528 of file specfun.h.

7.3.3.244 long double \_\_gnu\_cxx::sph\_bessel\_il ( unsigned int \_\_n, long double \_\_x ) [inline]

Return the regular modified spherical Bessel function  $i_n(x)$  of nonnegative order n and long double argument x >= 0.

See also

sph bessel i for details.

Definition at line 2543 of file specfun.h.

Return the irregular modified spherical Bessel function  $k_n(x)$  of nonnegative order n and real argument x >= 0.

The spherical Bessel function is defined by:

$$k_n(x) = \left(\frac{\pi}{2x}\right)^{1/2} K_{n+1/2}(x)$$

# **Template Parameters**

_Tp   The floating-point type of the argumentx.
---

#### **Parameters**

n	The integral order $n >= 0$
X	The real argument $x >= 0$

## **Exceptions**

$$std::domain\_error \mid if \__x < 0$$
 .

Definition at line 2624 of file specfun.h.

7.3.3.246 float \_\_gnu\_cxx::sph\_bessel\_kf ( unsigned int \_\_n, float \_\_x ) [inline]

Return the irregular modified spherical Bessel function  $k_n(x)$  of nonnegative order n and float argument x >= 0.

# See also

sph\_bessel\_k for more details.

Definition at line 2585 of file specfun.h.

7.3.3.247 long double \_\_gnu\_cxx::sph\_bessel\_kl( unsigned int \_\_n, long double \_\_x ) [inline]

Return the irregular modified spherical Bessel function  $k_n(x)$  of nonnegative order n and long double argument x>=0.

# See also

sph\_bessel\_k for more details.

Definition at line 2600 of file specfun.h.

7.3.3.248 template<typename \_Tp > std::complex<\_\_gnu\_cxx::\_\_promote\_num\_t<\_Tp>> \_\_gnu\_cxx::sph\_hankel\_1 ( unsigned int \_\_n, \_Tp \_\_z ) [inline]

Return the spherical Hankel function of the first kind  $h_n^{(1)}(x)$  of nonnegative order n and real argument x >= 0.

The spherical Hankel function of the first kind is defined by:

$$h_n^{(1)}(x) = \left(\frac{\pi}{2x}\right)^{1/2} H_{n+1/2}^{(1)}(x)$$

## **Template Parameters**

_ <i>Tp</i>	The real type of the argument

#### **Parameters**

n	The non-negative order
Z	The real argument

Definition at line 2470 of file specfun.h.

Return the complex spherical Hankel function of the first kind  $h_n^{(1)}(x)$  of non-negative integral n and complex argument x.

The spherical Hankel function of the first kind is defined by

$$h_n^{(1)}(x) = \left(\frac{\pi}{2x}\right)^{1/2} H_{n+1/2}^{(1)}(x) = j_n(x) + in_n(x)$$

where  $j_n(x)$  and  $n_n(x)$  are the spherical Bessel and Neumann functions respectively.

## **Parameters**

n	The integral order >= 0
X	The complex argument

Definition at line 4262 of file specfun.h.

Return the spherical Hankel function of the first kind  $h_n^{(1)}(x)$  of nonnegative order n and float argument x >= 0.

See also

sph\_hankel\_1 for details.

Definition at line 2442 of file specfun.h.

Return the complex spherical Hankel function of the first kind  $h_n^{(1)}(x)$  of non-negative integral n and  $std \leftarrow ::complex < float > argument <math>x$ .

See also

sph hankel 1 for more details.

Definition at line 4230 of file specfun.h.

```
7.3.3.252 std::complex < long double > __gnu_cxx::sph_hankel_1I ( unsigned int __n, long double __z ) [inline]
```

Return the spherical Hankel function of the first kind  $h_n^{(1)}(x)$  of nonnegative order n and long double argument x>=0.

See also

sph hankel 1 for details.

Definition at line 2452 of file specfun.h.

7.3.3.253 std::complex < long double >  $\_$ gnu\_cxx::sph\_hankel\_1I ( unsigned int  $\_$ n, std::complex < long double >  $\_$ x ) [inline]

Return the complex spherical Hankel function of the first kind  $h_n^{(1)}(x)$  of non-negative integral n and  $std \leftarrow ::complex < long double > argument <math>x$ .

See also

sph hankel 1 for more details.

Definition at line 4241 of file specfun.h.

7.3.3.254 template<typename \_Tp > std::complex< \_\_gnu\_cxx::\_promote\_num\_t<\_Tp> > \_\_gnu\_cxx::sph\_hankel\_2 ( unsigned int \_\_n, \_Tp \_\_z ) [inline]

Return the spherical Hankel function of the second kind  $h_n^{(2)}(x)$  of nonnegative order n and real argument x >= 0.

The spherical Hankel function of the second kind is defined by:

$$h_n^{(2)}(x) = \left(\frac{\pi}{2x}\right)^{1/2} H_{n+1/2}^{(2)}(x)$$

# **Template Parameters**

_ <i>Tp</i>	The real type of the argument

## **Parameters**

n	The non-negative order
Z	The real argument

Definition at line 2513 of file specfun.h.

7.3.3.255 template<typename \_Tp > std::complex<\_\_gnu\_cxx::\_\_promote\_num\_t<\_Tp> > \_\_gnu\_cxx::sph\_hankel\_2 ( unsigned int \_\_n, std::complex< \_Tp > \_\_x ) [inline]

Return the complex spherical Hankel function of the second kind  $h_n^{(2)}(x)$  of nonnegative order n and complex argument x.

The spherical Hankel function of the second kind is defined by

$$h_n^{(2)}(x) = \left(\frac{\pi}{2x}\right)^{1/2} H_{n+1/2}^{(2)}(x) = j_n(x) - in_n(x)$$

where  $j_n(x)$  and  $n_n(x)$  are the spherical Bessel and Neumann functions respectively.

#### **Parameters**

n	The integral order >= 0
X	The complex argument

Definition at line 4310 of file specfun.h.

```
7.3.3.256 std::complex<float> __gnu_cxx::sph_hankel_2f( unsigned int __n, float __z ) [inline]
```

Return the spherical Hankel function of the second kind  $h_n^{(2)}(x)$  of nonnegative order n and float argument x >= 0.

See also

```
sph_hankel_2 for details.
```

Definition at line 2485 of file specfun.h.

```
7.3.3.257 std::complex < float > __gnu_cxx::sph_hankel_2f ( unsigned int __n, std::complex < float > __x ) [inline]
```

Return the complex spherical Hankel function of the second kind  $h_n^{(2)}(x)$  of non-negative integral n and  $std \leftarrow ::complex < float > argument <math>x$ .

#### See also

```
sph_hankel_2 for more details.
```

Definition at line 4278 of file specfun.h.

```
7.3.3.258 std::complex < long double > __gnu_cxx::sph_hankel_2I ( unsigned int __n, long double __z ) [inline]
```

Return the spherical Hankel function of the second kind  $h_n^{(2)}(x)$  of nonnegative order n and long double argument x>=0.

# See also

```
sph hankel 2 for details.
```

Definition at line 2495 of file specfun.h.

```
7.3.3.259 std::complex < long double > \_gnu_cxx::sph_hankel_2I ( unsigned int \_n, std::complex < long double > \_x ) [inline]
```

Return the complex spherical Hankel function of the second kind  $h_n^{(2)}(x)$  of non-negative integral n and  $std \leftarrow ::complex < long double > argument <math>x$ .

# See also

```
sph hankel 2 for more details.
```

Definition at line 4289 of file specfun.h.

7.3.3.260 template<typename \_Ttheta , typename \_Tphi > std::complex<\_\_gnu\_cxx::\_\_promote\_num\_t<\_Ttheta, \_Tphi> > \_\_gnu\_cxx::sph\_harmonic ( unsigned int \_\_l, int \_\_m, \_Ttheta \_\_theta, \_Tphi \_\_phi ) [inline]

Return the complex spherical harmonic function of degree 1, order m, and real zenith angle  $\theta$ , and azimuth angle  $\phi$ .

The spherical harmonic function is defined by:

$$Y_l^m(\theta,\phi) = (-1)^m \left[ \frac{(2l+1)}{4\pi} \frac{(l-m)!}{(l+m)!} \right] P_l^{|m|}(\cos\theta) \exp^{im\phi}$$

#### **Parameters**

	The order
m	The degree
theta	The zenith angle in radians
phi	The azimuth angle in radians

Definition at line 4361 of file specfun.h.

7.3.3.261 std::complex < float > \_\_gnu\_cxx::sph\_harmonicf ( unsigned int \_\_I, int \_\_m, float \_\_theta, float \_\_phi ) [inline]

Return the complex spherical harmonic function of degree 1, order m, and float zenith angle  $\theta$ , and azimuth angle  $\phi$ .

#### See also

sph harmonic for details.

Definition at line 4325 of file specfun.h.

7.3.3.262 std::complex<long double> \_\_gnu\_cxx::sph\_harmonicl( unsigned int \_\_l, int \_\_m, long double \_\_theta, long double \_\_phi ) [inline]

Return the complex spherical harmonic function of degree 1, order m, and long double zenith angle  $\theta$ , and azimuth angle  $\phi$ .

# See also

sph harmonic for details.

Definition at line 4337 of file specfun.h.

7.3.3.263 template<typename\_Tpnu,typename\_Tp > \_\_gnu\_cxx::\_\_promote\_num\_t<\_Tpnu,\_Tp > \_\_gnu\_cxx::theta\_1 ( \_Tpnu \_\_nu, \_Tp \_\_x ) [inline]

Return the exponential theta-1 function  $\theta_1(\nu,x)$  of period nu and argument x.

The Neville theta-1 function is defined by

$$\theta_1(\nu, x) = \frac{1}{\sqrt{\pi x}} \sum_{j=-\infty}^{+\infty} (-1)^j \exp\left(\frac{-(\nu + j - 1/2)^2}{x}\right)$$

#### **Parameters**

nu	The periodic (period = 2) argument
X	The argument

Definition at line 4711 of file specfun.h.

```
7.3.3.264 float __gnu_cxx::theta_1f(float __nu, float __x) [inline]
```

Return the exponential theta-1 function  $\theta_1(\nu,x)$  of period nu and argument x.

See also

theta\_1 for details.

Definition at line 4683 of file specfun.h.

```
7.3.3.265 long double __gnu_cxx::theta_1I ( long double __nu, long double __x ) [inline]
```

Return the exponential theta-1 function  $\theta_1(\nu,x)$  of period nu and argument x.

See also

theta 1 for details.

Definition at line 4693 of file specfun.h.

Return the exponential theta-2 function  $\theta_2(\nu, x)$  of period nu and argument x.

The exponential theta-2 function is defined by

$$\theta_2(\nu, x) = \frac{1}{\sqrt{\pi x}} \sum_{j=-\infty}^{+\infty} (-1)^j \exp\left(\frac{-(\nu+j)^2}{x}\right)$$

# **Parameters**

nu	The periodic (period = 2) argument
x	The argument

Definition at line 4754 of file specfun.h.

Return the exponential theta-2 function  $\theta_2(\nu,x)$  of period nu and argument x.

See also

theta 2 for details.

Definition at line 4726 of file specfun.h.

7.3.3.268 long double \_\_gnu\_cxx::theta\_2l ( long double \_\_nu, long double \_\_x ) [inline]

Return the exponential theta-2 function  $\theta_2(\nu, x)$  of period nu and argument x.

See also

theta\_2 for details.

Definition at line 4736 of file specfun.h.

Return the exponential theta-3 function  $\theta_3(\nu, x)$  of period nu and argument x.

The exponential theta-3 function is defined by

$$\theta_3(\nu, x) = \frac{1}{\sqrt{\pi x}} \sum_{j=-\infty}^{+\infty} \exp\left(\frac{-(\nu+j)^2}{x}\right)$$

#### **Parameters**

nu	The periodic (period = 1) argument
X	The argument

Definition at line 4797 of file specfun.h.

Return the exponential theta-3 function  $\theta_3(\nu,x)$  of period nu and argument x.

See also

theta\_3 for details.

Definition at line 4769 of file specfun.h.

Return the exponential theta-3 function  $\theta_3(\nu, x)$  of period nu and argument x.

See also

theta 3 for details.

Definition at line 4779 of file specfun.h.

Return the exponential theta-4 function  $\theta_4(\nu, x)$  of period nu and argument x.

The exponential theta-4 function is defined by

$$\theta_4(\nu, x) = \frac{1}{\sqrt{\pi x}} \sum_{i=-\infty}^{+\infty} \exp\left(\frac{-(\nu + j + 1/2)^2}{x}\right)$$

## **Parameters**

nu	The periodic (period = 1) argument
X	The argument

Definition at line 4840 of file specfun.h.

```
7.3.3.273 float __gnu_cxx::theta_4f ( float __nu, float __x ) [inline]
```

Return the exponential theta-4 function  $\theta_4(\nu, x)$  of period nu and argument x.

See also

theta 4 for details.

Definition at line 4812 of file specfun.h.

Return the exponential theta-4 function  $\theta_4(\nu, x)$  of period nu and argument x.

See also

theta 4 for details.

Definition at line 4822 of file specfun.h.

Return the Neville theta-c function  $\theta_c(k,x)$  of modulus k and argument x.

The Neville theta-c function is defined by

# **Parameters**

k	The modulus $-1 <= k <= +1$
X	The argument

Definition at line 4964 of file specfun.h.

Return the Neville theta-c function  $\theta_c(k,x)$  of modulus k and argument x.

See also

theta\_c for details.

Definition at line 4937 of file specfun.h.

7.3.3.277 long double \_\_gnu\_cxx::theta\_cl ( long double \_\_k, long double \_\_x ) [inline]

Return the Neville theta-c function  $\theta_c(k,x)$  of long double modulus k and argument x.

See also

theta\_c for details.

Definition at line 4947 of file specfun.h.

Return the Neville theta-d function  $\theta_d(k,x)$  of modulus k and argument x.

The Neville theta-d function is defined by

$$\theta_d(k,x) =$$

## **Parameters**

k	The modulus $-1 <= k <= +1$
X	The argument

Definition at line 5006 of file specfun.h.

Return the Neville theta-d function  $\theta_d(k,x)$  of modulus k and argument x.

See also

theta\_d for details.

Definition at line 4979 of file specfun.h.

Return the Neville theta-d function  $\theta_d(k,x)$  of long double modulus k and argument x.

See also

theta d for details.

Definition at line 4989 of file specfun.h.

Return the Neville theta-n function  $\theta_n(k,x)$  of modulus k and argument x.

The Neville theta-n function is defined by

$$\theta_n(k,x) =$$

## **Parameters**

k	The modulus $-1 <= k <= +1$
X	The argument

Definition at line 5048 of file specfun.h.

```
7.3.3.282 float __gnu_cxx::theta_nf(float __k, float __x) [inline]
```

Return the Neville theta-n function  $\theta_n(k,x)$  of modulus k and argument x.

See also

theta\_n for details.

Definition at line 5021 of file specfun.h.

```
7.3.3.283 long double __gnu_cxx::theta_nl( long double __k, long double __x ) [inline]
```

Return the Neville theta-n function  $\theta_n(k,x)$  of long double modulus k and argument x.

See also

theta n for details.

Definition at line 5031 of file specfun.h.

Return the Neville theta-s function  $\theta_s(k,x)$  of modulus k and argument x.

The Neville theta-s function is defined by

# **Parameters**

k	The modulus $-1 <= k <= +1$
x	The argument

Definition at line 4922 of file specfun.h.

```
7.3.3.285 float __gnu_cxx::theta_sf(float __k, float __x) [inline]
```

Return the Neville theta-s function  $\theta_s(k,x)$  of modulus k and argument x.

See also

theta s for details.

Definition at line 4895 of file specfun.h.

7.3.3.286 long double \_\_gnu\_cxx::theta\_sl( long double \_\_k, long double \_\_x ) [inline]

Return the Neville theta-s function  $\theta_s(k,x)$  of long double modulus k and argument x.

See also

theta s for details.

Definition at line 4905 of file specfun.h.

Return the Zernicke polynomial  $Z_n^m(\rho,\phi)$  for non-negative degree n, signed order m, and real radial argument  $\rho$  and azimuthal angle  $\phi$ .

The even Zernicke polynomials are defined by:

$$Z_n^m(\rho,\phi) = R_n^m(\rho)\cos(m\phi)$$

and the odd Zernicke polynomials are defined by:

$$Z_n^{-m}(\rho,\phi) = R_n^m(\rho)\sin(m\phi)$$

for non-negative degree m and m <= n and where  $R_n^m(\rho)$  is the radial polynomial (

See also

radpoly).

# **Template Parameters**

_Trho	The real type of the radial coordinate
_Tphi	The real type of the azimuthal angle

## **Parameters**

n	The non-negative degree.
m	The (signed) azimuthal order
rho	The radial coordinate
phi	The azimuthal angle

Definition at line 2232 of file specfun.h.

Return the Zernicke polynomial  $Z_n^m(\rho,\phi)$  for non-negative degree n, signed order m, and real radial argument  $\rho$  and azimuthal angle  $\phi$ .

See also

zernike for details.

Definition at line 2193 of file specfun.h.

7.3.3.289 long double \_\_gnu\_cxx::zernikel( unsigned int \_\_n, int \_\_m, long double \_\_rho, long double \_\_phi ) [inline]

Return the Zernicke polynomial  $Z_n^m(\rho,\phi)$  for non-negative degree n, signed order m, and real radial argument  $\rho$  and azimuthal angle  $\phi$ .

See also

zernike for details.

Definition at line 2204 of file specfun.h.

# **Chapter 8**

# **Namespace Documentation**

# 8.1 \_\_gnu\_cxx Namespace Reference

# **Enumerations**

enum { \_GLIBCXX\_JACOBI\_SN, \_GLIBCXX\_JACOBI\_CN, \_GLIBCXX\_JACOBI\_DN }

# **Functions**

```
template<typename _Tp >
  __gnu_cxx::__promote_num_t< _Tp > airy_ai (_Tp __x)
float airy_aif (float __x)
• long double airy ail (long double x)
• template<typename _{\mathrm{Tp}} >
    _gnu_cxx::__promote_num_t< _Tp > airy_bi (_Tp __x)

    float airy bif (float x)

    long double airy_bil (long double __x)

template<typename _Tp >
   _gnu_cxx::__promote_num_t< _Tp > bernoulli (unsigned int __n)

    float bernoullif (unsigned int n)

    long double bernoullil (unsigned int __n)

template<typename</li>Tp >
   _gnu_cxx::__promote_num_t< _Tp > bincoef (unsigned int __n, unsigned int __k)
• float bincoeff (unsigned int __n, unsigned int __k)
• long double bincoefl (unsigned int n, unsigned int k)

    template<typename</li>
    Tp >

  __gnu_cxx::__promote_num_t< _Tp > chebyshev_t (unsigned int __n, _Tp __x)

    float chebyshev_tf (unsigned int __n, float __x)

    long double chebyshev_tl (unsigned int __n, long double __x)

• template<typename _{\mathrm{Tp}} >
    _gnu_cxx::__promote_num_t< _Tp > chebyshev_u (unsigned int __n, _Tp __x)

    float chebyshev uf (unsigned int n, float x)

    long double chebyshev_ul (unsigned int __n, long double __x)

template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > chebyshev_v (unsigned int __n, _Tp __x)

    float chebyshev_vf (unsigned int __n, float __x)
```

```
    long double chebyshev_vl (unsigned int __n, long double __x)

    template<typename</li>
    Tp >

    gnu cxx:: promote num t < Tp > chebyshev w (unsigned int n, Tp x)

    float chebyshev wf (unsigned int n, float x)

    long double chebyshev wl (unsigned int n, long double x)

template<typename_Tp>
   _gnu_cxx::__promote_num_t< _Tp > clausen (unsigned int __m, _Tp __w)
template<typename _Tp >
  std::complex < gnu cxx:: promote num t < Tp > > clausen (unsigned int m, std::complex < Tp > w)
template<typename_Tp>
   gnu cxx:: promote num t< Tp> clausen c (unsigned int m, Tp w)

    float clausen_cf (unsigned int __m, float __w)

• long double clausen_cl (unsigned int __m, long double __w)

    template<typename</li>
    Tp >

    gnu cxx:: promote num t< Tp > clausen s (unsigned int m, Tp w)

    float clausen_sf (unsigned int __m, float __w)

    long double clausen sl (unsigned int m, long double w)

• float clausenf (unsigned int m, float w)
• std::complex< float > clausenf (unsigned int m, std::complex< float > w)
• long double clausenl (unsigned int m, long double w)

    std::complex < long double > clausenl (unsigned int __m, std::complex < long double > __w)

• template<typename_Tk>
   _gnu_cxx::__promote_num_t< _Tk > comp_ellint_d (_Tk k)

    float comp ellint df (float k)

    long double comp_ellint_dl (long double __k)

    float comp ellint rf (float x, float y)

    long double comp_ellint_rf (long double __x, long double __y)

    template<typename _Tx , typename _Ty >

   _gnu_cxx::__promote_num_t< _Tx, _Ty > comp_ellint_rf (_Tx __x, _Ty __y)

    float comp_ellint_rg (float __x, float __y)

• long double comp_ellint_rg (long double __x, long double __y)

    template<typename _Tx , typename _Ty >

   gnu cxx:: promote num t< Tx, Ty> comp ellint rg (Txx, Tyy)

    template<typename _Tpa , typename _Tpc , typename _Tp >

   __gnu_cxx::__promote_3< _Tpa, _Tpc, _Tp >::__type conf_hyperg (_Tpa __a, _Tpc __c, _Tp __x)
• template < typename \_Tpc , typename \_Tp >
    _gnu_cxx::__promote_2< _Tpc, _Tp >::__type conf_hyperg_lim (_Tpc __c, _Tp __x)

    float conf hyperg limf (float c, float x)

    long double conf hyperg liml (long double c, long double x)

    float conf_hypergf (float __a, float __c, float __x)

    long double conf hypergl (long double a, long double c, long double x)

    template<typename</li>
    Tp >

    _gnu_cxx::__promote_num_t< _Tp > coshint (_Tp __x)

    float coshintf (float x)

    long double coshintl (long double x)

    template<typename</li>
    Tp >

   __gnu_cxx::__promote_num_t< _Tp > cosint (_Tp x)

    float cosintf (float x)

    long double cosintl (long double __x)

    template<typename _Tpnu , typename _Tp >

  std::complex< __gnu_cxx::__promote_num_t< _Tpnu, _Tp >> cyl_hankel_1 (_Tpnu __nu, _Tp __z)
```

```
template<typename _Tpnu , typename _Tp >
  std::complex< __gnu_cxx::__promote_num_t< _Tpnu, _Tp >> cyl_hankel_1 (std::complex< _Tpnu > __nu,
  std::complex < Tp > x)

    std::complex< float > cyl_hankel_1f (float __nu, float __z)

• std::complex< float > cyl_hankel_1f (std::complex< float > __nu, std::complex< float > __x)

    std::complex < long double > cyl hankel 1l (long double nu, long double z)

• std::complex < long double > cyl hankel 1l (std::complex < long double > nu, std::complex < long double >
   __x)

    template<typename _Tpnu , typename _Tp >

  std::complex< __gnu_cxx::__promote_num_t< _Tpnu, _Tp >> cyl_hankel_2 (_Tpnu __nu, _Tp __z)
• template<typename _Tpnu , typename _Tp >
  std::complex< __gnu_cxx::__promote_num_t< _Tpnu, _Tp >> cyl_hankel_2 (std::complex< _Tpnu > __nu,
  std::complex < _Tp > __x)

    std::complex< float > cyl_hankel_2f (float __nu, float __z)

    std::complex < float > cyl hankel 2f (std::complex < float > nu, std::complex < float > x)

    std::complex < long double > cyl hankel 2l (long double nu, long double z)

    std::complex < long double > cyl hankel 2l (std::complex < long double > nu, std::complex < long double >

   X)
template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > dawson (_Tp __x)

    float dawsonf (float x)

    long double dawsonl (long double x)

    template<typename</li>
    Tp >

    _gnu_cxx::__promote_num_t< _Tp > digamma (_Tp __z)
• float digammaf (float z)

    long double digammal (long double ___z)

    template<typename</li>
    Tp >

   _gnu_cxx::__promote_num_t< _Tp > dilog (_Tp __x)

 float dilogf (float __x)

    long double dilogl (long double x)

template<typename</li>Tp >
  _Tp dirichlet_beta (_Tp __s)

    float dirichlet_betaf (float __s)

    long double dirichlet betal (long double s)

template<typename _Tp >
  _Tp dirichlet_eta (_Tp __s)

    float dirichlet etaf (float s)

    long double dirichlet etal (long double s)

template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > double_factorial (int __n)

    float double factorialf (int n)

    long double double_factoriall (int __n)

- template<typename _Tk , typename _Tp , typename _Ta , typename _Tb >
   _gnu_cxx::_ promote_num_t< _Tk, _Tp, _Ta, _Tb > ellint_cel (_Tk __k_c, _Tp __p, _Ta __a, _Tb __b)

    float ellint_celf (float __k_c, float __p, float __a, float __b)

    long double ellint_cell (long double __k_c, long double __p, long double __a, long double __b)

    template<typename _Tk , typename _Tphi >

    gnu cxx:: promote num t< Tk, Tphi > ellint d (Tk k, Tphi phi)

    float ellint df (float k, float phi)

    long double ellint_dl (long double ___k, long double ___phi)

    template<typename _Tp , typename _Tk >

   _gnu_cxx::__promote_num_t< _Tp, _Tk > ellint_el1 (_Tp __x, _Tk __k_c)
```

```
• float ellint_el1f (float __x, float __k_c)
• long double ellint_el1l (long double __x, long double __k_c)
ullet template<typename _Tp , typename _Tk , typename _Ta , typename _Tb >
    _gnu_cxx::__promote_num_t< _Tp, _Tk, _Ta, _Tb > ellint_el2 (_Tp __x, _Tk __k_c, _Ta __a, _Tb __b)

    float ellint el2f (float x, float k c, float a, float b)

    long double ellint_el2l (long double __x, long double __k_c, long double __a, long double __b)

• template<typename _Tx , typename _Tk , typename _Tp >
    _gnu_cxx::__promote_num_t< _Tx, _Tk, _Tp > <u>ellint_el3</u> (_Tx __x, _Tk __k_c, _Tp __p)
• float ellint el3f (float x, float k c, float p)
• long double ellint_el3l (long double __x, long double __k_c, long double __p)
• template<typename _Tp , typename _Up >
    gnu cxx:: promote num t< Tp, Up > ellint rc (Tp x, Up y)

    float ellint rcf (float x, float y)

    long double ellint_rcl (long double __x, long double __y)

• template<typename _Tp , typename _Up , typename _Vp >
    gnu cxx:: promote num t< Tp, Up, Vp> ellint rd ( Tp x, Up y, Vp z)
• float ellint rdf (float x, float y, float z)

    long double ellint_rdl (long double __x, long double __y, long double __z)

    template<typename _Tp , typename _Up , typename _Vp >

    gnu cxx:: promote num t< Tp, Up, Vp> ellint rf ( Tp x, Up y, Vp z)

    float ellint_rff (float __x, float __y, float __z)

    long double ellint_rfl (long double __x, long double __y, long double __z)

• template<typename _Tp , typename _Up , typename _Vp >
    _gnu_cxx::__promote_num_t< _Tp, _Up, _Vp > ellint_rg (_Tp __x, _Up __y, _Vp __z)
float ellint_rgf (float __x, float __y, float __z)

    long double ellint rgl (long double x, long double y, long double z)

- template<typename _Tp , typename _Up , typename _Vp , typename _Wp >
    _gnu_cxx::__promote_num_t< _Tp, _Up, _Vp, _Wp > ellint_rj (_Tp __x, _Up __y, _Vp __z, _Wp __p)

    float ellint_rjf (float __x, float __y, float __z, float __p)

    long double ellint rjl (long double x, long double y, long double z, long double p)

template<typename_Tp>
  Tp ellnome (Tp k)

    float ellnomef (float k)

    long double ellnomel (long double k)

template<typename_Tp>
    gnu cxx:: promote num t < Tp > expint e1 (Tp x)

    float expint e1f (float x)

    long double expint e1l (long double x)

template<typename _Tp >
    gnu cxx:: promote num t < Tp > expint en (unsigned int n, Tp x)

    float expint enf (unsigned int n, float x)

    long double expint enl (unsigned int n, long double x)

template<typename _Tp >
    gnu cxx:: promote num t< Tp> factorial (unsigned int n)

    float factorialf (unsigned int n)

    long double factoriall (unsigned int n)

template<typename_Tp>
    gnu cxx:: promote num t < Tp > fresnel c (Tp x)

    float fresnel cf (float x)

    long double fresnel cl (long double x)

template<typename_Tp>
   gnu cxx:: promote num t < Tp > fresnel s (Tp x)
```

```
 float fresnel_sf (float __x)

    long double fresnel sl (long double x)

• template<typename _{\rm Tn}, typename _{\rm Tp} >
    _gnu_cxx::__promote_num_t< _Tn, _Tp > gamma_l (_Tn __n, _Tp __x)

    float gamma If (float n, float x)

    long double gamma_II (long double __n, long double __x)

    template<typename</li>
    Ta , typename
    Tp >

    _gnu_cxx::__promote_num_t< _Ta, _Tp > gamma_p (_Ta __a, _Tp __x)

 float gamma_pf (float __a, float __x)

    long double gamma pl (long double a, long double x)

• template<typename _Ta , typename _Tp >
    _gnu_cxx::__promote_num_t< _Ta, _Tp > gamma_q (_Ta __a, _Tp __x)

    float gamma_qf (float __a, float __x)

    long double gamma gl (long double a, long double x)

• template<typename _Tn , typename _Tp >
   _gnu_cxx::__promote_num_t< _Tn, _Tp > gamma_u (_Tn __n, _Tp __x)

 float gamma_uf (float __n, float __x)

    long double gamma_ul (long double ___n, long double ___x)

• template<typename _Talpha , typename _Tp >
    gnu cxx:: promote num t< Talpha, Tp > gegenbauer (unsigned int n, Talpha alpha, Tp x)

    float gegenbauerf (unsigned int __n, float __alpha, float __x)

    long double gegenbauerl (unsigned int __n, long double __alpha, long double __x)

• template<typename Tk, typename Tphi >
    _gnu_cxx::__promote_num_t< _Tk, _Tphi > heuman_lambda (_Tk __k, _Tphi __phi)

    float heuman lambdaf (float k, float phi)

    long double heuman lambdal (long double k, long double phi)

    template<typename _Tp , typename _Up >

    _gnu_cxx::__promote_num_t< _Tp, _Up > hurwitz_zeta (_Tp __s, _Up __a)
template<typename _Tp , typename _Up >
  std::complex< _Tp > hurwitz_zeta (_Tp __s, std::complex< _Up > __a)

    float hurwitz_zetaf (float __s, float __a)

    long double hurwitz_zetal (long double __s, long double __a)

• template<typename _Tpa , typename _Tpb , typename _Tpc , typename _Tp >
   _gnu_cxx::__promote_4< _Tpa, _Tpb, _Tpc, _Tp >::__type hyperg (_Tpa __a, _Tpb __b, _Tpc __c, _Tp __x)

    float hypergf (float __a, float __b, float __c, float __x)

• long double hypergl (long double __a, long double __b, long double __c, long double __x)
template<typename _Ta , typename _Tb , typename _Tp >
    _gnu_cxx::__promote_num_t< _Ta, _Tb, _Tp > ibeta (_Ta __a, _Tb __b, _Tp __x)
- template<typename _Ta , typename _Tb , typename _Tp >
   _gnu_cxx::__promote_num_t< _Ta, _Tb, _Tp > ibetac (_Ta __a, _Tb __b, _Tp __x)
• float ibetacf (float a, float b, float x)

    long double ibetacl (long double a, long double b, long double x)

 float ibetaf (float __a, float __b, float __x)

    long double ibetal (long double a, long double b, long double x)

    template<typename _Talpha , typename _Tbeta , typename _Tp >

    _gnu_cxx::__promote_num_t< _Talpha, _Tbeta, _Tp > jacobi (unsigned __n, _Talpha __alpha, _Tbeta __beta,
  _Tp __x)

    template<typename _Kp , typename _Up >

   __gnu_cxx::__promote_num_t< _Kp, _Up > jacobi_cn (_Kp __k, _Up __u)

    float jacobi_cnf (float __k, float __u)

    long double jacobi cnl (long double k, long double u)
```

```
    template<typename _Kp , typename _Up >

    gnu cxx:: promote num t< Kp, Up> jacobi dn ( Kp k, Up u)

    float jacobi dnf (float k, float u)

    long double jacobi_dnl (long double __k, long double __u)

    template<typename</li>
    Kp , typename
    Up >

    gnu cxx:: promote num t< Kp, Up> jacobi sn ( Kp k, Up u)

    float jacobi snf (float k, float u)

    long double jacobi_snl (long double __k, long double __u)

• template<typename _Tk , typename _Tphi >
    _gnu_cxx::__promote_num_t< _Tk, _Tphi > jacobi_zeta (_Tk __k, _Tphi __phi)

    float jacobi zetaf (float k, float phi)

• long double jacobi_zetal (long double __k, long double __phi)
• float jacobif (unsigned n, float alpha, float beta, float x)

    long double jacobil (unsigned __n, long double __alpha, long double __beta, long double __x)

template<typename _Tp >
    gnu cxx:: promote num t< Tp > lbincoef (unsigned int n, unsigned int k)

    float lbincoeff (unsigned int n, unsigned int k)

    long double lbincoefl (unsigned int __n, unsigned int __k)

template<typename _Tp >
    gnu cxx:: promote num t< Tp > Idouble factorial (int n)

    float Idouble factorialf (int n)

    long double ldouble_factoriall (int __n)

template<typename _Tp >
    gnu cxx:: promote num t< Tp> legendre q (unsigned int n, Tp x)
• float legendre qf (unsigned int n, float x)

    long double legendre ql (unsigned int n, long double x)

template<typename_Tp>
    gnu cxx:: promote num t< Tp> Ifactorial (unsigned int n)

    float Ifactorialf (unsigned int __n)

    long double lfactoriall (unsigned int n)

template<typename_Tp>
    gnu cxx:: promote num t < Tp > logint (Tp x)

    float logintf (float x)

    long double logintl (long double x)

• template<typename _Tp , typename _Tn >
    _gnu_cxx::__promote_num_t< _Tp, _Tn > lpochhammer_l (_Tp __a, _Tn __n)

    float lpochhammer If (float a, float n)

    long double lpochhammer II (long double a, long double n)

• template<typename _Tp , typename _Tn >
    gnu cxx:: promote num t< Tp, Tn> lpochhammer u (Tp a, Tn n)

    float lpochhammer uf (float a, float n)

    long double lpochhammer ul (long double a, long double n)

• template<typename _Tph , typename _Tpa >
    _gnu_cxx::__promote_num_t< _Tph, _Tpa > owens_t (_Tph __h, _Tpa __a)

    float owens tf (float h, float a)

• long double owens_tl (long double __h, long double __a)
• template<typename _Tp , typename _Tn >
    _gnu_cxx::__promote_num_t< _Tp, _Tn > pochhammer_l (_Tp __a, _Tn __n)

    float pochhammer If (float a, float n)

    long double pochhammer_II (long double __a, long double __n)

    template<typename _Tp , typename _Tn >

   __gnu_cxx::__promote_num_t< _Tp, _Tn > pochhammer_u (_Tp __a, _Tn __n)
```

```
    float pochhammer_uf (float __a, float __n)

    long double pochhammer_ul (long double __a, long double __n)

• template<typename _Tp , typename _Wp >
    _gnu_cxx::__promote_num_t< _Tp, _Wp > polylog (_Tp __s, _Wp __w)
template<typename _Tp , typename _Wp >
  std::complex< __gnu_cxx::__promote_num_t< _Tp, _Wp >> polylog (_Tp __s, std::complex< _Tp > __w)

    float polylogf (float s, float w)

    std::complex< float > polylogf (float __s, std::complex< float > __w)

    long double polylogi (long double s, long double w)

    std::complex < long double > polylogl (long double __s, std::complex < long double > __w)

template<typename _Tp >
    gnu cxx:: promote num t< Tp>psi ( Tpx)

    float psif (float x)

    long double psil (long double __x)

template<typename _Tp >
    gnu cxx:: promote num t< Tp> radpoly (unsigned int n, unsigned int m, Tp rho)

    float radpolyf (unsigned int n, unsigned int m, float rho)

    long double radpolyl (unsigned int __n, unsigned int __m, long double __rho)

template<typename _Tp >
    gnu cxx:: promote num t < Tp > sinc ( Tp x)
template<typename_Tp>
    _gnu_cxx::__promote_num_t< _Tp > sinc_pi (_Tp __x)

    float sinc pif (float x)

    long double sinc pil (long double x)

    float sincf (float x)

    long double sincl (long double x)

template<typename _Tp >
   _gnu_cxx::__promote_num_t< _Tp > sinhc (_Tp __x)
template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > sinhc_pi (_Tp __x)

    float sinhc pif (float x)

    long double sinhc pil (long double x)

    float sinhcf (float x)

    long double sinhcl (long double x)

template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > sinhint (_Tp __x)

    float sinhintf (float x)

    long double sinhintl (long double x)

template<typename _Tp >
    gnu cxx:: promote num t < Tp > sinint ( Tp x)

    float sinintf (float x)

    long double sinintl (long double x)

template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > sph_bessel_i (unsigned int __n, _Tp __x)

    float sph bessel if (unsigned int n, float x)

    long double sph_bessel_il (unsigned int __n, long double __x)

template<typename _Tp >
    gnu cxx:: promote num t< Tp> sph bessel k (unsigned int n, Tpx)

    float sph bessel kf (unsigned int n, float x)

    long double sph_bessel_kl (unsigned int __n, long double __x)

template<typename _Tp >
  std::complex< gnu cxx:: promote num t< Tp > > sph hankel 1 (unsigned int n, Tp z)
```

```
template<typename _Tp >
  std::complex < gnu cxx:: promote num t < Tp > sph hankel 1 (unsigned int n, std::complex < Tp > sph

    std::complex< float > sph_hankel_1f (unsigned int __n, float __z)

• std::complex < float > sph hankel 1f (unsigned int n, std::complex < float > x)

    std::complex < long double > sph hankel 1l (unsigned int n, long double z)

    std::complex < long double > sph hankel 1l (unsigned int n, std::complex < long double > x)

template<typename</li>Tp >
  std::complex< __gnu_cxx::__promote_num_t< _Tp >> sph_hankel_2 (unsigned int __n, _Tp __z)
template<typename Tp >
  std::complex < __gnu_cxx::__promote_num_t < _Tp >> sph_hankel_2 (unsigned int __n, std::complex < _Tp >
   __x)
• std::complex< float > sph_hankel_2f (unsigned int __n, float __z)

    std::complex < float > sph hankel 2f (unsigned int n, std::complex < float > x)

    std::complex < long double > sph_hankel_2l (unsigned int __n, long double __z)

    std::complex < long double > sph hankel 2l (unsigned int n, std::complex < long double > x)

• template<typename _Ttheta , typename _Tphi >
  std::complex < \underline{\quad} gnu\_cxx::\underline{\quad} promote\_num\_t < \underline{\quad} Ttheta, \underline{\quad} Tphi >> sph\_harmonic \ (unsigned \ int \underline{\quad} I, \ int \underline{\quad} m, \\
  _Ttheta __theta, _Tphi __phi)

    std::complex < float > sph_harmonicf (unsigned int __l, int __m, float __theta, float __phi)

• std::complex < long double > sph harmonicl (unsigned int I, int m, long double theta, long double phi)
• template<typename _Tpnu , typename _Tp >
    gnu cxx:: promote num t< Tpnu, Tp> theta 1 (Tpnu nu, Tpx)
• float theta 1f (float nu, float x)

    long double theta_1l (long double __nu, long double __x)

• template<typename _Tpnu , typename _Tp >
    gnu cxx:: promote num t< Tpnu, Tp > theta 2 (Tpnu nu, Tp x)
• float theta 2f (float nu, float x)

    long double theta_2l (long double __nu, long double __x)

\bullet \;\; {\sf template}{<} {\sf typename} \; {\sf \_Tpnu} \; , \\ {\sf typename} \; {\sf \_Tp} > \\
    gnu cxx:: promote num t< Tpnu, Tp > theta 3 (Tpnu nu, Tp x)
• float theta 3f (float nu, float x)

    long double theta_3l (long double __nu, long double __x)

• template<typename _Tpnu , typename _Tp >
    _gnu_cxx::__promote_num_t< _Tpnu, _Tp > theta_4 (_Tpnu __nu, _Tp __x)

 float theta_4f (float __nu, float __x)

    long double theta 4l (long double nu, long double x)

• template<typename _{\rm Tpk}, typename _{\rm Tp} >
    _gnu_cxx::__promote_num_t< _Tpk, _Tp > theta_c (_Tpk __k, _Tp __x)

    float theta_cf (float __k, float __x)

• long double theta cl (long double k, long double x)
• template<typename _{\rm Tpk}, typename _{\rm Tp} >
    _gnu_cxx::__promote_num_t< _Tpk, _Tp > theta_d (_Tpk __k, _Tp __x)

    float theta df (float k, float x)

    long double theta dl (long double k, long double x)

• template<typename _Tpk , typename _Tp >
    _gnu_cxx::__promote_num_t< _Tpk, _Tp > theta_n (_Tpk __k, _Tp __x)

    float theta nf (float k, float x)

    long double theta nl (long double k, long double x)

• template<typename _Tpk , typename _Tp >
    _gnu_cxx::__promote_num_t< _Tpk, _Tp > theta_s (_Tpk __k, _Tp __x)

    float theta sf (float k, float x)
```

```
    long double theta_sl (long double __k, long double __x)
    template<typename _Trho , typename _Tphi >
        __gnu_cxx::__promote_num_t< _Trho, _Tphi > zernike (unsigned int __n, int __m, _Trho __rho, _Tphi __phi)
    float zernikef (unsigned int __n, int __m, float __rho, float __phi)
    long double zernikel (unsigned int __n, int __m, long double __rho, long double __phi)
```

# 8.2 std Namespace Reference

# **Namespaces**

detail

## **Functions**

```
    template<typename</li>
    Tp >

  __gnu_cxx::__promote< _Tp >::__type assoc_laguerre (unsigned int __n, unsigned int __m, _Tp __x)

    float assoc_laguerref (unsigned int __n, unsigned int __m, float __x)

    long double assoc_laguerrel (unsigned int __n, unsigned int __m, long double __x)

template<typename _Tp >
    _gnu_cxx::__promote< _Tp >::__type assoc_legendre (unsigned int __I, unsigned int __m, _Tp __x)

    float assoc legendref (unsigned int I, unsigned int m, float x)

    long double assoc_legendrel (unsigned int __l, unsigned int __m, long double __x)

    template<typename _Tpa , typename _Tpb >

   __gnu_cxx::__promote_2< _Tpa, _Tpb >::__type beta (_Tpa __a, _Tpb __b)

    float betaf (float a, float b)

    long double betal (long double a, long double b)

template<typename _Tp >
    _gnu_cxx::__promote< _Tp >::__type comp_ellint_1 (_Tp __k)

    float comp_ellint_1f (float __k)

    long double comp_ellint_1l (long double __k)

template<typename _Tp >
    _gnu_cxx::__promote< _Tp >::__type comp_ellint_2 (_Tp __k)

    float comp ellint 2f (float k)

    long double comp ellint 2l (long double k)

• template<typename _Tp , typename _Tpn >
    _gnu_cxx::__promote_2< _Tp, _Tpn >::__type comp_ellint_3 (_Tp __k, _Tpn __nu)

    float comp_ellint_3f (float __k, float __nu)

      Return the complete elliptic integral of the third kind \Pi(k,\nu) for float modulus k.
• long double comp ellint 3l (long double k, long double nu)
      Return the complete elliptic integral of the third kind \Pi(k,\nu) for long double modulus k.
• template<typename _{\rm Tpnu}, typename _{\rm Tp}>
    _gnu_cxx::__promote_2< _Tpnu, _Tp >::__type cyl_bessel_i (_Tpnu __nu, _Tp __x)

    float cyl_bessel_if (float __nu, float __x)

• long double cyl_bessel_il (long double __nu, long double __x)
\bullet \;\; {\sf template}{<} {\sf typename} \; {\sf \_Tpnu} \; , \\ {\sf typename} \; {\sf \_Tp} > \\
    _gnu_cxx::__promote_2< _Tpnu, _Tp >::__type cyl_bessel_j (_Tpnu __nu, _Tp __x)

    float cyl bessel if (float nu, float x)

    long double cyl bessel jl (long double nu, long double x)
```

```
• template<typename _Tpnu , typename _Tp >
    _gnu_cxx::__promote_2< _Tpnu, _Tp >::__type cyl_bessel_k (_Tpnu __nu, _Tp __x)

    float cyl bessel kf (float nu, float x)

    long double cyl_bessel_kl (long double __nu, long double __x)

• template<typename Tpnu, typename Tp >
    _gnu_cxx::__promote_2< _Tpnu, _Tp >::__type cyl_neumann (_Tpnu __nu, _Tp __x)

    float cyl_neumannf (float __nu, float __x)

    long double cyl_neumannl (long double __nu, long double __x)

• template<typename Tp, typename Tpp>

    float ellint_1f (float __k, float __phi)

    long double ellint 11 (long double k, long double phi)

template<typename _Tp , typename _Tpp >
    _gnu_cxx::__promote_2< _Tp, _Tpp >::__type ellint_2 (_Tp __k, _Tpp __phi)

    float ellint 2f (float k, float phi)

      Return the incomplete elliptic integral of the second kind E(k, \phi) for float argument.

    long double ellint_2l (long double ___k, long double ___phi)

      Return the incomplete elliptic integral of the second kind E(k, \phi).

    template<typename _Tp , typename _Tpn , typename _Tpp >

   _gnu_cxx::__promote_3< _Tp, _Tpn, _Tpp >::__type ellint_3 (_Tp __k, _Tpn __nu, _Tpp __phi)
      Return the incomplete elliptic integral of the third kind \Pi(k, \nu, \phi).

    float ellint 3f (float k, float nu, float phi)

      Return the incomplete elliptic integral of the third kind \Pi(k, \nu, \phi) for float argument.
• long double ellint 3I (long double k, long double nu, long double phi)
      Return the incomplete elliptic integral of the third kind \Pi(k, \nu, \phi).

    template<typename</li>
    Tp >

   _gnu_cxx::__promote< _Tp >::__type expint (_Tp __x)

    float expintf (float __x)

    long double expintl (long double __x)

    template<typename</li>
    Tp >

   _gnu_cxx::__promote< _Tp >::__type hermite (unsigned int __n, _Tp __x)
• float hermitef (unsigned int __n, float __x)

    long double hermitel (unsigned int n, long double x)

template<typename _Tp >
    _gnu_cxx::__promote< _Tp >::__type laguerre (unsigned int __n, _Tp __x)

    float laguerref (unsigned int n, float x)

    long double laguerrel (unsigned int __n, long double __x)

• template<typename _Tp >
    _gnu_cxx::__promote< _Tp >::__type legendre (unsigned int __I, _Tp __x)

    float legendref (unsigned int I, float x)

    long double legendrel (unsigned int __l, long double __x)

template<typename _Tp >
    gnu cxx:: promote < Tp >:: type riemann zeta (Tp s)

    float riemann_zetaf (float __s)

    long double riemann zetal (long double s)

template<typename _Tp >
    gnu cxx:: promote < Tp >:: type sph bessel (unsigned int n, Tp x)

    float sph besself (unsigned int n, float x)

    long double sph_bessell (unsigned int __n, long double __x)

template<typename _Tp >
  gnu cxx:: promote < Tp >:: type sph legendre (unsigned int I, unsigned int m, Tp theta)
```

- float sph\_legendref (unsigned int \_\_l, unsigned int \_\_m, float \_\_theta)
- long double sph\_legendrel (unsigned int \_\_l, unsigned int \_\_m, long double \_\_theta)
- template<typename \_Tp >
   \_\_gnu\_cxx::\_\_promote< \_Tp >::\_\_type sph\_neumann (unsigned int \_\_n, \_Tp \_\_x)
- float sph\_neumannf (unsigned int \_\_n, float \_\_x)
- long double sph\_neumannl (unsigned int \_\_n, long double \_\_x)

# 8.3 std::\_\_detail Namespace Reference

#### Classes

· struct Factorial table

#### **Enumerations**

• enum { SININT, COSINT }

## **Functions**

template < typename \_Tp >
 void airy ( Tp z, Tp & Ai, Tp & Bi, Tp & Aip, Tp & Bip)

Compute the Airy functions Ai(x) and Bi(x) and their first derivatives Ai'(x) and Bi(x) respectively.

template<typename \_Tp >
 void \_\_airy (const std::complex< \_Tp > &\_z, \_Tp \_\_eps, std::complex< \_Tp > &\_Ai, std::complex< \_Tp >
 &\_Aip, std::complex< \_Tp > &\_Bi, std::complex< \_Tp > &\_Bip)

This function computes the Airy function Ai(z) and its first derivative in the complex z-plane.

template < typename \_Tp >
 std::complex < \_Tp > \_\_airy\_ai (std::complex < \_Tp > \_\_z)

Return the complex Airy Ai function.

template<typename\_Tp >
 void \_\_airy\_arg (std::complex< \_Tp > \_\_num2d3, std::complex< \_Tp > \_\_zeta, std::complex< \_Tp > &\_\_argp, std::complex< \_Tp > &\_\_argm)

Compute the arguments for the Airy function evaluations carefully to prevent premature overflow. Note that the major work here is in safe div. A faster, but less safe implementation can be obtained without use of safe div.

template<typename \_Tp >
 void \_\_airy\_asymp\_absarg\_ge\_pio3 (std::complex< \_Tp > \_\_z, std::complex< \_Tp > &\_Ai, std::complex< \_Tp > &\_Aip, int \_\_sign=-1)

This function evaluates Ai(z) and Ai'(z) from their asymptotic expansions for  $|arg(z)| < 2 * \pi/3$ . For speed, the number of terms needed to achieve about 16 decimals accuracy is tabled and determined from abs(z).

template<typename \_Tp >
 void \_\_airy\_asymp\_absarg\_lt\_pio3 (std::complex< \_Tp > \_\_z, std::complex< \_Tp > &\_Ai, std::complex< \_Tp > &\_Aip)

This function evaluates Ai(z) and Ai'(z) from their asymptotic expansions for |arg(-z)| < pi/3. For speed, the number of terms needed to achieve about 16 decimals accuracy is tabled and determined from |z|.

• template<typename \_Tp > void \_\_airy\_bessel\_i (const std::complex< \_Tp > &\_\_z, \_Tp \_\_eps, std::complex< \_Tp > &\_lp1d3, std  $\leftrightarrow$  ::complex< \_Tp > &\_lm1d3, std::complex< \_Tp > &\_lm2d3)

template<typename \_Tp >
 void \_\_airy\_bessel\_k (const std::complex< \_Tp > &\_\_z, \_Tp \_\_eps, std::complex< \_Tp > &\_Kp1d3, std
 ::complex< \_Tp > &\_Kp2d3)

Compute approximations to the modified Bessel functions of the second kind of orders 1/3 and 2/3 for moderate arguments.

• template<typename  $_{\rm Tp}>$ 

Return the complex Airy Bi function.

template<typename \_Tp >

```
\label{local_problem} $$\operatorname{void}$ $\_\operatorname{airy\_hyperg\_rational}$ (const std::complex < $\_Tp > \&\_z$, std::complex < $\_Tp > \&\_Ai, std::complex < $\_Tp > \&\_Bip$) $$
```

This function computes rational approximations to the hypergeometric functions related to the modified Bessel functions of orders  $\nu=+-1/3$  and  $\nu+-2/3$ . That is, A(z)/B(z), Where A(z) and B(z) are cubic polynomials with real coefficients, approximates

$$\frac{\Gamma(\nu+1)}{(z/2)^n u} I_{\nu}(z) =_0 F_1(;\nu+1;z^2/4),$$

where the function on the right is a confluent hypergeometric limit function. For |z| <= 1/4 and |arg(z)| <= pi/2, the approximations are accurate to about 16 decimals.

template<typename \_Tp >

This routine returns the associated Laguerre polynomial of order n, degree m:  $L_n^m(x)$ .

template<typename \_Tp >

Return the associated Legendre function by recursion on l and downward recursion on m.

template<typename</li>
 Tp >

This returns Bernoulli number  $B_n$ .

template<typename \_Tp >

This returns Bernoulli number  $B_n$ .

• template<typename  $_{\rm Tp}>$ 

This returns Bernoulli numbers from a table or by summation for larger values.

template<typename \_Tp >

Return the beta function B(a,b).

template<typename \_Tp >

Return the beta function: B(a,b).

template<typename \_Tp >

template<typename \_Tp >

template<typename\_Tp>

Return the beta function B(a,b) using the log gamma functions.

template<typename \_Tp >

Return the beta function B(x, y) using the product form.

template<typename\_Tp>

Return the binomial coefficient. The binomial coefficient is given by:

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

template<typename \_Tp >

\_GLIBCXX14\_CONSTEXPR \_Tp \_\_binomial\_cdf (\_Tp \_\_p, unsigned int \_\_n, unsigned int \_\_k)

Return the binomial cumulative distribution function.

• template<typename  $_{\rm Tp}>$ 

Return the complementary binomial cumulative distribution function.

template<typename\_Tp>

template<typename \_Tp >

template<typename\_Tp>

```
GLIBCXX14 CONSTEXPR Tp chi squared pdf (Tp chi2, unsigned int nu)
```

Return the chi-squared propability function. This returns the probability that the observed chi-squared for a correct model is less than the value  $\chi^2$ .

template<typename</li>Tp >

Return the complementary chi-squared propability function. This returns the probability that the observed chi-squared for a correct model is greater than the value  $\chi^2$ .

template<typename \_Tp >

$$std::pair < _Tp, _Tp > \underline{\hspace{0.5cm}} chshint (_Tp \__x, _Tp \&_Chi, _Tp \&_Shi)$$

This function returns the hyperbolic cosine Ci(x) and hyperbolic sine Si(x) integrals as a pair.

template<typename \_Tp >

This function computes the hyperbolic cosine Chi(x) and hyperbolic sine Shi(x) integrals by continued fraction for positive argument.

template<typename \_Tp >

This function computes the hyperbolic cosine Chi(x) and hyperbolic sine Shi(x) integrals by series summation for positive argument.

template<typename</li>
 Tp >

• template<typename  $_{\rm Tp}>$ 

template<typename\_Tp>

• template<typename  $_{\rm Tp}>$ 

```
_Tp __clausen (unsigned int __m, _Tp __w)
```

template<typename\_Tp>

$$_{\rm Tp}$$
  $_{\rm clausen\_c}$  (unsigned int  $_{\rm m}$ , std::complex<  $_{\rm Tp}$  >  $_{\rm w}$ )

```
template<typename _Tp >
  Tp clausen c (unsigned int m, Tp w)
template<typename_Tp>
  _Tp __clausen_s (unsigned int __m, std::complex< _Tp > __w)
template<typename _Tp >
  _Tp __clausen_s (unsigned int __m, _Tp __w)
• template<typename _{\mathrm{Tp}}>
  _Tp __comp_ellint_1 (_Tp __k)
      Return the complete elliptic integral of the first kind K(k) using the Carlson formulation.
template<typename _Tp >
  _Tp __comp_ellint_2 (_Tp __k)
      Return the complete elliptic integral of the second kind E(k) using the Carlson formulation.
template<typename _Tp >
  _Tp __comp_ellint_3 (_Tp __k, _Tp __nu)
      Return the complete elliptic integral of the third kind \Pi(k,\nu)=\Pi(k,\nu,\pi/2) using the Carlson formulation.
template<typename _Tp >
  _Tp __comp_ellint_d (_Tp __k)
template<typename_Tp>
  _Tp __comp_ellint_rf (_Tp __x, _Tp __y)

    template<typename</li>
    Tp >

  _Tp __comp_ellint_rg (_Tp __x, _Tp __y)
• template<typename _{\mathrm{Tp}} >
  _Tp __conf_hyperg (_Tp __a, _Tp __c, _Tp __x)
      Return the confluent hypergeometric function {}_1F_1(a;c;x).

    template<typename</li>
    Tp >

  _Tp __conf_hyperg_lim (_Tp __c, _Tp __x)
      Return the confluent hypergeometric limit function {}_{0}F_{1}(-;c;x).

    template<typename</li>
    Tp >

  _Tp __conf_hyperg_lim_series (_Tp __c, _Tp __x)
      This routine returns the confluent hypergeometric limit function by series expansion.
template<typename_Tp>
  _Tp __conf_hyperg_luke (_Tp __a, _Tp __c, _Tp __xin)
      Return the hypergeometric function _1F_1(a;c;x) by an iterative procedure described in Luke, Algorithms for the Compu-
      tation of Mathematical Functions.
template<typename_Tp>
  _Tp __conf_hyperg_series (_Tp __a, _Tp __c, _Tp __x)
      This routine returns the confluent hypergeometric function by series expansion.
template<typename _Tp >
  _Tp __coshint (const _Tp __x)
      Return the hyperbolic cosine integral li(x).

    template<typename</li>
    Tp >

  std::complex < _Tp > __cyl_bessel (std::complex < _Tp > __nu, std::complex < _Tp > __z)
      Return the complex cylindrical Bessel function.
template<typename_Tp>
  _Tp __cyl_bessel_i (_Tp __nu, _Tp __x)
      Return the regular modified Bessel function of order \nu: I_{\nu}(x).

    template<tvpename</li>
    Tp >

  _Tp __cyl_bessel_ij_series (_Tp __nu, _Tp __x, _Tp __sgn, unsigned int __max_iter)
      This routine returns the cylindrical Bessel functions of order \nu: J_{\nu} or I_{\nu} by series expansion.
```

```
template<typename _Tp >
  void cyl bessel ik (Tp nu, Tp x, Tp & Inu, Tp & Knu, Tp & Ipnu, Tp & Kpnu)
      Return the modified cylindrical Bessel functions and their derivatives of order \nu by various means.

    template<typename</li>
    Tp >

  void <u>cyl_bessel_ik_asymp</u> (_Tp __nu, _Tp __x, _Tp &_lnu, _Tp &_Knu, _Tp &_lpnu, _Tp &_Kpnu)
      This routine computes the asymptotic modified cylindrical Bessel and functions of order nu: I_{\nu}(x), N_{\nu}(x). Use this for
      x >> nu^2 + 1.
• template<typename _{\rm Tp}>
  void <u>cyl_bessel_ik_steed</u> (_Tp __nu, _Tp __x, _Tp &_Inu, _Tp &_Knu, _Tp &_Ipnu, _Tp &_Kpnu)
      Compute the modified Bessel functions I_{\nu}(x) and K_{\nu}(x) and their first derivatives I'_{\nu}(x) and K'_{\nu}(x) respectively. These
      four functions are computed together for numerical stability.

    template<typename</li>
    Tp >

  _Tp __cyl_bessel_j (_Tp __nu, _Tp __x)
      Return the Bessel function of order \nu: J_{\nu}(x).

    template<typename</li>
    Tp >

  void <u>__cyl_bessel_jn</u> (_Tp __nu, _Tp __x, _Tp &_Jnu, _Tp &_Nnu, _Tp &_Jpnu, _Tp &_Npnu)
      Return the cylindrical Bessel functions and their derivatives of order \nu by various means.
template<typename _Tp >
  void <u>cyl_bessel_jn_asymp</u> (_Tp __nu, _Tp __x, _Tp &_Jnu, _Tp &_Nnu, _Tp &_Jpnu, _Tp &_Npnu)
      This routine computes the asymptotic cylindrical Bessel and Neumann functions of order nu: J_{\nu}(x), N_{\nu}(x). Use this for
      x >> nu^2 + 1.

    template<typename</li>
    Tp >

  void cyl bessel in steed (Tp nu, Tp x, Tp & Jnu, Tp & Nnu, Tp & Jpnu, Tp & Npnu)
      Compute the Bessel J_{\nu}(x) and Neumann N_{\nu}(x) functions and their first derivatives J'_{\nu}(x) and N'_{\nu}(x) respectively. These
      four functions are computed together for numerical stability.
template<typename _Tp >
  _Tp __cyl_bessel_k (_Tp __nu, _Tp __x)
      Return the irregular modified Bessel function K_{\nu}(x) of order \nu.
template<typename _Tp >
  std::complex< _Tp > __cyl_hankel_1 (_Tp __nu, _Tp __x)
      Return the cylindrical Hankel function of the first kind H_{\nu}^{(1)}(x).

    template<tvpename</li>
    Tp >

  std::complex < Tp > \_cyl_hankel_1 (std::complex < Tp > \_nu, std::complex < Tp > \_z)
      Return the complex cylindrical Hankel function of the first kind.

    template<typename</li>
    Tp >

  std::complex< _Tp > __cyl_hankel_2 (_Tp __nu, _Tp __x)
      Return the cylindrical Hankel function of the second kind H_n^{(2)}u(x).

    template<typename</li>
    Tp >

  std::complex < _Tp > __cyl_hankel_2 (std::complex < _Tp > __nu, std::complex < _Tp > __z)
      Return the complex cylindrical Hankel function of the second kind.

    template<typename</li>
    Tp >

  std::complex < _Tp > __cyl_neumann (std::complex < _Tp > __nu, std::complex < _Tp > __z)
      Return the complex cylindrical Neumann function.
template<typename _Tp >
  _Tp <u>__cyl_neumann_n</u> (_Tp __nu, _Tp __x)
```

Return the Dawson integral, F(x), for real argument x.

Return the Neumann function of order  $\nu$ :  $N_{\nu}(x)$ .

template<typename \_Tp >\_Tp \_\_dawson (\_Tp \_\_x)

```
template<typename _Tp >
  _Tp __dawson_cont_frac (_Tp __x)
      Compute the Dawson integral using a sampling theorem representation.

    template<typename</li>
    Tp >

  _Tp __dawson_series (_Tp __x)
      Compute the Dawson integral using the series expansion.

    template<typename</li>
    Tp >

  void <u>debye_region</u> (std::complex< _Tp > __alpha, int &__indexr, char &__aorb)

    template<typename</li>
    Tp >

  _Tp <u>__dilog</u> (_Tp __x)
      Compute the dilogarithm function Li_2(x) by summation for x \le 1.

    template<tvpename</li>
    Tp >

  _Tp __dirichlet_beta (std::complex< _Tp > w)
template<typename _Tp >
  _Tp __dirichlet_beta (_Tp _ w)

    template<typename</li>
    Tp >

  std::complex < _Tp > __dirichlet_eta (std::complex < _Tp > __w)
• template<typename _{\rm Tp}>
  _Tp __dirichlet_eta (_Tp __w)
template<typename_Tp>
  GLIBCXX14 CONSTEXPR Tp double factorial (int n)
      Return the double factorial of the integer n.
template<typename_Tp>
  _Tp __ellint_1 (_Tp __k, _Tp __phi)
      Return the incomplete elliptic integral of the first kind F(k,\phi) using the Carlson formulation.
template<typename_Tp>
  _Tp __ellint_2 (_Tp __k, _Tp __phi)
      Return the incomplete elliptic integral of the second kind E(k,\phi) using the Carlson formulation.
template<typename_Tp>
  _Tp <u>__ellint_3</u> (_Tp __k, _Tp __nu, _Tp __phi)
      Return the incomplete elliptic integral of the third kind \Pi(k, \nu, \phi) using the Carlson formulation.

    template<typename</li>
    Tp >

  template<typename _Tp >
  _Tp <u>__ellint_d</u> (_Tp __k, _Tp __phi)
template<typename</li>Tp >
  _Tp <u>__ellint_el1</u> (_Tp __x, _Tp __k_c)
template<typename _Tp >
  _Tp <u>__ellint_el2</u> (_Tp __x, _Tp __k_c, _Tp __a, _Tp __b)

    template<typename</li>
    Tp >

  _Tp __ellint_el3 (_Tp __x, _Tp __k_c, _Tp __p)
template<typename _Tp >
  Tp ellint rc (Tp x, Tp y)
      Return the Carlson elliptic function R_C(x,y) = R_F(x,y,y) where R_F(x,y,z) is the Carlson elliptic function of the first
template<typename _Tp >
  _Tp __ellint_rd (_Tp __x, _Tp __y, _Tp __z)
      Return the Carlson elliptic function of the second kind R_D(x,y,z) = R_J(x,y,z,z) where R_J(x,y,z,p) is the Carlson
      elliptic function of the third kind.
template<typename _Tp >
  _{\rm Tp} _{\rm ellint\_rf} (_{\rm Tp} _{\rm x}, _{\rm Tp} _{\rm y}, _{\rm Tp} _{\rm z})
```

```
Return the Carlson elliptic function R_F(x, y, z) of the first kind.
template<typename _Tp >
  _Tp <u>__ellint_rg</u> (_Tp __x, _Tp __y, _Tp __z)
      Return the symmetric Carlson elliptic function of the second kind R_G(x, y, z).
• template<typename _Tp >
  _Tp __ellint_rj (_Tp __x, _Tp __y, _Tp __z, _Tp __p)
      Return the Carlson elliptic function R_J(x, y, z, p) of the third kind.
• template<typename _{\mathrm{Tp}} >
  _Tp __ellnome (_Tp __k)
template<typename_Tp>
  _Tp __ellnome_k (_Tp __k)
template<typename_Tp>
  _Tp __ellnome_series (_Tp __k)
template<typename _Tp >
  _Tp __expint (unsigned int __n, _Tp __x)
      Return the exponential integral E_n(x).
template<typename _Tp >
  _Tp __expint (_Tp __x)
      Return the exponential integral Ei(x).
template<typename _Tp >
  _Tp __expint_asymp (unsigned int __n, _Tp __x)
      Return the exponential integral E_n(x) for large argument.
template<typename _Tp >
  _Tp __expint_E1 (_Tp __x)
      Return the exponential integral E_1(x).
template<typename</li>Tp >
  _Tp __expint_E1_asymp (_Tp __x)
      Return the exponential integral E_1(x) by asymptotic expansion.

    template<typename</li>
    Tp >

  _Tp __expint_E1_series (_Tp __x)
      Return the exponential integral E_1(x) by series summation. This should be good for x < 1.
template<typename _Tp >
  _Tp __expint_Ei (_Tp __x)
      Return the exponential integral Ei(x).
template<typename _Tp >
  _Tp __expint_Ei_asymp (_Tp __x)
      Return the exponential integral Ei(x) by asymptotic expansion.
• template<typename _{\mathrm{Tp}} >
  _Tp __expint_Ei_series (_Tp __x)
      Return the exponential integral Ei(x) by series summation.
template<typename_Tp>
  _Tp __expint_En_cont_frac (unsigned int __n, _Tp __x)
      Return the exponential integral E_n(x) by continued fractions.
template<typename _Tp >
  _Tp __expint_En_recursion (unsigned int __n, _Tp __x)
      Return the exponential integral E_n(x) by recursion. Use upward recursion for x < n and downward recursion (Miller's
      algorithm) otherwise.

    template<typename</li>
    Tp >

  _Tp __expint_En_series (unsigned int __n, _Tp __x)
      Return the exponential integral E_n(x) by series summation.
```

```
template<typename _Tp >
  Tp expint large n (unsigned int n, Tp x)
      Return the exponential integral E_n(x) for large order.
template<typename_Tp>
  _GLIBCXX14_CONSTEXPR _Tp __factorial (unsigned int __n)
      Return the factorial of the integer n.

    template<typename</li>
    Tp >

  _Tp __fermi_dirac (_Tp __s, _Tp __x)

    template<typename</li>
    Tp >

  _GLIBCXX14_CONSTEXPR _Tp __fisher_f_cdf (_Tp __F, unsigned int __nu1, unsigned int __nu2)
      Return the F-distribution propability function. This returns the probability that the observed chi-square for a correct model
      exceeds the value \chi^2.
template<typename_Tp>
  _GLIBCXX14_CONSTEXPR _Tp __fisher_f_cdfc (_Tp __F, unsigned int __nu1, unsigned int __nu2)
      Return the F-distribution propability function. This returns the probability that the observed chi-square for a correct model
      exceeds the value \chi^2.
template<typename _Tp >
  void <u>fock_airy</u> (_Tp __x, std::complex< _Tp > &__w1, std::complex< _Tp > &__w2, std::complex< _Tp >
  _{\text{w1p, std::complex}} < _{\text{Tp}} > _{\text{w2p}}
      Compute the Fock-type Airy functions w_1(x) and w_2(x) and their first derivatives w_1'(x) and w_2'(x) respectively.
                                                 w_1(x) = \sqrt{\pi}(Ai(x) + iBi(x))
                                                 w_2(x) = \sqrt{\pi}(Ai(x) - iBi(x))
template<typename _Tp >
  bool __fpequal (const _Tp &__a, const _Tp &__b)
template<typename _Tp >
  bool fpimag (const std::complex < Tp > & w)
template<typename</li>Tp >
  bool __fpimag (const _Tp)
template<typename _Tp >
  bool fpreal (const std::complex< Tp > & w)
template<typename _Tp >
  bool __fpreal (const _Tp)

    template<typename</li>
    Tp >

  std::complex< _Tp > __fresnel (const _Tp __x)
      Return the Fresnel cosine and sine integrals as a complex number f(C(x)) + iS(x)

    template<typename</li>
    Tp >

  void <u>__fresnel_cont_frac</u> (const _Tp __ax, _Tp &_Cf, _Tp &_Sf)
      This function computes the Fresnel cosine and sine integrals by continued fractions for positive argument.
template<typename _Tp >
  void fresnel series (const Tp ax, Tp & Cf, Tp & Sf)
      This function returns the Fresnel cosine and sine integrals as a pair by series expansion for positive argument.
template<typename</li>Tp >
  _Tp <u>__gamma</u> (_Tp __x)
      Return \Gamma(x).

    template<typename</li>
    Tp >

  std::pair < \_Tp, \_Tp > \underline{\quad gamma\_cont\_frac} \ (\_Tp \ \underline{\quad } a, \_Tp \quad \  x)
template<typename _Tp >
  _Tp __gamma_I (_Tp __a, _Tp __x)
```

Return the lower incomplete gamma function. The lower incomplete gamma function is defined by

$$\gamma(a, x) = \int_0^x e^{-t} t^{a-1} dt (a > 0)$$

template<typename\_Tp>

Return the regularized lower incomplete gamma function. The regularized lower incomplete gamma function is defined by

$$P(a,x) = \frac{\gamma(a,x)}{\Gamma(a)}$$

where  $\Gamma(a)$  is the gamma function and

$$\gamma(a,x) = \int_0^x e^{-t} t^{a-1} dt (a > 0)$$

is the lower incomplete gamma function.

ullet template<typename\_Tp>

Return the regularized upper incomplete gamma function. The regularized upper incomplete gamma function is defined by

$$Q(a,x) = \frac{\Gamma(a,x)}{\Gamma(a)}$$

where  $\Gamma(a)$  is the gamma function and

$$\Gamma(a,x) = \int_{x}^{\infty} e^{-t} t^{a-1} dt (a > 0)$$

is the upper incomplete gamma function.

template<typename \_Tp >

template<typename\_Tp>

Compute the gamma functions required by the Temme series expansions of  $N_{\nu}(x)$  and  $K_{\nu}(x)$ .

$$\Gamma_1 = \frac{1}{2\mu} \left[ \frac{1}{\Gamma(1-\mu)} - \frac{1}{\Gamma(1+\mu)} \right]$$

and

$$\Gamma_2 = \frac{1}{2} \left[ \frac{1}{\Gamma(1-\mu)} + \frac{1}{\Gamma(1+\mu)} \right]$$

where  $-1/2 <= \mu <= 1/2$  is  $\mu = \nu - N$  and N. is the nearest integer to  $\nu$ . The values of  $\Gamma(1+\mu)$  and  $\Gamma(1-\mu)$  are returned as well.

 $\bullet \ \ template {<} typename \ \_Tp >$ 

$$_{\rm Tp}$$
  $_{\rm gamma}$   $_{\rm u}$   $(_{\rm Tp}$   $_{\rm u}$   $_{\rm a}$ ,  $_{\rm Tp}$   $_{\rm u}$   $_{\rm x})$ 

Return the upper incomplete gamma function. The lower incomplete gamma function is defined by

$$\Gamma(a,x) = \int_{x}^{\infty} e^{-t} t^{a-1} dt (a > 0)$$

• template<typename \_Tp >

template<typename\_Tp>

template<typename\_Tp>

void \_\_hankel (std::complex< \_Tp > \_\_nu, std::complex< \_Tp > \_\_z, std::complex< \_Tp > &\_H1, std  $\leftarrow$  ::complex< \_Tp > &\_H2, std::complex< \_Tp > &\_H1p, std::complex< \_Tp > &\_H2p)

- template<typename\_Tp >
   void \_\_hankel\_debye (std::complex< \_Tp > \_\_nu, std::complex< \_Tp > \_\_z, std::complex< \_Tp > \_\_alpha, int \_\_indexr, char &\_\_aorb, int &\_\_morn, std::complex< \_Tp > &\_H1, std::complex< \_Tp > &\_H2, std::complex< \_Tp > &\_H2, std::complex< \_Tp > &\_H2p)

Compute parameters depending on z and nu that appear in the uniform asymptotic expansions of the Hankel functions and their derivatives, except the arguments to the Airy functions.

template<typename \_Tp >
 void \_\_hankel\_uniform (std::complex< \_Tp > \_\_nu, std::complex< \_Tp > \_\_z, std::complex< \_Tp > &\_H1,

void  $\_$  nankel\_uniform (std::complex< \_1p >  $\_$ nu, std::complex< \_1p >  $\_$ z, std::complex< \_1p > &\_H1, std::complex< \_Tp > &\_H2, std::complex< \_Tp > &\_H2p)

This routine computes the uniform asymptotic approximations of the Hankel functions and their derivatives including a patch for the case when the order equals or nearly equals the argument. At such points, Olver's expressions have zero denominators (and numerators) resulting in numerical problems. This routine averages results from four surrounding points in the complex plane to obtain the result in such cases.

template<typename \_Tp >
 void \_\_hankel\_uniform\_olver (std::complex< \_Tp > \_\_nu, std::complex< \_Tp > \_\_z, std::complex< \_Tp > & \_\_
 \_H1, std::complex< \_Tp > & \_H2, std::complex< \_Tp > & \_H1p, std::complex< \_Tp > & \_H2p)

Compute approximate values for the Hankel functions of the first and second kinds using Olver's uniform asymptotic expansion to of order nu along with their derivatives.

template<typename \_Tp >
 void \_\_hankel\_uniform\_outer (std::complex < \_Tp > \_\_nu, std::complex < \_Tp > \_\_z, \_Tp \_\_eps, std::complex <
 \_Tp > &\_\_std::complex < \_Tp > &\_\_num1d3, std::complex < \_Tp >
 &\_\_num2d3, std::complex < \_Tp > &\_\_p, std::complex < \_Tp > &\_\_p2, std::complex < \_Tp > &\_\_etm3h, std 
 ::complex < \_Tp > &\_\_etrat, std::complex < \_Tp > &\_\_o4dp, std::complex < \_Tp > &

Compute outer factors and associated functions of z and nu appearing in Olver's uniform asymptotic expansions of the Hankel functions of the first and second kinds and their derivatives. The various functions of z and nu returned by  $bankel\_uniform\_outer$  are available for use in computing further terms in the expansions.

template<typename \_Tp >
 void \_\_hankel\_uniform\_sum (std::complex< \_Tp > \_\_p, std::complex< \_Tp > \_\_p2, std::complex< \_Tp > ←
 \_\_num2, std::complex< \_Tp > \_\_eatm3hf, std::complex< \_Tp > \_\_eatp, std::complex< \_Tp > \_\_o4dp, std
 ::complex< \_Tp > \_\_o4dp, std::complex< \_Tp > &
 \_\_o4dp, std::complex< \_Tp > \_\_o4dp, std::complex< \_Tp > &
 \_\_o4dp, std::complex< \_Tp > &

Compute the sums in appropriate linear combinations appearing in Olver's uniform asymptotic expansions for the Hankel functions of the first and second kinds and their derivatives, using up to nterms (less than 5) to achieve relative error eps.

template<typename \_Tp >
 \_Tp \_\_heuman\_lambda (\_Tp \_\_k, \_Tp \_\_phi)
template<typename \_Tp >
 \_Tp \_\_hurwitz\_zeta (\_Tp \_\_s, \_Tp \_\_a)
 Return the Hurwitz zeta function ζ(s, a) for all s != 1 and a > -1.
template<typename \_Tp >

std::complex< \_Tp > \_\_hurwitz\_zeta (\_Tp \_\_s, std::complex< \_Tp > \_\_a)

template < typename \_Tp >
 \_Tp \_ hurwitz \_zeta \_euler \_maclaurin (\_Tp \_\_s, \_Tp \_\_a)

Return the Hurwitz zeta function  $\zeta(s,a)$  for all s = 1 and a > -1.

template<typename \_Tp >

std::complex < Tp > hydrogen (const unsigned int n, const unsigned int I, const unsigned int m, const \_Tp \_Z, const \_Tp \_\_r, const \_Tp \_\_theta, const \_Tp \_\_phi)

template<typename \_Tp >

Return the hypergeometric function  ${}_{2}F_{1}(a,b;c;x)$ .

template<typename \_Tp >

Return the hypergeometric function  $_2F_1(a,b;c;x)$  by an iterative procedure described in Luke, Algorithms for the Computation of Mathematical Functions.

template<typename</li>
 Tp >

Return the hypergeometric function  ${}_2F_1(a,b;c;x)$  by the reflection formulae in Abramowitz & Stegun formula 15.3.6 for d e c - a - b not integral and formula 15.3.11 for d = c - a - b integral. This assumes a, b, c != negative integer.

template<typename \_Tp >

Return the hypergeometric function  ${}_{2}F_{1}(a,b;c;x)$  by series expansion.

template<typename\_Tp>

$$std::tuple < _Tp, _Tp, _Tp > \underline{_jacobi\_sncndn} (_Tp \underline{_k}, _Tp \underline{_u})$$

template<typename \_Tp >

template<typename \_Tp >

This routine returns the Laguerre polynomial of order n:  $L_n(x)$ .

template<typename \_Tp >

Return the Legendre function of the second kind by upward recursion on order l.

template<typename \_Tp >

Return the logarithm of the binomial coefficient. The binomial coefficient is given by:

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

template<typename \_Tp >

template<typename \_Tp >

Return the logarithm of the double factorial of the integer n.

template<typename \_Tp >

Return the logarithm of the factorial of the integer n.

template<typename \_Tp >

Return  $log(|\Gamma(x)|)$ . This will return values even for x < 0. To recover the sign of  $\Gamma(x)$  for any argument use  $\underline{\hspace{0.2cm}}log\_{\hookleftarrow}$ gamma\_sign.

template<typename \_Tp >

Return  $log(\Gamma(x))$  by asymptotic expansion with Bernoulli number coefficients. This is like Sterling's approximation.

template<typename\_Tp>

Return  $log(\Gamma(x))$  by the Lanczos method. This method dominates all others on the positive axis I think.

• template<typename\_Tp>

Return the sign of  $\Gamma(x)$ . At nonpositive integers zero is returned.

template<typename \_Tp >

Return  $\Gamma(z)$  by the Spouge algorithm:

$$\Gamma(z+1) = (z+a)^{z+1/2} e^{-z-a} \left[ \sqrt{2\pi} \sum_{k=1}^{\lceil a \rceil + 1} \frac{c_k(a)}{z+k} \right]$$

where

$$c_k(a) = \frac{(-1)^{k-1}}{(k-1)!} (a-k)^{k-1/2} e^{a-k}$$

and the error is bounded by

$$\epsilon(a) < a^{-1/2} (2\pi)^{-a-1/2}$$

•

ullet template<typename \_Tp >

Return the logarithm of the lower Pochhammer symbol or the falling factorial function. The lower Pochammer symbol is defined by

$$(a)_n = \prod_{k=0}^{n-1} (a-k), (a)_0 = 1 = \Gamma(a+1)/\Gamma(a-n+1)$$

In particular, f(n) = n! f. Thus this function returns

$$ln[(a)_n] = \Gamma(a+1) - \Gamma(a-n+1), ln[(a)_0] = 0$$

Many notations exist:

 $a^{\underline{n}}$ 

,

 $\left\{\begin{array}{c} a \\ n \end{array}\right\}$ 

, and others.

• template<typename\_Tp>

Return the logarithm of the (upper) Pochhammer symbol or the rising factorial function. The Pochammer symbol is defined by

$$(a)_n = \prod_{k=0}^{n-1} (a+k), (a)_0 = 1 = \Gamma(a+n)/\Gamma(n)$$

Thus this function returns

$$ln[(a)_n] = \Gamma(a+n) - \Gamma(n), ln[(a)_0] = 0$$

Many notations exist:

a'

, and others.

 $\bullet \ \ template {<} typename \ \_Tp >$ 

Return the logarithmic integral li(x).

 $\bullet \ \ template {<} typename \ \_Tp >$ 

template<typename \_Tp > \_Tp \_\_pochhammer\_l (\_Tp \_\_a, \_Tp \_\_n)

Return the logarithm of the lower Pochhammer symbol or the falling factorial function. The lower Pochammer symbol is defined by

$$(a)_n = \prod_{k=0}^{n-1} (a-k), (a)_0 = 1 = \Gamma(a+1)/\Gamma(a-n+1)$$

In particular,  $f[(n)_n = n! f]$ .

ullet template<typename \_Tp >

Return the (upper) Pochhammer function or the rising factorial function. The Pochammer symbol is defined by

$$(a)_n = \prod_{k=0}^{n-1} (a+k), (a)_0 = 1 = \Gamma(a+n)/\Gamma(n)$$

Many notations exist:

 $a^{\overline{n}}$ 

 $\left[\begin{array}{c} a \\ n \end{array}\right]$ 

, and others.

• template<typename \_Tp >

This routine returns the Hermite polynomial of order n:  $H_n(x)$ .

template<typename\_Tp>

This routine returns the Hermite polynomial of large order n:  $H_n(x)$ . We assume here that x >= 0.

template<typename\_Tp>

This routine returns the Hermite polynomial of order n:  $H_n(x)$  by recursion on n.

template<typename \_Tp >

• template<typename \_Tpa , typename \_Tp >

This routine returns the associated Laguerre polynomial of order n, degree  $\alpha$ :  $L_n^a lpha(x)$ .

• template<typename \_Tpa , typename \_Tp >

Evaluate the polynomial based on the confluent hypergeometric function in a safe way, with no restriction on the arguments.

• template<typename \_Tpa , typename \_Tp >

This routine returns the associated Laguerre polynomial of order n, degree  $\alpha > -1$  for large n. Abramowitz & Stegun, 13.5.21.

template<typename \_Tpa , typename \_Tp >

This routine returns the associated Laguerre polynomial of order n, degree  $\alpha$ :  $L_n^{\alpha}(x)$  by recursion.

template<typename</li>Tp >

Return the Legendre polynomial by upward recursion on order l.

template<typename\_Tp>

template<typename\_Tp>

```
template<typename _Tp >
  std::complex< _Tp > __polylog (_Tp __s, std::complex< _Tp > __w)

    template<typename _Tp , typename ArgType >

   __gnu_cxx::__promote_num_t< std::complex< _Tp >, ArgType > __polylog_exp (_Tp __s, ArgType __w)
template<typename</li>Tp >
  std::complex< _Tp > __polylog_exp_asymp (_Tp __s, std::complex< _Tp > __w)
template<typename</li>Tp >
  std::complex< _Tp > __polylog_exp_int_neg (int __s, std::complex< _Tp > __w)
template<typename _Tp >
  std::complex< _Tp > __polylog_exp_int_neg (const int __s, _Tp __w)
template<typename _Tp >
  std::complex< _Tp > __polylog_exp_int_pos (unsigned int __s, std::complex< _Tp > __w)

    template<typename</li>
    Tp >

  std::complex< _Tp > __polylog_exp_int_pos (unsigned int __s, _Tp __w)
template<typename Tp >
  std::complex< _Tp > __polylog_exp_neg (_Tp __s, std::complex< _Tp > __w)
template<typename_Tp>
  std::complex< _Tp > __polylog_exp_neg (int __s, std::complex< _Tp > __w)
template<typename _Tp , int __sigma>
  std::complex< _Tp > __polylog_exp_neg_even (unsigned int __n, std::complex< _Tp > __w)
• template<typename _Tp , int __sigma>
  std::complex< Tp > polylog exp neg odd (unsigned int n, std::complex< Tp > w)

    template<typename PowTp, typename Tp >

  _Tp __polylog_exp_negative_real_part (_PowTp __s, _Tp __w)
template<typename _Tp >
  std::complex< Tp > polylog exp pos (unsigned int s, std::complex< Tp > w)
template<typename</li>Tp >
  std::complex< _Tp > __polylog_exp_pos (unsigned int __s, _Tp __w)
template<typename</li>Tp >
  std::complex< _Tp > __polylog_exp_pos (_Tp __s, std::complex< _Tp > __w)
template<typename _Tp >
  std::complex < Tp > polylog exp real neg ( Tp s, std::complex < Tp > w)
template<typename_Tp>
  std::complex< _Tp > __polylog_exp_real_neg (_Tp __s, _Tp __w)
template<typename Tp >
  std::complex < Tp > polylog exp real pos (Tp s, std::complex < Tp > w)
template<typename</li>Tp >
  std::complex< _Tp > __polylog_exp_real_pos (_Tp __s, _Tp __w)
template<typename _Tp >
  _Tp __psi (_Tp __x)
     Return the digamma function. The digamma or \psi(x) function is defined by
                                                   \psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}
     For negative argument the reflection formula is used:
                                            \psi(x) = \psi(1-x) - \pi \cot(\pi x)
template<typename</li>Tp >
  _Tp __psi (unsigned int __n, Tp x)
```

Return the polygamma function  $\psi^{(n)}(x)$ .

template<typename \_Tp > \_Tp \_\_psi\_asymp (\_Tp \_\_x)

Return the digamma function for large argument. The digamma or  $\psi(x)$  function is defined by

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

.

• template<typename\_Tp>

Return the digamma function by series expansion. The digamma or  $\psi(x)$  function is defined by

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

.

• template<typename  $_{\rm Tp}>$ 

Return the Riemann zeta function  $\zeta(s)$ .

template<typename\_Tp>

Evaluate the Riemann zeta function  $\zeta(s)$  by an alternate series for s > 0.

template<typename</li>
 Tp >

Evaluate the Riemann zeta function  $\zeta(s)$  by an alternate series for s > 0.

template<typename\_Tp>

Evaluate the Riemann zeta function by series for all  $s \neq 1$ . Convergence is great until largish negative numbers. Then the convergence of the > 0 sum gets better.

• template<typename  $_{\mathrm{Tp}}$  >

Return the Riemann zeta function  $\zeta(s) - 1$ .

template<typename</li>
 Tp >

Return the Riemann zeta function  $\zeta(s)-1$  by summation for s>1. This is a small remainder for large s.

template<typename\_Tp>

Compute the Riemann zeta function  $\zeta(s)$  using the product over prime factors.

template<typename</li>
 Tp >

Compute the Riemann zeta function  $\zeta(s)$  by summation for s > 1.

template<typename\_Tp>

Return the generalized sinus cardinal function

$$sinc_a(x) = \frac{\sin(\pi x/a)}{(\pi x/a)}$$

.

template<typename\_Tp>

Return the normalized sinus cardinal function

$$sinc(x) = \frac{\sin(\pi x)}{\pi x}$$

.

template<typename \_Tp >
 \_\_gnu\_cxx::\_\_promote\_num\_t< \_Tp > \_\_sinc\_pi (\_Tp \_\_x)

Return the unnormalized sinus cardinal function

$$sinc_{\pi}(x) = \frac{\sin(x)}{x}$$

•

template<typename \_Tp >
 std::pair< Tp, Tp > sincosint ( Tp x)

This function returns the sine Si(x) and cosine Ci(x) integrals as a pair.

template<typename \_Tp >

This function computes the sine Si(x) and cosine Ci(x) integrals by asymptotic series summation for positive argument.

template<typename</li>Tp >

This function computes the sine Si(x) and cosine Ci(x) integrals by continued fraction for positive argument.

template<typename\_Tp>

This function computes the sine Si(x) and cosine Ci(x) integrals by series summation for positive argument.

template<typename\_Tp>

$$\_\_gnu\_cxx::\_promote\_num\_t < \_Tp > \_\_sinhc (\_Tp \_\_a, \_Tp \_\_x)$$

Return the generalized hyperbolic sinus cardinal function

$$sinhc_a(x) = \frac{\sinh(\pi x/a)}{\pi x/a}$$

.

template<typename\_Tp>

Return the normalized hyperbolic sinus cardinal function

$$sinhc(x) = \frac{\sinh(\pi x)}{\pi x}$$

.

template<typename</li>
 Tp >

Return the unnormalized hyperbolic sinus cardinal function

$$sinhc_{\pi}(x) = \frac{\sinh(x)}{x}$$

.

template<typename \_Tp >

Return the hyperbolic sine integral li(x).

template<typename \_Tp >

Return the spherical Bessel function  $j_n(x)$  of order n and non-negative real argument x.

template<typename \_Tp >

Return the complex spherical Bessel function.

template<typename\_Tp>

Compute the spherical modified Bessel functions  $i_n(x)$  and  $k_n(x)$  and their first derivatives  $i'_n(x)$  and  $k'_n(x)$  respectively.

```
template<typename _Tp >
  void sph bessel jn (unsigned int n, Tp x, Tp & j n, Tp & n n, Tp & jp n, Tp & np n)
      Compute the spherical Bessel j_n(x) and Neumann n_n(x) functions and their first derivatives j_n(x) and n'_n(x) respec-
template<typename _Tp >
  void <u>__sph_hankel</u> (unsigned int __n, std::complex < _Tp > __z, std::complex < _Tp > &_H1, std::complex < _Tp
  > & H1p, std::complex< Tp > & H2, std::complex< Tp > & H2p)
     Helper to compute complex spherical Hankel functions and their derivatives.
template<typename _Tp >
  std::complex< Tp > sph hankel 1 (unsigned int n, Tp x)
      Return the spherical Hankel function of the first kind h_n^{(1)}(x).

    template<typename</li>
    Tp >

  std::complex< Tp > sph hankel 1 (unsigned int n, std::complex< Tp > z)
      Return the complex spherical Hankel function of the first kind.

    template<typename</li>
    Tp >

  std::complex< Tp > sph hankel 2 (unsigned int n, Tp x)
      Return the spherical Hankel function of the second kind h_n^{(2)}(x).
template<typename _Tp >
  std::complex < _Tp > __sph_hankel_2 (unsigned int __n, std::complex < _Tp > __z)
      Return the complex spherical Hankel function of the second kind.
template<typename _Tp >
  std::complex< Tp > sph harmonic (unsigned int I, int m, Tp theta, Tp phi)
      Return the spherical harmonic function.
template<typename _Tp >
  _Tp __sph_legendre (unsigned int __l, unsigned int __m, _Tp __theta)
      Return the spherical associated Legendre function.
template<typename_Tp>
  _Tp __sph_neumann (unsigned int __n, _Tp __x)
      Return the spherical Neumann function n_n(x) of order n and non-negative real argument x.

    template<typename</li>
    Tp >

  std::complex < \_Tp > \_\_sph\_neumann \ (unsigned \ int \_\_n, \ std::complex < \_Tp > \quad z)
      Return the complex spherical Neumann function.

    template<typename</li>
    Tp >

  _GLIBCXX14_CONSTEXPR _Tp __student_t_cdf (_Tp __t, unsigned int __nu)
      Return the Students T probability function.

    template<tvpename</li>
    Tp >

  GLIBCXX14_CONSTEXPR _Tp __student_t_cdfc (_Tp __t, unsigned int __nu)
      Return the complement of the Students T probability function.
template<typename _Tp >
  Tp theta 1 (Tp nu, Tp x)

 template<typename _Tp >

  _Tp <u>__theta_2</u> (_Tp __nu, _Tp __x)
template<typename _Tp >
  _Tp __theta_2_asymp (_Tp __nu, _Tp __x)
template<typename</li>Tp >
  _Tp <u>__theta_2_sum</u> (_Tp __nu, _Tp __x)
template<typename _Tp >
  _Tp <u>__theta_3</u> (_Tp __nu, _Tp __x)

 template<typename _Tp >

  _Tp __theta_3_asymp (_Tp __nu, _Tp __x)
```

```
template<typename _Tp >
  _Tp __theta_3_sum (_Tp __nu, _Tp __x)

    template<typename</li>
    Tp >

  _Tp <u>__theta_4</u> (_Tp __nu, _Tp __x)
template<typename _Tp >
  _Tp <u>__theta_c</u> (_Tp __k, _Tp __x)
template<typename _Tp >
  _Tp <u>__theta_d</u> (_Tp __k, _Tp __x)

    template<typename _Tp >

  _Tp <u>__theta_</u>n (_Tp __k, _Tp __x)
• template<typename _{\rm Tp}>
  _Tp <u>__theta_s</u> (_Tp __k, _Tp __x)
template<typename _Tp >
   _gnu_cxx::__promote_num_t< _Tp > __zernike (unsigned int __n, int __m, _Tp __rho, _Tp __phi)

 template<typename _Tp >

  template<typename _Tp >
  Tp znorm2 (Tp x)
template<typename _Tp = double>
  _Tp evenzeta (unsigned int __k)
```

# **Variables**

```
    constexpr size t Num Euler Maclaurin zeta = 100

    constexpr Factorial table< long double > S double factorial table [301]

    constexpr long double S Euler Maclaurin zeta [ Num Euler Maclaurin zeta]

    constexpr Factorial table< long double > S factorial table [171]

    constexpr Factorial table < long double > S neg double factorial table [999]

• template<typename _{\mathrm{Tp}} >
  constexpr std::size t S num double factorials = 0
template<>
  constexpr std::size t S num double factorials < double > = 301

    template<>

  constexpr std::size_t _S_num_double_factorials< float > = 57
template<>
  constexpr std::size t S num double factorials < long double > = 301
template<typename_Tp>
  constexpr std::size_t _S_num_factorials = 0
template<>
  constexpr std::size t S num factorials < double > = 171
• template<>
  constexpr std::size_t _S_num_factorials< float > = 35
  constexpr std::size t S num factorials < long double > = 171
template<typename _Tp >
  constexpr std::size_t _S_num_neg_double_factorials = 0
template<>
  constexpr std::size_t _S_num_neg_double_factorials< double > = 150
template<>
  constexpr std::size_t _S_num_neg_double_factorials< float > = 27
template<>
  constexpr std::size t S num neg double factorials < long double > = 999
```

- constexpr size\_t \_S\_num\_zetam1 = 33
- constexpr long double \_S\_zetam1 [\_S\_num\_zetam1]

# 8.3.1 Enumeration Type Documentation

#### 8.3.1.1 anonymous enum

Enumerator

**SININT** 

COSINT

Definition at line 42 of file sf\_trigint.tcc.

## 8.3.2 Function Documentation

Compute the Airy functions Ai(x) and Bi(x) and their first derivatives Ai'(x) and Bi(x) respectively.

## **Parameters**

z	The argument of the Airy functions.
_Ai	The output Airy function of the first kind.
_Bi	The output Airy function of the second kind.
_Aip	The output derivative of the Airy function of the first kind.
_Bip	The output derivative of the Airy function of the second kind.

Definition at line 497 of file sf mod bessel.tcc.

References \_\_cyl\_bessel\_ik(), and \_\_cyl\_bessel\_jn().

This function computes the Airy function Ai(z) and its first derivative in the complex z-plane.

The algorithm used exploits numerous representations of the Airy function and its derivative. The representations are recorded here for reference:

$$(1a)Ai(z) = \frac{\sqrt{z}}{3}(I_{-1/3}(\zeta) - I_{1/3}(\zeta))$$

$$(1b)Bi(z) = \sqrt{\frac{z}{3}}(I_{-1/3}(\zeta) + I_{1/3}(\zeta))$$

$$(2)Ai(z) = \frac{\sqrt{z/3}}{\pi}K_{1/3}(\zeta) = \frac{2^{2/3}3^{-5/6}}{\sqrt{(\pi)}}z\exp(-\zeta)U(5/6;5/3;2\zeta)$$

$$(3a)Ai(-z) = \frac{\sqrt{z}}{3}(J_{-1/3}(\zeta) + J_{1/3}(\zeta))$$

$$(3b)Bi(-z) = \sqrt{\frac{z}{3}}(J_{-1/3}(\zeta) - J_{1/3}(\zeta))$$

$$(4a)Ai'(z) = \frac{z}{3}(I_{2/3}(\zeta) - I_{-2/3}(\zeta))$$

$$(4b)Bi'(z) = \frac{z}{\sqrt{3}}(I_{-2/3}(\zeta) + I_{2/3}(\zeta))$$

$$(5a)Ai'(z) = -\frac{z}{\pi\sqrt{(3)}}K_{(2/3)}(zeta) = -\frac{4^{2/3}3^{-7/6}}{\sqrt{(\pi)}}z^{2}\exp(-\zeta)U(7/6;7/3;2\zeta)$$

$$(6a)Ai'(-z) = \frac{z}{3}(J_{2/3}(\zeta) - J_{-2/3}(\zeta)),$$

$$(6b)Bi'(-z) = \frac{z}{\sqrt{3}}(J_{-2/3}(\zeta) + J_{2/3}(\zeta)),$$

Where  $\zeta = -\frac{2}{3}z^{3/2}$  and U(a;b;z) is the confluent hypergeometric function defined in

## See also

Stegun, I. A. and Abramowitz, M., Handbook of Mathematical Functions, Natl. Bureau of Standards, AMS 55, pp 504-515, 1964.

The asymptotic expansions derivable from these representations and Hankel's asymptotic expansions for the Bessel functions are used for large modulus of z. The implementation has taken advantage of the error bounds given in

#### See also

Olver, F. W. J., Error Bounds for Asymptotic Expansions, with an Application to Cylinder Functions of Large Argument, in Asymptotic Solutions of Differential Equations (Wilcox, Ed.), Wiley and Sons, pp 163-183, 1964 Olver, F. W. J., Asymptotics and Special Functions, Academic Press, pp 266-268, 1974.

For small modulus of z, a rational approximation is used. This approximant is derived from

Luke, Y. L., Mathematical Functions and their Approximations, Academic Press, pp 361-363, 1975.

The identities given below are for Bessel functions of the first kind in terms of modified Bessel functions of the first kind are also used with the rational approximant.

For moderate modulus of z, three techniques are used. Two use a backward recursion algorithm with (1), (3), (4), and (6). The third uses the confluent hypergeometric representations given by (2) and (5). The backward recursion algorithm generates values of the modified Bessel functions of the first kind of orders + or - 1/3 and + or - 2/3 for z in the right half plane. Values for the corresponding Bessel functions of the first kind are recovered via the identities

$$J_{\nu}(z) = exp(\nu \pi i/2)I_{\nu}(zexp(-\pi i/2)), 0 \le arg(z) \le \pi/2$$

and

$$J_{\nu}(z) = \exp(-\nu \pi i/2) I_{\nu}(z \exp(\pi i/2)), -\pi/2 <= \arg(z) < 0.$$

The particular backward recursion algorithm used is discussed in

#### See also

Olver, F. W. J, Numerical solution of second-order linear difference equations, NBS J. Res., Series B, VOL 71B, pp 111-129, 1967.

Olver, F. W. J. and Sookne, D. J., Note on backward recurrence algorithms, Math. Comp. Vol 26, No. 120, pp 941-947, Oct. 1972

Sookne, D. J., Bessel Functions I and J of Complex Argument and Integer Order, NBS J. Res., Series B, Vol 77B, Nos 3& 4, pp 111-113, July-December, 1973.

The following paper was also useful

#### See also

Cody, W. J., Preliminary report on software for the modified Bessel functions of the first kind, Applied Mathematics Division, Argonne National Laboratory, Tech. Memo. no. 357.

A backward recursion algorithm is also used to compute the confluent hypergeometric function. The recursion relations and a convergence theorem are given in

#### See also

Wimp, J., On the computation of Tricomi's psi function, Computing, Vol 13, pp 195-203, 1974.

## **Parameters**

in	Z	The argument at which the Airy function and its derivative are computed.
in	eps	Relative error required. Currently, eps is used only in the backward recursion
		algorithms.
out	_Ai	The value computed for Ai(z).
out	_Aip	The value computed for Ai'(z).
out	_Bi	The value computed for Bi(z).
out	_Bip	The value computed for Bi'(z).

Definition at line 1002 of file sf\_airy.tcc.

References \_\_airy\_asymp\_absarg\_ge\_pio3(), \_\_airy\_asymp\_absarg\_lt\_pio3(), \_\_airy\_bessel\_i(), \_\_airy\_bessel\_k(), and airy hyperg rational().

Referenced by \_\_airy\_ai(), \_\_airy\_bi(), \_\_hankel\_uniform\_outer(), and \_\_poly\_hermite\_asymp().

 $8.3.2.3 \quad template < typename \_Tp > std::\_detail::\_airy\_ai \ ( \ std::complex < \_Tp > \_\_z \ )$ 

Return the complex Airy Ai function.

Definition at line 1139 of file sf\_airy.tcc.

References \_\_airy().

8.3.2.4 template<typename \_Tp > void std::\_\_detail::\_\_airy\_arg ( std::complex< \_Tp > \_\_num2d3, std::complex< \_Tp > \_\_zeta, std::complex< \_Tp > & \_\_argp, std::complex< \_Tp > & \_\_argm )

Compute the arguments for the Airy function evaluations carefully to prevent premature overflow. Note that the major work here is in safe\_div. A faster, but less safe implementation can be obtained without use of safe\_div.

## **Parameters**

in	num2d3	$nu^{\wedge}(-2/3)$ - output from hankel_params.
in	zeta	zeta in the uniform asymptotic expansions - output from hankel_params.
out	argp	$\exp(+2*pi*i/3)*nu^{(2/3)}*zeta.$
out	argm	$\exp(-2*pi*i/3)*nu^{(2/3)}*zeta.$

## **Exceptions**

std::runtime_erro
-------------------

Definition at line 241 of file sf hankel.tcc.

Referenced by \_\_hankel\_uniform\_outer().

8.3.2.5 template < typename \_Tp > void std::\_\_detail::\_\_airy\_asymp\_absarg\_ge\_pio3 ( std::complex < \_Tp > \_\_z, std::complex < Tp > & Ai, std::complex < Tp > & Aip, int sign = -1 )

This function evaluates Ai(z) and Ai'(z) from their asymptotic expansions for  $|arg(z)| < 2 * \pi/3$ . For speed, the number of terms needed to achieve about 16 decimals accuracy is tabled and determined from abs(z).

Note that for speed and since this function is called by another, checks for valid arguments are not made.

#### See also

Digital Library of Mathematical Functions Sec. 9.7 Asymptotic Expansions http://dlmf.nist.gov/9.7

#### **Parameters**

in	z	Complex input variable set equal to the value at which $Ai(z)$ and $Bi(z)$ and their
		derivative are evaluated. This function assumes $ z >15$ and $ arg(z) <2\pi/3$ .
in,out	_Ai	The value computed for $Ai(z)$ .
in,out	_Aip	The value computed for $Ai'(z)$ .
in	sign	
		Ai functions for $ arg(z)  < \pi$ . The value +1 gives the Airy Bi functions for
		$ arg(z)  < \pi/3.$

Definition at line 71 of file sf airy.tcc.

Referenced by airy().

This function evaluates Ai(z) and Ai'(z) from their asymptotic expansions for |arg(-z)| < pi/3. For speed, the number of terms needed to achieve about 16 decimals accuracy is tabled and determined from |z|.

Note that for speed and since this function is called by another, checks for valid arguments are not made. This function assumes |z| > 15 and |arg(-z)| < pi/3.

# **Parameters**

in	Z	The value at which the Airy function and its derivative are evaluated.
out	_Ai	The computed value of the Airy function $Ai(z)$ .
out	_Aip	The computed value of the Airy function derivative $Ai'(z)$ .

Definition at line 186 of file sf airy.tcc.

Referenced by \_\_airy().

```
8.3.2.7 template<typename _Tp > void std::__detail::__airy_bessel_i ( const std::complex< _Tp > & _z, _Tp __eps, std::complex< _Tp > & _lp1d3, std::complex< _Tp > & _lp2d3, std::complex< _Tp > & _lm2d3 )
```

Compute the modified Bessel functions of the first kind of orders +-1/3 and +-2/3 needed to compute the Airy functions and their derivatives from their representation in terms of the modified Bessel functions. This function is only used for z less than two in modulus and in the closed right half plane. This stems from the fact that the values of the modified Bessel functions occuring in the representations of the Airy functions and their derivatives are almost equal for z large in the right half plane. This means that loss of significance occurs if these representations are used for z to large in magnitude. This algorithm is also not used for z too small, since a low order rational approximation can be used instead.

This routine is an implementation of a modified version of Miller's backward recurrence algorithm for computation by from the recurrence relation

$$I_{\nu-1} = (2\nu/z)I_{\nu} + I_{\nu+1}$$

satisfied by the modified Bessel functions of the first kind. the normalization relationship used is

$$\frac{z/2)^{\nu} e^{z}}{\Gamma(\nu+1)} = I_{\nu}(z) + 2\sum_{k=1}^{\infty} \frac{(k+\nu)\Gamma(2\nu+k)}{k!\Gamma(1+2\nu)} I_{\nu+k}(z).$$

This modification of the algorithm is given in part in

Olver, F. W. J. and Sookne, D. J., Note on Backward Recurrence Algorithms, Math. of Comp., Vol. 26, no. 120, Oct. 1972.

And further elaborated for the Bessel functions in

Sookne, D. J., Bessel Functions I and J of Complex Argument and Integer Order, J. Res. NBS - Series B, Vol 77B, Nos. 3 & 4, July-December, 1973.

Insight was also gained from

Cody, W. J., Preliminary Report on Software for the Modified Bessel Functions of the First Kind, Argonne National Laboratory, Applied Mathematics Division, Tech. Memo. No. 357, August, 1980.

Cody implements the algorithm of Sookne for fractional order and nonnegative real argument. Like Cody, we do not change the convergence testing mechanism in any substantial way. However, we do trim the overhead by making the additional assumption that performing the convergence test for the functions of order 2/3 will suffice for order 1/3 as well. This assumption has not been established by rigourous analysis at this time. For speed the convergence tests are performed in the 1-norm instead of the usual Euclidean norm used in the complex plane using the inequality

$$|x| + |y| \le \sqrt{(2)}\sqrt{(x^2 + y^2)}$$

#### **Parameters**

in	Z	The argument of the modified Bessel functions.
in	eps	The maximum relative error required in the results.
out	_lp1d3	The value of $I_{(}+1/3)(z)$ .
out	_lm1d3	The value of $I_{(-1/3)(z)$ .
out	_lp2d3	The value of $I_{(}+2/3)(z)$ .
out	_Im2d3	The value of $I_{(-2/3)(z)}$ .

Definition at line 390 of file sf airy.tcc.

Referenced by \_\_airy().

8.3.2.8 template<typename \_Tp > void std::\_\_detail::\_\_airy\_bessel\_k ( const std::complex< \_Tp > & \_z, \_Tp \_\_eps, std::complex< \_Tp > & \_Kp1d3, std::complex< \_Tp > & \_Kp2d3 )

Compute approximations to the modified Bessel functions of the second kind of orders 1/3 and 2/3 for moderate arguments.

This routine computes

$$E_{\nu}(z) = \exp z \sqrt{2z/\pi} K_{\nu}(z), for \nu = 1/3 and \nu = 2/3$$

using a rational approximation given in

Luke, Y. L., Mathematical functions and their approximations, Academic Press, pp 366-367, 1975.

Though the approximation converges in  $|\arg(z)| <= pi$ , The convergence weakens as abs(arg(z)) increases. Also, in the case of real order between 0 and 1, convergence weakens as the order approaches 1. For these reasons, we only use this code for  $|\arg(z)| <= 3pi/4$  and the convergence test is performed only for order 2/3.

The coding of this function is also influenced by the fact that it will only be used for about 2 <= |z| <= 15. Hence, certain considerations of overflow, underflow, and loss of significance are unimportant for our purpose.

#### **Parameters**

in	z	The value for which the quantity E_nu is to be computed. it is recommended that
		abs(z) not be too small and that $ \arg(z)  <= 3pi/4$ .
in	eps	The maximum relative error allowable in the computed results. The relative error
		test is based on the comparison of successive iterates.
out	_Kp1d3	The value computed for $E_{+1/3}(z)$ .
out	_Kp2d3	The value computed for $E_{\pm 2/3}(z)$ .

## Note

In the worst case, say, z=2 and arg(z)=3pi/4, 20 iterations should give 7 or 8 decimals of accuracy.

Definition at line 604 of file sf airy.tcc.

Referenced by \_\_airy().

8.3.2.9 template<typename \_Tp > std::complex<\_Tp> std::\_\_detail::\_\_airy\_bi ( std::complex< \_Tp > \_\_z )

Return the complex Airy Bi function.

Definition at line 1152 of file sf airy.tcc.

References \_\_airy().

8.3.2.10 template < typename \_Tp > void std::\_\_detail::\_\_airy\_hyperg\_rational ( const std::complex < \_Tp > & \_z, std::complex < \_Tp > & \_Ai, std::complex < \_Tp > & \_Bip )

This function computes rational approximations to the hypergeometric functions related to the modified Bessel functions of orders  $\nu = +-1/3$  and  $\nu + -2/3$ . That is, A(z)/B(z), Where A(z) and B(z) are cubic polynomials with real coefficients, approximates

$$\frac{\Gamma(\nu+1)}{(z/2)^n u} I_{\nu}(z) =_0 F_1(; \nu+1; z^2/4),$$

where the function on the right is a confluent hypergeometric limit function. For |z| <= 1/4 and |arg(z)| <= pi/2, the approximations are accurate to about 16 decimals.

For further details including the four term recurrence relation satisfied by the numerator and denominator poly-nomials in the higher order approximants, see

Luke, Y.L., Mathematical Functions and their Approximations, Academic Press, pp 361-363, 1975.

An asymptotic expression for the error is given as well as other useful expressions in the event one wants to extend this function to incorporate higher order approximants.

Note also that for speed and since this function is called by another, checks that are not absolutely necessary are not made.

#### **Parameters**

in	z	The argument at which the hypergeometric given above is to be evaluated. Since
		the approximation is of fixed order, $\left z\right $ must be small to ensure sufficient accuracy
		of the computed results.

out	_Ai	The Airy function $Ai(z)$ .
out	_Aip	The Airy function derivative $Ai'(z)$ .
out	_Bi	The Airy function $Bi(z)$ .
out	_Вір	The Airy function derivative $Bi'(z)$ .

Definition at line 787 of file sf\_airy.tcc.

Referenced by airy().

8.3.2.11 template < typename \_Tp > \_Tp std::\_\_detail::\_\_assoc\_laguerre ( unsigned int \_\_n, unsigned int \_\_m, \_Tp \_\_x )

This routine returns the associated Laguerre polynomial of order n, degree m:  $L_n^m(x)$ .

The associated Laguerre polynomial is defined for integral  $\alpha=m$  by:

$$L_n^m(x) = (-1)^m \frac{d^m}{dx^m} L_{n+m}(x)$$

where the Laguerre polynomial is defined by:

$$L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n} (x^n e^{-x})$$

## **Template Parameters**

_Tp	The type of the parameter

#### **Parameters**

n	The order
m	The degree
X	The argument

# Returns

The value of the associated Laguerre polynomial of order n, degree m, and argument x.

Definition at line 301 of file sf laguerre.tcc.

Referenced by \_\_hydrogen().

8.3.2.12 template < typename \_Tp > \_Tp std::\_\_detail::\_\_assoc\_legendre\_p ( unsigned int \_\_l, unsigned int \_\_m, \_Tp \_\_x )

Return the associated Legendre function by recursion on l and downward recursion on m.

The associated Legendre function is derived from the Legendre function  $P_l(x)$  by the Rodrigues formula:

$$P_l^m(x) = (1 - x^2)^{m/2} \frac{d^m}{dx^m} P_l(x)$$

# **Parameters**

I	The order of the associated Legendre function. $l>=0$ .
m	The order of the associated Legendre function. $m <= l$ .
X	The argument of the associated Legendre function. $ x  <= 1$ .

Definition at line 175 of file sf\_legendre.tcc.

References \_\_poly\_legendre\_p().

8.3.2.13 template < typename  $_{Tp} > _{GLIBCXX14}_{CONSTEXPR}_{Tp}$  std:: \_\_detail:: \_\_bernoulli ( int  $_{n}$  )

This returns Bernoulli number  $B_n$ .

## **Parameters**

n	the order n of the Bernoulli number.
---	--------------------------------------

## Returns

The Bernoulli number of order n.

Definition at line 1673 of file sf\_gamma.tcc.

```
8.3.2.14 template < typename _Tp > _GLIBCXX14_CONSTEXPR _Tp std::__detail::__bernoulli_2n ( int __n )
```

This returns Bernoulli number  $B_n$ .

## **Parameters**

n	the order n of the Bernoulli number.
---	--------------------------------------

#### Returns

The Bernoulli number of order n.

Definition at line 1685 of file sf gamma.tcc.

```
8.3.2.15 template < typename _Tp > _GLIBCXX14_CONSTEXPR _Tp std::__detail::__bernoulli_series ( unsigned int __n )
```

This returns Bernoulli numbers from a table or by summation for larger values.

Upward recursion is unstable.

## **Parameters**

n	the order n of the Bernoulli number.

## Returns

The Bernoulli number of order n.

Definition at line 1608 of file sf gamma.tcc.

8.3.2.16 template<typename\_Tp > \_Tp std::\_\_detail::\_\_beta ( \_Tp \_\_a, \_Tp \_\_b )

Return the beta function B(a, b).

The beta function is defined by

$$B(a,b) = \int_0^1 t^{a-1} (1-t)^{b-1} dt = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

#### **Parameters**

a	The first argument of the beta function.
b	The second argument of the beta function.

## Returns

The beta function.

Definition at line 173 of file sf beta.tcc.

References \_\_beta\_lgamma().

Referenced by \_\_poly\_jacobi(), \_\_gnu\_cxx::jacobi(), \_\_gnu\_cxx::jacobif(), and \_\_gnu\_cxx::jacobil().

8.3.2.17 template < typename \_Tp > \_Tp std::\_\_detail::\_\_beta\_gamma ( \_Tp \_\_a, \_Tp \_\_b )

Return the beta function: B(a,b).

The beta function is defined by

$$B(a,b) = \int_0^1 t^{a-1} (1-t)^{b-1} dt = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

## **Parameters**

a	The first argument of the beta function.
b	The second argument of the beta function.

# Returns

The beta function.

Definition at line 75 of file sf beta.tcc.

References \_\_gamma().

8.3.2.18 template<typename \_Tp > \_Tp std::\_\_detail::\_\_beta\_inc ( \_Tp \_\_a, \_Tp \_\_b, \_Tp \_\_x )

Return the regularized incomplete beta function,  $I_x(a,b)$ , of arguments a, b, and x.

The regularized incomplete beta function is defined by:

$$I_x(a,b) = \frac{B_x(a,b)}{B(a,b)}$$

where

$$B_x(a,b) = \int_0^x t^{a-1} (1-t)^{b-1} dt$$

is the non-regularized beta function and B(a,b) is the usual beta function.

#### **Parameters**

a	The first parameter
b	The second parameter
X	The argument

Definition at line 262 of file sf\_beta.tcc.

References \_\_beta\_inc\_cont\_frac().

Referenced by  $\_$ binomial\_cdf(),  $\_$ binomial\_cdfc(),  $\_$ fisher\_f\_cdf(),  $\_$ fisher\_f\_cdfc(),  $\_$ student\_t\_cdf(), and  $\_$   $\leftarrow$  student\_t\_cdfc().

 $8.3.2.19 \quad template < typename \_Tp > \_Tp \ std::\_detail::\_beta\_inc\_cont\_frac \ ( \ \_Tp \_\_a, \ \_Tp \_\_b, \ \_Tp \_\_x \ )$ 

Return the regularized incomplete beta function,  $I_x(a,b)$ , of arguments a, b, and x.

#### **Parameters**

a	The first parameter
b	The second parameter
X	The argument

Definition at line 193 of file sf beta.tcc.

Referenced by \_\_beta\_inc().

8.3.2.20 template < typename \_Tp > \_Tp std::\_\_detail::\_\_beta\_lgamma ( \_Tp \_\_a, \_Tp \_\_b )

Return the beta function B(a,b) using the log gamma functions.

The beta function is defined by

$$B(a,b) = \int_0^1 t^{a-1} (1-t)^{b-1} dt = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

# **Parameters**

a	The first argument of the beta function.
b	The second argument of the beta function.

# Returns

The beta function.

Definition at line 109 of file sf beta.tcc.

References log gamma().

Referenced by \_\_beta().

8.3.2.21 template < typename \_Tp > \_Tp std::\_\_detail::\_\_beta\_product ( \_Tp \_\_a, \_Tp \_\_b )

Return the beta function B(x, y) using the product form.

The beta function is defined by

$$B(a,b) = \int_0^1 t^{a-1} (1-t)^{b-1} dt = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

Here, we employ the product form:

$$B(a,b) = \frac{a+b}{ab} \prod_{k=1}^{\infty} \frac{1 + (a+b)/k}{(1+a/k)(1+b/k)}$$

#### **Parameters**

a	The first argument of the beta function.
b	The second argument of the beta function.

## Returns

The beta function.

Definition at line 140 of file sf\_beta.tcc.

8.3.2.22 template < typename  $_{\rm Tp} > _{\rm Tp}$  std::\_\_detail::\_\_bincoef ( unsigned int  $_{\rm n}$ , unsigned int  $_{\rm k}$  )

Return the binomial coefficient. The binomial coefficient is given by:

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

## **Parameters**

n	The first argument of the binomial coefficient.
k	The second argument of the binomial coefficient.

## Returns

The binomial coefficient.

Definition at line 1888 of file sf\_gamma.tcc.

8.3.2.23 template < typename \_Tp > \_GLIBCXX14\_CONSTEXPR \_Tp std::\_\_detail::\_\_binomial\_cdf ( \_Tp \_\_p, unsigned int \_\_n, unsigned int \_\_k )

Return the binomial cumulative distribution function.

The binomial cumulative distribution function is related to the incomplete beta function:

$$P(p|n,k) = I_n(k, n - k + 1)$$

#### **Parameters**

p	
n	
k	

Definition at line 405 of file sf\_beta.tcc.

References \_\_beta\_inc().

8.3.2.24 template < typename \_Tp > \_GLIBCXX14\_CONSTEXPR \_Tp std::\_\_detail::\_\_binomial\_cdfc ( \_Tp \_\_p, unsigned int \_\_n, unsigned int \_\_n,

Return the complementary binomial cumulative distribution function.

The binomial cumulative distribution function is related to the incomplete beta function:

$$Q(p|n,k) = I_{1-p}(n-k+1,k)$$

## **Parameters**

p	
n	
k	

Definition at line 435 of file sf\_beta.tcc.

References \_\_beta\_inc().

Return the Bose-Einstein integral of real order s and real argument x.

# See also

```
https://en.wikipedia.org/wiki/Clausen_function
http://dlmf.nist.gov/25.12#iii
```

## **Parameters**

s	The order $s \ge 0$ .
X	The real argument.

# Returns

The real Fermi-Dirac cosine sum  $G_s(x)$ ,

Definition at line 1401 of file sf\_polylog.tcc.

References polylog exp().

$$8.3.2.26 \quad template < typename \_Tp > \_Tp \ std::\_\_chebyshev\_recur \ ( \ unsigned \ int \_\_n, \ \_Tp \_\_x, \ \_Tp \_\_C0, \ \_Tp \_\_C1 \ )$$

Return a Chebyshev polynomial of non-negative order n and real argument x by the recursion

$$C_n(x) = 2xC_{n-1} - C_{n-2}$$

# **Template Parameters**

_ <i>Tp</i>	The real type of the argument

## **Parameters**

n	The non-negative integral order
X	The real argument $-1 \le x \le +1$
_C0	The value of the zeroth-order Chebyshev polynomial at $\boldsymbol{x}$
_C1	The value of the first-order Chebyshev polynomial at $\boldsymbol{x}$

Definition at line 57 of file sf\_chebyshev.tcc.

Referenced by \_\_chebyshev\_t(), \_\_chebyshev\_u(), \_\_chebyshev\_v(), and \_\_chebyshev\_w().

8.3.2.27 template < typename  $_{\rm Tp}$  >  $_{\rm Tp}$  std::\_\_detail::\_\_chebyshev\_t ( unsigned int  $_{\rm m}$ ,  $_{\rm Tp}$  \_\_x )

Return the Chebyshev polynomial of the first kind  $T_n(x)$  of non-negative order n and real argument x.

The Chebyshev polynomial of the first kind is defined by:

$$T_n(x) = \cos(n\theta)$$

where  $\theta = \arccos(x)$ ,  $-1 \le x \le +1$ .

# **Template Parameters**

_ <i>Tp</i>	The real type of the argument

# **Parameters**

n	The non-negative integral order
X	The real argument $-1 \le x \le +1$

Definition at line 85 of file sf chebyshev.tcc.

References \_\_chebyshev\_recur().

 $8.3.2.28 \quad template < typename \_Tp > \_Tp \ std:: \_detail:: \_chebyshev\_u \ ( \ unsigned \ int \_\_n, \ \_Tp \_\_x \ )$ 

Return the Chebyshev polynomial of the second kind  $U_n(x)$  of non-negative order n and real argument x.

The Chebyshev polynomial of the second kind is defined by:

$$U_n(x) = \frac{\sin[(n+1)\theta]}{\sin(\theta)}$$

where  $\theta = \arccos(x)$ ,  $-1 \le x \le +1$ .

# **Template Parameters**

_ <i>Tp</i>	The real type of the argument

# **Parameters**

n	The non-negative integral order
X	The real argument $-1 \le x \le +1$

Definition at line 114 of file sf\_chebyshev.tcc.

References \_\_chebyshev\_recur().

8.3.2.29 template<typename \_Tp > \_Tp std::\_\_chebyshev\_v ( unsigned int \_\_n, \_Tp \_\_x )

Return the Chebyshev polynomial of the third kind  $V_n(x)$  of non-negative order n and real argument x.

The Chebyshev polynomial of the third kind is defined by:

$$V_n(x) = \frac{\cos\left[\left(n + \frac{1}{2}\right)\theta\right]}{\cos\left(\frac{\theta}{2}\right)}$$

where  $\theta = \arccos(x)$ ,  $-1 \le x \le +1$ .

# **Template Parameters**

_ <i>Tp</i>	The real type of the argument

#### **Parameters**

n	The non-negative integral order
X	The real argument $-1 \le x \le +1$

Definition at line 144 of file sf chebyshev.tcc.

References \_\_chebyshev\_recur().

8.3.2.30 template<typename \_Tp > \_Tp std::\_\_detail::\_\_chebyshev\_w ( unsigned int \_\_n, \_Tp \_\_x )

Return the Chebyshev polynomial of the fourth kind  $W_n(x)$  of non-negative order n and real argument x.

The Chebyshev polynomial of the fourth kind is defined by:

$$W_n(x) = \frac{\sin\left[\left(n + \frac{1}{2}\right)\theta\right]}{\sin\left(\frac{\theta}{2}\right)}$$

where  $\theta = \arccos(x)$ ,  $-1 \le x \le +1$ .

# **Template Parameters**

_Tp   The real type of the argument
-------------------------------------

## **Parameters**

n	The non-negative integral order
X	The real argument $-1 \le x \le +1$

Definition at line 174 of file sf chebyshev.tcc.

References \_\_chebyshev\_recur().

8.3.2.31 template < typename \_Tp > \_GLIBCXX14\_CONSTEXPR \_Tp std::\_\_detail::\_\_chi\_squared\_pdf ( \_Tp \_\_chi2, unsigned int \_\_nu )

Return the chi-squared propability function. This returns the probability that the observed chi-squared for a correct model is less than the value  $\chi^2$ .

The chi-squared propability function is related to the normalized lower incomplete gamma function:

$$P(\chi^2|\nu) = \Gamma_P(\frac{\nu}{2}, \frac{\chi^2}{2})$$

Definition at line 2544 of file sf gamma.tcc.

References \_\_gamma\_p().

8.3.2.32 template < typename \_Tp > \_GLIBCXX14\_CONSTEXPR \_Tp std::\_\_detail::\_\_chi\_squared\_pdfc ( \_Tp \_\_chi2, unsigned int \_\_nu )

Return the complementary chi-squared propability function. This returns the probability that the observed chi-squared for a correct model is greater than the value  $\chi^2$ .

The complementary chi-squared propability function is related to the normalized upper incomplete gamma function:

$$Q(\chi^2|\nu) = \Gamma_Q(\frac{\nu}{2}, \frac{\chi^2}{2})$$

Definition at line 2568 of file sf\_gamma.tcc.

References \_\_gamma\_q().

This function returns the hyperbolic cosine Ci(x) and hyperbolic sine Si(x) integrals as a pair.

The hyperbolic cosine integral is defined by:

$$Chi(x) = \gamma_E + \log(x) + \int_0^x dt \frac{\cosh(t) - 1}{t}$$

The hyperbolic sine integral is defined by:

$$Shi(x) = \int_0^x dt \frac{\sinh(t)}{t}$$

Definition at line 162 of file sf\_hypint.tcc.

References chshint cont frac(), and chshint series().

This function computes the hyperbolic cosine Chi(x) and hyperbolic sine Shi(x) integrals by continued fraction for positive argument.

Definition at line 50 of file sf hypint.tcc.

Referenced by chshint().

This function computes the hyperbolic cosine Chi(x) and hyperbolic sine Shi(x) integrals by series summation for positive argument.

Definition at line 93 of file sf hypint.tcc.

Referenced by \_\_chshint().

Definition at line 136 of file sf\_polylog.tcc.

Referenced by \_\_polylog\_exp\_int\_neg(), \_\_polylog\_exp\_int\_pos(), \_\_polylog\_exp\_real\_neg(), and \_\_polylog\_exp\_\times real\_pos().

8.3.2.37 template<typename  $_{Tp}$  > std::complex< $_{Tp}$ > std::\_\_detail::\_\_clamp\_pi ( std::complex< $_{Tp}$ >  $_{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{I}}}}}}}}$  w )

Definition at line 123 of file sf polylog.tcc.

Referenced by \_\_polylog\_exp\_int\_neg(), \_\_polylog\_exp\_int\_pos(), \_\_polylog\_exp\_real\_neg(), and \_\_polylog\_exp\_\times real\_pos().

 $8.3.2.38 \quad template < typename \_Tp > std::complex < \_Tp > std::\_detail::\_clausen ( \ unsigned \ int \ \_m, \ std::complex < \_Tp > \_w )$ 

Return Clausen's function of integer order m and complex argument w. The notation and connection to polylog is from Wikipedia

# **Parameters**

m	The non-negative integral order.
W	The complex argument.

## Returns

The complex Clausen function.

Definition at line 1230 of file sf polylog.tcc.

References \_\_polylog\_exp().

8.3.2.39 template < typename \_Tp > \_Tp std::\_\_detail::\_\_clausen ( unsigned int \_\_m, \_Tp \_\_w )

Return Clausen's function of integer order m and real argument w. The notation and connection to polylog is from Wikipedia

## **Parameters**

m	The integer order $m \ge 1$ .
w	The real argument.

#### Returns

The Clausen function.

Definition at line 1254 of file sf\_polylog.tcc.

References \_\_polylog\_exp().

8.3.2.40 template < typename  $_{\rm Tp} > _{\rm Tp}$  std::\_\_detail::\_\_clausen\_c ( unsigned int  $_{\rm m}$ , std::complex <  $_{\rm Tp} > _{\rm w}$  )

Return Clausen's cosine sum Cl\_m for positive integer order m and complex argument w.

# See also

https://en.wikipedia.org/wiki/Clausen\_function

# **Parameters**

m	The integer order $m \ge 1$ .
w	The real argument.

#### Returns

The Clausen cosine sum Cl\_m(w),

Definition at line 1329 of file sf\_polylog.tcc.

References \_\_polylog\_exp().

Return Clausen's cosine sum Cl\_m for positive integer order m and real argument w.

## See also

```
https://en.wikipedia.org/wiki/Clausen_function
```

#### **Parameters**

m	The integer order $m \ge 1$ .
w	The real argument.

## Returns

The real Clausen cosine sum Cl\_m(w),

Definition at line 1354 of file sf\_polylog.tcc.

References \_\_polylog\_exp().

8.3.2.42 template < typename 
$$_{\rm Tp}$$
 >  $_{\rm Tp}$  std::\_\_detail::\_\_clausen\_s ( unsigned int  $_{\rm m}$ , std::complex <  $_{\rm Tp}$  >  $_{\rm w}$  )

Return Clausen's sine sum SI\_m for positive integer order m and complex argument w.

# See also

```
https://en.wikipedia.org/wiki/Clausen_function
```

# **Parameters**

m	The integer order $m \ge 1$ .
w	The complex argument.

# Returns

The Clausen sine sum SI m(w),

Definition at line 1279 of file sf\_polylog.tcc.

References \_\_polylog\_exp().

8.3.2.43 template < typename \_Tp > \_Tp std::\_\_detail::\_\_clausen\_s ( unsigned int \_\_m, \_Tp \_\_w )

Return Clausen's sine sum SI m for positive integer order m and real argument w.

#### See also

https://en.wikipedia.org/wiki/Clausen\_function

## **Parameters**

m	The integer order $m \ge 1$ .
W	The complex argument.

#### Returns

The Clausen sine sum SI m(w),

Definition at line 1304 of file sf\_polylog.tcc.

References \_\_polylog\_exp().

8.3.2.44 template<typename\_Tp > \_Tp std::\_\_detail::\_\_comp\_ellint\_1 ( \_Tp \_\_k )

Return the complete elliptic integral of the first kind K(k) using the Carlson formulation.

The complete elliptic integral of the first kind is defined as

$$K(k) = F(k, \pi/2) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - k^2 sin^2 \theta}}$$

where  $F(k,\phi)$  is the incomplete elliptic integral of the first kind.

## **Parameters**

k	The modulus of the complete elliptic function.

#### Returns

The complete elliptic function of the first kind.

Definition at line 565 of file sf\_ellint.tcc.

References \_\_comp\_ellint\_rf().

Referenced by \_\_ellint\_1(), \_\_ellnome\_k(), \_\_jacobi\_zeta(), \_\_theta\_c(), \_\_theta\_d(), \_\_theta\_n(), and \_\_theta\_s().

8.3.2.45 template < typename  $_{\text{Tp}} > _{\text{Tp}}$  std::\_\_detail::\_\_comp\_ellint\_2 (  $_{\text{Tp}}$  \_\_k )

Return the complete elliptic integral of the second kind E(k) using the Carlson formulation.

The complete elliptic integral of the second kind is defined as

$$E(k, \pi/2) = \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 \theta}$$

k	The modulus of the complete elliptic function.

#### Returns

The complete elliptic function of the second kind.

Definition at line 638 of file sf ellint.tcc.

References \_\_ellint\_rd(), and \_\_ellint\_rf().

Referenced by \_\_ellint\_2().

8.3.2.46 template> \_Tp std::\_\_detail::\_\_comp\_ellint\_3 ( \_Tp 
$$\_k$$
, \_Tp  $\_nu$  )

Return the complete elliptic integral of the third kind  $\Pi(k,\nu)=\Pi(k,\nu,\pi/2)$  using the Carlson formulation.

The complete elliptic integral of the third kind is defined as

$$\Pi(k,\nu) = \int_0^{\pi/2} \frac{d\theta}{(1-\nu\sin^2\theta)\sqrt{1-k^2\sin^2\theta}}$$

#### **Parameters**

k	The argument of the elliptic function.
nu	The second argument of the elliptic function.

## Returns

The complete elliptic function of the third kind.

Definition at line 727 of file sf\_ellint.tcc.

References \_\_ellint\_rf(), and \_\_ellint\_rj().

Referenced by \_\_ellint\_3().

Return the complete Legendre elliptic integral D.

Definition at line 832 of file sf ellint.tcc.

References \_\_ellint\_rd().

Definition at line 235 of file sf ellint.tcc.

Referenced by \_\_comp\_ellint\_1(), and \_\_ellint\_rf().

Definition at line 346 of file sf ellint.tcc.

Referenced by \_\_ellint\_rg().

Return the confluent hypergeometric function  ${}_{1}F_{1}(a;c;x)$ .

#### **Parameters**

a	The numerator parameter.
c	The <i>denominator</i> parameter.
X	The argument of the confluent hypergeometric function.

### Returns

The confluent hypergeometric function.

Definition at line 281 of file sf hyperg.tcc.

References conf hyperg luke(), and conf hyperg series().

Return the confluent hypergeometric limit function  ${}_0F_1(-;c;x)$ .

#### **Parameters**

c	The denominator parameter.
X	The argument of the confluent hypergeometric limit function.

## Returns

The confluent limit hypergeometric function.

Definition at line 109 of file sf hyperg.tcc.

References \_\_conf\_hyperg\_lim\_series().

8.3.2.52 template < typename 
$$_{\rm Tp} > _{\rm Tp}$$
 std::\_\_detail::\_\_conf\_hyperg\_lim\_series (  $_{\rm Tp}$  \_\_c,  $_{\rm Tp}$  \_\_x )

This routine returns the confluent hypergeometric limit function by series expansion.

$$_{0}F_{1}(-;c;x) = \Gamma(c) \sum_{n=0}^{\infty} \frac{1}{\Gamma(c+n)} \frac{x^{n}}{n!}$$

If a and b are integers and a < 0 and either b > 0 or b < a then the series is a polynomial with a finite number of terms.

### **Parameters**

c	The "denominator" parameter.
X	The argument of the confluent hypergeometric limit function.

### Returns

The confluent hypergeometric limit function.

Definition at line 76 of file sf hyperg.tcc.

Referenced by \_\_conf\_hyperg\_lim().

Return the hypergeometric function  ${}_1F_1(a;c;x)$  by an iterative procedure described in Luke, Algorithms for the Computation of Mathematical Functions.

Like the case of the 2F1 rational approximations, these are probably guaranteed to converge for x < 0, barring gross numerical instability in the pre-asymptotic regime.

Definition at line 176 of file sf hyperg.tcc.

Referenced by \_\_conf\_hyperg().

This routine returns the confluent hypergeometric function by series expansion.

$$_1F_1(a;c;x) = \frac{\Gamma(c)}{\Gamma(a)} \sum_{n=0}^{\infty} \frac{\Gamma(a+n)}{\Gamma(c+n)} \frac{x^n}{n!}$$

#### **Parameters**

a	The "numerator" parameter.
c	The "denominator" parameter.
x	The argument of the confluent hypergeometric function.

### Returns

The confluent hypergeometric function.

Definition at line 141 of file sf\_hyperg.tcc.

Referenced by conf hyperg().

8.3.2.55 template < typename \_Tp > \_Tp std::\_\_detail::\_\_coshint ( const \_Tp 
$$\underline{\hspace{0.1cm}} x$$
 )

Return the hyperbolic cosine integral li(x).

The hyperbolic cosine integral is given by

$$Chi(x) = (Ei(x) - E_1(x))/2$$

# **Parameters**

X	The argument of the hyperbolic cosine integral function.

### Returns

The hyperbolic cosine integral.

Definition at line 558 of file sf\_expint.tcc.

References \_\_expint\_E1(), and \_\_expint\_Ei().

Return the complex cylindrical Bessel function.

in	nu	The order for which the cylindrical Bessel function is evaluated.
in	z	The argument at which the cylindrical Bessel function is evaluated.

#### Returns

The complex cylindrical Bessel function.

Definition at line 1222 of file sf hankel.tcc.

References \_\_hankel().

Return the regular modified Bessel function of order  $\nu$ :  $I_{\nu}(x)$ .

The regular modified cylindrical Bessel function is:

$$I_{\nu}(x) = \sum_{k=0}^{\infty} \frac{(x/2)^{\nu+2k}}{k!\Gamma(\nu+k+1)}$$

#### **Parameters**

nu	The order of the regular modified Bessel function.
x	The argument of the regular modified Bessel function.

# Returns

The output regular modified Bessel function.

Definition at line 386 of file sf mod bessel.tcc.

References \_\_cyl\_bessel\_ij\_series(), and \_\_cyl\_bessel\_ik().

This routine returns the cylindrical Bessel functions of order  $\nu$ :  $J_{\nu}$  or  $I_{\nu}$  by series expansion.

The modified cylindrical Bessel function is:

$$Z_{\nu}(x) = \sum_{k=0}^{\infty} \frac{\sigma^{k} (x/2)^{\nu+2k}}{k! \Gamma(\nu+k+1)}$$

where  $\sigma = +1$  or -1 for Z = I or J respectively.

See Abramowitz & Stegun, 9.1.10 Abramowitz & Stegun, 9.6.7 (1) Handbook of Mathematical Functions, ed. Milton Abramowitz and Irene A. Stegun, Dover Publications, Equation 9.1.10 p. 360 and Equation 9.6.10 p. 375

# **Parameters**

nu	The order of the Bessel function.
X	The argument of the Bessel function.
sgn	The sign of the alternate terms -1 for the Bessel function of the first kind. +1 for the modified
	Bessel function of the first kind.
max_iter	The maximum number of iterations for sum.

### Returns

The output Bessel function.

Definition at line 413 of file sf bessel.tcc.

References \_\_log\_gamma().

Referenced by \_\_cyl\_bessel\_i(), and \_\_cyl\_bessel\_j().

8.3.2.59 template<typename\_Tp > void std::\_\_detail::\_\_cyl\_bessel\_ik ( \_Tp \_\_nu, \_Tp \_\_x, \_Tp & \_Inu, \_Tp & \_Knu, \_Tp & \_Ipnu, \_Tp & \_Kpnu )

Return the modified cylindrical Bessel functions and their derivatives of order  $\nu$  by various means.

## **Parameters**

nu	The order of the Bessel functions.
X	The argument of the Bessel functions.
_Inu	The output regular modified Bessel function.
_Knu	The output irregular modified Bessel function.
_lpnu	The output derivative of the regular modified Bessel function.
_Kpnu	The output derivative of the irregular modified Bessel function.

Definition at line 316 of file sf\_mod\_bessel.tcc.

References \_\_cyl\_bessel\_ik\_asymp(), and \_\_cyl\_bessel\_ik\_steed().

Referenced by \_\_airy(), \_\_cyl\_bessel\_i(), \_\_cyl\_bessel\_k(), and \_\_sph\_bessel\_ik().

8.3.2.60 template < typename \_Tp > void std::\_\_cyl\_bessel\_ik\_asymp ( \_Tp \_\_nu, \_Tp \_\_x, \_Tp & \_Inu, \_Tp & \_Knu, \_Tp & \_Inu, \_Tp & \_Kpnu )

This routine computes the asymptotic modified cylindrical Bessel and functions of order nu:  $I_{\nu}(x)$ ,  $N_{\nu}(x)$ . Use this for  $x >> nu^2 + 1$ .

References: (1) Handbook of Mathematical Functions, ed. Milton Abramowitz and Irene A. Stegun, Dover Publications, Section 9 p. 364, Equations 9.2.5-9.2.10

### **Parameters**

nu	The order of the Bessel functions.
X	The argument of the Bessel functions.
_Inu	The output regular modified Bessel function.
_Knu	The output irregular modified Bessel function.
_lpnu	The output derivative of the regular modified Bessel function.

.,	
Knnu	The output derivative of the irregular modified Bessel function.
	The datpat derivative of the irregular medined Beccor fanction.

Definition at line 81 of file sf mod bessel.tcc.

Referenced by \_\_cyl\_bessel\_ik(), and \_\_cyl\_bessel\_ik\_steed().

Compute the modified Bessel functions  $I_{\nu}(x)$  and  $K_{\nu}(x)$  and their first derivatives  $I'_{\nu}(x)$  and  $K'_{\nu}(x)$  respectively. These four functions are computed together for numerical stability.

### **Parameters**

nu	The order of the Bessel functions.
X	The argument of the Bessel functions.
_Inu	The output regular modified Bessel function.
_Knu	The output irregular modified Bessel function.
_lpnu	The output derivative of the regular modified Bessel function.
_Kpnu	The output derivative of the irregular modified Bessel function.

Definition at line 152 of file sf mod bessel.tcc.

References \_\_cyl\_bessel\_ik\_asymp(), and \_\_gamma\_temme().

Referenced by cyl bessel ik().

8.3.2.62 template < typename 
$$_{Tp} > _{Tp}$$
 std::\_\_detail::\_\_cyl\_bessel\_j (  $_{Tp}$  \_\_nu,  $_{Tp}$  \_\_x )

Return the Bessel function of order  $\nu$ :  $J_{\nu}(x)$ .

The cylindrical Bessel function is:

$$J_{\nu}(x) = \sum_{k=0}^{\infty} \frac{(-1)^k (x/2)^{\nu+2k}}{k! \Gamma(\nu+k+1)}$$

### **Parameters**

nu	The order of the Bessel function.
X	The argument of the Bessel function.

## Returns

The output Bessel function.

Definition at line 523 of file sf bessel.tcc.

References \_\_cyl\_bessel\_ij\_series(), and \_\_cyl\_bessel\_jn().

Return the cylindrical Bessel functions and their derivatives of order  $\nu$  by various means.

Definition at line 442 of file sf bessel.tcc.

References \_\_cyl\_bessel\_jn\_asymp(), and \_\_cyl\_bessel\_jn\_steed().

Referenced by  $\_airy()$ ,  $\_cyl\_bessel\_j()$ ,  $\_cyl\_hankel\_1()$ ,  $\_cyl\_hankel\_2()$ ,  $\_cyl\_neumann\_n()$ , and  $\_sph\_\hookleftarrow bessel\_jn()$ .

8.3.2.64 template < typename \_Tp > void std::\_\_detail::\_\_cyl\_bessel\_jn\_asymp ( \_Tp \_\_nu, \_Tp \_\_x, \_Tp & \_Jnu, \_Tp & \_Nnu, \_Tp & \_Jpnu, \_Tp & \_Npnu )

This routine computes the asymptotic cylindrical Bessel and Neumann functions of order nu:  $J_{\nu}(x)$ ,  $N_{\nu}(x)$ . Use this for  $x >> nu^2 + 1$ .

References: (1) Handbook of Mathematical Functions, ed. Milton Abramowitz and Irene A. Stegun, Dover Publications, Section 9 p. 364, Equations 9.2.5-9.2.10

#### **Parameters**

	nu	The order of the Bessel functions.
	x	The argument of the Bessel functions.
out	_Jnu	The Bessel function of the first kind.
out	_Nnu	The Neumann function (Bessel function of the second kind).
out	_Jpnu	The Bessel function of the first kind.
out	_Npnu	The Neumann function (Bessel function of the second kind).

Definition at line 79 of file sf bessel.tcc.

Referenced by cyl bessel in(), and cyl bessel in steed().

8.3.2.65 template < typename \_Tp > void std::\_\_detail::\_\_cyl\_bessel\_jn\_steed ( \_Tp \_\_nu, \_Tp \_\_x, \_Tp & \_Jnu, \_Tp & \_Nnu, \_Tp & \_Jpnu, \_Tp & \_Npnu )

Compute the Bessel  $J_{\nu}(x)$  and Neumann  $N_{\nu}(x)$  functions and their first derivatives  $J'_{\nu}(x)$  and  $N'_{\nu}(x)$  respectively. These four functions are computed together for numerical stability.

# **Parameters**

	nu	The order of the Bessel functions.
	X	The argument of the Bessel functions.
out	_Jnu	The output Bessel function of the first kind.
out	_Nnu	The output Neumann function (Bessel function of the second kind).
out	_Jpnu	The output derivative of the Bessel function of the first kind.
out	_Npnu	The output derivative of the Neumann function.

Definition at line 197 of file sf bessel.tcc.

References \_\_cyl\_bessel\_jn\_asymp(), and \_\_gamma\_temme().

Referenced by \_\_cyl\_bessel\_jn().

8.3.2.66 template<typename \_Tp > \_Tp std::\_\_detail::\_\_cyl\_bessel\_k ( \_Tp \_\_nu, \_Tp \_\_x )

Return the irregular modified Bessel function  $K_{\nu}(x)$  of order  $\nu$ .

The irregular modified Bessel function is defined by:

$$K_{\nu}(x) = \frac{\pi}{2} \frac{I_{-\nu}(x) - I_{\nu}(x)}{\sin \nu \pi}$$

where for integral  $\nu = n$  a limit is taken:  $lim_{\nu \to n}$ . For negative argument we have simply:

$$K_{-\nu}(x) = K_{\nu}(x)$$

nu	The order of the irregular modified Bessel function.
X	The argument of the irregular modified Bessel function.

## Returns

The output irregular modified Bessel function.

Definition at line 424 of file sf mod bessel.tcc.

References \_\_cyl\_bessel\_ik().

Return the cylindrical Hankel function of the first kind  $H_{\nu}^{(1)}(x).$ 

The cylindrical Hankel function of the first kind is defined by:

$$H_{\nu}^{(1)}(x) = J_{\nu}(x) + iN_{\nu}(x)$$

### **Parameters**

nu	The order of the spherical Neumann function.
X	The argument of the spherical Neumann function.

### Returns

The output spherical Neumann function.

Definition at line 588 of file sf bessel.tcc.

References \_\_cyl\_bessel\_jn().

Return the complex cylindrical Hankel function of the first kind.

# Parameters

in	nu	The order for which the cylindrical Hankel function of the first kind is evaluated.
in	z	The argument at which the cylindrical Hankel function of the first kind is evaluated.

## Returns

The complex cylindrical Hankel function of the first kind.

Definition at line 1190 of file sf\_hankel.tcc.

References hankel().

$$8.3.2.69 \quad template < typename \_Tp > std::complex < \_Tp > std::\_cyl\_hankel\_2 \left( \_Tp \_\_nu, \_Tp \_\_x \right)$$

Return the cylindrical Hankel function of the second kind  $H_n^{(2)} u(x)$ .

The cylindrical Hankel function of the second kind is defined by:

$$H_{\nu}^{(2)}(x) = J_{\nu}(x) - iN_{\nu}(x)$$

nu	The order of the spherical Neumann function.
X	The argument of the spherical Neumann function.

## Returns

The output spherical Neumann function.

Definition at line 623 of file sf bessel.tcc.

References \_\_cyl\_bessel\_jn().

Return the complex cylindrical Hankel function of the second kind.

#### **Parameters**

in	nu	The order for which the cylindrical Hankel function of the second kind is evaluated.
in	z	The argument at which the cylindrical Hankel function of the second kind is eval-
		uated.

## Returns

The complex cylindrical Hankel function of the second kind.

Definition at line 1206 of file sf hankel.tcc.

References \_\_hankel().

Return the complex cylindrical Neumann function.

#### **Parameters**

in	nu	The order for which the cylindrical Neumann function is evaluated.
in	z	The argument at which the cylindrical Neumann function is evaluated.

## Returns

The complex cylindrical Neumann function.

Definition at line 1238 of file sf hankel.tcc.

References \_\_hankel().

Return the Neumann function of order  $\nu$ :  $N_{\nu}(x)$ .

The Neumann function is defined by:

$$N_{\nu}(x) = \frac{J_{\nu}(x)\cos\nu\pi - J_{-\nu}(x)}{\sin\nu\pi}$$

where for integral  $\nu=n$  a limit is taken:  $lim_{\nu\to n}$ .

nu	The order of the Neumann function.	
X	The argument of the Neumann function.	

#### Returns

The output Neumann function.

Definition at line 558 of file sf bessel.tcc.

References \_\_cyl\_bessel\_jn().

Return the Dawson integral, F(x), for real argument x.

The Dawson integral is defined by:

$$F(x) = e^{-x^2} \int_0^x e^{y^2} dy$$

and it's derivative is:

$$F'(x) = 1 - 2xF(x)$$

### **Parameters**

X	The argument $-inf < x < inf$ .

Definition at line 233 of file sf dawson.tcc.

References \_\_dawson\_cont\_frac(), and \_\_dawson\_series().

$$8.3.2.74 \quad template < typename \_Tp > \_Tp \ std::\__detail::\__dawson\_cont\_frac \ ( \ \_Tp \ \_x \ )$$

Compute the Dawson integral using a sampling theorem representation.

Todo this needs some compile-time construction!

Definition at line 71 of file sf dawson.tcc.

Referenced by \_\_dawson().

8.3.2.75 template < typename \_Tp > \_Tp std::\_\_detail::\_\_dawson\_series ( \_Tp 
$$\_x$$
 )

Compute the Dawson integral using the series expansion.

Definition at line 47 of file sf\_dawson.tcc.

Referenced by \_\_dawson().

Compute the Debye region in te complex plane.

Definition at line 54 of file sf hankel.tcc.

Referenced by \_\_hankel().

8.3.2.77 template<typename \_Tp > \_Tp std::\_\_detail::\_\_dilog ( \_Tp \_\_x )

Compute the dilogarithm function  $Li_2(x)$  by summation for x <= 1.

The Riemann zeta function is defined by:

$$Li_2(x) = \sum_{k=1}^{\infty} \frac{1}{k^s} fors > 1$$

For |x| near 1 use the reflection formulae:

$$Li_2(-x) + Li_2(1-x) = \frac{\pi^2}{6} - \ln(x)\ln(1-x)$$
$$Li_2(-x) - Li_2(1-x) - \frac{1}{2}Li_2(1-x^2) = -\frac{\pi^2}{12} - \ln(x)\ln(1-x)$$

For x < 1 use the reflection formula:

$$Li_2(1-x) - Li_2(1-\frac{1}{1-x}) - \frac{1}{2}(\ln(x))^2$$

Definition at line 194 of file sf\_zeta.tcc.

 $8.3.2.78 \quad template < typename \_Tp > \_Tp \ std::\_detail::\_dirichlet\_beta \ ( \ std::complex < \_Tp > \_w \ )$ 

Return the Dirichlet beta function. Currently, w must be real (complex type but negligible imaginary part.) Otherwise std::domain error is thrown.

#### **Parameters**

\_\_w The complex (but on-real-axis) argument.

#### Returns

The Dirichlet Beta function of real argument.

## **Exceptions**

std::domain\_error | if the argument has a significant imaginary part.

Definition at line 1192 of file sf polylog.tcc.

References fpequal(), and polylog().

8.3.2.79 template < typename \_Tp > \_Tp std::\_\_detail::\_\_dirichlet\_beta ( \_Tp  $\_w$  )

Return the Dirichlet beta function for real argument.

#### **Parameters**

\_\_w The real argument.

### Returns

The Dirichlet Beta function of real argument.

Definition at line 1211 of file sf polylog.tcc.

References polylog().

 $8.3.2.80 \quad template < typename \_Tp > std::complex < \_Tp > std::\_detail::\_dirichlet\_eta \ ( \ std::complex < \_Tp > \_w \ )$ 

Return the Dirichlet eta function. Currently, w must be real (complex type but negligible imaginary part.) Otherwise std::domain\_error is thrown.

<i>W</i>	The complex (but on-real-axis) argument.

#### Returns

The complex Dirichlet eta function.

## **Exceptions**

std::domain_error	if the argument has a significant imaginary part.

Definition at line 1155 of file sf polylog.tcc.

References \_\_fpequal(), and \_\_polylog().

Return the Dirichlet eta function for real argument.

## **Parameters**

W	The real argument.

### Returns

The Dirichlet eta function.

Definition at line 1173 of file sf polylog.tcc.

References \_\_polylog().

Return the double factorial of the integer n.

The double factorial is defined for integral n by:

$$n!! = 135...(n-2)n, noddn!! = 246...(n-2)n, neven - 1!! = 10!! = 1$$

The double factorial is defined for odd negative integers in the obvious way:

$$(-2m-1)!! = 1/(1(-1)(-3)...(-2m+1)(-2m-1)) = \frac{(-1)^m}{(2m-1)!!}$$

for f[ n = -2m - 1 f].

Definition at line 2480 of file sf gamma.tcc.

References factorial(), log double factorial(), S double factorial table, and S neg double factorial table.

Return the incomplete elliptic integral of the first kind  $F(k,\phi)$  using the Carlson formulation.

The incomplete elliptic integral of the first kind is defined as

$$F(k,\phi) = \int_0^{\phi} \frac{d\theta}{\sqrt{1 - k^2 sin^2 \theta}}$$

k	The argument of the elliptic function.
phi	The integral limit argument of the elliptic function.

#### Returns

The elliptic function of the first kind.

Definition at line 594 of file sf ellint.tcc.

References \_\_comp\_ellint\_1(), and \_\_ellint\_rf().

8.3.2.84 template> \_Tp std::\_\_detail::\_\_ellint\_2 ( \_Tp 
$$\_k$$
, \_Tp  $\_phi$  )

Return the incomplete elliptic integral of the second kind  $E(k,\phi)$  using the Carlson formulation.

The incomplete elliptic integral of the second kind is defined as

$$E(k,\phi) = \int_0^\phi \sqrt{1 - k^2 sin^2 \theta}$$

### **Parameters**

k	The argument of the elliptic function.
phi	The integral limit argument of the elliptic function.

#### Returns

The elliptic function of the second kind.

Definition at line 673 of file sf\_ellint.tcc.

References \_\_comp\_ellint\_2(), \_\_ellint\_rd(), and \_\_ellint\_rf().

8.3.2.85 template < typename \_Tp > \_Tp std::\_\_detail::\_\_ellint\_3 ( \_Tp 
$$\_k$$
, \_Tp  $\_nu$ , \_Tp  $\_phi$  )

Return the incomplete elliptic integral of the third kind  $\Pi(k,\nu,\phi)$  using the Carlson formulation.

The incomplete elliptic integral of the third kind is defined as

$$\Pi(k,\nu,\phi) = \int_0^\phi \frac{d\theta}{(1-\nu\sin^2\theta)\sqrt{1-k^2\sin^2\theta}}$$

# Parameters

k	The argument of the elliptic function.
nu	The second argument of the elliptic function.
phi	The integral limit argument of the elliptic function.

### Returns

The elliptic function of the third kind.

Definition at line 768 of file sf ellint.tcc.

References \_\_comp\_ellint\_3(), \_\_ellint\_rf(), and \_\_ellint\_rj().

8.3.2.86 template < typename  $_{\rm Tp} > _{\rm Tp}$  std::\_\_detail::\_\_ellint\_cel (  $_{\rm Tp} \_k\_c, _{\rm Tp} \_p, _{\rm Tp} \_a, _{\rm Tp} \_b$  )

Return the Bulirsch complete elliptic integrals.

Definition at line 920 of file sf\_ellint.tcc.

References \_\_ellint\_rf(), and \_\_ellint\_rj().

8.3.2.87 template < typename  $_{\rm Tp} > _{\rm Tp}$  std::\_\_detail::\_\_ellint\_d (  $_{\rm Tp}$  \_\_k,  $_{\rm Tp}$  \_\_phi )

Return the Legendre elliptic integral D.

Definition at line 809 of file sf ellint.tcc.

References \_\_ellint\_rd().

8.3.2.88 template> \_Tp std::\_\_detail::\_\_ellint\_el1 ( \_Tp 
$$\_x$$
, \_Tp  $\_k\_c$  )

Return the Bulirsch elliptic integrals of the first kind.

Definition at line 848 of file sf ellint.tcc.

References \_\_ellint\_rf().

Return the Bulirsch elliptic integrals of the second kind.

Definition at line 869 of file sf ellint.tcc.

References \_\_ellint\_rd(), and \_\_ellint\_rf().

Return the Bulirsch elliptic integrals of the third kind.

Definition at line 894 of file sf ellint.tcc.

References \_\_ellint\_rf(), and \_\_ellint\_rj().

8.3.2.91 template < typename \_Tp > \_Tp std::\_\_detail::\_\_ellint\_rc ( \_Tp 
$$\underline{\hspace{0.1cm}}$$
x, \_Tp  $\underline{\hspace{0.1cm}}$ y )

Return the Carlson elliptic function  $R_C(x,y) = R_F(x,y,y)$  where  $R_F(x,y,z)$  is the Carlson elliptic function of the first kind.

The Carlson elliptic function is defined by:

$$R_C(x,y) = \frac{1}{2} \int_0^\infty \frac{dt}{(t+x)^{1/2}(t+y)}$$

Based on Carlson's algorithms:

- B. C. Carlson Numer. Math. 33, 1 (1979)
- B. C. Carlson, Special Functions of Applied Mathematics (1977)
- Numerical Recipes in C, 2nd ed, pp. 261-269, by Press, Teukolsky, Vetterling, Flannery (1992)

X	The first argument.
у	The second argument.

#### Returns

The Carlson elliptic function.

Definition at line 81 of file sf ellint.tcc.

Referenced by \_\_ellint\_rf(), and \_\_ellint\_rj().

$$8.3.2.92 \quad template < typename \_Tp > \_Tp \ std::\__detail::\__ellint\_rd \ ( \ \_Tp \_\_x, \ \_Tp \_\_y, \ \_Tp \_\_z \ )$$

Return the Carlson elliptic function of the second kind  $R_D(x,y,z) = R_J(x,y,z,z)$  where  $R_J(x,y,z,p)$  is the Carlson elliptic function of the third kind.

The Carlson elliptic function of the second kind is defined by:

$$R_D(x,y,z) = \frac{3}{2} \int_0^\infty \frac{dt}{(t+x)^{1/2}(t+y)^{1/2}(t+z)^{3/2}}$$

Based on Carlson's algorithms:

- B. C. Carlson Numer. Math. 33, 1 (1979)
- B. C. Carlson, Special Functions of Applied Mathematics (1977)
- Numerical Recipes in C, 2nd ed, pp. 261-269, by Press, Teukolsky, Vetterling, Flannery (1992)

# **Parameters**

X	The first of two symmetric arguments.
y	The second of two symmetric arguments.
Z	The third argument.

#### Returns

The Carlson elliptic function of the second kind.

Definition at line 163 of file sf ellint.tcc.

Referenced by  $\_$ comp\_ellint\_2(),  $\_$ comp\_ellint\_d(),  $\_$ ellint\_2(),  $\_$ ellint\_d(),  $\_$ ellint\_el2(),  $\_$ ellint\_rg(), and  $\_$  $\leftarrow$ ellint\_rj().

Return the Carlson elliptic function  $R_F(x, y, z)$  of the first kind.

The Carlson elliptic function of the first kind is defined by:

$$R_F(x,y,z) = \frac{1}{2} \int_0^\infty \frac{dt}{(t+x)^{1/2}(t+y)^{1/2}(t+z)^{1/2}}$$

X	The first of three symmetric arguments.
у	The second of three symmetric arguments.
z	The third of three symmetric arguments.

### Returns

The Carlson elliptic function of the first kind.

Definition at line 277 of file sf ellint.tcc.

References \_\_comp\_ellint\_rf(), and \_\_ellint\_rc().

Referenced by \_\_comp\_ellint\_2(), \_\_comp\_ellint\_3(), \_\_ellint\_1(), \_\_ellint\_2(), \_\_ellint\_3(), \_\_ellint\_cel(), \_\_ellint\_el1(), \_\_ellint\_el2(), \_\_ellint\_el3(), and \_\_heuman\_lambda().

Return the symmetric Carlson elliptic function of the second kind  $R_G(x, y, z)$ .

The Carlson symmetric elliptic function of the second kind is defined by:

$$R_G(x,y,z) = \frac{1}{4} \int_0^\infty dt t [(t+x)(t+y)(t+z)]^{-1/2} \left(\frac{x}{t+x} + \frac{y}{t+y} + \frac{z}{t+z}\right)$$

Based on Carlson's algorithms:

- B. C. Carlson Numer. Math. 33, 1 (1979)
- B. C. Carlson, Special Functions of Applied Mathematics (1977)
- · Numerical Recipes in C, 2nd ed, pp. 261-269, by Press, Teukolsky, Vetterling, Flannery (1992)

# **Parameters**

X	The first of three symmetric arguments.
y	The second of three symmetric arguments.
z	The third of three symmetric arguments.

## Returns

The Carlson symmetric elliptic function of the second kind.

Definition at line 408 of file sf\_ellint.tcc.

References comp ellint rg(), and ellint rd().

$$8.3.2.95 \quad template < typename \_Tp > \_Tp \ std::\_detail::\_ellint\_rj \ ( \ \_Tp \_x, \ \_Tp \_y, \ \_Tp \_z, \ \_Tp \_p \ )$$

Return the Carlson elliptic function  $R_J(x, y, z, p)$  of the third kind.

The Carlson elliptic function of the third kind is defined by:

$$R_J(x,y,z,p) = \frac{3}{2} \int_0^\infty \frac{dt}{(t+x)^{1/2}(t+y)^{1/2}(t+z)^{1/2}(t+p)}$$

Based on Carlson's algorithms:

- B. C. Carlson Numer. Math. 33, 1 (1979)
- B. C. Carlson, Special Functions of Applied Mathematics (1977)
- · Numerical Recipes in C, 2nd ed, pp. 261-269, by Press, Teukolsky, Vetterling, Flannery (1992)

X	The first of three symmetric arguments.
y	The second of three symmetric arguments.
Z	The third of three symmetric arguments.
p	The fourth argument.

#### Returns

The Carlson elliptic function of the fourth kind.

Definition at line 456 of file sf ellint.tcc.

References \_\_ellint\_rc(), and \_\_ellint\_rd().

Referenced by \_\_comp\_ellint\_3(), \_\_ellint\_cel(), \_\_ellint\_el3(), \_\_heuman\_lambda(), and \_\_jacobi\_zeta().

8.3.2.96 template<typename  $_{\text{Tp}} > _{\text{Tp}}$  std::\_\_detail::\_\_ellnome (  $_{\text{Tp}}$  \_\_k )

Return the elliptic nome given the modulus k.

Definition at line 292 of file sf\_theta.tcc.

References \_\_ellnome\_k(), and \_\_ellnome\_series().

Referenced by theta c(), theta d(), theta n(), and theta s().

8.3.2.97 template < typename \_Tp > \_Tp std::\_\_detail::\_\_ellnome\_k ( \_Tp  $\_k$  )

Use the arithmetic-geometric mean to calculate the elliptic nome given the , k.

Definition at line 278 of file sf theta.tcc.

References \_\_comp\_ellint\_1().

Referenced by \_\_ellnome().

8.3.2.98 template < typename \_Tp > \_Tp std::\_\_detail::\_\_ellnome\_series ( \_Tp  $\_k$  )

Use MacLaurin series to calculate the elliptic nome given the , k.

Definition at line 262 of file sf theta.tcc.

Referenced by \_\_ellnome().

8.3.2.99 template < typename \_Tp > \_Tp std::\_\_detail::\_\_expint ( unsigned int \_\_n, \_Tp \_\_x )

Return the exponential integral  $E_n(x)$ .

The exponential integral is given by

$$E_n(x) = \int_1^\infty \frac{e^{-xt}}{t^n} dt$$

This is something of an extension.

n	The order of the exponential integral function.
X	The argument of the exponential integral function.

### Returns

The exponential integral.

Definition at line 474 of file sf\_expint.tcc.

References \_\_expint\_E1(), and \_\_expint\_En\_recursion().

Referenced by \_\_logint().

8.3.2.100 template<typename \_Tp > \_Tp std::\_\_detail::\_\_expint ( \_Tp \_\_x )

Return the exponential integral Ei(x).

The exponential integral is given by

$$Ei(x) = -\int_{-x}^{\infty} \frac{e^t}{t} dt$$

### **Parameters**

Х	The argument of the exponential integral function.
	The digament of the experiental integral fallottern

# Returns

The exponential integral.

Definition at line 514 of file sf\_expint.tcc.

References \_\_expint\_Ei().

8.3.2.101 template<typename \_Tp > \_Tp std::\_\_expint\_asymp ( unsigned int \_\_n, \_Tp \_\_x )

Return the exponential integral  $E_n(x)$  for large argument.

The exponential integral is given by

$$E_n(x) = \int_1^\infty \frac{e^{-xt}}{t^n} dt$$

This is something of an extension.

#### **Parameters**

n	The order of the exponential integral function.
X	The argument of the exponential integral function.

## Returns

The exponential integral.

Definition at line 406 of file sf expint.tcc.

8.3.2.102 template<typename \_Tp > \_Tp std::\_\_detail::\_\_expint\_E1 ( \_Tp \_\_x )

Return the exponential integral  $E_1(x)$ .

The exponential integral is given by

$$E_1(x) = \int_1^\infty \frac{e^{-xt}}{t} dt$$

#### **Parameters**

\_\_x The argument of the exponential integral function.

### Returns

The exponential integral.

**Todo** Find a good asymptotic switch point in  $E_1(x)$ .

**Todo** Find a good asymptotic switch point in  $E_1(x)$ .

Definition at line 375 of file sf expint.tcc.

References \_\_expint\_E1\_asymp(), \_\_expint\_E1\_series(), \_\_expint\_Ei(), and \_\_expint\_En\_cont\_frac().

Referenced by \_\_coshint(), \_\_expint\_Ei(), \_\_expint\_Ei(), \_\_expint\_En\_recursion(), and \_\_sinhint().

8.3.2.103 template<typename \_Tp > \_Tp std::\_\_detail::\_\_expint\_E1\_asymp ( \_Tp \_\_x )

Return the exponential integral  $E_1(x)$  by asymptotic expansion.

The exponential integral is given by

$$E_1(x) = \int_1^\infty \frac{e^{-xt}}{t} dt$$

#### **Parameters**

\_\_x The argument of the exponential integral function.

# Returns

The exponential integral.

Definition at line 112 of file sf expint.tcc.

Referenced by \_\_expint\_E1().

8.3.2.104 template < typename \_Tp > \_Tp std::\_\_detail::\_\_expint\_E1\_series ( \_Tp \_\_x )

Return the exponential integral  $E_1(x)$  by series summation. This should be good for x < 1.

The exponential integral is given by

$$E_1(x) = \int_1^\infty \frac{e^{-xt}}{t} dt$$

\_\_x The argument of the exponential integral function.

#### Returns

The exponential integral.

Definition at line 75 of file sf expint.tcc.

Referenced by \_\_expint\_E1().

8.3.2.105 template<typename \_Tp > \_Tp std::\_\_detail::\_\_expint\_Ei ( \_Tp \_\_x )

Return the exponential integral Ei(x).

The exponential integral is given by

$$Ei(x) = -\int_{-x}^{\infty} \frac{e^t}{t} dt$$

## **Parameters**

\_\_x The argument of the exponential integral function.

### Returns

The exponential integral.

Definition at line 351 of file sf expint.tcc.

References \_\_expint\_E1(), \_\_expint\_Ei\_asymp(), and \_\_expint\_Ei\_series().

Referenced by \_\_coshint(), \_\_expint(), \_\_expint\_E1(), and \_\_sinhint().

8.3.2.106 template<typename \_Tp > \_Tp std::\_\_detail::\_\_expint\_Ei\_asymp ( \_Tp  $\_x$  )

Return the exponential integral Ei(x) by asymptotic expansion.

The exponential integral is given by

$$Ei(x) = -\int_{-x}^{\infty} \frac{e^t}{t} dt$$

# **Parameters**

\_\_x The argument of the exponential integral function.

# Returns

The exponential integral.

Definition at line 318 of file sf expint.tcc.

Referenced by \_\_expint\_Ei().

8.3.2.107 template<typename \_Tp > \_Tp std::\_\_detail::\_\_expint\_Ei\_series ( \_Tp \_\_x )

Return the exponential integral Ei(x) by series summation.

The exponential integral is given by

$$Ei(x) = -\int_{-x}^{\infty} \frac{e^t}{t} dt$$

### **Parameters**

X	The argument of the exponential integral function.

## Returns

The exponential integral.

Definition at line 286 of file sf\_expint.tcc.

Referenced by expint Ei().

 $8.3.2.108 \quad template < typename \_Tp > \_Tp \ std:: \_detail:: \_expint\_En\_cont\_frac \ ( \ unsigned \ int \_\_n, \ \_Tp \_\_x \ )$ 

Return the exponential integral  $E_n(x)$  by continued fractions.

The exponential integral is given by

$$E_n(x) = \int_1^\infty \frac{e^{-xt}}{t^n} dt$$

### **Parameters**

n	The order of the exponential integral function.
x	The argument of the exponential integral function.

## Returns

The exponential integral.

Definition at line 195 of file sf expint.tcc.

Referenced by \_\_expint\_E1().

 $8.3.2.109 \quad template < typename \_Tp > \_Tp \ std:: \_detail:: \_expint\_En\_recursion \ ( \ unsigned \ int \_\_n, \ \_Tp \_\_x \ )$ 

Return the exponential integral  $E_n(x)$  by recursion. Use upward recursion for x < n and downward recursion (Miller's algorithm) otherwise.

The exponential integral is given by

$$E_n(x) = \int_1^\infty \frac{e^{-xt}}{t^n} dt$$

n	The order of the exponential integral function.
X	The argument of the exponential integral function.

### Returns

The exponential integral.

**Todo** Find a principled starting number for the  $E_n(\boldsymbol{x})$  downward recursion.

Definition at line 240 of file sf\_expint.tcc.

References \_\_expint\_E1().

Referenced by \_\_expint().

 $8.3.2.110 \quad template < typename \_Tp > \_Tp \ std:: \_detail:: \_expint\_En\_series \ ( \ unsigned \ int \_\_n, \ \_Tp \_\_x \ )$ 

Return the exponential integral  $E_n(x)$  by series summation.

The exponential integral is given by

$$E_n(x) = \int_1^\infty \frac{e^{-xt}}{t^n} dt$$

### **Parameters**

n	The order of the exponential integral function.
X	The argument of the exponential integral function.

## Returns

The exponential integral.

Definition at line 149 of file sf\_expint.tcc.

References psi().

8.3.2.111 template<typename \_Tp > \_Tp std::\_\_expint\_large\_n ( unsigned int \_\_n, \_Tp \_\_x )

Return the exponential integral  $E_n(x)$  for large order.

The exponential integral is given by

$$E_n(x) = \int_1^\infty \frac{e^{-xt}}{t^n} dt$$

This is something of an extension.

# **Parameters**

n	The order of the exponential integral function.
X	The argument of the exponential integral function.

## Returns

The exponential integral.

Definition at line 440 of file sf expint.tcc.

8.3.2.112 template < typename \_Tp > \_GLIBCXX14\_CONSTEXPR \_Tp std::\_\_detail::\_\_factorial ( unsigned int  $\_n$  )

Return the factorial of the integer n.

The factorial is:

$$n! = 12...(n-1)n, 0! = 1$$

Definition at line 2422 of file sf\_gamma.tcc.

References \_S\_factorial\_table.

Referenced by \_\_double\_factorial().

8.3.2.113 template<typename \_Tp > \_Tp std::\_\_detail::\_\_fermi\_dirac ( \_Tp \_\_s, \_Tp \_\_x )

Return the Fermi-Dirac integral of real order s and real argument x.

#### See also

```
https://en.wikipedia.org/wiki/Clausen_function
http://dlmf.nist.gov/25.12#iii
```

## **Parameters**

s	The order $s \ge 0$ .
X	The real argument.

# Returns

The real Fermi-Dirac cosine sum  $F_s(x)$ ,

Definition at line 1379 of file sf\_polylog.tcc.

References polylog exp().

8.3.2.114 template<typename \_Tp > \_GLIBCXX14\_CONSTEXPR \_Tp std::\_\_detail::\_\_fisher\_f\_cdf ( \_Tp \_\_F, unsigned int \_\_nu1, unsigned int \_\_nu2 )

Return the F-distribution propability function. This returns the probability that the observed chi-square for a correct model exceeds the value  $\chi^2$ .

The f-distribution propability function is related to the incomplete beta function:

$$Q(F|\nu_1,\nu_2) = I_{\frac{\nu_2}{\nu_2 + \nu_1 F}}(\frac{\nu_2}{2}, \frac{\nu_1}{2})$$

#### **Parameters**

nu1	The number of degrees of freedom of sample 1
nu2	The number of degrees of freedom of sample 2
F	The F statistic

Definition at line 350 of file sf beta.tcc.

References beta inc().

8.3.2.115 template < typename \_Tp > \_GLIBCXX14\_CONSTEXPR \_Tp std::\_\_detail::\_\_fisher\_f\_cdfc ( \_Tp \_\_F, unsigned int \_\_nu1, unsigned int \_\_nu2 )

Return the F-distribution propability function. This returns the probability that the observed chi-square for a correct model exceeds the value  $\chi^2$ .

The f-distribution propability function is related to the incomplete beta function:

$$P(F|\nu_1,\nu_2) = 1 - I_{\frac{\nu_2}{\nu_2 + \nu_1 F}}(\frac{\nu_2}{2}, \frac{\nu_1}{2}) = 1 - Q(F|\nu_1,\nu_2)$$

#### **Parameters**

F	
nu1	
nu2	

Definition at line 379 of file sf\_beta.tcc.

References \_\_beta\_inc().

8.3.2.116 template<typename \_Tp > void std::\_\_detail::\_\_fock\_airy ( \_Tp \_\_x, std::complex< \_Tp > & \_\_w1, std::complex< \_Tp > & \_\_w2, std::complex< \_Tp > & \_\_w2p )

Compute the Fock-type Airy functions  $w_1(x)$  and  $w_2(x)$  and their first derivatives  $w_1'(x)$  and  $w_2'(x)$  respectively.

$$w_1(x) = \sqrt{\pi}(Ai(x) + iBi(x))$$

 $w_2(x) = \sqrt{\pi}(Ai(x) - iBi(x))$ 

### **Parameters**

X	The argument of the Airy functions.
w1	The output Fock-type Airy function of the first kind.
w2	The output Fock-type Airy function of the second kind.
w1p	The output derivative of the Fock-type Airy function of the first kind.
w2p	The output derivative of the Fock-type Airy function of the second kind.

Definition at line 580 of file sf mod bessel.tcc.

8.3.2.117 template<typename \_Tp > bool std::\_\_detail::\_\_fpequal ( const \_Tp & \_\_a, const \_Tp & \_\_b )

A function to reliably compare two floating point numbers.

### **Parameters**

ſ	a	the left hand side.
ſ	b	the right hand side

### Returns

returns true if a and b are equal to zero or differ only by max(a,b) \* 5 \* eps

Definition at line 62 of file sf\_polylog.tcc.

Referenced by  $\_$ dirichlet\_beta(),  $\_$ dirichlet\_eta(),  $\_$ fpimag(),  $\_$ polylog(),  $\_$ polylog(),  $\_$ polylog\_exp\_asymp(),  $\_$  $\hookrightarrow$ polylog\_exp\_int\_neg(),  $\_$ polylog\_exp\_int\_pos(),  $\_$ polylog\_exp\_neg(),  $\_$ polylog\_exp\_neg\_even(),  $\_$ polylog\_exp\_ $\hookleftarrow$ neg\_odd(),  $\_$ polylog\_exp\_negative\_real\_part(),  $\_$ polylog\_exp\_pos(), and  $\_$ polylog\_exp\_real\_pos().

8.3.2.118 template<typename \_Tp > bool std::\_\_detail::\_\_fpimag ( const std::complex < \_Tp > & \_\_w )

A function to reliably test a complex number for imaginaryness [?].

#### **Parameters**

\_\_w The complex argument.

### Returns

true if Re(w) is zero within 5\*epsilon, false otherwize.

Definition at line 107 of file sf polylog.tcc.

References \_\_fpequal().

8.3.2.119 template<typename \_Tp > bool std::\_\_detail::\_\_fpimag ( const \_Tp )

Definition at line 117 of file sf polylog.tcc.

8.3.2.120 template<typename\_Tp > bool std::\_\_detail::\_\_fpreal ( const std::complex< \_Tp > & \_\_w )

A function to reliably test a complex number for realness.

#### **Parameters**

W	The complex argument.
<del></del>	

#### Returns

true if Im(w) is zero within 5\*epsilon, false otherwize.

Definition at line 84 of file sf\_polylog.tcc.

References \_\_fpequal().

Referenced by \_\_polylog\_exp\_int\_pos(), and \_\_polylog\_exp\_real\_pos().

8.3.2.121 template<typename \_Tp > bool std::\_\_detail::\_\_fpreal ( const \_Tp )

Definition at line 94 of file sf polylog.tcc.

8.3.2.122 template < typename \_Tp > std::complex < \_Tp > std::\_\_detail::\_\_fresnel ( const \_Tp \_\_x )

Return the Fresnel cosine and sine integrals as a complex number f[C(x) + iS(x)].

The Fresnel cosine integral is defined by:

$$C(x) = \int_0^x \cos(\frac{\pi}{2}t^2)dt$$

The Fresnel sine integral is defined by:

$$S(x) = \int_0^x \sin(\frac{\pi}{2}t^2)dt$$

	The amount of the second of th
X	The argument
^	3

Definition at line 166 of file sf fresnel.tcc.

References \_\_fresnel\_cont\_frac(), and \_\_fresnel\_series().

8.3.2.123 template < typename \_Tp > void std::\_\_detail::\_\_fresnel\_cont\_frac ( const \_Tp \_\_ax, \_Tp & \_Cf, \_Tp & \_Sf )

This function computes the Fresnel cosine and sine integrals by continued fractions for positive argument.

Definition at line 105 of file sf fresnel.tcc.

Referenced by \_\_fresnel().

8.3.2.124 template < typename \_Tp > void std::\_\_detail::\_\_fresnel\_series ( const \_Tp \_\_ax, \_Tp & \_Cf, \_Tp & \_Sf )

This function returns the Fresnel cosine and sine integrals as a pair by series expansion for positive argument.

Definition at line 48 of file sf fresnel.tcc.

Referenced by \_\_fresnel().

8.3.2.125 template < typename  $_{\rm Tp}$  >  $_{\rm Tp}$  std::\_\_detail::\_\_gamma (  $_{\rm Tp}$  \_\_x )

Return  $\Gamma(x)$ .

**Parameters** 

\_\_x The argument of the gamma function.

# Returns

The gamma function.

Definition at line 1918 of file sf\_gamma.tcc.

References log gamma().

Referenced by \_\_beta\_gamma(), and \_\_riemann\_zeta().

8.3.2.126 template<typename\_Tp > std::pair<\_Tp, \_Tp > std::\_\_detail::\_\_gamma\_cont\_frac ( \_Tp \_\_a, \_Tp \_\_x )

Definition at line 1964 of file sf gamma.tcc.

Referenced by \_\_gamma\_l(), \_\_gamma\_p(), \_\_gamma\_q(), and \_\_gamma\_u().

8.3.2.127 template<typename \_Tp > \_Tp std::\_\_detail::\_\_gamma\_I ( \_Tp \_\_a, \_Tp \_\_x )

Return the lower incomplete gamma function. The lower incomplete gamma function is defined by

$$\gamma(a,x) = \int_0^x e^{-t} t^{a-1} dt (a>0)$$

.

Definition at line 2070 of file sf\_gamma.tcc.

References \_\_gamma\_cont\_frac(), and \_\_gamma\_series().

8.3.2.128 template < typename \_Tp > \_Tp std::\_\_detail::\_\_gamma\_p ( \_Tp \_\_a, \_Tp \_\_x )

Return the regularized lower incomplete gamma function. The regularized lower incomplete gamma function is defined by

$$P(a,x) = \frac{\gamma(a,x)}{\Gamma(a)}$$

where  $\Gamma(a)$  is the gamma function and

$$\gamma(a,x) = \int_0^x e^{-t} t^{a-1} dt (a > 0)$$

is the lower incomplete gamma function.

Definition at line 2013 of file sf gamma.tcc.

References \_\_gamma\_cont\_frac(), and \_\_gamma\_series().

Referenced by \_\_chi\_squared\_pdf().

8.3.2.129 template<typename \_Tp > \_Tp std::\_\_detail::\_\_gamma\_q ( \_Tp \_\_a, \_Tp \_\_x )

Return the regularized upper incomplete gamma function. The regularized upper incomplete gamma function is defined by

$$Q(a,x) = \frac{\Gamma(a,x)}{\Gamma(a)}$$

where  $\Gamma(a)$  is the gamma function and

$$\Gamma(a,x) = \int_{x}^{\infty} e^{-t} t^{a-1} dt (a > 0)$$

is the upper incomplete gamma function.

Definition at line 2044 of file sf\_gamma.tcc.

References \_\_gamma\_cont\_frac(), and \_\_gamma\_series().

Referenced by \_\_chi\_squared\_pdfc().

8.3.2.130 template<typename \_Tp > std::pair<\_Tp, \_Tp> std::\_\_detail::\_\_gamma\_series ( \_Tp \_\_a, \_Tp \_\_x )

Definition at line 1930 of file sf gamma.tcc.

Referenced by \_\_gamma\_l(), \_\_gamma\_p(), \_\_gamma\_q(), and \_\_gamma\_u().

8.3.2.131 template<typename \_Tp > void std::\_\_detail::\_\_gamma\_temme ( \_Tp \_\_mu, \_Tp & \_\_gam1, \_Tp & \_\_gam2, \_Tp & \_\_gammi )

Compute the gamma functions required by the Temme series expansions of  $N_{\nu}(x)$  and  $K_{\nu}(x)$ .

$$\Gamma_1 = \frac{1}{2\mu} \left[ \frac{1}{\Gamma(1-\mu)} - \frac{1}{\Gamma(1+\mu)} \right]$$

and

$$\Gamma_2 = \frac{1}{2} \left[ \frac{1}{\Gamma(1-\mu)} + \frac{1}{\Gamma(1+\mu)} \right]$$

where  $-1/2 <= \mu <= 1/2$  is  $\mu = \nu - N$  and N. is the nearest integer to  $\nu$ . The values of  $\Gamma(1+\mu)$  and  $\Gamma(1-\mu)$  are returned as well.

The accuracy requirements on this are exquisite.

#### **Parameters**

	mu	The input parameter of the gamma functions.
out	gam1	The output function $\Gamma_1(\mu)$
out	gam2	The output function $\Gamma_2(\mu)$
out	gampl	The output function $\Gamma(1+\mu)$
out	gammi	The output function $\Gamma(1-\mu)$

Definition at line 163 of file sf bessel.tcc.

Referenced by \_\_cyl\_bessel\_ik\_steed(), and \_\_cyl\_bessel\_jn\_steed().

Return the upper incomplete gamma function. The lower incomplete gamma function is defined by

$$\Gamma(a,x) = \int_{x}^{\infty} e^{-t} t^{a-1} dt (a > 0)$$

.

Definition at line 2102 of file sf gamma.tcc.

References \_\_gamma\_cont\_frac(), and \_\_gamma\_series().

8.3.2.133 template\_{\rm Tp} > 
$$_{\rm Tp}$$
 std::\_\_detail::\_\_gauss (  $_{\rm Tp}$  \_\_x )

The CDF of the normal distribution. i.e. the integrated lower tail of the normal PDF.

Definition at line 70 of file sf owens t.tcc.

Return the Gegenbauer polynomial  $C_n^{\alpha}(x)$  of degree n and real order  $\alpha$  and argument x.

The Gegenbauer polynomials are generated by a three-term recursion relation:

$$C_n^{\alpha}(x) = \frac{1}{n} \left[ 2x(n+\alpha-1)C_{n-1}^{\alpha}(x) - (n+2\alpha-2)C_{n-2}^{\alpha}(x) \right]$$

and  $C_0^{\alpha}(x) = 1$ ,  $C_1^{\alpha}(x) = 2\alpha x$ .

### **Template Parameters**

_Talpha	The real type of the order
_Tp	The real type of the argument

n	The non-negative integral degree	
alpha	The real order	
X	x The real argument	

Definition at line 61 of file sf\_gegenbauer.tcc.

```
8.3.2.135 template < typename _Tp > void std::__detail::__hankel ( std::complex < _Tp > __nu, std::complex < _Tp > __z, std::complex < _Tp > & _H1, std::complex < _Tp > & _H2, std::complex < _Tp > & _H1p, std::complex < _Tp > & _H2p )
```

### **Parameters**

in	nu	The order for which the Hankel functions are evaluated.
in	z	The argument at which the Hankel functions are evaluated.
out	_H1	The Hankel function of the first kind.
out	_H1p	The derivative of the Hankel function of the first kind.
out	_H2	The Hankel function of the second kind.
out	_H2p	The derivative of the Hankel function of the second kind.

Definition at line 1127 of file sf\_hankel.tcc.

References \_\_debye\_region(), \_\_hankel\_debye(), and \_\_hankel\_uniform().

Referenced by \_\_cyl\_bessel(), \_\_cyl\_hankel\_1(), \_\_cyl\_hankel\_2(), \_\_cyl\_neumann(), and \_\_sph\_hankel().

8.3.2.136 template < typename \_Tp > void std::\_\_detail::\_\_hankel\_debye ( std::complex < \_Tp > \_\_nu, std::complex < \_Tp > \_\_z, std::complex < \_Tp > \_\_alpha, int \_\_indexr, char & \_\_aorb, int & \_\_morn, std::complex < \_Tp > & \_H1, std::complex < \_Tp > & \_H2, std::complex < \_Tp > & \_H2p, std::complex < \_Tp > & \_H2p )

### **Parameters**

in	nu	The order for which the Hankel functions are evaluated.
in	z	The argument at which the Hankel functions are evaluated.
in	alpha	
in	indexr	
out	aorb	
out	morn	
out	_H1	The Hankel function of the first kind.
out	_H1p	The derivative of the Hankel function of the first kind.
out	_H2	The Hankel function of the second kind.
out	_H2p	The derivative of the Hankel function of the second kind.

Definition at line 959 of file sf hankel.tcc.

Referenced by \_\_hankel().

```
8.3.2.137 template < typename _Tp > void std::__detail::__hankel_params ( std::complex < _Tp > __nu, std::complex < _Tp > __zhat, std::complex < _Tp > & __p, std::complex < _Tp > & __p2, std::complex < _Tp > & __nup2, std::complex < _Tp > & __nup2, std::complex < _Tp > & __num2d3, std::complex < _Tp > & __num2d3, std::complex < _Tp > & __zetaphf, std::complex < _Tp > & __zetamhf, std::complex < _Tp > & __zetam3hf, std::complex < _Tp > & __zetat )
```

Compute parameters depending on z and nu that appear in the uniform asymptotic expansions of the Hankel functions and their derivatives, except the arguments to the Airy functions.

Definition at line 110 of file sf hankel.tcc.

Referenced by hankel uniform outer().

```
8.3.2.138 template<typename _Tp > void std::__detail::__hankel_uniform ( std::complex< _Tp > __nu, std::complex< _Tp > __z, std::complex< _Tp > & _H1, std::complex< _Tp > & _H2, std::complex< _Tp > & _H1p, std::complex< _Tp > & _H2p )
```

This routine computes the uniform asymptotic approximations of the Hankel functions and their derivatives including a patch for the case when the order equals or nearly equals the argument. At such points, Olver's expressions have zero denominators (and numerators) resulting in numerical problems. This routine averages results from four surrounding points in the complex plane to obtain the result in such cases.

#### **Parameters**

in	nu	The order for which the Hankel functions are evaluated.
in	z	The argument at which the Hankel functions are evaluated.
out	_H1	The Hankel function of the first kind.
out	_H1p	The derivative of the Hankel function of the first kind.
out	_H2	The Hankel function of the second kind.
out	_H2p	The derivative of the Hankel function of the second kind.

Definition at line 904 of file sf hankel.tcc.

References hankel uniform olver().

Referenced by \_\_hankel().

Compute approximate values for the Hankel functions of the first and second kinds using Olver's uniform asymptotic expansion to of order nu along with their derivatives.

### **Parameters**

in	nu	The order for which the Hankel functions are evaluated.
in	z	The argument at which the Hankel functions are evaluated.
out	_H1	The Hankel function of the first kind.
out	_H1p	The derivative of the Hankel function of the first kind.
out	_H2	The Hankel function of the second kind.
out	_H2p	The derivative of the Hankel function of the second kind.

Definition at line 818 of file sf hankel.tcc.

References \_\_hankel\_uniform\_outer(), and \_\_hankel\_uniform\_sum().

Referenced by hankel uniform().

```
8.3.2.140 template<typename _Tp > void std::__detail::__hankel_uniform_outer ( std::complex< _Tp > __nu, std::complex < _Tp > __z, _Tp __eps, std::complex < _Tp > & __zhat, std::complex < _Tp > & __num1d3, std::complex < _Tp > & __num2d3, std::complex < _Tp > & __p, std::complex < _Tp > & __p2, std::complex < _Tp > & __etrat, std::complex < _Tp > & __aip, std::complex < _Tp > & __aip, std::complex < _Tp > & __o4dp, std::complex < _Tp > & __o4dm, std::complex < _Tp > & __o4dm, std::complex < _Tp > & __o4dp, std::complex < _Tp > & __o4ddp, std::complex < __o4
```

Compute outer factors and associated functions of z and nu appearing in Olver's uniform asymptotic expansions of the Hankel functions of the first and second kinds and their derivatives. The various functions of z and nu returned by  $hankel\_uniform\_outer$  are available for use in computing further terms in the expansions.

Definition at line 273 of file sf hankel.tcc.

```
References __airy(), __airy_arg(), and __hankel_params().
```

Referenced by hankel uniform olver().

8.3.2.141 template < typename \_Tp > void std::\_\_detail::\_\_hankel\_uniform\_sum ( std::complex < \_Tp > \_\_p, std::complex < \_Tp > \_\_p, std::complex < \_Tp > \_\_p, std::complex < \_Tp > \_\_aip, std::complex < \_Tp > \_\_o4dp, std::complex < \_Tp > \_\_o4dm, \_\_rp \_\_eps, std::complex < \_Tp > \_\_o4dm, std::complex < \_Tp > \_\_o4dm, \_\_rp \_\_eps, std::complex < \_Tp > & \_\_H1sum, std::complex < \_Tp > & \_\_H2sum, std::complex < \_\_rp > & \_\_H2sum)

Compute the sums in appropriate linear combinations appearing in Olver's uniform asymptotic expansions for the Hankel functions of the first and second kinds and their derivatives, using up to nterms (less than 5) to achieve relative error eps.

## **Parameters**

in	p	
in	p2	
in	num2	
in	zetam3hf	
in	_Aip	The Airy function value $Ai()$ .
in	o4dp	
in	_Aim	The Airy function value $Ai()$ .
in	o4dm	
in	od2p	
in	od0dp	
in	od2m	
in	od0dm	
in	eps	The error tolerance
out	_H1sum	The Hankel function of the first kind.
out	_H1psum	The derivative of the Hankel function of the first kind.
out	_H2sum	The Hankel function of the second kind.
out	_H2psum	The derivative of the Hankel function of the second kind.

Definition at line 351 of file sf\_hankel.tcc.

Referenced by hankel uniform olver().

8.3.2.142 template < typename \_Tp > \_Tp std::\_\_detail::\_\_heuman\_lambda ( \_Tp \_\_k, \_Tp \_\_phi )

Return the Heuman lambda function.

Definition at line 941 of file sf\_ellint.tcc.

References \_\_ellint\_rf(), and \_\_ellint\_rj().

8.3.2.143 template<typename \_Tp > \_Tp std::\_\_detail::\_\_hurwitz\_zeta ( \_Tp \_\_s, \_Tp \_\_a )

Return the Hurwitz zeta function  $\zeta(s,a)$  for all s = 1 and a > -1.

The Hurwitz zeta function is defined by:

$$\zeta(s,a) = \sum_{n=0}^{\infty} \frac{1}{(n+a)^s}$$

The Riemann zeta function is a special case:

$$\zeta(s) = \zeta(s, 1)$$

#### **Parameters**

s	The argument $s! = 1$	
a	a The scale parameter $a>-1$	

Definition at line 702 of file sf\_zeta.tcc.

References hurwitz zeta euler maclaurin().

8.3.2.144 template < typename \_Tp > std::complex < \_Tp > std::\_\_detail::\_\_hurwitz\_zeta ( \_Tp \_\_s, std::complex < \_Tp > \_\_a )

Return the Hurwitz Zeta function for real s and complex a.

### **Parameters**

s	The real argument
a	The complex parameter

Todo This \_\_hurwitz\_zeta prefactor is prone to overflow. positive integer orders s?

Definition at line 1119 of file sf polylog.tcc.

References \_\_polylog\_exp().

Referenced by \_\_psi().

8.3.2.145 template < typename \_Tp > \_Tp std::\_\_detail::\_\_hurwitz\_zeta\_euler\_maclaurin ( \_Tp \_\_s, \_Tp \_\_a )

Return the Hurwitz zeta function  $\zeta(s,a)$  for all s = 1 and a > -1.

## See also

An efficient algorithm for accelerating the convergence of oscillatory series, useful for computing the polylogarithm and Hurwitz zeta functions, Linas Vep

# **Parameters**

s	The argument $s! = 1$
a	The scale parameter $a > -1$

Definition at line 560 of file sf\_zeta.tcc.

References \_S\_Euler\_Maclaurin\_zeta.

Referenced by \_\_hurwitz\_zeta().

8.3.2.146 template<typename \_Tp > std::complex<\_Tp> std::\_\_detail::\_\_hydrogen ( const unsigned int \_\_n, const unsigned int \_\_n, const unsigned int \_\_n, const \_Tp \_\_r, const \_Tp \_\_theta, const \_Tp \_\_phi )

Definition at line 44 of file sf\_hydrogen.tcc.

References \_\_assoc\_laguerre(), \_\_psi(), and \_\_sph\_legendre().

8.3.2.147 template<typename \_Tp > \_Tp std::\_\_detail::\_\_hyperg ( \_Tp \_\_a, \_Tp \_\_b, \_Tp \_\_c, \_Tp \_\_x )

Return the hypergeometric function  ${}_{2}F_{1}(a,b;c;x)$ .

The hypergeometric function is defined by

$$_{2}F_{1}(a,b;c;x) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)} \sum_{n=0}^{\infty} \frac{\Gamma(a+n)\Gamma(b+n)}{\Gamma(c+n)} \frac{x^{n}}{n!}$$

### **Parameters**

a	The first <i>numerator</i> parameter.
b	The second <i>numerator</i> parameter.
c	The denominator parameter.
X	The argument of the confluent hypergeometric function.

## Returns

The confluent hypergeometric function.

Definition at line 776 of file sf hyperg.tcc.

References \_\_hyperg\_luke(), \_\_hyperg\_reflect(), \_\_hyperg\_series(), \_\_log\_gamma(), and \_\_log\_gamma\_sign().

$$8.3.2.148 \quad template < typename \_Tp > \_Tp \ std::\_detail::\_hyperg\_luke \ ( \ \_Tp \_\_a, \ \_Tp \_\_b, \ \_Tp \_\_c, \ \_Tp \_\_xin \ )$$

Return the hypergeometric function  ${}_2F_1(a,b;c;x)$  by an iterative procedure described in Luke, Algorithms for the Computation of Mathematical Functions.

Definition at line 352 of file sf hyperg.tcc.

Referenced by \_\_hyperg().

Return the hypergeometric function  ${}_2F_1(a,b;c;x)$  by the reflection formulae in Abramowitz & Stegun formula 15.3.6 for d=c-a-b not integral and formula 15.3.11 for d=c-a-b integral. This assumes a,b,c!= negative integer.

The hypergeometric function is defined by

$${}_{2}F_{1}(a,b;c;x) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)} \sum_{n=0}^{\infty} \frac{\Gamma(a+n)\Gamma(b+n)}{\Gamma(c+n)} \frac{x^{n}}{n!}$$

The reflection formula for nonintegral d=c-a-b is:

$${}_{2}F_{1}(a,b;c;x) = \frac{\Gamma(c)\Gamma(d)}{\Gamma(c-a)\Gamma(c-b)} {}_{2}F_{1}(a,b;1-d;1-x) + \frac{\Gamma(c)\Gamma(-d)}{\Gamma(a)\Gamma(b)} {}_{2}F_{1}(c-a,c-b;1+d;1-x)$$

The reflection formula for integral m = c - a - b is:

$${}_{2}F_{1}(a,b;a+b+m;x) = \frac{\Gamma(m)\Gamma(a+b+m)}{\Gamma(a+m)\Gamma(b+m)} \sum_{k=0}^{m-1} \frac{(m+a)_{k}(m+b)_{k}}{k!(1-m)_{k}} -$$

Definition at line 486 of file sf\_hyperg.tcc.

References \_\_hyperg\_series(), \_\_log\_gamma(), \_\_log\_gamma\_sign(), and \_\_psi().

Referenced by \_\_hyperg().

Return the hypergeometric function  ${}_2F_1(a,b;c;x)$  by series expansion.

The hypergeometric function is defined by

$${}_{2}F_{1}(a,b;c;x) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)} \sum_{n=0}^{\infty} \frac{\Gamma(a+n)\Gamma(b+n)}{\Gamma(c+n)} \frac{x^{n}}{n!}$$

This works and it's pretty fast.

# **Parameters**

a	The first <i>numerator</i> parameter.
b	The second <i>numerator</i> parameter.
c	The denominator parameter.
X	The argument of the confluent hypergeometric function.

# Returns

The confluent hypergeometric function.

Definition at line 321 of file sf hyperg.tcc.

Referenced by \_\_hyperg(), and \_\_hyperg\_reflect().

8.3.2.151 template < typename  $\_$ Tp > std::tuple <  $\_$ Tp,  $\_$ Tp,  $\_$ Tp,  $\_$ std:: $\_$ detail:: $\_$ jacobi\_sncndn (  $\_$ Tp  $\_$ *k*,  $\_$ Tp  $\_$ *u* )

Return a tuple of the three primary Jacobi elliptic functions: sn(k, u), cn(k, u), dn(k, u).

Definition at line 414 of file sf theta.tcc.

8.3.2.152 template<typename \_Tp > \_Tp std::\_\_detail::\_\_jacobi\_zeta ( \_Tp \_\_k, \_Tp \_\_phi )

Return the Jacobi zeta function.

Definition at line 971 of file sf\_ellint.tcc.

References \_\_comp\_ellint\_1(), and \_\_ellint\_rj().

8.3.2.153 template<typename \_Tp > \_Tp std::\_\_detail::\_\_laguerre ( unsigned int \_\_n, \_Tp \_\_x )

This routine returns the Laguerre polynomial of order n:  $L_n(x)$ .

The Laguerre polynomial is defined by:

$$L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n} (x^n e^{-x})$$

#### **Parameters**

n	The order of the Laguerre polynomial.
X	The argument of the Laguerre polynomial.

## Returns

The value of the Laguerre polynomial of order n and argument x.

Definition at line 321 of file sf laguerre.tcc.

8.3.2.154 template<typename  $_{\rm Tp}$  >  $_{\rm Tp}$  std::\_\_detail::\_\_legendre\_q ( unsigned int  $_{\it Ll}$ ,  $_{\rm Tp}$   $_{\it Lx}$  )

Return the Legendre function of the second kind by upward recursion on order l.

The Legendre function of the second kind of order l and argument x,  $Q_l(x)$ , is defined by:

$$Q_l(x) = \frac{1}{2^l l!} \frac{d^l}{dx^l} (x^2 - 1)^l$$

## **Parameters**

	The order of the Legendre function. $l>=0$ .
x	The argument of the Legendre function. $ x  <= 1$ .

Definition at line 123 of file sf legendre.tcc.

8.3.2.155 template < typename  $_{\rm Tp} > _{\rm Tp}$  std:: $_{\rm detail}$ :: $_{\rm log}$  bincoef (unsigned int  $_{\rm m}$ , unsigned int  $_{\rm m}$ )

Return the logarithm of the binomial coefficient. The binomial coefficient is given by:

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

.

n	The first argument of the binomial coefficient.
k	The second argument of the binomial coefficient.

#### Returns

The logarithm of the binomial coefficient.

Definition at line 1862 of file sf gamma.tcc.

8.3.2.156 template < typename \_Tp > \_GLIBCXX14\_CONSTEXPR \_Tp std::\_\_detail::\_\_log\_double\_factorial ( \_Tp \_\_x )

Definition at line 2450 of file sf\_gamma.tcc.

References log gamma().

Referenced by \_\_double\_factorial(), and \_\_log\_double\_factorial().

 $8.3.2.157 \quad template < typename \_Tp > \_GLIBCXX14\_CONSTEXPR \_Tp \ std:: \_\_detail:: \_\_log\_double\_factorial \ ( \ int \_\_n \ )$ 

Return the logarithm of the double factorial of the integer n.

The double factorial is defined for integral n by:

$$n!! = 135...(n-2)n, noddn!! = 246...(n-2)n, neven - 1!! = 10!! = 1$$

The double factorial is defined for odd negative integers in the obvious way:

$$(-2m-1)!! = 1/(1(-1)(-3)...(-2m+1)(-2m-1)) = \frac{(-1)^m}{(2m-1)!!}$$

for f[ n = -2m - 1 f].

Definition at line 2516 of file sf gamma.tcc.

References \_\_log\_double\_factorial(), \_\_log\_factorial(), \_\_S\_double\_factorial\_table, and \_S\_neg\_double\_factorial\_table.

8.3.2.158 template < typename \_Tp > \_GLIBCXX14\_CONSTEXPR \_Tp std::\_\_detail::\_\_log\_factorial ( unsigned int \_\_n )

Return the logarithm of the factorial of the integer n.

The factorial is:

$$n! = 12...(n-1)n, 0! = 1$$

Definition at line 2440 of file sf\_gamma.tcc.

References \_\_log\_gamma(), and \_S\_factorial\_table.

Referenced by \_\_log\_double\_factorial().

8.3.2.159 template<typename \_Tp > \_Tp std::\_\_detail::\_\_log\_gamma ( \_Tp \_\_x )

Return  $log(|\Gamma(x)|)$ . This will return values even for x < 0. To recover the sign of  $\Gamma(x)$  for any argument use  $\underline{\hspace{0.5cm}}log\_{\hookleftarrow}$   $gamma\_sign$ .

\_\_x The argument of the log of the gamma function.

## Returns

The logarithm of the gamma function.

Definition at line 1800 of file sf gamma.tcc.

References log gamma lanczos().

Referenced by \_\_beta\_lgamma(), \_\_cyl\_bessel\_ij\_series(), \_\_gamma(), \_\_hyperg(), \_\_hyperg\_reflect(), \_\_log\_  $\hookleftarrow$  double\_factorial(), \_\_log\_factorial(), \_\_log\_pochhammer\_u(), \_\_poly\_laguerre\_large\_n(), \_\_psi(), \_\_riemann\_zeta(),  $\hookleftarrow$  \_\_riemann\_zeta\_glob(), and \_\_sph\_legendre().

8.3.2.160 template < typename \_Tp > \_GLIBCXX14\_CONSTEXPR \_Tp std::\_\_detail::\_\_log\_gamma\_bernoulli ( \_Tp \_\_x )

Return  $log(\Gamma(x))$  by asymptotic expansion with Bernoulli number coefficients. This is like Sterling's approximation.

#### **Parameters**

\_\_x The argument of the log of the gamma function.

## Returns

The logarithm of the gamma function.

Definition at line 1699 of file sf gamma.tcc.

8.3.2.161 template<typename\_Tp > \_GLIBCXX14\_CONSTEXPR\_Tp std::\_\_detail::\_\_log\_gamma\_lanczos ( \_Tp \_\_x )

Return  $log(\Gamma(x))$  by the Lanczos method. This method dominates all others on the positive axis I think.

## **Parameters**

\_\_x The argument of the log of the gamma function.

# Returns

The logarithm of the gamma function.

Definition at line 1755 of file sf gamma.tcc.

Referenced by \_\_log\_gamma().

8.3.2.162 template<typename \_Tp > \_Tp std::\_\_detail::\_\_log\_gamma\_sign ( \_Tp \_\_x )

Return the sign of  $\Gamma(x)$ . At nonpositive integers zero is returned.

# **Parameters**

 $\underline{\phantom{a}}$  The argument of the gamma function.

Returns

The sign of the gamma function.

Definition at line 1831 of file sf gamma.tcc.

Referenced by \_\_hyperg(), \_\_hyperg\_reflect(), and \_\_pochhammer\_l().

8.3.2.163 template<typename\_Tp > \_GLIBCXX14\_CONSTEXPR\_Tp std::\_\_detail::\_\_log\_gamma\_spouge( \_Tp \_\_z )

Return  $\Gamma(z)$  by the Spouge algorithm:

$$\Gamma(z+1) = (z+a)^{z+1/2} e^{-z-a} \left[ \sqrt{2\pi} \sum_{k=1}^{\lceil a \rceil + 1} \frac{c_k(a)}{z+k} \right]$$

where

$$c_k(a) = \frac{(-1)^{k-1}}{(k-1)!} (a-k)^{k-1/2} e^{a-k}$$

and the error is bounded by

$$\epsilon(a) < a^{-1/2} (2\pi)^{-a-1/2}$$

.

See also

Spouge, J.L., Computation of the gamma, digamma, and trigamma functions. SIAM Journal on Numerical Analysis 31, 3 (1994), pp. 931-944

### **Parameters**

\_\_z The argument of the gamma function.

#### Returns

The the gamma function.

Definition at line 1739 of file sf\_gamma.tcc.

8.3.2.164 template<typename \_Tp > \_Tp std::\_\_detail::\_\_log\_pochhammer\_I ( \_Tp \_\_a, \_Tp \_\_n )

Return the logarithm of the lower Pochhammer symbol or the falling factorial function. The lower Pochammer symbol is defined by

$$(a)_n = \prod_{k=0}^{n-1} (a-k), (a)_0 = 1 = \Gamma(a+1)/\Gamma(a-n+1)$$

In particular, f(n) = n! f. Thus this function returns

$$ln[(a)_n] = \Gamma(a+1) - \Gamma(a-n+1), ln[(a)_0] = 0$$

Many notations exist:

,

$$\left\{\begin{array}{c} a \\ n \end{array}\right\}$$

, and others.

Definition at line 2209 of file sf gamma.tcc.

8.3.2.165 template < typename \_Tp > \_Tp std::\_\_detail::\_\_log\_pochhammer\_u ( \_Tp 
$$\_a$$
, \_Tp  $\_n$  )

Return the logarithm of the (upper) Pochhammer symbol or the rising factorial function. The Pochammer symbol is defined by

$$(a)_n = \prod_{k=0}^{n-1} (a+k), (a)_0 = 1 = \Gamma(a+n)/\Gamma(n)$$

Thus this function returns

$$ln[(a)_n] = \Gamma(a+n) - \Gamma(n), ln[(a)_0] = 0$$

Many notations exist:

 $a^{\overline{n}}$ 

,

$$n = n$$

, and others.

Definition at line 2144 of file sf\_gamma.tcc.

References log gamma().

8.3.2.166 template<typename \_Tp > \_Tp std::\_\_detail::\_\_logint ( const \_Tp \_\_x )

Return the logarithmic integral li(x).

The logarithmic integral is given by

$$li(x) = Ei(\log(x))$$

#### **Parameters**

 $\underline{\phantom{a}}$  The argument of the logarithmic integral function.

## Returns

The logarithmic integral.

Definition at line 535 of file sf\_expint.tcc.

References \_\_expint().

8.3.2.167 template<typename \_Tp > \_Tp std::\_\_detail::\_\_owens\_t ( \_Tp \_\_h, \_Tp \_\_a )

Return the Owens T function:

$$T(h,a) = \frac{1}{2\pi} \int_0^a \frac{\exp[-\frac{1}{2}h^2(1+x^2)]}{1+x^2} dx$$

This implementation is a translation of the Fortran implementation in

See also

Patefield, M. and Tandy, D. "Fast and accurate Calculation of Owen's T-Function", Journal of Statistical Software, 5 (5), 1 - 25 (2000)

#### **Parameters**

in	h	The scale parameter.
in	a	The integration limit.

#### Returns

The owens T function.

Definition at line 92 of file sf owens t.tcc.

References \_\_znorm1(), and \_\_znorm2().

8.3.2.168 template < typename \_Tp > \_Tp std::\_\_detail::\_\_pochhammer\_I ( \_Tp \_\_a, \_Tp \_\_n )

Return the logarithm of the lower Pochhammer symbol or the falling factorial function. The lower Pochammer symbol is defined by

$$(a)_n = \prod_{k=0}^{n-1} (a-k), (a)_0 = 1 = \Gamma(a+1)/\Gamma(a-n+1)$$

In particular,  $f(n)_n = n! f$ .

Definition at line 2232 of file sf gamma.tcc.

References \_\_log\_gamma\_sign().

8.3.2.169 template<typename \_Tp > \_Tp std::\_\_detail::\_\_pochhammer\_u ( \_Tp  $\_a$ , \_Tp  $\_n$  )

Return the (upper) Pochhammer function or the rising factorial function. The Pochammer symbol is defined by

$$(a)_n = \prod_{k=0}^{n-1} (a+k), (a)_0 = 1 = \Gamma(a+n)/\Gamma(n)$$

Many notations exist:

$$a^{\overline{n}}$$

 $\begin{vmatrix} a \\ n \end{vmatrix}$ 

, and others.

Definition at line 2170 of file sf gamma.tcc.

8.3.2.170 template<typename\_Tp > \_Tp std::\_\_detail::\_\_poly\_hermite ( unsigned int \_\_n, \_Tp \_\_x )

This routine returns the Hermite polynomial of order n:  $H_n(x)$ .

The Hermite polynomial is defined by:

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2}$$

The Hermite polynomial obeys a reflection formula:

$$H_n(-x) = (-1)^n H_n(x)$$

n	The order of the Hermite polynomial.
X	The argument of the Hermite polynomial.

## Returns

The value of the Hermite polynomial of order n and argument x.

Definition at line 179 of file sf hermite.tcc.

References \_\_poly\_hermite\_asymp(), and \_\_poly\_hermite\_recursion().

 $8.3.2.171 \quad template < typename \_Tp > \_Tp \ std::\__detail::\__poly\_hermite\_asymp \ ( \ unsigned \ int \_\_n, \ \_Tp \_\_x \ )$ 

This routine returns the Hermite polynomial of large order n:  $H_n(x)$ . We assume here that  $x \ge 0$ .

The Hermite polynomial is defined by:

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2}$$

see "Asymptotic analysis of the Hermite polynomials from their differential-difference equation", Diego Dominici, arXiv ← :math/0601078v1 [math.CA] 4 Jan 2006

#### **Parameters**

n	The order of the Hermite polynomial.
X	The argument of the Hermite polynomial.

# Returns

The value of the Hermite polynomial of order n and argument x.

Definition at line 113 of file sf\_hermite.tcc.

References \_\_airy().

Referenced by \_\_poly\_hermite().

8.3.2.172 template < typename \_Tp > \_Tp std::\_\_detail::\_\_poly\_hermite\_recursion ( unsigned int  $\_n$ , \_Tp  $\_x$  )

This routine returns the Hermite polynomial of order n:  $H_n(x)$  by recursion on n.

The Hermite polynomial is defined by:

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2}$$

### **Parameters**

n	The order of the Hermite polynomial.
X	The argument of the Hermite polynomial.

## Returns

The value of the Hermite polynomial of order n and argument x.

Definition at line 69 of file sf hermite.tcc.

Referenced by \_\_poly\_hermite().

8.3.2.173 template<typename\_Tp > \_Tp std::\_\_detail::\_\_poly\_jacobi ( unsigned int \_\_n, \_Tp \_\_alpha, \_Tp \_\_beta, \_Tp \_\_x )

Compute the Jacobi polynomial by recursion on n:

$$2n(\alpha+\beta+n)(\alpha+\beta+2n-2)P_n^{(\alpha,\beta)}(x) = (\alpha+\beta+2n-1)((\alpha^2-\beta^2)+x(\alpha+\beta+2n-2)(\alpha+\beta+2n))P_{n-1}^{(\alpha,\beta)}(x) - 2(\alpha+n-1)(\beta+n-1)(\alpha+\beta+2n-2)P_n^{(\alpha,\beta)}(x) = (\alpha+\beta+2n-1)((\alpha^2-\beta^2)+x(\alpha+\beta+2n-2)(\alpha+\beta+2n))P_{n-1}^{(\alpha,\beta)}(x) = (\alpha+\beta+2n-1)((\alpha^2-\beta^2)+x(\alpha+\beta+2n-2)(\alpha+\beta+2n))P_{n-1}^{(\alpha,\beta)}(x) = (\alpha+\beta+2n-1)(\alpha+\beta+2n-2)(\alpha+2n-2)$$

Definition at line 57 of file sf jacobi.tcc.

References beta().

Referenced by \_\_poly\_radial\_jacobi().

This routine returns the associated Laguerre polynomial of order n, degree  $\alpha$ :  $L_n^a lpha(x)$ .

The associated Laguerre function is defined by

$$L_n^{\alpha}(x) = \frac{(\alpha+1)_n}{n!} F_1(-n; \alpha+1; x)$$

where  $(\alpha)_n$  is the Pochhammer symbol and  ${}_1F_1(a;c;x)$  is the confluent hypergeometric function.

The associated Laguerre polynomial is defined for integral  $\alpha=m$  by:

$$L_n^m(x) = (-1)^m \frac{d^m}{dx^m} L_{n+m}(x)$$

where the Laguerre polynomial is defined by:

$$L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n} (x^n e^{-x})$$

## **Template Parameters**

_Tpa	The type of the degree.
_Tp	The type of the parameter.

# **Parameters**

n	The order of the Laguerre function.
alpha1	The degree of the Laguerre function.
X	The argument of the Laguerre function.

# Returns

The value of the Laguerre function of order n, degree  $\alpha$ , and argument x.

Definition at line 248 of file sf laguerre.tcc.

References \_\_poly\_laguerre\_hyperg(), \_\_poly\_laguerre\_large\_n(), and \_\_poly\_laguerre\_recursion().

Evaluate the polynomial based on the confluent hypergeometric function in a safe way, with no restriction on the arguments.

The associated Laguerre function is defined by

$$L_n^{\alpha}(x) = \frac{(\alpha+1)_n}{n!} F_1(-n; \alpha+1; x)$$

where  $(\alpha)_n$  is the Pochhammer symbol and  ${}_1F_1(a;c;x)$  is the confluent hypergeometric function.

This function assumes x = 0.

This is from the GNU Scientific Library.

# **Template Parameters**

_Тра	The type of the degree.
_ <i>Tp</i>	The type of the parameter.

#### **Parameters**

n	The order of the Laguerre function.
alpha1	The degree of the Laguerre function.
X	The argument of the Laguerre function.

# Returns

The value of the Laguerre function of order n, degree  $\alpha$ , and argument x.

Definition at line 129 of file sf laguerre.tcc.

Referenced by \_\_poly\_laguerre().

8.3.2.176 template < typename \_Tpa , typename \_Tp > \_Tp std::\_\_detail::\_\_poly\_laguerre\_large\_n ( unsigned \_\_n, \_Tpa \_\_alpha1, \_\_Tp \_\_x )

This routine returns the associated Laguerre polynomial of order n, degree  $\alpha > -1$  for large n. Abramowitz & Stegun, 13.5.21.

# **Template Parameters**

_Tpa	The type of the degree.
_ <i>Tp</i>	The type of the parameter.

## **Parameters**

n	The order of the Laguerre function.
alpha1	The degree of the Laguerre function.
X	The argument of the Laguerre function.

# Returns

The value of the Laguerre function of order n, degree  $\alpha$ , and argument x.

This is from the GNU Scientific Library.

Definition at line 72 of file sf\_laguerre.tcc.

References \_\_log\_gamma().

Referenced by \_\_poly\_laguerre().

8.3.2.177 template<typename \_Tpa , typename \_Tp > \_Tp std::\_\_detail::\_\_poly\_laguerre\_recursion ( unsigned int \_\_n, \_Tpa \_\_alpha1, \_Tp \_\_x )

This routine returns the associated Laguerre polynomial of order n, degree  $\alpha$ :  $L_n^{\alpha}(x)$  by recursion.

The associated Laguerre function is defined by

$$L_n^{\alpha}(x) = \frac{(\alpha+1)_n}{n!} F_1(-n; \alpha+1; x)$$

where  $(\alpha)_n$  is the Pochhammer symbol and  ${}_1F_1(a;c;x)$  is the confluent hypergeometric function.

The associated Laguerre polynomial is defined for integral  $\alpha=m$  by:

$$L_n^m(x) = (-1)^m \frac{d^m}{dx^m} L_{n+m}(x)$$

where the Laguerre polynomial is defined by:

$$L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n} (x^n e^{-x})$$

## **Template Parameters**

_Tpa	The type of the degree.
_ <i>Tp</i>	The type of the parameter.

## **Parameters**

n	The order of the Laguerre function.
alpha1	The degree of the Laguerre function.
X	The argument of the Laguerre function.

# Returns

The value of the Laguerre function of order n, degree  $\alpha$ , and argument x.

Definition at line 187 of file sf\_laguerre.tcc.

Referenced by \_\_poly\_laguerre().

8.3.2.178 template < typename \_Tp > \_Tp std::\_\_detail::\_\_poly\_legendre\_p ( unsigned int  $\_I$ , \_Tp  $\_x$  )

Return the Legendre polynomial by upward recursion on order l.

The Legendre function of order l and argument x,  $P_l(x)$ , is defined by:

$$P_l(x) = \frac{1}{2^l l!} \frac{d^l}{dx^l} (x^2 - 1)^l$$

#### **Parameters**

	The order of the Legendre polynomial. $l>=0$ .

\_\_x | The argument of the Legendre polynomial. |x| <= 1.

Definition at line 73 of file sf legendre.tcc.

Referenced by \_\_assoc\_legendre\_p(), and \_\_sph\_legendre().

8.3.2.179 template<typename\_Tp > \_Tp std::\_\_detail::\_\_poly\_radial\_jacobi ( unsigned int \_\_n, unsigned int \_\_n, \_Tp \_\_rho )

Return the radial polynomial  $R_n^m(\rho)$  for non-negative degree n, order m <= n, and real radial argument  $\rho$ .

The radial polynomials are defined by

$$R_n^m(\rho) = \sum_{k=0}^{\frac{n-m}{2}} \frac{(-1)^k (n-k)!}{k!(\frac{n+m}{2}-k)!(\frac{n-m}{2}-k)!} \rho^{n-2k}$$

for n-m even and identically 0 for n-m odd. The radial polynomials can be related to the Zernike polynomials:

$$Z_n^m(\rho,\phi) = R_n^m(\rho)\cos(m\phi)$$

$$Z_n^{-m}(\rho,\phi) = R_n^m(\rho)\sin(m\phi)$$

for non-negative m, n.

See also

zernike for details on the Zernike polynomials.

## **Template Parameters**

_ <i>Tp</i>	The real type of the radial coordinate

# **Parameters**

n	The non-negative degree.
m	The non-negative azimuthal order
rho	The radial argument

Definition at line 138 of file sf\_jacobi.tcc.

References \_\_poly\_jacobi().

Referenced by \_\_zernike(), \_\_gnu\_cxx::radpolyf(), and \_\_gnu\_cxx::radpolyl().

8.3.2.180 template<typename \_Tp > \_Tp std::\_\_detail::\_\_polylog ( \_Tp \_\_s, \_Tp \_\_x )

Return the polylog  $Li_s(x)$  for two real arguments.

## **Parameters**

s	The real index.
x	The real argument.

#### Returns

The complex value of the polylogarithm.

Definition at line 1072 of file sf\_polylog.tcc.

References \_\_fpequal(), and \_\_polylog\_exp().

Referenced by \_\_dirichlet\_beta(), \_\_dirichlet\_eta(), and \_\_polylog().

8.3.2.181 template<typename\_Tp > std::complex<\_Tp > std::\_\_detail::\_\_polylog ( \_Tp \_\_s, std::complex<\_Tp > \_\_w )

Return the polylog in those cases where we can calculate it.

#### **Parameters**

s	The real index.
W	The complex argument.

#### Returns

The complex value of the polylogarithm.

Definition at line 1102 of file sf\_polylog.tcc.

References \_\_fpequal(), \_\_polylog(), and \_\_polylog\_exp().

This is the frontend function which calculates  $Li_s(e^w)$  First we branch into different parts depending on the properties of s. This function is the same irrespective of a real or complex w, hence the template parameter ArgType.

## Note

: I really wish we could return a variant<Tp, std::complex<Tp>>.

#### **Parameters**

s	The real order.
w	The real or complex argument.

# Returns

The real or complex value of Li\_s( $e^{\wedge}$ w).

Definition at line 1039 of file sf\_polylog.tcc.

References \_\_polylog\_exp\_int\_neg(), \_\_polylog\_exp\_int\_pos(), \_\_polylog\_exp\_negative\_real\_part(), \_\_polylog\_exp ← \_\_real\_neg(), and \_\_polylog\_exp\_real\_pos().

Referenced by \_\_bose\_einstein(), \_\_clausen(), \_\_clausen\_c(), \_\_clausen\_s(), \_\_fermi\_dirac(), \_\_hurwitz\_zeta(), and polylog().

This function implements the asymptotic series for the polylog. It is given by

$$2\sum_{k=0}^{\infty} \zeta(2k)w^{s-2k}/\Gamma(s-2k+1) - i\pi w^{(s-1)}/\Gamma(s)$$

for Re(w) >> 1

Don't check this against Mathematica 8. For real u the imaginary part of the polylog is given by  $Im(Li_s(e^u)) = -\pi u^{s-1}/\Gamma(s)$  Check this relation for any benchmark that you use. The use of evenzeta leads to a speedup of about 1000.

s	the real index s.
W	the large complex argument w.

#### Returns

the value of the polylogarithm.

Definition at line 686 of file sf\_polylog.tcc.

References \_\_fpequal().

Referenced by \_\_polylog\_exp\_int\_neg(), \_\_polylog\_exp\_int\_pos(), \_\_polylog\_exp\_real\_neg(), and \_\_polylog\_exp\_\times real\_pos().

8.3.2.184 template<typename \_Tp > std::complex<\_Tp> std::\_\_detail::\_\_polylog\_exp\_int\_neg ( int \_\_s, std::complex< \_Tp > \_\_w )

This treats the case where s is a negative integer.

## **Parameters**

s	a negative integer.
W	an arbitrary complex number

## Returns

the value of the polylogarith,.

Definition at line 856 of file sf polylog.tcc.

 $References \ \_\_clamp\_0\_m2pi(), \ \_\_clamp\_pi(), \ \_\_polylog\_exp\_asymp(), \ \_\_polylog\_exp\_neg(), \ and \ \_\_\hookleftarrow polylog\_exp\_negative\_real\_part().$ 

Referenced by \_\_polylog\_exp().

8.3.2.185 template<typename\_Tp > std::complex<\_Tp> std::\_\_detail::\_\_polylog\_exp\_int\_neg ( const int \_\_s, \_Tp \_\_w )

This treats the case where s is a negative integer and w is a real.

#### **Parameters**

s	a negative integer.
<i>W</i>	the argument.

#### Returns

the value of the polylogarithm.

Definition at line 898 of file sf\_polylog.tcc.

References \_\_fpequal(), \_\_polylog\_exp\_asymp(), \_\_polylog\_exp\_neg(), and \_\_polylog\_exp\_negative\_real\_part().

8.3.2.186 template<typename \_Tp > std::complex<\_Tp> std::\_\_detail::\_\_polylog\_exp\_int\_pos ( unsigned int \_\_s, std::complex< \_Tp > \_\_w )

Here s is a positive integer and the function descends into the different kernels depending on w.

s	a positive integer.
w	an arbitrary complex number.

## Returns

The value of the polylogarithm.

Definition at line 767 of file sf polylog.tcc.

Referenced by \_\_polylog\_exp().

8.3.2.187 template < typename \_Tp > std::complex < \_Tp > std::\_\_detail::\_\_polylog\_exp\_int\_pos ( unsigned int \_\_s, \_Tp \_\_w )

Here s is a positive integer and the function descends into the different kernels depending on w.

#### **Parameters**

s	a positive integer
w	an arbitrary real argument w

## Returns

the value of the polylogarithm.

Definition at line 815 of file sf\_polylog.tcc.

 $References \_\_fpequal(), \_\_polylog\_exp\_asymp(), \_\_polylog\_exp\_negative\_real\_part(), \_\_polylog\_exp\_pos(), and \_\_ \\ \leftarrow riemann\_zeta().$ 

8.3.2.188 template<typename \_Tp > std::complex<\_Tp> std::\_\_detail::\_\_polylog\_exp\_neg ( \_Tp \_\_s, std::complex< \_Tp > \_\_w )

This function treats the cases of negative real index s. Theoretical convergence is present for  $|w| < 2\pi$ . We use an optimized version of

$$Li_s(e^w) = \Gamma(1-s)(-w)^{(s-1)} + (2\pi)^{(-s)}/\pi A_p(w)$$
$$A_p(w) = \sum_k \Gamma(1+k-s)/k! \sin(\pi/2*(s-k))(w/2/\pi)^k \zeta(1+k-s)$$

## **Parameters**

	s	The real index
Ì	w	The complex argument

# Returns

The value of the polylogarithm.

Definition at line 346 of file sf polylog.tcc.

References \_\_fpequal(), \_\_riemann\_zeta(), and \_\_riemann\_zeta\_m\_1().

Referenced by polylog exp int neg(), and polylog exp real neg().

 $8.3.2.189 \quad template < typename \_Tp > std::complex < \_Tp > std::\_detail::\_polylog\_exp\_neg \ ( \ int \_s, \ std::complex < \_Tp > \_w \ )$ 

This function treats the cases of negative integer index s and branches accordingly

s	the integer index s.
w	The Argument w

## Returns

The value of the Polylogarithm evaluated by a suitable function.

Definition at line 564 of file sf polylog.tcc.

References polylog exp neg even(), and polylog exp neg odd().

8.3.2.190 template<typename \_Tp , int \_\_sigma> std::complex<\_Tp> std::\_\_detail::\_\_polylog\_exp\_neg\_even ( unsigned int \_\_n, std::complex< \_Tp > \_\_w )

This function treats the cases of negative integer index s which are multiples of two.

In that case the sine occurring in the expansion occasionally takes on the value zero. We use that to provide an optimized series for p = 2n:

In the template parameter sigma we transport whether p = 4k(sigma = 1) or p = 4k + 2(sigma = -1)

$$Li_p(e^w) = Gamma(1-p)(-w)^{p-1} - A_p(w) - \sigma * B_p(w)$$

with

$$A_p(w) = 2(2\pi)^{(p-1)(-p)!/(2\pi)^{(-p/2)(1+w^2/(4pi^2))^{-1/2+p/2}}\cos((1-p)ArcTan(2pi/w))$$

and

$$B_p(w) = -2(2\pi)^{\ell}(p-1)\sum_{k=0}^{\infty} \Gamma(2+2k-p)/(2k+1)!(-1)^k(w/2\pi)^{\ell}(2k+1)(\zeta(2+2k-p)-1)$$

This is suitable for  $|w| < 2\pi$  The original series is (This might be worthwhile if we use the already present table of the Bernoullis)

$$Li_p(e^w) = \Gamma(1-p)(-w)^{p-1} - \sigma(2\pi)^p/\pi \sum_{k=0}^{\infty} \Gamma(2+2k-p)/(2k+1)!(-1)^k (w/2\pi)^(2k+1)\zeta(2+2k-p)$$

## **Parameters**

n	the integral index $n=4k$ .
w	The complex argument w

#### Returns

the value of the Polylogarithm.

Definition at line 450 of file sf polylog.tcc.

References fpequal().

Referenced by \_\_polylog\_exp\_neg().

8.3.2.191 template<typename \_Tp , int \_\_sigma> std::complex<\_Tp> std::\_\_detail::\_\_polylog\_exp\_neg\_odd ( unsigned int \_\_n, std::complex< \_Tp > \_\_w )

This function treats the cases of negative integer index s which are odd.

In that case the sine occurring in the expansion occasionally vanishes. We use that to provide an optimized series for p=1+2k: In the template parameter sigma we transport whether  $p=1+4k(\sigma=1)$  or  $p=3+4k(\sigma=-1)$ 

$$Li_p(e^w) = \Gamma(1-p)(-w)^{p-1} + \sigma * A_p(w) - \sigma * B_p(w)$$

with

$$A_p(w) = 2(2\pi)^{(p-1)}\Gamma(1-p)(1+w^2/(4\pi^2))^{-1/2+p/2}\cos((1-p)ArcTan(2pi/w))$$

and

$$B_p(w) = 2(2pi)^{(p-1)} \sum_{k=0}^{\infty} \Gamma(1+2k-p)/(2k)! (-w^2/4/\pi^2)^k (\zeta(1+2k-p)-T) p1$$

This is suitable for  $|w| < 2\pi$ . The use of evenzeta gives a speedup of about 50 The original series is (This might be worthwhile if we use the already present table of the Bernoullis)

$$Li_p(e^w) = \Gamma(1-p) * (-w)^{p-1} - \sigma 2(2\pi)^{(p-1)} * \sum_{k=0}^{\infty} \Gamma(1+2k-p)/(2k)! (-1)^k (w/2/\pi)^{(2k)} \zeta(1+2k-p)$$

#### **Parameters**

n	the integral index n = 4k.
w	The complex argument w.

#### Returns

The value of the Polylogarithm.

Definition at line 517 of file sf polylog.tcc.

References \_\_fpequal().

Referenced by \_\_polylog\_exp\_neg().

8.3.2.192 template<typename \_PowTp , typename \_Tp > \_Tp std::\_\_detail::\_\_polylog\_exp\_negative\_real\_part ( \_PowTp \_\_s, \_Tp \_\_w )

Theoretical convergence for Re(w) < 0.

Seems to beat the other expansions for  $Re(w) < -\pi/2 - \pi/5$ . Note that this is an implementation of the basic series:

$$Li_s(e^z) = \sum_{k=1} e^(k * z) * k^(-s)$$

### **Parameters**

s	is an arbitrary type, integral or float.
w	something with a negative real part.

# Returns

the value of the polylogarithm.

Definition at line 737 of file sf polylog.tcc.

References \_\_fpequal().

Referenced by \_\_polylog\_exp(), \_\_polylog\_exp\_int\_neg(), \_\_polylog\_exp\_int\_pos(), \_\_polylog\_exp\_real\_neg(), and \cdot \_\_polylog\_exp\_real\_pos().

8.3.2.193 template<typename \_Tp > std::complex<\_Tp> std::\_\_detail::\_\_polylog\_exp\_pos ( unsigned int \_\_s, std::complex< \_Tp > w )

This function treats the cases of positive integer index s.

$$Li_s(e^w) = \sum_{k=0, k!=s-1} \zeta(s-k)w^k/k! + (H_{s-1} - \log(-w))w^(s-1)/(s-1)!$$

The radius of convergence is |w| < 2pi. Note that this series involves a  $\log(-x)$ . gcc and Mathematica differ in their implementation of  $\log(e^(i\pi))$ : gcc:  $\log(e^(+-i*\pi)) = +-i\pi$  whereas Mathematica doesn't preserve the sign in this case:  $\log(e^(+-i\pi)) = +i\pi$ 

## **Parameters**

-	s	the index s.
_	w	the argument w.

#### Returns

the value of the polylogarithm.

Definition at line 206 of file sf\_polylog.tcc.

References fpequal(), and riemann zeta().

Referenced by \_\_polylog\_exp\_int\_pos(), and \_\_polylog\_exp\_real\_pos().

8.3.2.194 template<typename\_Tp > std::complex<\_Tp> std::\_\_detail::\_\_polylog\_exp\_pos ( unsigned int \_\_s, \_Tp \_\_w )

This function treats the cases of positive integer index s for real w.

This specialization is worthwhile to catch the differing behaviour of log(x).

$$Li_s(e^w) = \sum_{k=0, k!=s-1} \zeta(s-k)w^k/k! + (H_{s-1} - \log(-w))w^(s-1)/(s-1)!$$

The radius of convergence is  $|w| < 2\pi$ . Note that this series involves a  $\log(-x)$ . The use of evenzeta yields a speedup of about 2.5. gcc and Mathematica differ in their implementation of  $\log(e^{(i\pi)})$ : gcc:  $\log(e^{(i\pi)}) = -i\pi$  whereas Mathematica doesn't preserve the sign in this case:  $\log(e^{(i\pi)}) = +i\pi$ 

## **Parameters**

s	the index.
W	the argument

# Returns

the value of the Polylogarithm

Definition at line 279 of file sf polylog.tcc.

References \_\_fpequal(), and \_\_riemann\_zeta().

8.3.2.195 template<typename \_Tp > std::complex<\_Tp> std::\_\_detail::\_\_polylog\_exp\_pos ( \_Tp \_\_s, std::complex< \_Tp > \_\_w )

This function treats the cases of positive real index s.

The defining series is

$$Li_s(e^w) = A_s(w) + B_s(w) + \Gamma(1-s)(-w)^{(s-1)}$$

with

$$A_s(w) = \sum_{k=0}^{m} \zeta(s-k)w^k/k!$$

$$B_s(w) = \sum_{k=m+1}^{\infty} \sin(\pi/2(s-k))\Gamma(1-s+k)\zeta(1-s+k)(w/2/\pi)^k/k!$$

#### **Parameters**

s	the positive real index s.
w	The complex argument w.

## Returns

the value of the polylogarithm.

Definition at line 603 of file sf polylog.tcc.

References \_\_fpequal(), and \_\_riemann\_zeta().

Return the polylog where s is a negative real value and for complex argument. Now we branch depending on the properties of w in the specific functions

## **Parameters**

s	A negative real value that does not reduce to a negative integer.
w	The complex argument.

## Returns

The value of the polylogarithm.

Definition at line 985 of file sf polylog.tcc.

References  $\_$ clamp $\_0$ \_m2pi(),  $\_$ clamp $\_$ pi(),  $\_$ polylog $\_$ exp $\_$ asymp(),  $\_$ polylog $\_$ exp $\_$ neg(), and  $\_$ polylog $\_$ exp $\_$ 

Referenced by \_\_polylog\_exp().

Return the polylog where s is a negative real value and for real argument. Now we branch depending on the properties of w in the specific functions.

#### **Parameters**

s	A negative real value.	
w A real argument.		

# Returns

The value of the polylogarithm.

Definition at line 1013 of file sf\_polylog.tcc.

References \_\_polylog\_exp\_asymp(), \_\_polylog\_exp\_neg(), and \_\_polylog\_exp\_negative\_real\_part().

Return the polylog where s is a positive real value and for complex argument.

#### **Parameters**

s	A positive real number.	
w	the complex argument.	

## Returns

The value of the polylogarithm.

Definition at line 922 of file sf\_polylog.tcc.

Referenced by \_\_polylog\_exp().

8.3.2.199 template<typename\_Tp > std::complex<\_Tp> std::\_\_detail::\_\_polylog\_exp\_real\_pos ( \_Tp \_\_s, \_Tp \_\_w )

Return the polylog where s is a positive real value and the argument is real.

# **Parameters**

s	A positive real number tht does not reduce to an integer.
w	The real argument w.

## Returns

The value of the polylogarithm.

Definition at line 956 of file sf polylog.tcc.

 $References \_\_fpequal(), \_\_polylog\_exp\_asymp(), \_\_polylog\_exp\_negative\_real\_part(), \_\_polylog\_exp\_pos(), and \_\_\leftarrow riemann\_zeta().$ 

8.3.2.200 template < typename  $_{\rm Tp}$  >  $_{\rm Tp}$  std::\_\_detail::\_\_psi (  $_{\rm Tp}$  \_\_x )

Return the digamma function. The digamma or  $\psi(x)$  function is defined by

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

For negative argument the reflection formula is used:

$$\psi(x) = \psi(1-x) - \pi \cot(\pi x)$$

.

Definition at line 2330 of file sf gamma.tcc.

References \_\_psi\_asymp().

Referenced by \_\_expint\_En\_series(), \_\_hydrogen(), \_\_hyperg\_reflect(), and \_\_psi().

8.3.2.201 template<typename \_Tp > \_Tp std::\_\_detail::\_\_psi ( unsigned int \_\_n, \_Tp \_\_x )

Return the polygamma function  $\psi^{(n)}(x)$ .

The polygamma function is related to the Hurwitz zeta function:

$$\psi^{(n)}(x) = (-1)^{n+1} m! \zeta(m+1, x)$$

Definition at line 2395 of file sf gamma.tcc.

References \_\_hurwitz\_zeta(), \_\_log\_gamma(), and \_\_psi().

8.3.2.202 template<typename \_Tp > \_Tp std::\_\_detail::\_\_psi\_asymp ( \_Tp  $\_x$  )

Return the digamma function for large argument. The digamma or  $\psi(x)$  function is defined by

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

.

The asymptotic series is given by:

$$\psi(x) = \ln(x) - \frac{1}{2x} - \sum_{n=1}^{\infty} \frac{B_{2n}}{2nx^{2n}}$$

Definition at line 2299 of file sf gamma.tcc.

Referenced by \_\_psi().

8.3.2.203 template<typename \_Tp > \_Tp std::\_\_detail::\_\_psi\_series ( \_Tp \_\_x )

Return the digamma function by series expansion. The digamma or  $\psi(x)$  function is defined by

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

.

The series is given by:

$$\psi(x) = -\gamma_E - \frac{1}{x} \sum_{k=1}^{\infty} \frac{x-1}{(k+1)(x+k)}$$

Definition at line 2268 of file sf gamma.tcc.

8.3.2.204 template<typename \_Tp > \_Tp std::\_\_detail::\_\_riemann\_zeta ( \_Tp \_\_s )

Return the Riemann zeta function  $\zeta(s)$ .

The Riemann zeta function is defined by:

$$\zeta(s) = \sum_{k=1}^\infty k^{-s} \text{ for } s > 1 \frac{(2\pi)^s}{\pi} \sin(\frac{\pi s}{2}) \Gamma(1-s) \zeta(1-s) \text{ for } s < 1$$

For s < 1 use the reflection formula:

$$\zeta(s) = 2^s \pi^{s-1} \Gamma(1-s) \zeta(1-s)$$

#### **Parameters**

\_\_s The argument

Definition at line 505 of file sf\_zeta.tcc.

 $References \underline{\quad gamma(), \quad log\underline{\quad gamma(), \quad riemann\underline{\quad zeta}\underline{\quad glob(), \quad riemann\underline{\quad zeta}\underline{\quad product(), \ and \quad \underline{\quad riemann}\underline{\quad zeta}\underline{\quad }constant \\ sum().}$ 

Referenced by \_\_polylog\_exp\_int\_pos(), \_\_polylog\_exp\_neg(), \_\_polylog\_exp\_pos(), \_\_polylog\_exp\_real\_pos(), and evenzeta().

8.3.2.205 template<typename \_Tp > \_Tp std::\_\_detail::\_\_riemann\_zeta\_alt ( \_Tp \_\_s )

Evaluate the Riemann zeta function  $\zeta(s)$  by an alternate series for s > 0.

The Riemann zeta function is defined by:

$$\zeta(s) = \sum_{k=1}^{\infty} \frac{1}{k^s} fors > 1$$

For s < 1 use the reflection formula:

$$\zeta(s) = 2^s \pi^{s-1} \Gamma(1-s) \zeta(1-s)$$

Definition at line 329 of file sf\_zeta.tcc.

8.3.2.206 template<typename \_Tp > \_Tp std::\_\_detail::\_\_riemann\_zeta\_euler\_maclaurin ( \_Tp \_\_s )

Evaluate the Riemann zeta function  $\zeta(s)$  by an alternate series for s>0.

This is a specialization of the code for the Hurwitz zeta function.

Definition at line 282 of file sf\_zeta.tcc.

References S Euler Maclaurin zeta.

8.3.2.207 template<typename \_Tp > \_Tp std::\_\_detail::\_\_riemann\_zeta\_glob ( \_Tp \_\_s )

Evaluate the Riemann zeta function by series for all s != 1. Convergence is great until largish negative numbers. Then the convergence of the > 0 sum gets better.

The series is:

$$\zeta(s) = \frac{1}{1 - 2^{1 - s}} \sum_{n = 0}^{\infty} \frac{1}{2^{n + 1}} \sum_{k = 0}^{n} (-1)^k \frac{n!}{(n - k)! k!} (k + 1)^{-s}$$

Havil 2003, p. 206.

The Riemann zeta function is defined by:

$$\zeta(s) = \sum_{k=1}^{\infty} \frac{1}{k^s} fors > 1$$

For s < 1 use the reflection formula:

$$\zeta(s) = 2^s \pi^{s-1} \Gamma(1-s) \zeta(1-s)$$

Definition at line 374 of file sf zeta.tcc.

References log gamma().

Referenced by \_\_riemann\_zeta().

8.3.2.208 template<typename \_Tp > \_Tp std::\_\_detail::\_\_riemann\_zeta\_m\_1 ( \_Tp \_\_s )

Return the Riemann zeta function  $\zeta(s) - 1$ .

#### **Parameters**

\_\_s | The argument s! = 1

Definition at line 672 of file sf zeta.tcc.

References \_\_riemann\_zeta\_m\_1\_sum(), \_S\_num\_zetam1, and \_S\_zetam1.

Referenced by \_\_polylog\_exp\_neg().

8.3.2.209 template<typename \_Tp > \_Tp std::\_\_detail::\_\_riemann\_zeta\_m\_1\_sum ( \_Tp  $\_s$  )

Return the Riemann zeta function  $\zeta(s)-1$  by summation for s>1. This is a small remainder for large s.

The Riemann zeta function is defined by:

$$\zeta(s) = \sum_{k=1}^{\infty} \frac{1}{k^s} fors > 1$$

### **Parameters**

$$\underline{\hspace{0.1cm}}$$
 The argument  $s!=1$ 

Definition at line 645 of file sf zeta.tcc.

Referenced by \_\_\_riemann\_zeta\_m\_1().

8.3.2.210 template<typename \_Tp > \_Tp std::\_\_detail::\_\_riemann\_zeta\_product ( \_Tp \_\_s )

Compute the Riemann zeta function  $\zeta(s)$  using the product over prime factors.

$$\zeta(s) = \prod_{i=1}^{\infty} \frac{1}{1 - p_i^{-s}}$$

where  $p_i$  are the prime numbers.

The Riemann zeta function is defined by:

$$\zeta(s) = \sum_{k=1}^{\infty} \frac{1}{k^s} fors > 1$$

For s < 1 use the reflection formula:

$$\zeta(s) = 2^s \pi^{s-1} \Gamma(1-s) \zeta(1-s)$$

\_\_s The argument

Definition at line 463 of file sf zeta.tcc.

Referenced by \_\_riemann\_zeta().

8.3.2.211 template<typename \_Tp > \_Tp std::\_\_detail::\_\_riemann\_zeta\_sum ( \_Tp \_\_s )

Compute the Riemann zeta function  $\zeta(s)$  by summation for s > 1.

The Riemann zeta function is defined by:

$$\zeta(s) = \sum_{k=1}^{\infty} \frac{1}{k^s} fors > 1$$

For s < 1 use the reflection formula:

$$\zeta(s) = 2^s \pi^{s-1} \Gamma(1-s) \zeta(1-s)$$

Definition at line 254 of file sf zeta.tcc.

Referenced by \_\_riemann\_zeta().

8.3.2.212 template<typename\_Tp > \_\_gnu\_cxx::\_\_promote\_num\_t<\_Tp> std::\_\_detail::\_\_sinc ( \_Tp \_\_a, \_Tp \_\_x )

Return the generalized sinus cardinal function

$$sinc_a(x) = \frac{\sin(\pi x/a)}{(\pi x/a)}$$

.

Definition at line 51 of file sf cardinal.tcc.

 $8.3.2.213 \quad template < typename \_Tp > \_\_gnu\_cxx::\_promote\_num\_t < \_Tp > std::\__detail::\_sinc \left( \ \_Tp \ \_x \ \right)$ 

Return the normalized sinus cardinal function

$$sinc(x) = \frac{\sin(\pi x)}{\pi x}$$

.

Definition at line 98 of file sf\_cardinal.tcc.

8.3.2.214 template<typename\_Tp > \_\_gnu\_cxx::\_\_promote\_num\_t<\_Tp> std::\_\_detail::\_\_sinc\_pi ( \_Tp \_\_x )

Return the unnormalized sinus cardinal function

$$sinc_{\pi}(x) = \frac{\sin(x)}{x}$$

.

Definition at line 78 of file sf cardinal.tcc.

8.3.2.215 template<typename \_Tp > std::pair<\_Tp, \_Tp> std::\_\_detail::\_\_sincosint( \_Tp \_\_x )

This function returns the sine Si(x) and cosine Ci(x) integrals as a pair.

The sine integral is defined by:

$$Si(x) = \int_0^x dt \frac{\sin(t)}{t}$$

The cosine integral is defined by:

$$Ci(x) = \gamma_E + \log(x) + \int_0^x dt \frac{\cos(t) - 1}{t}$$

Definition at line 227 of file sf trigint.tcc.

References \_\_sincosint\_asymp(), \_\_sincosint\_cont\_frac(), and \_\_sincosint\_series().

8.3.2.216 template<typename \_Tp > void std::\_\_detail::\_\_sincosint\_asymp ( \_Tp \_\_t, \_Tp & \_Si, \_Tp & \_Ci )

This function computes the sine Si(x) and cosine Ci(x) integrals by asymptotic series summation for positive argument.

The asymptotic series is very good for x > 50.

Definition at line 163 of file sf trigint.tcc.

Referenced by \_\_sincosint().

8.3.2.217 template<typename \_Tp > void std::\_\_detail::\_\_sincosint\_cont\_frac ( \_Tp \_\_t, \_Tp & \_Si, \_Tp & \_Ci )

This function computes the sine Si(x) and cosine Ci(x) integrals by continued fraction for positive argument.

Definition at line 55 of file sf trigint.tcc.

Referenced by \_\_sincosint().

8.3.2.218 template<typename\_Tp > void std::\_\_detail::\_\_sincosint\_series ( \_Tp \_\_t, \_Tp & \_Si, \_Tp & \_Ci )

This function computes the sine Si(x) and cosine Ci(x) integrals by series summation for positive argument.

Definition at line 98 of file sf\_trigint.tcc.

Referenced by \_\_sincosint().

8.3.2.219 template<typename\_Tp > \_\_gnu\_cxx::\_\_promote\_num\_t<\_Tp> std::\_\_detail::\_\_sinhc ( \_Tp \_\_a, \_Tp \_\_x )

Return the generalized hyperbolic sinus cardinal function

$$sinhc_a(x) = \frac{\sinh(\pi x/a)}{\pi x/a}$$

Definition at line 124 of file sf cardinal.tcc.

8.3.2.220 template<typename\_Tp > \_\_gnu\_cxx::\_promote\_num\_t<\_Tp> std::\_\_detail::\_\_sinhc ( \_Tp \_\_x )

Return the normalized hyperbolic sinus cardinal function

$$sinhc(x) = \frac{\sinh(\pi x)}{\pi x}$$

.

Definition at line 167 of file sf cardinal.tcc.

 $8.3.2.221 \quad template < typename \_Tp > \_gnu\_cxx::\_promote\_num\_t < \_Tp > std::\_detail::\_sinhc\_pi \left( \ \_Tp \_x \ \right)$ 

Return the unnormalized hyperbolic sinus cardinal function

$$sinhc_{\pi}(x) = \frac{\sinh(x)}{x}$$

.

Definition at line 149 of file sf\_cardinal.tcc.

8.3.2.222 template<typename \_Tp > \_Tp std::\_\_detail::\_\_sinhint ( const \_Tp \_\_x )

Return the hyperbolic sine integral li(x).

The hyperbolic sine integral is given by

$$Shi(x) = (Ei(x) - E_1(x))/2$$

#### **Parameters**

X	The argument of the hyperbolic sine integral function.

#### Returns

The hyperbolic sine integral.

Definition at line 581 of file sf expint.tcc.

References \_\_expint\_E1(), and \_\_expint\_Ei().

 $8.3.2.223 \quad template < typename \_Tp > \_Tp \ std::\_\_detail::\_\_sph\_bessel \ ( \ unsigned \ int \_\_n, \ \_Tp \_\_x \ )$ 

Return the spherical Bessel function  $j_n(x)$  of order n and non-negative real argument x.

The spherical Bessel function is defined by:

$$j_n(x) = \left(\frac{\pi}{2x}\right)^{1/2} J_{n+1/2}(x)$$

# **Parameters**

n	The non-negative integral order	
X	x The non-negative real argument	

## Returns

The output spherical Bessel function.

Definition at line 693 of file sf bessel.tcc.

References \_\_sph\_bessel\_jn().

8.3.2.224 template<typename \_Tp > std::complex<\_Tp> std::\_\_detail::\_\_sph\_bessel ( unsigned int \_\_n, std::complex< \_Tp > \_\_z )

Return the complex spherical Bessel function.

in	n	The order for which the spherical Bessel function is evaluated.
in	z	The argument at which the spherical Bessel function is evaluated.

#### Returns

The complex spherical Bessel function.

Definition at line 1314 of file sf\_hankel.tcc.

References \_\_sph\_hankel().

Compute the spherical modified Bessel functions  $i_n(x)$  and  $k_n(x)$  and their first derivatives  $i'_n(x)$  and  $k'_n(x)$  respectively.

## **Parameters**

n	The order of the modified spherical Bessel function.
x The argument of the modified spherical Bessel function.	
i_n	The output regular modified spherical Bessel function.
k_n	The output irregular modified spherical Bessel function.
ip_n	The output derivative of the regular modified spherical Bessel function.
kp_n	The output derivative of the irregular modified spherical Bessel function.

Definition at line 456 of file sf\_mod\_bessel.tcc.

References \_\_cyl\_bessel\_ik().

Compute the spherical Bessel  $j_n(x)$  and Neumann  $n_n(x)$  functions and their first derivatives  $j_n(x)$  and  $n'_n(x)$  respectively.

## **Parameters**

	n	The order of the spherical Bessel function.
	x	The argument of the spherical Bessel function.
out	j_n	The output spherical Bessel function.
out	n_n	The output spherical Neumann function.
out	jp_n	The output derivative of the spherical Bessel function.
out	np_n	The output derivative of the spherical Neumann function.

Definition at line 658 of file sf\_bessel.tcc.

References \_\_cyl\_bessel\_jn().

Referenced by \_\_sph\_bessel(), \_\_sph\_hankel\_1(), \_\_sph\_hankel\_2(), and \_\_sph\_neumann().

```
8.3.2.227 template < typename _Tp > void std::__detail::__sph_hankel ( unsigned int __n, std::complex < _Tp > __z, std::complex < _Tp > & _H1, std::complex < _Tp > & _H2, std::complex < _Tp > & _H2, std::complex < _Tp > & _H2p )
```

Helper to compute complex spherical Hankel functions and their derivatives.

in	n	The order for which the spherical Hankel functions are evaluated.
in	z	The argument at which the spherical Hankel functions are evaluated.
out	_H1	The spherical Hankel function of the first kind.
out	_H1p	The derivative of the spherical Hankel function of the first kind.
out	_H2	The spherical Hankel function of the second kind.
out	_H2p	The derivative of the spherical Hankel function of the second kind.

Definition at line 1258 of file sf\_hankel.tcc.

References \_\_hankel().

Referenced by \_\_sph\_bessel(), \_\_sph\_hankel\_1(), \_\_sph\_hankel\_2(), and \_\_sph\_neumann().

8.3.2.228 template<typename\_Tp > std::complex<\_Tp> std::\_\_detail::\_\_sph\_hankel\_1 ( unsigned int \_\_n, \_Tp \_\_x )

Return the spherical Hankel function of the first kind  $h_n^{(1)}(x)$ .

The spherical Hankel function of the first kind is defined by:

$$h_n^{(1)}(x) = j_n(x) + i n_n(x)$$

#### **Parameters**

n The order of the spherical Neumann function.	
X	The argument of the spherical Neumann function.

## Returns

The output spherical Neumann function.

Definition at line 762 of file sf\_bessel.tcc.

References \_\_sph\_bessel\_jn().

Return the complex spherical Hankel function of the first kind.

# **Parameters**

in	n	The order for which the spherical Hankel function of the first kind is evaluated.
in	z	The argument at which the spherical Hankel function of the first kind is evaluated.

## Returns

The complex spherical Hankel function of the first kind.

Definition at line 1282 of file sf\_hankel.tcc.

References \_\_sph\_hankel().

8.3.2.230 template<typename\_Tp > std::complex<\_Tp> std::\_detail::\_sph\_hankel\_2 ( unsigned int \_\_n, \_Tp \_\_x )

Return the spherical Hankel function of the second kind  $h_n^{(2)}(x)$ .

The spherical Hankel function of the second kind is defined by:

$$h_n^{(2)}(x) = j_n(x) - in_n(x)$$

# **Parameters**

n	The non-negative integral order	
X	The non-negative real argument	

#### Returns

The output spherical Neumann function.

Definition at line 794 of file sf\_bessel.tcc.

References \_\_sph\_bessel\_jn().

8.3.2.231 template<typename \_Tp > std::complex<\_Tp> std::\_\_detail::\_\_sph\_hankel\_2 ( unsigned int \_\_n, std::complex< \_Tp > \_\_z )

Return the complex spherical Hankel function of the second kind.

#### **Parameters**

in	n	The order for which the spherical Hankel function of the second kind is evaluated.
in	z	The argument at which the spherical Hankel function of the second kind is evalu-
		ated.

# Returns

The complex spherical Hankel function of the second kind.

Definition at line 1298 of file sf\_hankel.tcc.

References \_\_sph\_hankel().

8.3.2.232 template<typename \_Tp > std::complex<\_Tp> std::\_\_detail::\_\_sph\_harmonic ( unsigned int \_\_l, int \_\_m, \_Tp \_\_theta, \_\_Tp \_\_phi )

Return the spherical harmonic function.

The spherical harmonic function of l, m, and  $\theta, \phi$  is defined by:

$$Y_l^m(\theta,\phi) = (-1)^m \left[ \frac{(2l+1)}{4\pi} \frac{(l-m)!}{(l+m)!} \right] P_l^{|m|}(\cos\theta) \exp^{im\phi}$$

/	The order of the spherical harmonic function. $l>=0$ .
m	The order of the spherical harmonic function. $m <= l$ .
theta	The radian polar angle argument of the spherical harmonic function.
phi	The radian azimuthal angle argument of the spherical harmonic function.

Definition at line 350 of file sf\_legendre.tcc.

References \_\_sph\_legendre().

8.3.2.233 template < typename \_Tp > \_Tp std::\_\_detail::\_\_sph\_legendre ( unsigned int \_\_I, unsigned int \_\_m, \_Tp \_\_theta )

Return the spherical associated Legendre function.

The spherical associated Legendre function of l, m, and  $\theta$  is defined as  $Y_l^m(\theta, 0)$  where

$$Y_l^m(\theta,\phi) = (-1)^m \left[ \frac{(2l+1)}{4\pi} \frac{(l-m)!}{(l+m)!} \right] P_l^m(\cos\theta) \exp^{im\phi}$$

is the spherical harmonic function and  $P_l^m(\boldsymbol{x})$  is the associated Legendre function.

This function differs from the associated Legendre function by argument ( $x = \cos(\theta)$ ) and by a normalization factor but this factor is rather large for large l and m and so this function is stable for larger differences of l and m.

#### **Parameters**

	The order of the spherical associated Legendre function. $l>=0$ .
m	The order of the spherical associated Legendre function. $m <= l$ .
theta	The radian polar angle argument of the spherical associated Legendre function.

Definition at line 253 of file sf\_legendre.tcc.

References \_\_log\_gamma(), and \_\_poly\_legendre\_p().

Referenced by \_\_hydrogen(), and \_\_sph\_harmonic().

8.3.2.234 template < typename  $_{\rm Tp} > _{\rm Tp}$  std:: $_{\rm detail}$ :: $_{\rm sph}$  neumann (unsigned int  $_{\rm n}$ ,  $_{\rm Tp}$   $_{\rm x}$ )

Return the spherical Neumann function  $n_n(x)$  of order n and non-negative real argument x.

The spherical Neumann function is defined by:

$$n_n(x) = \left(\frac{\pi}{2x}\right)^{1/2} N_{n+1/2}(x)$$

# **Parameters**

n	The order of the spherical Neumann function.
X	The argument of the spherical Neumann function.

## Returns

The output spherical Neumann function.

Definition at line 730 of file sf bessel.tcc.

References sph bessel jn().

8.3.2.235 template < typename \_Tp > std::complex < \_Tp > std::\_\_detail::\_\_sph\_neumann ( unsigned int \_\_n, std::complex < \_Tp > \_\_z )

Return the complex spherical Neumann function.

in	n	The order for which the spherical Neumann function is evaluated.
in	z	The argument at which the spherical Neumann function is evaluated.

## Returns

The complex spherical Neumann function.

Definition at line 1330 of file sf hankel.tcc.

References \_\_sph\_hankel().

8.3.2.236 template < typename \_Tp > \_GLIBCXX14\_CONSTEXPR \_Tp std::\_\_detail::\_\_student\_t\_cdf ( \_Tp \_\_t, unsigned int \_\_nu )

Return the Students T probability function.

The students T propability function is related to the incomplete beta function:

$$A(t|\nu) = 1 - I_{\frac{\nu}{\nu + t^2}}(\frac{\nu}{2}, \frac{1}{2})A(t|\nu) =$$

## **Parameters**

t	
nu	

Definition at line 301 of file sf beta.tcc.

References beta inc().

8.3.2.237 template<typename\_Tp > \_GLIBCXX14\_CONSTEXPR \_Tp std::\_\_detail::\_\_student\_t\_cdfc ( \_Tp \_\_t, unsigned int \_\_nu )

Return the complement of the Students T probability function.

The complement of the students T propability function is:

$$A_c(t|\nu) = I_{\frac{\nu}{\nu+t^2}}(\frac{\nu}{2}, \frac{1}{2}) = 1 - A(t|\nu)$$

# **Parameters**

	t
n	

Definition at line 324 of file sf beta.tcc.

References \_\_beta\_inc().

Return the exponential theta-1 function of period nu and argument x.

The Neville theta-1 function is defined by

$$\theta_1(\nu, x) = \frac{1}{\sqrt{\pi x}} \sum_{j=-\infty}^{+\infty} (-1)^j \exp\left(\frac{-(\nu + j - 1/2)^2}{x}\right)$$

nu	The periodic (period = 2) argument
X	The argument

Definition at line 190 of file sf theta.tcc.

References \_\_theta\_2().

Referenced by \_\_theta\_s().

Return the exponential theta-2 function of period nu and argument x.

The exponential theta-2 function is defined by

$$\theta_2(\nu, x) = \frac{1}{\sqrt{\pi x}} \sum_{j=-\infty}^{+\infty} (-1)^j \exp\left(\frac{-(\nu+j)^2}{x}\right)$$

## **Parameters**

nu	The periodic (period = 2) argument
x	The argument

Definition at line 162 of file sf\_theta.tcc.

References \_\_theta\_2\_asymp(), and \_\_theta\_2\_sum().

Referenced by \_\_theta\_1(), and \_\_theta\_c().

Compute and return the  $\theta_2$  function by series expansion.

Definition at line 103 of file sf\_theta.tcc.

Referenced by \_\_theta\_2().

8.3.2.241 template < typename 
$$_{\rm Tp} > _{\rm Tp}$$
 std::\_\_detail::\_\_theta\_2\_sum (  $_{\rm Tp}$   $_{\rm nu}$ ,  $_{\rm Tp}$   $_{\rm x}$  )

Compute and return the  $\theta_1$  function by series expansion.

Definition at line 49 of file sf\_theta.tcc.

Referenced by \_\_theta\_2().

Return the exponential theta-3 function of period nu and argument x.

The exponential theta-3 function is defined by

$$\theta_3(\nu, x) = \frac{1}{\sqrt{\pi x}} \sum_{j=-\infty}^{+\infty} \exp\left(\frac{-(\nu+j)^2}{x}\right)$$

#### **Parameters**

nu	The periodic (period = 1) argument
X	The argument

Definition at line 216 of file sf theta.tcc.

References \_\_theta\_3\_asymp(), and \_\_theta\_3\_sum().

Referenced by \_\_theta\_4(), and \_\_theta\_d().

Compute and return the  $\theta_3$  function by asymptotic series expansion.

Definition at line 128 of file sf theta.tcc.

Referenced by \_\_theta\_3().

Compute and return the  $\theta_3$  function by series expansion.

Definition at line 77 of file sf\_theta.tcc.

Referenced by \_\_theta\_3().

Return the exponential theta-2 function of period nu and argument x.

The exponential theta-2 function is defined by

$$\theta_2(\nu, x) = \frac{1}{\sqrt{\pi x}} \sum_{j=-\infty}^{+\infty} (-1)^j \exp\left(\frac{-(\nu+j)^2}{x}\right)$$

#### **Parameters**

nu	The periodic (period = 2) argument
X	The argument

Definition at line 244 of file sf theta.tcc.

References \_\_theta\_3().

Referenced by \_\_theta\_n().

Return the Neville  $\theta_c$  function

Definition at line 337 of file sf\_theta.tcc.

References \_\_comp\_ellint\_1(), \_\_ellnome(), and \_\_theta\_2().

8.3.2.247 template<typename \_Tp > \_Tp std::\_\_detail::\_\_theta\_d ( \_Tp \_\_k, \_Tp \_\_x )

Return the Neville  $\theta_d$  function

Definition at line 362 of file sf\_theta.tcc.

References \_\_comp\_ellint\_1(), \_\_ellnome(), and \_\_theta\_3().

8.3.2.248 template<typename \_Tp > \_Tp std::\_\_detail::\_\_theta\_n ( \_Tp  $\_k$ , \_Tp  $\_x$  )

Return the Neville  $\theta_n$  function

Definition at line 387 of file sf\_theta.tcc.

References \_\_comp\_ellint\_1(), \_\_ellnome(), and \_\_theta\_4().

8.3.2.249 template<typename \_Tp > \_Tp std::\_\_detail::\_\_theta\_s ( \_Tp  $\_k$ , \_Tp  $\_x$  )

Return the Neville  $\theta_s$  function

Definition at line 311 of file sf\_theta.tcc.

References \_\_comp\_ellint\_1(), \_\_ellnome(), and \_\_theta\_1().

8.3.2.250 template<typename \_Tp > \_\_gnu\_cxx::\_\_promote\_num\_t<\_Tp> std::\_\_detail::\_\_zernike ( unsigned int \_\_n, int \_\_m, \_\_Tp \_\_rho, \_Tp \_\_phi )

Return the Zernicke polynomial  $Z_n^m(\rho,\phi)$  for non-negative degree n, signed order m, and real radial argument  $\rho$  and azimuthal angle  $\phi$ .

The even Zernicke polynomials are defined by:

$$Z_n^m(\rho,\phi) = R_n^m(\rho)\cos(m\phi)$$

and the odd Zernicke polynomials are defined by:

$$Z_n^{-m}(\rho,\phi) = R_n^m(\rho)\sin(m\phi)$$

for non-negative degree m and m <= n and where  $R_n^m(\rho)$  is the radial polynomial (

See also

poly radial jacobi).

## **Template Parameters**

	_ <i>Tp</i>	The real type of the radial coordinate and azimuthal angle
,		

#### **Parameters**

n	The non-negative degree.
m	The azimuthal order

rho	The radial coordinate
phi	The azimuthal angle

Definition at line 183 of file sf\_jacobi.tcc.

References \_\_poly\_radial\_jacobi().

8.3.2.251 template<typename \_Tp > \_Tp std::\_\_detail::\_\_znorm1 ( \_Tp  $\_x$  )

Definition at line 58 of file sf\_owens\_t.tcc.

Referenced by \_\_owens\_t().

8.3.2.252 template<typename \_Tp > \_Tp std::\_\_detail::\_\_znorm2 ( \_Tp  $\_x$  )

Definition at line 47 of file sf owens t.tcc.

Referenced by \_\_owens\_t().

8.3.2.253 template < typename  $_{Tp} = double > _{Tp} std::__detail::evenzeta (unsigned int <math>_{k}$ )

A function to calculate the values of zeta at even positive integers. For values smaller than thirty a table is used.

#### **Parameters**

1.	an interior at cubich we explicate the Diagrams and function
K	an integer at which we evaluate the Riemann zeta function.
<del></del> '	

#### Returns

zeta(k)

Definition at line 156 of file sf\_polylog.tcc.

References riemann zeta().

## 8.3.3 Variable Documentation

8.3.3.1 constexpr size\_t std::\_\_detail::\_Num\_Euler\_Maclaurin\_zeta = 100

Coefficients for Euler-Maclaurin summation of zeta functions.

$$B_{2j}/(2j)!$$

where  $\mathcal{B}_k$  are the Bernoulli numbers.

Definition at line 65 of file sf\_zeta.tcc.

8.3.3.2 constexpr Factorial\_table < long double > std::\_\_detail::\_S\_double\_factorial\_table[301]

Definition at line 274 of file sf\_gamma.tcc.

Referenced by \_\_double\_factorial(), and \_\_log\_double\_factorial().

```
8.3.3.3 constexpr long double std::__detail::_S_Euler_Maclaurin_zeta[_Num_Euler_Maclaurin_zeta]
Definition at line 68 of file sf zeta.tcc.
Referenced by __hurwitz_zeta_euler_maclaurin(), and __riemann_zeta_euler_maclaurin().
8.3.3.4 constexpr Factorial table<long double> std::_detail::_S_factorial_table[171]
Definition at line 84 of file sf gamma.tcc.
Referenced by __factorial(), and __log_factorial().
8.3.3.5 constexpr Factorial table<long double> std::__detail::_S_neg_double_factorial_table[999]
Definition at line 595 of file sf_gamma.tcc.
Referenced by __double_factorial(), and __log_double_factorial().
8.3.3.6 template < typename _Tp > constexpr std::size_t std::__detail::_S_num_double_factorials = 0
Definition at line 259 of file sf gamma.tcc.
8.3.3.7 template <> constexpr std::size_t std:: detail:: S num double factorials < double >= 301
Definition at line 264 of file sf_gamma.tcc.
8.3.3.8 template <> constexpr std::size_t std:: detail:: S num double factorials < float > = 57
Definition at line 262 of file sf gamma.tcc.
8.3.3.9 template <> constexpr std::size_t std::__detail::_S_num_double_factorials < long double >= 301
Definition at line 266 of file sf gamma.tcc.
8.3.3.10 template < typename _Tp > constexpr std::size t std:: __detail:: S _num_factorials = 0
Definition at line 69 of file sf_gamma.tcc.
8.3.3.11 template <> constexpr std::size_t std:: detail:: S num factorials < double > = 171
Definition at line 74 of file sf gamma.tcc.
8.3.3.12 template<> constexpr std::size_t std::__detail::_S_num_factorials< float > = 35
```

Definition at line 72 of file sf gamma.tcc.

8.3.3.13 template <> constexpr std::size\_t std::\_\_detail::\_S\_num\_factorials < long double > = 171

Definition at line 76 of file sf\_gamma.tcc.

8.3.3.14 template < typename \_Tp > constexpr std::size\_t std::\_\_detail::\_S\_num\_neg\_double\_factorials = 0

Definition at line 579 of file sf\_gamma.tcc.

8.3.3.15 template<> constexpr std::size\_t std::\_\_detail::\_S\_num\_neg\_double\_factorials< double > = 150

Definition at line 584 of file sf\_gamma.tcc.

8.3.3.16 template <> constexpr std::size\_t std::\_\_detail::\_S\_num\_neg\_double\_factorials < float > = 27

Definition at line 582 of file sf\_gamma.tcc.

8.3.3.17 template<> constexpr std::size\_t std::\_\_detail::\_S\_num\_neg\_double\_factorials< long double > = 999

Definition at line 586 of file sf\_gamma.tcc.

8.3.3.18 constexpr size\_t std::\_\_detail::\_S\_num\_zetam1 = 33

Table of zeta(n) - 1 from 2 - 32. MPFR - 128 bits.

Definition at line 592 of file sf\_zeta.tcc.

Referenced by \_\_\_riemann\_zeta\_m\_1().

8.3.3.19 constexpr long double std::\_\_detail::\_S\_zetam1[\_S\_num\_zetam1]

Definition at line 596 of file sf zeta.tcc.

Referenced by \_\_riemann\_zeta\_m\_1().

Namoenaco	<b>Documentation</b>
namespace	Documentation

## **Chapter 9**

## **Class Documentation**

9.1 std::\_\_detail::\_Factorial\_table< \_Tp > Struct Template Reference

## **Public Attributes**

- \_Tp \_\_factorial
- \_Tp \_\_log\_factorial
- unsigned int \_\_n

## 9.1.1 Detailed Description

template<typename \_Tp>struct std::\_\_detail::\_Factorial\_table< \_Tp>

Definition at line 61 of file sf\_gamma.tcc.

#### 9.1.2 Member Data Documentation

 $9.1.2.1 \quad template < typename \_Tp > \_Tp \ std::\_\_detail::\_Factorial\_table < \_Tp >::\_\_factorial$ 

Definition at line 64 of file sf gamma.tcc.

 $9.1.2.2 \quad template < typename \_Tp > \_Tp \ std::\_\_detail::\_Factorial\_table < \_Tp > ::\_\_log\_factorial$ 

Definition at line 65 of file sf\_gamma.tcc.

9.1.2.3 template < typename  $_{Tp} >$  unsigned int std:: $_{detail}$ :: $_{factorial}$  template <  $_{Tp} >$ :: $_{n}$ 

Definition at line 63 of file sf\_gamma.tcc.

The documentation for this struct was generated from the following file:

• bits/sf\_gamma.tcc

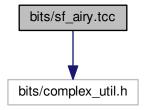
236	Class Documentation

# **Chapter 10**

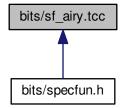
# **File Documentation**

## 10.1 bits/sf\_airy.tcc File Reference

#include <bits/complex\_util.h>
Include dependency graph for sf\_airy.tcc:



This graph shows which files directly or indirectly include this file:



## **Namespaces**

- std
- std:: detail

## **Macros**

#define \_GLIBCXX\_BITS\_SF\_AIRY\_TCC 1

#### **Functions**

template<typename \_Tp >
 void std::\_\_detail::\_\_airy (const std::complex< \_Tp > &\_\_z, \_Tp \_\_eps, std::complex< \_Tp > &\_Ai, std
 ::complex< \_Tp > &\_Aip, std::complex< \_Tp > &\_Bi, std::complex< \_Tp > &\_Bip)

This function computes the Airy function Ai(z) and its first derivative in the complex z-plane.

template<typename \_Tp >
 std::complex< \_Tp > std::\_\_detail::\_\_airy\_ai (std::complex< \_Tp > \_\_z)

Return the complex Airy Ai function.

template<typename \_Tp >
 void std::\_\_detail::\_\_airy\_asymp\_absarg\_ge\_pio3 (std::complex < \_Tp > \_\_z, std::complex < \_Tp > &\_Ai, std
 ::complex < \_Tp > &\_Aip, int \_\_sign=-1)

This function evaluates Ai(z) and Ai'(z) from their asymptotic expansions for  $|arg(z)| < 2 * \pi/3$ . For speed, the number of terms needed to achieve about 16 decimals accuracy is tabled and determined from abs(z).

template<typename \_Tp >
 void std::\_\_detail::\_\_airy\_asymp\_absarg\_lt\_pio3 (std::complex < \_Tp > \_\_z, std::complex < \_Tp > &\_Ai, std
 ::complex < Tp > & Aip)

This function evaluates Ai(z) and Ai'(z) from their asymptotic expansions for |arg(-z)| < pi/3. For speed, the number of terms needed to achieve about 16 decimals accuracy is tabled and determined from |z|.

- template<typename \_Tp >
   void std::\_\_detail::\_\_airy\_bessel\_i (const std::complex< \_Tp > &\_\_z, \_Tp \_\_eps, std::complex< \_Tp > &\_lp1d3, std::complex< \_Tp > &\_lm1d3, std::complex< \_Tp > &\_lm2d3)
- template<typename \_Tp >
   void std::\_\_detail::\_\_airy\_bessel\_k (const std::complex< \_Tp > &\_\_z, \_Tp \_\_eps, std::complex< \_Tp > &\_ 
   Kp1d3, std::complex< \_Tp > &\_Kp2d3)

Compute approximations to the modified Bessel functions of the second kind of orders 1/3 and 2/3 for moderate arguments.

template<typename \_Tp >
 std::complex< \_Tp > std::\_\_detail::\_\_airy\_bi (std::complex< \_Tp > \_\_z)
 Return the complex Airy Bi function.

template<typename \_Tp >
 void std::\_\_detail::\_\_airy\_hyperg\_rational (const std::complex< \_Tp > &\_\_z, std::complex< \_Tp > &\_Ai, std↔
 ::complex< Tp > & Aip, std::complex< Tp > & Bi, std::complex< Tp > & Bip)

This function computes rational approximations to the hypergeometric functions related to the modified Bessel functions of orders  $\nu=+-1/3$  and  $\nu+-2/3$ . That is, A(z)/B(z), Where A(z) and B(z) are cubic polynomials with real coefficients, approximates

$$\frac{\Gamma(\nu+1)}{(z/2)^n u} I_{\nu}(z) =_0 F_1(;\nu+1;z^2/4),$$

where the function on the right is a confluent hypergeometric limit function. For |z| <= 1/4 and |arg(z)| <= pi/2, the approximations are accurate to about 16 decimals.

## 10.1.1 Detailed Description

This is an internal header file, included by other library headers. You should not attempt to use it directly.

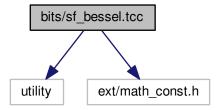
## 10.1.2 Macro Definition Documentation

10.1.2.1 #define \_GLIBCXX\_BITS\_SF\_AIRY\_TCC 1

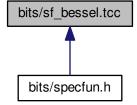
Definition at line 31 of file sf\_airy.tcc.

## 10.2 bits/sf\_bessel.tcc File Reference

```
#include <utility>
#include <ext/math_const.h>
Include dependency graph for sf bessel.tcc:
```



This graph shows which files directly or indirectly include this file:



## **Namespaces**

std

• std::\_\_detail

## **Macros**

#define \_GLIBCXX\_BITS\_SF\_BESSEL\_TCC 1

## **Functions**

template<typename \_Tp >
 \_Tp std::\_\_detail::\_\_cyl\_bessel\_ij\_series (\_Tp \_\_nu, \_Tp \_\_x, \_Tp \_\_sgn, unsigned int \_\_max\_iter)

This routine returns the cylindrical Bessel functions of order  $\nu$ :  $J_{\nu}$  or  $I_{\nu}$  by series expansion.

template<typename\_Tp>

```
_Tp std::__detail::__cyl_bessel_j (_Tp __nu, _Tp __x)
```

Return the Bessel function of order  $\nu$ :  $J_{\nu}(x)$ .

template<typename\_Tp>

```
void std::__detail::__cyl_bessel_jn (_Tp __nu, _Tp __x, _Tp &_Jnu, _Tp &_Nnu, _Tp &_Jpnu, _Tp &_Npnu)
```

Return the cylindrical Bessel functions and their derivatives of order  $\nu$  by various means.

template<typename\_Tp>

void std::\_\_detail::\_\_cyl\_bessel\_jn\_asymp (\_Tp \_\_nu, \_Tp \_\_x, \_Tp &\_Jnu, \_Tp &\_Nnu, \_Tp &\_Jpnu, \_Tp &\_↔ Npnu)

This routine computes the asymptotic cylindrical Bessel and Neumann functions of order nu:  $J_{\nu}(x)$ ,  $N_{\nu}(x)$ . Use this for  $x >> nu^2 + 1$ .

template<typename \_Tp >

void std::\_\_detail::\_\_cyl\_bessel\_jn\_steed (\_Tp \_\_nu, \_Tp \_\_x, \_Tp &\_Jnu, \_Tp &\_Nnu, \_Tp &\_Jpnu, \_Tp &\_↔ Npnu)

Compute the Bessel  $J_{\nu}(x)$  and Neumann  $N_{\nu}(x)$  functions and their first derivatives  $J'_{\nu}(x)$  and  $N'_{\nu}(x)$  respectively. These four functions are computed together for numerical stability.

template<typename\_Tp>

Return the cylindrical Hankel function of the first kind  $H_{\nu}^{(1)}(x)$ .

• template<typename  $_{\rm Tp}>$ 

Return the cylindrical Hankel function of the second kind  $H_n^{(2)}u(x)$ .

• template<typename \_Tp >

Return the Neumann function of order  $\nu$ :  $N_{\nu}(x)$ .

template<typename\_Tp>

Compute the gamma functions required by the Temme series expansions of  $N_{\nu}(x)$  and  $K_{\nu}(x)$ .

$$\Gamma_1 = \frac{1}{2\mu} \left[ \frac{1}{\Gamma(1-\mu)} - \frac{1}{\Gamma(1+\mu)} \right]$$

and

$$\Gamma_2 = \frac{1}{2} \left[ \frac{1}{\Gamma(1-\mu)} + \frac{1}{\Gamma(1+\mu)} \right]$$

where  $-1/2 <= \mu <= 1/2$  is  $\mu = \nu - N$  and N. is the nearest integer to  $\nu$ . The values of  $\Gamma(1 + \mu)$  and  $\Gamma(1 - \mu)$  are returned as well.

• template<typename  $_{\mathrm{Tp}}>$ 

Return the spherical Bessel function  $j_n(x)$  of order n and non-negative real argument x.

template<typename \_Tp >
 void std::\_\_detail::\_\_sph\_bessel\_jn (unsigned int \_\_n, \_Tp \_\_x, \_Tp &\_\_jn, \_Tp &\_\_nn, \_Tp &\_\_jp\_n, \_Tp &\_\_np\_n)

Compute the spherical Bessel  $j_n(x)$  and Neumann  $n_n(x)$  functions and their first derivatives  $j_n(x)$  and  $n'_n(x)$  respectively.

template < typename \_Tp >
 std::complex < \_Tp > std::\_\_detail::\_\_sph\_hankel\_1 (unsigned int \_\_n, \_Tp \_\_x)

Return the spherical Hankel function of the first kind  $h_n^{(1)}(x)$ .

template<typename \_Tp >
 std::complex< \_Tp > std::\_\_detail::\_\_sph\_hankel\_2 (unsigned int \_\_n, \_Tp \_\_x)

Return the spherical Hankel function of the second kind  $h_n^{(2)}(x)$ .

template<typename\_Tp >
 \_Tp std::\_\_detail::\_\_sph\_neumann (unsigned int \_\_n, \_Tp \_\_x)

Return the spherical Neumann function  $n_n(x)$  of order n and non-negative real argument x.

## 10.2.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

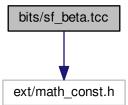
#### 10.2.2 Macro Definition Documentation

10.2.2.1 #define GLIBCXX BITS SF BESSEL TCC 1

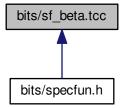
Definition at line 47 of file sf\_bessel.tcc.

## 10.3 bits/sf beta.tcc File Reference

#include <ext/math\_const.h>
Include dependency graph for sf\_beta.tcc:



This graph shows which files directly or indirectly include this file:



## **Namespaces**

- std
- std::\_\_detail

## **Macros**

• #define GLIBCXX BITS SF BETA TCC 1

## **Functions**

```
template<typename _Tp >
 _Tp std::__detail::__beta (_Tp __a, _Tp __b)
     Return the beta function B(a,b).
template<typename _Tp >
  _Tp std::__detail::__beta_gamma (_Tp __a, _Tp __b)
     Return the beta function: B(a, b).
template<typename _Tp >
  _Tp std::__detail::__beta_inc (_Tp __a, _Tp __b, _Tp __x)
template<typename _Tp >
  _Tp std::__detail::__beta_inc_cont_frac (_Tp __a, _Tp __b, _Tp __x)
template<typename _Tp >
 _Tp std::__detail::__beta_lgamma (_Tp __a, _Tp __b)
      Return the beta function B(a,b) using the log gamma functions.

    template<typename</li>
    Tp >

 _Tp std::__detail::__beta_product (_Tp __a, _Tp __b)
      Return the beta function B(x, y) using the product form.

    template<typename</li>
    Tp >

  _GLIBCXX14_CONSTEXPR _Tp std::__detail::__binomial_cdf (_Tp __p, unsigned int __n, unsigned int __k)
      Return the binomial cumulative distribution function.

    template<typename</li>
    Tp >

  _GLIBCXX14_CONSTEXPR _Tp std::__detail::__binomial_cdfc (_Tp __p, unsigned int __n, unsigned int __k)
     Return the complementary binomial cumulative distribution function.
```

template<typename \_Tp >
 \_GLIBCXX14\_CONSTEXPR \_Tp std::\_\_detail::\_\_fisher\_f\_cdf (\_Tp \_\_F, unsigned int \_\_nu1, unsigned int \_\_nu2)

Return the F-distribution propability function. This returns the probability that the observed chi-square for a correct model exceeds the value  $\chi^2$ .

Return the F-distribution propability function. This returns the probability that the observed chi-square for a correct model exceeds the value  $\chi^2$ .

template<typename\_Tp >
 \_GLIBCXX14\_CONSTEXPR \_Tp std::\_\_detail::\_\_student\_t\_cdf (\_Tp \_\_t, unsigned int \_\_nu)

Return the Students T probability function.

template < typename \_Tp >
 \_GLIBCXX14\_CONSTEXPR \_Tp std::\_\_detail::\_\_student\_t\_cdfc (\_Tp \_\_t, unsigned int \_\_nu)

Return the complement of the Students T probability function.

## 10.3.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <cmath>.

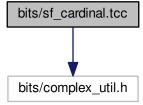
## 10.3.2 Macro Definition Documentation

10.3.2.1 #define \_GLIBCXX\_BITS\_SF\_BETA\_TCC 1

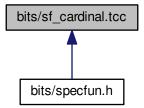
Definition at line 49 of file sf\_beta.tcc.

## 10.4 bits/sf cardinal.tcc File Reference

#include <bits/complex\_util.h>
Include dependency graph for sf cardinal.tcc:



This graph shows which files directly or indirectly include this file:



## **Namespaces**

- std
- std:: detail

#### **Macros**

• #define \_GLIBCXX\_BITS\_SF\_CARDINAL\_TCC 1

## **Functions**

template<typename \_Tp >
 \_\_gnu\_cxx::\_\_promote\_num\_t< \_Tp > std::\_\_detail::\_\_sinc (\_Tp \_\_a, \_Tp \_\_x)

Return the generalized sinus cardinal function

$$sinc_a(x) = \frac{\sin(\pi x/a)}{(\pi x/a)}$$

template<typename \_Tp >

\_\_gnu\_cxx::\_\_promote\_num\_t< \_Tp > std::\_\_detail::\_\_sinc (\_Tp \_\_x)

Return the normalized sinus cardinal function

$$sinc(x) = \frac{\sin(\pi x)}{\pi x}$$

• template<typename \_Tp >

Return the unnormalized sinus cardinal function

$$sinc_{\pi}(x) = \frac{\sin(x)}{x}$$

• template<typename  $_{\rm Tp}>$ 

Return the generalized hyperbolic sinus cardinal function

$$sinhc_a(x) = \frac{\sinh(\pi x/a)}{\pi x/a}$$

.

 $\bullet \ \ template {<} typename \ \_Tp >$ 

Return the normalized hyperbolic sinus cardinal function

$$sinhc(x) = \frac{\sinh(\pi x)}{\pi x}$$

.

 $\bullet \ \ template {<} typename \ \_Tp >$ 

Return the unnormalized hyperbolic sinus cardinal function

$$sinhc_{\pi}(x) = \frac{\sinh(x)}{x}$$

.

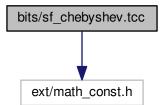
## 10.4.1 Macro Definition Documentation

10.4.1.1 #define \_GLIBCXX\_BITS\_SF\_CARDINAL\_TCC 1

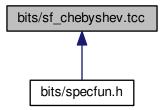
Definition at line 30 of file sf\_cardinal.tcc.

## 10.5 bits/sf\_chebyshev.tcc File Reference

#include <ext/math\_const.h>
Include dependency graph for sf\_chebyshev.tcc:



This graph shows which files directly or indirectly include this file:



## **Namespaces**

- std
- std::\_\_detail

## **Macros**

#define \_GLIBCXX\_SF\_CHEBYSHEV\_TCC 1

## **Functions**

```
template<typename _Tp >
    _Tp std::__detail::__chebyshev_recur (unsigned int __n, _Tp __x, _Tp _C0, _Tp _C1)
template<typename _Tp >
    _Tp std::__detail::__chebyshev_t (unsigned int __n, _Tp __x)
template<typename _Tp >
    _Tp std::__detail::__chebyshev_u (unsigned int __n, _Tp __x)
template<typename _Tp >
    _Tp std::__detail::__chebyshev_v (unsigned int __n, _Tp __x)
template<typename _Tp >
    _Tp std::__detail::__chebyshev_w (unsigned int __n, _Tp __x)
```

## 10.5.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

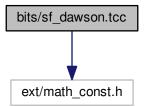
#### 10.5.2 Macro Definition Documentation

10.5.2.1 #define \_GLIBCXX\_SF\_CHEBYSHEV\_TCC 1

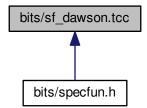
Definition at line 31 of file sf chebyshev.tcc.

## 10.6 bits/sf\_dawson.tcc File Reference

#include <ext/math\_const.h>
Include dependency graph for sf\_dawson.tcc:



This graph shows which files directly or indirectly include this file:



## **Namespaces**

- std
- std::\_\_detail

## **Macros**

#define \_GLIBCXX\_SF\_DAWSON\_TCC 1

## **Functions**

```
• template<typename _Tp > 
 _Tp std::__detail::__dawson (_Tp __x) 
 Return the Dawson integral, F(x), for real argument x.
```

```
    template<typename _Tp >
        _Tp std::__detail::__dawson_cont_frac (_Tp __x)
```

Compute the Dawson integral using a sampling theorem representation.

```
template<typename _Tp >
    _Tp std::__detail::__dawson_series (_Tp __x)
```

Compute the Dawson integral using the series expansion.

## 10.6.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

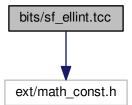
## 10.6.2 Macro Definition Documentation

10.6.2.1 #define \_GLIBCXX\_SF\_DAWSON\_TCC 1

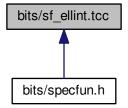
Definition at line 31 of file sf\_dawson.tcc.

## 10.7 bits/sf\_ellint.tcc File Reference

#include <ext/math\_const.h>
Include dependency graph for sf\_ellint.tcc:



This graph shows which files directly or indirectly include this file:



## **Namespaces**

- std
- std:: detail

#### **Macros**

#define \_GLIBCXX\_BITS\_SF\_ELLINT\_TCC 1

#### **Functions**

```
ullet template<typename _Tp >
  _Tp std::__detail::__comp_ellint_1 (_Tp __k)
      Return the complete elliptic integral of the first kind K(k) using the Carlson formulation.
template<typename _Tp >
  _Tp std::__detail::__comp_ellint_2 (_Tp __k)
      Return the complete elliptic integral of the second kind E(k) using the Carlson formulation.
template<typename _Tp >
  _Tp std::__detail::__comp_ellint_3 (_Tp __k, _Tp __nu)
      Return the complete elliptic integral of the third kind \Pi(k,\nu)=\Pi(k,\nu,\pi/2) using the Carlson formulation.
• template<typename _{\mathrm{Tp}} >
  _Tp std::__detail::__comp_ellint_d (_Tp __k)
template<typename _Tp >
  _Tp std::__detail::__comp_ellint_rf (_Tp __x, _Tp __y)
template<typename _Tp >
  _Tp std::__detail::__comp_ellint_rg (_Tp __x, _Tp __y)
template<typename _Tp >
  _Tp std::__detail::__ellint_1 (_Tp __k, _Tp __phi)
      Return the incomplete elliptic integral of the first kind F(k, \phi) using the Carlson formulation.

    template<typename</li>
    Tp >

  _Tp std::__detail::__ellint_2 (_Tp __k, _Tp __phi)
      Return the incomplete elliptic integral of the second kind E(k,\phi) using the Carlson formulation.
```

```
template<typename _Tp >
  _Tp std::__detail::__ellint_3 (_Tp __k, _Tp __nu, _Tp __phi)
      Return the incomplete elliptic integral of the third kind \Pi(k, \nu, \phi) using the Carlson formulation.
template<typename</li>Tp >
  _Tp std::__detail::__ellint_cel (_Tp __k_c, _Tp __p, _Tp __a, _Tp __b)

    template<typename</li>
    Tp >

  _Tp std::__detail::__ellint_d (_Tp __k, _Tp __phi)

    template<typename _Tp >

  _Tp std::__detail::__ellint_el1 (_Tp __x, _Tp __k_c)
template<typename _Tp >
  _Tp std::__detail::__ellint_el2 (_Tp __x, _Tp __k_c, _Tp __a, _Tp __b)
• template<typename _{\rm Tp}>
  _Tp std::__detail::__ellint_el3 (_Tp __x, _Tp __k_c, _Tp __p)
template<typename _Tp >
  _Tp std::__detail::__ellint_rc (_Tp __x, _Tp __y)
      Return the Carlson elliptic function R_C(x,y) = R_F(x,y,y) where R_F(x,y,z) is the Carlson elliptic function of the first
      kind.
template<typename _Tp >
  _Tp std::__detail::__ellint_rd (_Tp __x, _Tp __y, _Tp __z)
      Return the Carlson elliptic function of the second kind R_D(x,y,z) = R_J(x,y,z,z) where R_J(x,y,z,p) is the Carlson
      elliptic function of the third kind.
template<typename_Tp>
  _Tp std::__detail::__ellint_rf (_Tp __x, _Tp __y, _Tp __z)
      Return the Carlson elliptic function R_F(x,y,z) of the first kind.
template<typename _Tp >
  _Tp std::__detail::__ellint_rg (_Tp __x, _Tp __y, _Tp __z)
      Return the symmetric Carlson elliptic function of the second kind R_G(x, y, z).
template<typename _Tp >
  _Tp std::__detail::__ellint_rj (_Tp __x, _Tp __y, _Tp __z, _Tp __p)
      Return the Carlson elliptic function R_J(x, y, z, p) of the third kind.
template<typename _Tp >
  _Tp std::__detail::__heuman_lambda (_Tp __k, _Tp __phi)
template<typename</li>Tp >
  _Tp std::__detail::__jacobi_zeta (_Tp __k, _Tp __phi)
```

## 10.7.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <cmath>.

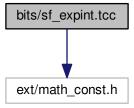
## 10.7.2 Macro Definition Documentation

10.7.2.1 #define \_GLIBCXX\_BITS\_SF\_ELLINT\_TCC 1

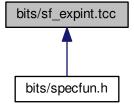
Definition at line 47 of file sf ellint.tcc.

## 10.8 bits/sf\_expint.tcc File Reference

#include <ext/math\_const.h>
Include dependency graph for sf\_expint.tcc:



This graph shows which files directly or indirectly include this file:



## **Namespaces**

- std
- std::\_\_detail

## **Macros**

#define \_GLIBCXX\_BITS\_SF\_EXPINT\_TCC 1

## **Functions**

```
• template<typename _Tp > 
 _Tp std::__detail::__coshint (const _Tp __x) 
 Return the hyperbolic cosine integral li(x).
```

```
template<typename _Tp >
  Tp std:: detail:: expint (unsigned int n, Tp x)
      Return the exponential integral E_n(x).
template<typename _Tp >
  _Tp std::__detail::__expint (_Tp __x)
      Return the exponential integral Ei(x).

    template<typename</li>
    Tp >

  _Tp std::__detail::__expint_asymp (unsigned int __n, _Tp __x)
      Return the exponential integral E_n(x) for large argument.
template<typename _Tp >
  Tp std:: detail:: expint E1 (Tp x)
      Return the exponential integral E_1(x).
template<typename _Tp >
  _Tp std::__detail::__expint_E1_asymp (_Tp __x)
      Return the exponential integral E_1(x) by asymptotic expansion.
template<typename _Tp >
  _Tp std::__detail::__expint_E1_series (_Tp __x)
      Return the exponential integral E_1(x) by series summation. This should be good for x < 1.

    template<typename</li>
    Tp >

  _Tp std::__detail::__expint_Ei (_Tp __x)
      Return the exponential integral Ei(x).
template<typename_Tp>
  _Tp std::__detail::__expint_Ei_asymp (_Tp __x)
      Return the exponential integral Ei(x) by asymptotic expansion.
template<typename _Tp >
  _Tp std::__detail::__expint_Ei_series (_Tp __x)
      Return the exponential integral Ei(x) by series summation.

    template<typename</li>
    Tp >

  _Tp std::__detail::__expint_En_cont_frac (unsigned int __n, _Tp __x)
      Return the exponential integral E_n(x) by continued fractions.

    template<typename</li>
    Tp >

  _Tp std::__detail::__expint_En_recursion (unsigned int __n, _Tp __x)
      Return the exponential integral E_n(x) by recursion. Use upward recursion for x < n and downward recursion (Miller's
      algorithm) otherwise.
template<typename _Tp >
  _Tp std::__detail::__expint_En_series (unsigned int __n, _Tp __x)
      Return the exponential integral E_n(x) by series summation.

    template<typename</li>
    Tp >

  _Tp std::__detail::__expint_large_n (unsigned int __n, _Tp __x)
      Return the exponential integral E_n(x) for large order.

    template<typename</li>
    Tp >

  _Tp std::__detail::__logint (const _Tp __x)
      Return the logarithmic integral li(x).
template<typename _Tp >
  _Tp std::__detail::__sinhint (const _Tp __x)
      Return the hyperbolic sine integral li(x).
```

## 10.8.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

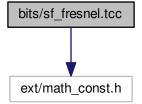
## 10.8.2 Macro Definition Documentation

10.8.2.1 #define \_GLIBCXX\_BITS\_SF\_EXPINT\_TCC 1

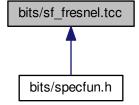
Definition at line 47 of file sf\_expint.tcc.

## 10.9 bits/sf\_fresnel.tcc File Reference

#include <ext/math\_const.h>
Include dependency graph for sf fresnel.tcc:



This graph shows which files directly or indirectly include this file:



## **Namespaces**

std

• std::\_\_detail

#### **Macros**

#define \_GLIBCXX\_SF\_FRESNEL\_TCC 1

## **Functions**

```
    template < typename _Tp >
    std::complex < _Tp > std::__detail::__fresnel (const _Tp __x)
```

Return the Fresnel cosine and sine integrals as a complex number f(C(x) + iS(x))

```
    template<typename _Tp >
        void std::__detail::__fresnel_cont_frac (const _Tp __ax, _Tp &_Cf, _Tp &_Sf)
```

This function computes the Fresnel cosine and sine integrals by continued fractions for positive argument.

```
    template<typename _Tp >
        void std::__detail::__fresnel_series (const _Tp __ax, _Tp &_Cf, _Tp &_Sf)
```

This function returns the Fresnel cosine and sine integrals as a pair by series expansion for positive argument.

## 10.9.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

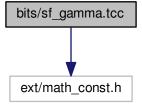
#### 10.9.2 Macro Definition Documentation

```
10.9.2.1 #define _GLIBCXX_SF_FRESNEL_TCC 1
```

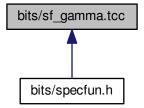
Definition at line 31 of file sf\_fresnel.tcc.

## 10.10 bits/sf gamma.tcc File Reference

```
#include <ext/math_const.h>
Include dependency graph for sf_gamma.tcc:
```



This graph shows which files directly or indirectly include this file:



## **Classes**

struct std::\_\_detail::\_Factorial\_table< \_Tp >

## **Namespaces**

- std
- std:: detail

#### **Macros**

#define \_GLIBCXX\_BITS\_SF\_GAMMA\_TCC 1

## **Functions**

template<typename\_Tp >
 GLIBCXX14\_CONSTEXPR\_Tp std::\_\_detail::\_\_chi\_squared\_pdf (\_Tp \_\_chi2, unsigned int \_\_nu)

Return the chi-squared propability function. This returns the probability that the observed chi-squared for a correct model is less than the value  $\chi^2$ .

template<typename</li>
 Tp >

Return the complementary chi-squared propability function. This returns the probability that the observed chi-squared for a correct model is greater than the value  $\chi^2$ .

template<typename\_Tp>

Return the double factorial of the integer n.

template<typename</li>
 Tp >

Return the factorial of the integer n.

template<typename \_Tp >

Return  $\Gamma(x)$ .

template<typename\_Tp>

template<typename \_Tp >

Return the lower incomplete gamma function. The lower incomplete gamma function is defined by

$$\gamma(a, x) = \int_0^x e^{-t} t^{a-1} dt (a > 0)$$

•

template<typename\_Tp>

Return the regularized lower incomplete gamma function. The regularized lower incomplete gamma function is defined by

$$P(a,x) = \frac{\gamma(a,x)}{\Gamma(a)}$$

where  $\Gamma(a)$  is the gamma function and

$$\gamma(a,x) = \int_0^x e^{-t} t^{a-1} dt (a > 0)$$

is the lower incomplete gamma function.

• template<typename  $_{\rm Tp}>$ 

Return the regularized upper incomplete gamma function. The regularized upper incomplete gamma function is defined by

$$Q(a,x) = \frac{\Gamma(a,x)}{\Gamma(a)}$$

where  $\Gamma(a)$  is the gamma function and

$$\Gamma(a,x) = \int_{-\infty}^{\infty} e^{-t} t^{a-1} dt (a > 0)$$

is the upper incomplete gamma function.

• template<typename\_Tp>

$$std::pair < _Tp, _Tp > std:: __detail:: __gamma_series (_Tp __a, _Tp __x)$$

template<typename \_Tp >

Return the upper incomplete gamma function. The lower incomplete gamma function is defined by

$$\Gamma(a,x) = \int_{-\infty}^{\infty} e^{-t} t^{a-1} dt (a > 0)$$

.

template<typename \_Tp >

Return the logarithm of the binomial coefficient. The binomial coefficient is given by:

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

template<typename\_Tp>

template<typename\_Tp>

Return the logarithm of the double factorial of the integer n.

template<typename \_Tp >

Return the logarithm of the factorial of the integer n.

template<typename\_Tp>

Return  $log(|\Gamma(x)|)$ . This will return values even for x < 0. To recover the sign of  $\Gamma(x)$  for any argument use  $\_log\_ \leftrightarrow gamma\_sign$ .

template<typename \_Tp >

Return  $log(\Gamma(x))$  by asymptotic expansion with Bernoulli number coefficients. This is like Sterling's approximation.

template<typename\_Tp>

Return  $log(\Gamma(x))$  by the Lanczos method. This method dominates all others on the positive axis I think.

template<typename \_Tp >

Return the sign of  $\Gamma(x)$ . At nonpositive integers zero is returned.

template<typename \_Tp >

Return  $\Gamma(z)$  by the Spouge algorithm:

$$\Gamma(z+1) = (z+a)^{z+1/2} e^{-z-a} \left[ \sqrt{2\pi} \sum_{k=1}^{\lceil a \rceil + 1} \frac{c_k(a)}{z+k} \right]$$

where

$$c_k(a) = \frac{(-1)^{k-1}}{(k-1)!} (a-k)^{k-1/2} e^{a-k}$$

and the error is bounded by

$$\epsilon(a) < a^{-1/2} (2\pi)^{-a-1/2}$$

template<typename\_Tp>

Return the logarithm of the lower Pochhammer symbol or the falling factorial function. The lower Pochammer symbol is defined by

$$(a)_n = \prod_{k=0}^{n-1} (a-k), (a)_0 = 1 = \Gamma(a+1)/\Gamma(a-n+1)$$

In particular, f(n) = n! f. Thus this function returns

$$ln[(a)_n] = \Gamma(a+1) - \Gamma(a-n+1), ln[(a)_0] = 0$$

Many notations exist:

 $a^{\underline{n}}$ 

$$\left\{\begin{array}{c} a \\ n \end{array}\right\}$$

, and others.

• template<typename  $_{\rm Tp}>$ 

Return the logarithm of the (upper) Pochhammer symbol or the rising factorial function. The Pochammer symbol is defined by

$$(a)_n = \prod_{k=0}^{n-1} (a+k), (a)_0 = 1 = \Gamma(a+n)/\Gamma(n)$$

Thus this function returns

$$ln[(a)_n] = \Gamma(a+n) - \Gamma(n), ln[(a)_0] = 0$$

Many notations exist:

 $a^{\overline{n}}$ 

,

 $\begin{bmatrix} a \\ n \end{bmatrix}$ 

, and others.

template<typename \_Tp >

Return the logarithm of the lower Pochhammer symbol or the falling factorial function. The lower Pochammer symbol is defined by

$$(a)_n = \prod_{k=0}^{n-1} (a-k), (a)_0 = 1 = \Gamma(a+1)/\Gamma(a-n+1)$$

In particular, f(n) = n! f(n)

template<typename \_Tp >

Return the (upper) Pochhammer function or the rising factorial function. The Pochammer symbol is defined by

$$(a)_n = \prod_{k=0}^{n-1} (a+k), (a)_0 = 1 = \Gamma(a+n)/\Gamma(n)$$

Many notations exist:

 $a^{\bar{\imath}}$ 

,

 $\left[\begin{array}{c} a \\ n \end{array}\right]$ 

, and others.

template<typename \_Tp >

Return the digamma function. The digamma or  $\psi(x)$  function is defined by

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

For negative argument the reflection formula is used:

$$\psi(x) = \psi(1-x) - \pi \cot(\pi x)$$

template<typename\_Tp>

Return the polygamma function  $\psi^{(n)}(x)$ .

template<typename \_Tp >
 \_Tp std::\_\_detail::\_\_psi\_asymp (\_Tp \_\_x)

Return the digamma function for large argument. The digamma or  $\psi(x)$  function is defined by

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

.

• template<typename\_Tp>

Return the digamma function by series expansion. The digamma or  $\psi(x)$  function is defined by

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

.

## **Variables**

```
• constexpr Factorial table < long double > std:: detail:: S double factorial table [301]
```

- constexpr \_Factorial\_table < long double > std::\_\_detail::\_S\_factorial\_table [171]
- constexpr\_Factorial\_table< long double > std::\_\_detail::\_S\_neg\_double\_factorial\_table [999]

```
    template<typename_Tp >
        constexpr std::size_t std::__detail::_S_num_double_factorials = 0
```

template<>

constexpr std::size\_t std::\_\_detail::\_S\_num\_double\_factorials< double > = 301

template<>

constexpr std::size\_t std::\_\_detail::\_S\_num\_double\_factorials< float > = 57

template<>

constexpr std::size t std:: detail:: S num double factorials < long double > = 301

ullet template<typename\_Tp>

constexpr std::size\_t std::\_\_detail::\_S\_num\_factorials = 0

• template<>

constexpr std::size\_t std::\_\_detail::\_S\_num\_factorials< double > = 171

template<>

constexpr std::size t std:: detail:: S num factorials < float > = 35

template<>

constexpr std::size\_t std::\_\_detail::\_S\_num\_factorials< long double > = 171

template<typename\_Tp>

constexpr std::size\_t std::\_\_detail::\_S\_num\_neg\_double\_factorials = 0

template<>

constexpr std::size\_t std::\_\_detail::\_S\_num\_neg\_double\_factorials< double > = 150

template<>

constexpr std::size\_t std::\_\_detail::\_S\_num\_neg\_double\_factorials< float > = 27

template<>

constexpr std::size\_t std::\_\_detail::\_S\_num\_neg\_double\_factorials< long double > = 999

## 10.10.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <cmath>.

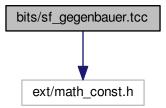
## 10.10.2 Macro Definition Documentation

10.10.2.1 #define \_GLIBCXX\_BITS\_SF\_GAMMA\_TCC 1

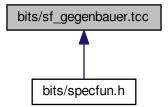
Definition at line 49 of file sf\_gamma.tcc.

## 10.11 bits/sf\_gegenbauer.tcc File Reference

#include <ext/math\_const.h>
Include dependency graph for sf\_gegenbauer.tcc:



This graph shows which files directly or indirectly include this file:



## **Namespaces**

- std
- std::\_\_detail

## **Macros**

#define \_GLIBCXX\_SF\_GEGENBAUER\_TCC 1

## **Functions**

```
    template<typename _Tp >
        _Tp std::__detail::__gegenbauer_poly (unsigned int __n, _Tp __alpha, _Tp __x)
```

## 10.11.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

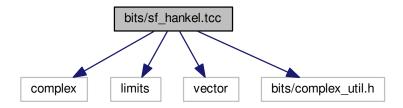
## 10.11.2 Macro Definition Documentation

10.11.2.1 #define \_GLIBCXX\_SF\_GEGENBAUER\_TCC 1

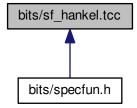
Definition at line 31 of file sf\_gegenbauer.tcc.

## 10.12 bits/sf hankel.tcc File Reference

```
#include <complex>
#include <limits>
#include <vector>
#include <bits/complex_util.h>
Include dependency graph for sf_hankel.tcc:
```



This graph shows which files directly or indirectly include this file:



## **Namespaces**

- std
- std:: detail

template<typename \_Tp >

## **Macros**

• #define GLIBCXX BITS SF HANKEL TCC 1

#### **Functions**

```
void std::__detail::__airy_arg (std::complex< _Tp > __num2d3, std::complex< _Tp > __zeta, std::complex<
  _Tp > &__argp, std::complex< _Tp > &__argm)
      Compute the arguments for the Airy function evaluations carefully to prevent premature overflow. Note that the major work
      here is in safe_div. A faster, but less safe implementation can be obtained without use of safe_div.
template<typename _Tp >
  std::complex< _Tp > std::__detail::__cyl_bessel (std::complex< _Tp > __nu, std::complex< _Tp > __z)
      Return the complex cylindrical Bessel function.
template<typename Tp >
  std::complex< _Tp > std::__detail::__cyl_hankel_1 (std::complex< _Tp > __nu, std::complex< _Tp > __z)
      Return the complex cylindrical Hankel function of the first kind.
template<typename _Tp >
  std::complex< _Tp > std::__detail::__cyl_hankel_2 (std::complex< _Tp > __nu, std::complex< _Tp > __z)
      Return the complex cylindrical Hankel function of the second kind.
template<typename</li>Tp >
  std::complex< _Tp > std::__detail::__cyl_neumann (std::complex< _Tp > __nu, std::complex< _Tp > __z)
      Return the complex cylindrical Neumann function.
template<typename _Tp >
  void std:: __detail:: __debye_region (std::complex < _Tp > __alpha, int &__indexr, char &__aorb)
template<typename _Tp >
  void std::__detail::__hankel (std::complex< _Tp > __nu, std::complex< _Tp > __z, std::complex< _Tp > &_H1,
```

std::complex< \_Tp > &\_H2, std::complex< \_Tp > &\_H1p, std::complex< \_Tp > &\_H2p)

- template<typename\_Tp >
   void std::\_\_detail::\_\_hankel\_debye (std::complex< \_Tp > \_\_nu, std::complex< \_Tp > \_\_z, std::complex< \_Tp >
   \_alpha, int \_\_indexr, char &\_\_aorb, int &\_\_morn, std::complex< \_Tp > &\_H1, std::complex< \_Tp > &\_H2, std::complex< \_Tp > &\_H1p, std::complex< \_Tp > &\_H2p)

Compute parameters depending on z and nu that appear in the uniform asymptotic expansions of the Hankel functions and their derivatives, except the arguments to the Airy functions.

template<typename\_Tp >
 void std::\_\_detail::\_\_hankel\_uniform (std::complex< \_Tp > \_\_nu, std::complex< \_Tp > \_\_z, std::complex< \_Tp > &\_H1, std::complex< \_Tp > &\_H2, std::complex< \_Tp > &\_H1p, std::complex< \_Tp > &\_H2p)

This routine computes the uniform asymptotic approximations of the Hankel functions and their derivatives including a patch for the case when the order equals or nearly equals the argument. At such points, Olver's expressions have zero denominators (and numerators) resulting in numerical problems. This routine averages results from four surrounding points in the complex plane to obtain the result in such cases.

• template<typename \_Tp > void std:: \_\_detail:: \_\_hankel\_uniform\_olver (std::complex< \_Tp > \_\_nu, std::complex< \_Tp > \_\_z, std  $\leftarrow$  ::complex< \_Tp > &\_H1, std::complex< \_Tp > &\_H1p, std::complex< \_Tp > &\_H2p)

Compute approximate values for the Hankel functions of the first and second kinds using Olver's uniform asymptotic expansion to of order nu along with their derivatives.

Compute outer factors and associated functions of z and nu appearing in Olver's uniform asymptotic expansions of the Hankel functions of the first and second kinds and their derivatives. The various functions of z and nu returned by  $hankel\_uniform\_outer$  are available for use in computing further terms in the expansions.

Compute the sums in appropriate linear combinations appearing in Olver's uniform asymptotic expansions for the Hankel functions of the first and second kinds and their derivatives, using up to nterms (less than 5) to achieve relative error eps.

template<typename \_Tp >
 std::complex< \_Tp > std::\_\_detail::\_\_sph\_bessel (unsigned int \_\_n, std::complex< \_Tp > \_\_z)

 Return the complex spherical Bessel function.

template<typename \_Tp >
 void std::\_\_detail::\_\_sph\_hankel (unsigned int \_\_n, std::complex< \_Tp > \_\_z, std::complex< \_Tp > &\_H1, std
 ::complex< \_Tp > &\_H1p, std::complex< \_Tp > &\_H2p)

Helper to compute complex spherical Hankel functions and their derivatives.

template<typename \_Tp >
 std::complex< \_Tp > std::\_\_detail::\_\_sph\_hankel\_1 (unsigned int \_\_n, std::complex< \_Tp > \_\_z)

Return the complex spherical Hankel function of the first kind.

```
    template<typename _Tp >
        std::complex< _Tp > std::__detail::__sph_hankel_2 (unsigned int __n, std::complex< _Tp > __z)
        Return the complex spherical Hankel function of the second kind.
    template<typename _Tp >
        std::complex< _Tp > std::__detail::__sph_neumann (unsigned int __n, std::complex< _Tp > __z)
        Return the complex spherical Neumann function.
```

## 10.12.1 Detailed Description

This is an internal header file, included by other library headers. You should not attempt to use it directly.

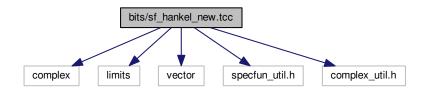
#### 10.12.2 Macro Definition Documentation

```
10.12.2.1 #define _GLIBCXX_BITS_SF_HANKEL_TCC 1
```

Definition at line 31 of file sf hankel.tcc.

## 10.13 bits/sf\_hankel\_new.tcc File Reference

```
#include <complex>
#include <limits>
#include <vector>
#include "specfun_util.h"
#include "complex_util.h"
Include dependency graph for sf_hankel_new.tcc:
```



## **Macros**

#define \_GLIBCXX\_BITS\_SF\_HANKEL\_NEW\_TCC 1

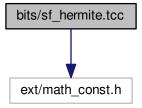
## 10.13.1 Macro Definition Documentation

10.13.1.1 #define \_GLIBCXX\_BITS\_SF\_HANKEL\_NEW\_TCC 1

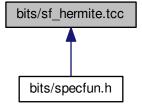
Definition at line 31 of file sf hankel new.tcc.

# 10.14 bits/sf\_hermite.tcc File Reference

#include <ext/math\_const.h>
Include dependency graph for sf\_hermite.tcc:



This graph shows which files directly or indirectly include this file:



## **Namespaces**

- std
- std::\_\_detail

# **Macros**

#define \_GLIBCXX\_BITS\_SF\_HERMITE\_TCC 1

```
• template<typename _Tp > 
 _Tp std::__detail::__poly_hermite (unsigned int __n, _Tp __x) 
 This routine returns the Hermite polynomial of order n: H_n(x).
```

```
    template<typename _Tp >
        _Tp std::__detail::__poly_hermite_asymp (unsigned int __n, _Tp __x)
        This routine returns the Hermite polynomial of large order n: H<sub>n</sub>(x). We assume here that x >= 0.
    template<typename _Tp >
        _Tp std::__detail::__poly_hermite_recursion (unsigned int __n, _Tp __x)
        This routine returns the Hermite polynomial of order n: H<sub>n</sub>(x) by recursion on n.
```

## 10.14.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

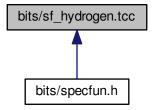
## 10.14.2 Macro Definition Documentation

```
10.14.2.1 #define _GLIBCXX_BITS_SF_HERMITE_TCC 1
```

Definition at line 42 of file sf\_hermite.tcc.

# 10.15 bits/sf\_hydrogen.tcc File Reference

This graph shows which files directly or indirectly include this file:



### **Namespaces**

- std
- std::\_\_detail

## **Macros**

#define \_GLIBCXX\_BITS\_SF\_HYDROGEN\_TCC 1

#### **Functions**

template<typename \_Tp >
 std::complex< \_Tp > std::\_\_detail::\_\_hydrogen (const unsigned int \_\_n, const unsigned int \_\_l, const unsigned int \_\_m, const \_Tp \_Z, const \_Tp \_\_r, const \_Tp \_\_theta, const \_Tp \_\_phi)

## 10.15.1 Detailed Description

This is an internal header file, included by other library headers. You should not attempt to use it directly.

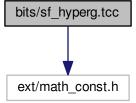
## 10.15.2 Macro Definition Documentation

10.15.2.1 #define \_GLIBCXX\_BITS\_SF\_HYDROGEN\_TCC 1

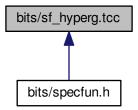
Definition at line 31 of file sf\_hydrogen.tcc.

# 10.16 bits/sf\_hyperg.tcc File Reference

#include <ext/math\_const.h>
Include dependency graph for sf\_hyperg.tcc:



This graph shows which files directly or indirectly include this file:



### **Namespaces**

- std
- · std:: detail

#### **Macros**

#define \_GLIBCXX\_BITS\_SF\_HYPERG\_TCC 1

```
template<typename _Tp >
  _Tp std::__detail::__conf_hyperg (_Tp __a, _Tp __c, _Tp __x)
      Return the confluent hypergeometric function _1F_1(a;c;x).
• template<typename _Tp >
  _Tp std::__detail::__conf_hyperg_lim (_Tp __c, _Tp __x)
      Return the confluent hypergeometric limit function {}_{0}F_{1}(-;c;x).
• template<typename _Tp >
  _Tp std::__detail::__conf_hyperg_lim_series (_Tp __c, _Tp __x)
      This routine returns the confluent hypergeometric limit function by series expansion.
template<typename _Tp >
  _Tp std::__detail::__conf_hyperg_luke (_Tp __a, _Tp __c, _Tp __xin)
      Return the hypergeometric function _1F_1(a;c;x) by an iterative procedure described in Luke, Algorithms for the Compu-
      tation of Mathematical Functions.
template<typename _Tp >
  _Tp std::__detail::__conf_hyperg_series (_Tp __a, _Tp __c, _Tp __x)
      This routine returns the confluent hypergeometric function by series expansion.

    template<typename</li>
    Tp >

  _Tp std::__detail::__hyperg (_Tp __a, _Tp __b, _Tp __c, _Tp __x)
      Return the hypergeometric function _2F_1(a,b;c;x).
template<typename _Tp >
  _Tp std::__detail::__hyperg_luke (_Tp __a, _Tp __b, _Tp __c, _Tp __xin)
```

Return the hypergeometric function  $_2F_1(a,b;c;x)$  by an iterative procedure described in Luke, Algorithms for the Computation of Mathematical Functions.

```
    template<typename _Tp >
        _Tp std::__detail::__hyperg_reflect (_Tp __a, _Tp __b, _Tp __c, _Tp __x)
```

Return the hypergeometric function  ${}_2F_1(a,b;c;x)$  by the reflection formulae in Abramowitz & Stegun formula 15.3.6 for d=c-a-b not integral and formula 15.3.11 for d=c-a-b integral. This assumes a,b,c!= negative integer.

```
    template<typename _Tp >
        _Tp std::__detail::__hyperg_series (_Tp __a, _Tp __b, _Tp __c, _Tp __x)
```

Return the hypergeometric function  ${}_2F_1(a,b;c;x)$  by series expansion.

## 10.16.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

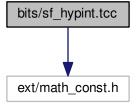
#### 10.16.2 Macro Definition Documentation

10.16.2.1 #define \_GLIBCXX\_BITS\_SF\_HYPERG\_TCC 1

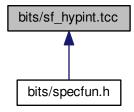
Definition at line 44 of file sf\_hyperg.tcc.

# 10.17 bits/sf\_hypint.tcc File Reference

#include <ext/math\_const.h>
Include dependency graph for sf hypint.tcc:



This graph shows which files directly or indirectly include this file:



## **Namespaces**

- std
- std:: detail

#### **Macros**

#define \_GLIBCXX\_SF\_HYPINT\_TCC 1

#### **Functions**

```
    template<typename_Tp >
        std::pair< _Tp, _Tp > std::__detail::__chshint (_Tp __x, _Tp &_Chi, _Tp &_Shi)
```

This function returns the hyperbolic cosine Ci(x) and hyperbolic sine Si(x) integrals as a pair.

• template<typename \_Tp >

```
void std::__detail::__chshint_cont_frac (_Tp __t, _Tp &_Chi, _Tp &_Shi)
```

This function computes the hyperbolic cosine Chi(x) and hyperbolic sine Shi(x) integrals by continued fraction for positive argument.

```
    template<typename _Tp >
        void std::__detail::__chshint_series (_Tp __t, _Tp &_Chi, _Tp &_Shi)
```

This function computes the hyperbolic cosine Chi(x) and hyperbolic sine Shi(x) integrals by series summation for positive argument.

#### 10.17.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

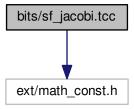
## 10.17.2 Macro Definition Documentation

10.17.2.1 #define \_GLIBCXX\_SF\_HYPINT\_TCC 1

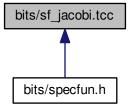
Definition at line 31 of file sf hypint.tcc.

# 10.18 bits/sf\_jacobi.tcc File Reference

#include <ext/math\_const.h>
Include dependency graph for sf\_jacobi.tcc:



This graph shows which files directly or indirectly include this file:



## **Namespaces**

- std
- std::\_\_detail

## **Macros**

• #define \_GLIBCXX\_SF\_JACOBI\_TCC 1

```
    template<typename _Tp >
        _Tp std::__detail::__poly_jacobi (unsigned int __n, _Tp __alpha, _Tp __beta, _Tp __x)
```

```
    template<typename _Tp >
        _Tp std::__detail::__poly_radial_jacobi (unsigned int __n, unsigned int __m, _Tp __rho)
```

```
    template<typename _Tp >
        __gnu_cxx::__promote_num_t< _Tp > std::__detail::__zernike (unsigned int __n, int __m, _Tp __rho, _Tp __phi)
```

# 10.18.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

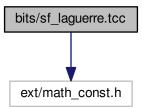
## 10.18.2 Macro Definition Documentation

10.18.2.1 #define GLIBCXX\_SF\_JACOBI\_TCC 1

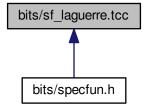
Definition at line 31 of file sf\_jacobi.tcc.

# 10.19 bits/sf\_laguerre.tcc File Reference

#include <ext/math\_const.h>
Include dependency graph for sf\_laguerre.tcc:



This graph shows which files directly or indirectly include this file:



#### **Namespaces**

- std
- · std:: detail

#### **Macros**

#define \_GLIBCXX\_BITS\_SF\_LAGUERRE\_TCC 1

```
template<typename _Tp >
  _Tp std::__detail::__assoc_laguerre (unsigned int __n, unsigned int __m, _Tp __x)
      This routine returns the associated Laguerre polynomial of order n, degree m: L_n^m(x).
template<typename _Tp >
  _Tp std::__detail::__laguerre (unsigned int __n, _Tp __x)
      This routine returns the Laguerre polynomial of order n: L_n(x).
• template<typename _Tpa , typename _Tp >
  _Tp std::__detail::__poly_laguerre (unsigned int __n, _Tpa __alpha1, _Tp __x)
      This routine returns the associated Laguerre polynomial of order n, degree \alpha: L_n^a lpha(x).
• template<typename _{\rm Tpa}, typename _{\rm Tp} >
  _Tp std::__detail::__poly_laguerre_hyperg (unsigned int __n, _Tpa __alpha1, _Tp __x)
      Evaluate the polynomial based on the confluent hypergeometric function in a safe way, with no restriction on the arguments.
• template<typename Tpa, typename Tp>
  _Tp std::__detail::__poly_laguerre_large_n (unsigned __n, _Tpa __alpha1, _Tp __x)
      This routine returns the associated Laguerre polynomial of order n, degree \alpha > -1 for large n. Abramowitz & Stegun,
      13.5.21.
• template<typename _Tpa , typename _Tp >
  _Tp std::__detail::__poly_laguerre_recursion (unsigned int __n, _Tpa __alpha1, _Tp __x)
      This routine returns the associated Laguerre polynomial of order n, degree \alpha: L_n^n(x) by recursion.
```

# 10.19.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

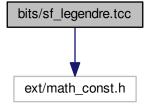
## 10.19.2 Macro Definition Documentation

10.19.2.1 #define \_GLIBCXX\_BITS\_SF\_LAGUERRE\_TCC 1

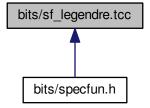
Definition at line 44 of file sf\_laguerre.tcc.

# 10.20 bits/sf\_legendre.tcc File Reference

#include <ext/math\_const.h>
Include dependency graph for sf\_legendre.tcc:



This graph shows which files directly or indirectly include this file:



## **Namespaces**

std

• std::\_\_detail

#### **Macros**

• #define \_GLIBCXX\_BITS\_SF\_LEGENDRE\_TCC 1

#### **Functions**

```
template < typename _Tp >
_Tp std::__detail::__assoc_legendre_p (unsigned int __I, unsigned int __m, _Tp __x)

Return the associated Legendre function by recursion on l and downward recursion on m.
template < typename _Tp >
_Tp std::__detail::__legendre_q (unsigned int __I, _Tp __x)

Return the Legendre function of the second kind by upward recursion on order l.
template < typename _Tp >
_Tp std::__detail::__poly_legendre_p (unsigned int __I, _Tp __x)

Return the Legendre polynomial by upward recursion on order l.
template < typename _Tp >
std::_complex < _Tp > std::__detail::__sph_harmonic (unsigned int __I, int __m, _Tp __theta, _Tp __phi)

Return the spherical harmonic function.
template < typename _Tp >
_Tp std::__detail::__sph_legendre (unsigned int __I, unsigned int __m, _Tp __theta)
```

## 10.20.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

#### 10.20.2 Macro Definition Documentation

```
10.20.2.1 #define _GLIBCXX_BITS_SF_LEGENDRE_TCC 1
```

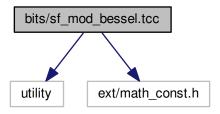
Return the spherical associated Legendre function.

Definition at line 47 of file sf\_legendre.tcc.

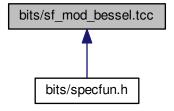
# 10.21 bits/sf mod bessel.tcc File Reference

```
#include <utility>
#include <ext/math_const.h>
```

Include dependency graph for sf\_mod\_bessel.tcc:



This graph shows which files directly or indirectly include this file:



# Namespaces

- std
- std::\_\_detail

## **Macros**

#define \_GLIBCXX\_BITS\_SF\_MOD\_BESSEL\_TCC 1

```
• template<typename _Tp > void std::__detail::__airy (_Tp __z, _Tp &_Ai, _Tp &_Bi, _Tp &_Aip, _Tp &_Bip) 
 Compute the Airy functions Ai(x) and Bi(x) and their first derivatives Ai'(x) and Bi(x) respectively. 
• template<typename _Tp > 
 _Tp std::__detail::__cyl_bessel_i (_Tp __nu, _Tp __x) 
 Return the regular modified Bessel function of order \nu: I_{\nu}(x).
```

template<typename\_Tp >
 void std::\_\_detail::\_\_cyl\_bessel\_ik (\_Tp \_\_nu, \_Tp \_\_x, \_Tp &\_Inu, \_Tp &\_Knu, \_Tp &\_Ipnu, \_Tp &\_Kpnu)

Return the modified cylindrical Bessel functions and their derivatives of order  $\nu$  by various means.

template<typename \_Tp >
 void std::\_\_detail::\_\_cyl\_bessel\_ik\_asymp (\_Tp \_\_nu, \_Tp \_\_x, \_Tp &\_Inu, \_Tp &\_Knu, \_Tp &\_Ipnu, \_Tp &\_
 Kpnu)

This routine computes the asymptotic modified cylindrical Bessel and functions of order nu:  $I_{\nu}(x)$ ,  $N_{\nu}(x)$ . Use this for  $x >> nu^2 + 1$ .

template<typename \_Tp >
 void std::\_\_detail::\_\_cyl\_bessel\_ik\_steed (\_Tp \_\_nu, \_Tp \_\_x, \_Tp &\_Inu, \_Tp &\_Knu, \_Tp &\_Ipnu, \_Tp &\_Kpnu)

Compute the modified Bessel functions  $I_{\nu}(x)$  and  $K_{\nu}(x)$  and their first derivatives  $I'_{\nu}(x)$  and  $K'_{\nu}(x)$  respectively. These four functions are computed together for numerical stability.

template < typename \_Tp >
 \_Tp std::\_\_detail::\_\_cyl\_bessel\_k (\_Tp \_\_nu, \_Tp \_\_x)

Return the irregular modified Bessel function  $K_{\nu}(x)$  of order  $\nu$ .

template<typename \_Tp >
 void std::\_\_detail::\_\_fock\_airy (\_Tp \_\_x, std::complex< \_Tp > &\_\_w1, std::complex< \_Tp > &\_\_w2, std
 ::complex< \_Tp > &\_\_w1p, std::complex< \_Tp > &\_\_w2p)

Compute the Fock-type Airy functions  $w_1(x)$  and  $w_2(x)$  and their first derivatives  $w_1'(x)$  and  $w_2'(x)$  respectively.

$$w_1(x) = \sqrt{\pi}(Ai(x) + iBi(x))$$

$$w_2(x) = \sqrt{\pi}(Ai(x) - iBi(x))$$

template<typename \_Tp >
 void std::\_\_detail::\_\_sph\_bessel\_ik (unsigned int \_\_n, \_Tp \_\_x, \_Tp &\_\_i\_n, \_Tp &\_\_k\_n, \_Tp &\_\_ip\_n, \_Tp &\_\_kp n)

Compute the spherical modified Bessel functions  $i_n(x)$  and  $k_n(x)$  and their first derivatives  $i'_n(x)$  and  $k'_n(x)$  respectively.

### 10.21.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <cmath>.

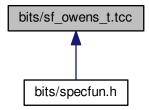
### 10.21.2 Macro Definition Documentation

10.21.2.1 #define \_GLIBCXX\_BITS\_SF\_MOD\_BESSEL\_TCC 1

Definition at line 47 of file sf mod bessel.tcc.

# 10.22 bits/sf\_owens\_t.tcc File Reference

This graph shows which files directly or indirectly include this file:



## **Namespaces**

- std
- std:: detail

#### **Macros**

#define \_GLIBCXX\_BITS\_SF\_OWENS\_T\_TCC 1

## **Functions**

```
template<typename _Tp >
    _Tp std::__detail::__gauss (_Tp __x)
template<typename _Tp >
    _Tp std::__detail::__owens_t (_Tp __h, _Tp __a)
template<typename _Tp >
    _Tp std::__detail::__znorm1 (_Tp __x)
template<typename _Tp >
    _Tp std::__detail::__znorm2 (_Tp __x)
```

#### 10.22.1 Detailed Description

This is an internal header file, included by other library headers. You should not attempt to use it directly.

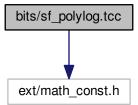
### 10.22.2 Macro Definition Documentation

10.22.2.1 #define \_GLIBCXX\_BITS\_SF\_OWENS\_T\_TCC 1

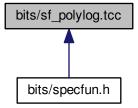
Definition at line 31 of file sf owens t.tcc.

# 10.23 bits/sf\_polylog.tcc File Reference

#include <ext/math\_const.h>
Include dependency graph for sf\_polylog.tcc:



This graph shows which files directly or indirectly include this file:



## **Namespaces**

- std
- std::\_\_detail

## **Macros**

• #define \_GLIBCXX\_BITS\_SF\_POLYLOG\_TCC 1

```
template<typename _Tp >
_Tp std::__detail::__bose_einstein (_Tp __s, _Tp __x)
```

```
template<typename _Tp >
  std::complex< _Tp > std::__detail::__clamp_0_m2pi (std::complex< _Tp > __w)
template<typename_Tp>
  std::complex< _Tp > std::__detail::__clamp_pi (std::complex< _Tp > __w)
template<typename _Tp >
  std::complex< Tp > std:: detail:: clausen (unsigned int m, std::complex< Tp > w)
template<typename _Tp >
  _Tp std::__detail::__clausen (unsigned int __m, _Tp __w)
template<typename _Tp >
  _Tp std::__detail::__clausen_c (unsigned int __m, std::complex< _Tp > w)
template<typename</li>Tp >
  _Tp std::__detail::__clausen_c (unsigned int __m, _Tp __w)

    template<typename _Tp >

  _Tp std::__detail::__clausen_s (unsigned int __m, std::complex< Tp > w)

    template<typename</li>
    Tp >

  _Tp std::__detail::__clausen_s (unsigned int __m, _Tp __w)
template<typename _Tp >
  Tp std:: detail:: dirichlet beta (std::complex < Tp > w)
template<typename _Tp >
  _Tp std::__detail::__dirichlet_beta (_Tp __w)
template<typename_Tp>
  std::complex< Tp > std:: detail:: dirichlet eta (std::complex< Tp > w)
template<typename _Tp >
  Tp std:: detail:: dirichlet eta (Tp w)
template<typename_Tp>
  _Tp std::__detail::__fermi_dirac (_Tp _ s, Tp x)

    template<typename</li>
    Tp >

  bool std::__detail::__fpequal (const _Tp &__a, const _Tp &__b)
template<typename _Tp >
  bool std::__detail::__fpimag (const std::complex < _Tp > &__w)
template<typename</li>Tp >
  bool std::__detail::__fpimag (const _Tp)

    template<typename</li>
    Tp >

  bool std::__detail::__fpreal (const std::complex < _Tp > &__w)
template<typename _Tp >
  bool std:: detail:: fpreal (const Tp)
template<typename _Tp >
  std::complex< _Tp > std::__detail::__hurwitz_zeta (_Tp __s, std::complex< _Tp > __a)

    template<typename</li>
    Tp >

  _Tp std::__detail::__polylog (_Tp __s, _Tp __x)
template<typename _Tp >
  std::complex< _Tp > std::__detail::__polylog (_Tp __s, std::complex< _Tp > __w)

    template<typename</li>
    Tp , typename
    ArgType >

    _gnu_cxx::__promote_num_t< std::complex< _Tp >, ArgType > std::__detail::__polylog_exp (_Tp __s, Arg↔
  Type __w)
template<typename _Tp >
  std::complex < _Tp > std:: __detail:: __polylog_exp_asymp (_Tp __s, std::complex < _Tp > __w)
template<typename</li>Tp >
  std::complex< _Tp > std::__detail::__polylog_exp_int_neg (int __s, std::complex< _Tp > __w)
template<typename_Tp>
  std::complex < _Tp > std::__detail::__polylog_exp_int_neg (const int __s, _Tp __w)
template<typename _Tp >
  std::complex< Tp > std:: detail:: polylog exp int pos (unsigned int s, std::complex< Tp > w)
```

```
template<typename _Tp >
  std::complex < _Tp > std::__detail::__polylog_exp_int_pos (unsigned int __s, _Tp __w)
template<typename _Tp >
  std::complex < \_Tp > std::\__detail::\__polylog\_exp\_neg \ (\_Tp \_\_s, \ std::complex < \_Tp > \_\_w)
• template<typename _{\rm Tp}>
  std::complex < Tp > std:: detail:: polylog exp neg (int s, std::complex < Tp > w)
• template<typename _Tp , int __sigma>
  std::complex< Tp > std:: detail:: polylog exp neg even (unsigned int n, std::complex< Tp > w)
• template<typename _Tp , int __sigma>
  std::complex< Tp > std:: detail:: polylog exp neg odd (unsigned int n, std::complex< Tp > w)
• template<typename _PowTp , typename _Tp >
  _Tp std::__detail::__polylog_exp_negative_real_part (_PowTp __s, _Tp __w)
template<typename _Tp >
  std::complex < Tp > std:: detail:: polylog exp pos (unsigned int s, std::complex < Tp > w)
template<typename _Tp >
  std::complex< Tp > std:: detail:: polylog exp pos (unsigned int s, Tp w)
template<typename _Tp >
  std::complex< _Tp > std::__detail::__polylog_exp_pos (_Tp __s, std::complex< _Tp > __w)
template<typename _Tp >
  std::complex < Tp > std:: detail:: polylog exp real neg ( Tp s, std::complex < Tp > w)
template<typename _Tp >
  std::complex< _Tp > std::__detail::__polylog_exp_real_neg (_Tp __s, _Tp __w)
template<typename _Tp >
  std::complex< _Tp > std::__detail::__polylog_exp_real_pos (_Tp __s, std::complex< _Tp > __w)
template<typename _Tp >
  std::complex< _Tp > std::__detail::__polylog_exp_real_pos (_Tp __s, _Tp __w)
• template<typename _Tp = double>
  _Tp std::__detail::evenzeta (unsigned int __k)
```

## 10.23.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <cmath>.

### 10.23.2 Macro Definition Documentation

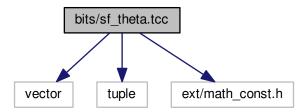
```
10.23.2.1 #define _GLIBCXX_BITS_SF_POLYLOG_TCC 1
```

Definition at line 41 of file sf\_polylog.tcc.

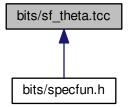
## 10.24 bits/sf theta.tcc File Reference

```
#include <vector>
#include <tuple>
#include <ext/math_const.h>
```

Include dependency graph for sf\_theta.tcc:



This graph shows which files directly or indirectly include this file:



# **Namespaces**

- std
- std::\_\_detail

# **Macros**

• #define \_GLIBCXX\_SF\_THETA\_TCC 1

```
template<typename _Tp >
    _Tp std::__detail::__ellnome (_Tp __k)
template<typename _Tp >
    _Tp std::__detail::__ellnome_k (_Tp __k)
template<typename _Tp >
    _Tp std::__detail::__ellnome_series (_Tp __k)
```

```
template<typename _Tp >
  std::tuple < _Tp, _Tp, _Tp > std::__detail::__jacobi_sncndn (_Tp __k, _Tp __u)
template<typename _Tp >
  _Tp std::__detail::__theta_1 (_Tp __nu, _Tp __x)
• template<typename _{\mathrm{Tp}} >
  _Tp std::__detail::__theta_2 (_Tp __nu, _Tp __x)
ullet template<typename _Tp >
  _Tp std::__detail::__theta_2_asymp (_Tp __nu, _Tp __x)
template<typename _Tp >
  _Tp std::__detail::__theta_2_sum (_Tp __nu, _Tp __x)
template<typename _Tp >
  _Tp std::__detail::__theta_3 (_Tp __nu, _Tp __x)
template<typename _Tp >
  _Tp std::__detail::__theta_3_asymp (_Tp __nu, _Tp __x)
template<typename _Tp >
  _Tp std::__detail::__theta_3_sum (_Tp __nu, _Tp __x)
template<typename_Tp>
  _Tp std::__detail::__theta_4 (_Tp __nu, _Tp __x)
template<typename _Tp >
  _Tp std::__detail::__theta_c (_Tp __k, _Tp __x)
template<typename _Tp >
  _Tp std::__detail::__theta_d (_Tp __k, _Tp __x)
template<typename Tp >
  _Tp std::__detail::__theta_n (_Tp __k, _Tp __x)
template<typename _Tp >
  _Tp std::__detail::__theta_s (_Tp __k, _Tp __x)
```

#### 10.24.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

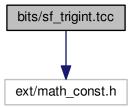
#### 10.24.2 Macro Definition Documentation

```
10.24.2.1 #define _GLIBCXX_SF_THETA_TCC 1
```

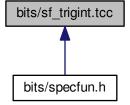
Definition at line 31 of file sf\_theta.tcc.

# 10.25 bits/sf\_trigint.tcc File Reference

#include <ext/math\_const.h>
Include dependency graph for sf\_trigint.tcc:



This graph shows which files directly or indirectly include this file:



## **Namespaces**

- std
- std::\_\_detail

#### **Macros**

• #define \_GLIBCXX\_SF\_TRIGINT\_TCC 1

## **Enumerations**

enum { std::\_\_detail::SININT, std::\_\_detail::COSINT }

#### **Functions**

template < typename \_Tp >
 std::pair < \_Tp, \_Tp > std::\_\_detail::\_\_sincosint (\_Tp \_\_x)

This function returns the sine Si(x) and cosine Ci(x) integrals as a pair.

template<typename \_Tp >
 void std:: \_\_detail:: \_\_sincosint\_asymp (\_Tp \_\_t, \_Tp &\_Si, \_Tp &\_Ci)

This function computes the sine Si(x) and cosine Ci(x) integrals by asymptotic series summation for positive argument.

template < typename \_Tp >
 void std::\_\_detail::\_\_sincosint\_cont\_frac (\_Tp \_\_t, \_Tp &\_Si, \_Tp &\_Ci)

This function computes the sine Si(x) and cosine Ci(x) integrals by continued fraction for positive argument.

template < typename \_Tp >
 void std:: \_\_detail:: \_\_sincosint\_series (\_Tp \_\_t, \_Tp &\_Si, \_Tp &\_Ci)

This function computes the sine Si(x) and cosine Ci(x) integrals by series summation for positive argument.

## 10.25.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

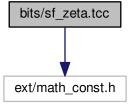
#### 10.25.2 Macro Definition Documentation

10.25.2.1 #define \_GLIBCXX\_SF\_TRIGINT\_TCC 1

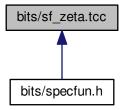
Definition at line 31 of file sf\_trigint.tcc.

# 10.26 bits/sf\_zeta.tcc File Reference

#include <ext/math\_const.h>
Include dependency graph for sf\_zeta.tcc:



This graph shows which files directly or indirectly include this file:



# **Namespaces**

- std
- std::\_\_detail

#### **Macros**

• #define GLIBCXX BITS SF ZETA TCC 1

convergence of the > 0 sum gets better.

```
template<typename</li>Tp >
  _Tp std::__detail::__dilog (_Tp __x)
      Compute the dilogarithm function Li_2(x) by summation for x \le 1.
template<typename _Tp >
  _Tp std::__detail::__hurwitz_zeta (_Tp __s, _Tp __a)
      Return the Hurwitz zeta function \zeta(s,a) for all s = 1 and a > -1.
template<typename _Tp >
  _Tp std::__detail::__hurwitz_zeta_euler_maclaurin (_Tp __s, _Tp __a)
      Return the Hurwitz zeta function \zeta(s,a) for all s \neq 1 and a > -1.
template<typename _Tp >
  _Tp std::__detail::__riemann_zeta (_Tp __s)
      Return the Riemann zeta function \zeta(s).

    template<typename</li>
    Tp >

  _Tp std::__detail::__riemann_zeta_alt (_Tp __s)
      Evaluate the Riemann zeta function \zeta(s) by an alternate series for s > 0.
template<typename _Tp >
  _Tp std::__detail::__riemann_zeta_euler_maclaurin (_Tp __s)
      Evaluate the Riemann zeta function \zeta(s) by an alternate series for s > 0.
template<typename _Tp >
  _Tp std::__detail::__riemann_zeta_glob (_Tp __s)
      Evaluate the Riemann zeta function by series for all s != 1. Convergence is great until largish negative numbers. Then the
```

```
template<typename _Tp >
    _Tp std::__detail::__riemann_zeta_m_1 (_Tp __s)
Return the Riemann zeta function ζ(s) - 1.
template<typename _Tp >
    _Tp std::__detail::__riemann_zeta_m_1_sum (_Tp __s)
Return the Riemann zeta function ζ(s) - 1 by summation for s > 1. This is a small remainder for large s.
template<typename _Tp >
    _Tp std::__detail::__riemann_zeta_product (_Tp __s)
Compute the Riemann zeta function ζ(s) using the product over prime factors.
template<typename _Tp >
    _Tp std::__detail::__riemann_zeta_sum (_Tp __s)
Compute the Riemann zeta function ζ(s) by summation for s > 1.
```

## **Variables**

- constexpr size\_t std::\_\_detail::\_Num\_Euler\_Maclaurin\_zeta = 100
- constexpr long double std:: \_\_detail:: S \_Euler \_Maclaurin \_zeta [ \_Num \_Euler \_Maclaurin \_zeta]
- constexpr size\_t std::\_\_detail::\_S\_num\_zetam1 = 33
- constexpr long double std::\_\_detail::\_S\_zetam1 [\_S\_num\_zetam1]

#### 10.26.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

#### 10.26.2 Macro Definition Documentation

10.26.2.1 #define \_GLIBCXX\_BITS\_SF\_ZETA\_TCC 1

Definition at line 46 of file sf\_zeta.tcc.

# 10.27 bits/specfun.h File Reference

```
#include <bits/c++config.h>
#include <limits>
#include <bits/stl_algobase.h>
#include <bits/specfun_util.h>
#include <type_traits>
#include <bits/numeric_limits.h>
#include <bits/complex_util.h>
#include <bits/sf_gamma.tcc>
#include <bits/sf_bessel.tcc>
#include <bits/sf_beta.tcc>
#include <bits/sf_cardinal.tcc>
#include <bits/sf_chebyshev.tcc>
#include <bits/sf dawson.tcc>
#include <bits/sf_ellint.tcc>
#include <bits/sf_expint.tcc>
#include <bits/sf_fresnel.tcc>
#include <bits/sf_gegenbauer.tcc>
#include <bits/sf_hyperg.tcc>
#include <bits/sf_hypint.tcc>
#include <bits/sf_jacobi.tcc>
#include <bits/sf_laguerre.tcc>
#include <bits/sf_legendre.tcc>
#include <bits/sf_hydrogen.tcc>
#include <bits/sf_mod_bessel.tcc>
#include <bits/sf_hermite.tcc>
#include <bits/sf_theta.tcc>
#include <bits/sf_trigint.tcc>
#include <bits/sf_zeta.tcc>
#include <bits/sf_owens_t.tcc>
#include <bits/sf_polylog.tcc>
#include <bits/sf_airy.tcc>
#include <bits/sf hankel.tcc>
Include dependency graph for specfun.h:
```



#### **Namespaces**

- \_\_gnu\_cxx
- std

#### **Macros**

- #define \_\_cpp\_lib\_math\_special\_functions 201603L
- #define STDCPP MATH SPEC FUNCS 201003L

#### **Enumerations**

enum { \_\_gnu\_cxx::\_GLIBCXX\_JACOBI\_SN, \_\_gnu\_cxx::\_GLIBCXX\_JACOBI\_CN, \_\_gnu\_cxx::\_GLIBCXX\_J
 ACOBI\_DN }

```
template<typename _Tp >
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::airy_ai (_Tp __x)

    float __gnu_cxx::airy_aif (float __x)

    long double <u>__gnu_cxx::airy_ail</u> (long double <u>__x</u>)

template<typename</li>Tp >
  __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::airy_bi (_Tp __x)

    float __gnu_cxx::airy_bif (float __x)

• long double gnu cxx::airy bil (long double x)
template<typename _Tp >
    gnu cxx:: promote< Tp >:: type std::assoc laguerre (unsigned int n, unsigned int m, Tp x)

    float std::assoc_laguerref (unsigned int __n, unsigned int __m, float __x)

    long double std::assoc_laguerrel (unsigned int __n, unsigned int __m, long double __x)

template<typename_Tp>
    _gnu_cxx::__promote< _Tp >::__type std::assoc_legendre (unsigned int __l, unsigned int __m, _Tp __x)
• float std::assoc legendref (unsigned int I, unsigned int m, float x)

    long double std::assoc legendrel (unsigned int I, unsigned int m, long double x)

template<typename_Tp>
    gnu cxx:: promote num t< Tp > gnu cxx::bernoulli (unsigned int n)

    float gnu cxx::bernoullif (unsigned int n)

    long double __gnu_cxx::bernoullil (unsigned int __n)

template<typename _Tpa , typename _Tpb >
   gnu cxx:: promote 2< Tpa, Tpb >:: type std::beta (Tpa a, Tpb b)

    float std::betaf (float a, float b)

    long double std::betal (long double __a, long double __b)

template<typename_Tp>
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::bincoef (unsigned int __n, unsigned int __k)
• float gnu cxx::bincoeff (unsigned int n, unsigned int k)

    long double gnu cxx::bincoefl (unsigned int n, unsigned int k)

template<typename</li>Tp >
  __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::chebyshev_t (unsigned int __n, _Tp __x)

    float gnu cxx::chebyshev tf (unsigned int n, float x)

    long double __gnu_cxx::chebyshev_tl (unsigned int __n, long double __x)

template<typename</li>Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::chebyshev_u (unsigned int __n, _Tp __x)

    float gnu cxx::chebyshev uf (unsigned int n, float x)

    long double __gnu_cxx::chebyshev_ul (unsigned int __n, long double __x)

template<typename _Tp >
    gnu cxx:: promote num t < Tp > gnu cxx::chebyshev v (unsigned int n, Tp x)

    float gnu cxx::chebyshev vf (unsigned int n, float x)

    long double gnu cxx::chebyshev vl (unsigned int n, long double x)

template<typename</li>Tp >
   __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::chebyshev_w (unsigned int __n, _Tp __x)

    float gnu cxx::chebyshev wf (unsigned int n, float x)

    long double gnu cxx::chebyshev wl (unsigned int n, long double x)
```

```
template<typename _Tp >
   gnu cxx:: promote num t < Tp > gnu cxx::clausen (unsigned int m, Tp w)

    template<typename</li>
    Tp >

  std::complex< __gnu_cxx::__promote_num_t< _Tp >> __gnu_cxx::clausen (unsigned int __m, std::complex<
  _{\mathsf{Tp}} > _{\mathsf{w}}

    template<typename</li>
    Tp >

   _gnu_cxx::_ promote_num_t< _Tp > __gnu_cxx::clausen_c (unsigned int __m, _Tp __w)
• float <u>gnu_cxx::clausen_cf</u> (unsigned int <u>m</u>, float <u>w</u>)
• long double gnu cxx::clausen cl (unsigned int m, long double w)
template<typename</li>Tp >
  __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::clausen_s (unsigned int __m, _Tp __w)

    float gnu cxx::clausen sf (unsigned int m, float w)

    long double gnu cxx::clausen sl (unsigned int m, long double w)

    float __gnu_cxx::clausenf (unsigned int __m, float __w)

• std::complex < float > gnu cxx::clausenf (unsigned int m, std::complex < float > w)

    long double __gnu_cxx::clausenl (unsigned int __m, long double __w)

    std::complex < long double > __gnu_cxx::clausenl (unsigned int __m, std::complex < long double > __w)

template<typename_Tp>
    gnu cxx:: promote < Tp >:: type std::comp ellint 1 (Tp k)

    float std::comp ellint 1f (float k)

    long double std::comp ellint 11 (long double k)

template<typename_Tp>
    _gnu_cxx::__promote< _Tp >::__type std::comp_ellint_2 (_Tp __k)

    float std::comp ellint 2f (float k)

    long double std::comp ellint 2l (long double k)

template<typename _Tp , typename _Tpn >
   _gnu_cxx::__promote_2< _Tp, _Tpn >::__type std::comp_ellint_3 (_Tp __k, _Tpn __nu)

    float std::comp ellint 3f (float k, float nu)

      Return the complete elliptic integral of the third kind \Pi(k,\nu) for float modulus k.

    long double std::comp ellint 3l (long double k, long double nu)

      Return the complete elliptic integral of the third kind \Pi(k,\nu) for long double modulus k.

    template<typename Tk >

    gnu cxx:: promote num t < Tk > gnu cxx::comp ellint d (Tk k)

    float gnu cxx::comp ellint df (float k)

    long double gnu cxx::comp ellint dl (long double k)

    float __gnu_cxx::comp_ellint_rf (float __x, float __y)

• long double gnu cxx::comp ellint rf (long double x, long double y)
• template<typename _{\rm Tx}, typename _{\rm Ty} >
   _gnu_cxx::__promote_num_t< _Tx, _Ty > __gnu_cxx::comp_ellint_rf (_Tx __x, _Ty __y)

    float __gnu_cxx::comp_ellint_rg (float __x, float __y)

    long double gnu cxx::comp ellint rg (long double x, long double y)

• template<typename _Tx , typename _Ty >
    _gnu_cxx::__promote_num_t< _Tx, _Ty > __gnu_cxx::comp_ellint_rg (_Tx __x, _Ty __y)
template<typename _Tpa , typename _Tpc , typename _Tp >
   _gnu_cxx::__promote_3< _Tpa, _Tpc, _Tp >::__type __gnu_cxx::conf_hyperg (_Tpa __a, _Tpc __c, _Tp __x)

    template<typename _Tpc , typename _Tp >

  __gnu_cxx::_promote_2< _Tpc, _Tp >::_type __gnu_cxx::conf_hyperg_lim (_Tpc __c, _Tp __x)

    float gnu cxx::conf hyperg limf (float c, float x)

    long double __gnu_cxx::conf_hyperg_liml (long double __c, long double __x)

    float __gnu_cxx::conf_hypergf (float __a, float __c, float __x)

    long double gnu cxx::conf hypergl (long double a, long double c, long double x)
```

```
template<typename _Tp >
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::coshint (_Tp __x)

    float gnu cxx::coshintf (float x)

    long double <u>gnu_cxx::coshintl</u> (long double <u>x</u>)

template<typename</li>Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::cosint (_Tp __x)

    float gnu cxx::cosintf (float x)

    long double gnu cxx::cosintl (long double x)

template<typename _Tpnu , typename _Tp >
   _gnu_cxx::__promote_2< _Tpnu, _Tp >::__type std::cyl_bessel_i (_Tpnu __nu, _Tp __x)

    float std::cyl_bessel_if (float __nu, float __x)

    long double std::cyl bessel il (long double nu, long double x)

• template<typename Tpnu, typename Tp>
    _gnu_cxx::__promote_2< _Tpnu, _Tp >::__type std::cyl_bessel_j (_Tpnu __nu, _Tp __x)

    float std::cyl bessel if (float nu, float x)

    long double std::cyl_bessel_jl (long double __nu, long double __x)

• template<typename _Tpnu , typename _Tp >
    _gnu_cxx::__promote_2< _Tpnu, _Tp >::__type std::cyl_bessel_k (_Tpnu __nu, _Tp __x)

    float std::cyl bessel kf (float nu, float x)

    long double std::cyl_bessel_kl (long double __nu, long double __x)

• template<typename _Tpnu , typename _Tp >
  std::complex< __gnu_cxx::__promote_num_t< _Tpnu, _Tp >> __gnu_cxx::cyl_hankel_1 (_Tpnu __nu, _Tp
  __z)
• template<typename _{\rm Tpnu}, typename _{\rm Tp} >
  std::complex < \underline{gnu\_cxx::\_promote\_num\_t} < \underline{Tpnu, \_Tp} > \underline{gnu\_cxx::cyl\_hankel\_1} (std::complex < \underline{\leftarrow}
  Tpnu > __nu, std::complex< _Tp > __x)

    std::complex< float > __gnu_cxx::cyl_hankel_1f (float __nu, float __z)

    std::complex < float > __gnu_cxx::cyl_hankel_1f (std::complex < float > __nu, std::complex < float > __x)

• std::complex < long double > gnu cxx::cyl hankel 1l (long double nu, long double z)

    std::complex < long double > gnu cxx::cyl hankel 1l (std::complex < long double > nu, std::complex < long</li>

  double > x)
• template<typename _Tpnu , typename _Tp >
  std::complex< gnu cxx:: promote num t< Tpnu, Tp >> gnu cxx::cyl hankel 2 (Tpnu nu, Tp
  __z)
• template<typename _Tpnu , typename _Tp >
  std::complex < \underline{gnu\_cxx::\_promote\_num\_t < \underline{Tpnu}, \underline{Tp} > \underline{gnu\_cxx::cyl\_hankel\_2} (std::complex < \underline{\leftarrow}
  Tpnu > __nu, std::complex< _Tp > __x)

    std::complex< float > __gnu_cxx::cyl_hankel_2f (float __nu, float __z)

    std::complex < float > __nu, std::complex < float > __nu, std::complex < float > __x)

• std::complex < long double > gnu cxx::cyl hankel 2l (long double nu, long double z)

    std::complex < long double > gnu cxx::cyl hankel 2l (std::complex < long double > nu, std::complex < long</li>

  double > x)
• template<typename _Tpnu , typename _Tp >
   gnu cxx:: promote 2< Tpnu, Tp >:: type std::cyl neumann (Tpnu nu, Tp x)

    float std::cyl neumannf (float nu, float x)

    long double std::cyl_neumannl (long double __nu, long double __x)

template<typename _Tp >
   gnu cxx:: promote num t < Tp > gnu cxx::dawson (Tp x)

    float __gnu_cxx::dawsonf (float __x)

    long double <u>__gnu_cxx::dawsonl</u> (long double <u>__x</u>)

template<typename _Tp >
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::digamma (_Tp __z)
```

```
    float __gnu_cxx::digammaf (float __z)

    long double __gnu_cxx::digammal (long double __z)

template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::dilog (_Tp __x)

    float gnu cxx::dilogf (float x)

    long double __gnu_cxx::dilogl (long double __x)

• template<typename _Tp >
  _Tp __gnu_cxx::dirichlet_beta (_Tp __s)

    float gnu cxx::dirichlet betaf (float s)

    long double gnu cxx::dirichlet betal (long double s)

template<typename _Tp >
  Tp gnu cxx::dirichlet eta (Tp s)

    float gnu cxx::dirichlet etaf (float s)

    long double __gnu_cxx::dirichlet_etal (long double __s)

template<typename</li>Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::double_factorial (int __n)

    float gnu cxx::double factorialf (int n)

    long double __gnu_cxx::double_factoriall (int __n)

• template<typename _Tp , typename _Tpp >
   _gnu_cxx::__promote_2< _Tp, _Tpp >::__type std::ellint_1 (_Tp __k, _Tpp __phi)

    float std::ellint_1f (float __k, float __phi)

    long double std::ellint 11 (long double k, long double phi)

    template<typename _Tp , typename _Tpp >

    _gnu_cxx::__promote_2< _Tp, _Tpp >::__type std::ellint_2 (_Tp __k, _Tpp __phi)

    float std::ellint 2f (float k, float phi)

      Return the incomplete elliptic integral of the second kind E(k,\phi) for float argument.

    long double std::ellint 2l (long double k, long double phi)

      Return the incomplete elliptic integral of the second kind E(k, \phi).

    template<typename Tp, typename Tpn, typename Tpp>

   _gnu_cxx::__promote_3< _Tp, _Tpn, _Tpp >::__type std::ellint_3 (_Tp __k, _Tpn __nu, _Tpp __phi)
      Return the incomplete elliptic integral of the third kind \Pi(k, \nu, \phi).

    float std::ellint 3f (float k, float nu, float phi)

      Return the incomplete elliptic integral of the third kind \Pi(k, \nu, \phi) for float argument.

    long double std::ellint_3l (long double __k, long double __nu, long double __phi)

      Return the incomplete elliptic integral of the third kind \Pi(k, \nu, \phi).

    template<typename _Tk , typename _Tp , typename _Ta , typename _Tb >

    _gnu_cxx::__promote_num_t< _Tk, _Tp, _Ta, _Tb > <u>__gnu_cxx::ellint_cel</u> (_Tk <u>__k_c, _</u>Tp <u>__p, _</u>Ta <u>__a, _</u>Tb
   b)

    float __gnu_cxx::ellint_celf (float __k_c, float __p, float _ a, float _ b)

    long double gnu cxx::ellint cell (long double k c, long double p, long double a, long double b)

• template<typename _Tk , typename _Tphi >
    _gnu_cxx::__promote_num_t< _Tk, _Tphi > __gnu_cxx::ellint_d (_Tk __k, _Tphi _ phi)

    float __gnu_cxx::ellint_df (float __k, float __phi)

    long double gnu cxx::ellint dl (long double k, long double phi)

    template<typename _Tp , typename _Tk >

    _gnu_cxx::__promote_num_t< _Tp, _Tk > __gnu_cxx::ellint_el1 (_Tp __x, _Tk __k_c)

    float __gnu_cxx::ellint_el1f (float __x, float __k_c)

    long double __gnu_cxx::ellint_el1l (long double __x, long double __k_c)

    template<typename _Tp , typename _Tk , typename _Ta , typename _Tb >

    _gnu_cxx::__promote_num_t< _Tp, _Tk, _Ta, _Tb > __gnu_cxx::ellint_el2 (_Tp __x, _Tk __k_c, _Ta __a, _Tb
  __b)
```

```
    float __gnu_cxx::ellint_el2f (float __x, float __k_c, float __a, float __b)

    long double __gnu_cxx::ellint_el2l (long double __x, long double __k_c, long double __a, long double __b)

• template<typename \_Tx, typename \_Tk, typename \_Tp>
    _gnu_cxx::__promote_num_t< _Tx, _Tk, _Tp > __gnu_cxx::ellint_el3 (_Tx __x, _Tk __k_c, _Tp __p)
• float gnu cxx::ellint el3f (float x, float k c, float p)
• long double gnu cxx::ellint el3l (long double x, long double k c, long double p)
• template<typename Tp, typename Up>
    _gnu_cxx::__promote_num_t< _Tp, _Up > __gnu_cxx::ellint_rc (_Tp __x, _Up __y)

    float __gnu_cxx::ellint_rcf (float __x, float __y)

    long double __gnu_cxx::ellint_rcl (long double __x, long double __y)

• template<typename _Tp , typename _Up , typename _Vp >
    gnu cxx:: promote num t< Tp, Up, Vp > gnu cxx::ellint rd (Tp x, Up y, Vp z)

    float __gnu_cxx::ellint_rdf (float __x, float __y, float __z)

    long double gnu cxx::ellint rdl (long double x, long double y, long double z)

template<typename _Tp , typename _Up , typename _Vp >
   _gnu_cxx::_promote_num_t< _Tp, _Up, _Vp > __gnu_cxx::ellint_rf (_Tp __x, _Up __y, _Vp __z)

    float __gnu_cxx::ellint_rff (float __x, float __y, float __z)

    long double gnu cxx::ellint rfl (long double x, long double y, long double z)

• template<typename _Tp , typename _Up , typename _Vp >
    _gnu_cxx::__promote_num_t< _Tp, _Up, _Vp > __gnu_cxx::ellint_rg (_Tp __x, _Up __y, _Vp __z)

    float __gnu_cxx::ellint_rgf (float __x, float __y, float __z)

    long double __gnu_cxx::ellint_rgl (long double __x, long double __y, long double __z)

template<typename _Tp , typename _Up , typename _Vp , typename _Wp >
   _gnu_cxx::__promote_num_t< _Tp, _Up, _Vp, _Wp > __gnu_cxx::ellint_rj (_Tp __x, _Up __y, _Vp __z, _Wp
  __p)
• float gnu cxx::ellint rif (float x, float y, float z, float p)

    long double __gnu_cxx::ellint_rjl (long double __x, long double __y, long double __z, long double __p)

template<typename _Tp >
  Tp gnu cxx::ellnome (Tp k)

    float gnu cxx::ellnomef (float k)

    long double gnu cxx::ellnomel (long double k)

template<typename</li>Tp >
   __gnu_cxx::__promote< _Tp >::__type std::expint (_Tp __x)
template<typename _Tp >
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::expint_e1 (_Tp __x)

    float gnu cxx::expint e1f (float x)

    long double gnu cxx::expint e1l (long double x)

template<typename</li>Tp >
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::expint_en (unsigned int __n, _Tp __x)
• float __gnu_cxx::expint_enf (unsigned int __n, float __x)

    long double __gnu_cxx::expint_enl (unsigned int __n, long double __x)

    float std::expintf (float x)

    long double std::expintl (long double x)

template<typename_Tp>
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::factorial (unsigned int __n)

    float gnu cxx::factorialf (unsigned int n)

    long double gnu cxx::factoriall (unsigned int n)

template<typename_Tp>
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::fresnel_c (_Tp __x)

    float <u>gnu_cxx::fresnel_cf</u> (float <u>x</u>)

    long double gnu cxx::fresnel cl (long double x)
```

```
template<typename _Tp >
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::fresnel_s (_Tp __x)

    float gnu cxx::fresnel sf (float x)

    long double __gnu_cxx::fresnel_sl (long double __x)

• template<typename Tn, typename Tp>
    _gnu_cxx::__promote_num_t< _Tn, _Tp > __gnu_cxx::gamma_l (_Tn __n, _Tp __x)
• float gnu cxx::gamma If (float n, float x)

    long double __gnu_cxx::gamma_ll (long double __n, long double __x)

• template<typename Ta, typename Tp>
   _gnu_cxx::__promote_num_t< _Ta, _Tp > __gnu_cxx::gamma_p (_Ta __a, _Tp __x)

    float gnu cxx::gamma pf (float a, float x)

    long double gnu cxx::gamma pl (long double a, long double x)

• template<typename _Ta , typename _Tp >
    _gnu_cxx::__promote_num_t< _Ta, _Tp > __gnu_cxx::gamma_q (_Ta __a, _Tp __x)

    float gnu cxx::gamma qf (float a, float x)

    long double gnu cxx::gamma ql (long double a, long double x)

template<typename _Tn , typename _Tp >
   _gnu_cxx::__promote_num_t< _Tn, _Tp > __gnu_cxx::gamma_u (_Tn __n, _Tp __x)

    float gnu cxx::gamma uf (float n, float x)

    long double gnu cxx::gamma ul (long double n, long double x)

• template<typename _Talpha , typename _Tp >
    _gnu_cxx::__promote_num_t< _Talpha, _Tp > __gnu_cxx::gegenbauer (unsigned int __n, _Talpha __alpha,
  Tp x)

    float gnu cxx::gegenbauerf (unsigned int n, float alpha, float x)

    long double __gnu_cxx::gegenbauerl (unsigned int __n, long double __alpha, long double __x)

template<typename _Tp >
    _gnu_cxx::__promote< _Tp >::__type std::hermite (unsigned int n, Tp x)

    float std::hermitef (unsigned int n, float x)

    long double std::hermitel (unsigned int __n, long double __x)

• template<typename Tk, typename Tphi >
    _gnu_cxx::__promote_num_t< _Tk, _Tphi > __gnu_cxx::heuman_lambda (_Tk __k, _Tphi __phi)

    float gnu cxx::heuman lambdaf (float k, float phi)

    long double gnu cxx::heuman lambdal (long double k, long double phi)

• template<typename _Tp , typename _Up >
    _gnu_cxx::__promote_num_t< _Tp, _Up > __gnu_cxx::hurwitz_zeta (_Tp __s, _Up __a)
• template<typename Tp, typename Up>
  std::complex< _Tp > __gnu_cxx::hurwitz_zeta (_Tp __s, std::complex< _Up > __a)

    float __gnu_cxx::hurwitz_zetaf (float __s, float __a)

    long double __gnu_cxx::hurwitz_zetal (long double __s, long double __a)

• template<typename Tpa, typename Tpb, typename Tpc, typename Tp>
    _gnu_cxx::__promote_4< _Tpa, _Tpb, _Tpc, _Tp >::__type __gnu_cxx::hyperg (_Tpa __a, _Tpb __b, _Tpc
   __c, _Tp ___x)

    float __gnu_cxx::hypergf (float __a, float __b, float _ c, float _ x)

• long double gnu cxx::hypergl (long double a, long double b, long double c, long double x)
template<typename _Ta , typename _Tb , typename _Tp >
    gnu_cxx::__promote_num_t< _Ta, _Tb, _Tp > __gnu_cxx::ibeta (_Ta __a, _Tb __b, _Tp __x)
- template<typename _Ta , typename _Tb , typename _Tp >
  __gnu_cxx::__promote_num_t< _Ta, _Tb, _Tp > __gnu_cxx::ibetac (_Ta __a, _Tb __b, _Tp __x)
• float gnu cxx::ibetacf (float a, float b, float x)

    long double __gnu_cxx::ibetacl (long double __a, long double __b, long double __x)

    float gnu cxx::ibetaf (float a, float b, float x)

    long double gnu cxx::ibetal (long double a, long double b, long double x)
```

```
    template<typename _Talpha , typename _Tbeta , typename _Tp >

   _gnu_cxx::__promote_num_t< _Talpha, _Tbeta, _Tp > __gnu_cxx::jacobi (unsigned __n, _Talpha __alpha,
  Tbeta beta, Tp x)
• template<typename _Kp , typename _Up >
   _gnu_cxx::__promote_num_t< _Kp, _Up > __gnu_cxx::jacobi_cn (_Kp __k, _Up __u)
• float gnu cxx::jacobi cnf (float k, float u)

    long double gnu cxx::jacobi cnl (long double k, long double u)

• template<typename _Kp , typename _Up >
    _gnu_cxx::__promote_num_t< _Kp, _Up > __gnu_cxx::jacobi_dn (_Kp __k, _Up __u)
• float gnu cxx::jacobi dnf (float k, float u)

    long double __gnu_cxx::jacobi_dnl (long double __k, long double __u)

• template<typename _Kp , typename _Up >
    _gnu_cxx::__promote_num_t< _Kp, _Up > __gnu_cxx::jacobi_sn (_Kp __k, _Up __u)

    float gnu cxx::jacobi snf (float k, float u)

    long double __gnu_cxx::jacobi_snl (long double __k, long double __u)

• template<typename Tk, typename Tphi >
    _gnu_cxx::__promote_num_t< _Tk, _Tphi > __gnu_cxx::jacobi_zeta (_Tk __k, _Tphi __phi)

    float gnu cxx::jacobi zetaf (float k, float phi)

    long double __gnu_cxx::jacobi_zetal (long double __k, long double __phi)

    float gnu cxx::jacobif (unsigned n, float alpha, float beta, float x)

    long double __gnu_cxx::jacobil (unsigned __n, long double __alpha, long double __beta, long double __x)

template<typename _Tp >
   gnu cxx:: promote < Tp >:: type std::laguerre (unsigned int n, Tp x)

    float std::laguerref (unsigned int n, float x)

    long double std::laguerrel (unsigned int __n, long double __x)

template<typename</li>Tp >
   gnu cxx:: promote num t < Tp > gnu cxx::lbincoef (unsigned int n, unsigned int k)
• float gnu cxx::lbincoeff (unsigned int n, unsigned int k)

    long double __gnu_cxx::lbincoefl (unsigned int __n, unsigned int __k)

template<typename</li>Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::ldouble_factorial (int __n)

    float __gnu_cxx::ldouble_factorialf (int __n)

    long double __gnu_cxx::ldouble_factoriall (int __n)

template<typename_Tp>
    _gnu_cxx::__promote< _Tp >::__type std::legendre (unsigned int __l, _Tp __x)
template<typename _Tp >
   __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::legendre_q (unsigned int __n, Tp x)

    float gnu cxx::legendre qf (unsigned int n, float x)

    long double gnu cxx::legendre ql (unsigned int n, long double x)

    float std::legendref (unsigned int I, float x)

    long double std::legendrel (unsigned int I, long double x)

template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::lfactorial (unsigned int __n)

    float gnu cxx::lfactorialf (unsigned int

    long double gnu cxx::lfactoriall (unsigned int n)

template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::logint (_Tp __x)

    float gnu cxx::logintf (float x)

    long double gnu cxx::logintl (long double x)

• template<typename _Tp , typename _Tn >
    _gnu_cxx::__promote_num_t< _Tp, _Tn > __gnu_cxx::lpochhammer_l (_Tp __a, _Tn __n)

    float gnu cxx::lpochhammer lf (float a, float n)
```

```
    long double __gnu_cxx::lpochhammer_ll (long double __a, long double __n)

 • template<typename _Tp , typename _Tn >
    _gnu_cxx::__promote_num_t< _Tp, _Tn > __gnu_cxx::lpochhammer_u( Tp = a, Tn = n)

    float __gnu_cxx::lpochhammer_uf (float __a, float __n)

• long double gnu cxx::lpochhammer ul (long double a, long double n)
• template<typename _Tph , typename _Tpa >
    _gnu_cxx::__promote_num_t< _Tph, _Tpa > __gnu_cxx::owens_t (_Tph __h, _Tpa __a)

    float __gnu_cxx::owens_tf (float __h, float __a)

    long double gnu cxx::owens tl (long double h, long double a)

• template<typename _Tp , typename _Tn >
    _gnu_cxx::__promote_num_t< _Tp, _Tn > __gnu_cxx::pochhammer_l (_Tp __a, _Tn __n)

    float gnu cxx::pochhammer lf (float a, float n)

• long double __gnu_cxx::pochhammer_ll (long double __a, long double __n)
• template<typename _Tp , typename _Tn >
    _gnu_cxx::__promote_num_t< _Tp, _Tn > __gnu_cxx::pochhammer_u (_Tp __a, _Tn __n)

    float gnu cxx::pochhammer uf (float a, float n)

    long double __gnu_cxx::pochhammer_ul (long double __a, long double __n)

• template<typename _Tp , typename _Wp >
    _gnu_cxx::__promote_num_t< _Tp, _Wp > __gnu_cxx::polylog (_Tp __s, _Wp __w)

    template<typename Tp, typename Wp>

  std::complex< __gnu_cxx::__promote_num_t< _Tp, _Wp >> __gnu_cxx::polylog (_Tp __s, std::complex< _Tp
  > w)

    float gnu cxx::polylogf (float s, float w)

    std::complex < float > gnu cxx::polylogf (float s, std::complex < float > w)

    long double __gnu_cxx::polylogl (long double __s, long double __w)

    std::complex < long double > __gnu_cxx::polylogl (long double __s, std::complex < long double > __w)

template<typename</li>Tp >
   __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::psi (_Tp __x)

    float __gnu_cxx::psif (float __x)

    long double __gnu_cxx::psil (long double __x)

template<typename</li>Tp >
   __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::radpoly (unsigned int __n, unsigned int __m, _Tp __rho)
• float __gnu_cxx::radpolyf (unsigned int __n, unsigned int __m, float __rho)

    long double gnu cxx::radpolyl (unsigned int n, unsigned int m, long double rho)

template<typename _Tp >
    _gnu_cxx::__promote< _Tp >::__type std::riemann_zeta (_Tp __s)

    float std::riemann zetaf (float s)

    long double std::riemann zetal (long double s)

template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::sinc (_Tp __x)
template<typename _Tp >
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::sinc_pi (_Tp __x)

    float __gnu_cxx::sinc_pif (float __x)

    long double gnu cxx::sinc pil (long double x)

    float gnu cxx::sincf (float x)

    long double <u>gnu_cxx::sincl</u> (long double <u>x</u>)

template<typename _Tp >
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::sinhc (_Tp __x)
template<typename _Tp >
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::sinhc_pi (_Tp __x)

    float gnu cxx::sinhc pif (float x)

    long double gnu cxx::sinhc pil (long double x)
```

```
    float __gnu_cxx::sinhcf (float __x)

    long double __gnu_cxx::sinhcl (long double __x)

template<typename_Tp>
       _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::sinhint (_Tp __x)

    float gnu cxx::sinhintf (float x)

    long double __gnu_cxx::sinhintl (long double __x)

template<typename _Tp >
      _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::sinint (_Tp __x)

    float gnu cxx::sinintf (float x)

    long double gnu cxx::sinintl (long double x)

template<typename _Tp >
      gnu cxx:: promote < Tp >:: type std::sph bessel (unsigned int n, Tp x)
template<typename_Tp>
       _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::sph_bessel_i (unsigned int __n, _Tp __x)

    float gnu cxx::sph bessel if (unsigned int n, float x)

• long double gnu cxx::sph_bessel_il (unsigned int __n, long double __x)
template<typename</li>Tp >
     _gnu_cxx::_promote_num_t< _Tp > __gnu_cxx::sph_bessel_k (unsigned int __n, _Tp __x)

    float __gnu_cxx::sph_bessel_kf (unsigned int __n, float __x)

    long double __gnu_cxx::sph_bessel_kl (unsigned int __n, long double __x)

    float std::sph besself (unsigned int n, float x)

    long double std::sph bessell (unsigned int n, long double x)

template<typename _Tp >
   std::complex< __gnu_cxx::_promote_num_t< _Tp >> __gnu_cxx::sph_hankel_1 (unsigned int __n, _Tp __z)
template<typename _Tp >
   std::complex< gnu cxx:: promote num t< Tp >> gnu cxx::sph hankel 1 (unsigned int n, std←
   ::complex < _Tp > __x)

    std::complex< float > __gnu_cxx::sph_hankel_1f (unsigned int __n, float __z)

    std::complex < float > gnu cxx::sph hankel 1f (unsigned int n, std::complex < float > x)

    std::complex < long double > gnu cxx::sph hankel 1l (unsigned int n, long double z)

    std::complex < long double > __gnu_cxx::sph_hankel_1l (unsigned int __n, std::complex < long double > __x)

template<typename _Tp >
   std::complex< gnu cxx:: promote num t< Tp>> gnu cxx::sph hankel 2 (unsigned int n, Tp z)
template<typename Tp >
   std::complex< \_\_gnu\_cxx::\_promote\_num\_t < \_Tp >> \_\_gnu\_cxx::sph\_hankel\_2 \ (unsigned \ int \ \_\_n, \ std \leftarrow 1.00 \ and \ below: below to be a complex < \_\_gnu\_cxx::sph\_hankel\_2 \ (unsigned \ int \ \_\_n, \ std \leftarrow 1.00 \ and \ below to be a complex < \_\_gnu\_cxx::sph\_hankel\_2 \ (unsigned \ int \ \_\_n, \ std \leftarrow 1.00 \ and \ below to be a complex < \_\_gnu\_cxx::sph\_hankel\_2 \ (unsigned \ int \ \_\_n, \ std \leftarrow 1.00 \ and \ below to be a complex < \_\_gnu\_cxx::sph\_hankel\_2 \ (unsigned \ int \ \_\_n, \ std \leftarrow 1.00 \ and \ below to be a complex < \_\_gnu\_cxx::sph\_hankel\_2 \ (unsigned \ int \ \_\_n, \ std \leftarrow 1.00 \ and \ below to be a complex < \_\_gnu\_cxx::sph\_hankel\_2 \ (unsigned \ int \ \_\_n, \ std \leftarrow 1.00 \ and \ below to be a complex < \_\_gnu\_cxx::sph\_hankel\_2 \ (unsigned \ int \ \_\_n, \ std \leftarrow 1.00 \ and \ below to be a complex < \_\_gnu\_cxx::sph\_hankel\_2 \ (unsigned \ int \ \_\_n, \ std \leftarrow 1.00 \ and \ below to be a complex < \_\_gnu\_cxx::sph\_hankel\_2 \ (unsigned \ int \ \_\_n, \ std \leftarrow 1.00 \ and \ below to be a complex < \_\_gnu\_cxx::sph\_hankel\_2 \ (unsigned \ int \ \_\_n, \ std \leftarrow 1.00 \ and \ below to be a complex < \_\_gnu\_cxx::sph\_hankel\_2 \ (unsigned \ int \ \_\_n, \ std \leftarrow 1.00 \ and \ below to be a complex < \_\_gnu\_cxx::sph\_hankel\_2 \ (unsigned \ int \ \_\_n, \ std \leftarrow 1.00 \ and \ below to be a complex < \_\_gnu\_cxx::sph\_hankel\_2 \ (unsigned \ int \ \_\_n, \ std \leftarrow 1.00 \ and \ below to be a complex < \_\_gnu\_cxx::sph\_hankel\_2 \ (unsigned \ int \ \_\_n, \ std \leftarrow 1.00 \ and \ below to be a complex < \_\_gnu\_cxx::sph\_hankel\_2 \ (unsigned \ int \ \_\_n, \ std \leftarrow 1.00 \ and \ below to be a complex < \_\_gnu\_cxx::sph\_hankel\_2 \ (unsigned \ int \ \_\_n, \ std \leftarrow 1.00 \ and \ below to be a complex < \_\_gnu\_cxx::sph\_hankel\_2 \ (unsigned \ int \ \_\_n, \ std \leftarrow 1.00 \ and \ below to be a complex < \_\_gnu\_cxx::sph\_hankel\_2 \ (unsigned \ int \ ) \ (unsig
   ::complex < _Tp > __x)

    std::complex < float > gnu cxx::sph hankel 2f (unsigned int n, float z)

    std::complex < float > __gnu_cxx::sph_hankel_2f (unsigned int __n, std::complex < float > __x)

    std::complex < long double > __gnu_cxx::sph_hankel_2l (unsigned int __n, long double __z)

    std::complex < long double > __gnu_cxx::sph_hankel_2l (unsigned int __n, std::complex < long double > __x)

• template<typename Ttheta, typename Tphi >
   std::complex< __gnu_cxx::_promote_num_t< _Ttheta, _Tphi >> __gnu_cxx::sph_harmonic (unsigned int ↔
     _l, int __m, _Ttheta __theta, _Tphi __phi)
• std::complex < float > __gnu_cxx::sph_harmonicf (unsigned int __l, int __m, float __theta, float __phi)
• std::complex < long double > gnu cxx::sph harmonicl (unsigned int I, int m, long double theta, long
   double __phi)
template<typename _Tp >
     gnu cxx:: promote < Tp >:: type std::sph legendre (unsigned int I, unsigned int m, Tp theta)
• float std::sph legendref (unsigned int I, unsigned int m, float theta)

    long double std::sph_legendrel (unsigned int __l, unsigned int __m, long double __theta)

template<typename _Tp >
     gnu cxx:: promote < Tp >:: type std::sph neumann (unsigned int n, Tp x)
```

```
    float std::sph_neumannf (unsigned int __n, float __x)

• long double std::sph_neumannl (unsigned int __n, long double x)
• template<typename _Tpnu , typename _Tp >
    _gnu_cxx::__promote_num_t< _Tpnu, _Tp > __gnu_cxx::theta_1 (_Tpnu __nu, _Tp __x)
• float gnu cxx::theta 1f (float nu, float x)

    long double __gnu_cxx::theta_1l (long double __nu, long double __x)

    template<typename _Tpnu , typename _Tp >

    _gnu_cxx::__promote_num_t< _Tpnu, _Tp > __gnu_cxx::theta_2 (_Tpnu __nu, _Tp __x)

    float __gnu_cxx::theta_2f (float __nu, float __x)

    long double __gnu_cxx::theta_2l (long double __nu, long double __x)

• template<typename Tpnu, typename Tp >
   _gnu_cxx::__promote_num_t< _Tpnu, _Tp > __gnu_cxx::theta_3 (_Tpnu __nu, _Tp __x)

    float __gnu_cxx::theta_3f (float __nu, float __x)

    long double __gnu_cxx::theta_3l (long double __nu, long double __x)

• template<typename _Tpnu , typename _Tp >
   _gnu_cxx::__promote_num_t< _Tpnu, _Tp > __gnu_cxx::theta_4 (_Tpnu __nu, _Tp __x)

    float gnu cxx::theta 4f (float nu, float x)

    long double gnu cxx::theta 4l (long double nu, long double x)

• template<typename \_\mathsf{Tpk} , typename \_\mathsf{Tp}>
    _gnu_cxx::__promote_num_t< _Tpk, _Tp > __gnu_cxx::theta_c (_Tpk __k, _Tp __x)

    float __gnu_cxx::theta_cf (float __k, float __x)

    long double __gnu_cxx::theta_cl (long double __k, long double __x)

template<typename _Tpk , typename _Tp >
   __gnu_cxx::__promote_num_t< _Tpk, _Tp > __gnu_cxx::theta_d (_Tpk __k, _Tp __x)

    float __gnu_cxx::theta_df (float __k, float __x)

    long double gnu cxx::theta dl (long double k, long double x)

template<typename _Tpk , typename _Tp >
    _gnu_cxx::__promote_num_t< _Tpk, _Tp > __gnu_cxx::theta_n (_Tpk __k, _Tp __x)

    float __gnu_cxx::theta_nf (float __k, float __x)

    long double gnu cxx::theta nl (long double k, long double x)

template<typename _Tpk , typename _Tp >
    _gnu_cxx::__promote_num_t< _Tpk, _Tp > __gnu_cxx::theta_s (_Tpk __k, _Tp __x)

    float __gnu_cxx::theta_sf (float __k, float __x)

    long double __gnu_cxx::theta_sl (long double __k, long double __x)

• template<typename \_Trho , typename \_Tphi >
    gnu cxx:: promote num t < Trho, Tphi > gnu cxx::zernike (unsigned int n, int m, Trho rho,
  (idg idgT

    float __gnu_cxx::zernikef (unsigned int __n, int __m, float __rho, float __phi)

    long double gnu cxx::zernikel (unsigned int n, int m, long double rho, long double phi)
```

#### 10.27.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

### 10.27.2 Macro Definition Documentation

10.27.2.1 #define \_\_cpp\_lib\_math\_special\_functions 201603L

Definition at line 39 of file specfun.h.

10.27.2.2 #define \_\_STDCPP\_MATH\_SPEC\_FUNCS\_\_ 201003L

Definition at line 37 of file specfun.h.

300	File Documentation

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