

C++ Special Math Functions

2.0

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Contents

1	Todo List	1
2	Module Index	3
2.1	Modules	3
3	Namespace Index	5
3.1	Namespace List	5
4	Hierarchical Index	7
4.1	Class Hierarchy	7
5	Class Index	9
5.1	Class List	9
6	File Index	11
6.1	File List	11
7	Module Documentation	13
7.1	C++17/IS29124 Mathematical Special Functions	13
7.1.1	Detailed Description	15
7.1.2	Function Documentation	15
7.1.2.1	assoc_laguerre() [1/2]	15
7.1.2.2	assoc_laguerre() [2/2]	16
7.1.2.3	assoc_laguerref()	18

7.1.2.4	<code>assoc_laguerrel()</code>	18
7.1.2.5	<code>assoc_legendre()</code>	19
7.1.2.6	<code>assoc_legendref()</code>	19
7.1.2.7	<code>assoc_legendrel()</code>	20
7.1.2.8	<code>beta()</code>	20
7.1.2.9	<code>betaf()</code>	21
7.1.2.10	<code>betal()</code>	21
7.1.2.11	<code>comp_ellint_1()</code>	22
7.1.2.12	<code>comp_ellint_1f()</code>	22
7.1.2.13	<code>comp_ellint_1l()</code>	23
7.1.2.14	<code>comp_ellint_2()</code>	23
7.1.2.15	<code>comp_ellint_2f()</code>	24
7.1.2.16	<code>comp_ellint_2l()</code>	24
7.1.2.17	<code>comp_ellint_3()</code>	24
7.1.2.18	<code>comp_ellint_3f()</code>	25
7.1.2.19	<code>comp_ellint_3l()</code>	25
7.1.2.20	<code>cyl_bessel_i()</code>	26
7.1.2.21	<code>cyl_bessel_if()</code>	26
7.1.2.22	<code>cyl_bessel_il()</code>	27
7.1.2.23	<code>cyl_bessel_j()</code>	27
7.1.2.24	<code>cyl_bessel_jf()</code>	28
7.1.2.25	<code>cyl_bessel_jl()</code>	28
7.1.2.26	<code>cyl_bessel_k()</code>	28
7.1.2.27	<code>cyl_bessel_kf()</code>	29
7.1.2.28	<code>cyl_bessel_kl()</code>	29
7.1.2.29	<code>cyl_neumann()</code>	30
7.1.2.30	<code>cyl_neumannf()</code>	30
7.1.2.31	<code>cyl_neumannl()</code>	31

7.1.2.32	ellint_1()	31
7.1.2.33	ellint_1f()	32
7.1.2.34	ellint_1l()	32
7.1.2.35	ellint_2()	32
7.1.2.36	ellint_2f()	33
7.1.2.37	ellint_2l()	33
7.1.2.38	ellint_3()	34
7.1.2.39	ellint_3f()	35
7.1.2.40	ellint_3l()	35
7.1.2.41	expint()	35
7.1.2.42	expintf()	36
7.1.2.43	expintl()	36
7.1.2.44	hermite()	37
7.1.2.45	hermitef()	37
7.1.2.46	hermitel()	38
7.1.2.47	laguerre()	38
7.1.2.48	laguerref()	39
7.1.2.49	laguerrel()	39
7.1.2.50	legendre()	39
7.1.2.51	legendref()	40
7.1.2.52	legendrel()	40
7.1.2.53	riemann_zeta()	41
7.1.2.54	riemann_zetaf()	41
7.1.2.55	riemann_zetal()	42
7.1.2.56	sph_bessel()	42
7.1.2.57	sph_besself()	43
7.1.2.58	sph_bessell()	43
7.1.2.59	sph_legendre()	43

7.1.2.60	<code>sph_legendref()</code>	44
7.1.2.61	<code>sph_legendrel()</code>	44
7.1.2.62	<code>sph_neumann()</code>	45
7.1.2.63	<code>sph_neumannf()</code>	45
7.1.2.64	<code>sph_neumannl()</code>	46
7.2	GNU Extended Mathematical Special Functions	47
7.2.1	Detailed Description	60
7.2.2	Function Documentation	60
7.2.2.1	<code>airy_ai()</code> $[1/2]$	60
7.2.2.2	<code>airy_ai()</code> $[2/2]$	61
7.2.2.3	<code>airy_aif()</code>	61
7.2.2.4	<code>airy_ail()</code>	62
7.2.2.5	<code>airy_bi()</code> $[1/2]$	62
7.2.2.6	<code>airy_bi()</code> $[2/2]$	62
7.2.2.7	<code>airy_bif()</code>	63
7.2.2.8	<code>airy_bil()</code>	63
7.2.2.9	<code>assoc_legendre_q()</code>	64
7.2.2.10	<code>assoc_legendre_qf()</code>	64
7.2.2.11	<code>assoc_legendre_ql()</code>	65
7.2.2.12	<code>bell()</code> $[1/2]$	65
7.2.2.13	<code>bell()</code> $[2/2]$	66
7.2.2.14	<code>bernoulli()</code> $[1/2]$	66
7.2.2.15	<code>bernoulli()</code> $[2/2]$	66
7.2.2.16	<code>bernoullif()</code>	67
7.2.2.17	<code>bernoullil()</code>	67
7.2.2.18	<code>binomial()</code>	68
7.2.2.19	<code>binomial_p()</code>	68
7.2.2.20	<code>binomial_pdf()</code>	69

7.2.2.21	binomialf()	69
7.2.2.22	binomiall()	70
7.2.2.23	bose_einstein()	70
7.2.2.24	bose_einsteinf()	71
7.2.2.25	bose_einsteinl()	71
7.2.2.26	chebyshev_t()	71
7.2.2.27	chebyshev_tf()	72
7.2.2.28	chebyshev_tl()	72
7.2.2.29	chebyshev_u()	73
7.2.2.30	chebyshev_uf()	73
7.2.2.31	chebyshev_ul()	74
7.2.2.32	chebyshev_v()	74
7.2.2.33	chebyshev_vf()	75
7.2.2.34	chebyshev_vl()	75
7.2.2.35	chebyshev_w()	75
7.2.2.36	chebyshev_wf()	76
7.2.2.37	chebyshev_wl()	76
7.2.2.38	clausen() ^[1/2]	77
7.2.2.39	clausen() ^[2/2]	77
7.2.2.40	clausen_cl()	78
7.2.2.41	clausen_clf()	78
7.2.2.42	clausen_cll()	79
7.2.2.43	clausen_sl()	79
7.2.2.44	clausen_slf()	80
7.2.2.45	clausen_sll()	80
7.2.2.46	clausenf() ^[1/2]	80
7.2.2.47	clausenf() ^[2/2]	81
7.2.2.48	clausenl() ^[1/2]	81

7.2.2.49	<code>clausenl()</code> [2/2]	81
7.2.2.50	<code>comp_ellint_d()</code>	81
7.2.2.51	<code>comp_ellint_df()</code>	82
7.2.2.52	<code>comp_ellint_dl()</code>	82
7.2.2.53	<code>comp_ellint_rf()</code> [1/3]	83
7.2.2.54	<code>comp_ellint_rf()</code> [2/3]	83
7.2.2.55	<code>comp_ellint_rf()</code> [3/3]	83
7.2.2.56	<code>comp_ellint_rg()</code> [1/3]	84
7.2.2.57	<code>comp_ellint_rg()</code> [2/3]	84
7.2.2.58	<code>comp_ellint_rg()</code> [3/3]	84
7.2.2.59	<code>conf_hyperg()</code>	85
7.2.2.60	<code>conf_hyperg_lim()</code>	85
7.2.2.61	<code>conf_hyperg_limf()</code>	86
7.2.2.62	<code>conf_hyperg_liml()</code>	86
7.2.2.63	<code>conf_hypergf()</code>	87
7.2.2.64	<code>conf_hypergl()</code>	87
7.2.2.65	<code>cos_pi()</code>	87
7.2.2.66	<code>cos_pif()</code>	88
7.2.2.67	<code>cos_pil()</code>	88
7.2.2.68	<code>cosh_pi()</code>	88
7.2.2.69	<code>cosh_pif()</code>	89
7.2.2.70	<code>cosh_pil()</code>	89
7.2.2.71	<code>coshint()</code>	89
7.2.2.72	<code>coshintf()</code>	90
7.2.2.73	<code>coshintl()</code>	90
7.2.2.74	<code>cosint()</code>	90
7.2.2.75	<code>cosintf()</code>	91
7.2.2.76	<code>cosintl()</code>	91

7.2.2.77	<code>cyl_hankel_1()</code> [1/2]	91
7.2.2.78	<code>cyl_hankel_1()</code> [2/2]	92
7.2.2.79	<code>cyl_hankel_1f()</code> [1/2]	92
7.2.2.80	<code>cyl_hankel_1f()</code> [2/2]	93
7.2.2.81	<code>cyl_hankel_1l()</code> [1/2]	93
7.2.2.82	<code>cyl_hankel_1l()</code> [2/2]	94
7.2.2.83	<code>cyl_hankel_2()</code> [1/2]	94
7.2.2.84	<code>cyl_hankel_2()</code> [2/2]	95
7.2.2.85	<code>cyl_hankel_2f()</code> [1/2]	95
7.2.2.86	<code>cyl_hankel_2f()</code> [2/2]	96
7.2.2.87	<code>cyl_hankel_2l()</code> [1/2]	96
7.2.2.88	<code>cyl_hankel_2l()</code> [2/2]	96
7.2.2.89	<code>dawson()</code>	97
7.2.2.90	<code>dawsonf()</code>	97
7.2.2.91	<code>dawsonl()</code>	97
7.2.2.92	<code>debye()</code>	98
7.2.2.93	<code>debyef()</code>	98
7.2.2.94	<code>debyel()</code>	99
7.2.2.95	<code>digamma()</code>	99
7.2.2.96	<code>digammaf()</code>	99
7.2.2.97	<code>digammal()</code>	100
7.2.2.98	<code>dilog()</code>	100
7.2.2.99	<code>dilogf()</code>	101
7.2.2.100	<code>dilogl()</code>	101
7.2.2.101	<code>dirichlet_beta()</code>	101
7.2.2.102	<code>dirichlet_betaf()</code>	102
7.2.2.103	<code>dirichlet_betall()</code>	102
7.2.2.104	<code>dirichlet_eta()</code>	102

7.2.2.105 <code>dirichlet_etaf()</code>	103
7.2.2.106 <code>dirichlet_etaf()</code>	103
7.2.2.107 <code>dirichlet_lambda()</code>	103
7.2.2.108 <code>dirichlet_lambdaf()</code>	104
7.2.2.109 <code>dirichlet_lambdal()</code>	104
7.2.2.110 <code>double_factorial()</code>	105
7.2.2.111 <code>double_factorialf()</code>	105
7.2.2.112 <code>double_factoriall()</code>	105
7.2.2.113 <code>ellint_cel()</code>	106
7.2.2.114 <code>ellint_celf()</code>	106
7.2.2.115 <code>ellint_cell()</code>	107
7.2.2.116 <code>ellint_d()</code>	107
7.2.2.117 <code>ellint_df()</code>	107
7.2.2.118 <code>ellint_dl()</code>	108
7.2.2.119 <code>ellint_el1()</code>	108
7.2.2.120 <code>ellint_el1f()</code>	109
7.2.2.121 <code>ellint_el1l()</code>	109
7.2.2.122 <code>ellint_el2()</code>	109
7.2.2.123 <code>ellint_el2f()</code>	110
7.2.2.124 <code>ellint_el2l()</code>	110
7.2.2.125 <code>ellint_el3()</code>	111
7.2.2.126 <code>ellint_el3f()</code>	111
7.2.2.127 <code>ellint_el3l()</code>	112
7.2.2.128 <code>ellint_rc()</code>	112
7.2.2.129 <code>ellint_rcf()</code>	113
7.2.2.130 <code>ellint_rcl()</code>	113
7.2.2.131 <code>ellint_rd()</code>	113
7.2.2.132 <code>ellint_rdf()</code>	114

7.2.2.133 ellint_rdl()	114
7.2.2.134 ellint_rf()	115
7.2.2.135 ellint_rff()	115
7.2.2.136 ellint_rfl()	116
7.2.2.137 ellint_rg()	116
7.2.2.138 ellint_rgf()	117
7.2.2.139 ellint_rgl()	117
7.2.2.140 ellint_rj()	118
7.2.2.141 ellint_rjf()	119
7.2.2.142 ellint_rjl()	119
7.2.2.143 ellnome()	120
7.2.2.144 ellnomef()	120
7.2.2.145 ellnomel()	121
7.2.2.146 euler()	121
7.2.2.147 eulerian_1() ^[1/2]	121
7.2.2.148 eulerian_1() ^[2/2]	122
7.2.2.149 eulerian_2()	122
7.2.2.150 expint()	122
7.2.2.151 expintf()	123
7.2.2.152 expintl()	123
7.2.2.153 exponential_p()	124
7.2.2.154 exponential_pdf()	124
7.2.2.155 factorial()	124
7.2.2.156 factorialf()	125
7.2.2.157 factoriall()	125
7.2.2.158 falling_factorial()	125
7.2.2.159 falling_factorialf()	126
7.2.2.160 falling_factoriall()	126

7.2.2.161	fermi_dirac()	126
7.2.2.162	fermi_diracf()	127
7.2.2.163	fermi_diracl()	127
7.2.2.164	fisher_f_p()	128
7.2.2.165	fisher_f_pdf()	128
7.2.2.166	fresnel_c()	129
7.2.2.167	fresnel_cf()	129
7.2.2.168	fresnel_cl()	129
7.2.2.169	fresnel_s()	130
7.2.2.170	fresnel_sf()	131
7.2.2.171	fresnel_sl()	131
7.2.2.172	gamma_p()	131
7.2.2.173	gamma_pdf()	132
7.2.2.174	gamma_pf()	132
7.2.2.175	gamma_pl()	132
7.2.2.176	gamma_q()	133
7.2.2.177	gamma_qf()	133
7.2.2.178	gamma_ql()	133
7.2.2.179	gamma_reciprocal()	133
7.2.2.180	gamma_reciprocalf()	134
7.2.2.181	gamma_reciprocall()	134
7.2.2.182	gegenbauer()	134
7.2.2.183	gegenbauerf()	135
7.2.2.184	gegenbauerl()	135
7.2.2.185	harmonic()	136
7.2.2.186	heuman_lambda()	136
7.2.2.187	heuman_lambdaf()	137
7.2.2.188	heuman_lambdal()	137

7.2.2.189 hurwitz_zeta() [1/2]	137
7.2.2.190 hurwitz_zeta() [2/2]	138
7.2.2.191 hurwitz_zetaf()	138
7.2.2.192 hurwitz_zetal()	138
7.2.2.193 hyperg()	139
7.2.2.194 hypergf()	139
7.2.2.195 hypergl()	140
7.2.2.196 ibeta()	140
7.2.2.197 ibetac()	141
7.2.2.198 ibetacf()	141
7.2.2.199 ibetacI()	141
7.2.2.200 ibetaf()	142
7.2.2.201 ibetal()	142
7.2.2.202 jacobi()	142
7.2.2.203 jacobi_cn()	143
7.2.2.204 jacobi_cnf()	144
7.2.2.205 jacobi_cnl()	144
7.2.2.206 jacobi_dn()	144
7.2.2.207 jacobi_dnf()	145
7.2.2.208 jacobi_dnl()	145
7.2.2.209 jacobi_sn()	146
7.2.2.210 jacobi_snf()	146
7.2.2.211 jacobi_snl()	147
7.2.2.212 jacobi_theta_1()	147
7.2.2.213 jacobi_theta_1f()	147
7.2.2.214 jacobi_theta_1I()	148
7.2.2.215 jacobi_theta_2()	148
7.2.2.216 jacobi_theta_2f()	149

7.2.2.217 jacobi_theta_2l()	149
7.2.2.218 jacobi_theta_3()	149
7.2.2.219 jacobi_theta_3f()	150
7.2.2.220 jacobi_theta_3l()	150
7.2.2.221 jacobi_theta_4()	150
7.2.2.222 jacobi_theta_4f()	151
7.2.2.223 jacobi_theta_4l()	151
7.2.2.224 jacobi_zeta()	151
7.2.2.225 jacobi_zetaf()	152
7.2.2.226 jacobi_zetal()	152
7.2.2.227 jacobif()	152
7.2.2.228 jacobil()	153
7.2.2.229 lah() $[1/2]$	153
7.2.2.230 lah() $[2/2]$	153
7.2.2.231 lbinomial()	154
7.2.2.232 lbinomialf()	154
7.2.2.233 lbinomiall()	155
7.2.2.234 ldouble_factorial()	155
7.2.2.235 ldouble_factorialf()	155
7.2.2.236 ldouble_factoriall()	156
7.2.2.237 legendre_q()	156
7.2.2.238 legendre_qf()	157
7.2.2.239 legendre_ql()	157
7.2.2.240 lerch_phi()	157
7.2.2.241 lerch_phif()	158
7.2.2.242 lerch_phil()	158
7.2.2.243 lfactorial()	159
7.2.2.244 lfactorialf()	159

7.2.2.245 lfactorial()	159
7.2.2.246 lfalling_factorial()	160
7.2.2.247 lfalling_factorialf()	160
7.2.2.248 lfalling_factoriall()	160
7.2.2.249 lgamma() $[1/2]$	161
7.2.2.250 lgamma() $[2/2]$	161
7.2.2.251 lgammaf() $[1/2]$	161
7.2.2.252 lgammaf() $[2/2]$	162
7.2.2.253 lgammal() $[1/2]$	162
7.2.2.254 lgammal() $[2/2]$	162
7.2.2.255 logint()	162
7.2.2.256 logintf()	163
7.2.2.257 logintl()	163
7.2.2.258 logistic_p()	164
7.2.2.259 logistic_pdf()	164
7.2.2.260 lognormal_p()	164
7.2.2.261 lognormal_pdf()	165
7.2.2.262 lrising_factorial()	165
7.2.2.263 lrising_factorialf()	165
7.2.2.264 lrising_factoriall()	166
7.2.2.265 mittag_leffler()	166
7.2.2.266 normal_p()	166
7.2.2.267 normal_pdf()	167
7.2.2.268 owens_t()	167
7.2.2.269 owens_tf()	168
7.2.2.270 owens_tl()	168
7.2.2.271 periodic_zeta() $[1/2]$	168
7.2.2.272 periodic_zeta() $[2/2]$	169

7.2.2.273 <code>periodic_zetaf()</code>	169
7.2.2.274 <code>periodic_zetal()</code>	170
7.2.2.275 <code>polygamma()</code>	170
7.2.2.276 <code>polygammaf()</code>	170
7.2.2.277 <code>polygammal()</code>	171
7.2.2.278 <code>polylog()</code> $[1/2]$	171
7.2.2.279 <code>polylog()</code> $[2/2]$	172
7.2.2.280 <code>polylogf()</code> $[1/2]$	172
7.2.2.281 <code>polylogf()</code> $[2/2]$	172
7.2.2.282 <code>polylogl()</code> $[1/2]$	173
7.2.2.283 <code>polylogl()</code> $[2/2]$	173
7.2.2.284 <code>radpoly()</code>	173
7.2.2.285 <code>radpolyf()</code>	174
7.2.2.286 <code>radpolyl()</code>	174
7.2.2.287 <code>rising_factorial()</code>	175
7.2.2.288 <code>rising_factorialf()</code>	175
7.2.2.289 <code>rising_factoriall()</code>	175
7.2.2.290 <code>sin_pi()</code>	176
7.2.2.291 <code>sin_pif()</code>	176
7.2.2.292 <code>sin_pil()</code>	176
7.2.2.293 <code>sinc()</code>	177
7.2.2.294 <code>sinc_pi()</code>	177
7.2.2.295 <code>sinc_pif()</code>	178
7.2.2.296 <code>sinc_pil()</code>	178
7.2.2.297 <code>sincf()</code>	178
7.2.2.298 <code>sincl()</code>	179
7.2.2.299 <code>sincos()</code> $[1/2]$	179
7.2.2.300 <code>sincos()</code> $[2/2]$	179

7.2.2.301 sincos_pi()	180
7.2.2.302 sincos_pif()	180
7.2.2.303 sincos_pil()	180
7.2.2.304 sincosf()	181
7.2.2.305 sincosl()	181
7.2.2.306 sinh_pi()	181
7.2.2.307 sinh_pif()	182
7.2.2.308 sinh_pil()	182
7.2.2.309 sinhc()	182
7.2.2.310 sinhc_pi()	183
7.2.2.311 sinhc_pif()	183
7.2.2.312 sinhc_pil()	184
7.2.2.313 sinhcf()	184
7.2.2.314 sinhcl()	184
7.2.2.315 sinhint()	185
7.2.2.316 sinhintf()	185
7.2.2.317 sinhintl()	185
7.2.2.318 sinint()	186
7.2.2.319 sinintf()	186
7.2.2.320 sinintl()	186
7.2.2.321 sph_bessel_i()	187
7.2.2.322 sph_bessel_if()	187
7.2.2.323 sph_bessel_il()	188
7.2.2.324 sph_bessel_k()	188
7.2.2.325 sph_bessel_kf()	189
7.2.2.326 sph_bessel_kl()	189
7.2.2.327 sph_hankel_1() [1/2]	189
7.2.2.328 sph_hankel_1() [2/2]	190

7.2.2.329 sph_hankel_1f() [1/2]	190
7.2.2.330 sph_hankel_1f() [2/2]	191
7.2.2.331 sph_hankel_1l() [1/2]	191
7.2.2.332 sph_hankel_1l() [2/2]	192
7.2.2.333 sph_hankel_2() [1/2]	192
7.2.2.334 sph_hankel_2() [2/2]	193
7.2.2.335 sph_hankel_2f() [1/2]	193
7.2.2.336 sph_hankel_2f() [2/2]	194
7.2.2.337 sph_hankel_2l() [1/2]	194
7.2.2.338 sph_hankel_2l() [2/2]	194
7.2.2.339 sph_harmonic()	195
7.2.2.340 sph_harmonicf()	195
7.2.2.341 sph_harmonicl()	196
7.2.2.342 stirling_1() [1/2]	196
7.2.2.343 stirling_1() [2/2]	197
7.2.2.344 stirling_2() [1/2]	197
7.2.2.345 stirling_2() [2/2]	197
7.2.2.346 student_t_p()	197
7.2.2.347 student_t_pdf()	198
7.2.2.348 tan_pi()	198
7.2.2.349 tan_pif()	199
7.2.2.350 tan_pil()	199
7.2.2.351 tanh_pi()	199
7.2.2.352 tanh_pif()	200
7.2.2.353 tanh_pil()	200
7.2.2.354 tgamma() [1/3]	201
7.2.2.355 tgamma() [2/3]	201
7.2.2.356 tgamma() [3/3]	201

7.2.2.357 <code>tgamma_lower()</code>	202
7.2.2.358 <code>tgamma_lowerf()</code>	202
7.2.2.359 <code>tgamma_lowerl()</code>	202
7.2.2.360 <code>tgammaf()</code> [1/3]	203
7.2.2.361 <code>tgammaf()</code> [2/3]	203
7.2.2.362 <code>tgammaf()</code> [3/3]	203
7.2.2.363 <code>tgammal()</code> [1/3]	204
7.2.2.364 <code>tgammal()</code> [2/3]	204
7.2.2.365 <code>tgammal()</code> [3/3]	204
7.2.2.366 <code>theta_1()</code>	204
7.2.2.367 <code>theta_1f()</code>	205
7.2.2.368 <code>theta_1l()</code>	205
7.2.2.369 <code>theta_2()</code>	205
7.2.2.370 <code>theta_2f()</code>	206
7.2.2.371 <code>theta_2l()</code>	206
7.2.2.372 <code>theta_3()</code>	206
7.2.2.373 <code>theta_3f()</code>	207
7.2.2.374 <code>theta_3l()</code>	207
7.2.2.375 <code>theta_4()</code>	207
7.2.2.376 <code>theta_4f()</code>	208
7.2.2.377 <code>theta_4l()</code>	208
7.2.2.378 <code>theta_c()</code>	209
7.2.2.379 <code>theta_cf()</code>	209
7.2.2.380 <code>theta_cl()</code>	210
7.2.2.381 <code>theta_d()</code>	210
7.2.2.382 <code>theta_df()</code>	211
7.2.2.383 <code>theta_dl()</code>	211
7.2.2.384 <code>theta_n()</code>	211

7.2.2.385	<code>theta_nf()</code>	212
7.2.2.386	<code>theta_nl()</code>	212
7.2.2.387	<code>theta_s()</code>	213
7.2.2.388	<code>theta_sf()</code>	213
7.2.2.389	<code>theta_sl()</code>	214
7.2.2.390	<code>tricomi_u()</code>	214
7.2.2.391	<code>tricomi_uf()</code>	215
7.2.2.392	<code>tricomi_ul()</code>	215
7.2.2.393	<code>weibull_p()</code>	215
7.2.2.394	<code>weibull_pdf()</code>	216
7.2.2.395	<code>zernike()</code>	216
7.2.2.396	<code>zernikef()</code>	217
7.2.2.397	<code>zernikel()</code>	217
7.3	Mathematical Special Functions	218
7.3.1	Detailed Description	218
7.3.2	Mathematical Special Functions	218
7.3.2.1	Introduction and History	218
7.3.2.2	Contents	219
7.3.2.3	Argument Promotion	223
7.3.2.4	NaN Arguments	223
7.3.2.5	Implementation	223
7.3.2.6	Testing	223
7.3.2.7	General Bibliography	224

8 Namespace Documentation	225
8.1 <code>__gnu_cxx</code> Namespace Reference	225
8.1.1 Enumeration Type Documentation	239
8.1.1.1 <code>gauss_quad_type</code>	239
8.1.2 Function Documentation	240
8.1.2.1 <code>__fp_is_equal()</code>	240
8.1.2.2 <code>__fp_is_even_integer()</code>	241
8.1.2.3 <code>__fp_is_half_integer()</code>	241
8.1.2.4 <code>__fp_is_half_odd_integer()</code>	242
8.1.2.5 <code>__fp_is_integer()</code>	242
8.1.2.6 <code>__fp_is_odd_integer()</code>	243
8.1.2.7 <code>__fp_is_zero()</code>	243
8.1.2.8 <code>__fp_max_abs()</code>	244
8.1.2.9 <code>__parity()</code>	244
8.2 <code>std</code> Namespace Reference	244
8.3 <code>std::__detail</code> Namespace Reference	246
8.3.1 Detailed Description	271
8.3.2 Function Documentation	271
8.3.2.1 <code>__airy()</code>	271
8.3.2.2 <code>__airy_ai()</code>	272
8.3.2.3 <code>__airy_arg()</code>	272
8.3.2.4 <code>__airy_bi()</code>	273
8.3.2.5 <code>__assoc_laguerre()</code>	273
8.3.2.6 <code>__assoc_legendre_p()</code>	274
8.3.2.7 <code>__assoc_legendre_q()</code>	275
8.3.2.8 <code>__bell()</code> ^[1/2]	275
8.3.2.9 <code>__bell()</code> ^[2/2]	275
8.3.2.10 <code>__bell_series()</code>	276

8.3.2.11	__bernoulli() [1/2]	276
8.3.2.12	__bernoulli() [2/2]	277
8.3.2.13	__bernoulli_2n()	277
8.3.2.14	__bernoulli_series()	278
8.3.2.15	__beta()	278
8.3.2.16	__beta_gamma()	279
8.3.2.17	__beta_inc()	280
8.3.2.18	__beta_lgamma()	280
8.3.2.19	__beta_p()	281
8.3.2.20	__beta_product()	281
8.3.2.21	__binomial() [1/2]	282
8.3.2.22	__binomial() [2/2]	283
8.3.2.23	__binomial_p()	283
8.3.2.24	__binomial_pdf()	284
8.3.2.25	__binomial_q()	284
8.3.2.26	__bose_einstein()	285
8.3.2.27	__cauchy_p()	286
8.3.2.28	__chebyshev_recur()	286
8.3.2.29	__chebyshev_t()	287
8.3.2.30	__chebyshev_u()	287
8.3.2.31	__chebyshev_v()	288
8.3.2.32	__chebyshev_w()	289
8.3.2.33	__chi_squared_pdf()	289
8.3.2.34	__chi_squared_pdfc()	290
8.3.2.35	__chshint()	290
8.3.2.36	__chshint_cont_frac()	291
8.3.2.37	__chshint_series()	291
8.3.2.38	__clamp_0_m2pi()	291

8.3.2.39	__clamp_pi()	292
8.3.2.40	__clausen() [1/2]	292
8.3.2.41	__clausen() [2/2]	292
8.3.2.42	__clausen_cl() [1/2]	293
8.3.2.43	__clausen_cl() [2/2]	294
8.3.2.44	__clausen_sl() [1/2]	294
8.3.2.45	__clausen_sl() [2/2]	295
8.3.2.46	__comp_ellint_1()	295
8.3.2.47	__comp_ellint_2()	297
8.3.2.48	__comp_ellint_3()	298
8.3.2.49	__comp_ellint_d()	298
8.3.2.50	__comp_ellint_rf()	299
8.3.2.51	__comp_ellint_rg()	299
8.3.2.52	__conf_hyperg()	299
8.3.2.53	__conf_hyperg_lim()	300
8.3.2.54	__conf_hyperg_lim_series()	300
8.3.2.55	__conf_hyperg_luke()	301
8.3.2.56	__conf_hyperg_series()	301
8.3.2.57	__cos_pi() [1/2]	302
8.3.2.58	__cos_pi() [2/2]	302
8.3.2.59	__cosh_pi() [1/2]	303
8.3.2.60	__cosh_pi() [2/2]	303
8.3.2.61	__coshint()	303
8.3.2.62	__coulomb_CF1()	304
8.3.2.63	__coulomb_CF2()	304
8.3.2.64	__coulomb_f_recur()	305
8.3.2.65	__coulomb_g_recur()	305
8.3.2.66	__coulomb_norm()	305

8.3.2.67	<code>__cyl_bessel()</code>	306
8.3.2.68	<code>__cyl_bessel_i()</code>	306
8.3.2.69	<code>__cyl_bessel_ij_series()</code>	307
8.3.2.70	<code>__cyl_bessel_ik()</code>	308
8.3.2.71	<code>__cyl_bessel_ik_asymp()</code>	308
8.3.2.72	<code>__cyl_bessel_ik_steel()</code>	309
8.3.2.73	<code>__cyl_bessel_j()</code>	309
8.3.2.74	<code>__cyl_bessel_jn()</code>	310
8.3.2.75	<code>__cyl_bessel_jn_asymp()</code>	310
8.3.2.76	<code>__cyl_bessel_jn_neg_arg()</code>	311
8.3.2.77	<code>__cyl_bessel_jn_steel()</code>	311
8.3.2.78	<code>__cyl_bessel_k()</code>	312
8.3.2.79	<code>__cyl_hankel_1()</code> $[1/2]$	313
8.3.2.80	<code>__cyl_hankel_1()</code> $[2/2]$	313
8.3.2.81	<code>__cyl_hankel_2()</code> $[1/2]$	314
8.3.2.82	<code>__cyl_hankel_2()</code> $[2/2]$	314
8.3.2.83	<code>__cyl_neumann()</code>	315
8.3.2.84	<code>__cyl_neumann_n()</code>	315
8.3.2.85	<code>__dawson()</code>	316
8.3.2.86	<code>__dawson_cont_frac()</code>	316
8.3.2.87	<code>__dawson_series()</code>	317
8.3.2.88	<code>__debye()</code>	317
8.3.2.89	<code>__debye_region()</code>	318
8.3.2.90	<code>__digamma()</code> $[1/2]$	318
8.3.2.91	<code>__digamma()</code> $[2/2]$	319
8.3.2.92	<code>__digamma_asymp()</code>	319
8.3.2.93	<code>__digamma_series()</code>	320
8.3.2.94	<code>__dilog()</code>	320

8.3.2.95	<code>__dirichlet_beta()</code> [1/2]	320
8.3.2.96	<code>__dirichlet_beta()</code> [2/2]	321
8.3.2.97	<code>__dirichlet_eta()</code> [1/2]	322
8.3.2.98	<code>__dirichlet_eta()</code> [2/2]	322
8.3.2.99	<code>__dirichlet_lambda()</code>	323
8.3.2.100	<code>__double_factorial()</code>	323
8.3.2.101	<code>__ellint_1()</code>	324
8.3.2.102	<code>__ellint_2()</code>	325
8.3.2.103	<code>__ellint_3()</code>	325
8.3.2.104	<code>__ellint_cel()</code>	326
8.3.2.105	<code>__ellint_d()</code>	326
8.3.2.106	<code>__ellint_el1()</code>	326
8.3.2.107	<code>__ellint_el2()</code>	327
8.3.2.108	<code>__ellint_el3()</code>	327
8.3.2.109	<code>__ellint_rc()</code>	327
8.3.2.110	<code>__ellint_rd()</code>	328
8.3.2.111	<code>__ellint_rf()</code>	329
8.3.2.112	<code>__ellint_rg()</code>	330
8.3.2.113	<code>__ellint_rj()</code>	330
8.3.2.114	<code>__ellnome()</code>	331
8.3.2.115	<code>__ellnome_k()</code>	332
8.3.2.116	<code>__ellnome_series()</code>	332
8.3.2.117	<code>__euler()</code> [1/2]	332
8.3.2.118	<code>__euler()</code> [2/2]	333
8.3.2.119	<code>__euler_series()</code>	333
8.3.2.120	<code>__eulerian_1()</code> [1/2]	334
8.3.2.121	<code>__eulerian_1()</code> [2/2]	334
8.3.2.122	<code>__eulerian_1_recur()</code> [1/2]	334

8.3.2.123 <code>__eulerian_1_recur()</code> [2/2]	335
8.3.2.124 <code>__eulerian_2()</code> [1/2]	335
8.3.2.125 <code>__eulerian_2()</code> [2/2]	335
8.3.2.126 <code>__eulerian_2_recur()</code> [1/2]	336
8.3.2.127 <code>__eulerian_2_recur()</code> [2/2]	336
8.3.2.128 <code>__exp2()</code>	336
8.3.2.129 <code>__expint()</code> [1/2]	336
8.3.2.130 <code>__expint()</code> [2/2]	337
8.3.2.131 <code>__expint_E1()</code>	338
8.3.2.132 <code>__expint_E1_asymp()</code>	338
8.3.2.133 <code>__expint_E1_series()</code>	339
8.3.2.134 <code>__expint_Ei()</code>	339
8.3.2.135 <code>__expint_Ei_asymp()</code>	340
8.3.2.136 <code>__expint_Ei_series()</code>	341
8.3.2.137 <code>__expint_En_asymp()</code>	341
8.3.2.138 <code>__expint_En_cont_frac()</code>	342
8.3.2.139 <code>__expint_En_large_n()</code>	342
8.3.2.140 <code>__expint_En_recursion()</code>	343
8.3.2.141 <code>__expint_En_series()</code>	344
8.3.2.142 <code>__exponential_p()</code>	344
8.3.2.143 <code>__exponential_pdf()</code>	345
8.3.2.144 <code>__exponential_q()</code>	345
8.3.2.145 <code>__factorial()</code>	345
8.3.2.146 <code>__falling_factorial()</code> [1/2]	346
8.3.2.147 <code>__falling_factorial()</code> [2/2]	346
8.3.2.148 <code>__fermi_dirac()</code>	346
8.3.2.149 <code>__fisher_f_p()</code>	347
8.3.2.150 <code>__fisher_f_pdf()</code>	348

8.3.2.151	<code>__fisher_f_q()</code>	348
8.3.2.152	<code>__fock_airy()</code>	349
8.3.2.153	<code>__fresnel()</code>	349
8.3.2.154	<code>__fresnel_cont_frac()</code>	350
8.3.2.155	<code>__fresnel_series()</code>	350
8.3.2.156	<code>__gamma()</code> $[1/2]$	350
8.3.2.157	<code>__gamma()</code> $[2/2]$	351
8.3.2.158	<code>__gamma_cont_frac()</code>	351
8.3.2.159	<code>__gamma_p()</code> $[1/2]$	352
8.3.2.160	<code>__gamma_p()</code> $[2/2]$	352
8.3.2.161	<code>__gamma_pdf()</code>	353
8.3.2.162	<code>__gamma_q()</code> $[1/2]$	353
8.3.2.163	<code>__gamma_q()</code> $[2/2]$	354
8.3.2.164	<code>__gamma_reciprocal()</code>	354
8.3.2.165	<code>__gamma_reciprocal_series()</code>	355
8.3.2.166	<code>__gamma_series()</code>	355
8.3.2.167	<code>__gamma_temme()</code>	356
8.3.2.168	<code>__gauss()</code>	356
8.3.2.169	<code>__gegenbauer_recur()</code>	357
8.3.2.170	<code>__gegenbauer_zeros()</code>	357
8.3.2.171	<code>__hankel()</code>	358
8.3.2.172	<code>__hankel_debye()</code>	358
8.3.2.173	<code>__hankel_params()</code>	359
8.3.2.174	<code>__hankel_uniform()</code>	360
8.3.2.175	<code>__hankel_uniform_olver()</code>	360
8.3.2.176	<code>__hankel_uniform_outer()</code>	361
8.3.2.177	<code>__hankel_uniform_sum()</code>	362
8.3.2.178	<code>__harmonic_number()</code>	363

8.3.2.179 <code>__hermite()</code>	363
8.3.2.180 <code>__hermite_asymp()</code>	364
8.3.2.181 <code>__hermite_recur()</code>	364
8.3.2.182 <code>__hermite_zeros()</code>	365
8.3.2.183 <code>__heuman_lambda()</code>	366
8.3.2.184 <code>__hurwitz_zeta()</code>	366
8.3.2.185 <code>__hurwitz_zeta_euler_maclaurin()</code>	367
8.3.2.186 <code>__hurwitz_zeta_polylog()</code>	367
8.3.2.187 <code>__hydrogen()</code>	368
8.3.2.188 <code>__hyperg()</code>	368
8.3.2.189 <code>__hyperg_luke()</code>	369
8.3.2.190 <code>__hyperg_recur()</code>	369
8.3.2.191 <code>__hyperg_reflect()</code>	370
8.3.2.192 <code>__hyperg_series()</code>	371
8.3.2.193 <code>__ibeta_cont_frac()</code>	371
8.3.2.194 <code>__jacobi_ellint()</code>	372
8.3.2.195 <code>__jacobi_recur()</code>	372
8.3.2.196 <code>__jacobi_theta_1()</code> [1/2]	373
8.3.2.197 <code>__jacobi_theta_1()</code> [2/2]	374
8.3.2.198 <code>__jacobi_theta_1_prod()</code>	374
8.3.2.199 <code>__jacobi_theta_1_sum()</code>	375
8.3.2.200 <code>__jacobi_theta_2()</code> [1/2]	376
8.3.2.201 <code>__jacobi_theta_2()</code> [2/2]	377
8.3.2.202 <code>__jacobi_theta_2_prod()</code>	377
8.3.2.203 <code>__jacobi_theta_2_sum()</code>	378
8.3.2.204 <code>__jacobi_theta_3()</code> [1/2]	378
8.3.2.205 <code>__jacobi_theta_3()</code> [2/2]	379
8.3.2.206 <code>__jacobi_theta_3_prod()</code>	380

8.3.2.207	<code>__jacobi_theta_3_sum()</code>	380
8.3.2.208	<code>__jacobi_theta_4()</code> [1/2]	381
8.3.2.209	<code>__jacobi_theta_4()</code> [2/2]	382
8.3.2.210	<code>__jacobi_theta_4_prod()</code>	382
8.3.2.211	<code>__jacobi_theta_4_sum()</code>	383
8.3.2.212	<code>__jacobi_zeros()</code>	383
8.3.2.213	<code>__jacobi_zeta()</code>	384
8.3.2.214	<code>__kolmogorov_p()</code>	384
8.3.2.215	<code>__laguerre()</code> [1/2]	385
8.3.2.216	<code>__laguerre()</code> [2/2]	386
8.3.2.217	<code>__laguerre_hyperr()</code>	386
8.3.2.218	<code>__laguerre_large_n()</code>	387
8.3.2.219	<code>__laguerre_recur()</code>	388
8.3.2.220	<code>__laguerre_zeros()</code>	389
8.3.2.221	<code>__lah()</code> [1/2]	389
8.3.2.222	<code>__lah()</code> [2/2]	389
8.3.2.223	<code>__lah_recur()</code> [1/2]	390
8.3.2.224	<code>__lah_recur()</code> [2/2]	390
8.3.2.225	<code>__lanczos_binet1p()</code>	390
8.3.2.226	<code>__lanczos_log_gamma1p()</code>	391
8.3.2.227	<code>__legendre_p()</code>	392
8.3.2.228	<code>__legendre_q()</code>	392
8.3.2.229	<code>__legendre_q_series()</code>	393
8.3.2.230	<code>__legendre_zeros()</code>	393
8.3.2.231	<code>__log_binomial()</code> [1/2]	393
8.3.2.232	<code>__log_binomial()</code> [2/2]	394
8.3.2.233	<code>__log_binomial_sign()</code> [1/2]	395
8.3.2.234	<code>__log_binomial_sign()</code> [2/2]	395

8.3.2.235 <code>__log_double_factorial()</code> [1/2]	396
8.3.2.236 <code>__log_double_factorial()</code> [2/2]	396
8.3.2.237 <code>__log_factorial()</code>	397
8.3.2.238 <code>__log_falling_factorial()</code>	397
8.3.2.239 <code>__log_gamma()</code> [1/2]	397
8.3.2.240 <code>__log_gamma()</code> [2/2]	398
8.3.2.241 <code>__log_gamma_bernoulli()</code>	399
8.3.2.242 <code>__log_gamma_sign()</code> [1/2]	399
8.3.2.243 <code>__log_gamma_sign()</code> [2/2]	400
8.3.2.244 <code>__log_rising_factorial()</code>	400
8.3.2.245 <code>__log_stirling_1()</code>	400
8.3.2.246 <code>__log_stirling_1_sign()</code>	401
8.3.2.247 <code>__log_stirling_2()</code>	401
8.3.2.248 <code>__logint()</code>	401
8.3.2.249 <code>__logistic_p()</code>	402
8.3.2.250 <code>__logistic_pdf()</code>	402
8.3.2.251 <code>__lognormal_p()</code>	403
8.3.2.252 <code>__lognormal_pdf()</code>	403
8.3.2.253 <code>__normal_p()</code>	403
8.3.2.254 <code>__normal_pdf()</code>	404
8.3.2.255 <code>__owens_t()</code>	404
8.3.2.256 <code>__periodic_zeta()</code>	405
8.3.2.257 <code>__polar_pi()</code> [1/2]	405
8.3.2.258 <code>__polar_pi()</code> [2/2]	406
8.3.2.259 <code>__polygamma()</code>	406
8.3.2.260 <code>__polylog()</code> [1/2]	406
8.3.2.261 <code>__polylog()</code> [2/2]	407
8.3.2.262 <code>__polylog_exp()</code>	408

8.3.2.263	<code>__polylog_exp_asymp()</code>	408
8.3.2.264	<code>__polylog_exp_neg()</code> $[1/2]$	409
8.3.2.265	<code>__polylog_exp_neg()</code> $[2/2]$	410
8.3.2.266	<code>__polylog_exp_neg_int()</code> $[1/2]$	410
8.3.2.267	<code>__polylog_exp_neg_int()</code> $[2/2]$	411
8.3.2.268	<code>__polylog_exp_neg_real()</code> $[1/2]$	412
8.3.2.269	<code>__polylog_exp_neg_real()</code> $[2/2]$	412
8.3.2.270	<code>__polylog_exp_pos()</code> $[1/3]$	413
8.3.2.271	<code>__polylog_exp_pos()</code> $[2/3]$	414
8.3.2.272	<code>__polylog_exp_pos()</code> $[3/3]$	414
8.3.2.273	<code>__polylog_exp_pos_int()</code> $[1/2]$	415
8.3.2.274	<code>__polylog_exp_pos_int()</code> $[2/2]$	416
8.3.2.275	<code>__polylog_exp_pos_real()</code> $[1/2]$	416
8.3.2.276	<code>__polylog_exp_pos_real()</code> $[2/2]$	417
8.3.2.277	<code>__polylog_exp_sum()</code>	417
8.3.2.278	<code>__prob_hermite_recur()</code>	418
8.3.2.279	<code>__radial_jacobi()</code>	419
8.3.2.280	<code>__radial_jacobi_zeros()</code>	420
8.3.2.281	<code>__rice_pdf()</code>	421
8.3.2.282	<code>__riemann_zeta()</code>	421
8.3.2.283	<code>__riemann_zeta_euler_maclaurin()</code>	422
8.3.2.284	<code>__riemann_zeta_glob()</code>	422
8.3.2.285	<code>__riemann_zeta_laurent()</code>	422
8.3.2.286	<code>__riemann_zeta_m_1()</code>	423
8.3.2.287	<code>__riemann_zeta_m_1_glob()</code>	423
8.3.2.288	<code>__riemann_zeta_product()</code>	424
8.3.2.289	<code>__riemann_zeta_sum()</code>	424
8.3.2.290	<code>__rising_factorial()</code> $[1/2]$	425

8.3.2.291	__rising_factorial() [2/2]	425
8.3.2.292	__sin_pi() [1/2]	426
8.3.2.293	__sin_pi() [2/2]	426
8.3.2.294	__sinc()	426
8.3.2.295	__sinc_pi()	427
8.3.2.296	__sincos() [1/4]	427
8.3.2.297	__sincos() [2/4]	427
8.3.2.298	__sincos() [3/4]	427
8.3.2.299	__sincos() [4/4]	428
8.3.2.300	__sincos_pi()	428
8.3.2.301	__sincosint()	428
8.3.2.302	__sincosint_asymp()	429
8.3.2.303	__sincosint_cont_frac()	429
8.3.2.304	__sincosint_series()	429
8.3.2.305	__sinh_pi() [1/2]	430
8.3.2.306	__sinh_pi() [2/2]	430
8.3.2.307	__sinhc()	430
8.3.2.308	__sinhc_pi()	431
8.3.2.309	__sinhint()	431
8.3.2.310	__sph_bessel() [1/2]	432
8.3.2.311	__sph_bessel() [2/2]	432
8.3.2.312	__sph_bessel_ik()	433
8.3.2.313	__sph_bessel_jn()	433
8.3.2.314	__sph_bessel_jn_neg_arg()	434
8.3.2.315	__sph_hankel()	434
8.3.2.316	__sph_hankel_1() [1/2]	435
8.3.2.317	__sph_hankel_1() [2/2]	436
8.3.2.318	__sph_hankel_2() [1/2]	436

8.3.2.319	<code>__sph_hankel_2()</code> [2/2]	437
8.3.2.320	<code>__sph_harmonic()</code>	437
8.3.2.321	<code>__sph_legendre()</code>	438
8.3.2.322	<code>__sph_neumann()</code> [1/2]	439
8.3.2.323	<code>__sph_neumann()</code> [2/2]	439
8.3.2.324	<code>__spouge_binet1p()</code>	440
8.3.2.325	<code>__spouge_log_gamma1p()</code>	441
8.3.2.326	<code>__stirling_1()</code> [1/2]	442
8.3.2.327	<code>__stirling_1()</code> [2/2]	442
8.3.2.328	<code>__stirling_1_recur()</code> [1/2]	443
8.3.2.329	<code>__stirling_1_recur()</code> [2/2]	443
8.3.2.330	<code>__stirling_2()</code> [1/2]	443
8.3.2.331	<code>__stirling_2()</code> [2/2]	444
8.3.2.332	<code>__stirling_2_recur()</code> [1/2]	444
8.3.2.333	<code>__stirling_2_recur()</code> [2/2]	445
8.3.2.334	<code>__stirling_2_series()</code>	445
8.3.2.335	<code>__student_t_p()</code>	445
8.3.2.336	<code>__student_t_pdf()</code>	446
8.3.2.337	<code>__student_t_q()</code>	446
8.3.2.338	<code>__tan_pi()</code> [1/2]	447
8.3.2.339	<code>__tan_pi()</code> [2/2]	447
8.3.2.340	<code>__tanh_pi()</code> [1/2]	448
8.3.2.341	<code>__tanh_pi()</code> [2/2]	448
8.3.2.342	<code>__tgamma()</code>	448
8.3.2.343	<code>__tgamma_lower()</code>	449
8.3.2.344	<code>__theta_1()</code>	449
8.3.2.345	<code>__theta_2()</code>	450
8.3.2.346	<code>__theta_2_asymp()</code>	450

8.3.2.347	<code>__theta_2_sum()</code>	451
8.3.2.348	<code>__theta_3()</code>	451
8.3.2.349	<code>__theta_3_asymp()</code>	452
8.3.2.350	<code>__theta_3_sum()</code>	452
8.3.2.351	<code>__theta_4()</code>	452
8.3.2.352	<code>__theta_c()</code>	453
8.3.2.353	<code>__theta_d()</code>	453
8.3.2.354	<code>__theta_n()</code>	454
8.3.2.355	<code>__theta_s()</code>	454
8.3.2.356	<code>__tricoli_u()</code>	454
8.3.2.357	<code>__tricoli_u_naive()</code>	455
8.3.2.358	<code>__weibull_p()</code>	456
8.3.2.359	<code>__weibull_pdf()</code>	456
8.3.2.360	<code>__zernike()</code>	457
8.3.2.361	<code>__znorm1()</code>	457
8.3.2.362	<code>__znorm2()</code>	458
8.3.3	Variable Documentation	458
8.3.3.1	<code>__max_FGH</code>	458
8.3.3.2	<code>__max_FGH< double ></code>	458
8.3.3.3	<code>__max_FGH< float ></code>	458
8.3.3.4	<code>_Num_Euler_Maclaurin_zeta</code>	459
8.3.3.5	<code>_Num_Stieltjes</code>	459
8.3.3.6	<code>_S_double_factorial_table</code>	459
8.3.3.7	<code>_S_Euler_Maclaurin_zeta</code>	459
8.3.3.8	<code>_S_factorial_table</code>	460
8.3.3.9	<code>_S_harmonic_denom</code>	460
8.3.3.10	<code>_S_harmonic_numer</code>	460
8.3.3.11	<code>_S_neg_double_factorial_table</code>	460

8.3.3.12	_S_num_double_factorials	460
8.3.3.13	_S_num_double_factorials< double >	461
8.3.3.14	_S_num_double_factorials< float >	461
8.3.3.15	_S_num_double_factorials< long double >	461
8.3.3.16	_S_num_factorials	461
8.3.3.17	_S_num_factorials< double >	461
8.3.3.18	_S_num_factorials< float >	462
8.3.3.19	_S_num_factorials< long double >	462
8.3.3.20	_S_num_harmonic_numer	462
8.3.3.21	_S_num_neg_double_factorials	462
8.3.3.22	_S_num_neg_double_factorials< double >	462
8.3.3.23	_S_num_neg_double_factorials< float >	463
8.3.3.24	_S_num_neg_double_factorials< long double >	463
8.3.3.25	_S_num_zetam1	463
8.3.3.26	_S_Stieltjes	463
8.3.3.27	_S_zetam1	464
9	Class Documentation	465
9.1	__gnu_cxx::__airy_t< _Tx, _Tp > Struct Template Reference	465
9.1.1	Detailed Description	465
9.1.2	Member Function Documentation	466
9.1.2.1	__Wronskian()	466
9.1.3	Member Data Documentation	466
9.1.3.1	__Ai_deriv	466
9.1.3.2	__Ai_value	466
9.1.3.3	__Bi_deriv	466
9.1.3.4	__Bi_value	467
9.1.3.5	__x_arg	467

9.2	__gnu_cxx::__assoc_legendre_p_t<_Tp> Struct Template Reference	467
9.2.1	Detailed Description	468
9.2.2	Member Function Documentation	468
9.2.2.1	deriv()	468
9.2.3	Member Data Documentation	468
9.2.3.1	__l	468
9.2.3.2	__m	468
9.2.3.3	__P_lm	469
9.2.3.4	__P_lm1m	469
9.2.3.5	__P_lm2m	469
9.2.3.6	__phase	469
9.2.3.7	__x	470
9.3	__gnu_cxx::__assoc_legendre_q_t<_Tp> Struct Template Reference	470
9.3.1	Detailed Description	470
9.3.2	Member Function Documentation	471
9.3.2.1	deriv()	471
9.3.3	Member Data Documentation	471
9.3.3.1	__l	471
9.3.3.2	__m	471
9.3.3.3	__phase	471
9.3.3.4	__Q_lm	472
9.3.3.5	__Q_lmm1	472
9.3.3.6	__Q_lmm2	472
9.3.3.7	__x	472
9.4	__gnu_cxx::__chebyshev_t_t<_Tp> Struct Template Reference	473
9.4.1	Detailed Description	473
9.4.2	Member Function Documentation	473
9.4.2.1	deriv()	473

9.4.2.2	deriv2()	473
9.4.3	Member Data Documentation	474
9.4.3.1	__n	474
9.4.3.2	__T_n	474
9.4.3.3	__T_nm1	474
9.4.3.4	__T_nm2	474
9.4.3.5	__x	475
9.5	__gnu_cxx::__chebyshev_u_t<_Tp> Struct Template Reference	475
9.5.1	Detailed Description	475
9.5.2	Member Function Documentation	475
9.5.2.1	deriv()	476
9.5.3	Member Data Documentation	476
9.5.3.1	__n	476
9.5.3.2	__U_n	476
9.5.3.3	__U_nm1	476
9.5.3.4	__U_nm2	476
9.5.3.5	__x	477
9.6	__gnu_cxx::__chebyshev_v_t<_Tp> Struct Template Reference	477
9.6.1	Detailed Description	477
9.6.2	Member Function Documentation	477
9.6.2.1	deriv()	478
9.6.3	Member Data Documentation	478
9.6.3.1	__n	478
9.6.3.2	__V_n	478
9.6.3.3	__V_nm1	478
9.6.3.4	__V_nm2	478
9.6.3.5	__x	479
9.7	__gnu_cxx::__chebyshev_w_t<_Tp> Struct Template Reference	479

9.7.1	Detailed Description	479
9.7.2	Member Function Documentation	479
9.7.2.1	deriv()	480
9.7.3	Member Data Documentation	480
9.7.3.1	__n	480
9.7.3.2	__W_n	480
9.7.3.3	__W_nm1	480
9.7.3.4	__W_nm2	480
9.7.3.5	__x	481
9.8	__gnu_cxx::__coulomb_t<_Teta, _Trho, _Tp> Struct Template Reference	481
9.8.1	Detailed Description	481
9.8.2	Member Function Documentation	482
9.8.2.1	__Wronskian()	482
9.8.3	Member Data Documentation	482
9.8.3.1	__eta_arg	482
9.8.3.2	__F_deriv	482
9.8.3.3	__F_value	482
9.8.3.4	__G_deriv	483
9.8.3.5	__G_value	483
9.8.3.6	__I	483
9.8.3.7	__rho_arg	483
9.9	__gnu_cxx::__cyl_bessel_t<_Tnu, _Tx, _Tp> Struct Template Reference	484
9.9.1	Detailed Description	484
9.9.2	Member Function Documentation	484
9.9.2.1	__Wronskian()	484
9.9.3	Member Data Documentation	485
9.9.3.1	__J_deriv	485
9.9.3.2	__J_value	485

9.9.3.3	__N_deriv	485
9.9.3.4	__N_value	485
9.9.3.5	__nu_arg	486
9.9.3.6	__x_arg	486
9.10	__gnu_cxx::__cyl_hankel_t<_Tnu, _Tx, _Tp> Struct Template Reference	486
9.10.1	Detailed Description	487
9.10.2	Member Function Documentation	487
9.10.2.1	__Wronskian()	487
9.10.3	Member Data Documentation	487
9.10.3.1	__H1_deriv	487
9.10.3.2	__H1_value	487
9.10.3.3	__H2_deriv	488
9.10.3.4	__H2_value	488
9.10.3.5	__nu_arg	488
9.10.3.6	__x_arg	488
9.11	__gnu_cxx::__cyl_mod_bessel_t<_Tnu, _Tx, _Tp> Struct Template Reference	489
9.11.1	Detailed Description	489
9.11.2	Member Function Documentation	489
9.11.2.1	__Wronskian()	489
9.11.3	Member Data Documentation	490
9.11.3.1	__I_deriv	490
9.11.3.2	__I_value	490
9.11.3.3	__K_deriv	490
9.11.3.4	__K_value	490
9.11.3.5	__nu_arg	491
9.11.3.6	__x_arg	491
9.12	__gnu_cxx::__fock_airy_t<_Tx, _Tp> Struct Template Reference	491
9.12.1	Detailed Description	492

9.12.2	Member Function Documentation	492
9.12.2.1	__Wronskian()	492
9.12.3	Member Data Documentation	492
9.12.3.1	__w1_deriv	492
9.12.3.2	__w1_value	492
9.12.3.3	__w2_deriv	493
9.12.3.4	__w2_value	493
9.12.3.5	__x_arg	493
9.13	__gnu_cxx::__fp_is_integer_t Struct Reference	493
9.13.1	Detailed Description	494
9.13.2	Member Function Documentation	494
9.13.2.1	operator bool()	494
9.13.2.2	operator>()()	494
9.13.3	Member Data Documentation	494
9.13.3.1	__is_integral	494
9.13.3.2	__value	495
9.14	__gnu_cxx::__gamma_inc_t<_Tp> Struct Template Reference	495
9.14.1	Detailed Description	495
9.14.2	Member Data Documentation	495
9.14.2.1	__lgamma_value	495
9.14.2.2	__tgamma_value	496
9.15	__gnu_cxx::__gamma_temme_t<_Tp> Struct Template Reference	496
9.15.1	Detailed Description	497
9.15.2	Member Data Documentation	497
9.15.2.1	__gamma_1_value	497
9.15.2.2	__gamma_2_value	497
9.15.2.3	__gamma_minus_value	498
9.15.2.4	__gamma_plus_value	498

9.15.2.5	<code>__mu_arg</code>	498
9.16	<code>__gnu_cxx::__gappa_pq_t<_Tp></code> Struct Template Reference	498
9.16.1	Detailed Description	499
9.16.2	Member Data Documentation	499
9.16.2.1	<code>__gappa_p_value</code>	499
9.16.2.2	<code>__gappa_q_value</code>	499
9.17	<code>__gnu_cxx::__gegenbauer_t<_Tp></code> Struct Template Reference	499
9.17.1	Detailed Description	500
9.17.2	Member Function Documentation	500
9.17.2.1	<code>deriv()</code>	500
9.17.3	Member Data Documentation	500
9.17.3.1	<code>__C_n</code>	500
9.17.3.2	<code>__C_nm1</code>	501
9.17.3.3	<code>__C_nm2</code>	501
9.17.3.4	<code>__lambda</code>	501
9.17.3.5	<code>__n</code>	501
9.17.3.6	<code>__x</code>	501
9.18	<code>__gnu_cxx::__hermite_he_t<_Tp></code> Struct Template Reference	502
9.18.1	Detailed Description	502
9.18.2	Member Function Documentation	502
9.18.2.1	<code>deriv()</code>	502
9.18.2.2	<code>deriv2()</code>	502
9.18.3	Member Data Documentation	503
9.18.3.1	<code>__He_n</code>	503
9.18.3.2	<code>__He_nm1</code>	503
9.18.3.3	<code>__He_nm2</code>	503
9.18.3.4	<code>__n</code>	503
9.18.3.5	<code>__x</code>	504

9.19	__gnu_cxx::__hermite_t<_Tp> Struct Template Reference	504
9.19.1	Detailed Description	504
9.19.2	Member Function Documentation	504
9.19.2.1	deriv()	505
9.19.2.2	deriv2()	505
9.19.3	Member Data Documentation	505
9.19.3.1	__H_n	505
9.19.3.2	__H_nm1	505
9.19.3.3	__H_nm2	505
9.19.3.4	__n	506
9.19.3.5	__x	506
9.20	__gnu_cxx::__jacobi_ellint_t<_Tp> Struct Template Reference	506
9.20.1	Detailed Description	507
9.20.2	Member Function Documentation	507
9.20.2.1	__am()	507
9.20.2.2	__cd()	507
9.20.2.3	__cn_deriv()	507
9.20.2.4	__cs()	507
9.20.2.5	__dc()	508
9.20.2.6	__ds()	508
9.20.2.7	__nc()	508
9.20.2.8	__nd()	508
9.20.2.9	__ns()	508
9.20.2.10	__sc()	509
9.20.2.11	__sd()	509
9.20.2.12	__sn_deriv()	509
9.20.3	Member Data Documentation	509
9.20.3.1	__cn_value	509

9.20.3.2	__dn_value	510
9.20.3.3	__sn_value	510
9.21	__gnu_cxx::__jacobi_t<_Tp> Struct Template Reference	510
9.21.1	Detailed Description	511
9.21.2	Member Function Documentation	511
9.21.2.1	deriv()	511
9.21.3	Member Data Documentation	511
9.21.3.1	__alpha1	511
9.21.3.2	__beta1	511
9.21.3.3	__n	512
9.21.3.4	__P_n	512
9.21.3.5	__P_nm1	512
9.21.3.6	__P_nm2	512
9.21.3.7	__x	512
9.22	__gnu_cxx::__laguerre_t<_Tpa, _Tp> Struct Template Reference	513
9.22.1	Detailed Description	513
9.22.2	Member Function Documentation	513
9.22.2.1	deriv()	513
9.22.3	Member Data Documentation	513
9.22.3.1	__alpha1	514
9.22.3.2	__L_n	514
9.22.3.3	__L_nm1	514
9.22.3.4	__L_nm2	514
9.22.3.5	__n	514
9.22.3.6	__x	515
9.23	__gnu_cxx::__legendre_p_t<_Tp> Struct Template Reference	515
9.23.1	Detailed Description	515
9.23.2	Member Function Documentation	516

9.23.2.1	deriv()	516
9.23.2.2	lobatto()	516
9.23.3	Member Data Documentation	516
9.23.3.1	__I	516
9.23.3.2	__P_I	516
9.23.3.3	__P_Im1	517
9.23.3.4	__P_Im2	517
9.23.3.5	__x	517
9.24	__gnu_cxx::__legendre_q_t<_Tp> Struct Template Reference	517
9.24.1	Detailed Description	518
9.24.2	Member Function Documentation	518
9.24.2.1	deriv()	518
9.24.3	Member Data Documentation	518
9.24.3.1	__I	518
9.24.3.2	__Q_I	519
9.24.3.3	__Q_Im1	519
9.24.3.4	__Q_Im2	519
9.24.3.5	__x	519
9.25	__gnu_cxx::__lgamma_t<_Tp> Struct Template Reference	519
9.25.1	Detailed Description	520
9.25.2	Member Data Documentation	520
9.25.2.1	__lgamma_sign	520
9.25.2.2	__lgamma_value	520
9.26	__gnu_cxx::__quadrature_point_t<_Tp> Struct Template Reference	521
9.26.1	Detailed Description	521
9.26.2	Constructor & Destructor Documentation	521
9.26.2.1	__quadrature_point_t() [1/2]	521
9.26.2.2	__quadrature_point_t() [2/2]	521

9.26.3	Member Data Documentation	522
9.26.3.1	__point	522
9.26.3.2	__weight	522
9.27	__gnu_cxx::__sincos_t<_Tp> Struct Template Reference	522
9.27.1	Detailed Description	522
9.27.2	Member Data Documentation	523
9.27.2.1	__cos_v	523
9.27.2.2	__sin_v	523
9.28	__gnu_cxx::__sph_bessel_t<_Tn, _Tx, _Tp> Struct Template Reference	523
9.28.1	Detailed Description	524
9.28.2	Member Function Documentation	524
9.28.2.1	__Wronskian()	524
9.28.3	Member Data Documentation	524
9.28.3.1	__j_deriv	525
9.28.3.2	__j_value	525
9.28.3.3	__n_arg	525
9.28.3.4	__n_deriv	525
9.28.3.5	__n_value	526
9.28.3.6	__x_arg	526
9.29	__gnu_cxx::__sph_hankel_t<_Tn, _Tx, _Tp> Struct Template Reference	526
9.29.1	Detailed Description	527
9.29.2	Member Function Documentation	527
9.29.2.1	__Wronskian()	527
9.29.3	Member Data Documentation	527
9.29.3.1	__h1_deriv	527
9.29.3.2	__h1_value	527
9.29.3.3	__h2_deriv	528
9.29.3.4	__h2_value	528

9.29.3.5	<code>__n_arg</code>	528
9.29.3.6	<code>__x_arg</code>	528
9.30	<code>__gnu_cxx::__sph_mod_bessel_t<_Tn, _Tx, _Tp></code> Struct Template Reference	529
9.30.1	Detailed Description	529
9.30.2	Member Function Documentation	529
9.30.2.1	<code>__Wronskian()</code>	529
9.30.3	Member Data Documentation	530
9.30.3.1	<code>__i_deriv</code>	530
9.30.3.2	<code>__i_value</code>	530
9.30.3.3	<code>__k_deriv</code>	530
9.30.3.4	<code>__k_value</code>	530
9.30.3.5	<code>__n_arg</code>	531
9.30.3.6	<code>__x_arg</code>	531
9.31	<code>__gnu_cxx::__stirling_1_t<_Tp></code> Struct Template Reference	531
9.31.1	Detailed Description	532
9.31.2	Member Typedef Documentation	532
9.31.2.1	<code>const_iterator</code>	532
9.31.2.2	<code>iterator</code>	532
9.31.3	Member Function Documentation	532
9.31.3.1	<code>begin()</code> [1/2]	532
9.31.3.2	<code>begin()</code> [2/2]	533
9.31.3.3	<code>degree()</code>	533
9.31.3.4	<code>end()</code> [1/2]	533
9.31.3.5	<code>end()</code> [2/2]	533
9.31.3.6	<code>operator>()</code>	533
9.31.3.7	<code>operator[]()</code>	534
9.31.4	Member Data Documentation	534
9.31.4.1	<code>__sigma</code>	534

9.32	__gnu_cxx::__stirling_2_t<_Tp> Struct Template Reference	534
9.32.1	Detailed Description	535
9.32.2	Member Typedef Documentation	535
9.32.2.1	const_iterator	535
9.32.2.2	iterator	535
9.32.3	Member Function Documentation	535
9.32.3.1	begin() [1/2]	535
9.32.3.2	begin() [2/2]	536
9.32.3.3	degree()	536
9.32.3.4	end() [1/2]	536
9.32.3.5	end() [2/2]	536
9.32.3.6	operator>()	536
9.32.3.7	operator[]()	537
9.32.4	Member Data Documentation	537
9.32.4.1	_S	537
9.33	std::__detail::__gamma_lanczos_data<_Tp> Struct Template Reference	537
9.33.1	Detailed Description	537
9.34	std::__detail::__gamma_lanczos_data<double> Struct Template Reference	537
9.34.1	Detailed Description	538
9.34.2	Member Data Documentation	538
9.34.2.1	_S_cheby	538
9.34.2.2	_S_g	538
9.35	std::__detail::__gamma_lanczos_data<float> Struct Template Reference	538
9.35.1	Detailed Description	539
9.35.2	Member Data Documentation	539
9.35.2.1	_S_cheby	539
9.35.2.2	_S_g	539
9.36	std::__detail::__gamma_lanczos_data<long double> Struct Template Reference	539

9.36.1 Detailed Description	540
9.36.2 Member Data Documentation	540
9.36.2.1 <code>_S_cheby</code>	540
9.36.2.2 <code>_S_g</code>	540
9.37 <code>std::__detail::__gamma_spouge_data< _Tp ></code> Struct Template Reference	541
9.37.1 Detailed Description	541
9.38 <code>std::__detail::__gamma_spouge_data< double ></code> Struct Template Reference	541
9.38.1 Detailed Description	541
9.38.2 Member Data Documentation	541
9.38.2.1 <code>_S_cheby</code>	542
9.39 <code>std::__detail::__gamma_spouge_data< float ></code> Struct Template Reference	542
9.39.1 Detailed Description	542
9.39.2 Member Data Documentation	542
9.39.2.1 <code>_S_cheby</code>	543
9.40 <code>std::__detail::__gamma_spouge_data< long double ></code> Struct Template Reference	543
9.40.1 Detailed Description	543
9.40.2 Member Data Documentation	543
9.40.2.1 <code>_S_cheby</code>	544
9.41 <code>std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 ></code> Struct Template Reference	544
9.41.1 Detailed Description	546
9.41.2 Member Typedef Documentation	546
9.41.2.1 <code>_Cmplx</code>	546
9.41.2.2 <code>_Real</code>	546
9.41.2.3 <code>_Real_Omega1</code>	546
9.41.2.4 <code>_Real_Omega3</code>	547
9.41.2.5 <code>_Tp_Nome</code>	547
9.41.3 Constructor & Destructor Documentation	547
9.41.3.1 <code>__jacobi_lattice_t()</code> [1/3]	547

9.41.3.2	__jacobi_lattice_t() [2/3]	547
9.41.3.3	__jacobi_lattice_t() [3/3]	548
9.41.4	Member Function Documentation	548
9.41.4.1	__ellnome()	548
9.41.4.2	__omega_1()	548
9.41.4.3	__omega_2()	549
9.41.4.4	__omega_3()	549
9.41.4.5	__reduce()	549
9.41.4.6	__tau()	550
9.41.5	Member Data Documentation	550
9.41.5.1	_M_omega_1	550
9.41.5.2	_M_omega_3	550
9.41.5.3	_S_pi	551
9.42	std::__detail::__jacobi_lattice_t<_Tp_Omega1,_Tp_Omega3>::__arg_t Struct Reference	551
9.42.1	Detailed Description	551
9.42.2	Member Data Documentation	551
9.42.2.1	__m	552
9.42.2.2	__n	552
9.42.2.3	__z	552
9.43	std::__detail::__jacobi_lattice_t<_Tp_Omega1,_Tp_Omega3>::__tau_t Struct Reference	552
9.43.1	Detailed Description	553
9.43.2	Constructor & Destructor Documentation	553
9.43.2.1	__tau_t()	553
9.43.3	Member Data Documentation	553
9.43.3.1	__val	553
9.44	std::__detail::__jacobi_theta_0_t<_Tp1,_Tp3> Struct Template Reference	554
9.44.1	Detailed Description	555
9.44.2	Member Typedef Documentation	555

9.44.2.1	<code>_Cmplx</code>	555
9.44.2.2	<code>_Real</code>	555
9.44.2.3	<code>_Type</code>	555
9.44.3	Constructor & Destructor Documentation	555
9.44.3.1	<code>__jacobi_theta_0_t()</code>	556
9.44.4	Member Function Documentation	556
9.44.4.1	<code>dedekind_eta()</code>	556
9.44.5	Member Data Documentation	556
9.44.5.1	<code>eta_1</code>	556
9.44.5.2	<code>eta_2</code>	557
9.44.5.3	<code>eta_3</code>	557
9.44.5.4	<code>th1p</code>	557
9.44.5.5	<code>th1ppp</code>	557
9.44.5.6	<code>th2</code>	557
9.44.5.7	<code>th2pp</code>	558
9.44.5.8	<code>th3</code>	558
9.44.5.9	<code>th3pp</code>	558
9.44.5.10	<code>th4</code>	558
9.44.5.11	<code>th4pp</code>	558
9.45	<code>std::__detail::__weierstrass_invariants_t<_Tp1,_Tp3></code> Struct Template Reference	559
9.45.1	Detailed Description	559
9.45.2	Member Typedef Documentation	560
9.45.2.1	<code>_Cmplx</code>	560
9.45.2.2	<code>_Real</code>	560
9.45.2.3	<code>_Type</code>	560
9.45.3	Constructor & Destructor Documentation	560
9.45.3.1	<code>__weierstrass_invariants_t()</code>	560
9.45.4	Member Function Documentation	561

9.45.4.1	__delta()	561
9.45.4.2	__klein_j()	561
9.45.5	Member Data Documentation	561
9.45.5.1	__g_2	561
9.45.5.2	__g_3	561
9.46	std::__detail::__weierstrass_roots_t<_Tp1, _Tp3> Struct Template Reference	562
9.46.1	Detailed Description	563
9.46.2	Member Typedef Documentation	563
9.46.2.1	_Cmplx	563
9.46.2.2	_Real	563
9.46.2.3	_Type	563
9.46.3	Constructor & Destructor Documentation	564
9.46.3.1	__weierstrass_roots_t() [1/2]	564
9.46.3.2	__weierstrass_roots_t() [2/2]	564
9.46.4	Member Function Documentation	564
9.46.4.1	__delta()	565
9.46.5	Member Data Documentation	565
9.46.5.1	__e1	565
9.46.5.2	__e2	565
9.46.5.3	__e3	565
9.47	std::__detail::_Airy<_Tp> Class Template Reference	566
9.47.1	Detailed Description	566
9.47.2	Member Typedef Documentation	566
9.47.2.1	scalar_type	566
9.47.2.2	value_type	567
9.47.3	Constructor & Destructor Documentation	567
9.47.3.1	_Airy() [1/3]	567
9.47.3.2	_Airy() [2/3]	567

9.47.3.3	_Airy() [3/3]	567
9.47.4	Member Function Documentation	567
9.47.4.1	operator>()	567
9.47.5	Member Data Documentation	568
9.47.5.1	inner_radius	568
9.47.5.2	outer_radius	568
9.48	std::__detail::_Airy_asymp< _Tp > Class Template Reference	568
9.48.1	Detailed Description	569
9.48.2	Member Typedef Documentation	570
9.48.2.1	_Cmplx	570
9.48.3	Constructor & Destructor Documentation	570
9.48.3.1	_Airy_asymp()	570
9.48.4	Member Function Documentation	570
9.48.4.1	_S_absarg_ge_pio3()	570
9.48.4.2	_S_absarg_lt_pio3()	571
9.48.4.3	operator>()	572
9.49	std::__detail::_Airy_asymp_data< _Tp > Struct Template Reference	572
9.49.1	Detailed Description	572
9.50	std::__detail::_Airy_asymp_data< double > Struct Template Reference	573
9.50.1	Detailed Description	573
9.50.2	Member Data Documentation	573
9.50.2.1	_S_c	573
9.50.2.2	_S_d	573
9.50.2.3	_S_max_cd	574
9.51	std::__detail::_Airy_asymp_data< float > Struct Template Reference	574
9.51.1	Detailed Description	574
9.51.2	Member Data Documentation	574
9.51.2.1	_S_c	574

9.51.2.2	_S_d	574
9.51.2.3	_S_max_cd	575
9.52	std::__detail::_Airy_asymp_data< long double > Struct Template Reference	575
9.52.1	Detailed Description	575
9.52.2	Member Data Documentation	575
9.52.2.1	_S_c	575
9.52.2.2	_S_d	575
9.52.2.3	_S_max_cd	576
9.53	std::__detail::_Airy_asymp_series< _Sum > Class Template Reference	576
9.53.1	Detailed Description	576
9.53.2	Member Typedef Documentation	577
9.53.2.1	scalar_type	577
9.53.2.2	value_type	577
9.53.3	Constructor & Destructor Documentation	577
9.53.3.1	_Airy_asymp_series() [1/3]	577
9.53.3.2	_Airy_asymp_series() [2/3]	577
9.53.3.3	_Airy_asymp_series() [3/3]	578
9.53.4	Member Function Documentation	578
9.53.4.1	operator>()	578
9.53.5	Member Data Documentation	578
9.53.5.1	_S_sqrt_pi	578
9.54	std::__detail::_Airy_default_radii< _Tp > Struct Template Reference	579
9.54.1	Detailed Description	579
9.55	std::__detail::_Airy_default_radii< double > Struct Template Reference	579
9.55.1	Detailed Description	579
9.55.2	Member Data Documentation	579
9.55.2.1	inner_radius	579
9.55.2.2	outer_radius	580

9.56	std::__detail::_Airy_default_radii< float > Struct Template Reference	580
9.56.1	Detailed Description	580
9.56.2	Member Data Documentation	580
9.56.2.1	inner_radius	580
9.56.2.2	outer_radius	580
9.57	std::__detail::_Airy_default_radii< long double > Struct Template Reference	581
9.57.1	Detailed Description	581
9.57.2	Member Data Documentation	581
9.57.2.1	inner_radius	581
9.57.2.2	outer_radius	581
9.58	std::__detail::_Airy_series< _Tp > Class Template Reference	581
9.58.1	Detailed Description	582
9.58.2	Member Typedef Documentation	582
9.58.2.1	_Cmplx	583
9.58.3	Member Function Documentation	583
9.58.3.1	_S_Ai()	583
9.58.3.2	_S_Airy()	583
9.58.3.3	_S_Bi()	584
9.58.3.4	_S_FGH()	585
9.58.3.5	_S_Fock()	585
9.58.3.6	_S_Scorer()	586
9.58.3.7	_S_Scorer2()	586
9.58.4	Member Data Documentation	587
9.58.4.1	_N_FGH	587
9.58.4.2	_S_Ai0	587
9.58.4.3	_S_Aip0	587
9.58.4.4	_S_Bi0	588
9.58.4.5	_S_Bip0	588

9.58.4.6	_S_eps	588
9.58.4.7	_S_Gi0	588
9.58.4.8	_S_Gip0	588
9.58.4.9	_S_Hi0	589
9.58.4.10	_S_Hip0	589
9.58.4.11	_S_i	589
9.58.4.12	_S_pi	589
9.58.4.13	_S_sqrt_pi	589
9.59	std::__detail::_AiryAuxilliaryState<_Tp> Struct Template Reference	590
9.59.1	Detailed Description	590
9.59.2	Member Typedef Documentation	590
9.59.2.1	_Val	590
9.59.3	Member Data Documentation	590
9.59.3.1	__fai_deriv	591
9.59.3.2	__fai_value	591
9.59.3.3	__gai_deriv	591
9.59.3.4	__gai_value	591
9.59.3.5	__hai_deriv	591
9.59.3.6	__hai_value	592
9.59.3.7	__z	592
9.60	std::__detail::_AiryState<_Tp> Struct Template Reference	592
9.60.1	Detailed Description	593
9.60.2	Member Typedef Documentation	593
9.60.2.1	_Real	593
9.60.3	Member Function Documentation	593
9.60.3.1	true_Wronskian()	593
9.60.3.2	Wronskian()	593
9.60.4	Member Data Documentation	594

9.60.4.1	__Ai_deriv	594
9.60.4.2	__Ai_value	594
9.60.4.3	__Bi_deriv	594
9.60.4.4	__Bi_value	594
9.60.4.5	__z	595
9.61	std::__detail::_AsympTerminator<_Tp> Class Template Reference	595
9.61.1	Detailed Description	595
9.61.2	Constructor & Destructor Documentation	595
9.61.2.1	_AsympTerminator()	596
9.61.3	Member Function Documentation	596
9.61.3.1	num_terms()	596
9.61.3.2	operator>()()	596
9.61.3.3	operator<<()	596
9.62	std::__detail::_Factorial_table<_Tp> Struct Template Reference	597
9.62.1	Detailed Description	597
9.62.2	Member Data Documentation	597
9.62.2.1	__factorial	597
9.62.2.2	__log_factorial	597
9.62.2.3	__n	598
9.63	std::__detail::_Terminator<_Tp> Class Template Reference	598
9.63.1	Detailed Description	598
9.63.2	Constructor & Destructor Documentation	598
9.63.2.1	_Terminator()	599
9.63.3	Member Function Documentation	599
9.63.3.1	num_terms()	599
9.63.3.2	operator>()()	599

10 File Documentation	601
10.1 cxx_fp_utils/include/ext/math_util.h File Reference	601
10.1.1 Detailed Description	603
10.2 include/bits/sfairy.tcc File Reference	603
10.2.1 Detailed Description	604
10.2.2 Macro Definition Documentation	604
10.2.2.1 _GLIBCXX_BITS_SF_AIRY_TCC	605
10.3 include/bits/sf_bernoulli.tcc File Reference	605
10.3.1 Detailed Description	606
10.3.2 Macro Definition Documentation	606
10.3.2.1 _GLIBCXX_BITS_SF_BERNOULLI_TCC	606
10.4 include/bits/sf_bessel.tcc File Reference	606
10.4.1 Detailed Description	608
10.4.2 Macro Definition Documentation	608
10.4.2.1 _GLIBCXX_BITS_SF_BESSEL_TCC	608
10.5 include/bits/sf_beta.tcc File Reference	609
10.5.1 Detailed Description	610
10.5.2 Macro Definition Documentation	610
10.5.2.1 _GLIBCXX_BITS_SF_BETA_TCC	610
10.6 include/bits/sf_cardinal.tcc File Reference	611
10.6.1 Macro Definition Documentation	612
10.6.1.1 _GLIBCXX_BITS_SF_CARDINAL_TCC	612
10.7 include/bits/sf_chebyshev.tcc File Reference	613
10.7.1 Detailed Description	614
10.7.2 Macro Definition Documentation	614
10.7.2.1 _GLIBCXX_BITS_SF_CHEBYSHEV_TCC	614
10.8 include/bits/sf_coulomb.tcc File Reference	614
10.8.1 Detailed Description	615

10.8.2	Macro Definition Documentation	616
10.8.2.1	<code>_GLIBCXX_BITS_SF_COULOMB_TCC</code>	616
10.9	<code>include/bits/sf_dawson.tcc</code> File Reference	616
10.9.1	Detailed Description	617
10.9.2	Macro Definition Documentation	617
10.9.2.1	<code>_GLIBCXX_BITS_SF_DAWSON_TCC</code>	617
10.10	<code>include/bits/sf_distributions.tcc</code> File Reference	618
10.10.1	Detailed Description	620
10.10.2	Macro Definition Documentation	621
10.10.2.1	<code>_GLIBCXX_BITS_SF_DISTRIBUTIONS_TCC</code>	621
10.11	<code>include/bits/sf_ellint.tcc</code> File Reference	621
10.11.1	Detailed Description	623
10.11.2	Macro Definition Documentation	623
10.11.2.1	<code>_GLIBCXX_BITS_SF_ELLINT_TCC</code>	623
10.12	<code>include/bits/sf_euler.tcc</code> File Reference	624
10.12.1	Detailed Description	625
10.12.2	Macro Definition Documentation	625
10.12.2.1	<code>_GLIBCXX_BITS_SF_EULER_TCC</code>	625
10.13	<code>include/bits/sf_expint.tcc</code> File Reference	626
10.13.1	Detailed Description	628
10.13.2	Macro Definition Documentation	628
10.13.2.1	<code>_GLIBCXX_BITS_SF_EXPINT_TCC</code>	628
10.14	<code>include/bits/sf_fresnel.tcc</code> File Reference	628
10.14.1	Detailed Description	629
10.14.2	Macro Definition Documentation	629
10.14.2.1	<code>_GLIBCXX_BITS_SF_FRESNEL_TCC</code>	629
10.15	<code>include/bits/sf_gamma.tcc</code> File Reference	630
10.15.1	Detailed Description	637

10.15.2 Macro Definition Documentation	637
10.15.2.1 _GLIBCXX_BITS_SF_GAMMA_TCC	637
10.16include/bits/sf_gegenbauer.tcc File Reference	638
10.16.1 Detailed Description	639
10.16.2 Macro Definition Documentation	639
10.16.2.1 _GLIBCXX_BITS_SF_GEGENBAUER_TCC	639
10.17include/bits/sf_hankel.tcc File Reference	639
10.17.1 Detailed Description	642
10.17.2 Macro Definition Documentation	642
10.17.2.1 _GLIBCXX_BITS_SF_HANKEL_TCC	642
10.18include/bits/sf_hermite.tcc File Reference	642
10.18.1 Detailed Description	643
10.18.2 Macro Definition Documentation	644
10.18.2.1 _GLIBCXX_BITS_SF_HERMITE_TCC	644
10.19include/bits/sf_hyperg.tcc File Reference	644
10.19.1 Detailed Description	646
10.19.2 Macro Definition Documentation	646
10.19.2.1 _GLIBCXX_BITS_SF_HYPERG_TCC	646
10.20include/bits/sf_hypint.tcc File Reference	646
10.20.1 Detailed Description	647
10.20.2 Macro Definition Documentation	648
10.20.2.1 _GLIBCXX_BITS_SF_HYPINT_TCC	648
10.21include/bits/sf_jacobi.tcc File Reference	648
10.21.1 Detailed Description	649
10.21.2 Macro Definition Documentation	649
10.21.2.1 _GLIBCXX_BITS_SF_JACOBI_TCC	649
10.22include/bits/sf_laguerre.tcc File Reference	650
10.22.1 Detailed Description	651

10.22.2 Macro Definition Documentation	651
10.22.2.1 _GLIBCXX_BITS_SF_LAGUERRE_TCC	651
10.23include/bits/sf_legendre.tcc File Reference	652
10.23.1 Detailed Description	653
10.23.2 Macro Definition Documentation	653
10.23.2.1 _GLIBCXX_BITS_SF_LEGENDRE_TCC	653
10.24include/bits/sf_mod_bessel.tcc File Reference	654
10.24.1 Detailed Description	655
10.24.2 Macro Definition Documentation	655
10.24.2.1 _GLIBCXX_BITS_SF_MOD_BESSEL_TCC	655
10.25include/bits/sf_owens_t.tcc File Reference	656
10.25.1 Detailed Description	656
10.25.2 Macro Definition Documentation	657
10.25.2.1 _GLIBCXX_BITS_SF_OWENS_T_TCC	657
10.26include/bits/sf_polylog.tcc File Reference	657
10.26.1 Detailed Description	659
10.26.2 Macro Definition Documentation	659
10.26.2.1 _GLIBCXX_BITS_SF_POLYLOG_TCC	660
10.27include/bits/sf_stirling.tcc File Reference	660
10.27.1 Detailed Description	661
10.27.2 Macro Definition Documentation	662
10.27.2.1 _GLIBCXX_BITS_SF_STIRLING_TCC	662
10.28include/bits/sf_theta.tcc File Reference	662
10.28.1 Detailed Description	664
10.28.2 Macro Definition Documentation	664
10.28.2.1 _GLIBCXX_BITS_SF_THETA_TCC	665
10.29include/bits/sf_trig.tcc File Reference	665
10.29.1 Detailed Description	666

10.29.2 Macro Definition Documentation	667
10.29.2.1 _GLIBCXX_BITS_SF_TRIG_TCC	667
10.30include/bits/sf_trigint.tcc File Reference	667
10.30.1 Detailed Description	668
10.30.2 Macro Definition Documentation	668
10.30.2.1 _GLIBCXX_BITS_SF_TRIGINT_TCC	668
10.31include/bits/sf_zeta.tcc File Reference	669
10.31.1 Detailed Description	671
10.31.2 Macro Definition Documentation	671
10.31.2.1 _GLIBCXX_BITS_SF_ZETA_TCC	671
10.32include/bits/specfun.h File Reference	671
10.32.1 Detailed Description	687
10.32.2 Macro Definition Documentation	687
10.32.2.1 __cpp_lib_math_special_functions	687
10.32.2.2 __STDCPP_MATH_SPEC_FUNCS__	688
10.33include/bits/specfun_state.h File Reference	688
10.33.1 Detailed Description	690
Index	691

Chapter 1

Todo List

Module `mathsf`

Provide accuracy comparisons on a per-function basis for a small number of targets.

Member `std::__detail::__bell_series` (unsigned int __n)

Test for blowup in Bell number summation.

Member `std::__detail::__debye` (unsigned int __n, _Tp __x)

: We should return both the Debye function and it's complement.

Find Debye for $x < -2\pi i$

Find Debye for $x < -2\pi i$

Member `std::__detail::__euler_series` (unsigned int __n)

Find a way to predict the maximum Euler number for a type.

Member `std::__detail::__expint` (unsigned int __n, _Tp __x)

Study arbitrary switch to large-n $E_n(x)$.

Find a good asymptotic switch point in $E_n(x)$.

Find a good asymptotic switch point in $E_n(x)$.

Member `std::__detail::__expint_E1` (_Tp __x)

Find a good asymptotic switch point in $E_1(x)$.

Member `std::__detail::__expint_En_recursion` (unsigned int __n, _Tp __x)

Find a principled starting number for the $E_n(x)$ downward recursion.

Member `std::__detail::__hermite_recur` (unsigned int __n, _Tp __x)

Find the sign of Hermite blowup values.

Member `std::__detail::__hurwitz_zeta_polylog` (_Tp __s, std::complex<_Tp> __a)

This `__hurwitz_zeta_polylog` prefactor is prone to overflow. positive integer orders s ?

Member `std::__detail::__log_stirling_2` (unsigned int __n, unsigned int __m)

Find asymptotic expressions for the Stirling numbers.

Member `std::__detail::__riemann_zeta` (_Tp __s)

Global double sum or MacLaurin series in `riemann_zeta`?

Member `std::__detail::__stirling_1` (unsigned int __n, unsigned int __m)

Find asymptotic expressions for the Stirling numbers.

Member `std::__detail::__stirling_2` (unsigned int __n, unsigned int __m)

Find asymptotic expressions for the Stirling numbers.

Member `std::__detail::__stirling_2` (unsigned int __n)

Find asymptotic expressions for the Stirling numbers.

Member `std::__detail::__stirling_2_series` (unsigned int __n, unsigned int __m)

Find a way to predict the maximum Stirling number supported for a given type.

Member `std::__detail::__Airy_asymp<_Tp>::__S_absarg_lt_pio3` (_Cmplx __z) const

Revisit these numbers of terms for the Airy asymptotic expansions.

Member `std::__detail::__Airy_series<_Tp>::__S_Scorer` (_Cmplx __t)

Find out what is wrong with the $H_i = f_{ai} + g_{ai} + h_{ai}$ scorer function.

Chapter 2

Module Index

2.1 Modules

Here is a list of all modules:

Mathematical Special Functions	218
C++17/IS29124 Mathematical Special Functions	13
GNU Extended Mathematical Special Functions	47

Chapter 3

Namespace Index

3.1 Namespace List

Here is a list of all namespaces with brief descriptions:

__gnu_cxx	225
std	244
std::__detail	
Implementation-space details	246

Chapter 4

Hierarchical Index

4.1 Class Hierarchy

This inheritance list is sorted roughly, but not completely, alphabetically:

__gnu_cxx::__airy_t<_Tx, _Tp>	465
__gnu_cxx::__assoc_legendre_p_t<_Tp>	467
__gnu_cxx::__assoc_legendre_q_t<_Tp>	470
__gnu_cxx::__chebyshev_t_t<_Tp>	473
__gnu_cxx::__chebyshev_u_t<_Tp>	475
__gnu_cxx::__chebyshev_v_t<_Tp>	477
__gnu_cxx::__chebyshev_w_t<_Tp>	479
__gnu_cxx::__coulomb_t<_Teta, _Trho, _Tp>	481
__gnu_cxx::__cyl_bessel_t<_Tnu, _Tx, _Tp>	484
__gnu_cxx::__cyl_hankel_t<_Tnu, _Tx, _Tp>	486
__gnu_cxx::__cyl_mod_bessel_t<_Tnu, _Tx, _Tp>	489
__gnu_cxx::__fock_airy_t<_Tx, _Tp>	491
__gnu_cxx::__fp_is_integer_t	493
__gnu_cxx::__gamma_inc_t<_Tp>	495
__gnu_cxx::__gamma_temme_t<_Tp>	496
__gnu_cxx::__gappa_pq_t<_Tp>	498
__gnu_cxx::__gegenbauer_t<_Tp>	499
__gnu_cxx::__hermite_he_t<_Tp>	502
__gnu_cxx::__hermite_t<_Tp>	504
__gnu_cxx::__jacobi_ellint_t<_Tp>	506
__gnu_cxx::__jacobi_t<_Tp>	510
__gnu_cxx::__laguerre_t<_Tpa, _Tp>	513
__gnu_cxx::__legendre_p_t<_Tp>	515
__gnu_cxx::__legendre_q_t<_Tp>	517
__gnu_cxx::__lgamma_t<_Tp>	519
__gnu_cxx::__quadrature_point_t<_Tp>	521
__gnu_cxx::__sincos_t<_Tp>	522
__gnu_cxx::__sph_bessel_t<_Tn, _Tx, _Tp>	523
__gnu_cxx::__sph_hankel_t<_Tn, _Tx, _Tp>	526
__gnu_cxx::__sph_mod_bessel_t<_Tn, _Tx, _Tp>	529
__gnu_cxx::__stirling_1_t<_Tp>	531

<code>__gnu_cxx::__stirling_2_t<_Tp></code>	534
<code>std::__detail::__jacobi_lattice_t<_Tp1,_Tp3></code>	544
<code>std::__detail::__gamma_lanczos_data<_Tp></code>	537
<code>std::__detail::__gamma_lanczos_data<double></code>	537
<code>std::__detail::__gamma_lanczos_data<float></code>	538
<code>std::__detail::__gamma_lanczos_data<long double></code>	539
<code>std::__detail::__gamma_spouge_data<_Tp></code>	541
<code>std::__detail::__gamma_spouge_data<double></code>	541
<code>std::__detail::__gamma_spouge_data<float></code>	542
<code>std::__detail::__gamma_spouge_data<long double></code>	543
<code>std::__detail::__jacobi_lattice_t<_Tp_Omega1,_Tp_Omega3></code>	544
<code>std::__detail::__jacobi_lattice_t<_Tp_Omega1,_Tp_Omega3>::__arg_t</code>	551
<code>std::__detail::__jacobi_lattice_t<_Tp_Omega1,_Tp_Omega3>::__tau_t</code>	552
<code>std::__detail::__jacobi_theta_0_t<_Tp1,_Tp3></code>	554
<code>std::__detail::__weierstrass_invariants_t<_Tp1,_Tp3></code>	559
<code>std::__detail::__weierstrass_roots_t<_Tp1,_Tp3></code>	562
<code>std::__detail::__Airy<_Tp></code>	566
<code>std::__detail::__Airy_asymp_data<_Tp></code>	572
<code>std::__detail::__Airy_asymp<_Tp></code>	568
<code>std::__detail::__Airy_asymp_data<double></code>	573
<code>std::__detail::__Airy_asymp_data<float></code>	574
<code>std::__detail::__Airy_asymp_data<long double></code>	575
<code>std::__detail::__Airy_asymp_series<_Sum></code>	576
<code>std::__detail::__Airy_default_radii<_Tp></code>	579
<code>std::__detail::__Airy_default_radii<double></code>	579
<code>std::__detail::__Airy_default_radii<float></code>	580
<code>std::__detail::__Airy_default_radii<long double></code>	581
<code>std::__detail::__Airy_series<_Tp></code>	581
<code>std::__detail::__AiryAuxilliaryState<_Tp></code>	590
<code>std::__detail::__AiryState<_Tp></code>	592
<code>std::__detail::__AsympTerminator<_Tp></code>	595
<code>std::__detail::__Factorial_table<_Tp></code>	597
<code>std::__detail::__Terminator<_Tp></code>	598

Chapter 5

Class Index

5.1 Class List

Here are the classes, structs, unions and interfaces with brief descriptions:

__gnu_cxx::__airy_t<_Tx, _Tp>	465
__gnu_cxx::__assoc_legendre_p_t<_Tp>	467
__gnu_cxx::__assoc_legendre_q_t<_Tp>	470
__gnu_cxx::__chebyshev_t_t<_Tp>	473
__gnu_cxx::__chebyshev_u_t<_Tp>	475
__gnu_cxx::__chebyshev_v_t<_Tp>	477
__gnu_cxx::__chebyshev_w_t<_Tp>	479
__gnu_cxx::__coulomb_t<_Teta, _Trho, _Tp>	481
__gnu_cxx::__cyl_bessel_t<_Tnu, _Tx, _Tp>	484
__gnu_cxx::__cyl_hankel_t<_Tnu, _Tx, _Tp>	486
__gnu_cxx::__cyl_mod_bessel_t<_Tnu, _Tx, _Tp>	489
__gnu_cxx::__fock_airy_t<_Tx, _Tp>	491
__gnu_cxx::__fp_is_integer_t	493
__gnu_cxx::__gamma_inc_t<_Tp>	495
__gnu_cxx::__gamma_temme_t<_Tp>	

A structure for the gamma functions required by the Temme series expansions of $N_\nu(x)$ and $K_\nu(x)$.

$$\Gamma_1 = \frac{1}{2\mu} \left[\frac{1}{\Gamma(1-\mu)} - \frac{1}{\Gamma(1+\mu)} \right]$$

and

$$\Gamma_2 = \frac{1}{2} \left[\frac{1}{\Gamma(1-\mu)} + \frac{1}{\Gamma(1+\mu)} \right]$$

where $-1/2 \leq \mu \leq 1/2$ is $\mu = \nu - N$ and N is the nearest integer to ν . The values of $\Gamma(1+\mu)$

and $\Gamma(1-\mu)$ are returned as well

__gnu_cxx::__gappa_pq_t<_Tp>	496
__gnu_cxx::__gegenbauer_t<_Tp>	499
__gnu_cxx::__hermite_he_t<_Tp>	502
__gnu_cxx::__hermite_t<_Tp>	504
__gnu_cxx::__jacobi_ellint_t<_Tp>	506
__gnu_cxx::__jacobi_t<_Tp>	510

__gnu_cxx::__laguerre_t<_Tpa, _Tp>	513
__gnu_cxx::__legendre_p_t<_Tp>	515
__gnu_cxx::__legendre_q_t<_Tp>	517
__gnu_cxx::__lgamma_t<_Tp>	519
__gnu_cxx::__quadrature_point_t<_Tp>	521
__gnu_cxx::__sincos_t<_Tp>	522
__gnu_cxx::__sph_bessel_t<_Tn, _Tx, _Tp>	523
__gnu_cxx::__sph_hankel_t<_Tn, _Tx, _Tp>	526
__gnu_cxx::__sph_mod_bessel_t<_Tn, _Tx, _Tp>	529
__gnu_cxx::__stirling_1_t<_Tp>	
A structure for Stirling numbers of the first kind	531
__gnu_cxx::__stirling_2_t<_Tp>	
A structure for Stirling numbers of the first kind	534
std::__detail::__gamma_lanczos_data<_Tp>	537
std::__detail::__gamma_lanczos_data<double>	537
std::__detail::__gamma_lanczos_data<float>	538
std::__detail::__gamma_lanczos_data<long double>	539
std::__detail::__gamma_spouge_data<_Tp>	541
std::__detail::__gamma_spouge_data<double>	541
std::__detail::__gamma_spouge_data<float>	542
std::__detail::__gamma_spouge_data<long double>	543
std::__detail::__jacobi_lattice_t<_Tp_Omega1, _Tp_Omega3>	544
std::__detail::__jacobi_lattice_t<_Tp_Omega1, _Tp_Omega3>::__arg_t	551
std::__detail::__jacobi_lattice_t<_Tp_Omega1, _Tp_Omega3>::__tau_t	552
std::__detail::__jacobi_theta_0_t<_Tp1, _Tp3>	554
std::__detail::__weierstrass_invariants_t<_Tp1, _Tp3>	559
std::__detail::__weierstrass_roots_t<_Tp1, _Tp3>	562
std::__detail::__Airy<_Tp>	566
std::__detail::__Airy_asymp<_Tp>	568
std::__detail::__Airy_asymp_data<_Tp>	572
std::__detail::__Airy_asymp_data<double>	573
std::__detail::__Airy_asymp_data<float>	574
std::__detail::__Airy_asymp_data<long double>	575
std::__detail::__Airy_asymp_series<_Sum>	576
std::__detail::__Airy_default_radii<_Tp>	579
std::__detail::__Airy_default_radii<double>	579
std::__detail::__Airy_default_radii<float>	580
std::__detail::__Airy_default_radii<long double>	581
std::__detail::__Airy_series<_Tp>	581
std::__detail::__AiryAuxilliaryState<_Tp>	590
std::__detail::__AiryState<_Tp>	592
std::__detail::__AsympTerminator<_Tp>	595
std::__detail::__Factorial_table<_Tp>	597
std::__detail::__Terminator<_Tp>	598

Chapter 6

File Index

6.1 File List

Here is a list of all files with brief descriptions:

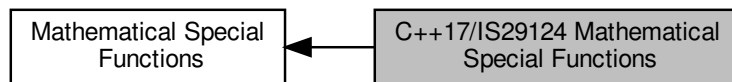
cxx_fp_utils/include/ext/math_util.h	601
include/bits/sf_airy.tcc	603
include/bits/sf_bernoulli.tcc	605
include/bits/sf_bessel.tcc	606
include/bits/sf_beta.tcc	609
include/bits/sf_cardinal.tcc	611
include/bits/sf_chebyshev.tcc	613
include/bits/sf_coulomb.tcc	614
include/bits/sf_dawson.tcc	616
include/bits/sf_distributions.tcc	618
include/bits/sf_ellint.tcc	621
include/bits/sf_euler.tcc	624
include/bits/sf_expint.tcc	626
include/bits/sf_fresnel.tcc	628
include/bits/sf_gamma.tcc	630
include/bits/sf_gegenbauer.tcc	638
include/bits/sf_hankel.tcc	639
include/bits/sf_hermite.tcc	642
include/bits/sf_hyperg.tcc	644
include/bits/sf_hypint.tcc	646
include/bits/sf_jacobi.tcc	648
include/bits/sf_laguerre.tcc	650
include/bits/sf_legendre.tcc	652
include/bits/sf_mod_bessel.tcc	654
include/bits/sf_owens_t.tcc	656
include/bits/sf_polylog.tcc	657
include/bits/sf_stirling.tcc	660
include/bits/sf_theta.tcc	662
include/bits/sf_trig.tcc	665
include/bits/sf_trigint.tcc	667
include/bits/sf_zeta.tcc	669
include/bits/specfun.h	671
include/bits/specfun_state.h	688

Chapter 7

Module Documentation

7.1 C++17/IS29124 Mathematical Special Functions

Collaboration diagram for C++17/IS29124 Mathematical Special Functions:



Functions

- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > std::assoc_laguerre (unsigned int __n, unsigned int __m, _Tp __x)`
- `template<typename _Talpha, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Talpha, _Tp > std::assoc_laguerre (unsigned int __n, _Talpha __alpha1, _Tp __x)`
- `float std::assoc_laguerref (unsigned int __n, unsigned int __m, float __x)`
- `long double std::assoc_laguerrel (unsigned int __n, unsigned int __m, long double __x)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > std::assoc_legendre (unsigned int __l, unsigned int __m, _Tp __x)`
- `float std::assoc_legendref (unsigned int __l, unsigned int __m, float __x)`
- `long double std::assoc_legendrel (unsigned int __l, unsigned int __m, long double __x)`
- `template<typename _Tpa, typename _Tpb >`
`__gnu_cxx::fp_promote_t< _Tpa, _Tpb > std::beta (_Tpa __a, _Tpb __b)`
- `float std::betaf (float __a, float __b)`
- `long double std::betal (long double __a, long double __b)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > std::comp_ellint_1 (_Tp __k)`

- float [std::comp_ellint_1f](#) (float __k)
- long double [std::comp_ellint_1l](#) (long double __k)
- template<typename _Tp >
__gnu_cxx::fp_promote_t< _Tp > [std::comp_ellint_2](#) (_Tp __k)
- float [std::comp_ellint_2f](#) (float __k)
- long double [std::comp_ellint_2l](#) (long double __k)
- template<typename _Tp, typename _Tpn >
__gnu_cxx::fp_promote_t< _Tp, _Tpn > [std::comp_ellint_3](#) (_Tp __k, _Tpn __nu)
- float [std::comp_ellint_3f](#) (float __k, float __nu)
- Return the complete elliptic integral of the third kind $\Pi(k, \nu)$ for float modulus k.*
- long double [std::comp_ellint_3l](#) (long double __k, long double __nu)
- Return the complete elliptic integral of the third kind $\Pi(k, \nu)$ for long double modulus k.*
- template<typename _Tpnu, typename _Tp >
__gnu_cxx::fp_promote_t< _Tpnu, _Tp > [std::cyl_bessel_i](#) (_Tpnu __nu, _Tp __x)
- float [std::cyl_bessel_if](#) (float __nu, float __x)
- long double [std::cyl_bessel_il](#) (long double __nu, long double __x)
- template<typename _Tpnu, typename _Tp >
__gnu_cxx::fp_promote_t< _Tpnu, _Tp > [std::cyl_bessel_j](#) (_Tpnu __nu, _Tp __x)
- float [std::cyl_bessel_jf](#) (float __nu, float __x)
- long double [std::cyl_bessel_jl](#) (long double __nu, long double __x)
- template<typename _Tpnu, typename _Tp >
__gnu_cxx::fp_promote_t< _Tpnu, _Tp > [std::cyl_bessel_k](#) (_Tpnu __nu, _Tp __x)
- float [std::cyl_bessel_kf](#) (float __nu, float __x)
- long double [std::cyl_bessel_kl](#) (long double __nu, long double __x)
- template<typename _Tpnu, typename _Tp >
__gnu_cxx::fp_promote_t< _Tpnu, _Tp > [std::cyl_neumann](#) (_Tpnu __nu, _Tp __x)
- float [std::cyl_neumannf](#) (float __nu, float __x)
- long double [std::cyl_neumannl](#) (long double __nu, long double __x)
- template<typename _Tp, typename _Tpp >
__gnu_cxx::fp_promote_t< _Tp, _Tpp > [std::ellint_1](#) (_Tp __k, _Tpp __phi)
- float [std::ellint_1f](#) (float __k, float __phi)
- long double [std::ellint_1l](#) (long double __k, long double __phi)
- template<typename _Tp, typename _Tpp >
__gnu_cxx::fp_promote_t< _Tp, _Tpp > [std::ellint_2](#) (_Tp __k, _Tpp __phi)
- float [std::ellint_2f](#) (float __k, float __phi)
- Return the incomplete elliptic integral of the second kind $E(k, \phi)$ for float argument.*
- long double [std::ellint_2l](#) (long double __k, long double __phi)
- Return the incomplete elliptic integral of the second kind $E(k, \phi)$.*
- template<typename _Tp, typename _Tpn, typename _Tpp >
__gnu_cxx::fp_promote_t< _Tp, _Tpn, _Tpp > [std::ellint_3](#) (_Tp __k, _Tpn __nu, _Tpp __phi)
- Return the incomplete elliptic integral of the third kind $\Pi(k, \nu, \phi)$.*
- float [std::ellint_3f](#) (float __k, float __nu, float __phi)
- Return the incomplete elliptic integral of the third kind $\Pi(k, \nu, \phi)$ for float argument.*
- long double [std::ellint_3l](#) (long double __k, long double __nu, long double __phi)
- Return the incomplete elliptic integral of the third kind $\Pi(k, \nu, \phi)$.*
- template<typename _Tp >
__gnu_cxx::fp_promote_t< _Tp > [std::expint](#) (_Tp __x)
- float [std::expintf](#) (float __x)
- long double [std::expintl](#) (long double __x)

- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > std::hermite (unsigned int __n, _Tp __x)`
- `float std::hermitef (unsigned int __n, float __x)`
- `long double std::hermitel (unsigned int __n, long double __x)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > std::laguerre (unsigned int __n, _Tp __x)`
- `float std::laguerref (unsigned int __n, float __x)`
- `long double std::laguerrel (unsigned int __n, long double __x)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > std::legendre (unsigned int __l, _Tp __x)`
- `float std::legendref (unsigned int __l, float __x)`
- `long double std::legendrel (unsigned int __l, long double __x)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > std::riemann_zeta (_Tp __s)`
- `float std::riemann_zetaf (float __s)`
- `long double std::riemann_zetal (long double __s)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > std::sph_bessel (unsigned int __n, _Tp __x)`
- `float std::sph_besself (unsigned int __n, float __x)`
- `long double std::sph_bessell (unsigned int __n, long double __x)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > std::sph_legendre (unsigned int __l, unsigned int __m, _Tp __theta)`
- `float std::sph_legendref (unsigned int __l, unsigned int __m, float __theta)`
- `long double std::sph_legendrel (unsigned int __l, unsigned int __m, long double __theta)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > std::sph_neumann (unsigned int __n, _Tp __x)`
- `float std::sph_neumannf (unsigned int __n, float __x)`
- `long double std::sph_neumannl (unsigned int __n, long double __x)`

7.1.1 Detailed Description

A collection of advanced mathematical special functions for C++17 and IS29124.

7.1.2 Function Documentation

7.1.2.1 `assoc_laguerre()` [1/2]

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> std::assoc_laguerre (
    unsigned int __n,
    unsigned int __m,
    _Tp __x ) [inline]
```

Return the associated Laguerre polynomial $L_n^{(m)}(x)$ of nonnegative degree n , nonnegative order m and real argument x .

The associated Laguerre function of real order α , $L_n^{(\alpha)}(x)$, is defined by

$$L_n^{(\alpha)}(x) = \frac{(\alpha+1)_n}{n!} {}_1F_1(-n; \alpha+1; x)$$

where $(\alpha)_n$ is the Pochhammer symbol and ${}_1F_1(a; c; x)$ is the confluent hypergeometric function.

The associated Laguerre polynomial is defined for integral order $\alpha = m$ by:

$$L_n^{(m)}(x) = (-1)^m \frac{d^m}{dx^m} L_{n+m}(x)$$

where the Laguerre polynomial is defined by:

$$L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n} (x^n e^{-x})$$

and $x \geq 0$.

See also

[laguerre](#) for details of the Laguerre function of degree n

Template Parameters

<code>_Tp</code>	The floating-point type of the argument <code>__x</code> .
------------------	--

Parameters

<code>__n</code>	The degree of the Laguerre function, <code>__n</code> ≥ 0 .
<code>__m</code>	The order of the Laguerre function, <code>__m</code> ≥ 0 .
<code>__x</code>	The argument of the Laguerre function, <code>__x</code> ≥ 0 .

Exceptions

<code>std::domain_error</code>	if <code>__x</code> < 0 .
--------------------------------	-----------------------------

Definition at line 475 of file specfun.h.

7.1.2.2 `assoc_laguerre()` [2/2]

```
template<typename _Talpha, typename _Tp>
__gnu_cxx::fp_promote_t<_Talpha, _Tp> std::assoc_laguerre (
    unsigned int __n,
```

```
_Talpha __alpha1,  
_Tp __x )    [inline]
```

Return the associated Laguerre polynomial $L_n^{(\alpha)}(x)$ of nonnegative degree n , order α and real argument x .

Template Parameters

<code>__Talpha</code>	The (signed integer or floating-point) type of the degree <code>__alpha1</code> .
<code>__Tp</code>	The floating-point type of the argument <code>__x</code> .

Definition at line 492 of file specfun.h.

7.1.2.3 `assoc_laguerref()`

```
float std::assoc_laguerref (
    unsigned int __n,
    unsigned int __m,
    float __x ) [inline]
```

Return the associated Laguerre polynomial $L_n^{(m)}(x)$ of order `n`, degree `m`, and `float` argument `x`.

See also

[assoc_laguerre](#) for more details.

Definition at line 427 of file specfun.h.

7.1.2.4 `assoc_laguerrel()`

```
long double std::assoc_laguerrel (
    unsigned int __n,
    unsigned int __m,
    long double __x ) [inline]
```

Return the associated Laguerre polynomial $L_n^{(m)}(x)$ of order `n`, degree `m` and `long double` argument `x`.

See also

[assoc_laguerre](#) for more details.

Definition at line 438 of file specfun.h.

7.1.2.5 `assoc_legendre()`

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> std::assoc_legendre (
    unsigned int __l,
    unsigned int __m,
    _Tp __x ) [inline]
```

Return the associated Legendre function $P_l^m(x)$ of degree l , order m , and real argument x .

The associated Legendre function is derived from the Legendre function $P_l(x)$ by the Rodrigues formula:

$$P_l^m(x) = (1 - x^2)^{m/2} \frac{d^m}{dx^m} P_l(x)$$

See also

[legendre](#) for details of the Legendre function of degree l

Note

$$P_l^m(x) = 0 \text{ if } m > l.$$

Template Parameters

<code>_Tp</code>	The floating-point type of the argument <code>__x</code> .
------------------	--

Parameters

<code>__l</code>	The degree <code>__l</code> ≥ 0 .
<code>__m</code>	The order <code>__m</code> .
<code>__x</code>	The argument, <code>abs (__x)</code> ≤ 1 .

Exceptions

<code>std::domain_error</code>	if <code>abs (__x)</code> > 1 .
--------------------------------	-----------------------------------

Definition at line 541 of file `specfun.h`.

7.1.2.6 `assoc_legendref()`

```
float std::assoc_legendref (
    unsigned int __l,
```

```
unsigned int __m,
float __x ) [inline]
```

Return the associated Legendre function $P_l^m(x)$ of degree `l`, order `m`, and `float` argument `x`.

See also

[assoc_legendre](#) for more details.

Definition at line 507 of file specfun.h.

7.1.2.7 assoc_legendrel()

```
long double std::assoc_legendrel (
    unsigned int __l,
    unsigned int __m,
    long double __x ) [inline]
```

Return the associated Legendre function $P_l^m(x)$ of degree `l`, order `m`, and `long double` argument `x`.

See also

[assoc_legendre](#) for more details.

Definition at line 518 of file specfun.h.

7.1.2.8 beta()

```
template<typename _Tpa , typename _Tpb >
__gnu_cxx::fp_promote_t<_Tpa, _Tpb> std::beta (
    _Tpa __a,
    _Tpb __b ) [inline]
```

Return the beta function, $B(a, b)$, for real parameters `a`, `b`.

The beta function is defined by

$$B(a, b) = \int_0^1 t^{a-1} (1-t)^{b-1} dt = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

where $a > 0$ and $b > 0$

Template Parameters

<code>_Tpa</code>	The floating-point type of the parameter <code>__a</code> .
<code>_Tpb</code>	The floating-point type of the parameter <code>__b</code> .

Parameters

\leftrightarrow <code>__a</code>	The first argument of the beta function, <code>__a > 0</code> .
\leftrightarrow <code>__b</code>	The second argument of the beta function, <code>__b > 0</code> .

Exceptions

<code>std::domain_error</code>	if <code>__a < 0</code> or <code>__b < 0</code> .
--------------------------------	---

Definition at line 588 of file `specfun.h`.

7.1.2.9 `betaf()`

```
float std::betaf (
    float __a,
    float __b ) [inline]
```

Return the beta function, $B(a, b)$, for `float` parameters `a`, `b`.

See also

[beta](#) for more details.

Definition at line 556 of file `specfun.h`.

7.1.2.10 `betal()`

```
long double std::betal (
    long double __a,
    long double __b ) [inline]
```

Return the beta function, $B(a, b)$, for long double parameters `a`, `b`.

See also

[beta](#) for more details.

Definition at line 566 of file `specfun.h`.

7.1.2.11 `comp_ellint_1()`

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> std::comp_ellint_1 (
    _Tp __k ) [inline]
```

Return the complete elliptic integral of the first kind $K(k)$ for real modulus k .

The complete elliptic integral of the first kind is defined as

$$K(k) = F(k, \pi/2) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - k^2 \sin^2 \theta}}$$

where $F(k, \phi)$ is the incomplete elliptic integral of the first kind and the modulus $|k| \leq 1$.

See also

[ellint_1](#) for details of the incomplete elliptic function of the first kind.

Template Parameters

<code>_Tp</code>	The floating-point type of the modulus <code>__k</code> .
------------------	---

Parameters

<code>__k</code>	The modulus, <code>abs (__k) <= 1</code>
------------------	---

Exceptions

<code>std::domain_error</code>	if <code>abs (__k) > 1</code> .
--------------------------------	------------------------------------

Definition at line 636 of file `specfun.h`.

7.1.2.12 `comp_ellint_1f()`

```
float std::comp_ellint_1f (
    float __k ) [inline]
```

Return the complete elliptic integral of the first kind $E(k)$ for `float` modulus k .

See also

[comp_ellint_1](#) for details.

Definition at line 603 of file `specfun.h`.

7.1.2.13 `comp_ellint_1l()`

```
long double std::comp_ellint_1l (
    long double __k ) [inline]
```

Return the complete elliptic integral of the first kind $E(k)$ for `long double` modulus `k`.

See also

[comp_ellint_1](#) for details.

Definition at line 613 of file `specfun.h`.

7.1.2.14 `comp_ellint_2()`

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> std::comp_ellint_2 (
    _Tp __k ) [inline]
```

Return the complete elliptic integral of the second kind $E(k)$ for real modulus `k`.

The complete elliptic integral of the second kind is defined as

$$E(k) = E(k, \pi/2) = \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 \theta} d\theta$$

where $E(k, \phi)$ is the incomplete elliptic integral of the second kind and the modulus $|k| \leq 1$.

See also

[ellint_2](#) for details of the incomplete elliptic function of the second kind.

Template Parameters

<code>_Tp</code>	The floating-point type of the modulus <code>__k</code> .
------------------	---

Parameters

<code>__k</code>	The modulus, <code>abs (__k) <= 1</code>
------------------	---

Exceptions

<code>std::domain_error</code>	if <code>abs (__k) > 1</code> .
--------------------------------	------------------------------------

Definition at line 683 of file specfun.h.

7.1.2.15 comp_ellint_2f()

```
float std::comp_ellint_2f (
    float __k ) [inline]
```

Return the complete elliptic integral of the second kind $E(k)$ for `float` modulus `k`.

See also

[comp_ellint_2](#) for details.

Definition at line 651 of file specfun.h.

7.1.2.16 comp_ellint_2l()

```
long double std::comp_ellint_2l (
    long double __k ) [inline]
```

Return the complete elliptic integral of the second kind $E(k)$ for `long double` modulus `k`.

See also

[comp_ellint_2](#) for details.

Definition at line 661 of file specfun.h.

7.1.2.17 comp_ellint_3()

```
template<typename _Tp , typename _Tpn >
__gnu_cxx::fp_promote_t<_Tp, _Tpn> std::comp_ellint_3 (
    _Tp __k,
    _Tpn __nu ) [inline]
```

Return the complete elliptic integral of the third kind $\Pi(k, \nu) = \Pi(k, \nu, \pi/2)$ for real modulus `k`.

The complete elliptic integral of the third kind is defined as

$$\Pi(k, \nu) = \Pi(k, \nu, \pi/2) = \int_0^{\pi/2} \frac{d\theta}{(1 - \nu \sin^2 \theta) \sqrt{1 - k^2 \sin^2 \theta}}$$

where $\Pi(k, \nu, \phi)$ is the incomplete elliptic integral of the second kind and the modulus $|k| \leq 1$.

See also

[ellint_3](#) for details of the incomplete elliptic function of the third kind.

Template Parameters

<code>__Tp</code>	The floating-point type of the modulus <code>__k</code> .
<code>__Tpn</code>	The floating-point type of the argument <code>__nu</code> .

Parameters

<code>__k</code>	The modulus, <code>abs (__k) <= 1</code> .
<code>__nu</code>	The characteristic.

Exceptions

<code>std::domain_error</code>	if <code>abs (__k) > 1</code> .
--------------------------------	------------------------------------

Definition at line 734 of file `specfun.h`.

7.1.2.18 `comp_ellint_3f()`

```
float std::comp_ellint_3f (
    float __k,
    float __nu ) [inline]
```

Return the complete elliptic integral of the third kind $\Pi(k, \nu)$ for `float` modulus `k`.

See also

[comp_ellint_3](#) for details.

Definition at line 698 of file `specfun.h`.

7.1.2.19 `comp_ellint_3l()`

```
long double std::comp_ellint_3l (
    long double __k,
    long double __nu ) [inline]
```

Return the complete elliptic integral of the third kind $\Pi(k, \nu)$ for `long double` modulus `k`.

See also

[comp_ellint_3](#) for details.

Definition at line 708 of file `specfun.h`.

7.1.2.20 `cyl_bessel_i()`

```
template<typename _Tpnu , typename _Tp >
__gnu_cxx::fp_promote_t<_Tpnu, _Tp> std::cyl_bessel_i (
    _Tpnu __nu,
    _Tp __x ) [inline]
```

Return the regular modified Bessel function $I_\nu(x)$ for real order ν and argument $x \geq 0$.

The regular modified cylindrical Bessel function is:

$$I_\nu(x) = i^{-\nu} J_\nu(ix) = \sum_{k=0}^{\infty} \frac{(x/2)^{\nu+2k}}{k! \Gamma(\nu + k + 1)}$$

Template Parameters

<code>_Tpnu</code>	The floating-point type of the order <code>__nu</code> .
<code>_Tp</code>	The floating-point type of the argument <code>__x</code> .

Parameters

<code>__nu</code>	The order
<code>__x</code>	The argument, <code>__x</code> ≥ 0

Exceptions

<code>std::domain_error</code>	if <code>__x</code> < 0 .
--------------------------------	-----------------------------

Definition at line 780 of file `specfun.h`.

7.1.2.21 `cyl_bessel_if()`

```
float std::cyl_bessel_if (
    float __nu,
    float __x ) [inline]
```

Return the regular modified Bessel function $I_\nu(x)$ for `float` order ν and argument $x \geq 0$.

See also

[cyl_bessel_i](#) for setails.

Definition at line 749 of file `specfun.h`.

7.1.2.22 `cyl_bessel_il()`

```
long double std::cyl_bessel_il (
    long double __nu,
    long double __x ) [inline]
```

Return the regular modified Bessel function $I_\nu(x)$ for `long double` order ν and argument $x \geq 0$.

See also

[cyl_bessel_i](#) for setails.

Definition at line 759 of file `specfun.h`.

7.1.2.23 `cyl_bessel_j()`

```
template<typename _Tpnu , typename _Tp >
__gnu_cxx::fp_promote_t<_Tpnu, _Tp> std::cyl_bessel_j (
    _Tpnu __nu,
    _Tp __x ) [inline]
```

Return the Bessel function $J_\nu(x)$ of real order ν and argument $x \geq 0$.

The cylindrical Bessel function is:

$$J_\nu(x) = \sum_{k=0}^{\infty} \frac{(-1)^k (x/2)^{\nu+2k}}{k! \Gamma(\nu + k + 1)}$$

Template Parameters

<code>_Tpnu</code>	The floating-point type of the order <code>__nu</code> .
<code>_Tp</code>	The floating-point type of the argument <code>__x</code> .

Parameters

<code>__nu</code>	The order
<code>__x</code>	The argument, <code>__x</code> ≥ 0

Exceptions

<code>std::domain_error</code>	if <code>__x</code> < 0 .
--------------------------------	-----------------------------

Definition at line 826 of file `specfun.h`.

7.1.2.24 `cyl_bessel_jf()`

```
float std::cyl_bessel_jf (
    float __nu,
    float __x ) [inline]
```

Return the Bessel function of the first kind $J_\nu(x)$ for `float` order ν and argument $x \geq 0$.

See also

[cyl_bessel_j](#) for setails.

Definition at line 795 of file `specfun.h`.

7.1.2.25 `cyl_bessel_jl()`

```
long double std::cyl_bessel_jl (
    long double __nu,
    long double __x ) [inline]
```

Return the Bessel function of the first kind $J_\nu(x)$ for `long double` order ν and argument $x \geq 0$.

See also

[cyl_bessel_j](#) for setails.

Definition at line 805 of file `specfun.h`.

7.1.2.26 `cyl_bessel_k()`

```
template<typename _Tpnu , typename _Tp >
__gnu_cxx::fp_promote_t<_Tpnu, _Tp> std::cyl_bessel_k (
    _Tpnu __nu,
    _Tp __x ) [inline]
```

Return the irregular modified Bessel function $K_\nu(x)$ of real order ν and argument x .

The irregular modified Bessel function is defined by:

$$K_\nu(x) = \frac{\pi}{2} \frac{I_{-\nu}(x) - I_\nu(x)}{\sin \nu\pi}$$

where for integral $\nu = n$ a limit is taken: $\lim_{\nu \rightarrow n}$. For negative argument we have simply:

$$K_{-\nu}(x) = K_\nu(x)$$

Template Parameters

<code>__Tpnu</code>	The floating-point type of the order <code>__nu</code> .
<code>__Tp</code>	The floating-point type of the argument <code>__x</code> .

Parameters

<code>__nu</code>	The order
<code>__x</code>	The argument, <code>__x >= 0</code>

Exceptions

<code>std::domain_error</code>	if <code>__x < 0</code> .
--------------------------------	------------------------------

Definition at line 878 of file `specfun.h`.

7.1.2.27 `cyl_bessel_kf()`

```
float std::cyl_bessel_kf (
    float __nu,
    float __x ) [inline]
```

Return the irregular modified Bessel function $K_\nu(x)$ for `float` order ν and argument $x \geq 0$.

See also

[cyl_bessel_k](#) for setails.

Definition at line 841 of file `specfun.h`.

7.1.2.28 `cyl_bessel_kl()`

```
long double std::cyl_bessel_kl (
    long double __nu,
    long double __x ) [inline]
```

Return the irregular modified Bessel function $K_\nu(x)$ for `long double` order ν and argument $x \geq 0$.

See also

[cyl_bessel_k](#) for setails.

Definition at line 851 of file `specfun.h`.

7.1.2.29 cyl_neumann()

```
template<typename _Tpnu , typename _Tp >
__gnu_cxx::fp_promote_t<_Tpnu, _Tp> std::cyl_neumann (
    _Tpnu __nu,
    _Tp __x ) [inline]
```

Return the Neumann function $N_\nu(x)$ of real order ν and argument $x \geq 0$.

The Neumann function is defined by:

$$N_\nu(x) = \frac{J_\nu(x) \cos \nu\pi - J_{-\nu}(x)}{\sin \nu\pi}$$

where $x \geq 0$ and for integral order $\nu = n$ a limit is taken: $\lim_{\nu \rightarrow n}$.

Template Parameters

<code>_Tpnu</code>	The floating-point type of the order <code>__nu</code> .
<code>_Tp</code>	The floating-point type of the argument <code>__x</code> .

Parameters

<code>__nu</code>	The order.
<code>__x</code>	The argument, <code>__x</code> ≥ 0 .

Exceptions

<code>std::domain_error</code>	if <code>__x</code> < 0 .
--------------------------------	-----------------------------

Definition at line 926 of file specfun.h.

7.1.2.30 cyl_neumannf()

```
float std::cyl_neumannf (
    float __nu,
    float __x ) [inline]
```

Return the Neumann function $N_\nu(x)$ of `float` order ν and argument `x`.

See also

[cyl_neumann](#) for setails.

Definition at line 893 of file specfun.h.

7.1.2.31 `cyl_neumannl()`

```
long double std::cyl_neumannl (
    long double __nu,
    long double __x ) [inline]
```

Return the Neumann function $N_\nu(x)$ of `long double` order ν and argument x .

See also

[cyl_neumann](#) for setails.

Definition at line 903 of file `specfun.h`.

7.1.2.32 `ellint_1()`

```
template<typename _Tp , typename _Tpp >
__gnu_cxx::fp_promote_t<_Tp, _Tpp> std::ellint_1 (
    _Tp __k,
    _Tpp __phi ) [inline]
```

Return the incomplete elliptic integral of the first kind $F(k, \phi)$ for `real` modulus k and angle ϕ .

The incomplete elliptic integral of the first kind is defined as

$$F(k, \phi) = \int_0^\phi \frac{d\theta}{\sqrt{1 - k^2 \sin^2 \theta}}$$

For $\phi = \pi/2$ this becomes the complete elliptic integral of the first kind, $K(k)$.

See also

[comp_ellint_1](#).

Template Parameters

<code>_Tp</code>	The floating-point type of the modulus <code>__k</code> .
<code>_Tpp</code>	The floating-point type of the angle <code>__phi</code> .

Parameters

<code>__k</code>	The modulus, <code>abs (__k) <= 1</code> .
<code>__phi</code>	The integral limit argument in radians.

Exceptions

<code>std::domain_error</code>	if <code>abs (__k) > 1</code> .
--------------------------------	------------------------------------

Definition at line 974 of file `specfun.h`.

7.1.2.33 `ellint_1f()`

```
float std::ellint_1f (
    float __k,
    float __phi ) [inline]
```

Return the incomplete elliptic integral of the first kind $E(k, \phi)$ for `float` modulus `k` and angle ϕ .

See also

[ellint_1](#) for details.

Definition at line 941 of file `specfun.h`.

7.1.2.34 `ellint_1l()`

```
long double std::ellint_1l (
    long double __k,
    long double __phi ) [inline]
```

Return the incomplete elliptic integral of the first kind $E(k, \phi)$ for `long double` modulus `k` and angle ϕ .

See also

[ellint_1](#) for details.

Definition at line 951 of file `specfun.h`.

7.1.2.35 `ellint_2()`

```
template<typename _Tp , typename _Tpp >
__gnu_cxx::fp_promote_t<_Tp, _Tpp> std::ellint_2 (
    _Tp __k,
    _Tpp __phi ) [inline]
```

Return the incomplete elliptic integral of the second kind $E(k, \phi)$.

The incomplete elliptic integral of the second kind is defined as

$$E(k, \phi) = \int_0^\phi \sqrt{1 - k^2 \sin^2 \theta}$$

For $\phi = \pi/2$ this becomes the complete elliptic integral of the second kind, $E(k)$.

See also

[comp_ellint_2](#).

Template Parameters

<code>__Tp</code>	The floating-point type of the modulus <code>__k</code> .
<code>__Tpp</code>	The floating-point type of the angle <code>__phi</code> .

Parameters

<code>__k</code>	The modulus, <code>abs (__k) <= 1</code>
<code>__phi</code>	The integral limit argument in radians

Returns

The elliptic function of the second kind.

Exceptions

<code>std::domain_error</code>	if <code>abs (__k) > 1</code> .
--------------------------------	------------------------------------

Definition at line 1022 of file `specfun.h`.

7.1.2.36 `ellint_2f()`

```
float std::ellint_2f (
    float __k,
    float __phi ) [inline]
```

Return the incomplete elliptic integral of the second kind $E(k, \phi)$ for `float` argument.

See also

[ellint_2](#) for details.

Definition at line 989 of file `specfun.h`.

7.1.2.37 `ellint_2l()`

```
long double std::ellint_2l (
    long double __k,
    long double __phi ) [inline]
```

Return the incomplete elliptic integral of the second kind $E(k, \phi)$.

See also

[ellint_2](#) for details.

Definition at line 999 of file `specfun.h`.

7.1.2.38 `ellint_3()`

```
template<typename _Tp , typename _Tpn , typename _Tpp >
__gnu_cxx::fp_promote_t<_Tp, _Tpn, _Tpp> std::ellint_3 (
    _Tp __k,
    _Tpn __nu,
    _Tpp __phi ) [inline]
```

Return the incomplete elliptic integral of the third kind $\Pi(k, \nu, \phi)$.

The incomplete elliptic integral of the third kind is defined by:

$$\Pi(k, \nu, \phi) = \int_0^\phi \frac{d\theta}{(1 - \nu \sin^2 \theta) \sqrt{1 - k^2 \sin^2 \theta}}$$

For $\phi = \pi/2$ this becomes the complete elliptic integral of the third kind, $\Pi(k, \nu)$.

See also

[comp_ellint_3](#).

Template Parameters

<code>_Tp</code>	The floating-point type of the modulus <code>__k</code> .
<code>_Tpn</code>	The floating-point type of the argument <code>__nu</code> .
<code>_Tpp</code>	The floating-point type of the angle <code>__phi</code> .

Parameters

<code>__k</code>	The modulus, <code>abs (__k) <= 1</code> .
<code>__nu</code>	The characteristic.
<code>__phi</code>	The integral limit argument in radians.

Returns

The elliptic function of the third kind.

Exceptions

<code>std::domain_error</code>	if <code>abs (__k) > 1</code> .
--------------------------------	------------------------------------

Definition at line 1075 of file `specfun.h`.

7.1.2.39 `ellint_3f()`

```
float std::ellint_3f (
    float __k,
    float __nu,
    float __phi ) [inline]
```

Return the incomplete elliptic integral of the third kind $\Pi(k, \nu, \phi)$ for `float` argument.

See also

[ellint_3](#) for details.

Definition at line 1037 of file `specfun.h`.

7.1.2.40 `ellint_3l()`

```
long double std::ellint_3l (
    long double __k,
    long double __nu,
    long double __phi ) [inline]
```

Return the incomplete elliptic integral of the third kind $\Pi(k, \nu, \phi)$.

See also

[ellint_3](#) for details.

Definition at line 1047 of file `specfun.h`.

7.1.2.41 `expint()`

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> std::expint (
    _Tp __x ) [inline]
```

Return the exponential integral $Ei(x)$ for `real` argument `x`.

The exponential integral is given by

$$Ei(x) = - \int_{-x}^{\infty} \frac{e^t}{t} dt$$

Template Parameters

<code>__Tp</code>	The floating-point type of the argument <code>__x</code> .
-------------------	--

Parameters

<code>__x</code>	The argument of the exponential integral function.
------------------	--

Definition at line 1115 of file `specfun.h`.

7.1.2.42 `expintf()`

```
float std::expintf (
    float __x ) [inline]
```

Return the exponential integral $Ei(x)$ for `float` argument `x`.

See also

[expint](#) for details.

Definition at line 1089 of file `specfun.h`.

7.1.2.43 `expintl()`

```
long double std::expintl (
    long double __x ) [inline]
```

Return the exponential integral $Ei(x)$ for `long double` argument `x`.

See also

[expint](#) for details.

Definition at line 1099 of file `specfun.h`.

7.1.2.44 `hermite()`

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> std::hermite (
    unsigned int __n,
    _Tp __x ) [inline]
```

Return the Hermite polynomial $H_n(x)$ of order `n` and `real` argument `x`.

The Hermite polynomial is defined by:

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2}$$

The Hermite polynomial obeys a reflection formula:

$$H_n(-x) = (-1)^n H_n(x)$$

Template Parameters

<code>_Tp</code>	The floating-point type of the argument <code>__x</code> .
------------------	--

Parameters

<code>__n</code>	The order
<code>__x</code>	The argument

Definition at line 1163 of file `specfun.h`.

7.1.2.45 `hermitef()`

```
float std::hermitef (
    unsigned int __n,
    float __x ) [inline]
```

Return the Hermite polynomial $H_n(x)$ of nonnegative order `n` and `float` argument `x`.

See also

[hermite](#) for details.

Definition at line 1130 of file `specfun.h`.

7.1.2.46 `hermitel()`

```
long double std::hermitel (
    unsigned int __n,
    long double __x ) [inline]
```

Return the Hermite polynomial $H_n(x)$ of nonnegative order n and `long double` argument x .

See also

[hermite](#) for details.

Definition at line 1140 of file `specfun.h`.

7.1.2.47 `laguerre()`

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> std::laguerre (
    unsigned int __n,
    _Tp __x ) [inline]
```

Returns the Laguerre polynomial $L_n(x)$ of nonnegative degree n and real argument $x \geq 0$.

The Laguerre polynomial is defined by:

$$L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n} (x^n e^{-x})$$

Template Parameters

<code>_Tp</code>	The floating-point type of the argument <code>__x</code> .
------------------	--

Parameters

<code>__n</code>	The nonnegative order
<code>__x</code>	The argument <code>__x</code> ≥ 0

Exceptions

<code>std::domain_error</code>	if <code>__x</code> < 0 .
--------------------------------	-----------------------------

Definition at line 1207 of file `specfun.h`.

7.1.2.48 `laguerref()`

```
float std::laguerref (
    unsigned int __n,
    float __x ) [inline]
```

Returns the Laguerre polynomial $L_n(x)$ of nonnegative degree `n` and `float` argument $x \geq 0$.

See also

[laguerre](#) for more details.

Definition at line 1178 of file `specfun.h`.

7.1.2.49 `laguerrel()`

```
long double std::laguerrel (
    unsigned int __n,
    long double __x ) [inline]
```

Returns the Laguerre polynomial $L_n(x)$ of nonnegative degree `n` and `long double` argument $x \geq 0$.

See also

[laguerre](#) for more details.

Definition at line 1188 of file `specfun.h`.

7.1.2.50 `legendre()`

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> std::legendre (
    unsigned int __l,
    _Tp __x ) [inline]
```

Return the Legendre polynomial $P_l(x)$ of nonnegative degree `l` and real argument $|x| \leq 1$.

The Legendre function of order `l` and argument x , $P_l(x)$, is defined by:

$$P_l(x) = \frac{1}{2^l l!} \frac{d^l}{dx^l} (x^2 - 1)^l$$

Template Parameters

<code>__Tp</code>	The floating-point type of the argument <code>__x</code> .
-------------------	--

Parameters

<code>__l</code>	The degree $l \geq 0$
<code>__x</code>	The argument $\text{abs}(\text{__x}) \leq 1$

Exceptions

<code>std::domain_error</code>	if $\text{abs}(\text{__x}) > 1$
--------------------------------	---------------------------------

Definition at line 1252 of file `specfun.h`.

7.1.2.51 `legendref()`

```
float std::legendref (
    unsigned int __l,
    float __x ) [inline]
```

Return the Legendre polynomial $P_l(x)$ of nonnegative degree `l` and `float` argument $|x| \leq 0$.

See also

[legendre](#) for more details.

Definition at line 1222 of file `specfun.h`.

7.1.2.52 `legendrel()`

```
long double std::legendrel (
    unsigned int __l,
    long double __x ) [inline]
```

Return the Legendre polynomial $P_l(x)$ of nonnegative degree `l` and `long double` argument $|x| \leq 0$.

See also

[legendre](#) for more details.

Definition at line 1232 of file `specfun.h`.

7.1.2.53 `riemann_zeta()`

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> std::riemann_zeta (
    _Tp __s ) [inline]
```

Return the Riemann zeta function $\zeta(s)$ for real argument s .

The Riemann zeta function is defined by:

$$\zeta(s) = \sum_{k=1}^{\infty} k^{-s} \text{ for } s > 1$$

and

$$\zeta(s) = \frac{1}{1-2^{1-s}} \sum_{k=1}^{\infty} (-1)^{k-1} k^{-s} \text{ for } 0 <= s < 1$$

For $s < 1$ use the reflection formula:

$$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s) \zeta(1-s)$$

Template Parameters

<code>_Tp</code>	The floating-point type of the argument <code>__s</code> .
------------------	--

Parameters

<code>__s</code>	The order $s \neq 1$
------------------	----------------------

Definition at line 1303 of file `specfun.h`.

7.1.2.54 `riemann_zetaf()`

```
float std::riemann_zetaf (
    float __s ) [inline]
```

Return the Riemann zeta function $\zeta(s)$ for `float` argument s .

See also

[riemann_zeta](#) for more details.

Definition at line 1267 of file `specfun.h`.

7.1.2.55 `riemann_zetal()`

```
long double std::riemann_zetal (
    long double __s ) [inline]
```

Return the Riemann zeta function $\zeta(s)$ for `long double` argument `s`.

See also

[riemann_zeta](#) for more details.

Definition at line 1277 of file `specfun.h`.

7.1.2.56 `sph_bessel()`

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> std::sph_bessel (
    unsigned int __n,
    _Tp __x ) [inline]
```

Return the spherical Bessel function $j_n(x)$ of nonnegative order `n` and real argument $x \geq 0$.

The spherical Bessel function is defined by:

$$j_n(x) = \left(\frac{\pi}{2x}\right)^{1/2} J_{n+1/2}(x)$$

Template Parameters

<code>_Tp</code>	The floating-point type of the argument <code>__x</code> .
------------------	--

Parameters

<code>__n</code>	The integral order $n \geq 0$
<code>__x</code>	The real argument $x \geq 0$

Exceptions

<code>std::domain_error</code>	if <code>__x < 0</code> .
--------------------------------	------------------------------

Definition at line 1347 of file `specfun.h`.

7.1.2.57 sph_besself()

```
float std::sph_besself (
    unsigned int __n,
    float __x ) [inline]
```

Return the spherical Bessel function $j_n(x)$ of nonnegative order n and `float` argument $x \geq 0$.

See also

[sph_bessel](#) for more details.

Definition at line 1318 of file specfun.h.

7.1.2.58 sph_bessell()

```
long double std::sph_bessell (
    unsigned int __n,
    long double __x ) [inline]
```

Return the spherical Bessel function $j_n(x)$ of nonnegative order n and `long double` argument $x \geq 0$.

See also

[sph_bessel](#) for more details.

Definition at line 1328 of file specfun.h.

7.1.2.59 sph_legendre()

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> std::sph_legendre (
    unsigned int __l,
    unsigned int __m,
    _Tp __theta ) [inline]
```

Return the spherical Legendre function of nonnegative integral degree l and order m and real angle θ in radians.

The spherical Legendre function is defined by

$$Y_l^m(\theta, 0) = (-1)^m \frac{(2l+1)}{4\pi} \frac{(l-m)!}{(l+m)!} P_l^m(\cos \theta) e^{im\phi}$$

where $P_l^m(x)$ is the associated Legendre polynomial. The full (complex) spherical harmonic function includes a phase factor in the azimuthal angle ϕ :

$$Y_l^m(\theta, \phi) = Y_l^m(\theta, 0) e^{im\phi}$$

Template Parameters

<code>__Tp</code>	The floating-point type of the angle <code>__theta</code> .
-------------------	---

Parameters

<code>__l</code>	The order <code>__l >= 0</code>
<code>__m</code>	The degree <code>__m >= 0</code> and <code>__m <= __l</code>
<code>__theta</code>	The radian polar angle argument

See also

[assoc_legendre](#) for the unnormalized associated Legendre polynomial.

Definition at line 1401 of file `specfun.h`.

7.1.2.60 `sph_legendref()`

```
float std::sph_legendref (
    unsigned int __l,
    unsigned int __m,
    float __theta ) [inline]
```

Return the spherical Legendre function of nonnegative integral degree `l` and order `m` and float angle θ in radians.

See also

[sph_legendre](#) for details.

Definition at line 1362 of file `specfun.h`.

7.1.2.61 `sph_legendrel()`

```
long double std::sph_legendrel (
    unsigned int __l,
    unsigned int __m,
    long double __theta ) [inline]
```

Return the spherical Legendre function of nonnegative integral degree `l` and order `m` and `long double` angle θ in radians.

See also

[sph_legendre](#) for details.

Definition at line 1373 of file `specfun.h`.

7.1.2.62 sph_neumann()

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> std::sph_neumann (
    unsigned int __n,
    _Tp __x ) [inline]
```

Return the spherical Neumann function of integral order $n \geq 0$ and real argument $x \geq 0$.

The spherical Neumann function is defined by

$$n_n(x) = \left(\frac{\pi}{2x}\right)^{1/2} N_{n+1/2}(x)$$

Template Parameters

<code>_Tp</code>	The floating-point type of the argument <code>__x</code> .
------------------	--

Parameters

<code>__n</code>	The integral order $n \geq 0$
<code>__x</code>	The real argument $__x \geq 0$

Exceptions

<code>std::domain_error</code>	if <code>__x < 0</code> .
--------------------------------	------------------------------

Definition at line 1445 of file specfun.h.

7.1.2.63 sph_neumannf()

```
float std::sph_neumannf (
    unsigned int __n,
    float __x ) [inline]
```

Return the spherical Neumann function of integral order $n \geq 0$ and `float` argument $x \geq 0$.

See also

[sph_neumann](#) for details.

Definition at line 1416 of file specfun.h.

7.1.2.64 sph_neumannl()

```
long double std::sph_neumannl (
    unsigned int __n,
    long double __x ) [inline]
```

Return the spherical Neumann function of integral order $n \geq 0$ and long double $x \geq 0$.

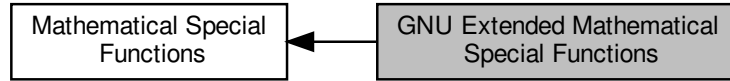
See also

[sph_neumann](#) for details.

Definition at line 1426 of file specfun.h.

7.2 GNU Extended Mathematical Special Functions

Collaboration diagram for GNU Extended Mathematical Special Functions:



Functions

- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::airy_ai (_Tp __x)`
- `template<typename _Tp >`
`std::complex< __gnu_cxx::fp_promote_t< _Tp > > __gnu_cxx::airy_ai (std::complex< _Tp > __x)`
- `float __gnu_cxx::airy_aif (float __x)`
- `long double __gnu_cxx::airy_ail (long double __x)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::airy_bi (_Tp __x)`
- `template<typename _Tp >`
`std::complex< __gnu_cxx::fp_promote_t< _Tp > > __gnu_cxx::airy_bi (std::complex< _Tp > __x)`
- `float __gnu_cxx::airy_bif (float __x)`
- `long double __gnu_cxx::airy_bil (long double __x)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::assoc_legendre_q (unsigned int __l, unsigned int __m, _Tp __x)`
- `float __gnu_cxx::assoc_legendre_qf (unsigned int __l, unsigned int __m, float __x)`
- `long double __gnu_cxx::assoc_legendre_ql (unsigned int __l, unsigned int __m, long double __x)`
- `template<typename _Tp >`
`std::vector< _Tp > __gnu_cxx::bell (unsigned int __n)`
- `template<typename _Tp, typename _Up >`
`_Up __gnu_cxx::bell (unsigned int __n, _Up __x)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::bernoulli (unsigned int __n)`
- `template<typename _Tp >`
`_Tp __gnu_cxx::bernoulli (unsigned int __n, _Tp __x)`
- `float __gnu_cxx::bernoullif (unsigned int __n)`
- `long double __gnu_cxx::bernoullil (unsigned int __n)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::binomial (unsigned int __n, unsigned int __k)`

Return the binomial coefficient as a real number. The binomial coefficient is given by:

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

The binomial coefficients are generated by:

$$(1+t)^n = \sum_{k=0}^n \binom{n}{k} t^k$$

- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::binomial_p (_Tp __p, unsigned int __n, unsigned int __k)`
Return the binomial cumulative distribution function.
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::binomial_pdf (_Tp __p, unsigned int __n, unsigned int __k)`
Return the binomial probability mass function.
- `float __gnu_cxx::binomialf (unsigned int __n, unsigned int __k)`
- `long double __gnu_cxx::binomiall (unsigned int __n, unsigned int __k)`
- `template<typename _Tps, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tps, _Tp > __gnu_cxx::bose_einstein (_Tps __s, _Tp __x)`
- `float __gnu_cxx::bose_einsteinf (float __s, float __x)`
- `long double __gnu_cxx::bose_einsteinl (long double __s, long double __x)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::chebyshev_t (unsigned int __n, _Tp __x)`
- `float __gnu_cxx::chebyshev_tf (unsigned int __n, float __x)`
- `long double __gnu_cxx::chebyshev_tl (unsigned int __n, long double __x)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::chebyshev_u (unsigned int __n, _Tp __x)`
- `float __gnu_cxx::chebyshev_uf (unsigned int __n, float __x)`
- `long double __gnu_cxx::chebyshev_ul (unsigned int __n, long double __x)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::chebyshev_v (unsigned int __n, _Tp __x)`
- `float __gnu_cxx::chebyshev_vf (unsigned int __n, float __x)`
- `long double __gnu_cxx::chebyshev_vl (unsigned int __n, long double __x)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::chebyshev_w (unsigned int __n, _Tp __x)`
- `float __gnu_cxx::chebyshev_wf (unsigned int __n, float __x)`
- `long double __gnu_cxx::chebyshev_wl (unsigned int __n, long double __x)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::clausen (unsigned int __m, _Tp __x)`
- `template<typename _Tp >`
`std::complex< __gnu_cxx::fp_promote_t< _Tp > > __gnu_cxx::clausen (unsigned int __m, std::complex< _Tp > __z)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::clausen_cl (unsigned int __m, _Tp __x)`
- `float __gnu_cxx::clausen_clf (unsigned int __m, float __x)`
- `long double __gnu_cxx::clausen_cll (unsigned int __m, long double __x)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::clausen_sl (unsigned int __m, _Tp __x)`
- `float __gnu_cxx::clausen_slf (unsigned int __m, float __x)`
- `long double __gnu_cxx::clausen_sll (unsigned int __m, long double __x)`
- `float __gnu_cxx::clausenf (unsigned int __m, float __x)`
- `std::complex< float > __gnu_cxx::clausenf (unsigned int __m, std::complex< float > __z)`
- `long double __gnu_cxx::clausenl (unsigned int __m, long double __x)`
- `std::complex< long double > __gnu_cxx::clausenl (unsigned int __m, std::complex< long double > __z)`
- `template<typename _Tk >`
`__gnu_cxx::fp_promote_t< _Tk > __gnu_cxx::comp_ellint_d (_Tk __k)`
- `float __gnu_cxx::comp_ellint_df (float __k)`
- `long double __gnu_cxx::comp_ellint_dl (long double __k)`
- `float __gnu_cxx::comp_ellint_rf (float __x, float __y)`
- `long double __gnu_cxx::comp_ellint_rf (long double __x, long double __y)`

- `template<typename _Tx, typename _Ty >`
`__gnu_cxx::fp_promote_t< _Tx, _Ty > __gnu_cxx::comp_ellint_rf (_Tx __x, _Ty __y)`
- `float __gnu_cxx::comp_ellint_rg (float __x, float __y)`
- `long double __gnu_cxx::comp_ellint_rg (long double __x, long double __y)`
- `template<typename _Tx, typename _Ty >`
`__gnu_cxx::fp_promote_t< _Tx, _Ty > __gnu_cxx::comp_ellint_rg (_Tx __x, _Ty __y)`
- `template<typename _Tpa, typename _Tpc, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tpa, _Tpc, _Tp > __gnu_cxx::conf_hyperg (_Tpa __a, _Tpc __c, _Tp __x)`
- `template<typename _Tpc, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tpc, _Tp > __gnu_cxx::conf_hyperg_lim (_Tpc __c, _Tp __x)`
- `float __gnu_cxx::conf_hyperg_limf (float __c, float __x)`
- `long double __gnu_cxx::conf_hyperg_liml (long double __c, long double __x)`
- `float __gnu_cxx::conf_hypergf (float __a, float __c, float __x)`
- `long double __gnu_cxx::conf_hypergl (long double __a, long double __c, long double __x)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::cos_pi (_Tp __x)`
- `float __gnu_cxx::cos_pif (float __x)`
- `long double __gnu_cxx::cos_pil (long double __x)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::cosh_pi (_Tp __x)`
- `float __gnu_cxx::cosh_pif (float __x)`
- `long double __gnu_cxx::cosh_pil (long double __x)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::coshint (_Tp __x)`
- `float __gnu_cxx::coshintf (float __x)`
- `long double __gnu_cxx::coshintl (long double __x)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::cosint (_Tp __x)`
- `float __gnu_cxx::cosintf (float __x)`
- `long double __gnu_cxx::cosintl (long double __x)`
- `template<typename _Tpnu, typename _Tp >`
`std::complex< __gnu_cxx::fp_promote_t< _Tpnu, _Tp > > __gnu_cxx::cyl_hankel_1 (_Tpnu __nu, _Tp __z)`
- `template<typename _Tpnu, typename _Tp >`
`std::complex< __gnu_cxx::fp_promote_t< _Tpnu, _Tp > > __gnu_cxx::cyl_hankel_1 (std::complex< _Tpnu > __nu, std::complex< _Tp > __x)`
- `std::complex< float > __gnu_cxx::cyl_hankel_1f (float __nu, float __z)`
- `std::complex< float > __gnu_cxx::cyl_hankel_1f (std::complex< float > __nu, std::complex< float > __x)`
- `std::complex< long double > __gnu_cxx::cyl_hankel_1l (long double __nu, long double __z)`
- `std::complex< long double > __gnu_cxx::cyl_hankel_1l (std::complex< long double > __nu, std::complex< long double > __x)`
- `template<typename _Tpnu, typename _Tp >`
`std::complex< __gnu_cxx::fp_promote_t< _Tpnu, _Tp > > __gnu_cxx::cyl_hankel_2 (_Tpnu __nu, _Tp __z)`
- `template<typename _Tpnu, typename _Tp >`
`std::complex< __gnu_cxx::fp_promote_t< _Tpnu, _Tp > > __gnu_cxx::cyl_hankel_2 (std::complex< _Tpnu > __nu, std::complex< _Tp > __x)`
- `std::complex< float > __gnu_cxx::cyl_hankel_2f (float __nu, float __z)`
- `std::complex< float > __gnu_cxx::cyl_hankel_2f (std::complex< float > __nu, std::complex< float > __x)`
- `std::complex< long double > __gnu_cxx::cyl_hankel_2l (long double __nu, long double __z)`
- `std::complex< long double > __gnu_cxx::cyl_hankel_2l (std::complex< long double > __nu, std::complex< long double > __x)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::dawson (_Tp __x)`

- float [__gnu_cxx::dawsonf](#) (float __x)
- long double [__gnu_cxx::dawsonl](#) (long double __x)
- template<typename _Tp >
__gnu_cxx::fp_promote_t< _Tp > [__gnu_cxx::debye](#) (unsigned int __n, _Tp __x)
- float [__gnu_cxx::debyef](#) (unsigned int __n, float __x)
- long double [__gnu_cxx::debyel](#) (unsigned int __n, long double __x)
- template<typename _Tp >
__gnu_cxx::fp_promote_t< _Tp > [__gnu_cxx::digamma](#) (_Tp __x)
- float [__gnu_cxx::digammaf](#) (float __x)
- long double [__gnu_cxx::digammal](#) (long double __x)
- template<typename _Tp >
__gnu_cxx::fp_promote_t< _Tp > [__gnu_cxx::dilog](#) (_Tp __x)
- float [__gnu_cxx::dilogf](#) (float __x)
- long double [__gnu_cxx::dilogl](#) (long double __x)
- template<typename _Tp >
_Tp [__gnu_cxx::dirichlet_beta](#) (_Tp __s)
- float [__gnu_cxx::dirichlet_betaf](#) (float __s)
- long double [__gnu_cxx::dirichlet_betel](#) (long double __s)
- template<typename _Tp >
_Tp [__gnu_cxx::dirichlet_eta](#) (_Tp __s)
- float [__gnu_cxx::dirichlet_etaf](#) (float __s)
- long double [__gnu_cxx::dirichlet_etel](#) (long double __s)
- template<typename _Tp >
_Tp [__gnu_cxx::dirichlet_lambda](#) (_Tp __s)
- float [__gnu_cxx::dirichlet_lambdaf](#) (float __s)
- long double [__gnu_cxx::dirichlet_lambdal](#) (long double __s)
- template<typename _Tp >
__gnu_cxx::fp_promote_t< _Tp > [__gnu_cxx::double_factorial](#) (int __n)

Return the double factorial $n!!$ of the argument as a real number.

$$n!! = n(n-2)\dots(2), 0!! = 1$$

for even n and

$$n!! = n(n-2)\dots(1), (-1)!! = 1$$

for odd n .

- float [__gnu_cxx::double_factorialf](#) (int __n)
- long double [__gnu_cxx::double_factoriall](#) (int __n)
- template<typename _Tk, typename _Tp, typename _Ta, typename _Tb >
__gnu_cxx::fp_promote_t< _Tk, _Tp, _Ta, _Tb > [__gnu_cxx::ellint_cel](#) (_Tk __k_c, _Tp __p, _Ta __a, _Tb __b)
- float [__gnu_cxx::ellint_celf](#) (float __k_c, float __p, float __a, float __b)
- long double [__gnu_cxx::ellint_cel](#) (long double __k_c, long double __p, long double __a, long double __b)
- template<typename _Tk, typename _Tphi >
__gnu_cxx::fp_promote_t< _Tk, _Tphi > [__gnu_cxx::ellint_d](#) (_Tk __k, _Tphi __phi)
- float [__gnu_cxx::ellint_df](#) (float __k, float __phi)
- long double [__gnu_cxx::ellint_d](#) (long double __k, long double __phi)
- template<typename _Tp, typename _Tk >
__gnu_cxx::fp_promote_t< _Tp, _Tk > [__gnu_cxx::ellint_el1](#) (_Tp __x, _Tk __k_c)
- float [__gnu_cxx::ellint_el1f](#) (float __x, float __k_c)
- long double [__gnu_cxx::ellint_el1](#) (long double __x, long double __k_c)
- template<typename _Tp, typename _Tk, typename _Ta, typename _Tb >
__gnu_cxx::fp_promote_t< _Tp, _Tk, _Ta, _Tb > [__gnu_cxx::ellint_el2](#) (_Tp __x, _Tk __k_c, _Ta __a, _Tb __b)
- float [__gnu_cxx::ellint_el2f](#) (float __x, float __k_c, float __a, float __b)

- long double [__gnu_cxx::ellint_el2l](#) (long double __x, long double __k_c, long double __a, long double __b)
- template<typename _Tx, typename _Tk, typename _Tp >
[__gnu_cxx::fp_promote_t<_Tx, _Tk, _Tp>](#) [__gnu_cxx::ellint_el3](#) (_Tx __x, _Tk __k_c, _Tp __p)
- float [__gnu_cxx::ellint_el3f](#) (float __x, float __k_c, float __p)
- long double [__gnu_cxx::ellint_el3l](#) (long double __x, long double __k_c, long double __p)
- template<typename _Tp, typename _Up >
[__gnu_cxx::fp_promote_t<_Tp, _Up>](#) [__gnu_cxx::ellint_rc](#) (_Tp __x, _Up __y)
- float [__gnu_cxx::ellint_rcf](#) (float __x, float __y)
- long double [__gnu_cxx::ellint_rcl](#) (long double __x, long double __y)
- template<typename _Tp, typename _Up, typename _Vp >
[__gnu_cxx::fp_promote_t<_Tp, _Up, _Vp>](#) [__gnu_cxx::ellint_rd](#) (_Tp __x, _Up __y, _Vp __z)
- float [__gnu_cxx::ellint_rdf](#) (float __x, float __y, float __z)
- long double [__gnu_cxx::ellint_rdl](#) (long double __x, long double __y, long double __z)
- template<typename _Tp, typename _Up, typename _Vp >
[__gnu_cxx::fp_promote_t<_Tp, _Up, _Vp>](#) [__gnu_cxx::ellint_rf](#) (_Tp __x, _Up __y, _Vp __z)
- float [__gnu_cxx::ellint_rff](#) (float __x, float __y, float __z)
- long double [__gnu_cxx::ellint_rfl](#) (long double __x, long double __y, long double __z)
- template<typename _Tp, typename _Up, typename _Vp >
[__gnu_cxx::fp_promote_t<_Tp, _Up, _Vp>](#) [__gnu_cxx::ellint_rg](#) (_Tp __x, _Up __y, _Vp __z)
- float [__gnu_cxx::ellint_rgf](#) (float __x, float __y, float __z)
- long double [__gnu_cxx::ellint_rgl](#) (long double __x, long double __y, long double __z)
- template<typename _Tp, typename _Up, typename _Vp, typename _Wp >
[__gnu_cxx::fp_promote_t<_Tp, _Up, _Vp, _Wp>](#) [__gnu_cxx::ellint_rj](#) (_Tp __x, _Up __y, _Vp __z, _Wp __p)
- float [__gnu_cxx::ellint_rjf](#) (float __x, float __y, float __z, float __p)
- long double [__gnu_cxx::ellint_rjl](#) (long double __x, long double __y, long double __z, long double __p)
- template<typename _Tp >
[_Tp __gnu_cxx::ellnome](#) (_Tp __k)
- float [__gnu_cxx::ellnomef](#) (float __k)
- long double [__gnu_cxx::ellnomel](#) (long double __k)
- template<typename _Tp >
[_Tp __gnu_cxx::euler](#) (unsigned int __n)
This returns Euler number E_n .
- template<typename _Tp >
[_Tp __gnu_cxx::eulerian_1](#) (unsigned int __n, unsigned int __m)
- template<typename _Tp >
std::vector<_Tp> [__gnu_cxx::eulerian_1](#) (unsigned int __n)
- template<typename _Tp >
[_Tp __gnu_cxx::eulerian_2](#) (unsigned int __n, unsigned int __m)
- template<typename _Tp >
[__gnu_cxx::fp_promote_t<_Tp>](#) [__gnu_cxx::expint](#) (unsigned int __n, _Tp __x)
- float [__gnu_cxx::expintf](#) (unsigned int __n, float __x)
- long double [__gnu_cxx::expintl](#) (unsigned int __n, long double __x)
- template<typename _Tlam, typename _Tp >
[__gnu_cxx::fp_promote_t<_Tlam, _Tp>](#) [__gnu_cxx::exponential_p](#) (_Tlam __lambda, _Tp __x)
Return the exponential cumulative probability density function.
- template<typename _Tlam, typename _Tp >
[__gnu_cxx::fp_promote_t<_Tlam, _Tp>](#) [__gnu_cxx::exponential_pdf](#) (_Tlam __lambda, _Tp __x)
Return the exponential probability density function.
- template<typename _Tp >
[__gnu_cxx::fp_promote_t<_Tp>](#) [__gnu_cxx::factorial](#) (unsigned int __n)

Return the factorial $n!$ of the argument as a real number.

$$n! = 1 \times 2 \times \dots \times n, 0! = 1$$

- float [__gnu_cxx::factorialf](#) (unsigned int __n)
- long double [__gnu_cxx::factoriall](#) (unsigned int __n)
- template<typename _Tp, typename _Tnu >
[__gnu_cxx::fp_promote_t<_Tp, _Tnu> __gnu_cxx::falling_factorial](#) (_Tp __a, _Tnu __nu)

Return the falling factorial function or the lower Pochhammer symbol for real argument a and integral order n . The falling factorial function is defined by

$$a^{\overline{n}} = \prod_{k=0}^{n-1} (a - k) = \Gamma(a + 1) / \Gamma(a - n + 1)$$

where $a^{\overline{0}} \equiv 1$. In particular, $n^{\overline{n}} = n!$.

- float [__gnu_cxx::falling_factorialf](#) (float __a, float __nu)
- long double [__gnu_cxx::falling_factoriall](#) (long double __a, long double __nu)
- template<typename _Tps, typename _Tp >
[__gnu_cxx::fp_promote_t<_Tps, _Tp> __gnu_cxx::fermi_dirac](#) (_Tps __s, _Tp __x)
- float [__gnu_cxx::fermi_diracf](#) (float __s, float __x)
- long double [__gnu_cxx::fermi_diracl](#) (long double __s, long double __x)
- template<typename _Tp >
[__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::fisher_f_p](#) (_Tp __F, unsigned int __nu1, unsigned int __nu2)
- Return the F -distribution propability function. This returns the probability that the observed chi-square for a correct model exceeds the value χ^2 .
- template<typename _Tp >
[__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::fisher_f_pdf](#) (_Tp __F, unsigned int __nu1, unsigned int __nu2)
- Return the F -distribution propability function. This returns the probability that the observed chi-square for a correct model exceeds the value χ^2 .
- template<typename _Tp >
[__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::fresnel_c](#) (_Tp __x)
- float [__gnu_cxx::fresnel_cf](#) (float __x)
- long double [__gnu_cxx::fresnel_cl](#) (long double __x)
- template<typename _Tp >
[__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::fresnel_s](#) (_Tp __x)
- float [__gnu_cxx::fresnel_sf](#) (float __x)
- long double [__gnu_cxx::fresnel_sl](#) (long double __x)
- template<typename _Ta, typename _Tp >
[__gnu_cxx::fp_promote_t<_Ta, _Tp> __gnu_cxx::gamma_p](#) (_Ta __a, _Tp __x)
- Return the gamma cumulative propability distribution function or the regularized lower incomplete gamma function.
- template<typename _Ta, typename _Tb, typename _Tp >
[__gnu_cxx::fp_promote_t<_Ta, _Tb, _Tp> __gnu_cxx::gamma_pdf](#) (_Ta __alpha, _Tb __beta, _Tp __x)
- Return the gamma propability distribution function.
- float [__gnu_cxx::gamma_pf](#) (float __a, float __x)
- long double [__gnu_cxx::gamma_pl](#) (long double __a, long double __x)
- template<typename _Ta, typename _Tp >
[__gnu_cxx::fp_promote_t<_Ta, _Tp> __gnu_cxx::gamma_q](#) (_Ta __a, _Tp __x)
- Return the gamma complementary cumulative propability distribution (or survival) function or the regularized upper incomplete gamma function.
- float [__gnu_cxx::gamma_qf](#) (float __a, float __x)
- long double [__gnu_cxx::gamma_ql](#) (long double __a, long double __x)
- template<typename _Ta >
[__gnu_cxx::fp_promote_t<_Ta> __gnu_cxx::gamma_reciprocal](#) (_Ta __a)

- float `__gnu_cxx::gamma_reciprocalf` (float __a)
- long double `__gnu_cxx::gamma_reciprocall` (long double __a)
- template<typename _Tlam, typename _Tp >
`__gnu_cxx::fp_promote_t<_Tlam, _Tp>` `__gnu_cxx::gegenbauer` (unsigned int __n, _Tlam __lambda, _Tp __x)
- float `__gnu_cxx::gegenbauerf` (unsigned int __n, float __lambda, float __x)
- long double `__gnu_cxx::gegenbauerl` (unsigned int __n, long double __lambda, long double __x)
- template<typename _Tp >
`__gnu_cxx::fp_promote_t<_Tp>` `__gnu_cxx::harmonic` (unsigned int __n)
- template<typename _Tk, typename _Tphi >
`__gnu_cxx::fp_promote_t<_Tk, _Tphi>` `__gnu_cxx::heuman_lambda` (_Tk __k, _Tphi __phi)
- float `__gnu_cxx::heuman_lambdaf` (float __k, float __phi)
- long double `__gnu_cxx::heuman_lambdal` (long double __k, long double __phi)
- template<typename _Tp, typename _Up >
`__gnu_cxx::fp_promote_t<_Tp, _Up>` `__gnu_cxx::hurwitz_zeta` (_Tp __s, _Up __a)
- template<typename _Tp, typename _Up >
`std::complex<_Tp>` `__gnu_cxx::hurwitz_zeta` (_Tp __s, std::complex<_Up> __a)
- float `__gnu_cxx::hurwitz_zetaf` (float __s, float __a)
- long double `__gnu_cxx::hurwitz_zetal` (long double __s, long double __a)
- template<typename _Tpa, typename _Tpb, typename _Tpc, typename _Tp >
`__gnu_cxx::fp_promote_t<_Tpa, _Tpb, _Tpc, _Tp>` `__gnu_cxx::hyperg` (_Tpa __a, _Tpb __b, _Tpc __c, _Tp __x)
- float `__gnu_cxx::hypergf` (float __a, float __b, float __c, float __x)
- long double `__gnu_cxx::hypergl` (long double __a, long double __b, long double __c, long double __x)
- template<typename _Ta, typename _Tb, typename _Tp >
`__gnu_cxx::fp_promote_t<_Ta, _Tb, _Tp>` `__gnu_cxx::ibeta` (_Ta __a, _Tb __b, _Tp __x)
- template<typename _Ta, typename _Tb, typename _Tp >
`__gnu_cxx::fp_promote_t<_Ta, _Tb, _Tp>` `__gnu_cxx::ibetac` (_Ta __a, _Tb __b, _Tp __x)
- float `__gnu_cxx::ibetaf` (float __a, float __b, float __x)
- long double `__gnu_cxx::ibetac` (long double __a, long double __b, long double __x)
- float `__gnu_cxx::ibetaf` (float __a, float __b, float __x)
- long double `__gnu_cxx::ibetal` (long double __a, long double __b, long double __x)
- template<typename _Talpha, typename _Tbeta, typename _Tp >
`__gnu_cxx::fp_promote_t<_Talpha, _Tbeta, _Tp>` `__gnu_cxx::jacobi` (unsigned __n, _Talpha __alpha, _Tbeta __beta, _Tp __x)
- template<typename _Kp, typename _Up >
`__gnu_cxx::fp_promote_t<_Kp, _Up>` `__gnu_cxx::jacobi_cn` (_Kp __k, _Up __u)
- float `__gnu_cxx::jacobi_cnf` (float __k, float __u)
- long double `__gnu_cxx::jacobi_cnl` (long double __k, long double __u)
- template<typename _Kp, typename _Up >
`__gnu_cxx::fp_promote_t<_Kp, _Up>` `__gnu_cxx::jacobi_dn` (_Kp __k, _Up __u)
- float `__gnu_cxx::jacobi_dnf` (float __k, float __u)
- long double `__gnu_cxx::jacobi_dnl` (long double __k, long double __u)
- template<typename _Kp, typename _Up >
`__gnu_cxx::fp_promote_t<_Kp, _Up>` `__gnu_cxx::jacobi_sn` (_Kp __k, _Up __u)
- float `__gnu_cxx::jacobi_snf` (float __k, float __u)
- long double `__gnu_cxx::jacobi_snl` (long double __k, long double __u)
- template<typename _Tpq, typename _Tp >
`__gnu_cxx::fp_promote_t<_Tpq, _Tp>` `__gnu_cxx::jacobi_theta_1` (_Tpq __q, _Tp __x)
- float `__gnu_cxx::jacobi_theta_1f` (float __q, float __x)
- long double `__gnu_cxx::jacobi_theta_1l` (long double __q, long double __x)
- template<typename _Tpq, typename _Tp >
`__gnu_cxx::fp_promote_t<_Tpq, _Tp>` `__gnu_cxx::jacobi_theta_2` (_Tpq __q, _Tp __x)

- float [__gnu_cxx::jacobi_theta_2f](#) (float __q, float __x)
- long double [__gnu_cxx::jacobi_theta_2l](#) (long double __q, long double __x)
- template<typename _Tpq, typename _Tp >
__gnu_cxx::fp_promote_t< _Tpq, _Tp > [__gnu_cxx::jacobi_theta_3](#) (_Tpq __q, _Tp __x)
- float [__gnu_cxx::jacobi_theta_3f](#) (float __q, float __x)
- long double [__gnu_cxx::jacobi_theta_3l](#) (long double __q, long double __x)
- template<typename _Tpq, typename _Tp >
__gnu_cxx::fp_promote_t< _Tpq, _Tp > [__gnu_cxx::jacobi_theta_4](#) (_Tpq __q, _Tp __x)
- float [__gnu_cxx::jacobi_theta_4f](#) (float __q, float __x)
- long double [__gnu_cxx::jacobi_theta_4l](#) (long double __q, long double __x)
- template<typename _Tk, typename _Tphi >
__gnu_cxx::fp_promote_t< _Tk, _Tphi > [__gnu_cxx::jacobi_zeta](#) (_Tk __k, _Tphi __phi)
- float [__gnu_cxx::jacobi_zetaf](#) (float __k, float __phi)
- long double [__gnu_cxx::jacobi_zetal](#) (long double __k, long double __phi)
- float [__gnu_cxx::jacobi_f](#) (unsigned __n, float __alpha, float __beta, float __x)
- long double [__gnu_cxx::jacobi_l](#) (unsigned __n, long double __alpha, long double __beta, long double __x)
- template<typename _Tp >
_Tp [__gnu_cxx::lah](#) (unsigned int __n, unsigned int __k)
- template<typename _Tp >
std::vector< _Tp > [__gnu_cxx::lah](#) (unsigned int __n)
- template<typename _Tp >
__gnu_cxx::fp_promote_t< _Tp > [__gnu_cxx::lbinomial](#) (unsigned int __n, unsigned int __k)

Return the logarithm of the binomial coefficient as a real number. The binomial coefficient is given by:

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

The binomial coefficients are generated by:

$$(1+t)^n = \sum_{k=0}^n \binom{n}{k} t^k$$

- float [__gnu_cxx::lbinomialf](#) (unsigned int __n, unsigned int __k)
- long double [__gnu_cxx::lbinomiall](#) (unsigned int __n, unsigned int __k)
- template<typename _Tp >
__gnu_cxx::fp_promote_t< _Tp > [__gnu_cxx::ldouble_factorial](#) (int __n)

Return the logarithm of the double factorial $\ln(n!!)$ of the argument as a real number.

$$n!! = n(n-2)\dots(2), 0!! = 1$$

for even n and

$$n!! = n(n-2)\dots(1), (-1)!! = 1$$

for odd n .

- float [__gnu_cxx::ldouble_factorialf](#) (int __n)
- long double [__gnu_cxx::ldouble_factoriall](#) (int __n)
- template<typename _Tp >
__gnu_cxx::fp_promote_t< _Tp > [__gnu_cxx::legendre_q](#) (unsigned int __l, _Tp __x)
- float [__gnu_cxx::legendre_qf](#) (unsigned int __l, float __x)
- long double [__gnu_cxx::legendre_ql](#) (unsigned int __l, long double __x)
- template<typename _Tp, typename _Ts, typename _Ta >
__gnu_cxx::fp_promote_t< _Tp, _Ts, _Ta > [__gnu_cxx::lerch_phi](#) (_Tp __z, _Ts __s, _Ta __a)
- float [__gnu_cxx::lerch_phif](#) (float __z, float __s, float __a)
- long double [__gnu_cxx::lerch_phil](#) (long double __z, long double __s, long double __a)

- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::lfactorial (unsigned int __n)`

Return the logarithm of the factorial $\ln(n!)$ of the argument as a real number.

$$n! = 1 \times 2 \times \dots \times n, 0! = 1$$

- `float __gnu_cxx::lfactorialf (unsigned int __n)`
- `long double __gnu_cxx::lfactoriall (unsigned int __n)`
- `template<typename _Tp, typename _Tnu >`
`__gnu_cxx::fp_promote_t< _Tp, _Tnu > __gnu_cxx::lfalling_factorial (_Tp __a, _Tnu __nu)`

Return the logarithm of the falling factorial function or the lower Pochhammer symbol. The falling factorial function is defined by

$$a^{\underline{n}} = \frac{\Gamma(a+1)}{\Gamma(a-\nu+1)} = \prod_{k=0}^{n-1} (a-k)$$

where $a^{\underline{0}} \equiv 1$. In particular, $n^{\underline{n}} = n!$. Thus this function returns

$$\ln[a^{\underline{n}}] = \ln[\Gamma(a+1)] - \ln[\Gamma(a-\nu+1)]$$

where $\ln[a^{\underline{0}}] \equiv 0$. Many notations exist for this function: $(a)_{\nu}$,

$$\left\{ \begin{matrix} a \\ \nu \end{matrix} \right\}$$

, and others.

- `float __gnu_cxx::lfalling_factorialf (float __a, float __nu)`
- `long double __gnu_cxx::lfalling_factoriall (long double __a, long double __nu)`
- `template<typename _Ta >`
`__gnu_cxx::fp_promote_t< _Ta > __gnu_cxx::lgamma (_Ta __a)`
- `template<typename _Ta >`
`std::complex< __gnu_cxx::fp_promote_t< _Ta > > __gnu_cxx::lgamma (std::complex< _Ta > __a)`
- `float __gnu_cxx::lgammaf (float __a)`
- `std::complex< float > __gnu_cxx::lgammaf (std::complex< float > __a)`
- `long double __gnu_cxx::lgammal (long double __a)`
- `std::complex< long double > __gnu_cxx::lgammal (std::complex< long double > __a)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::logint (_Tp __x)`
- `float __gnu_cxx::logintf (float __x)`
- `long double __gnu_cxx::logintl (long double __x)`
- `template<typename _Ta, typename _Tb, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Ta, _Tb, _Tp > __gnu_cxx::logistic_p (_Ta __a, _Tb __b, _Tp __x)`
Return the logistic cumulative distribution function.
- `template<typename _Ta, typename _Tb, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Ta, _Tb, _Tp > __gnu_cxx::logistic_pdf (_Ta __a, _Tb __b, _Tp __x)`
Return the logistic probability density function.
- `template<typename _Tmu, typename _Tsig, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tmu, _Tsig, _Tp > __gnu_cxx::lognormal_p (_Tmu __mu, _Tsig __sigma, _Tp __x)`
Return the lognormal cumulative probability density function.
- `template<typename _Tmu, typename _Tsig, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tmu, _Tsig, _Tp > __gnu_cxx::lognormal_pdf (_Tmu __mu, _Tsig __sigma, _Tp __x)`
Return the lognormal probability density function.
- `template<typename _Tp, typename _Tnu >`
`__gnu_cxx::fp_promote_t< _Tp, _Tnu > __gnu_cxx::lrising_factorial (_Tp __a, _Tnu __nu)`

Return the logarithm of the rising factorial function or the (upper) Pochhammer symbol. The rising factorial function is defined for integer order by

$$a^{\overline{\nu}} = \Gamma(a + \nu) / \Gamma(a) = \prod_{k=0}^{\nu-1} (a + k), \overline{0} \equiv 1$$

Thus this function returns

$$\ln[a^{\overline{\nu}}] = \ln[\Gamma(a + \nu)] - \ln[\Gamma(a)], \ln[a^{\overline{0}}] \equiv 0$$

Many notations exist for this function: $(a)_{\nu}$, called the Pochhammer function (esp. in the literature of special functions), and

$$\left[\begin{matrix} a \\ \nu \end{matrix} \right]$$

, and others.

- float [__gnu_cxx::lrising_factorialf](#) (float __a, float __nu)
- long double [__gnu_cxx::lrising_factoriall](#) (long double __a, long double __nu)
- template<typename _Tp, typename _Ta, typename _Tb >
std::complex< __gnu_cxx::fp_promote_t< _Tp, _Ta, _Tb > > [__gnu_cxx::mittag_leffler](#) (_Ta __alpha, _Tb __beta, const std::complex< _Tp > &__z)
- template<typename _Tmu, typename _Tsig, typename _Tp >
__gnu_cxx::fp_promote_t< _Tmu, _Tsig, _Tp > [__gnu_cxx::normal_p](#) (_Tmu __mu, _Tsig __sigma, _Tp __x)
Return the normal cumulative probability density function.
- template<typename _Tmu, typename _Tsig, typename _Tp >
__gnu_cxx::fp_promote_t< _Tmu, _Tsig, _Tp > [__gnu_cxx::normal_pdf](#) (_Tmu __mu, _Tsig __sigma, _Tp __x)
Return the gamma cumulative probability distribution function.
- template<typename _Tph, typename _Tpa >
__gnu_cxx::fp_promote_t< _Tph, _Tpa > [__gnu_cxx::owens_t](#) (_Tph __h, _Tpa __a)
- float [__gnu_cxx::owens_tf](#) (float __h, float __a)
- long double [__gnu_cxx::owens_tl](#) (long double __h, long double __a)
- template<typename _Tp, typename _Up >
__gnu_cxx::fp_promote_t< std::complex< _Tp >, _Up > [__gnu_cxx::periodic_zeta](#) (_Tp __x, _Up __s)
- template<typename _Tp, typename _Up >
__gnu_cxx::fp_promote_t< std::complex< _Tp >, std::complex< _Up > > [__gnu_cxx::periodic_zeta](#) (std::complex< _Up > __z, _Tp __s)
- std::complex< float > [__gnu_cxx::periodic_zetaf](#) (float __x, float __s)
- std::complex< long double > [__gnu_cxx::periodic_zetal](#) (long double __x, long double __s)
- template<typename _Tp >
__gnu_cxx::fp_promote_t< _Tp > [__gnu_cxx::polygamma](#) (unsigned int __m, _Tp __x)
- float [__gnu_cxx::polygammaf](#) (unsigned int __m, float __x)
- long double [__gnu_cxx::polygammal](#) (unsigned int __m, long double __x)
- template<typename _Tp, typename _Wp >
__gnu_cxx::fp_promote_t< _Tp, _Wp > [__gnu_cxx::polylog](#) (_Tp __s, _Wp __w)
- template<typename _Tp, typename _Wp >
std::complex< __gnu_cxx::fp_promote_t< _Tp, _Wp > > [__gnu_cxx::polylog](#) (_Tp __s, std::complex< _Tp > __w)
- float [__gnu_cxx::polylogf](#) (float __s, float __w)
- std::complex< float > [__gnu_cxx::polylogf](#) (float __s, std::complex< float > __w)
- long double [__gnu_cxx::polylogl](#) (long double __s, long double __w)
- std::complex< long double > [__gnu_cxx::polylogl](#) (long double __s, std::complex< long double > __w)
- template<typename _Tp >
__gnu_cxx::fp_promote_t< _Tp > [__gnu_cxx::radpoly](#) (unsigned int __n, unsigned int __m, _Tp __rho)
- float [__gnu_cxx::radpolyf](#) (unsigned int __n, unsigned int __m, float __rho)
- long double [__gnu_cxx::radpolyl](#) (unsigned int __n, unsigned int __m, long double __rho)
- template<typename _Tp, typename _Tnu >
__gnu_cxx::fp_promote_t< _Tp, _Tnu > [__gnu_cxx::rising_factorial](#) (_Tp __a, _Tnu __nu)

Return the rising factorial function or the (upper) Pochhammer function. The rising factorial function is defined by

$$a^{\overline{\nu}} = \Gamma(a + \nu) / \Gamma(\nu)$$

Many notations exist for this function: $(a)_{\nu}$, called the Pochhammer function (esp. in the literature of special functions), and

$$\begin{bmatrix} a \\ \nu \end{bmatrix}$$

, and others.

- float [__gnu_cxx::rising_factorialf](#) (float __a, float __nu)
- long double [__gnu_cxx::rising_factoriall](#) (long double __a, long double __nu)
- template<typename _Tp >
 [__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::sin_pi](#) (_Tp __x)
- float [__gnu_cxx::sin_pif](#) (float __x)
- long double [__gnu_cxx::sin_pil](#) (long double __x)
- template<typename _Tp >
 [__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::sinc](#) (_Tp __x)
- template<typename _Tp >
 [__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::sinc_pi](#) (_Tp __x)
- float [__gnu_cxx::sinc_pif](#) (float __x)
- long double [__gnu_cxx::sinc_pil](#) (long double __x)
- float [__gnu_cxx::sincf](#) (float __x)
- long double [__gnu_cxx::sincl](#) (long double __x)
- [__gnu_cxx::__sincos_t<double> __gnu_cxx::sincos](#) (double __x)
- template<typename _Tp >
 [__gnu_cxx::__sincos_t<__gnu_cxx::fp_promote_t<_Tp>> __gnu_cxx::sincos](#) (_Tp __x)
- template<typename _Tp >
 [__gnu_cxx::__sincos_t<__gnu_cxx::fp_promote_t<_Tp>> __gnu_cxx::sincos_pi](#) (_Tp __x)
- [__gnu_cxx::__sincos_t<float> __gnu_cxx::sincos_pif](#) (float __x)
- [__gnu_cxx::__sincos_t<long double> __gnu_cxx::sincos_pil](#) (long double __x)
- [__gnu_cxx::__sincos_t<float> __gnu_cxx::sincosf](#) (float __x)
- [__gnu_cxx::__sincos_t<long double> __gnu_cxx::sincosl](#) (long double __x)
- template<typename _Tp >
 [__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::sinh_pi](#) (_Tp __x)
- float [__gnu_cxx::sinh_pif](#) (float __x)
- long double [__gnu_cxx::sinh_pil](#) (long double __x)
- template<typename _Tp >
 [__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::sinhc](#) (_Tp __x)
- template<typename _Tp >
 [__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::sinhc_pi](#) (_Tp __x)
- float [__gnu_cxx::sinhc_pif](#) (float __x)
- long double [__gnu_cxx::sinhc_pil](#) (long double __x)
- float [__gnu_cxx::sinhcf](#) (float __x)
- long double [__gnu_cxx::sinhcl](#) (long double __x)
- template<typename _Tp >
 [__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::sinhint](#) (_Tp __x)
- float [__gnu_cxx::sinhintf](#) (float __x)
- long double [__gnu_cxx::sinhintl](#) (long double __x)
- template<typename _Tp >
 [__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::sinint](#) (_Tp __x)
- float [__gnu_cxx::sinintf](#) (float __x)
- long double [__gnu_cxx::sinintl](#) (long double __x)

- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::sph_bessel_i (unsigned int __n, _Tp __x)`
- `float __gnu_cxx::sph_bessel_if (unsigned int __n, float __x)`
- `long double __gnu_cxx::sph_bessel_il (unsigned int __n, long double __x)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::sph_bessel_k (unsigned int __n, _Tp __x)`
- `float __gnu_cxx::sph_bessel_kf (unsigned int __n, float __x)`
- `long double __gnu_cxx::sph_bessel_kl (unsigned int __n, long double __x)`
- `template<typename _Tp >`
`std::complex< __gnu_cxx::fp_promote_t< _Tp > > __gnu_cxx::sph_hankel_1 (unsigned int __n, _Tp __z)`
- `template<typename _Tp >`
`std::complex< __gnu_cxx::fp_promote_t< _Tp > > __gnu_cxx::sph_hankel_1 (unsigned int __n, std::complex< __Tp > __x)`
- `std::complex< float > __gnu_cxx::sph_hankel_1f (unsigned int __n, float __z)`
- `std::complex< float > __gnu_cxx::sph_hankel_1f (unsigned int __n, std::complex< float > __x)`
- `std::complex< long double > __gnu_cxx::sph_hankel_1l (unsigned int __n, long double __z)`
- `std::complex< long double > __gnu_cxx::sph_hankel_1l (unsigned int __n, std::complex< long double > __x)`
- `template<typename _Tp >`
`std::complex< __gnu_cxx::fp_promote_t< _Tp > > __gnu_cxx::sph_hankel_2 (unsigned int __n, _Tp __z)`
- `template<typename _Tp >`
`std::complex< __gnu_cxx::fp_promote_t< _Tp > > __gnu_cxx::sph_hankel_2 (unsigned int __n, std::complex< __Tp > __x)`
- `std::complex< float > __gnu_cxx::sph_hankel_2f (unsigned int __n, float __z)`
- `std::complex< float > __gnu_cxx::sph_hankel_2f (unsigned int __n, std::complex< float > __x)`
- `std::complex< long double > __gnu_cxx::sph_hankel_2l (unsigned int __n, long double __z)`
- `std::complex< long double > __gnu_cxx::sph_hankel_2l (unsigned int __n, std::complex< long double > __x)`
- `template<typename _Ttheta, typename _Tphi >`
`std::complex< __gnu_cxx::fp_promote_t< _Ttheta, _Tphi > > __gnu_cxx::sph_harmonic (unsigned int __l, int __m, _Ttheta __theta, _Tphi __phi)`
- `std::complex< float > __gnu_cxx::sph_harmonicf (unsigned int __l, int __m, float __theta, float __phi)`
- `std::complex< long double > __gnu_cxx::sph_harmonicl (unsigned int __l, int __m, long double __theta, long double __phi)`
- `template<typename _Tp >`
`_Tp __gnu_cxx::stirling_1 (unsigned int __n, unsigned int __m)`
- `template<typename _Tp >`
`std::vector< _Tp > __gnu_cxx::stirling_1 (unsigned int __n)`
- `template<typename _Tp >`
`_Tp __gnu_cxx::stirling_2 (unsigned int __n, unsigned int __m)`
- `template<typename _Tp >`
`std::vector< _Tp > __gnu_cxx::stirling_2 (unsigned int __n)`
- `template<typename _Tt, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::student_t_p (_Tt __t, unsigned int __nu)`
Return the Students T probability function.
- `template<typename _Tt, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::student_t_pdf (_Tt __t, unsigned int __nu)`
Return the complement of the Students T probability function.
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::tan_pi (_Tp __x)`
- `float __gnu_cxx::tan_pif (float __x)`
- `long double __gnu_cxx::tan_pil (long double __x)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::tanh_pi (_Tp __x)`

- float [__gnu_cxx::tanh_pif](#) (float __x)
- long double [__gnu_cxx::tanh_pil](#) (long double __x)
- template<typename _Ta >
 [__gnu_cxx::fp_promote_t<_Ta>](#) [__gnu_cxx::tgamma](#) (_Ta __a)
- template<typename _Ta >
 std::complex< [__gnu_cxx::fp_promote_t<_Ta>](#) > [__gnu_cxx::tgamma](#) (std::complex< _Ta > __a)
- template<typename _Ta, typename _Tp >
 [__gnu_cxx::fp_promote_t<_Ta, _Tp>](#) [__gnu_cxx::tgamma](#) (_Ta __a, _Tp __x)
- template<typename _Ta, typename _Tp >
 [__gnu_cxx::fp_promote_t<_Ta, _Tp>](#) [__gnu_cxx::tgamma_lower](#) (_Ta __a, _Tp __x)
- float [__gnu_cxx::tgamma_lowerf](#) (float __a, float __x)
- long double [__gnu_cxx::tgamma_lowerl](#) (long double __a, long double __x)
- float [__gnu_cxx::tgammaf](#) (float __a)
- std::complex< float > [__gnu_cxx::tgammaf](#) (std::complex< float > __a)
- float [__gnu_cxx::tgammaf](#) (float __a, float __x)
- long double [__gnu_cxx::tgammal](#) (long double __a)
- std::complex< long double > [__gnu_cxx::tgammal](#) (std::complex< long double > __a)
- long double [__gnu_cxx::tgammal](#) (long double __a, long double __x)
- template<typename _Tpnu, typename _Tp >
 [__gnu_cxx::fp_promote_t<_Tpnu, _Tp>](#) [__gnu_cxx::theta_1](#) (_Tpnu __nu, _Tp __x)
- float [__gnu_cxx::theta_1f](#) (float __nu, float __x)
- long double [__gnu_cxx::theta_1l](#) (long double __nu, long double __x)
- template<typename _Tpnu, typename _Tp >
 [__gnu_cxx::fp_promote_t<_Tpnu, _Tp>](#) [__gnu_cxx::theta_2](#) (_Tpnu __nu, _Tp __x)
- float [__gnu_cxx::theta_2f](#) (float __nu, float __x)
- long double [__gnu_cxx::theta_2l](#) (long double __nu, long double __x)
- template<typename _Tpnu, typename _Tp >
 [__gnu_cxx::fp_promote_t<_Tpnu, _Tp>](#) [__gnu_cxx::theta_3](#) (_Tpnu __nu, _Tp __x)
- float [__gnu_cxx::theta_3f](#) (float __nu, float __x)
- long double [__gnu_cxx::theta_3l](#) (long double __nu, long double __x)
- template<typename _Tpnu, typename _Tp >
 [__gnu_cxx::fp_promote_t<_Tpnu, _Tp>](#) [__gnu_cxx::theta_4](#) (_Tpnu __nu, _Tp __x)
- float [__gnu_cxx::theta_4f](#) (float __nu, float __x)
- long double [__gnu_cxx::theta_4l](#) (long double __nu, long double __x)
- template<typename _Tp_k, typename _Tp >
 [__gnu_cxx::fp_promote_t<_Tp_k, _Tp>](#) [__gnu_cxx::theta_c](#) (_Tp_k __k, _Tp __x)
- float [__gnu_cxx::theta_cf](#) (float __k, float __x)
- long double [__gnu_cxx::theta_cl](#) (long double __k, long double __x)
- template<typename _Tp_k, typename _Tp >
 [__gnu_cxx::fp_promote_t<_Tp_k, _Tp>](#) [__gnu_cxx::theta_d](#) (_Tp_k __k, _Tp __x)
- float [__gnu_cxx::theta_df](#) (float __k, float __x)
- long double [__gnu_cxx::theta_dl](#) (long double __k, long double __x)
- template<typename _Tp_k, typename _Tp >
 [__gnu_cxx::fp_promote_t<_Tp_k, _Tp>](#) [__gnu_cxx::theta_n](#) (_Tp_k __k, _Tp __x)
- float [__gnu_cxx::theta_nf](#) (float __k, float __x)
- long double [__gnu_cxx::theta_nl](#) (long double __k, long double __x)
- template<typename _Tp_k, typename _Tp >
 [__gnu_cxx::fp_promote_t<_Tp_k, _Tp>](#) [__gnu_cxx::theta_s](#) (_Tp_k __k, _Tp __x)
- float [__gnu_cxx::theta_sf](#) (float __k, float __x)
- long double [__gnu_cxx::theta_sl](#) (long double __k, long double __x)
- template<typename _Tpa, typename _Tpc, typename _Tp >
 [__gnu_cxx::fp_promote_t<_Tpa, _Tpc, _Tp>](#) [__gnu_cxx::tricomu](#) (_Tpa __a, _Tpc __c, _Tp __x)

- float [__gnu_cxx::tricomi_uf](#) (float __a, float __c, float __x)
- long double [__gnu_cxx::tricomi_ul](#) (long double __a, long double __c, long double __x)
- template<typename _Ta, typename _Tb, typename _Tp >
[__gnu_cxx::fp_promote_t<_Ta, _Tb, _Tp>](#) [__gnu_cxx::weibull_p](#) (_Ta __a, _Tb __b, _Tp __x)
Return the Weibull cumulative probability density function.
- template<typename _Ta, typename _Tb, typename _Tp >
[__gnu_cxx::fp_promote_t<_Ta, _Tb, _Tp>](#) [__gnu_cxx::weibull_pdf](#) (_Ta __a, _Tb __b, _Tp __x)
Return the Weibull probability density function.
- template<typename _Trho, typename _Tphi >
[__gnu_cxx::fp_promote_t<_Trho, _Tphi>](#) [__gnu_cxx::zernike](#) (unsigned int __n, int __m, _Trho __rho, _Tphi __phi)
- float [__gnu_cxx::zernikef](#) (unsigned int __n, int __m, float __rho, float __phi)
- long double [__gnu_cxx::zernikel](#) (unsigned int __n, int __m, long double __rho, long double __phi)

7.2.1 Detailed Description

An extended collection of advanced mathematical special functions for GNU.

7.2.2 Function Documentation

7.2.2.1 [airy_ai\(\)](#) [1/2]

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::airy_ai (
    _Tp __x ) [inline]
```

Return the Airy function $Ai(x)$ of real argument x .

The Airy function is defined by:

$$Ai(x) = \frac{1}{\pi} \int_0^{\infty} \cos\left(\frac{t^3}{3} + xt\right) dt$$

Template Parameters

<code>_Tp</code>	The real type of the argument
------------------	-------------------------------

Parameters

<code>__x</code>	The argument
------------------	--------------

Definition at line 2902 of file specfun.h.

7.2.2.2 `airy_ai()` [2/2]

```
template<typename _Tp >
std::complex<__gnu_cxx::fp_promote_t<_Tp> > __gnu_cxx::airy_ai (
    std::complex< _Tp > __x ) [inline]
```

Return the Airy function $Ai(x)$ of complex argument `x`.

The Airy function is defined by:

$$Ai(x) = \frac{1}{\pi} \int_0^{\infty} \cos\left(\frac{t^3}{3} + xt\right) dt$$

Template Parameters

<code>_Tp</code>	The real type of the argument
------------------	-------------------------------

Parameters

<code>↔ __x</code>	The complex argument
------------------------	----------------------

Definition at line 2922 of file `specfun.h`.

7.2.2.3 `airy_aif()`

```
float __gnu_cxx::airy_aif (
    float __x ) [inline]
```

Return the Airy function $Ai(x)$ for `float` argument `x`.

See also

[airy_ai](#) for details.

Definition at line 2875 of file `specfun.h`.

7.2.2.4 `airy_ail()`

```
long double __gnu_cxx::airy_ail (
    long double __x ) [inline]
```

Return the Airy function $Ai(x)$ for long double argument `x`.

See also

[airy_ai](#) for details.

Definition at line 2885 of file `specfun.h`.

7.2.2.5 `airy_bi()` [1/2]

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::airy_bi (
    _Tp __x ) [inline]
```

Return the Airy function $Bi(x)$ of real argument `x`.

The Airy function is defined by:

$$Bi(x) = \frac{1}{\pi} \int_0^{\infty} \left[\exp\left(-\frac{t^3}{3} + xt\right) + \sin\left(\frac{t^3}{3} + xt\right) \right] dt$$

Template Parameters

<code>_Tp</code>	The real type of the argument
------------------	-------------------------------

Parameters

<code>__x</code>	The argument
------------------	--------------

Definition at line 2964 of file `specfun.h`.

7.2.2.6 `airy_bi()` [2/2]

```
template<typename _Tp >
std::complex<__gnu_cxx::fp_promote_t<_Tp> > __gnu_cxx::airy_bi (
    std::complex< _Tp > __x ) [inline]
```

Return the Airy function $Bi(x)$ of complex argument x .

The Airy function is defined by:

$$Bi(x) = \frac{1}{\pi} \int_0^{\infty} \left[\exp\left(-\frac{t^3}{3} + xt\right) + \sin\left(\frac{t^3}{3} + xt\right) \right] dt$$

Template Parameters

<code>_Tp</code>	The real type of the argument
------------------	-------------------------------

Parameters

<code>↔ _x</code>	The complex argument
-----------------------	----------------------

Definition at line 2985 of file specfun.h.

7.2.2.7 `airy_bif()`

```
float __gnu_cxx::airy_bif (
    float __x ) [inline]
```

Return the Airy function $Bi(x)$ for `float` argument x .

See also

[airy_bi](#) for details.

Definition at line 2936 of file specfun.h.

7.2.2.8 `airy_bil()`

```
long double __gnu_cxx::airy_bil (
    long double __x ) [inline]
```

Return the Airy function $Bi(x)$ for `long double` argument x .

See also

[airy_bi](#) for details.

Definition at line 2946 of file specfun.h.

7.2.2.9 assoc_legendre_q()

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::assoc_legendre_q (
    unsigned int __l,
    unsigned int __m,
    _Tp __x ) [inline]
```

Return the associated Legendre function $Q_l^m(x)$ of degree l , order m , and real argument x .

The associated Legendre function is derived from the Legendre function $Q_l(x)$ by the Rodrigues formula:

$$Q_l^m(x) = (1 - x^2)^{m/2} \frac{d^m}{dx^m} Q_l(x)$$

See also

legendre for details of the Legendre function of degree l

Note

$Q_l^m(x) = 0$ if $m > l$.

Template Parameters

<code>_Tp</code>	The floating-point type of the argument <code>__x</code> .
------------------	--

Parameters

<code>__l</code>	The degree <code>__l</code> ≥ 0 .
<code>__m</code>	The order <code>__m</code> .
<code>__x</code>	The argument, $\text{abs}(\text{__x}) \leq 1$.

Exceptions

<code>std::domain_error</code>	if $\text{abs}(\text{__x}) > 1$.
--------------------------------	-----------------------------------

Definition at line 4563 of file specfun.h.

7.2.2.10 assoc_legendre_qf()

```
float __gnu_cxx::assoc_legendre_qf (
    unsigned int __l,
```

```
unsigned int __m,
float __x ) [inline]
```

Return the associated Legendre function $Q_l^m(x)$ of degree `l`, order `m`, and `float` argument `x`.

See also

[assoc_legendre_q](#) for more details.

Definition at line 4527 of file `specfun.h`.

7.2.2.11 `assoc_legendre_ql()`

```
long double __gnu_cxx::assoc_legendre_ql (
    unsigned int __l,
    unsigned int __m,
    long double __x ) [inline]
```

Return the associated Legendre function $Q_l^m(x)$ of degree `l`, order `m`, and `long double` argument `x`.

See also

[assoc_legendre_q](#) for more details.

Definition at line 4538 of file `specfun.h`.

7.2.2.12 `bell()` [1/2]

```
template<typename _Tp >
std::vector<_Tp> __gnu_cxx::bell (
    unsigned int __n ) [inline]
```

Return a vector of the Bell numbers

$$B(n) = \sum_{k=0}^n S_n^{(k)}$$

where $S_n^{(k)}$ are the Stirling numbers of the second kind.

Definition at line 7226 of file `specfun.h`.

7.2.2.13 bell() [2/2]

```
template<typename _Tp , typename _Up >
_Up __gnu_cxx::bell (
    unsigned int __n,
    _Up __x ) [inline]
```

Evaluate the Bell polynomial

$$B(n, x) = \sum_{k=0}^n S_n^{(k)} x^k$$

where $S_n^{(k)}$ are the Stirling numbers of the second kind.

Definition at line 7238 of file specfun.h.

References std::__detail::__bell().

7.2.2.14 bernoulli() [1/2]

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::bernoulli (
    unsigned int __n ) [inline]
```

Return the Bernoulli number of integer order n.

The Bernoulli numbers are defined by

$$B_{2n} = (-1)^{n+1} 2 \frac{(2n)!}{(2\pi)^{2n}} \zeta(2n), B_1 = -1/2$$

All odd Bernoulli numbers except B_1 are zero.

Parameters

\leftrightarrow	The order.
$_n$	

Definition at line 4461 of file specfun.h.

7.2.2.15 bernoulli() [2/2]

```
template<typename _Tp >
_Tp __gnu_cxx::bernoulli (
    unsigned int __n,
    _Tp __x ) [inline]
```


Return the Bernoulli polynomial $B_n(x)$ of order n at argument x .

The values at 0 and 1 are equal to the corresponding Bernoulli number:

$$B_n(0) = B_n(1) = B_n$$

The derivative is proportional to the previous polynomial:

$$B'_n(x) = nB_{n-1}(x)$$

The series expansion for the Bernoulli polynomials is:

$$B_n(x) = \sum_{k=0}^n B_k \binom{n}{k} x^{n-k}$$

A useful argument promotion is:

$$B_n(x+1) - B_n(x) = nx^{n-1}$$

Definition at line 7073 of file specfun.h.

References std::__detail::__bernoulli().

7.2.2.16 bernoullif()

```
float __gnu_cxx::bernoullif (
    unsigned int __n ) [inline]
```

Return the Bernoulli number of integer order n as a `float`.

See also

[bernoulli](#) for details.

Definition at line 4434 of file specfun.h.

7.2.2.17 bernoullil()

```
long double __gnu_cxx::bernoullil (
    unsigned int __n ) [inline]
```

Return the Bernoulli number of integer order n as a `long double`.

See also

[bernoulli](#) for details.

Definition at line 4444 of file specfun.h.

7.2.2.18 binomial()

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::binomial (
    unsigned int __n,
    unsigned int __k ) [inline]
```

Return the binomial coefficient as a real number. The binomial coefficient is given by:

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

The binomial coefficients are generated by:

$$(1+t)^n = \sum_{k=0}^n \binom{n}{k} t^k$$

Parameters

\leftrightarrow __n	The first argument of the binomial coefficient.
\leftrightarrow __k	The second argument of the binomial coefficient.

Returns

The binomial coefficient.

Definition at line 4377 of file specfun.h.

7.2.2.19 binomial_p()

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::binomial_p (
    _Tp __p,
    unsigned int __n,
    unsigned int __k )
```

Return the binomial cumulative distribution function.

The binomial cumulative distribution function is related to the incomplete beta function:

$$P(k|n, p) = I_p(k, n - k + 1)$$

Parameters

\overleftarrow{p}	
\overleftarrow{n}	
\overleftarrow{k}	

Definition at line 6926 of file specfun.h.

7.2.2.20 `binomial_pdf()`

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::binomial_pdf (
    _Tp __p,
    unsigned int __n,
    unsigned int __k )
```

Return the binomial probability mass function.

The binomial cumulative distribution function is related to the incomplete beta function:

$$f(k|n, p) = \binom{n}{k} p^k (1 - p)^{n-k}$$

Parameters

\overleftarrow{p}	
\overleftarrow{n}	
\overleftarrow{k}	

Definition at line 6905 of file specfun.h.

7.2.2.21 `binomialf()`

```
float __gnu_cxx::binomialf (
    unsigned int __n,
    unsigned int __k ) [inline]
```

Return the binomial coefficient as a float.

See also

[binomial](#) for details.

Definition at line 4348 of file specfun.h.

7.2.2.22 binomiall()

```
long double __gnu_cxx::binomiall (
    unsigned int __n,
    unsigned int __k ) [inline]
```

Return the binomial coefficient as a long double.

See also

[binomial](#) for details.

Definition at line 4357 of file specfun.h.

7.2.2.23 bose_einstein()

```
template<typename _Tps , typename _Tp >
__gnu_cxx::fp_promote_t<_Tps, _Tp> __gnu_cxx::bose_einstein (
    _Tps __s,
    _Tp __x ) [inline]
```

Return the Bose-Einstein integral of integer or real order s and real argument x.

See also

https://en.wikipedia.org/wiki/Clausen_function
<http://dlmf.nist.gov/25.12.16>

$$G_s(x) = \frac{1}{\Gamma(s+1)} \int_0^\infty \frac{t^s}{e^{t-x} - 1} dt = Li_{s+1}(e^x)$$

Parameters

<code>__s</code>	The order s >= 0.
<code>__x</code>	The real argument.

Returns

The real Bose-Einstein integral $G_s(x)$,

Definition at line 6303 of file specfun.h.

7.2.2.24 `bose_einsteinf()`

```
float __gnu_cxx::bose_einsteinf (
    float __s,
    float __x ) [inline]
```

Return the Bose-Einstein integral of `float` order `s` and argument `x`.

See also

[bose_einstein](#) for details.

Definition at line 6273 of file specfun.h.

7.2.2.25 `bose_einsteinl()`

```
long double __gnu_cxx::bose_einsteinl (
    long double __s,
    long double __x ) [inline]
```

Return the Bose-Einstein integral of `long double` order `s` and argument `x`.

See also

[bose_einstein](#) for details.

Definition at line 6283 of file specfun.h.

7.2.2.26 `chebyshev_t()`

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::chebyshev_t (
    unsigned int __n,
    _Tp __x ) [inline]
```

Return the Chebyshev polynomial of the first kind $T_n(x)$ of non-negative order `n` and real argument `x`.

The Chebyshev polynomial of the first kind is defined by:

$$T_n(x) = \cos(n\theta)$$

where $\theta = \arccos(x)$, $-1 \leq x \leq +1$.

Template Parameters

<code>__Tp</code>	The real type of the argument
-------------------	-------------------------------

Parameters

<code>__n</code>	The non-negative integral order
<code>__x</code>	The real argument $-1 \leq x \leq +1$

Definition at line 2133 of file specfun.h.

7.2.2.27 `chebyshev_tf()`

```
float __gnu_cxx::chebyshev_tf (
    unsigned int __n,
    float __x ) [inline]
```

Return the Chebyshev polynomials of the first kind $T_n(x)$ of non-negative order `n` and `float` argument `x`.

See also

[chebyshev_t](#) for details.

Definition at line 2104 of file specfun.h.

7.2.2.28 `chebyshev_tl()`

```
long double __gnu_cxx::chebyshev_tl (
    unsigned int __n,
    long double __x ) [inline]
```

Return the Chebyshev polynomials of the first kind $T_n(x)$ of non-negative order `n` and real argument `x`.

See also

[chebyshev_t](#) for details.

Definition at line 2114 of file specfun.h.

7.2.2.29 `chebyshev_u()`

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::chebyshev_u (
    unsigned int __n,
    _Tp __x ) [inline]
```

Return the Chebyshev polynomial of the second kind $U_n(x)$ of non-negative order `n` and real argument `x`.

The Chebyshev polynomial of the second kind is defined by:

$$U_n(x) = \frac{\sin[(n+1)\theta]}{\sin(\theta)}$$

where $\theta = \arccos(x)$, $-1 \leq x \leq +1$.

Template Parameters

<code>_Tp</code>	The real type of the argument
------------------	-------------------------------

Parameters

<code>↵ _n</code>	The non-negative integral order
<code>↵ _x</code>	The real argument $-1 \leq x \leq +1$

Definition at line 2177 of file `specfun.h`.

7.2.2.30 `chebyshev_uf()`

```
float __gnu_cxx::chebyshev_uf (
    unsigned int __n,
    float __x ) [inline]
```

Return the Chebyshev polynomials of the second kind $U_n(x)$ of non-negative order `n` and `float` argument `x`.

See also

[chebyshev_u](#) for details.

Definition at line 2148 of file `specfun.h`.

7.2.2.31 `chebyshev_ul()`

```
long double __gnu_cxx::chebyshev_ul (
    unsigned int __n,
    long double __x ) [inline]
```

Return the Chebyshev polynomials of the second kind $U_n(x)$ of non-negative order n and real argument x .

See also

[chebyshev_u](#) for details.

Definition at line 2158 of file `specfun.h`.

7.2.2.32 `chebyshev_v()`

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::chebyshev_v (
    unsigned int __n,
    _Tp __x ) [inline]
```

Return the Chebyshev polynomial of the third kind $V_n(x)$ of non-negative order n and real argument x .

The Chebyshev polynomial of the third kind is defined by:

$$V_n(x) = \frac{\cos \left[\left(n + \frac{1}{2} \right) \theta \right]}{\cos \left(\frac{\theta}{2} \right)}$$

where $\theta = \arccos(x)$, $-1 \leq x \leq +1$.

Template Parameters

<code>_Tp</code>	The real type of the argument
------------------	-------------------------------

Parameters

<code>↵ _n</code>	The non-negative integral order
<code>↵ _x</code>	The real argument $-1 \leq x \leq +1$

Definition at line 2222 of file `specfun.h`.

7.2.2.33 chebyshev_vf()

```
float __gnu_cxx::chebyshev_vf (
    unsigned int __n,
    float __x ) [inline]
```

Return the Chebyshev polynomials of the third kind $V_n(x)$ of non-negative order `n` and `float` argument `x`.

See also

[chebyshev_v](#) for details.

Definition at line 2192 of file specfun.h.

7.2.2.34 chebyshev_vl()

```
long double __gnu_cxx::chebyshev_vl (
    unsigned int __n,
    long double __x ) [inline]
```

Return the Chebyshev polynomials of the third kind $V_n(x)$ of non-negative order `n` and real argument `x`.

See also

[chebyshev_v](#) for details.

Definition at line 2202 of file specfun.h.

7.2.2.35 chebyshev_w()

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::chebyshev_w (
    unsigned int __n,
    _Tp __x ) [inline]
```

Return the Chebyshev polynomial of the fourth kind $W_n(x)$ of non-negative order `n` and real argument `x`.

The Chebyshev polynomial of the fourth kind is defined by:

$$W_n(x) = \frac{\sin \left[\left(n + \frac{1}{2} \right) \theta \right]}{\sin \left(\frac{\theta}{2} \right)}$$

where $\theta = \arccos(x)$, $-1 \leq x \leq +1$.

Template Parameters

<code>__Tp</code>	The real type of the argument
-------------------	-------------------------------

Parameters

<code>__n</code>	The non-negative integral order
<code>__x</code>	The real argument $-1 \leq x \leq +1$

Definition at line 2267 of file specfun.h.

7.2.2.36 `chebyshev_wf()`

```
float __gnu_cxx::chebyshev_wf (
    unsigned int __n,
    float __x ) [inline]
```

Return the Chebyshev polynomials of the fourth kind $W_n(x)$ of non-negative order `n` and `float` argument `x`.

See also

[chebyshev_w](#) for details.

Definition at line 2237 of file specfun.h.

7.2.2.37 `chebyshev_wl()`

```
long double __gnu_cxx::chebyshev_wl (
    unsigned int __n,
    long double __x ) [inline]
```

Return the Chebyshev polynomials of the fourth kind $W_n(x)$ of non-negative order `n` and real argument `x`.

See also

[chebyshev_w](#) for details.

Definition at line 2247 of file specfun.h.

7.2.2.38 `clausen()` [1/2]

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::clausen (
    unsigned int __m,
    _Tp __x ) [inline]
```

Return the Clausen function $C_m(x)$ of integer order m and real argument x .

The Clausen function is defined by

$$C_m(x) = Sl_m(x) = \sum_{k=1}^{\infty} \frac{\sin(kx)}{k^m} \text{ for even } m = Cl_m(x) = \sum_{k=1}^{\infty} \frac{\cos(kx)}{k^m} \text{ for odd } m$$

Template Parameters

<code>_Tp</code>	The real type of the argument
------------------	-------------------------------

Parameters

<code>↵ _m</code>	The integral order
<code>↵ _x</code>	The real argument

Definition at line 5554 of file `specfun.h`.

7.2.2.39 `clausen()` [2/2]

```
template<typename _Tp >
std::complex<__gnu_cxx::fp_promote_t<_Tp> > __gnu_cxx::clausen (
    unsigned int __m,
    std::complex< _Tp > __z ) [inline]
```

Return the Clausen function $C_m(z)$ of integer order m and complex argument z .

The Clausen function is defined by

$$C_m(z) = Sl_m(z) = \sum_{k=1}^{\infty} \frac{\sin(kz)}{k^m} \text{ for even } m = Cl_m(z) = \sum_{k=1}^{\infty} \frac{\cos(kz)}{k^m} \text{ for odd } m$$

Template Parameters

<code>_Tp</code>	The real type of the complex components
------------------	---

Parameters

\leftrightarrow _m	The integral order
\leftrightarrow _z	The complex argument

Definition at line 5598 of file specfun.h.

7.2.2.40 clausen_cl()

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::clausen_cl (
    unsigned int __m,
    _Tp __x ) [inline]
```

Return the Clausen cosine function $Cl_m(x)$ of order `m` and real argument `x`.

The Clausen cosine function is defined by

$$Cl_m(x) = \sum_{k=1}^{\infty} \frac{\cos(kx)}{k^m}$$

Template Parameters

_Tp	The real type of the argument
-----	-------------------------------

Parameters

\leftrightarrow _m	The unsigned integer order
\leftrightarrow _x	The real argument

Definition at line 5509 of file specfun.h.

7.2.2.41 clausen_clf()

```
float __gnu_cxx::clausen_clf (
    unsigned int __m,
    float __x ) [inline]
```

Return the Clausen cosine function $Cl_m(x)$ of order `m` and `float` argument `x`.

See also

[clausen_cl](#) for details.

Definition at line 5481 of file specfun.h.

7.2.2.42 clausen_cll()

```
long double __gnu_cxx::clausen_cll (
    unsigned int __m,
    long double __x ) [inline]
```

Return the Clausen cosine function $Cl_m(x)$ of order `m` and `long double` argument `x`.

See also

[clausen_cl](#) for details.

Definition at line 5491 of file specfun.h.

7.2.2.43 clausen_sl()

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::clausen_sl (
    unsigned int __m,
    _Tp __x ) [inline]
```

Return the Clausen sine function $Sl_m(x)$ of order `m` and real argument `x`.

The Clausen sine function is defined by

$$Sl_m(x) = \sum_{k=1}^{\infty} \frac{\sin(kx)}{k^m}$$

Template Parameters

<code>_Tp</code>	The real type of the argument
------------------	-------------------------------

Parameters

<code>__m</code>	The unsigned integer order
<code>__x</code>	The real argument

Definition at line 5466 of file specfun.h.

7.2.2.44 `clausen_slf()`

```
float __gnu_cxx::clausen_slf (
    unsigned int __m,
    float __x ) [inline]
```

Return the Clausen sine function $Sl_m(x)$ of order `m` and `float` argument `x`.

See also

[clausen_sl](#) for details.

Definition at line 5438 of file specfun.h.

7.2.2.45 `clausen_sll()`

```
long double __gnu_cxx::clausen_sll (
    unsigned int __m,
    long double __x ) [inline]
```

Return the Clausen sine function $Sl_m(x)$ of order `m` and `long double` argument `x`.

See also

[clausen_sl](#) for details.

Definition at line 5448 of file specfun.h.

7.2.2.46 `clausenf()` [1/2]

```
float __gnu_cxx::clausenf (
    unsigned int __m,
    float __x ) [inline]
```

Return the Clausen function $C_m(x)$ of integer order `m` and `float` argument `x`.

See also

[clausen](#) for details.

Definition at line 5524 of file specfun.h.

7.2.2.47 clausenf() [2/2]

```
std::complex<float> __gnu_cxx::clausenf (
    unsigned int __m,
    std::complex< float > __z ) [inline]
```

Return the Clausen function $C_m(z)$ of integer order m and `std::complex<float>` argument z .

See also

[clausen](#) for details.

Definition at line 5569 of file `specfun.h`.

7.2.2.48 clausenl() [1/2]

```
long double __gnu_cxx::clausenl (
    unsigned int __m,
    long double __x ) [inline]
```

Return the Clausen function $C_m(x)$ of integer order m and `long double` argument x .

See also

[clausen](#) for details.

Definition at line 5534 of file `specfun.h`.

7.2.2.49 clausenl() [2/2]

```
std::complex<long double> __gnu_cxx::clausenl (
    unsigned int __m,
    std::complex< long double > __z ) [inline]
```

Return the Clausen function $C_m(z)$ of integer order m and `std::complex<long double>` argument z .

See also

[clausen](#) for details.

Definition at line 5579 of file `specfun.h`.

7.2.2.50 comp_ellint_d()

```
template<typename _Tk >
__gnu_cxx::fp_promote_t<_Tk> __gnu_cxx::comp_ellint_d (
    _Tk __k ) [inline]
```

Return the complete Legendre elliptic integral $D(k)$ of real modulus k .

The complete Legendre elliptic integral D is defined by

$$D(k) = \int_0^{\pi/2} \frac{\sin^2 \theta d\theta}{\sqrt{1 - k^2 \sin^2 \theta}}$$

Template Parameters

<code>__Tk</code>	The type of the modulus k .
-------------------	-------------------------------

Parameters

<code>__k</code>	The modulus $-1 \leq \text{__k} \leq +1$.
------------------	--

Definition at line 4730 of file `specfun.h`.

7.2.2.51 `comp_ellint_df()`

```
float __gnu_cxx::comp_ellint_df (
    float __k ) [inline]
```

Return the complete Legendre elliptic integral $D(k)$ of `float` modulus k .

See also

[comp_ellint_d](#) for details.

Definition at line 4703 of file `specfun.h`.

7.2.2.52 `comp_ellint_dl()`

```
long double __gnu_cxx::comp_ellint_dl (
    long double __k ) [inline]
```

Return the complete Legendre elliptic integral $D(k)$ of `long double` modulus k .

See also

[comp_ellint_d](#) for details.

Definition at line 4713 of file `specfun.h`.

7.2.2.53 comp_ellint_rf() [1/3]

```
float __gnu_cxx::comp_ellint_rf (
    float __x,
    float __y ) [inline]
```

Return the complete Carlson elliptic function $R_F(x, y, z)$ for `float` arguments.

See also

[comp_ellint_rf](#) for details.

Definition at line 3245 of file specfun.h.

7.2.2.54 comp_ellint_rf() [2/3]

```
long double __gnu_cxx::comp_ellint_rf (
    long double __x,
    long double __y ) [inline]
```

Return the complete Carlson elliptic function $R_F(x, y)$ for `long double` arguments.

See also

[comp_ellint_rf](#) for details.

Definition at line 3255 of file specfun.h.

7.2.2.55 comp_ellint_rf() [3/3]

```
template<typename _Tx , typename _Ty >
__gnu_cxx::fp_promote_t<_Tx, _Ty> __gnu_cxx::comp_ellint_rf (
    _Tx __x,
    _Ty __y ) [inline]
```

Return the complete Carlson elliptic function $R_F(x, y)$ for real arguments.

The complete Carlson elliptic function of the first kind is defined by:

$$R_F(x, y) = R_F(x, y, y) = \frac{1}{2} \int_0^\infty \frac{dt}{(t+x)^{1/2}(t+y)}$$

Parameters

\leftrightarrow _x	The first argument.
\leftrightarrow _y	The second argument.

Definition at line 3273 of file specfun.h.

7.2.2.56 comp_ellint_rg() [1/3]

```
float __gnu_cxx::comp_ellint_rg (
    float __x,
    float __y ) [inline]
```

Return the Carlson complementary elliptic function $R_G(x, y)$.

See also

[comp_ellint_rg](#) for details.

Definition at line 3478 of file specfun.h.

7.2.2.57 comp_ellint_rg() [2/3]

```
long double __gnu_cxx::comp_ellint_rg (
    long double __x,
    long double __y ) [inline]
```

Return the Carlson complementary elliptic function $R_G(x, y)$.

See also

[comp_ellint_rg](#) for details.

Definition at line 3487 of file specfun.h.

7.2.2.58 comp_ellint_rg() [3/3]

```
template<typename _Tx , typename _Ty >
__gnu_cxx::fp_promote_t<_Tx, _Ty> __gnu_cxx::comp_ellint_rg (
    _Tx __x,
    _Ty __y ) [inline]
```

Return the complete Carlson elliptic function $R_G(x, y)$ for real arguments.

The complete Carlson elliptic function is defined by:

$$R_G(x, y) = R_G(x, y, y) = \frac{1}{4} \int_0^\infty dt t(t+x)^{-1/2}(t+y)^{-1} \left(\frac{x}{t+x} + \frac{2y}{t+y} \right)$$

Parameters

\leftrightarrow _x	The first argument.
\leftrightarrow _y	The second argument.

Definition at line 3506 of file specfun.h.

7.2.2.59 conf_hyperg()

```
template<typename _Tpa , typename _Tpc , typename _Tp >
__gnu_cxx::fp_promote_t<_Tpa, _Tpc, _Tp> __gnu_cxx::conf_hyperg (
    _Tpa __a,
    _Tpc __c,
    _Tp __x ) [inline]
```

Return the confluent hypergeometric function ${}_1F_1(a; c; x)$ of real numerator parameter a , denominator parameter c , and argument x .

The confluent hypergeometric function is defined by

$${}_1F_1(a; c; x) = \sum_{n=0}^{\infty} \frac{(a)_n x^n}{(c)_n n!}$$

where the Pochhammer symbol is $(x)_k = (x)(x+1)\dots(x+k-1)$, $(x)_0 = 1$

Parameters

\leftrightarrow _a	The numerator parameter
\leftrightarrow _c	The denominator parameter
\leftrightarrow _x	The argument

Definition at line 1511 of file specfun.h.

7.2.2.60 conf_hyperg_lim()

```
template<typename _Tpc , typename _Tp >
__gnu_cxx::fp_promote_t<_Tpc, _Tp> __gnu_cxx::conf_hyperg_lim (
```

```

_Tpc __c,
_Tp __x ) [inline]

```

Return the confluent hypergeometric limit function ${}_0F_1(; c; x)$ of real numerator parameter `c` and argument `x`.

The confluent hypergeometric limit function is defined by

$${}_0F_1(; c; x) = \sum_{n=0}^{\infty} \frac{x^n}{(c)_n n!}$$

where the Pochhammer symbol is $(x)_k = (x)(x+1)\dots(x+k-1)$, $(x)_0 = 1$

Parameters

<code>__c</code>	The denominator parameter
<code>__x</code>	The argument

Definition at line 1656 of file specfun.h.

7.2.2.61 `conf_hyperg_limf()`

```

float __gnu_cxx::conf_hyperg_limf (
    float __c,
    float __x ) [inline]

```

Return the confluent hypergeometric limit function ${}_0F_1(; c; x)$ of `float` numerator parameter `c` and argument `x`.

See also

[conf_hyperg_lim](#) for details.

Definition at line 1627 of file specfun.h.

7.2.2.62 `conf_hyperg_liml()`

```

long double __gnu_cxx::conf_hyperg_liml (
    long double __c,
    long double __x ) [inline]

```

Return the confluent hypergeometric limit function ${}_0F_1(; c; x)$ of `long double` numerator parameter `c` and argument `x`.

See also

[conf_hyperg_lim](#) for details.

Definition at line 1637 of file specfun.h.

7.2.2.63 `conf_hypergf()`

```
float __gnu_cxx::conf_hypergf (
    float __a,
    float __c,
    float __x ) [inline]
```

Return the confluent hypergeometric function ${}_1F_1(a; c; x)$ of `float` numerator parameter `a`, denominator parameter `c`, and argument `x`.

See also

[conf_hyperg](#) for details.

Definition at line 1479 of file `specfun.h`.

7.2.2.64 `conf_hypergl()`

```
long double __gnu_cxx::conf_hypergl (
    long double __a,
    long double __c,
    long double __x ) [inline]
```

Return the confluent hypergeometric function ${}_1F_1(a; c; x)$ of `long double` numerator parameter `a`, denominator parameter `c`, and argument `x`.

See also

[conf_hyperg](#) for details.

Definition at line 1490 of file `specfun.h`.

7.2.2.65 `cos_pi()`

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::cos_pi (
    _Tp __x ) [inline]
```

Return the reperiodized cosine function $\cos_\pi(x)$ for real argument `x`.

The reperiodized cosine function is defined by:

$$\cos_\pi(x) = \cos(\pi x)$$

Template Parameters

<code>_Tp</code>	The floating-point type of the argument <code>__x</code> .
------------------	--

Parameters

<code>__x</code>	The argument
------------------	--------------

Definition at line 6429 of file `specfun.h`.

7.2.2.66 `cos_pif()`

```
float __gnu_cxx::cos_pif (
    float __x ) [inline]
```

Return the reperiodized cosine function $\cos_\pi(x)$ for `float` argument `x`.

See also

[cos_pi](#) for more details.

Definition at line 6402 of file `specfun.h`.

7.2.2.67 `cos_pil()`

```
long double __gnu_cxx::cos_pil (
    long double __x ) [inline]
```

Return the reperiodized cosine function $\cos_\pi(x)$ for `long double` argument `x`.

See also

[cos_pi](#) for more details.

Definition at line 6412 of file `specfun.h`.

7.2.2.68 `cosh_pi()`

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::cosh_pi (
    _Tp __x ) [inline]
```

Return the reperiodized hyperbolic cosine function $\cosh_\pi(x)$ for real argument `x`.

The reperiodized hyperbolic cosine function is defined by:

$$\cosh_\pi(x) = \cosh(\pi x)$$

Template Parameters

<code>_Tp</code>	The floating-point type of the argument <code>__x</code> .
------------------	--

Parameters

<code>__x</code>	The argument
------------------	--------------

Definition at line 6471 of file `specfun.h`.

7.2.2.69 `cosh_pif()`

```
float __gnu_cxx::cosh_pif (
    float __x ) [inline]
```

Return the reperiodized hyperbolic cosine function $\cosh_{\pi}(x)$ for `float` argument `x`.

See also

[cosh_pi](#) for more details.

Definition at line 6444 of file `specfun.h`.

7.2.2.70 `cosh_pil()`

```
long double __gnu_cxx::cosh_pil (
    long double __x ) [inline]
```

Return the reperiodized hyperbolic cosine function $\cosh_{\pi}(x)$ for `long double` argument `x`.

See also

[cosh_pi](#) for more details.

Definition at line 6454 of file `specfun.h`.

7.2.2.71 `coshint()`

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::coshint (
    _Tp __x ) [inline]
```

Return the hyperbolic cosine integral $Chi(x)$ of real argument `x`.

The hyperbolic cosine integral is defined by

$$Chi(x) = - \int_x^{\infty} \frac{\cosh(t)}{t} dt = \gamma_E + \ln(x) + \int_0^x \frac{\cosh(t) - 1}{t} dt$$

Template Parameters

<code>_Tp</code>	The type of the real argument
------------------	-------------------------------

Parameters

<code>_↔ _x</code>	The real argument
------------------------	-------------------

Definition at line 1939 of file `specfun.h`.

7.2.2.72 `coshintf()`

```
float __gnu_cxx::coshintf (
    float __x ) [inline]
```

Return the hyperbolic cosine integral of `float` argument `x`.

See also

[coshint](#) for details.

Definition at line 1911 of file `specfun.h`.

7.2.2.73 `coshintl()`

```
long double __gnu_cxx::coshintl (
    long double __x ) [inline]
```

Return the hyperbolic cosine integral $Chi(x)$ of `long double` argument `x`.

See also

[coshint](#) for details.

Definition at line 1921 of file `specfun.h`.

7.2.2.74 `cosint()`

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::cosint (
    _Tp __x ) [inline]
```

Return the cosine integral $Ci(x)$ of real argument `x`.

The cosine integral is defined by

$$Ci(x) = - \int_x^\infty \frac{\cos(t)}{t} dt = \gamma_E + \ln(x) + \int_0^x \frac{\cos(t) - 1}{t} dt$$

Parameters

<code>_↔</code>	The real upper integration limit
<code>_x</code>	

Definition at line 1856 of file specfun.h.

7.2.2.75 `cosintf()`

```
float __gnu_cxx::cosintf (
    float __x ) [inline]
```

Return the cosine integral $Ci(x)$ of float argument `x`.

See also

[cosint](#) for details.

Definition at line 1830 of file specfun.h.

7.2.2.76 `cosintl()`

```
long double __gnu_cxx::cosintl (
    long double __x ) [inline]
```

Return the cosine integral $Ci(x)$ of long double argument `x`.

See also

[cosint](#) for details.

Definition at line 1840 of file specfun.h.

7.2.2.77 `cyl_hankel_1()` [1/2]

```
template<typename _Tpnu , typename _Tp >
std::complex<__gnu_cxx::fp_promote_t<_Tpnu, _Tp> > __gnu_cxx::cyl_hankel_1 (
    _Tpnu __nu,
    _Tp __z ) [inline]
```

Return the cylindrical Hankel function of the first kind $H_n^{(1)}(x)$ of real order ν and argument $x \geq 0$.

The spherical Hankel function of the first kind is defined by:

$$H_\nu^{(1)}(x) = J_\nu(x) + iN_\nu(x)$$

where $J_\nu(x)$ and $N_\nu(x)$ are the cylindrical Bessel and Neumann functions respectively (see `cyl_bessel` and `cyl_↔neumann`).

Template Parameters

<code>_Tp</code>	The real type of the argument
------------------	-------------------------------

Parameters

<code>__nu</code>	The real order
<code>__z</code>	The real argument

Definition at line 2629 of file specfun.h.

7.2.2.78 `cyl_hankel_1()` [2/2]

```
template<typename _Tpnu , typename _Tp >
std::complex<__gnu_cxx::fp_promote_t<_Tpnu, _Tp> > __gnu_cxx::cyl_hankel_1 (
    std::complex< _Tpnu > __nu,
    std::complex< _Tp > __x ) [inline]
```

Return the complex cylindrical Hankel function of the first kind $H_\nu^{(1)}(x)$ of complex order ν and argument x .

The cylindrical Hankel function of the first kind is defined by

$$H_\nu^{(1)}(x) = J_\nu(x) + iN_\nu(x)$$

Template Parameters

<code>_Tpnu</code>	The complex type of the order
<code>_Tp</code>	The complex type of the argument

Parameters

<code>__nu</code>	The complex order
<code>__x</code>	The complex argument

Definition at line 5007 of file specfun.h.

7.2.2.79 `cyl_hankel_1f()` [1/2]

```
std::complex<float> __gnu_cxx::cyl_hankel_1f (
    float __nu,
    float __z ) [inline]
```

Return the cylindrical Hankel function of the first kind $H_\nu^{(1)}(x)$ of `float` order ν and argument $x \geq 0$.

See also

[cyl_hankel_1](#) for details.

Definition at line 2597 of file specfun.h.

7.2.2.80 `cyl_hankel_1f()` [2/2]

```
std::complex<float> __gnu_cxx::cyl_hankel_1f (
    std::complex< float > __nu,
    std::complex< float > __x ) [inline]
```

Return the complex cylindrical Hankel function of the first kind $H_\nu^{(1)}(x)$ of `std::complex<float>` order ν and argument x .

See also

[cyl_hankel_1](#) for more details.

Definition at line 4976 of file specfun.h.

7.2.2.81 `cyl_hankel_1l()` [1/2]

```
std::complex<long double> __gnu_cxx::cyl_hankel_1l (
    long double __nu,
    long double __z ) [inline]
```

Return the cylindrical Hankel function of the first kind $H_\nu^{(1)}(x)$ of `long double` order ν and argument $x \geq 0$.

See also

[cyl_hankel_1](#) for details.

Definition at line 2608 of file specfun.h.

7.2.2.82 `cyl_hankel_1l()` [2/2]

```
std::complex<long double> __gnu_cxx::cyl_hankel_1l (
    std::complex< long double > __nu,
    std::complex< long double > __x ) [inline]
```

Return the complex cylindrical Hankel function of the first kind $H_\nu^{(1)}(x)$ of `std::complex<long double>` order ν and argument x .

See also

[cyl_hankel_1](#) for more details.

Definition at line 4987 of file `specfun.h`.

7.2.2.83 `cyl_hankel_2()` [1/2]

```
template<typename _Tpnu , typename _Tp >
std::complex<__gnu_cxx::fp_promote_t<_Tpnu, _Tp> > __gnu_cxx::cyl_hankel_2 (
    _Tpnu __nu,
    _Tp __z ) [inline]
```

Return the cylindrical Hankel function of the second kind $H_n^{(2)}(x)$ of real order ν and argument $x \geq 0$.

The cylindrical Hankel function of the second kind is defined by:

$$H_\nu^{(2)}(x) = J_\nu(x) - iN_\nu(x)$$

where $J_\nu(x)$ and $N_\nu(x)$ are the cylindrical Bessel and Neumann functions respectively (see `cyl_bessel` and `cyl_neumann`).

Template Parameters

<code>_Tp</code>	The real type of the argument
------------------	-------------------------------

Parameters

<code>__nu</code>	The real order
<code>__z</code>	The real argument

Definition at line 2677 of file `specfun.h`.

7.2.2.84 `cyl_hankel_2()` [2/2]

```
template<typename _Tpnu , typename _Tp >
std::complex<__gnu_cxx::fp_promote_t<_Tpnu, _Tp> > __gnu_cxx::cyl_hankel_2 (
    std::complex< _Tpnu > __nu,
    std::complex< _Tp > __x ) [inline]
```

Return the complex cylindrical Hankel function of the second kind $H_\nu^{(2)}(x)$ of complex order ν and argument x .

The cylindrical Hankel function of the second kind is defined by

$$H_\nu^{(2)}(x) = J_\nu(x) - iN_\nu(x)$$

Template Parameters

<code>_Tpnu</code>	The complex type of the order
<code>_Tp</code>	The complex type of the argument

Parameters

<code>__nu</code>	The complex order
<code>__x</code>	The complex argument

Definition at line 5054 of file `specfun.h`.

7.2.2.85 `cyl_hankel_2f()` [1/2]

```
std::complex<float> __gnu_cxx::cyl_hankel_2f (
    float __nu,
    float __z ) [inline]
```

Return the cylindrical Hankel function of the second kind $H_\nu^{(2)}(x)$ of `float` order ν and argument $x \geq 0$.

See also

[cyl_hankel_2](#) for details.

Definition at line 2645 of file `specfun.h`.

7.2.2.86 `cyl_hankel_2f()` [2/2]

```
std::complex<float> __gnu_cxx::cyl_hankel_2f (
    std::complex< float > __nu,
    std::complex< float > __x ) [inline]
```

Return the complex cylindrical Hankel function of the second kind $H_\nu^{(2)}(x)$ of `std::complex<float>` order ν and argument x .

See also

[cyl_hankel_2](#) for more details.

Definition at line 5023 of file specfun.h.

7.2.2.87 `cyl_hankel_2l()` [1/2]

```
std::complex<long double> __gnu_cxx::cyl_hankel_2l (
    long double __nu,
    long double __z ) [inline]
```

Return the cylindrical Hankel function of the second kind $H_\nu^{(2)}(x)$ of `long double` order ν and argument $x \geq 0$.

See also

[cyl_hankel_2](#) for details.

Definition at line 2656 of file specfun.h.

7.2.2.88 `cyl_hankel_2l()` [2/2]

```
std::complex<long double> __gnu_cxx::cyl_hankel_2l (
    std::complex< long double > __nu,
    std::complex< long double > __x ) [inline]
```

Return the complex cylindrical Hankel function of the second kind $H_\nu^{(2)}(x)$ of `std::complex<long double>` order ν and argument x .

See also

[cyl_hankel_2](#) for more details.

Definition at line 5034 of file specfun.h.

7.2.2.89 dawson()

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::dawson (
    _Tp __x ) [inline]
```

Return the Dawson integral, $F(x)$, for real argument x .

The Dawson integral is defined by:

$$F(x) = e^{-x^2} \int_0^x e^{y^2} dy$$

and it's derivative is:

$$F'(x) = 1 - 2xF(x)$$

Parameters

\leftrightarrow	The argument $-inf < x < inf$.
<code>__x</code>	

Definition at line 3946 of file specfun.h.

7.2.2.90 dawsonf()

```
float __gnu_cxx::dawsonf (
    float __x ) [inline]
```

Return the Dawson integral, $F(x)$, for `float` argument x .

See also

[dawson](#) for details.

Definition at line 3917 of file specfun.h.

7.2.2.91 dawsonl()

```
long double __gnu_cxx::dawsonl (
    long double __x ) [inline]
```

Return the Dawson integral, $F(x)$, for `long double` argument x .

See also

[dawson](#) for details.

Definition at line 3927 of file specfun.h.

7.2.2.92 `debye()`

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::debye (
    unsigned int __n,
    _Tp __x ) [inline]
```

Return the Debye function $D_n(x)$ of positive order `n` and real argument `x`.

The Debye function is defined by:

$$D_n(x) = \frac{n}{x^n} \int_0^x \frac{t^n}{e^t - 1} dt$$

Template Parameters

<code>_Tp</code>	The real type of the argument
------------------	-------------------------------

Parameters

<code>↔ _n</code>	The positive integral order
<code>↔ _x</code>	The real argument $x \geq 0$

Definition at line 7042 of file `specfun.h`.

7.2.2.93 `debyef()`

```
float __gnu_cxx::debyef (
    unsigned int __n,
    float __x ) [inline]
```

Return the Debye function $D_n(x)$ of positive order `n` and `float` argument `x`.

See also

[debye](#) for details.

Definition at line 7014 of file `specfun.h`.

7.2.2.94 `debyel()`

```
long double __gnu_cxx::debyel (
    unsigned int __n,
    long double __x ) [inline]
```

Return the Debye function $D_n(x)$ of positive order n and real argument x .

See also

[debye](#) for details.

Definition at line 7024 of file `specfun.h`.

7.2.2.95 `digamma()`

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::digamma (
    _Tp __x ) [inline]
```

Return the digamma or psi function of argument x .

The the digamma or psi function is defined by

$$\psi(x) = \frac{d}{dx} \log(\Gamma(x)) = \frac{\Gamma'(x)}{\Gamma(x)},$$

the logarithmic derivative of the gamma function.

Parameters

<code>__x</code>	The parameter
------------------	---------------

Definition at line 3708 of file `specfun.h`.

7.2.2.96 `digammaf()`

```
float __gnu_cxx::digammaf (
    float __x ) [inline]
```

Return the digamma or psi function of `float` argument x .

See also

[digamma](#) for details.

Definition at line 3681 of file specfun.h.

7.2.2.97 digammal()

```
long double __gnu_cxx::digammal (
    long double __x ) [inline]
```

Return the digamma or psi function of `long double` argument `x`.

See also

[digamma](#) for details.

Definition at line 3691 of file specfun.h.

7.2.2.98 dilog()

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::dilog (
    _Tp __x ) [inline]
```

Return the dilogarithm function $Li_2(z)$ for real argument.

The dilogarithm is defined by:

$$Li_2(x) = \sum_{k=1}^{\infty} \frac{x^k}{k^2}$$

Parameters

<code>__x</code>	The argument.
------------------	---------------

Definition at line 3230 of file specfun.h.

7.2.2.99 `dilogf()`

```
float __gnu_cxx::dilogf (
    float __x ) [inline]
```

Return the dilogarithm function $Li_2(z)$ for `float` argument.

See also

[dilog](#) for details.

Definition at line 3204 of file `specfun.h`.

7.2.2.100 `dilogl()`

```
long double __gnu_cxx::dilogl (
    long double __x ) [inline]
```

Return the dilogarithm function $Li_2(z)$ for `long double` argument.

See also

[dilog](#) for details.

Definition at line 3214 of file `specfun.h`.

7.2.2.101 `dirichlet_beta()`

```
template<typename _Tp >
_Tp __gnu_cxx::dirichlet_beta (
    _Tp __s ) [inline]
```

Return the Dirichlet beta function of real argument `s`.

The Dirichlet beta function is defined by:

$$\beta(s) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)^s}$$

An important reflection formula is:

$$\beta(1-s) = \left(\frac{2}{\pi}\right)^s \sin\left(\frac{\pi s}{2}\right) \Gamma(s) \beta(s)$$

The Dirichlet beta function, in terms of the polylogarithm, is

$$\beta(s) = \Im[Li_s(i)]$$

Parameters

<code>__s</code>	The order.
------------------	------------

Definition at line 5380 of file specfun.h.

7.2.2.102 `dirichlet_betaf()`

```
float __gnu_cxx::dirichlet_betaf (
    float __s ) [inline]
```

Return the Dirichlet beta function of real argument `s`.

See also

[dirichlet_beta](#) for details.

Definition at line 5347 of file specfun.h.

7.2.2.103 `dirichlet_betal()`

```
long double __gnu_cxx::dirichlet_betal (
    long double __s ) [inline]
```

Return the Dirichlet beta function of real argument `s`.

See also

[dirichlet_beta](#) for details.

Definition at line 5356 of file specfun.h.

7.2.2.104 `dirichlet_eta()`

```
template<typename _Tp >
_Tp __gnu_cxx::dirichlet_eta (
    _Tp __s ) [inline]
```

Return the Dirichlet eta function of real argument `s`.

The Dirichlet eta function is defined by

$$\eta(s) = \sum_{k=1}^{\infty} \frac{(-1)^k}{k^s} = (1 - 2^{1-s}) \zeta(s)$$

An important reflection formula is:

$$\eta(-s) = 2 \frac{1 - 2^{-s-1}}{1 - 2^{-s}} \pi^{-s-1} s \sin\left(\frac{\pi s}{2}\right) \Gamma(s) \eta(s+1)$$

The Dirichlet eta function, in terms of the polylogarithm, is

$$\eta(s) = -\Re[Li_s(-1)]$$

Parameters

<code>_↔</code>	The order.
<code>_s</code>	

Definition at line 5333 of file specfun.h.

7.2.2.105 `dirichlet_etaf()`

```
float __gnu_cxx::dirichlet_etaf (
    float __s ) [inline]
```

Return the Dirichlet eta function of real argument `s`.

See also

[dirichlet_eta](#) for details.

Definition at line 5299 of file specfun.h.

7.2.2.106 `dirichlet_etal()`

```
long double __gnu_cxx::dirichlet_etal (
    long double __s ) [inline]
```

Return the Dirichlet eta function of real argument `s`.

See also

[dirichlet_eta](#) for details.

Definition at line 5308 of file specfun.h.

7.2.2.107 `dirichlet_lambda()`

```
template<typename _Tp >
_Tp __gnu_cxx::dirichlet_lambda (
    _Tp __s ) [inline]
```

Return the Dirichlet lambda function of real argument `s`.

The Dirichlet lambda function is defined by

$$\lambda(s) = \sum_{k=0}^{\infty} \frac{1}{(2k+1)^s} = (1 - 2^{-s}) \zeta(s)$$

In terms of the Riemann zeta and the Dirichlet eta functions

$$\lambda(s) = \frac{1}{2}(\zeta(s) + \eta(s))$$

Parameters

\leftrightarrow	The order.
s	

Definition at line 5423 of file specfun.h.

7.2.2.108 `dirichlet_lambdaf()`

```
float __gnu_cxx::dirichlet_lambdaf (  
    float __s ) [inline]
```

Return the Dirichlet lambda function of real argument s .

See also

[dirichlet_lambda](#) for details.

Definition at line 5394 of file specfun.h.

7.2.2.109 `dirichlet_lambdal()`

```
long double __gnu_cxx::dirichlet_lambdal (  
    long double __s ) [inline]
```

Return the Dirichlet lambda function of real argument s .

See also

[dirichlet_lambda](#) for details.

Definition at line 5403 of file specfun.h.

7.2.2.110 double_factorial()

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::double_factorial (
    int __n )    [inline]
```

Return the double factorial $n!!$ of the argument as a real number.

$$n!! = n(n-2)\dots(2), 0!! = 1$$

for even n and

$$n!! = n(n-2)\dots(1), (-1)!! = 1$$

for odd n .

Definition at line 4255 of file specfun.h.

7.2.2.111 double_factorialf()

```
float __gnu_cxx::double_factorialf (
    int __n )    [inline]
```

Return the double factorial $n!!$ of the argument as a `float`.

See also

[double_factorial](#) for more details

Definition at line 4228 of file specfun.h.

7.2.2.112 double_factoriall()

```
long double __gnu_cxx::double_factoriall (
    int __n )    [inline]
```

Return the double factorial $n!!$ of the argument as a `long double`.

See also

[double_factorial](#) for more details

Definition at line 4238 of file specfun.h.

7.2.2.113 `ellint_cel()`

```
template<typename _Tk , typename _Tp , typename _Ta , typename _Tb >
__gnu_cxx::fp_promote_t<_Tk, _Tp, _Ta, _Tb> __gnu_cxx::ellint_cel (
    _Tk __k_c,
    _Tp __p,
    _Ta __a,
    _Tb __b ) [inline]
```

Return the Bulirsch complete elliptic integral $cel(k_c, p, a, b)$ of real complementary modulus k_c , and parameters p , a , and b .

The Bulirsch complete elliptic integral is defined by

$$cel(k_c, p, a, b) = \int_0^{\pi/2} \frac{a \cos^2 \theta + b \sin^2 \theta}{\cos^2 \theta + p \sin^2 \theta} \frac{d\theta}{\sqrt{\cos^2 \theta + k_c^2 \sin^2 \theta}}$$

Parameters

<code>__k_c</code>	The complementary modulus $k_c = \sqrt{1 - k^2}$
<code>__p</code>	The parameter
<code>__a</code>	The parameter
<code>__b</code>	The parameter

Definition at line 4960 of file `specfun.h`.

7.2.2.114 `ellint_celf()`

```
float __gnu_cxx::ellint_celf (
    float __k_c,
    float __p,
    float __a,
    float __b ) [inline]
```

Return the Bulirsch complete elliptic integral $cel(k_c, p, a, b)$ of real complementary modulus k_c , and parameters p , a , and b .

See also

[ellint_cel](#) for details.

Definition at line 4928 of file `specfun.h`.

7.2.2.115 ellint_cell()

```
long double __gnu_cxx::ellint_cell (
    long double __k_c,
    long double __p,
    long double __a,
    long double __b ) [inline]
```

Return the Bulirsch complete elliptic integral $cel(k_c, p, a, b)$.

See also

[ellint_cel](#) for details.

Definition at line 4937 of file specfun.h.

7.2.2.116 ellint_d()

```
template<typename _Tk , typename _Tphi >
__gnu_cxx::fp_promote_t<_Tk, _Tphi> __gnu_cxx::ellint_d (
    _Tk __k,
    _Tphi __phi ) [inline]
```

Return the incomplete Legendre elliptic integral $D(k, \phi)$ of real modulus k and angular limit ϕ .

The Legendre elliptic integral D is defined by

$$D(k, \phi) = \int_0^\phi \frac{\sin^2 \theta d\theta}{\sqrt{1 - k^2 \sin^2 \theta}}$$

Parameters

<code>__k</code>	The modulus $-1 \leq \text{__k} \leq +1$
<code>__phi</code>	The angle

Definition at line 4773 of file specfun.h.

7.2.2.117 ellint_df()

```
float __gnu_cxx::ellint_df (
    float __k,
    float __phi ) [inline]
```

Return the incomplete Legendre elliptic integral $D(k, \phi)$ of `float` modulus k and angular limit ϕ .

See also

[ellint_d](#) for details.

Definition at line 4745 of file specfun.h.

7.2.2.118 ellint_dl()

```
long double __gnu_cxx::ellint_dl (
    long double __k,
    long double __phi ) [inline]
```

Return the incomplete Legendre elliptic integral $D(k, \phi)$ of long double modulus k and angular limit ϕ .

See also

[ellint_d](#) for details.

Definition at line 4755 of file specfun.h.

7.2.2.119 ellint_el1()

```
template<typename _Tp , typename _Tk >
__gnu_cxx::fp_promote_t<_Tp, _Tk> __gnu_cxx::ellint_el1 (
    _Tp __x,
    _Tk __k_c ) [inline]
```

Return the Bulirsch elliptic integral $el1(x, k_c)$ of the first kind of real tangent limit x and complementary modulus k_c .

The Bulirsch elliptic integral of the first kind is defined by

$$el1(x, k_c) = el2(x, k_c, 1, 1) = \int_0^{\arctan x} \frac{1 + \tan^2 \theta}{\sqrt{(1 + \tan^2 \theta)(1 + k_c^2 \tan^2 \theta)}} d\theta$$

Parameters

<code>__x</code>	The tangent of the angular integration limit
<code>__k_c</code>	The complementary modulus $k_c = \sqrt{1 - k^2}$

Definition at line 4819 of file specfun.h.

7.2.2.120 ellint_el1f()

```
float __gnu_cxx::ellint_el1f (
    float __x,
    float __k_c ) [inline]
```

Return the Bulirsch elliptic integral $el1(x, k_c)$ of the first kind of `float` tangent limit x and complementary modulus k_c .

See also

[ellint_el1](#) for details.

Definition at line 4789 of file `specfun.h`.

7.2.2.121 ellint_el1l()

```
long double __gnu_cxx::ellint_el1l (
    long double __x,
    long double __k_c ) [inline]
```

Return the Bulirsch elliptic integral $el1(x, k_c)$ of the first kind of real tangent limit x and complementary modulus k_c .

See also

[ellint_el1](#) for details.

Definition at line 4800 of file `specfun.h`.

7.2.2.122 ellint_el2()

```
template<typename _Tp , typename _Tk , typename _Ta , typename _Tb >
__gnu_cxx::fp_promote_t<_Tp, _Tk, _Ta, _Tb> __gnu_cxx::ellint_el2 (
    _Tp __x,
    _Tk __k_c,
    _Ta __a,
    _Tb __b ) [inline]
```

Return the Bulirsch elliptic integral of the second kind $el2(x, k_c, a, b)$.

The Bulirsch elliptic integral of the second kind is defined by

$$el2(x, k_c, a, b) = \int_0^{\arctan x} \frac{a + b \tan^2 \theta}{\sqrt{(1 + \tan^2 \theta)(1 + k_c^2 \tan^2 \theta)}} d\theta$$

Parameters

<code>__x</code>	The tangent of the angular integration limit
<code>__k_c</code> <code>__c</code>	The complementary modulus $k_c = \sqrt{1 - k^2}$
<code>__a</code>	The parameter
<code>__b</code>	The parameter

Definition at line 4865 of file specfun.h.

7.2.2.123 `ellint_el2f()`

```
float __gnu_cxx::ellint_el2f (
    float __x,
    float __k_c,
    float __a,
    float __b ) [inline]
```

Return the Bulirsch elliptic integral of the second kind $el2(x, k_c, a, b)$.

See also

[ellint_el2](#) for details.

Definition at line 4834 of file specfun.h.

7.2.2.124 `ellint_el2l()`

```
long double __gnu_cxx::ellint_el2l (
    long double __x,
    long double __k_c,
    long double __a,
    long double __b ) [inline]
```

Return the Bulirsch elliptic integral of the second kind $el2(x, k_c, a, b)$.

See also

[ellint_el2](#) for details.

Definition at line 4844 of file specfun.h.

7.2.2.125 `ellint_el3()`

```
template<typename _Tx , typename _Tk , typename _Tp >
__gnu_cxx::fp_promote_t<_Tx, _Tk, _Tp> __gnu_cxx::ellint_el3 (
    _Tx __x,
    _Tk __k_c,
    _Tp __p ) [inline]
```

Return the Bulirsch elliptic integral of the third kind $el3(x, k_c, p)$ of real tangent limit x , complementary modulus k_c , and parameter p .

The Bulirsch elliptic integral of the third kind is defined by

$$el3(x, k_c, p) = \int_0^{\arctan x} \frac{d\theta}{(\cos^2 \theta + p \sin^2 \theta) \sqrt{\cos^2 \theta + k_c^2 \sin^2 \theta}}$$

Parameters

<code>__x</code>	The tangent of the angular integration limit
<code>__k_c</code>	The complementary modulus $k_c = \sqrt{1 - k^2}$
<code>__c</code>	
<code>__p</code>	The parameter

Definition at line 4912 of file `specfun.h`.

7.2.2.126 `ellint_el3f()`

```
float __gnu_cxx::ellint_el3f (
    float __x,
    float __k_c,
    float __p ) [inline]
```

Return the Bulirsch elliptic integral of the third kind $el3(x, k_c, p)$ of `float` tangent limit x , complementary modulus k_c , and parameter p .

See also

[ellint_el3](#) for details.

Definition at line 4881 of file `specfun.h`.

7.2.2.127 ellint_el3l()

```
long double __gnu_cxx::ellint_el3l (
    long double __x,
    long double __k_c,
    long double __p ) [inline]
```

Return the Bulirsch elliptic integral of the third kind $el3(x, k_c, p)$ of long double tangent limit x , complementary modulus k_c , and parameter p .

See also

[ellint_el3](#) for details.

Definition at line 4892 of file specfun.h.

7.2.2.128 ellint_rc()

```
template<typename _Tp , typename _Up >
__gnu_cxx::fp_promote_t<_Tp, _Up> __gnu_cxx::ellint_rc (
    _Tp __x,
    _Up __y ) [inline]
```

Return the Carlson elliptic function $R_C(x, y) = R_F(x, y, y)$ where $R_F(x, y, z)$ is the Carlson elliptic function of the first kind.

The Carlson elliptic function is defined by:

$$R_C(x, y) = \frac{1}{2} \int_0^\infty \frac{dt}{(t+x)^{1/2}(t+y)}$$

Based on Carlson's algorithms:

- B. C. Carlson Numer. Math. 33, 1 (1979)
- B. C. Carlson, Special Functions of Applied Mathematics (1977)
- Numerical Recipes in C, 2nd ed, pp. 261-269, by Press, Teukolsky, Vetterling, Flannery (1992)

Parameters

\leftarrow __x	The first argument.
\leftarrow __y	The second argument.

Definition at line 3365 of file specfun.h.

7.2.2.129 `ellint_rcf()`

```
float __gnu_cxx::ellint_rcf (
    float __x,
    float __y ) [inline]
```

Return the Carlson elliptic function $R_C(x, y)$.

See also

[ellint_rc](#) for details.

Definition at line 3331 of file specfun.h.

7.2.2.130 `ellint_rcl()`

```
long double __gnu_cxx::ellint_rcl (
    long double __x,
    long double __y ) [inline]
```

Return the Carlson elliptic function $R_C(x, y)$.

See also

[ellint_rc](#) for details.

Definition at line 3340 of file specfun.h.

7.2.2.131 `ellint_rd()`

```
template<typename _Tp , typename _Up , typename _Vp >
__gnu_cxx::fp_promote_t<_Tp, _Up, _Vp> __gnu_cxx::ellint_rd (
    _Tp __x,
    _Up __y,
    _Vp __z ) [inline]
```

Return the Carlson elliptic function of the second kind $R_D(x, y, z) = R_J(x, y, z, z)$ where $R_J(x, y, z, p)$ is the Carlson elliptic function of the third kind.

The Carlson elliptic function of the second kind is defined by:

$$R_D(x, y, z) = \frac{3}{2} \int_0^\infty \frac{dt}{(t+x)^{1/2}(t+y)^{1/2}(t+z)^{3/2}}$$

Based on Carlson's algorithms:

- B. C. Carlson Numer. Math. 33, 1 (1979)
- B. C. Carlson, Special Functions of Applied Mathematics (1977)
- Numerical Recipes in C, 2nd ed, pp. 261-269, by Press, Teukolsky, Vetterling, Flannery (1992)

Parameters

$_x$	The first of two symmetric arguments.
$_y$	The second of two symmetric arguments.
$_z$	The third argument.

Definition at line 3464 of file specfun.h.

7.2.2.132 `ellint_rdf()`

```
float __gnu_cxx::ellint_rdf (
    float __x,
    float __y,
    float __z ) [inline]
```

Return the Carlson elliptic function $R_D(x, y, z)$.

See also

[ellint_rd](#) for details.

Definition at line 3428 of file specfun.h.

7.2.2.133 `ellint_rdl()`

```
long double __gnu_cxx::ellint_rdl (
    long double __x,
    long double __y,
    long double __z ) [inline]
```

Return the Carlson elliptic function $R_D(x, y, z)$.

See also

[ellint_rd](#) for details.

Definition at line 3437 of file specfun.h.

7.2.2.134 `ellint_rf()`

```
template<typename _Tp , typename _Up , typename _Vp >
__gnu_cxx::fp_promote_t<_Tp, _Up, _Vp> __gnu_cxx::ellint_rf (
    _Tp __x,
    _Up __y,
    _Vp __z ) [inline]
```

Return the Carlson elliptic function $R_F(x, y, z)$ of the first kind for real arguments.

The Carlson elliptic function of the first kind is defined by:

$$R_F(x, y, z) = \frac{1}{2} \int_0^\infty \frac{dt}{(t+x)^{1/2}(t+y)^{1/2}(t+z)^{1/2}}$$

Parameters

<code>__x</code>	The first of three symmetric arguments.
<code>__y</code>	The second of three symmetric arguments.
<code>__z</code>	The third of three symmetric arguments.

Definition at line 3317 of file `specfun.h`.

7.2.2.135 `ellint_rff()`

```
float __gnu_cxx::ellint_rff (
    float __x,
    float __y,
    float __z ) [inline]
```

Return the Carlson elliptic function $R_F(x, y, z)$ of the first kind for `float` arguments.

See also

[ellint_rf](#) for details.

Definition at line 3288 of file `specfun.h`.

7.2.2.136 `ellint_rfl()`

```
long double __gnu_cxx::ellint_rfl (
    long double __x,
    long double __y,
    long double __z ) [inline]
```

Return the Carlson elliptic function $R_F(x, y, z)$ of the first kind for `long double` arguments.

See also

[ellint_rf](#) for details.

Definition at line 3298 of file `specfun.h`.

7.2.2.137 `ellint_rg()`

```
template<typename _Tp , typename _Up , typename _Vp >
__gnu_cxx::fp_promote_t<_Tp, _Up, _Vp> __gnu_cxx::ellint_rg (
    _Tp __x,
    _Up __y,
    _Vp __z ) [inline]
```

Return the symmetric Carlson elliptic function of the second kind $R_G(x, y, z)$.

The Carlson symmetric elliptic function of the second kind is defined by:

$$R_G(x, y, z) = \frac{1}{4} \int_0^\infty dt [(t+x)(t+y)(t+z)]^{-1/2} \left(\frac{x}{t+x} + \frac{y}{t+y} + \frac{z}{t+z} \right)$$

Based on Carlson's algorithms:

- B. C. Carlson Numer. Math. 33, 1 (1979)
- B. C. Carlson, Special Functions of Applied Mathematics (1977)
- Numerical Recipes in C, 2nd ed, pp. 261-269, by Press, Teukolsky, Vetterling, Flannery (1992)

Parameters

<code>__x</code>	The first of three symmetric arguments.
<code>__y</code>	The second of three symmetric arguments.
<code>__z</code>	The third of three symmetric arguments.

Definition at line 3555 of file specfun.h.

7.2.2.138 `ellint_rgf()`

```
float __gnu_cxx::ellint_rgf (
    float __x,
    float __y,
    float __z ) [inline]
```

Return the Carlson elliptic function $R_G(x, y)$.

See also

[ellint_rg](#) for details.

Definition at line 3520 of file specfun.h.

7.2.2.139 `ellint_rgl()`

```
long double __gnu_cxx::ellint_rgl (
    long double __x,
    long double __y,
    long double __z ) [inline]
```

Return the Carlson elliptic function $R_G(x, y)$.

See also

[ellint_rg](#) for details.

Definition at line 3529 of file specfun.h.

7.2.2.140 `ellint_rj()`

```
template<typename _Tp , typename _Up , typename _Vp , typename _Wp >
__gnu_cxx::fp_promote_t<_Tp, _Up, _Vp, _Wp> __gnu_cxx::ellint_rj (
    _Tp __x,
    _Up __y,
    _Vp __z,
    _Wp __p ) [inline]
```

Return the Carlson elliptic function $R_J(x, y, z, p)$ of the third kind.

The Carlson elliptic function of the third kind is defined by:

$$R_J(x, y, z, p) = \frac{3}{2} \int_0^\infty \frac{dt}{(t+x)^{1/2}(t+y)^{1/2}(t+z)^{1/2}(t+p)}$$

Based on Carlson's algorithms:

- B. C. Carlson Numer. Math. 33, 1 (1979)
- B. C. Carlson, Special Functions of Applied Mathematics (1977)
- Numerical Recipes in C, 2nd ed, pp. 261-269, by Press, Teukolsky, Vetterling, Flannery (1992)

Parameters

\leftrightarrow _x	The first of three symmetric arguments.
\leftrightarrow _y	The second of three symmetric arguments.
\leftrightarrow _z	The third of three symmetric arguments.
\leftrightarrow _p	The fourth argument.

Definition at line 3414 of file specfun.h.

7.2.2.141 `ellint_rjf()`

```
float __gnu_cxx::ellint_rjf (
    float __x,
    float __y,
    float __z,
    float __p ) [inline]
```

Return the Carlson elliptic function $R_J(x, y, z, p)$.

See also

[ellint_rj](#) for details.

Definition at line 3379 of file specfun.h.

7.2.2.142 `ellint_rjl()`

```
long double __gnu_cxx::ellint_rjl (
    long double __x,
    long double __y,
    long double __z,
    long double __p ) [inline]
```

Return the Carlson elliptic function $R_J(x, y, z, p)$.

See also

[ellint_rj](#) for details.

Definition at line 3388 of file specfun.h.

7.2.2.143 ellnome()

```
template<typename _Tp >
_Tp __gnu_cxx::ellnome (
    _Tp __k ) [inline]
```

Return the elliptic nome function $q(k)$ of modulus k .

The elliptic nome function is defined by

$$q(k) = \exp\left(-\pi \frac{K(\sqrt{1-k^2})}{K(k)}\right)$$

where $K(k)$ is the complete elliptic function of the first kind.

Template Parameters

<code>_Tp</code>	The real type of the modulus
------------------	------------------------------

Parameters

<code>↔ _k</code>	The modulus $-1 \leq k \leq +1$
-----------------------	---------------------------------

Definition at line 5812 of file specfun.h.

7.2.2.144 ellnomef()

```
float __gnu_cxx::ellnomef (
    float __k ) [inline]
```

Return the elliptic nome function $q(k)$ of modulus k .

See also

[ellnome](#) for details.

Definition at line 5785 of file specfun.h.

7.2.2.145 `ellnome1()`

```
long double __gnu_cxx::ellnome1 (
    long double __k ) [inline]
```

Return the elliptic nome function $q(k)$ of `long double` modulus `k`.

See also

[ellnome](#) for details.

Definition at line 5795 of file `specfun.h`.

7.2.2.146 `euler()`

```
template<typename _Tp >
_Tp __gnu_cxx::euler (
    unsigned int __n ) [inline]
```

This returns Euler number E_n .

Parameters

<code>__n</code>	the order <code>n</code> of the Euler number.
------------------	---

Returns

The Euler number of order `n`.

Definition at line 7084 of file `specfun.h`.

7.2.2.147 `eulerian_1()` [1/2]

```
template<typename _Tp >
_Tp __gnu_cxx::eulerian_1 (
    unsigned int __n,
    unsigned int __m ) [inline]
```

Return the Eulerian number of the first kind. The Eulerian numbers of the first kind are defined by recursion:

$$\left\langle \begin{matrix} n \\ m \end{matrix} \right\rangle = (n - m) \left\langle \begin{matrix} n - 1 \\ m - 1 \end{matrix} \right\rangle + (m + 1) \left\langle \begin{matrix} n - 1 \\ m \end{matrix} \right\rangle \text{ for } n > 0$$

Note that $A(n, m)$ is a common older notation.

Definition at line 7100 of file `specfun.h`.

7.2.2.148 `eulerian_1()` [2/2]

```
template<typename _Tp >
std::vector<_Tp> __gnu_cxx::eulerian_1 (
    unsigned int __n ) [inline]
```

Return a vector of Eulerian numbers of the first kind.

Definition at line 7108 of file specfun.h.

7.2.2.149 `eulerian_2()`

```
template<typename _Tp >
_Tp __gnu_cxx::eulerian_2 (
    unsigned int __n,
    unsigned int __m ) [inline]
```

Return the Eulerian number of the second kind. The Eulerian numbers of the second kind are defined by recursion:

$$\left\langle\left\langle n \right\rangle\right\rangle_m = (2n - m - 1) \left\langle\left\langle n - 1 \right\rangle\right\rangle_{m-1} + (m + 1) \left\langle\left\langle n - 1 \right\rangle\right\rangle_m \text{ for } n > 0$$

Definition at line 7125 of file specfun.h.

7.2.2.150 `expint()`

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::expint (
    unsigned int __n,
    _Tp __x ) [inline]
```

Return the exponential integral $E_n(x)$ of integral order n and real argument x . The exponential integral is defined by:

$$E_n(x) = \int_1^\infty \frac{e^{-tx}}{t^n} dt$$

In particular

$$E_1(x) = \int_1^\infty \frac{e^{-tx}}{t} dt = -Ei(-x)$$

Template Parameters

<code>_Tp</code>	The real type of the argument
------------------	-------------------------------

Parameters

\leftrightarrow _n	The integral order
\leftrightarrow _x	The real argument

Definition at line 3992 of file specfun.h.

7.2.2.151 expintf()

```
float __gnu_cxx::expintf (
    unsigned int __n,
    float __x ) [inline]
```

Return the exponential integral $E_n(x)$ for integral order `n` and `float` argument `x`.

See also

[expint](#) for details.

Definition at line 3961 of file specfun.h.

7.2.2.152 expintl()

```
long double __gnu_cxx::expintl (
    unsigned int __n,
    long double __x ) [inline]
```

Return the exponential integral $E_n(x)$ for integral order `n` and `long double` argument `x`.

See also

[expint](#) for details.

Definition at line 3971 of file specfun.h.

7.2.2.153 exponential_p()

```
template<typename _Tlam , typename _Tp >
__gnu_cxx::fp_promote_t<_Tlam, _Tp> __gnu_cxx::exponential_p (
    _Tlam __lambda,
    _Tp __x ) [inline]
```

Return the exponential cumulative probability density function.

The formula for the exponential cumulative probability density function is

$$F(x|\lambda) = 1 - e^{-\lambda x} \text{ for } x \geq 0$$

Definition at line 6761 of file specfun.h.

7.2.2.154 exponential_pdf()

```
template<typename _Tlam , typename _Tp >
__gnu_cxx::fp_promote_t<_Tlam, _Tp> __gnu_cxx::exponential_pdf (
    _Tlam __lambda,
    _Tp __x ) [inline]
```

Return the exponential probability density function.

The formula for the exponential probability density function is

$$f(x|\lambda) = \lambda e^{-\lambda x} \text{ for } x \geq 0$$

Definition at line 6745 of file specfun.h.

7.2.2.155 factorial()

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::factorial (
    unsigned int __n ) [inline]
```

Return the factorial $n!$ of the argument as a real number.

$$n! = 1 \times 2 \times \dots \times n, 0! = 1$$

.

Definition at line 4214 of file specfun.h.

7.2.2.156 factorialf()

```
float __gnu_cxx::factorialf (
    unsigned int __n ) [inline]
```

Return the factorial $n!$ of the argument as a `float`.

See also

[factorial](#) for more details

Definition at line 4194 of file `specfun.h`.

7.2.2.157 factoriall()

```
long double __gnu_cxx::factoriall (
    unsigned int __n ) [inline]
```

Return the factorial $n!$ of the argument as a `long double`.

See also

[factorial](#) for more details

Definition at line 4203 of file `specfun.h`.

7.2.2.158 falling_factorial()

```
template<typename _Tp , typename _Tnu >
__gnu_cxx::fp_promote_t<_Tp, _Tnu> __gnu_cxx::falling_factorial (
    _Tp __a,
    _Tnu __nu ) [inline]
```

Return the falling factorial function or the lower Pochhammer symbol for real argument a and integral order n . The falling factorial function is defined by

$$a^n = \prod_{k=0}^{n-1} (a - k) = \Gamma(a + 1) / \Gamma(a - n + 1)$$

where $a^0 \equiv 1$. In particular, $n^n = n!$.

Definition at line 4180 of file `specfun.h`.

7.2.2.159 falling_factorialf()

```
float __gnu_cxx::falling_factorialf (
    float __a,
    float __nu ) [inline]
```

Return the falling factorial $a^{\underline{n}}$ for float arguments.

See also

[falling_factorial](#) for details.

Definition at line 4153 of file specfun.h.

7.2.2.160 falling_factoriall()

```
long double __gnu_cxx::falling_factoriall (
    long double __a,
    long double __nu ) [inline]
```

Return the falling factorial $a^{\underline{n}}$ for long double arguments.

See also

[falling_factorial](#) for details.

Definition at line 4163 of file specfun.h.

7.2.2.161 fermi_dirac()

```
template<typename _Tps , typename _Tp >
__gnu_cxx::fp_promote_t<_Tps, _Tp> __gnu_cxx::fermi_dirac (
    _Tps __s,
    _Tp __x ) [inline]
```

Return the Fermi-Dirac integral of integer or real order s and real argument x.

See also

https://en.wikipedia.org/wiki/Clausen_function
<http://dlmf.nist.gov/25.12.16>

$$F_s(x) = \frac{1}{\Gamma(s+1)} \int_0^\infty \frac{t^s}{e^{t-x} + 1} dt = -Li_{s+1}(-e^x)$$

Parameters

$_s$	The order $s > -1$.
$_x$	The real argument.

Returns

The real Fermi-Dirac integral $F_s(x)$,

Definition at line 6259 of file specfun.h.

7.2.2.162 `fermi_diracf()`

```
float __gnu_cxx::fermi_diracf (
    float __s,
    float __x ) [inline]
```

Return the Fermi-Dirac integral of `float` order `s` and argument `x`.

See also

[fermi_dirac](#) for details.

Definition at line 6229 of file specfun.h.

7.2.2.163 `fermi_diracl()`

```
long double __gnu_cxx::fermi_diracl (
    long double __s,
    long double __x ) [inline]
```

Return the Fermi-Dirac integral of `long double` order `s` and argument `x`.

See also

[fermi_dirac](#) for details.

Definition at line 6239 of file specfun.h.

7.2.2.164 `fisher_f_p()`

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::fisher_f_p (
    _Tp __F,
    unsigned int __nu1,
    unsigned int __nu2 )
```

Return the F-distribution propability function. This returns the probability that the observed chi-square for a correct model exceeds the value χ^2 .

The f-distribution propability function is related to the incomplete beta function:

$$Q(F|\nu_1, \nu_2) = I_{\frac{\nu_2}{\nu_2 + \nu_1 F}}\left(\frac{\nu_2}{2}, \frac{\nu_1}{2}\right)$$

Parameters

<code>__nu1</code>	The number of degrees of freedom of sample 1
<code>__nu2</code>	The number of degrees of freedom of sample 2
<code>__F</code>	The F statistic

Definition at line 6884 of file `specfun.h`.

7.2.2.165 `fisher_f_pdf()`

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::fisher_f_pdf (
    _Tp __F,
    unsigned int __nu1,
    unsigned int __nu2 )
```

Return the F-distribution propability function. This returns the probability that the observed chi-square for a correct model exceeds the value χ^2 .

The f-distribution propability function is related to the incomplete beta function:

$$P(F|\nu_1, \nu_2) = 1 - I_{\frac{\nu_2}{\nu_2 + \nu_1 F}}\left(\frac{\nu_2}{2}, \frac{\nu_1}{2}\right) = 1 - Q(F|\nu_1, \nu_2)$$

Parameters

<code>__F</code>	
<code>__nu1</code>	
<code>__nu2</code>	

Definition at line 6861 of file specfun.h.

7.2.2.166 fresnel_c()

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::fresnel_c (
    _Tp __x ) [inline]
```

Return the Fresnel cosine integral of argument x .

The Fresnel cosine integral is defined by

$$C(x) = \int_0^x \cos\left(\frac{\pi}{2}t^2\right)dt$$

Parameters

\leftrightarrow	The argument
x	

Definition at line 3903 of file specfun.h.

7.2.2.167 fresnel_cf()

```
float __gnu_cxx::fresnel_cf (
    float __x ) [inline]
```

Definition at line 3884 of file specfun.h.

7.2.2.168 fresnel_cl()

```
long double __gnu_cxx::fresnel_cl (
    long double __x ) [inline]
```

Definition at line 3888 of file specfun.h.

7.2.2.169 fresnel_s()

```
template<typename _Tp >  
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::fresnel_s (  
    _Tp __x )    [inline]
```

Return the Fresnel sine integral of argument x .

The Fresnel sine integral is defined by

$$S(x) = \int_0^x \sin\left(\frac{\pi}{2}t^2\right)dt$$

Parameters

<code>_↔</code>	The argument
<code>_x</code>	

Definition at line 3875 of file specfun.h.

7.2.2.170 `fresnel_sf()`

```
float __gnu_cxx::fresnel_sf (
    float __x ) [inline]
```

Definition at line 3856 of file specfun.h.

7.2.2.171 `fresnel_sl()`

```
long double __gnu_cxx::fresnel_sl (
    long double __x ) [inline]
```

Definition at line 3860 of file specfun.h.

7.2.2.172 `gamma_p()`

```
template<typename _Ta , typename _Tp >
__gnu_cxx::fp_promote_t<_Ta, _Tp> __gnu_cxx::gamma_p (
    _Ta __a,
    _Tp __x ) [inline]
```

Return the gamma cumulative propability distribution function or the regularized lower incomplete gamma function.

The formula for the gamma probability density function is:

$$\Gamma(x|\alpha, \beta) = \frac{1}{\beta\Gamma(\alpha)} (x/\beta)^{\alpha-1} e^{-x/\beta}$$

Definition at line 4591 of file specfun.h.

7.2.2.173 gamma_pdf()

```
template<typename _Ta , typename _Tb , typename _Tp >
__gnu_cxx::fp_promote_t<_Ta, _Tb, _Tp> __gnu_cxx::gamma_pdf (
    _Ta __alpha,
    _Tb __beta,
    _Tp __x ) [inline]
```

Return the gamma propability distribution function.

The formula for the gamma probability density function is:

$$\Gamma(x|\alpha, \beta) = \frac{1}{\beta\Gamma(\alpha)} (x/\beta)^{\alpha-1} e^{-x/\beta}$$

Definition at line 6645 of file specfun.h.

References std::__detail::__beta().

7.2.2.174 gamma_pf()

```
float __gnu_cxx::gamma_pf (
    float __a,
    float __x ) [inline]
```

Definition at line 4572 of file specfun.h.

7.2.2.175 gamma_pl()

```
long double __gnu_cxx::gamma_pl (
    long double __a,
    long double __x ) [inline]
```

Definition at line 4576 of file specfun.h.

7.2.2.176 gamma_q()

```
template<typename _Ta , typename _Tp >
__gnu_cxx::fp_promote_t<_Ta, _Tp> __gnu_cxx::gamma_q (
    _Ta __a,
    _Tp __x ) [inline]
```

Return the gamma complementary cumulative propability distribution (or survival) function or the regularized upper incomplete gamma function.

The formula for the gamma probability density function is:

$$\Gamma(x|\alpha, \beta) = \frac{1}{\beta\Gamma(\alpha)} (x/\beta)^{\alpha-1} e^{-x/\beta}$$

Definition at line 4619 of file specfun.h.

7.2.2.177 gamma_qf()

```
float __gnu_cxx::gamma_qf (
    float __a,
    float __x ) [inline]
```

Definition at line 4600 of file specfun.h.

7.2.2.178 gamma_ql()

```
long double __gnu_cxx::gamma_ql (
    long double __a,
    long double __x ) [inline]
```

Definition at line 4604 of file specfun.h.

7.2.2.179 gamma_reciprocal()

```
template<typename _Ta >
__gnu_cxx::fp_promote_t<_Ta> __gnu_cxx::gamma_reciprocal (
    _Ta __a ) [inline]
```

Return the reciprocal gamma function for real argument.

The reciprocal of the Gamma function is what you'd expect:

$$\Gamma_r(a) = \frac{1}{\Gamma(a)}$$

But unlike the Gamma function this function has no singularities and is exponentially decreasing for increasing argument.

Definition at line 6999 of file specfun.h.

7.2.2.180 gamma_reciproalf()

```
float __gnu_cxx::gamma_reciproalf (
    float __a ) [inline]
```

Return the reciprocal gamma function for `float` argument.

See also

[gamma_reciprocal](#) for details.

Definition at line 6974 of file `specfun.h`.

7.2.2.181 gamma_reciprocall()

```
long double __gnu_cxx::gamma_reciprocall (
    long double __a ) [inline]
```

Return the reciprocal gamma function for `long double` argument.

See also

[gamma_reciprocal](#) for details.

Definition at line 6984 of file `specfun.h`.

7.2.2.182 gegenbauer()

```
template<typename _Tlam , typename _Tp >
__gnu_cxx::fp_promote_t<_Tlam, _Tp> __gnu_cxx::gegenbauer (
    unsigned int __n,
    _Tlam __lambda,
    _Tp __x ) [inline]
```

Return the Gegenbauer polynomial $C_n^\lambda(x)$ of degree `n` and real order $\lambda > -1/2$, $\lambda \neq 0$ and argument `x`.

The Gegenbauer polynomial is generated by a three-term recursion relation:

$$C_n^\lambda(x) = \frac{1}{n} [2x(n + \lambda - 1)C_{n-1}^\lambda(x) - (n + 2\lambda - 2)C_{n-2}^\lambda(x)]$$

and $C_0^\lambda(x) = 1$, $C_1^\lambda(x) = 2\lambda x$.

Template Parameters

<code>__Tlam</code>	The real type of the order
<code>__Tp</code>	The real type of the argument

Parameters

<code>__n</code>	The non-negative integral degree
<code>__lambda</code>	The real order
<code>__x</code>	The real argument

Definition at line 2388 of file specfun.h.

7.2.2.183 `gegenbauerf()`

```
float __gnu_cxx::gegenbauerf (
    unsigned int __n,
    float __lambda,
    float __x ) [inline]
```

Return the Gegenbauer polynomial $C_n^{(\lambda)}(x)$ of degree `n` and `float` order $\lambda > -1/2, \lambda \neq 0$ and argument `x`.

See also

[gegenbauer](#) for details.

Definition at line 2351 of file specfun.h.

7.2.2.184 `gegenbauerl()`

```
long double __gnu_cxx::gegenbauerl (
    unsigned int __n,
    long double __lambda,
    long double __x ) [inline]
```

Return the Gegenbauer polynomial $C_n^{(\lambda)}(x)$ of degree `n` and `long double` order $\lambda > -1/2, \lambda \neq 0$ and argument `x`.

See also

[gegenbauer](#) for details.

Definition at line 2362 of file specfun.h.

7.2.2.185 harmonic()

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::harmonic (
    unsigned int __n ) [inline]
```

Return the harmonic number H_n .

The harmonic number is defined by

$$H_n = \sum_{k=1}^n \frac{1}{k}$$

Parameters

\leftrightarrow	The parameter
<code>__n</code>	

Definition at line 3767 of file specfun.h.

7.2.2.186 heuman_lambda()

```
template<typename _Tk , typename _Tphi >
__gnu_cxx::fp_promote_t<_Tk, _Tphi> __gnu_cxx::heuman_lambda (
    _Tk __k,
    _Tphi __phi ) [inline]
```

Return the Heuman lambda function $\Lambda(k, \phi)$ of modulus k and angular limit ϕ .

The complete Heuman lambda function is defined by

$$\Lambda(k, \phi) = \frac{F(1-m, \phi)}{K(1-m)} + \frac{2}{\pi} K(m) Z(1-m, \phi)$$

where $m = k^2$, $K(k)$ is the complete elliptic function of the first kind, and $Z(k, \phi)$ is the Jacobi zeta function.

Template Parameters

<code>_Tk</code>	the floating-point type of the modulus
<code>_Tphi</code>	the floating-point type of the angular limit argument

Parameters

<code>__k</code>	The modulus
<code>__phi</code>	The angle

Definition at line 4688 of file specfun.h.

7.2.2.187 `heuman_lambdaf()`

```
float __gnu_cxx::heuman_lambdaf (
    float __k,
    float __phi ) [inline]
```

Definition at line 4662 of file specfun.h.

7.2.2.188 `heuman_lambdal()`

```
long double __gnu_cxx::heuman_lambdal (
    long double __k,
    long double __phi ) [inline]
```

Definition at line 4666 of file specfun.h.

7.2.2.189 `hurwitz_zeta()` [1/2]

```
template<typename _Tp , typename _Up >
__gnu_cxx::fp_promote_t<_Tp, _Up> __gnu_cxx::hurwitz_zeta (
    _Tp __s,
    _Up __a ) [inline]
```

Return the Hurwitz zeta function of real argument s , and parameter a .

The the Hurwitz zeta function is defined by

$$\zeta(s, a) = \sum_{n=0}^{\infty} \frac{1}{(a+n)^s}$$

Parameters

<code>__s</code>	The order.
<code>__a</code>	The parameter.

Definition at line 3597 of file specfun.h.

7.2.2.190 `hurwitz_zeta()` [2/2]

```
template<typename _Tp , typename _Up >
std::complex<_Tp> __gnu_cxx::hurwitz_zeta (
    _Tp __s,
    std::complex< _Up > __a )
```

Return the Hurwitz zeta function of real order `s`, and complex parameter `a`.

See also

[hurwitz_zeta](#) for details.

Definition at line 3611 of file `specfun.h`.

7.2.2.191 `hurwitz_zetaf()`

```
float __gnu_cxx::hurwitz_zetaf (
    float __s,
    float __a ) [inline]
```

Return the Hurwitz zeta function of `float` argument `s`, and parameter `a`.

See also

[hurwitz_zeta](#) for details.

Definition at line 3570 of file `specfun.h`.

7.2.2.192 `hurwitz_zetal()`

```
long double __gnu_cxx::hurwitz_zetal (
    long double __s,
    long double __a ) [inline]
```

Return the Hurwitz zeta function of `long double` argument `s`, and parameter `a`.

See also

[hurwitz_zeta](#) for details.

Definition at line 3580 of file `specfun.h`.

7.2.2.193 `hyperg()`

```
template<typename _Tpa , typename _Tpb , typename _Tpc , typename _Tp >
__gnu_cxx::fp_promote_t<_Tpa, _Tpb, _Tpc, _Tp> __gnu_cxx::hyperg (
    _Tpa __a,
    _Tpb __b,
    _Tpc __c,
    _Tp __x ) [inline]
```

Return the hypergeometric function ${}_2F_1(a, b; c; x)$ of real numerator parameters a and b , denominator parameter c , and argument x .

The hypergeometric function is defined by

$${}_2F_1(a, b; c; x) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n x^n}{(c)_n n!}$$

where the Pochhammer symbol is $(x)_k = (x)(x+1)\dots(x+k-1)$, $(x)_0 = 1$

Parameters

<code>↔ _a</code>	The first numerator parameter
<code>↔ _b</code>	The second numerator parameter
<code>↔ _c</code>	The denominator parameter
<code>↔ _x</code>	The argument

Definition at line 1610 of file `specfun.h`.

7.2.2.194 `hypergf()`

```
float __gnu_cxx::hypergf (
    float __a,
    float __b,
    float __c,
    float __x ) [inline]
```

Return the hypergeometric function ${}_2F_1(a, b; c; x)$ of `float` numerator parameters a and b , denominator parameter c , and argument x .

See also

[hyperg](#) for details.

Definition at line 1577 of file `specfun.h`.

7.2.2.195 hypergl()

```
long double __gnu_cxx::hypergl (
    long double __a,
    long double __b,
    long double __c,
    long double __x ) [inline]
```

Return the hypergeometric function ${}_2F_1(a, b; c; x)$ of long double numerator parameters a and b, denominator parameter c, and argument x.

See also

[hyperg](#) for details.

Definition at line 1588 of file specfun.h.

7.2.2.196 ibeta()

```
template<typename _Ta , typename _Tb , typename _Tp >
__gnu_cxx::fp_promote_t<_Ta, _Tb, _Tp> __gnu_cxx::ibeta (
    _Ta __a,
    _Tb __b,
    _Tp __x ) [inline]
```

Return the regularized incomplete beta function of parameters a, b, and argument x.

The regularized incomplete beta function is defined by

$$I_x(a, b) = \frac{B_x(a, b)}{B(a, b)}$$

where

$$B_x(a, b) = \int_0^x t^{a-1} (1-t)^{b-1} dt$$

is the non-regularized incomplete beta function and $B(a, b)$ is the usual beta function.

Parameters

$_a$	The first parameter
$_b$	The second parameter
$_x$	The argument

Definition at line 3816 of file specfun.h.

7.2.2.197 ibetac()

```
template<typename _Ta , typename _Tb , typename _Tp >
__gnu_cxx::fp_promote_t<_Ta, _Tb, _Tp> __gnu_cxx::ibetac (
    _Ta __a,
    _Tb __b,
    _Tp __x ) [inline]
```

Return the regularized complementary incomplete beta function of parameters a , b , and argument x .

The regularized complementary incomplete beta function is defined by

$$I_x(a, b) = I_x(a, b)$$

Parameters

\leftrightarrow _a	The parameter
\leftrightarrow _b	The parameter
\leftrightarrow _x	The argument

Definition at line 3847 of file specfun.h.

7.2.2.198 ibetacf()

```
float __gnu_cxx::ibetacf (
    float __a,
    float __b,
    float __x ) [inline]
```

Definition at line 3825 of file specfun.h.

References `__gnu_cxx::ibetaf()`.

7.2.2.199 ibetac1()

```
long double __gnu_cxx::ibetac1 (
    long double __a,
    long double __b,
    long double __x ) [inline]
```

Definition at line 3829 of file specfun.h.

References `__gnu_cxx::ibetal()`.

7.2.2.200 ibetaf()

```
float __gnu_cxx::ibetaf (
    float __a,
    float __b,
    float __x ) [inline]
```

Return the regularized incomplete beta function of parameters *a*, *b*, and argument *x*.

See `ibeta` for details.

Definition at line 3782 of file `specfun.h`.

Referenced by `__gnu_cxx::ibetaf()`.

7.2.2.201 ibetal()

```
long double __gnu_cxx::ibetal (
    long double __a,
    long double __b,
    long double __x ) [inline]
```

Return the regularized incomplete beta function of parameters *a*, *b*, and argument *x*.

See `ibeta` for details.

Definition at line 3792 of file `specfun.h`.

Referenced by `__gnu_cxx::ibetac()`.

7.2.2.202 jacobi()

```
template<typename _Talpha , typename _Tbeta , typename _Tp >
__gnu_cxx::fp_promote_t<_Talpha, _Tbeta, _Tp> __gnu_cxx::jacobi (
    unsigned __n,
    _Talpha __alpha,
    _Tbeta __beta,
    _Tp __x ) [inline]
```

Return the Jacobi polynomial $P_n^{(\alpha, \beta)}(x)$ of degree *n* and float orders $\alpha, \beta > -1$ and argument *x*.

The Jacobi polynomials are generated by a three-term recursion relation:

$$2n(\alpha + \beta + n)(\alpha + \beta + 2n - 2)P_n^{(\alpha, \beta)}(x) = (\alpha + \beta + 2n - 1)[(\alpha^2 - \beta^2) + x(\alpha + \beta + 2n - 2)(\alpha + \beta + 2n)]P_{n-1}^{(\alpha, \beta)}(x) - 2(\alpha + n - 1)(\beta + n - 1)(\alpha + \beta + 2n)P_{n-2}^{(\alpha, \beta)}(x)$$

where $P_0^{(\alpha, \beta)}(x) = 1$ and $P_1^{(\alpha, \beta)}(x) = [(\alpha - \beta) + (\alpha + \beta + 2)x]/2$.

Template Parameters

<code>_Talpha</code>	The real type of the order α
<code>_Tbeta</code>	The real type of the order β
<code>_Tp</code>	The real type of the argument

Parameters

<code>__n</code>	The non-negative integral degree
<code>__alpha</code>	The real order
<code>__beta</code>	The real order
<code>__x</code>	The real argument

Definition at line 2334 of file `specfun.h`.

References `std::__detail::__beta()`.

7.2.2.203 `jacobi_cn()`

```
template<typename _Kp , typename _Up >
__gnu_cxx::fp_promote_t<_Kp, _Up> __gnu_cxx::jacobi_cn (
    _Kp __k,
    _Up __u ) [inline]
```

Return the Jacobi elliptic cosine amplitude function $cn(k, u)$ of real modulus k and argument u .

The Jacobi elliptic `cn` integral is defined by

$$\cos(\phi) = cn(k, F(k, \phi))$$

where $F(k, \phi)$ is the Legendre elliptic integral of the first kind (see `ellint_1`).

Template Parameters

<code>_Kp</code>	The type of the real modulus
<code>_Up</code>	The type of the real argument

Parameters

<code>↔ _k</code>	The real modulus
<code>↔ _u</code>	The real argument

Definition at line 2039 of file specfun.h.

7.2.2.204 `jacobi_cnf()`

```
float __gnu_cxx::jacobi_cnf (
    float __k,
    float __u ) [inline]
```

Return the Jacobi elliptic cosine amplitude function $cn(k, u)$ of `float` modulus `k` and argument `u`.

See also

[jacobi_cn](#) for details.

Definition at line 2004 of file specfun.h.

7.2.2.205 `jacobi_cnl()`

```
long double __gnu_cxx::jacobi_cnl (
    long double __k,
    long double __u ) [inline]
```

Return the Jacobi elliptic cosine amplitude function $cn(k, u)$ of `long double` modulus `k` and argument `u`.

See also

[jacobi_cn](#) for details.

Definition at line 2016 of file specfun.h.

7.2.2.206 `jacobi_dn()`

```
template<typename _Kp , typename _Up >
__gnu_cxx::fp_promote_t<_Kp, _Up> __gnu_cxx::jacobi_dn (
    _Kp __k,
    _Up __u ) [inline]
```

Return the Jacobi elliptic delta amplitude function $dn(k, u)$ of real modulus `k` and argument `u`.

The Jacobi elliptic `dn` integral is defined by

$$\sqrt{1 - k^2 \sin(\phi)} = dn(k, F(k, \phi))$$

where $F(k, \phi)$ is the Legendre elliptic integral of the first kind (see `ellint_1`).

Template Parameters

<code>_Kp</code>	The type of the real modulus
<code>_Up</code>	The type of the real argument

Parameters

<code>↔ _k</code>	The real modulus
<code>↔ _u</code>	The real argument

Definition at line 2089 of file `specfun.h`.

7.2.2.207 `jacobi_dnf()`

```
float __gnu_cxx::jacobi_dnf (
    float __k,
    float __u ) [inline]
```

Return the Jacobi elliptic delta amplitude function $dn(k, u)$ of `float` modulus `k` and argument `u`.

See also

[jacobi_dn](#) for details.

Definition at line 2054 of file `specfun.h`.

7.2.2.208 `jacobi_dnl()`

```
long double __gnu_cxx::jacobi_dnl (
    long double __k,
    long double __u ) [inline]
```

Return the Jacobi elliptic delta amplitude function $dn(k, u)$ of `long double` modulus `k` and argument `u`.

See also

[jacobi_dn](#) for details.

Definition at line 2066 of file `specfun.h`.

7.2.2.209 `jacobi_sn()`

```
template<typename _Kp , typename _Up >
__gnu_cxx::fp_promote_t<_Kp, _Up> __gnu_cxx::jacobi_sn (
    _Kp __k,
    _Up __u ) [inline]
```

Return the Jacobi elliptic sine amplitude function $sn(k, u)$ of real modulus k and argument u .

The Jacobi elliptic sn integral is defined by

$$\sin(\phi) = sn(k, F(k, \phi))$$

where $F(k, \phi)$ is the Legendre elliptic integral of the first kind (see `ellint_1`).

Template Parameters

<code>_Kp</code>	The type of the real modulus
<code>_Up</code>	The type of the real argument

Parameters

<code>↔ _k</code>	The real modulus
<code>↔ _u</code>	The real argument

Definition at line 1989 of file `specfun.h`.

7.2.2.210 `jacobi_snf()`

```
float __gnu_cxx::jacobi_snf (
    float __k,
    float __u ) [inline]
```

Return the Jacobi elliptic sine amplitude function $sn(k, u)$ of `float` modulus k and argument u .

See also

[jacobi_sn](#) for details.

Definition at line 1954 of file `specfun.h`.

7.2.2.211 `jacobi_sn()`

```
long double __gnu_cxx::jacobi_sn1 (
    long double __k,
    long double __u ) [inline]
```

Return the Jacobi elliptic sine amplitude function $sn(k, u)$ of `long double` modulus `k` and argument `u`.

See also

[jacobi_sn](#) for details.

Definition at line 1966 of file `specfun.h`.

7.2.2.212 `jacobi_theta_1()`

```
template<typename _Tpq , typename _Tp >
__gnu_cxx::fp_promote_t<_Tpq, _Tp> __gnu_cxx::jacobi_theta_1 (
    _Tpq __q,
    _Tp __x ) [inline]
```

Return the Jacobi theta-1 function $\theta_1(q, x)$ of nome `q` and argument `x`.

The Jacobi theta-1 function is defined by

$$\theta_1(q, x) = \frac{1}{\sqrt{\pi x}} \sum_{j=-\infty}^{+\infty} (-1)^j \exp\left(\frac{-(q + j - 1/2)^2}{x}\right)$$

Parameters

<code>__q</code>	The periodic (period = 2) argument
<code>__x</code>	The argument

Definition at line 6043 of file `specfun.h`.

7.2.2.213 `jacobi_theta_1f()`

```
float __gnu_cxx::jacobi_theta_1f (
    float __q,
    float __x ) [inline]
```

Return the Jacobi theta-1 function $\theta_1(q, x)$ of nome `q` and argument `x`.

See also

[jacobi_theta_1](#) for details.

Definition at line 6015 of file specfun.h.

7.2.2.214 `jacobi_theta_1l()`

```
long double __gnu_cxx::jacobi_theta_1l (
    long double __q,
    long double __x ) [inline]
```

Return the Jacobi theta-1 function $\theta_1(q, x)$ of nome q and argument x .

See also

[jacobi_theta_1](#) for details.

Definition at line 6025 of file specfun.h.

7.2.2.215 `jacobi_theta_2()`

```
template<typename _Tpq , typename _Tp >
__gnu_cxx::fp_promote_t<_Tpq, _Tp> __gnu_cxx::jacobi_theta_2 (
    _Tpq __q,
    _Tp __x ) [inline]
```

Return the Jacobi theta-2 function $\theta_2(q, x)$ of nome q and argument x .

The Jacobi theta-2 function is defined by

$$\theta_2(q, x) = \frac{1}{\sqrt{\pi x}} \sum_{j=-\infty}^{+\infty} (-1)^j \exp\left(\frac{-(q+j)^2}{x}\right)$$

Parameters

\leftrightarrow <code>__q</code>	The periodic (period = 2) argument
\leftrightarrow <code>__x</code>	The argument

Definition at line 6086 of file specfun.h.

7.2.2.216 jacobi_theta_2f()

```
float __gnu_cxx::jacobi_theta_2f (
    float __q,
    float __x ) [inline]
```

Return the Jacobi theta-2 function $\theta_2(q, x)$ of nome q and argument x .

See also

[jacobi_theta_2](#) for details.

Definition at line 6058 of file specfun.h.

7.2.2.217 jacobi_theta_2l()

```
long double __gnu_cxx::jacobi_theta_2l (
    long double __q,
    long double __x ) [inline]
```

Return the Jacobi theta-2 function $\theta_2(q, x)$ of nome q and argument x .

See also

[jacobi_theta_2](#) for details.

Definition at line 6068 of file specfun.h.

7.2.2.218 jacobi_theta_3()

```
template<typename _Tpq , typename _Tp >
__gnu_cxx::fp_promote_t<_Tpq, _Tp> __gnu_cxx::jacobi_theta_3 (
    _Tpq __q,
    _Tp __x ) [inline]
```

Return the Jacobi theta-3 function $\theta_3(q, x)$ of nome q and argument x .

The Jacobi theta-3 function is defined by

$$\theta_3(q, x) = \frac{1}{\sqrt{\pi x}} \sum_{j=-\infty}^{+\infty} \exp\left(\frac{-(q+j)^2}{x}\right)$$

Parameters

\leftrightarrow _q	The elliptic nome
\leftrightarrow _x	The argument

Definition at line 6129 of file specfun.h.

7.2.2.219 jacobi_theta_3f()

```
float __gnu_cxx::jacobi_theta_3f (
    float __q,
    float __x ) [inline]
```

Return the Jacobi theta-3 function $\theta_3(q, x)$ of nome q and argument x .

See also

[jacobi_theta_3](#) for details.

Definition at line 6101 of file specfun.h.

7.2.2.220 jacobi_theta_3l()

```
long double __gnu_cxx::jacobi_theta_3l (
    long double __q,
    long double __x ) [inline]
```

Return the Jacobi theta-3 function $\theta_3(q, x)$ of nome q and argument x .

See also

[jacobi_theta_3](#) for details.

Definition at line 6111 of file specfun.h.

7.2.2.221 jacobi_theta_4()

```
template<typename _Tpq , typename _Tp >
__gnu_cxx::fp_promote_t<_Tpq, _Tp> __gnu_cxx::jacobi_theta_4 (
    _Tpq __q,
    _Tp __x ) [inline]
```

Return the Jacobi theta-4 function $\theta_4(q, x)$ of nome q and argument x .

The Jacobi theta-4 function is defined by

$$\theta_4(q, x) = \frac{1}{\sqrt{\pi x}} \sum_{j=-\infty}^{+\infty} \exp\left(\frac{-(q + j + 1/2)^2}{x}\right)$$

Parameters

\leftrightarrow _q	The elliptic nome
\leftrightarrow _x	The argument

Definition at line 6172 of file specfun.h.

7.2.2.222 jacobi_theta_4f()

```
float __gnu_cxx::jacobi_theta_4f (
    float __q,
    float __x ) [inline]
```

Return the Jacobi theta-4 function $\theta_4(q, x)$ of nome q and argument x .

See also

[jacobi_theta_4](#) for details.

Definition at line 6144 of file specfun.h.

7.2.2.223 jacobi_theta_4l()

```
long double __gnu_cxx::jacobi_theta_4l (
    long double __q,
    long double __x ) [inline]
```

Return the Jacobi theta-4 function $\theta_4(q, x)$ of nome q and argument x .

See also

[jacobi_theta_4](#) for details.

Definition at line 6154 of file specfun.h.

7.2.2.224 jacobi_zeta()

```
template<typename _Tk , typename _Tphi >
__gnu_cxx::fp_promote_t<_Tk, _Tphi> __gnu_cxx::jacobi_zeta (
    _Tk __k,
    _Tphi __phi ) [inline]
```

Return the Jacobi zeta function of k and ϕ .

The Jacobi zeta function is defined by

$$Z(m, \phi) = E(m, \phi) - \frac{E(m)F(m, \phi)}{K(m)}$$

where $E(m, \phi)$ is the elliptic function of the second kind, $E(m)$ is the complete elliptic function of the second kind, and $F(m, \phi)$ is the elliptic function of the first kind.

Template Parameters

<code>_Tk</code>	the real type of the modulus
<code>_Tphi</code>	the real type of the angle limit

Parameters

<code>__k</code>	The modulus
<code>__phi</code>	The angle

Definition at line 4653 of file `specfun.h`.

7.2.2.225 `jacobi_zetaf()`

```
float __gnu_cxx::jacobi_zetaf (
    float __k,
    float __phi ) [inline]
```

Definition at line 4628 of file `specfun.h`.

7.2.2.226 `jacobi_zetal()`

```
long double __gnu_cxx::jacobi_zetal (
    long double __k,
    long double __phi ) [inline]
```

Definition at line 4632 of file `specfun.h`.

7.2.2.227 `jacobif()`

```
float __gnu_cxx::jacobif (
    unsigned __n,
    float __alpha,
    float __beta,
    float __x ) [inline]
```

Return the Jacobi polynomial $P_n^{(\alpha, \beta)}(x)$ of degree `n` and float orders $\alpha, \beta > -1$ and argument `x`.

See also

[jacobi](#) for details.

Definition at line 2283 of file `specfun.h`.

References `std::__detail::__beta()`.

7.2.2.228 `jacobil()`

```
long double __gnu_cxx::jacobil (
    unsigned __n,
    long double __alpha,
    long double __beta,
    long double __x ) [inline]
```

Return the Jacobi polynomial $P_n^{(\alpha,\beta)}(x)$ of degree `n` and long double orders $\alpha, \beta > -1$ and argument `x`.

See also

[jacobi](#) for details.

Definition at line 2297 of file `specfun.h`.

References `std::__detail::__beta()`.

7.2.2.229 `lah()` [1/2]

```
template<typename _Tp >
_Tp __gnu_cxx::lah (
    unsigned int __n,
    unsigned int __k ) [inline]
```

Return the Lah number. Lah numbers are defined by downward recurrence:

$$L(n, k-1) = \frac{k(k-1)}{n-k+1} L(n, k); L(n, n) = 1$$

Definition at line 7202 of file `specfun.h`.

7.2.2.230 `lah()` [2/2]

```
template<typename _Tp >
std::vector<_Tp> __gnu_cxx::lah (
    unsigned int __n ) [inline]
```

Return a vector of Lah numbers. Lah numbers are defined by downward recurrence:

$$L(n, k-1) = \frac{k(k-1)}{n-k+1} L(n, k); L(n, n) = 1$$

Definition at line 7214 of file `specfun.h`.

7.2.2.231 lbinomial()

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::lbinomial (
    unsigned int __n,
    unsigned int __k ) [inline]
```

Return the logarithm of the binomial coefficient as a real number. The binomial coefficient is given by:

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

The binomial coefficients are generated by:

$$(1+t)^n = \sum_{k=0}^n \binom{n}{k} t^k$$

Parameters

\leftrightarrow __n	The first argument of the binomial coefficient.
\leftrightarrow __k	The second argument of the binomial coefficient.

Returns

The logarithm of the binomial coefficient.

Definition at line 4420 of file specfun.h.

7.2.2.232 lbinomialf()

```
float __gnu_cxx::lbinomialf (
    unsigned int __n,
    unsigned int __k ) [inline]
```

Return the logarithm of the binomial coefficient as a float.

See also

[lbinomial](#) for details.

Definition at line 4391 of file specfun.h.

7.2.2.233 lbinomial()

```
long double __gnu_cxx::lbinomial (
    unsigned int __n,
    unsigned int __k ) [inline]
```

Return the logarithm of the binomial coefficient as a `long double`.

See also

[lbinomial](#) for details.

Definition at line 4400 of file `specfun.h`.

7.2.2.234 ldouble_factorial()

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::ldouble_factorial (
    int __n ) [inline]
```

Return the logarithm of the double factorial $\ln(n!!)$ of the argument as a real number.

$$n!! = n(n-2)\dots(2), 0!! = 1$$

for even n and

$$n!! = n(n-2)\dots(1), (-1)!! = 1$$

for odd n .

Definition at line 4334 of file `specfun.h`.

7.2.2.235 ldouble_factorialf()

```
float __gnu_cxx::ldouble_factorialf (
    int __n ) [inline]
```

Return the logarithm of the double factorial $\ln(n!!)$ of the argument as a `float`.

See also

[ldouble_factorial](#) for more details

Definition at line 4307 of file `specfun.h`.

7.2.2.236 ldoube_factoriall()

```
long double __gnu_cxx::ldouble_factoriall (
    int __n ) [inline]
```

Return the logarithm of the double factorial $ln(n!!)$ of the argument as a long double .

See also

[double_factorial](#) for more details

Definition at line 4317 of file specfun.h.

7.2.2.237 legendre_q()

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::legendre_q (
    unsigned int __l,
    _Tp __x ) [inline]
```

Return the Legendre function of the second kind $Q_l(x)$ of nonnegative degree l and real argument $|x| \leq 0$.

The Legendre function of the second kind of order l and argument x , $Q_l(x)$, is defined by:

$$Q_l(x) = \frac{1}{2} \log \frac{x+1}{x-1} P_l(x) - \sum_{k=0}^{l-1} \frac{(l+k)!}{(l-k)!(k!)^2 s^k} [\psi(l+1) - \psi(k+1)] (x-1)^k$$

where $P_l(x)$ is the Legendre polynomial of degree l and $\psi(x)$ is the digamma or psi function which for integral argument is related to the harmonic number: $\psi(n) = -\gamma_E + H_n$.

Template Parameters

<code>_Tp</code>	The floating-point type of the argument <code>__x</code> .
------------------	--

Parameters

<code>__l</code>	The degree $l \geq 0$
<code>__x</code>	The argument $\text{abs}(\text{__x}) \leq 1$

Exceptions

<code>std::domain_error</code>	if $\text{abs}(\text{__x}) > 1$
--------------------------------	---------------------------------

Definition at line 4512 of file specfun.h.

7.2.2.238 `legendre_qf()`

```
float __gnu_cxx::legendre_qf (
    unsigned int __l,
    float __x ) [inline]
```

Return the Legendre function of the second kind $Q_l(x)$ of nonnegative degree `l` and `float` argument.

See also

[legendre_q](#) for details.

Definition at line 4476 of file specfun.h.

7.2.2.239 `legendre_ql()`

```
long double __gnu_cxx::legendre_ql (
    unsigned int __l,
    long double __x ) [inline]
```

Return the Legendre function of the second kind $Q_l(x)$ of nonnegative degree `l` and `long double` argument.

See also

[legendre_q](#) for details.

Definition at line 4486 of file specfun.h.

7.2.2.240 `lerch_phi()`

```
template<typename _Tp , typename _Ts , typename _Ta >
__gnu_cxx::fp_promote_t<_Tp, _Ts, _Ta> __gnu_cxx::lerch_phi (
    _Tp __z,
    _Ts __s,
    _Ta __a ) [inline]
```

Return the Lerch transcendent $\Phi(z, s, a)$.

The series is:

$$\Phi(z, s, a) = \sum_{k=0}^{\infty} \frac{z^k}{(k+a)^s}$$

Parameters

$_z$	The argument.
$_s$	The order $s! = 1$.
$_a$	The scale parameter $a > -1$.

Definition at line 7274 of file specfun.h.

7.2.2.241 `lerch_phif()`

```
float __gnu_cxx::lerch_phif (
    float __z,
    float __s,
    float __a ) [inline]
```

Return the Lerch transcendent $\Phi(z, s, a)$ for `float` arguments.

See also

[lerch_phi](#) for details.

Definition at line 7247 of file specfun.h.

7.2.2.242 `lerch_phil()`

```
long double __gnu_cxx::lerch_phil (
    long double __z,
    long double __s,
    long double __a ) [inline]
```

Return the Lerch transcendent $\Phi(z, s, a)$ for `long double` arguments.

See also

[lerch_phi](#) for details.

Definition at line 7257 of file specfun.h.

7.2.2.243 `lfactorial()`

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::lfactorial (
    unsigned int __n ) [inline]
```

Return the logarithm of the factorial $\ln(n!)$ of the argument as a real number.

$$n! = 1 \times 2 \times \dots \times n, 0! = 1$$

.

Definition at line 4292 of file `specfun.h`.

7.2.2.244 `lfactorialf()`

```
float __gnu_cxx::lfactorialf (
    unsigned int __n ) [inline]
```

Return the logarithm of the factorial $\ln(n!)$ of the argument as a `float`.

See also

[lfactorial](#) for more details

Definition at line 4270 of file `specfun.h`.

7.2.2.245 `lfactoriall()`

```
long double __gnu_cxx::lfactoriall (
    unsigned int __n ) [inline]
```

Return the logarithm of the factorial $\ln(n!)$ of the argument as a `long double`.

See also

[lfactorial](#) for more details

Definition at line 4280 of file `specfun.h`.

7.2.2.246 lfalling_factorial()

```
template<typename _Tp , typename _Tnu >
__gnu_cxx::fp_promote_t<_Tp, _Tnu> __gnu_cxx::lfalling_factorial (
    _Tp __a,
    _Tnu __nu ) [inline]
```

Return the logarithm of the falling factorial function or the lower Pochhammer symbol. The falling factorial function is defined by

$$a^{\underline{n}} = \frac{\Gamma(a+1)}{\Gamma(a-\nu+1)} = \prod_{k=0}^{n-1} (a-k)$$

where $a^{\underline{0}} \equiv 1$. In particular, $n^{\underline{n}} = n!$. Thus this function returns

$$\ln[a^{\underline{n}}] = \ln[\Gamma(a+1)] - \ln[\Gamma(a-\nu+1)]$$

where $\ln[a^{\underline{0}}] \equiv 0$. Many notations exist for this function: $(a)_{\nu}$,

$$\left\{ \begin{matrix} a \\ \nu \end{matrix} \right\}$$

, and others.

Definition at line 4094 of file specfun.h.

7.2.2.247 lfalling_factorialf()

```
float __gnu_cxx::lfalling_factorialf (
    float __a,
    float __nu ) [inline]
```

Return the logarithm of the falling factorial $\ln(a^{\underline{\nu}})$ for float arguments.

See also

[lfalling_factorial](#) for details.

Definition at line 4057 of file specfun.h.

7.2.2.248 lfalling_factoriall()

```
long double __gnu_cxx::lfalling_factoriall (
    long double __a,
    long double __nu ) [inline]
```

Return the logarithm of the falling factorial $\ln(a^{\underline{\nu}})$ for float arguments.

See also

[lfalling_factorial](#) for details.

Definition at line 4067 of file specfun.h.

7.2.2.249 lgamma() [1/2]

```
template<typename _Ta >
__gnu_cxx::fp_promote_t<_Ta> __gnu_cxx::lgamma (
    _Ta __a ) [inline]
```

Return the logarithm of the gamma function for real argument.

Definition at line 3018 of file specfun.h.

Referenced by `std::__detail::__gegenbauer_zeros()`, `std::__detail::__jacobi_zeros()`, and `std::__detail::__laguerre_↵
zeros()`.

7.2.2.250 lgamma() [2/2]

```
template<typename _Ta >
std::complex<__gnu_cxx::fp_promote_t<_Ta> > __gnu_cxx::lgamma (
    std::complex< _Ta > __a ) [inline]
```

Return the logarithm of the gamma function for complex argument.

Definition at line 3051 of file specfun.h.

7.2.2.251 lgammaf() [1/2]

```
float __gnu_cxx::lgammaf (
    float __a ) [inline]
```

Return the logarithm of the gamma function for `float` argument.

See also

[lgamma](#) for details.

Definition at line 3000 of file specfun.h.

7.2.2.252 lgammaf() [2/2]

```
std::complex<float> __gnu_cxx::lgammaf (
    std::complex< float > __a ) [inline]
```

Return the logarithm of the gamma function for `std::complex<float>` argument.

See also

[lgamma](#) for details.

Definition at line 3033 of file `specfun.h`.

7.2.2.253 lgammal() [1/2]

```
long double __gnu_cxx::lgammal (
    long double __a ) [inline]
```

Return the logarithm of the gamma function for `long double` argument.

See also

[lgamma](#) for details.

Definition at line 3010 of file `specfun.h`.

7.2.2.254 lgammal() [2/2]

```
std::complex<long double> __gnu_cxx::lgammal (
    std::complex< long double > __a ) [inline]
```

Return the logarithm of the gamma function for `std::complex<long double>` argument.

See also

[lgamma](#) for details.

Definition at line 3043 of file `specfun.h`.

7.2.2.255 logint()

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::logint (
    _Tp __x ) [inline]
```

Return the logarithmic integral of argument `x`.

The logarithmic integral is defined by

$$li(x) = \int_0^x \frac{dt}{\ln(t)} = Ei(\ln(x))$$

where $Ei(x)$ is the exponential integral (see [std::expint](#)).

Parameters

$_x$	The real upper integration limit
-------	----------------------------------

Definition at line 1777 of file specfun.h.

7.2.2.256 logintf()

```
float __gnu_cxx::logintf (  
    float __x ) [inline]
```

Return the logarithmic integral of argument x .

See also

[logint](#) for details.

Definition at line 1752 of file specfun.h.

7.2.2.257 logintl()

```
long double __gnu_cxx::logintl (  
    long double __x ) [inline]
```

Return the logarithmic integral of argument x .

See also

[logint](#) for details.

Definition at line 1761 of file specfun.h.

7.2.2.258 logistic_p()

```
template<typename _Ta , typename _Tb , typename _Tp >
__gnu_cxx::fp_promote_t<_Ta, _Tb, _Tp> __gnu_cxx::logistic_p (
    _Ta __a,
    _Tb __b,
    _Tp __x ) [inline]
```

Return the logistic cumulative distribution function.

The formula for the logistic probability function is

$$P(x|a, b) = \frac{e^{(x-a)/b}}{1 + e^{(x-a)/b}}$$

where $b > 0$.

Definition at line 6960 of file specfun.h.

7.2.2.259 logistic_pdf()

```
template<typename _Ta , typename _Tb , typename _Tp >
__gnu_cxx::fp_promote_t<_Ta, _Tb, _Tp> __gnu_cxx::logistic_pdf (
    _Ta __a,
    _Tb __b,
    _Tp __x ) [inline]
```

Return the logistic probability density function.

The formula for the logistic probability density function is

$$f(x|a, b) = \frac{e^{(x-a)/b}}{b[1 + e^{(x-a)/b}]^2}$$

where $b > 0$.

Definition at line 6943 of file specfun.h.

7.2.2.260 lognormal_p()

```
template<typename _Tmu , typename _Tsig , typename _Tp >
__gnu_cxx::fp_promote_t<_Tmu, _Tsig, _Tp> __gnu_cxx::lognormal_p (
    _Tmu __mu,
    _Tsig __sigma,
    _Tp __x ) [inline]
```

Return the lognormal cumulative probability density function.

The formula for the lognormal cumulative probability density function is

$$F(x|\mu, \sigma) = \frac{1}{2} \left[1 - \operatorname{erf} \left(\frac{\ln x - \mu}{\sqrt{2}\sigma} \right) \right]$$

Definition at line 6729 of file specfun.h.

7.2.2.261 lognormal_pdf()

```
template<typename _Tmu , typename _Tsig , typename _Tp >
__gnu_cxx::fp_promote_t<_Tmu, _Tsig, _Tp> __gnu_cxx::lognormal_pdf (
    _Tmu __mu,
    _Tsig __sigma,
    _Tp __x ) [inline]
```

Return the lognormal probability density function.

The formula for the lognormal probability density function is

$$f(x|\mu, \sigma) = \frac{e^{(\ln x - \mu)^2 / 2\sigma^2}}{\sigma\sqrt{2\pi}}$$

Definition at line 6711 of file specfun.h.

7.2.2.262 lrising_factorial()

```
template<typename _Tp , typename _Tnu >
__gnu_cxx::fp_promote_t<_Tp, _Tnu> __gnu_cxx::lrising_factorial (
    _Tp __a,
    _Tnu __nu ) [inline]
```

Return the logarithm of the rising factorial function or the (upper) Pochhammer symbol. The rising factorial function is defined for integer order by

$$a^{\overline{\nu}} = \Gamma(a + \nu) / \Gamma(a) = \prod_{k=0}^{\nu-1} (a + k), \overline{0} \equiv 1$$

Thus this function returns

$$\ln[a^{\overline{\nu}}] = \ln[\Gamma(a + \nu)] - \ln[\Gamma(a)], \ln[a^{\overline{0}}] \equiv 0$$

Many notations exist for this function: $(a)_{\nu}$, called the Pochhammer function (esp. in the literature of special functions), and

$$\left[\begin{matrix} a \\ \nu \end{matrix} \right]$$

, and others.

Definition at line 4042 of file specfun.h.

7.2.2.263 lrising_factorialf()

```
float __gnu_cxx::lrising_factorialf (
    float __a,
    float __nu ) [inline]
```

Return the logarithm of the rising factorial $a^{\overline{\nu}}$ for float arguments.

See also

[lrising_factorial](#) for details.

Definition at line 4007 of file specfun.h.

7.2.2.264 `lrising_factorial()`

```
long double __gnu_cxx::lrising_factorial (
    long double __a,
    long double __nu ) [inline]
```

Return the logarithm of the rising factorial $\ln(a^{\overline{v}})$ for long double arguments.

See also

[lrising_factorial](#) for details.

Definition at line 4017 of file specfun.h.

7.2.2.265 `mittag_leffler()`

```
template<typename _Tp , typename _Ta , typename _Tb >
std::complex<__gnu_cxx::fp_promote_t<_Tp, _Ta, _Tb> > __gnu_cxx::mittag_leffler (
    _Ta __alpha,
    _Tb __beta,
    const std::complex< _Tp > & __z ) [inline]
```

Compute the Mittag-Leffler function:

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\beta + \alpha k)}, \quad \alpha > 0, \beta \in \mathbb{C}, z \in \mathbb{C}$$

See also

COMPUTATION OF THE MITTAG-LEFFLER FUNCTION $E_{\alpha,\beta}(z)$ AND ITS DERIVATIVE, Rudolf Gorenflo, Joulia Loutchko & Yuri Luchko

Definition at line 7292 of file specfun.h.

References `std::__detail::__beta()`.

7.2.2.266 `normal_p()`

```
template<typename _Tmu , typename _Tsig , typename _Tp >
__gnu_cxx::fp_promote_t<_Tmu, _Tsig, _Tp> __gnu_cxx::normal_p (
    _Tmu __mu,
    _Tsig __sigma,
    _Tp __x ) [inline]
```

Return the normal cumulative probability density function.

The formula for the normal cumulative probability density function is

$$F(x|\mu, \sigma) = \frac{1}{2} \left[1 - \operatorname{erf}\left(\frac{x - \mu}{\sqrt{2}\sigma}\right) \right]$$

Definition at line 6695 of file specfun.h.

7.2.2.267 `normal_pdf()`

```
template<typename _Tmu , typename _Tsig , typename _Tp >
__gnu_cxx::fp_promote_t<_Tmu, _Tsig, _Tp> __gnu_cxx::normal_pdf (
    _Tmu __mu,
    _Tsig __sigma,
    _Tp __x ) [inline]
```

Return the gamma cumulative propability distribution function.

The formula for the gamma probability density function is:

$$\Gamma(x|\alpha, \beta) = \frac{1}{\beta\Gamma(\alpha)} (x/\beta)^{\alpha-1} e^{-x/\beta}$$

```
template<typename _Ta, typename _Tb, typename _Tp> inline __gnu_cxx::fp_promote_t<_Ta, _Tb, _Tp> gamma_
_p(_Ta __alpha, _Tb __beta, _Tp __x) { using __type = __gnu_cxx::fp_promote_t<_Ta, _Tb, _Tp>; return std::__
detail::__gamma_p<__type>(__alpha, __beta, __x); } Return the normal probability density function.
```

The formula for the normal probability density function is

$$f(x|\mu, \sigma) = \frac{e^{(x-\mu)^2/2\sigma^2}}{\sigma\sqrt{2\pi}}$$

Definition at line 6678 of file specfun.h.

7.2.2.268 `owens_t()`

```
template<typename _Tph , typename _Tpa >
__gnu_cxx::fp_promote_t<_Tph, _Tpa> __gnu_cxx::owens_t (
    _Tph __h,
    _Tpa __a ) [inline]
```

Return the Owens T function $T(h, a)$ of shape factor h and integration limit a .

The Owens T function is defined by

$$T(h, a) = \frac{1}{2\pi} \int_0^a \frac{\exp[-\frac{1}{2}h^2(1+x^2)]}{1+x^2} dx$$

Parameters

\hookleftarrow _h	The shape factor
\hookleftarrow _a	The integration limit

Definition at line 6215 of file specfun.h.

7.2.2.269 `owens_tf()`

```
float __gnu_cxx::owens_tf (
    float __h,
    float __a ) [inline]
```

Return the Owens T function $T(h, a)$ of shape factor `h` and integration limit `a`.

See also

[owens_t](#) for details.

Definition at line 6187 of file specfun.h.

7.2.2.270 `owens_tl()`

```
long double __gnu_cxx::owens_tl (
    long double __h,
    long double __a ) [inline]
```

Return the Owens T function $T(h, a)$ of long double shape factor `h` and integration limit `a`.

See also

[owens_t](#) for details.

Definition at line 6197 of file specfun.h.

7.2.2.271 `periodic_zeta()` [1/2]

```
template<typename _Tp , typename _Up >
__gnu_cxx::fp_promote_t<std::complex<_Tp>, _Up> __gnu_cxx::periodic_zeta (
    _Tp __x,
    _Up __s ) [inline]
```

Return the periodic zeta function of real argument `x`, and parameter `s`.

The the periodic zeta function is defined by

$$F(x, s) = \sum_{n=1}^{\infty} \frac{e^{i2\pi nx}}{n^s}$$

Parameters

\leftrightarrow _x	The argument.
\leftrightarrow _s	The order.

Definition at line 3653 of file specfun.h.

7.2.2.272 `periodic_zeta()` [2/2]

```
template<typename _Tp , typename _Up >
__gnu_cxx::fp_promote_t<std::complex<_Tp>, std::complex<_Up> > __gnu_cxx::periodic_zeta (
    std::complex< _Up > __z,
    _Tp __s ) [inline]
```

Return the periodic zeta function of complex argument `z`, and real parameter `s`.

See also

[periodic_zeta](#) for details.

Definition at line 3667 of file specfun.h.

7.2.2.273 `periodic_zetaf()`

```
std::complex<float> __gnu_cxx::periodic_zetaf (
    float __x,
    float __s ) [inline]
```

Return the periodic zeta function of `float` argument `x`, and parameter `s`.

See also

[periodic_zeta](#) for details.

Definition at line 3626 of file specfun.h.

7.2.2.274 periodic_zetal()

```
std::complex<long double> __gnu_cxx::periodic_zetal (
    long double __x,
    long double __s ) [inline]
```

Return the periodic zeta function of `long double` argument `x`, and parameter `s`.

See also

[periodic_zeta](#) for details.

Definition at line 3636 of file `specfun.h`.

7.2.2.275 polygamma()

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::polygamma (
    unsigned int __m,
    _Tp __x ) [inline]
```

Return the polygamma function of argument `x`.

The the polygamma or digamma function is defined by

$$\psi(x) = \frac{d}{dx} \log(\Gamma(x)) = \frac{\Gamma'(x)}{\Gamma(x)}$$

Parameters

<code>__m</code>	The order
<code>__x</code>	The parameter

Definition at line 3749 of file `specfun.h`.

7.2.2.276 polygammaf()

```
float __gnu_cxx::polygammaf (
    unsigned int __m,
    float __x ) [inline]
```

Return the polygamma function of `float` argument `x`.

See also

[polygamma](#) for details.

Definition at line 3722 of file specfun.h.

7.2.2.277 polygamma()

```
long double __gnu_cxx::polygamma1 (
    unsigned int __m,
    long double __x ) [inline]
```

Return the polygamma function of long double argument x.

See also

[polygamma](#) for details.

Definition at line 3732 of file specfun.h.

7.2.2.278 polylog() [1/2]

```
template<typename _Tp , typename _Wp >
__gnu_cxx::fp_promote_t<_Tp, _Wp> __gnu_cxx::polylog (
    _Tp __s,
    _Wp __w ) [inline]
```

Return the polylogarithm function of real order s and real argument w.

The polylogarithm function is defined by

$$Li_s(w) = \sum_{k=1}^{\infty} \frac{w^k}{k^s}$$

Parameters

\leftrightarrow _s	The order.
\leftrightarrow _w	Argument.

Definition at line 5245 of file specfun.h.

7.2.2.279 polylog() [2/2]

```
template<typename _Tp , typename _Wp >
std::complex<__gnu_cxx::fp_promote_t<_Tp, _Wp> > __gnu_cxx::polylog (
    _Tp __s,
    std::complex< _Tp > __w ) [inline]
```

Return the complex polylogarithm function of real order s and complex argument w .

The polylogarithm function is defined by

$$Li_s(w) = \sum_{k=1}^{\infty} \frac{w^k}{k^s}$$

Parameters

\leftarrow _s	The order.
\leftarrow _w	Argument.

Definition at line 5285 of file specfun.h.

7.2.2.280 polylogf() [1/2]

```
float __gnu_cxx::polylogf (
    float __s,
    float __w ) [inline]
```

Return the real polylogarithm function of real order s and real argument w .

See also

[polylog](#) for details.

Definition at line 5218 of file specfun.h.

7.2.2.281 polylogf() [2/2]

```
std::complex<float> __gnu_cxx::polylogf (
    float __s,
    std::complex< float > __w ) [inline]
```

Return the complex polylogarithm function of real order s and complex argument w .

See also

[polylog](#) for details.

Definition at line 5258 of file specfun.h.

7.2.2.282 polylog() [1/2]

```
long double __gnu_cxx::polylogl (
    long double __s,
    long double __w ) [inline]
```

Return the complex polylogarithm function of real order s and argument w .

See also

[polylog](#) for details.

Definition at line 5228 of file specfun.h.

7.2.2.283 polylog() [2/2]

```
std::complex<long double> __gnu_cxx::polylogl (
    long double __s,
    std::complex< long double > __w ) [inline]
```

Return the complex polylogarithm function of real order s and complex argument w .

See also

[polylog](#) for details.

Definition at line 5268 of file specfun.h.

7.2.2.284 radpoly()

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::radpoly (
    unsigned int __n,
    unsigned int __m,
    _Tp __rho ) [inline]
```

Return the radial polynomial $R_n^m(\rho)$ for non-negative degree n , order $m \leq n$, and real radial argument ρ .

The radial polynomials are defined by

$$R_n^m(\rho) = \sum_{k=0}^{\frac{n-m}{2}} \frac{(-1)^k (n-k)!}{k! \left(\frac{n+m}{2} - k\right)! \left(\frac{n-m}{2} - k\right)!} \rho^{n-2k}$$

for $n - m$ even and identically 0 for $n - m$ odd. The radial polynomials can be related to the Jacobi polynomials:

$$R_n^m(\rho) = (-1)^{(n-m)/2} \rho^m P_{(n-m)/2}^{(m,0)}$$

See also

[jacobi](#) for details on the Jacobi polynomials (see [jacobi](#)).

Template Parameters

<code>__Tp</code>	The real type of the radial coordinate
-------------------	--

Parameters

<code>__n</code>	The non-negative degree.
<code>__m</code>	The non-negative azimuthal order
<code>__rho</code>	The radial argument

Definition at line 2499 of file `specfun.h`.

7.2.2.285 `radpolyf()`

```
float __gnu_cxx::radpolyf (
    unsigned int __n,
    unsigned int __m,
    float __rho ) [inline]
```

Return the radial polynomial $R_n^m(\rho)$ for non-negative degree `n`, order $m \leq n$, and `float` radial argument ρ .

See also

[radpoly](#) for details.

Definition at line 2460 of file `specfun.h`.

References `std::__detail::__radial_jacobi()`.

7.2.2.286 `radpolyl()`

```
long double __gnu_cxx::radpolyl (
    unsigned int __n,
    unsigned int __m,
    long double __rho ) [inline]
```

Return the radial polynomial $R_n^m(\rho)$ for non-negative degree `n`, order $m \leq n$, and `long double` radial argument ρ .

See also

[radpoly](#) for details.

Definition at line 2471 of file `specfun.h`.

References `std::__detail::__radial_jacobi()`.

7.2.2.287 rising_factorial()

```
template<typename _Tp , typename _Tnu >
__gnu_cxx::fp_promote_t<_Tp, _Tnu> __gnu_cxx::rising_factorial (
    _Tp __a,
    _Tnu __nu ) [inline]
```

Return the rising factorial function or the (upper) Pochhammer function. The rising factorial function is defined by

$$a^{\overline{\nu}} = \Gamma(a + \nu) / \Gamma(\nu)$$

Many notations exist for this function: $(a)_{\nu}$, called the Pochhammer function (esp. in the literature of special functions), and

$$\left[\begin{matrix} a \\ \nu \end{matrix} \right]$$

, and others.

Definition at line 4138 of file specfun.h.

7.2.2.288 rising_factorialf()

```
float __gnu_cxx::rising_factorialf (
    float __a,
    float __nu ) [inline]
```

Return the rising factorial $a^{\overline{\nu}}$ for float arguments.

See also

[rising_factorial](#) for details.

Definition at line 4109 of file specfun.h.

7.2.2.289 rising_factoriall()

```
long double __gnu_cxx::rising_factoriall (
    long double __a,
    long double __nu ) [inline]
```

Return the rising factorial $a^{\overline{\nu}}$ for long double arguments.

See also

[rising_factorial](#) for details.

Definition at line 4119 of file specfun.h.

7.2.2.290 `sin_pi()`

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::sin_pi (
    _Tp __x ) [inline]
```

Return the reperiodized sine function $\sin_{\pi}(x)$ for real argument `x`.

The reperiodized sine function is defined by:

$$\sin_{\pi}(x) = \sin(\pi x)$$

Template Parameters

<code>_Tp</code>	The floating-point type of the argument <code>__x</code> .
------------------	--

Parameters

<code>__x</code>	The argument
------------------	--------------

Definition at line 6345 of file `specfun.h`.

7.2.2.291 `sin_pif()`

```
float __gnu_cxx::sin_pif (
    float __x ) [inline]
```

Return the reperiodized sine function $\sin_{\pi}(x)$ for `float` argument `x`.

See also

[sin_pi](#) for more details.

Definition at line 6318 of file `specfun.h`.

7.2.2.292 `sin_pil()`

```
long double __gnu_cxx::sin_pil (
    long double __x ) [inline]
```

Return the reperiodized sine function $\sin_{\pi}(x)$ for `long double` argument `x`.

See also

[sin_pi](#) for more details.

Definition at line 6328 of file `specfun.h`.

7.2.2.293 sinc()

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::sinc (
    _Tp __x ) [inline]
```

Return the sinus cardinal function $\text{sinc}_{\pi}(x)$ for real argument `__x`. The sinus cardinal function is defined by:

$$\text{sinc}(x) = \frac{\sin(x)}{x}$$

Template Parameters

<code>_Tp</code>	The real type of the argument
------------------	-------------------------------

Parameters

<code>__x</code>	The argument
------------------	--------------

Definition at line 1697 of file `specfun.h`.

7.2.2.294 sinc_pi()

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::sinc_pi (
    _Tp __x ) [inline]
```

Return the reperiodized sinus cardinal function $\text{sinc}(x)$ for real argument `__x`. The normalized sinus cardinal function is defined by:

$$\text{sinc}_{\pi}(x) = \frac{\sin(\pi x)}{\pi x}$$

Template Parameters

<code>_Tp</code>	The real type of the argument
------------------	-------------------------------

Parameters

<code>__x</code>	The argument
------------------	--------------

Definition at line 1738 of file `specfun.h`.

7.2.2.295 sinc_pif()

```
float __gnu_cxx::sinc_pif (
    float __x ) [inline]
```

Return the reperiodized sinus cardinal function $\text{sinc}(x)$ for float argument `__x`.

See also

[sinc](#) for details.

Definition at line 1712 of file specfun.h.

7.2.2.296 sinc_pil()

```
long double __gnu_cxx::sinc_pil (
    long double __x ) [inline]
```

Return the reperiodized sinus cardinal function $\text{sinc}(x)$ for long double argument `__x`.

See also

[sinc](#) for details.

Definition at line 1722 of file specfun.h.

7.2.2.297 sincf()

```
float __gnu_cxx::sincf (
    float __x ) [inline]
```

Return the sinus cardinal function $\text{sinc}_\pi(x)$ for float argument `__x`.

See also

[sinc_pi](#) for details.

Definition at line 1671 of file specfun.h.

7.2.2.298 `sinc()`

```
long double __gnu_cxx::sinc (
    long double __x ) [inline]
```

Return the sinus cardinal function $\text{sinc}_\pi(x)$ for long double argument `__x`.

See also

[sinc_pi](#) for details.

Definition at line 1681 of file `specfun.h`.

7.2.2.299 `sincos()` [1/2]

```
__gnu_cxx::__sincos_t<double> __gnu_cxx::sincos (
    double __x ) [inline]
```

Return both the sine and the cosine of a double argument.

See also

[sincos](#) for details.

Definition at line 6583 of file `specfun.h`.

7.2.2.300 `sincos()` [2/2]

```
template<typename _Tp >
__gnu_cxx::__sincos_t<__gnu_cxx::fp_promote_t<_Tp> > __gnu_cxx::sincos (
    _Tp __x ) [inline]
```

Return both the sine and the cosine of a reperiodized argument.

$$\text{sincos}(x) = \sin(x), \cos(x)$$

Definition at line 6594 of file `specfun.h`.

7.2.2.301 sincos_pi()

```
template<typename _Tp >
__gnu_cxx::__sincos_t<__gnu_cxx::fp_promote_t<_Tp> > __gnu_cxx::sincos_pi (
    _Tp __x ) [inline]
```

Return both the sine and the cosine of a reperiodized real argument.

$$\text{sincos}_{\pi}(x) = \sin(\pi x), \cos(\pi x)$$

Definition at line 6628 of file specfun.h.

7.2.2.302 sincos_pif()

```
__gnu_cxx::__sincos_t<float> __gnu_cxx::sincos_pif (
    float __x ) [inline]
```

Return both the sine and the cosine of a reperiodized float argument.

See also

[sincos_pi](#) for details.

Definition at line 6606 of file specfun.h.

7.2.2.303 sincos_pil()

```
__gnu_cxx::__sincos_t<long double> __gnu_cxx::sincos_pil (
    long double __x ) [inline]
```

Return both the sine and the cosine of a reperiodized long double argument.

See also

[sincos_pi](#) for details.

Definition at line 6616 of file specfun.h.

7.2.2.304 sincosf()

```
__gnu_cxx::__sincos_t<float> __gnu_cxx::sincosf (
    float __x ) [inline]
```

Return both the sine and the cosine of a `float` argument.

Definition at line 6565 of file `specfun.h`.

7.2.2.305 sincosl()

```
__gnu_cxx::__sincos_t<long double> __gnu_cxx::sincosl (
    long double __x ) [inline]
```

Return both the sine and the cosine of a `long double` argument.

See also

[sincos](#) for details.

Definition at line 6574 of file `specfun.h`.

7.2.2.306 sinh_pi()

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::sinh_pi (
    _Tp __x ) [inline]
```

Return the reperiodized hyperbolic sine function $\sinh_{\pi}(x)$ for real argument `x`.

The reperiodized hyperbolic sine function is defined by:

$$\sinh_{\pi}(x) = \sinh(\pi x)$$

Template Parameters

<code>_Tp</code>	The floating-point type of the argument <code>__x</code> .
------------------	--

Parameters

<code>__x</code>	The argument
------------------	--------------

Definition at line 6387 of file specfun.h.

7.2.2.307 `sinh_pif()`

```
float __gnu_cxx::sinh_pif (
    float __x ) [inline]
```

Return the reperiodized hyperbolic sine function $\sinh_{\pi}(x)$ for `float` argument `x`.

See also

[sinh_pi](#) for more details.

Definition at line 6360 of file specfun.h.

7.2.2.308 `sinh_pil()`

```
long double __gnu_cxx::sinh_pil (
    long double __x ) [inline]
```

Return the reperiodized hyperbolic sine function $\sinh_{\pi}(x)$ for `long double` argument `x`.

See also

[sinh_pi](#) for more details.

Definition at line 6370 of file specfun.h.

7.2.2.309 `sinhc()`

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::sinhc (
    _Tp __x ) [inline]
```

Return the normalized hyperbolic sinus cardinal function $\operatorname{sinhc}(x)$ for real argument `__x`. The normalized hyperbolic sinus cardinal function is defined by:

$$\operatorname{sinhc}(x) = \frac{\sinh(\pi x)}{\pi x}$$

Template Parameters

<code>_Tp</code>	The real type of the argument
------------------	-------------------------------

Parameters

<code>__x</code>	The argument
------------------	--------------

Definition at line 2581 of file `specfun.h`.

7.2.2.310 `sinhc_pi()`

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::sinhc_pi (
    _Tp __x ) [inline]
```

Return the hyperbolic sinus cardinal function $\operatorname{sinhc}_{\pi}(x)$ for real argument `__x`. The sinus cardinal function is defined by:

$$\operatorname{sinhc}_{\pi}(x) = \frac{\sinh(x)}{x}$$

Template Parameters

<code>_Tp</code>	The real type of the argument
------------------	-------------------------------

Parameters

<code>__x</code>	The argument
------------------	--------------

Definition at line 2540 of file `specfun.h`.

7.2.2.311 `sinhc_pif()`

```
float __gnu_cxx::sinhc_pif (
    float __x ) [inline]
```

Return the hyperbolic sinus cardinal function $\operatorname{sinhc}_{\pi}(x)$ for `float` argument `__x`.

See also

[sinhc_pi](#) for details.

Definition at line 2514 of file specfun.h.

7.2.2.312 `sinhc_pil()`

```
long double __gnu_cxx::sinhc_pil (  
    long double __x ) [inline]
```

Return the hyperbolic sinus cardinal function $\operatorname{sinhc}_{\pi}(x)$ for long double argument `__x`.

See also

[sinhc_pi](#) for details.

Definition at line 2524 of file specfun.h.

7.2.2.313 `sinhcf()`

```
float __gnu_cxx::sinhcf (  
    float __x ) [inline]
```

Return the normalized hyperbolic sinus cardinal function $\operatorname{sinhc}(x)$ for float argument `__x`.

See also

[sinhc](#) for details.

Definition at line 2555 of file specfun.h.

7.2.2.314 `sinhcl()`

```
long double __gnu_cxx::sinhcl (  
    long double __x ) [inline]
```

Return the normalized hyperbolic sinus cardinal function $\operatorname{sinhc}(x)$ for long double argument `__x`.

See also

[sinhc](#) for details.

Definition at line 2565 of file specfun.h.

7.2.2.315 `sinhint()`

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::sinhint (
    _Tp __x ) [inline]
```

Return the hyperbolic sine integral $Shi(x)$ of real argument `x`.

The hyperbolic sine integral is defined by

$$Shi(x) = \int_0^x \frac{\sinh(t)}{t} dt$$

Template Parameters

<code>_Tp</code>	The type of the real argument
------------------	-------------------------------

Parameters

<code>__x</code>	The argument
------------------	--------------

Definition at line 1897 of file `specfun.h`.

7.2.2.316 `sinhintf()`

```
float __gnu_cxx::sinhintf (
    float __x ) [inline]
```

Return the hyperbolic sine integral of `float` argument `x`.

See also

[sinhint](#) for details.

Definition at line 1870 of file `specfun.h`.

7.2.2.317 `sinhintl()`

```
long double __gnu_cxx::sinhintl (
    long double __x ) [inline]
```

Return the hyperbolic sine integral $Shi(x)$ of `long double` argument `x`.

See also

[sinhint](#) for details.

Definition at line 1880 of file `specfun.h`.

7.2.2.318 `sinint()`

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::sinint (
    _Tp __x )    [inline]
```

Return the sine integral $Si(x)$ of real argument `x`.

The sine integral is defined by

$$Si(x) = \int_0^x \frac{\sin(t)}{t} dt$$

Parameters

<code>__x</code>	The real upper integration limit
------------------	----------------------------------

Definition at line 1816 of file `specfun.h`.

7.2.2.319 `sinintf()`

```
float __gnu_cxx::sinintf (
    float __x )    [inline]
```

Return the sine integral $Si(x)$ of `float` argument `x`.

See also

[sinint](#) for details.

Definition at line 1791 of file `specfun.h`.

7.2.2.320 `sinintl()`

```
long double __gnu_cxx::sinintl (
    long double __x )    [inline]
```

Return the sine integral $Si(x)$ of `long double` argument `x`.

See also

[sinint](#) for details.

Definition at line 1801 of file `specfun.h`.

7.2.2.321 `sph_bessel_i()`

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::sph_bessel_i (
    unsigned int __n,
    _Tp __x ) [inline]
```

Return the regular modified spherical Bessel function $i_n(x)$ of nonnegative order n and real argument $x \geq 0$.

The spherical Bessel function is defined by:

$$i_n(x) = \left(\frac{\pi}{2x}\right)^{1/2} I_{n+1/2}(x)$$

Template Parameters

<code>_Tp</code>	The floating-point type of the argument <code>__x</code> .
------------------	--

Parameters

<code>__n</code>	The integral order $n \geq 0$
<code>__x</code>	The real argument $x \geq 0$

Exceptions

<code>std::domain_error</code>	if <code>__x < 0</code> .
--------------------------------	------------------------------

Definition at line 2817 of file `specfun.h`.

7.2.2.322 `sph_bessel_if()`

```
float __gnu_cxx::sph_bessel_if (
    unsigned int __n,
    float __x ) [inline]
```

Return the regular modified spherical Bessel function $i_n(x)$ of nonnegative order n and `float` argument $x \geq 0$.

See also

[sph_bessel_i](#) for details.

Definition at line 2788 of file `specfun.h`.

7.2.2.323 sph_bessel_il()

```
long double __gnu_cxx::sph_bessel_il (
    unsigned int __n,
    long double __x ) [inline]
```

Return the regular modified spherical Bessel function $i_n(x)$ of nonnegative order n and `long double` argument $x \geq 0$.

See also

[sph_bessel_i](#) for details.

Definition at line 2798 of file specfun.h.

7.2.2.324 sph_bessel_k()

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::sph_bessel_k (
    unsigned int __n,
    _Tp __x ) [inline]
```

Return the irregular modified spherical Bessel function $k_n(x)$ of nonnegative order n and real argument $x \geq 0$.

The spherical Bessel function is defined by:

$$k_n(x) = \left(\frac{\pi}{2x}\right)^{1/2} K_{n+1/2}(x)$$

Template Parameters

<code>_Tp</code>	The floating-point type of the argument <code>__x</code> .
------------------	--

Parameters

<code>__n</code>	The integral order $n \geq 0$
<code>__x</code>	The real argument $x \geq 0$

Exceptions

<code>std::domain_error</code>	if <code>__x < 0</code> .
--------------------------------	------------------------------

Definition at line 2861 of file specfun.h.

7.2.2.325 sph_bessel_kf()

```
float __gnu_cxx::sph_bessel_kf (
    unsigned int __n,
    float __x ) [inline]
```

Return the irregular modified spherical Bessel function $k_n(x)$ of nonnegative order n and `float` argument $x \geq 0$.

See also

[sph_bessel_k](#) for more details.

Definition at line 2832 of file specfun.h.

7.2.2.326 sph_bessel_kl()

```
long double __gnu_cxx::sph_bessel_kl (
    unsigned int __n,
    long double __x ) [inline]
```

Return the irregular modified spherical Bessel function $k_n(x)$ of nonnegative order n and `long double` argument $x \geq 0$.

See also

[sph_bessel_k](#) for more details.

Definition at line 2842 of file specfun.h.

7.2.2.327 sph_hankel_1() [1/2]

```
template<typename _Tp >
std::complex<__gnu_cxx::fp_promote_t<_Tp> > __gnu_cxx::sph_hankel_1 (
    unsigned int __n,
    _Tp __z ) [inline]
```

Return the spherical Hankel function of the first kind $h_n^{(1)}(x)$ of nonnegative order n and real argument $x \geq 0$.

The spherical Hankel function of the first kind is defined by:

$$h_n^{(1)}(x) = \left(\frac{\pi}{2x}\right)^{1/2} H_{n+1/2}^{(1)}(x)$$

or in terms of the cylindrical Bessel and Neumann functions by:

$$h_n^{(1)}(x) = \left(\frac{\pi}{2x}\right)^{1/2} [J_{n+1/2}(x) + iN_{n+1/2}(x)]$$

Template Parameters

<code>_Tp</code>	The real type of the argument
------------------	-------------------------------

Parameters

<code>__n</code>	The non-negative order
<code>__z</code>	The real argument

Definition at line 2725 of file `specfun.h`.

7.2.2.328 `sph_hankel_1()` [2/2]

```
template<typename _Tp >
std::complex<__gnu_cxx::fp_promote_t<_Tp> > __gnu_cxx::sph_hankel_1 (
    unsigned int __n,
    std::complex< _Tp > __x ) [inline]
```

Return the complex spherical Hankel function of the first kind $h_n^{(1)}(x)$ of non-negative integral `n` and complex argument `x`.

The spherical Hankel function of the first kind is defined by

$$h_n^{(1)}(x) = \left(\frac{\pi}{2x}\right)^{1/2} H_{n+1/2}^{(1)}(x) = j_n(x) + in_n(x)$$

where $j_n(x)$ and $n_n(x)$ are the spherical Bessel and Neumann functions respectively.

Parameters

<code>__n</code>	The integral order ≥ 0
<code>__x</code>	The complex argument

Definition at line 5102 of file `specfun.h`.

7.2.2.329 `sph_hankel_1f()` [1/2]

```
std::complex<float> __gnu_cxx::sph_hankel_1f (
    unsigned int __n,
    float __z ) [inline]
```

Return the spherical Hankel function of the first kind $h_n^{(1)}(x)$ of nonnegative order `n` and `float` argument `x` ≥ 0 .

See also

[sph_hankel_1](#) for details.

Definition at line 2692 of file specfun.h.

7.2.2.330 sph_hankel_1f() [2/2]

```
std::complex<float> __gnu_cxx::sph_hankel_1f (
    unsigned int __n,
    std::complex< float > __x ) [inline]
```

Return the complex spherical Hankel function of the first kind $h_n^{(1)}(x)$ of non-negative integral n and $\text{std::complex<float>}$ argument x .

See also

[sph_hankel_1](#) for more details.

Definition at line 5070 of file specfun.h.

7.2.2.331 sph_hankel_1l() [1/2]

```
std::complex<long double> __gnu_cxx::sph_hankel_1l (
    unsigned int __n,
    long double __z ) [inline]
```

Return the spherical Hankel function of the first kind $h_n^{(1)}(x)$ of nonnegative order n and long double argument $x \geq 0$.

See also

[sph_hankel_1](#) for details.

Definition at line 2702 of file specfun.h.

7.2.2.332 sph_hankel_1l() [2/2]

```
std::complex<long double> __gnu_cxx::sph_hankel_1l (
    unsigned int __n,
    std::complex< long double > __x ) [inline]
```

Return the complex spherical Hankel function of the first kind $h_n^{(1)}(x)$ of non-negative integral n and $\text{std::complex<long double>}$ argument x .

See also

[sph_hankel_1](#) for more details.

Definition at line 5081 of file specfun.h.

7.2.2.333 sph_hankel_2() [1/2]

```
template<typename _Tp >
std::complex<__gnu_cxx::fp_promote_t<_Tp> > __gnu_cxx::sph_hankel_2 (
    unsigned int __n,
    _Tp __z ) [inline]
```

Return the spherical Hankel function of the second kind $h_n^{(2)}(x)$ of nonnegative order n and real argument $x \geq 0$.

The spherical Hankel function of the second kind is defined by:

$$h_n^{(2)}(x) = \left(\frac{\pi}{2x}\right)^{1/2} H_{n+1/2}^{(2)}(x)$$

or in terms of the cylindrical Bessel and Neumann functions by:

$$h_n^{(2)}(x) = \left(\frac{\pi}{2x}\right)^{1/2} [J_{n+1/2}(x) - iN_{n+1/2}(x)]$$

Template Parameters

<code>_Tp</code>	The real type of the argument
------------------	-------------------------------

Parameters

<code>↔ _n</code>	The non-negative order
<code>↔ _z</code>	The real argument

Definition at line 2773 of file specfun.h.

7.2.2.334 sph_hankel_2() [2/2]

```
template<typename _Tp >
std::complex<__gnu_cxx::fp_promote_t<_Tp> > __gnu_cxx::sph_hankel_2 (
    unsigned int __n,
    std::complex< _Tp > __x ) [inline]
```

Return the complex spherical Hankel function of the second kind $h_n^{(2)}(x)$ of nonnegative order `n` and complex argument `x`.

The spherical Hankel function of the second kind is defined by

$$h_n^{(2)}(x) = \left(\frac{\pi}{2x}\right)^{1/2} H_{n+1/2}^{(2)}(x) = j_n(x) - in_n(x)$$

where $j_n(x)$ and $n_n(x)$ are the spherical Bessel and Neumann functions respectively.

Parameters

<code>__n</code>	The integral order ≥ 0
<code>__x</code>	The complex argument

Definition at line 5150 of file `specfun.h`.

7.2.2.335 sph_hankel_2f() [1/2]

```
std::complex<float> __gnu_cxx::sph_hankel_2f (
    unsigned int __n,
    float __z ) [inline]
```

Return the spherical Hankel function of the second kind $h_n^{(2)}(x)$ of nonnegative order `n` and `float` argument `x` ≥ 0 .

See also

[sph_hankel_2](#) for details.

Definition at line 2740 of file `specfun.h`.

7.2.2.336 sph_hankel_2f() [2/2]

```
std::complex<float> __gnu_cxx::sph_hankel_2f (
    unsigned int __n,
    std::complex< float > __x ) [inline]
```

Return the complex spherical Hankel function of the second kind $h_n^{(2)}(x)$ of non-negative integral n and `std::complex<float>` argument x .

See also

[sph_hankel_2](#) for more details.

Definition at line 5118 of file specfun.h.

7.2.2.337 sph_hankel_2l() [1/2]

```
std::complex<long double> __gnu_cxx::sph_hankel_2l (
    unsigned int __n,
    long double __z ) [inline]
```

Return the spherical Hankel function of the second kind $h_n^{(2)}(x)$ of nonnegative order n and `long double` argument $x \geq 0$.

See also

[sph_hankel_2](#) for details.

Definition at line 2750 of file specfun.h.

7.2.2.338 sph_hankel_2l() [2/2]

```
std::complex<long double> __gnu_cxx::sph_hankel_2l (
    unsigned int __n,
    std::complex< long double > __x ) [inline]
```

Return the complex spherical Hankel function of the second kind $h_n^{(2)}(x)$ of non-negative integral n and `std::complex<long double>` argument x .

See also

[sph_hankel_2](#) for more details.

Definition at line 5129 of file specfun.h.

7.2.2.339 sph_harmonic()

```
template<typename _Ttheta , typename _Tphi >
std::complex<__gnu_cxx::fp_promote_t<_Ttheta, _Tphi> > __gnu_cxx::sph_harmonic (
    unsigned int __l,
    int __m,
    _Ttheta __theta,
    _Tphi __phi ) [inline]
```

Return the complex spherical harmonic function of degree l , order m , and real zenith angle θ , and azimuth angle ϕ .

The spherical harmonic function is defined by:

$$Y_l^m(\theta, \phi) = (-1)^m \frac{(2l+1)}{4\pi} \frac{(l-m)!}{(l+m)!} P_l^{|m|}(\cos \theta) \exp^{im\phi}$$

Note

$$Y_l^m(\theta, \phi) = 0 @cf |m| > l.$$

Parameters

<code>__l</code>	The order
<code>__m</code>	The degree
<code>__theta</code>	The zenith angle in radians
<code>__phi</code>	The azimuth angle in radians

Definition at line 5203 of file specfun.h.

7.2.2.340 sph_harmonicf()

```
std::complex<float> __gnu_cxx::sph_harmonicf (
    unsigned int __l,
    int __m,
    float __theta,
    float __phi ) [inline]
```

Return the complex spherical harmonic function of degree l , order m , and float zenith angle θ , and azimuth angle ϕ .

See also

[sph_harmonic](#) for details.

Definition at line 5166 of file specfun.h.

7.2.2.341 sph_harmonic()

```
std::complex<long double> __gnu_cxx::sph_harmonic1 (
    unsigned int __l,
    int __m,
    long double __theta,
    long double __phi ) [inline]
```

Return the complex spherical harmonic function of degree l , order m , and long double zenith angle θ , and azimuth angle ϕ .

See also

[sph_harmonic](#) for details.

Definition at line 5178 of file specfun.h.

7.2.2.342 stirling_1() [1/2]

```
template<typename _Tp >
_Tp __gnu_cxx::stirling_1 (
    unsigned int __n,
    unsigned int __m ) [inline]
```

Return the Stirling number of the first kind.

The Stirling numbers of the first kind are the coefficients of the Pochhammer polynomials or the rising factorials:

$$(x)_n = \sum_{k=0}^n \begin{bmatrix} n \\ k \end{bmatrix} x^k$$

The recursion is

$$\begin{bmatrix} n+1 \\ m \end{bmatrix} = \begin{bmatrix} n \\ m-1 \end{bmatrix} - n \begin{bmatrix} n \\ m \end{bmatrix}$$

with starting values

$$\begin{bmatrix} 0 \\ 0 \rightarrow m \end{bmatrix} = 1, 0, 0, \dots, 0$$

and

$$\begin{bmatrix} 0 \rightarrow n \\ 0 \end{bmatrix} = 1, 0, 0, \dots, 0$$

The Stirling number of the first kind is denoted by other symbols in the literature, usually $S_n^{(m)}$.

Definition at line 7155 of file specfun.h.

7.2.2.343 `stirling_1()` [2/2]

```
template<typename _Tp >
std::vector<_Tp> __gnu_cxx::stirling_1 (
    unsigned int __n ) [inline]
```

Return a vector of Stirling numbers of the first kind.

Definition at line 7163 of file specfun.h.

7.2.2.344 `stirling_2()` [1/2]

```
template<typename _Tp >
_Tp __gnu_cxx::stirling_2 (
    unsigned int __n,
    unsigned int __m ) [inline]
```

Return the Stirling number of the second kind by series expansion or by recursion.

The series is:

$$\sigma_n^{(m)} = \left\{ \begin{matrix} n \\ m \end{matrix} \right\} = \sum_{k=0}^m \frac{(-1)^{m-k} k^n}{(m-k)! k!}$$

The Stirling number of the second kind is denoted by other symbols in the literature: $\sigma_n^{(m)}$, $S_n^{(m)}$ and others.

Definition at line 7182 of file specfun.h.

7.2.2.345 `stirling_2()` [2/2]

```
template<typename _Tp >
std::vector<_Tp> __gnu_cxx::stirling_2 (
    unsigned int __n ) [inline]
```

Return a vector of Stirling numbers of the second kind.

Definition at line 7190 of file specfun.h.

7.2.2.346 `student_t_p()`

```
template<typename _Tt , typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::student_t_p (
    _Tt __t,
    unsigned int __nu )
```

Return the Students T probability function.

The students T propability function is related to the incomplete beta function:

$$A(t|\nu) = 1 - I_{\frac{\nu}{\nu+t^2}}\left(\frac{\nu}{2}, \frac{1}{2}\right)$$

(see ibeta).

Parameters

<code>__t</code>	
<code>__nu</code>	

Definition at line 6836 of file specfun.h.

7.2.2.347 student_t_pdf()

```
template<typename _Tt , typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::student_t_pdf (
    _Tt __t,
    unsigned int __nu )
```

Return the complement of the Students T probability function.

The complement of the students T propability function is:

$$A_c(t|\nu) = I_{\frac{\nu}{\nu+t^2}}\left(\frac{\nu}{2}, \frac{1}{2}\right) = 1 - A(t|\nu)$$

Parameters

<code>__t</code>	
<code>__nu</code>	

Definition at line 6816 of file specfun.h.

7.2.2.348 tan_pi()

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::tan_pi (
    _Tp __x ) [inline]
```

Return the reperiodized tangent function $\tan_{\pi}(x)$ for real argument x.

The reperiodized tangent function is defined by:

$$\tan_{\pi}(x) = \tan(\pi x)$$

Template Parameters

<code>_Tp</code>	The floating-point type of the argument <code>__x</code> .
------------------	--

Parameters

<code>__x</code>	The argument
------------------	--------------

Definition at line 6513 of file specfun.h.

7.2.2.349 `tan_pif()`

```
float __gnu_cxx::tan_pif (
    float __x ) [inline]
```

Return the reperiodized tangent function $\tan_{\pi}(x)$ for `float` argument `x`.

See also

[tan_pi](#) for more details.

Definition at line 6486 of file specfun.h.

7.2.2.350 `tan_pil()`

```
long double __gnu_cxx::tan_pil (
    long double __x ) [inline]
```

Return the reperiodized tangent function $\tan_{\pi}(x)$ for `long double` argument `x`.

See also

[tan_pi](#) for more details.

Definition at line 6496 of file specfun.h.

7.2.2.351 `tanh_pi()`

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::tanh_pi (
    _Tp __x ) [inline]
```

Return the reperiodized hyperbolic tangent function $\tanh_{\pi}(x)$ for real argument `x`.

The reperiodized hyperbolic tangent function is defined by:

$$\tanh_{\pi}(x) = \tanh(\pi x)$$

Template Parameters

<code>__Tp</code>	The floating-point type of the argument <code>__x</code> .
-------------------	--

Parameters

<code>__x</code>	The argument
------------------	--------------

Definition at line 6555 of file `specfun.h`.

7.2.2.352 `tanh_pif()`

```
float __gnu_cxx::tanh_pif (
    float __x ) [inline]
```

Return the reperiodized hyperbolic tangent function $\tanh_{\pi}(x)$ for `float` argument `x`.

See also

[tanh_pi](#) for more details.

Definition at line 6528 of file `specfun.h`.

7.2.2.353 `tanh_pil()`

```
long double __gnu_cxx::tanh_pil (
    long double __x ) [inline]
```

Return the reperiodized hyperbolic tangent function $\tanh_{\pi}(x)$ for `long double` argument `x`.

See also

[tanh_pi](#) for more details.

Definition at line 6538 of file `specfun.h`.

7.2.2.354 `tgamma()` [1/3]

```
template<typename _Ta >
__gnu_cxx::fp_promote_t<_Ta> __gnu_cxx::tgamma (
    _Ta __a ) [inline]
```

Return the gamma function for real argument.

Definition at line 3083 of file `specfun.h`.

Referenced by `std::__detail::__tricoli_u_naive()`.

7.2.2.355 `tgamma()` [2/3]

```
template<typename _Ta >
std::complex<__gnu_cxx::fp_promote_t<_Ta> > __gnu_cxx::tgamma (
    std::complex< _Ta > __a ) [inline]
```

Return the gamma function for complex argument.

Definition at line 3115 of file `specfun.h`.

7.2.2.356 `tgamma()` [3/3]

```
template<typename _Ta , typename _Tp >
__gnu_cxx::fp_promote_t<_Ta, _Tp> __gnu_cxx::tgamma (
    _Ta __a,
    _Tp __x ) [inline]
```

Return the upper incomplete gamma function $\Gamma(a, x)$. The (upper) incomplete gamma function is defined by

$$\Gamma(a, x) = \int_x^{\infty} t^{a-1} e^{-t} dt$$

Definition at line 3152 of file `specfun.h`.

7.2.2.357 tgamma_lower()

```
template<typename _Ta , typename _Tp >
__gnu_cxx::fp_promote_t<_Ta, _Tp> __gnu_cxx::tgamma_lower (
    _Ta __a,
    _Tp __x ) [inline]
```

Return the lower incomplete gamma function $\gamma(a, x)$. The lower incomplete gamma function is defined by

$$\gamma(a, x) = \int_0^x t^{a-1} e^{-t} dt$$

Definition at line 3189 of file specfun.h.

7.2.2.358 tgamma_lowerf()

```
float __gnu_cxx::tgamma_lowerf (
    float __a,
    float __x ) [inline]
```

Return the lower incomplete gamma function $\gamma(a, x)$ for `float` argument.

See also

[tgamma_lower](#) for details.

Definition at line 3167 of file specfun.h.

7.2.2.359 tgamma_lowerl()

```
long double __gnu_cxx::tgamma_lowerl (
    long double __a,
    long double __x ) [inline]
```

Return the lower incomplete gamma function $\gamma(a, x)$ for `long double` argument.

See also

[tgamma_lower](#) for details.

Definition at line 3177 of file specfun.h.

7.2.2.360 `tgammaf()` [1/3]

```
float __gnu_cxx::tgammaf (
    float __a ) [inline]
```

Return the gamma function for `float` argument.

See also

[lgamma](#) for details.

Definition at line 3065 of file `specfun.h`.

7.2.2.361 `tgammaf()` [2/3]

```
std::complex<float> __gnu_cxx::tgammaf (
    std::complex< float > __a ) [inline]
```

Return the gamma function for `std::complex<float>` argument.

See also

[lgamma](#) for details.

Definition at line 3097 of file `specfun.h`.

7.2.2.362 `tgammaf()` [3/3]

```
float __gnu_cxx::tgammaf (
    float __a,
    float __x ) [inline]
```

Return the upper incomplete gamma function $\Gamma(a, x)$ for `float` argument.

See also

[tgamma](#) for details.

Definition at line 3130 of file `specfun.h`.

7.2.2.363 `tgammal()` [1/3]

```
long double __gnu_cxx::tgammal (
    long double __a ) [inline]
```

Return the gamma function for `long double` argument.

See also

[lgamma](#) for details.

Definition at line 3075 of file `specfun.h`.

7.2.2.364 `tgammal()` [2/3]

```
std::complex<long double> __gnu_cxx::tgammal (
    std::complex< long double > __a ) [inline]
```

Return the gamma function for `std::complex<long double>` argument.

See also

[lgamma](#) for details.

Definition at line 3107 of file `specfun.h`.

7.2.2.365 `tgammal()` [3/3]

```
long double __gnu_cxx::tgammal (
    long double __a,
    long double __x ) [inline]
```

Return the upper incomplete gamma function $\Gamma(a, x)$ for `long double` argument.

See also

[tgamma](#) for details.

Definition at line 3140 of file `specfun.h`.

7.2.2.366 `theta_1()`

```
template<typename _Tpnu , typename _Tp >
__gnu_cxx::fp_promote_t<_Tpnu, _Tp> __gnu_cxx::theta_1 (
    _Tpnu __nu,
    _Tp __x ) [inline]
```

Return the exponential theta-1 function $\theta_1(\nu, x)$ of period ν and argument x .

The exponential theta-1 function is defined by

$$\theta_1(\nu, x) = \frac{1}{\sqrt{\pi x}} \sum_{j=-\infty}^{+\infty} (-1)^j \exp\left(\frac{-(\nu + j - 1/2)^2}{x}\right)$$

Parameters

<code>__nu</code>	The periodic (period = 2) argument
<code>__x</code>	The argument

Definition at line 5641 of file specfun.h.

7.2.2.367 `theta_1f()`

```
float __gnu_cxx::theta_1f (
    float __nu,
    float __x ) [inline]
```

Return the exponential theta-1 function $\theta_1(\nu, x)$ of period ν and argument x .

See also

[theta_1](#) for details.

Definition at line 5613 of file specfun.h.

7.2.2.368 `theta_1l()`

```
long double __gnu_cxx::theta_1l (
    long double __nu,
    long double __x ) [inline]
```

Return the exponential theta-1 function $\theta_1(\nu, x)$ of period ν and argument x .

See also

[theta_1](#) for details.

Definition at line 5623 of file specfun.h.

7.2.2.369 `theta_2()`

```
template<typename _Tpnu , typename _Tp >
__gnu_cxx::fp_promote_t<_Tpnu, _Tp> __gnu_cxx::theta_2 (
    _Tpnu __nu,
    _Tp __x ) [inline]
```

Return the exponential theta-2 function $\theta_2(\nu, x)$ of period ν and argument x .

The exponential theta-2 function is defined by

$$\theta_2(\nu, x) = \frac{1}{\sqrt{\pi x}} \sum_{j=-\infty}^{+\infty} (-1)^j \exp\left(\frac{-(\nu + j)^2}{x}\right)$$

Parameters

<code>__nu</code>	The periodic (period = 2) argument
<code>__x</code>	The argument

Definition at line 5684 of file specfun.h.

7.2.2.370 `theta_2f()`

```
float __gnu_cxx::theta_2f (
    float __nu,
    float __x ) [inline]
```

Return the exponential theta-2 function $\theta_2(\nu, x)$ of period ν and argument x .

See also

[theta_2](#) for details.

Definition at line 5656 of file specfun.h.

7.2.2.371 `theta_2l()`

```
long double __gnu_cxx::theta_2l (
    long double __nu,
    long double __x ) [inline]
```

Return the exponential theta-2 function $\theta_2(\nu, x)$ of period ν and argument x .

See also

[theta_2](#) for details.

Definition at line 5666 of file specfun.h.

7.2.2.372 `theta_3()`

```
template<typename _Tpnu , typename _Tp >
__gnu_cxx::fp_promote_t<_Tpnu, _Tp> __gnu_cxx::theta_3 (
    _Tpnu __nu,
    _Tp __x ) [inline]
```

Return the exponential theta-3 function $\theta_3(\nu, x)$ of period ν and argument x .

The exponential theta-3 function is defined by

$$\theta_3(\nu, x) = \frac{1}{\sqrt{\pi x}} \sum_{j=-\infty}^{+\infty} \exp\left(\frac{-(\nu + j)^2}{x}\right)$$

Parameters

<code>__nu</code>	The periodic (period = 1) argument
<code>__x</code>	The argument

Definition at line 5727 of file specfun.h.

7.2.2.373 `theta_3f()`

```
float __gnu_cxx::theta_3f (
    float __nu,
    float __x ) [inline]
```

Return the exponential theta-3 function $\theta_3(\nu, x)$ of period ν and argument x .

See also

[theta_3](#) for details.

Definition at line 5699 of file specfun.h.

7.2.2.374 `theta_3l()`

```
long double __gnu_cxx::theta_3l (
    long double __nu,
    long double __x ) [inline]
```

Return the exponential theta-3 function $\theta_3(\nu, x)$ of period ν and argument x .

See also

[theta_3](#) for details.

Definition at line 5709 of file specfun.h.

7.2.2.375 `theta_4()`

```
template<typename _Tpnu , typename _Tp >
__gnu_cxx::fp_promote_t<_Tpnu, _Tp> __gnu_cxx::theta_4 (
    _Tpnu __nu,
    _Tp __x ) [inline]
```

Return the exponential theta-4 function $\theta_4(\nu, x)$ of period ν and argument x .

The exponential theta-4 function is defined by

$$\theta_4(\nu, x) = \frac{1}{\sqrt{\pi x}} \sum_{j=-\infty}^{+\infty} \exp\left(\frac{-(\nu + j + 1/2)^2}{x}\right)$$

Parameters

<code>__nu</code>	The periodic (period = 1) argument
<code>__x</code>	The argument

Definition at line 5770 of file specfun.h.

7.2.2.376 `theta_4f()`

```
float __gnu_cxx::theta_4f (
    float __nu,
    float __x ) [inline]
```

Return the exponential theta-4 function $\theta_4(\nu, x)$ of period ν and argument x .

See also

[theta_4](#) for details.

Definition at line 5742 of file specfun.h.

7.2.2.377 `theta_4l()`

```
long double __gnu_cxx::theta_4l (
    long double __nu,
    long double __x ) [inline]
```

Return the exponential theta-4 function $\theta_4(\nu, x)$ of period ν and argument x .

See also

[theta_4](#) for details.

Definition at line 5752 of file specfun.h.

7.2.2.378 `theta_c()`

```
template<typename _Tp, typename _Tp>
__gnu_cxx::fp_promote_t<_Tp, _Tp> __gnu_cxx::theta_c (
    _Tp __k,
    _Tp __x ) [inline]
```

Return the Neville theta-c function $\theta_c(k, x)$ of modulus `k` and argument `x`.

The Neville theta-c function is defined by

$$\theta_c(k, x) = \sqrt{\frac{\pi}{2kK(k)}} \theta_1\left(q(k), \frac{\pi x}{2K(k)}\right)$$

where $q(k)$ is the elliptic nome, $K(k)$ is the complete Legendre elliptic integral of the first kind, and $\theta_1(\nu, x)$ is the exponential theta-1 function.

See also

[ellnome](#), [std::comp_ellint_1](#), and [theta_1](#) for details.

Parameters

<code>↔ _k</code>	The modulus $-1 \leq k \leq +1$
<code>↔ _x</code>	The argument

Definition at line 5906 of file `specfun.h`.

7.2.2.379 `theta_cf()`

```
float __gnu_cxx::theta_cf (
    float __k,
    float __x ) [inline]
```

Return the Neville theta-c function $\theta_c(k, x)$ of modulus `k` and argument `x`.

See also

[theta_c](#) for details.

Definition at line 5874 of file `specfun.h`.

7.2.2.380 `theta_cl()`

```
long double __gnu_cxx::theta_cl (
    long double __k,
    long double __x ) [inline]
```

Return the Neville theta-c function $\theta_c(k, x)$ of long double modulus k and argument x .

See also

[theta_c](#) for details.

Definition at line 5884 of file `specfun.h`.

7.2.2.381 `theta_d()`

```
template<typename _TpK , typename _Tp >
__gnu_cxx::fp_promote_t<_TpK, _Tp> __gnu_cxx::theta_d (
    _TpK __k,
    _Tp __x ) [inline]
```

Return the Neville theta-d function $\theta_d(k, x)$ of modulus k and argument x .

The Neville theta-d function is defined by

$$\theta_d(k, x) = \sqrt{\frac{\pi}{2K(k)}} \theta_3 \left(q(k), \frac{\pi x}{2K(k)} \right)$$

where $q(k)$ is the elliptic nome, $K(k)$ is the complete Legendre elliptic integral of the first kind, and $\theta_3(\nu, x)$ is the exponential theta-3 function.

See also

[ellnome](#), [std::comp_ellint_1](#), and [theta_3](#) for details.

Parameters

\hookleftarrow <code>_k</code>	The modulus $-1 \leq k \leq +1$
\hookleftarrow <code>_x</code>	The argument

Definition at line 5953 of file `specfun.h`.

7.2.2.382 theta_df()

```
float __gnu_cxx::theta_df (
    float __k,
    float __x ) [inline]
```

Return the Neville theta-d function $\theta_d(k, x)$ of modulus k and argument x .

See also

[theta_d](#) for details.

Definition at line 5921 of file specfun.h.

7.2.2.383 theta_dl()

```
long double __gnu_cxx::theta_dl (
    long double __k,
    long double __x ) [inline]
```

Return the Neville theta-d function $\theta_d(k, x)$ of `long double` modulus k and argument x .

See also

[theta_d](#) for details.

Definition at line 5931 of file specfun.h.

7.2.2.384 theta_n()

```
template<typename _TpK , typename _Tp >
__gnu_cxx::fp_promote_t<_TpK, _Tp> __gnu_cxx::theta_n (
    _TpK __k,
    _Tp __x ) [inline]
```

Return the Neville theta-n function $\theta_n(k, x)$ of modulus k and argument x .

The Neville theta-n function is defined by

$$\theta_n(k, x) = \sqrt{\frac{\pi}{2k'K(k)}} \theta_4 \left(q(k), \frac{\pi x}{2K(k)} \right)$$

where $q(k)$ is the elliptic nome, $K(k)$ is the complete Legendre elliptic integral of the first kind, and $\theta_4(\nu, x)$ is the exponential theta-4 function.

See also

[ellnome](#), [std::comp_ellint_1](#), and [theta_4](#) for details.

Parameters

\leftrightarrow _k	The modulus $-1 \leq k \leq +1$
\leftrightarrow _x	The argument

Definition at line 6000 of file specfun.h.

7.2.2.385 theta_nf()

```
float __gnu_cxx::theta_nf (
    float __k,
    float __x ) [inline]
```

Return the Neville theta-n function $\theta_n(k, x)$ of modulus k and argument x .

See also

[theta_n](#) for details.

Definition at line 5968 of file specfun.h.

7.2.2.386 theta_nl()

```
long double __gnu_cxx::theta_nl (
    long double __k,
    long double __x ) [inline]
```

Return the Neville theta-n function $\theta_n(k, x)$ of long double modulus k and argument x .

See also

[theta_n](#) for details.

Definition at line 5978 of file specfun.h.

7.2.2.387 `theta_s()`

```
template<typename _TpK , typename _Tp >
__gnu_cxx::fp_promote_t<_TpK, _Tp> __gnu_cxx::theta_s (
    _TpK __k,
    _Tp __x ) [inline]
```

Return the Neville theta-s function $\theta_s(k, x)$ of modulus `k` and argument `x`.

The Neville theta-s function is defined by

$$\theta_s(k, x) = \sqrt{\frac{\pi}{2kk'K(k)}} \theta_1\left(q(k), \frac{\pi x}{2K(k)}\right)$$

where $q(k)$ is the elliptic nome, $K(k)$ is the complete Legendre elliptic integral of the first kind, and $\theta_1(\nu, x)$ is the exponential theta-1 function.

See also

[ellnome](#), [std::comp_ellint_1](#), and [theta_1](#) for details.

Parameters

\leftrightarrow <code>_k</code>	The modulus $-1 \leq k \leq +1$
\leftrightarrow <code>_x</code>	The argument

Definition at line 5859 of file `specfun.h`.

7.2.2.388 `theta_sf()`

```
float __gnu_cxx::theta_sf (
    float __k,
    float __x ) [inline]
```

Return the Neville theta-s function $\theta_s(k, x)$ of modulus `k` and argument `x`.

See also

[theta_s](#) for details.

Definition at line 5827 of file `specfun.h`.

7.2.2.389 `theta_sl()`

```
long double __gnu_cxx::theta_sl (
    long double __k,
    long double __x ) [inline]
```

Return the Neville theta-s function $\theta_s(k, x)$ of `long double` modulus `k` and argument `x`.

See also

[theta_s](#) for details.

Definition at line 5837 of file `specfun.h`.

7.2.2.390 `tricoli_u()`

```
template<typename _Tpa , typename _Tpc , typename _Tp >
__gnu_cxx::fp_promote_t<_Tpa, _Tpc, _Tp> __gnu_cxx::tricoli_u (
    _Tpa __a,
    _Tpc __c,
    _Tp __x ) [inline]
```

Return the Tricomi confluent hypergeometric function $U(a, c, x)$ of real numerator parameter `a`, denominator parameter `c`, and argument `x`.

The Tricomi confluent hypergeometric function is defined by

$$U(a, c, x) = \frac{\Gamma(1-c)}{\Gamma(a-c+1)} {}_1F_1(a; c; x) + \frac{\Gamma(c-1)}{\Gamma(a)} x^{1-c} {}_1F_1(a-c+1; 2-c; x)$$

where ${}_1F_1(a; c; x)$ is the confluent hypergeometric function.

See also

[conf_hyperg](#).

Parameters

<code>__a</code>	The numerator parameter
<code>__c</code>	The denominator parameter
<code>__x</code>	The argument

Definition at line 1561 of file `specfun.h`.

7.2.2.391 `tricoli_uf()`

```
float __gnu_cxx::tricoli_uf (
    float __a,
    float __c,
    float __x ) [inline]
```

Return the Tricoli confluent hypergeometric function $U(a, c, x)$ of `float` numerator parameter `a`, denominator parameter `c`, and argument `x`.

See also

[tricoli_u](#) for details.

Definition at line 1527 of file `specfun.h`.

7.2.2.392 `tricoli_ul()`

```
long double __gnu_cxx::tricoli_ul (
    long double __a,
    long double __c,
    long double __x ) [inline]
```

Return the Tricoli confluent hypergeometric function $U(a, c, x)$ of `long double` numerator parameter `a`, denominator parameter `c`, and argument `x`.

See also

[tricoli_u](#) for details.

Definition at line 1538 of file `specfun.h`.

7.2.2.393 `weibull_p()`

```
template<typename _Ta , typename _Tb , typename _Tp >
__gnu_cxx::fp_promote_t<_Ta, _Tb, _Tp> __gnu_cxx::weibull_p (
    _Ta __a,
    _Tb __b,
    _Tp __x ) [inline]
```

Return the Weibull cumulative probability density function.

The formula for the Weibull cumulative probability density function is

$$F(x|\lambda) = 1 - e^{-(x/b)^a} \text{ for } x \geq 0$$

Definition at line 6796 of file `specfun.h`.

7.2.2.394 weibull_pdf()

```
template<typename _Ta , typename _Tb , typename _Tp >
__gnu_cxx::fp_promote_t<_Ta, _Tb, _Tp> __gnu_cxx::weibull_pdf (
    _Ta __a,
    _Tb __b,
    _Tp __x ) [inline]
```

Return the Weibull probability density function.

The formula for the Weibull probability density function is

$$f(x|a, b) = \frac{a}{b} \left(\frac{x}{b}\right)^{a-1} \exp - \left(\frac{x}{b}\right)^a \text{ for } x \geq 0$$

Definition at line 6780 of file specfun.h.

7.2.2.395 zernike()

```
template<typename _Trho , typename _Tphi >
__gnu_cxx::fp_promote_t<_Trho, _Tphi> __gnu_cxx::zernike (
    unsigned int __n,
    int __m,
    _Trho __rho,
    _Tphi __phi ) [inline]
```

Return the Zernike polynomial $Z_n^m(\rho, \phi)$ for non-negative degree n , signed order m , and real radial argument ρ and azimuthal angle ϕ .

The even Zernike polynomials are defined by:

$$Z_n^m(\rho, \phi) = R_n^m(\rho) \cos(m\phi)$$

and the odd Zernike polynomials are defined by:

$$Z_n^{-m}(\rho, \phi) = R_n^m(\rho) \sin(m\phi)$$

for non-negative degree m and $m \leq n$ and where $R_n^m(\rho)$ is the radial polynomial (see radpoly).

Template Parameters

<code>_Trho</code>	The real type of the radial coordinate
<code>_Tphi</code>	The real type of the azimuthal angle

Parameters

<code>__n</code>	The non-negative degree.
<code>__m</code>	The (signed) azimuthal order

Parameters

<code>__rho</code>	The radial coordinate
<code>__phi</code>	The azimuthal angle

Definition at line 2444 of file specfun.h.

7.2.2.396 `zernikef()`

```
float __gnu_cxx::zernikef (
    unsigned int __n,
    int __m,
    float __rho,
    float __phi ) [inline]
```

Return the Zernike polynomial $Z_n^m(\rho, \phi)$ for non-negative degree n , signed order m , and real radial argument ρ and azimuthal angle ϕ .

See also

[zernike](#) for details.

Definition at line 2405 of file specfun.h.

7.2.2.397 `zernikel()`

```
long double __gnu_cxx::zernikel (
    unsigned int __n,
    int __m,
    long double __rho,
    long double __phi ) [inline]
```

Return the Zernike polynomial $Z_n^m(\rho, \phi)$ for non-negative degree n , signed order m , and real radial argument ρ and azimuthal angle ϕ .

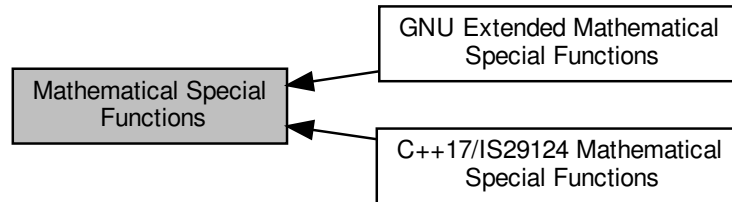
See also

[zernike](#) for details.

Definition at line 2416 of file specfun.h.

7.3 Mathematical Special Functions

Collaboration diagram for Mathematical Special Functions:



Modules

- [C++17/IS29124 Mathematical Special Functions](#)
- [GNU Extended Mathematical Special Functions](#)

7.3.1 Detailed Description

7.3.2 Mathematical Special Functions

A collection of advanced mathematical special functions, defined by ISO/IEC IS 29124 and then added to ISO C++ 2017.

7.3.2.1 Introduction and History

The first significant library upgrade on the road to C++2011, [TR1](#), included a set of 23 mathematical functions that significantly extended the standard transcendental functions inherited from C and declared in `<cmath>`.

Although most components from TR1 were eventually adopted for C++11 these math functions were left behind out of concern for implementability. The math functions were published as a separate international standard [IS 29124 - Extensions to the C++ Library to Support Mathematical Special Functions](#).

Follow-up proposals for new special functions have also been published: [A proposal to add special mathematical functions according to the ISO/IEC 80000-2:2009 standard](#), Vincent Reverdý.

[A Proposal to add Mathematical Functions for Statistics to the C++ Standard Library](#), Paul A Bristow.

[A proposal to add sincos to the standard library](#), Paul Dreik.

For C++17 these functions were incorporated into the main standard.

7.3.2.2 Contents

The following functions are implemented in namespace `std`:

- [assoc_laguerre](#) - Associated Laguerre functions
- [assoc_legendre](#) - Associated Legendre functions
- [assoc_legendre_q](#) - Associated Legendre functions of the second kind
- [beta](#) - Beta functions
- [comp_ellint_1](#) - Complete elliptic functions of the first kind
- [comp_ellint_2](#) - Complete elliptic functions of the second kind
- [comp_ellint_3](#) - Complete elliptic functions of the third kind
- [cyl_bessel_i](#) - Regular modified cylindrical Bessel functions
- [cyl_bessel_j](#) - Cylindrical Bessel functions of the first kind
- [cyl_bessel_k](#) - Irregular modified cylindrical Bessel functions
- [cyl_neumann](#) - Cylindrical Neumann functions or Cylindrical Bessel functions of the second kind
- [ellint_1](#) - Incomplete elliptic functions of the first kind
- [ellint_2](#) - Incomplete elliptic functions of the second kind
- [ellint_3](#) - Incomplete elliptic functions of the third kind
- [expint](#) - The exponential integral
- [hermite](#) - Hermite polynomials
- [laguerre](#) - Laguerre functions
- [legendre](#) - Legendre polynomials
- [riemann_zeta](#) - The Riemann zeta function
- [sph_bessel](#) - Spherical Bessel functions
- [sph_legendre](#) - Spherical Legendre functions
- [sph_neumann](#) - Spherical Neumann functions

The hypergeometric functions were stricken from the TR29124 and C++17 versions of this math library because of implementation concerns. However, since they were in the TR1 version and since they are popular we kept them as an extension in namespace `__gnu_cxx`:

- [conf_hyperg](#) - Confluent hypergeometric functions
- [hyperg](#) - Hypergeometric functions

In addition a large number of new functions are added as extensions:

- [airy_ai](#) - Airy functions of the first kind

- [airy_bi](#) - Airy functions of the second kind
- [bell](#) - Bell numbers and polynomials
- [bernoulli](#) - Bernoulli polynomials
- [binomial](#) - Binomial coefficients
- [bose_einstein](#) - Bose-Einstein integrals
- [chebyshev_t](#) - Chebyshev polynomials of the first kind
- [chebyshev_u](#) - Chebyshev polynomials of the second kind
- [chebyshev_v](#) - Chebyshev polynomials of the third kind
- [chebyshev_w](#) - Chebyshev polynomials of the fourth kind
- [clausen](#) - Clausen integrals
- [clausen_cl](#) - Clausen cosine integrals
- [clausen_sl](#) - Clausen sine integrals
- [comp_ellint_d](#) - Incomplete Legendre D elliptic integral
- [conf_hyperg_lim](#) - Confluent hypergeometric limit functions
- [cos_pi](#) - Reperiodized cosine function.
- [cosh_pi](#) - Reperiodized hyperbolic cosine function.
- [coshint](#) - Hyperbolic cosine integral
- [cosint](#) - Cosine integral
- [cyl_hankel_1](#) - Cylindrical Hankel functions of the first kind
- [cyl_hankel_2](#) - Cylindrical Hankel functions of the second kind
- [dawson](#) - Dawson integrals
- [debye](#) - Debye functions
- [digamma](#) - Digamma or psi function
- [dilog](#) - Dilogarithm functions
- [dirichlet_beta](#) - Dirichlet beta function
- [dirichlet_eta](#) - Dirichlet beta function
- [dirichlet_lambda](#) - Dirichlet lambda function
- [double_factorial](#) - Double factorials
- [ellint_d](#) - Legendre D elliptic integrals
- [ellint_rc](#) - Carlson elliptic functions R_C
- [ellint_rd](#) - Carlson elliptic functions R_D
- [ellint_rf](#) - Carlson elliptic functions R_F
- [ellint_rg](#) - Carlson elliptic functions R_G

- [ellint_rj](#) - Carlson elliptic functions R_J
- [ellnome](#) - Elliptic nome
- [euler](#) - Euler numbers
- [euler](#) - Euler polynomials
- [eulerian_1](#) - Eulerian numbers of the first kind
- [eulerian_2](#) - Eulerian numbers of the second kind
- [expint](#) - Exponential integrals
- [factorial](#) - Factorials
- [falling_factorial](#) - Falling factorials
- [fermi_dirac](#) - Fermi-Dirac integrals
- [fresnel_c](#) - Fresnel cosine integrals
- [fresnel_s](#) - Fresnel sine integrals
- [gamma_p](#) - Regularized lower incomplete gamma functions
- [gamma_q](#) - Regularized upper incomplete gamma functions
- [gamma_reciprocal](#) - Reciprocal gamma function
- [gegenbauer](#) - Gegenbauer polynomials
- [heuman_lambda](#) - Heuman lambda functions
- [hurwitz_zeta](#) - Hurwitz zeta functions
- [ibeta](#) - Regularized incomplete beta functions
- [jacobi](#) - Jacobi polynomials
- [jacobi_sn](#) - Jacobi sine amplitude functions
- [jacobi_cn](#) - Jacobi cosine amplitude functions
- [jacobi_dn](#) - Jacobi delta amplitude functions
- [theta_1](#) - Jacobi theta function 1
- [theta_2](#) - Jacobi theta function 2
- [theta_3](#) - Jacobi theta function 3
- [theta_4](#) - Jacobi theta function 4
- [jacobi_zeta](#) - Jacobi zeta functions
- [lah](#) - Lah numbers
- [lbinomial](#) - Log binomial coefficients
- [ldouble_factorial](#) - Log double factorials
- [legendre_q](#) - Legendre functions of the second kind
- [lerch_phi](#) - The Lerch transcendent

- [lfactorial](#) - Log factorials
- [lfalling_factorial](#) - Log falling factorials
- [lgamma](#) - Log gamma for complex arguments
- [lrising_factorial](#) - Log rising factorials
- [mittag_leffler](#) - Mittag-Leffler functions
- [owens_t](#) - Owens T functions
- [periodic_zeta](#) - Periodic zeta functions
- [radpoly](#) - Radial polynomials
- [rising_factorial](#) - Rising factorials
- [sinhc](#) - Hyperbolic sinus cardinal function
- [sinhc_pi](#) - Reperiodized hyperbolic sinus cardinal function
- [sinc](#) - Normalized sinus cardinal function
- [sincos](#) - Sine + cosine function
- [sincos_pi](#) - Reperiodized sine + cosine function
- [sin_pi](#) - Reperiodized sine function.
- [sinh_pi](#) - Reperiodized hyperbolic sine function.
- [sinc_pi](#) - Sinus cardinal function
- [sinhint](#) - Hyperbolic sine integral
- [sinint](#) - Sine integral
- [sph_bessel_i](#) - Spherical regular modified Bessel functions
- [sph_bessel_k](#) - Spherical iregular modified Bessel functions
- [sph_hankel_1](#) - Spherical Hankel functions of the first kind
- [sph_hankel_2](#) - Spherical Hankel functions of the first kind
- [sph_harmonic](#) - Spherical
- [stirling_1](#) - Stirling numbers of the first kind
- [stirling_2](#) - Stirling numbers of the second kind
- [tan_pi](#) - Reperiodized tangent function.
- [tanh_pi](#) - Reperiodized hyperbolic tangent function.
- [tgamma](#) - Gamma for complex arguments
- [tgamma](#) - Upper incomplete gamma functions
- [tgamma_lower](#) - Lower incomplete gamma functions
- [theta_1](#) - Exponential theta function 1
- [theta_2](#) - Exponential theta function 2

- [theta_3](#) - Exponential theta function 3
- [theta_4](#) - Exponential theta function 4
- [tricomi_u](#) - Tricomi confluent hypergeometric function
- [zernike](#) - Zernike polynomials

7.3.2.3 Argument Promotion

The arguments supplied to the non-suffixed functions will be promoted according to the following rules:

1. If any argument intended to be floating point is given an integral value That integral value is promoted to double.
2. All floating point arguments are promoted up to the largest floating point precision among them.

7.3.2.4 NaN Arguments

If any of the floating point arguments supplied to these functions is invalid or NaN (`std::numeric_limits<Tp>::quiet_NaN`), the value NaN is returned.

7.3.2.5 Implementation

We strive to implement the underlying math with type generic algorithms to the greatest extent possible. In practice, the functions are thin wrappers that dispatch to function templates. Type dependence is controlled with `std::numeric_limits` and functions thereof.

We don't promote `float` to `double` or `double` to `long double` reflexively. The goal is for `float` functions to operate more quickly, at the cost of `float` accuracy and possibly a smaller domain of validity. Similarly, `long double` should give you more dynamic range and slightly more precision than `double` on many systems.

7.3.2.6 Testing

These functions have been tested against equivalent implementations from the [Gnu Scientific Library](http://www.gnu.org/software/scientific/), [GSL](http://www.gnu.org/software/gsl/) and [Boost](http://www.boost.org/doc/libs/1_60_0/libs/math/doc/html/index.html) and the ratio

$$\frac{|f - f_{test}|}{|f_{test}|}$$

is generally found to be within 10^{-15} for 64-bit double on linux-x86_64 systems over most of the ranges of validity.

Todo Provide accuracy comparisons on a per-function basis for a small number of targets.

7.3.2.7 General Bibliography

See also

Abramowitz and Stegun: Handbook of Mathematical Functions, with Formulas, Graphs, and Mathematical Tables Edited by Milton Abramowitz and Irene A. Stegun, National Bureau of Standards Applied Mathematics Series - 55 Issued June 1964, Tenth Printing, December 1972, with corrections Electronic versions of A&S abound including both pdf and navigable html.

for example <http://people.math.sfu.ca/~cbm/aands/>

The old A&S has been redone as the NIST Digital Library of Mathematical Functions: <http://dlmf.nist.gov/> This version is far more navigable and includes more recent work.

The PlanetMath website has a wealth of material. In particular https://planetmath.org/msc.html#33_Special_functions

An Atlas of Functions: with Equator, the Atlas Function Calculator 2nd Edition, by Oldham, Keith B., Myland, Jan, Spanier, Jerome

Asymptotics and Special Functions by Frank W. J. Olver, Academic Press, 1974

Numerical Recipes in C, The Art of Scientific Computing, by William H. Press, Second Ed., Saul A. Teukolsky, William T. Vetterling, and Brian P. Flannery, Cambridge University Press, 1992

The Special Functions and Their Approximations: Volumes 1 and 2, by Yudell L. Luke, Academic Press, 1969

Chapter 8

Namespace Documentation

8.1 `__gnu_cxx` Namespace Reference

Classes

- struct [__airy_t](#)
- struct [__assoc_legendre_p_t](#)
- struct [__assoc_legendre_q_t](#)
- struct [__chebyshev_t_t](#)
- struct [__chebyshev_u_t](#)
- struct [__chebyshev_v_t](#)
- struct [__chebyshev_w_t](#)
- struct [__coulomb_t](#)
- struct [__cyl_bessel_t](#)
- struct [__cyl_hankel_t](#)
- struct [__cyl_mod_bessel_t](#)
- struct [__fock_airy_t](#)
- struct [__fp_is_integer_t](#)
- struct [__gamma_inc_t](#)
- struct [__gamma_temme_t](#)

A structure for the gamma functions required by the Temme series expansions of $N_\nu(x)$ and $K_\nu(x)$.

$$\Gamma_1 = \frac{1}{2\mu} \left[\frac{1}{\Gamma(1-\mu)} - \frac{1}{\Gamma(1+\mu)} \right]$$

and

$$\Gamma_2 = \frac{1}{2} \left[\frac{1}{\Gamma(1-\mu)} + \frac{1}{\Gamma(1+\mu)} \right]$$

where $-1/2 \leq \mu \leq 1/2$ is $\mu = \nu - N$ and N is the nearest integer to ν . The values of $\Gamma(1+\mu)$ and $\Gamma(1-\mu)$ are returned as well.

- struct [__gappa_pq_t](#)
- struct [__gegenbauer_t](#)
- struct [__hermite_he_t](#)
- struct [__hermite_t](#)
- struct [__jacobi_ellint_t](#)
- struct [__jacobi_t](#)

- struct [__laguerre_t](#)
- struct [__legendre_p_t](#)
- struct [__legendre_q_t](#)
- struct [__lgamma_t](#)
- struct [__quadrature_point_t](#)
- struct [__sincos_t](#)
- struct [__sph_bessel_t](#)
- struct [__sph_hankel_t](#)
- struct [__sph_mod_bessel_t](#)
- struct [__stirling_1_t](#)

A structure for Stirling numbers of the first kind.

- struct [__stirling_2_t](#)

A structure for Stirling numbers of the first kind.

Enumerations

- enum [gauss_quad_type](#) { [Gauss](#), [Gauss_Lobatto](#), [Gauss_Radau_lower](#), [Gauss_Radau_upper](#) }

Enumeration for differing types of Gauss quadrature. The `gauss_quad_type` is used to determine the boundary condition modifications applied to orthogonal polynomials for quadrature rules.

Functions

- template<typename `_Tp` >
constexpr bool [__fp_is_equal](#) (`_Tp` `__a`, `_Tp` `__b`, `_Tp` `__mul=_Tp{1}`) noexcept
- template<typename `_Tp` >
constexpr [__fp_is_integer_t](#) [__fp_is_even_integer](#) (`_Tp` `__a`, `_Tp` `__mul=_Tp{1}`) noexcept
- template<typename `_Tp` >
constexpr [__fp_is_integer_t](#) [__fp_is_half_integer](#) (`_Tp` `__a`, `_Tp` `__mul=_Tp{1}`) noexcept
- template<typename `_Tp` >
constexpr [__fp_is_integer_t](#) [__fp_is_half_odd_integer](#) (`_Tp` `__a`, `_Tp` `__mul=_Tp{1}`) noexcept
- template<typename `_Tp` >
constexpr [__fp_is_integer_t](#) [__fp_is_integer](#) (`_Tp` `__a`, `_Tp` `__mul=_Tp{1}`) noexcept
- template<typename `_Tp` >
constexpr [__fp_is_integer_t](#) [__fp_is_odd_integer](#) (`_Tp` `__a`, `_Tp` `__mul=_Tp{1}`) noexcept
- template<typename `_Tp` >
constexpr bool [__fp_is_zero](#) (`_Tp` `__a`, `_Tp` `__mul=_Tp{1}`) noexcept
- template<typename `_Tp` >
constexpr `_Tp` [__fp_max_abs](#) (`_Tp` `__a`, `_Tp` `__b`) noexcept
- template<typename `_Tp`, typename `_IntTp` >
constexpr `_Tp` [__parity](#) (`_IntTp` `__k`) noexcept
- template<typename `_Tp` >
`__gnu_cxx::fp_promote_t`< `_Tp` > [airy_ai](#) (`_Tp` `__x`)
- template<typename `_Tp` >
`std::complex`< `__gnu_cxx::fp_promote_t`< `_Tp` > > [airy_ai](#) (`std::complex`< `_Tp` > `__x`)
- float [airy_aif](#) (float `__x`)
- long double [airy_ail](#) (long double `__x`)
- template<typename `_Tp` >
`__gnu_cxx::fp_promote_t`< `_Tp` > [airy_bi](#) (`_Tp` `__x`)

- `template<typename _Tp >`
`std::complex< __gnu_cxx::fp_promote_t< _Tp > >` `airy_bi` (`std::complex< _Tp > __x`)
- `float` `airy_bif` (`float __x`)
- `long double` `airy_bil` (`long double __x`)
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp >` `assoc_legendre_q` (`unsigned int __l, unsigned int __m, _Tp __x`)
- `float` `assoc_legendre_qf` (`unsigned int __l, unsigned int __m, float __x`)
- `long double` `assoc_legendre_ql` (`unsigned int __l, unsigned int __m, long double __x`)
- `template<typename _Tp >`
`std::vector< _Tp >` `bell` (`unsigned int __n`)
- `template<typename _Tp, typename _Up >`
`_Up` `bell` (`unsigned int __n, _Up __x`)
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp >` `bernoulli` (`unsigned int __n`)
- `template<typename _Tp >`
`_Tp` `bernoulli` (`unsigned int __n, _Tp __x`)
- `float` `bernoullif` (`unsigned int __n`)
- `long double` `bernoullil` (`unsigned int __n`)
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp >` `binomial` (`unsigned int __n, unsigned int __k`)

Return the binomial coefficient as a real number. The binomial coefficient is given by:

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

The binomial coefficients are generated by:

$$(1+t)^n = \sum_{k=0}^n \binom{n}{k} t^k$$

- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp >` `binomial_p` (`_Tp __p, unsigned int __n, unsigned int __k`)
Return the binomial cumulative distribution function.
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp >` `binomial_pdf` (`_Tp __p, unsigned int __n, unsigned int __k`)
Return the binomial probability mass function.
- `float` `binomialf` (`unsigned int __n, unsigned int __k`)
- `long double` `binomiall` (`unsigned int __n, unsigned int __k`)
- `template<typename _Tps, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tps, _Tp >` `bose_einstein` (`_Tps __s, _Tp __x`)
- `float` `bose_einsteinf` (`float __s, float __x`)
- `long double` `bose_einsteinl` (`long double __s, long double __x`)
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp >` `chebyshev_t` (`unsigned int __n, _Tp __x`)
- `float` `chebyshev_tf` (`unsigned int __n, float __x`)
- `long double` `chebyshev_tl` (`unsigned int __n, long double __x`)
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp >` `chebyshev_u` (`unsigned int __n, _Tp __x`)
- `float` `chebyshev_uf` (`unsigned int __n, float __x`)
- `long double` `chebyshev_ul` (`unsigned int __n, long double __x`)
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp >` `chebyshev_v` (`unsigned int __n, _Tp __x`)
- `float` `chebyshev_vf` (`unsigned int __n, float __x`)

- long double [chebyshev_vl](#) (unsigned int __n, long double __x)
- template<typename _Tp >
__gnu_cxx::fp_promote_t< _Tp > [chebyshev_w](#) (unsigned int __n, _Tp __x)
- float [chebyshev_wf](#) (unsigned int __n, float __x)
- long double [chebyshev_wl](#) (unsigned int __n, long double __x)
- template<typename _Tp >
__gnu_cxx::fp_promote_t< _Tp > [clausen](#) (unsigned int __m, _Tp __x)
- template<typename _Tp >
std::complex< __gnu_cxx::fp_promote_t< _Tp > > [clausen](#) (unsigned int __m, std::complex< _Tp > __z)
- template<typename _Tp >
__gnu_cxx::fp_promote_t< _Tp > [clausen_cl](#) (unsigned int __m, _Tp __x)
- float [clausen_clf](#) (unsigned int __m, float __x)
- long double [clausen_cli](#) (unsigned int __m, long double __x)
- template<typename _Tp >
__gnu_cxx::fp_promote_t< _Tp > [clausen_sl](#) (unsigned int __m, _Tp __x)
- float [clausen_slf](#) (unsigned int __m, float __x)
- long double [clausen_sll](#) (unsigned int __m, long double __x)
- float [clausenf](#) (unsigned int __m, float __x)
- std::complex< float > [clausenf](#) (unsigned int __m, std::complex< float > __z)
- long double [clausenl](#) (unsigned int __m, long double __x)
- std::complex< long double > [clausenl](#) (unsigned int __m, std::complex< long double > __z)
- template<typename _Tk >
__gnu_cxx::fp_promote_t< _Tk > [comp_ellint_d](#) (_Tk __k)
- float [comp_ellint_df](#) (float __k)
- long double [comp_ellint_dl](#) (long double __k)
- float [comp_ellint_rf](#) (float __x, float __y)
- long double [comp_ellint_rf](#) (long double __x, long double __y)
- template<typename _Tx, typename _Ty >
__gnu_cxx::fp_promote_t< _Tx, _Ty > [comp_ellint_rf](#) (_Tx __x, _Ty __y)
- float [comp_ellint_rg](#) (float __x, float __y)
- long double [comp_ellint_rg](#) (long double __x, long double __y)
- template<typename _Tx, typename _Ty >
__gnu_cxx::fp_promote_t< _Tx, _Ty > [comp_ellint_rg](#) (_Tx __x, _Ty __y)
- template<typename _Tpa, typename _Tpc, typename _Tp >
__gnu_cxx::fp_promote_t< _Tpa, _Tpc, _Tp > [conf_hyperg](#) (_Tpa __a, _Tpc __c, _Tp __x)
- template<typename _Tpc, typename _Tp >
__gnu_cxx::fp_promote_t< _Tpc, _Tp > [conf_hyperg_lim](#) (_Tpc __c, _Tp __x)
- float [conf_hyperg_limf](#) (float __c, float __x)
- long double [conf_hyperg_liml](#) (long double __c, long double __x)
- float [conf_hypergf](#) (float __a, float __c, float __x)
- long double [conf_hypergl](#) (long double __a, long double __c, long double __x)
- template<typename _Tp >
__gnu_cxx::fp_promote_t< _Tp > [cos_pi](#) (_Tp __x)
- float [cos_pif](#) (float __x)
- long double [cos_pil](#) (long double __x)
- template<typename _Tp >
__gnu_cxx::fp_promote_t< _Tp > [cosh_pi](#) (_Tp __x)
- float [cosh_pif](#) (float __x)
- long double [cosh_pil](#) (long double __x)
- template<typename _Tp >
__gnu_cxx::fp_promote_t< _Tp > [coshint](#) (_Tp __x)

- float [coshintf](#) (float __x)
- long double [coshintl](#) (long double __x)
- template<typename _Tp >
__gnu_cxx::fp_promote_t< _Tp > [cosint](#) (_Tp __x)
- float [cosintf](#) (float __x)
- long double [cosintl](#) (long double __x)
- template<typename _Tpnu, typename _Tp >
std::complex< __gnu_cxx::fp_promote_t< _Tpnu, _Tp > > [cyl_hankel_1](#) (_Tpnu __nu, _Tp __z)
- template<typename _Tpnu, typename _Tp >
std::complex< __gnu_cxx::fp_promote_t< _Tpnu, _Tp > > [cyl_hankel_1](#) (std::complex< _Tpnu > __nu, std::complex< _Tp > __x)
- std::complex< float > [cyl_hankel_1f](#) (float __nu, float __z)
- std::complex< float > [cyl_hankel_1f](#) (std::complex< float > __nu, std::complex< float > __x)
- std::complex< long double > [cyl_hankel_1l](#) (long double __nu, long double __z)
- std::complex< long double > [cyl_hankel_1l](#) (std::complex< long double > __nu, std::complex< long double > __x)
- template<typename _Tpnu, typename _Tp >
std::complex< __gnu_cxx::fp_promote_t< _Tpnu, _Tp > > [cyl_hankel_2](#) (_Tpnu __nu, _Tp __z)
- template<typename _Tpnu, typename _Tp >
std::complex< __gnu_cxx::fp_promote_t< _Tpnu, _Tp > > [cyl_hankel_2](#) (std::complex< _Tpnu > __nu, std::complex< _Tp > __x)
- std::complex< float > [cyl_hankel_2f](#) (float __nu, float __z)
- std::complex< float > [cyl_hankel_2f](#) (std::complex< float > __nu, std::complex< float > __x)
- std::complex< long double > [cyl_hankel_2l](#) (long double __nu, long double __z)
- std::complex< long double > [cyl_hankel_2l](#) (std::complex< long double > __nu, std::complex< long double > __x)
- template<typename _Tp >
__gnu_cxx::fp_promote_t< _Tp > [dawson](#) (_Tp __x)
- float [dawsonf](#) (float __x)
- long double [dawsonl](#) (long double __x)
- template<typename _Tp >
__gnu_cxx::fp_promote_t< _Tp > [debye](#) (unsigned int __n, _Tp __x)
- float [debyef](#) (unsigned int __n, float __x)
- long double [debyel](#) (unsigned int __n, long double __x)
- template<typename _Tp >
__gnu_cxx::fp_promote_t< _Tp > [digamma](#) (_Tp __x)
- float [digammaf](#) (float __x)
- long double [digammal](#) (long double __x)
- template<typename _Tp >
__gnu_cxx::fp_promote_t< _Tp > [dilog](#) (_Tp __x)
- float [dilogf](#) (float __x)
- long double [dilogl](#) (long double __x)
- template<typename _Tp >
_Tp [dirichlet_beta](#) (_Tp __s)
- float [dirichlet_betaf](#) (float __s)
- long double [dirichlet_betel](#) (long double __s)
- template<typename _Tp >
_Tp [dirichlet_eta](#) (_Tp __s)
- float [dirichlet_etaf](#) (float __s)
- long double [dirichlet_etel](#) (long double __s)
- template<typename _Tp >
_Tp [dirichlet_lambda](#) (_Tp __s)

- float [dirichlet_lambdaf](#) (float __s)
- long double [dirichlet_lambdal](#) (long double __s)
- template<typename _Tp >
[__gnu_cxx::fp_promote_t< _Tp > double_factorial](#) (int __n)
Return the double factorial $n!!$ of the argument as a real number.

$$n!! = n(n-2)\dots(2), 0!! = 1$$
for even n and

$$n!! = n(n-2)\dots(1), (-1)!! = 1$$
for odd n .
- float [double_factorialf](#) (int __n)
- long double [double_factoriall](#) (int __n)
- template<typename _Tk, typename _Tp, typename _Ta, typename _Tb >
[__gnu_cxx::fp_promote_t< _Tk, _Tp, _Ta, _Tb > ellint_cel](#) (_Tk __k_c, _Tp __p, _Ta __a, _Tb __b)
- float [ellint_celf](#) (float __k_c, float __p, float __a, float __b)
- long double [ellint_cell](#) (long double __k_c, long double __p, long double __a, long double __b)
- template<typename _Tk, typename _Tphi >
[__gnu_cxx::fp_promote_t< _Tk, _Tphi > ellint_d](#) (_Tk __k, _Tphi __phi)
- float [ellint_df](#) (float __k, float __phi)
- long double [ellint_dl](#) (long double __k, long double __phi)
- template<typename _Tp, typename _Tk >
[__gnu_cxx::fp_promote_t< _Tp, _Tk > ellint_el1](#) (_Tp __x, _Tk __k_c)
- float [ellint_el1f](#) (float __x, float __k_c)
- long double [ellint_el1l](#) (long double __x, long double __k_c)
- template<typename _Tp, typename _Tk, typename _Ta, typename _Tb >
[__gnu_cxx::fp_promote_t< _Tp, _Tk, _Ta, _Tb > ellint_el2](#) (_Tp __x, _Tk __k_c, _Ta __a, _Tb __b)
- float [ellint_el2f](#) (float __x, float __k_c, float __a, float __b)
- long double [ellint_el2l](#) (long double __x, long double __k_c, long double __a, long double __b)
- template<typename _Tx, typename _Tk, typename _Tp >
[__gnu_cxx::fp_promote_t< _Tx, _Tk, _Tp > ellint_el3](#) (_Tx __x, _Tk __k_c, _Tp __p)
- float [ellint_el3f](#) (float __x, float __k_c, float __p)
- long double [ellint_el3l](#) (long double __x, long double __k_c, long double __p)
- template<typename _Tp, typename _Up >
[__gnu_cxx::fp_promote_t< _Tp, _Up > ellint_rc](#) (_Tp __x, _Up __y)
- float [ellint_rcf](#) (float __x, float __y)
- long double [ellint_rcl](#) (long double __x, long double __y)
- template<typename _Tp, typename _Up, typename _Vp >
[__gnu_cxx::fp_promote_t< _Tp, _Up, _Vp > ellint_rd](#) (_Tp __x, _Up __y, _Vp __z)
- float [ellint_rdf](#) (float __x, float __y, float __z)
- long double [ellint_rdl](#) (long double __x, long double __y, long double __z)
- template<typename _Tp, typename _Up, typename _Vp >
[__gnu_cxx::fp_promote_t< _Tp, _Up, _Vp > ellint_rf](#) (_Tp __x, _Up __y, _Vp __z)
- float [ellint_rff](#) (float __x, float __y, float __z)
- long double [ellint_rfl](#) (long double __x, long double __y, long double __z)
- template<typename _Tp, typename _Up, typename _Vp >
[__gnu_cxx::fp_promote_t< _Tp, _Up, _Vp > ellint_rg](#) (_Tp __x, _Up __y, _Vp __z)
- float [ellint_rgf](#) (float __x, float __y, float __z)
- long double [ellint_rgl](#) (long double __x, long double __y, long double __z)
- template<typename _Tp, typename _Up, typename _Vp, typename _Wp >
[__gnu_cxx::fp_promote_t< _Tp, _Up, _Vp, _Wp > ellint_rj](#) (_Tp __x, _Up __y, _Vp __z, _Wp __p)
- float [ellint_rjf](#) (float __x, float __y, float __z, float __p)

- long double [ellint_rjl](#) (long double __x, long double __y, long double __z, long double __p)
- template<typename _Tp >
_Tp [ellnome](#) (_Tp __k)
- float [ellnomef](#) (float __k)
- long double [ellnomel](#) (long double __k)
- template<typename _Tp >
_Tp [euler](#) (unsigned int __n)

This returns Euler number E_n .

- template<typename _Tp >
_Tp [eulerian_1](#) (unsigned int __n, unsigned int __m)
- template<typename _Tp >
std::vector< _Tp > [eulerian_1](#) (unsigned int __n)
- template<typename _Tp >
_Tp [eulerian_2](#) (unsigned int __n, unsigned int __m)
- template<typename _Tp >
__gnu_cxx::fp_promote_t< _Tp > [expint](#) (unsigned int __n, _Tp __x)
- float [expintf](#) (unsigned int __n, float __x)
- long double [expintl](#) (unsigned int __n, long double __x)
- template<typename _Tlam, typename _Tp >
__gnu_cxx::fp_promote_t< _Tlam, _Tp > [exponential_p](#) (_Tlam __lambda, _Tp __x)
- *Return the exponential cumulative probability density function.*
- template<typename _Tlam, typename _Tp >
__gnu_cxx::fp_promote_t< _Tlam, _Tp > [exponential_pdf](#) (_Tlam __lambda, _Tp __x)
- *Return the exponential probability density function.*
- template<typename _Tp >
__gnu_cxx::fp_promote_t< _Tp > [factorial](#) (unsigned int __n)
- *Return the factorial $n!$ of the argument as a real number.*

$$n! = 1 \times 2 \times \dots \times n, 0! = 1$$

- float [factorialf](#) (unsigned int __n)
- long double [factoriall](#) (unsigned int __n)
- template<typename _Tp, typename _Tnu >
__gnu_cxx::fp_promote_t< _Tp, _Tnu > [falling_factorial](#) (_Tp __a, _Tnu __nu)
- *Return the falling factorial function or the lower Pochhammer symbol for real argument a and integral order n . The falling factorial function is defined by*

$$a^{\underline{n}} = \prod_{k=0}^{n-1} (a - k) = \Gamma(a + 1) / \Gamma(a - n + 1)$$

where $a^{\underline{0}} \equiv 1$. In particular, $n^{\underline{n}} = n!$.

- float [falling_factorialf](#) (float __a, float __nu)
- long double [falling_factoriall](#) (long double __a, long double __nu)
- template<typename _Tps, typename _Tp >
__gnu_cxx::fp_promote_t< _Tps, _Tp > [fermi_dirac](#) (_Tps __s, _Tp __x)
- float [fermi_diracf](#) (float __s, float __x)
- long double [fermi_diracl](#) (long double __s, long double __x)
- template<typename _Tp >
__gnu_cxx::fp_promote_t< _Tp > [fisher_f_p](#) (_Tp __F, unsigned int __nu1, unsigned int __nu2)
- *Return the F-distribution propability function. This returns the probability that the observed chi-square for a correct model exceeds the value χ^2 .*
- template<typename _Tp >
__gnu_cxx::fp_promote_t< _Tp > [fisher_f_pdf](#) (_Tp __F, unsigned int __nu1, unsigned int __nu2)

Return the F-distribution propability function. This returns the probability that the observed chi-square for a correct model exceeds the value χ^2 .

- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > fresnel_c (_Tp __x)`
- `float fresnel_cf (float __x)`
- `long double fresnel_cl (long double __x)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > fresnel_s (_Tp __x)`
- `float fresnel_sf (float __x)`
- `long double fresnel_sl (long double __x)`
- `template<typename _Ta, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Ta, _Tp > gamma_p (_Ta __a, _Tp __x)`

Return the gamma cumulative propability distribution function or the regularized lower incomplete gamma function.

- `template<typename _Ta, typename _Tb, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Ta, _Tb, _Tp > gamma_pdf (_Ta __alpha, _Tb __beta, _Tp __x)`

Return the gamma propability distribution function.

- `float gamma_pf (float __a, float __x)`
- `long double gamma_pl (long double __a, long double __x)`
- `template<typename _Ta, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Ta, _Tp > gamma_q (_Ta __a, _Tp __x)`

Return the gamma complementary cumulative propability distribution (or survival) function or the regularized upper incomplete gamma function.

- `float gamma_qf (float __a, float __x)`
- `long double gamma_ql (long double __a, long double __x)`
- `template<typename _Ta >`
`__gnu_cxx::fp_promote_t< _Ta > gamma_reciprocal (_Ta __a)`
- `float gamma_reciprocalf (float __a)`
- `long double gamma_reciprocall (long double __a)`
- `template<typename _Tlam, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tlam, _Tp > gegenbauer (unsigned int __n, _Tlam __lambda, _Tp __x)`
- `float gegenbauerf (unsigned int __n, float __lambda, float __x)`
- `long double gegenbauerl (unsigned int __n, long double __lambda, long double __x)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > harmonic (unsigned int __n)`
- `template<typename _Tk, typename _Tphi >`
`__gnu_cxx::fp_promote_t< _Tk, _Tphi > heuman_lambda (_Tk __k, _Tphi __phi)`
- `float heuman_lambdaf (float __k, float __phi)`
- `long double heuman_lambdal (long double __k, long double __phi)`
- `template<typename _Tp, typename _Up >`
`__gnu_cxx::fp_promote_t< _Tp, _Up > hurwitz_zeta (_Tp __s, _Up __a)`
- `template<typename _Tp, typename _Up >`
`std::complex< _Tp > hurwitz_zeta (_Tp __s, std::complex< _Up > __a)`
- `float hurwitz_zetaf (float __s, float __a)`
- `long double hurwitz_zetal (long double __s, long double __a)`
- `template<typename _Tpa, typename _Tpb, typename _Tpc, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tpa, _Tpb, _Tpc, _Tp > hyperg (_Tpa __a, _Tpb __b, _Tpc __c, _Tp __x)`
- `float hypergf (float __a, float __b, float __c, float __x)`
- `long double hypergl (long double __a, long double __b, long double __c, long double __x)`
- `template<typename _Ta, typename _Tb, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Ta, _Tb, _Tp > ibeta (_Ta __a, _Tb __b, _Tp __x)`
- `template<typename _Ta, typename _Tb, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Ta, _Tb, _Tp > ibetac (_Ta __a, _Tb __b, _Tp __x)`

- float `ibetacf` (float `__a`, float `__b`, float `__x`)
- long double `ibetacdl` (long double `__a`, long double `__b`, long double `__x`)
- float `ibetaf` (float `__a`, float `__b`, float `__x`)
- long double `ibetadl` (long double `__a`, long double `__b`, long double `__x`)
- template<typename `_Talpha`, typename `_Tbeta`, typename `_Tp` >
`__gnu_cxx::fp_promote_t<_Talpha, _Tbeta, _Tp>` `jacobi` (unsigned `__n`, `_Talpha` `__alpha`, `_Tbeta` `__beta`, `_Tp` `__x`)
- template<typename `_Kp`, typename `_Up` >
`__gnu_cxx::fp_promote_t<_Kp, _Up>` `jacobi_cn` (`_Kp` `__k`, `_Up` `__u`)
- float `jacobi_cnf` (float `__k`, float `__u`)
- long double `jacobi_cndl` (long double `__k`, long double `__u`)
- template<typename `_Kp`, typename `_Up` >
`__gnu_cxx::fp_promote_t<_Kp, _Up>` `jacobi_dn` (`_Kp` `__k`, `_Up` `__u`)
- float `jacobi_dnf` (float `__k`, float `__u`)
- long double `jacobi_dndl` (long double `__k`, long double `__u`)
- template<typename `_Kp`, typename `_Up` >
`__gnu_cxx::fp_promote_t<_Kp, _Up>` `jacobi_sn` (`_Kp` `__k`, `_Up` `__u`)
- float `jacobi_snf` (float `__k`, float `__u`)
- long double `jacobi_snld` (long double `__k`, long double `__u`)
- template<typename `_Tpq`, typename `_Tp` >
`__gnu_cxx::fp_promote_t<_Tpq, _Tp>` `jacobi_theta_1` (`_Tpq` `__q`, `_Tp` `__x`)
- float `jacobi_theta_1f` (float `__q`, float `__x`)
- long double `jacobi_theta_1ld` (long double `__q`, long double `__x`)
- template<typename `_Tpq`, typename `_Tp` >
`__gnu_cxx::fp_promote_t<_Tpq, _Tp>` `jacobi_theta_2` (`_Tpq` `__q`, `_Tp` `__x`)
- float `jacobi_theta_2f` (float `__q`, float `__x`)
- long double `jacobi_theta_2ld` (long double `__q`, long double `__x`)
- template<typename `_Tpq`, typename `_Tp` >
`__gnu_cxx::fp_promote_t<_Tpq, _Tp>` `jacobi_theta_3` (`_Tpq` `__q`, `_Tp` `__x`)
- float `jacobi_theta_3f` (float `__q`, float `__x`)
- long double `jacobi_theta_3ld` (long double `__q`, long double `__x`)
- template<typename `_Tpq`, typename `_Tp` >
`__gnu_cxx::fp_promote_t<_Tpq, _Tp>` `jacobi_theta_4` (`_Tpq` `__q`, `_Tp` `__x`)
- float `jacobi_theta_4f` (float `__q`, float `__x`)
- long double `jacobi_theta_4ld` (long double `__q`, long double `__x`)
- template<typename `_Tk`, typename `_Tphi` >
`__gnu_cxx::fp_promote_t<_Tk, _Tphi>` `jacobi_zeta` (`_Tk` `__k`, `_Tphi` `__phi`)
- float `jacobi_zetaf` (float `__k`, float `__phi`)
- long double `jacobi_zetadl` (long double `__k`, long double `__phi`)
- float `jacobif` (unsigned `__n`, float `__alpha`, float `__beta`, float `__x`)
- long double `jacobild` (unsigned `__n`, long double `__alpha`, long double `__beta`, long double `__x`)
- template<typename `_Tp` >
`_Tp` `lah` (unsigned int `__n`, unsigned int `__k`)
- template<typename `_Tp` >
`std::vector<_Tp>` `lah` (unsigned int `__n`)
- template<typename `_Tp` >
`__gnu_cxx::fp_promote_t<_Tp>` `lbinomial` (unsigned int `__n`, unsigned int `__k`)

Return the logarithm of the binomial coefficient as a real number. The binomial coefficient is given by:

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

The binomial coefficients are generated by:

$$(1+t)^n = \sum_{k=0}^n \binom{n}{k} t^k$$

- float [lbinomialf](#) (unsigned int __n, unsigned int __k)
- long double [lbinomiall](#) (unsigned int __n, unsigned int __k)
- template<typename _Tp >
__gnu_cxx::fp_promote_t< _Tp > [ldouble_factorial](#) (int __n)

Return the logarithm of the double factorial $\ln(n!!)$ of the argument as a real number.

$$n!! = n(n-2)\dots(2), 0!! = 1$$

for even n and

$$n!! = n(n-2)\dots(1), (-1)!! = 1$$

for odd n .

- float [ldouble_factorialf](#) (int __n)
- long double [ldouble_factoriall](#) (int __n)
- template<typename _Tp >
__gnu_cxx::fp_promote_t< _Tp > [legendre_q](#) (unsigned int __l, _Tp __x)
- float [legendre_qf](#) (unsigned int __l, float __x)
- long double [legendre_ql](#) (unsigned int __l, long double __x)
- template<typename _Tp, typename _Ts, typename _Ta >
__gnu_cxx::fp_promote_t< _Tp, _Ts, _Ta > [lerch_phi](#) (_Tp __z, _Ts __s, _Ta __a)
- float [lerch_phif](#) (float __z, float __s, float __a)
- long double [lerch_phil](#) (long double __z, long double __s, long double __a)
- template<typename _Tp >
__gnu_cxx::fp_promote_t< _Tp > [lfactorial](#) (unsigned int __n)

Return the logarithm of the factorial $\ln(n!)$ of the argument as a real number.

$$n! = 1 \times 2 \times \dots \times n, 0! = 1$$

- float [lfactorialf](#) (unsigned int __n)
- long double [lfactoriall](#) (unsigned int __n)
- template<typename _Tp, typename _Tnu >
__gnu_cxx::fp_promote_t< _Tp, _Tnu > [lfalling_factorial](#) (_Tp __a, _Tnu __nu)

Return the logarithm of the falling factorial function or the lower Pochhammer symbol. The falling factorial function is defined by

$$a^{\underline{n}} = \frac{\Gamma(a+1)}{\Gamma(a-\nu+1)} = \prod_{k=0}^{n-1} (a-k)$$

where $a^{\underline{0}} \equiv 1$. In particular, $n^{\underline{n}} = n!$. Thus this function returns

$$\ln[a^{\underline{n}}] = \ln[\Gamma(a+1)] - \ln[\Gamma(a-\nu+1)]$$

where $\ln[a^{\underline{0}}] \equiv 0$. Many notations exist for this function: $(a)_{\nu}$,

$$\left\{ \begin{matrix} a \\ \nu \end{matrix} \right\}$$

, and others.

- float [lfalling_factorialf](#) (float __a, float __nu)
- long double [lfalling_factoriall](#) (long double __a, long double __nu)
- template<typename _Ta >
__gnu_cxx::fp_promote_t< _Ta > [lgamma](#) (_Ta __a)

- `template<typename _Ta >`
`std::complex< __gnu_cxx::fp_promote_t< _Ta > > lgamma (std::complex< _Ta > __a)`
- `float lgammaf (float __a)`
- `std::complex< float > lgammaf (std::complex< float > __a)`
- `long double lgammal (long double __a)`
- `std::complex< long double > lgammal (std::complex< long double > __a)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > logint (_Tp __x)`
- `float logintf (float __x)`
- `long double logintl (long double __x)`
- `template<typename _Ta, typename _Tb, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Ta, _Tb, _Tp > logistic_p (_Ta __a, _Tb __b, _Tp __x)`
Return the logistic cumulative distribution function.
- `template<typename _Ta, typename _Tb, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Ta, _Tb, _Tp > logistic_pdf (_Ta __a, _Tb __b, _Tp __x)`
Return the logistic probability density function.
- `template<typename _Tmu, typename _Tsig, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tmu, _Tsig, _Tp > lognormal_p (_Tmu __mu, _Tsig __sigma, _Tp __x)`
Return the lognormal cumulative probability density function.
- `template<typename _Tmu, typename _Tsig, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tmu, _Tsig, _Tp > lognormal_pdf (_Tmu __mu, _Tsig __sigma, _Tp __x)`
Return the lognormal probability density function.
- `template<typename _Tp, typename _Tnu >`
`__gnu_cxx::fp_promote_t< _Tp, _Tnu > lrising_factorial (_Tp __a, _Tnu __nu)`
Return the logarithm of the rising factorial function or the (upper) Pochhammer symbol. The rising factorial function is defined for integer order by

$$a^{\overline{\nu}} = \Gamma(a + \nu) / \Gamma(a) = \prod_{k=0}^{\nu-1} (a + k), \overline{0} \equiv 1$$
Thus this function returns

$$\ln[a^{\overline{\nu}}] = \ln[\Gamma(a + \nu)] - \ln[\Gamma(a)], \ln[a^{\overline{0}}] \equiv 0$$
Many notations exist for this function: $(a)_{\nu}$, called the Pochhammer function (esp. in the literature of special functions), and

$$\left[\begin{matrix} a \\ \nu \end{matrix} \right]$$
, and others.
- `float lrising_factorialf (float __a, float __nu)`
- `long double lrising_factoriall (long double __a, long double __nu)`
- `template<typename _Tp, typename _Ta, typename _Tb >`
`std::complex< __gnu_cxx::fp_promote_t< _Tp, _Ta, _Tb > > mittag_leffler (_Ta __alpha, _Tb __beta, const std::complex< _Tp > & __z)`
- `template<typename _Tmu, typename _Tsig, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tmu, _Tsig, _Tp > normal_p (_Tmu __mu, _Tsig __sigma, _Tp __x)`
Return the normal cumulative probability density function.
- `template<typename _Tmu, typename _Tsig, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tmu, _Tsig, _Tp > normal_pdf (_Tmu __mu, _Tsig __sigma, _Tp __x)`
Return the gamma cumulative probability distribution function.
- `template<typename _Tph, typename _Tpa >`
`__gnu_cxx::fp_promote_t< _Tph, _Tpa > owens_t (_Tph __h, _Tpa __a)`
- `float owens_tf (float __h, float __a)`
- `long double owens_tl (long double __h, long double __a)`

- `template<typename _Tp, typename _Up >`
`__gnu_cxx::fp_promote_t< std::complex< _Tp >, _Up > periodic_zeta (_Tp __x, _Up __s)`
- `template<typename _Tp, typename _Up >`
`__gnu_cxx::fp_promote_t< std::complex< _Tp >, std::complex< _Up > > periodic_zeta (std::complex< _Up > __z, _Tp __s)`
- `std::complex< float > periodic_zetaf (float __x, float __s)`
- `std::complex< long double > periodic_zetal (long double __x, long double __s)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > polygamma (unsigned int __m, _Tp __x)`
- `float polygammaf (unsigned int __m, float __x)`
- `long double polygammal (unsigned int __m, long double __x)`
- `template<typename _Tp, typename _Wp >`
`__gnu_cxx::fp_promote_t< _Tp, _Wp > polylog (_Tp __s, _Wp __w)`
- `template<typename _Tp, typename _Wp >`
`std::complex< __gnu_cxx::fp_promote_t< _Tp, _Wp > > polylog (_Tp __s, std::complex< _Tp > __w)`
- `float polylogf (float __s, float __w)`
- `std::complex< float > polylogf (float __s, std::complex< float > __w)`
- `long double polylogl (long double __s, long double __w)`
- `std::complex< long double > polylogl (long double __s, std::complex< long double > __w)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > radpoly (unsigned int __n, unsigned int __m, _Tp __rho)`
- `float radpolyf (unsigned int __n, unsigned int __m, float __rho)`
- `long double radpolyl (unsigned int __n, unsigned int __m, long double __rho)`
- `template<typename _Tp, typename _Tnu >`
`__gnu_cxx::fp_promote_t< _Tp, _Tnu > rising_factorial (_Tp __a, _Tnu __nu)`

Return the rising factorial function or the (upper) Pochhammer function. The rising factorial function is defined by

$$a^{\overline{\nu}} = \Gamma(a + \nu) / \Gamma(\nu)$$

Many notations exist for this function: $(a)_{\nu}$, called the Pochhammer function (esp. in the literature of special functions), and

$$\left[\begin{matrix} a \\ \nu \end{matrix} \right]$$

, and others.

- `float rising_factorialf (float __a, float __nu)`
- `long double rising_factoriall (long double __a, long double __nu)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > sin_pi (_Tp __x)`
- `float sin_pif (float __x)`
- `long double sin_pil (long double __x)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > sinc (_Tp __x)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > sinc_pi (_Tp __x)`
- `float sinc_pif (float __x)`
- `long double sinc_pil (long double __x)`
- `float sincf (float __x)`
- `long double sincl (long double __x)`
- `__gnu_cxx::__sincos_t< double > sincos (double __x)`
- `template<typename _Tp >`
`__gnu_cxx::__sincos_t< __gnu_cxx::fp_promote_t< _Tp > > sincos (_Tp __x)`
- `template<typename _Tp >`
`__gnu_cxx::__sincos_t< __gnu_cxx::fp_promote_t< _Tp > > sincos_pi (_Tp __x)`

- `__gnu_cxx::__sincos_t< float > sincos_pif` (float __x)
- `__gnu_cxx::__sincos_t< long double > sincos_pil` (long double __x)
- `__gnu_cxx::__sincos_t< float > sincosf` (float __x)
- `__gnu_cxx::__sincos_t< long double > sincosl` (long double __x)
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > sinh_pi` (_Tp __x)
- `float sinh_pif` (float __x)
- `long double sinh_pil` (long double __x)
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > sinhc` (_Tp __x)
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > sinhc_pi` (_Tp __x)
- `float sinhc_pif` (float __x)
- `long double sinhc_pil` (long double __x)
- `float sinhcf` (float __x)
- `long double sinhcl` (long double __x)
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > sinhint` (_Tp __x)
- `float sinhintf` (float __x)
- `long double sinhintl` (long double __x)
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > sinint` (_Tp __x)
- `float sinintf` (float __x)
- `long double sinintl` (long double __x)
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > sph_bessel_i` (unsigned int __n, _Tp __x)
- `float sph_bessel_if` (unsigned int __n, float __x)
- `long double sph_bessel_il` (unsigned int __n, long double __x)
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > sph_bessel_k` (unsigned int __n, _Tp __x)
- `float sph_bessel_kf` (unsigned int __n, float __x)
- `long double sph_bessel_kl` (unsigned int __n, long double __x)
- `template<typename _Tp >`
`std::complex< __gnu_cxx::fp_promote_t< _Tp > > sph_hankel_1` (unsigned int __n, _Tp __z)
- `template<typename _Tp >`
`std::complex< __gnu_cxx::fp_promote_t< _Tp > > sph_hankel_1` (unsigned int __n, std::complex< _Tp > __x)
- `std::complex< float > sph_hankel_1f` (unsigned int __n, float __z)
- `std::complex< float > sph_hankel_1f` (unsigned int __n, std::complex< float > __x)
- `std::complex< long double > sph_hankel_1l` (unsigned int __n, long double __z)
- `std::complex< long double > sph_hankel_1l` (unsigned int __n, std::complex< long double > __x)
- `template<typename _Tp >`
`std::complex< __gnu_cxx::fp_promote_t< _Tp > > sph_hankel_2` (unsigned int __n, _Tp __z)
- `template<typename _Tp >`
`std::complex< __gnu_cxx::fp_promote_t< _Tp > > sph_hankel_2` (unsigned int __n, std::complex< _Tp > __x)
- `std::complex< float > sph_hankel_2f` (unsigned int __n, float __z)
- `std::complex< float > sph_hankel_2f` (unsigned int __n, std::complex< float > __x)
- `std::complex< long double > sph_hankel_2l` (unsigned int __n, long double __z)
- `std::complex< long double > sph_hankel_2l` (unsigned int __n, std::complex< long double > __x)
- `template<typename _Ttheta, typename _Tphi >`
`std::complex< __gnu_cxx::fp_promote_t< _Ttheta, _Tphi > > sph_harmonic` (unsigned int __l, int __m, _Ttheta __theta, _Tphi __phi)

- `std::complex< float > sph_harmonicf` (unsigned int __l, int __m, float __theta, float __phi)
- `std::complex< long double > sph_harmonicl` (unsigned int __l, int __m, long double __theta, long double __phi)
- `template<typename _Tp >`
`_Tp stirling_1` (unsigned int __n, unsigned int __m)
- `template<typename _Tp >`
`std::vector< _Tp > stirling_1` (unsigned int __n)
- `template<typename _Tp >`
`_Tp stirling_2` (unsigned int __n, unsigned int __m)
- `template<typename _Tp >`
`std::vector< _Tp > stirling_2` (unsigned int __n)
- `template<typename _Tt, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > student_t_p` (_Tt __t, unsigned int __nu)
Return the Students T probability function.
- `template<typename _Tt, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > student_t_pdf` (_Tt __t, unsigned int __nu)
Return the complement of the Students T probability function.
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > tan_pi` (_Tp __x)
- `float tan_pif` (float __x)
- `long double tan_pil` (long double __x)
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > tanh_pi` (_Tp __x)
- `float tanh_pif` (float __x)
- `long double tanh_pil` (long double __x)
- `template<typename _Ta >`
`__gnu_cxx::fp_promote_t< _Ta > tgamma` (_Ta __a)
- `template<typename _Ta >`
`std::complex< __gnu_cxx::fp_promote_t< _Ta > > tgamma` (std::complex< _Ta > __a)
- `template<typename _Ta, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Ta, _Tp > tgamma` (_Ta __a, _Tp __x)
- `template<typename _Ta, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Ta, _Tp > tgamma_lower` (_Ta __a, _Tp __x)
- `float tgamma_lowerf` (float __a, float __x)
- `long double tgamma_lowerl` (long double __a, long double __x)
- `float tgammaf` (float __a)
- `std::complex< float > tgammaf` (std::complex< float > __a)
- `float tgammaf` (float __a, float __x)
- `long double tgammaf` (long double __a)
- `std::complex< long double > tgammaf` (std::complex< long double > __a)
- `long double tgammaf` (long double __a, long double __x)
- `template<typename _Tpnu, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tpnu, _Tp > theta_1` (_Tpnu __nu, _Tp __x)
- `float theta_1f` (float __nu, float __x)
- `long double theta_1l` (long double __nu, long double __x)
- `template<typename _Tpnu, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tpnu, _Tp > theta_2` (_Tpnu __nu, _Tp __x)
- `float theta_2f` (float __nu, float __x)
- `long double theta_2l` (long double __nu, long double __x)
- `template<typename _Tpnu, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tpnu, _Tp > theta_3` (_Tpnu __nu, _Tp __x)
- `float theta_3f` (float __nu, float __x)

- long double [theta_3l](#) (long double __nu, long double __x)
- template<typename _Tpnu, typename _Tp >
__gnu_cxx::fp_promote_t< _Tpnu, _Tp > [theta_4](#) (_Tpnu __nu, _Tp __x)
- float [theta_4f](#) (float __nu, float __x)
- long double [theta_4l](#) (long double __nu, long double __x)
- template<typename _Tp_k, typename _Tp >
__gnu_cxx::fp_promote_t< _Tp_k, _Tp > [theta_c](#) (_Tp_k __k, _Tp __x)
- float [theta_cf](#) (float __k, float __x)
- long double [theta_cl](#) (long double __k, long double __x)
- template<typename _Tp_k, typename _Tp >
__gnu_cxx::fp_promote_t< _Tp_k, _Tp > [theta_d](#) (_Tp_k __k, _Tp __x)
- float [theta_df](#) (float __k, float __x)
- long double [theta_dl](#) (long double __k, long double __x)
- template<typename _Tp_k, typename _Tp >
__gnu_cxx::fp_promote_t< _Tp_k, _Tp > [theta_n](#) (_Tp_k __k, _Tp __x)
- float [theta_nf](#) (float __k, float __x)
- long double [theta_nl](#) (long double __k, long double __x)
- template<typename _Tp_k, typename _Tp >
__gnu_cxx::fp_promote_t< _Tp_k, _Tp > [theta_s](#) (_Tp_k __k, _Tp __x)
- float [theta_sf](#) (float __k, float __x)
- long double [theta_sl](#) (long double __k, long double __x)
- template<typename _Tpa, typename _Tpc, typename _Tp >
__gnu_cxx::fp_promote_t< _Tpa, _Tpc, _Tp > [tricorni_u](#) (_Tpa __a, _Tpc __c, _Tp __x)
- float [tricorni_uf](#) (float __a, float __c, float __x)
- long double [tricorni_ul](#) (long double __a, long double __c, long double __x)
- template<typename _Ta, typename _Tb, typename _Tp >
__gnu_cxx::fp_promote_t< _Ta, _Tb, _Tp > [weibull_p](#) (_Ta __a, _Tb __b, _Tp __x)
Return the Weibull cumulative probability density function.
- template<typename _Ta, typename _Tb, typename _Tp >
__gnu_cxx::fp_promote_t< _Ta, _Tb, _Tp > [weibull_pdf](#) (_Ta __a, _Tb __b, _Tp __x)
Return the Weibull probability density function.
- template<typename _Trho, typename _Tphi >
__gnu_cxx::fp_promote_t< _Trho, _Tphi > [zernike](#) (unsigned int __n, int __m, _Trho __rho, _Tphi __phi)
- float [zernikef](#) (unsigned int __n, int __m, float __rho, float __phi)
- long double [zernikel](#) (unsigned int __n, int __m, long double __rho, long double __phi)

8.1.1 Enumeration Type Documentation

8.1.1.1 gauss_quad_type

```
enum __gnu_cxx::gauss_quad_type
```

Enumeration for differing types of Gauss quadrature. The `gauss_quad_type` is used to determine the boundary condition modifications applied to orthogonal polynomials for quadrature rules.

Enumerator

Gauss	Gauss quadrature.
Gauss_Lobatto	Gauss-Lobatto quadrature.
Gauss_Radau_lower	Gauss-Radau quadrature including the node -1.
Gauss_Radau_upper	Gauss-Radau quadrature including the node +1.

Definition at line 48 of file specfun_state.h.

8.1.2 Function Documentation

8.1.2.1 __fp_is_equal()

```
template<typename _Tp >
constexpr bool __gnu_cxx::__fp_is_equal (
    _Tp __a,
    _Tp __b,
    _Tp __mul = _Tp{1} )    [inline], [noexcept]
```

A function to reliably compare two floating point numbers.

Parameters

<code>__a</code>	The left hand side
<code>__b</code>	The right hand side
<code>__mul</code>	The multiplier for numeric epsilon for comparison

Returns

`true` if `a` and `b` are equal to zero or differ only by $\max(a, b) * \text{mul} * \text{epsilon}$

Definition at line 85 of file math_util.h.

References `__fp_max_abs()`.

Referenced by `__fp_is_half_integer()`, `__fp_is_half_odd_integer()`, `__fp_is_integer()`, `std::__detail::__polylog()`, `std::__detail::__polylog_exp_neg()`, `std::__detail::__polylog_exp_neg_int()`, `std::__detail::__polylog_exp_pos_int()`, and `std::__detail::__polylog_exp_pos_real()`.

8.1.2.2 __fp_is_even_integer()

```
template<typename _Tp >
constexpr __fp_is_integer_t __gnu_cxx::__fp_is_even_integer (
    _Tp __a,
    _Tp __mul = _Tp{1} ) [inline], [noexcept]
```

A function to reliably detect if a floating point number is an even integer.

Parameters

<code>__a</code>	The floating point number
<code>__mul</code>	The multiplier of machine epsilon for the tolerance

Returns

`true` if `a` is an even integer within `mul * epsilon`.

Definition at line 221 of file `math_util.h`.

References `__fp_is_integer()`.

Referenced by `std::__detail::__riemann_zeta_glob()`.

8.1.2.3 __fp_is_half_integer()

```
template<typename _Tp >
constexpr __fp_is_integer_t __gnu_cxx::__fp_is_half_integer (
    _Tp __a,
    _Tp __mul = _Tp{1} ) [inline], [noexcept]
```

A function to reliably detect if a floating point number is a half-integer.

Parameters

<code>__a</code>	The floating point number
<code>__mul</code>	The multiplier of machine epsilon for the tolerance

Returns

`true` if `2a` is an integer within `mul * epsilon` and the returned value is half the integer, `int(a) / 2`.

Definition at line 176 of file `math_util.h`.

References `__fp_is_equal()`.

8.1.2.4 `__fp_is_half_odd_integer()`

```
template<typename _Tp >
constexpr __fp_is_integer_t __gnu_cxx::__fp_is_half_odd_integer (
    _Tp __a,
    _Tp __mul = _Tp{1} ) [inline], [noexcept]
```

A function to reliably detect if a floating point number is a half-odd-integer.

Parameters

<code>__a</code>	The floating point number
<code>__mul</code>	The multiplier of machine epsilon for the tolerance

Returns

`true` if $2a$ is an odd integer within $mul * \epsilon$ and the returned value is $\text{int}(a - 1) / 2$.

Definition at line 199 of file `math_util.h`.

References `__fp_is_equal()`.

Referenced by `std::__detail::__digamma()`.

8.1.2.5 `__fp_is_integer()`

```
template<typename _Tp >
constexpr __fp_is_integer_t __gnu_cxx::__fp_is_integer (
    _Tp __a,
    _Tp __mul = _Tp{1} ) [inline], [noexcept]
```

A function to reliably detect if a floating point number is an integer.

Parameters

<code>__a</code>	The floating point number
<code>__mul</code>	The multiplier of machine epsilon for the tolerance

Returns

`true` if a is an integer within $mul * \epsilon$.

Definition at line 154 of file `math_util.h`.

References `__fp_is_equal()`.

Referenced by `std::__detail::__conf_hyperg()`, `std::__detail::__conf_hyperg_lim()`, `std::__detail::__digamma()`, `std::__detail::__dirichlet_eta()`, `std::__detail::__falling_factorial()`, `__fp_is_even_integer()`, `__fp_is_odd_integer()`, `std::__detail::__gamma()`, `std::__detail::__gamma_p()`, `std::__detail::__gamma_q()`, `std::__detail::__gamma_reciprocal()`, `std::__detail::__gamma_series()`, `std::__detail::__hyperg()`, `std::__detail::__hyperg_reflect()`, `std::__detail::__log_falling_factorial()`, `std::__detail::__log_gamma()`, `std::__detail::__polygamma()`, `std::__detail::__polylog()`, `std::__detail::__polylog_exp()`, `std::__detail::__riemann_zeta()`, `std::__detail::__riemann_zeta_m_1()`, `std::__detail::__tgamma()`, `std::__detail::__tgamma_lower()`, and `std::__detail::__tricoli_u_naive()`.

8.1.2.6 `__fp_is_odd_integer()`

```
template<typename _Tp >
constexpr __fp_is_integer_t __gnu_cxx::__fp_is_odd_integer (
    _Tp __a,
    _Tp __mul = _Tp{1} ) [inline], [noexcept]
```

A function to reliably detect if a floating point number is an odd integer.

Parameters

<code>__a</code>	The floating point number
<code>__mul</code>	The multiplier of machine epsilon for the tolerance

Returns

`true` if `a` is an odd integer within `mul * epsilon`.

Definition at line 241 of file `math_util.h`.

References `__fp_is_integer()`.

8.1.2.7 `__fp_is_zero()`

```
template<typename _Tp >
constexpr bool __gnu_cxx::__fp_is_zero (
    _Tp __a,
    _Tp __mul = _Tp{1} ) [inline], [noexcept]
```

A function to reliably compare a floating point number with zero.

Parameters

<code>__a</code>	The floating point number
<code>__mul</code>	The multiplier for numeric epsilon for comparison

Returns

`true` if `a` and `b` are equal to zero or differ only by $\max(a, b) * mul * epsilon$

Definition at line 110 of file `math_util.h`.

Referenced by `std::__detail::__periodic_zeta()`, `std::__detail::__polylog()`, `std::__detail::__polylog_exp_neg()`, `std::__detail::__polylog_exp_neg_int()`, `std::__detail::__polylog_exp_pos_int()`, `std::__detail::__polylog_exp_pos_real()`, and `std::__detail::__theta_1()`.

8.1.2.8 __fp_max_abs()

```
template<typename _Tp >
constexpr _Tp __gnu_cxx::__fp_max_abs (
    _Tp __a,
    _Tp __b ) [inline], [noexcept]
```

A function to return the maximum of the absolute values of two numbers ... so we won't include everything.

Parameters

<code>__a</code>	The left hand side
<code>__b</code>	The right hand side

Definition at line 62 of file `math_util.h`.

Referenced by `__fp_is_equal()`.

8.1.2.9 __parity()

```
template<typename _Tp , typename _IntTp >
constexpr _Tp __gnu_cxx::__parity (
    _IntTp __k ) [inline], [noexcept]
```

Return -1 if the integer argument is odd and +1 if it is even.

Definition at line 51 of file `math_util.h`.

8.2 std Namespace Reference**Namespaces**

- [__detail](#)

Implementation-space details.

Functions

- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > assoc_laguerre (unsigned int __n, unsigned int __m, _Tp __x)`
- `template<typename _Talpha, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Talpha, _Tp > assoc_laguerre (unsigned int __n, _Talpha __alpha1, _Tp __x)`
- `float assoc_laguerref (unsigned int __n, unsigned int __m, float __x)`
- `long double assoc_laguerrel (unsigned int __n, unsigned int __m, long double __x)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > assoc_legendre (unsigned int __l, unsigned int __m, _Tp __x)`
- `float assoc_legendref (unsigned int __l, unsigned int __m, float __x)`
- `long double assoc_legendrel (unsigned int __l, unsigned int __m, long double __x)`
- `template<typename _Tpa, typename _Tpb >`
`__gnu_cxx::fp_promote_t< _Tpa, _Tpb > beta (_Tpa __a, _Tpb __b)`
- `float betaf (float __a, float __b)`
- `long double betal (long double __a, long double __b)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > comp_ellint_1 (_Tp __k)`
- `float comp_ellint_1f (float __k)`
- `long double comp_ellint_1l (long double __k)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > comp_ellint_2 (_Tp __k)`
- `float comp_ellint_2f (float __k)`
- `long double comp_ellint_2l (long double __k)`
- `template<typename _Tp, typename _Tpn >`
`__gnu_cxx::fp_promote_t< _Tp, _Tpn > comp_ellint_3 (_Tp __k, _Tpn __nu)`
- `float comp_ellint_3f (float __k, float __nu)`
Return the complete elliptic integral of the third kind $\Pi(k, \nu)$ for float modulus k.
- `long double comp_ellint_3l (long double __k, long double __nu)`
Return the complete elliptic integral of the third kind $\Pi(k, \nu)$ for long double modulus k.
- `template<typename _Tpnu, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tpnu, _Tp > cyl_bessel_i (_Tpnu __nu, _Tp __x)`
- `float cyl_bessel_if (float __nu, float __x)`
- `long double cyl_bessel_il (long double __nu, long double __x)`
- `template<typename _Tpnu, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tpnu, _Tp > cyl_bessel_j (_Tpnu __nu, _Tp __x)`
- `float cyl_bessel_jf (float __nu, float __x)`
- `long double cyl_bessel_jl (long double __nu, long double __x)`
- `template<typename _Tpnu, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tpnu, _Tp > cyl_bessel_k (_Tpnu __nu, _Tp __x)`
- `float cyl_bessel_kf (float __nu, float __x)`
- `long double cyl_bessel_kl (long double __nu, long double __x)`
- `template<typename _Tpnu, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tpnu, _Tp > cyl_neumann (_Tpnu __nu, _Tp __x)`
- `float cyl_neumannf (float __nu, float __x)`
- `long double cyl_neumannl (long double __nu, long double __x)`
- `template<typename _Tp, typename _Tpp >`
`__gnu_cxx::fp_promote_t< _Tp, _Tpp > ellint_1 (_Tp __k, _Tpp __phi)`
- `float ellint_1f (float __k, float __phi)`
- `long double ellint_1l (long double __k, long double __phi)`
- `template<typename _Tp, typename _Tpp >`
`__gnu_cxx::fp_promote_t< _Tp, _Tpp > ellint_2 (_Tp __k, _Tpp __phi)`

- float [ellint_2f](#) (float __k, float __phi)
Return the incomplete elliptic integral of the second kind $E(k, \phi)$ for float argument.
- long double [ellint_2l](#) (long double __k, long double __phi)
Return the incomplete elliptic integral of the second kind $E(k, \phi)$.
- template<typename _Tp, typename _Tpn, typename _Tpp >
[__gnu_cxx::fp_promote_t<_Tp, _Tpn, _Tpp > ellint_3](#) (_Tp __k, _Tpn __nu, _Tpp __phi)
Return the incomplete elliptic integral of the third kind $\Pi(k, \nu, \phi)$.
- float [ellint_3f](#) (float __k, float __nu, float __phi)
Return the incomplete elliptic integral of the third kind $\Pi(k, \nu, \phi)$ for float argument.
- long double [ellint_3l](#) (long double __k, long double __nu, long double __phi)
Return the incomplete elliptic integral of the third kind $\Pi(k, \nu, \phi)$.
- template<typename _Tp >
[__gnu_cxx::fp_promote_t<_Tp > expint](#) (_Tp __x)
- float [expintf](#) (float __x)
- long double [expintl](#) (long double __x)
- template<typename _Tp >
[__gnu_cxx::fp_promote_t<_Tp > hermite](#) (unsigned int __n, _Tp __x)
- float [hermitef](#) (unsigned int __n, float __x)
- long double [hermitel](#) (unsigned int __n, long double __x)
- template<typename _Tp >
[__gnu_cxx::fp_promote_t<_Tp > laguerre](#) (unsigned int __n, _Tp __x)
- float [laguerref](#) (unsigned int __n, float __x)
- long double [laguerrel](#) (unsigned int __n, long double __x)
- template<typename _Tp >
[__gnu_cxx::fp_promote_t<_Tp > legendre](#) (unsigned int __l, _Tp __x)
- float [legendref](#) (unsigned int __l, float __x)
- long double [legendrel](#) (unsigned int __l, long double __x)
- template<typename _Tp >
[__gnu_cxx::fp_promote_t<_Tp > riemann_zeta](#) (_Tp __s)
- float [riemann_zetaf](#) (float __s)
- long double [riemann_zetal](#) (long double __s)
- template<typename _Tp >
[__gnu_cxx::fp_promote_t<_Tp > sph_bessel](#) (unsigned int __n, _Tp __x)
- float [sph_besself](#) (unsigned int __n, float __x)
- long double [sph_bessell](#) (unsigned int __n, long double __x)
- template<typename _Tp >
[__gnu_cxx::fp_promote_t<_Tp > sph_legendre](#) (unsigned int __l, unsigned int __m, _Tp __theta)
- float [sph_legendref](#) (unsigned int __l, unsigned int __m, float __theta)
- long double [sph_legendrel](#) (unsigned int __l, unsigned int __m, long double __theta)
- template<typename _Tp >
[__gnu_cxx::fp_promote_t<_Tp > sph_neumann](#) (unsigned int __n, _Tp __x)
- float [sph_neumannf](#) (unsigned int __n, float __x)
- long double [sph_neumannl](#) (unsigned int __n, long double __x)

8.3 std::__detail Namespace Reference

Implementation-space details.

Classes

- struct [__gamma_lanczos_data](#)
- struct [__gamma_lanczos_data< double >](#)
- struct [__gamma_lanczos_data< float >](#)
- struct [__gamma_lanczos_data< long double >](#)
- struct [__gamma_spouge_data](#)
- struct [__gamma_spouge_data< double >](#)
- struct [__gamma_spouge_data< float >](#)
- struct [__gamma_spouge_data< long double >](#)
- struct [__jacobi_lattice_t](#)
- struct [__jacobi_theta_0_t](#)
- struct [__weierstrass_invariants_t](#)
- struct [__weierstrass_roots_t](#)
- class [_Airy](#)
- class [_Airy_asymp](#)
- struct [_Airy_asymp_data](#)
- struct [_Airy_asymp_data< double >](#)
- struct [_Airy_asymp_data< float >](#)
- struct [_Airy_asymp_data< long double >](#)
- class [_Airy_asymp_series](#)
- struct [_Airy_default_radII](#)
- struct [_Airy_default_radII< double >](#)
- struct [_Airy_default_radII< float >](#)
- struct [_Airy_default_radII< long double >](#)
- class [_Airy_series](#)
- struct [_AiryAuxilliaryState](#)
- struct [_AiryState](#)
- class [_AsympTerminator](#)
- struct [_Factorial_table](#)
- class [_Terminator](#)

Functions

- template<typename [_Tp](#) >
[__gnu_cxx::__airy_t](#)< [_Tp](#), [_Tp](#) > [__airy](#) ([_Tp](#) [__z](#))
Compute the Airy functions $Ai(x)$ and $Bi(x)$ and their first derivatives $Ai'(x)$ and $Bi'(x)$ respectively.
- template<typename [_Tp](#) >
[std::complex](#)< [_Tp](#) > [__airy_ai](#) ([std::complex](#)< [_Tp](#) > [__z](#))
Return the complex Airy Ai function.
- template<typename [_Tp](#) >
[void](#) [__airy_arg](#) ([std::complex](#)< [_Tp](#) > [__num2d3](#), [std::complex](#)< [_Tp](#) > [__zeta](#), [std::complex](#)< [_Tp](#) > &[__argp](#),
[std::complex](#)< [_Tp](#) > &[__argm](#))
Compute the arguments for the Airy function evaluations carefully to prevent premature overflow. Note that the major work here is in `safe_div`. A faster, but less safe implementation can be obtained without use of `safe_div`.
- template<typename [_Tp](#) >
[std::complex](#)< [_Tp](#) > [__airy_bi](#) ([std::complex](#)< [_Tp](#) > [__z](#))
Return the complex Airy Bi function.
- template<typename [_Tpa](#) , typename [_Tp](#) >
[_Tp](#) [__assoc_laguerre](#) (unsigned int [__n](#), [_Tpa](#) [__alpha](#), [_Tp](#) [__x](#))

This routine returns the associated Laguerre polynomial of degree n , order m : $L_n^{(m)}(x)$.

- `template<typename _Tp >`
`__gnu_cxx::__assoc_legendre_p_t< _Tp > __assoc_legendre_p (unsigned int __l, unsigned int __m, _Tp __x,`
`_Tp __phase=__Tp{+1})`

Return the associated Legendre function by recursion on l and downward recursion on m .

- `template<typename _Tp >`
`__gnu_cxx::__assoc_legendre_q_t< _Tp > __assoc_legendre_q (unsigned int __l, unsigned int __m, _Tp __x,`
`_Tp __phase=__Tp{+1})`
- `template<typename _Tp >`
`std::vector< _Tp > __bell (unsigned int __n)`
- `template<typename _Tp, typename _Up >`
`_Up __bell (unsigned int __n, _Up __x)`
- `template<typename _Tp >`
`std::vector< _Tp > __bell_series (unsigned int __n)`
- `template<typename _Tp >`
`_GLIBCXX14_CONSTEXPR _Tp __bernoulli (unsigned int __n)`

This returns Bernoulli number B_n .

- `template<typename _Tp >`
`_Tp __bernoulli (unsigned int __n, _Tp __x)`
- `template<typename _Tp >`
`_GLIBCXX14_CONSTEXPR _Tp __bernoulli_2n (unsigned int __n)`

This returns Bernoulli number B_{2n} at even integer arguments $2n$.

- `template<typename _Tp >`
`_GLIBCXX14_CONSTEXPR _Tp __bernoulli_series (unsigned int __n)`

This returns Bernoulli numbers from a table or by summation for larger values.

$$B_{2n} = (-1)^{n+1} 2 \frac{(2n)!}{(2\pi)^{2n}} \zeta(2n)$$

- `template<typename _Tp >`
`_Tp __beta (_Tp __a, _Tp __b)`
- `template<typename _Tp >`
`_Tp __beta_gamma (_Tp __a, _Tp __b)`
- `template<typename _Tp >`
`_Tp __beta_inc (_Tp __a, _Tp __b, _Tp __x)`
- `template<typename _Tp >`
`_Tp __beta_lgamma (_Tp __a, _Tp __b)`

Return the beta function $B(a, b)$.

Return the beta function: $B(a, b)$.

Return the beta function $B(a, b)$ using the log gamma functions.

- `template<typename _Tp >`
`_Tp __beta_p (_Tp __a, _Tp __b, _Tp __x)`
- `template<typename _Tp >`
`_Tp __beta_product (_Tp __a, _Tp __b)`

Return the beta function $B(x, y)$ using the product form.

- `template<typename _Tp >`
`_Tp __binomial (unsigned int __n, unsigned int __k)`

Return the binomial coefficient. The binomial coefficient is given by:

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

The binomial coefficients are generated by:

$$(1+t)^n = \sum_{k=0}^n \binom{n}{k} t^k$$

- `template<typename _Tp >`
`_Tp __binomial (_Tp __nu, unsigned int __k)`

Return the binomial coefficient for non-integral degree. The binomial coefficient is given by:

$$\binom{\nu}{k} = \frac{\Gamma(\nu+1)}{\Gamma(\nu-k+1)\Gamma(k+1)}$$

The binomial coefficients are generated by:

$$(1+t)^\nu = \sum_{k=0}^{\infty} \binom{\nu}{k} t^k$$

- `template<typename _Tp >`
`_Tp __binomial_p (_Tp __p, unsigned int __n, unsigned int __k)`
Return the binomial cumulative distribution function.
- `template<typename _Tp >`
`_Tp __binomial_pdf (_Tp __p, unsigned int __n, unsigned int __k)`
Return the binomial probability mass function.
- `template<typename _Tp >`
`_Tp __binomial_q (_Tp __p, unsigned int __n, unsigned int __k)`
Return the complementary binomial cumulative distribution function.
- `template<typename _Sp, typename _Tp >`
`_Tp __bose_einstein (_Sp __s, _Tp __x)`
- `template<typename _Tp >`
`_Tp __cauchy_p (_Tp __a, _Tp __b, _Tp __x)`
- `template<typename _Tp >`
`std::tuple< _Tp, _Tp, _Tp > __chebyshev_recur (unsigned int __n, _Tp __x, _Tp __C0, _Tp __C1)`
- `template<typename _Tp >`
`__gnu_cxx::__chebyshev_t_t< _Tp > __chebyshev_t (unsigned int __n, _Tp __x)`
- `template<typename _Tp >`
`__gnu_cxx::__chebyshev_u_t< _Tp > __chebyshev_u (unsigned int __n, _Tp __x)`
- `template<typename _Tp >`
`__gnu_cxx::__chebyshev_v_t< _Tp > __chebyshev_v (unsigned int __n, _Tp __x)`
- `template<typename _Tp >`
`__gnu_cxx::__chebyshev_w_t< _Tp > __chebyshev_w (unsigned int __n, _Tp __x)`
- `template<typename _Tp >`
`_Tp __chi_squared_pdf (_Tp __chi2, unsigned int __nu)`
Return the chi-squared probability function. This returns the probability that the observed chi-squared for a correct model is less than the value χ^2 .
- `template<typename _Tp >`
`_Tp __chi_squared_pdfc (_Tp __chi2, unsigned int __nu)`
Return the complementary chi-squared probability function. This returns the probability that the observed chi-squared for a correct model is greater than the value χ^2 .
- `template<typename _Tp >`
`std::pair< _Tp, _Tp > __chshint (_Tp __x, _Tp &_Chi, _Tp &_Shi)`
This function returns the hyperbolic cosine $Chi(x)$ and hyperbolic sine $Shi(x)$ integrals as a pair.
- `template<typename _Tp >`
`void __chshint_cont_frac (_Tp __t, _Tp &_Chi, _Tp &_Shi)`

This function computes the hyperbolic cosine $\text{Chi}(x)$ and hyperbolic sine $\text{Shi}(x)$ integrals by continued fraction for positive argument.

- `template<typename _Tp >`
`void __chshint_series (_Tp __t, _Tp &_Chi, _Tp &_Shi)`

This function computes the hyperbolic cosine $\text{Chi}(x)$ and hyperbolic sine $\text{Shi}(x)$ integrals by series summation for positive argument.

- `template<typename _Tp >`
`std::complex< _Tp > __clamp_0_m2pi (std::complex< _Tp > __z)`
- `template<typename _Tp >`
`std::complex< _Tp > __clamp_pi (std::complex< _Tp > __z)`
- `template<typename _Tp >`
`std::complex< _Tp > __clausen (unsigned int __m, std::complex< _Tp > __z)`
- `template<typename _Tp >`
`_Tp __clausen (unsigned int __m, _Tp __x)`
- `template<typename _Tp >`
`_Tp __clausen_cl (unsigned int __m, std::complex< _Tp > __z)`
- `template<typename _Tp >`
`_Tp __clausen_cl (unsigned int __m, _Tp __x)`
- `template<typename _Tp >`
`_Tp __clausen_sl (unsigned int __m, std::complex< _Tp > __z)`
- `template<typename _Tp >`
`_Tp __clausen_sl (unsigned int __m, _Tp __x)`
- `template<typename _Tp >`
`_Tp __comp_ellint_1 (_Tp __k)`

Return the complete elliptic integral of the first kind $K(k)$ using the Carlson formulation.

- `template<typename _Tp >`
`_Tp __comp_ellint_2 (_Tp __k)`

Return the complete elliptic integral of the second kind $E(k)$ using the Carlson formulation.

- `template<typename _Tp >`
`_Tp __comp_ellint_3 (_Tp __k, _Tp __nu)`

Return the complete elliptic integral of the third kind $\Pi(k, \nu) = \Pi(k, \nu, \pi/2)$ using the Carlson formulation.

- `template<typename _Tp >`
`_Tp __comp_ellint_d (_Tp __k)`
- `template<typename _Tp >`
`_Tp __comp_ellint_rf (_Tp __x, _Tp __y)`
- `template<typename _Tp >`
`_Tp __comp_ellint_rg (_Tp __x, _Tp __y)`
- `template<typename _Tp >`
`_Tp __conf_hyperg (_Tp __a, _Tp __c, _Tp __x)`

Return the confluent hypergeometric function ${}_1F_1(a; c; x) = M(a, c, x)$.

- `template<typename _Tp >`
`_Tp __conf_hyperg_lim (_Tp __c, _Tp __x)`

Return the confluent hypergeometric limit function ${}_0F_1(-; c; x)$.

- `template<typename _Tp >`
`_Tp __conf_hyperg_lim_series (_Tp __c, _Tp __x)`

This routine returns the confluent hypergeometric limit function by series expansion.

- `template<typename _Tp >`
`_Tp __conf_hyperg_luke (_Tp __a, _Tp __c, _Tp __xin)`

Return the hypergeometric function ${}_1F_1(a; c; x)$ by an iterative procedure described in Luke, Algorithms for the Computation of Mathematical Functions.

- `template<typename _Tp >`
`_Tp __conf_hyperg_series (_Tp __a, _Tp __c, _Tp __x)`
This routine returns the confluent hypergeometric function by series expansion.
- `template<typename _Tp >`
`_Tp __cos_pi (_Tp __x)`
- `template<typename _Tp >`
`std::complex< _Tp > __cos_pi (std::complex< _Tp > __z)`
- `template<typename _Tp >`
`_Tp __cosh_pi (_Tp __x)`
- `template<typename _Tp >`
`std::complex< _Tp > __cosh_pi (std::complex< _Tp > __z)`
- `template<typename _Tp >`
`_Tp __coshint (const _Tp __x)`
Return the hyperbolic cosine integral $Chi(x)$.
- `template<typename _Tp >`
`std::pair< _Tp, _Tp > __coulomb_CF1 (unsigned int __l, _Tp __eta, _Tp __x)`
- `template<typename _Tp >`
`std::complex< _Tp > __coulomb_CF2 (unsigned int __l, _Tp __eta, _Tp __x)`
- `template<typename _Tp >`
`std::pair< _Tp, _Tp > __coulomb_f_recur (unsigned int __l_min, unsigned int __k_max, _Tp __eta, _Tp __x, _Tp __F_l_max, _Tp __Fp_l_max)`
- `template<typename _Tp >`
`std::pair< _Tp, _Tp > __coulomb_g_recur (unsigned int __l_min, unsigned int __k_max, _Tp __eta, _Tp __x, _Tp __G_l_min, _Tp __Gp_l_min)`
- `template<typename _Tp >`
`_Tp __coulomb_norm (unsigned int __l, _Tp __eta)`
- `template<typename _Tp >`
`std::complex< _Tp > __cyl_bessel (std::complex< _Tp > __nu, std::complex< _Tp > __z)`
Return the complex cylindrical Bessel function.
- `template<typename _Tp >`
`_Tp __cyl_bessel_i (_Tp __nu, _Tp __x)`
Return the regular modified Bessel function of order ν : $I_\nu(x)$.
- `template<typename _Tp >`
`_Tp __cyl_bessel_ij_series (_Tp __nu, _Tp __x, _Tp __sgn, unsigned int __max_iter)`
This routine returns the cylindrical Bessel functions of order ν : J_ν or I_ν by series expansion.
- `template<typename _Tp >`
`__gnu_cxx::__cyl_mod_bessel_t< _Tp, _Tp, _Tp > __cyl_bessel_ik (_Tp __nu, _Tp __x)`
Return the modified cylindrical Bessel functions and their derivatives of order ν by various means.
- `template<typename _Tp >`
`__gnu_cxx::__cyl_mod_bessel_t< _Tp, _Tp, _Tp > __cyl_bessel_ik_asymp (_Tp __nu, _Tp __x)`
This routine computes the asymptotic modified cylindrical Bessel and functions of order ν : $I_\nu(x)$, $N_\nu(x)$. Use this for $x \gg nu^2 + 1$.
- `template<typename _Tp >`
`__gnu_cxx::__cyl_mod_bessel_t< _Tp, _Tp, _Tp > __cyl_bessel_ik_steel (_Tp __nu, _Tp __x)`
Compute the modified Bessel functions $I_\nu(x)$ and $K_\nu(x)$ and their first derivatives $I'_\nu(x)$ and $K'_\nu(x)$ respectively. These four functions are computed together for numerical stability.
- `template<typename _Tp >`
`_Tp __cyl_bessel_j (_Tp __nu, _Tp __x)`
Return the Bessel function of order ν : $J_\nu(x)$.
- `template<typename _Tp >`
`__gnu_cxx::__cyl_bessel_t< _Tp, _Tp, _Tp > __cyl_bessel_jn (_Tp __nu, _Tp __x)`

Return the cylindrical Bessel functions and their derivatives of order ν by various means.

- `template<typename _Tp >`
`__gnu_cxx::__cyl_bessel_t< _Tp, _Tp, _Tp > __cyl_bessel_jn_asymp (_Tp __nu, _Tp __x)`
This routine computes the asymptotic cylindrical Bessel and Neumann functions of order ν : $J_\nu(x)$, $N_\nu(x)$. Use this for $x \gg \nu^2 + 1$.
- `template<typename _Tp >`
`__gnu_cxx::__cyl_bessel_t< _Tp, _Tp, std::complex< _Tp > > __cyl_bessel_jn_neg_arg (_Tp __nu, _Tp __x)`
Return the cylindrical Bessel functions and their derivatives of order ν and argument $x < 0$.
- `template<typename _Tp >`
`__gnu_cxx::__cyl_bessel_t< _Tp, _Tp, _Tp > __cyl_bessel_jn_steel (_Tp __nu, _Tp __x)`
Compute the Bessel $J_\nu(x)$ and Neumann $N_\nu(x)$ functions and their first derivatives $J'_\nu(x)$ and $N'_\nu(x)$ respectively. These four functions are computed together for numerical stability.
- `template<typename _Tp >`
`_Tp __cyl_bessel_k (_Tp __nu, _Tp __x)`
Return the irregular modified Bessel function $K_\nu(x)$ of order ν .
- `template<typename _Tp >`
`std::complex< _Tp > __cyl_hankel_1 (_Tp __nu, _Tp __x)`
Return the cylindrical Hankel function of the first kind $H_\nu^{(1)}(x)$.
- `template<typename _Tp >`
`std::complex< _Tp > __cyl_hankel_1 (std::complex< _Tp > __nu, std::complex< _Tp > __z)`
Return the complex cylindrical Hankel function of the first kind.
- `template<typename _Tp >`
`std::complex< _Tp > __cyl_hankel_2 (_Tp __nu, _Tp __x)`
Return the cylindrical Hankel function of the second kind $H_\nu^{(2)}(x)$.
- `template<typename _Tp >`
`std::complex< _Tp > __cyl_hankel_2 (std::complex< _Tp > __nu, std::complex< _Tp > __z)`
Return the complex cylindrical Hankel function of the second kind.
- `template<typename _Tp >`
`std::complex< _Tp > __cyl_neumann (std::complex< _Tp > __nu, std::complex< _Tp > __z)`
Return the complex cylindrical Neumann function.
- `template<typename _Tp >`
`_Tp __cyl_neumann_n (_Tp __nu, _Tp __x)`
Return the Neumann function of order ν : $N_\nu(x)$.
- `template<typename _Tp >`
`_Tp __dawson (_Tp __x)`
Return the Dawson integral, $F(x)$, for real argument x .
- `template<typename _Tp >`
`_Tp __dawson_cont_frac (_Tp __x)`
Compute the Dawson integral using a sampling theorem representation.
- `template<typename _Tp >`
`_Tp __dawson_series (_Tp __x)`
Compute the Dawson integral using the series expansion.
- `template<typename _Tp >`
`_Tp __debye (unsigned int __n, _Tp __x)`
- `template<typename _Tp >`
`void __debye_region (std::complex< _Tp > __alpha, int &__indexr, char &__aorb)`
- `template<typename _Tp >`
`_Tp __digamma (unsigned int __n)`

Return the digamma function of integral argument. The digamma or $\psi(x)$ function is defined as the logarithmic derivative of the gamma function:

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

The digamma series for integral argument is given by:

$$\psi(n) = -\gamma_E + \sum_{k=1}^{n-1} \frac{1}{k}$$

The latter sum is called the harmonic number, H_n .

- `template<typename _Tp >`
`_Tp __digamma (_Tp __x)`

Return the digamma function. The digamma or $\psi(x)$ function is defined by

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

For negative argument the reflection formula is used:

$$\psi(x) = \psi(1-x) - \pi \cot(\pi x)$$

- `template<typename _Tp >`
`_Tp __digamma_asymp (_Tp __x)`

Return the digamma function for large argument. The digamma or $\psi(x)$ function is defined by

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

- `template<typename _Tp >`
`_Tp __digamma_series (_Tp __x)`

Return the digamma function by series expansion. The digamma or $\psi(x)$ function is defined by

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

- `template<typename _Tp >`
`_Tp __dilog (_Tp __x)`

Compute the dilogarithm function $Li_2(x)$ by summation for $x \leq 1$.

- `template<typename _Tp >`
`_Tp __dirichlet_beta (std::complex< _Tp > __s)`
- `template<typename _Tp >`
`_Tp __dirichlet_beta (_Tp __s)`
- `template<typename _Tp >`
`std::complex< _Tp > __dirichlet_eta (std::complex< _Tp > __s)`
- `template<typename _Tp >`
`_Tp __dirichlet_eta (_Tp __s)`
- `template<typename _Tp >`
`_Tp __dirichlet_lambda (_Tp __s)`
- `template<typename _Tp >`
`_GLIBCXX14_CONSTEXPR _Tp __double_factorial (int __n)`

Return the double factorial of the integer n .

- `template<typename _Tp >`
`_Tp __ellint_1 (_Tp __k, _Tp __phi)`

Return the incomplete elliptic integral of the first kind $F(k, \phi)$ using the Carlson formulation.

- `template<typename _Tp >`
`_Tp __ellint_2 (_Tp __k, _Tp __phi)`
Return the incomplete elliptic integral of the second kind $E(k, \phi)$ using the Carlson formulation.
- `template<typename _Tp >`
`_Tp __ellint_3 (_Tp __k, _Tp __nu, _Tp __phi)`
Return the incomplete elliptic integral of the third kind $\Pi(k, \nu, \phi)$ using the Carlson formulation.
- `template<typename _Tp >`
`_Tp __ellint_cel (_Tp __k_c, _Tp __p, _Tp __a, _Tp __b)`
- `template<typename _Tp >`
`_Tp __ellint_d (_Tp __k, _Tp __phi)`
- `template<typename _Tp >`
`_Tp __ellint_el1 (_Tp __x, _Tp __k_c)`
- `template<typename _Tp >`
`_Tp __ellint_el2 (_Tp __x, _Tp __k_c, _Tp __a, _Tp __b)`
- `template<typename _Tp >`
`_Tp __ellint_el3 (_Tp __x, _Tp __k_c, _Tp __p)`
- `template<typename _Tp >`
`_Tp __ellint_rc (_Tp __x, _Tp __y)`
Return the Carlson elliptic function $R_C(x, y) = R_F(x, y, y)$ where $R_F(x, y, z)$ is the Carlson elliptic function of the first kind.
- `template<typename _Tp >`
`_Tp __ellint_rd (_Tp __x, _Tp __y, _Tp __z)`
Return the Carlson elliptic function of the second kind $R_D(x, y, z) = R_J(x, y, z, z)$ where $R_J(x, y, z, p)$ is the Carlson elliptic function of the third kind.
- `template<typename _Tp >`
`_Tp __ellint_rf (_Tp __x, _Tp __y, _Tp __z)`
Return the Carlson elliptic function $R_F(x, y, z)$ of the first kind.
- `template<typename _Tp >`
`_Tp __ellint_rg (_Tp __x, _Tp __y, _Tp __z)`
Return the symmetric Carlson elliptic function of the second kind $R_G(x, y, z)$.
- `template<typename _Tp >`
`_Tp __ellint_rj (_Tp __x, _Tp __y, _Tp __z, _Tp __p)`
Return the Carlson elliptic function $R_J(x, y, z, p)$ of the third kind.
- `template<typename _Tp >`
`_Tp __ellnome (_Tp __k)`
- `template<typename _Tp >`
`_Tp __ellnome_k (_Tp __k)`
- `template<typename _Tp >`
`_Tp __ellnome_series (_Tp __k)`
- `template<typename _Tp >`
`_Tp __euler (unsigned int __n)`
This returns Euler number E_n .
- `template<typename _Tp >`
`_Tp __euler (unsigned int __n, _Tp __x)`
- `template<typename _Tp >`
`_Tp __euler_series (unsigned int __n)`
- `template<typename _Tp >`
`_Tp __eulerian_1 (unsigned int __n, unsigned int __m)`
- `template<typename _Tp >`
`std::vector< _Tp > __eulerian_1 (unsigned int __n)`

- `template<typename _Tp >`
`_Tp __eulerian_1_recur (unsigned int __n, unsigned int __m)`
- `template<typename _Tp >`
`std::vector< _Tp > __eulerian_1_recur (unsigned int __n)`
- `template<typename _Tp >`
`_Tp __eulerian_2 (unsigned int __n, unsigned int __m)`
- `template<typename _Tp >`
`std::vector< _Tp > __eulerian_2 (unsigned int __n)`
- `template<typename _Tp >`
`_Tp __eulerian_2_recur (unsigned int __n, unsigned int __m)`
- `template<typename _Tp >`
`std::vector< _Tp > __eulerian_2_recur (unsigned int __n)`
- `template<typename _Tp >`
`_Tp __exp2 (_Tp __x)`
- `template<typename _Tp >`
`_Tp __expint (unsigned int __n, _Tp __x)`
Return the exponential integral $E_n(x)$.
- `template<typename _Tp >`
`_Tp __expint (_Tp __x)`
Return the exponential integral $Ei(x)$.
- `template<typename _Tp >`
`_Tp __expint_E1 (_Tp __x)`
Return the exponential integral $E_1(x)$.
- `template<typename _Tp >`
`_Tp __expint_E1_asymp (_Tp __x)`
Return the exponential integral $E_1(x)$ by asymptotic expansion.
- `template<typename _Tp >`
`_Tp __expint_E1_series (_Tp __x)`
Return the exponential integral $E_1(x)$ by series summation. This should be good for $x < 1$.
- `template<typename _Tp >`
`_Tp __expint_Ei (_Tp __x)`
Return the exponential integral $Ei(x)$.
- `template<typename _Tp >`
`_Tp __expint_Ei_asymp (_Tp __x)`
Return the exponential integral $Ei(x)$ by asymptotic expansion.
- `template<typename _Tp >`
`_Tp __expint_Ei_series (_Tp __x)`
Return the exponential integral $Ei(x)$ by series summation.
- `template<typename _Tp >`
`_Tp __expint_En_asymp (unsigned int __n, _Tp __x)`
Return the exponential integral $E_n(x)$ for large argument.
- `template<typename _Tp >`
`_Tp __expint_En_cont_frac (unsigned int __n, _Tp __x)`
Return the exponential integral $E_n(x)$ by continued fractions.
- `template<typename _Tp >`
`_Tp __expint_En_large_n (unsigned int __n, _Tp __x)`
Return the exponential integral $E_n(x)$ for large order.
- `template<typename _Tp >`
`_Tp __expint_En_recursion (unsigned int __n, _Tp __x)`

Return the exponential integral $E_n(x)$ by recursion. Use upward recursion for $x < n$ and downward recursion (Miller's algorithm) otherwise.

- `template<typename _Tp >`
`_Tp __expint_En_series (unsigned int __n, _Tp __x)`
 Return the exponential integral $E_n(x)$ by series summation.

- `template<typename _Tp >`
`_Tp __exponential_p (_Tp __lambda, _Tp __x)`
 Return the exponential cumulative probability density function.

- `template<typename _Tp >`
`_Tp __exponential_pdf (_Tp __lambda, _Tp __x)`
 Return the exponential probability density function.

- `template<typename _Tp >`
`_Tp __exponential_q (_Tp __lambda, _Tp __x)`
 Return the complement of the exponential cumulative probability density function.

- `template<typename _Tp >`
`_GLIBCXX14_CONSTEXPR _Tp __factorial (unsigned int __n)`
 Return the factorial of the integer n .

- `template<typename _Tp >`
`_Tp __falling_factorial (_Tp __a, int __n)`
 Return the logarithm of the falling factorial function or the lower Pochhammer symbol for real argument a and integral order n . The falling factorial function is defined by

$$a^{\underline{n}} = \prod_{k=0}^{n-1} (a - k), (a)_0 = 1 = \Gamma(a + 1) / \Gamma(a - n + 1)$$

In particular, $n^{\underline{n}} = n!$.

- `template<typename _Tp >`
`_Tp __falling_factorial (_Tp __a, _Tp __nu)`
 Return the logarithm of the falling factorial function or the lower Pochhammer symbol for real argument a and order ν . The falling factorial function is defined by

$$a^{\underline{\nu}} = \Gamma(a + 1) / \Gamma(a - \nu + 1)$$

- `template<typename _Sp, typename _Tp >`
`_Tp __fermi_dirac (_Sp __s, _Tp __x)`
- `template<typename _Tp >`
`_Tp __fisher_f_p (_Tp __F, unsigned int __nu1, unsigned int __nu2)`
 Return the F -distribution propability function. This returns the probability that the observed chi-square for a correct model exceeds the value χ^2 .

- `template<typename _Tp >`
`_Tp __fisher_f_pdf (_Tp __F, unsigned int __nu1, unsigned int __nu2)`
 Return the F -distribution propability function. This returns the probability that the observed chi-square for a correct model exceeds the value χ^2 .

- `template<typename _Tp >`
`_Tp __fisher_f_q (_Tp __F, unsigned int __nu1, unsigned int __nu2)`
 Return the F -distribution propability function. This returns the probability that the observed chi-square for a correct model exceeds the value χ^2 .

- `template<typename _Tp >`
`__gnu_cxx::__fock_airy_t< _Tp, std::complex< _Tp > > __fock_airy (_Tp __x)`
 Compute the Fock-type Airy functions $w_1(x)$ and $w_2(x)$ and their first derivatives $w'_1(x)$ and $w'_2(x)$ respectively.

$$w_1(x) = \sqrt{\pi}(Ai(x) + iBi(x))$$

$$w_2(x) = \sqrt{\pi}(Ai(x) - iBi(x))$$

- `template<typename _Tp >`
`std::complex< _Tp > __fresnel (const _Tp __x)`
Return the Fresnel cosine and sine integrals as a complex number $\$f[C(x) + iS(x) \$f]$.
- `template<typename _Tp >`
`void __fresnel_cont_frac (const _Tp __ax, _Tp &_Cf, _Tp &_Sf)`
This function computes the Fresnel cosine and sine integrals by continued fractions for positive argument.
- `template<typename _Tp >`
`void __fresnel_series (const _Tp __ax, _Tp &_Cf, _Tp &_Sf)`
This function returns the Fresnel cosine and sine integrals as a pair by series expansion for positive argument.
- `template<typename _Tp >`
`_Tp __gamma (_Tp __a)`
Return the gamma function $\Gamma(a)$. The gamma function is defined by:

$$\Gamma(a) = \int_0^{\infty} e^{-t} t^{a-1} dt (a > 0)$$

- `template<typename _Tp >`
`std::pair< _Tp, _Tp > __gamma (_Tp __a, _Tp __x)`
Return the incomplete gamma functions.
- `template<typename _Tp >`
`std::pair< _Tp, _Tp > __gamma_cont_frac (_Tp __a, _Tp __x)`
Return the incomplete gamma function by continued fraction.
- `template<typename _Tp >`
`_Tp __gamma_p (_Tp __alpha, _Tp __beta, _Tp __x)`
Return the gamma cumulative propability distribution function.
- `template<typename _Tp >`
`_Tp __gamma_p (_Tp __a, _Tp __x)`
Return the regularized lower incomplete gamma function. The regularized lower incomplete gamma function is defined by

$$P(a, x) = \frac{\gamma(a, x)}{\Gamma(a)}$$

where $\Gamma(a)$ is the gamma function and

$$\gamma(a, x) = \int_0^x e^{-t} t^{a-1} dt (a > 0)$$

is the lower incomplete gamma function.

- `template<typename _Tp >`
`_Tp __gamma_pdf (_Tp __alpha, _Tp __beta, _Tp __x)`
Return the gamma probability distribution function.
- `template<typename _Tp >`
`_Tp __gamma_q (_Tp __alpha, _Tp __beta, _Tp __x)`
Return the gamma complementary cumulative propability distribution function.
- `template<typename _Tp >`
`_Tp __gamma_q (_Tp __a, _Tp __x)`
Return the regularized upper incomplete gamma function. The regularized upper incomplete gamma function is defined by

$$Q(a, x) = \frac{\Gamma(a, x)}{\Gamma(a)}$$

where $\Gamma(a)$ is the gamma function and

$$\Gamma(a, x) = \int_x^{\infty} e^{-t} t^{a-1} dt (a > 0)$$

is the upper incomplete gamma function.

- `template<typename _Tp >`
`_Tp __gamma_reciprocal (_Tp __a)`
- `template<typename _Tp >`
`_Tp __gamma_reciprocal_series (_Tp __a)`
- `template<typename _Tp >`
`std::pair< _Tp, _Tp > __gamma_series (_Tp __a, _Tp __x)`

Return the incomplete gamma function by series summation.

$$\gamma(a, x) = x^a e^{-x} \sum_{k=1}^{\infty} \frac{x^k}{(a)_k}$$

- `template<typename _Tp >`
`__gnu_cxx::__gamma_temme_t< _Tp > __gamma_temme (_Tp __mu)`
Compute the gamma functions required by the Temme series expansions of $N_\nu(x)$ and $K_\nu(x)$.

$$\Gamma_1 = \frac{1}{2\mu} \left[\frac{1}{\Gamma(1-\mu)} - \frac{1}{\Gamma(1+\mu)} \right]$$

and

$$\Gamma_2 = \frac{1}{2} \left[\frac{1}{\Gamma(1-\mu)} + \frac{1}{\Gamma(1+\mu)} \right]$$

where $-1/2 \leq \mu \leq 1/2$ is $\mu = \nu - N$ and N is the nearest integer to ν . The values of $\Gamma(1+\mu)$ and $\Gamma(1-\mu)$ are returned as well.

- `template<typename _Tp >`
`_Tp __gauss (_Tp __x)`
- `template<typename _Tp >`
`__gnu_cxx::__gegenbauer_t< _Tp > __gegenbauer_recur (unsigned int __n, _Tp __lambda, _Tp __x)`
- `template<typename _Tp >`
`std::vector< __gnu_cxx::__quadrature_point_t< _Tp > > __gegenbauer_zeros (unsigned int __n, _Tp __lambda)`
- `template<typename _Tp >`
`__gnu_cxx::__cyl_hankel_t< std::complex< _Tp >, std::complex< _Tp >, std::complex< _Tp > > __hankel (std::complex< _Tp > __nu, std::complex< _Tp > __z)`
- `template<typename _Tp >`
`__gnu_cxx::__cyl_hankel_t< std::complex< _Tp >, std::complex< _Tp >, std::complex< _Tp > > __hankel_debye (std::complex< _Tp > __nu, std::complex< _Tp > __z, std::complex< _Tp > __alpha, int __indexr, char &__aorb, int &__morn)`
- `template<typename _Tp >`
`void __hankel_params (std::complex< _Tp > __nu, std::complex< _Tp > __zhat, std::complex< _Tp > &__p, std::complex< _Tp > &__p2, std::complex< _Tp > &__nup2, std::complex< _Tp > &__num2, std::complex< _Tp > &__num1d3, std::complex< _Tp > &__num2d3, std::complex< _Tp > &__num4d3, std::complex< _Tp > &__zeta, std::complex< _Tp > &__zetaphf, std::complex< _Tp > &__zetamhf, std::complex< _Tp > &__zetam3hf, std::complex< _Tp > &__zetrat)`
Compute parameters depending on z and nu that appear in the uniform asymptotic expansions of the Hankel functions and their derivatives, except the arguments to the Airy functions.
- `template<typename _Tp >`
`__gnu_cxx::__cyl_hankel_t< std::complex< _Tp >, std::complex< _Tp >, std::complex< _Tp > > __hankel_uniform (std::complex< _Tp > __nu, std::complex< _Tp > __z)`
This routine computes the uniform asymptotic approximations of the Hankel functions and their derivatives including a patch for the case when the order equals or nearly equals the argument. At such points, Olver's expressions have zero denominators (and numerators) resulting in numerical problems. This routine averages results from four surrounding points in the complex plane to obtain the result in such cases.
- `template<typename _Tp >`
`__gnu_cxx::__cyl_hankel_t< std::complex< _Tp >, std::complex< _Tp >, std::complex< _Tp > > __hankel_uniform_olver (std::complex< _Tp > __nu, std::complex< _Tp > __z)`

Compute approximate values for the Hankel functions of the first and second kinds using Olver's uniform asymptotic expansion to of order `nu` along with their derivatives.

- `template<typename _Tp >`
`void __hankel_uniform_outer (std::complex< _Tp > __nu, std::complex< _Tp > __z, _Tp __eps, std::complex< _Tp > &__zhat, std::complex< _Tp > &__1dnsq, std::complex< _Tp > &__num1d3, std::complex< _Tp > &__num2d3, std::complex< _Tp > &__p, std::complex< _Tp > &__p2, std::complex< _Tp > &__etm3h, std::complex< _Tp > &__etrat, std::complex< _Tp > &__Aip, std::complex< _Tp > &__o4dp, std::complex< _Tp > &__Aim, std::complex< _Tp > &__o4dm, std::complex< _Tp > &__od2p, std::complex< _Tp > &__od0dp, std::complex< _Tp > &__od2m, std::complex< _Tp > &__od0dm)`

Compute outer factors and associated functions of `z` and `nu` appearing in Olver's uniform asymptotic expansions of the Hankel functions of the first and second kinds and their derivatives. The various functions of `z` and `nu` returned by `hankel_uniform_outer` are available for use in computing further terms in the expansions.

- `template<typename _Tp >`
`void __hankel_uniform_sum (std::complex< _Tp > __p, std::complex< _Tp > __p2, std::complex< _Tp > &__num2, std::complex< _Tp > __zetam3hf, std::complex< _Tp > __Aip, std::complex< _Tp > __o4dp, std::complex< _Tp > __Aim, std::complex< _Tp > __o4dm, std::complex< _Tp > __od2p, std::complex< _Tp > __od0dp, std::complex< _Tp > __od2m, std::complex< _Tp > __od0dm, _Tp __eps, std::complex< _Tp > &__H1sum, std::complex< _Tp > &__H1psum, std::complex< _Tp > &__H2sum, std::complex< _Tp > &__H2psum)`

Compute the sums in appropriate linear combinations appearing in Olver's uniform asymptotic expansions for the Hankel functions of the first and second kinds and their derivatives, using up to `n` terms (less than 5) to achieve relative error `eps`.

- `template<typename _Tp >`
`_Tp __harmonic_number (unsigned int __n)`

- `template<typename _Tp >`
`_Tp __hermite (unsigned int __n, _Tp __x)`

This routine returns the Hermite polynomial of order n : $H_n(x)$.

- `template<typename _Tp >`
`_Tp __hermite_asymp (unsigned int __n, _Tp __x)`

This routine returns the Hermite polynomial of large order n : $H_n(x)$. We assume here that $x \geq 0$.

- `template<typename _Tp >`
`__gnu_cxx::__hermite_t< _Tp > __hermite_recur (unsigned int __n, _Tp __x)`

This routine returns the Hermite polynomial of order n : $H_n(x)$ by recursion on n .

- `template<typename _Tp >`
`std::vector< __gnu_cxx::__quadrature_point_t< _Tp > > __hermite_zeros (unsigned int __n, _Tp __proto=__Tp{})`

- `template<typename _Tp >`
`_Tp __heuman_lambda (_Tp __k, _Tp __phi)`

- `template<typename _Tp >`
`_Tp __hurwitz_zeta (_Tp __s, _Tp __a)`

Return the Hurwitz zeta function $\zeta(s, a)$ for all $s \neq 1$ and $a > -1$.

- `template<typename _Tp >`
`_Tp __hurwitz_zeta_euler_maclaurin (_Tp __s, _Tp __a)`

Return the Hurwitz zeta function $\zeta(s, a)$ for all $s \neq 1$ and $a > -1$.

- `template<typename _Tp >`
`std::complex< _Tp > __hurwitz_zeta_polylog (_Tp __s, std::complex< _Tp > __a)`

- `template<typename _Tp >`
`std::complex< _Tp > __hydrogen (unsigned int __n, unsigned int __l, unsigned int __m, _Tp __Z, _Tp __r, _Tp __theta, _Tp __phi)`

- `template<typename _Tp >`
`_Tp __hyperg (_Tp __a, _Tp __b, _Tp __c, _Tp __x)`

Return the hypergeometric function ${}_2F_1(a, b; c; x)$.

- `template<typename _Tp >`
`_Tp __hyperg_luke (_Tp __a, _Tp __b, _Tp __c, _Tp __xin)`

Return the hypergeometric function ${}_2F_1(a, b; c; x)$ by an iterative procedure described in Luke, *Algorithms for the Computation of Mathematical Functions*.

- `template<typename _Tp >`
`_Tp __hyperg_recur (int __m, _Tp __b, _Tp __c, _Tp __x)`
 Return the hypergeometric polynomial ${}_2F_1(-m, b; c; x)$ by Holm recursion.
- `template<typename _Tp >`
`_Tp __hyperg_reflect (_Tp __a, _Tp __b, _Tp __c, _Tp __x)`
 Return the hypergeometric function ${}_2F_1(a, b; c; x)$ by the reflection formulae in Abramowitz & Stegun formula 15.3.6 for $d = c - a - b$ not integral and formula 15.3.11 for $d = c - a - b$ integral. This assumes $a, b, c \neq$ negative integer.
- `template<typename _Tp >`
`_Tp __hyperg_series (_Tp __a, _Tp __b, _Tp __c, _Tp __x)`
 Return the hypergeometric function ${}_2F_1(a, b; c; x)$ by series expansion.
- `template<typename _Tp >`
`_Tp __ibeta_cont_frac (_Tp __a, _Tp __b, _Tp __x)`
- `template<typename _Tp >`
`__gnu_cxx::__jacobi_ellint_t< _Tp > __jacobi_ellint (_Tp __k, _Tp __u)`
- `template<typename _Tp >`
`__gnu_cxx::__jacobi_t< _Tp > __jacobi_recur (unsigned int __n, _Tp __alpha1, _Tp __beta1, _Tp __x)`
- `template<typename _Tp >`
`std::complex< _Tp > __jacobi_theta_1 (std::complex< _Tp > __q, std::complex< _Tp > __x)`
- `template<typename _Tp >`
`_Tp __jacobi_theta_1 (_Tp __q, const _Tp __x)`
- `template<typename _Tp >`
`_Tp __jacobi_theta_1_prod (_Tp __q, _Tp __x)`
- `template<typename _Tp >`
`_Tp __jacobi_theta_1_sum (_Tp __q, _Tp __x)`
- `template<typename _Tp >`
`std::complex< _Tp > __jacobi_theta_2 (std::complex< _Tp > __q, std::complex< _Tp > __x)`
- `template<typename _Tp >`
`_Tp __jacobi_theta_2 (_Tp __q, const _Tp __x)`
- `template<typename _Tp >`
`_Tp __jacobi_theta_2_prod (_Tp __q, _Tp __x)`
- `template<typename _Tp >`
`_Tp __jacobi_theta_2_sum (_Tp __q, _Tp __x)`
- `template<typename _Tp >`
`std::complex< _Tp > __jacobi_theta_3 (std::complex< _Tp > __q, std::complex< _Tp > __x)`
- `template<typename _Tp >`
`_Tp __jacobi_theta_3 (_Tp __q, const _Tp __x)`
- `template<typename _Tp >`
`_Tp __jacobi_theta_3_prod (_Tp __q, _Tp __x)`
- `template<typename _Tp >`
`_Tp __jacobi_theta_3_sum (_Tp __q, _Tp __x)`
- `template<typename _Tp >`
`std::complex< _Tp > __jacobi_theta_4 (std::complex< _Tp > __q, std::complex< _Tp > __x)`
- `template<typename _Tp >`
`_Tp __jacobi_theta_4 (_Tp __q, const _Tp __x)`
- `template<typename _Tp >`
`_Tp __jacobi_theta_4_prod (_Tp __q, _Tp __x)`
- `template<typename _Tp >`
`_Tp __jacobi_theta_4_sum (_Tp __q, _Tp __x)`
- `template<typename _Tp >`
`std::vector< __gnu_cxx::__quadrature_point_t< _Tp > > __jacobi_zeros (unsigned int __n, _Tp __alpha1, _Tp __beta1)`

- `template<typename _Tp >`
`_Tp __jacobi_zeta (_Tp __k, _Tp __phi)`
- `template<typename _Tp >`
`_Tp __kolmogorov_p (_Tp __a, _Tp __b, _Tp __x)`
- `template<typename _Tpa, typename _Tp >`
`_Tp __laguerre (unsigned int __n, _Tpa __alpha1, _Tp __x)`
This routine returns the associated Laguerre polynomial of degree n , order α : $L_n^{(\alpha)}(x)$.
- `template<typename _Tp >`
`_Tp __laguerre (unsigned int __n, _Tp __x)`
This routine returns the Laguerre polynomial of degree n : $L_n(x)$.
- `template<typename _Tpa, typename _Tp >`
`_Tp __laguerre_hyperg (unsigned int __n, _Tpa __alpha1, _Tp __x)`
Evaluate the polynomial based on the confluent hypergeometric function in a safe way, with no restriction on the arguments.
- `template<typename _Tpa, typename _Tp >`
`_Tp __laguerre_large_n (unsigned __n, _Tpa __alpha1, _Tp __x)`
This routine returns the associated Laguerre polynomial of degree n , order $\alpha > -1$ for large n . Abramowitz & Stegun, 13.5.21.
- `template<typename _Tpa, typename _Tp >`
`__gnu_cxx::__laguerre_t<_Tpa, _Tp> __laguerre_recur (unsigned int __n, _Tpa __alpha1, _Tp __x)`
This routine returns the associated Laguerre polynomial of degree n , order α : $L_n^{(\alpha)}(x)$ by recursion.
- `template<typename _Tp >`
`std::vector< __gnu_cxx::__quadrature_point_t<_Tp> > __laguerre_zeros (unsigned int __n, _Tp __alpha1)`
- `template<typename _Tp >`
`_Tp __lah (unsigned int __n, unsigned int __k)`
- `template<typename _Tp >`
`std::vector<_Tp> __lah (unsigned int __n)`
- `template<typename _Tp >`
`_Tp __lah_recur (unsigned int __n, unsigned int __k)`
- `template<typename _Tp >`
`std::vector<_Tp> __lah_recur (unsigned int __n)`
- `template<typename _Tp >`
`_GLIBCXX14_CONSTEXPR _Tp __lanczos_binet1p (_Tp __z)`
Return the Binet function $J(1+z)$ by the Lanczos method. The Binet function is the log of the scaled Gamma function $\log(\Gamma^(z))$ defined by*

$$J(z) = \log(\Gamma^*(z)) = \log(\Gamma(z)) + z - \left(z - \frac{1}{2}\right) \log(z) - \log(2\pi)$$
or

$$\Gamma(z) = \sqrt{2\pi} z^{z-\frac{1}{2}} e^{-z} e^{J(z)}$$
where $\Gamma(z)$ is the gamma function.
- `template<typename _Tp >`
`_GLIBCXX14_CONSTEXPR _Tp __lanczos_log_gamma1p (_Tp __z)`
Return the logarithm of the gamma function $\log(\Gamma(1+z))$ by the Lanczos method.
- `template<typename _Tp >`
`__gnu_cxx::__legendre_p_t<_Tp> __legendre_p (unsigned int __l, _Tp __x)`
Return the Legendre polynomial by upward recursion on degree l .
- `template<typename _Tp >`
`__gnu_cxx::__legendre_q_t<_Tp> __legendre_q (unsigned int __l, _Tp __x)`
Return the Legendre function of the second kind by upward recursion on degree l .
- `template<typename _Tp >`
`_Tp __legendre_q_series (unsigned int __l, _Tp __x)`

- `template<typename _Tp >`
`std::vector< __gnu_cxx::__quadrature_point_t< _Tp > > __legendre_zeros` (unsigned int __l, _Tp proto=_Tp{})
- `template<typename _Tp >`
`_Tp __log_binomial` (unsigned int __n, unsigned int __k)

Return the logarithm of the binomial coefficient. The binomial coefficient is given by:

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

The binomial coefficients are generated by:

$$(1+t)^n = \sum_{k=0}^n \binom{n}{k} t^k$$

- `template<typename _Tp >`
`_Tp __log_binomial` (_Tp __nu, unsigned int __k)

Return the logarithm of the binomial coefficient for non-integral degree. The binomial coefficient is given by:

$$\binom{\nu}{k} = \frac{\Gamma(\nu+1)}{\Gamma(\nu-k+1)\Gamma(k+1)}$$

The binomial coefficients are generated by:

$$(1+t)^\nu = \sum_{k=0}^{\infty} \binom{\nu}{k} t^k$$

- `template<typename _Tp >`
`_Tp __log_binomial_sign` (_Tp __nu, unsigned int __k)

Return the sign of the exponentiated logarithm of the binomial coefficient for non-integral degree. The binomial coefficient is given by:

$$\binom{\nu}{k} = \frac{\Gamma(\nu+1)}{\Gamma(\nu-k+1)\Gamma(k+1)}$$

The binomial coefficients are generated by:

$$(1+t)^\nu = \sum_{k=0}^{\infty} \binom{\nu}{k} t^k$$

- `template<typename _Tp >`
`std::complex< _Tp > __log_binomial_sign` (std::complex< _Tp > __nu, unsigned int __k)
- `template<typename _Tp >`
`_GLIBCXX14_CONSTEXPR _Tp __log_double_factorial` (_Tp __nu)
- `template<typename _Tp >`
`_GLIBCXX14_CONSTEXPR _Tp __log_double_factorial` (int __n)

Return the logarithm of the double factorial of the integer n.

- `template<typename _Tp >`
`_GLIBCXX14_CONSTEXPR _Tp __log_factorial` (unsigned int __n)

Return the logarithm of the factorial of the integer n.

- `template<typename _Tp >`
`_Tp __log_falling_factorial` (_Tp __a, _Tp __nu)

Return the logarithm of the falling factorial function or the lower Pochhammer symbol. The lower Pochhammer symbol is defined by

$$a^{\underline{n}} = \Gamma(a+1)/\Gamma(a-\nu+1) = \prod_{k=0}^{n-1} (a-k), (a)_0 = 1$$

In particular, $n^{\underline{n}} = n!$. Thus this function returns

$$\ln[a^{\underline{n}}] = \ln[\Gamma(a+1)] - \ln[\Gamma(a-\nu+1)], \ln[a^{\underline{0}}] = 0$$

Many notations exist for this function:

$$(a)_\nu$$

,

$$\left\{ \begin{array}{c} a \\ \nu \end{array} \right\}$$

, and others.

- `template<typename _Tp >`
`_Tp __log_gamma (_Tp __a)`
Return $\log(|\Gamma(a)|)$. This will return values even for $a < 0$. To recover the sign of $\Gamma(a)$ for any argument use `__log_gamma_sign`.
- `template<typename _Tp >`
`std::complex< _Tp > __log_gamma (std::complex< _Tp > __a)`
Return $\log(\Gamma(a))$ for complex argument.
- `template<typename _Tp >`
`_GLIBCXX14_CONSTEXPR _Tp __log_gamma_bernoulli (_Tp __x)`
Return $\log(\Gamma(x))$ by asymptotic expansion with Bernoulli number coefficients. This is like Sterling's approximation.
- `template<typename _Tp >`
`_Tp __log_gamma_sign (_Tp __a)`
Return the sign of $\Gamma(x)$. At nonpositive integers zero is returned indicating $\Gamma(x)$ is undefined.
- `template<typename _Tp >`
`std::complex< _Tp > __log_gamma_sign (std::complex< _Tp > __a)`
- `template<typename _Tp >`
`_Tp __log_rising_factorial (_Tp __a, _Tp __nu)`
Return the logarithm of the rising factorial function or the (upper) Pochhammer symbol. The Pochhammer symbol is defined for integer order by

$$a^{\overline{\nu}} = \Gamma(a + \nu) / \Gamma(a) = \prod_{k=0}^{\nu-1} (a + k), (a)_0 = 1$$

Thus this function returns

$$\ln[a^{\overline{\nu}}] = \ln[\Gamma(a + \nu)] - \ln[\Gamma(a)], \ln[(a)_0] = 0$$

Many notations exist for this function:

$$(a)_\nu$$

(especially in the literature of special functions),

$$\left[\begin{array}{c} a \\ \nu \end{array} \right]$$

, and others.

- `template<typename _Tp >`
`_Tp __log_stirling_1 (unsigned int __n, unsigned int __m)`
- `template<typename _Tp >`
`_Tp __log_stirling_1_sign (unsigned int __n, unsigned int __m)`
- `template<typename _Tp >`
`_Tp __log_stirling_2 (unsigned int __n, unsigned int __m)`
- `template<typename _Tp >`
`_Tp __logint (const _Tp __x)`
Return the logarithmic integral $li(x)$.
- `template<typename _Tp >`
`_Tp __logistic_p (_Tp __a, _Tp __b, _Tp __x)`
Return the logistic cumulative distribution function.
- `template<typename _Tp >`
`_Tp __logistic_pdf (_Tp __a, _Tp __b, _Tp __x)`
Return the logistic probability density function.

- `template<typename _Tp >`
`_Tp __lognormal_p (_Tp __mu, _Tp __sigma, _Tp __x)`
Return the lognormal cumulative probability density function.
- `template<typename _Tp >`
`_Tp __lognormal_pdf (_Tp __nu, _Tp __sigma, _Tp __x)`
Return the lognormal probability density function.
- `template<typename _Tp >`
`_Tp __normal_p (_Tp __mu, _Tp __sigma, _Tp __x)`
Return the normal cumulative probability density function.
- `template<typename _Tp >`
`_Tp __normal_pdf (_Tp __mu, _Tp __sigma, _Tp __x)`
Return the normal probability density function.
- `template<typename _Tp >`
`_Tp __owens_t (_Tp __h, _Tp __a)`
- `template<typename _Tp, typename _ArgType >`
`__gnu_cxx::fp_promote_t< std::complex< _Tp >, _ArgType > __periodic_zeta (_ArgType __z, _Tp __s)`
- `template<typename _Tp >`
`std::complex< _Tp > __polar_pi (_Tp __rho, _Tp __phi_pi)`
- `template<typename _Tp >`
`std::complex< _Tp > __polar_pi (_Tp __rho, const std::complex< _Tp > &__phi_pi)`
- `template<typename _Tp >`
`_Tp __polygamma (unsigned int __m, _Tp __x)`
Return the polygamma function $\psi^{(m)}(x)$.
- `template<typename _Tp >`
`_Tp __polylog (_Tp __s, _Tp __x)`
- `template<typename _Tp >`
`std::complex< _Tp > __polylog (_Tp __s, std::complex< _Tp > __w)`
- `template<typename _Tp, typename _ArgType >`
`__gnu_cxx::fp_promote_t< std::complex< _Tp >, _ArgType > __polylog_exp (_Tp __s, _ArgType __w)`
- `template<typename _Tp >`
`std::complex< _Tp > __polylog_exp_asymp (_Tp __s, std::complex< _Tp > __w)`
- `template<typename _Tp >`
`std::complex< _Tp > __polylog_exp_neg (_Tp __s, std::complex< _Tp > __w)`
- `template<typename _Tp >`
`std::complex< _Tp > __polylog_exp_neg (int __n, std::complex< _Tp > __w)`
- `template<typename _Tp >`
`std::complex< _Tp > __polylog_exp_neg_int (int __s, std::complex< _Tp > __w)`
- `template<typename _Tp >`
`std::complex< _Tp > __polylog_exp_neg_int (int __s, _Tp __w)`
- `template<typename _Tp >`
`std::complex< _Tp > __polylog_exp_neg_real (_Tp __s, std::complex< _Tp > __w)`
- `template<typename _Tp >`
`std::complex< _Tp > __polylog_exp_neg_real (_Tp __s, _Tp __w)`
- `template<typename _Tp >`
`std::complex< _Tp > __polylog_exp_pos (unsigned int __s, std::complex< _Tp > __w)`
- `template<typename _Tp >`
`std::complex< _Tp > __polylog_exp_pos (unsigned int __s, _Tp __w)`
- `template<typename _Tp >`
`std::complex< _Tp > __polylog_exp_pos (_Tp __s, std::complex< _Tp > __w)`
- `template<typename _Tp >`
`std::complex< _Tp > __polylog_exp_pos_int (unsigned int __s, std::complex< _Tp > __w)`

- `template<typename _Tp >`
`std::complex< _Tp > __polylog_exp_pos_int (unsigned int __s, _Tp __w)`
- `template<typename _Tp >`
`std::complex< _Tp > __polylog_exp_pos_real (_Tp __s, std::complex< _Tp > __w)`
- `template<typename _Tp >`
`std::complex< _Tp > __polylog_exp_pos_real (_Tp __s, _Tp __w)`
- `template<typename _PowTp, typename _Tp >`
`_Tp __polylog_exp_sum (_PowTp __s, _Tp __w)`
- `template<typename _Tp >`
`__gnu_cxx::__hermite_he_t< _Tp > __prob_hermite_recur (unsigned int __n, _Tp __x)`
This routine returns the Probabilists Hermite polynomial of order n: $He_n(x)$ by recursion on n.
- `template<typename _Tp >`
`_Tp __radial_jacobi (unsigned int __n, unsigned int __m, _Tp __rho)`
- `template<typename _Tp >`
`std::vector< __gnu_cxx::__quadrature_point_t< _Tp > > __radial_jacobi_zeros (unsigned int __n, unsigned int __m)`
- `template<typename _Tp >`
`_Tp __rice_pdf (_Tp __nu, _Tp __sigma, _Tp __x)`
Return the Rice probability density function.
- `template<typename _Tp >`
`_Tp __riemann_zeta (_Tp __s)`
Return the Riemann zeta function $\zeta(s)$.
- `template<typename _Tp >`
`_Tp __riemann_zeta_euler_maclaurin (_Tp __s)`
Evaluate the Riemann zeta function $\zeta(s)$ by an alternate series for $s > 0$.
- `template<typename _Tp >`
`_Tp __riemann_zeta_glob (_Tp __s)`
- `template<typename _Tp >`
`_Tp __riemann_zeta_laurent (_Tp __s)`
Compute the Riemann zeta function $\zeta(s)$ by Laurent expansion about $s = 1$.
- `template<typename _Tp >`
`_Tp __riemann_zeta_m_1 (_Tp __s)`
Return the Riemann zeta function $\zeta(s) - 1$.
- `template<typename _Tp >`
`_Tp __riemann_zeta_m_1_glob (_Tp __s)`
Evaluate the Riemann zeta function by series for all $s \neq 1$. Convergence is great until largish negative numbers. Then the convergence of the > 0 sum gets better.
- `template<typename _Tp >`
`_Tp __riemann_zeta_product (_Tp __s)`
Compute the Riemann zeta function $\zeta(s)$ using the product over prime factors.
- `template<typename _Tp >`
`_Tp __riemann_zeta_sum (_Tp __s)`
Compute the Riemann zeta function $\zeta(s)$ by summation for $s > 1$.
- `template<typename _Tp >`
`_Tp __rising_factorial (_Tp __a, int __n)`
Return the (upper) Pochhammer function or the rising factorial function. The Pochhammer symbol is defined by

$$a^{\overline{n}} = \Gamma(a + \nu) / \Gamma(\nu) = \prod_{k=0}^{n-1} (a + k), (a)_0 = 1$$

Many notations exist for this function:

$$(a)_{\nu}$$

, (especially in the literature of special functions),

$$\begin{bmatrix} a \\ n \end{bmatrix}$$

, and others.

- `template<typename _Tp >`
`_Tp __rising_factorial (_Tp __a, _Tp __nu)`

Return the rising factorial function or the (upper) Pochhammer function. The rising factorial function is defined by

$$a^{\overline{\nu}} = \Gamma(a + \nu) / \Gamma(\nu)$$

Many notations exist for this function:

$$(a)_{\nu}$$

, (especially in the literature of special functions),

$$\begin{bmatrix} a \\ n \end{bmatrix}$$

, and others.

- `template<typename _Tp >`
`_Tp __sin_pi (_Tp __x)`
- `template<typename _Tp >`
`std::complex< _Tp > __sin_pi (std::complex< _Tp > __z)`
- `template<typename _Tp >`
`_gnu_cxx::fp_promote_t< _Tp > __sinc (_Tp __x)`

Return the sinus cardinal function

$$\text{sinc}(x) = \frac{\sin(x)}{x}$$

- `template<typename _Tp >`
`_gnu_cxx::fp_promote_t< _Tp > __sinc_pi (_Tp __x)`

Return the reperiodized sinus cardinal function

$$\text{sinc}_{\pi}(x) = \frac{\sin(\pi x)}{\pi x}$$

- `template<typename _Tp >`
`_gnu_cxx::__sincos_t< _Tp > __sincos (_Tp __x)`
- `template<>`
`_gnu_cxx::__sincos_t< float > __sincos (float __x)`
- `template<>`
`_gnu_cxx::__sincos_t< double > __sincos (double __x)`
- `template<>`
`_gnu_cxx::__sincos_t< long double > __sincos (long double __x)`
- `template<typename _Tp >`
`_gnu_cxx::__sincos_t< _Tp > __sincos_pi (_Tp __x)`
- `template<typename _Tp >`
`std::pair< _Tp, _Tp > __sincosint (_Tp __x)`

This function returns the sine $Si(x)$ and cosine $Ci(x)$ integrals as a pair.

- `template<typename _Tp >`
`void __sincosint_asymp (_Tp __t, _Tp &_Si, _Tp &_Ci)`

This function computes the sine $Si(x)$ and cosine $Ci(x)$ integrals by asymptotic series summation for positive argument.

- `template<typename _Tp >`
`void __sincosint_cont_frac (_Tp __t, _Tp &_Si, _Tp &_Ci)`

This function computes the sine $Si(x)$ and cosine $Ci(x)$ integrals by continued fraction for positive argument.

- `template<typename _Tp >`
`void __sincosint_series (_Tp __t, _Tp &_Si, _Tp &_Ci)`

This function computes the sine $Si(x)$ and cosine $Ci(x)$ integrals by series summation for positive argument.

- `template<typename _Tp >`
`_Tp __sinh_pi (_Tp __x)`
- `template<typename _Tp >`
`std::complex< _Tp > __sinh_pi (std::complex< _Tp > __z)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __sinhc (_Tp __x)`

Return the hyperbolic sinus cardinal function

$$\operatorname{sinhc}(x) = \frac{\sinh(x)}{x}$$

- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __sinhc_pi (_Tp __x)`

Return the reperiodized hyperbolic sinus cardinal function

$$\operatorname{sinhc}_\pi(x) = \frac{\sinh(\pi x)}{\pi x}$$

- `template<typename _Tp >`
`_Tp __sinhint (const _Tp __x)`

Return the hyperbolic sine integral $Shi(x)$.

- `template<typename _Tp >`
`_Tp __sph_bessel (unsigned int __n, _Tp __x)`

Return the spherical Bessel function $j_n(x)$ of order n and non-negative real argument x .

- `template<typename _Tp >`
`std::complex< _Tp > __sph_bessel (unsigned int __n, std::complex< _Tp > __z)`

Return the complex spherical Bessel function.

- `template<typename _Tp >`
`__gnu_cxx::__sph_mod_bessel_t< unsigned int, _Tp, _Tp > __sph_bessel_ik (unsigned int __n, _Tp __x)`

Compute the spherical modified Bessel functions $i_n(x)$ and $k_n(x)$ and their first derivatives $i'_n(x)$ and $k'_n(x)$ respectively.

- `template<typename _Tp >`
`__gnu_cxx::__sph_bessel_t< unsigned int, _Tp, _Tp > __sph_bessel_jn (unsigned int __n, _Tp __x)`

Compute the spherical Bessel $j_n(x)$ and Neumann $n_n(x)$ functions and their first derivatives $j'_n(x)$ and $n'_n(x)$ respectively.

- `template<typename _Tp >`
`__gnu_cxx::__sph_bessel_t< unsigned int, _Tp, std::complex< _Tp > > __sph_bessel_jn_neg_arg (unsigned int __n, _Tp __x)`

- `template<typename _Tp >`
`__gnu_cxx::__sph_hankel_t< unsigned int, std::complex< _Tp >, std::complex< _Tp > > __sph_hankel (unsigned int __n, std::complex< _Tp > __z)`

Helper to compute complex spherical Hankel functions and their derivatives.

- `template<typename _Tp >`
`std::complex< _Tp > __sph_hankel_1 (unsigned int __n, _Tp __x)`

Return the spherical Hankel function of the first kind $h_n^{(1)}(x)$.

- `template<typename _Tp >`
`std::complex< _Tp > __sph_hankel_1 (unsigned int __n, std::complex< _Tp > __z)`

Return the complex spherical Hankel function of the first kind.

- `template<typename _Tp >`
`std::complex< _Tp > __sph_hankel_2 (unsigned int __n, _Tp __x)`

Return the spherical Hankel function of the second kind $h_n^{(2)}(x)$.

- `template<typename _Tp >`
`std::complex< _Tp > __sph_hankel_2 (unsigned int __n, std::complex< _Tp > __z)`

Return the complex spherical Hankel function of the second kind.

- `template<typename _Tp >`
`std::complex< _Tp > __sph_harmonic` (unsigned int __l, int __m, _Tp __theta, _Tp __phi)

Return the spherical harmonic function.

- `template<typename _Tp >`
`_Tp __sph_legendre` (unsigned int __l, unsigned int __m, _Tp __theta)

Return the spherical associated Legendre function.

- `template<typename _Tp >`
`_Tp __sph_neumann` (unsigned int __n, _Tp __x)

Return the spherical Neumann function $n_n(x)$ of order n and non-negative real argument x .

- `template<typename _Tp >`
`std::complex< _Tp > __sph_neumann` (unsigned int __n, std::complex< _Tp > __z)

Return the complex spherical Neumann function.

- `template<typename _Tp >`
`_GLIBCXX14_CONSTEXPR _Tp __spouge_binet1p` (_Tp __z)

Return the Binet function $J(1+z)$ by the Spouge method. The Binet function is the log of the scaled Gamma function $\log(\Gamma^(z))$ defined by*

$$J(z) = \log(\Gamma^*(z)) = \log(\Gamma(z)) + z - \left(z - \frac{1}{2}\right) \log(z) - \log(2\pi)$$

or

$$\Gamma(z) = \sqrt{2\pi} z^{z-\frac{1}{2}} e^{-z} e^{J(z)}$$

where $\Gamma(z)$ is the gamma function.

- `template<typename _Tp >`
`_GLIBCXX14_CONSTEXPR _Tp __spouge_log_gamma1p` (_Tp __z)

Return the logarithm of the gamma function $\log(\Gamma(1+z))$ by the Spouge algorithm:

$$\Gamma(z+1) = (z+a)^{z+1/2} e^{-z-a} \left[\sqrt{2\pi} + \sum_{k=1}^{\lceil a \rceil + 1} \frac{c_k(a)}{z+k} \right]$$

where

$$c_k(a) = \frac{(-1)^{k-1}}{(k-1)!} (a-k)^{k-1/2} e^{a-k}$$

and the error is bounded by

$$\epsilon(a) < a^{-1/2} (2\pi)^{-a-1/2}$$

- `template<typename _Tp >`
`_Tp __stirling_1` (unsigned int __n, unsigned int __m)
- `template<typename _Tp >`
`std::vector< _Tp > __stirling_1` (unsigned int __n)
- `template<typename _Tp >`
`_Tp __stirling_1_recur` (unsigned int __n, unsigned int __m)
- `template<typename _Tp >`
`std::vector< _Tp > __stirling_1_recur` (unsigned int __n)
- `template<typename _Tp >`
`_Tp __stirling_2` (unsigned int __n, unsigned int __m)
- `template<typename _Tp >`
`std::vector< _Tp > __stirling_2` (unsigned int __n)
- `template<typename _Tp >`
`_Tp __stirling_2_recur` (unsigned int __n, unsigned int __m)
- `template<typename _Tp >`
`std::vector< _Tp > __stirling_2_recur` (unsigned int __n)

- `template<typename _Tp >`
`_Tp __stirling_2_series` (unsigned int __n, unsigned int __m)
- `template<typename _Tp >`
`_Tp __student_t_p` (_Tp __t, unsigned int __nu)
Return the Students T probability function.
- `template<typename _Tp >`
`_Tp __student_t_pdf` (_Tp __t, unsigned int __nu)
Return the Students T probability density.
- `template<typename _Tp >`
`_Tp __student_t_q` (_Tp __t, unsigned int __nu)
Return the complement of the Students T probability function.
- `template<typename _Tp >`
`_Tp __tan_pi` (_Tp __x)
- `template<typename _Tp >`
`std::complex< _Tp > __tan_pi` (std::complex< _Tp > __z)
- `template<typename _Tp >`
`_Tp __tanh_pi` (_Tp __x)
- `template<typename _Tp >`
`std::complex< _Tp > __tanh_pi` (std::complex< _Tp > __z)
- `template<typename _Tp >`
`_Tp __tgamma` (_Tp __a, _Tp __x)
Return the upper incomplete gamma function. The lower incomplete gamma function is defined by

$$\Gamma(a, x) = \int_x^{\infty} e^{-t} t^{a-1} dt (a > 0)$$

- `template<typename _Tp >`
`_Tp __tgamma_lower` (_Tp __a, _Tp __x)
Return the lower incomplete gamma function. The lower incomplete gamma function is defined by

$$\gamma(a, x) = \int_0^x e^{-t} t^{a-1} dt (a > 0)$$

- `template<typename _Tp >`
`_Tp __theta_1` (_Tp __nu, _Tp __x)
- `template<typename _Tp >`
`_Tp __theta_2` (_Tp __nu, _Tp __x)
- `template<typename _Tp >`
`_Tp __theta_2_asymp` (_Tp __nu, _Tp __x)
- `template<typename _Tp >`
`_Tp __theta_2_sum` (_Tp __nu, _Tp __x)
- `template<typename _Tp >`
`_Tp __theta_3` (_Tp __nu, _Tp __x)
- `template<typename _Tp >`
`_Tp __theta_3_asymp` (_Tp __nu, _Tp __x)
- `template<typename _Tp >`
`_Tp __theta_3_sum` (_Tp __nu, _Tp __x)
- `template<typename _Tp >`
`_Tp __theta_4` (_Tp __nu, _Tp __x)
- `template<typename _Tp >`
`_Tp __theta_c` (_Tp __k, _Tp __x)
- `template<typename _Tp >`
`_Tp __theta_d` (_Tp __k, _Tp __x)

- `template<typename _Tp >`
`_Tp __theta_n (_Tp __k, _Tp __x)`
- `template<typename _Tp >`
`_Tp __theta_s (_Tp __k, _Tp __x)`
- `template<typename _Tp >`
`_Tp __tricomi_u (_Tp __a, _Tp __c, _Tp __x)`

Return the Tricomi confluent hypergeometric function

$$U(a, c, x) = \frac{\Gamma(1-c)}{\Gamma(a-c+1)} {}_1F_1(a; c; x) + \frac{\Gamma(c-1)}{\Gamma(a)} x^{1-c} {}_1F_1(a-c+1; 2-c; x)$$

- `template<typename _Tp >`
`_Tp __tricomi_u_naive (_Tp __a, _Tp __c, _Tp __x)`

Return the Tricomi confluent hypergeometric function

$$U(a, c, x) = \frac{\Gamma(1-c)}{\Gamma(a-c+1)} {}_1F_1(a; c; x) + \frac{\Gamma(c-1)}{\Gamma(a)} x^{1-c} {}_1F_1(a-c+1; 2-c; x)$$

- `template<typename _Tp >`
`_Tp __weibull_p (_Tp __a, _Tp __b, _Tp __x)`
Return the Weibull cumulative probability density function.
- `template<typename _Tp >`
`_Tp __weibull_pdf (_Tp __a, _Tp __b, _Tp __x)`
Return the Weibull probability density function.
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t<_Tp> __zernike (unsigned int __n, int __m, _Tp __rho, _Tp __phi)`
- `template<typename _Tp >`
`_Tp __znorm1 (_Tp __x)`
- `template<typename _Tp >`
`_Tp __znorm2 (_Tp __x)`

Variables

- `template<typename _Tp >`
`constexpr int __max_FGH = _Airy_series<_Tp>::_N_FGH`
- `template<>`
`constexpr int __max_FGH< double > = 79`
- `template<>`
`constexpr int __max_FGH< float > = 15`
- `constexpr size_t _Num_Euler_Maclaurin_zeta = 100`
- `constexpr size_t _Num_Stieltjes = 21`
- `constexpr _Factorial_table< long double > _S_double_factorial_table [301]`
- `constexpr long double _S_Euler_Maclaurin_zeta [_Num_Euler_Maclaurin_zeta]`
- `constexpr _Factorial_table< long double > _S_factorial_table [171]`
- `constexpr unsigned long long _S_harmonic_denom [_S_num_harmonic_numer]`
- `constexpr unsigned long long _S_harmonic_numer [_S_num_harmonic_numer]`
- `constexpr _Factorial_table< long double > _S_neg_double_factorial_table [999]`
- `template<typename _Tp >`
`constexpr std::size_t _S_num_double_factorials = 0`
- `template<>`
`constexpr std::size_t _S_num_double_factorials< double > = 301`

- `template<>`
`constexpr std::size_t _S_num_double_factorials< float > = 57`
- `template<>`
`constexpr std::size_t _S_num_double_factorials< long double > = 301`
- `template<typename _Tp >`
`constexpr std::size_t _S_num_factorials = 0`
- `template<>`
`constexpr std::size_t _S_num_factorials< double > = 171`
- `template<>`
`constexpr std::size_t _S_num_factorials< float > = 35`
- `template<>`
`constexpr std::size_t _S_num_factorials< long double > = 171`
- `constexpr unsigned long long _S_num_harmonic_numer = 29`
- `template<typename _Tp >`
`constexpr std::size_t _S_num_neg_double_factorials = 0`
- `template<>`
`constexpr std::size_t _S_num_neg_double_factorials< double > = 150`
- `template<>`
`constexpr std::size_t _S_num_neg_double_factorials< float > = 27`
- `template<>`
`constexpr std::size_t _S_num_neg_double_factorials< long double > = 999`
- `constexpr size_t _S_num_zetam1 = 121`
- `constexpr long double _S_Stieltjes [_Num_Stieltjes]`
- `constexpr long double _S_zetam1 [_S_num_zetam1]`

8.3.1 Detailed Description

Implementation-space details.

8.3.2 Function Documentation

8.3.2.1 __airy()

```
template<typename _Tp >
__gnu_cxx::__airy_t<_Tp, _Tp> std::__detail::__airy (
    _Tp __z )
```

Compute the Airy functions $Ai(x)$ and $Bi(x)$ and their first derivatives $Ai'(x)$ and $Bi'(x)$ respectively.

Parameters

<code>__z</code>	The argument of the Airy functions.
------------------	-------------------------------------

Returns

A struct containing the Airy functions of the first and second kinds and their derivatives.

Definition at line 475 of file sf_mod_bessel.tcc.

References `__cyl_bessel_ik()`, and `__cyl_bessel_jn()`.

Referenced by `__airy_ai()`, `__airy_bi()`, `__fock_airy()`, and `__hermite_asymp()`.

8.3.2.2 __airy_ai()

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__airy_ai (
    std::complex< _Tp > __z )
```

Return the complex Airy Ai function.

Definition at line 2629 of file sf_airy.tcc.

References `__airy()`.

8.3.2.3 __airy_arg()

```
template<typename _Tp >
void std::__detail::__airy_arg (
    std::complex< _Tp > __num2d3,
    std::complex< _Tp > __zeta,
    std::complex< _Tp > & __argp,
    std::complex< _Tp > & __argm )
```

Compute the arguments for the Airy function evaluations carefully to prevent premature overflow. Note that the major work here is in `safe_div`. A faster, but less safe implementation can be obtained without use of `safe_div`.

Parameters

in	<code>__num2d3</code>	$\nu^{-2/3}$ - output from <code>hankel_params</code>
in	<code>__zeta</code>	zeta in the uniform asymptotic expansions - output from <code>hankel_params</code>
out	<code>__argp</code>	$e^{+i2\pi/3}\nu^{2/3}\zeta$
out	<code>__argm</code>	$e^{-i2\pi/3}\nu^{2/3}\zeta$

Exceptions

<code>std::runtime_error</code>	if unable to compute Airy function arguments
---------------------------------	--

Definition at line 214 of file sf_hankel.tcc.

Referenced by __hankel_uniform_outer().

8.3.2.4 __airy_bi()

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__airy_bi (
    std::complex< _Tp > __z )
```

Return the complex Airy Bi function.

Definition at line 2641 of file sf_airy.tcc.

References __airy().

8.3.2.5 __assoc_laguerre()

```
template<typename _Tpa , typename _Tp >
_Tp std::__detail::__assoc_laguerre (
    unsigned int __n,
    _Tpa __alpha,
    _Tp __x )
```

This routine returns the associated Laguerre polynomial of degree n, order m: $L_n^{(m)}(x)$.

The associated Laguerre polynomial is defined for integral order $\alpha = m$ by:

$$L_n^{(m)}(x) = (-1)^m \frac{d^m}{dx^m} L_{n+m}(x)$$

where the Laguerre polynomial is defined by:

$$L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n} (x^n e^{-x})$$

Template Parameters

<code>_Tpa</code>	The type of the order.
<code>_Tp</code>	The type of the parameter

Parameters

<code>__n</code>	The degree
<code>__alpha</code>	The order

Parameters

<code>__x</code>	The argument
------------------	--------------

Returns

The value of the associated Laguerre polynomial of order n , degree m , and argument x .

Definition at line 367 of file `sf_laguerre.tcc`.

Referenced by `__hydrogen()`.

8.3.2.6 `__assoc_legendre_p()`

```
template<typename _Tp >
__gnu_cxx::__assoc_legendre_p_t<_Tp> std::__detail::__assoc_legendre_p (
    unsigned int __l,
    unsigned int __m,
    _Tp __x,
    _Tp __phase = _Tp{+1} )
```

Return the associated Legendre function by recursion on l and downward recursion on m .

The associated Legendre function is derived from the Legendre function $P_l(x)$ by the Rodrigues formula:

$$P_l^m(x) = (1 - x^2)^{m/2} \frac{d^m}{dx^m} P_l(x)$$

Note

The Condon-Shortley phase factor $(-1)^m$ is absent by default.

$P_l^m(x) = 0$ if $m > l$.

Parameters

<code>__l</code>	The degree of the associated Legendre function. $l \geq 0$.
<code>__m</code>	The order of the associated Legendre function.
<code>__x</code>	The argument of the associated Legendre function.
<code>__phase</code>	The phase of the associated Legendre function. Use -1 for the Condon-Shortley phase convention.

Definition at line 241 of file `sf_legendre.tcc`.

References `__legendre_p()`.

8.3.2.7 __assoc_legendre_q()

```
template<typename _Tp >
__gnu_cxx::__assoc_legendre_q_t<_Tp> std::__detail::__assoc_legendre_q (
    unsigned int __l,
    unsigned int __m,
    _Tp __x,
    _Tp __phase = _Tp{+1} )
```

Definition at line 299 of file sf_legendre.tcc.

8.3.2.8 __bell() [1/2]

```
template<typename _Tp >
std::vector<_Tp> std::__detail::__bell (
    unsigned int __n ) [inline]
```

Return a vector of the Bell numbers.

Definition at line 509 of file sf_stirling.tcc.

Referenced by __bell(), __bell_series(), and __gnu_cxx::bell().

8.3.2.9 __bell() [2/2]

```
template<typename _Tp , typename _Up >
_Up std::__detail::__bell (
    unsigned int __n,
    _Up __x ) [inline]
```

Evaluate the Bell polynomial

$$B(n) = \sum_{k=0}^n S_n^{(k)} x^k$$

where $S_n^{(k)}$ are the Stirling numbers of the second kind.

Definition at line 521 of file sf_stirling.tcc.

References __bell().

8.3.2.10 `__bell_series()`

```
template<typename _Tp >
std::vector<_Tp> std::__detail::__bell_series (
    unsigned int __n )
```

Return a vector of the Bell numbers by summation.

$$B(n) = \sum_{k=0}^n S_n^{(k)} = \sum_{k=1}^n \binom{n-1}{k-1} B(n-k)$$

where $S_n^{(k)}$ are the Stirling numbers of the second kind.

Todo Test for blowup in Bell number summation.

Definition at line 490 of file `sf_stirling.tcc`.

References `__bell()`.

8.3.2.11 `__bernoulli()` [1/2]

```
template<typename _Tp >
_GLIBCXX14_CONSTEXPR _Tp std::__detail::__bernoulli (
    unsigned int __n )
```

This returns Bernoulli number B_n .

Parameters

<code>__n</code>	the order n of the Bernoulli number.
------------------	--------------------------------------

Returns

The Bernoulli number of order n.

Definition at line 128 of file `sf_bernoulli.tcc`.

Referenced by `__euler()`, and `__gnu_cxx::bernoulli()`.

8.3.2.12 __bernoulli() [2/2]

```
template<typename _Tp >
_Tp std::__detail::__bernoulli (
    unsigned int __n,
    _Tp __x )
```

Return the Bernoulli polynomial $B_n(x)$ of order n at argument x .

The values at 0 and 1 are equal to the corresponding Bernoulli number:

$$B_n(0) = B_n(1) = B_n$$

The derivative is proportional to the previous polynomial:

$$B'_n(x) = n * B_{n-1}(x)$$

The series expansion is:

$$B_n(x) = \sum_{k=0}^n B_k \binom{n}{k} x^{n-k}$$

A useful argument promotion is:

$$B_n(x+1) - B_n(x) = n * x^{n-1}$$

Definition at line 168 of file sf_bernoulli.tcc.

References `__binomial()`.

8.3.2.13 __bernoulli_2n()

```
template<typename _Tp >
_GLIBCXX14_CONSTEXPR _Tp std::__detail::__bernoulli_2n (
    unsigned int __n )
```

This returns Bernoulli number B_{2n} at even integer arguments $2n$.

Parameters

<code>__n</code>	the half-order n of the Bernoulli number.
------------------	---

Returns

The Bernoulli number of order $2n$.

Definition at line 140 of file sf_bernoulli.tcc.

8.3.2.14 __bernoulli_series()

```
template<typename _Tp >
_GLIBCXX14_CONSTEXPR _Tp std::__detail::__bernoulli_series (
    unsigned int __n )
```

This returns Bernoulli numbers from a table or by summation for larger values.

$$B_{2n} = (-1)^{n+1} 2 \frac{(2n)!}{(2\pi)^{2n}} \zeta(2n)$$

.

Note that

$$\zeta(2n) - 1 = (-1)^{n+1} \frac{(2\pi)^{2n}}{(2n)!} B_{2n} - 2$$

are small and rapidly decreasing functions of n.

Parameters

\longleftrightarrow	the order n of the Bernoulli number.
$_n$	

Returns

The Bernoulli number of order n.

Definition at line 65 of file sf_bernoulli.tcc.

8.3.2.15 __beta()

```
template<typename _Tp >
_Tp std::__detail::__beta (
    _Tp __a,
    _Tp __b )
```

Return the beta function $B(a, b)$.

The beta function is defined by

$$B(a, b) = \int_0^1 t^{a-1} (1-t)^{b-1} dt = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

Parameters

\leftrightarrow _a	The first argument of the beta function.
\leftrightarrow _b	The second argument of the beta function.

Returns

The beta function.

Definition at line 215 of file sf_beta.tcc.

References `__beta_gamma()`, and `__beta_lgamma()`.

Referenced by `__fisher_f_pdf()`, `__gnu_cxx::gamma_pdf()`, `__gnu_cxx::jacobi()`, `__gnu_cxx::jacobif()`, `__gnu_cxx::jacobil()`, `__gnu_cxx::mittag_leffler()`, and `std::__detail::Airy<_Tp>::operator()()`.

8.3.2.16 __beta_gamma()

```
template<typename _Tp >
_Tp std::__detail::__beta_gamma (
    _Tp __a,
    _Tp __b )
```

Return the beta function: $B(a, b)$.

The beta function is defined by

$$B(a, b) = \int_0^1 t^{a-1} (1-t)^{b-1} dt = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

Parameters

\leftrightarrow _a	The first argument of the beta function.
\leftrightarrow _b	The second argument of the beta function.

Returns

The beta function.

Definition at line 77 of file sf_beta.tcc.

References `__gamma()`.

Referenced by `__beta()`.

8.3.2.17 `__beta_inc()`

```
template<typename _Tp >
_Tp std::__detail::__beta_inc (
    _Tp __a,
    _Tp __b,
    _Tp __x )
```

Return the regularized incomplete beta function, $I_x(a, b)$, of arguments a , b , and x .

The regularized incomplete beta function is defined by:

$$I_x(a, b) = \frac{B_x(a, b)}{B(a, b)}$$

where

$$B_x(a, b) = \int_0^x t^{a-1}(1-t)^{b-1} dt$$

is the non-regularized beta function and $B(a, b)$ is the usual beta function.

Parameters

<code>__a</code>	The first parameter
<code>__b</code>	The second parameter
<code>__x</code>	The argument

Definition at line 311 of file `sf_beta.tcc`.

References `__ibeta_cont_frac()`, `__log_gamma()`, and `__log_gamma_sign()`.

Referenced by `__beta_p()`, `__binomial_p()`, `__binomial_q()`, `__fisher_f_p()`, `__fisher_f_q()`, `__student_t_p()`, and `__student_t_q()`.

8.3.2.18 `__beta_lgamma()`

```
template<typename _Tp >
_Tp std::__detail::__beta_lgamma (
    _Tp __a,
    _Tp __b )
```

Return the beta function $B(a, b)$ using the log gamma functions.

The beta function is defined by

$$B(a, b) = \int_0^1 t^{a-1}(1-t)^{b-1} dt = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

Parameters

\leftrightarrow _a	The first argument of the beta function.
\leftrightarrow _b	The second argument of the beta function.

Returns

The beta function.

Definition at line 125 of file sf_beta.tcc.

References `__log_gamma()`, and `__log_gamma_sign()`.

Referenced by `__beta()`.

8.3.2.19 __beta_p()

```
template<typename _Tp >
_Tp std::__detail::__beta_p (
    _Tp __a,
    _Tp __b,
    _Tp __x )
```

Definition at line 705 of file sf_distributions.tcc.

References `__beta_inc()`.

8.3.2.20 __beta_product()

```
template<typename _Tp >
_Tp std::__detail::__beta_product (
    _Tp __a,
    _Tp __b )
```

Return the beta function $B(x, y)$ using the product form.

The beta function is defined by

$$B(a, b) = \int_0^1 t^{a-1} (1-t)^{b-1} dt = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

Here, we employ the product form:

$$B(a, b) = \frac{a+b}{ab} \prod_{k=1}^{\infty} \frac{1 + (a+b)/k}{(1+a/k)(1+b/k)} = \frac{a+b}{ab} \prod_{k=1}^{\infty} \left[1 - \frac{ab}{(a+k)(b+k)} \right]$$

Parameters

\leftrightarrow _a	The first argument of the beta function.
\leftrightarrow _b	The second argument of the beta function.

Returns

The beta function.

Definition at line 179 of file sf_beta.tcc.

8.3.2.21 __binomial() [1/2]

```
template<typename _Tp >
_Tp std::__detail::__binomial (
    unsigned int __n,
    unsigned int __k )
```

Return the binomial coefficient. The binomial coefficient is given by:

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

The binomial coefficients are generated by:

$$(1+t)^n = \sum_{k=0}^n \binom{n}{k} t^k$$

.

Parameters

\leftrightarrow _n	The first argument of the binomial coefficient.
\leftrightarrow _k	The second argument of the binomial coefficient.

Returns

The binomial coefficient.

Definition at line 2540 of file sf_gamma.tcc.

References std::__detail::_Factorial_table<_Tp>::__n.

Referenced by __bernoulli().

8.3.2.22 __binomial() [2/2]

```
template<typename _Tp >
_Tp std::__detail::__binomial (
    _Tp __nu,
    unsigned int __k )
```

Return the binomial coefficient for non-integral degree. The binomial coefficient is given by:

$$\binom{\nu}{k} = \frac{\Gamma(\nu + 1)}{\Gamma(\nu - k + 1)\Gamma(k + 1)}$$

The binomial coefficients are generated by:

$$(1 + t)^\nu = \sum_{k=0}^{\infty} \binom{\nu}{k} t^k$$

Parameters

<code>__nu</code>	The real first argument of the binomial coefficient.
<code>__k</code>	The second argument of the binomial coefficient.

Returns

The binomial coefficient.

Definition at line 2600 of file sf_gamma.tcc.

References `__gamma()`, `__log_binomial()`, `__log_binomial_sign()`, and `std::__detail::_Factorial_table<_Tp>::__n`.

8.3.2.23 __binomial_p()

```
template<typename _Tp >
_Tp std::__detail::__binomial_p (
    _Tp __p,
    unsigned int __n,
    unsigned int __k )
```

Return the binomial cumulative distribution function.

The binomial cumulative distribution function is related to the incomplete beta function:

$$P(k|n, p) = I_p(k, n - k + 1)$$

Parameters

\leftrightarrow _p	
\leftrightarrow _n	
\leftrightarrow _k	

Definition at line 614 of file sf_distributions.tcc.

References `__beta_inc()`.

8.3.2.24 `__binomial_pdf()`

```
template<typename _Tp >
_Tp std::__detail::__binomial_pdf (
    _Tp __p,
    unsigned int __n,
    unsigned int __k )
```

Return the binomial probability mass function.

The binomial cumulative distribution function is related to the incomplete beta function:

$$f(k|n, p) = \binom{n}{k} p^k (1 - p)^{n-k}$$

Parameters

\leftrightarrow _p	
\leftrightarrow _n	
\leftrightarrow _k	

Definition at line 578 of file sf_distributions.tcc.

8.3.2.25 `__binomial_q()`

```
template<typename _Tp >
_Tp std::__detail::__binomial_q (
```

```

_Tp __p,
unsigned int __n,
unsigned int __k )

```

Return the complementary binomial cumulative distribution function.

The binomial cumulative distribution function is related to the incomplete beta function:

$$Q(k|n, p) = I_{1-p}(n - k + 1, k)$$

Parameters

\leftrightarrow __p	
\leftrightarrow __n	
\leftrightarrow __k	

Definition at line 644 of file sf_distributions.tcc.

References `__beta_inc()`.

8.3.2.26 __bose_einstein()

```

template<typename _Sp , typename _Tp >
_Tp std::__detail::__bose_einstein (
    _Sp __s,
    _Tp __x )

```

Return the Bose-Einstein integral of integer or real order s and real argument x.

See also

https://en.wikipedia.org/wiki/Clausen_function
<http://dlmf.nist.gov/25.12.16>

$$G_s(x) = \frac{1}{\Gamma(s+1)} \int_0^\infty \frac{t^s}{e^{t-x} - 1} dt = Li_{s+1}(e^x)$$

Parameters

\leftrightarrow __s	The order s >= 0.
\leftrightarrow __x	The real argument.

Returns

The real Bose-Einstein integral $G_s(x)$,

Definition at line 1493 of file sf_polylog.tcc.

References `__polylog_exp()`.

8.3.2.27 `__cauchy_p()`

```
template<typename _Tp >
_Tp std::__detail::__cauchy_p (
    _Tp __a,
    _Tp __b,
    _Tp __x )
```

Definition at line 697 of file sf_distributions.tcc.

8.3.2.28 `__chebyshev_recur()`

```
template<typename _Tp >
std::tuple<_Tp, _Tp, _Tp> std::__detail::__chebyshev_recur (
    unsigned int __n,
    _Tp __x,
    _Tp __C0,
    _Tp __C1 )
```

Return a Chebyshev polynomial of non-negative order n and real argument x by the recursion

$$C_n(x) = 2xC_{n-1} - C_{n-2}$$

Template Parameters

<code>_Tp</code>	The real type of the argument
------------------	-------------------------------

Parameters

<code>__n</code>	The non-negative integral order
<code>__x</code>	The real argument $-1 \leq x \leq +1$
<code>__C0</code>	The value of the zeroth-order Chebyshev polynomial at x
<code>__C1</code>	The value of the first-order Chebyshev polynomial at x

Definition at line 60 of file sf_chebyshev.tcc.

Referenced by __chebyshev_t(), __chebyshev_u(), __chebyshev_v(), and __chebyshev_w().

8.3.2.29 __chebyshev_t()

```
template<typename _Tp >
__gnu_cxx::__chebyshev_t_t<_Tp> std::__detail::__chebyshev_t (
    unsigned int __n,
    _Tp __x )
```

Return the Chebyshev polynomial of the first kind $T_n(x)$ of non-negative order n and real argument x .

The Chebyshev polynomial of the first kind is defined by:

$$T_n(x) = \cos(n\theta)$$

where $\theta = \arccos(x)$, $-1 \leq x \leq +1$.

Template Parameters

<code>_Tp</code>	The real type of the argument
------------------	-------------------------------

Parameters

<code>↵ __n</code>	The non-negative integral order
<code>↵ __x</code>	The real argument $-1 \leq x \leq +1$

Definition at line 88 of file sf_chebyshev.tcc.

References __chebyshev_recur().

8.3.2.30 __chebyshev_u()

```
template<typename _Tp >
__gnu_cxx::__chebyshev_u_t<_Tp> std::__detail::__chebyshev_u (
    unsigned int __n,
    _Tp __x )
```

Return the Chebyshev polynomial of the second kind $U_n(x)$ of non-negative order n and real argument x .

The Chebyshev polynomial of the second kind is defined by:

$$U_n(x) = \frac{\sin[(n+1)\theta]}{\sin(\theta)}$$

where $\theta = \arccos(x)$, $-1 \leq x \leq +1$.

Template Parameters

<code>_Tp</code>	The real type of the argument
------------------	-------------------------------

Parameters

<code>↔ _n</code>	The non-negative integral order
<code>↔ _x</code>	The real argument $-1 \leq x \leq +1$

Definition at line 118 of file sf_chebyshev.tcc.

References `__chebyshev_recur()`.

8.3.2.31 `__chebyshev_v()`

```
template<typename _Tp >
__gnu_cxx::__chebyshev_v_t<_Tp> std::__detail::__chebyshev_v (
    unsigned int __n,
    _Tp __x )
```

Return the Chebyshev polynomial of the third kind $V_n(x)$ of non-negative order n and real argument x .

The Chebyshev polynomial of the third kind is defined by:

$$V_n(x) = \frac{\cos \left[\left(n + \frac{1}{2} \right) \theta \right]}{\cos \left(\frac{\theta}{2} \right)}$$

where $\theta = \arccos(x)$, $-1 \leq x \leq +1$.

Template Parameters

<code>_Tp</code>	The real type of the argument
------------------	-------------------------------

Parameters

<code>↔ _n</code>	The non-negative integral order
<code>↔ _x</code>	The real argument $-1 \leq x \leq +1$

Definition at line 149 of file sf_chebyshev.tcc.

References `__chebyshev_recur()`.

8.3.2.32 __chebyshev_w()

```
template<typename _Tp >
__gnu_cxx::__chebyshev_w_t<_Tp> std::__detail::__chebyshev_w (
    unsigned int __n,
    _Tp __x )
```

Return the Chebyshev polynomial of the fourth kind $W_n(x)$ of non-negative order n and real argument x .

The Chebyshev polynomial of the fourth kind is defined by:

$$W_n(x) = \frac{\sin \left[\left(n + \frac{1}{2} \right) \theta \right]}{\sin \left(\frac{\theta}{2} \right)}$$

where $\theta = \arccos(x)$, $-1 \leq x \leq +1$.

Template Parameters

<code>_Tp</code>	The real type of the argument
------------------	-------------------------------

Parameters

<code>↔ _n</code>	The non-negative integral order
<code>↔ _x</code>	The real argument $-1 \leq x \leq +1$

Definition at line 180 of file sf_chebyshev.tcc.

References `__chebyshev_recur()`.

8.3.2.33 __chi_squared_pdf()

```
template<typename _Tp >
_Tp std::__detail::__chi_squared_pdf (
    _Tp __chi2,
    unsigned int __nu )
```

Return the chi-squared propability function. This returns the probability that the observed chi-squared for a correct model is less than the value χ^2 .

The chi-squared propability function is related to the normalized lower incomplete gamma function:

$$P(\chi^2|\nu) = \Gamma_P\left(\frac{\nu}{2}, \frac{\chi^2}{2}\right)$$

Definition at line 75 of file sf_distributions.tcc.

References `__gamma_p()`.

8.3.2.34 `__chi_squared_pdfc()`

```
template<typename _Tp >
_Tp std::__detail::__chi_squared_pdfc (
    _Tp __chi2,
    unsigned int __nu )
```

Return the complementary chi-squared propability function. This returns the probability that the observed chi-squared for a correct model is greater than the value χ^2 .

The complementary chi-squared propability function is related to the normalized upper incomplete gamma function:

$$Q(\chi^2|\nu) = \Gamma_Q\left(\frac{\nu}{2}, \frac{\chi^2}{2}\right)$$

Definition at line 99 of file `sf_distributions.tcc`.

References `__gamma_q()`.

8.3.2.35 `__chshint()`

```
template<typename _Tp >
std::pair<_Tp, _Tp> std::__detail::__chshint (
    _Tp __x,
    _Tp & __Chi,
    _Tp & __Shi )
```

This function returns the hyperbolic cosine $Chi(x)$ and hyperbolic sine $Shi(x)$ integrals as a pair.

The hyperbolic cosine integral is defined by:

$$Chi(x) = \gamma_E + \log(x) + \int_0^x dt \frac{\cosh(t) - 1}{t}$$

The hyperbolic sine integral is defined by:

$$Shi(x) = \int_0^x dt \frac{\sinh(t)}{t}$$

Definition at line 166 of file `sf_hypint.tcc`.

References `__chshint_cont_frac()`, and `__chshint_series()`.

8.3.2.36 __chshint_cont_frac()

```
template<typename _Tp >
void std::__detail::__chshint_cont_frac (
    _Tp __t,
    _Tp & _Chi,
    _Tp & _Shi )
```

This function computes the hyperbolic cosine $Chi(x)$ and hyperbolic sine $Shi(x)$ integrals by continued fraction for positive argument.

Definition at line 53 of file sf_hypint.tcc.

Referenced by __chshint().

8.3.2.37 __chshint_series()

```
template<typename _Tp >
void std::__detail::__chshint_series (
    _Tp __t,
    _Tp & _Chi,
    _Tp & _Shi )
```

This function computes the hyperbolic cosine $Chi(x)$ and hyperbolic sine $Shi(x)$ integrals by series summation for positive argument.

Definition at line 96 of file sf_hypint.tcc.

Referenced by __chshint().

8.3.2.38 __clamp_0_m2pi()

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__clamp_0_m2pi (
    std::complex< _Tp > __z )
```

Definition at line 185 of file sf_polylog.tcc.

Referenced by __polylog_exp_neg_int(), __polylog_exp_neg_real(), __polylog_exp_pos_int(), and __polylog_exp_pos_real().

8.3.2.39 `__clamp_pi()`

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__clamp_pi (
    std::complex< _Tp > __z )
```

Definition at line 171 of file `sf_polylog.tcc`.

Referenced by `__polylog_exp_neg_int()`, `__polylog_exp_neg_real()`, `__polylog_exp_pos_int()`, and `__polylog_exp_pos_real()`.

8.3.2.40 `__clausen()` [1/2]

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__clausen (
    unsigned int __m,
    std::complex< _Tp > __z )
```

Return Clausen's function of integer order m and complex argument z . The notation and connection to polylog is from Wikipedia

Parameters

<code>__m</code>	The non-negative integral order.
<code>__z</code>	The complex argument.

Returns

The complex Clausen function.

Definition at line 1288 of file `sf_polylog.tcc`.

References `__polylog_exp()`.

8.3.2.41 `__clausen()` [2/2]

```
template<typename _Tp >
_Tp std::__detail::__clausen (
    unsigned int __m,
    _Tp __x )
```

Return Clausen's function of integer order m and real argument x . The notation and connection to polylog is from Wikipedia

Parameters

\leftrightarrow _m	The integer order $m \geq 1$.
\leftrightarrow _x	The real argument.

Returns

The Clausen function.

Definition at line 1315 of file sf_polylog.tcc.

References `__polylog_exp()`.

8.3.2.42 `__clausen_cl()` [1/2]

```
template<typename _Tp >
_Tp std::__detail::__clausen_cl (
    unsigned int __m,
    std::complex< _Tp > __z )
```

Return Clausen's cosine sum Cl_m for positive integer order m and complex argument w .

See also

https://en.wikipedia.org/wiki/Clausen_function

Parameters

\leftrightarrow _m	The integer order $m \geq 1$.
\leftrightarrow _z	The complex argument.

Returns

The Clausen cosine sum $Cl_m(w)$,

Definition at line 1399 of file sf_polylog.tcc.

References `__polylog_exp()`.

8.3.2.43 `__clausen_cl()` [2/2]

```
template<typename _Tp >
_Tp std::__detail::__clausen_cl (
    unsigned int __m,
    _Tp __x )
```

Return Clausen's cosine sum Cl_m for positive integer order m and real argument w .

See also

https://en.wikipedia.org/wiki/Clausen_function

Parameters

\leftrightarrow <code>__m</code>	The integer order $m \geq 1$.
\leftrightarrow <code>__x</code>	The real argument.

Returns

The real Clausen cosine sum $Cl_m(w)$,

Definition at line 1427 of file `sf_polylog.tcc`.

References `__polylog_exp()`.

8.3.2.44 `__clausen_sl()` [1/2]

```
template<typename _Tp >
_Tp std::__detail::__clausen_sl (
    unsigned int __m,
    std::complex< _Tp > __z )
```

Return Clausen's sine sum Sl_m for positive integer order m and complex argument z .

See also

https://en.wikipedia.org/wiki/Clausen_function

Parameters

\leftrightarrow <code>__m</code>	The integer order $m \geq 1$.
\leftrightarrow <code>__z</code>	The complex argument.

Returns

The Clausen sine sum $Sl_m(w)$,

Definition at line 1343 of file sf_polylog.tcc.

References `__polylog_exp()`.

8.3.2.45 __clausen_sl() [2/2]

```
template<typename _Tp >
_Tp std::__detail::__clausen_sl (
    unsigned int __m,
    _Tp __x )
```

Return Clausen's sine sum Sl_m for positive integer order m and real argument x .

See also

https://en.wikipedia.org/wiki/Clausen_function

Parameters

\leftarrow <code>__m</code>	The integer order $m \geq 1$.
\leftarrow <code>__x</code>	The real argument.

Returns

The Clausen sine sum $Sl_m(w)$,

Definition at line 1371 of file sf_polylog.tcc.

References `__polylog_exp()`.

8.3.2.46 __comp_ellint_1()

```
template<typename _Tp >
_Tp std::__detail::__comp_ellint_1 (
    _Tp __k )
```

Return the complete elliptic integral of the first kind $K(k)$ using the Carlson formulation.

The complete elliptic integral of the first kind is defined as

$$K(k) = F(k, \pi/2) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - k^2 \sin^2 \theta}}$$

where $F(k, \phi)$ is the incomplete elliptic integral of the first kind.

Parameters

\leftarrow _k	The modulus of the complete elliptic function.
--------------------	--

Returns

The complete elliptic function of the first kind.

Definition at line 592 of file sf_ellint.tcc.

References `__comp_ellint_rf()`.

Referenced by `__ellint_1()`, `__ellnome_k()`, `__heuman_lambda()`, `__jacobi_zeta()`, `__theta_c()`, `__theta_d()`, `__theta_ \leftarrow n()`, and `__theta_s()`.

8.3.2.47 `__comp_ellint_2()`

```
template<typename _Tp >
_Tp std::__detail::__comp_ellint_2 (
    _Tp __k )
```

Return the complete elliptic integral of the second kind $E(k)$ using the Carlson formulation.

The complete elliptic integral of the second kind is defined as

$$E(k, \pi/2) = \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 \theta} d\theta$$

Parameters

\leftarrow _k	The modulus of the complete elliptic function.
--------------------	--

Returns

The complete elliptic function of the second kind.

Definition at line 666 of file sf_ellint.tcc.

References `__ellint_rd()`, and `__ellint_rf()`.

Referenced by `__ellint_2()`.

8.3.2.48 __comp_ellint_3()

```
template<typename _Tp >
_Tp std::__detail::__comp_ellint_3 (
    _Tp __k,
    _Tp __nu )
```

Return the complete elliptic integral of the third kind $\Pi(k, \nu) = \Pi(k, \nu, \pi/2)$ using the Carlson formulation.

The complete elliptic integral of the third kind is defined as

$$\Pi(k, \nu) = \int_0^{\pi/2} \frac{d\theta}{(1 - \nu \sin^2 \theta) \sqrt{1 - k^2 \sin^2 \theta}}$$

Parameters

<code>__k</code>	The elliptic modulus.
<code>__nu</code>	The characteristic.

Returns

The complete elliptic function of the third kind.

Definition at line 756 of file `sf_ellint.tcc`.

References `__ellint_rf()`, and `__ellint_rj()`.

Referenced by `__ellint_3()`.

8.3.2.49 __comp_ellint_d()

```
template<typename _Tp >
_Tp std::__detail::__comp_ellint_d (
    _Tp __k )
```

Return the complete Legendre elliptic integral D.

Definition at line 862 of file `sf_ellint.tcc`.

References `__ellint_rd()`.

8.3.2.50 __comp_ellint_rf()

```
template<typename _Tp >
_Tp std::__detail::__comp_ellint_rf (
    _Tp __x,
    _Tp __y )
```

Definition at line 252 of file sf_ellint.tcc.

Referenced by __comp_ellint_1(), and __ellint_rf().

8.3.2.51 __comp_ellint_rg()

```
template<typename _Tp >
_Tp std::__detail::__comp_ellint_rg (
    _Tp __x,
    _Tp __y )
```

Definition at line 368 of file sf_ellint.tcc.

Referenced by __ellint_rg().

8.3.2.52 __conf_hyperg()

```
template<typename _Tp >
_Tp std::__detail::__conf_hyperg (
    _Tp __a,
    _Tp __c,
    _Tp __x )
```

Return the confluent hypergeometric function ${}_1F_1(a; c; x) = M(a, c, x)$.

Parameters

\longleftrightarrow __a	The <i>numerator</i> parameter.
\longleftrightarrow __c	The <i>denominator</i> parameter.
\longleftrightarrow __x	The argument of the confluent hypergeometric function.

Returns

The confluent hypergeometric function.

Definition at line 337 of file sf_hyperg.tcc.

References `__conf_hyperg_luke()`, `__conf_hyperg_series()`, and `__gnu_cxx::__fp_is_integer()`.

Referenced by `__tricomi_u_naive()`.

8.3.2.53 `__conf_hyperg_lim()`

```
template<typename _Tp >
_Tp std::__detail::__conf_hyperg_lim (
    _Tp __c,
    _Tp __x )
```

Return the confluent hypergeometric limit function ${}_0F_1(-; c; x)$.

Parameters

<code>__c</code>	The <i>denominator</i> parameter.
<code>__x</code>	The argument of the confluent hypergeometric limit function.

Returns

The confluent limit hypergeometric function.

Definition at line 163 of file sf_hyperg.tcc.

References `__conf_hyperg_lim_series()`, and `__gnu_cxx::__fp_is_integer()`.

8.3.2.54 `__conf_hyperg_lim_series()`

```
template<typename _Tp >
_Tp std::__detail::__conf_hyperg_lim_series (
    _Tp __c,
    _Tp __x )
```

This routine returns the confluent hypergeometric limit function by series expansion.

$${}_0F_1(-; c; x) = \Gamma(c) \sum_{n=0}^{\infty} \frac{1}{\Gamma(c+n)} \frac{x^n}{n!}$$

If a and b are integers and a < 0 and either b > 0 or b < a then the series is a polynomial with a finite number of terms.

Parameters

\leftrightarrow _c	The "denominator" parameter.
\leftrightarrow _x	The argument of the confluent hypergeometric limit function.

Returns

The confluent hypergeometric limit function.

Definition at line 130 of file sf_hyperg.tcc.

Referenced by __conf_hyperg_lim().

8.3.2.55 __conf_hyperg_luke()

```
template<typename _Tp >
_Tp std::__detail::__conf_hyperg_luke (
    _Tp __a,
    _Tp __c,
    _Tp __xin )
```

Return the hypergeometric function ${}_1F_1(a; c; x)$ by an iterative procedure described in Luke, Algorithms for the Computation of Mathematical Functions.

Like the case of the ${}_2F_1$ rational approximations, these are probably guaranteed to converge for $x < 0$, barring gross numerical instability in the pre-asymptotic regime.

Definition at line 231 of file sf_hyperg.tcc.

Referenced by __conf_hyperg().

8.3.2.56 __conf_hyperg_series()

```
template<typename _Tp >
_Tp std::__detail::__conf_hyperg_series (
    _Tp __a,
    _Tp __c,
    _Tp __x )
```

This routine returns the confluent hypergeometric function by series expansion.

$${}_1F_1(a; c; x) = \frac{\Gamma(c)}{\Gamma(a)} \sum_{n=0}^{\infty} \frac{\Gamma(a+n)}{\Gamma(c+n)} \frac{x^n}{n!}$$

Parameters

\leftrightarrow _a	The "numerator" parameter.
\leftrightarrow _c	The "denominator" parameter.
\leftrightarrow _x	The argument of the confluent hypergeometric function.

Returns

The confluent hypergeometric function.

Definition at line 196 of file sf_hyperg.tcc.

Referenced by __conf_hyperg().

8.3.2.57 __cos_pi() [1/2]

```
template<typename _Tp >
_Tp std::__detail::__cos_pi (
    _Tp __x )
```

Return the reperiodized cosine of argument x:

$$\cos_{\pi}(x) = \cos(\pi x)$$

Definition at line 104 of file sf_trig.tcc.

Referenced by __cos_pi(), __cosh_pi(), __cyl_bessel_jn(), __cyl_bessel_jn_neg_arg(), __log_double_factorial(), \leftrightarrow sin_pi(), and __sinh_pi().

8.3.2.58 __cos_pi() [2/2]

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__cos_pi (
    std::complex< _Tp > __z )
```

Return the reperiodized cosine of complex argument z:

$$\cos_{\pi}(z) = \cos(\pi z) = \cos_{\pi}(x)\cosh_{\pi}(y) - i\sin_{\pi}(x)\sinh_{\pi}(y)$$

Definition at line 231 of file sf_trig.tcc.

References __cos_pi(), and __sin_pi().

8.3.2.59 `__cosh_pi()` [1/2]

```
template<typename _Tp >
_Tp std::__detail::__cosh_pi (
    _Tp __x )
```

Return the reperiodized hyperbolic cosine of argument x:

$$\cosh_{\pi}(x) = \cosh(\pi x)$$

Definition at line 133 of file sf_trig.tcc.

8.3.2.60 `__cosh_pi()` [2/2]

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__cosh_pi (
    std::complex< _Tp > __z )
```

Return the reperiodized hyperbolic cosine of complex argument z:

$$\cosh_{\pi}(z) = \cosh_{\pi}(x) \cos_{\pi}(y) + i \sinh_{\pi}(x) \sin_{\pi}(y)$$

Definition at line 253 of file sf_trig.tcc.

References `__cos_pi()`, and `__sin_pi()`.

8.3.2.61 `__coshint()`

```
template<typename _Tp >
_Tp std::__detail::__coshint (
    const _Tp __x )
```

Return the hyperbolic cosine integral $Chi(x)$.

The hyperbolic cosine integral is given by

$$Chi(x) = (Ei(x) - E_1(x))/2 = (Ei(x) + Ei(-x))/2$$

Parameters

<code>__x</code>	The argument of the hyperbolic cosine integral function.
------------------	--

Returns

The hyperbolic cosine integral.

Definition at line 561 of file sf_expint.tcc.

References `__expint_E1()`, and `__expint_Ei()`.

8.3.2.62 `__coulomb_CF1()`

```
template<typename _Tp >
std::pair<_Tp, _Tp> std::__detail::__coulomb_CF1 (
    unsigned int __l,
    _Tp __eta,
    _Tp __x )
```

Evaluate the first continued fraction, giving the ratio F'/F at the upper l value. We also determine the sign of F at that point, since it is the sign of the last denominator in the continued fraction.

Definition at line 146 of file sf_coulomb.tcc.

8.3.2.63 `__coulomb_CF2()`

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__coulomb_CF2 (
    unsigned int __l,
    _Tp __eta,
    _Tp __x )
```

Evaluate the second continued fraction to obtain the ratio

$$(G' + iF')/(G + iF) := P + iQ$$

at the specified l value.

Definition at line 204 of file sf_coulomb.tcc.

8.3.2.64 __coulomb_f_recur()

```
template<typename _Tp >
std::pair<_Tp, _Tp> std::__detail::__coulomb_f_recur (
    unsigned int __l_min,
    unsigned int __k_max,
    _Tp __eta,
    _Tp __x,
    _Tp _F_l_max,
    _Tp _Fp_l_max )
```

Evolve the backwards recurrence for F, F'.

$$F_{l-1} = (S_l F_l + F'_l) / R_l F'_{l-1} = (S_l F_{l-1} - R_l F_l)$$

where

$$R_l = \sqrt{1 + (\eta/l)^2} S_l = l/x + \eta/l$$

Definition at line 77 of file sf_coulomb.tcc.

8.3.2.65 __coulomb_g_recur()

```
template<typename _Tp >
std::pair<_Tp, _Tp> std::__detail::__coulomb_g_recur (
    unsigned int __l_min,
    unsigned int __k_max,
    _Tp __eta,
    _Tp __x,
    _Tp _G_l_min,
    _Tp _Gp_l_min )
```

Evolve the forward recurrence for G, G'.

$$G_{l+1} = (S_l G_l - G'_l) / R_l G'_{l+1} = R_{l+1} G_l - S_l G_{l+1}$$

where

$$R_l = \sqrt{1 + (\eta/l)^2} S_l = l/x + \eta/l$$

Definition at line 115 of file sf_coulomb.tcc.

8.3.2.66 __coulomb_norm()

```
template<typename _Tp >
_Tp std::__detail::__coulomb_norm (
    unsigned int __l,
    _Tp __eta )
```

Definition at line 49 of file sf_coulomb.tcc.

8.3.2.67 `__cyl_bessel()`

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__cyl_bessel (
    std::complex< _Tp > __nu,
    std::complex< _Tp > __z )
```

Return the complex cylindrical Bessel function.

Parameters

in	<code>__nu</code>	The order for which the cylindrical Bessel function is evaluated.
in	<code>__z</code>	The argument at which the cylindrical Bessel function is evaluated.

Returns

The complex cylindrical Bessel function.

Definition at line 1173 of file `sf_hankel.tcc`.

References `__hankel()`.

8.3.2.68 `__cyl_bessel_i()`

```
template<typename _Tp >
_Tp std::__detail::__cyl_bessel_i (
    _Tp __nu,
    _Tp __x )
```

Return the regular modified Bessel function of order ν : $I_\nu(x)$.

The regular modified cylindrical Bessel function is:

$$I_\nu(x) = \sum_{k=0}^{\infty} \frac{(x/2)^{\nu+2k}}{k! \Gamma(\nu + k + 1)}$$

Parameters

<code>__nu</code>	The order of the regular modified Bessel function.
<code>__x</code>	The argument of the regular modified Bessel function.

Returns

The output regular modified Bessel function.

Definition at line 371 of file sf_mod_bessel.tcc.

References `__cyl_bessel_ij_series()`, and `__cyl_bessel_ik()`.

Referenced by `__rice_pdf()`.

8.3.2.69 __cyl_bessel_ij_series()

```
template<typename _Tp >
_Tp std::__detail::__cyl_bessel_ij_series (
    _Tp __nu,
    _Tp __x,
    _Tp __sgn,
    unsigned int __max_iter )
```

This routine returns the cylindrical Bessel functions of order ν : J_ν or I_ν by series expansion.

The modified cylindrical Bessel function is:

$$Z_\nu(x) = \sum_{k=0}^{\infty} \frac{\sigma^k (x/2)^{\nu+2k}}{k! \Gamma(\nu + k + 1)}$$

where $\sigma = +1$ or -1 for $Z = I$ or J respectively.

See Abramowitz & Stegun, 9.1.10 Abramowitz & Stegun, 9.6.7 (1) Handbook of Mathematical Functions, ed. Milton Abramowitz and Irene A. Stegun, Dover Publications, Equation 9.1.10 p. 360 and Equation 9.6.10 p. 375

Parameters

<code>__nu</code>	The order of the Bessel function.
<code>__x</code>	The argument of the Bessel function.
<code>__sgn</code>	The sign of the alternate terms -1 for the Bessel function of the first kind. +1 for the modified Bessel function of the first kind.
<code>__max_iter</code>	The maximum number of iterations for sum.

Returns

The output Bessel function.

Definition at line 434 of file sf_bessel.tcc.

References `__log_gamma()`.

Referenced by `__cyl_bessel_i()`, and `__cyl_bessel_j()`.

8.3.2.70 `__cyl_bessel_ik()`

```
template<typename _Tp >
__gnu_cxx::__cyl_mod_bessel_t<_Tp, _Tp, _Tp> std::__detail::__cyl_bessel_ik (
    _Tp __nu,
    _Tp __x )
```

Return the modified cylindrical Bessel functions and their derivatives of order ν by various means.

Parameters

<code>__nu</code>	The order of the Bessel functions.
<code>__x</code>	The argument of the Bessel functions.

Returns

A struct containing the modified cylindrical Bessel functions of the first and second kinds and their derivatives.

Definition at line 309 of file `sf_mod_bessel.tcc`.

References `__cyl_bessel_ik_asymp()`, `__cyl_bessel_ik_stepped()`, and `__sin_pi()`.

Referenced by `__airy()`, `__cyl_bessel_i()`, `__cyl_bessel_k()`, and `__sph_bessel_ik()`.

8.3.2.71 `__cyl_bessel_ik_asymp()`

```
template<typename _Tp >
__gnu_cxx::__cyl_mod_bessel_t<_Tp, _Tp, _Tp> std::__detail::__cyl_bessel_ik_asymp (
    _Tp __nu,
    _Tp __x )
```

This routine computes the asymptotic modified cylindrical Bessel and functions of order ν : $I_\nu(x)$, $N_\nu(x)$. Use this for $x \gg \nu^2 + 1$.

References: (1) Handbook of Mathematical Functions, ed. Milton Abramowitz and Irene A. Stegun, Dover Publications, Section 9 p. 364, Equations 9.2.5-9.2.10

Parameters

<code>__nu</code>	The order of the Bessel functions.
<code>__x</code>	The argument of the Bessel functions.

Returns

A struct containing the modified cylindrical Bessel functions of the first and second kinds and their derivatives.

Definition at line 79 of file sf_mod_bessel.tcc.

Referenced by `__cyl_bessel_ik()`, and `__cyl_bessel_ik_steel()`.

8.3.2.72 `__cyl_bessel_ik_steel()`

```
template<typename _Tp >
__gnu_cxx::__cyl_mod_bessel_t<_Tp, _Tp, _Tp> std::__detail::__cyl_bessel_ik_steel (
    _Tp __nu,
    _Tp __x )
```

Compute the modified Bessel functions $I_\nu(x)$ and $K_\nu(x)$ and their first derivatives $I'_\nu(x)$ and $K'_\nu(x)$ respectively. These four functions are computed together for numerical stability.

Parameters

<code>__nu</code>	The order of the Bessel functions.
<code>__x</code>	The argument of the Bessel functions.

Returns

A struct containing the modified cylindrical Bessel functions of the first and second kinds and their derivatives.

Definition at line 153 of file sf_mod_bessel.tcc.

References `__cyl_bessel_ik_asymp()`, and `__gamma_temme()`.

Referenced by `__cyl_bessel_ik()`.

8.3.2.73 `__cyl_bessel_j()`

```
template<typename _Tp >
_Tp std::__detail::__cyl_bessel_j (
    _Tp __nu,
    _Tp __x )
```

Return the Bessel function of order ν : $J_\nu(x)$.

The cylindrical Bessel function is:

$$J_\nu(x) = \sum_{k=0}^{\infty} \frac{(-1)^k (x/2)^{\nu+2k}}{k! \Gamma(\nu + k + 1)}$$

Parameters

<code>__nu</code>	The order of the Bessel function.
<code>__x</code>	The argument of the Bessel function.

Returns

The output Bessel function.

Definition at line 581 of file `sf_bessel.tcc`.

References `__cyl_bessel_ij_series()`, and `__cyl_bessel_jn()`.

8.3.2.74 `__cyl_bessel_jn()`

```
template<typename _Tp >
__gnu_cxx::__cyl_bessel_t<_Tp, _Tp, _Tp> std::__detail::__cyl_bessel_jn (
    _Tp __nu,
    _Tp __x )
```

Return the cylindrical Bessel functions and their derivatives of order ν by various means.

Definition at line 473 of file `sf_bessel.tcc`.

References `__cos_pi()`, `__cyl_bessel_jn_asymp()`, `__cyl_bessel_jn_steel()`, and `__sin_pi()`.

Referenced by `__airy()`, `__cyl_bessel_j()`, `__cyl_bessel_jn_neg_arg()`, `__cyl_hankel_1()`, `__cyl_hankel_2()`, `__cyl_neumann_n()`, and `__sph_bessel_jn()`.

8.3.2.75 `__cyl_bessel_jn_asymp()`

```
template<typename _Tp >
__gnu_cxx::__cyl_bessel_t<_Tp, _Tp, _Tp> std::__detail::__cyl_bessel_jn_asymp (
    _Tp __nu,
    _Tp __x )
```

This routine computes the asymptotic cylindrical Bessel and Neumann functions of order ν : $J_\nu(x)$, $N_\nu(x)$. Use this for $x \gg \nu^2 + 1$.

$$J_\nu(z) = \left(\frac{2}{\pi z}\right)^{1/2} \left(\cos(\omega) \sum_{k=0}^{\infty} (-1)^k \frac{a_{2k}(\nu)}{z^{2k}} - \sin(\omega) \sum_{k=0}^{\infty} (-1)^k \frac{a_{2k+1}(\nu)}{z^{2k+1}} \right)$$

and

$$N_\nu(z) = \left(\frac{2}{\pi z}\right)^{1/2} \left(\sin(\omega) \sum_{k=0}^{\infty} (-1)^k \frac{a_{2k}(\nu)}{z^{2k}} + \cos(\omega) \sum_{k=0}^{\infty} (-1)^k \frac{a_{2k+1}(\nu)}{z^{2k+1}} \right)$$

where $\omega = z - \nu\pi/2 - \pi/4$ and

$$a_k(\nu) = \frac{(4\nu^2 - 1^2)(4\nu^2 - 3^2) \dots (4\nu^2 - (2k-1)^2)}{8^k k!}$$

There sums work everywhere but on the negative real axis: $|ph(z)| < \pi - \delta$.

References: (1) Handbook of Mathematical Functions, ed. Milton Abramowitz and Irene A. Stegun, Dover Publications, Section 9 p. 364, Equations 9.2.5-9.2.10

Parameters

<code>__nu</code>	The order of the Bessel functions.
<code>__x</code>	The argument of the Bessel functions.

Returns

A struct containing the cylindrical Bessel functions of the first and second kinds and their derivatives.

Definition at line 100 of file `sf_bessel.tcc`.

Referenced by `__cyl_bessel_jn()`, and `__cyl_bessel_jn_steel()`.

8.3.2.76 `__cyl_bessel_jn_neg_arg()`

```
template<typename _Tp >
__gnu_cxx::__cyl_bessel_t<_Tp, _Tp, std::complex<_Tp> > std::__detail::__cyl_bessel_jn_neg_arg (
    _Tp __nu,
    _Tp __x )
```

Return the cylindrical Bessel functions and their derivatives of order ν and argument $x < 0$.

Definition at line 539 of file `sf_bessel.tcc`.

References `__cos_pi()`, `__cyl_bessel_jn()`, and `__polar_pi()`.

Referenced by `__cyl_hankel_1()`, `__cyl_hankel_2()`, and `__sph_bessel_jn_neg_arg()`.

8.3.2.77 `__cyl_bessel_jn_steel()`

```
template<typename _Tp >
__gnu_cxx::__cyl_bessel_t<_Tp, _Tp, _Tp> std::__detail::__cyl_bessel_jn_steel (
    _Tp __nu,
    _Tp __x )
```

Compute the Bessel $J_\nu(x)$ and Neumann $N_\nu(x)$ functions and their first derivatives $J'_\nu(x)$ and $N'_\nu(x)$ respectively. These four functions are computed together for numerical stability.

Parameters

<code>__nu</code>	The order of the Bessel functions.
<code>__x</code>	The argument of the Bessel functions.

Returns

A struct containing the cylindrical Bessel functions of the first and second kinds and their derivatives.

Definition at line 229 of file sf_bessel.tcc.

References `__cyl_bessel_jn_asymp()`, and `__gamma_temme()`.

Referenced by `__cyl_bessel_jn()`.

8.3.2.78 `__cyl_bessel_k()`

```
template<typename _Tp >
_Tp std::__detail::__cyl_bessel_k (
    _Tp __nu,
    _Tp __x )
```

Return the irregular modified Bessel function $K_\nu(x)$ of order ν .

The irregular modified Bessel function is defined by:

$$K_\nu(x) = \frac{\pi}{2} \frac{I_{-\nu}(x) - I_\nu(x)}{\sin \nu\pi}$$

where for integral $\nu = n$ a limit is taken: $\lim_{\nu \rightarrow n}$. For negative argument we have simply:

$$K_{-\nu}(x) = K_\nu(x)$$

Parameters

<code>__nu</code>	The order of the irregular modified Bessel function.
<code>__x</code>	The argument of the irregular modified Bessel function.

Returns

The output irregular modified Bessel function.

Definition at line 405 of file sf_mod_bessel.tcc.

References `__cyl_bessel_ik()`.

8.3.2.79 `__cyl_hankel_1()` [1/2]

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__cyl_hankel_1 (
    _Tp __nu,
    _Tp __x )
```

Return the cylindrical Hankel function of the first kind $H_\nu^{(1)}(x)$.

The cylindrical Hankel function of the first kind is defined by:

$$H_\nu^{(1)}(x) = J_\nu(x) + iN_\nu(x)$$

Parameters

<code>__nu</code>	The order of the spherical Neumann function.
<code>__x</code>	The argument of the spherical Neumann function.

Returns

The output spherical Neumann function.

Definition at line 638 of file `sf_bessel.tcc`.

References `__cyl_bessel_jn()`, `__cyl_bessel_jn_neg_arg()`, and `__polar_pi()`.

8.3.2.80 `__cyl_hankel_1()` [2/2]

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__cyl_hankel_1 (
    std::complex<_Tp> __nu,
    std::complex<_Tp> __z )
```

Return the complex cylindrical Hankel function of the first kind.

Parameters

<code>in</code>	<code>__nu</code>	The order for which the cylindrical Hankel function of the first kind is evaluated.
<code>in</code>	<code>__z</code>	The argument at which the cylindrical Hankel function of the first kind is evaluated.

Returns

The complex cylindrical Hankel function of the first kind.

Definition at line 1139 of file `sf_hankel.tcc`.

References `__hankel()`.

8.3.2.81 `__cyl_hankel_2()` [1/2]

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__cyl_hankel_2 (
    _Tp __nu,
    _Tp __x )
```

Return the cylindrical Hankel function of the second kind $H_n^{(2)}(x)$.

The cylindrical Hankel function of the second kind is defined by:

$$H_\nu^{(2)}(x) = J_\nu(x) - iN_\nu(x)$$

Parameters

<code>__nu</code>	The order of the spherical Neumann function.
<code>__x</code>	The argument of the spherical Neumann function.

Returns

The output spherical Neumann function.

Definition at line 677 of file `sf_bessel.tcc`.

References `__cyl_bessel_jn()`, `__cyl_bessel_jn_neg_arg()`, and `__polar_pi()`.

8.3.2.82 `__cyl_hankel_2()` [2/2]

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__cyl_hankel_2 (
    std::complex<_Tp> __nu,
    std::complex<_Tp> __z )
```

Return the complex cylindrical Hankel function of the second kind.

Parameters

in	<code>__nu</code>	The order for which the cylindrical Hankel function of the second kind is evaluated.
in	<code>__z</code>	The argument at which the cylindrical Hankel function of the second kind is evaluated.

Returns

The complex cylindrical Hankel function of the second kind.

Definition at line 1156 of file sf_hankel.tcc.

References `__hankel()`.

8.3.2.83 `__cyl_neumann()`

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__cyl_neumann (
    std::complex< _Tp > __nu,
    std::complex< _Tp > __z )
```

Return the complex cylindrical Neumann function.

Parameters

in	<code>__nu</code>	The order for which the cylindrical Neumann function is evaluated.
in	<code>__z</code>	The argument at which the cylindrical Neumann function is evaluated.

Returns

The complex cylindrical Neumann function.

Definition at line 1190 of file sf_hankel.tcc.

References `__hankel()`.

8.3.2.84 `__cyl_neumann_n()`

```
template<typename _Tp >
_Tp std::__detail::__cyl_neumann_n (
    _Tp __nu,
    _Tp __x )
```

Return the Neumann function of order ν : $N_\nu(x)$.

The Neumann function is defined by:

$$N_\nu(x) = \frac{J_\nu(x) \cos \nu\pi - J_{-\nu}(x)}{\sin \nu\pi}$$

where for integral $\nu = n$ a limit is taken: $\lim_{\nu \rightarrow n}$.

Parameters

<code>__nu</code>	The order of the Neumann function.
<code>__x</code>	The argument of the Neumann function.

Returns

The output Neumann function.

Definition at line 612 of file `sf_bessel.tcc`.

References `__cyl_bessel_jn()`.

8.3.2.85 `__dawson()`

```
template<typename _Tp >
_Tp std::__detail::__dawson (
    _Tp __x )
```

Return the Dawson integral, $F(x)$, for real argument `x`.

The Dawson integral is defined by:

$$F(x) = e^{-x^2} \int_0^x e^{y^2} dy$$

and it's derivative is:

$$F'(x) = 1 - 2xF(x)$$

Parameters

<code>__x</code>	The argument $-inf < x < inf$.
------------------	---------------------------------

Definition at line 235 of file `sf_dawson.tcc`.

References `__dawson_cont_frac()`, and `__dawson_series()`.

8.3.2.86 `__dawson_cont_frac()`

```
template<typename _Tp >
_Tp std::__detail::__dawson_cont_frac (
    _Tp __x )
```

Compute the Dawson integral using a sampling theorem representation.

This array could be built on a thread-local basis.

Definition at line 73 of file sf_dawson.tcc.

Referenced by __dawson().

8.3.2.87 __dawson_series()

```
template<typename _Tp >
_Tp std::__detail::__dawson_series (
    _Tp __x )
```

Compute the Dawson integral using the series expansion.

Definition at line 49 of file sf_dawson.tcc.

Referenced by __dawson().

8.3.2.88 __debye()

```
template<typename _Tp >
_Tp std::__detail::__debye (
    unsigned int __n,
    _Tp __x )
```

Return the Debye function. The Debye functions are related to the incomplete Riemann zeta function:

$$\zeta_x(s) = \frac{1}{\Gamma(s)} \int_0^x \frac{t^{s-1}}{e^t - 1} dt = \sum_{k=1}^{\infty} \frac{P(s, kx)}{k^s}$$

$$Z_x(s) = \frac{1}{\Gamma(s)} \int_x^{\infty} \frac{t^{s-1}}{e^t - 1} dt = \sum_{k=1}^{\infty} \frac{Q(s, kx)}{k^s}$$

where $P(a, x)$, $Q(a, x)$ is the incomplete gamma function ratios. The Debye function is:

$$D_n(x) = \frac{n}{x^n} \int_0^x \frac{t^n}{e^t - 1} dt = \Gamma(n+1) \zeta_x(n+1)$$

Note the infinite limit:

$$D_n(\infty) = \int_0^{\infty} \frac{t^n}{e^t - 1} dt = n! \zeta(n+1)$$

Todo : We should return both the Debye function and it's complement.

Compute the Debye function:

$$D_n(x) = 1 - \sum_{k=1}^{\infty} e^{-kx} \frac{n}{k} \sum_{m=0}^n \frac{n!}{(n-m)!} \frac{1}{k^m} (kx)^m$$

Abramowitz & Stegun 27.1.2

Compute the Debye function:

$$D_n(x) = 1 - \frac{nx}{2(n+1)} + n \sum_{k=1}^{\infty} \frac{B_{2k} x^{2k}}{(2k+n)(2k)!}$$

for $|x| < 2\pi$. Abramowitz-Stegun 27.1.1

Todo Find Debye for $x < -2\pi$!

Definition at line 916 of file `sf_zeta.tcc`.

8.3.2.89 `__debye_region()`

```
template<typename _Tp >
void std::__detail::__debye_region (
    std::complex< _Tp > __alpha,
    int & __indexr,
    char & __aorb )
```

Compute the Debye region in the complex plane.

Definition at line 53 of file `sf_hankel.tcc`.

Referenced by `__hankel()`.

8.3.2.90 `__digamma()` [1/2]

```
template<typename _Tp >
_Tp std::__detail::__digamma (
    unsigned int __n )
```

Return the digamma function of integral argument. The digamma or $\psi(x)$ function is defined as the logarithmic derivative of the gamma function:

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

The digamma series for integral argument is given by:

$$\psi(n) = -\gamma_E + \sum_{k=1}^{n-1} \frac{1}{k}$$

The latter sum is called the harmonic number, H_n .

Definition at line 3319 of file `sf_gamma.tcc`.

Referenced by `__digamma()`, `__hyperg_reflect()`, and `__polygamma()`.

8.3.2.91 __digamma() [2/2]

```
template<typename _Tp >
_Tp std::__detail::__digamma (
    _Tp __x )
```

Return the digamma function. The digamma or $\psi(x)$ function is defined by

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

For negative argument the reflection formula is used:

$$\psi(x) = \psi(1-x) - \pi \cot(\pi x)$$

.

Definition at line 3409 of file sf_gamma.tcc.

References __digamma(), __digamma_asyp(), __gnu_cxx::__fp_is_half_odd_integer(), __gnu_cxx::__fp_is_integer(), __hurwitz_zeta(), std::__detail::_Factorial_table<_Tp>::__n, and __tan_pi().

8.3.2.92 __digamma_asyp()

```
template<typename _Tp >
_Tp std::__detail::__digamma_asyp (
    _Tp __x )
```

Return the digamma function for large argument. The digamma or $\psi(x)$ function is defined by

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

.

The asymptotic series is given by:

$$\psi(x) = \ln(x) - \frac{1}{2x} - \sum_{n=1}^{\infty} \frac{B_{2n}}{2nx^{2n}}$$

Definition at line 3376 of file sf_gamma.tcc.

Referenced by __digamma().

8.3.2.93 __digamma_series()

```
template<typename _Tp >
_Tp std::__detail::__digamma_series (
    _Tp __x )
```

Return the digamma function by series expansion. The digamma or $\psi(x)$ function is defined by

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

.

The series is given by:

$$\psi(x) = -\gamma_E - \frac{1}{x} \sum_{k=1}^{\infty} \frac{x-1}{(k+1)(x+k)}$$

Definition at line 3344 of file sf_gamma.tcc.

8.3.2.94 __dilog()

```
template<typename _Tp >
_Tp std::__detail::__dilog (
    _Tp __x )
```

Compute the dilogarithm function $Li_2(x)$ by summation for $x \leq 1$.

The dilogarithm function is defined by:

$$Li_2(x) = \sum_{k=1}^{\infty} \frac{1}{k^s} \text{ for } s > 1$$

For $|x|$ near 1 use the reflection formulae:

$$Li_2(-x) + Li_2(1-x) = \frac{\pi^2}{6} - \ln(x) \ln(1-x)$$

$$Li_2(-x) - Li_2(1-x) - \frac{1}{2} Li_2(1-x^2) = -\frac{\pi^2}{12} - \ln(x) \ln(1-x)$$

For $x < -1$ use the reflection formula:

$$Li_2(1-x) - Li_2\left(1 - \frac{1}{1-x}\right) - \frac{1}{2} (\ln(x))^2$$

Definition at line 246 of file sf_zeta.tcc.

8.3.2.95 __dirichlet_beta() [1/2]

```
template<typename _Tp >
_Tp std::__detail::__dirichlet_beta (
    std::complex< _Tp > __s )
```

Return the Dirichlet beta function. Currently, s must be real (complex type but negligible imaginary part.) Otherwise `std::domain_error` is thrown. The Dirichlet beta function, in terms of the polylogarithm, is

$$\beta(s) = \Im[Li_s(i)]$$

Parameters

<code>_↔</code> <code>_s</code>	The complex (but on-real-axis) argument.
------------------------------------	--

Returns

The Dirichlet Beta function of real argument.

Exceptions

<code>std::domain_error</code>	if the argument has a significant imaginary part.
--------------------------------	---

Definition at line 1227 of file sf_polylog.tcc.

References `__polylog()`.

8.3.2.96 __dirichlet_beta() [2/2]

```
template<typename _Tp >
_Tp std::__detail::__dirichlet_beta (
    _Tp __s )
```

Return the Dirichlet beta function for real argument. The Dirichlet beta function, in terms of the polylogarithm, is

$$\beta(s) = \Im[Li_s(i)]$$

Parameters

<code>_↔</code> <code>_s</code>	The real argument.
------------------------------------	--------------------

Returns

The Dirichlet Beta function of real argument.

Definition at line 1250 of file sf_polylog.tcc.

References `__polylog()`.

8.3.2.97 `__dirichlet_eta()` [1/2]

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__dirichlet_eta (
    std::complex< _Tp > __s )
```

Return the Dirichlet eta function. Currently, s must be real (complex type but negligible imaginary part.) Otherwise `std::domain_error` is thrown. The Dirichlet eta function, in terms of the polylogarithm, is

$$\eta(s) = -\Re[Li_s(-1)]$$

Parameters

<code>__s</code>	The complex (but on-real-axis) argument.
------------------	--

Returns

The complex Dirichlet eta function.

Exceptions

<code>std::domain_error</code>	if the argument has a significant imaginary part.
--------------------------------	---

Definition at line 1167 of file `sf_polylog.tcc`.

References `__polylog()`.

Referenced by `__dirichlet_eta()`, and `__dirichlet_lambda()`.

8.3.2.98 `__dirichlet_eta()` [2/2]

```
template<typename _Tp >
_Tp std::__detail::__dirichlet_eta (
    _Tp __s )
```

Return the Dirichlet eta function for real argument. The Dirichlet eta function, in terms of the polylogarithm, is

$$\eta(s) = -\Re[Li_s(-1)]$$

Parameters

<code>__s</code>	The real argument.
------------------	--------------------

Returns

The Dirichlet eta function.

Definition at line 1189 of file sf_polylog.tcc.

References `__dirichlet_eta()`, `__gnu_cxx::__fp_is_integer()`, `__gamma()`, `__polylog()`, and `__sin_pi()`.

8.3.2.99 __dirichlet_lambda()

```
template<typename _Tp >
_Tp std::__detail::__dirichlet_lambda (
    _Tp __s )
```

Return the Dirichlet lambda function for real argument.

$$\lambda(s) = \frac{1}{2}(\zeta(s) + \eta(s))$$

Parameters

<code>__s</code>	The real argument.
------------------	--------------------

Returns

The Dirichlet lambda function.

Definition at line 1270 of file sf_polylog.tcc.

References `__dirichlet_eta()`, and `__riemann_zeta()`.

8.3.2.100 __double_factorial()

```
template<typename _Tp >
__GLIBCXX14_CONSTEXPR _Tp std::__detail::__double_factorial (
    int __n )
```

Return the double factorial of the integer n.

The double factorial is defined for integral n by:

$$n!! = 135\dots(n-2)n, \text{ odd } n!! = 246\dots(n-2)n, \text{ even } -1!! = 10!! = 1$$

The double factorial is defined for odd negative integers in the obvious way:

$$(-2m-1)!! = 1/(1(-1)(-3)\dots(-2m+1)(-2m-1)) = \frac{(-1)^m}{(2m-1)!!}$$

for $n = -2m - 1$.

Definition at line 1687 of file sf_gamma.tcc.

References `std::__detail::_Factorial_table<_Tp>::_factorial`, `__log_double_factorial()`, `std::__detail::_Factorial_table<_Tp>::_n`, `_S_double_factorial_table`, and `_S_neg_double_factorial_table`.

8.3.2.101 __ellint_1()

```
template<typename _Tp>
_Tp std::__detail::__ellint_1 (
    _Tp __k,
    _Tp __phi )
```

Return the incomplete elliptic integral of the first kind $F(k, \phi)$ using the Carlson formulation.

The incomplete elliptic integral of the first kind is defined as

$$F(k, \phi) = \int_0^\phi \frac{d\theta}{\sqrt{1 - k^2 \sin^2 \theta}}$$

Parameters

<code>__k</code>	The elliptic modulus.
<code>__phi</code>	The integral limit argument of the elliptic function.

Returns

The elliptic function of the first kind.

Definition at line 621 of file sf_ellint.tcc.

References `__comp_ellint_1()`, and `__ellint_rf()`.

Referenced by `__heuman_lambda()`.

8.3.2.102 __ellint_2()

```
template<typename _Tp >
_Tp std::__detail::__ellint_2 (
    _Tp __k,
    _Tp __phi )
```

Return the incomplete elliptic integral of the second kind $E(k, \phi)$ using the Carlson formulation.

The incomplete elliptic integral of the second kind is defined as

$$E(k, \phi) = \int_0^\phi \sqrt{1 - k^2 \sin^2 \theta}$$

Parameters

<code>__k</code>	The elliptic modulus.
<code>__phi</code>	The integral limit argument of the elliptic function.

Returns

The elliptic function of the second kind.

Definition at line 702 of file sf_ellint.tcc.

References `__comp_ellint_2()`, `__ellint_rd()`, and `__ellint_rf()`.

8.3.2.103 __ellint_3()

```
template<typename _Tp >
_Tp std::__detail::__ellint_3 (
    _Tp __k,
    _Tp __nu,
    _Tp __phi )
```

Return the incomplete elliptic integral of the third kind $\Pi(k, \nu, \phi)$ using the Carlson formulation.

The incomplete elliptic integral of the third kind is defined as

$$\Pi(k, \nu, \phi) = \int_0^\phi \frac{d\theta}{(1 - \nu \sin^2 \theta) \sqrt{1 - k^2 \sin^2 \theta}}$$

Parameters

<code>__k</code>	The elliptic modulus.
<code>__nu</code>	The characteristic.
<code>__phi</code>	The integral limit argument of the elliptic function.

Returns

The elliptic function of the third kind.

Definition at line 795 of file `sf_ellint.tcc`.

References `__comp_ellint_3()`, `__ellint_rf()`, and `__ellint_rj()`.

8.3.2.104 `__ellint_cel()`

```
template<typename _Tp >
_Tp std::__detail::__ellint_cel (
    _Tp __k_c,
    _Tp __p,
    _Tp __a,
    _Tp __b )
```

Return the Bulirsch complete elliptic integrals.

Definition at line 950 of file `sf_ellint.tcc`.

References `__ellint_rf()`, and `__ellint_rj()`.

8.3.2.105 `__ellint_d()`

```
template<typename _Tp >
_Tp std::__detail::__ellint_d (
    _Tp __k,
    _Tp __phi )
```

Return the Legendre elliptic integral D.

Definition at line 836 of file `sf_ellint.tcc`.

References `__ellint_rd()`.

8.3.2.106 `__ellint_el1()`

```
template<typename _Tp >
_Tp std::__detail::__ellint_el1 (
    _Tp __x,
    _Tp __k_c )
```

Return the Bulirsch elliptic integrals of the first kind.

Definition at line 878 of file `sf_ellint.tcc`.

References `__ellint_rf()`.

8.3.2.107 __ellint_el2()

```
template<typename _Tp >
_Tp std::__detail::__ellint_el2 (
    _Tp __x,
    _Tp __k_c,
    _Tp __a,
    _Tp __b )
```

Return the Bulirsch elliptic integrals of the second kind.

Definition at line 899 of file sf_ellint.tcc.

References __ellint_rd(), and __ellint_rf().

8.3.2.108 __ellint_el3()

```
template<typename _Tp >
_Tp std::__detail::__ellint_el3 (
    _Tp __x,
    _Tp __k_c,
    _Tp __p )
```

Return the Bulirsch elliptic integrals of the third kind.

Definition at line 924 of file sf_ellint.tcc.

References __ellint_rf(), and __ellint_rj().

8.3.2.109 __ellint_rc()

```
template<typename _Tp >
_Tp std::__detail::__ellint_rc (
    _Tp __x,
    _Tp __y )
```

Return the Carlson elliptic function $R_C(x, y) = R_F(x, y, y)$ where $R_F(x, y, z)$ is the Carlson elliptic function of the first kind.

The Carlson elliptic function is defined by:

$$R_C(x, y) = \frac{1}{2} \int_0^\infty \frac{dt}{(t+x)^{1/2}(t+y)}$$

Based on Carlson's algorithms:

- B. C. Carlson Numer. Math. 33, 1 (1979)
- B. C. Carlson, Special Functions of Applied Mathematics (1977)
- Numerical Recipes in C, 2nd ed, pp. 261-269, by Press, Teukolsky, Vetterling, Flannery (1992)

Parameters

\leftrightarrow _x	The first argument.
\leftrightarrow _y	The second argument.

Returns

The Carlson elliptic function.

Definition at line 84 of file sf_ellint.tcc.

Referenced by __ellint_rf(), and __ellint_rj().

8.3.2.110 __ellint_rd()

```
template<typename _Tp >
_Tp std::__detail::__ellint_rd (
    _Tp __x,
    _Tp __y,
    _Tp __z )
```

Return the Carlson elliptic function of the second kind $R_D(x, y, z) = R_J(x, y, z, z)$ where $R_J(x, y, z, p)$ is the Carlson elliptic function of the third kind.

The Carlson elliptic function of the second kind is defined by:

$$R_D(x, y, z) = \frac{3}{2} \int_0^\infty \frac{dt}{(t+x)^{1/2}(t+y)^{1/2}(t+z)^{3/2}}$$

Based on Carlson's algorithms:

- B. C. Carlson Numer. Math. 33, 1 (1979)
- B. C. Carlson, Special Functions of Applied Mathematics (1977)
- Numerical Recipes in C, 2nd ed, pp. 261-269, by Press, Teukolsky, Vetterling, Flannery (1992)

Parameters

\leftrightarrow _x	The first of two symmetric arguments.
\leftrightarrow _y	The second of two symmetric arguments.
\leftrightarrow _z	The third argument.

Returns

The Carlson elliptic function of the second kind.

Definition at line 175 of file sf_ellint.tcc.

Referenced by __comp_ellint_2(), __comp_ellint_d(), __ellint_2(), __ellint_d(), __ellint_el2(), __ellint_rg(), and __ellint_rj().

8.3.2.111 __ellint_rf()

```
template<typename _Tp >
_Tp std::__detail::__ellint_rf (
    _Tp __x,
    _Tp __y,
    _Tp __z )
```

Return the Carlson elliptic function $R_F(x, y, z)$ of the first kind.

The Carlson elliptic function of the first kind is defined by:

$$R_F(x, y, z) = \frac{1}{2} \int_0^\infty \frac{dt}{(t+x)^{1/2}(t+y)^{1/2}(t+z)^{1/2}}$$

Parameters

$_x$	The first of three symmetric arguments.
$_y$	The second of three symmetric arguments.
$_z$	The third of three symmetric arguments.

Returns

The Carlson elliptic function of the first kind.

Definition at line 294 of file sf_ellint.tcc.

References __comp_ellint_rf(), and __ellint_rc().

Referenced by __comp_ellint_2(), __comp_ellint_3(), __ellint_1(), __ellint_2(), __ellint_3(), __ellint_cel(), __ellint_el1(), __ellint_el2(), __ellint_el3(), and __heuman_lambda().

8.3.2.112 `__ellint_rg()`

```
template<typename _Tp >
_Tp std::__detail::__ellint_rg (
    _Tp __x,
    _Tp __y,
    _Tp __z )
```

Return the symmetric Carlson elliptic function of the second kind $R_G(x, y, z)$.

The Carlson symmetric elliptic function of the second kind is defined by:

$$R_G(x, y, z) = \frac{1}{4} \int_0^\infty dt [(t+x)(t+y)(t+z)]^{-1/2} \left(\frac{x}{t+x} + \frac{y}{t+y} + \frac{z}{t+z} \right)$$

Based on Carlson's algorithms:

- B. C. Carlson Numer. Math. 33, 1 (1979)
- B. C. Carlson, Special Functions of Applied Mathematics (1977)
- Numerical Recipes in C, 2nd ed, pp. 261-269, by Press, Teukolsky, Vetterling, Flannery (1992)

Parameters

<code>__x</code>	The first of three symmetric arguments.
<code>__y</code>	The second of three symmetric arguments.
<code>__z</code>	The third of three symmetric arguments.

Returns

The Carlson symmetric elliptic function of the second kind.

Definition at line 430 of file `sf_ellint.tcc`.

References `__comp_ellint_rg()`, and `__ellint_rd()`.

8.3.2.113 `__ellint_rj()`

```
template<typename _Tp >
_Tp std::__detail::__ellint_rj (
    _Tp __x,
    _Tp __y,
```



```

_Tp __z,
_Tp __p )

```

Return the Carlson elliptic function $R_J(x, y, z, p)$ of the third kind.

The Carlson elliptic function of the third kind is defined by:

$$R_J(x, y, z, p) = \frac{3}{2} \int_0^\infty \frac{dt}{(t+x)^{1/2}(t+y)^{1/2}(t+z)^{1/2}(t+p)}$$

Based on Carlson's algorithms:

- B. C. Carlson Numer. Math. 33, 1 (1979)
- B. C. Carlson, Special Functions of Applied Mathematics (1977)
- Numerical Recipes in C, 2nd ed, pp. 261-269, by Press, Teukolsky, Vetterling, Flannery (1992)

Parameters

<code>__x</code>	The first of three symmetric arguments.
<code>__y</code>	The second of three symmetric arguments.
<code>__z</code>	The third of three symmetric arguments.
<code>__p</code>	The fourth argument.

Returns

The Carlson elliptic function of the fourth kind.

Definition at line 478 of file sf_ellint.tcc.

References `__ellint_rc()`, and `__ellint_rd()`.

Referenced by `__comp_ellint_3()`, `__ellint_3()`, `__ellint_cel()`, `__ellint_el3()`, `__heuman_lambda()`, and `__jacobi_zeta()`.

8.3.2.114 __ellnome()

```

template<typename _Tp >
_Tp std::__detail::__ellnome (
    _Tp __k )

```

Return the elliptic nome given the modulus k .

$$q(k) = \exp\left(-\pi \frac{K(k')}{K(k)}\right)$$

Definition at line 329 of file sf_theta.tcc.

References `__ellnome_k()`, and `__ellnome_series()`.

Referenced by `__theta_c()`, `__theta_d()`, `__theta_n()`, and `__theta_s()`.

8.3.2.115 `__ellnome_k()`

```
template<typename _Tp >
_Tp std::__detail::__ellnome_k (
    _Tp __k )
```

Use the arithmetic-geometric mean to calculate the elliptic nome given the elliptic argument k .

$$q(k) = \exp\left(-\pi \frac{K(k')}{K(k)}\right)$$

where $k' = \sqrt{1 - k^2}$ is the complementary elliptic argument and K is the Legendre elliptic integral of the first kind.

Definition at line 312 of file sf_theta.tcc.

References `__comp_ellint_1()`.

Referenced by `__ellnome()`.

8.3.2.116 `__ellnome_series()`

```
template<typename _Tp >
_Tp std::__detail::__ellnome_series (
    _Tp __k )
```

Use MacLaurin series to calculate the elliptic nome given the elliptic argument k .

$$q(k) = \exp\left(-\pi \frac{K(k')}{K(k)}\right)$$

where $k' = \sqrt{1 - k^2}$ is the complementary elliptic argument and K is the Legendre elliptic integral of the first kind.

Definition at line 291 of file sf_theta.tcc.

Referenced by `__ellnome()`.

8.3.2.117 `__euler()` [1/2]

```
template<typename _Tp >
_Tp std::__detail::__euler (
    unsigned int __n ) [inline]
```

This returns Euler number E_n .

Parameters

<code>__n</code>	the order n of the Euler number.
------------------	----------------------------------

Returns

The Euler number of order n.

Definition at line 119 of file sf_euler.tcc.

8.3.2.118 `__euler()` [2/2]

```
template<typename _Tp >
_Tp std::__detail::__euler (
    unsigned int __n,
    _Tp __x )
```

Return the Euler polynomial $E_n(x)$ of order n at argument x.

The derivative is proportional to the previous polynomial:

$$E'_n(x) = nE_{n-1}(x)$$

$$E_n(1/2) = \frac{E_n}{2^n}, \text{ where } E_n \text{ is the n-th Euler number.}$$

Definition at line 137 of file sf_euler.tcc.

References `__bernoulli()`.

8.3.2.119 `__euler_series()`

```
template<typename _Tp >
_Tp std::__detail::__euler_series (
    unsigned int __n )
```

Return the Euler number from lookup or by series expansion.

The Euler numbers are given by the recursive sum:

$$E_n = B_n(1) = B_n$$

where $E_0 = 1$, $E_1 = 0$, $E_2 = -1$

Todo Find a way to predict the maximum Euler number for a type.

Definition at line 61 of file sf_euler.tcc.

8.3.2.120 `__eulerian_1()` [1/2]

```
template<typename _Tp >
_Tp std::__detail::__eulerian_1 (
    unsigned int __n,
    unsigned int __m ) [inline]
```

Return the Eulerian number of the first kind. The Eulerian numbers of the first kind are defined by recursion:

$$\left\langle \begin{matrix} n \\ m \end{matrix} \right\rangle = (n - m) \left\langle \begin{matrix} n - 1 \\ m - 1 \end{matrix} \right\rangle + (m + 1) \left\langle \begin{matrix} n - 1 \\ m \end{matrix} \right\rangle \text{ for } n > 0$$

Note that $A(n, m)$ is a common older notation.

Definition at line 207 of file `sf_euler.tcc`.

8.3.2.121 `__eulerian_1()` [2/2]

```
template<typename _Tp >
std::vector<_Tp> std::__detail::__eulerian_1 (
    unsigned int __n ) [inline]
```

Return a vector Eulerian numbers of the first kind. The Eulerian numbers are defined by recursion:

$$A(n, m) = (n - m)A(n - 1, m - 1) + (m + 1)A(n - 1, m) \text{ for } n > 0$$

Definition at line 253 of file `sf_euler.tcc`.

8.3.2.122 `__eulerian_1_recur()` [1/2]

```
template<typename _Tp >
_Tp std::__detail::__eulerian_1_recur (
    unsigned int __n,
    unsigned int __m )
```

Return the Eulerian number of the first kind. The Eulerian numbers of the first kind are defined by recursion:

$$\left\langle \begin{matrix} n \\ m \end{matrix} \right\rangle = (n - m) \left\langle \begin{matrix} n - 1 \\ m - 1 \end{matrix} \right\rangle + (m + 1) \left\langle \begin{matrix} n - 1 \\ m \end{matrix} \right\rangle \text{ for } n > 0$$

Note that $A(n, m)$ is a common older notation.

Definition at line 166 of file `sf_euler.tcc`.

8.3.2.123 `__eulerian_1_recur()` [2/2]

```
template<typename _Tp >
std::vector<_Tp> std::__detail::__eulerian_1_recur (
    unsigned int __n )
```

Return a vector Eulerian numbers of the first kind by recursion. The recursion is

$$A(n, m) = (n - m)A(n - 1, m - 1) + (m + 1)A(n - 1, m) \text{ for } n > 0$$

Definition at line 219 of file sf_euler.tcc.

8.3.2.124 `__eulerian_2()` [1/2]

```
template<typename _Tp >
_Tp std::__detail::__eulerian_2 (
    unsigned int __n,
    unsigned int __m ) [inline]
```

Return the Eulerian number of the second kind. The Eulerian numbers of the second kind are defined by recursion:

$$\left\langle \left\langle n \right\rangle \right\rangle_m = (2n - m - 1) \left\langle \left\langle n - 1 \right\rangle \right\rangle_{m - 1} + (m + 1) \left\langle \left\langle n - 1 \right\rangle \right\rangle_m \text{ for } n > 0$$

Definition at line 309 of file sf_euler.tcc.

8.3.2.125 `__eulerian_2()` [2/2]

```
template<typename _Tp >
std::vector<_Tp> std::__detail::__eulerian_2 (
    unsigned int __n ) [inline]
```

Return a vector of Eulerian numbers of the second kind.

$$\left\langle \left\langle n \right\rangle \right\rangle_m = (2n - m - 1) \left\langle \left\langle n - 1 \right\rangle \right\rangle_{m - 1} + (m + 1) \left\langle \left\langle n - 1 \right\rangle \right\rangle_m \text{ for } n > 0$$

Definition at line 363 of file sf_euler.tcc.

8.3.2.126 `__eulerian_2_recur()` [1/2]

```
template<typename _Tp >
_Tp std::__detail::__eulerian_2_recur (
    unsigned int __n,
    unsigned int __m )
```

Return the Eulerian number of the second kind by recursion:

$$\left\langle \left\langle n \right\rangle \right\rangle_m = (2n - m - 1) \left\langle \left\langle n - 1 \right\rangle \right\rangle_{m-1} + (m + 1) \left\langle \left\langle n - 1 \right\rangle \right\rangle_m \text{ for } n > 0$$

Definition at line 269 of file `sf_euler.tcc`.

8.3.2.127 `__eulerian_2_recur()` [2/2]

```
template<typename _Tp >
std::vector<_Tp> std::__detail::__eulerian_2_recur (
    unsigned int __n )
```

Return a vector of Eulerian numbers of the second kind.

$$\left\langle \left\langle n \right\rangle \right\rangle_m = (2n - m - 1) \left\langle \left\langle n - 1 \right\rangle \right\rangle_{m-1} + (m + 1) \left\langle \left\langle n - 1 \right\rangle \right\rangle_m \text{ for } n > 0$$

Definition at line 325 of file `sf_euler.tcc`.

8.3.2.128 `__exp2()`

```
template<typename _Tp >
_Tp std::__detail::__exp2 (
    _Tp __x )
```

Make `exp2` available to complex and real types.

Definition at line 64 of file `sf_zeta.tcc`.

Referenced by `__riemann_zeta()`.

8.3.2.129 `__expint()` [1/2]

```
template<typename _Tp >
_Tp std::__detail::__expint (
    unsigned int __n,
    _Tp __x )
```

Return the exponential integral $E_n(x)$.

The exponential integral is given by

$$E_n(x) = \int_1^\infty \frac{e^{-xt}}{t^n} dt$$

Parameters

$_n$	The order of the exponential integral function.
$_x$	The argument of the exponential integral function.

Returns

The exponential integral.

Todo Study arbitrary switch to large-n $E_n(x)$.

Todo Find a good asymptotic switch point in $E_n(x)$.

Definition at line 476 of file sf_expint.tcc.

References `__expint_E1()`, `__expint_En_asymp()`, `__expint_En_cont_frac()`, `__expint_En_large_n()`, and `__expint_ $_n$ _series()`.

Referenced by `__logint()`.

8.3.2.130 `__expint()` [2/2]

```
template<typename _Tp >
_Tp std::__detail::__expint (
    _Tp __x )
```

Return the exponential integral $Ei(x)$.

The exponential integral is given by

$$Ei(x) = - \int_{-x}^{\infty} \frac{e^t}{t} dt$$

Parameters

$_x$	The argument of the exponential integral function.
-------	--

Returns

The exponential integral.

Definition at line 517 of file sf_expint.tcc.

References `__expint_Ei()`.

8.3.2.131 `__expint_E1()`

```
template<typename _Tp >
_Tp std::__detail::__expint_E1 (
    _Tp __x )
```

Return the exponential integral $E_1(x)$.

The exponential integral is given by

$$E_1(x) = \int_1^{\infty} \frac{e^{-xt}}{t} dt$$

Parameters

<code>__x</code>	The argument of the exponential integral function.
------------------	--

Returns

The exponential integral.

Todo Find a good asymptotic switch point in $E_1(x)$.

Todo Find a good asymptotic switch point in $E_1(x)$.

Definition at line 381 of file `sf_expint.tcc`.

References `__expint_E1_asymp()`, `__expint_E1_series()`, `__expint_Ei()`, and `__expint_En_cont_frac()`.

Referenced by `__coshint()`, `__expint()`, `__expint_Ei()`, `__expint_En_recursion()`, and `__sinhint()`.

8.3.2.132 `__expint_E1_asymp()`

```
template<typename _Tp >
_Tp std::__detail::__expint_E1_asymp (
    _Tp __x )
```

Return the exponential integral $E_1(x)$ by asymptotic expansion.

The exponential integral is given by

$$E_1(x) = \int_1^{\infty} \frac{e^{-xt}}{t} dt$$

Parameters

\leftrightarrow _X	The argument of the exponential integral function.
-------------------------	--

Returns

The exponential integral.

Definition at line 114 of file sf_expint.tcc.

Referenced by __expint_E1().

8.3.2.133 __expint_E1_series()

```
template<typename _Tp >
_Tp std::__detail::__expint_E1_series (
    _Tp __x )
```

Return the exponential integral $E_1(x)$ by series summation. This should be good for $x < 1$.

The exponential integral is given by

$$E_1(x) = \int_1^{\infty} \frac{e^{-xt}}{t} dt$$

Parameters

\leftrightarrow _X	The argument of the exponential integral function.
-------------------------	--

Returns

The exponential integral.

Definition at line 76 of file sf_expint.tcc.

Referenced by __expint_E1().

8.3.2.134 __expint_Ei()


```
template<typename _Tp >
_Tp std::__detail::__expint_Ei (
    _Tp __x )
```

Return the exponential integral $Ei(x)$.

The exponential integral is given by

$$Ei(x) = - \int_{-x}^{\infty} \frac{e^t}{t} dt$$

Parameters

	The argument of the exponential integral function.
<code>__x</code>	

Returns

The exponential integral.

Definition at line 356 of file sf_expint.tcc.

References `__expint_E1()`, `__expint_Ei_asyp()`, and `__expint_Ei_series()`.

Referenced by `__coshint()`, `__expint()`, `__expint_E1()`, and `__sinhint()`.

8.3.2.135 `__expint_Ei_asyp()`


```
template<typename _Tp >
_Tp std::__detail::__expint_Ei_asyp (
    _Tp __x )
```

Return the exponential integral $Ei(x)$ by asymptotic expansion.

The exponential integral is given by

$$Ei(x) = - \int_{-x}^{\infty} \frac{e^t}{t} dt$$

Parameters

	The argument of the exponential integral function.
<code>__x</code>	

Returns

The exponential integral.

Definition at line 322 of file sf_expint.tcc.

Referenced by `__expint_Ei()`.

8.3.2.136 __expint_Ei_series()

```
template<typename _Tp >
_Tp std::__detail::__expint_Ei_series (
    _Tp __x )
```

Return the exponential integral $Ei(x)$ by series summation.

The exponential integral is given by

$$Ei(x) = - \int_{-x}^{\infty} \frac{e^t}{t} dt$$

Parameters

\leftrightarrow __x	The argument of the exponential integral function.
--------------------------	--

Returns

The exponential integral.

Definition at line 289 of file sf_expint.tcc.

Referenced by __expint_Ei().

8.3.2.137 __expint_En_asymp()

```
template<typename _Tp >
_Tp std::__detail::__expint_En_asymp (
    unsigned int __n,
    _Tp __x )
```

Return the exponential integral $E_n(x)$ for large argument.

The exponential integral is given by

$$E_n(x) = \int_1^{\infty} \frac{e^{-xt}}{t^n} dt$$

Parameters

\leftrightarrow __n	The order of the exponential integral function.
\leftrightarrow __x	The argument of the exponential integral function.

Returns

The exponential integral.

Definition at line 410 of file sf_expint.tcc.

Referenced by __expint().

8.3.2.138 __expint_En_cont_frac()

```
template<typename _Tp >
_Tp std::__detail::__expint_En_cont_frac (
    unsigned int __n,
    _Tp __x )
```

Return the exponential integral $E_n(x)$ by continued fractions.

The exponential integral is given by

$$E_n(x) = \int_1^{\infty} \frac{e^{-xt}}{t^n} dt$$

Parameters

\leftrightarrow __n	The order of the exponential integral function.
\leftrightarrow __x	The argument of the exponential integral function.

Returns

The exponential integral.

Definition at line 198 of file sf_expint.tcc.

Referenced by __expint(), and __expint_E1().

8.3.2.139 __expint_En_large_n()

```
template<typename _Tp >
_Tp std::__detail::__expint_En_large_n (
    unsigned int __n,
    _Tp __x )
```

Return the exponential integral $E_n(x)$ for large order.

The exponential integral is given by

$$E_n(x) = \int_1^{\infty} \frac{e^{-xt}}{t^n} dt$$

Parameters

\longleftrightarrow _n	The order of the exponential integral function.
\longleftrightarrow _x	The argument of the exponential integral function.

Returns

The exponential integral.

Definition at line 442 of file sf_expint.tcc.

Referenced by __expint().

8.3.2.140 __expint_En_recursion()

```
template<typename _Tp >
_Tp std::__detail::__expint_En_recursion (
    unsigned int __n,
    _Tp __x )
```

Return the exponential integral $E_n(x)$ by recursion. Use upward recursion for $x < n$ and downward recursion (Miller's algorithm) otherwise.

The exponential integral is given by

$$E_n(x) = \int_1^{\infty} \frac{e^{-xt}}{t^n} dt$$

Parameters

\longleftrightarrow _n	The order of the exponential integral function.
\longleftrightarrow _x	The argument of the exponential integral function.

Returns

The exponential integral.

Todo Find a principled starting number for the $E_n(x)$ downward recursion.

Definition at line 244 of file sf_expint.tcc.

References __expint_E1().

8.3.2.141 `__expint_En_series()`

```
template<typename _Tp >
_Tp std::__detail::__expint_En_series (
    unsigned int __n,
    _Tp __x )
```

Return the exponential integral $E_n(x)$ by series summation.

The exponential integral is given by

$$E_n(x) = \int_1^{\infty} \frac{e^{-xt}}{t^n} dt$$

Parameters

<code>__n</code>	The order of the exponential integral function.
<code>__x</code>	The argument of the exponential integral function.

Returns

The exponential integral.

Definition at line 150 of file `sf_expint.tcc`.

Referenced by `__expint()`.

8.3.2.142 `__exponential_p()`

```
template<typename _Tp >
_Tp std::__detail::__exponential_p (
    _Tp __lambda,
    _Tp __x )
```

Return the exponential cumulative probability density function.

The formula for the exponential cumulative probability density function is

$$F(x|\lambda) = 1 - e^{-\lambda x} \text{ for } x \geq 0$$

Definition at line 328 of file `sf_distributions.tcc`.

8.3.2.143 __exponential_pdf()

```
template<typename _Tp >
_Tp std::__detail::__exponential_pdf (
    _Tp __lambda,
    _Tp __x )
```

Return the exponential probability density function.

The formula for the exponential probability density function is

$$f(x|\lambda) = \lambda e^{-\lambda x} \text{ for } x \geq 0$$

Definition at line 308 of file sf_distributions.tcc.

8.3.2.144 __exponential_q()

```
template<typename _Tp >
_Tp std::__detail::__exponential_q (
    _Tp __lambda,
    _Tp __x )
```

Return the complement of the exponential cumulative probability density function.

The formula for the complement of the exponential cumulative probability density function is

$$F(x|\lambda) = e^{-\lambda x} \text{ for } x \geq 0$$

Definition at line 350 of file sf_distributions.tcc.

8.3.2.145 __factorial()

```
template<typename _Tp >
_GLIBCXX14_CONSTEXPR _Tp std::__detail::__factorial (
    unsigned int __n )
```

Return the factorial of the integer n.

The factorial is:

$$n! = 12...(n-1)n, 0! = 1$$

Definition at line 1617 of file sf_gamma.tcc.

References std::__detail::_Factorial_table<_Tp>::__n, and _S_factorial_table.

8.3.2.146 `__falling_factorial()` [1/2]

```
template<typename _Tp >
_Tp std::__detail::__falling_factorial (
    _Tp __a,
    int __n )
```

Return the logarithm of the falling factorial function or the lower Pochhammer symbol for real argument a and integral order n . The falling factorial function is defined by

$$a^{\underline{n}} = \prod_{k=0}^{n-1} (a - k), (a)_0 = 1 = \Gamma(a + 1) / \Gamma(a - n + 1)$$

In particular, $n^{\underline{n}} = n!$.

Definition at line 2943 of file `sf_gamma.tcc`.

References `__gnu_cxx::__fp_is_integer()`, `__log_gamma()`, `__log_gamma_sign()`, and `std::__detail::_Factorial_table<_Tp>::__n`.

Referenced by `__falling_factorial()`, and `__log_falling_factorial()`.

8.3.2.147 `__falling_factorial()` [2/2]

```
template<typename _Tp >
_Tp std::__detail::__falling_factorial (
    _Tp __a,
    _Tp __nu )
```

Return the logarithm of the falling factorial function or the lower Pochhammer symbol for real argument a and order ν . The falling factorial function is defined by

$$a^{\underline{\nu}} = \Gamma(a + 1) / \Gamma(a - \nu + 1)$$

.

Definition at line 2998 of file `sf_gamma.tcc`.

References `__falling_factorial()`, `__gnu_cxx::__fp_is_integer()`, `__log_gamma()`, and `__log_gamma_sign()`.

8.3.2.148 `__fermi_dirac()`

```
template<typename _Sp , typename _Tp >
_Tp std::__detail::__fermi_dirac (
    _Sp __s,
    _Tp __x )
```

Return the Fermi-Dirac integral of integer or real order s and real argument x .

See also

https://en.wikipedia.org/wiki/Clausen_function
<http://dlmf.nist.gov/25.12.16>

$$F_s(x) = \frac{1}{\Gamma(s+1)} \int_0^\infty \frac{t^s}{e^{t-x} + 1} dt = -Li_{s+1}(-e^x)$$

Parameters

\leftrightarrow _s	The order $s > -1$.
\leftrightarrow _x	The real argument.

Returns

The real Fermi-Dirac integral $F_s(x)$,

Definition at line 1461 of file sf_polylog.tcc.

References `__polylog_exp()`.

8.3.2.149 __fisher_f_p()

```
template<typename _Tp >
_Tp std::__detail::__fisher_f_p (
    _Tp __F,
    unsigned int __nu1,
    unsigned int __nu2 )
```

Return the F-distribution propability function. This returns the probability that the observed chi-square for a correct model exceeds the value χ^2 .

The f-distribution propability function is related to the incomplete beta function:

$$Q(F|\nu_1, \nu_2) = I_{\frac{\nu_2}{\nu_2 + \nu_1 F}}\left(\frac{\nu_2}{2}, \frac{\nu_1}{2}\right)$$

Parameters

__nu1	The number of degrees of freedom of sample 1
__nu2	The number of degrees of freedom of sample 2
__F	The F statistic

Definition at line 523 of file sf_distributions.tcc.

References `__beta_inc()`.

8.3.2.150 `__fisher_f_pdf()`

```
template<typename _Tp >
_Tp std::__detail::__fisher_f_pdf (
    _Tp __F,
    unsigned int __nu1,
    unsigned int __nu2 )
```

Return the F-distribution propability function. This returns the probability that the observed chi-square for a correct model exceeds the value χ^2 .

The f-distribution propability function is related to the incomplete beta function:

$$Q(F|\nu_1, \nu_2) = I_{\frac{\nu_2}{\nu_2 + \nu_1 F}}\left(\frac{\nu_2}{2}, \frac{\nu_1}{2}\right)$$

Parameters

<code>__nu1</code>	The number of degrees of freedom of sample 1
<code>__nu2</code>	The number of degrees of freedom of sample 2
<code>__F</code>	The F statistic

Definition at line 493 of file `sf_distributions.tcc`.

References `__beta()`.

8.3.2.151 `__fisher_f_q()`

```
template<typename _Tp >
_Tp std::__detail::__fisher_f_q (
    _Tp __F,
    unsigned int __nu1,
    unsigned int __nu2 )
```

Return the F-distribution propability function. This returns the probability that the observed chi-square for a correct model exceeds the value χ^2 .

The f-distribution propability function is related to the incomplete beta function:

$$P(F|\nu_1, \nu_2) = 1 - I_{\frac{\nu_2}{\nu_2 + \nu_1 F}}\left(\frac{\nu_2}{2}, \frac{\nu_1}{2}\right) = 1 - Q(F|\nu_1, \nu_2)$$

Parameters

<code>__F</code>	
<code>__nu1</code>	
<code>__nu2</code>	

Definition at line 552 of file sf_distributions.tcc.

References `__beta_inc()`.

8.3.2.152 `__fock_airy()`

```
template<typename _Tp >
__gnu_cxx::__fock_airy_t<_Tp, std::complex<_Tp> > std::__detail::__fock_airy (
    _Tp __x )
```

Compute the Fock-type Airy functions $w_1(x)$ and $w_2(x)$ and their first derivatives $w'_1(x)$ and $w'_2(x)$ respectively.

$$w_1(x) = \sqrt{\pi}(Ai(x) + iBi(x))$$

$$w_2(x) = \sqrt{\pi}(Ai(x) - iBi(x))$$

.

Parameters

<code>__x</code>	The argument of the Airy functions.
------------------	-------------------------------------

Returns

A struct containing the Fock-type Airy functions of the first and second kinds and their derivatives.

Definition at line 560 of file sf_mod_bessel.tcc.

References `__airy()`.

8.3.2.153 `__fresnel()`

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__fresnel (
    const _Tp __x )
```

Return the Fresnel cosine and sine integrals as a complex number $C(x) + iS(x)$.

The Fresnel cosine integral is defined by:

$$C(x) = \int_0^x \cos\left(\frac{\pi}{2}t^2\right)dt$$

The Fresnel sine integral is defined by:

$$S(x) = \int_0^x \sin\left(\frac{\pi}{2}t^2\right)dt$$

Parameters

\leftrightarrow	The argument
x	

Definition at line 170 of file sf_fresnel.tcc.

References `__fresnel_cont_frac()`, and `__fresnel_series()`.

8.3.2.154 `__fresnel_cont_frac()`

```
template<typename _Tp >
void std::__detail::__fresnel_cont_frac (
    const _Tp __ax,
    _Tp & _Cf,
    _Tp & _Sf )
```

This function computes the Fresnel cosine and sine integrals by continued fractions for positive argument.

Definition at line 109 of file sf_fresnel.tcc.

Referenced by `__fresnel()`.

8.3.2.155 `__fresnel_series()`

```
template<typename _Tp >
void std::__detail::__fresnel_series (
    const _Tp __ax,
    _Tp & _Cf,
    _Tp & _Sf )
```

This function returns the Fresnel cosine and sine integrals as a pair by series expansion for positive argument.

Definition at line 51 of file sf_fresnel.tcc.

Referenced by `__fresnel()`.

8.3.2.156 `__gamma()` [1/2]

```
template<typename _Tp >
_Tp std::__detail::__gamma (
    _Tp __a )
```

Return the gamma function $\Gamma(a)$. The gamma function is defined by:

$$\Gamma(a) = \int_0^{\infty} e^{-t} t^{a-1} dt (a > 0)$$

.

Parameters

\leftrightarrow	The argument of the gamma function.
\leftarrow	

Returns

The gamma function.

Definition at line 2641 of file sf_gamma.tcc.

References `__gnu_cxx::__fp_is_integer()`, `__gamma_reciprocal_series()`, `__log_gamma()`, `__log_gamma_sign()`, `std::__detail::_Factorial_table<_Tp>::__n`, and `_S_factorial_table`.

Referenced by `__beta_gamma()`, `__binomial()`, `__dirichlet_eta()`, `__gamma_p()`, `__gamma_pdf()`, `__gamma_q()`, \leftrightarrow `__gamma_reciprocal()`, `__gamma_reciprocal_series()`, `__hurwitz_zeta_polylog()`, `__polylog_exp_pos()`, \leftrightarrow `__riemann_zeta()`, `__riemann_zeta_glob()`, `__riemann_zeta_m_1()`, `__riemann_zeta_sum()`, `__student_t_pdf()`, and `std::__detail::__Airy_series<_Tp>::__S_Scorer2()`.

8.3.2.157 `__gamma()` [2/2]

```
template<typename _Tp >
std::pair<_Tp, _Tp> std::__detail::__gamma (
    _Tp __a,
    _Tp __x )
```

Return the incomplete gamma functions.

Definition at line 2768 of file sf_gamma.tcc.

References `__gnu_cxx::__fp_is_integer()`, `__gamma_cont_frac()`, and `__gamma_series()`.

8.3.2.158 `__gamma_cont_frac()`

```
template<typename _Tp >
std::pair<_Tp, _Tp> std::__detail::__gamma_cont_frac (
    _Tp __a,
    _Tp __x )
```

Return the incomplete gamma function by continued fraction.

Definition at line 2723 of file sf_gamma.tcc.

References `__log_gamma()`, `__log_gamma_sign()`, and `std::__detail::_Factorial_table<_Tp>::__n`.

Referenced by `__gamma()`, `__gamma_p()`, `__gamma_q()`, `__tgamma()`, and `__tgamma_lower()`.

8.3.2.159 `__gamma_p()` [1/2]

```
template<typename _Tp >
_Tp std::__detail::__gamma_p (
    _Tp __alpha,
    _Tp __beta,
    _Tp __x )
```

Return the gamma cumulative propability distribution function.

The formula for the gamma probability density function is:

$$\Gamma(x|\alpha, \beta) = \frac{1}{\beta\Gamma(\alpha)} (x/\beta)^{\alpha-1} e^{-x/\beta}$$

Definition at line 141 of file `sf_distributions.tcc`.

References `__gamma()`, and `__tgamma_lower()`.

Referenced by `__chi_squared_pdf()`.

8.3.2.160 `__gamma_p()` [2/2]

```
template<typename _Tp >
_Tp std::__detail::__gamma_p (
    _Tp __a,
    _Tp __x )
```

Return the regularized lower incomplete gamma function. The regularized lower incomplete gamma function is defined by

$$P(a, x) = \frac{\gamma(a, x)}{\Gamma(a)}$$

where $\Gamma(a)$ is the gamma function and

$$\gamma(a, x) = \int_0^x e^{-t} t^{a-1} dt (a > 0)$$

is the lower incomplete gamma function.

Definition at line 2807 of file `sf_gamma.tcc`.

References `__gnu_cxx::__fp_is_integer()`, `__gamma_cont_frac()`, and `__gamma_series()`.

8.3.2.161 `__gamma_pdf()`

```
template<typename _Tp >
_Tp std::__detail::__gamma_pdf (
    _Tp __alpha,
    _Tp __beta,
    _Tp __x )
```

Return the gamma propability distribution function.

The formula for the gamma probability density function is:

$$\Gamma(x|\alpha, \beta) = \frac{1}{\beta\Gamma(\alpha)} (x/\beta)^{\alpha-1} e^{-x/\beta}$$

Definition at line 121 of file `sf_distributions.tcc`.

References `__gamma()`.

8.3.2.162 `__gamma_q()` [1/2]

```
template<typename _Tp >
_Tp std::__detail::__gamma_q (
    _Tp __alpha,
    _Tp __beta,
    _Tp __x )
```

Return the gamma complementary cumulative propability distribution function.

The formula for the gamma probability density function is:

$$\Gamma(x|\alpha, \beta) = \frac{1}{\beta\Gamma(\alpha)} (x/\beta)^{\alpha-1} e^{-x/\beta}$$

Definition at line 162 of file `sf_distributions.tcc`.

References `__gamma()`, and `__tgamma()`.

Referenced by `__chi_squared_pdfc()`.

8.3.2.163 `__gamma_q()` [2/2]

```
template<typename _Tp >
_Tp std::__detail::__gamma_q (
    _Tp __a,
    _Tp __x )
```

Return the regularized upper incomplete gamma function. The regularized upper incomplete gamma function is defined by

$$Q(a, x) = \frac{\Gamma(a, x)}{\Gamma(a)}$$

where $\Gamma(a)$ is the gamma function and

$$\Gamma(a, x) = \int_x^\infty e^{-t} t^{a-1} dt (a > 0)$$

is the upper incomplete gamma function.

Definition at line 2841 of file `sf_gamma.tcc`.

References `__gnu_cxx::__fp_is_integer()`, `__gamma_cont_frac()`, and `__gamma_series()`.

8.3.2.164 `__gamma_reciprocal()`

```
template<typename _Tp >
_Tp std::__detail::__gamma_reciprocal (
    _Tp __a )
```

Return the reciprocal of the Gamma function:

$$\frac{1}{\Gamma(a)}$$

Parameters

<code>__a</code>	The argument of the reciprocal of the gamma function.
------------------	---

Returns

The reciprocal of the gamma function.

Definition at line 2271 of file `sf_gamma.tcc`.

References `std::__detail::_Factorial_table<_Tp>::__factorial`, `__gnu_cxx::__fp_is_integer()`, `__gamma()`, `__gamma_reciprocal_series()`, `std::__detail::_Factorial_table<_Tp>::__n`, `__sin_pi()`, and `_S_factorial_table`.

Referenced by `__polylog_exp_asymp()`.

8.3.2.165 __gamma_reciprocal_series()

```
template<typename _Tp >
_Tp std::__detail::__gamma_reciprocal_series (
    _Tp __a )
```

Return the reciprocal of the Gamma function by series. The reciprocal of the Gamma function is given by


$$\frac{1}{\Gamma(a)} = \sum_{k=1}^{\infty} c_k a^k$$

where the coefficients are defined by recursion:

$$c_{k+1} = \frac{1}{k} \left[\gamma_E c_k + (-1)^k \sum_{j=1}^{k-1} (-1)^j \zeta(j+1-k) c_j \right]$$

where $c_1 = 1$

Parameters

 __a	The argument of the reciprocal of the gamma function.
--	---

Returns

The reciprocal of the gamma function.

Definition at line 2203 of file sf_gamma.tcc.

References __gamma().

Referenced by __gamma(), __gamma_reciprocal(), and __gamma_temme().

8.3.2.166 __gamma_series()

```
template<typename _Tp >
std::pair<_Tp, _Tp> std::__detail::__gamma_series (
    _Tp __a,
    _Tp __x )
```

Return the incomplete gamma function by series summation.

$$\gamma(a, x) = x^a e^{-x} \sum_{k=1}^{\infty} \frac{x^k}{(a)_k}$$

.

Definition at line 2678 of file sf_gamma.tcc.

References __gnu_cxx::__fp_is_integer(), __log_gamma(), __log_gamma_sign(), and std::__detail::_Factorial_table<_Tp>::__n.

Referenced by __gamma(), __gamma_p(), __gamma_q(), __tgamma(), and __tgamma_lower().

8.3.2.167 `__gamma_temme()`

```
template<typename _Tp >
__gnu_cxx::__gamma_temme_t<_Tp> std::__detail::__gamma_temme (
    _Tp __mu )
```

Compute the gamma functions required by the Temme series expansions of $N_\nu(x)$ and $K_\nu(x)$.

$$\Gamma_1 = \frac{1}{2\mu} \left[\frac{1}{\Gamma(1-\mu)} - \frac{1}{\Gamma(1+\mu)} \right]$$

and

$$\Gamma_2 = \frac{1}{2} \left[\frac{1}{\Gamma(1-\mu)} + \frac{1}{\Gamma(1+\mu)} \right]$$

where $-1/2 \leq \mu \leq 1/2$ is $\mu = \nu - N$ and N is the nearest integer to ν . The values of $\Gamma(1+\mu)$ and $\Gamma(1-\mu)$ are returned as well.

The accuracy requirements on this are exquisite.

Parameters

<code>__mu</code>	The input parameter of the gamma functions.
-------------------	---

Returns

An output structure containing four gamma functions.

Definition at line 188 of file `sf_bessel.tcc`.

References `__gamma_reciprocal_series()`.

Referenced by `__cyl_bessel_ik_steel()`, and `__cyl_bessel_jn_steel()`.

8.3.2.168 `__gauss()`

```
template<typename _Tp >
_Tp std::__detail::__gauss (
    _Tp __x )
```

The CDF of the normal distribution. i.e. the integrated lower tail of the normal PDF.

Definition at line 70 of file `sf_owens_t.tcc`.

8.3.2.169 __gegenbauer_recur()

```
template<typename _Tp >
__gnu_cxx::__gegenbauer_t<_Tp> std::__detail::__gegenbauer_recur (
    unsigned int __n,
    _Tp __lambda,
    _Tp __x )
```

Return the Gegenbauer polynomial $C_n^{(\lambda)}(x)$ of degree n and real order λ and argument x .

The Gegenbauer polynomials are generated by a three-term recursion relation:

$$C_n^{(\lambda)}(x) = \frac{1}{n} \left[2x(n + \lambda - 1)C_{n-1}^{(\lambda)}(x) - (n + 2\lambda - 2)C_{n-2}^{(\lambda)}(x) \right]$$

and $C_0^{(\lambda)}(x) = 1$, $C_1^{(\lambda)}(x) = 2\lambda x$. This works for $\lambda > -1/2$

Template Parameters

<code>_Tp</code>	The real type of the argument and order
------------------	---

Parameters

<code>__n</code>	The non-negative integral degree
<code>__lambda</code>	The order of the Gegenbauer polynomial
<code>__x</code>	The real argument

Definition at line 65 of file `sf_gegenbauer.tcc`.

8.3.2.170 __gegenbauer_zeros()

```
template<typename _Tp >
std::vector<__gnu_cxx::__quadrature_point_t<_Tp> > std::__detail::__gegenbauer_zeros (
    unsigned int __n,
    _Tp __lambda )
```

Return a vector containing the zeros of the Gegenbauer or ultraspherical polynomial $C_n^{(\lambda)}$. This works for $\lambda > -1/2$

Template Parameters

<code>_Tp</code>	The real type of the order
------------------	----------------------------

Parameters

<code>in</code>	<code>__n</code>	The degree of the Gegenbauer polynomial
-----------------	------------------	---

Parameters

in	<code>__lambda</code>	The order of the Gegenbauer polynomial
----	-----------------------	--

Definition at line 104 of file `sf_gegenbauer.tcc`.

References `__gnu_cxx::lgamma()`.

8.3.2.171 `__hankel()`

```
template<typename _Tp >
__gnu_cxx::__cyl_hankel_t<std::complex<_Tp>, std::complex<_Tp>, std::complex<_Tp> > std::__↵
detail::__hankel (
    std::complex< _Tp > __nu,
    std::complex< _Tp > __z )
```

Parameters

in	<code>__nu</code>	The order for which the Hankel functions are evaluated.
in	<code>__z</code>	The argument at which the Hankel functions are evaluated.

Returns

A struct containing the cylindrical Hankel functions of the first and second kinds and their derivatives.

Definition at line 1080 of file `sf_hankel.tcc`.

References `__debye_region()`, `__hankel_debye()`, and `__hankel_uniform()`.

Referenced by `__cyl_bessel()`, `__cyl_hankel_1()`, `__cyl_hankel_2()`, `__cyl_neumann()`, and `__sph_hankel()`.

8.3.2.172 `__hankel_debye()`

```
template<typename _Tp >
__gnu_cxx::__cyl_hankel_t<std::complex<_Tp>, std::complex<_Tp>, std::complex<_Tp> > std::__↵
detail::__hankel_debye (
    std::complex< _Tp > __nu,
    std::complex< _Tp > __z,
    std::complex< _Tp > __alpha,
    int __indexr,
    char & __aorb,
    int & __morn )
```

Parameters

in	<code>__nu</code>	The order for which the Hankel functions are evaluated.
in	<code>__z</code>	The argument at which the Hankel functions are evaluated.
in	<code>__alpha</code>	
in	<code>__indexr</code>	
out	<code>__aorb</code>	
out	<code>__morn</code>	

Returns

A struct containing the cylindrical Hankel functions of the first and second kinds and their derivatives.

Definition at line 913 of file `sf_hankel.tcc`.

References `__sin_pi()`.

Referenced by `__hankel()`.

8.3.2.173 `__hankel_params()`

```
template<typename _Tp >
void std::__detail::__hankel_params (
    std::complex< _Tp > __nu,
    std::complex< _Tp > __zhat,
    std::complex< _Tp > & __p,
    std::complex< _Tp > & __p2,
    std::complex< _Tp > & __nup2,
    std::complex< _Tp > & __num2,
    std::complex< _Tp > & __num1d3,
    std::complex< _Tp > & __num2d3,
    std::complex< _Tp > & __num4d3,
    std::complex< _Tp > & __zeta,
    std::complex< _Tp > & __zetaphf,
    std::complex< _Tp > & __zetamhf,
    std::complex< _Tp > & __zetam3hf,
    std::complex< _Tp > & __zetrat )
```

Compute parameters depending on z and nu that appear in the uniform asymptotic expansions of the Hankel functions and their derivatives, except the arguments to the Airy functions.

Definition at line 108 of file `sf_hankel.tcc`.

Referenced by `__hankel_uniform_outer()`.

8.3.2.174 `__hankel_uniform()`

```
template<typename _Tp >
__gnu_cxx::__cyl_hankel_t<std::complex<_Tp>, std::complex<_Tp>, std::complex<_Tp> > std::__↵
detail::__hankel_uniform (
    std::complex< _Tp > __nu,
    std::complex< _Tp > __z )
```

This routine computes the uniform asymptotic approximations of the Hankel functions and their derivatives including a patch for the case when the order equals or nearly equals the argument. At such points, Olver's expressions have zero denominators (and numerators) resulting in numerical problems. This routine averages results from four surrounding points in the complex plane to obtain the result in such cases.

Parameters

in	<code>__nu</code>	The order for which the Hankel functions are evaluated.
in	<code>__z</code>	The argument at which the Hankel functions are evaluated.

Returns

A struct containing the cylindrical Hankel functions of the first and second kinds and their derivatives.

Definition at line 860 of file `sf_hankel.tcc`.

References `__hankel_uniform_olver()`.

Referenced by `__hankel()`.

8.3.2.175 `__hankel_uniform_olver()`

```
template<typename _Tp >
__gnu_cxx::__cyl_hankel_t<std::complex<_Tp>, std::complex<_Tp>, std::complex<_Tp> > std::__↵
detail::__hankel_uniform_olver (
    std::complex< _Tp > __nu,
    std::complex< _Tp > __z )
```

Compute approximate values for the Hankel functions of the first and second kinds using Olver's uniform asymptotic expansion to of order `nu` along with their derivatives.

Parameters

in	<code>__nu</code>	The order for which the Hankel functions are evaluated.
in	<code>__z</code>	The argument at which the Hankel functions are evaluated.

Returns

A struct containing the cylindrical Hankel functions of the first and second kinds and their derivatives.

Definition at line 777 of file sf_hankel.tcc.

References `__hankel_uniform_outer()`, and `__hankel_uniform_sum()`.

Referenced by `__hankel_uniform()`.

8.3.2.176 __hankel_uniform_outer()

```
template<typename _Tp >
void std::__detail::__hankel_uniform_outer (
    std::complex< _Tp > __nu,
    std::complex< _Tp > __z,
    _Tp __eps,
    std::complex< _Tp > & __zhat,
    std::complex< _Tp > & __1dnsq,
    std::complex< _Tp > & __num1d3,
    std::complex< _Tp > & __num2d3,
    std::complex< _Tp > & __p,
    std::complex< _Tp > & __p2,
    std::complex< _Tp > & __etm3h,
    std::complex< _Tp > & __etrat,
    std::complex< _Tp > & __Aip,
    std::complex< _Tp > & __o4dp,
    std::complex< _Tp > & __Aim,
    std::complex< _Tp > & __o4dm,
    std::complex< _Tp > & __od2p,
    std::complex< _Tp > & __od0dp,
    std::complex< _Tp > & __od2m,
    std::complex< _Tp > & __od0dm )
```

Compute outer factors and associated functions of `z` and `nu` appearing in Olver's uniform asymptotic expansions of the Hankel functions of the first and second kinds and their derivatives. The various functions of `z` and `nu` returned by `hankel_uniform_outer` are available for use in computing further terms in the expansions.

Definition at line 247 of file sf_hankel.tcc.

References `__airy_arg()`, and `__hankel_params()`.

Referenced by `__hankel_uniform_olver()`.

8.3.2.177 `__hankel_uniform_sum()`

```

template<typename _Tp >
void std::__detail::__hankel_uniform_sum (
    std::complex< _Tp > __p,
    std::complex< _Tp > __p2,
    std::complex< _Tp > __num2,
    std::complex< _Tp > __zetam3hf,
    std::complex< _Tp > __Aip,
    std::complex< _Tp > __o4dp,
    std::complex< _Tp > __Aim,
    std::complex< _Tp > __o4dm,
    std::complex< _Tp > __od2p,
    std::complex< _Tp > __od0dp,
    std::complex< _Tp > __od2m,
    std::complex< _Tp > __od0dm,
    _Tp __eps,
    std::complex< _Tp > & __H1sum,
    std::complex< _Tp > & __H1psum,
    std::complex< _Tp > & __H2sum,
    std::complex< _Tp > & __H2psum )

```

Compute the sums in appropriate linear combinations appearing in Olver's uniform asymptotic expansions for the Hankel functions of the first and second kinds and their derivatives, using up to `nterms` (less than 5) to achieve relative error `eps`.

Parameters

in	<code>__p</code>	
in	<code>__p2</code>	
in	<code>__num2</code>	
in	<code>__zetam3hf</code>	
in	<code>__Aip</code>	The Airy function value $Ai()$.
in	<code>__o4dp</code>	
in	<code>__Aim</code>	The Airy function value $Ai()$.
in	<code>__o4dm</code>	
in	<code>__od2p</code>	
in	<code>__od0dp</code>	
in	<code>__od2m</code>	
in	<code>__od0dm</code>	
in	<code>__eps</code>	The error tolerance
out	<code>__H1sum</code>	The Hankel function of the first kind.
out	<code>__H1psum</code>	The derivative of the Hankel function of the first kind.
out	<code>__H2sum</code>	The Hankel function of the second kind.
out	<code>__H2psum</code>	The derivative of the Hankel function of the second kind.

Definition at line 324 of file `sf_hankel.tcc`.

Referenced by `__hankel_uniform_olver()`.

8.3.2.178 __harmonic_number()

```
template<typename _Tp >
_Tp std::__detail::__harmonic_number (
    unsigned int __n )
```

Definition at line 3288 of file sf_gamma.tcc.

References std::__detail::_Factorial_table<_Tp>::__n, _S_harmonic_denom, _S_harmonic_numer, and _S_num_↵harmonic_numer.

8.3.2.179 __hermite()

```
template<typename _Tp >
_Tp std::__detail::__hermite (
    unsigned int __n,
    _Tp __x )
```

This routine returns the Hermite polynomial of order n: $H_n(x)$.

The Hermite polynomial is defined by:

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2}$$

An explicit series formula is:

$$H_n(x) = \sum_{k=0}^m \frac{(-1)^k}{k!(n-2k)!} (2x)^{n-2k} \text{ where } m = \left\lfloor \frac{n}{2} \right\rfloor$$

The Hermite polynomial obeys a reflection formula:

$$H_n(-x) = (-1)^n H_n(x)$$

Parameters

$_↵n$	The order of the Hermite polynomial.
$_↵x$	The argument of the Hermite polynomial.

Returns

The value of the Hermite polynomial of order n and argument x.

Definition at line 212 of file sf_hermite.tcc.

References __hermite_asymp(), and __hermite_recur().

8.3.2.180 `__hermite_asymp()`

```
template<typename _Tp >
_Tp std::__detail::__hermite_asymp (
    unsigned int __n,
    _Tp __x )
```

This routine returns the Hermite polynomial of large order n : $H_n(x)$. We assume here that $x \geq 0$.

The Hermite polynomial is defined by:

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2}$$

See also

"Asymptotic analysis of the Hermite polynomials from their differential-difference equation", Diego Dominici, arXiv:math/0601078v1 [math.CA] 4 Jan 2006

Parameters

<code>__n</code>	The order of the Hermite polynomial.
<code>__x</code>	The argument of the Hermite polynomial.

Returns

The value of the Hermite polynomial of order n and argument x .

Definition at line 143 of file `sf_hermite.tcc`.

References `__airy()`.

Referenced by `__hermite()`.

8.3.2.181 `__hermite_recur()`

```
template<typename _Tp >
__gnu_cxx::__hermite_t<_Tp> std::__detail::__hermite_recur (
    unsigned int __n,
    _Tp __x )
```

This routine returns the Hermite polynomial of order n : $H_n(x)$ by recursion on n .

The Hermite polynomial is defined by:

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2}$$

The Hermite polynomial has first and second derivatives:

$$H'_n(x) = 2nH_{n-1}(x)$$

and

$$H''_n(x) = 4n(n-1)H_{n-2}(x)$$

The Physicists Hermite polynomials have highest-order coefficient 2^n and are orthogonal with respect to the weight function

$$w(x) = e^{-x^2}$$

Parameters

\leftarrow __n	The order of the Hermite polynomial.
\leftarrow __x	The argument of the Hermite polynomial.

Returns

The value of the Hermite polynomial of order n and argument x.

Todo Find the sign of Hermite blowup values.

Definition at line 86 of file sf_hermite.tcc.

Referenced by __hermite().

8.3.2.182 __hermite_zeros()

```
template<typename _Tp>
std::vector<__gnu_cxx::__quadrature_point_t<_Tp> > std::__detail::__hermite_zeros (
    unsigned int __n,
    _Tp __proto = _Tp{} )
```

Build a vector of the Gauss-Hermite integration rule abscissae and weights.

Definition at line 289 of file sf_hermite.tcc.

8.3.2.183 `__heuman_lambda()`

```
template<typename _Tp >
_Tp std::__detail::__heuman_lambda (
    _Tp __k,
    _Tp __phi )
```

Return the Heuman lambda function.

Definition at line 1008 of file `sf_ellint.tcc`.

References `__comp_ellint_1()`, `__ellint_1()`, `__ellint_rf()`, `__ellint_rj()`, and `__jacobi_zeta()`.

8.3.2.184 `__hurwitz_zeta()`

```
template<typename _Tp >
_Tp std::__detail::__hurwitz_zeta (
    _Tp __s,
    _Tp __a )
```

Return the Hurwitz zeta function $\zeta(s, a)$ for all $s \neq 1$ and $a > -1$.

The Hurwitz zeta function is defined by:

$$\zeta(s, a) = \sum_{n=0}^{\infty} \frac{1}{(n+a)^s}$$

The Riemann zeta function is a special case:

$$\zeta(s) = \zeta(s, 1)$$

Parameters

\leftarrow <code>__s</code>	The order $s \neq 1$
\leftarrow <code>__a</code>	The scale parameter $a > -1$

Definition at line 871 of file `sf_zeta.tcc`.

References `__hurwitz_zeta_euler_maclaurin()`, and `__riemann_zeta()`.

Referenced by `__digamma()`, and `__polygamma()`.

8.3.2.185 __hurwitz_zeta_euler_maclaurin()

```
template<typename _Tp >
_Tp std::__detail::__hurwitz_zeta_euler_maclaurin (
    _Tp __s,
    _Tp __a )
```

Return the Hurwitz zeta function $\zeta(s, a)$ for all $s \neq 1$ and $a > -1$.

See also

An efficient algorithm for accelerating the convergence of oscillatory series, useful for computing the polylogarithm and Hurwitz zeta functions, Linas Vep"0160tas

Parameters

\leftarrow __s	The order $s \neq 1$
\leftarrow __a	The scale parameter $a > -1$

Definition at line 823 of file sf_zeta.tcc.

References `_S_Euler_Maclaurin_zeta`.

Referenced by `__hurwitz_zeta()`.

8.3.2.186 __hurwitz_zeta_polylog()

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__hurwitz_zeta_polylog (
    _Tp __s,
    std::complex<_Tp> __a )
```

Return the Hurwitz Zeta function for real s and complex a . This uses Jonquiere's identity:

$$\frac{(i2\pi)^s}{\Gamma(s)} \zeta(a, 1-s) = Li_s(e^{i2\pi a}) + (-1)^s Li_s(e^{-i2\pi a})$$

Parameters

\leftarrow __s	The real argument
\leftarrow __a	The complex parameter

Todo This `__hurwitz_zeta_polylog` prefactor is prone to overflow. positive integer orders s ?

Definition at line 1127 of file `sf_polylog.tcc`.

References `__gamma()`, and `__polylog_exp()`.

8.3.2.187 `__hydrogen()`

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__hydrogen (
    unsigned int __n,
    unsigned int __l,
    unsigned int __m,
    _Tp __Z,
    _Tp __r,
    _Tp __theta,
    _Tp __phi )
```

Return the bound-state Coulomb wave-function.

Definition at line 248 of file `sf_coulomb.tcc`.

References `__assoc_laguerre()`, `__log_gamma()`, and `__sph_legendre()`.

8.3.2.188 `__hyperg()`

```
template<typename _Tp >
_Tp std::__detail::__hyperg (
    _Tp __a,
    _Tp __b,
    _Tp __c,
    _Tp __x )
```

Return the hypergeometric function ${}_2F_1(a, b; c; x)$.

The hypergeometric function is defined by

$${}_2F_1(a, b; c; x) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)} \sum_{n=0}^{\infty} \frac{\Gamma(a+n)\Gamma(b+n)}{\Gamma(c+n)} \frac{x^n}{n!}$$

Parameters

<code>__a</code>	The first <i>numerator</i> parameter.
<code>__b</code>	The second <i>numerator</i> parameter.
<code>__c</code>	The <i>denominator</i> parameter.
<code>__x</code>	The argument of the confluent hypergeometric function.

Returns

The confluent hypergeometric function.

Definition at line 927 of file sf_hyperg.tcc.

References `__gnu_cxx::__fp_is_integer()`, `__hyperg_luke()`, `__hyperg_reflect()`, `__hyperg_series()`, `__log_gamma()`, and `__log_gamma_sign()`.

Referenced by `__legendre_q_series()`.

8.3.2.189 __hyperg_luke()

```
template<typename _Tp >
_Tp std::__detail::__hyperg_luke (
    _Tp __a,
    _Tp __b,
    _Tp __c,
    _Tp __xin )
```

Return the hypergeometric function ${}_2F_1(a, b; c; x)$ by an iterative procedure described in Luke, Algorithms for the Computation of Mathematical Functions.

Definition at line 501 of file sf_hyperg.tcc.

Referenced by `__hyperg()`.

8.3.2.190 __hyperg_recur()

```
template<typename _Tp >
_Tp std::__detail::__hyperg_recur (
    int __m,
    _Tp __b,
    _Tp __c,
    _Tp __x )
```

Return the hypergeometric polynomial ${}_2F_1(-m, b; c; x)$ by Holm recursion.

The hypergeometric function is defined by

$${}_2F_1(-m, b; c; x) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)} \sum_{n=0}^{\infty} \frac{\Gamma(n-m)\Gamma(b+n)}{\Gamma(c+n)} \frac{x^n}{n!}$$

Parameters

\leftarrow _m	The first <i>numerator</i> parameter.
\leftarrow _b	The second <i>numerator</i> parameter.
\leftarrow _c	The <i>denominator</i> parameter.
\leftarrow _x	The argument of the confluent hypergeometric function.

Returns

The confluent hypergeometric function.

: go recur!

Definition at line 478 of file sf_hyperg.tcc.

8.3.2.191 __hyperg_reflect()

```
template<typename _Tp >
_Tp std::__detail::__hyperg_reflect (
    _Tp __a,
    _Tp __b,
    _Tp __c,
    _Tp __x )
```

Return the hypergeometric function ${}_2F_1(a, b; c; x)$ by the reflection formulae in Abramowitz & Stegun formula 15.3.6 for $d = c - a - b$ not integral and formula 15.3.11 for $d = c - a - b$ integral. This assumes $a, b, c \neq$ negative integer.

The hypergeometric function is defined by

$${}_2F_1(a, b; c; x) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)} \sum_{n=0}^{\infty} \frac{\Gamma(a+n)\Gamma(b+n)}{\Gamma(c+n)} \frac{x^n}{n!}$$

The reflection formula for nonintegral $d = c - a - b$ is:

$${}_2F_1(a, b; c; x) = \frac{\Gamma(c)\Gamma(d)}{\Gamma(c-a)\Gamma(c-b)} {}_2F_1(a, b; 1-d; 1-x) + \frac{\Gamma(c)\Gamma(-d)}{\Gamma(a)\Gamma(b)} {}_2F_1(c-a, c-b; 1+d; 1-x)$$

The reflection formula for integral $m = c - a - b$ is:

$${}_2F_1(a, b; a+b+m; x) = \frac{\Gamma(m)\Gamma(a+b+m)}{\Gamma(a+m)\Gamma(b+m)} \sum_{k=0}^{m-1} \frac{(m+a)_k(m+b)_k}{k!(1-m)_k} (1-x)^k + (-1)^m$$

Definition at line 637 of file sf_hyperg.tcc.

References __digamma(), __gnu_cxx::__fp_is_integer(), __hyperg_series(), __log_gamma(), and __log_gamma_ \leftarrow sign().

Referenced by __hyperg().

8.3.2.192 __hyperg_series()

```
template<typename _Tp >
_Tp std::__detail::__hyperg_series (
    _Tp __a,
    _Tp __b,
    _Tp __c,
    _Tp __x )
```

Return the hypergeometric function ${}_2F_1(a, b; c; x)$ by series expansion.

The hypergeometric function is defined by

$${}_2F_1(a, b; c; x) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)} \sum_{n=0}^{\infty} \frac{\Gamma(a+n)\Gamma(b+n)}{\Gamma(c+n)} \frac{x^n}{n!}$$

This works and it's pretty fast.

Parameters

\leftrightarrow __a	The first <i>numerator</i> parameter.
\leftrightarrow __b	The second <i>numerator</i> parameter.
\leftrightarrow __c	The <i>denominator</i> parameter.
\leftrightarrow __x	The argument of the confluent hypergeometric function.

Returns

The confluent hypergeometric function.

Definition at line 430 of file sf_hyperg.tcc.

Referenced by __hyperg(), and __hyperg_reflect().

8.3.2.193 __ibeta_cont_frac()

```
template<typename _Tp >
_Tp std::__detail::__ibeta_cont_frac (
    _Tp __a,
    _Tp __b,
    _Tp __x )
```

Return the regularized incomplete beta function, $I_x(a, b)$, of arguments a, b, and x.

Parameters

\leftrightarrow _a	The first parameter
\leftrightarrow _b	The second parameter
\leftrightarrow _x	The argument

Definition at line 239 of file sf_beta.tcc.

Referenced by __beta_inc().

8.3.2.194 __jacobi_ellint()

```
template<typename _Tp >
__gnu_cxx::__jacobi_ellint_t<_Tp> std::__detail::__jacobi_ellint (
    _Tp __k,
    _Tp __u )
```

Return a structure containing the three primary Jacobi elliptic functions: $sn(k, u)$, $cn(k, u)$, $dn(k, u)$.

Parameters

\leftrightarrow _k	The elliptic modulus $ k < 1$.
\leftrightarrow _u	The argument.

Returns

An object containing the three principal Jacobi elliptic functions, $sn(k, u)$, $cn(k, u)$, $dn(k, u)$ and the means to compute the remaining nine as well as the amplitude.

Definition at line 1649 of file sf_theta.tcc.

8.3.2.195 __jacobi_recur()

```
template<typename _Tp >
__gnu_cxx::__jacobi_t<_Tp> std::__detail::__jacobi_recur (
    unsigned int __n,
    _Tp __alpha1,
```

```

    _Tp __beta1,
    _Tp __x )

```

Compute the Jacobi polynomial by recursion on n :

$$2n(\alpha+\beta+n)(\alpha+\beta+2n-2)P_n^{(\alpha,\beta)}(x) = (\alpha+\beta+2n-1)((\alpha^2-\beta^2)+x(\alpha+\beta+2n-2)(\alpha+\beta+2n))P_{n-1}^{(\alpha,\beta)}(x) - 2(\alpha+n-1)(\beta+n-1)(\alpha+\beta+2n-2)P_{n-2}^{(\alpha,\beta)}(x)$$

This works for $\alpha, \beta > -1$

Template Parameters

<code>_Tp</code>	The real type of the argument
------------------	-------------------------------

Parameters

in	<code>__n</code>	The degree of the Jacobi polynomial
in	<code>__alpha1</code>	The first order parameter of the Jacobi polynomial
in	<code>__beta1</code>	The second order parameter of the Jacobi polynomial
in	<code>__x</code>	The argument

Definition at line 68 of file `sf_jacobi.tcc`.

Referenced by `__radial_jacobi()`.

8.3.2.196 __jacobi_theta_1() [1/2]

```

template<typename _Tp >
std::complex<_Tp> std::__detail::__jacobi_theta_1 (
    std::complex<_Tp > __q,
    std::complex<_Tp > __x )

```

Return the Jacobi θ_1 function by summation of the series.

The Jacobi or elliptic theta function is defined by

$$\theta_1(q, x) = 2 \sum_{n=1}^{\infty} (-1)^n q^{(n+\frac{1}{2})^2} \sin(2n+1)x$$

Regarding the nome and the theta function as functions of the lattice parameter $\tau - i \log(q)/\pi$ or $q = e^{i\pi\tau}$ the lattice parameter is transformed to maximize its imaginary part:

$$\theta_1(\tau+1, x) = -ie^{i\pi/4} \theta_1(\tau, x)$$

and

$$\sqrt{-i\tau} \theta_1(\tau, x) = e^{(i\tau x^2/\pi)} \theta_1(\tau', \tau' x)$$

where the new lattice parameter is $\tau' = -1/\tau$.

The argument is reduced with

$$\theta_1(q, x + (m+n\tau)\pi) = (-1)^{m+n} q^{-n^2} e^{-2inx} \theta_1(q, x)$$

Parameters

\leftarrow _q	The elliptic nome, $ q < 1$.
\leftarrow _x	The argument.

Definition at line 980 of file sf_theta.tcc.

References `__jacobi_theta_1_prod()`, `__jacobi_theta_1_sum()`, `__polar_pi()`, `std::__detail::__jacobi_lattice_t< _Tp_ \leftarrow Omega1, _Tp_Omega3 >::__reduce()`, `std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::__tau()`, and `std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::__S_pi`.

Referenced by `__jacobi_theta_1()`.

8.3.2.197 `__jacobi_theta_1()` [2/2]

```
template<typename _Tp >
_Tp std::__detail::__jacobi_theta_1 (
    _Tp _q,
    const _Tp _x )
```

Return the Jacobi θ_1 function for real nome and argument.

The Jacobi or elliptic theta function is defined by

$$\theta_1(q, x) = 2 \sum_{n=1}^{\infty} (-1)^n q^{(n+\frac{1}{2})^2} \sin(2n+1)x$$

Parameters

\leftarrow _q	The elliptic nome, $ q < 1$.
\leftarrow _x	The argument.

Definition at line 1048 of file sf_theta.tcc.

References `__jacobi_theta_1()`.

8.3.2.198 `__jacobi_theta_1_prod()`

```
template<typename _Tp >
_Tp std::__detail::__jacobi_theta_1_prod (
```

```

    _Tp __q,
    _Tp __x )

```

Return the Jacobi θ_1 function by accumulation of the product.

The Jacobi or elliptic theta-1 function is defined by

$$\theta_1(q, x) = 2q^{1/4} \sin(x) \prod_{n=1}^{\infty} (1 - q^{2n})(1 - 2q^{2n} \cos(2x) + q^{4n})$$

Parameters

\leftrightarrow _q	The elliptic nome, $ q < 1$.
\leftrightarrow _x	The argument.

Definition at line 923 of file sf_theta.tcc.

Referenced by __jacobi_theta_1().

8.3.2.199 __jacobi_theta_1_sum()

```

template<typename _Tp >
_Tp std::__detail::__jacobi_theta_1_sum (
    _Tp __q,
    _Tp __x )

```

Return the Jacobi θ_1 function by summation of the series.

The Jacobi or elliptic theta-1 function is defined by

$$\theta_1(q, x) = 2 \sum_{n=1}^{\infty} (-1)^n q^{(n+\frac{1}{2})^2} \sin(2n+1)x$$

Parameters

\leftrightarrow _q	The elliptic nome, $ q < 1$.
\leftrightarrow _x	The argument.

Definition at line 888 of file sf_theta.tcc.

Referenced by __jacobi_theta_1().

8.3.2.200 `__jacobi_theta_2()` [1/2]

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__jacobi_theta_2 (
    std::complex< _Tp > __q,
    std::complex< _Tp > __x )
```

Return the Jacobi θ_2 function by summation of the series.

The Jacobi or elliptic theta function is defined by

$$\theta_2(q, x) = 2 \sum_{n=1}^{\infty} q^{(n+\frac{1}{2})^2} \cos(2n+1)x$$

Regarding the nome and the theta function as functions of the lattice parameter $\tau = i \log(q)/\pi$ or $q = e^{i\pi\tau}$ the lattice parameter is transformed to maximize its imaginary part:

$$\theta_2(\tau + 1, x) = e^{i\pi/4} \theta_2(\tau, x)$$

and

$$\sqrt{-i\tau} \theta_2(\tau, x) = e^{(i\tau x^2/\pi)} \theta_4(\tau', \tau'x)$$

where the new lattice parameter is $\tau' = -1/\tau$.

The argument is reduced with

$$\theta_2(q, x + (m + n\tau)\pi) = (-1)^m q^{-n^2} e^{-2inx} \theta_2(q, x)$$

Parameters

\leftarrow __q	The elliptic nome, $ q < 1$.
\leftarrow __x	The argument.

Definition at line 1176 of file sf_theta.tcc.

References `__jacobi_theta_2_prod()`, `__jacobi_theta_2_sum()`, `__jacobi_theta_4_sum()`, `__polar_pi()`, `std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::__reduce()`, `std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::__tau()`, `std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::__S_pi`, and `std::__detail::__jacobi_theta_0_t< _Tp1, _Tp3 >::th2`.

Referenced by `__jacobi_theta_2()`.

8.3.2.201 __jacobi_theta_2() [2/2]

```
template<typename _Tp >
_Tp std::__detail::__jacobi_theta_2 (
    _Tp __q,
    const _Tp __x )
```

Return the Jacobi θ_2 function for real nome and argument.

The Jacobi or elliptic theta function is defined by

$$\theta_2(q, x) = 2 \sum_{n=1}^{\infty} q^{(n+\frac{1}{2})^2} \cos(2n+1)x$$

Parameters

\leftarrow _q	The elliptic nome, $ q < 1$.
\leftarrow _x	The argument.

Definition at line 1249 of file sf_theta.tcc.

References __jacobi_theta_2().

8.3.2.202 __jacobi_theta_2_prod()

```
template<typename _Tp >
_Tp std::__detail::__jacobi_theta_2_prod (
    _Tp __q,
    _Tp __x )
```

Return the Jacobi θ_2 function by accumulation of the product.

The Jacobi or elliptic theta-2 function is defined by

$$\theta_2(q, x) = 2q^{1/4} \sin(x) \prod_{n=1}^{\infty} (1 - q^{2n})(1 + 2q^{2n} \cos(2x) + q^{4n})$$

Parameters

\leftarrow _q	The elliptic nome, $ q < 1$.
\leftarrow _x	The argument.

Definition at line 1109 of file sf_theta.tcc.

References `__jacobi_theta_4_prod()`, and `__jacobi_theta_4_sum()`.

Referenced by `__jacobi_theta_2()`.

8.3.2.203 `__jacobi_theta_2_sum()`

```
template<typename _Tp >
_Tp std::__detail::__jacobi_theta_2_sum (
    _Tp __q,
    _Tp __x )
```

Return the Jacobi θ_2 function by summation of the series.

The Jacobi or elliptic theta-2 function is defined by

$$\theta_2(q, x) = 2 \sum_{n=1}^{\infty} q^{(n+\frac{1}{2})^2} \cos(2n+1)x$$

Parameters

\leftarrow <code>__q</code>	The elliptic nome, $ q < 1$.
\leftarrow <code>__x</code>	The argument.

Definition at line 1077 of file sf_theta.tcc.

Referenced by `__jacobi_theta_2()`, and `__jacobi_theta_4()`.

8.3.2.204 `__jacobi_theta_3()` [1/2]

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__jacobi_theta_3 (
    std::complex<_Tp> __q,
    std::complex<_Tp> __x )
```

Return the Jacobi θ_3 function by summation of the series.

The Jacobi or elliptic theta function is defined by

$$\theta_3(q, x) = 1 + 2 \sum_{n=1}^{\infty} q^{n^2} \cos 2nx$$

Regarding the nome and the theta function as functions of the lattice parameter $\tau - i\log(q)/\pi$ or $q = e^{i\pi\tau}$ the lattice parameter is transformed to maximize its imaginary part:

$$\theta_3(\tau + 1, x) = \theta_3(\tau, x)$$

and

$$\sqrt{-i\tau}\theta_3(\tau, x) = e^{(i\tau x^2/\pi)}\theta_3(\tau', \tau'x)$$

where the new lattice parameter is $\tau' = -1/\tau$.

The argument is reduced with

$$\theta_3(q, x + (m + n\tau)\pi) = q^{-n^2} e^{-2inx} \theta_3(q, x)$$

Parameters

\hookleftarrow _q	The elliptic nome, $ q < 1$.
\hookleftarrow _x	The argument.

Definition at line 1365 of file sf_theta.tcc.

References `__jacobi_theta_3_prod()`, `__jacobi_theta_3_sum()`, `std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::__reduce()`, `std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::__tau()`, `std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::__S_pi`, and `std::__detail::__jacobi_theta_0_t< _Tp1, _Tp3 >::th3`.

Referenced by `__jacobi_theta_3()`.

8.3.2.205 __jacobi_theta_3() [2/2]

```
template<typename _Tp >
_Tp std::__detail::__jacobi_theta_3 (
    _Tp __q,
    const _Tp __x )
```

Return the Jacobi θ_3 function for real nome and argument.

The Jacobi or elliptic theta function is defined by

$$\theta_3(q, x) = 1 + 2 \sum_{n=1}^{\infty} q^{n^2} \cos 2nx$$

Parameters

\hookleftarrow _q	The elliptic nome, $ q < 1$.
\hookleftarrow _x	The argument.

Definition at line 1433 of file sf_theta.tcc.

References `__jacobi_theta_3()`.

8.3.2.206 `__jacobi_theta_3_prod()`

```
template<typename _Tp >
_Tp std::__detail::__jacobi_theta_3_prod (
    _Tp __q,
    _Tp __x )
```

Return the Jacobi θ_3 function by accumulation of the product.

The Jacobi or elliptic theta-3 function is defined by

$$\theta_3(q, x) = \prod_{n=1}^{\infty} (1 - q^{2n})(1 + 2q^{2n-1} \cos(2x) + q^{4n-2})$$

Parameters

\leftarrow <code>__q</code>	The elliptic nome, $ q < 1$.
\leftarrow <code>__x</code>	The argument.

Definition at line 1309 of file sf_theta.tcc.

Referenced by `__jacobi_theta_3()`.

8.3.2.207 `__jacobi_theta_3_sum()`

```
template<typename _Tp >
_Tp std::__detail::__jacobi_theta_3_sum (
    _Tp __q,
    _Tp __x )
```

Return the Jacobi θ_3 function by summation of the series.

The Jacobi or elliptic theta-3 function is defined by

$$\theta_3(q, x) = 1 + 2 \sum_{n=1}^{\infty} q^{n^2} \cos 2nx$$

Parameters

\leftrightarrow _q	The elliptic nome, $ q < 1$.
\leftrightarrow _x	The argument.

Definition at line 1277 of file sf_theta.tcc.

Referenced by __jacobi_theta_3().

8.3.2.208 __jacobi_theta_4() [1/2]

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__jacobi_theta_4 (
    std::complex<_Tp> _q,
    std::complex<_Tp> _x )
```

Return the Jacobi θ_4 function by summation of the series.

The Jacobi or elliptic theta-4 function is defined by

$$\theta_4(q, x) = 1 + 2 \sum_{n=1}^{\infty} (-1)^n q^{n^2} \cos 2nx$$

Regarding the nome and the theta function as functions of the lattice parameter $\tau - i \log(q)/\pi$ or $q = e^{i\pi\tau}$ the lattice parameter is transformed to maximize its imaginary part:

$$\theta_4(\tau + 1, x) = \theta_4(\tau, x)$$

and

$$\sqrt{-i\tau} \theta_4(\tau, x) = e^{(i\tau x^2/\pi)} \theta_2(\tau', \tau' x)$$

where the new lattice parameter is $\tau' = -1/\tau$.

The argument is reduced with

$$\theta_4(q, z + (m + n\tau)\pi) = (-1)^n q^{-n^2} e^{-2inz} \theta_4(q, z)$$

Parameters

\leftrightarrow _q	The elliptic nome, $ q < 1$.
\leftrightarrow _x	The argument.

Definition at line 1551 of file sf_theta.tcc.

References `__jacobi_theta_2_sum()`, `__jacobi_theta_4_prod()`, `__jacobi_theta_4_sum()`, `std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::__reduce()`, `std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::__tau()`, `std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::__S_pi`, and `std::__detail::__jacobi_theta_0_t< _Tp1, _Tp3 >::th4`.

Referenced by `__jacobi_theta_4()`.

8.3.2.209 `__jacobi_theta_4()` [2/2]

```
template<typename _Tp >
_Tp std::__detail::__jacobi_theta_4 (
    _Tp __q,
    const _Tp __x )
```

Return the Jacobi θ_4 function for real nome and argument.

The Jacobi or elliptic theta function is defined by

$$\theta_4(q, x) = 1 + 2 \sum_{n=1}^{\infty} (-1)^n q^{n^2} \cos 2nx$$

Parameters

<code>__q</code>	The elliptic nome, $ q < 1$.
<code>__x</code>	The argument.

Definition at line 1622 of file sf_theta.tcc.

References `__jacobi_theta_4()`.

8.3.2.210 `__jacobi_theta_4_prod()`

```
template<typename _Tp >
_Tp std::__detail::__jacobi_theta_4_prod (
    _Tp __q,
    _Tp __x )
```

Return the Jacobi θ_4 function by accumulation of the product.

The Jacobi or elliptic theta-4 function is defined by

$$\theta_4(q, x) = \prod_{n=1}^{\infty} (1 - q^{2n})(1 - 2q^{2n-1} \cos(2x) + q^{4n-2})$$

Parameters

\leftrightarrow _q	The elliptic nome, $ q < 1$.
\leftrightarrow _x	The argument.

Definition at line 1495 of file sf_theta.tcc.

Referenced by __jacobi_theta_2_prod(), and __jacobi_theta_4().

8.3.2.211 __jacobi_theta_4_sum()

```
template<typename _Tp >
_Tp std::__detail::__jacobi_theta_4_sum (
    _Tp __q,
    _Tp __x )
```

Return the Jacobi θ_4 function by summation of the series.

The Jacobi or elliptic theta function is defined by

$$\theta_4(q, x) = 1 + 2 \sum_{n=1}^{\infty} (-1)^n q^{n^2} \cos 2nx$$

Parameters

\leftrightarrow _q	The elliptic nome, $ q < 1$.
\leftrightarrow _x	The argument.

Definition at line 1461 of file sf_theta.tcc.

Referenced by __jacobi_theta_2(), __jacobi_theta_2_prod(), and __jacobi_theta_4().

8.3.2.212 __jacobi_zeros()

```
template<typename _Tp >
std::vector<__gnu_cxx::__quadrature_point_t<_Tp> > std::__detail::__jacobi_zeros (
    unsigned int __n,
    _Tp __alpha1,
    _Tp __beta1 )
```

Return a vector containing the zeros of the Jacobi polynomial $P_n^{(\alpha, \beta)}(x)$. This works for $\alpha, \beta > -1$.

Template Parameters

<code>_Tp</code>	The real type of the parameters
------------------	---------------------------------

Parameters

in	<code>__n</code>	The degree of the Jacobi polynomial
in	<code>__alpha1</code>	The first order parameter of the Jacobi polynomial
in	<code>__beta1</code>	The second order parameter of the Jacobi polynomial

Definition at line 142 of file `sf_jacobi.tcc`.

References `__gnu_cxx::lgamma()`.

Referenced by `__radial_jacobi_zeros()`.

8.3.2.213 `__jacobi_zeta()`

```
template<typename _Tp >
_Tp std::__detail::__jacobi_zeta (
    _Tp __k,
    _Tp __phi )
```

Return the Jacobi zeta function.

Definition at line 971 of file `sf_ellint.tcc`.

References `__comp_ellint_1()`, and `__ellint_rj()`.

Referenced by `__heuman_lambda()`.

8.3.2.214 `__kolmogorov_p()`

```
template<typename _Tp >
_Tp std::__detail::__kolmogorov_p (
    _Tp __a,
    _Tp __b,
    _Tp __x )
```

$$P(K \leq x) = 1 - e^{-2x^2} + e^{-2 \cdot 4x^2} + e^{-2 \cdot 9x^2} - e^{-2 \cdot 16x^2} + \dots$$

Definition at line 723 of file `sf_distributions.tcc`.

8.3.2.215 __laguerre() [1/2]

```
template<typename _Tpa , typename _Tp >
_Tp std::__detail::__laguerre (
    unsigned int __n,
    _Tpa __alpha1,
    _Tp __x )
```

This routine returns the associated Laguerre polynomial of degree n , order α : $L_n^{(\alpha)}(x)$.

The associated Laguerre function is defined by

$$L_n^{(\alpha)}(x) = \frac{(\alpha + 1)_n}{n!} {}_1F_1(-n; \alpha + 1; x)$$

where $(\alpha)_n$ is the Pochhammer symbol and ${}_1F_1(a; c; x)$ is the confluent hypergeometric function.

The associated Laguerre polynomial is defined for integral order $\alpha = m$ by:

$$L_n^{(m)}(x) = (-1)^m \frac{d^m}{dx^m} L_{n+m}(x)$$

where the Laguerre polynomial is defined by:

$$L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n} (x^n e^{-x})$$

Template Parameters

<code>_Tpa</code>	The type of the order.
<code>_Tp</code>	The type of the parameter.

Parameters

<code>__n</code>	The degree of the Laguerre function.
<code>__alpha1</code>	The order of the Laguerre function.
<code>__x</code>	The argument of the Laguerre function.

Returns

The value of the Laguerre function of degree n , order α , and argument x .

Definition at line 316 of file `sf_laguerre.tcc`.

References `__laguerre_hyperg()`, `__laguerre_large_n()`, and `__laguerre_recur()`.

8.3.2.216 `__laguerre()` [2/2]

```
template<typename _Tp >
_Tp std::__detail::__laguerre (
    unsigned int __n,
    _Tp __x )
```

This routine returns the Laguerre polynomial of degree n: $L_n(x)$.

The Laguerre polynomial is defined by:

$$L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n} (x^n e^{-x})$$

Parameters

<code>__n</code>	The degree of the Laguerre polynomial.
<code>__x</code>	The argument of the Laguerre polynomial.

Returns

The value of the Laguerre polynomial of order n and argument x.

Definition at line 387 of file `sf_laguerre.tcc`.

8.3.2.217 `__laguerre_hyperg()`

```
template<typename _Tpa , typename _Tp >
_Tp std::__detail::__laguerre_hyperg (
    unsigned int __n,
    _Tpa __alpha1,
    _Tp __x )
```

Evaluate the polynomial based on the confluent hypergeometric function in a safe way, with no restriction on the arguments.

The associated Laguerre function is defined by

$$L_n^{(\alpha)}(x) = \frac{(\alpha + 1)_n}{n!} {}_1F_1(-n; \alpha + 1; x)$$

where $(\alpha)_n$ is the Pochhammer symbol and ${}_1F_1(a; c; x)$ is the confluent hypergeometric function.

This function assumes $x \neq 0$.

This is from the GNU Scientific Library.

Template Parameters

<code>_Tpa</code>	The type of the order.
<code>_Tp</code>	The type of the parameter.

Parameters

<code>__n</code>	The degree of the Laguerre function.
<code>__alpha1</code>	The order of the Laguerre function.
<code>__x</code>	The argument of the Laguerre function.

Returns

The value of the Laguerre function of degree n , order α , and argument x .

Definition at line 131 of file `sf_laguerre.tcc`.

Referenced by `__laguerre()`.

8.3.2.218 `__laguerre_large_n()`

```
template<typename _Tpa , typename _Tp >
_Tp std::__detail::__laguerre_large_n (
    unsigned __n,
    _Tpa __alpha1,
    _Tp __x )
```

This routine returns the associated Laguerre polynomial of degree n , order $\alpha > -1$ for large n . Abramowitz & Stegun, 13.5.21.

Template Parameters

<code>_Tpa</code>	The type of the order.
<code>_Tp</code>	The type of the parameter.

Parameters

<code>__n</code>	The degree of the Laguerre function.
<code>__alpha1</code>	The order of the Laguerre function.
<code>__x</code>	The argument of the Laguerre function.

Returns

The value of the Laguerre function of degree n , order α , and argument x .

This is from the GNU Scientific Library.

Definition at line 75 of file `sf_laguerre.tcc`.

References `__log_gamma()`, and `__sin_pi()`.

Referenced by `__laguerre()`.

8.3.2.219 __laguerre_recur()

```
template<typename _Tpa , typename _Tp >
__gnu_cxx::__laguerre_t<_Tpa, _Tp> std::__detail::__laguerre_recur (
    unsigned int __n,
    _Tpa __alpha1,
    _Tp __x )
```

This routine returns the associated Laguerre polynomial of degree n , order α : $L_n^{(\alpha)}(x)$ by recursion.

The associated Laguerre function is defined by

$$L_n^{(\alpha)}(x) = \frac{(\alpha+1)_n}{n!} {}_1F_1(-n; \alpha+1; x)$$

where $(\alpha)_n$ is the Pochhammer symbol and ${}_1F_1(a; c; x)$ is the confluent hypergeometric function.

The associated Laguerre polynomial is defined for integral order $\alpha = m$ by:

$$L_n^{(m)}(x) = (-1)^m \frac{d^m}{dx^m} L_{n+m}(x)$$

where the Laguerre polynomial is defined by:

$$L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n} (x^n e^{-x})$$

Template Parameters

<code>_Tpa</code>	The type of the order.
<code>_Tp</code>	The type of the parameter.

Parameters

<code>__n</code>	The degree of the Laguerre function.
<code>__alpha1</code>	The order of the Laguerre function.
<code>__x</code>	The argument of the Laguerre function.

Returns

The value of the Laguerre function of order n , degree α , and argument x .

Definition at line 189 of file sf_laguerre.tcc.

Referenced by __laguerre().

8.3.2.220 __laguerre_zeros()

```
template<typename _Tp >
std::vector<__gnu_cxx::__quadrature_point_t<_Tp> > std::__detail::__laguerre_zeros (
    unsigned int __n,
    _Tp __alpha1 )
```

Return an array of abscissae and weights for the Gauss-Laguerre rule.

Definition at line 225 of file sf_laguerre.tcc.

References __gnu_cxx::lgamma().

8.3.2.221 __lah() [1/2]

```
template<typename _Tp >
_Tp std::__detail::__lah (
    unsigned int __n,
    unsigned int __k ) [inline]
```

Return the Lah number. Lah numbers are defined by downward recurrence:

$$L(n, k - 1) = \frac{k(k - 1)}{n - k + 1} L(n, k); L(n, n) = 1$$

Definition at line 440 of file sf_stirling.tcc.

8.3.2.222 __lah() [2/2]

```
template<typename _Tp >
std::vector<_Tp> std::__detail::__lah (
    unsigned int __n ) [inline]
```

Return a vector of Lah numbers. Lah numbers are defined by downward recurrence:

$$L(n, k - 1) = \frac{k(k - 1)}{n - k + 1} L(n, k); L(n, n) = 1$$

Definition at line 478 of file sf_stirling.tcc.

8.3.2.223 `__lah_recur()` [1/2]

```
template<typename _Tp >
_Tp std::__detail::__lah_recur (
    unsigned int __n,
    unsigned int __k )
```

Return the Lah number by downward recurrence:

$$L(n, k - 1) = \frac{k(k - 1)}{n - k + 1} L(n, k); L(n, n) = 1$$

Definition at line 416 of file `sf_stirling.tcc`.

8.3.2.224 `__lah_recur()` [2/2]

```
template<typename _Tp >
std::vector<_Tp> std::__detail::__lah_recur (
    unsigned int __n )
```

Return a vector of Lah numbers defined by downward recurrence:

$$L(n, k - 1) = \frac{k(k - 1)}{n - k + 1} L(n, k); L(n, n) = 1$$

Definition at line 451 of file `sf_stirling.tcc`.

8.3.2.225 `__lanczos_binet1p()`

```
template<typename _Tp >
_GLIBCXX14_CONSTEXPR _Tp std::__detail::__lanczos_binet1p (
    _Tp __z )
```

Return the Binet function $J(1 + z)$ by the Lanczos method. The Binet function is the log of the scaled Gamma function $\log(\Gamma^*(z))$ defined by

$$J(z) = \log(\Gamma^*(z)) = \log(\Gamma(z)) + z - \left(z - \frac{1}{2}\right) \log(z) - \log(2\pi)$$

or

$$\Gamma(z) = \sqrt{2\pi} z^{z-\frac{1}{2}} e^{-z} e^{J(z)}$$

where $\Gamma(z)$ is the gamma function.

Parameters

<code>_↔ _z</code>	The argument of the log of the gamma function.
------------------------	--

Returns

The logarithm of the gamma function.

Definition at line 2125 of file sf_gamma.tcc.

References `std::__detail::_Factorial_table<_Tp>::__n`.

Referenced by `__lanczos_log_gamma1p()`.

8.3.2.226 `__lanczos_log_gamma1p()`

```
template<typename _Tp >
_GLIBCXX14_CONSTEXPR _Tp std::__detail::__lanczos_log_gamma1p (
    _Tp __z )
```

Return the logarithm of the gamma function $\log(\Gamma(1+z))$ by the Lanczos method.

If the argument is real, the log of the absolute value of the Gamma function is returned. The sign to be applied to the exponential of this log Gamma can be recovered with a call to `__log_gamma_sign`.

For complex argument the fully complex log of the gamma function is returned.

Parameters

<code>_↔ _z</code>	The argument of the log of the gamma function.
------------------------	--

Returns

The logarithm of the gamma function.

Definition at line 2159 of file sf_gamma.tcc.

References `__lanczos_binet1p()`, and `__sin_pi()`.

8.3.2.227 `__legendre_p()`

```
template<typename _Tp >
__gnu_cxx::__legendre_p_t<_Tp> std::__detail::__legendre_p (
    unsigned int __l,
    _Tp __x )
```

Return the Legendre polynomial by upward recursion on degree l .

The Legendre function of degree l and argument x , $P_l(x)$, is defined by:

$$P_l(x) = \frac{1}{2^l l!} \frac{d^l}{dx^l} (x^2 - 1)^l$$

This can be expressed as a series:

$$P_l(x) = \frac{1}{2^l l!} \sum_{k=0}^{\lfloor l/2 \rfloor} \frac{(-1)^k (2l - 2k)!}{k! (l - k)! (l - 2k)!} x^{l-2k}$$

Parameters

<code>__l</code>	The degree of the Legendre polynomial. $l \geq 0$.
<code>__x</code>	The argument of the Legendre polynomial.

Definition at line 85 of file `sf_legendre.tcc`.

Referenced by `__assoc_legendre_p()`, and `__sph_legendre()`.

8.3.2.228 `__legendre_q()`

```
template<typename _Tp >
__gnu_cxx::__legendre_q_t<_Tp> std::__detail::__legendre_q (
    unsigned int __l,
    _Tp __x )
```

Return the Legendre function of the second kind by upward recursion on degree l .

The Legendre function of the second kind of degree l and argument x , $Q_l(x)$, is defined by:

$$Q_l(x) = \frac{1}{2^l l!} \frac{d^l}{dx^l} (x^2 - 1)^l$$

Parameters

\leftrightarrow __l	The degree of the Legendre function. $l \geq 0$.
\leftrightarrow __x	The argument of the Legendre function. $ x \leq 1$.

Definition at line 165 of file sf_legendre.tcc.

References `__legendre_q_series()`.

8.3.2.229 `__legendre_q_series()`

```
template<typename _Tp >
_Tp std::__detail::__legendre_q_series (
    unsigned int __l,
    _Tp __x )
```

Legendre q series.

Definition at line 133 of file sf_legendre.tcc.

References `__hyperg()`.

Referenced by `__legendre_q()`.

8.3.2.230 `__legendre_zeros()`

```
template<typename _Tp >
std::vector<__gnu_cxx::__quadrature_point_t<_Tp> > std::__detail::__legendre_zeros (
    unsigned int __l,
    _Tp proto = _Tp{} )
```

Build a list of zeros and weights for the Gauss-Legendre integration rule for the Legendre polynomial of degree `l`.

Definition at line 521 of file sf_legendre.tcc.

8.3.2.231 `__log_binomial()` [1/2]

```
template<typename _Tp >
_Tp std::__detail::__log_binomial (
    unsigned int __n,
    unsigned int __k )
```

Return the logarithm of the binomial coefficient. The binomial coefficient is given by:

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

The binomial coefficients are generated by:

$$(1+t)^n = \sum_{k=0}^n \binom{n}{k} t^k$$

.

Parameters

\leftrightarrow __n	The first argument of the binomial coefficient.
\leftrightarrow __k	The second argument of the binomial coefficient.

Returns

The logarithm of the binomial coefficient.

Definition at line 2436 of file sf_gamma.tcc.

References `__log_gamma()`, and `std::__detail::_Factorial_table<_Tp>::__n`.

Referenced by `__binomial()`.

8.3.2.232 __log_binomial() [2/2]

```
template<typename _Tp >
_Tp std::__detail::__log_binomial (
    _Tp __nu,
    unsigned int __k )
```

Return the logarithm of the binomial coefficient for non-integral degree. The binomial coefficient is given by:

$$\binom{\nu}{k} = \frac{\Gamma(\nu + 1)}{\Gamma(\nu - k + 1)\Gamma(k + 1)}$$

The binomial coefficients are generated by:

$$(1 + t)^\nu = \sum_{k=0}^{\infty} \binom{\nu}{k} t^k$$

.

Parameters

__nu	The first argument of the binomial coefficient.
__k	The second argument of the binomial coefficient.

Returns

The logarithm of the binomial coefficient.

Definition at line 2473 of file sf_gamma.tcc.

References `__log_gamma()`, and `std::__detail::_Factorial_table<_Tp>::__n`.

8.3.2.233 __log_binomial_sign() [1/2]

```
template<typename _Tp >
_Tp std::__detail::__log_binomial_sign (
    _Tp __nu,
    unsigned int __k )
```

Return the sign of the exponentiated logarithm of the binomial coefficient for non-integral degree. The binomial coefficient is given by:

$$\binom{\nu}{k} = \frac{\Gamma(\nu + 1)}{\Gamma(\nu - k + 1)\Gamma(k + 1)}$$

The binomial coefficients are generated by:

$$(1 + t)^\nu = \sum_{k=0}^{\infty} \binom{\nu}{k} t^k$$

Parameters

<code>__nu</code>	The first argument of the binomial coefficient.
<code>__k</code>	The second argument of the binomial coefficient.

Returns

The sign of the gamma function.

Definition at line 2504 of file `sf_gamma.tcc`.

References `__log_gamma_sign()`, and `std::__detail::_Factorial_table<_Tp>::__n`.

Referenced by `__binomial()`.

8.3.2.234 __log_binomial_sign() [2/2]

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__log_binomial_sign (
    std::complex<_Tp> __nu,
    unsigned int __k )
```

Definition at line 2519 of file `sf_gamma.tcc`.

8.3.2.235 `__log_double_factorial()` [1/2]

```
template<typename _Tp >
_GLIBCXX14_CONSTEXPR _Tp std::__detail::__log_double_factorial (
    _Tp __nu )
```

Extend double factorial to non-integer arguments. Arkken,

$$\log(\nu!!) = \frac{\nu}{2} \log(2) + (\cos(\pi\nu) - 1) \log(\pi/2)/4 + \log(\Gamma(1 + \nu/2))$$

Definition at line 1657 of file `sf_gamma.tcc`.

References `__cos_pi()`, and `__log_gamma()`.

Referenced by `__double_factorial()`, and `__log_double_factorial()`.

8.3.2.236 `__log_double_factorial()` [2/2]

```
template<typename _Tp >
_GLIBCXX14_CONSTEXPR _Tp std::__detail::__log_double_factorial (
    int __n )
```

Return the logarithm of the double factorial of the integer `n`.

The double factorial is defined for integral `n` by:

$$n!! = 135\dots(n-2)n, \text{ odd } n!! = 246\dots(n-2)n, \text{ even } -1!! = 10!! = 1$$

The double factorial is defined for odd negative integers in the obvious way:

$$(-2m-1)!! = 1/(1(-1)(-3)\dots(-2m+1)(-2m-1)) = \frac{(-1)^m}{(2m-1)!!}$$

for $n = -2m - 1$.

Definition at line 1727 of file `sf_gamma.tcc`.

References `__log_double_factorial()`, `std::__detail::_Factorial_table<_Tp>::__log_factorial`, `std::__detail::_Factorial←_table<_Tp>::__n`, `_S_double_factorial_table`, and `_S_neg_double_factorial_table`.

8.3.2.237 __log_factorial()

```
template<typename _Tp >
_GLIBCXX14_CONSTEXPR _Tp std::__detail::__log_factorial (
    unsigned int __n )
```

Return the logarithm of the factorial of the integer n.

The factorial is:

$$n! = 12...(n-1)n, 0! = 1$$

Definition at line 1635 of file sf_gamma.tcc.

References `__log_gamma()`, `std::__detail::_Factorial_table< _Tp >::__n`, `_S_double_factorial_table`, and `_S_factorial_table`.

8.3.2.238 __log_falling_factorial()

```
template<typename _Tp >
_Tp std::__detail::__log_falling_factorial (
    _Tp __a,
    _Tp __nu )
```

Return the logarithm of the falling factorial function or the lower Pochhammer symbol. The lower Pochhammer symbol is defined by

$$a^{\underline{n}} = \Gamma(a+1)/\Gamma(a-\nu+1) = \prod_{k=0}^{n-1} (a-k), (a)_0 = 1$$

In particular, $n^{\underline{n}} = n!$. Thus this function returns

$$\ln[a^{\underline{n}}] = \ln[\Gamma(a+1)] - \ln[\Gamma(a-\nu+1)], \ln[a^{\underline{0}}] = 0$$

Many notations exist for this function:

$$(a)_{\nu}$$

,

$$\left\{ \begin{matrix} a \\ \nu \end{matrix} \right\}$$

, and others.

Definition at line 3052 of file sf_gamma.tcc.

References `__falling_factorial()`, `__gnu_cxx::__fp_is_integer()`, and `__log_gamma()`.

8.3.2.239 __log_gamma() [1/2]

```
template<typename _Tp >
_Tp std::__detail::__log_gamma (
    _Tp __a )
```

Return $\log(|\Gamma(a)|)$. This will return values even for $a < 0$. To recover the sign of $\Gamma(a)$ for any argument use `__log_gamma_sign`.

Parameters

<code>↔ _a</code>	The argument of the log of the gamma function.
-----------------------	--

Returns

The logarithm of the gamma function.

Definition at line 2327 of file `sf_gamma.tcc`.

References `__sin_pi()`, and `__spouge_log_gamma1p()`.

Referenced by `__beta_inc()`, `__beta_lgamma()`, `__cyl_bessel_ij_series()`, `__falling_factorial()`, `__gamma()`, `↔
gamma_cont_frac()`, `__gamma_series()`, `__hydrogen()`, `__hyperg()`, `__hyperg_reflect()`, `__laguerre_large_n()`, `↔
log_binomial()`, `__log_double_factorial()`, `__log_factorial()`, `__log_falling_factorial()`, `__log_gamma()`, `__log_rising_↔
factorial()`, `__polygamma()`, `__polylog_exp_neg()`, `__polylog_exp_pos()`, `__riemann_zeta()`, `__rising_factorial()`, and `__sph_legendre()`.

8.3.2.240 __log_gamma() [2/2]

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__log_gamma (
    std::complex< _Tp > __a )
```

Return $\log(\Gamma(a))$ for complex argument.

Parameters

<code>↔ _a</code>	The complex argument of the log of the gamma function.
-----------------------	--

Returns

The complex logarithm of the gamma function.

Definition at line 2362 of file `sf_gamma.tcc`.

References `__gnu_cxx::__fp_is_integer()`, `std::__detail::_Factorial_table< _Tp >::__log_factorial`, `__log_gamma()`, `std::__detail::_Factorial_table< _Tp >::__n`, `__sin_pi()`, `__spouge_log_gamma1p()`, and `_S_factorial_table`.

8.3.2.241 __log_gamma_bernoulli()

```
template<typename _Tp >
_GLIBCXX14_CONSTEXPR _Tp std::__detail::__log_gamma_bernoulli (
    _Tp __x )
```

Return $\log(\Gamma(x))$ by asymptotic expansion with Bernoulli number coefficients. This is like Sterling's approximation.

Parameters

\leftarrow __x	The argument of the log of the gamma function.
---------------------	--

Returns

The logarithm of the gamma function.

Definition at line 1759 of file sf_gamma.tcc.

8.3.2.242 __log_gamma_sign() [1/2]

```
template<typename _Tp >
_Tp std::__detail::__log_gamma_sign (
    _Tp __a )
```

Return the sign of $\Gamma(x)$. At nonpositive integers zero is returned indicating $\Gamma(x)$ is undefined.

Parameters

\leftarrow __a	The argument of the gamma function.
---------------------	-------------------------------------

Returns

The sign of the gamma function.

Definition at line 2403 of file sf_gamma.tcc.

Referenced by __beta_inc(), __beta_lgamma(), __falling_factorial(), __gamma(), __gamma_cont_frac(), __gamma_ \leftarrow series(), __hyperg(), __hyperg_reflect(), __log_binomial_sign(), and __rising_factorial().

8.3.2.243 `__log_gamma_sign()` [2/2]

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__log_gamma_sign (
    std::complex< _Tp > __a )
```

Definition at line 2415 of file `sf_gamma.tcc`.

8.3.2.244 `__log_rising_factorial()`

```
template<typename _Tp >
_Tp std::__detail::__log_rising_factorial (
    _Tp __a,
    _Tp __nu )
```

Return the logarithm of the rising factorial function or the (upper) Pochhammer symbol. The Pochhammer symbol is defined for integer order by

$$a^{\overline{\nu}} = \Gamma(a + \nu) / \Gamma(a) = \prod_{k=0}^{\nu-1} (a + k), (a)_0 = 1$$

Thus this function returns

$$\ln[a^{\overline{\nu}}] = \ln[\Gamma(a + \nu)] - \ln[\Gamma(a)], \ln[(a)_0] = 0$$

Many notations exist for this function:

$$(a)_{\nu}$$

(especially in the literature of special functions),

$$\left[\begin{matrix} a \\ \nu \end{matrix} \right]$$

, and others.

Definition at line 3201 of file `sf_gamma.tcc`.

References `__log_gamma()`, and `__rising_factorial()`.

8.3.2.245 `__log_stirling_1()`

```
template<typename _Tp >
_Tp std::__detail::__log_stirling_1 (
    unsigned int __n,
    unsigned int __m )
```

Return the logarithm of the absolute value of Stirling number of the first kind.

Definition at line 387 of file `sf_stirling.tcc`.

8.3.2.246 __log_stirling_1_sign()

```
template<typename _Tp >
_Tp std::__detail::__log_stirling_1_sign (
    unsigned int __n,
    unsigned int __m ) [inline]
```

Return the sign of the exponent of the logarithm of the Stirling number of the first kind.

Definition at line 405 of file sf_stirling.tcc.

8.3.2.247 __log_stirling_2()

```
template<typename _Tp >
_Tp std::__detail::__log_stirling_2 (
    unsigned int __n,
    unsigned int __m )
```

Return the Stirling number of the second kind.

Todo Find asymptotic expressions for the Stirling numbers.

Definition at line 234 of file sf_stirling.tcc.

8.3.2.248 __logint()

```
template<typename _Tp >
_Tp std::__detail::__logint (
    const _Tp __x )
```

Return the logarithmic integral $li(x)$.

The logarithmic integral is given by

$$li(x) = Ei(\log(x))$$

Parameters

$_x$	The argument of the logarithmic integral function.
-------	--

Returns

The logarithmic integral.

Definition at line 538 of file sf_expint.tcc.

References `__expint()`.

8.3.2.249 `__logistic_p()`

```
template<typename _Tp >
_Tp std::__detail::__logistic_p (
    _Tp __a,
    _Tp __b,
    _Tp __x )
```

Return the logistic cumulative distribution function.

The formula for the logistic probability function is

$$cdf(x|a, b) = \frac{e^{(x-a)/b}}{1 + e^{(x-a)/b}}$$

where $b > 0$.

Definition at line 688 of file sf_distributions.tcc.

8.3.2.250 `__logistic_pdf()`

```
template<typename _Tp >
_Tp std::__detail::__logistic_pdf (
    _Tp __a,
    _Tp __b,
    _Tp __x )
```

Return the logistic probability density function.

The formula for the logistic probability density function is

$$p(x|a, b) = \frac{e^{(x-a)/b}}{b[1 + e^{(x-a)/b}]^2}$$

where $b > 0$.

Definition at line 670 of file sf_distributions.tcc.

8.3.2.251 __lognormal_p()

```
template<typename _Tp >
_Tp std::__detail::__lognormal_p (
    _Tp __mu,
    _Tp __sigma,
    _Tp __x )
```

Return the lognormal cumulative probability density function.

The formula for the lognormal cumulative probability density function is

$$F(x|\mu, \sigma) = \frac{1}{2} \left[1 - \operatorname{erf}\left(\frac{\ln x - \mu}{\sqrt{2}\sigma}\right) \right]$$

Definition at line 287 of file sf_distributions.tcc.

8.3.2.252 __lognormal_pdf()

```
template<typename _Tp >
_Tp std::__detail::__lognormal_pdf (
    _Tp __nu,
    _Tp __sigma,
    _Tp __x )
```

Return the lognormal probability density function.

The formula for the lognormal probability density function is

$$f(x|\mu, \sigma) = \frac{e^{(\ln x - \mu)^2 / 2\sigma^2}}{\sigma\sqrt{2\pi}}$$

Definition at line 259 of file sf_distributions.tcc.

8.3.2.253 __normal_p()

```
template<typename _Tp >
_Tp std::__detail::__normal_p (
    _Tp __mu,
    _Tp __sigma,
    _Tp __x )
```

Return the normal cumulative probability density function.

The formula for the normal cumulative probability density function is

$$F(x|\mu, \sigma) = \frac{1}{2} \left[1 - \operatorname{erf}\left(\frac{x - \mu}{\sqrt{2}\sigma}\right) \right]$$

Definition at line 238 of file sf_distributions.tcc.

8.3.2.254 `__normal_pdf()`

```
template<typename _Tp >
_Tp std::__detail::__normal_pdf (
    _Tp __mu,
    _Tp __sigma,
    _Tp __x )
```

Return the normal probability density function.

The formula for the normal probability density function is

$$f(x|\mu, \sigma) = \frac{e^{(x-\mu)^2/2\sigma^2}}{\sigma\sqrt{2\pi}}$$

Definition at line 210 of file `sf_distributions.tcc`.

8.3.2.255 `__owens_t()`

```
template<typename _Tp >
_Tp std::__detail::__owens_t (
    _Tp __h,
    _Tp __a )
```

Return the Owens T function:

$$T(h, a) = \frac{1}{2\pi} \int_0^a \frac{\exp[-\frac{1}{2}h^2(1+x^2)]}{1+x^2} dx$$

This implementation is a translation of the Fortran implementation in

See also

Patefield, M. and Tandy, D. "Fast and accurate Calculation of Owen's T-Function", Journal of Statistical Software, 5 (5), 1 - 25 (2000)

Parameters

in	<code>__h</code>	The scale parameter.
in	<code>__a</code>	The integration limit.

Returns

The owens T function.

Definition at line 92 of file sf_owens_t.tcc.

References `__znorm1()`, and `__znorm2()`.

8.3.2.256 `__periodic_zeta()`

```
template<typename _Tp , typename _ArgType >
__gnu_cxx::fp_promote_t<std::complex<_Tp>, _ArgType> std::__detail::__periodic_zeta (
    _ArgType __z,
    _Tp __s )
```

Return the periodic zeta function $F(z,s)$ for two real arguments.

The periodic zeta function is defined by

$$F(z, s) = \sum_{k=1}^{\infty} \frac{e^{i2\pi kz}}{k^s} = Li_s(e^{i2\pi kz})$$

Parameters

<code>__s</code>	The real order.
<code>__z</code>	The real or complex argument.

Returns

The complex value of the periodic zeta function.

Definition at line 1102 of file sf_polylog.tcc.

References `__gnu_cxx::__fp_is_zero()`, `__polylog_exp()`, and `__riemann_zeta()`.

8.3.2.257 `__polar_pi()` ^[1/2]

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__polar_pi (
    _Tp __rho,
    _Tp __phi_pi ) [inline]
```

Reperiodized complex constructor.

Definition at line 401 of file sf_trig.tcc.

References `__gnu_cxx::__sincos_t<_Tp>::__cos_v`, `__gnu_cxx::__sincos_t<_Tp>::__sin_v`, and `__sincos_pi()`.

Referenced by `__cyl_bessel_jn_neg_arg()`, `__cyl_hankel_1()`, `__cyl_hankel_2()`, `__jacobi_theta_1()`, `__jacobi_theta_2()`, `__polylog_exp_neg()`, and `__polylog_exp_pos()`.

8.3.2.258 `__polar_pi()` [2/2]

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__polar_pi (
    _Tp __rho,
    const std::complex< _Tp > & __phi_pi ) [inline]
```

Reperiodized complex constructor.

Definition at line 413 of file `sf_trig.tcc`.

References `__gnu_cxx::__sincos_t< _Tp >::__cos_v`, `__gnu_cxx::__sincos_t< _Tp >::__sin_v`, and `__sincos_pi()`.

8.3.2.259 `__polygamma()`

```
template<typename _Tp >
_Tp std::__detail::__polygamma (
    unsigned int __m,
    _Tp __x )
```

Return the polygamma function $\psi^{(m)}(x)$.

The polygamma function is related to the Hurwitz zeta function:

$$\psi^{(m)}(x) = (-1)^{m+1} m! \zeta(m+1, x)$$

Definition at line 3467 of file `sf_gamma.tcc`.

References `__digamma()`, `__gnu_cxx::__fp_is_integer()`, `__hurwitz_zeta()`, `__log_gamma()`, and `std::__detail::__Factorial_table< _Tp >::__n`.

8.3.2.260 `__polylog()` [1/2]

```
template<typename _Tp >
_Tp std::__detail::__polylog (
    _Tp __s,
    _Tp __x )
```

Return the polylog $Li_s(x)$ for two real arguments.

The polylog is defined by

$$Li_s(x) = \sum_{k=1}^{\infty} \frac{x^k}{k^s}$$

Parameters

$_s$	The real order.
$_x$	The real argument.

Returns

The complex value of the polylogarithm.

Definition at line 1037 of file sf_polylog.tcc.

References `__gnu_cxx::__fp_is_equal()`, `__gnu_cxx::__fp_is_integer()`, `__gnu_cxx::__fp_is_zero()`, and `__polylog_exp()`.

Referenced by `__dirichlet_beta()`, `__dirichlet_eta()`, and `__polylog()`.

8.3.2.261 `__polylog()` [2/2]

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__polylog (
    _Tp __s,
    std::complex< _Tp > __w )
```

Return the polylog in those cases where we can calculate it.

Parameters

$_s$	The real order.
$_w$	The complex argument.

Returns

The complex value of the polylogarithm.

Definition at line 1078 of file sf_polylog.tcc.

References `__polylog()`, and `__polylog_exp()`.

8.3.2.262 `__polylog_exp()`

```
template<typename _Tp , typename _ArgType >
__gnu_cxx::__fp_promote_t<std::complex<_Tp>, _ArgType> std::__detail::__polylog_exp (
    _Tp __s,
    _ArgType __w )
```

This is the frontend function which calculates $Li_s(e^w)$. First we branch into different parts depending on the properties of s . This function is the same irrespective of a real or complex w , hence the template parameter `ArgType`.

Note

: I *really* wish we could return a `variant<Tp, std::complex<Tp>>`.

Parameters

<code>__s</code>	The real order.
<code>__w</code>	The real or complex argument.

Returns

The real or complex value of $Li_s(e^w)$.

Definition at line 996 of file `sf_polylog.tcc`.

References `__gnu_cxx::__fp_is_integer()`, `__polylog_exp_neg_int()`, `__polylog_exp_neg_real()`, `__polylog_exp_pos_int()`, `__polylog_exp_pos_real()`, and `__polylog_exp_sum()`.

Referenced by `__bose_einstein()`, `__clausen()`, `__clausen_cl()`, `__clausen_sl()`, `__fermi_dirac()`, `__hurwitz_zeta()`, `__polylog()`, `__periodic_zeta()`, and `__polylog()`.

8.3.2.263 `__polylog_exp_asymp()`

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__polylog_exp_asymp (
    _Tp __s,
    std::complex<_Tp> __w )
```

This function implements the asymptotic series for the polylog. It is given by

$$2 \sum_{k=0}^{\infty} \zeta(2k) w^{s-2k} / \Gamma(s-2k+1) - i\pi w^{s-1} / \Gamma(s)$$

for $Re(w) \gg 1$

Don't check this against Mathematica 8. For real w the imaginary part of the polylog is given by $Im(Li_s(e^w)) = -\pi w^{s-1} / \Gamma(s)$. Check this relation for any benchmark that you use.

Parameters

$_s$	the real index s.
$_w$	the large complex argument w.

Returns

the value of the polylogarithm.

Definition at line 603 of file sf_polylog.tcc.

References `__gamma_reciprocal()`.

Referenced by `__polylog_exp_neg_int()`, `__polylog_exp_neg_real()`, `__polylog_exp_pos_int()`, and `__polylog_exp_pos_real()`.

8.3.2.264 `__polylog_exp_neg()` [1/2]

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__polylog_exp_neg (
    _Tp __s,
    std::complex< _Tp > __w )
```

This function treats the cases of negative real index s. Theoretical convergence is present for $|w| < 2\pi$. We use an optimized version of

$$Li_s(e^w) = \Gamma(1-s)(-w)^{s-1} + \frac{(2\pi)^{-s}}{\pi} A_p(w)$$

$$A_p(w) = \sum_k \frac{\Gamma(1+k-s)}{k!} \sin\left(\frac{\pi}{2}(s-k)\right) \left(\frac{w}{2\pi}\right)^k \zeta(1+k-s)$$

Parameters

$_s$	The negative real index
$_w$	The complex argument

Returns

The value of the polylogarithm.

Definition at line 367 of file sf_polylog.tcc.

References `__log_gamma()`, `__polar_pi()`, and `__riemann_zeta_m_1()`.

Referenced by `__polylog_exp_neg_int()`, and `__polylog_exp_neg_real()`.

8.3.2.265 `__polylog_exp_neg()` [2/2]

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__polylog_exp_neg (
    int __n,
    std::complex< _Tp > __w )
```

Compute the polylogarithm for negative integer order.

$$Li_{-p}(e^w) = p!(-w)^{-(p+1)} - \sum_{k=0}^{\infty} \frac{B_{p+2k+q+1}}{(p+2k+q+1)!} \frac{(p+2k+q)!}{(2k+q)!} w^{2k+q}$$

where $q = (p+1) \bmod 2$.

Parameters

<code>__n</code>	the negative integer index $n = -p$.
<code>__w</code>	the argument w .

Returns

the value of the polylogarithm.

Definition at line 453 of file `sf_polylog.tcc`.

References `__gnu_cxx::__fp_is_equal()`, `__gnu_cxx::__fp_is_zero()`, `_Num_Euler_Maclaurin_zeta`, and `_S_Euler_Maclaurin_zeta`.

8.3.2.266 `__polylog_exp_neg_int()` [1/2]

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__polylog_exp_neg_int (
    int __s,
    std::complex< _Tp > __w )
```

This treats the case where s is a negative integer.

Parameters

$_s$	a negative integer.
$_w$	an arbitrary complex number

Returns

the value of the polylogarithm.

Definition at line 789 of file sf_polylog.tcc.

References `__clamp_0_m2pi()`, `__clamp_pi()`, `__gnu_cxx::__fp_is_equal()`, `__polylog_exp_asymp()`, `__polylog_exp_neg()`, and `__polylog_exp_sum()`.

Referenced by `__polylog_exp()`.

8.3.2.267 __polylog_exp_neg_int() [2/2]

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__polylog_exp_neg_int (
    int __s,
    _Tp __w )
```

This treats the case where `s` is a negative integer and `w` is a real.

Parameters

$_s$	a negative integer.
$_w$	the argument.

Returns

the value of the polylogarithm.

Definition at line 835 of file sf_polylog.tcc.

References `__gnu_cxx::__fp_is_zero()`, `__polylog_exp_asymp()`, `__polylog_exp_neg()`, and `__polylog_exp_sum()`.

8.3.2.268 `__polylog_exp_neg_real()` [1/2]

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__polylog_exp_neg_real (
    _Tp __s,
    std::complex< _Tp > __w )
```

Return the polylog where s is a negative real value and for complex argument. Now we branch depending on the properties of w in the specific functions

Parameters

\leftarrow __s	A negative real value that does not reduce to a negative integer.
\leftarrow __w	The complex argument.

Returns

The value of the polylogarithm.

Definition at line 936 of file `sf_polylog.tcc`.

References `__clamp_0_m2pi()`, `__clamp_pi()`, `__polylog_exp_asymp()`, `__polylog_exp_neg()`, and `__polylog_exp_ \leftarrow sum()`.

Referenced by `__polylog_exp()`.

8.3.2.269 `__polylog_exp_neg_real()` [2/2]

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__polylog_exp_neg_real (
    _Tp __s,
    _Tp __w )
```

Return the polylog where s is a negative real value and for real argument. Now we branch depending on the properties of w in the specific functions.

Parameters

\leftarrow __s	A negative real value.
\leftarrow __w	A real argument.

Returns

The value of the polylogarithm.

Definition at line 967 of file sf_polylog.tcc.

References `__polylog_exp_asymp()`, `__polylog_exp_neg()`, and `__polylog_exp_sum()`.

8.3.2.270 __polylog_exp_pos() [1/3]

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__polylog_exp_pos (
    unsigned int __s,
    std::complex< _Tp > __w )
```

This function treats the cases of positive integer index s for complex argument w .

$$Li_s(e^w) = \sum_{k=0, k! = s-1} \zeta(s-k) \frac{w^k}{k!} + [H_{s-1} - \log(-w)] \frac{w^{s-1}}{(s-1)!}$$

The radius of convergence is $|w| < 2\pi$. Note that this series involves a $\log(-x)$. gcc and Mathematica differ in their implementation of $\log(e^{i\pi})$: gcc: $\log(e^{+i\pi}) = + - i\pi$ whereas Mathematica doesn't preserve the sign in this case: $\log(e^{+i\pi}) = +i\pi$

Parameters

\leftrightarrow __s	the positive integer index.
\leftrightarrow __w	the argument.

Returns

the value of the polylogarithm.

Definition at line 219 of file sf_polylog.tcc.

References `__riemann_zeta()`.

Referenced by `__polylog_exp_pos_int()`, and `__polylog_exp_pos_real()`.

8.3.2.271 `__polylog_exp_pos()` [2/3]

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__polylog_exp_pos (
    unsigned int __s,
    _Tp __w )
```

This function treats the cases of positive integer index s for real argument w .

This specialization is worthwhile to catch the differing behaviour of $\log(x)$.

$$Li_s(e^w) = \sum_{k=0, k! \neq s-1} \zeta(s-k) \frac{w^k}{k!} + [H_{s-1} - \log(-w)] \frac{w^{s-1}}{(s-1)!}$$

The radius of convergence is $|w| < 2\pi$. Note that this series involves a $\log(-x)$. gcc and Mathematica differ in their implementation of $\log(e^{i\pi})$: gcc: $\log(e^{+i\pi}) = + - i\pi$ whereas Mathematica doesn't preserve the sign in this case: $\log(e^{+i\pi}) = +i\pi$

Parameters

<code>__s</code>	the positive integer index.
<code>__w</code>	the argument.

Returns

the value of the polylogarithm.

Definition at line 295 of file `sf_polylog.tcc`.

References `__riemann_zeta()`.

8.3.2.272 `__polylog_exp_pos()` [3/3]

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__polylog_exp_pos (
    _Tp __s,
    std::complex<_Tp> __w )
```

This function treats the cases of positive real index s .

The defining series is

$$Li_s(e^w) = A_s(w) + B_s(w) + \Gamma(1-s)(-w)^{s-1}$$

with

$$A_s(w) = \sum_{k=0}^m \zeta(s-k) w^k / k!$$

$$B_s(w) = \sum_{k=m+1}^{\infty} \sin(\pi/2(s-k)) \Gamma(1-s+k) \zeta(1-s+k) (w/2/\pi)^k / k!$$

Parameters

$_s$	the positive real index s.
$_w$	The complex argument w.

Returns

the value of the polylogarithm.

Definition at line 516 of file sf_polylog.tcc.

References `__gamma()`, `__log_gamma()`, `__polar_pi()`, and `__riemann_zeta()`.

8.3.2.273 `__polylog_exp_pos_int()` [1/2]

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__polylog_exp_pos_int (
    unsigned int __s,
    std::complex< _Tp > __w )
```

Here s is a positive integer and the function descends into the different kernels depending on w.

Parameters

$_s$	a positive integer.
$_w$	an arbitrary complex number.

Returns

The value of the polylogarithm.

Definition at line 678 of file sf_polylog.tcc.

References `__clamp_0_m2pi()`, `__clamp_pi()`, `__gnu_cxx::__fp_is_equal()`, `__gnu_cxx::__fp_is_zero()`, `__polylog_exp_asymp()`, `__polylog_exp_pos()`, and `__polylog_exp_sum()`.

Referenced by `__polylog_exp()`.

8.3.2.274 `__polylog_exp_pos_int()` [2/2]

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__polylog_exp_pos_int (
    unsigned int __s,
    _Tp __w )
```

Here *s* is a positive integer and the function descends into the different kernels depending on *w*.

Parameters

<code>__s</code>	a positive integer
<code>__w</code>	an arbitrary real argument <i>w</i>

Returns

the value of the polylogarithm.

Definition at line 739 of file `sf_polylog.tcc`.

References `__gnu_cxx::__fp_is_zero()`, `__polylog_exp_asymp()`, `__polylog_exp_pos()`, and `__polylog_exp_sum()`.

8.3.2.275 `__polylog_exp_pos_real()` [1/2]

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__polylog_exp_pos_real (
    _Tp __s,
    std::complex< _Tp > __w )
```

Return the polylog where *s* is a positive real value and for complex argument.

Parameters

<code>__s</code>	A positive real number.
<code>__w</code>	the complex argument.

Returns

The value of the polylogarithm.

Definition at line 862 of file `sf_polylog.tcc`.

References `__clamp_0_m2pi()`, `__clamp_pi()`, `__gnu_cxx::__fp_is_equal()`, `__gnu_cxx::__fp_is_zero()`, `__polylog_↵exp_asymp()`, `__polylog_exp_pos()`, `__polylog_exp_sum()`, and `__riemann_zeta()`.

Referenced by `__polylog_exp()`.

8.3.2.276 __polylog_exp_pos_real() [2/2]

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__polylog_exp_pos_real (
    _Tp __s,
    _Tp __w )
```

Return the polylog where s is a positive real value and the argument is real.

Parameters

<code>↵ __s</code>	A positive real number tht does not reduce to an integer.
<code>↵ __w</code>	The real argument w.

Returns

The value of the polylogarithm.

Definition at line 902 of file `sf_polylog.tcc`.

References `__gnu_cxx::__fp_is_equal()`, `__gnu_cxx::__fp_is_zero()`, `__polylog_exp_asymp()`, `__polylog_exp_pos()`, `↵__polylog_exp_sum()`, and `__riemann_zeta()`.

8.3.2.277 __polylog_exp_sum()

```
template<typename _PowTp , typename _Tp >
_Tp std::__detail::__polylog_exp_sum (
    _PowTp __s,
    _Tp __w )
```

Theoretical convergence for $\text{Re}(w) < 0$.

Seems to beat the other expansions for $\text{Re}(w) < -\pi/2 - \pi/5$. Note that this is an implementation of the basic series:

$$Li_s(e^z) = \sum_{k=1}^{\infty} e^{kz} k^{-s}$$

Parameters

<code>↔ _s</code>	is an arbitrary type, integral or float.
<code>↔ _w</code>	something with a negative real part.

Returns

the value of the polylogarithm.

Definition at line 647 of file sf_polylog.tcc.

Referenced by `__polylog_exp()`, `__polylog_exp_neg_int()`, `__polylog_exp_neg_real()`, `__polylog_exp_pos_int()`, and `↔
__polylog_exp_pos_real()`.

8.3.2.278 __prob_hermite_recur()

```
template<typename _Tp >
__gnu_cxx::__hermite_he_t<_Tp> std::__detail::__prob_hermite_recur (
    unsigned int __n,
    _Tp __x )
```

This routine returns the Probabilists Hermite polynomial of order n: $He_n(x)$ by recursion on n.

The Probabilists Hermite polynomial is defined by:

$$He_n(x) = (-1)^n e^{x^2/2} \frac{d^n}{dx^n} e^{-x^2/2}$$

or

$$He_n(x) = \frac{1}{2^{-n/2}} H_n \left(\frac{x}{\sqrt{2}} \right)$$

where $H_n(x)$ is the Physicists Hermite function.

The Probabilists Hermite polynomial has first and second derivatives:

$$He'_n(x) = n He_{n-1}(x)$$

and

$$He''_n(x) = n(n-1) He_{n-2}(x)$$

The Probabilists Hermite polynomial are monic and are orthogonal with respect to the weight function

$$w(x) = e^{x^2/2}$$

Parameters

\leftrightarrow _n	The order of the Hermite polynomial.
\leftrightarrow _x	The argument of the Hermite polynomial.

Returns

The value of the Hermite polynomial of order n and argument x.

Definition at line 260 of file sf_hermite.tcc.

8.3.2.279 __radial_jacobi()

```
template<typename _Tp >
_Tp std::__detail::__radial_jacobi (
    unsigned int __n,
    unsigned int __m,
    _Tp __rho )
```

Return the radial polynomial $R_n^m(\rho)$ for non-negative n and m , and real radial argument ρ is a polynomial of degree $m + 2n$ in ρ .

The radial polynomials are defined by

$$R_n^m(\rho) = \sum_{k=0}^{\frac{n-m}{2}} \frac{(-1)^k (n-k)!}{k! (\frac{n+m}{2} - k)! (\frac{n-m}{2} - k)!} \rho^{n-2k}$$

for $n - m$ even and identically 0 for $n - m$ odd.

The radial polynomials are related to the Jacobi polynomials:

$$R_n^m(\rho) = (-1)^n x^m P_n^{(m,0)}(1 - 2\rho^2)$$

for $0 \leq \rho \leq 1$

The radial polynomials can be related to the Zernike polynomials:

$$Z_n^m(\rho, \phi) = R_n^m(\rho) \cos(m\phi)$$

$$Z_n^{-m}(\rho, \phi) = R_n^m(\rho) \sin(m\phi)$$

for non-negative m, n .

See also

zernike for details on the Zernike polynomials.

Principals of Optics, 7th edition, Max Born and Emil Wolf, Cambridge University Press, 1999, pp 523-525 and 905-910.

Zernike Polynomials: Evaluation, Quadrature, and Interpolation Philip Greengard, Kirill Serkh, Technical Report YALEU/DCS/TR-1539, February 20, 2018

Template Parameters

<code>_Tp</code>	The real type of the radial coordinate
------------------	--

Parameters

<code>__n</code>	The non-negative degree.
<code>__m</code>	The non-negative azimuthal order
<code>__rho</code>	The radial argument

Definition at line 295 of file `sf_jacobi.tcc`.

References `__jacobi_recur()`.

Referenced by `__zernike()`, `__gnu_cxx::radpolyf()`, and `__gnu_cxx::radpolyl()`.

8.3.2.280 `__radial_jacobi_zeros()`

```
template<typename _Tp >
std::vector<__gnu_cxx::__quadrature_point_t<_Tp> > std::__detail::__radial_jacobi_zeros (
    unsigned int __n,
    unsigned int __m )
```

Return a vector containing the zeros of the radial Jacobi polynomial $P_n^{(\alpha, \beta)}(1 - 2\rho^2)$.

Template Parameters

<code>_Tp</code>	The real type of the radial coordinate
------------------	--

Parameters

in	<code>↔</code> <code>__n</code>	The degree of the radial Jacobi polynomial
in	<code>↔</code> <code>__m</code>	The order of the radial Jacobi polynomial

Definition at line 326 of file `sf_jacobi.tcc`.

References `__jacobi_zeros()`.

8.3.2.281 __rice_pdf()

```
template<typename _Tp >
_Tp std::__detail::__rice_pdf (
    _Tp __nu,
    _Tp __sigma,
    _Tp __x )
```

Return the Rice probability density function.

The formula for the Rice probability density function is

$$p(x|\nu, \sigma) = \frac{x}{\sigma^2} \exp\left(-\frac{x^2 + \nu^2}{2\sigma^2}\right) I_0\left(\frac{x\nu}{\sigma^2}\right)$$

where $I_0(x)$ is the modified Bessel function of the first kind of order 0 and $\nu \geq 0$ and $\sigma > 0$.

Definition at line 186 of file sf_distributions.tcc.

References __cyl_bessel_i().

8.3.2.282 __riemann_zeta()

```
template<typename _Tp >
_Tp std::__detail::__riemann_zeta (
    _Tp __s )
```

Return the Riemann zeta function $\zeta(s)$.

The Riemann zeta function is defined by:

$$\zeta(s) = \sum_{k=1}^{\infty} k^{-s} \text{ for } \Re(s) > 1 \quad \frac{(2\pi)^s}{\pi} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s) \zeta(1-s) \text{ for } \Re(s) < 1$$

Parameters

\leftarrow	The order
s	

Todo Global double sum or MacLaurin series in riemann_zeta?

Definition at line 761 of file sf_zeta.tcc.

References __exp2(), __gnu_cxx::__fp_is_integer(), __gamma(), __log_gamma(), __riemann_zeta_glob(), \leftarrow riemann_zeta_m_1(), __riemann_zeta_product(), __riemann_zeta_sum(), and __sin_pi().

Referenced by `__dirichlet_lambda()`, `__hurwitz_zeta()`, `__periodic_zeta()`, `__polylog_exp_pos()`, and `__polylog_exp_pos_real()`.

8.3.2.283 `__riemann_zeta_euler_maclaurin()`

```
template<typename _Tp >
_Tp std::__detail::__riemann_zeta_euler_maclaurin (
    _Tp __s )
```

Evaluate the Riemann zeta function $\zeta(s)$ by an alternate series for $s > 0$.

This is a specialization of the code for the Hurwitz zeta function.

Definition at line 391 of file `sf_zeta.tcc`.

References `_S_Euler_Maclaurin_zeta`.

8.3.2.284 `__riemann_zeta_glob()`

```
template<typename _Tp >
_Tp std::__detail::__riemann_zeta_glob (
    _Tp __s )
```

Definition at line 501 of file `sf_zeta.tcc`.

References `__gnu_cxx::__fp_is_even_integer()`, `__gamma()`, `__riemann_zeta_m_1_glob()`, and `__sin_pi()`.

Referenced by `__riemann_zeta()`.

8.3.2.285 `__riemann_zeta_laurent()`

```
template<typename _Tp >
_Tp std::__detail::__riemann_zeta_laurent (
    _Tp __s )
```

Compute the Riemann zeta function $\zeta(s)$ by Laurent expansion about $s = 1$.

The Laurent expansion of the Riemann zeta function is given by:

$$\zeta(s) = \frac{1}{s-1} + \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \gamma_k (s-1)^k$$

Where γ_k are the Stieltjes constants, $\gamma_0 = \gamma_E$ the Euler-Mascheroni constant.

The Stieltjes constants can be found from a limiting process:

$$\gamma_k = \lim_{n \rightarrow \infty} \left\{ \sum_{i=1}^n \frac{(\ln i)^k}{i} - \frac{(\ln n)^{k+1}}{k+1} \right\}$$

Definition at line 314 of file `sf_zeta.tcc`.

References `_Num_Stieltjes`, and `_S_Stieltjes`.

Referenced by `__riemann_zeta_m_1()`.

8.3.2.286 __riemann_zeta_m_1()

```
template<typename _Tp >
_Tp std::__detail::__riemann_zeta_m_1 (
    _Tp __s )
```

Return the Riemann zeta function $\zeta(s) - 1$.

Parameters

\leftarrow _s	The order $s! = 1$
--------------------	--------------------

Definition at line 717 of file sf_zeta.tcc.

References __gnu_cxx::__fp_is_integer(), __gamma(), __riemann_zeta_laurent(), __riemann_zeta_m_1_glob(), \leftarrow sin_pi(), _S_num_zetam1, and _S_zetam1.

Referenced by __polylog_exp_neg(), and __riemann_zeta().

8.3.2.287 __riemann_zeta_m_1_glob()

```
template<typename _Tp >
_Tp std::__detail::__riemann_zeta_m_1_glob (
    _Tp __s )
```

Evaluate the Riemann zeta function by series for all $s \neq 1$. Convergence is great until largish negative numbers. Then the convergence of the > 0 sum gets better.

The series is:

$$\zeta(s) = \frac{1}{1 - 2^{1-s}} \sum_{n=0}^{\infty} \frac{1}{2^{n+1}} \sum_{k=0}^n (-1)^k \frac{n!}{(n-k)!k!} (k+1)^{-s}$$

Havil 2003, p. 206.

The Riemann zeta function is defined by:

$$\zeta(s) = \sum_{k=1}^{\infty} \frac{1}{k^s} \text{ for } s > 1$$

For $s < 1$ use the reflection formula:

$$\zeta(s) = (2\pi)^s \Gamma(1-s) \zeta(1-s) / \pi$$

Definition at line 450 of file sf_zeta.tcc.

Referenced by __riemann_zeta_glob(), and __riemann_zeta_m_1().

8.3.2.288 __riemann_zeta_product()

```
template<typename _Tp >
_Tp std::__detail::__riemann_zeta_product (
    _Tp __s )
```

Compute the Riemann zeta function $\zeta(s)$ using the product over prime factors.

$$\zeta(s) = \prod_{i=1}^{\infty} \frac{1}{1 - p_i^{-s}}$$

where p_i are the prime numbers.

The Riemann zeta function is defined by:

$$\zeta(s) = \sum_{k=1}^{\infty} \frac{1}{k^s} \text{ for } \Re(s) > 1$$

For $(s) < 1$ use the reflection formula:

$$\zeta(s) = (2\pi)^s \Gamma(1-s) \zeta(1-s) / \pi$$

Parameters

$_ \leftrightarrow$	The order.
$_s$	

Definition at line 551 of file sf_zeta.tcc.

Referenced by __riemann_zeta().

8.3.2.289 __riemann_zeta_sum()

```
template<typename _Tp >
_Tp std::__detail::__riemann_zeta_sum (
    _Tp __s )
```

Compute the Riemann zeta function $\zeta(s)$ by summation for $s > 1$.

The Riemann zeta function is defined by:

$$\zeta(s) = \sum_{k=1}^{\infty} \frac{1}{k^s} \text{ for } s > 1$$

For $s < 1$ use the reflection formula:

$$\zeta(s) = (2\pi)^s \Gamma(1-s) \zeta(1-s) / \pi$$

Definition at line 348 of file sf_zeta.tcc.

References __gamma(), and __sin_pi().

Referenced by __riemann_zeta().

8.3.2.290 __rising_factorial() [1/2]

```
template<typename _Tp >
_Tp std::__detail::__rising_factorial (
    _Tp __a,
    int __n )
```

Return the (upper) Pochhammer function or the rising factorial function. The Pochhammer symbol is defined by

$$a^{\overline{n}} = \Gamma(a + \nu) / \Gamma(\nu) = \prod_{k=0}^{n-1} (a + k), (a)_0 = 1$$

Many notations exist for this function:

$$(a)_{\nu}$$

, (especially in the literature of special functions),

$$\left[\begin{matrix} a \\ n \end{matrix} \right]$$

, and others.

Definition at line 3102 of file sf_gamma.tcc.

References `__log_gamma()`, `__log_gamma_sign()`, and `std::__detail::_Factorial_table<_Tp>::__n`.

Referenced by `__log_rising_factorial()`, and `__rising_factorial()`.

8.3.2.291 __rising_factorial() [2/2]

```
template<typename _Tp >
_Tp std::__detail::__rising_factorial (
    _Tp __a,
    _Tp __nu )
```

Return the rising factorial function or the (upper) Pochhammer function. The rising factorial function is defined by

$$a^{\overline{\nu}} = \Gamma(a + \nu) / \Gamma(\nu)$$

Many notations exist for this function:

$$(a)_{\nu}$$

, (especially in the literature of special functions),

$$\left[\begin{matrix} a \\ n \end{matrix} \right]$$

, and others.

Definition at line 3157 of file sf_gamma.tcc.

References `__log_gamma()`, `__log_gamma_sign()`, `std::__detail::_Factorial_table<_Tp>::__n`, and `__rising_factorial()`.

8.3.2.292 `__sin_pi()` [1/2]

```
template<typename _Tp >
_Tp std::__detail::__sin_pi (
    _Tp __x )
```

Return the reperiodized sine of argument x:

$$\sin_{\pi}(x) = \sin(\pi x)$$

Definition at line 52 of file sf_trig.tcc.

Referenced by `__cos_pi()`, `__cosh_pi()`, `__cyl_bessel_ik()`, `__cyl_bessel_jn()`, `__dirichlet_eta()`, `__gamma_reciprocal()`, `__hankel_debye()`, `__laguerre_large_n()`, `__lanczos_log_gamma1p()`, `__log_gamma()`, `__riemann_zeta()`, `__riemann_zeta_glob()`, `__riemann_zeta_m_1()`, `__riemann_zeta_sum()`, `__sin_pi()`, `__sinc_pi()`, `__sinh_pi()`, and `__spouge_log_gamma1p()`.

8.3.2.293 `__sin_pi()` [2/2]

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__sin_pi (
    std::complex< _Tp > __z )
```

Return the reperiodized sine of complex argument z:

$$\sin_{\pi}(z) = \sin(\pi z) = \sin_{\pi}(x)\cosh_{\pi}(y) + i\cos_{\pi}(x)\sinh_{\pi}(y)$$

Definition at line 187 of file sf_trig.tcc.

References `__cos_pi()`, and `__sin_pi()`.

8.3.2.294 `__sinc()`

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> std::__detail::__sinc (
    _Tp __x )
```

Return the sinus cardinal function

$$\text{sinc}(x) = \frac{\sin(x)}{x}$$

.

Definition at line 52 of file sf_cardinal.tcc.

8.3.2.295 __sinc_pi()

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> std::__detail::__sinc_pi (
    _Tp __x )
```

Return the reperiodized sinus cardinal function

$$\text{sinc}_{\pi}(x) = \frac{\sin(\pi x)}{\pi x}$$

.

Definition at line 72 of file sf_cardinal.tcc.

References __sin_pi().

8.3.2.296 __sincos() [1/4]

```
template<typename _Tp >
__gnu_cxx::__sincos_t<_Tp> std::__detail::__sincos (
    _Tp __x ) [inline]
```

Definition at line 316 of file sf_trig.tcc.

Referenced by __sincos_pi().

8.3.2.297 __sincos() [2/4]

```
template<>
__gnu_cxx::__sincos_t<float> std::__detail::__sincos (
    float __x ) [inline]
```

Definition at line 324 of file sf_trig.tcc.

8.3.2.298 __sincos() [3/4]

```
template<>
__gnu_cxx::__sincos_t<double> std::__detail::__sincos (
    double __x ) [inline]
```

Definition at line 336 of file sf_trig.tcc.

8.3.2.299 `__sincos()` [4 / 4]

```
template<>
__gnu_cxx::__sincos_t<long double> std::__detail::__sincos (
    long double __x ) [inline]
```

Definition at line 348 of file `sf_trig.tcc`.

8.3.2.300 `__sincos_pi()`

```
template<typename _Tp >
__gnu_cxx::__sincos_t<_Tp> std::__detail::__sincos_pi (
    _Tp __x )
```

Reperiodized sincos.

Definition at line 360 of file `sf_trig.tcc`.

References `__gnu_cxx::__sincos_t<_Tp>::__cos_v`, `__gnu_cxx::__sincos_t<_Tp>::__sin_v`, and `__sincos()`.

Referenced by `__polar_pi()`.

8.3.2.301 `__sincosint()`

```
template<typename _Tp >
std::pair<_Tp, _Tp> std::__detail::__sincosint (
    _Tp __x )
```

This function returns the sine $Si(x)$ and cosine $Ci(x)$ integrals as a `pair`.

The sine integral is defined by:

$$Si(x) = \int_0^x dt \frac{\sin(t)}{t}$$

The cosine integral is defined by:

$$Ci(x) = \gamma_E + \log(x) + \int_0^x dt \frac{\cos(t) - 1}{t}$$

Definition at line 226 of file `sf_trigint.tcc`.

References `__sincosint_asymp()`, `__sincosint_cont_frac()`, and `__sincosint_series()`.

8.3.2.302 __sincosint_asyp()

```
template<typename _Tp >
void std::__detail::__sincosint_asyp (
    _Tp __t,
    _Tp & __Si,
    _Tp & __Ci )
```

This function computes the sine $Si(x)$ and cosine $Ci(x)$ integrals by asymptotic series summation for positive argument.

The asymptotic series is very good for $x > 50$.

Definition at line 159 of file sf_trigint.tcc.

Referenced by __sincosint().

8.3.2.303 __sincosint_cont_frac()

```
template<typename _Tp >
void std::__detail::__sincosint_cont_frac (
    _Tp __t,
    _Tp & __Si,
    _Tp & __Ci )
```

This function computes the sine $Si(x)$ and cosine $Ci(x)$ integrals by continued fraction for positive argument.

Definition at line 52 of file sf_trigint.tcc.

Referenced by __sincosint().

8.3.2.304 __sincosint_series()

```
template<typename _Tp >
void std::__detail::__sincosint_series (
    _Tp __t,
    _Tp & __Si,
    _Tp & __Ci )
```

This function computes the sine $Si(x)$ and cosine $Ci(x)$ integrals by series summation for positive argument.

Definition at line 95 of file sf_trigint.tcc.

Referenced by __sincosint().

8.3.2.305 `__sinh_pi()` [1/2]

```
template<typename _Tp >
_Tp std::__detail::__sinh_pi (
    _Tp __x )
```

Return the reperiodized hyperbolic sine of argument x:

$$\sinh_{\pi}(x) = \sinh(\pi x)$$

Definition at line 84 of file sf_trig.tcc.

Referenced by `__sinhc_pi()`.

8.3.2.306 `__sinh_pi()` [2/2]

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__sinh_pi (
    std::complex< _Tp > __z )
```

Return the reperiodized hyperbolic sine of complex argument z:

$$\sinh_{\pi}(z) = \sinh(\pi z) = \sinh_{\pi}(x)\cos_{\pi}(y) + i\cosh_{\pi}(x)\sin_{\pi}(y)$$

Definition at line 209 of file sf_trig.tcc.

References `__cos_pi()`, and `__sin_pi()`.

8.3.2.307 `__sinhc()`

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> std::__detail::__sinhc (
    _Tp __x )
```

Return the hyperbolic sinus cardinal function

$$\operatorname{sinhc}(x) = \frac{\sinh(x)}{x}$$

.

Definition at line 97 of file sf_cardinal.tcc.

8.3.2.308 __sinhc_pi()

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> std::__detail::__sinhc_pi (
    _Tp __x )
```

Return the reperiodized hyperbolic sinus cardinal function

$$\operatorname{sinhc}_{\pi}(x) = \frac{\sinh(\pi x)}{\pi x}$$

.

Definition at line 115 of file sf_cardinal.tcc.

References `__sinh_pi()`.

8.3.2.309 __sinhint()

```
template<typename _Tp >
_Tp std::__detail::__sinhint (
    const _Tp __x )
```

Return the hyperbolic sine integral $Shi(x)$.

The hyperbolic sine integral is given by

$$Shi(x) = (Ei(x) + E_1(x))/2 = (Ei(x) - Ei(-x))/2$$

Parameters

<code>__x</code>	The argument of the hyperbolic sine integral function.
------------------	--

Returns

The hyperbolic sine integral.

Definition at line 584 of file sf_expint.tcc.

References `__expint_E1()`, and `__expint_Ei()`.

8.3.2.310 `__sph_bessel()` [1/2]

```
template<typename _Tp >
_Tp std::__detail::__sph_bessel (
    unsigned int __n,
    _Tp __x )
```

Return the spherical Bessel function $j_n(x)$ of order n and non-negative real argument x .

The spherical Bessel function is defined by:

$$j_n(x) = \left(\frac{\pi}{2x}\right)^{1/2} J_{n+1/2}(x)$$

Parameters

<code>__n</code>	The non-negative integral order
<code>__x</code>	The non-negative real argument

Returns

The output spherical Bessel function.

Definition at line 781 of file `sf_bessel.tcc`.

References `__sph_bessel_jn()`.

8.3.2.311 `__sph_bessel()` [2/2]

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__sph_bessel (
    unsigned int __n,
    std::complex<_Tp> __z )
```

Return the complex spherical Bessel function.

Parameters

in	<code>__n</code>	The order for which the spherical Bessel function is evaluated.
in	<code>__z</code>	The argument at which the spherical Bessel function is evaluated.

Returns

The complex spherical Bessel function.

Definition at line 1273 of file sf_hankel.tcc.

References `__sph_hankel()`.

8.3.2.312 __sph_bessel_ik()

```
template<typename _Tp >
__gnu_cxx::__sph_mod_bessel_t<unsigned int, _Tp, _Tp> std::__detail::__sph_bessel_ik (
    unsigned int __n,
    _Tp __x )
```

Compute the spherical modified Bessel functions $i_n(x)$ and $k_n(x)$ and their first derivatives $i'_n(x)$ and $k'_n(x)$ respectively.

Parameters

\leftarrow <code>__n</code>	The order of the modified spherical Bessel function.
\leftarrow <code>__x</code>	The argument of the modified spherical Bessel function.

Returns

A struct containing the modified spherical Bessel functions of the first and second kinds and their derivatives.

Definition at line 428 of file sf_mod_bessel.tcc.

References `__cyl_bessel_ik()`.

8.3.2.313 __sph_bessel_jn()

```
template<typename _Tp >
__gnu_cxx::__sph_bessel_t<unsigned int, _Tp, _Tp> std::__detail::__sph_bessel_jn (
    unsigned int __n,
    _Tp __x )
```

Compute the spherical Bessel $j_n(x)$ and Neumann $n_n(x)$ functions and their first derivatives $j'_n(x)$ and $n'_n(x)$ respectively.

Parameters

\leftrightarrow _n	The order of the spherical Bessel function.
\leftrightarrow _x	The argument of the spherical Bessel function.

Returns

The output derivative of the spherical Neumann function.

Definition at line 713 of file sf_bessel.tcc.

References `__cyl_bessel_jn()`.

Referenced by `__sph_bessel()`, `__sph_hankel_1()`, `__sph_hankel_2()`, and `__sph_neumann()`.

8.3.2.314 `__sph_bessel_jn_neg_arg()`

```
template<typename _Tp >
__gnu_cxx::__sph_bessel_t<unsigned int, _Tp, std::complex<_Tp> > std::__detail::__sph_bessel_↵
jn_neg_arg (
    unsigned int __n,
    _Tp __x )
```

Return the spherical Bessel functions and their derivatives of order ν and argument $x < 0$.

Definition at line 737 of file sf_bessel.tcc.

References `__cyl_bessel_jn_neg_arg()`.

Referenced by `__sph_hankel_1()`, and `__sph_hankel_2()`.

8.3.2.315 `__sph_hankel()`

```
template<typename _Tp >
__gnu_cxx::__sph_hankel_t<unsigned int, std::complex<_Tp>, std::complex<_Tp> > std::__detail::__↵
__sph_hankel (
    unsigned int __n,
    std::complex< _Tp > __z )
```

Helper to compute complex spherical Hankel functions and their derivatives.

Parameters

in	\leftrightarrow _n	The order for which the spherical Hankel functions are evaluated.
in	\leftrightarrow _z	The argument at which the spherical Hankel functions are evaluated.

Returns

A struct containing the spherical Hankel functions of the first and second kinds and their derivatives.

Definition at line 1209 of file sf_hankel.tcc.

References `__hankel()`.

Referenced by `__sph_bessel()`, `__sph_hankel_1()`, `__sph_hankel_2()`, and `__sph_neumann()`.

8.3.2.316 __sph_hankel_1() [1/2]

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__sph_hankel_1 (
    unsigned int __n,
    _Tp __x )
```

Return the spherical Hankel function of the first kind $h_n^{(1)}(x)$.

The spherical Hankel function of the first kind is defined by:

$$h_n^{(1)}(x) = j_n(x) + in_n(x)$$

Parameters

\leftrightarrow _n	The order of the spherical Neumann function.
\leftrightarrow _x	The argument of the spherical Neumann function.

Returns

The output spherical Neumann function.

Definition at line 842 of file sf_bessel.tcc.

References `__sph_bessel_jn()`, and `__sph_bessel_jn_neg_arg()`.

8.3.2.317 `__sph_hankel_1()` [2/2]

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__sph_hankel_1 (
    unsigned int __n,
    std::complex< _Tp > __z )
```

Return the complex spherical Hankel function of the first kind.

Parameters

in	\longleftrightarrow <code>__n</code>	The order for which the spherical Hankel function of the first kind is evaluated.
in	\longleftrightarrow <code>__z</code>	The argument at which the spherical Hankel function of the first kind is evaluated.

Returns

The complex spherical Hankel function of the first kind.

Definition at line 1239 of file `sf_hankel.tcc`.

References `__sph_hankel()`.

8.3.2.318 `__sph_hankel_2()` [1/2]

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__sph_hankel_2 (
    unsigned int __n,
    _Tp __x )
```

Return the spherical Hankel function of the second kind $h_n^{(2)}(x)$.

The spherical Hankel function of the second kind is defined by:

$$h_n^{(2)}(x) = j_n(x) - in_n(x)$$

Parameters

\longleftrightarrow <code>__n</code>	The non-negative integral order
\longleftrightarrow <code>__x</code>	The non-negative real argument

Returns

The output spherical Neumann function.

Definition at line 877 of file sf_bessel.tcc.



References `__sph_bessel_jn()`, and `__sph_bessel_jn_neg_arg()`.

8.3.2.319 __sph_hankel_2() [2/2]

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__sph_hankel_2 (
    unsigned int __n,
    std::complex< _Tp > __z )
```

Return the complex spherical Hankel function of the second kind.

Parameters

in	 __n	The order for which the spherical Hankel function of the second kind is evaluated.
in	 __z	The argument at which the spherical Hankel function of the second kind is evaluated.

Returns

The complex spherical Hankel function of the second kind.

Definition at line 1256 of file sf_hankel.tcc.

References `__sph_hankel()`.

8.3.2.320 __sph_harmonic()

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__sph_harmonic (
    unsigned int __l,
    int __m,
    _Tp __theta,
    _Tp __phi )
```

Return the spherical harmonic function.

The spherical harmonic function of l , m , and θ , ϕ is defined by:

$$Y_l^m(\theta, \phi) = (-1)^m \left[\frac{(2l+1)}{4\pi} \frac{(l-m)!}{(l+m)!} \right] P_l^{|m|}(\cos \theta) \exp^{im\phi}$$

Note

$$Y_l^m(\theta, \phi) = 0 \text{ if } |m| > l.$$

Parameters

<code>__l</code>	The degree of the spherical harmonic function. $l \geq 0$.
<code>__m</code>	The order of the spherical harmonic function.
<code>__theta</code>	The radian polar angle argument of the spherical harmonic function.
<code>__phi</code>	The radian azimuthal angle argument of the spherical harmonic function.

Definition at line 502 of file `sf_legendre.tcc`.

References `__sph_legendre()`.

8.3.2.321 `__sph_legendre()`

```
template<typename _Tp >
_Tp std::__detail::__sph_legendre (
    unsigned int __l,
    unsigned int __m,
    _Tp __theta )
```

Return the spherical associated Legendre function.

The spherical associated Legendre function of l , m , and θ is defined as $Y_l^m(\theta, 0)$ where

$$Y_l^m(\theta, \phi) = (-1)^m \left[\frac{(2l+1)}{4\pi} \frac{(l-m)!}{(l+m)!} \right] P_l^m(\cos \theta) \exp^{im\phi}$$

is the spherical harmonic function and $P_l^m(x)$ is the associated Legendre function.

This function differs from the associated Legendre function by argument ($x = \cos(\theta)$) and by a normalization factor but this factor is rather large for large l and m and so this function is stable for larger differences of l and m .

Note

Unlike the case for `__assoc_legendre_p` the Condon-Shortley phase factor $(-1)^m$ is present here.

$$Y_l^m(\theta) = 0 \text{ if } m > l.$$

Parameters

<code>__l</code>	The degree of the spherical associated Legendre function. $l \geq 0$.
<code>__m</code>	The order of the spherical associated Legendre function.
<code>__theta</code>	The radian polar angle argument of the spherical associated Legendre function.

Definition at line 409 of file sf_legendre.tcc.

References `__legendre_p()`, and `__log_gamma()`.

Referenced by `__hydrogen()`, and `__sph_harmonic()`.

8.3.2.322 `__sph_neumann()` [1/2]

```
template<typename _Tp >
_Tp std::__detail::__sph_neumann (
    unsigned int __n,
    _Tp __x )
```

Return the spherical Neumann function $n_n(x)$ of order n and non-negative real argument x .

The spherical Neumann function is defined by:

$$n_n(x) = \left(\frac{\pi}{2x}\right)^{1/2} N_{n+1/2}(x)$$

Parameters

<code>↵ __n</code>	The order of the spherical Neumann function.
<code>↵ __x</code>	The argument of the spherical Neumann function.

Returns

The output spherical Neumann function.

Definition at line 814 of file sf_bessel.tcc.

References `__sph_bessel_jn()`.

8.3.2.323 `__sph_neumann()` [2/2]

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__sph_neumann (
    unsigned int __n,
    std::complex< _Tp > __z )
```

Return the complex spherical Neumann function.

Parameters

in	\leftrightarrow _n	The order for which the spherical Neumann function is evaluated.
in	\leftrightarrow _z	The argument at which the spherical Neumann function is evaluated.

Returns

The complex spherical Neumann function.

Definition at line 1290 of file sf_hankel.tcc.

References __sph_hankel().

8.3.2.324 __spouge_binet1p()

```
template<typename _Tp >
_GLIBCXX14_CONSTEXPR _Tp std::__detail::__spouge_binet1p (
    _Tp __z )
```

Return the Binet function $J(1 + z)$ by the Spouge method. The Binet function is the log of the scaled Gamma function $\log(\Gamma^*(z))$ defined by

$$J(z) = \log(\Gamma^*(z)) = \log(\Gamma(z)) + z - \left(z - \frac{1}{2}\right) \log(z) - \log(2\pi)$$

or

$$\Gamma(z) = \sqrt{2\pi} z^{z-\frac{1}{2}} e^{-z} e^{J(z)}$$

where $\Gamma(z)$ is the gamma function.

Parameters

\leftrightarrow _z	The argument of the log of the gamma function.
-------------------------	--

Returns

The logarithm of the gamma function.

Definition at line 1941 of file sf_gamma.tcc.

Referenced by __spouge_log_gamma1p().

8.3.2.325 __spouge_log_gamma1p()

```
template<typename _Tp >
_GLIBCXX14_CONSTEXPR _Tp std::__detail::__spouge_log_gamma1p (
    _Tp __z )
```

Return the logarithm of the gamma function $\log(\Gamma(1+z))$ by the Spouge algorithm:

$$\Gamma(z+1) = (z+a)^{z+1/2} e^{-z-a} \left[\sqrt{2\pi} + \sum_{k=1}^{\lceil a \rceil + 1} \frac{c_k(a)}{z+k} \right]$$

where

$$c_k(a) = \frac{(-1)^{k-1}}{(k-1)!} (a-k)^{k-1/2} e^{a-k}$$

and the error is bounded by

$$\epsilon(a) < a^{-1/2} (2\pi)^{-a-1/2}$$

.

If the argument is real, the log of the absolute value of the Gamma function is returned. The sign to be applied to the exponential of this log Gamma can be recovered with a call to `__log_gamma_sign`.

For complex argument the fully complex log of the gamma function is returned.

See also

Spouge, J. L., Computation of the gamma, digamma, and trigamma functions. SIAM Journal on Numerical Analysis 31, 3 (1994), pp. 931-944

Parameters

<code>__z</code>	The argument of the gamma function.
------------------	-------------------------------------

Returns

The the gamma function.

Definition at line 1985 of file `sf_gamma.tcc`.

References `__sin_pi()`, and `__spouge_binet1p()`.

Referenced by `__log_gamma()`.

8.3.2.326 `__stirling_1()` [1/2]

```
template<typename _Tp >
_Tp std::__detail::__stirling_1 (
    unsigned int __n,
    unsigned int __m )
```

Return the Stirling number of the first kind.

The Stirling numbers of the first kind are the coefficients of the Pochhammer polynomials:

$$(x)_n = \sum_{k=0}^n S_n^{(k)} x^k$$

The recursion is

$$S_{n+1}^{(m)} = S_n^{(m-1)} - n S_n^{(m)} \text{ or}$$

with starting values

$$S_0^{(0 \rightarrow m)} = 1, 0, 0, \dots, 0$$

and

$$S_{0 \rightarrow n}^{(0)} = 1, 0, 0, \dots, 0$$

Todo Find asymptotic expressions for the Stirling numbers.

Definition at line 311 of file `sf_stirling.tcc`.

8.3.2.327 `__stirling_1()` [2/2]

```
template<typename _Tp >
std::vector<_Tp> std::__detail::__stirling_1 (
    unsigned int __n )
```

Return a vector of Stirling numbers of the first kind. The recursion is

$$S_{n+1}^{(m)} = S_n^{(m-1)} - n S_n^{(m)} \text{ or}$$

with starting values

$$S_0^{(0 \rightarrow m)} = 1, 0, 0, \dots, 0$$

and

$$S_{0 \rightarrow n}^{(0)} = 1, 0, 0, \dots, 0$$

Definition at line 378 of file `sf_stirling.tcc`.

8.3.2.328 __stirling_1_recur() [1/2]

```
template<typename _Tp >
_Tp std::__detail::__stirling_1_recur (
    unsigned int __n,
    unsigned int __m )
```

Return the Stirling number of the first kind by recursion. The recursion is

$$S_{n+1}^{(m)} = S_n^{(m-1)} - nS_n^{(m)} \text{ or}$$

with starting values

$$S_0^{(0 \rightarrow m)} = 1, 0, 0, \dots, 0$$

and

$$S_{0 \rightarrow n}^{(0)} = 1, 0, 0, \dots, 0$$

Definition at line 263 of file sf_stirling.tcc.

8.3.2.329 __stirling_1_recur() [2/2]

```
template<typename _Tp >
std::vector<_Tp> std::__detail::__stirling_1_recur (
    unsigned int __n )
```

Return a vector of Stirling numbers of the first kind by recursion. The recursion is

$$S_{n+1}^{(m)} = S_n^{(m-1)} - nS_n^{(m)} \text{ or}$$

with starting values

$$S_0^{(0 \rightarrow m)} = 1, 0, 0, \dots, 0$$

and

$$S_{0 \rightarrow n}^{(0)} = 1, 0, 0, \dots, 0$$

Definition at line 340 of file sf_stirling.tcc.

8.3.2.330 __stirling_2() [1/2]

```
template<typename _Tp >
_Tp std::__detail::__stirling_2 (
    unsigned int __n,
    unsigned int __m )
```

Return the Stirling number of the second kind from lookup or by series expansion.

The series is:

$$\sigma_n^{(m)} = \sum_{k=0}^m \frac{(-1)^{m-k} k^n}{(m-k)! k!}$$

Todo Find asymptotic expressions for the Stirling numbers.

Definition at line 155 of file sf_stirling.tcc.

8.3.2.331 `__stirling_2()` [2/2]

```
template<typename _Tp >
std::vector<_Tp> std::__detail::__stirling_2 (
    unsigned int __n )
```

Return a vector of Stirling numbers of the second kind. or by series expansion.

The series is:

$$\sigma_n^{(m)} = \sum_{k=0}^m \frac{(-1)^{m-k} k^n}{(m-k)! k!}$$

Todo Find asymptotic expressions for the Stirling numbers.

Definition at line 224 of file sf_stirling.tcc.

8.3.2.332 `__stirling_2_recur()` [1/2]

```
template<typename _Tp >
_Tp std::__detail::__stirling_2_recur (
    unsigned int __n,
    unsigned int __m )
```

Return the Stirling number of the second kind by recursion. The recursion is

$$\left\{ \begin{matrix} n \\ m \end{matrix} \right\} = m \left\{ \begin{matrix} n-1 \\ m \end{matrix} \right\} + \left\{ \begin{matrix} n-1 \\ m-1 \end{matrix} \right\}$$

with starting values

$$\left\{ \begin{matrix} 0 \\ 0 \rightarrow m \end{matrix} \right\} = 1, 0, 0, \dots, 0$$

and

$$\left\{ \begin{matrix} 0 \rightarrow n \\ 0 \end{matrix} \right\} = 1, 0, 0, \dots, 0$$

The Stirling number of the second kind is denoted by other symbols in the literature: $\sigma_n^{(m)}$, $S_n^{(m)}$ and others.

Definition at line 119 of file sf_stirling.tcc.

8.3.2.333 __stirling_2_recur() [2/2]

```
template<typename _Tp >
std::vector<_Tp> std::__detail::__stirling_2_recur (
    unsigned int __n )
```

Return a vector of Stirling numbers of the second kind by recursion. The recursion is

$$\left\{ \begin{matrix} n \\ m \end{matrix} \right\} = m \left\{ \begin{matrix} n-1 \\ m \end{matrix} \right\} + \left\{ \begin{matrix} n-1 \\ m-1 \end{matrix} \right\}$$

with starting values

$$\left\{ \begin{matrix} 0 \\ 0 \rightarrow m \end{matrix} \right\} = 1, 0, 0, \dots, 0$$

and

$$\left\{ \begin{matrix} 0 \rightarrow n \\ 0 \end{matrix} \right\} = 1, 0, 0, \dots, 0$$

The Stirling number of the second kind is denoted by other symbols in the literature: $\sigma_n^{(m)}$, $S_n^{(m)}$ and others.

Definition at line 189 of file sf_stirling.tcc.

8.3.2.334 __stirling_2_series()

```
template<typename _Tp >
_Tp std::__detail::__stirling_2_series (
    unsigned int __n,
    unsigned int __m )
```

Return the Stirling number of the second kind from lookup or by series expansion.

The series is:

$$\sigma_n^{(m)} = \left\{ \begin{matrix} n \\ m \end{matrix} \right\} = \sum_{k=0}^m \frac{(-1)^{m-k} k^n}{(m-k)!k!}$$

The Stirling number of the second kind is denoted by other symbols in the literature: $\sigma_n^{(m)}$, $S_n^{(m)}$ and others.

Todo Find a way to predict the maximum Stirling number supported for a given type.

Definition at line 66 of file sf_stirling.tcc.

8.3.2.335 __student_t_p()

```
template<typename _Tp >
_Tp std::__detail::__student_t_p (
    _Tp __t,
    unsigned int __nu )
```

Return the Students T probability function.

The students T propability function is related to the incomplete beta function:

$$A(t|\nu) = 1 - I_{\frac{\nu}{\nu+t^2}}\left(\frac{\nu}{2}, \frac{1}{2}\right)A(t|\nu) =$$

Parameters

<code>__t</code>	
<code>__nu</code>	

Definition at line 444 of file sf_distributions.tcc.

References `__beta_inc()`.

8.3.2.336 `__student_t_pdf()`

```
template<typename _Tp >
_Tp std::__detail::__student_t_pdf (
    _Tp __t,
    unsigned int __nu )
```

Return the Students T probability density.

The students T propability density is:

$$A(t|\nu) = 1 - I_{\frac{\nu}{\nu+t^2}}\left(\frac{\nu}{2}, \frac{1}{2}\right)A(t|\nu) =$$

Parameters

<code>__t</code>	
<code>__nu</code>	

Definition at line 419 of file sf_distributions.tcc.

References `__gamma()`.

8.3.2.337 `__student_t_q()`

```
template<typename _Tp >
_Tp std::__detail::__student_t_q (
    _Tp __t,
    unsigned int __nu )
```

Return the complement of the Students T probability function.

The complement of the students T propability function is:

$$A_c(t|\nu) = I_{\frac{\nu}{\nu+t^2}}\left(\frac{\nu}{2}, \frac{1}{2}\right) = 1 - A(t|\nu)$$

Parameters

<code>__t</code>	
<code>__nu</code>	

Definition at line 467 of file sf_distributions.tcc.

References `__beta_inc()`.

8.3.2.338 `__tan_pi()` [1/2]

```
template<typename _Tp >
_Tp std::__detail::__tan_pi (
    _Tp __x )
```

Return the reperiodized tangent of argument x:

$$\tan_p i(x) = \tan(\pi x)$$

Definition at line 153 of file sf_trig.tcc.

Referenced by `__digamma()`, `__tan_pi()`, and `__tanh_pi()`.

8.3.2.339 `__tan_pi()` [2/2]

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__tan_pi (
    std::complex< _Tp > __z )
```

Return the reperiodized tangent of complex argument z:

$$\tan_{\pi}(z) = \tan(\pi z) = \frac{\tan_{\pi}(x) + i \tanh_{\pi}(y)}{1 - i \tan_{\pi}(x) \tanh_{\pi}(y)}$$

Definition at line 275 of file sf_trig.tcc.

References `__tan_pi()`.

8.3.2.340 `__tanh_pi()` [1/2]

```
template<typename _Tp >
_Tp std::__detail::__tanh_pi (
    _Tp __x )
```

Return the reperiodized hyperbolic tangent of argument x:

$$\tanh_{\pi}(x) = \tanh(\pi x)$$

Definition at line 169 of file `sf_trig.tcc`.

8.3.2.341 `__tanh_pi()` [2/2]

```
template<typename _Tp >
std::complex<_Tp> std::__detail::__tanh_pi (
    std::complex< _Tp > __z )
```

Return the reperiodized hyperbolic tangent of complex argument z:

$$\tanh_{\pi}(z) = \tanh(\pi z) = \frac{\tanh_{\pi}(x) + i \tan_{\pi}(y)}{1 + i \tanh_{\pi}(x) \tan_{\pi}(y)}$$

Definition at line 298 of file `sf_trig.tcc`.

References `__tan_pi()`.

8.3.2.342 `__tgamma()`

```
template<typename _Tp >
_Tp std::__detail::__tgamma (
    _Tp __a,
    _Tp __x )
```

Return the upper incomplete gamma function. The lower incomplete gamma function is defined by

$$\Gamma(a, x) = \int_x^{\infty} e^{-t} t^{a-1} dt (a > 0)$$

.

Definition at line 2905 of file `sf_gamma.tcc`.

References `__gnu_cxx::__fp_is_integer()`, `__gamma_cont_frac()`, and `__gamma_series()`.

Referenced by `__gamma_q()`.

8.3.2.343 __tgamma_lower()

```
template<typename _Tp >
_Tp std::__detail::__tgamma_lower (
    _Tp __a,
    _Tp __x )
```

Return the lower incomplete gamma function. The lower incomplete gamma function is defined by

$$\gamma(a, x) = \int_0^x e^{-t} t^{a-1} dt (a > 0)$$

.

Definition at line 2870 of file sf_gamma.tcc.

References `__gnu_cxx::__fp_is_integer()`, `__gamma_cont_frac()`, and `__gamma_series()`.

Referenced by `__gamma_p()`.

8.3.2.344 __theta_1()

```
template<typename _Tp >
_Tp std::__detail::__theta_1 (
    _Tp __nu,
    _Tp __x )
```

Return the exponential theta-1 function of period `nu` and argument `x`.

The exponential theta-1 function is defined by

$$\theta_1(\nu, x) = \frac{1}{\sqrt{\pi x}} \sum_{k=-\infty}^{+\infty} (-1)^k \exp\left(\frac{-(\nu + k - 1/2)^2}{x}\right)$$

Parameters

<code>__nu</code>	The periodic (period = 2) argument
<code>__x</code>	The argument

Definition at line 212 of file sf_theta.tcc.

References `__gnu_cxx::__fp_is_zero()`, and `__theta_2()`.

Referenced by `__theta_s()`.

8.3.2.345 __theta_2()

```
template<typename _Tp >
_Tp std::__detail::__theta_2 (
    _Tp __nu,
    _Tp __x )
```

Return the exponential theta-2 function of period `nu` and argument `x`.

The exponential theta-2 function is defined by

$$\theta_2(\nu, x) = \frac{1}{\sqrt{\pi x}} \sum_{k=-\infty}^{+\infty} (-1)^k \exp\left(\frac{-(\nu + k)^2}{x}\right)$$

Parameters

<code>__nu</code>	The periodic (period = 2) argument
<code>__x</code>	The argument

Definition at line 184 of file `sf_theta.tcc`.

References `__theta_2_asymp()`, and `__theta_2_sum()`.

Referenced by `__theta_1()`, and `__theta_c()`.

8.3.2.346 __theta_2_asymp()

```
template<typename _Tp >
_Tp std::__detail::__theta_2_asymp (
    _Tp __nu,
    _Tp __x )
```

Compute and return the exponential θ_2 function by asymptotic series expansion:

$$\theta_2(\nu, x) = 2 \sum_{k=0}^{\infty} e^{-((k+1/2)\pi)^2 x} \cos((2k+1)\nu\pi)$$

Definition at line 120 of file `sf_theta.tcc`.

Referenced by `__theta_2()`.

8.3.2.347 __theta_2_sum()

```
template<typename _Tp >
_Tp std::__detail::__theta_2_sum (
    _Tp __nu,
    _Tp __x )
```

Compute and return the exponential θ_2 function by series expansion:

$$\theta_2(\nu, x) = \frac{1}{\sqrt{\pi x}} \sum_{k=-\infty}^{\infty} (-1)^k e^{-(\nu+k)^2/x}$$

Definition at line 56 of file sf_theta.tcc.

Referenced by __theta_2().

8.3.2.348 __theta_3()

```
template<typename _Tp >
_Tp std::__detail::__theta_3 (
    _Tp __nu,
    _Tp __x )
```

Return the exponential theta-3 function of period `nu` and argument `x`.

The exponential theta-3 function is defined by

$$\theta_3(\nu, x) = \frac{1}{\sqrt{\pi x}} \sum_{k=-\infty}^{+\infty} \exp\left(\frac{-(\nu+k)^2}{x}\right)$$

Parameters

<code>__nu</code>	The periodic (period = 1) argument
<code>__x</code>	The argument

Definition at line 240 of file sf_theta.tcc.

References __theta_3_asymp(), and __theta_3_sum().

Referenced by __theta_4(), and __theta_d().

8.3.2.349 __theta_3_asymp()

```
template<typename _Tp >
_Tp std::__detail::__theta_3_asymp (
    _Tp __nu,
    _Tp __x )
```

Compute and return the exponential θ_3 function by asymptotic series expansion:

$$\theta_3(\nu, x) = 1 + 2 \sum_{k=1}^{\infty} e^{-(k\pi)^2 x} \cos(2k\nu\pi)$$

Definition at line 150 of file sf_theta.tcc.

Referenced by __theta_3().

8.3.2.350 __theta_3_sum()

```
template<typename _Tp >
_Tp std::__detail::__theta_3_sum (
    _Tp __nu,
    _Tp __x )
```

Compute and return the exponential θ_3 function by series expansion:

$$\theta_3(\nu, x) = \frac{1}{\sqrt{\pi x}} \sum_{k=-\infty}^{\infty} e^{-(\nu+k)^2/x}$$

Definition at line 89 of file sf_theta.tcc.

Referenced by __theta_3().

8.3.2.351 __theta_4()

```
template<typename _Tp >
_Tp std::__detail::__theta_4 (
    _Tp __nu,
    _Tp __x )
```

Return the exponential theta-4 function of period `nu` and argument `x`.

The exponential theta-4 function is defined by

$$\theta_4(\nu, x) = \frac{1}{\sqrt{\pi x}} \sum_{k=-\infty}^{+\infty} (-1)^k \exp\left(\frac{-(\nu+k)^2}{x}\right)$$

Parameters

<code>__nu</code>	The periodic (period = 2) argument
<code>__x</code>	The argument

Definition at line 268 of file sf_theta.tcc.

References `__theta_3()`.

Referenced by `__theta_n()`.

8.3.2.352 `__theta_c()`

```
template<typename _Tp >
_Tp std::__detail::__theta_c (
    _Tp __k,
    _Tp __x )
```

Return the Neville θ_c function

$$\theta_c(k, x) = \sqrt{\frac{\pi}{2kK(k)}} \theta_1 \left(q(k), \frac{\pi x}{2K(k)} \right)$$

Definition at line 382 of file sf_theta.tcc.

References `__comp_ellint_1()`, `__ellnome()`, and `__theta_2()`.

8.3.2.353 `__theta_d()`

```
template<typename _Tp >
_Tp std::__detail::__theta_d (
    _Tp __k,
    _Tp __x )
```

Return the Neville θ_d function

$$\theta_d(k, x) = \sqrt{\frac{\pi}{2K(k)}} \theta_3 \left(q(k), \frac{\pi x}{2K(k)} \right)$$

Definition at line 411 of file sf_theta.tcc.

References `__comp_ellint_1()`, `__ellnome()`, and `__theta_3()`.

8.3.2.354 __theta_n()

```
template<typename _Tp >
_Tp std::__detail::__theta_n (
    _Tp __k,
    _Tp __x )
```

Return the Neville θ_n function

The Neville theta-n function is defined by

$$\theta_n(k, x) = \sqrt{\frac{\pi}{2k'K(k)}} \theta_4 \left(q(k), \frac{\pi x}{2K(k)} \right)$$

Definition at line 442 of file sf_theta.tcc.

References `__comp_ellint_1()`, `__ellnome()`, and `__theta_4()`.

8.3.2.355 __theta_s()

```
template<typename _Tp >
_Tp std::__detail::__theta_s (
    _Tp __k,
    _Tp __x )
```

Return the Neville θ_s function

$$\theta_s(k, x) = \sqrt{\frac{\pi}{2kk'K(k)}} \theta_1 \left(q(k), \frac{\pi x}{2K(k)} \right)$$

Definition at line 352 of file sf_theta.tcc.

References `__comp_ellint_1()`, `__ellnome()`, and `__theta_1()`.

8.3.2.356 __tricomi_u()

```
template<typename _Tp >
_Tp std::__detail::__tricomi_u (
    _Tp __a,
    _Tp __c,
    _Tp __x )
```

Return the Tricomi confluent hypergeometric function

$$U(a, c, x) = \frac{\Gamma(1-c)}{\Gamma(a-c+1)} {}_1F_1(a; c; x) + \frac{\Gamma(c-1)}{\Gamma(a)} x^{1-c} {}_1F_1(a-c+1; 2-c; x)$$

.

Parameters

\leftrightarrow _a	The <i>numerator</i> parameter.
\leftrightarrow _c	The <i>denominator</i> parameter.
\leftrightarrow _x	The argument of the confluent hypergeometric function.

Returns

The Tricomi confluent hypergeometric function.

Definition at line 402 of file sf_hyperg.tcc.

References `__tricomi_u_naive()`.

8.3.2.357 __tricomi_u_naive()

```
template<typename _Tp >
_Tp std::__detail::__tricomi_u_naive (
    _Tp __a,
    _Tp __c,
    _Tp __x )
```

Return the Tricomi confluent hypergeometric function

$$U(a, c, x) = \frac{\Gamma(1-c)}{\Gamma(a-c+1)} {}_1F_1(a; c; x) + \frac{\Gamma(c-1)}{\Gamma(a)} x^{1-c} {}_1F_1(a-c+1; 2-c; x)$$

.

Parameters

\leftrightarrow _a	The <i>numerator</i> parameter.
\leftrightarrow _c	The <i>denominator</i> parameter.
\leftrightarrow _x	The argument of the confluent hypergeometric function.

Returns

The Tricomi confluent hypergeometric function.

Definition at line 368 of file sf_hyperg.tcc.

References `__conf_hyperg()`, `__gnu_cxx::__fp_is_integer()`, and `__gnu_cxx::tgamma()`.

Referenced by `__tricoli_u()`.

8.3.2.358 `__weibull_p()`

```
template<typename _Tp >
_Tp std::__detail::__weibull_p (
    _Tp __a,
    _Tp __b,
    _Tp __x )
```

Return the Weibull cumulative probability density function.

The formula for the Weibull cumulative probability density function is

$$F(x|\lambda) = 1 - e^{-(x/b)^a} \text{ for } x \geq 0$$

Definition at line 395 of file `sf_distributions.tcc`.

8.3.2.359 `__weibull_pdf()`

```
template<typename _Tp >
_Tp std::__detail::__weibull_pdf (
    _Tp __a,
    _Tp __b,
    _Tp __x )
```

Return the Weibull probability density function.

The formula for the Weibull probability density function is

$$f(x|a, b) = \frac{a}{b} \left(\frac{x}{b}\right)^{a-1} \exp - \left(\frac{x}{b}\right)^a \text{ for } x \geq 0$$

Definition at line 374 of file `sf_distributions.tcc`.

8.3.2.360 __zernike()

```
template<typename _Tp >
__gnu_cxx::fp_promote_t<_Tp> std::__detail::__zernike (
    unsigned int __n,
    int __m,
    _Tp __rho,
    _Tp __phi )
```

Return the Zernike polynomial $Z_n^m(\rho, \phi)$ for non-negative integral degree n , signed integral order m , and real radial argument ρ and azimuthal angle ϕ .

The even Zernike polynomials are defined by:

$$Z_n^m(\rho, \phi) = R_n^m(\rho) \cos(m\phi)$$

and the odd Zernike polynomials are defined by:

$$Z_n^{-m}(\rho, \phi) = R_n^m(\rho) \sin(m\phi)$$

for non-negative degree m and $m \leq n$ and where $R_n^m(\rho)$ is the radial polynomial (

See also

[__radial_jacobi](#)).

Principals of Optics, 7th edition, Max Born and Emil Wolf, Cambridge University Press, 1999, pp 523-525 and 905-910.

Template Parameters

<code>_Tp</code>	The real type of the radial coordinate and azimuthal angle
------------------	--

Parameters

<code>__n</code>	The non-negative integral degree.
<code>__m</code>	The integral azimuthal order
<code>__rho</code>	The radial coordinate
<code>__phi</code>	The azimuthal angle

Definition at line 373 of file `sf_jacobi.tcc`.

References [__radial_jacobi](#)).

8.3.2.361 __znorm1()

```
template<typename _Tp >
_Tp std::__detail::__znorm1 (
    _Tp __x )
```

Definition at line 58 of file sf_owens_t.tcc.

Referenced by `__owens_t()`.

8.3.2.362 `__znorm2()`

```
template<typename _Tp >
_Tp std::__detail::__znorm2 (
    _Tp __x )
```

Definition at line 47 of file sf_owens_t.tcc.

Referenced by `__owens_t()`.

8.3.3 Variable Documentation

8.3.3.1 `__max_FGH`

```
template<typename _Tp >
constexpr int std::__detail::__max_FGH = \_Airy\_series<_Tp>::_N_FGH
```

Definition at line 178 of file sf_airy.tcc.

8.3.3.2 `__max_FGH< double >`

```
template<>
constexpr int std::\_\_detail::\_\_max\_FGH< double > = 79
```

Definition at line 184 of file sf_airy.tcc.

8.3.3.3 `__max_FGH< float >`

```
template<>
constexpr int std::\_\_detail::\_\_max\_FGH< float > = 15
```

Definition at line 181 of file sf_airy.tcc.

8.3.3.4 _Num_Euler_Maclaurin_zeta

```
constexpr size_t std::__detail::_Num_Euler_Maclaurin_zeta = 100
```

Coefficients for Euler-Maclaurin summation of zeta functions.

$$B_{2j}/(2j)!$$

where B_k are the Bernoulli numbers.

Definition at line 117 of file sf_zeta.tcc.

Referenced by __polylog_exp_neg().

8.3.3.5 _Num_Stieltjes

```
constexpr size_t std::__detail::_Num_Stieltjes = 21
```

Coefficients for the expansion of the Riemann zeta function:

$$\zeta(s) = \frac{1}{s-1} + \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \gamma_n (s-1)^n$$

$\gamma_0 = \gamma_E$ the Euler-Masceroni constant.

<http://www.plouffe.fr/simon/constants/stieltjesgamma.txt>

Definition at line 83 of file sf_zeta.tcc.

Referenced by __riemann_zeta_laurent().

8.3.3.6 _S_double_factorial_table

```
constexpr _Factorial_table<long double> std::__detail::_S_double_factorial_table[301]
```

Definition at line 280 of file sf_gamma.tcc.

Referenced by __double_factorial(), __log_double_factorial(), and __log_factorial().

8.3.3.7 _S_Euler_Maclaurin_zeta

```
constexpr long double std::__detail::_S_Euler_Maclaurin_zeta[_Num_Euler_Maclaurin_zeta]
```

Definition at line 120 of file sf_zeta.tcc.

Referenced by __hurwitz_zeta_euler_maclaurin(), __polylog_exp_neg(), and __riemann_zeta_euler_maclaurin().

8.3.3.8 `_S_factorial_table`

```
constexpr \_Factorial\_table<long double> std::__detail::_S_factorial_table[171]
```

Definition at line 90 of file `sf_gamma.tcc`.

Referenced by `__factorial()`, `__gamma()`, `__gamma_reciprocal()`, `__log_factorial()`, and `__log_gamma()`.

8.3.3.9 `_S_harmonic_denom`

```
constexpr unsigned long long std::__detail::_S_harmonic_denom[\_S\_num\_harmonic\_number]
```

Definition at line 3254 of file `sf_gamma.tcc`.

Referenced by `__harmonic_number()`.

8.3.3.10 `_S_harmonic_numer`

```
constexpr unsigned long long std::__detail::_S_harmonic_numer[\_S\_num\_harmonic\_number]
```

Definition at line 3221 of file `sf_gamma.tcc`.

Referenced by `__harmonic_number()`.

8.3.3.11 `_S_neg_double_factorial_table`

```
constexpr \_Factorial\_table<long double> std::__detail::_S_neg_double_factorial_table[999]
```

Definition at line 601 of file `sf_gamma.tcc`.

Referenced by `__double_factorial()`, and `__log_double_factorial()`.

8.3.3.12 `_S_num_double_factorials`

```
template<typename _Tp >  
constexpr std::size_t std::__detail::_S_num_double_factorials = 0
```

Definition at line 265 of file `sf_gamma.tcc`.

8.3.3.13 `_S_num_double_factorials< double >`

```
template<>
constexpr std::size_t std::__detail::_S_num_double_factorials< double > = 301
```

Definition at line 270 of file sf_gamma.tcc.

8.3.3.14 `_S_num_double_factorials< float >`

```
template<>
constexpr std::size_t std::__detail::_S_num_double_factorials< float > = 57
```

Definition at line 268 of file sf_gamma.tcc.

8.3.3.15 `_S_num_double_factorials< long double >`

```
template<>
constexpr std::size_t std::__detail::_S_num_double_factorials< long double > = 301
```

Definition at line 272 of file sf_gamma.tcc.

8.3.3.16 `_S_num_factorials`

```
template<typename _Tp >
constexpr std::size_t std::__detail::_S_num_factorials = 0
```

Definition at line 75 of file sf_gamma.tcc.

8.3.3.17 `_S_num_factorials< double >`

```
template<>
constexpr std::size_t std::__detail::_S_num_factorials< double > = 171
```

Definition at line 80 of file sf_gamma.tcc.

8.3.3.18 `_S_num_factorials< float >`

```
template<>
constexpr std::size_t std::__detail::_S_num_factorials< float > = 35
```

Definition at line 78 of file `sf_gamma.tcc`.

8.3.3.19 `_S_num_factorials< long double >`

```
template<>
constexpr std::size_t std::__detail::_S_num_factorials< long double > = 171
```

Definition at line 82 of file `sf_gamma.tcc`.

8.3.3.20 `_S_num_harmonic_numer`

```
constexpr unsigned long long std::__detail::_S_num_harmonic_numer = 29
```

Definition at line 3218 of file `sf_gamma.tcc`.

Referenced by `__harmonic_number()`.

8.3.3.21 `_S_num_neg_double_factorials`

```
template<typename _Tp >
constexpr std::size_t std::__detail::_S_num_neg_double_factorials = 0
```

Definition at line 585 of file `sf_gamma.tcc`.

8.3.3.22 `_S_num_neg_double_factorials< double >`

```
template<>
constexpr std::size_t std::__detail::_S_num_neg_double_factorials< double > = 150
```

Definition at line 590 of file `sf_gamma.tcc`.

8.3.3.23 `_S_num_neg_double_factorials< float >`

```
template<>
constexpr std::size_t std::__detail::_S_num_neg_double_factorials< float > = 27
```

Definition at line 588 of file `sf_gamma.tcc`.

8.3.3.24 `_S_num_neg_double_factorials< long double >`

```
template<>
constexpr std::size_t std::__detail::_S_num_neg_double_factorials< long double > = 999
```

Definition at line 592 of file `sf_gamma.tcc`.

8.3.3.25 `_S_num_zetam1`

```
constexpr size_t std::__detail::_S_num_zetam1 = 121
```

Table of $\zeta(n) - 1$ from 0 - 120. MPFR @ 128 bits precision.

Definition at line 580 of file `sf_zeta.tcc`.

Referenced by `__riemann_zeta_m_1()`.

8.3.3.26 `_S_Stieltjes`

```
constexpr long double std::__detail::_S_Stieltjes[_Num_Stieltjes]
```

Initial value:

```
{
    +0.5772156649015328606065120900824024310421593359L,
    -0.0728158454836767248605863758749013191377363383L,
    -0.0096903631928723184845303860352125293590658061L,
    +0.0020538344203033458661600465427533842857158044L,
    +0.0023253700654673000574681701775260680009044694L,
    +0.0007933238173010627017533348774444448307315394L,
    -0.0002387693454301996098724218419080042777837151L,
    -0.0005272895670577510460740975054788582819962534L,
    -0.0003521233538030395096020521650012087417291805L,
    -0.0000343947744180880481779146237982273906207895L,
    +0.0002053328149090647946837222892370653029598537L,
    +0.0002701844395439035266729020820679556738278420L,
    +0.0001672729121051401933535015433411834466078066L,
    -0.0000274638066037601588600076036933551815267853L,
    -0.0002092092620592999458371396973445849578315442L,
    -0.0002834686553202414466429344749971269770687029L,
    -0.0001996968583089697747077845632032403919157649L,
    +0.0000262770371099183366994665976305101228160786L,
    +0.0003073684081492528265927547519486256455238112L,
    +0.0005036054530473556290555964377171600353212698L,
    +0.0004663435615115594494005948244335505251131434L,
}
```

Definition at line 86 of file `sf_zeta.tcc`.

Referenced by `__riemann_zeta_laurent()`.

8.3.3.27 `_S_zetam1`

```
constexpr long double std::__detail::_S_zetam1[\_S\_num\_zetam1]
```

Definition at line 584 of file `sf_zeta.tcc`.

Referenced by `__riemann_zeta_m_1()`.

Chapter 9

Class Documentation

9.1 `__gnu_cxx::__airy_t<_Tx, _Tp>` Struct Template Reference

```
#include <specfun_state.h>
```

Public Member Functions

- `_Tp __Wronskian () const`
Return the Wronskian of this Airy function state.

Public Attributes

- `_Tp __Ai_deriv`
The derivative of the Airy function Ai.
- `_Tp __Ai_value`
The value of the Airy function Ai.
- `_Tp __Bi_deriv`
The derivative of the Airy function Bi.
- `_Tp __Bi_value`
The value of the Airy function Bi.
- `_Tx __x_arg`
The argument of the Airy fuctions.

9.1.1 Detailed Description

```
template<typename _Tx, typename _Tp>  
struct __gnu_cxx::__airy_t<_Tx, _Tp>
```

Definition at line 472 of file `specfun_state.h`.

9.1.2 Member Function Documentation

9.1.2.1 __Wronskian()

```
template<typename _Tx , typename _Tp >
_Tp __gnu_cxx::__airy_t< _Tx, _Tp >::__Wronskian ( ) const [inline]
```

Return the Wronskian of this Airy function state.

Definition at line 490 of file specfun_state.h.

9.1.3 Member Data Documentation

9.1.3.1 __Ai_deriv

```
template<typename _Tx , typename _Tp >
_Tp __gnu_cxx::__airy_t< _Tx, _Tp >::__Ai_deriv
```

The derivative of the Airy function Ai.

Definition at line 481 of file specfun_state.h.

9.1.3.2 __Ai_value

```
template<typename _Tx , typename _Tp >
_Tp __gnu_cxx::__airy_t< _Tx, _Tp >::__Ai_value
```

The value of the Airy function Ai.

Definition at line 478 of file specfun_state.h.

9.1.3.3 __Bi_deriv

```
template<typename _Tx , typename _Tp >
_Tp __gnu_cxx::__airy_t< _Tx, _Tp >::__Bi_deriv
```

The derivative of the Airy function Bi.

Definition at line 487 of file specfun_state.h.

9.1.3.4 __Bi_value

```
template<typename _Tx , typename _Tp >
_Tp __gnu_cxx::__airy_t< _Tx, _Tp >::__Bi_value
```

The value of the Airy function Bi.

Definition at line 484 of file specfun_state.h.

9.1.3.5 __x_arg

```
template<typename _Tx , typename _Tp >
_Tx __gnu_cxx::__airy_t< _Tx, _Tp >::__x_arg
```

The argument of the Airy fuctions.

Definition at line 475 of file specfun_state.h.

The documentation for this struct was generated from the following file:

- [include/bits/specfun_state.h](#)

9.2 __gnu_cxx::__assoc_legendre_p_t<_Tp> Struct Template Reference

```
#include <specfun_state.h>
```

Public Member Functions

- [_Tp deriv](#) () const

Public Attributes

- unsigned int [__l](#)
- unsigned int [__m](#)
- [_Tp __P_lm](#)
- [_Tp __P_lm1m](#)

$$P_l^{\{(m)\}}(x)$$
- [_Tp __P_lm2m](#)

$$P_{\{l-1\}}^{\{(m)\}}(x)$$
- [_Tp __phase](#) = 1

$$P_{\{l-2\}}^{\{(m)\}}(x)$$
- [_Tp __x](#)

9.2.1 Detailed Description

```
template<typename _Tp>
struct __gnu_cxx::__assoc_legendre_p_t<_Tp>
```

A type describing the state of an associated Legendre function.

Definition at line 157 of file specfun_state.h.

9.2.2 Member Function Documentation

9.2.2.1 deriv()

```
template<typename _Tp>
_Tp __gnu_cxx::__assoc_legendre_p_t<_Tp>::deriv ( ) const [inline]
```

Definition at line 168 of file specfun_state.h.

9.2.3 Member Data Documentation

9.2.3.1 __l

```
template<typename _Tp>
unsigned int __gnu_cxx::__assoc_legendre_p_t<_Tp>::__l
```

Definition at line 159 of file specfun_state.h.

9.2.3.2 __m

```
template<typename _Tp>
unsigned int __gnu_cxx::__assoc_legendre_p_t<_Tp>::__m
```

Definition at line 160 of file specfun_state.h.

9.2.3.3 __P_lm

```
template<typename _Tp >
_Tp __gnu_cxx::__assoc_legendre_p_t< _Tp >::__P_lm
```

Definition at line 162 of file specfun_state.h.

9.2.3.4 __P_lm1m

```
template<typename _Tp >
_Tp __gnu_cxx::__assoc_legendre_p_t< _Tp >::__P_lm1m
```

$$P_l^{(m)}(x)$$

Definition at line 163 of file specfun_state.h.

9.2.3.5 __P_lm2m

```
template<typename _Tp >
_Tp __gnu_cxx::__assoc_legendre_p_t< _Tp >::__P_lm2m
```

$$P_{l-1}^{(m)}(x)$$

Definition at line 164 of file specfun_state.h.

9.2.3.6 __phase

```
template<typename _Tp >
_Tp __gnu_cxx::__assoc_legendre_p_t< _Tp >::__phase = 1
```

$$P_{l-2}^{(m)}(x)$$

Definition at line 165 of file specfun_state.h.

9.2.3.7 __x

```
template<typename _Tp >
_Tp __gnu_cxx::__assoc_legendre_p_t< _Tp >::__x
```

Definition at line 161 of file specfun_state.h.

The documentation for this struct was generated from the following file:

- include/bits/specfun_state.h

9.3 __gnu_cxx::__assoc_legendre_q_t< _Tp > Struct Template Reference

```
#include <specfun_state.h>
```

Public Member Functions

- `_Tp deriv () const`

Public Attributes

- unsigned int `__l`
- unsigned int `__m`
degree
- `_Tp __phase = 1`
 $Q_l^{\{m-2\}}(x)$
- `_Tp __Q_lm`
argument
- `_Tp __Q_lmm1`
 $Q_l^{\{m\}}(x)$
- `_Tp __Q_lmm2`
 $Q_l^{\{m-1\}}(x)$
- `_Tp __x`
order

9.3.1 Detailed Description

```
template<typename _Tp>
struct __gnu_cxx::__assoc_legendre_q_t< _Tp >
```

A type describing the state of an associated Legendre function of the second kind.

Definition at line 225 of file specfun_state.h.

9.3.2 Member Function Documentation

9.3.2.1 deriv()

```
template<typename _Tp >
_Tp __gnu_cxx::__assoc_legendre_q_t<_Tp>::deriv ( ) const [inline]
```

Definition at line 236 of file specfun_state.h.

9.3.3 Member Data Documentation

9.3.3.1 __l

```
template<typename _Tp >
unsigned int __gnu_cxx::__assoc_legendre_q_t<_Tp>::__l
```

Definition at line 227 of file specfun_state.h.

9.3.3.2 __m

```
template<typename _Tp >
unsigned int __gnu_cxx::__assoc_legendre_q_t<_Tp>::__m
```

degree

Definition at line 228 of file specfun_state.h.

9.3.3.3 __phase

```
template<typename _Tp >
_Tp __gnu_cxx::__assoc_legendre_q_t<_Tp>::__phase = 1
```

$Q_l^{(m-2)}(x)$

Definition at line 233 of file specfun_state.h.

9.3.3.4 __Q_lm

```
template<typename _Tp >
_Tp __gnu_cxx::__assoc_legendre_q_t< _Tp >::__Q_lm
```

argument

Definition at line 230 of file specfun_state.h.

9.3.3.5 __Q_lmm1

```
template<typename _Tp >
_Tp __gnu_cxx::__assoc_legendre_q_t< _Tp >::__Q_lmm1
```

$Q_l^{(m)}(x)$

Definition at line 231 of file specfun_state.h.

9.3.3.6 __Q_lmm2

```
template<typename _Tp >
_Tp __gnu_cxx::__assoc_legendre_q_t< _Tp >::__Q_lmm2
```

$Q_l^{(m-1)}(x)$

Definition at line 232 of file specfun_state.h.

9.3.3.7 __x

```
template<typename _Tp >
_Tp __gnu_cxx::__assoc_legendre_q_t< _Tp >::__x
```

order

Definition at line 229 of file specfun_state.h.

The documentation for this struct was generated from the following file:

- [include/bits/specfun_state.h](#)

9.4 __gnu_cxx::__chebyshev_t_t<_Tp> Struct Template Reference

```
#include <specfun_state.h>
```

Public Member Functions

- [_Tp deriv](#) () const
- [_Tp deriv2](#) () const

Public Attributes

- unsigned int [__n](#)
- [_Tp __T_n](#)
- [_Tp __T_nm1](#)
- [_Tp __T_nm2](#)
- [_Tp __x](#)

9.4.1 Detailed Description

```
template<typename _Tp>  
struct __gnu_cxx::__chebyshev_t_t<_Tp>
```

A type describing the state of a Chebyshev polynomial of the first kind.

Definition at line 321 of file specfun_state.h.

9.4.2 Member Function Documentation

9.4.2.1 deriv()

```
template<typename _Tp>  
_Tp __gnu_cxx::__chebyshev_t_t<_Tp>::deriv ( ) const [inline]
```

Definition at line 330 of file specfun_state.h.

9.4.2.2 deriv2()

```
template<typename _Tp>  
_Tp __gnu_cxx::__chebyshev_t_t<_Tp>::deriv2 ( ) const [inline]
```

Definition at line 334 of file specfun_state.h.

9.4.3 Member Data Documentation

9.4.3.1 __n

```
template<typename _Tp >  
unsigned int __gnu_cxx::__chebyshev_t_t< _Tp >::__n
```

Definition at line 323 of file specfun_state.h.

9.4.3.2 __T_n

```
template<typename _Tp >  
_Tp __gnu_cxx::__chebyshev_t_t< _Tp >::__T_n
```

Definition at line 325 of file specfun_state.h.

9.4.3.3 __T_nm1

```
template<typename _Tp >  
_Tp __gnu_cxx::__chebyshev_t_t< _Tp >::__T_nm1
```

Definition at line 326 of file specfun_state.h.

9.4.3.4 __T_nm2

```
template<typename _Tp >  
_Tp __gnu_cxx::__chebyshev_t_t< _Tp >::__T_nm2
```

Definition at line 327 of file specfun_state.h.

9.4.3.5 `__x`

```
template<typename _Tp >
_Tp __gnu_cxx::__chebyshev_t_t< _Tp >::__x
```

Definition at line 324 of file `specfun_state.h`.

The documentation for this struct was generated from the following file:

- `include/bits/specfun_state.h`

9.5 `__gnu_cxx::__chebyshev_u_t<_Tp>` Struct Template Reference

```
#include <specfun_state.h>
```

Public Member Functions

- `_Tp deriv () const`

Public Attributes

- `unsigned int __n`
- `_Tp __U_n`
- `_Tp __U_nm1`
- `_Tp __U_nm2`
- `_Tp __x`

9.5.1 Detailed Description

```
template<typename _Tp>
struct __gnu_cxx::__chebyshev_u_t<_Tp>
```

A type describing the state of a Chebyshev polynomial of the second kind.

Definition at line 348 of file `specfun_state.h`.

9.5.2 Member Function Documentation

9.5.2.1 deriv()

```
template<typename _Tp >  
_Tp __gnu_cxx::__chebyshev_u_t< _Tp >::deriv ( ) const [inline]
```

Definition at line 357 of file specfun_state.h.

9.5.3 Member Data Documentation

9.5.3.1 __n

```
template<typename _Tp >  
unsigned int __gnu_cxx::__chebyshev_u_t< _Tp >::__n
```

Definition at line 350 of file specfun_state.h.

9.5.3.2 __U_n

```
template<typename _Tp >  
_Tp __gnu_cxx::__chebyshev_u_t< _Tp >::__U_n
```

Definition at line 352 of file specfun_state.h.

9.5.3.3 __U_nm1

```
template<typename _Tp >  
_Tp __gnu_cxx::__chebyshev_u_t< _Tp >::__U_nm1
```

Definition at line 353 of file specfun_state.h.

9.5.3.4 __U_nm2

```
template<typename _Tp >  
_Tp __gnu_cxx::__chebyshev_u_t< _Tp >::__U_nm2
```

Definition at line 354 of file specfun_state.h.

9.5.3.5 `__x`

```
template<typename _Tp >
_Tp __gnu_cxx::__chebyshev_u_t< _Tp >::__x
```

Definition at line 351 of file `specfun_state.h`.

The documentation for this struct was generated from the following file:

- `include/bits/specfun_state.h`

9.6 `__gnu_cxx::__chebyshev_v_t<_Tp>` Struct Template Reference

```
#include <specfun_state.h>
```

Public Member Functions

- `_Tp deriv () const`

Public Attributes

- `unsigned int __n`
- `_Tp __V_n`
- `_Tp __V_nm1`
- `_Tp __V_nm2`
- `_Tp __x`

9.6.1 Detailed Description

```
template<typename _Tp>
struct __gnu_cxx::__chebyshev_v_t<_Tp>
```

A type describing the state of a Chebyshev polynomial of the third kind.

Definition at line 368 of file `specfun_state.h`.

9.6.2 Member Function Documentation

9.6.2.1 deriv()

```
template<typename _Tp >
_Tp __gnu_cxx::__chebyshev_v_t< _Tp >::deriv ( ) const [inline]
```

Definition at line 377 of file specfun_state.h.

9.6.3 Member Data Documentation

9.6.3.1 __n

```
template<typename _Tp >
unsigned int __gnu_cxx::__chebyshev_v_t< _Tp >::__n
```

Definition at line 370 of file specfun_state.h.

9.6.3.2 __V_n

```
template<typename _Tp >
_Tp __gnu_cxx::__chebyshev_v_t< _Tp >::__V_n
```

Definition at line 372 of file specfun_state.h.

9.6.3.3 __V_nm1

```
template<typename _Tp >
_Tp __gnu_cxx::__chebyshev_v_t< _Tp >::__V_nm1
```

Definition at line 373 of file specfun_state.h.

9.6.3.4 __V_nm2

```
template<typename _Tp >
_Tp __gnu_cxx::__chebyshev_v_t< _Tp >::__V_nm2
```

Definition at line 374 of file specfun_state.h.

9.6.3.5 `__x`

```
template<typename _Tp >
_Tp __gnu_cxx::__chebyshev_v_t< _Tp >::__x
```

Definition at line 371 of file `specfun_state.h`.

The documentation for this struct was generated from the following file:

- `include/bits/specfun_state.h`

9.7 `__gnu_cxx::__chebyshev_w_t<_Tp>` Struct Template Reference

```
#include <specfun_state.h>
```

Public Member Functions

- `_Tp deriv () const`

Public Attributes

- `unsigned int __n`
- `_Tp __W_n`
- `_Tp __W_nm1`
- `_Tp __W_nm2`
- `_Tp __x`

9.7.1 Detailed Description

```
template<typename _Tp>
struct __gnu_cxx::__chebyshev_w_t<_Tp>
```

A type describing the state of a Chebyshev polynomial of the fourth kind.

Definition at line 390 of file `specfun_state.h`.

9.7.2 Member Function Documentation

9.7.2.1 deriv()

```
template<typename _Tp >
_Tp __gnu_cxx::__chebyshev_w_t< _Tp >::deriv ( ) const [inline]
```

Definition at line 399 of file specfun_state.h.

9.7.3 Member Data Documentation

9.7.3.1 __n

```
template<typename _Tp >
unsigned int __gnu_cxx::__chebyshev_w_t< _Tp >::__n
```

Definition at line 392 of file specfun_state.h.

9.7.3.2 __W_n

```
template<typename _Tp >
_Tp __gnu_cxx::__chebyshev_w_t< _Tp >::__W_n
```

Definition at line 394 of file specfun_state.h.

9.7.3.3 __W_nm1

```
template<typename _Tp >
_Tp __gnu_cxx::__chebyshev_w_t< _Tp >::__W_nm1
```

Definition at line 395 of file specfun_state.h.

9.7.3.4 __W_nm2

```
template<typename _Tp >
_Tp __gnu_cxx::__chebyshev_w_t< _Tp >::__W_nm2
```

Definition at line 396 of file specfun_state.h.

9.7.3.5 `__x`

```
template<typename _Tp >
_Tp __gnu_cxx::__chebyshev_w_t<_Tp>::__x
```

Definition at line 393 of file `specfun_state.h`.

The documentation for this struct was generated from the following file:

- [include/bits/specfun_state.h](#)

9.8 `__gnu_cxx::__coulomb_t<_Teta, _Trho, _Tp>` Struct Template Reference

```
#include <specfun_state.h>
```

Public Member Functions

- `_Tp __Wronskian () const`
Return the Wronskian of this Coulomb function state.

Public Attributes

- `_Teta __eta_arg`
The real parameter of the Coulomb functions.
- `_Tp __F_deriv`
The derivative of the regular Coulomb function.
- `_Tp __F_value`
The value of the regular Coulomb function.
- `_Tp __G_deriv`
The derivative of the irregular Coulomb function.
- `_Tp __G_value`
The value of the irregular Coulomb function.
- `unsigned int __l`
The nonnegative order of the Coulomb functions.
- `_Trho __rho_arg`
The argument of the Coulomb functions.

9.8.1 Detailed Description

```
template<typename _Teta, typename _Trho, typename _Tp>
struct __gnu_cxx::__coulomb_t<_Teta, _Trho, _Tp>
```

This struct captures the state of the Coulomb functions at a given order and argument.

Definition at line 555 of file `specfun_state.h`.

9.8.2 Member Function Documentation

9.8.2.1 __Wronskian()

```
template<typename _Teta , typename _Trho , typename _Tp >
_Tp __gnu_cxx::__coulomb_t< _Teta, _Trho, _Tp >::__Wronskian ( ) const [inline]
```

Return the Wronskian of this Coulomb function state.

Definition at line 579 of file specfun_state.h.

9.8.3 Member Data Documentation

9.8.3.1 __eta_arg

```
template<typename _Teta , typename _Trho , typename _Tp >
_Teta __gnu_cxx::__coulomb_t< _Teta, _Trho, _Tp >::__eta_arg
```

The real parameter of the Coulomb functions.

Definition at line 561 of file specfun_state.h.

9.8.3.2 __F_deriv

```
template<typename _Teta , typename _Trho , typename _Tp >
_Tp __gnu_cxx::__coulomb_t< _Teta, _Trho, _Tp >::__F_deriv
```

The derivative of the regular Coulomb function.

Definition at line 570 of file specfun_state.h.

9.8.3.3 __F_value

```
template<typename _Teta , typename _Trho , typename _Tp >
_Tp __gnu_cxx::__coulomb_t< _Teta, _Trho, _Tp >::__F_value
```

The value of the regular Coulomb function.

Definition at line 567 of file specfun_state.h.

9.8.3.4 `__G_deriv`

```
template<typename _Teta , typename _Trho , typename _Tp >
_Tp __gnu_cxx::__coulomb_t< _Teta, _Trho, _Tp >::__G_deriv
```

The derivative of the irregular Coulomb function.

Definition at line 576 of file `specfun_state.h`.

9.8.3.5 `__G_value`

```
template<typename _Teta , typename _Trho , typename _Tp >
_Tp __gnu_cxx::__coulomb_t< _Teta, _Trho, _Tp >::__G_value
```

The value of the irregular Coulomb function.

Definition at line 573 of file `specfun_state.h`.

9.8.3.6 `__l`

```
template<typename _Teta , typename _Trho , typename _Tp >
unsigned int __gnu_cxx::__coulomb_t< _Teta, _Trho, _Tp >::__l
```

The nonnegative order of the Coulomb functions.

Definition at line 558 of file `specfun_state.h`.

9.8.3.7 `__rho_arg`

```
template<typename _Teta , typename _Trho , typename _Tp >
_Trho __gnu_cxx::__coulomb_t< _Teta, _Trho, _Tp >::__rho_arg
```

The argument of the Coulomb functions.

Definition at line 564 of file `specfun_state.h`.

The documentation for this struct was generated from the following file:

- `include/bits/specfun_state.h`

9.9 `__gnu_cxx::__cyl_bessel_t<_Tnu, _Tx, _Tp>` Struct Template Reference

```
#include <specfun_state.h>
```

Public Member Functions

- `_Tp __Wronskian () const`
Return the Wronskian of this cylindrical Bessel function state.

Public Attributes

- `_Tp __J_deriv`
The derivative of the Bessel function of the first kind.
- `_Tp __J_value`
The value of the Bessel function of the first kind.
- `_Tp __N_deriv`
The derivative of the Bessel function of the second kind.
- `_Tp __N_value`
The value of the Bessel function of the second kind.
- `_Tnu __nu_arg`
The real order of the cylindrical Bessel functions.
- `_Tx __x_arg`
The argument of the cylindrical Bessel functions.

9.9.1 Detailed Description

```
template<typename _Tnu, typename _Tx, typename _Tp>
struct __gnu_cxx::__cyl_bessel_t<_Tnu, _Tx, _Tp>
```

This struct captures the state of the cylindrical Bessel functions at a given order and argument.

Definition at line 525 of file `specfun_state.h`.

9.9.2 Member Function Documentation

9.9.2.1 `__Wronskian()`

```
template<typename _Tnu , typename _Tx , typename _Tp >
_Tp __gnu_cxx::__cyl_bessel_t<_Tnu, _Tx, _Tp>::__Wronskian ( ) const [inline]
```

Return the Wronskian of this cylindrical Bessel function state.

Definition at line 546 of file `specfun_state.h`.

9.9.3 Member Data Documentation

9.9.3.1 __J_deriv

```
template<typename _Tnu , typename _Tx , typename _Tp >  
_Tp __gnu_cxx::__cyl_bessel_t< _Tnu, _Tx, _Tp >::__J_deriv
```

The derivative of the Bessel function of the first kind.

Definition at line 537 of file specfun_state.h.

9.9.3.2 __J_value

```
template<typename _Tnu , typename _Tx , typename _Tp >  
_Tp __gnu_cxx::__cyl_bessel_t< _Tnu, _Tx, _Tp >::__J_value
```

The value of the Bessel function of the first kind.

Definition at line 534 of file specfun_state.h.

9.9.3.3 __N_deriv

```
template<typename _Tnu , typename _Tx , typename _Tp >  
_Tp __gnu_cxx::__cyl_bessel_t< _Tnu, _Tx, _Tp >::__N_deriv
```

The derivative of the Bessel function of the second kind.

Definition at line 543 of file specfun_state.h.

9.9.3.4 __N_value

```
template<typename _Tnu , typename _Tx , typename _Tp >  
_Tp __gnu_cxx::__cyl_bessel_t< _Tnu, _Tx, _Tp >::__N_value
```

The value of the Bessel function of the second kind.

Definition at line 540 of file specfun_state.h.

9.9.3.5 `__nu_arg`

```
template<typename _Tnu , typename _Tx , typename _Tp >
_Tnu __gnu_cxx::__cyl_bessel_t< _Tnu, _Tx, _Tp >::__nu_arg
```

The real order of the cylindrical Bessel functions.

Definition at line 528 of file `specfun_state.h`.

9.9.3.6 `__x_arg`

```
template<typename _Tnu , typename _Tx , typename _Tp >
_Tx __gnu_cxx::__cyl_bessel_t< _Tnu, _Tx, _Tp >::__x_arg
```

The argument of the cylindrical Bessel functions.

Definition at line 531 of file `specfun_state.h`.

The documentation for this struct was generated from the following file:

- `include/bits/specfun_state.h`

9.10 `__gnu_cxx::__cyl_hankel_t< _Tnu, _Tx, _Tp >` Struct Template Reference

```
#include <specfun_state.h>
```

Public Member Functions

- `_Tp __Wronskian () const`
Return the Wronskian of this cylindrical Hankel function state.

Public Attributes

- `_Tp __H1_deriv`
The derivative of the cylindrical Hankel function of the first kind.
- `_Tp __H1_value`
The value of the cylindrical Hankel function of the first kind.
- `_Tp __H2_deriv`
The derivative of the cylindrical Hankel function of the second kind.
- `_Tp __H2_value`
The value of the cylindrical Hankel function of the second kind.
- `_Tnu __nu_arg`
The real order of the cylindrical Hankel functions.
- `_Tx __x_arg`
The argument of the modified Hankel functions.

9.10.1 Detailed Description

```
template<typename _Tnu, typename _Tx, typename _Tp>
struct __gnu_cxx::__cyl_hankel_t< _Tnu, _Tx, _Tp >
```

`_Tp` pretty much has to be complex.

Definition at line 622 of file `specfun_state.h`.

9.10.2 Member Function Documentation

9.10.2.1 `__Wronskian()`

```
template<typename _Tnu, typename _Tx, typename _Tp>
_Tp __gnu_cxx::__cyl_hankel_t< _Tnu, _Tx, _Tp >::__Wronskian ( ) const [inline]
```

Return the Wronskian of this cylindrical Hankel function state.

Definition at line 643 of file `specfun_state.h`.

9.10.3 Member Data Documentation

9.10.3.1 `__H1_deriv`

```
template<typename _Tnu, typename _Tx, typename _Tp>
_Tp __gnu_cxx::__cyl_hankel_t< _Tnu, _Tx, _Tp >::__H1_deriv
```

The derivative of the cylindrical Hankel function of the first kind.

Definition at line 634 of file `specfun_state.h`.

9.10.3.2 `__H1_value`

```
template<typename _Tnu, typename _Tx, typename _Tp>
_Tp __gnu_cxx::__cyl_hankel_t< _Tnu, _Tx, _Tp >::__H1_value
```

The value of the cylindrical Hankel function of the first kind.

Definition at line 631 of file `specfun_state.h`.

9.10.3.3 __H2_deriv

```
template<typename _Tnu, typename _Tx, typename _Tp>
_Tp __gnu_cxx::__cyl_hankel_t< _Tnu, _Tx, _Tp >::__H2_deriv
```

The derivative of the cylindrical Hankel function of the second kind.

Definition at line 640 of file specfun_state.h.

9.10.3.4 __H2_value

```
template<typename _Tnu, typename _Tx, typename _Tp>
_Tp __gnu_cxx::__cyl_hankel_t< _Tnu, _Tx, _Tp >::__H2_value
```

The value of the cylindrical Hankel function of the second kind.

Definition at line 637 of file specfun_state.h.

9.10.3.5 __nu_arg

```
template<typename _Tnu, typename _Tx, typename _Tp>
_Tnu __gnu_cxx::__cyl_hankel_t< _Tnu, _Tx, _Tp >::__nu_arg
```

The real order of the cylindrical Hankel functions.

Definition at line 625 of file specfun_state.h.

9.10.3.6 __x_arg

```
template<typename _Tnu, typename _Tx, typename _Tp>
_Tx __gnu_cxx::__cyl_hankel_t< _Tnu, _Tx, _Tp >::__x_arg
```

The argument of the modified Hankel functions.

Definition at line 628 of file specfun_state.h.

The documentation for this struct was generated from the following file:

- [include/bits/specfun_state.h](#)

9.11 `__gnu_cxx::__cyl_mod_bessel_t<_Tnu, _Tx, _Tp>` Struct Template Reference

```
#include <specfun_state.h>
```

Public Member Functions

- `_Tp __Wronskian () const`
Return the Wronskian of this modified cylindrical Bessel function state.

Public Attributes

- `_Tp __I_deriv`
The derivative of the modified cylindrical Bessel function of the first kind.
- `_Tp __I_value`
The value of the modified cylindrical Bessel function of the first kind.
- `_Tp __K_deriv`
The derivative of the modified cylindrical Bessel function of the second kind.
- `_Tp __K_value`
The value of the modified cylindrical Bessel function of the second kind.
- `_Tnu __nu_arg`
The real order of the modified cylindrical Bessel functions.
- `_Tx __x_arg`
The argument of the modified cylindrical Bessel functions.

9.11.1 Detailed Description

```
template<typename _Tnu, typename _Tx, typename _Tp>
struct __gnu_cxx::__cyl_mod_bessel_t<_Tnu, _Tx, _Tp>
```

This struct captures the state of the modified cylindrical Bessel functions at a given order and argument.

Definition at line 588 of file `specfun_state.h`.

9.11.2 Member Function Documentation

9.11.2.1 `__Wronskian()`

```
template<typename _Tnu , typename _Tx , typename _Tp >
_Tp __gnu_cxx::__cyl_mod_bessel_t<_Tnu, _Tx, _Tp>::__Wronskian ( ) const [inline]
```

Return the Wronskian of this modified cylindrical Bessel function state.

Definition at line 614 of file `specfun_state.h`.

9.11.3 Member Data Documentation

9.11.3.1 __I_deriv

```
template<typename _Tnu , typename _Tx , typename _Tp >  
_Tp __gnu_cxx::__cyl_mod_bessel_t< _Tnu, _Tx, _Tp >::__I_deriv
```

The derivative of the modified cylindrical Bessel function of the first kind.

Definition at line 602 of file specfun_state.h.

9.11.3.2 __I_value

```
template<typename _Tnu , typename _Tx , typename _Tp >  
_Tp __gnu_cxx::__cyl_mod_bessel_t< _Tnu, _Tx, _Tp >::__I_value
```

The value of the modified cylindrical Bessel function of the first kind.

Definition at line 598 of file specfun_state.h.

9.11.3.3 __K_deriv

```
template<typename _Tnu , typename _Tx , typename _Tp >  
_Tp __gnu_cxx::__cyl_mod_bessel_t< _Tnu, _Tx, _Tp >::__K_deriv
```

The derivative of the modified cylindrical Bessel function of the second kind.

Definition at line 610 of file specfun_state.h.

9.11.3.4 __K_value

```
template<typename _Tnu , typename _Tx , typename _Tp >  
_Tp __gnu_cxx::__cyl_mod_bessel_t< _Tnu, _Tx, _Tp >::__K_value
```

The value of the modified cylindrical Bessel function of the second kind.

Definition at line 606 of file specfun_state.h.

9.11.3.5 `__nu_arg`

```
template<typename _Tnu , typename _Tx , typename _Tp >
_Tnu __gnu_cxx::__cyl_mod_bessel_t< _Tnu, _Tx, _Tp >::__nu_arg
```

The real order of the modified cylindrical Bessel functions.

Definition at line 591 of file `specfun_state.h`.

9.11.3.6 `__x_arg`

```
template<typename _Tnu , typename _Tx , typename _Tp >
_Tx __gnu_cxx::__cyl_mod_bessel_t< _Tnu, _Tx, _Tp >::__x_arg
```

The argument of the modified cylindrical Bessel functions.

Definition at line 594 of file `specfun_state.h`.

The documentation for this struct was generated from the following file:

- [include/bits/specfun_state.h](#)

9.12 `__gnu_cxx::__fock_airy_t<_Tx, _Tp>` Struct Template Reference

```
#include <specfun_state.h>
```

Public Member Functions

- `_Tp __Wronskian () const`
Return the Wronskian of this Fock-type Airy function state.

Public Attributes

- `_Tp __w1_deriv`
The derivative of the Fock-type Airy function w1.
- `_Tp __w1_value`
The value of the Fock-type Airy function w1.
- `_Tp __w2_deriv`
The derivative of the Fock-type Airy function w2.
- `_Tp __w2_value`
The value of the Fock-type Airy function w2.
- `_Tx __x_arg`
The argument of the Fock-type Airy fuctions.

9.12.1 Detailed Description

```
template<typename _Tx, typename _Tp>
struct __gnu_cxx::__fock_airy_t< _Tx, _Tp >
```

_Tp pretty much has to be complex.

Definition at line 498 of file specfun_state.h.

9.12.2 Member Function Documentation

9.12.2.1 __Wronskian()

```
template<typename _Tx , typename _Tp >
_Tp __gnu_cxx::__fock_airy_t< _Tx, _Tp >::__Wronskian ( ) const [inline]
```

Return the Wronskian of this Fock-type Airy function state.

Definition at line 516 of file specfun_state.h.

9.12.3 Member Data Documentation

9.12.3.1 __w1_deriv

```
template<typename _Tx , typename _Tp >
_Tp __gnu_cxx::__fock_airy_t< _Tx, _Tp >::__w1_deriv
```

The derivative of the Fock-type Airy function w1.

Definition at line 507 of file specfun_state.h.

9.12.3.2 __w1_value

```
template<typename _Tx , typename _Tp >
_Tp __gnu_cxx::__fock_airy_t< _Tx, _Tp >::__w1_value
```

The value of the Fock-type Airy function w1.

Definition at line 504 of file specfun_state.h.

9.12.3.3 __w2_deriv

```
template<typename _Tx , typename _Tp >
_Tp __gnu_cxx::__fock_airy_t< _Tx, _Tp >::__w2_deriv
```

The derivative of the Fock-type Airy function w2.

Definition at line 513 of file specfun_state.h.

9.12.3.4 __w2_value

```
template<typename _Tx , typename _Tp >
_Tp __gnu_cxx::__fock_airy_t< _Tx, _Tp >::__w2_value
```

The value of the Fock-type Airy function w2.

Definition at line 510 of file specfun_state.h.

9.12.3.5 __x_arg

```
template<typename _Tx , typename _Tp >
_Tx __gnu_cxx::__fock_airy_t< _Tx, _Tp >::__x_arg
```

The argument of the Fock-type Airy fuctions.

Definition at line 501 of file specfun_state.h.

The documentation for this struct was generated from the following file:

- [include/bits/specfun_state.h](#)

9.13 __gnu_cxx::__fp_is_integer_t Struct Reference

```
#include <math_util.h>
```

Public Member Functions

- constexpr [operator bool](#) () const noexcept
- constexpr int [operator\(\)](#) () const noexcept

Public Attributes

- bool `__is_integral` = false
- int `__value` = 0

9.13.1 Detailed Description

A struct returned by floating point integer queries.

Definition at line 127 of file `math_util.h`.

9.13.2 Member Function Documentation

9.13.2.1 `operator bool()`

```
constexpr __gnu_cxx::__fp_is_integer_t::operator bool ( ) const [inline], [noexcept]
```

Definition at line 136 of file `math_util.h`.

References `__is_integral`.

9.13.2.2 `operator()()`

```
constexpr int __gnu_cxx::__fp_is_integer_t::operator() ( ) const [inline], [noexcept]
```

Definition at line 141 of file `math_util.h`.

References `__value`.

9.13.3 Member Data Documentation

9.13.3.1 `__is_integral`

```
bool __gnu_cxx::__fp_is_integer_t::__is_integral = false
```

Definition at line 130 of file `math_util.h`.

Referenced by `operator bool()`.

9.13.3.2 `__value`

```
int __gnu_cxx::__fp_is_integer_t::__value = 0
```

Definition at line 133 of file `math_util.h`.

Referenced by `operator()()`.

The documentation for this struct was generated from the following file:

- `cxx_fp_utils/include/ext/math_util.h`

9.14 `__gnu_cxx::__gamma_inc_t<_Tp>` Struct Template Reference

```
#include <specfun_state.h>
```

Public Attributes

- `_Tp __lgamma_value`
The value of the log of the incomplete gamma function.
- `_Tp __tgamma_value`
The value of the total gamma function.

9.14.1 Detailed Description

```
template<typename _Tp>
struct __gnu_cxx::__gamma_inc_t<_Tp>
```

The sign of the exponentiated `log(gamma)` is applied to the `tgamma` value.

Definition at line 761 of file `specfun_state.h`.

9.14.2 Member Data Documentation

9.14.2.1 `__lgamma_value`

```
template<typename _Tp>
_Tp __gnu_cxx::__gamma_inc_t<_Tp>::__lgamma_value
```

The value of the log of the incomplete gamma function.

Definition at line 766 of file `specfun_state.h`.

9.14.2.2 __tgamma_value

```
template<typename _Tp >
_Tp __gnu_cxx::__gamma_inc_t< _Tp >::__tgamma_value
```

The value of the total gamma function.

Definition at line 764 of file specfun_state.h.

The documentation for this struct was generated from the following file:

- [include/bits/specfun_state.h](#)

9.15 __gnu_cxx::__gamma_temme_t< _Tp > Struct Template Reference

A structure for the gamma functions required by the Temme series expansions of $N_\nu(x)$ and $K_\nu(x)$.

$$\Gamma_1 = \frac{1}{2\mu} \left[\frac{1}{\Gamma(1-\mu)} - \frac{1}{\Gamma(1+\mu)} \right]$$

and

$$\Gamma_2 = \frac{1}{2} \left[\frac{1}{\Gamma(1-\mu)} + \frac{1}{\Gamma(1+\mu)} \right]$$

where $-1/2 \leq \mu \leq 1/2$ is $\mu = \nu - N$ and N is the nearest integer to ν . The values of $\Gamma(1+\mu)$ and $\Gamma(1-\mu)$ are returned as well.

```
#include <specfun_state.h>
```

Public Attributes

- [_Tp __gamma_1_value](#)
The output function $\Gamma_1(\mu)$.
- [_Tp __gamma_2_value](#)
The output function $\Gamma_2(\mu)$.
- [_Tp __gamma_minus_value](#)
The output function $1/\Gamma(1-\mu)$.
- [_Tp __gamma_plus_value](#)
The output function $1/\Gamma(1+\mu)$.
- [_Tp __mu_arg](#)
The input parameter of the gamma functions.

9.15.1 Detailed Description

```
template<typename _Tp>
struct __gnu_cxx::__gamma_temme_t<_Tp>
```

A structure for the gamma functions required by the Temme series expansions of $N_\nu(x)$ and $K_\nu(x)$.

$$\Gamma_1 = \frac{1}{2\mu} \left[\frac{1}{\Gamma(1-\mu)} - \frac{1}{\Gamma(1+\mu)} \right]$$

and

$$\Gamma_2 = \frac{1}{2} \left[\frac{1}{\Gamma(1-\mu)} + \frac{1}{\Gamma(1+\mu)} \right]$$

where $-1/2 \leq \mu \leq 1/2$ is $\mu = \nu - N$ and N is the nearest integer to ν . The values of $\Gamma(1+\mu)$ and $\Gamma(1-\mu)$ are returned as well.

The accuracy requirements on this are high for $|\mu| < 0$.

Definition at line 789 of file `specfun_state.h`.

9.15.2 Member Data Documentation

9.15.2.1 `__gamma_1_value`

```
template<typename _Tp>
_Tp __gnu_cxx::__gamma_temme_t<_Tp>::__gamma_1_value
```

The output function $\Gamma_1(\mu)$.

Definition at line 801 of file `specfun_state.h`.

9.15.2.2 `__gamma_2_value`

```
template<typename _Tp>
_Tp __gnu_cxx::__gamma_temme_t<_Tp>::__gamma_2_value
```

The output function $\Gamma_2(\mu)$.

Definition at line 804 of file `specfun_state.h`.

9.15.2.3 `__gamma_minus_value`

```
template<typename _Tp >
_Tp __gnu_cxx::__gamma_temme_t< _Tp >::__gamma_minus_value
```

The output function $1/\Gamma(1 - \mu)$.

Definition at line 798 of file `specfun_state.h`.

9.15.2.4 `__gamma_plus_value`

```
template<typename _Tp >
_Tp __gnu_cxx::__gamma_temme_t< _Tp >::__gamma_plus_value
```

The output function $1/\Gamma(1 + \mu)$.

Definition at line 795 of file `specfun_state.h`.

9.15.2.5 `__mu_arg`

```
template<typename _Tp >
_Tp __gnu_cxx::__gamma_temme_t< _Tp >::__mu_arg
```

The input parameter of the gamma functions.

Definition at line 792 of file `specfun_state.h`.

The documentation for this struct was generated from the following file:

- [include/bits/specfun_state.h](#)

9.16 `__gnu_cxx::__gappa_pq_t< _Tp >` Struct Template Reference

```
#include <specfun_state.h>
```

Public Attributes

- `_Tp __gappa_p_value`
- `_Tp __gappa_q_value`

9.16.1 Detailed Description

```
template<typename _Tp>
struct __gnu_cxx::__gappa_pq_t<_Tp>
```

Definition at line 734 of file `specfun_state.h`.

9.16.2 Member Data Documentation

9.16.2.1 `__gappa_p_value`

```
template<typename _Tp>
_Tp __gnu_cxx::__gappa_pq_t<_Tp>::__gappa_p_value
```

Definition at line 737 of file `specfun_state.h`.

9.16.2.2 `__gappa_q_value`

```
template<typename _Tp>
_Tp __gnu_cxx::__gappa_pq_t<_Tp>::__gappa_q_value
```

Definition at line 740 of file `specfun_state.h`.

The documentation for this struct was generated from the following file:

- `include/bits/specfun_state.h`

9.17 `__gnu_cxx::__gegenbauer_t<_Tp>` Struct Template Reference

```
#include <specfun_state.h>
```

Public Member Functions

- `_Tp deriv () const`

Public Attributes

- [_Tp __C_n](#)
- [_Tp __C_nm1](#)
- [_Tp __C_nm2](#)
- [_Tp __lambda](#)
- unsigned int [__n](#)
- [_Tp __x](#)

9.17.1 Detailed Description

```
template<typename _Tp>  
struct __gnu_cxx::__gegenbauer_t< _Tp >
```

A type describing the state of a Gegenbauer polynomial.

Definition at line 298 of file specfun_state.h.

9.17.2 Member Function Documentation

9.17.2.1 `deriv()`

```
template<typename _Tp >  
_Tp __gnu_cxx::__gegenbauer_t< _Tp >::deriv ( ) const [inline]
```

Definition at line 308 of file specfun_state.h.

9.17.3 Member Data Documentation

9.17.3.1 `__C_n`

```
template<typename _Tp >  
_Tp __gnu_cxx::__gegenbauer_t< _Tp >::__C_n
```

Definition at line 303 of file specfun_state.h.

9.17.3.2 `__C_nm1`

```
template<typename _Tp >
_Tp __gnu_cxx::__gegenbauer_t<_Tp >::__C_nm1
```

Definition at line 304 of file `specfun_state.h`.

9.17.3.3 `__C_nm2`

```
template<typename _Tp >
_Tp __gnu_cxx::__gegenbauer_t<_Tp >::__C_nm2
```

Definition at line 305 of file `specfun_state.h`.

9.17.3.4 `__lambda`

```
template<typename _Tp >
_Tp __gnu_cxx::__gegenbauer_t<_Tp >::__lambda
```

Definition at line 301 of file `specfun_state.h`.

9.17.3.5 `__n`

```
template<typename _Tp >
unsigned int __gnu_cxx::__gegenbauer_t<_Tp >::__n
```

Definition at line 300 of file `specfun_state.h`.

9.17.3.6 `__x`

```
template<typename _Tp >
_Tp __gnu_cxx::__gegenbauer_t<_Tp >::__x
```

Definition at line 302 of file `specfun_state.h`.

The documentation for this struct was generated from the following file:

- `include/bits/specfun_state.h`

9.18 __gnu_cxx::__hermite_he_t< _Tp > Struct Template Reference

```
#include <specfun_state.h>
```

Public Member Functions

- [_Tp deriv](#) () const
- [_Tp deriv2](#) () const

Public Attributes

- [_Tp __He_n](#)
- [_Tp __He_nm1](#)
- [_Tp __He_nm2](#)
- unsigned int [__n](#)
- [_Tp __x](#)

9.18.1 Detailed Description

```
template<typename _Tp>  
struct __gnu_cxx::__hermite_he_t< _Tp >
```

A type describing the state of a probabilists Hermite polynomial.

Definition at line 98 of file specfun_state.h.

9.18.2 Member Function Documentation

9.18.2.1 [deriv\(\)](#)

```
template<typename _Tp >  
_Tp __gnu_cxx::__hermite_he_t< _Tp >::deriv ( ) const [inline]
```

Definition at line 107 of file specfun_state.h.

9.18.2.2 [deriv2\(\)](#)

```
template<typename _Tp >  
_Tp __gnu_cxx::__hermite_he_t< _Tp >::deriv2 ( ) const [inline]
```

Definition at line 111 of file specfun_state.h.

9.18.3 Member Data Documentation

9.18.3.1 __He_n

```
template<typename _Tp >  
_Tp __gnu_cxx::__hermite_he_t< _Tp >::__He_n
```

Definition at line 102 of file specfun_state.h.

9.18.3.2 __He_nm1

```
template<typename _Tp >  
_Tp __gnu_cxx::__hermite_he_t< _Tp >::__He_nm1
```

Definition at line 103 of file specfun_state.h.

9.18.3.3 __He_nm2

```
template<typename _Tp >  
_Tp __gnu_cxx::__hermite_he_t< _Tp >::__He_nm2
```

Definition at line 104 of file specfun_state.h.

9.18.3.4 __n

```
template<typename _Tp >  
unsigned int __gnu_cxx::__hermite_he_t< _Tp >::__n
```

Definition at line 100 of file specfun_state.h.

9.18.3.5 __x

```
template<typename _Tp >
__Tp __gnu_cxx::__hermite_he_t< _Tp >::__x
```

Definition at line 101 of file specfun_state.h.

The documentation for this struct was generated from the following file:

- [include/bits/specfun_state.h](#)

9.19 __gnu_cxx::__hermite_t<_Tp> Struct Template Reference

```
#include <specfun_state.h>
```

Public Member Functions

- [_Tp deriv](#) () const
- [_Tp deriv2](#) () const

Public Attributes

- [_Tp __H_n](#)
- [_Tp __H_nm1](#)
- [_Tp __H_nm2](#)
- unsigned int [__n](#)
- [_Tp __x](#)

9.19.1 Detailed Description

```
template<typename _Tp>
struct __gnu_cxx::__hermite_t< _Tp >
```

A type describing the state of a Hermite polynomial.

Definition at line 77 of file specfun_state.h.

9.19.2 Member Function Documentation

9.19.2.1 `deriv()`

```
template<typename _Tp >
_Tp __gnu_cxx::__hermite_t<_Tp >::deriv ( ) const [inline]
```

Definition at line 86 of file `specfun_state.h`.

9.19.2.2 `deriv2()`

```
template<typename _Tp >
_Tp __gnu_cxx::__hermite_t<_Tp >::deriv2 ( ) const [inline]
```

Definition at line 90 of file `specfun_state.h`.

9.19.3 Member Data Documentation

9.19.3.1 `__H_n`

```
template<typename _Tp >
_Tp __gnu_cxx::__hermite_t<_Tp >::__H_n
```

Definition at line 81 of file `specfun_state.h`.

9.19.3.2 `__H_nm1`

```
template<typename _Tp >
_Tp __gnu_cxx::__hermite_t<_Tp >::__H_nm1
```

Definition at line 82 of file `specfun_state.h`.

9.19.3.3 `__H_nm2`

```
template<typename _Tp >
_Tp __gnu_cxx::__hermite_t<_Tp >::__H_nm2
```

Definition at line 83 of file `specfun_state.h`.

9.19.3.4 __n

```
template<typename _Tp >
unsigned int __gnu_cxx::__hermite_t< _Tp >::__n
```

Definition at line 79 of file specfun_state.h.

9.19.3.5 __x

```
template<typename _Tp >
_Tp __gnu_cxx::__hermite_t< _Tp >::__x
```

Definition at line 80 of file specfun_state.h.

The documentation for this struct was generated from the following file:

- [include/bits/specfun_state.h](#)

9.20 __gnu_cxx::__jacobi_ellint_t< _Tp > Struct Template Reference

```
#include <specfun_state.h>
```

Public Member Functions

- [_Tp __am](#) () const
- [_Tp __cd](#) () const
- [_Tp __cn_deriv](#) () const
- [_Tp __cs](#) () const
- [_Tp __dc](#) () const
- [_Tp __ds](#) () const
- [_Tp __nc](#) () const
- [_Tp __nd](#) () const
- [_Tp __ns](#) () const
- [_Tp __sc](#) () const
- [_Tp __sd](#) () const
- [_Tp __sn_deriv](#) () const

Public Attributes

- [_Tp __cn_value](#)
Jacobi cosine amplitude value.
- [_Tp __dn_value](#)
Jacobi delta amplitude value.
- [_Tp __sn_value](#)
Jacobi sine amplitude value.

9.20.1 Detailed Description

```
template<typename _Tp>
struct __gnu_cxx::__jacobi_ellint_t<_Tp>
```

Slots for Jacobi elliptic function tuple.

Definition at line 423 of file specfun_state.h.

9.20.2 Member Function Documentation

9.20.2.1 __am()

```
template<typename _Tp >
_Tp __gnu_cxx::__jacobi_ellint_t<_Tp>::__am ( ) const [inline]
```

Definition at line 434 of file specfun_state.h.

9.20.2.2 __cd()

```
template<typename _Tp >
_Tp __gnu_cxx::__jacobi_ellint_t<_Tp>::__cd ( ) const [inline]
```

Definition at line 452 of file specfun_state.h.

9.20.2.3 __cn_deriv()

```
template<typename _Tp >
_Tp __gnu_cxx::__jacobi_ellint_t<_Tp>::__cn_deriv ( ) const [inline]
```

Definition at line 467 of file specfun_state.h.

9.20.2.4 __cs()

```
template<typename _Tp >
_Tp __gnu_cxx::__jacobi_ellint_t<_Tp>::__cs ( ) const [inline]
```

Definition at line 455 of file specfun_state.h.

9.20.2.5 __dc()

```
template<typename _Tp >  
_Tp __gnu_cxx::__jacobi_ellint_t< _Tp >::__dc ( ) const [inline]
```

Definition at line 461 of file specfun_state.h.

9.20.2.6 __ds()

```
template<typename _Tp >  
_Tp __gnu_cxx::__jacobi_ellint_t< _Tp >::__ds ( ) const [inline]
```

Definition at line 458 of file specfun_state.h.

9.20.2.7 __nc()

```
template<typename _Tp >  
_Tp __gnu_cxx::__jacobi_ellint_t< _Tp >::__nc ( ) const [inline]
```

Definition at line 440 of file specfun_state.h.

9.20.2.8 __nd()

```
template<typename _Tp >  
_Tp __gnu_cxx::__jacobi_ellint_t< _Tp >::__nd ( ) const [inline]
```

Definition at line 443 of file specfun_state.h.

9.20.2.9 __ns()

```
template<typename _Tp >  
_Tp __gnu_cxx::__jacobi_ellint_t< _Tp >::__ns ( ) const [inline]
```

Definition at line 437 of file specfun_state.h.

9.20.2.10 __sc()

```
template<typename _Tp >
_Tp __gnu_cxx::__jacobi_ellint_t<_Tp>::__sc ( ) const [inline]
```

Definition at line 446 of file specfun_state.h.

9.20.2.11 __sd()

```
template<typename _Tp >
_Tp __gnu_cxx::__jacobi_ellint_t<_Tp>::__sd ( ) const [inline]
```

Definition at line 449 of file specfun_state.h.

9.20.2.12 __sn_deriv()

```
template<typename _Tp >
_Tp __gnu_cxx::__jacobi_ellint_t<_Tp>::__sn_deriv ( ) const [inline]
```

Definition at line 464 of file specfun_state.h.

9.20.3 Member Data Documentation

9.20.3.1 __cn_value

```
template<typename _Tp >
_Tp __gnu_cxx::__jacobi_ellint_t<_Tp>::__cn_value
```

Jacobi cosine amplitude value.

Definition at line 429 of file specfun_state.h.

9.20.3.2 __dn_value

```
template<typename _Tp >
_Tp __gnu_cxx::__jacobi_ellint_t< _Tp >::__dn_value
```

Jacobi delta amplitude value.

Definition at line 432 of file `specfun_state.h`.

9.20.3.3 __sn_value

```
template<typename _Tp >
_Tp __gnu_cxx::__jacobi_ellint_t< _Tp >::__sn_value
```

Jacobi sine amplitude value.

Definition at line 426 of file `specfun_state.h`.

The documentation for this struct was generated from the following file:

- `include/bits/specfun_state.h`

9.21 __gnu_cxx::__jacobi_t< _Tp > Struct Template Reference

```
#include <specfun_state.h>
```

Public Member Functions

- `_Tp deriv () const`

Public Attributes

- `_Tp __alpha1`
- `_Tp __beta1`
- `unsigned int __n`
- `_Tp __P_n`
- `_Tp __P_nm1`
- `_Tp __P_nm2`
- `_Tp __x`

9.21.1 Detailed Description

```
template<typename _Tp>  
struct __gnu_cxx::__jacobi_t<_Tp>
```

A type describing the state of a Jacobi polynomial.

Definition at line 274 of file `specfun_state.h`.

9.21.2 Member Function Documentation

9.21.2.1 `deriv()`

```
template<typename _Tp>  
_Tp __gnu_cxx::__jacobi_t<_Tp>::deriv ( ) const [inline]
```

Definition at line 285 of file `specfun_state.h`.

9.21.3 Member Data Documentation

9.21.3.1 `__alpha1`

```
template<typename _Tp>  
_Tp __gnu_cxx::__jacobi_t<_Tp>::__alpha1
```

Definition at line 277 of file `specfun_state.h`.

9.21.3.2 `__beta1`

```
template<typename _Tp>  
_Tp __gnu_cxx::__jacobi_t<_Tp>::__beta1
```

Definition at line 278 of file `specfun_state.h`.

9.21.3.3 __n

```
template<typename _Tp >
unsigned int __gnu_cxx::__jacobi_t< _Tp >::__n
```

Definition at line 276 of file specfun_state.h.

9.21.3.4 __P_n

```
template<typename _Tp >
_Tp __gnu_cxx::__jacobi_t< _Tp >::__P_n
```

Definition at line 280 of file specfun_state.h.

9.21.3.5 __P_nm1

```
template<typename _Tp >
_Tp __gnu_cxx::__jacobi_t< _Tp >::__P_nm1
```

Definition at line 281 of file specfun_state.h.

9.21.3.6 __P_nm2

```
template<typename _Tp >
_Tp __gnu_cxx::__jacobi_t< _Tp >::__P_nm2
```

Definition at line 282 of file specfun_state.h.

9.21.3.7 __x

```
template<typename _Tp >
_Tp __gnu_cxx::__jacobi_t< _Tp >::__x
```

Definition at line 279 of file specfun_state.h.

The documentation for this struct was generated from the following file:

- [include/bits/specfun_state.h](#)

9.22 __gnu_cxx::__laguerre_t<_Tpa, _Tp> Struct Template Reference

```
#include <specfun_state.h>
```

Public Member Functions

- [_Tp deriv](#) () const

Public Attributes

- [_Tpa __alpha1](#)
- [_Tp __L_n](#)
- [_Tp __L_nm1](#)
- [_Tp __L_nm2](#)
- unsigned int [__n](#)
- [_Tp __x](#)

9.22.1 Detailed Description

```
template<typename _Tpa, typename _Tp>  
struct __gnu_cxx::__laguerre_t<_Tpa, _Tp>
```

A type describing the state of a Laguerre polynomial.

Definition at line 256 of file specfun_state.h.

9.22.2 Member Function Documentation

9.22.2.1 [deriv\(\)](#)

```
template<typename _Tpa , typename _Tp >  
_Tp \_\_gnu\_cxx::\_\_laguerre\_t<\_Tpa, \_Tp>::deriv ( ) const [inline]
```

Definition at line 266 of file specfun_state.h.

9.22.3 Member Data Documentation

9.22.3.1 __alpha1

```
template<typename _Tpa , typename _Tp >  
_Tpa __gnu_cxx::__laguerre_t< _Tpa, _Tp >::__alpha1
```

Definition at line 259 of file specfun_state.h.

9.22.3.2 __L_n

```
template<typename _Tpa , typename _Tp >  
_Tp __gnu_cxx::__laguerre_t< _Tpa, _Tp >::__L_n
```

Definition at line 261 of file specfun_state.h.

9.22.3.3 __L_nm1

```
template<typename _Tpa , typename _Tp >  
_Tp __gnu_cxx::__laguerre_t< _Tpa, _Tp >::__L_nm1
```

Definition at line 262 of file specfun_state.h.

9.22.3.4 __L_nm2

```
template<typename _Tpa , typename _Tp >  
_Tp __gnu_cxx::__laguerre_t< _Tpa, _Tp >::__L_nm2
```

Definition at line 263 of file specfun_state.h.

9.22.3.5 __n

```
template<typename _Tpa , typename _Tp >  
unsigned int __gnu_cxx::__laguerre_t< _Tpa, _Tp >::__n
```

Definition at line 258 of file specfun_state.h.

9.22.3.6 `__x`

```
template<typename _Tpa , typename _Tp >
_Tp __gnu_cxx::__laguerre_t< _Tpa, _Tp >::__x
```

Definition at line 260 of file `specfun_state.h`.

The documentation for this struct was generated from the following file:

- `include/bits/specfun_state.h`

9.23 `__gnu_cxx::__legendre_p_t<_Tp>` Struct Template Reference

```
#include <specfun_state.h>
```

Public Member Functions

- `_Tp deriv () const`
- `_Tp lobatto () const`
 $P_{\{l-2\}}(x)$

Public Attributes

- unsigned int `__l`
- `_Tp __P_l`
- `_Tp __P_lm1`
 $P_l(x)$
- `_Tp __P_lm2`
 $P_{\{l-1\}}(x)$
- `_Tp __x`

9.23.1 Detailed Description

```
template<typename _Tp>
struct __gnu_cxx::__legendre_p_t<_Tp>
```

A type describing the state of a Legendre polynomial.

The method `lobatto()` will return the Lobatto polynomial:

$$L_{0l}(x) = (1 - x^2)P_l'(x) = l[P_{l-1}(x) - xP_l(x)]$$

Definition at line 124 of file `specfun_state.h`.

9.23.2 Member Function Documentation

9.23.2.1 deriv()

```
template<typename _Tp >  
_Tp __gnu_cxx::__legendre_p_t< _Tp >::deriv ( ) const [inline]
```

Definition at line 138 of file specfun_state.h.

9.23.2.2 lobatto()

```
template<typename _Tp >  
_Tp __gnu_cxx::__legendre_p_t< _Tp >::lobatto ( ) const [inline]
```

$P_{l-2}(x)$

Definition at line 134 of file specfun_state.h.

9.23.3 Member Data Documentation

9.23.3.1 __l

```
template<typename _Tp >  
unsigned int __gnu_cxx::__legendre_p_t< _Tp >::__l
```

Definition at line 126 of file specfun_state.h.

9.23.3.2 __P_l

```
template<typename _Tp >  
_Tp __gnu_cxx::__legendre_p_t< _Tp >::__P_l
```

Definition at line 128 of file specfun_state.h.

9.23.3.3 __P_lm1

```
template<typename _Tp >
_Tp __gnu_cxx::__legendre_p_t< _Tp >::__P_lm1
```

$P_l(x)$

Definition at line 129 of file specfun_state.h.

9.23.3.4 __P_lm2

```
template<typename _Tp >
_Tp __gnu_cxx::__legendre_p_t< _Tp >::__P_lm2
```

$P_{l-1}(x)$

Definition at line 130 of file specfun_state.h.

9.23.3.5 __x

```
template<typename _Tp >
_Tp __gnu_cxx::__legendre_p_t< _Tp >::__x
```

Definition at line 127 of file specfun_state.h.

The documentation for this struct was generated from the following file:

- [include/bits/specfun_state.h](#)

9.24 __gnu_cxx::__legendre_q_t< _Tp > Struct Template Reference

```
#include <specfun_state.h>
```

Public Member Functions

- $Q_{l-2}(x)$ `_Tp deriv () const`

Public Attributes

- unsigned int `__l`
- `_Tp __Q_l`
- `_Tp __Q_lm1`
 $Q_l(x)$
- `_Tp __Q_lm2`
 $Q_{l-1}(x)$
- `_Tp __x`

9.24.1 Detailed Description

```
template<typename _Tp>
struct __gnu_cxx::__legendre_q_t< _Tp >
```

A type describing the state of a Legendre function of the second kind.

Definition at line 200 of file `specfun_state.h`.

9.24.2 Member Function Documentation

9.24.2.1 `deriv()`

```
template<typename _Tp >
_Tp __gnu_cxx::__legendre_q_t< _Tp >::deriv ( ) const [inline]
```

$Q_{l-2}(x)$

Definition at line 209 of file `specfun_state.h`.

9.24.3 Member Data Documentation

9.24.3.1 `__l`

```
template<typename _Tp >
unsigned int __gnu_cxx::__legendre_q_t< _Tp >::__l
```

Definition at line 202 of file `specfun_state.h`.

9.24.3.2 `__Q_l`

```
template<typename _Tp >
__Tp __gnu_cxx::__legendre_q_t< _Tp >::__Q_l
```

Definition at line 204 of file `specfun_state.h`.

9.24.3.3 `__Q_lm1`

```
template<typename _Tp >
__Tp __gnu_cxx::__legendre_q_t< _Tp >::__Q_lm1
```

`Q_l(x)`

Definition at line 205 of file `specfun_state.h`.

9.24.3.4 `__Q_lm2`

```
template<typename _Tp >
__Tp __gnu_cxx::__legendre_q_t< _Tp >::__Q_lm2
```

`Q_{l-1}(x)`

Definition at line 206 of file `specfun_state.h`.

9.24.3.5 `__x`

```
template<typename _Tp >
__Tp __gnu_cxx::__legendre_q_t< _Tp >::__x
```

Definition at line 203 of file `specfun_state.h`.

The documentation for this struct was generated from the following file:

- `include/bits/specfun_state.h`

9.25 `__gnu_cxx::__lgamma_t<_Tp>` Struct Template Reference

```
#include <specfun_state.h>
```

Public Attributes

- [int __lgamma_sign](#)
The sign of the exponent of the log gamma value.
- [_Tp __lgamma_value](#)
The value log gamma function.

9.25.1 Detailed Description

```
template<typename _Tp>  
struct __gnu_cxx::__lgamma_t< _Tp >
```

The log of the absolute value of the gamma function The sign of the exponentiated log(gamma) is stored in sign.

Definition at line 748 of file specfun_state.h.

9.25.2 Member Data Documentation

9.25.2.1 __lgamma_sign

```
template<typename _Tp >  
int __gnu_cxx::__lgamma_t< _Tp >::__lgamma_sign
```

The sign of the exponent of the log gamma value.

Definition at line 754 of file specfun_state.h.

9.25.2.2 __lgamma_value

```
template<typename _Tp >  
_Tp __gnu_cxx::__lgamma_t< _Tp >::__lgamma_value
```

The value log gamma function.

Definition at line 751 of file specfun_state.h.

The documentation for this struct was generated from the following file:

- [include/bits/specfun_state.h](#)

9.26 `__gnu_cxx::__quadrature_point_t<_Tp>` Struct Template Reference

```
#include <specfun_state.h>
```

Public Member Functions

- `__quadrature_point_t()`=default
- `__quadrature_point_t`(`_Tp __pt`, `_Tp __wt`)

Public Attributes

- `_Tp __point`
- `_Tp __weight`

9.26.1 Detailed Description

```
template<typename _Tp>
struct __gnu_cxx::__quadrature_point_t<_Tp>
```

A structure to store quadrature rules.

Definition at line 60 of file `specfun_state.h`.

9.26.2 Constructor & Destructor Documentation

9.26.2.1 `__quadrature_point_t()` [1/2]

```
template<typename _Tp>
__gnu_cxx::__quadrature_point_t<_Tp>::__quadrature_point_t ( ) [default]
```

9.26.2.2 `__quadrature_point_t()` [2/2]

```
template<typename _Tp>
__gnu_cxx::__quadrature_point_t<_Tp>::__quadrature_point_t (
    _Tp __pt,
    _Tp __wt ) [inline]
```

Definition at line 67 of file `specfun_state.h`.

9.26.3 Member Data Documentation

9.26.3.1 __point

```
template<typename _Tp >
_Tp __gnu_cxx::__quadrature_point_t< _Tp >::__point
```

Definition at line 62 of file specfun_state.h.

9.26.3.2 __weight

```
template<typename _Tp >
_Tp __gnu_cxx::__quadrature_point_t< _Tp >::__weight
```

Definition at line 63 of file specfun_state.h.

The documentation for this struct was generated from the following file:

- [include/bits/specfun_state.h](#)

9.27 __gnu_cxx::__sincos_t< _Tp > Struct Template Reference

```
#include <specfun_state.h>
```

Public Attributes

- [_Tp __cos_v](#)
- [_Tp __sin_v](#)

9.27.1 Detailed Description

```
template<typename _Tp>
struct __gnu_cxx::__sincos_t< _Tp >
```

A type describing a cosine and a sine value. A return for sincos-type functions.

Definition at line 413 of file specfun_state.h.

9.27.2 Member Data Documentation

9.27.2.1 `__cos_v`

```
template<typename _Tp>
_Tp __gnu_cxx::__sincos_t<_Tp>::__cos_v
```

Definition at line 416 of file `specfun_state.h`.

Referenced by `std::__detail::__polar_pi()`, and `std::__detail::__sincos_pi()`.

9.27.2.2 `__sin_v`

```
template<typename _Tp>
_Tp __gnu_cxx::__sincos_t<_Tp>::__sin_v
```

Definition at line 415 of file `specfun_state.h`.

Referenced by `std::__detail::__polar_pi()`, and `std::__detail::__sincos_pi()`.

The documentation for this struct was generated from the following file:

- `include/bits/specfun_state.h`

9.28 `__gnu_cxx::__sph_bessel_t<_Tn,_Tx,_Tp>` Struct Template Reference

```
#include <specfun_state.h>
```

Public Member Functions

- `_Tp __Wronskian () const`
Return the Wronskian of this spherical Bessel function state.

Public Attributes

- [_Tp __j_deriv](#)
The derivative of the spherical Bessel function of the first kind.
- [_Tp __j_value](#)
The value of the spherical Bessel function of the first kind.
- [_Tn __n_arg](#)
The integral order of the spherical Bessel functions.
- [_Tp __n_deriv](#)
The derivative of the spherical Bessel function of the second kind.
- [_Tp __n_value](#)
The value of the spherical Bessel function of the second kind.
- [_Tx __x_arg](#)
The argument of the spherical Bessel functions.

9.28.1 Detailed Description

```
template<typename _Tn, typename _Tx, typename _Tp>
struct __gnu_cxx::__sph_bessel_t< _Tn, _Tx, _Tp >
```

Definition at line 648 of file specfun_state.h.

9.28.2 Member Function Documentation

9.28.2.1 __Wronskian()

```
template<typename _Tn , typename _Tx , typename _Tp >
_Tp __gnu_cxx::__sph_bessel_t< _Tn, _Tx, _Tp >::__Wronskian ( ) const [inline]
```

Return the Wronskian of this spherical Bessel function state.

Definition at line 669 of file specfun_state.h.

9.28.3 Member Data Documentation

9.28.3.1 `__j_deriv`

```
template<typename _Tn , typename _Tx , typename _Tp >  
_Tp __gnu_cxx::__sph_bessel_t< _Tn, _Tx, _Tp >::__j_deriv
```

The derivative of the spherical Bessel function of the first kind.

Definition at line 660 of file `specfun_state.h`.

9.28.3.2 `__j_value`

```
template<typename _Tn , typename _Tx , typename _Tp >  
_Tp __gnu_cxx::__sph_bessel_t< _Tn, _Tx, _Tp >::__j_value
```

The value of the spherical Bessel function of the first kind.

Definition at line 657 of file `specfun_state.h`.

9.28.3.3 `__n_arg`

```
template<typename _Tn , typename _Tx , typename _Tp >  
_Tn __gnu_cxx::__sph_bessel_t< _Tn, _Tx, _Tp >::__n_arg
```

The integral order of the spherical Bessel functions.

Definition at line 651 of file `specfun_state.h`.

9.28.3.4 `__n_deriv`

```
template<typename _Tn , typename _Tx , typename _Tp >  
_Tp __gnu_cxx::__sph_bessel_t< _Tn, _Tx, _Tp >::__n_deriv
```

The derivative of the spherical Bessel function of the second kind.

Definition at line 666 of file `specfun_state.h`.

9.28.3.5 `__n_value`

```
template<typename _Tn , typename _Tx , typename _Tp >
_Tp __gnu_cxx::__sph_bessel_t< _Tn, _Tx, _Tp >::__n_value
```

The value of the spherical Bessel function of the second kind.

Definition at line 663 of file `specfun_state.h`.

9.28.3.6 `__x_arg`

```
template<typename _Tn , typename _Tx , typename _Tp >
_Tx __gnu_cxx::__sph_bessel_t< _Tn, _Tx, _Tp >::__x_arg
```

The argument of the spherical Bessel functions.

Definition at line 654 of file `specfun_state.h`.

The documentation for this struct was generated from the following file:

- `include/bits/specfun_state.h`

9.29 `__gnu_cxx::__sph_hankel_t< _Tn, _Tx, _Tp >` Struct Template Reference

```
#include <specfun_state.h>
```

Public Member Functions

- `_Tp __Wronskian () const`
Return the Wronskian of this cylindrical Hankel function state.

Public Attributes

- `_Tp __h1_deriv`
The derivative of the spherical Hankel function of the first kind.
- `_Tp __h1_value`
The value of the spherical Hankel function of the first kind.
- `_Tp __h2_deriv`
The derivative of the spherical Hankel function of the second kind.
- `_Tp __h2_value`
The value of the spherical Hankel function of the second kind.
- `_Tn __n_arg`
The integral order of the spherical Hankel functions.
- `_Tx __x_arg`
The argument of the spherical Hankel functions.

9.29.1 Detailed Description

```
template<typename _Tn, typename _Tx, typename _Tp>
struct __gnu_cxx::__sph_hankel_t<_Tn, _Tx, _Tp >
```

`_Tp` pretty much has to be complex.

Definition at line 708 of file `specfun_state.h`.

9.29.2 Member Function Documentation

9.29.2.1 `__Wronskian()`

```
template<typename _Tn , typename _Tx , typename _Tp >
_Tp __gnu_cxx::__sph_hankel_t<_Tn, _Tx, _Tp >::__Wronskian ( ) const [inline]
```

Return the Wronskian of this cylindrical Hankel function state.

Definition at line 729 of file `specfun_state.h`.

9.29.3 Member Data Documentation

9.29.3.1 `__h1_deriv`

```
template<typename _Tn , typename _Tx , typename _Tp >
_Tp __gnu_cxx::__sph_hankel_t<_Tn, _Tx, _Tp >::__h1_deriv
```

The derivative of the spherical Hankel function of the first kind.

Definition at line 720 of file `specfun_state.h`.

9.29.3.2 `__h1_value`

```
template<typename _Tn , typename _Tx , typename _Tp >
_Tp __gnu_cxx::__sph_hankel_t<_Tn, _Tx, _Tp >::__h1_value
```

The value of the spherical Hankel function of the first kind.

Definition at line 717 of file `specfun_state.h`.

9.29.3.3 __h2_deriv

```
template<typename _Tn , typename _Tx , typename _Tp >
_Tp __gnu_cxx::__sph_hankel_t< _Tn, _Tx, _Tp >::__h2_deriv
```

The derivative of the spherical Hankel function of the second kind.

Definition at line 726 of file specfun_state.h.

9.29.3.4 __h2_value

```
template<typename _Tn , typename _Tx , typename _Tp >
_Tp __gnu_cxx::__sph_hankel_t< _Tn, _Tx, _Tp >::__h2_value
```

The value of the spherical Hankel function of the second kind.

Definition at line 723 of file specfun_state.h.

9.29.3.5 __n_arg

```
template<typename _Tn , typename _Tx , typename _Tp >
_Tn __gnu_cxx::__sph_hankel_t< _Tn, _Tx, _Tp >::__n_arg
```

The integral order of the spherical Hankel functions.

Definition at line 711 of file specfun_state.h.

9.29.3.6 __x_arg

```
template<typename _Tn , typename _Tx , typename _Tp >
_Tx __gnu_cxx::__sph_hankel_t< _Tn, _Tx, _Tp >::__x_arg
```

The argument of the spherical Hankel functions.

Definition at line 714 of file specfun_state.h.

The documentation for this struct was generated from the following file:

- [include/bits/specfun_state.h](#)

9.30 `__gnu_cxx::__sph_mod_bessel_t<_Tn, _Tx, _Tp>` Struct Template Reference

```
#include <specfun_state.h>
```

Public Member Functions

- `_Tp __Wronskian () const`
Return the Wronskian of this modified cylindrical Bessel function state.

Public Attributes

- `_Tp __i_deriv`
The derivative of the modified spherical Bessel function of the first kind.
- `_Tp __i_value`
The value of the modified spherical Bessel function of the first kind.
- `_Tp __k_deriv`
The derivative of the modified spherical Bessel function of the second kind.
- `_Tp __k_value`
The value of the modified spherical Bessel function of the second kind.
- `_Tn __n_arg`
The integral order of the modified spherical Bessel functions.
- `_Tx __x_arg`
The argument of the modified spherical Bessel functions.

9.30.1 Detailed Description

```
template<typename _Tn, typename _Tx, typename _Tp>
struct __gnu_cxx::__sph_mod_bessel_t<_Tn, _Tx, _Tp>
```

Definition at line 674 of file `specfun_state.h`.

9.30.2 Member Function Documentation

9.30.2.1 `__Wronskian()`

```
template<typename _Tn , typename _Tx , typename _Tp >
_Tp __gnu_cxx::__sph_mod_bessel_t<_Tn, _Tx, _Tp>::__Wronskian ( ) const [inline]
```

Return the Wronskian of this modified cylindrical Bessel function state.

Definition at line 700 of file `specfun_state.h`.

9.30.3 Member Data Documentation

9.30.3.1 __i_deriv

```
template<typename _Tn , typename _Tx , typename _Tp >  
_Tp __gnu_cxx::__sph_mod_bessel_t< _Tn, _Tx, _Tp >::__i_deriv
```

The derivative of the modified spherical Bessel function of the first kind.

Definition at line 688 of file specfun_state.h.

9.30.3.2 __i_value

```
template<typename _Tn , typename _Tx , typename _Tp >  
_Tp __gnu_cxx::__sph_mod_bessel_t< _Tn, _Tx, _Tp >::__i_value
```

The value of the modified spherical Bessel function of the first kind.

Definition at line 684 of file specfun_state.h.

9.30.3.3 __k_deriv

```
template<typename _Tn , typename _Tx , typename _Tp >  
_Tp __gnu_cxx::__sph_mod_bessel_t< _Tn, _Tx, _Tp >::__k_deriv
```

The derivative of the modified spherical Bessel function of the second kind.

Definition at line 696 of file specfun_state.h.

9.30.3.4 __k_value

```
template<typename _Tn , typename _Tx , typename _Tp >  
_Tp __gnu_cxx::__sph_mod_bessel_t< _Tn, _Tx, _Tp >::__k_value
```

The value of the modified spherical Bessel function of the second kind.

Definition at line 692 of file specfun_state.h.

9.30.3.5 `__n_arg`

```
template<typename _Tn , typename _Tx , typename _Tp >
_Tn __gnu_cxx::__sph_mod_bessel_t< _Tn, _Tx, _Tp >::__n_arg
```

The integral order of the modified spherical Bessel functions.

Definition at line 680 of file `specfun_state.h`.

9.30.3.6 `__x_arg`

```
template<typename _Tn , typename _Tx , typename _Tp >
_Tx __gnu_cxx::__sph_mod_bessel_t< _Tn, _Tx, _Tp >::__x_arg
```

The argument of the modified spherical Bessel functions.

Definition at line 677 of file `specfun_state.h`.

The documentation for this struct was generated from the following file:

- `include/bits/specfun_state.h`

9.31 `__gnu_cxx::__stirling_1_t<_Tp>` Struct Template Reference

A structure for Stirling numbers of the first kind.

```
#include <specfun_state.h>
```

Public Types

- using `const_iterator` = `typename std::vector<_Tp>::const_iterator`
- using `iterator` = `typename std::vector<_Tp>::iterator`

Public Member Functions

- `iterator begin` () noexcept
- `const_iterator begin` () const noexcept
- unsigned int `degree` () const noexcept
- `iterator end` () noexcept
- `const_iterator end` () const noexcept
- `template<typename _Up >`
`auto operator()` (_Up __x) const noexcept
- `_Tp operator[]` (unsigned int __k) const noexcept

Public Attributes

- `std::vector<_Tp> __sigma`

9.31.1 Detailed Description

```
template<typename _Tp>
struct __gnu_cxx::__stirling_1_t<_Tp>
```

A structure for Stirling numbers of the first kind.

Definition at line 811 of file `specfun_state.h`.

9.31.2 Member Typedef Documentation

9.31.2.1 `const_iterator`

```
template<typename _Tp>
using __gnu_cxx::__stirling_1_t<_Tp>::const_iterator = typename std::vector<_Tp>::const_↵
iterator
```

Definition at line 814 of file `specfun_state.h`.

9.31.2.2 `iterator`

```
template<typename _Tp>
using __gnu_cxx::__stirling_1_t<_Tp>::iterator = typename std::vector<_Tp>::iterator
```

Definition at line 813 of file `specfun_state.h`.

9.31.3 Member Function Documentation

9.31.3.1 `begin()` [1/2]

```
template<typename _Tp>
iterator __gnu_cxx::__stirling_1_t<_Tp>::begin ( ) [inline], [noexcept]
```

Definition at line 837 of file `specfun_state.h`.

9.31.3.2 `begin()` [2/2]

```
template<typename _Tp >
const_iterator __gnu_cxx::__stirling_1_t<_Tp >::begin ( ) const [inline], [noexcept]
```

Definition at line 845 of file `specfun_state.h`.

9.31.3.3 `degree()`

```
template<typename _Tp >
unsigned int __gnu_cxx::__stirling_1_t<_Tp >::degree ( ) const [inline], [noexcept]
```

Definition at line 819 of file `specfun_state.h`.

9.31.3.4 `end()` [1/2]

```
template<typename _Tp >
iterator __gnu_cxx::__stirling_1_t<_Tp >::end ( ) [inline], [noexcept]
```

Definition at line 841 of file `specfun_state.h`.

9.31.3.5 `end()` [2/2]

```
template<typename _Tp >
const_iterator __gnu_cxx::__stirling_1_t<_Tp >::end ( ) const [inline], [noexcept]
```

Definition at line 849 of file `specfun_state.h`.

9.31.3.6 `operator>()()`

```
template<typename _Tp >
template<typename _Up >
auto __gnu_cxx::__stirling_1_t<_Tp >::operator() (
    _Up __x ) const [inline], [noexcept]
```

Definition at line 828 of file `specfun_state.h`.

9.31.3.7 operator[]()

```
template<typename _Tp >
_Tp __gnu_cxx::__stirling_1_t< _Tp >::operator[] (
    unsigned int __k ) const [inline], [noexcept]
```

Definition at line 823 of file specfun_state.h.

9.31.4 Member Data Documentation

9.31.4.1 __sigma

```
template<typename _Tp >
std::vector<_Tp> __gnu_cxx::__stirling_1_t< _Tp >::__sigma
```

Definition at line 816 of file specfun_state.h.

The documentation for this struct was generated from the following file:

- include/bits/specfun_state.h

9.32 __gnu_cxx::__stirling_2_t< _Tp > Struct Template Reference

A structure for Stirling numbers of the first kind.

```
#include <specfun_state.h>
```

Public Types

- using [const_iterator](#) = typename std::vector< _Tp >::const_iterator
- using [iterator](#) = typename std::vector< _Tp >::iterator

Public Member Functions

- [iterator begin](#) () noexcept
- [const_iterator begin](#) () const noexcept
- unsigned int [degree](#) () const noexcept
- [iterator end](#) () noexcept
- [const_iterator end](#) () const noexcept
- template<typename _Up >
auto [operator](#)() (_Up __x) const noexcept
Return the Bell polynomial.
- _Tp [operator\[\]](#) (unsigned int __k) const noexcept

Public Attributes

- `std::vector<_Tp>` [_S](#)

9.32.1 Detailed Description

```
template<typename _Tp>
struct __gnu_cxx::__stirling_2_t<_Tp>
```

A structure for Stirling numbers of the first kind.

Definition at line 857 of file `specfun_state.h`.

9.32.2 Member Typedef Documentation

9.32.2.1 `const_iterator`

```
template<typename _Tp>
using __gnu_cxx::__stirling_2_t<_Tp>::const_iterator = typename std::vector<_Tp>::const_↵
iterator
```

Definition at line 860 of file `specfun_state.h`.

9.32.2.2 `iterator`

```
template<typename _Tp>
using __gnu_cxx::__stirling_2_t<_Tp>::iterator = typename std::vector<_Tp>::iterator
```

Definition at line 859 of file `specfun_state.h`.

9.32.3 Member Function Documentation

9.32.3.1 `begin()` [1/2]

```
template<typename _Tp>
iterator __gnu_cxx::__stirling_2_t<_Tp>::begin ( ) [inline], [noexcept]
```

Definition at line 884 of file `specfun_state.h`.

9.32.3.2 begin() [2/2]

```
template<typename _Tp >
const_iterator __gnu_cxx::__stirling_2_t< _Tp >::begin ( ) const [inline], [noexcept]
```

Definition at line 892 of file specfun_state.h.

9.32.3.3 degree()

```
template<typename _Tp >
unsigned int __gnu_cxx::__stirling_2_t< _Tp >::degree ( ) const [inline], [noexcept]
```

Definition at line 865 of file specfun_state.h.

9.32.3.4 end() [1/2]

```
template<typename _Tp >
iterator __gnu_cxx::__stirling_2_t< _Tp >::end ( ) [inline], [noexcept]
```

Definition at line 888 of file specfun_state.h.

9.32.3.5 end() [2/2]

```
template<typename _Tp >
const_iterator __gnu_cxx::__stirling_2_t< _Tp >::end ( ) const [inline], [noexcept]
```

Definition at line 896 of file specfun_state.h.

9.32.3.6 operator()()

```
template<typename _Tp >
template<typename _Up >
auto __gnu_cxx::__stirling_2_t< _Tp >::operator() (
    _Up __x ) const [inline], [noexcept]
```

Return the Bell polynomial.

Definition at line 875 of file specfun_state.h.

9.32.3.7 operator[]()

```
template<typename _Tp >
_Tp __gnu_cxx::__stirling_2_t<_Tp>::operator[] (
    unsigned int __k ) const [inline], [noexcept]
```

Definition at line 869 of file specfun_state.h.

9.32.4 Member Data Documentation

9.32.4.1 _S

```
template<typename _Tp >
std::vector<_Tp> __gnu_cxx::__stirling_2_t<_Tp>::_S
```

Definition at line 862 of file specfun_state.h.

The documentation for this struct was generated from the following file:

- [include/bits/specfun_state.h](#)

9.33 std::__detail::__gamma_lanczos_data<_Tp> Struct Template Reference

9.33.1 Detailed Description

```
template<typename _Tp>
struct std::__detail::__gamma_lanczos_data<_Tp>
```

A struct for Lanczos algorithm Chebyshev arrays of coefficients.

Definition at line 2018 of file sf_gamma.tcc.

The documentation for this struct was generated from the following file:

- [include/bits/sf_gamma.tcc](#)

9.34 std::__detail::__gamma_lanczos_data<double> Struct Template Reference

Static Public Attributes

- static constexpr std::array<double, 10> [_S_cheby](#)
- static constexpr double [_S_g](#) = 9.5

9.34.1 Detailed Description

```
template<>
struct std::__detail::__gamma_lanczos_data< double >
```

Definition at line 2040 of file sf_gamma.tcc.

9.34.2 Member Data Documentation

9.34.2.1 `_S_cheby`

```
constexpr std::array<double, 10> std::__detail::__gamma_lanczos_data< double >::_S_cheby [static]
```

Initial value:

```
{
    5.557569219204146e+03,
   -4.248114953727554e+03,
    1.881719608233706e+03,
   -4.705537221412237e+02,
    6.325224688788239e+01,
   -4.206901076213398e+00,
    1.202512485324405e-01,
   -1.141081476816908e-03,
    2.055079676210880e-06,
    1.280568540096283e-09,
}
```

Definition at line 2045 of file sf_gamma.tcc.

9.34.2.2 `_S_g`

```
constexpr double std::__detail::__gamma_lanczos_data< double >::_S_g = 9.5 [static]
```

Definition at line 2042 of file sf_gamma.tcc.

The documentation for this struct was generated from the following file:

- [include/bits/sf_gamma.tcc](#)

9.35 `std::__detail::__gamma_lanczos_data< float >` Struct Template Reference

Static Public Attributes

- static constexpr std::array< float, 7 > [_S_cheby](#)
- static constexpr float [_S_g](#) = 6.5F

9.35.1 Detailed Description

```
template<>
struct std::__detail::__gamma_lanczos_data< float >
```

Definition at line 2023 of file sf_gamma.tcc.

9.35.2 Member Data Documentation

9.35.2.1 _S_cheby

```
constexpr std::array<float, 7> std::__detail::__gamma_lanczos_data< float >::_S_cheby [static]
```

Initial value:

```
{
    3.307139e+02F,
   -2.255998e+02F,
    6.989520e+01F,
   -9.058929e+00F,
    4.110107e-01F,
   -4.150391e-03F,
   -3.417969e-03F,
}
```

Definition at line 2028 of file sf_gamma.tcc.

9.35.2.2 _S_g

```
constexpr float std::__detail::__gamma_lanczos_data< float >::_S_g = 6.5F [static]
```

Definition at line 2025 of file sf_gamma.tcc.

The documentation for this struct was generated from the following file:

- include/bits/sf_gamma.tcc

9.36 std::__detail::__gamma_lanczos_data< long double > Struct Template Reference

Static Public Attributes

- static constexpr std::array< long double, 11 > [_S_cheby](#)
- static constexpr long double [_S_g](#) = 10.5L

9.36.1 Detailed Description

```
template<>
struct std::__detail::__gamma_lanczos_data< long double >
```

Definition at line 2060 of file sf_gamma.tcc.

9.36.2 Member Data Documentation

9.36.2.1 _S_cheby

```
constexpr std::array<long double, 11> std::__detail::__gamma_lanczos_data< long double >::_S_cheby [static]
```

Initial value:

```
{
    1.440399692024250728e+04L,
    -1.128006201837065341e+04L,
    5.384108670160999829e+03L,
    -1.536234184127325861e+03L,
    2.528551924697309561e+02L,
    -2.265389090278717887e+01L,
    1.006663776178612579e+00L,
    -1.900805731354182626e-02L,
    1.150508317664389324e-04L,
    -1.208915136885480024e-07L,
    -1.518856151960790157e-10L,
}
```

Definition at line 2065 of file sf_gamma.tcc.

9.36.2.2 _S_g

```
constexpr long double std::__detail::__gamma_lanczos_data< long double >::_S_g = 10.5L [static]
```

Definition at line 2062 of file sf_gamma.tcc.

The documentation for this struct was generated from the following file:

- [include/bits/sf_gamma.tcc](#)

9.37 std::__detail::__gamma_spouge_data< _Tp > Struct Template Reference

9.37.1 Detailed Description

```
template<typename _Tp>
struct std::__detail::__gamma_spouge_data< _Tp >
```

A struct for Spouge algorithm Chebyshev arrays of coefficients.

Definition at line 1792 of file sf_gamma.tcc.

The documentation for this struct was generated from the following file:

- [include/bits/sf_gamma.tcc](#)

9.38 std::__detail::__gamma_spouge_data< double > Struct Template Reference

Static Public Attributes

- static constexpr std::array< double, 18 > [_S_cheby](#)

9.38.1 Detailed Description

```
template<>
struct std::__detail::__gamma_spouge_data< double >
```

Definition at line 1813 of file sf_gamma.tcc.

9.38.2 Member Data Documentation

9.38.2.1 `_S_cheby`

```
constexpr std::array<double, 18> std::__detail::__gamma_spouge_data< double >::_S_cheby [static]
```

Initial value:

```
{
    2.785716565770350e+08,
    -1.693088166941517e+09,
    4.549688586500031e+09,
    -7.121728036151557e+09,
    7.202572947273274e+09,
    -4.935548868770376e+09,
    2.338187776097503e+09,
    -7.678102458920741e+08,
    1.727524819329867e+08,
    -2.595321377008346e+07,
    2.494811203993971e+06,
    -1.437252641338402e+05,
    4.490767356961276e+03,
    -6.505596924745029e+01,
    3.362323142416327e-01,
    -3.817361443986454e-04,
    3.273137866873352e-08,
    -7.642333165976788e-15,
}
```

Definition at line 1817 of file `sf_gamma.tcc`.

The documentation for this struct was generated from the following file:

- `include/bits/sf_gamma.tcc`

9.39 `std::__detail::__gamma_spouge_data< float >` Struct Template Reference

Static Public Attributes

- static constexpr `std::array< float, 7 > _S_cheby`

9.39.1 Detailed Description

```
template<>
struct std::__detail::__gamma_spouge_data< float >
```

Definition at line 1797 of file `sf_gamma.tcc`.

9.39.2 Member Data Documentation

9.39.2.1 _S_cheby

```
constexpr std::array<float, 7> std::__detail::__gamma_spouge_data< float >::_S_cheby [static]
```

Initial value:

```
{
    2.901419e+03F,
   -5.929168e+03F,
    4.148274e+03F,
   -1.164761e+03F,
    1.174135e+02F,
   -2.786588e+00F,
    3.775392e-03F,
}
```

Definition at line 1801 of file sf_gamma.tcc.

The documentation for this struct was generated from the following file:

- [include/bits/sf_gamma.tcc](#)

9.40 std::__detail::__gamma_spouge_data< long double > Struct Template Reference

Static Public Attributes

- static constexpr std::array< long double, 22 > [_S_cheby](#)

9.40.1 Detailed Description

```
template<>
struct std::__detail::__gamma_spouge_data< long double >
```

Definition at line 1840 of file sf_gamma.tcc.

9.40.2 Member Data Documentation

9.40.2.1 `_S_cheby`

```
constexpr std::array<long double, 22> std::__detail::__gamma_spouge_data< long double >::_S_cheby [static]
```

Initial value:

```
{
    1.681473171108908244e+10L,
    -1.269150315503303974e+11L,
    4.339449429013039995e+11L,
    -8.893680202692714895e+11L,
    1.218472425867950986e+12L,
    -1.178403473259353616e+12L,
    8.282455311246278274e+11L,
    -4.292112878930625978e+11L,
    1.646988347276488710e+11L,
    -4.661514921989111004e+10L,
    9.619972564515443397e+09L,
    -1.419382551781042824e+09L,
    1.454145470816386107e+08L,
    -9.923020719435758179e+06L,
    4.253557563919127284e+05L,
    -1.053371059784341875e+04L,
    1.332425479537961437e+02L,
    -7.118343974029489132e-01L,
    1.172051640057979518e-03L,
    -3.323940885824119041e-07L,
    4.503801674404338524e-12L,
    -5.320477002211632680e-20L,
}
```

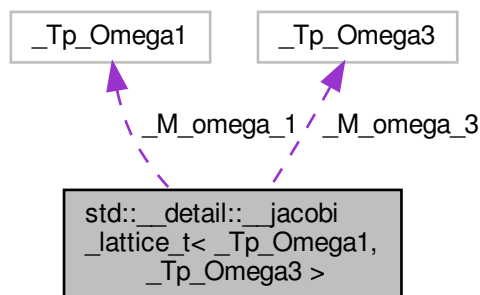
Definition at line 1844 of file `sf_gamma.tcc`.

The documentation for this struct was generated from the following file:

- `include/bits/sf_gamma.tcc`

9.41 `std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >` Struct Template Reference

Collaboration diagram for `std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >`:



Classes

- struct [__arg_t](#)
- struct [__tau_t](#)

Public Types

- using [_Cmplx](#) = std::complex< [_Real](#) >
- using [_Real](#) = __gnu_cxx::fp_promote_t< [_Real_Omega1](#), [_Real_Omega3](#) >
- using [_Real_Omega1](#) = __num_traits_t< _Tp_Omega1 >
- using [_Real_Omega3](#) = __num_traits_t< _Tp_Omega3 >
- using [_Tp_Nome](#) = std::conditional_t< __gnu_cxx::is_complex_v< _Tp_Omega1 > && __gnu_cxx::is_←
complex_v< _Tp_Omega3 >, [_Cmplx](#), [_Real](#) >

Public Member Functions

- [__jacobi_lattice_t](#) (const _Tp_Omega1 &__omega1, const _Tp_Omega3 &__omega3)
Construct the lattice from two complex lattice frequencies.
- [__jacobi_lattice_t](#) (const [__tau_t](#) &__tau)
Construct the lattice from a single complex lattice parameter or half period ratio.
- [__jacobi_lattice_t](#) ([_Tp_Nome](#) __q)
Construct the lattice from a single scalar elliptic nome.
- [_Tp_Nome](#) [__ellnome](#) () const
- [_Tp_Omega1](#) [__omega_1](#) () const
Return the first lattice frequency.
- [_Cmplx](#) [__omega_2](#) () const
Return the second lattice frequency.
- [_Tp_Omega3](#) [__omega_3](#) () const
Return the third lattice frequency.
- [__arg_t](#) [__reduce](#) (const [_Cmplx](#) &__z) const
- [__tau_t](#) [__tau](#) () const
Return the acalar lattice parameter or half period ratio.

Public Attributes

- [_Tp_Omega1](#) [_M_omega_1](#)
- [_Tp_Omega3](#) [_M_omega_3](#)

Static Public Attributes

- static constexpr auto [_S_pi](#) = __gnu_cxx::__const_pi<[_Real](#)>()

9.41.1 Detailed Description

```
template<typename _Tp_Omega1, typename _Tp_Omega3 = std::complex<_Tp_Omega1>>
struct std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >
```

A struct representing the Jacobi and Weierstrass lattice. The two types for the frequencies and the subsequent type calculus allow us to treat the rectangular lattice (real nome, pure imaginary lattice parameter) specially.

Definition at line 470 of file sf_theta.tcc.

9.41.2 Member Typedef Documentation

9.41.2.1 _Cmplx

```
template<typename _Tp_Omega1, typename _Tp_Omega3 = std::complex<_Tp_Omega1>>
using std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::__Cmplx = std::complex<_Real>
```

Definition at line 478 of file sf_theta.tcc.

9.41.2.2 _Real

```
template<typename _Tp_Omega1, typename _Tp_Omega3 = std::complex<_Tp_Omega1>>
using std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::__Real = __gnu_cxx::fp_promote←
_t<_Real_Omega1, _Real_Omega3>
```

Definition at line 477 of file sf_theta.tcc.

9.41.2.3 _Real_Omega1

```
template<typename _Tp_Omega1, typename _Tp_Omega3 = std::complex<_Tp_Omega1>>
using std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::__Real_Omega1 = __num_traits←
t<_Tp_Omega1>
```

Definition at line 475 of file sf_theta.tcc.

9.41.2.4 _Real_Omega3

```
template<typename _Tp_Omega1, typename _Tp_Omega3 = std::complex<_Tp_Omega1>>
using std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::__Real_Omega3 = __num_traits_↵
t<_Tp_Omega3>
```

Definition at line 476 of file sf_theta.tcc.

9.41.2.5 _Tp_Nome

```
template<typename _Tp_Omega1, typename _Tp_Omega3 = std::complex<_Tp_Omega1>>
using std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::__Tp_Nome = std::conditional_↵
t<__gnu_cxx::is_complex_v<_Tp_Omega1> && __gnu_cxx::is_complex_v<_Tp_Omega3>, _Cmplx, _Real>
```

Definition at line 481 of file sf_theta.tcc.

9.41.3 Constructor & Destructor Documentation

9.41.3.1 __jacobi_lattice_t() [1/3]

```
template<typename _Tp_Omega1, typename _Tp_Omega3 = std::complex<_Tp_Omega1>>
std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::__jacobi_lattice_t (
    const _Tp_Omega1 & __omega1,
    const _Tp_Omega3 & __omega3 ) [inline]
```

Construct the lattice from two complex lattice frequencies.

Definition at line 508 of file sf_theta.tcc.

9.41.3.2 __jacobi_lattice_t() [2/3]

```
template<typename _Tp_Omega1, typename _Tp_Omega3 = std::complex<_Tp_Omega1>>
std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::__jacobi_lattice_t (
    const __tau_t & __tau ) [inline], [explicit]
```

Construct the lattice from a single complex lattice parameter or half period ratio.

Definition at line 530 of file sf_theta.tcc.

9.41.3.3 `__jacobi_lattice_t()` [3/3]

```
template<typename _Tp_Omega1, typename _Tp_Omega3 = std::complex<_Tp_Omega1>>
std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::__jacobi_lattice_t (
    _Tp_Nome __q ) [inline], [explicit]
```

Construct the lattice from a single scalar elliptic nome.

Definition at line 549 of file `sf_theta.tcc`.

9.41.4 Member Function Documentation

9.41.4.1 `__ellnome()`

```
template<typename _Tp_Omega1 , typename _Tp_Omega3 >
__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::__Tp_Nome std::__detail::__jacobi_lattice_t< _Tp_↵
Omega1, _Tp_Omega3 >::__ellnome ( ) const
```

Return the elliptic nome corresponding to the lattice parameter.

Definition at line 593 of file `sf_theta.tcc`.

Referenced by `std::__detail::__jacobi_theta_0_t< _Tp1, _Tp3 >::__jacobi_theta_0_t()`, and `std::__detail::__jacobi_↵
lattice_t< _Tp1, _Tp3 >::__omega_3()`.

9.41.4.2 `__omega_1()`

```
template<typename _Tp_Omega1, typename _Tp_Omega3 = std::complex<_Tp_Omega1>>
_Tp_Omega1 std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::__omega_1 ( ) const [inline]
```

Return the first lattice frequency.

Definition at line 564 of file `sf_theta.tcc`.

Referenced by `std::__detail::__jacobi_theta_0_t< _Tp1, _Tp3 >::__jacobi_theta_0_t()`, and `std::__detail::__↵
weierstrass_roots_t< _Tp1, _Tp3 >::__weierstrass_roots_t()`.

9.41.4.3 __omega_2()

```
template<typename _Tp_Omega1, typename _Tp_Omega3 = std::complex<_Tp_Omega1>>
_Cmplx std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::__omega_2 ( ) const [inline]
```

Return the second lattice frequency.

Definition at line 569 of file sf_theta.tcc.

Referenced by std::__detail::__jacobi_theta_0_t< _Tp1, _Tp3 >::__jacobi_theta_0_t().

9.41.4.4 __omega_3()

```
template<typename _Tp_Omega1, typename _Tp_Omega3 = std::complex<_Tp_Omega1>>
_Tp_Omega3 std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::__omega_3 ( ) const [inline]
```

Return the third lattice frequency.

Definition at line 574 of file sf_theta.tcc.

Referenced by std::__detail::__jacobi_theta_0_t< _Tp1, _Tp3 >::__jacobi_theta_0_t().

9.41.4.5 __reduce()

```
template<typename _Tp_Omega1, typename _Tp_Omega3 = std::complex<_Tp_Omega1>>
__jacobi_lattice_t< _Tp1, _Tp3 >::__arg_t std::__detail::__jacobi_lattice_t< _Tp1, _Tp3 >::__↵
reduce (
    const _Cmplx & __z ) const
```

Reduce the argument to the fundamental lattice parallelogram $(0, 2\pi, 2\pi(1 + \tau), 2\pi\tau)$. This is sort of like a 2D lattice remquo.

Parameters

__↵	The argument to be reduced.
__z	

Returns

A struct containing the argument reduced to the interior of the fundamental parallelogram and two integers indicating the number of periods in the 'real' and 'tau' directions.

Definition at line 616 of file sf_theta.tcc.

Referenced by `std::__detail::__jacobi_lattice_t< _Tp1, _Tp3 >::__ellnome()`, `std::__detail::__jacobi_theta_1()`, `std::__detail::__jacobi_theta_2()`, `std::__detail::__jacobi_theta_3()`, `std::__detail::__jacobi_theta_4()`, and `std::__detail::__jacobi_lattice_t< _Tp1, _Tp3 >::__omega_3()`.

9.41.4.6 `__tau()`

```
template<typename _Tp_Omega1, typename _Tp_Omega3 = std::complex<_Tp_Omega1>>
__tau_t std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::__tau ( ) const [inline]
```

Return the acalar lattice parameter or half period ratio.

Definition at line 559 of file `sf_theta.tcc`.

Referenced by `std::__detail::__jacobi_lattice_t< _Tp1, _Tp3 >::__ellnome()`, `std::__detail::__jacobi_lattice_t< _Tp1, _Tp3 >::__jacobi_lattice_t()`, `std::__detail::__jacobi_theta_1()`, `std::__detail::__jacobi_theta_2()`, `std::__detail::__jacobi_theta_3()`, `std::__detail::__jacobi_theta_4()`, and `std::__detail::__jacobi_lattice_t< _Tp1, _Tp3 >::__reduce()`.

9.41.5 Member Data Documentation

9.41.5.1 `_M_omega_1`

```
template<typename _Tp_Omega1, typename _Tp_Omega3 = std::complex<_Tp_Omega1>>
_Tp_Omega1 std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::_M_omega_1
```

Definition at line 584 of file `sf_theta.tcc`.

Referenced by `std::__detail::__jacobi_lattice_t< _Tp1, _Tp3 >::__jacobi_lattice_t()`, `std::__detail::__jacobi_lattice_t< _Tp1, _Tp3 >::__omega_1()`, `std::__detail::__jacobi_lattice_t< _Tp1, _Tp3 >::__omega_2()`, and `std::__detail::__jacobi_lattice_t< _Tp1, _Tp3 >::__tau()`.

9.41.5.2 `_M_omega_3`

```
template<typename _Tp_Omega1, typename _Tp_Omega3 = std::complex<_Tp_Omega1>>
_Tp_Omega3 std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::_M_omega_3
```

Definition at line 585 of file `sf_theta.tcc`.

Referenced by `std::__detail::__jacobi_lattice_t< _Tp1, _Tp3 >::__jacobi_lattice_t()`, `std::__detail::__jacobi_lattice_t< _Tp1, _Tp3 >::__omega_2()`, `std::__detail::__jacobi_lattice_t< _Tp1, _Tp3 >::__omega_3()`, and `std::__detail::__jacobi_lattice_t< _Tp1, _Tp3 >::__tau()`.

9.41.5.3 _S_pi

```
template<typename _Tp_Omega1, typename _Tp_Omega3 = std::complex<_Tp_Omega1>>
constexpr auto std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::_S_pi = __gnu_cxx::__
__const_pi<_Real>() [static]
```

Definition at line 583 of file sf_theta.tcc.

Referenced by std::__detail::__jacobi_lattice_t< _Tp1, _Tp3 >::__ellnome(), std::__detail::__jacobi_lattice_t< _Tp1, _Tp3 >::__jacobi_lattice_t(), std::__detail::__jacobi_theta_0_t< _Tp1, _Tp3 >::__jacobi_theta_0_t(), std::__detail::__jacobi_theta_1_t(), std::__detail::__jacobi_theta_2_t(), std::__detail::__jacobi_theta_3_t(), std::__detail::__jacobi_theta_4_t(), std::__detail::__jacobi_lattice_t< _Tp1, _Tp3 >::__reduce(), and std::__detail::__weierstrass_roots_t< _Tp1, _Tp3 >::__weierstrass_roots_t().

The documentation for this struct was generated from the following file:

- include/bits/sf_theta.tcc

9.42 std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::__arg_t Struct Reference

Public Attributes

- int __m
- int __n
- _Cmplx __z

9.42.1 Detailed Description

```
template<typename _Tp_Omega1, typename _Tp_Omega3 = std::complex<_Tp_Omega1>>
struct std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::__arg_t
```

A struct representing a complex argument reduced to the 'central' lattice cell.

Definition at line 500 of file sf_theta.tcc.

9.42.2 Member Data Documentation

9.42.2.1 `__m`

```
template<typename _Tp_Omega1, typename _Tp_Omega3 = std::complex<_Tp_Omega1>>
int std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::__arg_t::__m
```

Definition at line 502 of file `sf_theta.tcc`.

9.42.2.2 `__n`

```
template<typename _Tp_Omega1, typename _Tp_Omega3 = std::complex<_Tp_Omega1>>
int std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::__arg_t::__n
```

Definition at line 503 of file `sf_theta.tcc`.

9.42.2.3 `__z`

```
template<typename _Tp_Omega1, typename _Tp_Omega3 = std::complex<_Tp_Omega1>>
_Cmplx std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::__arg_t::__z
```

Definition at line 504 of file `sf_theta.tcc`.

The documentation for this struct was generated from the following file:

- [include/bits/sf_theta.tcc](#)

9.43 `std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::__tau_t` Struct Reference

Public Member Functions

- [__tau_t](#)([_Cmplx](#) [__tau](#))

Public Attributes

- [_Cmplx](#) [__val](#)

9.43.1 Detailed Description

```
template<typename _Tp_Omega1, typename _Tp_Omega3 = std::complex<_Tp_Omega1>>
struct std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::__tau_t
```

A struct representing a complex scalar lattice parameter or half period ratio.

Definition at line 487 of file sf_theta.tcc.

9.43.2 Constructor & Destructor Documentation

9.43.2.1 __tau_t()

```
template<typename _Tp_Omega1, typename _Tp_Omega3 = std::complex<_Tp_Omega1>>
std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::__tau_t::__tau_t (
    _Cmplx __tau ) [inline], [explicit]
```

Definition at line 491 of file sf_theta.tcc.

Referenced by std::__detail::__jacobi_lattice_t< _Tp1, _Tp3 >::__jacobi_lattice_t(), and std::__detail::__jacobi_lattice_t< _Tp1, _Tp3 >::__tau_t().

9.43.3 Member Data Documentation

9.43.3.1 __val

```
template<typename _Tp_Omega1, typename _Tp_Omega3 = std::complex<_Tp_Omega1>>
_Cmplx std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::__tau_t::__val
```

Definition at line 489 of file sf_theta.tcc.

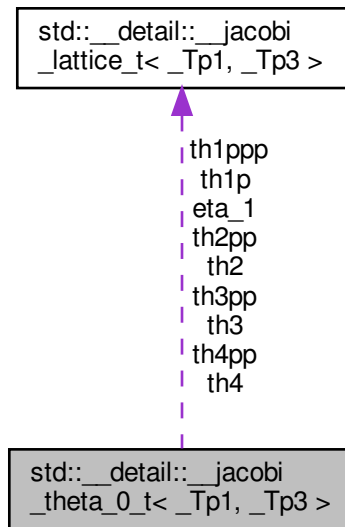
Referenced by std::__detail::__jacobi_lattice_t< _Tp1, _Tp3 >::__ellnome(), std::__detail::__jacobi_lattice_t< _Tp1, _Tp3 >::__jacobi_lattice_t(), and std::__detail::__jacobi_lattice_t< _Tp1, _Tp3 >::__reduce().

The documentation for this struct was generated from the following file:

- [include/bits/sf_theta.tcc](#)

9.44 `std::__detail::__jacobi_theta_0_t<_Tp1, _Tp3 >` Struct Template Reference

Collaboration diagram for `std::__detail::__jacobi_theta_0_t<_Tp1, _Tp3 >`:



Public Types

- using `_Cmplx` = `std::complex<_Real >`
- using `_Real` = `__num_traits_t<_Type >`
- using `_Type` = `typename __jacobi_lattice_t<_Tp1, _Tp3 >::_Tp_Nome`

Public Member Functions

- `__jacobi_theta_0_t` (const `__jacobi_lattice_t<_Tp1, _Tp3 > &__lattice`)
- `_Type dedekind_eta` () const

Public Attributes

- `_Type eta_1`
- `_Cmplx eta_2`
- `_Cmplx eta_3`
- `_Type th1p`
- `_Type th1ppp`
- `_Type th2`
- `_Type th2pp`
- `_Type th3`
- `_Type th3pp`
- `_Type th4`
- `_Type th4pp`

9.44.1 Detailed Description

```
template<typename _Tp1, typename _Tp3 = std::complex<_Tp1>>
struct std::__detail::__jacobi_theta_0_t<_Tp1, _Tp3>
```

A struct for the non-zero theta functions and their derivatives at zero argument.

Definition at line 643 of file sf_theta.tcc.

9.44.2 Member Typedef Documentation

9.44.2.1 _Cmplx

```
template<typename _Tp1, typename _Tp3 = std::complex<_Tp1>>
using std::__detail::__jacobi_theta_0_t<_Tp1, _Tp3>::__Cmplx = std::complex<_Real>
```

Definition at line 649 of file sf_theta.tcc.

9.44.2.2 _Real

```
template<typename _Tp1, typename _Tp3 = std::complex<_Tp1>>
using std::__detail::__jacobi_theta_0_t<_Tp1, _Tp3>::__Real = __num_traits_t<_Type>
```

Definition at line 648 of file sf_theta.tcc.

9.44.2.3 _Type

```
template<typename _Tp1, typename _Tp3 = std::complex<_Tp1>>
using std::__detail::__jacobi_theta_0_t<_Tp1, _Tp3>::__Type = typename __jacobi_lattice_t<_Tp1,
_Tp3>::__Tp_None
```

Definition at line 647 of file sf_theta.tcc.

9.44.3 Constructor & Destructor Documentation

9.44.3.1 `__jacobi_theta_0_t()`

```
template<typename _Tp1 , typename _Tp3 >
std::__detail::__jacobi_theta_0_t< _Tp1, _Tp3 >::__jacobi_theta_0_t (
    const __jacobi_lattice_t< _Tp1, _Tp3 > & __lattice )
```

Return a struct of the Jacobi theta functions and up to three non-zero derivatives evaluated at zero argument.

Definition at line 674 of file `sf_theta.tcc`.

References `std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::__ellnome()`, `std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::__omega_1()`, `std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::__omega_2()`, `std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::__omega_3()`, and `std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::__S_pi`.

Referenced by `std::__detail::__jacobi_theta_0_t< _Tp1, _Tp3 >::dedekind_eta()`.

9.44.4 Member Function Documentation

9.44.4.1 `dedekind_eta()`

```
template<typename _Tp1, typename _Tp3 = std::complex<_Tp1>>
_Type std::__detail::__jacobi_theta_0_t< _Tp1, _Tp3 >::dedekind_eta ( ) const [inline]
```

Definition at line 664 of file `sf_theta.tcc`.

References `std::__detail::__jacobi_theta_0_t< _Tp1, _Tp3 >::__jacobi_theta_0_t()`.

9.44.5 Member Data Documentation

9.44.5.1 `eta_1`

```
template<typename _Tp1, typename _Tp3 = std::complex<_Tp1>>
_Type std::__detail::__jacobi_theta_0_t< _Tp1, _Tp3 >::eta_1
```

Definition at line 659 of file `sf_theta.tcc`.

9.44.5.2 eta_2

```
template<typename _Tp1, typename _Tp3 = std::complex<_Tp1>>
_Cmplx std::__detail::__jacobi_theta_0_t< _Tp1, _Tp3 >::eta_2
```

Definition at line 660 of file sf_theta.tcc.

9.44.5.3 eta_3

```
template<typename _Tp1, typename _Tp3 = std::complex<_Tp1>>
_Cmplx std::__detail::__jacobi_theta_0_t< _Tp1, _Tp3 >::eta_3
```

Definition at line 661 of file sf_theta.tcc.

9.44.5.4 th1p

```
template<typename _Tp1, typename _Tp3 = std::complex<_Tp1>>
_Type std::__detail::__jacobi_theta_0_t< _Tp1, _Tp3 >::th1p
```

Definition at line 651 of file sf_theta.tcc.

9.44.5.5 th1ppp

```
template<typename _Tp1, typename _Tp3 = std::complex<_Tp1>>
_Type std::__detail::__jacobi_theta_0_t< _Tp1, _Tp3 >::th1ppp
```

Definition at line 652 of file sf_theta.tcc.

9.44.5.6 th2

```
template<typename _Tp1, typename _Tp3 = std::complex<_Tp1>>
_Type std::__detail::__jacobi_theta_0_t< _Tp1, _Tp3 >::th2
```

Definition at line 653 of file sf_theta.tcc.

Referenced by std::__detail::__jacobi_theta_2(), and std::__detail::__weierstrass_roots_t< _Tp1, _Tp3 >::__weierstrass_roots_t().

9.44.5.7 th2pp

```
template<typename _Tp1, typename _Tp3 = std::complex<_Tp1>>
_Type std::__detail::__jacobi_theta_0_t< _Tp1, _Tp3 >::th2pp
```

Definition at line 654 of file sf_theta.tcc.

9.44.5.8 th3

```
template<typename _Tp1, typename _Tp3 = std::complex<_Tp1>>
_Type std::__detail::__jacobi_theta_0_t< _Tp1, _Tp3 >::th3
```

Definition at line 655 of file sf_theta.tcc.

Referenced by std::__detail::__jacobi_theta_3().

9.44.5.9 th3pp

```
template<typename _Tp1, typename _Tp3 = std::complex<_Tp1>>
_Type std::__detail::__jacobi_theta_0_t< _Tp1, _Tp3 >::th3pp
```

Definition at line 656 of file sf_theta.tcc.

9.44.5.10 th4

```
template<typename _Tp1, typename _Tp3 = std::complex<_Tp1>>
_Type std::__detail::__jacobi_theta_0_t< _Tp1, _Tp3 >::th4
```

Definition at line 657 of file sf_theta.tcc.

Referenced by std::__detail::__jacobi_theta_4(), and std::__detail::__weierstrass_roots_t< _Tp1, _Tp3 >::__weierstrass_roots_t().

9.44.5.11 th4pp

```
template<typename _Tp1, typename _Tp3 = std::complex<_Tp1>>
_Type std::__detail::__jacobi_theta_0_t< _Tp1, _Tp3 >::th4pp
```

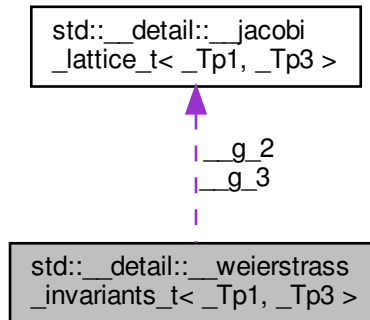
Definition at line 658 of file sf_theta.tcc.

The documentation for this struct was generated from the following file:

- [include/bits/sf_theta.tcc](#)

9.45 std::__detail::__weierstrass_invariants_t< _Tp1, _Tp3 > Struct Template Reference

Collaboration diagram for std::__detail::__weierstrass_invariants_t< _Tp1, _Tp3 >:



Public Types

- using `_Cmplx` = `std::complex< _Real >`
- using `_Real` = `__num_traits_t< _Type >`
- using `_Type` = `typename __jacobi_lattice_t< _Tp1, _Tp3 >::Tp_Nome`

Public Member Functions

- `__weierstrass_invariants_t` (const `__jacobi_lattice_t< _Tp1, _Tp3 > &`)
- `_Type __delta` () const
Return the discriminant $\Delta = g_2^3 - 27g_3^2$.
- `_Type __klein_j` () const
Return Klein's invariant $J = 1738g_2^3/(g_2^3 - 27g_3^2)$.

Public Attributes

- `_Type __g_2`
- `_Type __g_3`

9.45.1 Detailed Description

```
template<typename _Tp1, typename _Tp3>
struct std::__detail::__weierstrass_invariants_t< _Tp1, _Tp3 >
```

A struct of the Weierstrass elliptic function invariants.

$$g_2 = 2(e_1e_2 + e_2e_3 + e_3e_1)$$

$$g_3 = 4(e_1e_2e_3)$$

Definition at line 827 of file sf_theta.tcc.

9.45.2 Member Typedef Documentation

9.45.2.1 `_Cmplx`

```
template<typename _Tp1 , typename _Tp3 >
using std::__detail::__weierstrass_invariants_t< _Tp1, _Tp3 >::__Cmplx = std::complex<_Real>
```

Definition at line 831 of file sf_theta.tcc.

9.45.2.2 `_Real`

```
template<typename _Tp1 , typename _Tp3 >
using std::__detail::__weierstrass_invariants_t< _Tp1, _Tp3 >::__Real = __num_traits_t<_Type>
```

Definition at line 830 of file sf_theta.tcc.

9.45.2.3 `_Type`

```
template<typename _Tp1 , typename _Tp3 >
using std::__detail::__weierstrass_invariants_t< _Tp1, _Tp3 >::__Type = typename __jacobi_lattice_t<_Tp1, _Tp3>::__Tp_Nome
```

Definition at line 829 of file sf_theta.tcc.

9.45.3 Constructor & Destructor Documentation

9.45.3.1 `__weierstrass_invariants_t()`

```
template<typename _Tp1 , typename _Tp3 >
std::__detail::__weierstrass_invariants_t< _Tp1, _Tp3 >::__weierstrass_invariants_t (
    const __jacobi_lattice_t< _Tp1, _Tp3 > & __lattice )
```

Constructor for the Weierstrass invariants.

$$g_2 = 2(e_1e_2 + e_2e_3 + e_3e_1)$$

$$g_3 = 4(e_1e_2e_3)$$

Definition at line 865 of file sf_theta.tcc.

References `std::__detail::__weierstrass_roots_t< _Tp1, _Tp3 >::__e1`.

Referenced by `std::__detail::__weierstrass_invariants_t< _Tp1, _Tp3 >::__klein_j()`.

9.45.4 Member Function Documentation

9.45.4.1 __delta()

```
template<typename _Tp1 , typename _Tp3 >
_Type std::__detail::__weierstrass_invariants_t< _Tp1, _Tp3 >::__delta ( ) const [inline]
```

Return the discriminant $\Delta = g_2^3 - 27g_3^2$.

Definition at line 839 of file sf_theta.tcc.

9.45.4.2 __klein_j()

```
template<typename _Tp1 , typename _Tp3 >
_Type std::__detail::__weierstrass_invariants_t< _Tp1, _Tp3 >::__klein_j ( ) const [inline]
```

Return Klein's invariant $J = 1738g_2^3/(g_2^3 - 27g_3^2)$.

Definition at line 847 of file sf_theta.tcc.

References `std::__detail::__weierstrass_invariants_t<_Tp1, _Tp3>::__weierstrass_invariants_t()`.

9.45.5 Member Data Documentation

9.45.5.1 __g_2

```
template<typename _Tp1 , typename _Tp3 >
_Type std::__detail::__weierstrass_invariants_t< _Tp1, _Tp3 >::__g_2
```

Definition at line 833 of file sf_theta.tcc.

9.45.5.2 __g_3

```
template<typename _Tp1 , typename _Tp3 >
_Type std::__detail::__weierstrass_invariants_t< _Tp1, _Tp3 >::__g_3
```

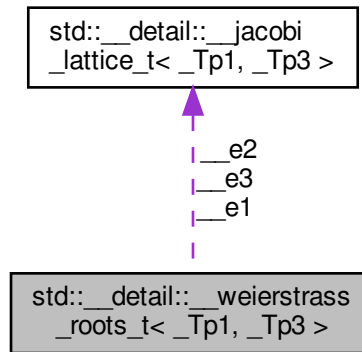
Definition at line 833 of file sf_theta.tcc.

The documentation for this struct was generated from the following file:

- [include/bits/sf_theta.tcc](#)

9.46 std::__detail::__weierstrass_roots_t< _Tp1, _Tp3 > Struct Template Reference

Collaboration diagram for std::__detail::__weierstrass_roots_t< _Tp1, _Tp3 >:



Public Types

- using [_Cmplx](#) = std::complex< [_Real](#) >
- using [_Real](#) = __num_traits_t< [_Type](#) >
- using [_Type](#) = typename [__jacobi_lattice_t](#)< _Tp1, _Tp3 >::__Tp_Nome

Public Member Functions

- [__weierstrass_roots_t](#) (const [__jacobi_lattice_t](#)< _Tp1, _Tp3 > &__lattice)
- [__weierstrass_roots_t](#) (const [__jacobi_theta_0_t](#)< _Tp1, _Tp3 > &__theta0, _Tp1 __omega1)
- [_Type](#) [__delta](#) () const

Return the discriminant $\Delta = 16(e_2 - e_3)^2(e_3 - e_1)^2(e_1 - e_2)^2$.

Public Attributes

- [_Type](#) [__e1](#)
- [_Type](#) [__e2](#)
- [_Type](#) [__e3](#)

9.46.1 Detailed Description

```
template<typename _Tp1, typename _Tp3 = std::complex<_Tp1>>
struct std::__detail::__weierstrass_roots_t<_Tp1, _Tp3>
```

A struct of the Weierstrass elliptic function roots.

$$e_1 = \frac{\pi^2}{12\omega_1^2}(\theta_2^4(q, 0) + 2\theta_4^4(q, 0))$$

$$e_2 = \frac{\pi^2}{12\omega_1^2}(\theta_2^4(q, 0) - \theta_4^4(q, 0))$$

$$e_3 = \frac{\pi^2}{12\omega_1^2}(-2\theta_2^4(q, 0) - \theta_4^4(q, 0))$$

Note that $e_1 + e_2 + e_3 = 0$

Definition at line 747 of file sf_theta.tcc.

9.46.2 Member Typedef Documentation

9.46.2.1 _Cmplx

```
template<typename _Tp1, typename _Tp3 = std::complex<_Tp1>>
using std::__detail::__weierstrass_roots_t<_Tp1, _Tp3>::__Cmplx = std::complex<_Real>
```

Definition at line 751 of file sf_theta.tcc.

9.46.2.2 _Real

```
template<typename _Tp1, typename _Tp3 = std::complex<_Tp1>>
using std::__detail::__weierstrass_roots_t<_Tp1, _Tp3>::__Real = __num_traits_t<_Type>
```

Definition at line 750 of file sf_theta.tcc.

9.46.2.3 _Type

```
template<typename _Tp1, typename _Tp3 = std::complex<_Tp1>>
using std::__detail::__weierstrass_roots_t<_Tp1, _Tp3>::__Type = typename __jacobi_lattice_t<_Tp1, _Tp3>::__Tp_None
```

Definition at line 749 of file sf_theta.tcc.

9.46.3 Constructor & Destructor Documentation

9.46.3.1 `__weierstrass_roots_t()` [1/2]

```
template<typename _Tp1 , typename _Tp3 >
std::__detail::__weierstrass_roots_t< _Tp1, _Tp3 >::__weierstrass_roots_t (
    const __jacobi_lattice_t< _Tp1, _Tp3 > & __lattice ) [explicit]
```

Constructor for the Weierstrass roots.

Parameters

<code>__lattice</code>	The Jacobi lattice.
------------------------	---------------------

Definition at line 781 of file `sf_theta.tcc`.

Referenced by `std::__detail::__weierstrass_roots_t< _Tp1, _Tp3 >::__delta()`.

9.46.3.2 `__weierstrass_roots_t()` [2/2]

```
template<typename _Tp1 , typename _Tp3 >
std::__detail::__weierstrass_roots_t< _Tp1, _Tp3 >::__weierstrass_roots_t (
    const __jacobi_theta_0_t< _Tp1, _Tp3 > & __theta0,
    _Tp1 __omega_1 )
```

Constructor for the Weierstrass roots.

Parameters

<code>__theta0</code>	Exponential theta functions of argument 0.
<code>__omega_1</code>	The first lattice parameter.

Definition at line 800 of file `sf_theta.tcc`.

References `std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::__omega_1()`, `std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_Omega3 >::__S_pi`, `std::__detail::__jacobi_theta_0_t< _Tp1, _Tp3 >::th2`, and `std::__detail::__jacobi_theta_0_t< _Tp1, _Tp3 >::th4`.

9.46.4 Member Function Documentation

9.46.4.1 __delta()

```
template<typename _Tp1, typename _Tp3 = std::complex<_Tp1>>
_Type std::__detail::__weierstrass_roots_t< _Tp1, _Tp3 >::__delta ( ) const [inline]
```

Return the discriminant $\Delta = 16(e_2 - e_3)^2(e_3 - e_1)^2(e_1 - e_2)^2$.

Definition at line 764 of file sf_theta.tcc.

References std::__detail::__weierstrass_roots_t<_Tp1, _Tp3>::__weierstrass_roots_t().

9.46.5 Member Data Documentation

9.46.5.1 __e1

```
template<typename _Tp1, typename _Tp3 = std::complex<_Tp1>>
_Type std::__detail::__weierstrass_roots_t< _Tp1, _Tp3 >::__e1
```

Definition at line 753 of file sf_theta.tcc.

Referenced by std::__detail::__weierstrass_invariants_t<_Tp1, _Tp3>::__weierstrass_invariants_t().

9.46.5.2 __e2

```
template<typename _Tp1, typename _Tp3 = std::complex<_Tp1>>
_Type std::__detail::__weierstrass_roots_t< _Tp1, _Tp3 >::__e2
```

Definition at line 753 of file sf_theta.tcc.

9.46.5.3 __e3

```
template<typename _Tp1, typename _Tp3 = std::complex<_Tp1>>
_Type std::__detail::__weierstrass_roots_t< _Tp1, _Tp3 >::__e3
```

Definition at line 753 of file sf_theta.tcc.

The documentation for this struct was generated from the following file:

- [include/bits/sf_theta.tcc](#)

9.47 std::__detail::_Airy<_Tp> Class Template Reference

Public Types

- using [scalar_type](#) = __num_traits_t< [value_type](#) >
- using [value_type](#) = _Tp

Public Member Functions

- constexpr [_Airy](#) ()=default
- [_Airy](#) (const [_Airy](#) &)=default
- [_Airy](#) ([_Airy](#) &&)=default
- constexpr [_AiryState](#)< [value_type](#) > [operator\(\)](#) ([value_type](#) __y) const

Public Attributes

- [scalar_type](#) [inner_radius](#) { [_Airy_default_radII](#)<[scalar_type](#)>::inner_radius }
- [scalar_type](#) [outer_radius](#) { [_Airy_default_radII](#)<[scalar_type](#)>::outer_radius }

9.47.1 Detailed Description

```
template<typename _Tp>
class std::__detail::_Airy<_Tp>
```

Class to manage the asymptotic expansions for Airy functions. The parameters describing the various regions are adjustable.

Definition at line 2504 of file sf_airy.tcc.

9.47.2 Member Typedef Documentation

9.47.2.1 scalar_type

```
template<typename _Tp>
using std::__detail::_Airy<_Tp>::scalar_type = __num_traits_t<value_type>
```

Definition at line 2509 of file sf_airy.tcc.

9.47.2.2 value_type

```
template<typename _Tp>
using std::__detail::_Airy<_Tp>::value_type = _Tp
```

Definition at line 2508 of file sf_airy.tcc.

9.47.3 Constructor & Destructor Documentation

9.47.3.1 _Airy() [1/3]

```
template<typename _Tp>
constexpr std::__detail::_Airy<_Tp>::_Airy ( ) [default]
```

9.47.3.2 _Airy() [2/3]

```
template<typename _Tp>
std::__detail::_Airy<_Tp>::_Airy (
    const _Airy<_Tp> & ) [default]
```

9.47.3.3 _Airy() [3/3]

```
template<typename _Tp>
std::__detail::_Airy<_Tp>::_Airy (
    _Airy<_Tp> && ) [default]
```

9.47.4 Member Function Documentation

9.47.4.1 operator>()()

```
template<typename _Tp>
constexpr _AiryState<_Tp> std::__detail::_Airy<_Tp>::operator() (
    value_type __y ) const
```

Return the Airy functions for complex argument.

Definition at line 2527 of file sf_airy.tcc.

References `std::__detail::__beta()`, `std::__detail::_Airy_series<_Tp>::_S_Ai()`, and `std::__detail::_Airy_series<_Tp>::_S_Bi()`.

9.47.5 Member Data Documentation

9.47.5.1 inner_radius

```
template<typename _Tp>
scalar_type std::__detail::_Airy< _Tp >::inner_radius {__Airy_default_radii<scalar_type>::inner←
_radius}
```

Definition at line 2518 of file sf_airy.tcc.

9.47.5.2 outer_radius

```
template<typename _Tp>
scalar_type std::__detail::_Airy< _Tp >::outer_radius {__Airy_default_radii<scalar_type>::outer←
_radius}
```

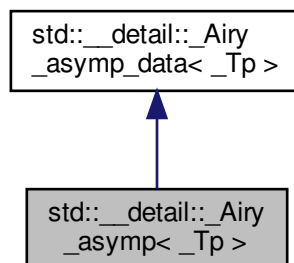
Definition at line 2519 of file sf_airy.tcc.

The documentation for this class was generated from the following file:

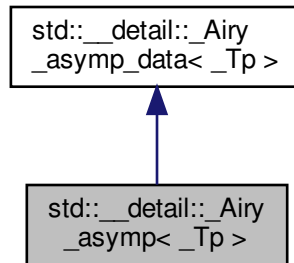
- [include/bits/sf_airy.tcc](#)

9.48 std::__detail::_Airy_asymp< _Tp > Class Template Reference

Inheritance diagram for std::__detail::_Airy_asymp< _Tp >:



Collaboration diagram for std::__detail::_Airy_asymp<_Tp>:



Public Types

- using `_Cmplx` = `std::complex<_Tp>`

Public Member Functions

- `constexpr _Airy_asymp()` = default
- `_AiryState<_Cmplx> _S_absarg_ge_pio3(_Cmplx __z) const`
*This function evaluates $Ai(z)$, $Ai'(z)$ and $Bi(z)$, $Bi'(z)$ from their asymptotic expansions for $|\arg(z)| < 2 * \pi/3$ i.e. roughly along the negative real axis.*
- `_AiryState<_Cmplx> _S_absarg_lt_pio3(_Cmplx __z) const`
This function evaluates $Ai(z)$ and $Ai'(z)$ from their asymptotic expansions for $|\arg(-z)| < \pi/3$ i.e. roughly along the negative real axis.
- `_AiryState<_Cmplx> operator()(_Cmplx __t, bool __return_fock_airy=false) const`

9.48.1 Detailed Description

```
template<typename _Tp>
class std::__detail::_Airy_asymp<_Tp>
```

A class encapsulating the asymptotic expansions of Airy functions and their derivatives.

Template Parameters

<code>_Tp</code>	A real type
------------------	-------------

Definition at line 1997 of file `sf_airy.tcc`.

9.48.2 Member Typedef Documentation

9.48.2.1 _Cmplx

```
template<typename _Tp >
using std::__detail::_Airy_asymp< _Tp >::_Cmplx = std::complex<_Tp>
```

Definition at line 2002 of file sf_airy.tcc.

9.48.3 Constructor & Destructor Documentation

9.48.3.1 _Airy_asymp()

```
template<typename _Tp >
constexpr std::__detail::_Airy_asymp< _Tp >::_Airy_asymp ( ) [default]
```

9.48.4 Member Function Documentation

9.48.4.1 _S_absarg_ge_pio3()

```
template<typename _Tp >
_AiryState< std::complex< _Tp > > std::__detail::_Airy_asymp< _Tp >::_S_absarg_ge_pio3 (
    _Cmplx __z ) const
```

This function evaluates $Ai(z)$, $Ai'(z)$ and $Bi(z)$, $Bi'(z)$ from their asymptotic expansions for $|arg(z)| < 2 * \pi/3$ i.e. roughly along the negative real axis.

Template Parameters

<code>_Tp</code>	A real type
------------------	-------------

Parameters

in	<code>__z</code>	Complex argument at which $Ai(z)$ and $Bi(z)$ and their derivative are evaluated. This function assumes $ z > 15$ and $ arg(z) < 2\pi/3$.
----	------------------	--

Returns

A struct containing z , $Ai(z)$, $Ai'(z)$, $Bi(z)$, $Bi'(z)$.

Definition at line 2271 of file sf_airy.tcc.

References std::__detail::_AiryState<_Tp>::__z.

9.48.4.2 _S_absarg_lt_pio3()

```
template<typename _Tp >
_AiryState< std::complex< _Tp > > std::__detail::_Airy_asymp< _Tp >::_S_absarg_lt_pio3 (
    _Cmplx __z ) const
```

This function evaluates $Ai(z)$ and $Ai'(z)$ from their asymptotic expansions for $|arg(-z)| < \pi/3$ i.e. roughly along the negative real axis.

For speed, the number of terms needed to achieve about 16 decimals accuracy is tabled and determined for $|z|$. This function assumes $|z| > 15$ and $|arg(-z)| < \pi/3$.

Note that for speed and since this function is called by another, checks for valid arguments are not made. Hence, an error return is not needed.

Template Parameters

<code>_Tp</code>	A real type
------------------	-------------

Parameters

in	<code>__z</code>	The value at which the Airy function and their derivatives are evaluated.
----	------------------	---

Returns

A struct containing z , $Ai(z)$, $Ai'(z)$, $Bi(z)$, $Bi'(z)$.

Todo Revisit these numbers of terms for the Airy asymptotic expansions.

Definition at line 2301 of file sf_airy.tcc.

References std::__detail::_AiryState<_Tp>::__z.

9.48.4.3 operator()

```
template<typename _Tp >
_AiryState< std::complex< _Tp > > std::__detail::_Airy_asymp< _Tp >::operator() (
    _Cmplx __t,
    bool __return_fock_airy = false ) const
```

Return the Airy functions for a given argument using asymptotic series.

Template Parameters

<code>_Tp</code>	A real type
------------------	-------------

Definition at line 2028 of file `sf_airy.tcc`.

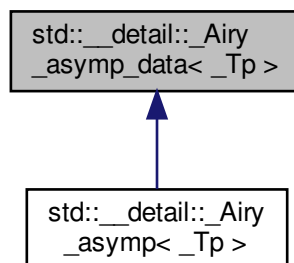
References `std::__detail::_AiryState< _Tp >::__z`.

The documentation for this class was generated from the following file:

- `include/bits/sf_airy.tcc`

9.49 std::__detail::_Airy_asymp_data< _Tp > Struct Template Reference

Inheritance diagram for `std::__detail::_Airy_asymp_data< _Tp >`:



9.49.1 Detailed Description

```
template<typename _Tp>
struct std::__detail::_Airy_asymp_data< _Tp >
```

A class encapsulating data for the asymptotic expansions of Airy functions and their derivatives.

Template Parameters

<code>_Tp</code>	A real type
------------------	-------------

Definition at line 631 of file `sf_airy.tcc`.

The documentation for this struct was generated from the following file:

- [include/bits/sf_airy.tcc](#)

9.50 `std::__detail::_Airy_asymp_data< double >` Struct Template Reference

Static Public Attributes

- static constexpr double `_S_c` [`_S_max_cd`]
- static constexpr double `_S_d` [`_S_max_cd`]
- static constexpr int `_S_max_cd` = 198

9.50.1 Detailed Description

```
template<>
struct std::__detail::_Airy_asymp_data< double >
```

Definition at line 738 of file `sf_airy.tcc`.

9.50.2 Member Data Documentation

9.50.2.1 `_S_c`

```
constexpr double std::__detail::_Airy_asymp_data< double >::_S_c[_S_max_cd] [static]
```

Definition at line 744 of file `sf_airy.tcc`.

9.50.2.2 `_S_d`

```
constexpr double std::__detail::_Airy_asymp_data< double >::_S_d[_S_max_cd] [static]
```

Definition at line 947 of file `sf_airy.tcc`.

9.50.2.3 `_S_max_cd`

```
constexpr int std::__detail::_Airy_asymp_data< double >::_S_max_cd = 198 [static]
```

Definition at line 740 of file `sf_airy.tcc`.

The documentation for this struct was generated from the following file:

- `include/bits/sf_airy.tcc`

9.51 `std::__detail::_Airy_asymp_data< float >` Struct Template Reference

Static Public Attributes

- static constexpr float `_S_c` [`_S_max_cd`]
- static constexpr float `_S_d` [`_S_max_cd`]
- static constexpr int `_S_max_cd` = 43

9.51.1 Detailed Description

```
template<>
struct std::__detail::_Airy_asymp_data< float >
```

Definition at line 635 of file `sf_airy.tcc`.

9.51.2 Member Data Documentation

9.51.2.1 `_S_c`

```
constexpr float std::__detail::_Airy_asymp_data< float >::_S_c[_S_max_cd] [static]
```

Definition at line 641 of file `sf_airy.tcc`.

9.51.2.2 `_S_d`

```
constexpr float std::__detail::_Airy_asymp_data< float >::_S_d[_S_max_cd] [static]
```

Definition at line 689 of file `sf_airy.tcc`.

9.51.2.3 _S_max_cd

```
constexpr int std::__detail::_Airy_asymp_data< float >::_S_max_cd = 43 [static]
```

Definition at line 637 of file sf_airy.tcc.

The documentation for this struct was generated from the following file:

- [include/bits/sf_airy.tcc](#)

9.52 std::__detail::_Airy_asymp_data< long double > Struct Template Reference

Static Public Attributes

- static constexpr long double [_S_c](#) [[_S_max_cd](#)]
- static constexpr long double [_S_d](#) [[_S_max_cd](#)]
- static constexpr int [_S_max_cd](#) = 201

9.52.1 Detailed Description

```
template<>
struct std::__detail::_Airy_asymp_data< long double >
```

Definition at line 1151 of file sf_airy.tcc.

9.52.2 Member Data Documentation

9.52.2.1 _S_c

```
constexpr long double std::__detail::_Airy_asymp_data< long double >::_S_c[\_S\_max\_cd] [static]
```

Definition at line 1157 of file sf_airy.tcc.

9.52.2.2 _S_d

```
constexpr long double std::__detail::_Airy_asymp_data< long double >::_S_d[\_S\_max\_cd] [static]
```

Definition at line 1363 of file sf_airy.tcc.

9.52.2.3 `_S_max_cd`

```
constexpr int std::__detail::_Airy_asymp_data< long double >::_S_max_cd = 201 [static]
```

Definition at line 1153 of file `sf_airy.tcc`.

The documentation for this struct was generated from the following file:

- `include/bits/sf_airy.tcc`

9.53 `std::__detail::_Airy_asymp_series< _Sum >` Class Template Reference

Public Types

- using `scalar_type` = `__num_traits_t< value_type >`
- using `value_type` = `typename _Sum::value_type`

Public Member Functions

- `_Airy_asymp_series` (`_Sum __proto`)
- `_Airy_asymp_series` (`const _Airy_asymp_series &`)=default
- `_Airy_asymp_series` (`_Airy_asymp_series &&`)=default
- `_AiryState< value_type > operator()` (`value_type __y`)

Static Public Attributes

- static constexpr `scalar_type _S_sqrt_pi` = `__gnu_cxx::__const_root_pi(scalar_type{})`

9.53.1 Detailed Description

```
template<typename _Sum>
class std::__detail::_Airy_asymp_series< _Sum >
```

Class to manage the asymptotic series for Airy functions.

Template Parameters

<code>_Sum</code>	A sum type
-------------------	------------

Definition at line 2364 of file `sf_airy.tcc`.

9.53.2 Member Typedef Documentation

9.53.2.1 scalar_type

```
template<typename _Sum>
using std::__detail::_Airy_asymp_series<_Sum>::scalar_type = __num_traits_t<value_type>
```

Definition at line 2369 of file sf_airy.tcc.

9.53.2.2 value_type

```
template<typename _Sum>
using std::__detail::_Airy_asymp_series<_Sum>::value_type = typename _Sum::value_type
```

Definition at line 2368 of file sf_airy.tcc.

9.53.3 Constructor & Destructor Documentation

9.53.3.1 _Airy_asymp_series() [1/3]

```
template<typename _Sum>
std::__detail::_Airy_asymp_series<_Sum>::_Airy_asymp_series (
    _Sum __proto ) [inline]
```

Definition at line 2373 of file sf_airy.tcc.

9.53.3.2 _Airy_asymp_series() [2/3]

```
template<typename _Sum>
std::__detail::_Airy_asymp_series<_Sum>::_Airy_asymp_series (
    const _Airy_asymp_series<_Sum> & ) [default]
```

9.53.3.3 `_Airy_asymp_series()` [3/3]

```
template<typename _Sum>
std::__detail::_Airy_asymp_series< _Sum >::_Airy_asymp_series (
    _Airy_asymp_series< _Sum > && ) [default]
```

9.53.4 Member Function Documentation

9.53.4.1 `operator>()`

```
template<typename _Sum>
_AiryState< typename _Airy_asymp_series< _Sum >::value_type > std::__detail::_Airy_asymp_series<
_Sum >::operator() (
    value_type __y )
```

Return an `_AiryState` containing, not actual Airy functions, but four asymptotic Airy components:

Template Parameters

<code>_Sum</code>	A sum type
-------------------	------------

Definition at line 2418 of file `sf_airy.tcc`.

9.53.5 Member Data Documentation

9.53.5.1 `_S_sqrt_pi`

```
template<typename _Sum>
constexpr _Airy_asymp_series< _Sum >::scalar_type std::__detail::_Airy_asymp_series< _Sum >::_S_sqrt_pi =
__gnu_cxx::__const_root_pi(scalar_type{}) [static]
```

Definition at line 2371 of file `sf_airy.tcc`.

The documentation for this class was generated from the following file:

- `include/bits/sf_airy.tcc`

9.54 `std::__detail::_Airy_default_radii<_Tp>` Struct Template Reference

9.54.1 Detailed Description

```
template<typename _Tp>
struct std::__detail::_Airy_default_radii<_Tp>
```

Definition at line 2475 of file `sf_airy.tcc`.

The documentation for this struct was generated from the following file:

- `include/bits/sf_airy.tcc`

9.55 `std::__detail::_Airy_default_radii<double>` Struct Template Reference

Static Public Attributes

- static constexpr double `inner_radius` {4.0}
- static constexpr double `outer_radius` {12.0}

9.55.1 Detailed Description

```
template<>
struct std::__detail::_Airy_default_radii<double>
```

Definition at line 2486 of file `sf_airy.tcc`.

9.55.2 Member Data Documentation

9.55.2.1 `inner_radius`

```
constexpr double std::__detail::_Airy_default_radii<double>::inner_radius {4.0} [static]
```

Definition at line 2488 of file `sf_airy.tcc`.

9.55.2.2 `outer_radius`

```
constexpr double std::__detail::_Airy_default_radii< double >::outer_radius {12.0} [static]
```

Definition at line 2489 of file `sf_airy.tcc`.

The documentation for this struct was generated from the following file:

- `include/bits/sf_airy.tcc`

9.56 `std::__detail::_Airy_default_radii< float >` Struct Template Reference

Static Public Attributes

- static constexpr float `inner_radius` {2.0F}
- static constexpr float `outer_radius` {6.0F}

9.56.1 Detailed Description

```
template<>
struct std::__detail::_Airy_default_radii< float >
```

Definition at line 2479 of file `sf_airy.tcc`.

9.56.2 Member Data Documentation

9.56.2.1 `inner_radius`

```
constexpr float std::__detail::_Airy_default_radii< float >::inner_radius {2.0F} [static]
```

Definition at line 2481 of file `sf_airy.tcc`.

9.56.2.2 `outer_radius`

```
constexpr float std::__detail::_Airy_default_radii< float >::outer_radius {6.0F} [static]
```

Definition at line 2482 of file `sf_airy.tcc`.

The documentation for this struct was generated from the following file:

- `include/bits/sf_airy.tcc`

9.57 `std::__detail::_Airy_default_radII< long double >` Struct Template Reference

Static Public Attributes

- static constexpr long double `inner_radius` {5.0L}
- static constexpr long double `outer_radius` {15.0L}

9.57.1 Detailed Description

```
template<>
struct std::__detail::_Airy_default_radII< long double >
```

Definition at line 2493 of file `sf_airy.tcc`.

9.57.2 Member Data Documentation

9.57.2.1 `inner_radius`

```
constexpr long double std::__detail::_Airy_default_radII< long double >::inner_radius {5.0L}
[static]
```

Definition at line 2495 of file `sf_airy.tcc`.

9.57.2.2 `outer_radius`

```
constexpr long double std::__detail::_Airy_default_radII< long double >::outer_radius {15.0L}
[static]
```

Definition at line 2496 of file `sf_airy.tcc`.

The documentation for this struct was generated from the following file:

- `include/bits/sf_airy.tcc`

9.58 `std::__detail::_Airy_series< _Tp >` Class Template Reference

Public Types

- using `_Cmplx` = `std::complex< _Tp >`

Static Public Member Functions

- static std::pair< [_Cmplx](#), [_Cmplx](#) > [_S_Ai](#) ([_Cmplx](#) __t)
- static [_AiryState](#)< [_Cmplx](#) > [_S_Airy](#) ([_Cmplx](#) __t)
- static std::pair< [_Cmplx](#), [_Cmplx](#) > [_S_Bi](#) ([_Cmplx](#) __t)
- static [_AiryAuxilliaryState](#)< [_Cmplx](#) > [_S_FGH](#) ([_Cmplx](#) __t)
- static [_AiryState](#)< [_Cmplx](#) > [_S_Fock](#) ([_Cmplx](#) __t)
- static [_AiryState](#)< [_Cmplx](#) > [_S_Scorer](#) ([_Cmplx](#) __t)
- static [_AiryState](#)< [_Cmplx](#) > [_S_Scorer2](#) ([_Cmplx](#) __t)

Static Public Attributes

- static constexpr int [_N_FGH](#) = 200
- static constexpr _Tp [_S_Ai0](#) = _Tp{3.550280538878172392600631860041831763980e-1L}
- static constexpr _Tp [_S_Aip0](#) = _Tp{-2.588194037928067984051835601892039634793e-1L}
- static constexpr _Tp [_S_Bi0](#) = _Tp{6.149266274460007351509223690936135535960e-1L}
- static constexpr _Tp [_S_Bip0](#) = _Tp{4.482883573538263579148237103988283908668e-1L}
- static constexpr _Tp [_S_eps](#) = __gnu_cxx::__epsilon(_Tp{})
- static constexpr _Tp [_S_Gi0](#) = _Tp{2.049755424820002450503074563645378511979e-1L}
- static constexpr _Tp [_S_Gip0](#) = _Tp{1.494294524512754526382745701329427969551e-1L}
- static constexpr _Tp [_S_Hi0](#) = _Tp{4.099510849640004901006149127290757023959e-1L}
- static constexpr _Tp [_S_Hip0](#) = _Tp{2.988589049025509052765491402658855939102e-1L}
- static constexpr [_Cmplx](#) [_S_i](#) {_Tp{0}, _Tp{1}}
- static constexpr _Tp [_S_pi](#) = __gnu_cxx::__const_pi(_Tp{})
- static constexpr _Tp [_S_sqrt_pi](#) = __gnu_cxx::__const_root_pi(_Tp{})

9.58.1 Detailed Description

```
template<typename _Tp>
class std::__detail::_Airy_series< _Tp >
```

This class organizes series solutions of the Airy function.

$$fai(x) = \sum_{k=0}^{\infty} \frac{(2k+1)!!!x^{3k}}{(2k+1)!}$$

$$gai(x) = \sum_{k=0}^{\infty} \frac{(2k+2)!!!x^{3k+1}}{(2k+2)!}$$

$$hai(x) = \sum_{k=0}^{\infty} \frac{(2k+3)!!!x^{3k+2}}{(2k+3)!}$$

This class contains tabulations of the factors appearing in the sums above.

Definition at line 107 of file sf_airy.tcc.

9.58.2 Member Typedef Documentation

9.58.2.1 _Cmplx

```
template<typename _Tp >
using std::__detail::_Airy_series<_Tp>::__Cmplx = std::complex<_Tp>
```

Definition at line 111 of file sf_airy.tcc.

9.58.3 Member Function Documentation

9.58.3.1 _S_Ai()

```
template<typename _Tp >
std::pair< std::complex<_Tp>, std::complex<_Tp> > std::__detail::_Airy_series<_Tp>::_S_Ai
(
    __Cmplx __t ) [static]
```

Return the Airy function of the first kind and its derivative by using the series expansions of the auxilliary Airy functions:

$$fai(x) = \sum_{k=0}^{\infty} \frac{(2k+1)!!!x^{3k}}{(2k+1)!}$$

$$gai(x) = \sum_{k=0}^{\infty} \frac{(2k+2)!!!x^{3k+1}}{(2k+2)!}$$

The Airy function of the first kind is then defined by:

$$Ai(x) = Ai(0)fai(x) + Ai'(0)gai(x)$$

where $Ai(0) = 3^{-2/3}/\Gamma(2/3)$, $Ai'(0) = -3^{-1/2}Bi'(0)$ and $Bi(0) = 3^{1/2}Ai(0)$, $Bi'(0) = 3^{1/6}/\Gamma(1/3)$

Template Parameters

<code>_Tp</code>	A real type
------------------	-------------

Definition at line 340 of file sf_airy.tcc.

Referenced by std::__detail::_Airy<_Tp>::operator()().

9.58.3.2 _S_Airy()

```
template<typename _Tp >
_AiryState< std::complex<_Tp> > std::__detail::_Airy_series<_Tp>::_S_Airy (
    __Cmplx __t ) [static]
```

Return the Fock-type Airy functions $Ai(t)$, and $Bi(t)$ and their derivatives of complex argument.

Template Parameters

<code>_Tp</code>	A real type
------------------	-------------

Parameters

<code>←</code>	The complex argument
<code>←</code>	
<code>←</code>	
<code>←</code>	
<code>t</code>	

Definition at line 608 of file `sf_airy.tcc`.

9.58.3.3 `_S_Bi()`

```
template<typename _Tp >
std::pair< std::complex< _Tp >, std::complex< _Tp > > std::__detail::_Airy_series< _Tp >::_S_Bi
(
    _Cmplx __t ) [static]
```

Return the Airy function of the second kind and its derivative by using the series expansions of the auxilliary Airy functions:

$$fai(x) = \sum_{k=0}^{\infty} \frac{(2k+1)!!!x^{3k}}{(2k+1)!}$$

$$gai(x) = \sum_{k=0}^{\infty} \frac{(2k+2)!!!x^{3k+1}}{(2k+2)!}$$

The Airy function of the second kind is then defined by:

$$Bi(x) = Bi(0)fai(x) + Bi'(0)gai(x)$$

where $Ai(0) = 3^{-2/3}/\Gamma(2/3)$, $Ai'(0) = -3-1/2Bi'(0)$ and $Bi(0) = 3^{1/2}Ai(0)$, $Bi'(0) = 3^{1/6}/\Gamma(1/3)$

Template Parameters

<code>_Tp</code>	A real type
------------------	-------------

Definition at line 363 of file `sf_airy.tcc`.

Referenced by `std::__detail::_Airy< _Tp >::operator()()`.

9.58.3.4 _S_FGH()

```
template<typename _Tp >
_AiryAuxilliaryState< std::complex< _Tp > > std::__detail::_Airy_series< _Tp >::_S_FGH (
    _Cmplx __t ) [static]
```

Return the auxilliary Airy functions:

$$fai(x) = \sum_{k=0}^{\infty} \frac{(2k+1)!!!x^{3k}}{(2k+1)!}$$

$$gai(x) = \sum_{k=0}^{\infty} \frac{(2k+2)!!!x^{3k+1}}{(2k+2)!}$$

$$hai(x) = \sum_{k=0}^{\infty} \frac{(2k+3)!!!x^{3k+2}}{(2k+3)!}$$

Template Parameters

<code>_Tp</code>	A real type
------------------	-------------

Definition at line 382 of file sf_airy.tcc.

9.58.3.5 _S_Fock()

```
template<typename _Tp >
_AiryState< std::complex< _Tp > > std::__detail::_Airy_series< _Tp >::_S_Fock (
    _Cmplx __t ) [static]
```

Return the Fock-type Airy functions $w_1(t)$, and $w_2(t)$ and their derivatives of complex argument.

Template Parameters

<code>_Tp</code>	A real type
------------------	-------------

Parameters

\leftarrow	The complex argument
\leftarrow	
\leftarrow	
\leftarrow	
t	

Definition at line 620 of file sf_airy.tcc.

9.58.3.6 `_S_Scorer()`

```
template<typename _Tp >
_AiryState< std::complex< _Tp > > std::__detail::_Airy_series< _Tp >::_S_Scorer (
    _Cmplx __t ) [static]
```

Return the Scorer functions by using the series expansions of the auxilliary Airy functions:

$$fai(x) = \sum_{k=0}^{\infty} \frac{(2k+1)!!!x^{3k}}{(2k+1)!}$$

$$gai(x) = \sum_{k=0}^{\infty} \frac{(2k+2)!!!x^{3k+1}}{(2k+2)!}$$

$$hai(x) = \sum_{k=0}^{\infty} \frac{(2k+3)!!!x^{3k+2}}{(2k+3)!}$$

The Scorer function is then defined by:

$$Hi(x) = Hi(0) (fai(x) + gai(x) + hai(x))$$

where $Hi(0) = 2/(3^{7/6}\Gamma(2/3))$ and $Hi'(0) = 2/(3^{5/6}\Gamma(1/3))$. The other Scorer function is found from the identity

$$Gi(x) + Hi(x) = Bi(x)$$

Todo Find out what is wrong with the $Hi = fai + gai + hai$ scorer function.

Template Parameters

<code>_Tp</code>	A real type
------------------	-------------

Definition at line 463 of file `sf_airy.tcc`.

9.58.3.7 `_S_Scorer2()`

```
template<typename _Tp >
_AiryState< std::complex< _Tp > > std::__detail::_Airy_series< _Tp >::_S_Scorer2 (
    _Cmplx __t ) [static]
```

Return the Scorer functions by using the series expansions:

$$Hi(x) = \frac{3^{-2/3}}{\pi} \sum_{k=0}^{\infty} \Gamma\left(\frac{k+1}{3}\right) \frac{3^{1/3}x}{k!}$$

$$Hi'(x) = \frac{3^{-1/3}}{\pi} \sum_{k=0}^{\infty} \Gamma\left(\frac{k+2}{3}\right) \frac{3^{1/3}x}{k!}$$

$$Gi(x) = \frac{3^{-2/3}}{\pi} \sum_{k=0}^{\infty} \cos\left(\frac{2k-1}{3}\pi\right) \Gamma\left(\frac{k+1}{3}\right) \frac{3^{1/3}x}{k!}$$

$$Gi'(x) = \frac{3^{-1/3}}{\pi} \sum_{k=0}^{\infty} \cos\left(\frac{2k+1}{3}\pi\right) \Gamma\left(\frac{k+2}{3}\right) \frac{3^{1/3}x}{k!}$$

Definition at line 500 of file sf_airy.tcc.

References std::__detail::__gamma().

9.58.4 Member Data Documentation

9.58.4.1 _N_FGH

```
template<typename _Tp >
constexpr int std::__detail::_Airy_series<_Tp>::_N_FGH = 200 [static]
```

Definition at line 113 of file sf_airy.tcc.

9.58.4.2 _S_Ai0

```
template<typename _Tp >
constexpr _Tp std::__detail::_Airy_series<_Tp>::_S_Ai0 = _Tp{3.550280538878172392600631860041831763980e-1↵
L} [static]
```

Definition at line 129 of file sf_airy.tcc.

9.58.4.3 _S_Aip0

```
template<typename _Tp >
constexpr _Tp std::__detail::_Airy_series<_Tp>::_S_Aip0 = _Tp{-2.588194037928067984051835601892039634793e-1↵
L} [static]
```

Definition at line 131 of file sf_airy.tcc.

9.58.4.4 `_S_Bi0`

```
template<typename _Tp >
constexpr _Tp std::__detail::_Airy_series< _Tp >::_S_Bi0 = _Tp{6.149266274460007351509223690936135535960e-1↵
L} [static]
```

Definition at line 133 of file `sf_airy.tcc`.

9.58.4.5 `_S_Bip0`

```
template<typename _Tp >
constexpr _Tp std::__detail::_Airy_series< _Tp >::_S_Bip0 = _Tp{4.482883573538263579148237103988283908668e-1↵
L} [static]
```

Definition at line 135 of file `sf_airy.tcc`.

9.58.4.6 `_S_eps`

```
template<typename _Tp >
constexpr _Tp std::__detail::_Airy_series< _Tp >::_S_eps = __gnu_cxx::__epsilon(_Tp{}) [static]
```

Definition at line 124 of file `sf_airy.tcc`.

9.58.4.7 `_S_Gi0`

```
template<typename _Tp >
constexpr _Tp std::__detail::_Airy_series< _Tp >::_S_Gi0 = _Tp{2.049755424820002450503074563645378511979e-1↵
L} [static]
```

Definition at line 141 of file `sf_airy.tcc`.

9.58.4.8 `_S_Gip0`

```
template<typename _Tp >
constexpr _Tp std::__detail::_Airy_series< _Tp >::_S_Gip0 = _Tp{1.494294524512754526382745701329427969551e-1↵
L} [static]
```

Definition at line 143 of file `sf_airy.tcc`.

9.58.4.9 _S_Hi0

```
template<typename _Tp >
constexpr _Tp std::__detail::_Airy_series<_Tp>::_S_Hi0 = _Tp{4.099510849640004901006149127290757023959e-1↵
L} [static]
```

Definition at line 137 of file sf_airy.tcc.

9.58.4.10 _S_Hip0

```
template<typename _Tp >
constexpr _Tp std::__detail::_Airy_series<_Tp>::_S_Hip0 = _Tp{2.988589049025509052765491402658855939102e-1↵
L} [static]
```

Definition at line 139 of file sf_airy.tcc.

9.58.4.11 _S_i

```
template<typename _Tp >
constexpr std::complex<_Tp> std::__detail::_Airy_series<_Tp>::_S_i {_Tp{0}, _Tp{1}} [static]
```

Definition at line 144 of file sf_airy.tcc.

9.58.4.12 _S_pi

```
template<typename _Tp >
constexpr _Tp std::__detail::_Airy_series<_Tp>::_S_pi = __gnu_cxx::__const_pi(_Tp{}) [static]
```

Definition at line 125 of file sf_airy.tcc.

9.58.4.13 _S_sqrt_pi

```
template<typename _Tp >
constexpr _Tp std::__detail::_Airy_series<_Tp>::_S_sqrt_pi = __gnu_cxx::__const_root_pi(_Tp{})
[static]
```

Definition at line 127 of file sf_airy.tcc.

The documentation for this class was generated from the following file:

- [include/bits/sf_airy.tcc](#)

9.59 std::__detail::_AiryAuxilliaryState< _Tp > Struct Template Reference

Public Types

- using [_Val](#) = __num_traits_t< _Tp >

Public Attributes

- [_Tp __fai_deriv](#)
- [_Tp __fai_value](#)
- [_Tp __gai_deriv](#)
- [_Tp __gai_value](#)
- [_Tp __hai_deriv](#)
- [_Tp __hai_value](#)
- [_Tp __z](#)

9.59.1 Detailed Description

```
template<typename _Tp>  
struct std::__detail::_AiryAuxilliaryState< _Tp >
```

A structure containing three auxilliary Airy functions and their derivatives.

Definition at line 79 of file sf_airy.tcc.

9.59.2 Member Typedef Documentation

9.59.2.1 _Val

```
template<typename _Tp>  
using std::\_\_detail::\_AiryAuxilliaryState< _Tp >::\_Val = __num_traits_t<_Tp>
```

Definition at line 81 of file sf_airy.tcc.

9.59.3 Member Data Documentation

9.59.3.1 __fai_deriv

```
template<typename _Tp>
_Tp std::__detail::__AiryAuxilliaryState< _Tp >::__fai_deriv
```

Definition at line 85 of file sf_airy.tcc.

9.59.3.2 __fai_value

```
template<typename _Tp>
_Tp std::__detail::__AiryAuxilliaryState< _Tp >::__fai_value
```

Definition at line 84 of file sf_airy.tcc.

9.59.3.3 __gai_deriv

```
template<typename _Tp>
_Tp std::__detail::__AiryAuxilliaryState< _Tp >::__gai_deriv
```

Definition at line 87 of file sf_airy.tcc.

9.59.3.4 __gai_value

```
template<typename _Tp>
_Tp std::__detail::__AiryAuxilliaryState< _Tp >::__gai_value
```

Definition at line 86 of file sf_airy.tcc.

9.59.3.5 __hai_deriv

```
template<typename _Tp>
_Tp std::__detail::__AiryAuxilliaryState< _Tp >::__hai_deriv
```

Definition at line 89 of file sf_airy.tcc.

9.59.3.6 `__hai_value`

```
template<typename _Tp>
_Tp std::__detail::_AiryAuxilliaryState< _Tp >::__hai_value
```

Definition at line 88 of file `sf_airy.tcc`.

9.59.3.7 `__z`

```
template<typename _Tp>
_Tp std::__detail::_AiryAuxilliaryState< _Tp >::__z
```

Definition at line 83 of file `sf_airy.tcc`.

The documentation for this struct was generated from the following file:

- `include/bits/sf_airy.tcc`

9.60 `std::__detail::_AiryState< _Tp >` Struct Template Reference

Public Types

- using `_Real` = `__num_traits_t< _Tp >`

Public Member Functions

- `_Real true_Wronskian ()`
- `_Tp Wronskian () const`

Public Attributes

- `_Tp __Ai_deriv`
- `_Tp __Ai_value`
- `_Tp __Bi_deriv`
- `_Tp __Bi_value`
- `_Tp __z`

9.60.1 Detailed Description

```
template<typename _Tp>
struct std::__detail::_AiryState<_Tp>
```

This struct defines the Airy function state with two presumably numerically useful Airy functions and their derivatives. The data members are directly accessible. The lone method computes the Wronskian from the stored functions. A static method returns the correct Wronskian.

Definition at line 54 of file sf_airy.tcc.

9.60.2 Member Typedef Documentation

9.60.2.1 _Real

```
template<typename _Tp>
using std::__detail::_AiryState<_Tp>::_Real = __num_traits_t<_Tp>
```

Definition at line 56 of file sf_airy.tcc.

9.60.3 Member Function Documentation

9.60.3.1 true_Wronskian()

```
template<typename _Tp>
_Real std::__detail::_AiryState<_Tp>::true_Wronskian ( ) [inline]
```

Definition at line 69 of file sf_airy.tcc.

9.60.3.2 Wronskian()

```
template<typename _Tp>
_Tp std::__detail::_AiryState<_Tp>::Wronskian ( ) const [inline]
```

Definition at line 65 of file sf_airy.tcc.

References `std::__detail::_AiryState<_Tp>::__Ai_deriv`.

9.60.4 Member Data Documentation

9.60.4.1 __Ai_deriv

```
template<typename _Tp>  
_Tp std::__detail::_AiryState< _Tp >::__Ai_deriv
```

Definition at line 60 of file sf_airy.tcc.

Referenced by `std::__detail::_AiryState< _Tp >::Wronskian()`.

9.60.4.2 __Ai_value

```
template<typename _Tp>  
_Tp std::__detail::_AiryState< _Tp >::__Ai_value
```

Definition at line 59 of file sf_airy.tcc.

9.60.4.3 __Bi_deriv

```
template<typename _Tp>  
_Tp std::__detail::_AiryState< _Tp >::__Bi_deriv
```

Definition at line 62 of file sf_airy.tcc.

9.60.4.4 __Bi_value

```
template<typename _Tp>  
_Tp std::__detail::_AiryState< _Tp >::__Bi_value
```

Definition at line 61 of file sf_airy.tcc.

9.60.4.5 __z

```
template<typename _Tp>
_Tp std::__detail::_AiryState< _Tp >::__z
```

Definition at line 58 of file sf_airy.tcc.

Referenced by std::__detail::_Airy_asymp<_Tp>::_S_absarg_ge_pio3(), std::__detail::_Airy_asymp<_Tp>::_S_↵absarg_lt_pio3(), and std::__detail::_Airy_asymp<_Tp>::operator().

The documentation for this struct was generated from the following file:

- include/bits/sf_airy.tcc

9.61 std::__detail::_AsympTerminator<_Tp> Class Template Reference

Public Member Functions

- [_AsympTerminator](#) (std::size_t __max_iter, _Real __mul=_Real{1})
- std::size_t [num_terms](#) () const
Return the current number of terms summed.
- bool [operator\(\)](#) (_Tp __term, _Tp __sum)
Detect if the sum should terminate either because the incoming term is small enough or because the terms are starting to grow or.
- _Tp [operator<<](#) (_Tp __term)
Filter a term before applying it to the sum.

9.61.1 Detailed Description

```
template<typename _Tp>
class std::__detail::_AsympTerminator<_Tp>
```

This class manages the termination of asymptotic series. In particular, this termination watches for the growth of the sequence of terms to stop the series.

Termination conditions involve both a maximum iteration count and a relative precision.

Definition at line 107 of file sf_polylog.tcc.

9.61.2 Constructor & Destructor Documentation

9.61.2.1 `_AsympTerminator()`

```
template<typename _Tp>
std::__detail::_AsympTerminator< _Tp >::_AsympTerminator (
    std::size_t __max_iter,
    _Real __mul = _Real{1} ) [inline]
```

Definition at line 120 of file `sf_polylog.tcc`.

9.61.3 Member Function Documentation

9.61.3.1 `num_terms()`

```
template<typename _Tp>
std::size_t std::__detail::_AsympTerminator< _Tp >::num_terms ( ) const [inline]
```

Return the current number of terms summed.

Definition at line 140 of file `sf_polylog.tcc`.

9.61.3.2 `operator()()`

```
template<typename _Tp>
bool std::__detail::_AsympTerminator< _Tp >::operator() (
    _Tp __term,
    _Tp __sum ) [inline]
```

Detect if the sum should terminate either because the incoming term is small enough or because the terms are starting to grow or.

Definition at line 147 of file `sf_polylog.tcc`.

9.61.3.3 `operator<<()`

```
template<typename _Tp>
_Tp std::__detail::_AsympTerminator< _Tp >::operator<< (
    _Tp __term ) [inline]
```

Filter a term before applying it to the sum.

Definition at line 127 of file `sf_polylog.tcc`.

The documentation for this class was generated from the following file:

- [include/bits/sf_polylog.tcc](#)

9.62 std::__detail::_Factorial_table< _Tp > Struct Template Reference

Public Attributes

- [_Tp __factorial](#)
- [_Tp __log_factorial](#)
- [int __n](#)

9.62.1 Detailed Description

```
template<typename _Tp>
struct std::__detail::_Factorial_table< _Tp >
```

Definition at line 67 of file sf_gamma.tcc.

9.62.2 Member Data Documentation

9.62.2.1 __factorial

```
template<typename _Tp >
_Tp std::__detail::_Factorial_table< _Tp >::__factorial
```

Definition at line 70 of file sf_gamma.tcc.

Referenced by `std::__detail::__double_factorial()`, and `std::__detail::__gamma_reciprocal()`.

9.62.2.2 __log_factorial

```
template<typename _Tp >
_Tp std::__detail::_Factorial_table< _Tp >::__log_factorial
```

Definition at line 71 of file sf_gamma.tcc.

Referenced by `std::__detail::__log_double_factorial()`, and `std::__detail::__log_gamma()`.

9.62.2.3 `__n`

```
template<typename _Tp >
int std::__detail::_Factorial_table< _Tp >::__n
```

Definition at line 69 of file `sf_gamma.tcc`.

Referenced by `std::__detail::__binomial()`, `std::__detail::__digamma()`, `std::__detail::__double_factorial()`, `std::__detail::__factorial()`, `std::__detail::__falling_factorial()`, `std::__detail::__gamma()`, `std::__detail::__gamma_cont_frac()`, `std::__detail::__gamma_reciprocal()`, `std::__detail::__gamma_series()`, `std::__detail::__harmonic_number()`, `std::__detail::__lanczos_binet1p()`, `std::__detail::__log_binomial()`, `std::__detail::__log_binomial_sign()`, `std::__detail::__log_double_factorial()`, `std::__detail::__log_factorial()`, `std::__detail::__log_gamma()`, `std::__detail::__polygamma()`, and `std::__detail::__rising_factorial()`.

The documentation for this struct was generated from the following file:

- `include/bits/sf_gamma.tcc`

9.63 `std::__detail::_Terminator< _Tp >` Class Template Reference

Public Member Functions

- `_Terminator` (`std::size_t __max_iter`, `_Real __mul=_Real{1}`)
- `std::size_t num_terms ()` const
Return the current number of terms summed.
- `bool operator() (_Tp __term, _Tp __sum)`
Detect if the sum should terminate either because the incoming term is small enough or the maximum number of terms has been reached.

9.63.1 Detailed Description

```
template<typename _Tp>
class std::__detail::_Terminator< _Tp >
```

This class manages the termination of series. Termination conditions involve both a maximum iteration count and a relative precision.

Definition at line 62 of file `sf_polylog.tcc`.

9.63.2 Constructor & Destructor Documentation

9.63.2.1 _Terminator()

```
template<typename _Tp>
std::__detail::_Terminator<_Tp>::_Terminator (
    std::size_t __max_iter,
    _Real __mul = _Real{1} ) [inline]
```

Definition at line 73 of file sf_polylog.tcc.

9.63.3 Member Function Documentation

9.63.3.1 num_terms()

```
template<typename _Tp>
std::size_t std::__detail::_Terminator<_Tp>::num_terms ( ) const [inline]
```

Return the current number of terms summed.

Definition at line 80 of file sf_polylog.tcc.

9.63.3.2 operator>()()

```
template<typename _Tp>
bool std::__detail::_Terminator<_Tp>::operator() (
    _Tp __term,
    _Tp __sum ) [inline]
```

Detect if the sum should terminate either because the incoming term is small enough or the maximum number of terms has been reached.

Definition at line 86 of file sf_polylog.tcc.

The documentation for this class was generated from the following file:

- [include/bits/sf_polylog.tcc](#)

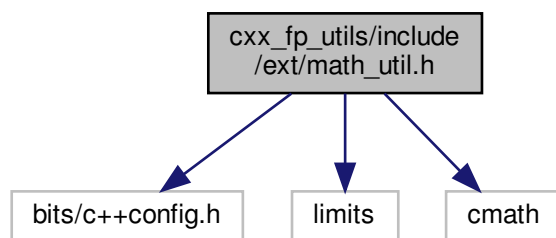
Chapter 10

File Documentation

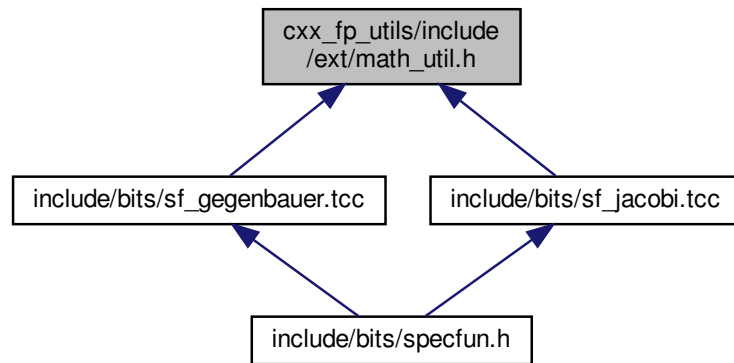
10.1 cxx_fp_utils/include/ext/math_util.h File Reference

```
#include <bits/c++config.h>  
#include <limits>  
#include <cmath>
```

Include dependency graph for math_util.h:



This graph shows which files directly or indirectly include this file:



Classes

- struct [__gnu_cxx::__fp_is_integer_t](#)

Namespaces

- [__gnu_cxx](#)

Functions

- `template<typename _Tp >`
`constexpr bool __gnu_cxx::__fp_is_equal (_Tp __a, _Tp __b, _Tp __mul=_Tp{1}) noexcept`
- `template<typename _Tp >`
`constexpr __fp_is_integer_t __gnu_cxx::__fp_is_even_integer (_Tp __a, _Tp __mul=_Tp{1}) noexcept`
- `template<typename _Tp >`
`constexpr __fp_is_integer_t __gnu_cxx::__fp_is_half_integer (_Tp __a, _Tp __mul=_Tp{1}) noexcept`
- `template<typename _Tp >`
`constexpr __fp_is_integer_t __gnu_cxx::__fp_is_half_odd_integer (_Tp __a, _Tp __mul=_Tp{1}) noexcept`
- `template<typename _Tp >`
`constexpr __fp_is_integer_t __gnu_cxx::__fp_is_integer (_Tp __a, _Tp __mul=_Tp{1}) noexcept`
- `template<typename _Tp >`
`constexpr __fp_is_integer_t __gnu_cxx::__fp_is_odd_integer (_Tp __a, _Tp __mul=_Tp{1}) noexcept`
- `template<typename _Tp >`
`constexpr bool __gnu_cxx::__fp_is_zero (_Tp __a, _Tp __mul=_Tp{1}) noexcept`
- `template<typename _Tp >`
`constexpr _Tp __gnu_cxx::__fp_max_abs (_Tp __a, _Tp __b) noexcept`
- `template<typename _Tp, typename _IntTp >`
`constexpr _Tp __gnu_cxx::__parity (_IntTp __k) noexcept`

10.1.1 Detailed Description

This file is a GNU extension to the Standard C++ Library.

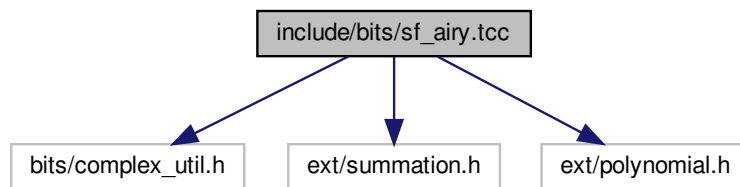
10.2 include/bits/sf_airy.tcc File Reference

```
#include <bits/complex_util.h>
```

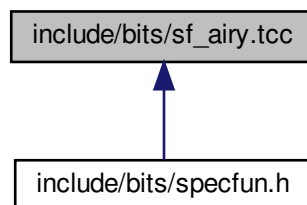
```
#include <ext/summation.h>
```

```
#include <ext/polynomial.h>
```

Include dependency graph for sf_airy.tcc:



This graph shows which files directly or indirectly include this file:



Classes

- class `std::__detail::_Airy<_Tp>`
- class `std::__detail::_Airy_asymp<_Tp>`
- struct `std::__detail::_Airy_asymp_data<_Tp>`
- struct `std::__detail::_Airy_asymp_data<double>`
- struct `std::__detail::_Airy_asymp_data<float>`

- struct [std::__detail::_Airy_asymp_data](#)< long double >
- class [std::__detail::_Airy_asymp_series](#)< _Sum >
- struct [std::__detail::_Airy_default_radii](#)< _Tp >
- struct [std::__detail::_Airy_default_radii](#)< double >
- struct [std::__detail::_Airy_default_radii](#)< float >
- struct [std::__detail::_Airy_default_radii](#)< long double >
- class [std::__detail::_Airy_series](#)< _Tp >
- struct [std::__detail::_AiryAuxilliaryState](#)< _Tp >
- struct [std::__detail::_AiryState](#)< _Tp >

Namespaces

- [std](#)
- [std::__detail](#)

Implementation-space details.

Macros

- [#define _GLIBCXX_BITS_SF_AIRY_TCC](#) 1

Functions

- [template<typename _Tp > std::complex< _Tp > std::__detail::__airy_ai](#) (std::complex< _Tp > __z)
Return the complex Airy Ai function.
- [template<typename _Tp > std::complex< _Tp > std::__detail::__airy_bi](#) (std::complex< _Tp > __z)
Return the complex Airy Bi function.

Variables

- [template<typename _Tp > constexpr int std::__detail::__max_FGH](#) = [_Airy_series<_Tp>::__N_FGH](#)
- [template<> constexpr int std::__detail::__max_FGH< double >](#) = 79
- [template<> constexpr int std::__detail::__max_FGH< float >](#) = 15

10.2.1 Detailed Description

This is an internal header file, included by other library headers. You should not attempt to use it directly.

10.2.2 Macro Definition Documentation

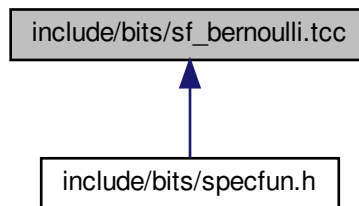
10.2.2.1 _GLIBCXX_BITS_SF_AIRY_TCC

```
#define _GLIBCXX_BITS_SF_AIRY_TCC 1
```

Definition at line 31 of file sf_airy.tcc.

10.3 include/bits/sf_bernoulli.tcc File Reference

This graph shows which files directly or indirectly include this file:



Namespaces

- [std](#)
- [std::__detail](#)

Implementation-space details.

Macros

- [#define _GLIBCXX_BITS_SF_BERNOULLI_TCC 1](#)

Functions

- `template<typename _Tp >
_GLIBCXX14_CONSTEXPR _Tp std::__detail::__bernoulli (unsigned int __n)`
This returns Bernoulli number B_n .
- `template<typename _Tp >
_Tp std::__detail::__bernoulli (unsigned int __n, _Tp __x)`
- `template<typename _Tp >
_GLIBCXX14_CONSTEXPR _Tp std::__detail::__bernoulli_2n (unsigned int __n)`
This returns Bernoulli number B_{2n} at even integer arguments $2n$.
- `template<typename _Tp >
_GLIBCXX14_CONSTEXPR _Tp std::__detail::__bernoulli_series (unsigned int __n)`
This returns Bernoulli numbers from a table or by summation for larger values.

$$B_{2n} = (-1)^{n+1} 2 \frac{(2n)!}{(2\pi)^{2n}} \zeta(2n)$$

10.3.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include `<cmath>`.

10.3.2 Macro Definition Documentation

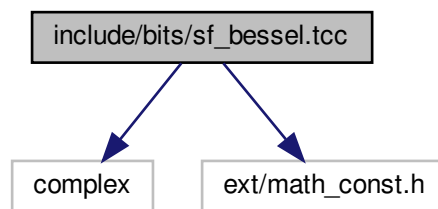
10.3.2.1 `_GLIBCXX_BITS_SF_BERNOULLI_TCC`

```
#define _GLIBCXX_BITS_SF_BERNOULLI_TCC 1
```

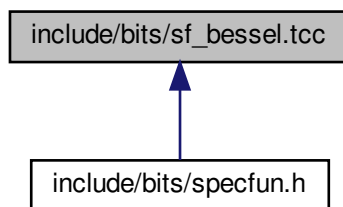
Definition at line 35 of file `sf_bernoulli.tcc`.

10.4 `include/bits/sf_bessel.tcc` File Reference

```
#include <complex>
#include <ext/math_const.h>
Include dependency graph for sf_bessel.tcc:
```



This graph shows which files directly or indirectly include this file:



Namespaces

- [std](#)
- [std::__detail](#)

Implementation-space details.

Macros

- `#define _GLIBCXX_BITS_SF_BESSEL_TCC 1`

Functions

- `template<typename _Tp >`
`_Tp std::__detail::__cyl_bessel_ij_series (_Tp __nu, _Tp __x, _Tp __sgn, unsigned int __max_iter)`
This routine returns the cylindrical Bessel functions of order ν : J_ν or I_ν by series expansion.
- `template<typename _Tp >`
`_Tp std::__detail::__cyl_bessel_j (_Tp __nu, _Tp __x)`
Return the Bessel function of order ν : $J_\nu(x)$.
- `template<typename _Tp >`
`_gnu_cxx::__cyl_bessel_t<_Tp, _Tp, _Tp> std::__detail::__cyl_bessel_jn (_Tp __nu, _Tp __x)`
Return the cylindrical Bessel functions and their derivatives of order ν by various means.
- `template<typename _Tp >`
`_gnu_cxx::__cyl_bessel_t<_Tp, _Tp, _Tp> std::__detail::__cyl_bessel_jn_asymp (_Tp __nu, _Tp __x)`
This routine computes the asymptotic cylindrical Bessel and Neumann functions of order ν : $J_\nu(x)$, $N_\nu(x)$. Use this for $x \gg nu^2 + 1$.
- `template<typename _Tp >`
`_gnu_cxx::__cyl_bessel_t<_Tp, _Tp, std::complex<_Tp>> std::__detail::__cyl_bessel_jn_neg_arg (_Tp __nu, _Tp __x)`
Return the cylindrical Bessel functions and their derivatives of order ν and argument $x < 0$.
- `template<typename _Tp >`
`_gnu_cxx::__cyl_bessel_t<_Tp, _Tp, _Tp> std::__detail::__cyl_bessel_jn_steel (_Tp __nu, _Tp __x)`
Compute the Bessel $J_\nu(x)$ and Neumann $N_\nu(x)$ functions and their first derivatives $J'_\nu(x)$ and $N'_\nu(x)$ respectively. These four functions are computed together for numerical stability.
- `template<typename _Tp >`
`std::complex<_Tp> std::__detail::__cyl_hankel_1 (_Tp __nu, _Tp __x)`
Return the cylindrical Hankel function of the first kind $H_\nu^{(1)}(x)$.
- `template<typename _Tp >`
`std::complex<_Tp> std::__detail::__cyl_hankel_2 (_Tp __nu, _Tp __x)`
Return the cylindrical Hankel function of the second kind $H_n^{(2)}u(x)$.
- `template<typename _Tp >`
`_Tp std::__detail::__cyl_neumann_n (_Tp __nu, _Tp __x)`
Return the Neumann function of order ν : $N_\nu(x)$.
- `template<typename _Tp >`
`_gnu_cxx::__gamma_temme_t<_Tp> std::__detail::__gamma_temme (_Tp __mu)`

Compute the gamma functions required by the Temme series expansions of $N_\nu(x)$ and $K_\nu(x)$.

$$\Gamma_1 = \frac{1}{2\mu} \left[\frac{1}{\Gamma(1-\mu)} - \frac{1}{\Gamma(1+\mu)} \right]$$

and

$$\Gamma_2 = \frac{1}{2} \left[\frac{1}{\Gamma(1-\mu)} + \frac{1}{\Gamma(1+\mu)} \right]$$

where $-1/2 \leq \mu \leq 1/2$ is $\mu = \nu - N$ and N is the nearest integer to ν . The values of $\Gamma(1+\mu)$ and $\Gamma(1-\mu)$ are returned as well.

- `template<typename _Tp >`
`_Tp std::__detail::__sph_bessel` (unsigned int __n, _Tp __x)
Return the spherical Bessel function $j_n(x)$ of order n and non-negative real argument x .
- `template<typename _Tp >`
`__gnu_cxx::__sph_bessel_t< unsigned int, _Tp, _Tp > std::__detail::__sph_bessel_jn` (unsigned int __n, _Tp __x)
Compute the spherical Bessel $j_n(x)$ and Neumann $n_n(x)$ functions and their first derivatives $j'_n(x)$ and $n'_n(x)$ respectively.
- `template<typename _Tp >`
`__gnu_cxx::__sph_bessel_t< unsigned int, _Tp, std::complex< _Tp > > std::__detail::__sph_bessel_jn_neg<_Tp>` (unsigned int __n, _Tp __x)
Compute the spherical Bessel $j_n(x)$ and Neumann $n_n(x)$ functions and their first derivatives $j'_n(x)$ and $n'_n(x)$ respectively.
- `template<typename _Tp >`
`std::complex< _Tp > std::__detail::__sph_hankel_1` (unsigned int __n, _Tp __x)
Return the spherical Hankel function of the first kind $h_n^{(1)}(x)$.
- `template<typename _Tp >`
`std::complex< _Tp > std::__detail::__sph_hankel_2` (unsigned int __n, _Tp __x)
Return the spherical Hankel function of the second kind $h_n^{(2)}(x)$.
- `template<typename _Tp >`
`_Tp std::__detail::__sph_neumann` (unsigned int __n, _Tp __x)
Return the spherical Neumann function $n_n(x)$ of order n and non-negative real argument x .

10.4.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include `<cmath>`.

10.4.2 Macro Definition Documentation

10.4.2.1 _GLIBCXX_BITS_SF_BESSEL_TCC

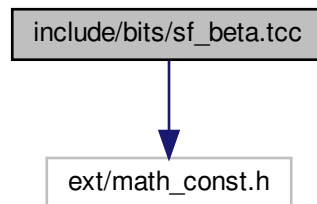
```
#define _GLIBCXX_BITS_SF_BESSEL_TCC 1
```

Definition at line 47 of file `sf_bessel.tcc`.

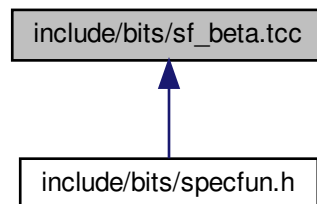
10.5 include/bits/sf_beta.tcc File Reference

```
#include <ext/math_const.h>
```

Include dependency graph for sf_beta.tcc:



This graph shows which files directly or indirectly include this file:



Namespaces

- [std](#)
- [std::__detail](#)

Implementation-space details.

Macros

- [#define _GLIBCXX_BITS_SF_BETA_TCC 1](#)

Functions

- `template<typename _Tp >`
`_Tp std::__detail::__beta (_Tp __a, _Tp __b)`
Return the beta function $B(a, b)$.
- `template<typename _Tp >`
`_Tp std::__detail::__beta_gamma (_Tp __a, _Tp __b)`
Return the beta function: $B(a, b)$.
- `template<typename _Tp >`
`_Tp std::__detail::__beta_inc (_Tp __a, _Tp __b, _Tp __x)`
- `template<typename _Tp >`
`_Tp std::__detail::__beta_lgamma (_Tp __a, _Tp __b)`
Return the beta function $B(a, b)$ using the log gamma functions.
- `template<typename _Tp >`
`_Tp std::__detail::__beta_product (_Tp __a, _Tp __b)`
Return the beta function $B(x, y)$ using the product form.
- `template<typename _Tp >`
`_Tp std::__detail::__ibeta_cont_frac (_Tp __a, _Tp __b, _Tp __x)`

10.5.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include `<cmath>`.

10.5.2 Macro Definition Documentation

10.5.2.1 `_GLIBCXX_BITS_SF_BETA_TCC`

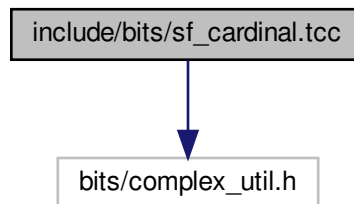
```
#define _GLIBCXX_BITS_SF_BETA_TCC 1
```

Definition at line 49 of file `sf_beta.tcc`.

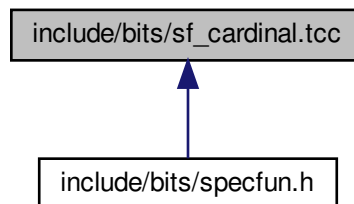
10.6 include/bits/sf_cardinal.tcc File Reference

```
#include <bits/complex_util.h>
```

Include dependency graph for sf_cardinal.tcc:



This graph shows which files directly or indirectly include this file:



Namespaces

- [std](#)
- [std::__detail](#)

Implementation-space details.

Macros

- [#define _GLIBCXX_BITS_SF_CARDINAL_TCC](#) 1

Functions

- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > std::__detail::__sinc (_Tp __x)`

Return the sinus cardinal function

$$\text{sinc}(x) = \frac{\sin(x)}{x}$$

- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > std::__detail::__sinc_pi (_Tp __x)`

Return the reperiodized sinus cardinal function

$$\text{sinc}_{\pi}(x) = \frac{\sin(\pi x)}{\pi x}$$

- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > std::__detail::__sinhc (_Tp __x)`

Return the hyperbolic sinus cardinal function

$$\text{sinhc}(x) = \frac{\sinh(x)}{x}$$

- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > std::__detail::__sinhc_pi (_Tp __x)`

Return the reperiodized hyperbolic sinus cardinal function

$$\text{sinhc}_{\pi}(x) = \frac{\sinh(\pi x)}{\pi x}$$

10.6.1 Macro Definition Documentation

10.6.1.1 _GLIBCXX_BITS_SF_CARDINAL_TCC

```
#define _GLIBCXX_BITS_SF_CARDINAL_TCC 1
```

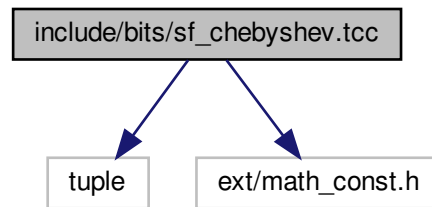
Definition at line 31 of file sf_cardinal.tcc.

10.7 include/bits/sf_chebyshev.tcc File Reference

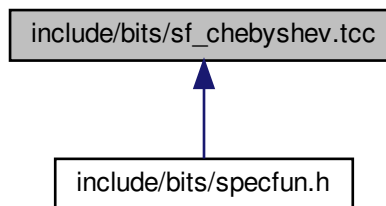
```
#include <tuple>
```

```
#include <ext/math_const.h>
```

Include dependency graph for sf_chebyshev.tcc:



This graph shows which files directly or indirectly include this file:



Namespaces

- [std](#)
- [std::__detail](#)

Implementation-space details.

Macros

- [#define _GLIBCXX_BITS_SF_CHEBYSHEV_TCC 1](#)

Functions

- `template<typename _Tp >`
`std::tuple< _Tp, _Tp, _Tp > std::__detail::__chebyshev_recur (unsigned int __n, _Tp __x, _Tp _C0, _Tp _C1)`
- `template<typename _Tp >`
`__gnu_cxx::__chebyshev_t_t< _Tp > std::__detail::__chebyshev_t (unsigned int __n, _Tp __x)`
- `template<typename _Tp >`
`__gnu_cxx::__chebyshev_u_t< _Tp > std::__detail::__chebyshev_u (unsigned int __n, _Tp __x)`
- `template<typename _Tp >`
`__gnu_cxx::__chebyshev_v_t< _Tp > std::__detail::__chebyshev_v (unsigned int __n, _Tp __x)`
- `template<typename _Tp >`
`__gnu_cxx::__chebyshev_w_t< _Tp > std::__detail::__chebyshev_w (unsigned int __n, _Tp __x)`

10.7.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include `<cmath>`.

10.7.2 Macro Definition Documentation

10.7.2.1 `_GLIBCXX_BITS_SF_CHEBYSHEV_TCC`

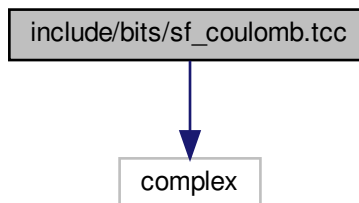
```
#define _GLIBCXX_BITS_SF_CHEBYSHEV_TCC 1
```

Definition at line 31 of file `sf_chebyshev.tcc`.

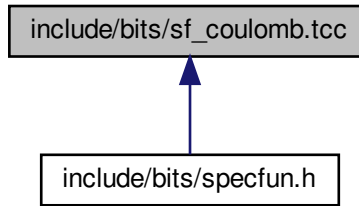
10.8 `include/bits/sf_coulomb.tcc` File Reference

```
#include <complex>
```

Include dependency graph for `sf_coulomb.tcc`:



This graph shows which files directly or indirectly include this file:



Namespaces

- [std](#)
- [std::__detail](#)

Implementation-space details.

Macros

- [#define _GLIBCXX_BITS_SF_COULOMB_TCC 1](#)

Functions

- [template<typename _Tp > std::pair< _Tp, _Tp > std::__detail::__coulomb_CF1 \(unsigned int __l, _Tp __eta, _Tp __x\)](#)
- [template<typename _Tp > std::complex< _Tp > std::__detail::__coulomb_CF2 \(unsigned int __l, _Tp __eta, _Tp __x\)](#)
- [template<typename _Tp > std::pair< _Tp, _Tp > std::__detail::__coulomb_f_recur \(unsigned int __l_min, unsigned int __k_max, _Tp __eta, _Tp __x, _Tp _F_l_max, _Tp _Fp_l_max\)](#)
- [template<typename _Tp > std::pair< _Tp, _Tp > std::__detail::__coulomb_g_recur \(unsigned int __l_min, unsigned int __k_max, _Tp __eta, _Tp __x, _Tp _G_l_min, _Tp _Gp_l_min\)](#)
- [template<typename _Tp > _Tp std::__detail::__coulomb_norm \(unsigned int __l, _Tp __eta\)](#)
- [template<typename _Tp > std::complex< _Tp > std::__detail::__hydrogen \(unsigned int __n, unsigned int __l, unsigned int __m, _Tp __Z, _Tp __r, _Tp __theta, _Tp __phi\)](#)

10.8.1 Detailed Description

This is an internal header file, included by other library headers. You should not attempt to use it directly.

10.8.2 Macro Definition Documentation

10.8.2.1 _GLIBCXX_BITS_SF_COULOMB_TCC

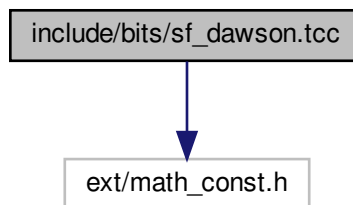
```
#define _GLIBCXX_BITS_SF_COULOMB_TCC 1
```

Definition at line 31 of file sf_coulomb.tcc.

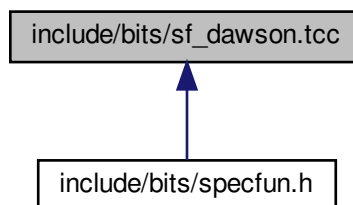
10.9 include/bits/sf_dawson.tcc File Reference

```
#include <ext/math_const.h>
```

Include dependency graph for sf_dawson.tcc:



This graph shows which files directly or indirectly include this file:



Namespaces

- [std](#)
- [std::__detail](#)

Implementation-space details.

Macros

- `#define _GLIBCXX_BITS_SF_DAWSON_TCC 1`

Functions

- `template<typename _Tp >`
`_Tp std::__detail::__dawson (_Tp __x)`
Return the Dawson integral, $F(x)$, for real argument x .
- `template<typename _Tp >`
`_Tp std::__detail::__dawson_cont_frac (_Tp __x)`
Compute the Dawson integral using a sampling theorem representation.
- `template<typename _Tp >`
`_Tp std::__detail::__dawson_series (_Tp __x)`
Compute the Dawson integral using the series expansion.

10.9.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include `<cmath>`.

10.9.2 Macro Definition Documentation

10.9.2.1 `_GLIBCXX_BITS_SF_DAWSON_TCC`

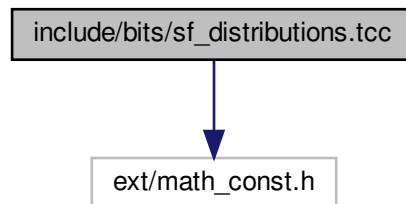
```
#define _GLIBCXX_BITS_SF_DAWSON_TCC 1
```

Definition at line 31 of file `sf_dawson.tcc`.

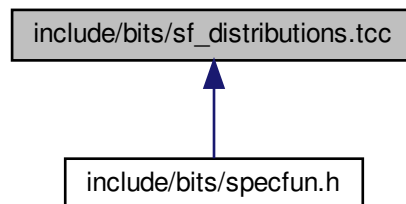
10.10 include/bits/sf_distributions.tcc File Reference

```
#include <ext/math_const.h>
```

Include dependency graph for sf_distributions.tcc:



This graph shows which files directly or indirectly include this file:



Namespaces

- [std](#)
- [std::__detail](#)

Implementation-space details.

Macros

- [#define _GLIBCXX_BITS_SF_DISTRIBUTIONS_TCC](#) 1

Functions

- `template<typename _Tp >`
`_Tp std::__detail::__beta_p (_Tp __a, _Tp __b, _Tp __x)`
- `template<typename _Tp >`
`_Tp std::__detail::__binomial_p (_Tp __p, unsigned int __n, unsigned int __k)`
Return the binomial cumulative distribution function.
- `template<typename _Tp >`
`_Tp std::__detail::__binomial_pdf (_Tp __p, unsigned int __n, unsigned int __k)`
Return the binomial probability mass function.
- `template<typename _Tp >`
`_Tp std::__detail::__binomial_q (_Tp __p, unsigned int __n, unsigned int __k)`
Return the complementary binomial cumulative distribution function.
- `template<typename _Tp >`
`_Tp std::__detail::__cauchy_p (_Tp __a, _Tp __b, _Tp __x)`
- `template<typename _Tp >`
`_Tp std::__detail::__chi_squared_pdf (_Tp __chi2, unsigned int __nu)`
Return the chi-squared propability function. This returns the probability that the observed chi-squared for a correct model is less than the value χ^2 .
- `template<typename _Tp >`
`_Tp std::__detail::__chi_squared_pdfc (_Tp __chi2, unsigned int __nu)`
Return the complementary chi-squared probability function. This returns the probability that the observed chi-squared for a correct model is greater than the value χ^2 .
- `template<typename _Tp >`
`_Tp std::__detail::__exponential_p (_Tp __lambda, _Tp __x)`
Return the exponential cumulative probability density function.
- `template<typename _Tp >`
`_Tp std::__detail::__exponential_pdf (_Tp __lambda, _Tp __x)`
Return the exponential probability density function.
- `template<typename _Tp >`
`_Tp std::__detail::__exponential_q (_Tp __lambda, _Tp __x)`
Return the complement of the exponential cumulative probability density function.
- `template<typename _Tp >`
`_Tp std::__detail::__fisher_f_p (_Tp __F, unsigned int __nu1, unsigned int __nu2)`
Return the F-distribution propability function. This returns the probability that the observed chi-square for a correct model exceeds the value χ^2 .
- `template<typename _Tp >`
`_Tp std::__detail::__fisher_f_pdf (_Tp __F, unsigned int __nu1, unsigned int __nu2)`
Return the F-distribution propability function. This returns the probability that the observed chi-square for a correct model exceeds the value χ^2 .
- `template<typename _Tp >`
`_Tp std::__detail::__fisher_f_q (_Tp __F, unsigned int __nu1, unsigned int __nu2)`
Return the F-distribution propability function. This returns the probability that the observed chi-square for a correct model exceeds the value χ^2 .
- `template<typename _Tp >`
`_Tp std::__detail::__gamma_p (_Tp __alpha, _Tp __beta, _Tp __x)`
Return the gamma cumulative propability distribution function.
- `template<typename _Tp >`
`_Tp std::__detail::__gamma_pdf (_Tp __alpha, _Tp __beta, _Tp __x)`
Return the gamma propability distribution function.

- `template<typename _Tp >`
`_Tp std::__detail::__gamma_q (_Tp __alpha, _Tp __beta, _Tp __x)`
Return the gamma complementary cumulative propability distribution function.
- `template<typename _Tp >`
`_Tp std::__detail::__kolmogorov_p (_Tp __a, _Tp __b, _Tp __x)`
- `template<typename _Tp >`
`_Tp std::__detail::__logistic_p (_Tp __a, _Tp __b, _Tp __x)`
Return the logistic cumulative distribution function.
- `template<typename _Tp >`
`_Tp std::__detail::__logistic_pdf (_Tp __a, _Tp __b, _Tp __x)`
Return the logistic probability density function.
- `template<typename _Tp >`
`_Tp std::__detail::__lognormal_p (_Tp __mu, _Tp __sigma, _Tp __x)`
Return the lognormal cumulative probability density function.
- `template<typename _Tp >`
`_Tp std::__detail::__lognormal_pdf (_Tp __nu, _Tp __sigma, _Tp __x)`
Return the lognormal probability density function.
- `template<typename _Tp >`
`_Tp std::__detail::__normal_p (_Tp __mu, _Tp __sigma, _Tp __x)`
Return the normal cumulative probability density function.
- `template<typename _Tp >`
`_Tp std::__detail::__normal_pdf (_Tp __mu, _Tp __sigma, _Tp __x)`
Return the normal probability density function.
- `template<typename _Tp >`
`_Tp std::__detail::__rice_pdf (_Tp __nu, _Tp __sigma, _Tp __x)`
Return the Rice probability density function.
- `template<typename _Tp >`
`_Tp std::__detail::__student_t_p (_Tp __t, unsigned int __nu)`
Return the Students T probability function.
- `template<typename _Tp >`
`_Tp std::__detail::__student_t_pdf (_Tp __t, unsigned int __nu)`
Return the Students T probability density.
- `template<typename _Tp >`
`_Tp std::__detail::__student_t_q (_Tp __t, unsigned int __nu)`
Return the complement of the Students T probability function.
- `template<typename _Tp >`
`_Tp std::__detail::__weibull_p (_Tp __a, _Tp __b, _Tp __x)`
Return the Weibull cumulative probability density function.
- `template<typename _Tp >`
`_Tp std::__detail::__weibull_pdf (_Tp __a, _Tp __b, _Tp __x)`
Return the Weibull probability density function.

10.10.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include `<cmath>`.

10.10.2 Macro Definition Documentation

10.10.2.1 _GLIBCXX_BITS_SF_DISTRIBUTIONS_TCC

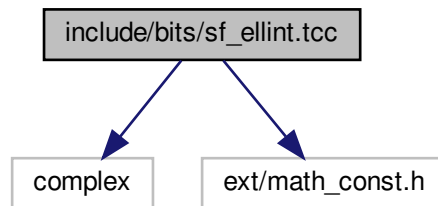
```
#define _GLIBCXX_BITS_SF_DISTRIBUTIONS_TCC 1
```

Definition at line 49 of file sf_distributions.tcc.

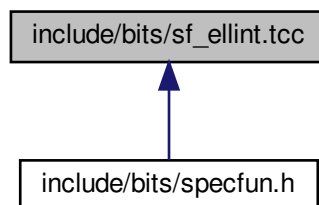
10.11 include/bits/sf_ellint.tcc File Reference

```
#include <complex>
#include <ext/math_const.h>
```

Include dependency graph for sf_ellint.tcc:



This graph shows which files directly or indirectly include this file:



Namespaces

- [std](#)
- [std::__detail](#)

Implementation-space details.

Macros

- `#define _GLIBCXX_BITS_SF_ELLINT_TCC 1`

Functions

- `template<typename _Tp >`
`_Tp std::__detail::__comp_ellint_1 (_Tp __k)`
Return the complete elliptic integral of the first kind $K(k)$ using the Carlson formulation.
- `template<typename _Tp >`
`_Tp std::__detail::__comp_ellint_2 (_Tp __k)`
Return the complete elliptic integral of the second kind $E(k)$ using the Carlson formulation.
- `template<typename _Tp >`
`_Tp std::__detail::__comp_ellint_3 (_Tp __k, _Tp __nu)`
Return the complete elliptic integral of the third kind $\Pi(k, \nu) = \Pi(k, \nu, \pi/2)$ using the Carlson formulation.
- `template<typename _Tp >`
`_Tp std::__detail::__comp_ellint_d (_Tp __k)`
- `template<typename _Tp >`
`_Tp std::__detail::__comp_ellint_rf (_Tp __x, _Tp __y)`
- `template<typename _Tp >`
`_Tp std::__detail::__comp_ellint_rg (_Tp __x, _Tp __y)`
- `template<typename _Tp >`
`_Tp std::__detail::__ellint_1 (_Tp __k, _Tp __phi)`
Return the incomplete elliptic integral of the first kind $F(k, \phi)$ using the Carlson formulation.
- `template<typename _Tp >`
`_Tp std::__detail::__ellint_2 (_Tp __k, _Tp __phi)`
Return the incomplete elliptic integral of the second kind $E(k, \phi)$ using the Carlson formulation.
- `template<typename _Tp >`
`_Tp std::__detail::__ellint_3 (_Tp __k, _Tp __nu, _Tp __phi)`
Return the incomplete elliptic integral of the third kind $\Pi(k, \nu, \phi)$ using the Carlson formulation.
- `template<typename _Tp >`
`_Tp std::__detail::__ellint_cel (_Tp __k_c, _Tp __p, _Tp __a, _Tp __b)`
- `template<typename _Tp >`
`_Tp std::__detail::__ellint_d (_Tp __k, _Tp __phi)`
- `template<typename _Tp >`
`_Tp std::__detail::__ellint_el1 (_Tp __x, _Tp __k_c)`
- `template<typename _Tp >`
`_Tp std::__detail::__ellint_el2 (_Tp __x, _Tp __k_c, _Tp __a, _Tp __b)`
- `template<typename _Tp >`
`_Tp std::__detail::__ellint_el3 (_Tp __x, _Tp __k_c, _Tp __p)`
- `template<typename _Tp >`
`_Tp std::__detail::__ellint_rc (_Tp __x, _Tp __y)`

Return the Carlson elliptic function $R_C(x, y) = R_F(x, y, y)$ where $R_F(x, y, z)$ is the Carlson elliptic function of the first kind.

- `template<typename _Tp >`
`_Tp std::__detail::__ellint_rd (_Tp __x, _Tp __y, _Tp __z)`

Return the Carlson elliptic function of the second kind $R_D(x, y, z) = R_J(x, y, z, z)$ where $R_J(x, y, z, p)$ is the Carlson elliptic function of the third kind.

- `template<typename _Tp >`
`_Tp std::__detail::__ellint_rf (_Tp __x, _Tp __y, _Tp __z)`

Return the Carlson elliptic function $R_F(x, y, z)$ of the first kind.

- `template<typename _Tp >`
`_Tp std::__detail::__ellint_rg (_Tp __x, _Tp __y, _Tp __z)`

Return the symmetric Carlson elliptic function of the second kind $R_G(x, y, z)$.

- `template<typename _Tp >`
`_Tp std::__detail::__ellint_rj (_Tp __x, _Tp __y, _Tp __z, _Tp __p)`

Return the Carlson elliptic function $R_J(x, y, z, p)$ of the third kind.

- `template<typename _Tp >`
`_Tp std::__detail::__heuman_lambda (_Tp __k, _Tp __phi)`

- `template<typename _Tp >`
`_Tp std::__detail::__jacobi_zeta (_Tp __k, _Tp __phi)`

10.11.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include `<cmath>`.

10.11.2 Macro Definition Documentation

10.11.2.1 _GLIBCXX_BITS_SF_ELLINT_TCC

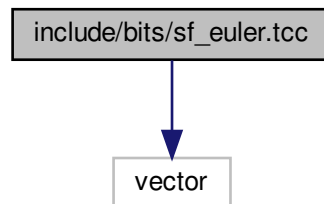
```
#define _GLIBCXX_BITS_SF_ELLINT_TCC 1
```

Definition at line 47 of file sf_ellint.tcc.

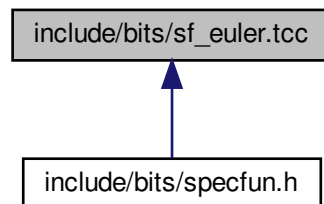
10.12 include/bits/sf_euler.tcc File Reference

```
#include <vector>
```

Include dependency graph for sf_euler.tcc:



This graph shows which files directly or indirectly include this file:



Namespaces

- [std](#)
- [std::__detail](#)

Implementation-space details.

Macros

- [#define _GLIBCXX_BITS_SF_EULER_TCC 1](#)

Functions

- `template<typename _Tp >`
`_Tp std::__detail::__euler` (unsigned int __n)
This returns Euler number E_n .
- `template<typename _Tp >`
`_Tp std::__detail::__euler` (unsigned int __n, _Tp __x)
- `template<typename _Tp >`
`_Tp std::__detail::__euler_series` (unsigned int __n)
- `template<typename _Tp >`
`_Tp std::__detail::__eulerian_1` (unsigned int __n, unsigned int __m)
- `template<typename _Tp >`
`std::vector< _Tp > std::__detail::__eulerian_1` (unsigned int __n)
- `template<typename _Tp >`
`_Tp std::__detail::__eulerian_1_recur` (unsigned int __n, unsigned int __m)
- `template<typename _Tp >`
`std::vector< _Tp > std::__detail::__eulerian_1_recur` (unsigned int __n)
- `template<typename _Tp >`
`_Tp std::__detail::__eulerian_2` (unsigned int __n, unsigned int __m)
- `template<typename _Tp >`
`std::vector< _Tp > std::__detail::__eulerian_2` (unsigned int __n)
- `template<typename _Tp >`
`_Tp std::__detail::__eulerian_2_recur` (unsigned int __n, unsigned int __m)
- `template<typename _Tp >`
`std::vector< _Tp > std::__detail::__eulerian_2_recur` (unsigned int __n)

10.12.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include `<cmath>`.

10.12.2 Macro Definition Documentation

10.12.2.1 _GLIBCXX_BITS_SF_EULER_TCC

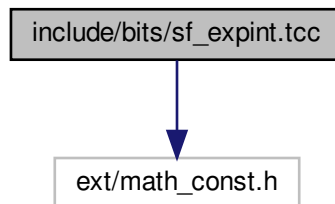
```
#define _GLIBCXX_BITS_SF_EULER_TCC 1
```

Definition at line 35 of file `sf_euler.tcc`.

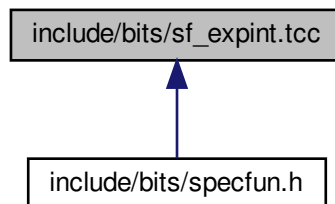
10.13 include/bits/sf_expint.tcc File Reference

```
#include <ext/math_const.h>
```

Include dependency graph for sf_expint.tcc:



This graph shows which files directly or indirectly include this file:



Namespaces

- [std](#)
- [std::__detail](#)

Implementation-space details.

Macros

- [#define _GLIBCXX_BITS_SF_EXPINT_TCC 1](#)

Functions

- `template<typename _Tp >`
`_Tp std::__detail::__coshint (const _Tp __x)`
Return the hyperbolic cosine integral $Chi(x)$.
- `template<typename _Tp >`
`_Tp std::__detail::__expint (unsigned int __n, _Tp __x)`
Return the exponential integral $E_n(x)$.
- `template<typename _Tp >`
`_Tp std::__detail::__expint (_Tp __x)`
Return the exponential integral $Ei(x)$.
- `template<typename _Tp >`
`_Tp std::__detail::__expint_E1 (_Tp __x)`
Return the exponential integral $E_1(x)$.
- `template<typename _Tp >`
`_Tp std::__detail::__expint_E1_asymp (_Tp __x)`
Return the exponential integral $E_1(x)$ by asymptotic expansion.
- `template<typename _Tp >`
`_Tp std::__detail::__expint_E1_series (_Tp __x)`
Return the exponential integral $E_1(x)$ by series summation. This should be good for $x < 1$.
- `template<typename _Tp >`
`_Tp std::__detail::__expint_Ei (_Tp __x)`
Return the exponential integral $Ei(x)$.
- `template<typename _Tp >`
`_Tp std::__detail::__expint_Ei_asymp (_Tp __x)`
Return the exponential integral $Ei(x)$ by asymptotic expansion.
- `template<typename _Tp >`
`_Tp std::__detail::__expint_Ei_series (_Tp __x)`
Return the exponential integral $Ei(x)$ by series summation.
- `template<typename _Tp >`
`_Tp std::__detail::__expint_En_asymp (unsigned int __n, _Tp __x)`
Return the exponential integral $E_n(x)$ for large argument.
- `template<typename _Tp >`
`_Tp std::__detail::__expint_En_cont_frac (unsigned int __n, _Tp __x)`
Return the exponential integral $E_n(x)$ by continued fractions.
- `template<typename _Tp >`
`_Tp std::__detail::__expint_En_large_n (unsigned int __n, _Tp __x)`
Return the exponential integral $E_n(x)$ for large order.
- `template<typename _Tp >`
`_Tp std::__detail::__expint_En_recursion (unsigned int __n, _Tp __x)`
Return the exponential integral $E_n(x)$ by recursion. Use upward recursion for $x < n$ and downward recursion (Miller's algorithm) otherwise.
- `template<typename _Tp >`
`_Tp std::__detail::__expint_En_series (unsigned int __n, _Tp __x)`
Return the exponential integral $E_n(x)$ by series summation.
- `template<typename _Tp >`
`_Tp std::__detail::__logint (const _Tp __x)`
Return the logarithmic integral $li(x)$.
- `template<typename _Tp >`
`_Tp std::__detail::__sinhint (const _Tp __x)`
Return the hyperbolic sine integral $Shi(x)$.

10.13.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include `<cmath>`.

10.13.2 Macro Definition Documentation

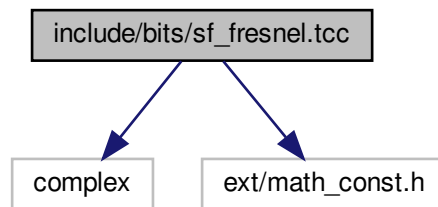
10.13.2.1 `_GLIBCXX_BITS_SF_EXPINT_TCC`

```
#define _GLIBCXX_BITS_SF_EXPINT_TCC 1
```

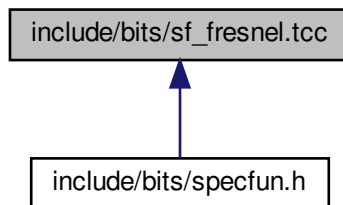
Definition at line 47 of file `sf_expint.tcc`.

10.14 `include/bits/sf_fresnel.tcc` File Reference

```
#include <complex>
#include <ext/math_const.h>
Include dependency graph for sf_fresnel.tcc:
```



This graph shows which files directly or indirectly include this file:



Namespaces

- [std](#)
- [std::__detail](#)

Implementation-space details.

Macros

- [#define _GLIBCXX_BITS_SF_FRESNEL_TCC 1](#)

Functions

- `template<typename _Tp >
std::complex< _Tp > std::__detail::__fresnel (const _Tp __x)`
Return the Fresnel cosine and sine integrals as a complex number $\text{Cf}(x) + i\text{Sf}(x)$.
- `template<typename _Tp >
void std::__detail::__fresnel_cont_frac (const _Tp __ax, _Tp &_Cf, _Tp &_Sf)`
This function computes the Fresnel cosine and sine integrals by continued fractions for positive argument.
- `template<typename _Tp >
void std::__detail::__fresnel_series (const _Tp __ax, _Tp &_Cf, _Tp &_Sf)`
This function returns the Fresnel cosine and sine integrals as a pair by series expansion for positive argument.

10.14.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include `<cmath>`.

10.14.2 Macro Definition Documentation

10.14.2.1 _GLIBCXX_BITS_SF_FRESNEL_TCC

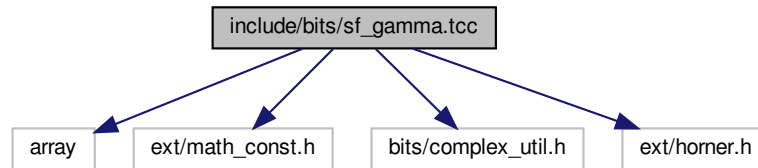
```
#define _GLIBCXX_BITS_SF_FRESNEL_TCC 1
```

Definition at line 31 of file `sf_fresnel.tcc`.

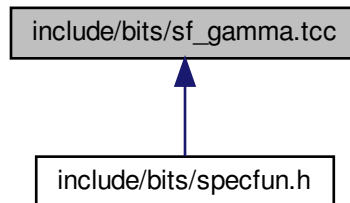
10.15 include/bits/sf_gamma.tcc File Reference

```
#include <array>
#include <ext/math_const.h>
#include <bits/complex_util.h>
#include <ext/horner.h>
```

Include dependency graph for sf_gamma.tcc:



This graph shows which files directly or indirectly include this file:



Classes

- struct [std::__detail::__gamma_lanczos_data< _Tp >](#)
- struct [std::__detail::__gamma_lanczos_data< double >](#)
- struct [std::__detail::__gamma_lanczos_data< float >](#)
- struct [std::__detail::__gamma_lanczos_data< long double >](#)
- struct [std::__detail::__gamma_spouge_data< _Tp >](#)
- struct [std::__detail::__gamma_spouge_data< double >](#)
- struct [std::__detail::__gamma_spouge_data< float >](#)
- struct [std::__detail::__gamma_spouge_data< long double >](#)
- struct [std::__detail::__Factorial_table< _Tp >](#)

Namespaces

- [std](#)
- [std::__detail](#)

Implementation-space details.

Macros

- `#define _GLIBCXX_BITS_SF_GAMMA_TCC 1`

Functions

- `template<typename _Tp >`
`_Tp std::__detail::__binomial (unsigned int __n, unsigned int __k)`

Return the binomial coefficient. The binomial coefficient is given by:

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

The binomial coefficients are generated by:

$$(1+t)^n = \sum_{k=0}^n \binom{n}{k} t^k$$

- `template<typename _Tp >`
`_Tp std::__detail::__binomial (_Tp __nu, unsigned int __k)`

Return the binomial coefficient for non-integral degree. The binomial coefficient is given by:

$$\binom{\nu}{k} = \frac{\Gamma(\nu+1)}{\Gamma(\nu-k+1)\Gamma(k+1)}$$

The binomial coefficients are generated by:

$$(1+t)^\nu = \sum_{k=0}^{\infty} \binom{\nu}{k} t^k$$

- `template<typename _Tp >`
`_Tp std::__detail::__digamma (unsigned int __n)`

Return the digamma function of integral argument. The digamma or $\psi(x)$ function is defined as the logarithmic derivative of the gamma function:

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

The digamma series for integral argument is given by:

$$\psi(n) = -\gamma_E + \sum_{k=1}^{n-1} \frac{1}{k}$$

The latter sum is called the harmonic number, H_n .

- `template<typename _Tp >`
`_Tp std::__detail::__digamma (_Tp __x)`

Return the digamma function. The digamma or $\psi(x)$ function is defined by

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

For negative argument the reflection formula is used:

$$\psi(x) = \psi(1-x) - \pi \cot(\pi x)$$

- `template<typename _Tp >`
`_Tp std::__detail::__digamma_asymp (_Tp __x)`

Return the digamma function for large argument. The digamma or $\psi(x)$ function is defined by

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

- `template<typename _Tp >`
`_Tp std::__detail::__digamma_series (_Tp __x)`

Return the digamma function by series expansion. The digamma or $\psi(x)$ function is defined by

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

- `template<typename _Tp >`
`_GLIBCXX14_CONSTEXPR _Tp std::__detail::__double_factorial (int __n)`

Return the double factorial of the integer n .

- `template<typename _Tp >`
`_GLIBCXX14_CONSTEXPR _Tp std::__detail::__factorial (unsigned int __n)`

Return the factorial of the integer n .

- `template<typename _Tp >`
`_Tp std::__detail::__falling_factorial (_Tp __a, int __n)`

Return the logarithm of the falling factorial function or the lower Pochhammer symbol for real argument a and integral order n . The falling factorial function is defined by

$$a^{\underline{n}} = \prod_{k=0}^{n-1} (a - k), (a)_0 = 1 = \Gamma(a+1)/\Gamma(a-n+1)$$

In particular, $n^{\underline{n}} = n!$.

- `template<typename _Tp >`
`_Tp std::__detail::__falling_factorial (_Tp __a, _Tp __nu)`

Return the logarithm of the falling factorial function or the lower Pochhammer symbol for real argument a and order ν . The falling factorial function is defined by

$$a^{\underline{\nu}} = \Gamma(a+1)/\Gamma(a-\nu+1)$$

- `template<typename _Tp >`
`_Tp std::__detail::__gamma (_Tp __a)`

Return the gamma function $\Gamma(a)$. The gamma function is defined by:

$$\Gamma(a) = \int_0^{\infty} e^{-t} t^{a-1} dt (a > 0)$$

- `template<typename _Tp >`
`std::pair<_Tp, _Tp> std::__detail::__gamma (_Tp __a, _Tp __x)`

Return the incomplete gamma functions.

- `template<typename _Tp >`
`std::pair< _Tp, _Tp > std::__detail::__gamma_cont_frac (_Tp __a, _Tp __x)`

Return the incomplete gamma function by continued fraction.

- `template<typename _Tp >`
`_Tp std::__detail::__gamma_p (_Tp __a, _Tp __x)`

Return the regularized lower incomplete gamma function. The regularized lower incomplete gamma function is defined by

$$P(a, x) = \frac{\gamma(a, x)}{\Gamma(a)}$$

where $\Gamma(a)$ is the gamma function and

$$\gamma(a, x) = \int_0^x e^{-t} t^{a-1} dt (a > 0)$$

is the lower incomplete gamma function.

- `template<typename _Tp >`
`_Tp std::__detail::__gamma_q (_Tp __a, _Tp __x)`

Return the regularized upper incomplete gamma function. The regularized upper incomplete gamma function is defined by

$$Q(a, x) = \frac{\Gamma(a, x)}{\Gamma(a)}$$

where $\Gamma(a)$ is the gamma function and

$$\Gamma(a, x) = \int_x^\infty e^{-t} t^{a-1} dt (a > 0)$$

is the upper incomplete gamma function.

- `template<typename _Tp >`
`_Tp std::__detail::__gamma_reciprocal (_Tp __a)`
- `template<typename _Tp >`
`_Tp std::__detail::__gamma_reciprocal_series (_Tp __a)`
- `template<typename _Tp >`
`std::pair< _Tp, _Tp > std::__detail::__gamma_series (_Tp __a, _Tp __x)`

Return the incomplete gamma function by series summation.

$$\gamma(a, x) = x^a e^{-x} \sum_{k=1}^{\infty} \frac{x^k}{(a)_k}$$

- `template<typename _Tp >`
`_Tp std::__detail::__harmonic_number (unsigned int __n)`

- `template<typename _Tp >`
`_Tp std::__detail::__hurwitz_zeta (_Tp __s, _Tp __a)`

Return the Hurwitz zeta function $\zeta(s, a)$ for all $s \neq 1$ and $a > -1$.

- `template<typename _Tp >`
`_GLIBCXX14_CONSTEXPR _Tp std::__detail::__lanczos_binet1p (_Tp __z)`

Return the Binet function $J(1 + z)$ by the Lanczos method. The Binet function is the log of the scaled Gamma function $\log(\Gamma^(z))$ defined by*

$$J(z) = \log(\Gamma^*(z)) = \log(\Gamma(z)) + z - \left(z - \frac{1}{2}\right) \log(z) - \log(2\pi)$$

or

$$\Gamma(z) = \sqrt{2\pi} z^{z-\frac{1}{2}} e^{-z} e^{J(z)}$$

where $\Gamma(z)$ is the gamma function.

- `template<typename _Tp >`
`_GLIBCXX14_CONSTEXPR _Tp std::__detail::__lanczos_log_gamma1p (_Tp __z)`

Return the logarithm of the gamma function $\log(\Gamma(1+z))$ by the Lanczos method.

- `template<typename _Tp >`
`_Tp std::__detail::__log_binomial (unsigned int __n, unsigned int __k)`

Return the logarithm of the binomial coefficient. The binomial coefficient is given by:

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

The binomial coefficients are generated by:

$$(1+t)^n = \sum_{k=0}^n \binom{n}{k} t^k$$

- `template<typename _Tp >`
`_Tp std::__detail::__log_binomial (_Tp __nu, unsigned int __k)`

Return the logarithm of the binomial coefficient for non-integral degree. The binomial coefficient is given by:

$$\binom{\nu}{k} = \frac{\Gamma(\nu+1)}{\Gamma(\nu-k+1)\Gamma(k+1)}$$

The binomial coefficients are generated by:

$$(1+t)^\nu = \sum_{k=0}^{\infty} \binom{\nu}{k} t^k$$

- `template<typename _Tp >`
`_Tp std::__detail::__log_binomial_sign (_Tp __nu, unsigned int __k)`

Return the sign of the exponentiated logarithm of the binomial coefficient for non-integral degree. The binomial coefficient is given by:

$$\binom{\nu}{k} = \frac{\Gamma(\nu+1)}{\Gamma(\nu-k+1)\Gamma(k+1)}$$

The binomial coefficients are generated by:

$$(1+t)^\nu = \sum_{k=0}^{\infty} \binom{\nu}{k} t^k$$

- `template<typename _Tp >`
`std::complex< _Tp > std::__detail::__log_binomial_sign (std::complex< _Tp > __nu, unsigned int __k)`

- `template<typename _Tp >`
`_GLIBCXX14_CONSTEXPR _Tp std::__detail::__log_double_factorial (_Tp __nu)`

- `template<typename _Tp >`
`_GLIBCXX14_CONSTEXPR _Tp std::__detail::__log_double_factorial (int __n)`

Return the logarithm of the double factorial of the integer n .

- `template<typename _Tp >`
`_GLIBCXX14_CONSTEXPR _Tp std::__detail::__log_factorial (unsigned int __n)`

Return the logarithm of the factorial of the integer n .

- `template<typename _Tp >`
`_Tp std::__detail::__log_falling_factorial (_Tp __a, _Tp __nu)`

Return the logarithm of the falling factorial function or the lower Pochhammer symbol. The lower Pochhammer symbol is defined by

$$a^{\underline{n}} = \Gamma(a+1)/\Gamma(a-\nu+1) = \prod_{k=0}^{n-1} (a-k), (a)_0 = 1$$

In particular, $n^{\underline{n}} = n!$. Thus this function returns

$$\ln[a^{\underline{n}}] = \ln[\Gamma(a+1)] - \ln[\Gamma(a-\nu+1)], \ln[a^{\underline{0}}] = 0$$

Many notations exist for this function:

$$(a)_\nu$$

,

$$\left\{ \begin{matrix} a \\ \nu \end{matrix} \right\}$$

, and others.

- `template<typename _Tp >`
`_Tp std::__detail::__log_gamma (_Tp __a)`
Return $\log(|\Gamma(a)|)$. This will return values even for $a < 0$. To recover the sign of $\Gamma(a)$ for any argument use `__log_gamma_sign`.
- `template<typename _Tp >`
`std::complex< _Tp > std::__detail::__log_gamma (std::complex< _Tp > __a)`
Return $\log(\Gamma(a))$ for complex argument.
- `template<typename _Tp >`
`_GLIBCXX14_CONSTEXPR _Tp std::__detail::__log_gamma_bernoulli (_Tp __x)`
Return $\log(\Gamma(x))$ by asymptotic expansion with Bernoulli number coefficients. This is like Sterling's approximation.
- `template<typename _Tp >`
`_Tp std::__detail::__log_gamma_sign (_Tp __a)`
Return the sign of $\Gamma(x)$. At nonpositive integers zero is returned indicating $\Gamma(x)$ is undefined.
- `template<typename _Tp >`
`std::complex< _Tp > std::__detail::__log_gamma_sign (std::complex< _Tp > __a)`
- `template<typename _Tp >`
`_Tp std::__detail::__log_rising_factorial (_Tp __a, _Tp __nu)`
Return the logarithm of the rising factorial function or the (upper) Pochhammer symbol. The Pochhammer symbol is defined for integer order by

$$a^{\overline{\nu}} = \Gamma(a + \nu) / \Gamma(a) = \prod_{k=0}^{\nu-1} (a + k), (a)_0 = 1$$

Thus this function returns

$$\ln[a^{\overline{\nu}}] = \ln[\Gamma(a + \nu)] - \ln[\Gamma(a)], \ln[(a)_0] = 0$$

Many notations exist for this function:

$$(a)_\nu$$

(especially in the literature of special functions),

$$\left[\begin{matrix} a \\ \nu \end{matrix} \right]$$

, and others.

- `template<typename _Tp >`
`_Tp std::__detail::__polygamma (unsigned int __m, _Tp __x)`
Return the polygamma function $\psi^{(m)}(x)$.
- `template<typename _Tp >`
`_Tp std::__detail::__rising_factorial (_Tp __a, int __n)`
Return the (upper) Pochhammer function or the rising factorial function. The Pochhammer symbol is defined by

$$a^{\overline{n}} = \Gamma(a + n) / \Gamma(a) = \prod_{k=0}^{n-1} (a + k), (a)_0 = 1$$

Many notations exist for this function:

$$(a)_\nu$$

, (especially in the literature of special functions),

$$\left[\begin{matrix} a \\ n \end{matrix} \right]$$

, and others.

- `template<typename _Tp >`
`_Tp std::__detail::__rising_factorial (_Tp __a, _Tp __nu)`

Return the rising factorial function or the (upper) Pochhammer function. The rising factorial function is defined by

$$a^{\overline{\nu}} = \Gamma(a + \nu) / \Gamma(\nu)$$

Many notations exist for this function:

$$(a)_{\nu}$$

, (especially in the literature of special functions),

$$\begin{bmatrix} a \\ n \end{bmatrix}$$

, and others.

- `template<typename _Tp >`
`_GLIBCXX14_CONSTEXPR _Tp std::__detail::__spouge_binet1p (_Tp __z)`

Return the Binet function $J(1 + z)$ by the Spouge method. The Binet function is the log of the scaled Gamma function $\log(\Gamma^(z))$ defined by*

$$J(z) = \log(\Gamma^*(z)) = \log(\Gamma(z)) + z - \left(z - \frac{1}{2}\right) \log(z) - \log(2\pi)$$

or

$$\Gamma(z) = \sqrt{2\pi} z^{z-\frac{1}{2}} e^{-z} e^{J(z)}$$

where $\Gamma(z)$ is the gamma function.

- `template<typename _Tp >`
`_GLIBCXX14_CONSTEXPR _Tp std::__detail::__spouge_log_gamma1p (_Tp __z)`

Return the logarithm of the gamma function $\log(\Gamma(1 + z))$ by the Spouge algorithm:

$$\Gamma(z + 1) = (z + a)^{z+1/2} e^{-z-a} \left[\sqrt{2\pi} + \sum_{k=1}^{\lceil a \rceil + 1} \frac{c_k(a)}{z + k} \right]$$

where

$$c_k(a) = \frac{(-1)^{k-1}}{(k-1)!} (a - k)^{k-1/2} e^{a-k}$$

and the error is bounded by

$$\epsilon(a) < a^{-1/2} (2\pi)^{-a-1/2}$$

- `template<typename _Tp >`
`_Tp std::__detail::__tgamma (_Tp __a, _Tp __x)`

Return the upper incomplete gamma function. The lower incomplete gamma function is defined by

$$\Gamma(a, x) = \int_x^{\infty} e^{-t} t^{a-1} dt (a > 0)$$

- `template<typename _Tp >`
`_Tp std::__detail::__tgamma_lower (_Tp __a, _Tp __x)`

Return the lower incomplete gamma function. The lower incomplete gamma function is defined by

$$\gamma(a, x) = \int_0^x e^{-t} t^{a-1} dt (a > 0)$$

Variables

- constexpr _Factorial_table< long double > [std::__detail::_S_double_factorial_table](#) [301]
- constexpr _Factorial_table< long double > [std::__detail::_S_factorial_table](#) [171]
- constexpr unsigned long long [std::__detail::_S_harmonic_denom](#) [_S_num_harmonic_number]
- constexpr unsigned long long [std::__detail::_S_harmonic_number](#) [_S_num_harmonic_number]
- constexpr _Factorial_table< long double > [std::__detail::_S_neg_double_factorial_table](#) [999]
- template<typename _Tp >
constexpr std::size_t [std::__detail::_S_num_double_factorials](#) = 0
- template<>
constexpr std::size_t [std::__detail::_S_num_double_factorials](#)< double > = 301
- template<>
constexpr std::size_t [std::__detail::_S_num_double_factorials](#)< float > = 57
- template<>
constexpr std::size_t [std::__detail::_S_num_double_factorials](#)< long double > = 301
- template<typename _Tp >
constexpr std::size_t [std::__detail::_S_num_factorials](#) = 0
- template<>
constexpr std::size_t [std::__detail::_S_num_factorials](#)< double > = 171
- template<>
constexpr std::size_t [std::__detail::_S_num_factorials](#)< float > = 35
- template<>
constexpr std::size_t [std::__detail::_S_num_factorials](#)< long double > = 171
- constexpr unsigned long long [std::__detail::_S_num_harmonic_number](#) = 29
- template<typename _Tp >
constexpr std::size_t [std::__detail::_S_num_neg_double_factorials](#) = 0
- template<>
constexpr std::size_t [std::__detail::_S_num_neg_double_factorials](#)< double > = 150
- template<>
constexpr std::size_t [std::__detail::_S_num_neg_double_factorials](#)< float > = 27
- template<>
constexpr std::size_t [std::__detail::_S_num_neg_double_factorials](#)< long double > = 999

10.15.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include `<cmath>`.

10.15.2 Macro Definition Documentation

10.15.2.1 _GLIBCXX_BITS_SF_GAMMA_TCC

```
#define _GLIBCXX_BITS_SF_GAMMA_TCC 1
```

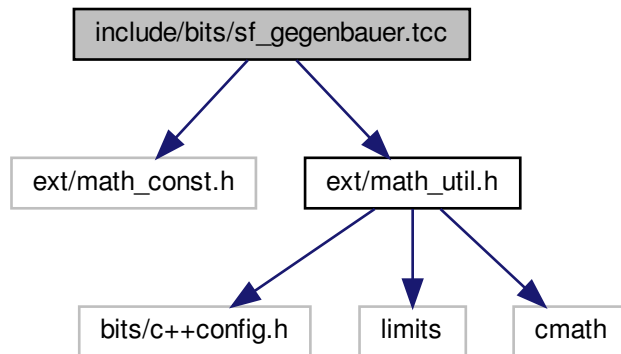
Definition at line 49 of file sf_gamma.tcc.

10.16 include/bits/sf_gegenbauer.tcc File Reference

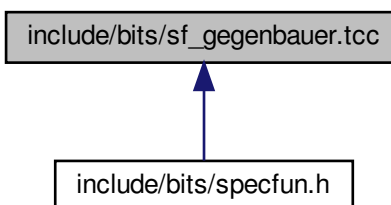
```
#include <ext/math_const.h>
```

```
#include <ext/math_util.h>
```

Include dependency graph for sf_gegenbauer.tcc:



This graph shows which files directly or indirectly include this file:



Namespaces

- [std](#)
- [std::__detail](#)

Implementation-space details.

Macros

- [#define _GLIBCXX_BITS_SF_GEGENBAUER_TCC](#) 1

Functions

- `template<typename _Tp > __gnu_cxx::__gegenbauer_t< _Tp > std::__detail::__gegenbauer_recur (unsigned int __n, _Tp __lambda, _Tp __x)`
- `template<typename _Tp > std::vector< __gnu_cxx::__quadrature_point_t< _Tp > > std::__detail::__gegenbauer_zeros (unsigned int __n, _Tp __lambda)`

10.16.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include `<cmath>`.

10.16.2 Macro Definition Documentation

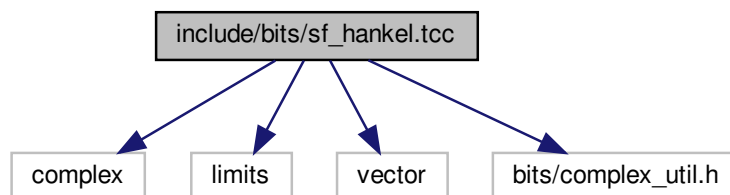
10.16.2.1 `_GLIBCXX_BITS_SF_GEGENBAUER_TCC`

```
#define _GLIBCXX_BITS_SF_GEGENBAUER_TCC 1
```

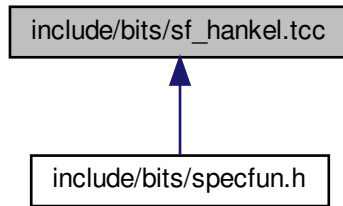
Definition at line 31 of file `sf_gegenbauer.tcc`.

10.17 include/bits/sf_hankel.tcc File Reference

```
#include <complex>
#include <limits>
#include <vector>
#include <bits/complex_util.h>
Include dependency graph for sf_hankel.tcc:
```



This graph shows which files directly or indirectly include this file:



Namespaces

- [std](#)
- [std::__detail](#)

Implementation-space details.

Macros

- [#define _GLIBCXX_BITS_SF_HANKEL_TCC 1](#)

Functions

- `template<typename _Tp >`
`void std::__detail::__airy_arg (std::complex< _Tp > __num2d3, std::complex< _Tp > __zeta, std::complex< _Tp > &__argp, std::complex< _Tp > &__argm)`
Compute the arguments for the Airy function evaluations carefully to prevent premature overflow. Note that the major work here is in `safe_div`. A faster, but less safe implementation can be obtained without use of `safe_div`.
- `template<typename _Tp >`
`std::complex< _Tp > std::__detail::__cyl_bessel (std::complex< _Tp > __nu, std::complex< _Tp > __z)`
Return the complex cylindrical Bessel function.
- `template<typename _Tp >`
`std::complex< _Tp > std::__detail::__cyl_hankel_1 (std::complex< _Tp > __nu, std::complex< _Tp > __z)`
Return the complex cylindrical Hankel function of the first kind.
- `template<typename _Tp >`
`std::complex< _Tp > std::__detail::__cyl_hankel_2 (std::complex< _Tp > __nu, std::complex< _Tp > __z)`
Return the complex cylindrical Hankel function of the second kind.
- `template<typename _Tp >`
`std::complex< _Tp > std::__detail::__cyl_neumann (std::complex< _Tp > __nu, std::complex< _Tp > __z)`
Return the complex cylindrical Neumann function.
- `template<typename _Tp >`
`void std::__detail::__debye_region (std::complex< _Tp > __alpha, int &__indexr, char &__aorb)`

- `template<typename _Tp >`
`__gnu_cxx::__cyl_hankel_t< std::complex< _Tp >, std::complex< _Tp >, std::complex< _Tp > > std::__detail::__hankel (std::complex< _Tp > __nu, std::complex< _Tp > __z)`
- `template<typename _Tp >`
`__gnu_cxx::__cyl_hankel_t< std::complex< _Tp >, std::complex< _Tp >, std::complex< _Tp > > std::__detail::__hankel_debye (std::complex< _Tp > __nu, std::complex< _Tp > __z, std::complex< _Tp > __alpha, int __indexr, char & __aorb, int & __morn)`
- `template<typename _Tp >`
`void std::__detail::__hankel_params (std::complex< _Tp > __nu, std::complex< _Tp > __zhat, std::complex< _Tp > & __p, std::complex< _Tp > & __p2, std::complex< _Tp > & __nup2, std::complex< _Tp > & __num2, std::complex< _Tp > & __num1d3, std::complex< _Tp > & __num2d3, std::complex< _Tp > & __num4d3, std::complex< _Tp > & __zeta, std::complex< _Tp > & __zetaphf, std::complex< _Tp > & __zetamhf, std::complex< _Tp > & __zetam3hf, std::complex< _Tp > & __zetrat)`
Compute parameters depending on z and nu that appear in the uniform asymptotic expansions of the Hankel functions and their derivatives, except the arguments to the Airy functions.
- `template<typename _Tp >`
`__gnu_cxx::__cyl_hankel_t< std::complex< _Tp >, std::complex< _Tp >, std::complex< _Tp > > std::__detail::__hankel_uniform (std::complex< _Tp > __nu, std::complex< _Tp > __z)`
This routine computes the uniform asymptotic approximations of the Hankel functions and their derivatives including a patch for the case when the order equals or nearly equals the argument. At such points, Olver's expressions have zero denominators (and numerators) resulting in numerical problems. This routine averages results from four surrounding points in the complex plane to obtain the result in such cases.
- `template<typename _Tp >`
`__gnu_cxx::__cyl_hankel_t< std::complex< _Tp >, std::complex< _Tp >, std::complex< _Tp > > std::__detail::__hankel_uniform_olver (std::complex< _Tp > __nu, std::complex< _Tp > __z)`
Compute approximate values for the Hankel functions of the first and second kinds using Olver's uniform asymptotic expansion to of order nu along with their derivatives.
- `template<typename _Tp >`
`void std::__detail::__hankel_uniform_outer (std::complex< _Tp > __nu, std::complex< _Tp > __z, _Tp __eps, std::complex< _Tp > & __zhat, std::complex< _Tp > & __1dnsq, std::complex< _Tp > & __num1d3, std::complex< _Tp > & __num2d3, std::complex< _Tp > & __p, std::complex< _Tp > & __p2, std::complex< _Tp > & __etm3h, std::complex< _Tp > & __etrat, std::complex< _Tp > & __Aip, std::complex< _Tp > & __o4dp, std::complex< _Tp > & __Aim, std::complex< _Tp > & __o4dm, std::complex< _Tp > & __od2p, std::complex< _Tp > & __od0dp, std::complex< _Tp > & __od2m, std::complex< _Tp > & __od0dm)`
Compute outer factors and associated functions of z and nu appearing in Olver's uniform asymptotic expansions of the Hankel functions of the first and second kinds and their derivatives. The various functions of z and nu returned by `hankel_uniform_outer` are available for use in computing further terms in the expansions.
- `template<typename _Tp >`
`void std::__detail::__hankel_uniform_sum (std::complex< _Tp > __p, std::complex< _Tp > __p2, std::complex< _Tp > __num2, std::complex< _Tp > __zetam3hf, std::complex< _Tp > __Aip, std::complex< _Tp > __o4dp, std::complex< _Tp > __Aim, std::complex< _Tp > __o4dm, std::complex< _Tp > __od2p, std::complex< _Tp > __od0dp, std::complex< _Tp > __od2m, std::complex< _Tp > __od0dm, _Tp __eps, std::complex< _Tp > & __H1sum, std::complex< _Tp > & __H1psum, std::complex< _Tp > & __H2sum, std::complex< _Tp > & __H2psum)`
Compute the sums in appropriate linear combinations appearing in Olver's uniform asymptotic expansions for the Hankel functions of the first and second kinds and their derivatives, using up to nterms (less than 5) to achieve relative error eps.
- `template<typename _Tp >`
`std::complex< _Tp > std::__detail::__sph_bessel (unsigned int __n, std::complex< _Tp > __z)`
Return the complex spherical Bessel function.
- `template<typename _Tp >`
`__gnu_cxx::__sph_hankel_t< unsigned int, std::complex< _Tp >, std::complex< _Tp > > std::__detail::__sph_hankel (unsigned int __n, std::complex< _Tp > __z)`
Helper to compute complex spherical Hankel functions and their derivatives.

- `template<typename _Tp >`
`std::complex< _Tp > std::__detail::__sph_hankel_1 (unsigned int __n, std::complex< _Tp > __z)`
Return the complex spherical Hankel function of the first kind.
- `template<typename _Tp >`
`std::complex< _Tp > std::__detail::__sph_hankel_2 (unsigned int __n, std::complex< _Tp > __z)`
Return the complex spherical Hankel function of the second kind.
- `template<typename _Tp >`
`std::complex< _Tp > std::__detail::__sph_neumann (unsigned int __n, std::complex< _Tp > __z)`
Return the complex spherical Neumann function.

10.17.1 Detailed Description

This is an internal header file, included by other library headers. You should not attempt to use it directly.

10.17.2 Macro Definition Documentation

10.17.2.1 `_GLIBCXX_BITS_SF_HANKEL_TCC`

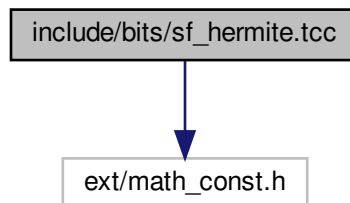
```
#define _GLIBCXX_BITS_SF_HANKEL_TCC 1
```

Definition at line 31 of file `sf_hankel.tcc`.

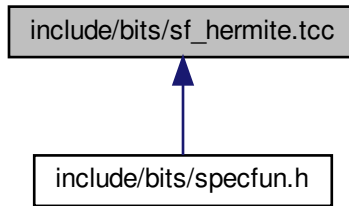
10.18 `include/bits/sf_hermite.tcc` File Reference

```
#include <ext/math_const.h>
```

Include dependency graph for `sf_hermite.tcc`:



This graph shows which files directly or indirectly include this file:



Namespaces

- [std](#)
- [std::__detail](#)

Implementation-space details.

Macros

- `#define _GLIBCXX_BITS_SF_HERMITE_TCC 1`

Functions

- `template<typename _Tp >`
`_Tp std::__detail::__hermite (unsigned int __n, _Tp __x)`
This routine returns the Hermite polynomial of order n : $H_n(x)$.
- `template<typename _Tp >`
`_Tp std::__detail::__hermite_asymp (unsigned int __n, _Tp __x)`
This routine returns the Hermite polynomial of large order n : $H_n(x)$. We assume here that $x \geq 0$.
- `template<typename _Tp >`
`__gnu_cxx::__hermite_t<_Tp> std::__detail::__hermite_recur (unsigned int __n, _Tp __x)`
This routine returns the Hermite polynomial of order n : $H_n(x)$ by recursion on n .
- `template<typename _Tp >`
`std::vector< __gnu_cxx::__quadrature_point_t<_Tp> > std::__detail::__hermite_zeros (unsigned int __n, _Tp __proto=_Tp{})`
- `template<typename _Tp >`
`__gnu_cxx::__hermite_he_t<_Tp> std::__detail::__prob_hermite_recur (unsigned int __n, _Tp __x)`
This routine returns the Probabilists Hermite polynomial of order n : $He_n(x)$ by recursion on n .

10.18.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include `<cmath>`.

10.18.2 Macro Definition Documentation

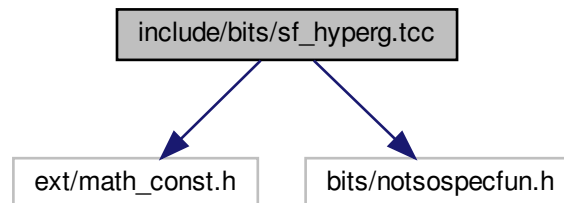
10.18.2.1 _GLIBCXX_BITS_SF_HERMITE_TCC

```
#define _GLIBCXX_BITS_SF_HERMITE_TCC 1
```

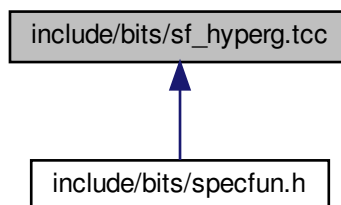
Definition at line 42 of file sf_hermite.tcc.

10.19 include/bits/sf_hyperg.tcc File Reference

```
#include <ext/math_const.h>  
#include <bits/notsospecfun.h>  
Include dependency graph for sf_hyperg.tcc:
```



This graph shows which files directly or indirectly include this file:



Namespaces

- [std](#)
- [std::__detail](#)

Implementation-space details.

Macros

- [#define _GLIBCXX_BITS_SF_HYPERG_TCC 1](#)

Functions

- [template<typename _Tp > _Tp std::__detail::__conf_hyperg \(_Tp __a, _Tp __c, _Tp __x\)](#)
Return the confluent hypergeometric function ${}_1F_1(a; c; x) = M(a, c, x)$.
- [template<typename _Tp > _Tp std::__detail::__conf_hyperg_lim \(_Tp __c, _Tp __x\)](#)
Return the confluent hypergeometric limit function ${}_0F_1(-; c; x)$.
- [template<typename _Tp > _Tp std::__detail::__conf_hyperg_lim_series \(_Tp __c, _Tp __x\)](#)
This routine returns the confluent hypergeometric limit function by series expansion.
- [template<typename _Tp > _Tp std::__detail::__conf_hyperg_luke \(_Tp __a, _Tp __c, _Tp __xin\)](#)
Return the hypergeometric function ${}_1F_1(a; c; x)$ by an iterative procedure described in Luke, Algorithms for the Computation of Mathematical Functions.
- [template<typename _Tp > _Tp std::__detail::__conf_hyperg_series \(_Tp __a, _Tp __c, _Tp __x\)](#)
This routine returns the confluent hypergeometric function by series expansion.
- [template<typename _Tp > _Tp std::__detail::__hyperg \(_Tp __a, _Tp __b, _Tp __c, _Tp __x\)](#)
Return the hypergeometric function ${}_2F_1(a, b; c; x)$.
- [template<typename _Tp > _Tp std::__detail::__hyperg_luke \(_Tp __a, _Tp __b, _Tp __c, _Tp __xin\)](#)
Return the hypergeometric function ${}_2F_1(a, b; c; x)$ by an iterative procedure described in Luke, Algorithms for the Computation of Mathematical Functions.
- [template<typename _Tp > _Tp std::__detail::__hyperg_recur \(int __m, _Tp __b, _Tp __c, _Tp __x\)](#)
Return the hypergeometric polynomial ${}_2F_1(-m, b; c; x)$ by Holm recursion.
- [template<typename _Tp > _Tp std::__detail::__hyperg_reflect \(_Tp __a, _Tp __b, _Tp __c, _Tp __x\)](#)
Return the hypergeometric function ${}_2F_1(a, b; c; x)$ by the reflection formulae in Abramowitz & Stegun formula 15.3.6 for $d = c - a - b$ not integral and formula 15.3.11 for $d = c - a - b$ integral. This assumes $a, b, c \neq$ negative integer.
- [template<typename _Tp > _Tp std::__detail::__hyperg_series \(_Tp __a, _Tp __b, _Tp __c, _Tp __x\)](#)
Return the hypergeometric function ${}_2F_1(a, b; c; x)$ by series expansion.
- [template<typename _Tp > _Tp std::__detail::__tricomi_u \(_Tp __a, _Tp __c, _Tp __x\)](#)

Return the Tricomi confluent hypergeometric function

$$U(a, c, x) = \frac{\Gamma(1-c)}{\Gamma(a-c+1)} {}_1F_1(a; c; x) + \frac{\Gamma(c-1)}{\Gamma(a)} x^{1-c} {}_1F_1(a-c+1; 2-c; x)$$

- `template<typename _Tp >`
`_Tp std::__detail::__tricomi_u_naive (_Tp __a, _Tp __c, _Tp __x)`

Return the Tricomi confluent hypergeometric function

$$U(a, c, x) = \frac{\Gamma(1-c)}{\Gamma(a-c+1)} {}_1F_1(a; c; x) + \frac{\Gamma(c-1)}{\Gamma(a)} x^{1-c} {}_1F_1(a-c+1; 2-c; x)$$

10.19.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include `<cmath>`.

10.19.2 Macro Definition Documentation

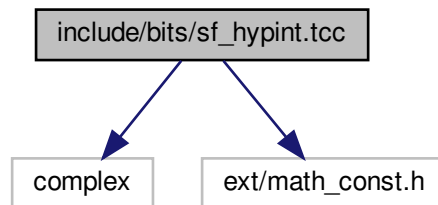
10.19.2.1 _GLIBCXX_BITS_SF_HYPERG_TCC

```
#define _GLIBCXX_BITS_SF_HYPERG_TCC 1
```

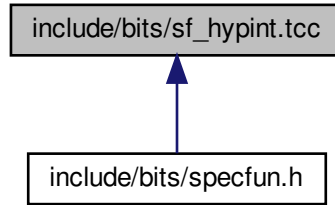
Definition at line 44 of file `sf_hyperg.tcc`.

10.20 include/bits/sf_hypint.tcc File Reference

```
#include <complex>
#include <ext/math_const.h>
Include dependency graph for sf_hypint.tcc:
```



This graph shows which files directly or indirectly include this file:



Namespaces

- [std](#)
- [std::__detail](#)

Implementation-space details.

Macros

- [#define _GLIBCXX_BITS_SF_HYPINT_TCC 1](#)

Functions

- `template<typename _Tp >`
`std::pair< _Tp, _Tp > std::__detail::__chshint (_Tp __x, _Tp &_Chi, _Tp &_Shi)`
This function returns the hyperbolic cosine $Chi(x)$ and hyperbolic sine $Shi(x)$ integrals as a pair.
- `template<typename _Tp >`
`void std::__detail::__chshint_cont_frac (_Tp __t, _Tp &_Chi, _Tp &_Shi)`
This function computes the hyperbolic cosine $Chi(x)$ and hyperbolic sine $Shi(x)$ integrals by continued fraction for positive argument.
- `template<typename _Tp >`
`void std::__detail::__chshint_series (_Tp __t, _Tp &_Chi, _Tp &_Shi)`
This function computes the hyperbolic cosine $Chi(x)$ and hyperbolic sine $Shi(x)$ integrals by series summation for positive argument.

10.20.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include `<cmath>`.

10.20.2 Macro Definition Documentation

10.20.2.1 _GLIBCXX_BITS_SF_HYPINT_TCC

```
#define _GLIBCXX_BITS_SF_HYPINT_TCC 1
```

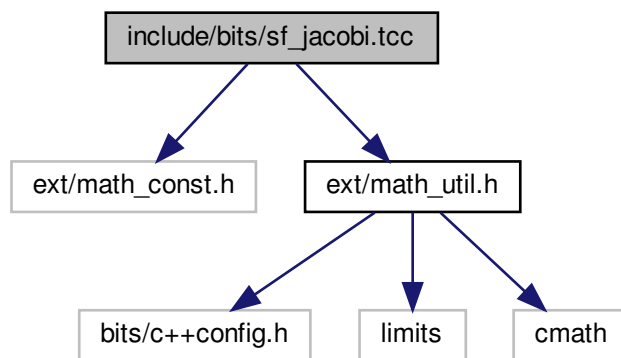
Definition at line 31 of file sf_hypint.tcc.

10.21 include/bits/sf_jacobi.tcc File Reference

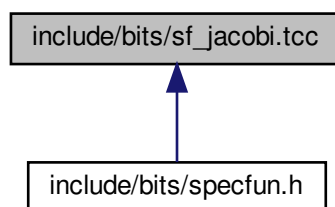
```
#include <ext/math_const.h>
```

```
#include <ext/math_util.h>
```

Include dependency graph for sf_jacobi.tcc:



This graph shows which files directly or indirectly include this file:



Namespaces

- [std](#)
- [std::__detail](#)

Implementation-space details.

Macros

- [#define _GLIBCXX_BITS_SF_JACOBI_TCC 1](#)

Functions

- [template<typename _Tp > \[__gnu_cxx::__jacobi_t\]\(#\)< _Tp > \[std::__detail::__jacobi_recur\]\(#\) \(unsigned int __n, _Tp __alpha1, _Tp __beta1, _Tp __x\)](#)
- [template<typename _Tp > \[std::vector\]\(#\)< \[__gnu_cxx::__quadrature_point_t\]\(#\)< _Tp > > \[std::__detail::__jacobi_zeros\]\(#\) \(unsigned int __n, _Tp __alpha1, _Tp __beta1\)](#)
- [template<typename _Tp > \[_Tp\]\(#\) \[std::__detail::__radial_jacobi\]\(#\) \(unsigned int __n, unsigned int __m, _Tp __rho\)](#)
- [template<typename _Tp > \[std::vector\]\(#\)< \[__gnu_cxx::__quadrature_point_t\]\(#\)< _Tp > > \[std::__detail::__radial_jacobi_zeros\]\(#\) \(unsigned int __n, unsigned int __m\)](#)
- [template<typename _Tp > \[__gnu_cxx::fp_promote_t\]\(#\)< _Tp > \[std::__detail::__zernike\]\(#\) \(unsigned int __n, int __m, _Tp __rho, _Tp __phi\)](#)

10.21.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include `<cmath>`.

10.21.2 Macro Definition Documentation

10.21.2.1 [_GLIBCXX_BITS_SF_JACOBI_TCC](#)

```
#define _GLIBCXX_BITS_SF_JACOBI_TCC 1
```

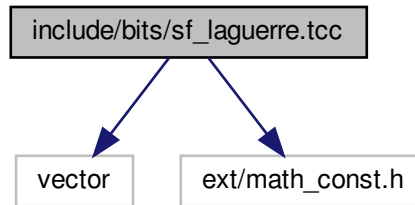
Definition at line 31 of file `sf_jacobi.tcc`.

10.22 include/bits/sf_laguerre.tcc File Reference

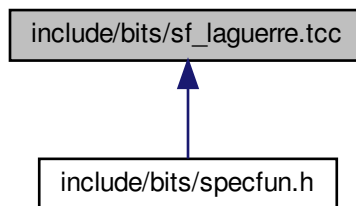
```
#include <vector>
```

```
#include <ext/math_const.h>
```

Include dependency graph for sf_laguerre.tcc:



This graph shows which files directly or indirectly include this file:



Namespaces

- [std](#)
- [std::__detail](#)

Implementation-space details.

Macros

- [#define _GLIBCXX_BITS_SF_LAGUERRE_TCC](#) 1

Functions

- `template<typename _Tpa, typename _Tp >`
`_Tp std::__detail::__assoc_laguerre` (unsigned int __n, _Tpa __alpha, _Tp __x)
This routine returns the associated Laguerre polynomial of degree n, order m: $L_n^{(m)}(x)$.
- `template<typename _Tpa, typename _Tp >`
`_Tp std::__detail::__laguerre` (unsigned int __n, _Tpa __alpha1, _Tp __x)
This routine returns the associated Laguerre polynomial of degree n, order α : $L_n^{(\alpha)}(x)$.
- `template<typename _Tp >`
`_Tp std::__detail::__laguerre` (unsigned int __n, _Tp __x)
This routine returns the Laguerre polynomial of degree n: $L_n(x)$.
- `template<typename _Tpa, typename _Tp >`
`_Tp std::__detail::__laguerre_hyperg` (unsigned int __n, _Tpa __alpha1, _Tp __x)
Evaluate the polynomial based on the confluent hypergeometric function in a safe way, with no restriction on the arguments.
- `template<typename _Tpa, typename _Tp >`
`_Tp std::__detail::__laguerre_large_n` (unsigned __n, _Tpa __alpha1, _Tp __x)
This routine returns the associated Laguerre polynomial of degree n, order $\alpha > -1$ for large n. Abramowitz & Stegun, 13.5.21.
- `template<typename _Tpa, typename _Tp >`
`__gnu_cxx::__laguerre_t<_Tpa, _Tp> std::__detail::__laguerre_recur` (unsigned int __n, _Tpa __alpha1, _Tp __x)
This routine returns the associated Laguerre polynomial of degree n, order α : $L_n^{(\alpha)}(x)$ by recursion.
- `template<typename _Tp >`
`std::vector<__gnu_cxx::__quadrature_point_t<_Tp>> std::__detail::__laguerre_zeros` (unsigned int __n, _Tp __alpha1)

10.22.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include `<cmath>`.

10.22.2 Macro Definition Documentation

10.22.2.1 _GLIBCXX_BITS_SF_LAGUERRE_TCC

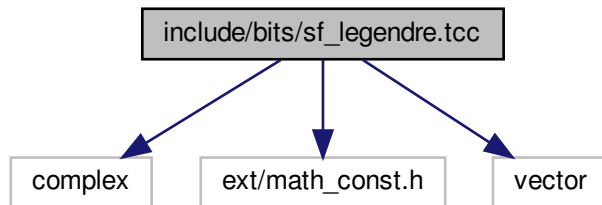
```
#define _GLIBCXX_BITS_SF_LAGUERRE_TCC 1
```

Definition at line 44 of file sf_laguerre.tcc.

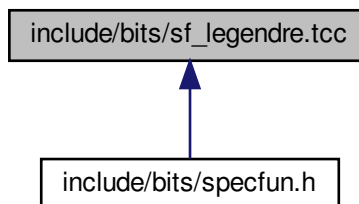
10.23 include/bits/sf_legendre.tcc File Reference

```
#include <complex>
#include <ext/math_const.h>
#include <vector>
```

Include dependency graph for sf_legendre.tcc:



This graph shows which files directly or indirectly include this file:



Namespaces

- [std](#)
- [std::__detail](#)

Implementation-space details.

Macros

- [#define _GLIBCXX_BITS_SF_LEGENDRE_TCC](#) 1

Functions

- `template<typename _Tp >`
`__gnu_cxx::__assoc_legendre_p_t< _Tp > std::__detail::__assoc_legendre_p` (unsigned int __l, unsigned int __m, _Tp __x, _Tp __phase=__Tp{+1})
Return the associated Legendre function by recursion on l and downward recursion on m.
- `template<typename _Tp >`
`__gnu_cxx::__assoc_legendre_q_t< _Tp > std::__detail::__assoc_legendre_q` (unsigned int __l, unsigned int __m, _Tp __x, _Tp __phase=__Tp{+1})
- `template<typename _Tp >`
`__gnu_cxx::__legendre_p_t< _Tp > std::__detail::__legendre_p` (unsigned int __l, _Tp __x)
Return the Legendre polynomial by upward recursion on degree l.
- `template<typename _Tp >`
`__gnu_cxx::__legendre_q_t< _Tp > std::__detail::__legendre_q` (unsigned int __l, _Tp __x)
Return the Legendre function of the second kind by upward recursion on degree l.
- `template<typename _Tp >`
`_Tp std::__detail::__legendre_q_series` (unsigned int __l, _Tp __x)
- `template<typename _Tp >`
`std::vector< __gnu_cxx::__quadrature_point_t< _Tp > > std::__detail::__legendre_zeros` (unsigned int __l, _Tp proto=_Tp{})
- `template<typename _Tp >`
`std::complex< _Tp > std::__detail::__sph_harmonic` (unsigned int __l, int __m, _Tp __theta, _Tp __phi)
Return the spherical harmonic function.
- `template<typename _Tp >`
`_Tp std::__detail::__sph_legendre` (unsigned int __l, unsigned int __m, _Tp __theta)
Return the spherical associated Legendre function.

10.23.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include `<cmath>`.

10.23.2 Macro Definition Documentation

10.23.2.1 _GLIBCXX_BITS_SF_LEGENDRE_TCC

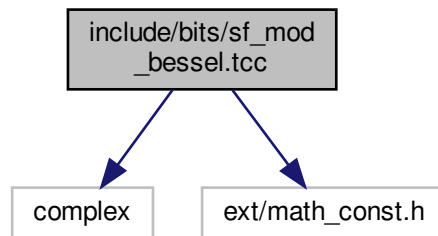
```
#define _GLIBCXX_BITS_SF_LEGENDRE_TCC 1
```

Definition at line 50 of file sf_legendre.tcc.

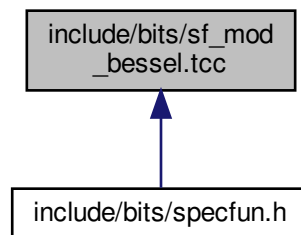
10.24 include/bits/sf_mod_bessel.tcc File Reference

```
#include <complex>
#include <ext/math_const.h>
```

Include dependency graph for sf_mod_bessel.tcc:



This graph shows which files directly or indirectly include this file:



Namespaces

- [std](#)
- [std::__detail](#)

Implementation-space details.

Macros

- [#define _GLIBCXX_BITS_SF_MOD_BESSEL_TCC](#) 1

Functions

- `template<typename _Tp >`
`__gnu_cxx::__airy_t< _Tp, _Tp > std::__detail::__airy (_Tp __z)`
Compute the Airy functions $Ai(x)$ and $Bi(x)$ and their first derivatives $Ai'(x)$ and $Bi'(x)$ respectively.
- `template<typename _Tp >`
`_Tp std::__detail::__cyl_bessel_i (_Tp __nu, _Tp __x)`
Return the regular modified Bessel function of order ν : $I_\nu(x)$.
- `template<typename _Tp >`
`__gnu_cxx::__cyl_mod_bessel_t< _Tp, _Tp, _Tp > std::__detail::__cyl_bessel_ik (_Tp __nu, _Tp __x)`
Return the modified cylindrical Bessel functions and their derivatives of order ν by various means.
- `template<typename _Tp >`
`__gnu_cxx::__cyl_mod_bessel_t< _Tp, _Tp, _Tp > std::__detail::__cyl_bessel_ik_asymp (_Tp __nu, _Tp __x)`
This routine computes the asymptotic modified cylindrical Bessel and functions of order ν : $I_\nu(x)$, $N_\nu(x)$. Use this for $x \gg nu^2 + 1$.
- `template<typename _Tp >`
`__gnu_cxx::__cyl_mod_bessel_t< _Tp, _Tp, _Tp > std::__detail::__cyl_bessel_ik_steel (_Tp __nu, _Tp __x)`
Compute the modified Bessel functions $I_\nu(x)$ and $K_\nu(x)$ and their first derivatives $I'_\nu(x)$ and $K'_\nu(x)$ respectively. These four functions are computed together for numerical stability.
- `template<typename _Tp >`
`_Tp std::__detail::__cyl_bessel_k (_Tp __nu, _Tp __x)`
Return the irregular modified Bessel function $K_\nu(x)$ of order ν .
- `template<typename _Tp >`
`__gnu_cxx::__fock_airy_t< _Tp, std::complex< _Tp > > std::__detail::__fock_airy (_Tp __x)`
Compute the Fock-type Airy functions $w_1(x)$ and $w_2(x)$ and their first derivatives $w'_1(x)$ and $w'_2(x)$ respectively.

$$w_1(x) = \sqrt{\pi}(Ai(x) + iBi(x))$$

$$w_2(x) = \sqrt{\pi}(Ai(x) - iBi(x))$$
- `template<typename _Tp >`
`__gnu_cxx::__sph_mod_bessel_t< unsigned int, _Tp, _Tp > std::__detail::__sph_bessel_ik (unsigned int __n, _Tp __x)`
Compute the spherical modified Bessel functions $i_n(x)$ and $k_n(x)$ and their first derivatives $i'_n(x)$ and $k'_n(x)$ respectively.

10.24.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include `<cmath>`.

10.24.2 Macro Definition Documentation

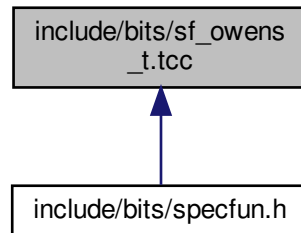
10.24.2.1 _GLIBCXX_BITS_SF_MOD_BESSEL_TCC

```
#define _GLIBCXX_BITS_SF_MOD_BESSEL_TCC 1
```

Definition at line 47 of file `sf_mod_bessel.tcc`.

10.25 include/bits/sf_owens_t.tcc File Reference

This graph shows which files directly or indirectly include this file:



Namespaces

- [std](#)
- [std::__detail](#)

Implementation-space details.

Macros

- [#define _GLIBCXX_BITS_SF_OWENS_T_TCC 1](#)

Functions

- [template<typename _Tp > _Tp std::__detail::__gauss \(_Tp __x\)](#)
- [template<typename _Tp > _Tp std::__detail::__owens_t \(_Tp __h, _Tp __a\)](#)
- [template<typename _Tp > _Tp std::__detail::__znorm1 \(_Tp __x\)](#)
- [template<typename _Tp > _Tp std::__detail::__znorm2 \(_Tp __x\)](#)

10.25.1 Detailed Description

This is an internal header file, included by other library headers. You should not attempt to use it directly.

10.25.2 Macro Definition Documentation

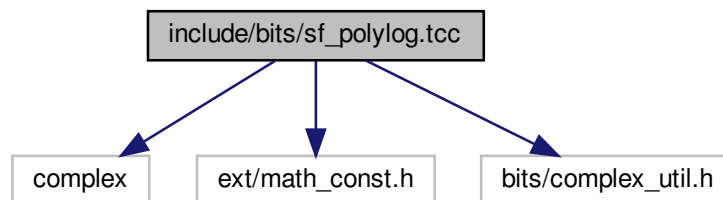
10.25.2.1 _GLIBCXX_BITS_SF_OWENS_T_TCC

```
#define _GLIBCXX_BITS_SF_OWENS_T_TCC 1
```

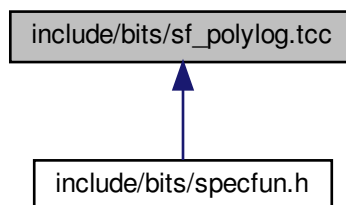
Definition at line 31 of file sf_owens_t.tcc.

10.26 include/bits/sf_polylog.tcc File Reference

```
#include <complex>
#include <ext/math_const.h>
#include <bits/complex_util.h>
Include dependency graph for sf_polylog.tcc:
```



This graph shows which files directly or indirectly include this file:



Classes

- class [std::__detail::__AsympTerminator<_Tp>](#)
- class [std::__detail::__Terminator<_Tp>](#)

Namespaces

- [std](#)
- [std::__detail](#)

Implementation-space details.

Macros

- [#define _GLIBCXX_BITS_SF_POLYLOG_TCC 1](#)

Functions

- [template<typename _Sp, typename _Tp> _Tp std::__detail::__bose_einstein\(_Sp __s, _Tp __x\)](#)
- [template<typename _Tp> std::complex<_Tp> std::__detail::__clamp_0_m2pi\(std::complex<_Tp> __z\)](#)
- [template<typename _Tp> std::complex<_Tp> std::__detail::__clamp_pi\(std::complex<_Tp> __z\)](#)
- [template<typename _Tp> std::complex<_Tp> std::__detail::__clausen\(unsigned int __m, std::complex<_Tp> __z\)](#)
- [template<typename _Tp> _Tp std::__detail::__clausen\(unsigned int __m, _Tp __x\)](#)
- [template<typename _Tp> _Tp std::__detail::__clausen_cl\(unsigned int __m, std::complex<_Tp> __z\)](#)
- [template<typename _Tp> _Tp std::__detail::__clausen_cl\(unsigned int __m, _Tp __x\)](#)
- [template<typename _Tp> _Tp std::__detail::__clausen_sl\(unsigned int __m, std::complex<_Tp> __z\)](#)
- [template<typename _Tp> _Tp std::__detail::__clausen_sl\(unsigned int __m, _Tp __x\)](#)
- [template<typename _Tp> _Tp std::__detail::__dirichlet_beta\(std::complex<_Tp> __s\)](#)
- [template<typename _Tp> _Tp std::__detail::__dirichlet_beta\(_Tp __s\)](#)
- [template<typename _Tp> std::complex<_Tp> std::__detail::__dirichlet_eta\(std::complex<_Tp> __s\)](#)
- [template<typename _Tp> _Tp std::__detail::__dirichlet_eta\(_Tp __s\)](#)
- [template<typename _Tp> _Tp std::__detail::__dirichlet_lambda\(_Tp __s\)](#)
- [template<typename _Sp, typename _Tp> _Tp std::__detail::__fermi_dirac\(_Sp __s, _Tp __x\)](#)
- [template<typename _Tp> std::complex<_Tp> std::__detail::__hurwitz_zeta_polylog\(_Tp __s, std::complex<_Tp> __a\)](#)

- `template<typename _Tp, typename _ArgType >`
`__gnu_cxx::fp_promote_t< std::complex< _Tp >, _ArgType > std::__detail::__periodic_zeta (_ArgType __z, ↵`
`_Tp __s)`
- `template<typename _Tp >`
`_Tp std::__detail::__polylog (_Tp __s, _Tp __x)`
- `template<typename _Tp >`
`std::complex< _Tp > std::__detail::__polylog (_Tp __s, std::complex< _Tp > __w)`
- `template<typename _Tp, typename _ArgType >`
`__gnu_cxx::fp_promote_t< std::complex< _Tp >, _ArgType > std::__detail::__polylog_exp (_Tp __s, _ArgType`
`__w)`
- `template<typename _Tp >`
`std::complex< _Tp > std::__detail::__polylog_exp_asymp (_Tp __s, std::complex< _Tp > __w)`
- `template<typename _Tp >`
`std::complex< _Tp > std::__detail::__polylog_exp_neg (_Tp __s, std::complex< _Tp > __w)`
- `template<typename _Tp >`
`std::complex< _Tp > std::__detail::__polylog_exp_neg (int __n, std::complex< _Tp > __w)`
- `template<typename _Tp >`
`std::complex< _Tp > std::__detail::__polylog_exp_neg_int (int __s, std::complex< _Tp > __w)`
- `template<typename _Tp >`
`std::complex< _Tp > std::__detail::__polylog_exp_neg_int (int __s, _Tp __w)`
- `template<typename _Tp >`
`std::complex< _Tp > std::__detail::__polylog_exp_neg_real (_Tp __s, std::complex< _Tp > __w)`
- `template<typename _Tp >`
`std::complex< _Tp > std::__detail::__polylog_exp_neg_real (_Tp __s, _Tp __w)`
- `template<typename _Tp >`
`std::complex< _Tp > std::__detail::__polylog_exp_pos (unsigned int __s, std::complex< _Tp > __w)`
- `template<typename _Tp >`
`std::complex< _Tp > std::__detail::__polylog_exp_pos (unsigned int __s, _Tp __w)`
- `template<typename _Tp >`
`std::complex< _Tp > std::__detail::__polylog_exp_pos (_Tp __s, std::complex< _Tp > __w)`
- `template<typename _Tp >`
`std::complex< _Tp > std::__detail::__polylog_exp_pos_int (unsigned int __s, std::complex< _Tp > __w)`
- `template<typename _Tp >`
`std::complex< _Tp > std::__detail::__polylog_exp_pos_int (unsigned int __s, _Tp __w)`
- `template<typename _Tp >`
`std::complex< _Tp > std::__detail::__polylog_exp_pos_real (_Tp __s, std::complex< _Tp > __w)`
- `template<typename _Tp >`
`std::complex< _Tp > std::__detail::__polylog_exp_pos_real (_Tp __s, _Tp __w)`
- `template<typename _PowTp, typename _Tp >`
`_Tp std::__detail::__polylog_exp_sum (_PowTp __s, _Tp __w)`

10.26.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include `<cmath>`.

10.26.2 Macro Definition Documentation

10.26.2.1 _GLIBCXX_BITS_SF_POLYLOG_TCC

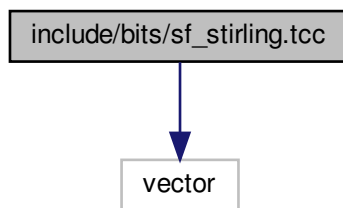
```
#define _GLIBCXX_BITS_SF_POLYLOG_TCC 1
```

Definition at line 41 of file sf_polylog.tcc.

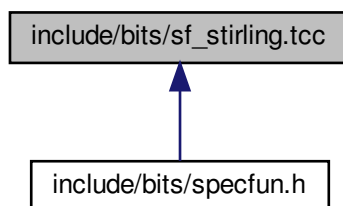
10.27 include/bits/sf_stirling.tcc File Reference

```
#include <vector>
```

Include dependency graph for sf_stirling.tcc:



This graph shows which files directly or indirectly include this file:



Namespaces

- [std](#)
- [std::__detail](#)

Implementation-space details.

Macros

- `#define _GLIBCXX_BITS_SF_STIRLING_TCC 1`

Functions

- `template<typename _Tp >`
`std::vector< _Tp > std::__detail::__bell (unsigned int __n)`
- `template<typename _Tp, typename _Up >`
`_Up std::__detail::__bell (unsigned int __n, _Up __x)`
- `template<typename _Tp >`
`std::vector< _Tp > std::__detail::__bell_series (unsigned int __n)`
- `template<typename _Tp >`
`_Tp std::__detail::__lah (unsigned int __n, unsigned int __k)`
- `template<typename _Tp >`
`std::vector< _Tp > std::__detail::__lah (unsigned int __n)`
- `template<typename _Tp >`
`_Tp std::__detail::__lah_recur (unsigned int __n, unsigned int __k)`
- `template<typename _Tp >`
`std::vector< _Tp > std::__detail::__lah_recur (unsigned int __n)`
- `template<typename _Tp >`
`_Tp std::__detail::__log_stirling_1 (unsigned int __n, unsigned int __m)`
- `template<typename _Tp >`
`_Tp std::__detail::__log_stirling_1_sign (unsigned int __n, unsigned int __m)`
- `template<typename _Tp >`
`_Tp std::__detail::__log_stirling_2 (unsigned int __n, unsigned int __m)`
- `template<typename _Tp >`
`_Tp std::__detail::__stirling_1 (unsigned int __n, unsigned int __m)`
- `template<typename _Tp >`
`std::vector< _Tp > std::__detail::__stirling_1 (unsigned int __n)`
- `template<typename _Tp >`
`_Tp std::__detail::__stirling_1_recur (unsigned int __n, unsigned int __m)`
- `template<typename _Tp >`
`std::vector< _Tp > std::__detail::__stirling_1_recur (unsigned int __n)`
- `template<typename _Tp >`
`_Tp std::__detail::__stirling_2 (unsigned int __n, unsigned int __m)`
- `template<typename _Tp >`
`std::vector< _Tp > std::__detail::__stirling_2 (unsigned int __n)`
- `template<typename _Tp >`
`_Tp std::__detail::__stirling_2_recur (unsigned int __n, unsigned int __m)`
- `template<typename _Tp >`
`std::vector< _Tp > std::__detail::__stirling_2_recur (unsigned int __n)`
- `template<typename _Tp >`
`_Tp std::__detail::__stirling_2_series (unsigned int __n, unsigned int __m)`

10.27.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include `<cmath>`.

10.27.2 Macro Definition Documentation

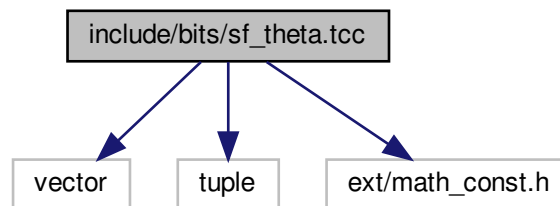
10.27.2.1 _GLIBCXX_BITS_SF_STIRLING_TCC

```
#define _GLIBCXX_BITS_SF_STIRLING_TCC 1
```

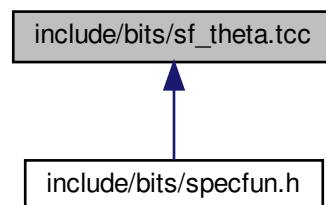
Definition at line 35 of file sf_stirling.tcc.

10.28 include/bits/sf_theta.tcc File Reference

```
#include <vector>
#include <tuple>
#include <ext/math_const.h>
Include dependency graph for sf_theta.tcc:
```



This graph shows which files directly or indirectly include this file:



Classes

- struct [std::__detail::__jacobi_lattice_t](#)< _Tp_Omega1, _Tp_Omega3 >
- struct [std::__detail::__jacobi_lattice_t](#)< _Tp_Omega1, _Tp_Omega3 >::__arg_t
- struct [std::__detail::__jacobi_lattice_t](#)< _Tp_Omega1, _Tp_Omega3 >::__tau_t
- struct [std::__detail::__jacobi_theta_0_t](#)< _Tp1, _Tp3 >
- struct [std::__detail::__weierstrass_invariants_t](#)< _Tp1, _Tp3 >
- struct [std::__detail::__weierstrass_roots_t](#)< _Tp1, _Tp3 >

Namespaces

- [std](#)
- [std::__detail](#)

Implementation-space details.

Macros

- `#define _GLIBCXX_BITS_SF_THETA_TCC 1`

Functions

- [template<typename _Tp > _Tp std::__detail::__ellnome](#) (_Tp __k)
- [template<typename _Tp > _Tp std::__detail::__ellnome_k](#) (_Tp __k)
- [template<typename _Tp > _Tp std::__detail::__ellnome_series](#) (_Tp __k)
- [template<typename _Tp > \[__gnu_cxx::__jacobi_ellint_t\]\(#\)< _Tp > std::__detail::__jacobi_ellint](#) (_Tp __k, _Tp __u)
- [template<typename _Tp > std::complex< _Tp > std::__detail::__jacobi_theta_1](#) (std::complex< _Tp > __q, std::complex< _Tp > __x)
- [template<typename _Tp > _Tp std::__detail::__jacobi_theta_1](#) (_Tp __q, const _Tp __x)
- [template<typename _Tp > _Tp std::__detail::__jacobi_theta_1_prod](#) (_Tp __q, _Tp __x)
- [template<typename _Tp > _Tp std::__detail::__jacobi_theta_1_sum](#) (_Tp __q, _Tp __x)
- [template<typename _Tp > std::complex< _Tp > std::__detail::__jacobi_theta_2](#) (std::complex< _Tp > __q, std::complex< _Tp > __x)
- [template<typename _Tp > _Tp std::__detail::__jacobi_theta_2](#) (_Tp __q, const _Tp __x)
- [template<typename _Tp > _Tp std::__detail::__jacobi_theta_2_prod](#) (_Tp __q, _Tp __x)
- [template<typename _Tp > _Tp std::__detail::__jacobi_theta_2_sum](#) (_Tp __q, _Tp __x)
- [template<typename _Tp > std::complex< _Tp > std::__detail::__jacobi_theta_3](#) (std::complex< _Tp > __q, std::complex< _Tp > __x)
- [template<typename _Tp > _Tp std::__detail::__jacobi_theta_3](#) (_Tp __q, const _Tp __x)

- `template<typename _Tp >`
`_Tp std::__detail::__jacobi_theta_3_prod (_Tp __q, _Tp __x)`
- `template<typename _Tp >`
`_Tp std::__detail::__jacobi_theta_3_sum (_Tp __q, _Tp __x)`
- `template<typename _Tp >`
`std::complex< _Tp > std::__detail::__jacobi_theta_4 (std::complex< _Tp > __q, std::complex< _Tp > __x)`
- `template<typename _Tp >`
`_Tp std::__detail::__jacobi_theta_4 (_Tp __q, const _Tp __x)`
- `template<typename _Tp >`
`_Tp std::__detail::__jacobi_theta_4_prod (_Tp __q, _Tp __x)`
- `template<typename _Tp >`
`_Tp std::__detail::__jacobi_theta_4_sum (_Tp __q, _Tp __x)`
- `template<typename _Tp >`
`_Tp std::__detail::__theta_1 (_Tp __nu, _Tp __x)`
- `template<typename _Tp >`
`_Tp std::__detail::__theta_2 (_Tp __nu, _Tp __x)`
- `template<typename _Tp >`
`_Tp std::__detail::__theta_2_asymp (_Tp __nu, _Tp __x)`
- `template<typename _Tp >`
`_Tp std::__detail::__theta_2_sum (_Tp __nu, _Tp __x)`
- `template<typename _Tp >`
`_Tp std::__detail::__theta_3 (_Tp __nu, _Tp __x)`
- `template<typename _Tp >`
`_Tp std::__detail::__theta_3_asymp (_Tp __nu, _Tp __x)`
- `template<typename _Tp >`
`_Tp std::__detail::__theta_3_sum (_Tp __nu, _Tp __x)`
- `template<typename _Tp >`
`_Tp std::__detail::__theta_4 (_Tp __nu, _Tp __x)`
- `template<typename _Tp >`
`_Tp std::__detail::__theta_c (_Tp __k, _Tp __x)`
- `template<typename _Tp >`
`_Tp std::__detail::__theta_d (_Tp __k, _Tp __x)`
- `template<typename _Tp >`
`_Tp std::__detail::__theta_n (_Tp __k, _Tp __x)`
- `template<typename _Tp >`
`_Tp std::__detail::__theta_s (_Tp __k, _Tp __x)`

10.28.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include `<cmath>`.

10.28.2 Macro Definition Documentation

10.28.2.1 _GLIBCXX_BITS_SF_THETA_TCC

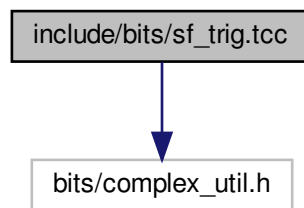
```
#define _GLIBCXX_BITS_SF_THETA_TCC 1
```

Definition at line 31 of file sf_theta.tcc.

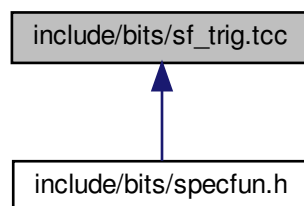
10.29 include/bits/sf_trig.tcc File Reference

```
#include <bits/complex_util.h>
```

Include dependency graph for sf_trig.tcc:



This graph shows which files directly or indirectly include this file:



Namespaces

- [std](#)
- [std::__detail](#)

Implementation-space details.

Macros

- `#define _GLIBCXX_BITS_SF_TRIG_TCC 1`

Functions

- `template<typename _Tp >`
`_Tp std::__detail::__cos_pi (_Tp __x)`
- `template<typename _Tp >`
`std::complex< _Tp > std::__detail::__cos_pi (std::complex< _Tp > __z)`
- `template<typename _Tp >`
`_Tp std::__detail::__cosh_pi (_Tp __x)`
- `template<typename _Tp >`
`std::complex< _Tp > std::__detail::__cosh_pi (std::complex< _Tp > __z)`
- `template<typename _Tp >`
`std::complex< _Tp > std::__detail::__polar_pi (_Tp __rho, _Tp __phi_pi)`
- `template<typename _Tp >`
`std::complex< _Tp > std::__detail::__polar_pi (_Tp __rho, const std::complex< _Tp > &__phi_pi)`
- `template<typename _Tp >`
`_Tp std::__detail::__sin_pi (_Tp __x)`
- `template<typename _Tp >`
`std::complex< _Tp > std::__detail::__sin_pi (std::complex< _Tp > __z)`
- `template<typename _Tp >`
`__gnu_cxx::__sincos_t< _Tp > std::__detail::__sincos (_Tp __x)`
- `template<>`
`__gnu_cxx::__sincos_t< float > std::__detail::__sincos (float __x)`
- `template<>`
`__gnu_cxx::__sincos_t< double > std::__detail::__sincos (double __x)`
- `template<>`
`__gnu_cxx::__sincos_t< long double > std::__detail::__sincos (long double __x)`
- `template<typename _Tp >`
`__gnu_cxx::__sincos_t< _Tp > std::__detail::__sincos_pi (_Tp __x)`
- `template<typename _Tp >`
`_Tp std::__detail::__sinh_pi (_Tp __x)`
- `template<typename _Tp >`
`std::complex< _Tp > std::__detail::__sinh_pi (std::complex< _Tp > __z)`
- `template<typename _Tp >`
`_Tp std::__detail::__tan_pi (_Tp __x)`
- `template<typename _Tp >`
`std::complex< _Tp > std::__detail::__tan_pi (std::complex< _Tp > __z)`
- `template<typename _Tp >`
`_Tp std::__detail::__tanh_pi (_Tp __x)`
- `template<typename _Tp >`
`std::complex< _Tp > std::__detail::__tanh_pi (std::complex< _Tp > __z)`

10.29.1 Detailed Description

This is an internal header file, included by other library headers. You should not attempt to use it directly.

10.29.2 Macro Definition Documentation

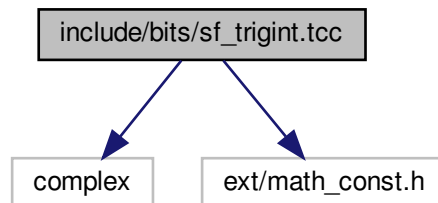
10.29.2.1 _GLIBCXX_BITS_SF_TRIG_TCC

```
#define _GLIBCXX_BITS_SF_TRIG_TCC 1
```

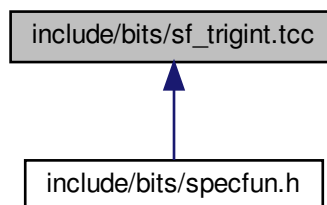
Definition at line 31 of file sf_trig.tcc.

10.30 include/bits/sf_trigint.tcc File Reference

```
#include <complex>
#include <ext/math_const.h>
Include dependency graph for sf_trigint.tcc:
```



This graph shows which files directly or indirectly include this file:



Namespaces

- [std](#)
- [std::__detail](#)

Implementation-space details.

Macros

- [#define _GLIBCXX_BITS_SF_TRIGINT_TCC 1](#)

Functions

- `template<typename _Tp >
std::pair<_Tp, _Tp> std::__detail::__sincosint (_Tp __x)`
This function returns the sine $Si(x)$ and cosine $Ci(x)$ integrals as a pair.
- `template<typename _Tp >
void std::__detail::__sincosint_asymp (_Tp __t, _Tp &_Si, _Tp &_Ci)`
This function computes the sine $Si(x)$ and cosine $Ci(x)$ integrals by asymptotic series summation for positive argument.
- `template<typename _Tp >
void std::__detail::__sincosint_cont_frac (_Tp __t, _Tp &_Si, _Tp &_Ci)`
This function computes the sine $Si(x)$ and cosine $Ci(x)$ integrals by continued fraction for positive argument.
- `template<typename _Tp >
void std::__detail::__sincosint_series (_Tp __t, _Tp &_Si, _Tp &_Ci)`
This function computes the sine $Si(x)$ and cosine $Ci(x)$ integrals by series summation for positive argument.

10.30.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include `<cmath>`.

10.30.2 Macro Definition Documentation

10.30.2.1 _GLIBCXX_BITS_SF_TRIGINT_TCC

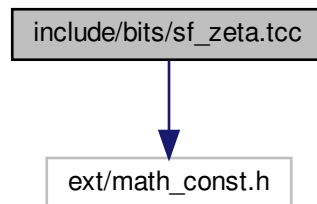
```
#define _GLIBCXX_BITS_SF_TRIGINT_TCC 1
```

Definition at line 31 of file `sf_trigint.tcc`.

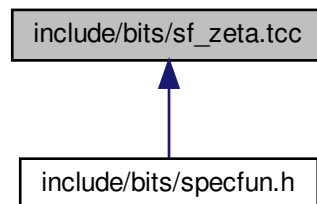
10.31 include/bits/sf_zeta.tcc File Reference

```
#include <ext/math_const.h>
```

Include dependency graph for sf_zeta.tcc:



This graph shows which files directly or indirectly include this file:



Namespaces

- [std](#)
- [std::__detail](#)

Implementation-space details.

Macros

- [#define _GLIBCXX_BITS_SF_ZETA_TCC 1](#)

Functions

- `template<typename _Tp >`
`_Tp std::__detail::__debye (unsigned int __n, _Tp __x)`
- `template<typename _Tp >`
`_Tp std::__detail::__dilog (_Tp __x)`
Compute the dilogarithm function $Li_2(x)$ by summation for $x \leq 1$.
- `template<typename _Tp >`
`_Tp std::__detail::__exp2 (_Tp __x)`
- `template<typename _Tp >`
`_Tp std::__detail::__hurwitz_zeta (_Tp __s, _Tp __a)`
Return the Hurwitz zeta function $\zeta(s, a)$ for all $s \neq 1$ and $a > -1$.
- `template<typename _Tp >`
`_Tp std::__detail::__hurwitz_zeta_euler_maclaurin (_Tp __s, _Tp __a)`
Return the Hurwitz zeta function $\zeta(s, a)$ for all $s \neq 1$ and $a > -1$.
- `template<typename _Tp >`
`_Tp std::__detail::__riemann_zeta (_Tp __s)`
Return the Riemann zeta function $\zeta(s)$.
- `template<typename _Tp >`
`_Tp std::__detail::__riemann_zeta_euler_maclaurin (_Tp __s)`
Evaluate the Riemann zeta function $\zeta(s)$ by an alternate series for $s > 0$.
- `template<typename _Tp >`
`_Tp std::__detail::__riemann_zeta_glob (_Tp __s)`
- `template<typename _Tp >`
`_Tp std::__detail::__riemann_zeta_laurent (_Tp __s)`
Compute the Riemann zeta function $\zeta(s)$ by Laurent expansion about $s = 1$.
- `template<typename _Tp >`
`_Tp std::__detail::__riemann_zeta_m_1 (_Tp __s)`
Return the Riemann zeta function $\zeta(s) - 1$.
- `template<typename _Tp >`
`_Tp std::__detail::__riemann_zeta_m_1_glob (_Tp __s)`
Evaluate the Riemann zeta function by series for all $s \neq 1$. Convergence is great until largish negative numbers. Then the convergence of the > 0 sum gets better.
- `template<typename _Tp >`
`_Tp std::__detail::__riemann_zeta_product (_Tp __s)`
Compute the Riemann zeta function $\zeta(s)$ using the product over prime factors.
- `template<typename _Tp >`
`_Tp std::__detail::__riemann_zeta_sum (_Tp __s)`
Compute the Riemann zeta function $\zeta(s)$ by summation for $s > 1$.

Variables

- `constexpr size_t std::__detail::__Num_Euler_Maclaurin_zeta = 100`
- `constexpr size_t std::__detail::__Num_Stieltjes = 21`
- `constexpr long double std::__detail::__S_Euler_Maclaurin_zeta [_Num_Euler_Maclaurin_zeta]`
- `constexpr size_t std::__detail::__S_num_zetam1 = 121`
- `constexpr long double std::__detail::__S_Stieltjes [_Num_Stieltjes]`
- `constexpr long double std::__detail::__S_zetam1 [_S_num_zetam1]`

10.31.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include `<cmath>`.

10.31.2 Macro Definition Documentation

10.31.2.1 _GLIBCXX_BITS_SF_ZETA_TCC

```
#define _GLIBCXX_BITS_SF_ZETA_TCC 1
```

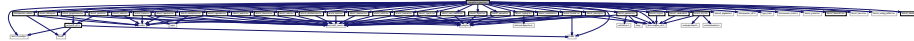
Definition at line 46 of file `sf_zeta.tcc`.

10.32 include/bits/specfun.h File Reference

```
#include <bits/c++config.h>
#include <limits>
#include <bits/stl_algobase.h>
#include <bits/specfun_state.h>
#include <bits/specfun_util.h>
#include <type_traits>
#include <bits/numeric_limits.h>
#include <bits/complex_util.h>
#include <bits/sf_prime.tcc>
#include <bits/sf_trig.tcc>
#include <bits/sf_bernoulli.tcc>
#include <bits/sf_gamma.tcc>
#include <bits/sf_euler.tcc>
#include <bits/sf_stirling.tcc>
#include <bits/sf_bessel.tcc>
#include <bits/sf_beta.tcc>
#include <bits/sf_cardinal.tcc>
#include <bits/sf_chebyshev.tcc>
#include <bits/sf_coulomb.tcc>
#include <bits/sf_dawson.tcc>
#include <bits/sf_ellint.tcc>
#include <bits/sf_expint.tcc>
#include <bits/sf_fresnel.tcc>
#include <bits/sf_gegenbauer.tcc>
#include <bits/sf_hyperg.tcc>
#include <bits/sf_hypint.tcc>
#include <bits/sf_jacobi.tcc>
#include <bits/sf_laguerre.tcc>
#include <bits/sf_legendre.tcc>
#include <bits/sf_lerch.tcc>
```

```
#include <bits/sf_mittag_leffler.tcc>
#include <bits/sf_mod_bessel.tcc>
#include <bits/sf_hermite.tcc>
#include <bits/sf_theta.tcc>
#include <bits/sf_trigint.tcc>
#include <bits/sf_zeta.tcc>
#include <bits/sf_owens_t.tcc>
#include <bits/sf_polylog.tcc>
#include <bits/sf_airy.tcc>
#include <bits/sf_hankel.tcc>
#include <bits/sf_distributions.tcc>
```

Include dependency graph for specfun.h:



Namespaces

- [__gnu_cxx](#)
- [std](#)

Macros

- [#define __cpp_lib_math_special_functions 201603L](#)
- [#define __STDCPP_MATH_SPEC_FUNCS__ 201003L](#)

Functions

- [template<typename _Tp > __gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::airy_ai \(_Tp __x\)](#)
- [template<typename _Tp > std::complex< __gnu_cxx::fp_promote_t< _Tp > > __gnu_cxx::airy_ai \(std::complex< _Tp > __x\)](#)
- [float __gnu_cxx::airy_aif \(float __x\)](#)
- [long double __gnu_cxx::airy_ail \(long double __x\)](#)
- [template<typename _Tp > __gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::airy_bi \(_Tp __x\)](#)
- [template<typename _Tp > std::complex< __gnu_cxx::fp_promote_t< _Tp > > __gnu_cxx::airy_bi \(std::complex< _Tp > __x\)](#)
- [float __gnu_cxx::airy_bif \(float __x\)](#)
- [long double __gnu_cxx::airy_bil \(long double __x\)](#)
- [template<typename _Tp > __gnu_cxx::fp_promote_t< _Tp > std::assoc_laguerre \(unsigned int __n, unsigned int __m, _Tp __x\)](#)
- [template<typename _Talpha, typename _Tp > __gnu_cxx::fp_promote_t< _Talpha, _Tp > std::assoc_laguerre \(unsigned int __n, _Talpha __alpha1, _Tp __x\)](#)
- [float std::assoc_laguerref \(unsigned int __n, unsigned int __m, float __x\)](#)
- [long double std::assoc_laguerrel \(unsigned int __n, unsigned int __m, long double __x\)](#)
- [template<typename _Tp > __gnu_cxx::fp_promote_t< _Tp > std::assoc_legendre \(unsigned int __l, unsigned int __m, _Tp __x\)](#)

- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::assoc_legendre_q` (unsigned int __l, unsigned int __m, _Tp __x)
- `float __gnu_cxx::assoc_legendre_qf` (unsigned int __l, unsigned int __m, float __x)
- `long double __gnu_cxx::assoc_legendre_ql` (unsigned int __l, unsigned int __m, long double __x)
- `float std::assoc_legendref` (unsigned int __l, unsigned int __m, float __x)
- `long double std::assoc_legendrel` (unsigned int __l, unsigned int __m, long double __x)
- `template<typename _Tp >`
`std::vector< _Tp > __gnu_cxx::bell` (unsigned int __n)
- `template<typename _Tp, typename _Up >`
`_Up __gnu_cxx::bell` (unsigned int __n, _Up __x)
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::bernoulli` (unsigned int __n)
- `template<typename _Tp >`
`_Tp __gnu_cxx::bernoulli` (unsigned int __n, _Tp __x)
- `float __gnu_cxx::bernoullif` (unsigned int __n)
- `long double __gnu_cxx::bernoullil` (unsigned int __n)
- `template<typename _Tpa, typename _Tpb >`
`__gnu_cxx::fp_promote_t< _Tpa, _Tpb > std::beta` (_Tpa __a, _Tpb __b)
- `float std::betaf` (float __a, float __b)
- `long double std::betal` (long double __a, long double __b)
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::binomial` (unsigned int __n, unsigned int __k)

Return the binomial coefficient as a real number. The binomial coefficient is given by:

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

The binomial coefficients are generated by:

$$(1+t)^n = \sum_{k=0}^n \binom{n}{k} t^k$$

- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::binomial_p` (_Tp __p, unsigned int __n, unsigned int __k)
Return the binomial cumulative distribution function.
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::binomial_pdf` (_Tp __p, unsigned int __n, unsigned int __k)
Return the binomial probability mass function.
- `float __gnu_cxx::binomialf` (unsigned int __n, unsigned int __k)
- `long double __gnu_cxx::binomiall` (unsigned int __n, unsigned int __k)
- `template<typename _Tps, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tps, _Tp > __gnu_cxx::bose_einstein` (_Tps __s, _Tp __x)
- `float __gnu_cxx::bose_einsteinf` (float __s, float __x)
- `long double __gnu_cxx::bose_einsteinl` (long double __s, long double __x)
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::chebyshev_t` (unsigned int __n, _Tp __x)
- `float __gnu_cxx::chebyshev_tf` (unsigned int __n, float __x)
- `long double __gnu_cxx::chebyshev_tl` (unsigned int __n, long double __x)
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::chebyshev_u` (unsigned int __n, _Tp __x)
- `float __gnu_cxx::chebyshev_uf` (unsigned int __n, float __x)
- `long double __gnu_cxx::chebyshev_ul` (unsigned int __n, long double __x)

- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::chebyshev_v (unsigned int __n, _Tp __x)`
- `float __gnu_cxx::chebyshev_vf (unsigned int __n, float __x)`
- `long double __gnu_cxx::chebyshev_vl (unsigned int __n, long double __x)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::chebyshev_w (unsigned int __n, _Tp __x)`
- `float __gnu_cxx::chebyshev_wf (unsigned int __n, float __x)`
- `long double __gnu_cxx::chebyshev_wl (unsigned int __n, long double __x)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::clausen (unsigned int __m, _Tp __x)`
- `template<typename _Tp >`
`std::complex< __gnu_cxx::fp_promote_t< _Tp > > __gnu_cxx::clausen (unsigned int __m, std::complex< _Tp > __z)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::clausen_cl (unsigned int __m, _Tp __x)`
- `float __gnu_cxx::clausen_clf (unsigned int __m, float __x)`
- `long double __gnu_cxx::clausen_cll (unsigned int __m, long double __x)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::clausen_sl (unsigned int __m, _Tp __x)`
- `float __gnu_cxx::clausen_slf (unsigned int __m, float __x)`
- `long double __gnu_cxx::clausen_sll (unsigned int __m, long double __x)`
- `float __gnu_cxx::clausenf (unsigned int __m, float __x)`
- `std::complex< float > __gnu_cxx::clausenf (unsigned int __m, std::complex< float > __z)`
- `long double __gnu_cxx::clausenl (unsigned int __m, long double __x)`
- `std::complex< long double > __gnu_cxx::clausenl (unsigned int __m, std::complex< long double > __z)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > std::comp_ellint_1 (_Tp __k)`
- `float std::comp_ellint_1f (float __k)`
- `long double std::comp_ellint_1l (long double __k)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > std::comp_ellint_2 (_Tp __k)`
- `float std::comp_ellint_2f (float __k)`
- `long double std::comp_ellint_2l (long double __k)`
- `template<typename _Tp, typename _Tpn >`
`__gnu_cxx::fp_promote_t< _Tp, _Tpn > std::comp_ellint_3 (_Tp __k, _Tpn __nu)`
- `float std::comp_ellint_3f (float __k, float __nu)`
Return the complete elliptic integral of the third kind $\Pi(k, \nu)$ for float modulus k.
- `long double std::comp_ellint_3l (long double __k, long double __nu)`
Return the complete elliptic integral of the third kind $\Pi(k, \nu)$ for long double modulus k.
- `template<typename _Tk >`
`__gnu_cxx::fp_promote_t< _Tk > __gnu_cxx::comp_ellint_d (_Tk __k)`
- `float __gnu_cxx::comp_ellint_df (float __k)`
- `long double __gnu_cxx::comp_ellint_dl (long double __k)`
- `float __gnu_cxx::comp_ellint_rf (float __x, float __y)`
- `long double __gnu_cxx::comp_ellint_rf (long double __x, long double __y)`
- `template<typename _Tx, typename _Ty >`
`__gnu_cxx::fp_promote_t< _Tx, _Ty > __gnu_cxx::comp_ellint_rf (_Tx __x, _Ty __y)`
- `float __gnu_cxx::comp_ellint_rg (float __x, float __y)`
- `long double __gnu_cxx::comp_ellint_rg (long double __x, long double __y)`
- `template<typename _Tx, typename _Ty >`
`__gnu_cxx::fp_promote_t< _Tx, _Ty > __gnu_cxx::comp_ellint_rg (_Tx __x, _Ty __y)`

- `template<typename _Tpa, typename _Tpc, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tpa, _Tpc, _Tp > __gnu_cxx::conf_hyperg (_Tpa __a, _Tpc __c, _Tp __x)`
- `template<typename _Tpc, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tpc, _Tp > __gnu_cxx::conf_hyperg_lim (_Tpc __c, _Tp __x)`
- `float __gnu_cxx::conf_hyperg_limf (float __c, float __x)`
- `long double __gnu_cxx::conf_hyperg_liml (long double __c, long double __x)`
- `float __gnu_cxx::conf_hypergf (float __a, float __c, float __x)`
- `long double __gnu_cxx::conf_hypergl (long double __a, long double __c, long double __x)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::cos_pi (_Tp __x)`
- `float __gnu_cxx::cos_pif (float __x)`
- `long double __gnu_cxx::cos_pil (long double __x)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::cosh_pi (_Tp __x)`
- `float __gnu_cxx::cosh_pif (float __x)`
- `long double __gnu_cxx::cosh_pil (long double __x)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::coshint (_Tp __x)`
- `float __gnu_cxx::coshintf (float __x)`
- `long double __gnu_cxx::coshintl (long double __x)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::cosint (_Tp __x)`
- `float __gnu_cxx::cosintf (float __x)`
- `long double __gnu_cxx::cosintl (long double __x)`
- `template<typename _Tpnu, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tpnu, _Tp > std::cyl_bessel_i (_Tpnu __nu, _Tp __x)`
- `float std::cyl_bessel_if (float __nu, float __x)`
- `long double std::cyl_bessel_il (long double __nu, long double __x)`
- `template<typename _Tpnu, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tpnu, _Tp > std::cyl_bessel_j (_Tpnu __nu, _Tp __x)`
- `float std::cyl_bessel_jf (float __nu, float __x)`
- `long double std::cyl_bessel_jl (long double __nu, long double __x)`
- `template<typename _Tpnu, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tpnu, _Tp > std::cyl_bessel_k (_Tpnu __nu, _Tp __x)`
- `float std::cyl_bessel_kf (float __nu, float __x)`
- `long double std::cyl_bessel_kl (long double __nu, long double __x)`
- `template<typename _Tpnu, typename _Tp >`
`std::complex< __gnu_cxx::fp_promote_t< _Tpnu, _Tp > > __gnu_cxx::cyl_hankel_1 (_Tpnu __nu, _Tp __z)`
- `template<typename _Tpnu, typename _Tp >`
`std::complex< __gnu_cxx::fp_promote_t< _Tpnu, _Tp > > __gnu_cxx::cyl_hankel_1 (std::complex< _Tpnu > __nu, std::complex< _Tp > __x)`
- `std::complex< float > __gnu_cxx::cyl_hankel_1f (float __nu, float __z)`
- `std::complex< float > __gnu_cxx::cyl_hankel_1f (std::complex< float > __nu, std::complex< float > __x)`
- `std::complex< long double > __gnu_cxx::cyl_hankel_1l (long double __nu, long double __z)`
- `std::complex< long double > __gnu_cxx::cyl_hankel_1l (std::complex< long double > __nu, std::complex< long double > __x)`
- `template<typename _Tpnu, typename _Tp >`
`std::complex< __gnu_cxx::fp_promote_t< _Tpnu, _Tp > > __gnu_cxx::cyl_hankel_2 (_Tpnu __nu, _Tp __z)`
- `template<typename _Tpnu, typename _Tp >`
`std::complex< __gnu_cxx::fp_promote_t< _Tpnu, _Tp > > __gnu_cxx::cyl_hankel_2 (std::complex< _Tpnu > __nu, std::complex< _Tp > __x)`
- `std::complex< float > __gnu_cxx::cyl_hankel_2f (float __nu, float __z)`

- `std::complex< float > __gnu_cxx::cyl_hankel_2f` (`std::complex< float > __nu`, `std::complex< float > __x`)
- `std::complex< long double > __gnu_cxx::cyl_hankel_2l` (`long double __nu`, `long double __z`)
- `std::complex< long double > __gnu_cxx::cyl_hankel_2l` (`std::complex< long double > __nu`, `std::complex< long double > __x`)
- `template<typename _Tpnu, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tpnu, _Tp > std::cyl_neumann` (`_Tpnu __nu`, `_Tp __x`)
- `float std::cyl_neumannf` (`float __nu`, `float __x`)
- `long double std::cyl_neumannl` (`long double __nu`, `long double __x`)
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::dawson` (`_Tp __x`)
- `float __gnu_cxx::dawsonf` (`float __x`)
- `long double __gnu_cxx::dawsonl` (`long double __x`)
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::debye` (`unsigned int __n`, `_Tp __x`)
- `float __gnu_cxx::debyef` (`unsigned int __n`, `float __x`)
- `long double __gnu_cxx::debyel` (`unsigned int __n`, `long double __x`)
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::digamma` (`_Tp __x`)
- `float __gnu_cxx::digammaf` (`float __x`)
- `long double __gnu_cxx::digammal` (`long double __x`)
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::dilog` (`_Tp __x`)
- `float __gnu_cxx::dilogf` (`float __x`)
- `long double __gnu_cxx::dilogl` (`long double __x`)
- `template<typename _Tp >`
`_Tp __gnu_cxx::dirichlet_beta` (`_Tp __s`)
- `float __gnu_cxx::dirichlet_betaf` (`float __s`)
- `long double __gnu_cxx::dirichlet_betall` (`long double __s`)
- `template<typename _Tp >`
`_Tp __gnu_cxx::dirichlet_eta` (`_Tp __s`)
- `float __gnu_cxx::dirichlet_etaf` (`float __s`)
- `long double __gnu_cxx::dirichlet_etaall` (`long double __s`)
- `template<typename _Tp >`
`_Tp __gnu_cxx::dirichlet_lambda` (`_Tp __s`)
- `float __gnu_cxx::dirichlet_lambdaf` (`float __s`)
- `long double __gnu_cxx::dirichlet_lambdaall` (`long double __s`)
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::double_factorial` (`int __n`)

Return the double factorial $n!!$ of the argument as a real number.

$$n!! = n(n-2)\dots(2), 0!! = 1$$

for even n and

$$n!! = n(n-2)\dots(1), (-1)!! = 1$$

for odd n .

- `float __gnu_cxx::double_factorialf` (`int __n`)
- `long double __gnu_cxx::double_factoriall` (`int __n`)
- `template<typename _Tp, typename _Tpp >`
`__gnu_cxx::fp_promote_t< _Tp, _Tpp > std::ellint_1` (`_Tp __k`, `_Tpp __phi`)
- `float std::ellint_1f` (`float __k`, `float __phi`)
- `long double std::ellint_1l` (`long double __k`, `long double __phi`)
- `template<typename _Tp, typename _Tpp >`
`__gnu_cxx::fp_promote_t< _Tp, _Tpp > std::ellint_2` (`_Tp __k`, `_Tpp __phi`)

- float [std::ellint_2f](#) (float __k, float __phi)
Return the incomplete elliptic integral of the second kind $E(k, \phi)$ for float argument.
- long double [std::ellint_2l](#) (long double __k, long double __phi)
Return the incomplete elliptic integral of the second kind $E(k, \phi)$.
- template<typename _Tp, typename _Tpn, typename _Tpp >
[__gnu_cxx::fp_promote_t](#)< _Tp, _Tpn, _Tpp > [std::ellint_3](#) (_Tp __k, _Tpn __nu, _Tpp __phi)
Return the incomplete elliptic integral of the third kind $\Pi(k, \nu, \phi)$.
- float [std::ellint_3f](#) (float __k, float __nu, float __phi)
Return the incomplete elliptic integral of the third kind $\Pi(k, \nu, \phi)$ for float argument.
- long double [std::ellint_3l](#) (long double __k, long double __nu, long double __phi)
Return the incomplete elliptic integral of the third kind $\Pi(k, \nu, \phi)$.
- template<typename _Tk, typename _Tp, typename _Ta, typename _Tb >
[__gnu_cxx::fp_promote_t](#)< _Tk, _Tp, _Ta, _Tb > [__gnu_cxx::ellint_cel](#) (_Tk __k_c, _Tp __p, _Ta __a, _Tb __b)
- float [__gnu_cxx::ellint_celf](#) (float __k_c, float __p, float __a, float __b)
- long double [__gnu_cxx::ellint_cell](#) (long double __k_c, long double __p, long double __a, long double __b)
- template<typename _Tk, typename _Tphi >
[__gnu_cxx::fp_promote_t](#)< _Tk, _Tphi > [__gnu_cxx::ellint_d](#) (_Tk __k, _Tphi __phi)
- float [__gnu_cxx::ellint_df](#) (float __k, float __phi)
- long double [__gnu_cxx::ellint_dl](#) (long double __k, long double __phi)
- template<typename _Tp, typename _Tk >
[__gnu_cxx::fp_promote_t](#)< _Tp, _Tk > [__gnu_cxx::ellint_el1](#) (_Tp __x, _Tk __k_c)
- float [__gnu_cxx::ellint_el1f](#) (float __x, float __k_c)
- long double [__gnu_cxx::ellint_el1l](#) (long double __x, long double __k_c)
- template<typename _Tp, typename _Tk, typename _Ta, typename _Tb >
[__gnu_cxx::fp_promote_t](#)< _Tp, _Tk, _Ta, _Tb > [__gnu_cxx::ellint_el2](#) (_Tp __x, _Tk __k_c, _Ta __a, _Tb __b)
- float [__gnu_cxx::ellint_el2f](#) (float __x, float __k_c, float __a, float __b)
- long double [__gnu_cxx::ellint_el2l](#) (long double __x, long double __k_c, long double __a, long double __b)
- template<typename _Tx, typename _Tk, typename _Tp >
[__gnu_cxx::fp_promote_t](#)< _Tx, _Tk, _Tp > [__gnu_cxx::ellint_el3](#) (_Tx __x, _Tk __k_c, _Tp __p)
- float [__gnu_cxx::ellint_el3f](#) (float __x, float __k_c, float __p)
- long double [__gnu_cxx::ellint_el3l](#) (long double __x, long double __k_c, long double __p)
- template<typename _Tp, typename _Up >
[__gnu_cxx::fp_promote_t](#)< _Tp, _Up > [__gnu_cxx::ellint_rc](#) (_Tp __x, _Up __y)
- float [__gnu_cxx::ellint_rcf](#) (float __x, float __y)
- long double [__gnu_cxx::ellint_rcl](#) (long double __x, long double __y)
- template<typename _Tp, typename _Up, typename _Vp >
[__gnu_cxx::fp_promote_t](#)< _Tp, _Up, _Vp > [__gnu_cxx::ellint_rd](#) (_Tp __x, _Up __y, _Vp __z)
- float [__gnu_cxx::ellint_rdf](#) (float __x, float __y, float __z)
- long double [__gnu_cxx::ellint_rdl](#) (long double __x, long double __y, long double __z)
- template<typename _Tp, typename _Up, typename _Vp >
[__gnu_cxx::fp_promote_t](#)< _Tp, _Up, _Vp > [__gnu_cxx::ellint_rf](#) (_Tp __x, _Up __y, _Vp __z)
- float [__gnu_cxx::ellint_rff](#) (float __x, float __y, float __z)
- long double [__gnu_cxx::ellint_rfl](#) (long double __x, long double __y, long double __z)
- template<typename _Tp, typename _Up, typename _Vp >
[__gnu_cxx::fp_promote_t](#)< _Tp, _Up, _Vp > [__gnu_cxx::ellint_rg](#) (_Tp __x, _Up __y, _Vp __z)
- float [__gnu_cxx::ellint_rgf](#) (float __x, float __y, float __z)
- long double [__gnu_cxx::ellint_rgl](#) (long double __x, long double __y, long double __z)
- template<typename _Tp, typename _Up, typename _Vp, typename _Wp >
[__gnu_cxx::fp_promote_t](#)< _Tp, _Up, _Vp, _Wp > [__gnu_cxx::ellint_rj](#) (_Tp __x, _Up __y, _Vp __z, _Wp __p)
- float [__gnu_cxx::ellint_rjf](#) (float __x, float __y, float __z, float __p)
- long double [__gnu_cxx::ellint_rjl](#) (long double __x, long double __y, long double __z, long double __p)

- `template<typename _Tp >`
`_Tp __gnu_cxx::ellnome (_Tp __k)`
- `float __gnu_cxx::ellnomef (float __k)`
- `long double __gnu_cxx::ellnome1 (long double __k)`
- `template<typename _Tp >`
`_Tp __gnu_cxx::euler (unsigned int __n)`
This returns Euler number E_n .
- `template<typename _Tp >`
`_Tp __gnu_cxx::eulerian_1 (unsigned int __n, unsigned int __m)`
- `template<typename _Tp >`
`std::vector< _Tp > __gnu_cxx::eulerian_1 (unsigned int __n)`
- `template<typename _Tp >`
`_Tp __gnu_cxx::eulerian_2 (unsigned int __n, unsigned int __m)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > std::expint (_Tp __x)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::expint (unsigned int __n, _Tp __x)`
- `float std::expintf (float __x)`
- `float __gnu_cxx::expintf (unsigned int __n, float __x)`
- `long double std::expintl (long double __x)`
- `long double __gnu_cxx::expintl (unsigned int __n, long double __x)`
- `template<typename _Tlam, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tlam, _Tp > __gnu_cxx::exponential_p (_Tlam __lambda, _Tp __x)`
Return the exponential cumulative probability density function.
- `template<typename _Tlam, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tlam, _Tp > __gnu_cxx::exponential_pdf (_Tlam __lambda, _Tp __x)`
Return the exponential probability density function.
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::factorial (unsigned int __n)`
Return the factorial $n!$ of the argument as a real number.

$$n! = 1 \times 2 \times \dots \times n, 0! = 1$$

- `float __gnu_cxx::factorialf (unsigned int __n)`
- `long double __gnu_cxx::factoriall (unsigned int __n)`
- `template<typename _Tp, typename _Tnu >`
`__gnu_cxx::fp_promote_t< _Tp, _Tnu > __gnu_cxx::falling_factorial (_Tp __a, _Tnu __nu)`
Return the falling factorial function or the lower Pochhammer symbol for real argument a and integral order n . The falling factorial function is defined by

$$a^{\underline{n}} = \prod_{k=0}^{n-1} (a - k) = \Gamma(a + 1) / \Gamma(a - n + 1)$$

where $a^{\underline{0}} \equiv 1$. In particular, $n^{\underline{n}} = n!$.

- `float __gnu_cxx::falling_factorialf (float __a, float __nu)`
- `long double __gnu_cxx::falling_factoriall (long double __a, long double __nu)`
- `template<typename _Tps, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tps, _Tp > __gnu_cxx::fermi_dirac (_Tps __s, _Tp __x)`
- `float __gnu_cxx::fermi_diracf (float __s, float __x)`
- `long double __gnu_cxx::fermi_diracl (long double __s, long double __x)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::fisher_f_p (_Tp __F, unsigned int __nu1, unsigned int __nu2)`

Return the F-distribution propability function. This returns the probability that the observed chi-square for a correct model exceeds the value χ^2 .

- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::fisher_f_pdf (_Tp __F, unsigned int __nu1, unsigned int __nu2)`

Return the F-distribution propability function. This returns the probability that the observed chi-square for a correct model exceeds the value χ^2 .

- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::fresnel_c (_Tp __x)`

- `float __gnu_cxx::fresnel_cf (float __x)`
- `long double __gnu_cxx::fresnel_cl (long double __x)`

- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::fresnel_s (_Tp __x)`

- `float __gnu_cxx::fresnel_sf (float __x)`
- `long double __gnu_cxx::fresnel_sl (long double __x)`

- `template<typename _Ta, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Ta, _Tp > __gnu_cxx::gamma_p (_Ta __a, _Tp __x)`

Return the gamma cumulative probability distribution function or the regularized lower incomplete gamma function.

- `template<typename _Ta, typename _Tb, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Ta, _Tb, _Tp > __gnu_cxx::gamma_pdf (_Ta __alpha, _Tb __beta, _Tp __x)`

Return the gamma propability distribution function.

- `float __gnu_cxx::gamma_pf (float __a, float __x)`
- `long double __gnu_cxx::gamma_pl (long double __a, long double __x)`

- `template<typename _Ta, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Ta, _Tp > __gnu_cxx::gamma_q (_Ta __a, _Tp __x)`

Return the gamma complementary cumulative probability distribution (or survival) function or the regularized upper incomplete gamma function.

- `float __gnu_cxx::gamma_qf (float __a, float __x)`
- `long double __gnu_cxx::gamma_ql (long double __a, long double __x)`

- `template<typename _Ta >`
`__gnu_cxx::fp_promote_t< _Ta > __gnu_cxx::gamma_reciprocal (_Ta __a)`

- `float __gnu_cxx::gamma_reciprocalf (float __a)`
- `long double __gnu_cxx::gamma_reciprocall (long double __a)`

- `template<typename _Tlam, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tlam, _Tp > __gnu_cxx::gegenbauer (unsigned int __n, _Tlam __lambda, _Tp __x)`

- `float __gnu_cxx::gegenbauerf (unsigned int __n, float __lambda, float __x)`
- `long double __gnu_cxx::gegenbauerl (unsigned int __n, long double __lambda, long double __x)`

- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::harmonic (unsigned int __n)`

- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > std::hermite (unsigned int __n, _Tp __x)`

- `float std::hermitef (unsigned int __n, float __x)`
- `long double std::hermitel (unsigned int __n, long double __x)`

- `template<typename _Tk, typename _Tphi >`
`__gnu_cxx::fp_promote_t< _Tk, _Tphi > __gnu_cxx::heuman_lambda (_Tk __k, _Tphi __phi)`

- `float __gnu_cxx::heuman_lambdaf (float __k, float __phi)`
- `long double __gnu_cxx::heuman_lambdal (long double __k, long double __phi)`

- `template<typename _Tp, typename _Up >`
`__gnu_cxx::fp_promote_t< _Tp, _Up > __gnu_cxx::hurwitz_zeta (_Tp __s, _Up __a)`

- `template<typename _Tp, typename _Up >`
`std::complex< _Tp > __gnu_cxx::hurwitz_zeta (_Tp __s, std::complex< _Up > __a)`
- `float __gnu_cxx::hurwitz_zetaf (float __s, float __a)`

- long double [__gnu_cxx::hurwitz_zetal](#) (long double __s, long double __a)
- template<typename _Tpa, typename _Tpb, typename _Tpc, typename _Tp >
[__gnu_cxx::fp_promote_t](#)< _Tpa, _Tpb, _Tpc, _Tp > [__gnu_cxx::hyperg](#) (_Tpa __a, _Tpb __b, _Tpc __c, _Tp __x)
- float [__gnu_cxx::hypergf](#) (float __a, float __b, float __c, float __x)
- long double [__gnu_cxx::hypergl](#) (long double __a, long double __b, long double __c, long double __x)
- template<typename _Ta, typename _Tb, typename _Tp >
[__gnu_cxx::fp_promote_t](#)< _Ta, _Tb, _Tp > [__gnu_cxx::ibeta](#) (_Ta __a, _Tb __b, _Tp __x)
- template<typename _Ta, typename _Tb, typename _Tp >
[__gnu_cxx::fp_promote_t](#)< _Ta, _Tb, _Tp > [__gnu_cxx::ibetac](#) (_Ta __a, _Tb __b, _Tp __x)
- float [__gnu_cxx::ibetacf](#) (float __a, float __b, float __x)
- long double [__gnu_cxx::ibetacl](#) (long double __a, long double __b, long double __x)
- float [__gnu_cxx::ibetaf](#) (float __a, float __b, float __x)
- long double [__gnu_cxx::ibetal](#) (long double __a, long double __b, long double __x)
- template<typename _Talpha, typename _Tbeta, typename _Tp >
[__gnu_cxx::fp_promote_t](#)< _Talpha, _Tbeta, _Tp > [__gnu_cxx::jacobi](#) (unsigned __n, _Talpha __alpha, _Tbeta __beta, _Tp __x)
- template<typename _Kp, typename _Up >
[__gnu_cxx::fp_promote_t](#)< _Kp, _Up > [__gnu_cxx::jacobi_cn](#) (_Kp __k, _Up __u)
- float [__gnu_cxx::jacobi_cnf](#) (float __k, float __u)
- long double [__gnu_cxx::jacobi_cnl](#) (long double __k, long double __u)
- template<typename _Kp, typename _Up >
[__gnu_cxx::fp_promote_t](#)< _Kp, _Up > [__gnu_cxx::jacobi_dn](#) (_Kp __k, _Up __u)
- float [__gnu_cxx::jacobi_dnf](#) (float __k, float __u)
- long double [__gnu_cxx::jacobi_dnl](#) (long double __k, long double __u)
- template<typename _Kp, typename _Up >
[__gnu_cxx::fp_promote_t](#)< _Kp, _Up > [__gnu_cxx::jacobi_sn](#) (_Kp __k, _Up __u)
- float [__gnu_cxx::jacobi_snf](#) (float __k, float __u)
- long double [__gnu_cxx::jacobi_snl](#) (long double __k, long double __u)
- template<typename _Tpq, typename _Tp >
[__gnu_cxx::fp_promote_t](#)< _Tpq, _Tp > [__gnu_cxx::jacobi_theta_1](#) (_Tpq __q, _Tp __x)
- float [__gnu_cxx::jacobi_theta_1f](#) (float __q, float __x)
- long double [__gnu_cxx::jacobi_theta_1l](#) (long double __q, long double __x)
- template<typename _Tpq, typename _Tp >
[__gnu_cxx::fp_promote_t](#)< _Tpq, _Tp > [__gnu_cxx::jacobi_theta_2](#) (_Tpq __q, _Tp __x)
- float [__gnu_cxx::jacobi_theta_2f](#) (float __q, float __x)
- long double [__gnu_cxx::jacobi_theta_2l](#) (long double __q, long double __x)
- template<typename _Tpq, typename _Tp >
[__gnu_cxx::fp_promote_t](#)< _Tpq, _Tp > [__gnu_cxx::jacobi_theta_3](#) (_Tpq __q, _Tp __x)
- float [__gnu_cxx::jacobi_theta_3f](#) (float __q, float __x)
- long double [__gnu_cxx::jacobi_theta_3l](#) (long double __q, long double __x)
- template<typename _Tpq, typename _Tp >
[__gnu_cxx::fp_promote_t](#)< _Tpq, _Tp > [__gnu_cxx::jacobi_theta_4](#) (_Tpq __q, _Tp __x)
- float [__gnu_cxx::jacobi_theta_4f](#) (float __q, float __x)
- long double [__gnu_cxx::jacobi_theta_4l](#) (long double __q, long double __x)
- template<typename _Tk, typename _Tphi >
[__gnu_cxx::fp_promote_t](#)< _Tk, _Tphi > [__gnu_cxx::jacobi_zeta](#) (_Tk __k, _Tphi __phi)
- float [__gnu_cxx::jacobi_zetaf](#) (float __k, float __phi)
- long double [__gnu_cxx::jacobi_zetal](#) (long double __k, long double __phi)
- float [__gnu_cxx::jacobif](#) (unsigned __n, float __alpha, float __beta, float __x)
- long double [__gnu_cxx::jacobil](#) (unsigned __n, long double __alpha, long double __beta, long double __x)

- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > std::laguerre` (unsigned int __n, _Tp __x)
- float `std::laguerref` (unsigned int __n, float __x)
- long double `std::laguerrel` (unsigned int __n, long double __x)
- `template<typename _Tp >`
`_Tp __gnu_cxx::lah` (unsigned int __n, unsigned int __k)
- `template<typename _Tp >`
`std::vector< _Tp > __gnu_cxx::lah` (unsigned int __n)
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::lbinomial` (unsigned int __n, unsigned int __k)

Return the logarithm of the binomial coefficient as a real number. The binomial coefficient is given by:

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

The binomial coefficients are generated by:

$$(1+t)^n = \sum_{k=0}^n \binom{n}{k} t^k$$

- float `__gnu_cxx::lbinomialf` (unsigned int __n, unsigned int __k)
 - long double `__gnu_cxx::lbinomiall` (unsigned int __n, unsigned int __k)
 - `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::ldouble_factorial` (int __n)
- Return the logarithm of the double factorial $\ln(n!!)$ of the argument as a real number.*

$$n!! = n(n-2)\dots(2), 0!! = 1$$

for even n and

$$n!! = n(n-2)\dots(1), (-1)!! = 1$$

for odd n.

- float `__gnu_cxx::ldouble_factorialf` (int __n)
- long double `__gnu_cxx::ldouble_factoriall` (int __n)
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > std::legendre` (unsigned int __l, _Tp __x)
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::legendre_q` (unsigned int __l, _Tp __x)
- float `__gnu_cxx::legendre_qf` (unsigned int __l, float __x)
- long double `__gnu_cxx::legendre_ql` (unsigned int __l, long double __x)
- float `std::legendref` (unsigned int __l, float __x)
- long double `std::legendrel` (unsigned int __l, long double __x)
- `template<typename _Tp, typename _Ts, typename _Ta >`
`__gnu_cxx::fp_promote_t< _Tp, _Ts, _Ta > __gnu_cxx::lerch_phi` (_Tp __z, _Ts __s, _Ta __a)
- float `__gnu_cxx::lerch_phif` (float __z, float __s, float __a)
- long double `__gnu_cxx::lerch_phil` (long double __z, long double __s, long double __a)
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::lfactorial` (unsigned int __n)

Return the logarithm of the factorial $\ln(n!)$ of the argument as a real number.

$$n! = 1 \times 2 \times \dots \times n, 0! = 1$$

- float `__gnu_cxx::lfactorialf` (unsigned int __n)
- long double `__gnu_cxx::lfactoriall` (unsigned int __n)

- `template<typename _Tp, typename _Tnu >`
`__gnu_cxx::fp_promote_t< _Tp, _Tnu > __gnu_cxx::lfalling_factorial (_Tp __a, _Tnu __nu)`

Return the logarithm of the falling factorial function or the lower Pochhammer symbol. The falling factorial function is defined by

$$a^{\underline{n}} = \frac{\Gamma(a+1)}{\Gamma(a-\nu+1)} = \prod_{k=0}^{n-1} (a-k)$$

where $a^{\underline{0}} \equiv 1$. In particular, $n^{\underline{n}} = n!$. Thus this function returns

$$\ln[a^{\underline{n}}] = \ln[\Gamma(a+1)] - \ln[\Gamma(a-\nu+1)]$$

where $\ln[a^{\underline{0}}] \equiv 0$. Many notations exist for this function: $(a)_{\nu}$,

$$\left\{ \begin{matrix} a \\ \nu \end{matrix} \right\}$$

, and others.

- `float __gnu_cxx::lfalling_factorialf (float __a, float __nu)`
- `long double __gnu_cxx::lfalling_factoriall (long double __a, long double __nu)`
- `template<typename _Ta >`
`__gnu_cxx::fp_promote_t< _Ta > __gnu_cxx::lgamma (_Ta __a)`
- `template<typename _Ta >`
`std::complex< __gnu_cxx::fp_promote_t< _Ta > > __gnu_cxx::lgamma (std::complex< _Ta > __a)`
- `float __gnu_cxx::lgammaf (float __a)`
- `std::complex< float > __gnu_cxx::lgammaf (std::complex< float > __a)`
- `long double __gnu_cxx::lgammal (long double __a)`
- `std::complex< long double > __gnu_cxx::lgammal (std::complex< long double > __a)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::logint (_Tp __x)`
- `float __gnu_cxx::logintf (float __x)`
- `long double __gnu_cxx::logintl (long double __x)`
- `template<typename _Ta, typename _Tb, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Ta, _Tb, _Tp > __gnu_cxx::logistic_p (_Ta __a, _Tb __b, _Tp __x)`
Return the logistic cumulative distribution function.
- `template<typename _Ta, typename _Tb, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Ta, _Tb, _Tp > __gnu_cxx::logistic_pdf (_Ta __a, _Tb __b, _Tp __x)`
Return the logistic probability density function.
- `template<typename _Tmu, typename _Tsig, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tmu, _Tsig, _Tp > __gnu_cxx::lognormal_p (_Tmu __mu, _Tsig __sigma, _Tp __x)`
Return the lognormal cumulative probability density function.
- `template<typename _Tmu, typename _Tsig, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tmu, _Tsig, _Tp > __gnu_cxx::lognormal_pdf (_Tmu __mu, _Tsig __sigma, _Tp __x)`
Return the lognormal probability density function.
- `template<typename _Tp, typename _Tnu >`
`__gnu_cxx::fp_promote_t< _Tp, _Tnu > __gnu_cxx::lrising_factorial (_Tp __a, _Tnu __nu)`

Return the logarithm of the rising factorial function or the (upper) Pochhammer symbol. The rising factorial function is defined for integer order by

$$a^{\overline{\nu}} = \Gamma(a+\nu)/\Gamma(a) = \prod_{k=0}^{\nu-1} (a+k), \overline{0} \equiv 1$$

Thus this function returns

$$\ln[a^{\overline{\nu}}] = \ln[\Gamma(a+\nu)] - \ln[\Gamma(a)], \ln[a^{\overline{0}}] \equiv 0$$

Many notations exist for this function: $(a)_\nu$, called the Pochhammer function (esp. in the literature of special functions), and

$$\begin{bmatrix} a \\ \nu \end{bmatrix}$$

, and others.

- float [__gnu_cxx::lrising_factorialf](#) (float __a, float __nu)
- long double [__gnu_cxx::lrising_factoriall](#) (long double __a, long double __nu)
- template<typename _Tp, typename _Ta, typename _Tb >
std::complex< __gnu_cxx::fp_promote_t< _Tp, _Ta, _Tb > > [__gnu_cxx::mittag_leffler](#) (_Ta __alpha, _Tb __beta, const std::complex< _Tp > &__z)
- template<typename _Tmu, typename _Tsig, typename _Tp >
__gnu_cxx::fp_promote_t< _Tmu, _Tsig, _Tp > [__gnu_cxx::normal_p](#) (_Tmu __mu, _Tsig __sigma, _Tp __x)
Return the normal cumulative probability density function.
- template<typename _Tmu, typename _Tsig, typename _Tp >
__gnu_cxx::fp_promote_t< _Tmu, _Tsig, _Tp > [__gnu_cxx::normal_pdf](#) (_Tmu __mu, _Tsig __sigma, _Tp __x)
Return the gamma cumulative propability distribution function.
- template<typename _Tph, typename _Tpa >
__gnu_cxx::fp_promote_t< _Tph, _Tpa > [__gnu_cxx::owens_t](#) (_Tph __h, _Tpa __a)
- float [__gnu_cxx::owens_tf](#) (float __h, float __a)
- long double [__gnu_cxx::owens_tl](#) (long double __h, long double __a)
- template<typename _Tp, typename _Up >
__gnu_cxx::fp_promote_t< std::complex< _Tp >, _Up > [__gnu_cxx::periodic_zeta](#) (_Tp __x, _Up __s)
- template<typename _Tp, typename _Up >
__gnu_cxx::fp_promote_t< std::complex< _Tp >, std::complex< _Up > > [__gnu_cxx::periodic_zeta](#) (std::complex< _Up > __z, _Tp __s)
- std::complex< float > [__gnu_cxx::periodic_zetaf](#) (float __x, float __s)
- std::complex< long double > [__gnu_cxx::periodic_zetal](#) (long double __x, long double __s)
- template<typename _Tp >
__gnu_cxx::fp_promote_t< _Tp > [__gnu_cxx::polygamma](#) (unsigned int __m, _Tp __x)
- float [__gnu_cxx::polygammaf](#) (unsigned int __m, float __x)
- long double [__gnu_cxx::polygammal](#) (unsigned int __m, long double __x)
- template<typename _Tp, typename _Wp >
__gnu_cxx::fp_promote_t< _Tp, _Wp > [__gnu_cxx::polylog](#) (_Tp __s, _Wp __w)
- template<typename _Tp, typename _Wp >
std::complex< __gnu_cxx::fp_promote_t< _Tp, _Wp > > [__gnu_cxx::polylog](#) (_Tp __s, std::complex< _Tp > __w)
- float [__gnu_cxx::polylogf](#) (float __s, float __w)
- std::complex< float > [__gnu_cxx::polylogf](#) (float __s, std::complex< float > __w)
- long double [__gnu_cxx::polylogl](#) (long double __s, long double __w)
- std::complex< long double > [__gnu_cxx::polylogl](#) (long double __s, std::complex< long double > __w)
- template<typename _Tp >
__gnu_cxx::fp_promote_t< _Tp > [__gnu_cxx::radpoly](#) (unsigned int __n, unsigned int __m, _Tp __rho)
- float [__gnu_cxx::radpolyf](#) (unsigned int __n, unsigned int __m, float __rho)
- long double [__gnu_cxx::radpolyl](#) (unsigned int __n, unsigned int __m, long double __rho)
- template<typename _Tp >
__gnu_cxx::fp_promote_t< _Tp > [std::riemann_zeta](#) (_Tp __s)
- float [std::riemann_zetaf](#) (float __s)
- long double [std::riemann_zetal](#) (long double __s)
- template<typename _Tp, typename _Tnu >
__gnu_cxx::fp_promote_t< _Tp, _Tnu > [__gnu_cxx::rising_factorial](#) (_Tp __a, _Tnu __nu)

Return the rising factorial function or the (upper) Pochhammer function. The rising factorial function is defined by

$$a^{\overline{\nu}} = \Gamma(a + \nu) / \Gamma(\nu)$$

Many notations exist for this function: $(a)_{\nu}$, called the Pochhammer function (esp. in the literature of special functions), and

$$\begin{bmatrix} a \\ \nu \end{bmatrix}$$

, and others.

- float [__gnu_cxx::rising_factorialf](#) (float __a, float __nu)
- long double [__gnu_cxx::rising_factoriall](#) (long double __a, long double __nu)
- template<typename _Tp >
 [__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::sin_pi](#) (_Tp __x)
- float [__gnu_cxx::sin_pif](#) (float __x)
- long double [__gnu_cxx::sin_pil](#) (long double __x)
- template<typename _Tp >
 [__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::sinc](#) (_Tp __x)
- template<typename _Tp >
 [__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::sinc_pi](#) (_Tp __x)
- float [__gnu_cxx::sinc_pif](#) (float __x)
- long double [__gnu_cxx::sinc_pil](#) (long double __x)
- float [__gnu_cxx::sincf](#) (float __x)
- long double [__gnu_cxx::sincl](#) (long double __x)
- [__gnu_cxx::__sincos_t<double> __gnu_cxx::sincos](#) (double __x)
- template<typename _Tp >
 [__gnu_cxx::__sincos_t<__gnu_cxx::fp_promote_t<_Tp>> __gnu_cxx::sincos](#) (_Tp __x)
- template<typename _Tp >
 [__gnu_cxx::__sincos_t<__gnu_cxx::fp_promote_t<_Tp>> __gnu_cxx::sincos_pi](#) (_Tp __x)
- [__gnu_cxx::__sincos_t<float> __gnu_cxx::sincos_pif](#) (float __x)
- [__gnu_cxx::__sincos_t<long double> __gnu_cxx::sincos_pil](#) (long double __x)
- [__gnu_cxx::__sincos_t<float> __gnu_cxx::sincosf](#) (float __x)
- [__gnu_cxx::__sincos_t<long double> __gnu_cxx::sincosl](#) (long double __x)
- template<typename _Tp >
 [__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::sinh_pi](#) (_Tp __x)
- float [__gnu_cxx::sinh_pif](#) (float __x)
- long double [__gnu_cxx::sinh_pil](#) (long double __x)
- template<typename _Tp >
 [__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::sinhc](#) (_Tp __x)
- template<typename _Tp >
 [__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::sinhc_pi](#) (_Tp __x)
- float [__gnu_cxx::sinhc_pif](#) (float __x)
- long double [__gnu_cxx::sinhc_pil](#) (long double __x)
- float [__gnu_cxx::sinhcf](#) (float __x)
- long double [__gnu_cxx::sinhcl](#) (long double __x)
- template<typename _Tp >
 [__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::sinhint](#) (_Tp __x)
- float [__gnu_cxx::sinhintf](#) (float __x)
- long double [__gnu_cxx::sinhintl](#) (long double __x)
- template<typename _Tp >
 [__gnu_cxx::fp_promote_t<_Tp> __gnu_cxx::sinint](#) (_Tp __x)
- float [__gnu_cxx::sinintf](#) (float __x)
- long double [__gnu_cxx::sinintl](#) (long double __x)

- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > std::sph_bessel (unsigned int __n, _Tp __x)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::sph_bessel_i (unsigned int __n, _Tp __x)`
- `float __gnu_cxx::sph_bessel_if (unsigned int __n, float __x)`
- `long double __gnu_cxx::sph_bessel_il (unsigned int __n, long double __x)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::sph_bessel_k (unsigned int __n, _Tp __x)`
- `float __gnu_cxx::sph_bessel_kf (unsigned int __n, float __x)`
- `long double __gnu_cxx::sph_bessel_kl (unsigned int __n, long double __x)`
- `float std::sph_besself (unsigned int __n, float __x)`
- `long double std::sph_bessell (unsigned int __n, long double __x)`
- `template<typename _Tp >`
`std::complex< __gnu_cxx::fp_promote_t< _Tp > > __gnu_cxx::sph_hankel_1 (unsigned int __n, _Tp __z)`
- `template<typename _Tp >`
`std::complex< __gnu_cxx::fp_promote_t< _Tp > > __gnu_cxx::sph_hankel_1 (unsigned int __n, std::complex< __Tp > __x)`
- `std::complex< float > __gnu_cxx::sph_hankel_1f (unsigned int __n, float __z)`
- `std::complex< float > __gnu_cxx::sph_hankel_1f (unsigned int __n, std::complex< float > __x)`
- `std::complex< long double > __gnu_cxx::sph_hankel_1l (unsigned int __n, long double __z)`
- `std::complex< long double > __gnu_cxx::sph_hankel_1l (unsigned int __n, std::complex< long double > __x)`
- `template<typename _Tp >`
`std::complex< __gnu_cxx::fp_promote_t< _Tp > > __gnu_cxx::sph_hankel_2 (unsigned int __n, _Tp __z)`
- `template<typename _Tp >`
`std::complex< __gnu_cxx::fp_promote_t< _Tp > > __gnu_cxx::sph_hankel_2 (unsigned int __n, std::complex< __Tp > __x)`
- `std::complex< float > __gnu_cxx::sph_hankel_2f (unsigned int __n, float __z)`
- `std::complex< float > __gnu_cxx::sph_hankel_2f (unsigned int __n, std::complex< float > __x)`
- `std::complex< long double > __gnu_cxx::sph_hankel_2l (unsigned int __n, long double __z)`
- `std::complex< long double > __gnu_cxx::sph_hankel_2l (unsigned int __n, std::complex< long double > __x)`
- `template<typename _Ttheta, typename _Tphi >`
`std::complex< __gnu_cxx::fp_promote_t< _Ttheta, _Tphi > > __gnu_cxx::sph_harmonic (unsigned int __l, int __m, _Ttheta __theta, _Tphi __phi)`
- `std::complex< float > __gnu_cxx::sph_harmonicf (unsigned int __l, int __m, float __theta, float __phi)`
- `std::complex< long double > __gnu_cxx::sph_harmonicl (unsigned int __l, int __m, long double __theta, long double __phi)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > std::sph_legendre (unsigned int __l, unsigned int __m, _Tp __theta)`
- `float std::sph_legendref (unsigned int __l, unsigned int __m, float __theta)`
- `long double std::sph_legendrel (unsigned int __l, unsigned int __m, long double __theta)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > std::sph_neumann (unsigned int __n, _Tp __x)`
- `float std::sph_neumannf (unsigned int __n, float __x)`
- `long double std::sph_neumannl (unsigned int __n, long double __x)`
- `template<typename _Tp >`
`_Tp __gnu_cxx::stirling_1 (unsigned int __n, unsigned int __m)`
- `template<typename _Tp >`
`std::vector< _Tp > __gnu_cxx::stirling_1 (unsigned int __n)`
- `template<typename _Tp >`
`_Tp __gnu_cxx::stirling_2 (unsigned int __n, unsigned int __m)`
- `template<typename _Tp >`
`std::vector< _Tp > __gnu_cxx::stirling_2 (unsigned int __n)`

- `template<typename _Tt, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::student_t_p (_Tt __t, unsigned int __nu)`
Return the Students T probability function.
- `template<typename _Tt, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::student_t_pdf (_Tt __t, unsigned int __nu)`
Return the complement of the Students T probability function.
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::tan_pi (_Tp __x)`
- `float __gnu_cxx::tan_pif (float __x)`
- `long double __gnu_cxx::tan_pil (long double __x)`
- `template<typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tp > __gnu_cxx::tanh_pi (_Tp __x)`
- `float __gnu_cxx::tanh_pif (float __x)`
- `long double __gnu_cxx::tanh_pil (long double __x)`
- `template<typename _Ta >`
`__gnu_cxx::fp_promote_t< _Ta > __gnu_cxx::tgamma (_Ta __a)`
- `template<typename _Ta >`
`std::complex< __gnu_cxx::fp_promote_t< _Ta > > __gnu_cxx::tgamma (std::complex< _Ta > __a)`
- `template<typename _Ta, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Ta, _Tp > __gnu_cxx::tgamma (_Ta __a, _Tp __x)`
- `template<typename _Ta, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Ta, _Tp > __gnu_cxx::tgamma_lower (_Ta __a, _Tp __x)`
- `float __gnu_cxx::tgamma_lowerf (float __a, float __x)`
- `long double __gnu_cxx::tgamma_lowerl (long double __a, long double __x)`
- `float __gnu_cxx::tgammaf (float __a)`
- `std::complex< float > __gnu_cxx::tgammaf (std::complex< float > __a)`
- `float __gnu_cxx::tgammaf (float __a, float __x)`
- `long double __gnu_cxx::tgammal (long double __a)`
- `std::complex< long double > __gnu_cxx::tgammal (std::complex< long double > __a)`
- `long double __gnu_cxx::tgammal (long double __a, long double __x)`
- `template<typename _Tpnu, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tpnu, _Tp > __gnu_cxx::theta_1 (_Tpnu __nu, _Tp __x)`
- `float __gnu_cxx::theta_1f (float __nu, float __x)`
- `long double __gnu_cxx::theta_1l (long double __nu, long double __x)`
- `template<typename _Tpnu, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tpnu, _Tp > __gnu_cxx::theta_2 (_Tpnu __nu, _Tp __x)`
- `float __gnu_cxx::theta_2f (float __nu, float __x)`
- `long double __gnu_cxx::theta_2l (long double __nu, long double __x)`
- `template<typename _Tpnu, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tpnu, _Tp > __gnu_cxx::theta_3 (_Tpnu __nu, _Tp __x)`
- `float __gnu_cxx::theta_3f (float __nu, float __x)`
- `long double __gnu_cxx::theta_3l (long double __nu, long double __x)`
- `template<typename _Tpnu, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tpnu, _Tp > __gnu_cxx::theta_4 (_Tpnu __nu, _Tp __x)`
- `float __gnu_cxx::theta_4f (float __nu, float __x)`
- `long double __gnu_cxx::theta_4l (long double __nu, long double __x)`
- `template<typename _Tpk, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tpk, _Tp > __gnu_cxx::theta_c (_Tpk __k, _Tp __x)`
- `float __gnu_cxx::theta_cf (float __k, float __x)`
- `long double __gnu_cxx::theta_cl (long double __k, long double __x)`

- `template<typename _Tpk, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tpk, _Tp > __gnu_cxx::theta_d (_Tpk __k, _Tp __x)`
- `float __gnu_cxx::theta_df (float __k, float __x)`
- `long double __gnu_cxx::theta_dl (long double __k, long double __x)`
- `template<typename _Tpk, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tpk, _Tp > __gnu_cxx::theta_n (_Tpk __k, _Tp __x)`
- `float __gnu_cxx::theta_nf (float __k, float __x)`
- `long double __gnu_cxx::theta_nl (long double __k, long double __x)`
- `template<typename _Tpk, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tpk, _Tp > __gnu_cxx::theta_s (_Tpk __k, _Tp __x)`
- `float __gnu_cxx::theta_sf (float __k, float __x)`
- `long double __gnu_cxx::theta_sl (long double __k, long double __x)`
- `template<typename _Tpa, typename _Tpc, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Tpa, _Tpc, _Tp > __gnu_cxx::tricomi_u (_Tpa __a, _Tpc __c, _Tp __x)`
- `float __gnu_cxx::tricomi_uf (float __a, float __c, float __x)`
- `long double __gnu_cxx::tricomi_ul (long double __a, long double __c, long double __x)`
- `template<typename _Ta, typename _Tb, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Ta, _Tb, _Tp > __gnu_cxx::weibull_p (_Ta __a, _Tb __b, _Tp __x)`
Return the Weibull cumulative probability density function.
- `template<typename _Ta, typename _Tb, typename _Tp >`
`__gnu_cxx::fp_promote_t< _Ta, _Tb, _Tp > __gnu_cxx::weibull_pdf (_Ta __a, _Tb __b, _Tp __x)`
Return the Weibull probability density function.
- `template<typename _Trho, typename _Tphi >`
`__gnu_cxx::fp_promote_t< _Trho, _Tphi > __gnu_cxx::zernike (unsigned int __n, int __m, _Trho __rho, _Tphi __phi)`
- `float __gnu_cxx::zernikef (unsigned int __n, int __m, float __rho, float __phi)`
- `long double __gnu_cxx::zernikel (unsigned int __n, int __m, long double __rho, long double __phi)`

10.32.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include `<cmath>`.

10.32.2 Macro Definition Documentation

10.32.2.1 __cpp_lib_math_special_functions

```
#define __cpp_lib_math_special_functions 201603L
```

Definition at line 39 of file `specfun.h`.

10.32.2.2 __STDCPP_MATH_SPEC_FUNCS__

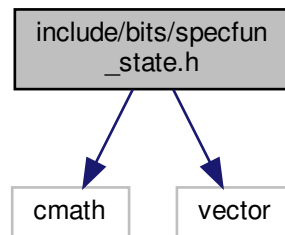
```
#define __STDCPP_MATH_SPEC_FUNCS__ 201003L
```

Definition at line 37 of file specfun.h.

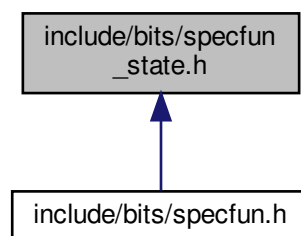
10.33 include/bits/specfun_state.h File Reference

```
#include <cmath>  
#include <vector>
```

Include dependency graph for specfun_state.h:



This graph shows which files directly or indirectly include this file:



Classes

- struct [__gnu_cxx::__airy_t](#)< _Tx, _Tp >
- struct [__gnu_cxx::__assoc_legendre_p_t](#)< _Tp >
- struct [__gnu_cxx::__assoc_legendre_q_t](#)< _Tp >
- struct [__gnu_cxx::__chebyshev_t](#)< _Tp >
- struct [__gnu_cxx::__chebyshev_u_t](#)< _Tp >
- struct [__gnu_cxx::__chebyshev_v_t](#)< _Tp >
- struct [__gnu_cxx::__chebyshev_w_t](#)< _Tp >
- struct [__gnu_cxx::__coulomb_t](#)< _Teta, _Trho, _Tp >
- struct [__gnu_cxx::__cyl_bessel_t](#)< _Tnu, _Tx, _Tp >
- struct [__gnu_cxx::__cyl_hankel_t](#)< _Tnu, _Tx, _Tp >
- struct [__gnu_cxx::__cyl_mod_bessel_t](#)< _Tnu, _Tx, _Tp >
- struct [__gnu_cxx::__fock_airy_t](#)< _Tx, _Tp >
- struct [__gnu_cxx::__gamma_inc_t](#)< _Tp >
- struct [__gnu_cxx::__gamma_temme_t](#)< _Tp >

A structure for the gamma functions required by the Temme series expansions of $N_\nu(x)$ and $K_\nu(x)$.

$$\Gamma_1 = \frac{1}{2\mu} \left[\frac{1}{\Gamma(1-\mu)} - \frac{1}{\Gamma(1+\mu)} \right]$$

and

$$\Gamma_2 = \frac{1}{2} \left[\frac{1}{\Gamma(1-\mu)} + \frac{1}{\Gamma(1+\mu)} \right]$$

where $-1/2 \leq \mu \leq 1/2$ is $\mu = \nu - N$ and N is the nearest integer to ν . The values of $\Gamma(1+\mu)$ and $\Gamma(1-\mu)$ are returned as well.

- struct [__gnu_cxx::__gappa_pq_t](#)< _Tp >
- struct [__gnu_cxx::__gegenbauer_t](#)< _Tp >
- struct [__gnu_cxx::__hermite_he_t](#)< _Tp >
- struct [__gnu_cxx::__hermite_t](#)< _Tp >
- struct [__gnu_cxx::__jacobi_ellint_t](#)< _Tp >
- struct [__gnu_cxx::__jacobi_t](#)< _Tp >
- struct [__gnu_cxx::__laguerre_t](#)< _Tpa, _Tp >
- struct [__gnu_cxx::__legendre_p_t](#)< _Tp >
- struct [__gnu_cxx::__legendre_q_t](#)< _Tp >
- struct [__gnu_cxx::__lgamma_t](#)< _Tp >
- struct [__gnu_cxx::__quadrature_point_t](#)< _Tp >
- struct [__gnu_cxx::__sincos_t](#)< _Tp >
- struct [__gnu_cxx::__sph_bessel_t](#)< _Tn, _Tx, _Tp >
- struct [__gnu_cxx::__sph_hankel_t](#)< _Tn, _Tx, _Tp >
- struct [__gnu_cxx::__sph_mod_bessel_t](#)< _Tn, _Tx, _Tp >
- struct [__gnu_cxx::__stirling_1_t](#)< _Tp >

A structure for Stirling numbers of the first kind.

- struct [__gnu_cxx::__stirling_2_t](#)< _Tp >

A structure for Stirling numbers of the first kind.

Namespaces

- [__gnu_cxx](#)

Enumerations

- enum `__gnu_cxx::gauss_quad_type` { `__gnu_cxx::Gauss`, `__gnu_cxx::Gauss_Lobatto`, `__gnu_cxx::Gauss_Radau_lower`, `__gnu_cxx::Gauss_Radau_upper` }

Enumeration for differing types of Gauss quadrature. The `gauss_quad_type` is used to determine the boundary condition modifications applied to orthogonal polynomials for quadrature rules.

10.33.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include `<cmath>`.

Index

`_S`
 `__gnu_cxx::__stirling_2_t`, [537](#)
`_Airy`
 `std::__detail::_Airy`, [567](#)
`_Airy_asymp`
 `std::__detail::_Airy_asymp`, [570](#)
`_Airy_asymp_series`
 `std::__detail::_Airy_asymp_series`, [577](#)
`_AsympTerminator`
 `std::__detail::_AsympTerminator`, [595](#)
`_Cmplx`
 `std::__detail::_jacobi_lattice_t`, [546](#)
 `std::__detail::_jacobi_theta_0_t`, [555](#)
 `std::__detail::_weierstrass_invariants_t`, [560](#)
 `std::__detail::_weierstrass_roots_t`, [563](#)
 `std::__detail::_Airy_asymp`, [570](#)
 `std::__detail::_Airy_series`, [582](#)
`_GLIBCXX_BITS_SF_AIRY_TCC`
 `sf_airy.tcc`, [604](#)
`_GLIBCXX_BITS_SF_BERNOULLI_TCC`
 `sf_bernoulli.tcc`, [606](#)
`_GLIBCXX_BITS_SF_BESSEL_TCC`
 `sf_bessel.tcc`, [608](#)
`_GLIBCXX_BITS_SF_BETA_TCC`
 `sf_beta.tcc`, [610](#)
`_GLIBCXX_BITS_SF_CARDINAL_TCC`
 `sf_cardinal.tcc`, [612](#)
`_GLIBCXX_BITS_SF_CHEBYSHEV_TCC`
 `sf_chebyshev.tcc`, [614](#)
`_GLIBCXX_BITS_SF_COULOMB_TCC`
 `sf_coulomb.tcc`, [616](#)
`_GLIBCXX_BITS_SF_DAWSON_TCC`
 `sf_dawson.tcc`, [617](#)
`_GLIBCXX_BITS_SF_DISTRIBUTIONS_TCC`
 `sf_distributions.tcc`, [621](#)
`_GLIBCXX_BITS_SF_ELLINT_TCC`
 `sf_ellint.tcc`, [623](#)
`_GLIBCXX_BITS_SF_EULER_TCC`
 `sf_euler.tcc`, [625](#)
`_GLIBCXX_BITS_SF_EXPINT_TCC`
 `sf_expint.tcc`, [628](#)
`_GLIBCXX_BITS_SF_FRESNEL_TCC`
 `sf_fresnel.tcc`, [629](#)
`_GLIBCXX_BITS_SF_GAMMA_TCC`
 `sf_gamma.tcc`, [637](#)
`_GLIBCXX_BITS_SF_GEGENBAUER_TCC`
 `sf_gegenbauer.tcc`, [639](#)
`_GLIBCXX_BITS_SF_HANKEL_TCC`
 `sf_hankel.tcc`, [642](#)
`_GLIBCXX_BITS_SF_HERMITE_TCC`
 `sf_hermite.tcc`, [644](#)
`_GLIBCXX_BITS_SF_HYPERG_TCC`
 `sf_hyperg.tcc`, [646](#)
`_GLIBCXX_BITS_SF_HYPINT_TCC`
 `sf_hypint.tcc`, [648](#)
`_GLIBCXX_BITS_SF_JACOBI_TCC`
 `sf_jacobi.tcc`, [649](#)
`_GLIBCXX_BITS_SF_LAGUERRE_TCC`
 `sf_laguerre.tcc`, [651](#)
`_GLIBCXX_BITS_SF_LEGENDRE_TCC`
 `sf_legendre.tcc`, [653](#)
`_GLIBCXX_BITS_SF_MOD_BESSEL_TCC`
 `sf_mod_bessel.tcc`, [655](#)
`_GLIBCXX_BITS_SF_OWENS_T_TCC`
 `sf_owens.t.tcc`, [657](#)
`_GLIBCXX_BITS_SF_POLYLOG_TCC`
 `sf_polylog.tcc`, [659](#)
`_GLIBCXX_BITS_SF_STIRLING_TCC`
 `sf_stirling.tcc`, [662](#)
`_GLIBCXX_BITS_SF_THETA_TCC`
 `sf_theta.tcc`, [664](#)
`_GLIBCXX_BITS_SF_TRIGINT_TCC`
 `sf_trigint.tcc`, [668](#)
`_GLIBCXX_BITS_SF_TRIG_TCC`
 `sf_trig.tcc`, [667](#)
`_GLIBCXX_BITS_SF_ZETA_TCC`
 `sf_zeta.tcc`, [671](#)
`_M_omega_1`
 `std::__detail::_jacobi_lattice_t`, [550](#)
`_M_omega_3`
 `std::__detail::_jacobi_lattice_t`, [550](#)
`_N_FGH`
 `std::__detail::_Airy_series`, [587](#)
`_Num_Euler_Maclaurin_zeta`
 `std::__detail`, [458](#)
`_Num_Stieltjes`
 `std::__detail`, [459](#)
`_Real`
 `std::__detail::_jacobi_lattice_t`, [546](#)
 `std::__detail::_jacobi_theta_0_t`, [555](#)

- std::__detail::__weierstrass_invariants_t, [560](#)
- std::__detail::__weierstrass_roots_t, [563](#)
- std::__detail::__AiryState, [593](#)
- _Real_Omega1
 - std::__detail::__jacobi_lattice_t, [546](#)
- _Real_Omega3
 - std::__detail::__jacobi_lattice_t, [546](#)
- _S_Ai
 - std::__detail::__Airy_series, [583](#)
- _S_Ai0
 - std::__detail::__Airy_series, [587](#)
- _S_Aip0
 - std::__detail::__Airy_series, [587](#)
- _S_Airy
 - std::__detail::__Airy_series, [583](#)
- _S_Bi
 - std::__detail::__Airy_series, [584](#)
- _S_Bi0
 - std::__detail::__Airy_series, [587](#)
- _S_Bip0
 - std::__detail::__Airy_series, [588](#)
- _S_Euler_Maclaurin_zeta
 - std::__detail, [459](#)
- _S_FGH
 - std::__detail::__Airy_series, [584](#)
- _S_Fock
 - std::__detail::__Airy_series, [585](#)
- _S_Gi0
 - std::__detail::__Airy_series, [588](#)
- _S_Gip0
 - std::__detail::__Airy_series, [588](#)
- _S_Hi0
 - std::__detail::__Airy_series, [588](#)
- _S_Hip0
 - std::__detail::__Airy_series, [589](#)
- _S_Scorer
 - std::__detail::__Airy_series, [585](#)
- _S_Scorer2
 - std::__detail::__Airy_series, [586](#)
- _S_Stieltjes
 - std::__detail, [463](#)
- _S_absarg_ge_pio3
 - std::__detail::__Airy_asymp, [570](#)
- _S_absarg_lt_pio3
 - std::__detail::__Airy_asymp, [571](#)
- _S_c
 - std::__detail::__Airy_asymp_data< double >, [573](#)
 - std::__detail::__Airy_asymp_data< float >, [574](#)
 - std::__detail::__Airy_asymp_data< long double >, [575](#)
- _S_cheby
 - std::__detail::__gamma_lanczos_data< double >, [538](#)
 - std::__detail::__gamma_lanczos_data< float >, [539](#)
- std::__detail::__gamma_lanczos_data< long double >, [540](#)
- std::__detail::__gamma_spouge_data< double >, [541](#)
- std::__detail::__gamma_spouge_data< float >, [542](#)
- std::__detail::__gamma_spouge_data< long double >, [543](#)
- _S_d
 - std::__detail::__Airy_asymp_data< double >, [573](#)
 - std::__detail::__Airy_asymp_data< float >, [574](#)
 - std::__detail::__Airy_asymp_data< long double >, [575](#)
- _S_double_factorial_table
 - std::__detail, [459](#)
- _S_eps
 - std::__detail::__Airy_series, [588](#)
- _S_factorial_table
 - std::__detail, [459](#)
- _S_g
 - std::__detail::__gamma_lanczos_data< double >, [538](#)
 - std::__detail::__gamma_lanczos_data< float >, [539](#)
 - std::__detail::__gamma_lanczos_data< long double >, [540](#)
- _S_harmonic_denom
 - std::__detail, [460](#)
- _S_harmonic_numer
 - std::__detail, [460](#)
- _S_i
 - std::__detail::__Airy_series, [589](#)
- _S_max_cd
 - std::__detail::__Airy_asymp_data< double >, [573](#)
 - std::__detail::__Airy_asymp_data< float >, [574](#)
 - std::__detail::__Airy_asymp_data< long double >, [575](#)
- _S_neg_double_factorial_table
 - std::__detail, [460](#)
- _S_num_double_factorials
 - std::__detail, [460](#)
- _S_num_double_factorials< double >
 - std::__detail, [460](#)
- _S_num_double_factorials< float >
 - std::__detail, [461](#)
- _S_num_double_factorials< long double >
 - std::__detail, [461](#)
- _S_num_factorials
 - std::__detail, [461](#)
- _S_num_factorials< double >
 - std::__detail, [461](#)
- _S_num_factorials< float >
 - std::__detail, [461](#)
- _S_num_factorials< long double >
 - std::__detail, [462](#)
- _S_num_harmonic_numer

std::__detail, [462](#)
 _S_num_neg_double_factorials
 std::__detail, [462](#)
 _S_num_neg_double_factorials< double >
 std::__detail, [462](#)
 _S_num_neg_double_factorials< float >
 std::__detail, [462](#)
 _S_num_neg_double_factorials< long double >
 std::__detail, [463](#)
 _S_num_zetam1
 std::__detail, [463](#)
 _S_pi
 std::__detail::__jacobi_lattice_t, [550](#)
 std::__detail::__Airy_series, [589](#)
 _S_sqrt_pi
 std::__detail::__Airy_asymp_series, [578](#)
 std::__detail::__Airy_series, [589](#)
 _S_zetam1
 std::__detail, [463](#)
 _Terminator
 std::__detail::__Terminator, [598](#)
 _Tp_Nome
 std::__detail::__jacobi_lattice_t, [547](#)
 _Type
 std::__detail::__jacobi_theta_0_t, [555](#)
 std::__detail::__weierstrass_invariants_t, [560](#)
 std::__detail::__weierstrass_roots_t, [563](#)
 _Val
 std::__detail::__AiryAuxilliaryState, [590](#)
 __Ai_deriv
 __gnu_cxx::__airy_t, [466](#)
 std::__detail::__AiryState, [594](#)
 __Ai_value
 __gnu_cxx::__airy_t, [466](#)
 std::__detail::__AiryState, [594](#)
 __Bi_deriv
 __gnu_cxx::__airy_t, [466](#)
 std::__detail::__AiryState, [594](#)
 __Bi_value
 __gnu_cxx::__airy_t, [466](#)
 std::__detail::__AiryState, [594](#)
 __C_n
 __gnu_cxx::__gegenbauer_t, [500](#)
 __C_nm1
 __gnu_cxx::__gegenbauer_t, [500](#)
 __C_nm2
 __gnu_cxx::__gegenbauer_t, [501](#)
 __F_deriv
 __gnu_cxx::__coulomb_t, [482](#)
 __F_value
 __gnu_cxx::__coulomb_t, [482](#)
 __G_deriv
 __gnu_cxx::__coulomb_t, [482](#)
 __G_value
 __gnu_cxx::__coulomb_t, [483](#)
 __H1_deriv
 __gnu_cxx::__cyl_hankel_t, [487](#)
 __H1_value
 __gnu_cxx::__cyl_hankel_t, [487](#)
 __H2_deriv
 __gnu_cxx::__cyl_hankel_t, [487](#)
 __H2_value
 __gnu_cxx::__cyl_hankel_t, [488](#)
 __H_n
 __gnu_cxx::__hermite_t, [505](#)
 __H_nm1
 __gnu_cxx::__hermite_t, [505](#)
 __H_nm2
 __gnu_cxx::__hermite_t, [505](#)
 __He_n
 __gnu_cxx::__hermite_he_t, [503](#)
 __He_nm1
 __gnu_cxx::__hermite_he_t, [503](#)
 __He_nm2
 __gnu_cxx::__hermite_he_t, [503](#)
 __I_deriv
 __gnu_cxx::__cyl_mod_bessel_t, [490](#)
 __I_value
 __gnu_cxx::__cyl_mod_bessel_t, [490](#)
 __J_deriv
 __gnu_cxx::__cyl_bessel_t, [485](#)
 __J_value
 __gnu_cxx::__cyl_bessel_t, [485](#)
 __K_deriv
 __gnu_cxx::__cyl_mod_bessel_t, [490](#)
 __K_value
 __gnu_cxx::__cyl_mod_bessel_t, [490](#)
 __L_n
 __gnu_cxx::__laguerre_t, [514](#)
 __L_nm1
 __gnu_cxx::__laguerre_t, [514](#)
 __L_nm2
 __gnu_cxx::__laguerre_t, [514](#)
 __N_deriv
 __gnu_cxx::__cyl_bessel_t, [485](#)
 __N_value
 __gnu_cxx::__cyl_bessel_t, [485](#)
 __P_l
 __gnu_cxx::__legendre_p_t, [516](#)
 __P_lm
 __gnu_cxx::__assoc_legendre_p_t, [468](#)
 __P_lm1
 __gnu_cxx::__legendre_p_t, [516](#)
 __P_lm1m
 __gnu_cxx::__assoc_legendre_p_t, [469](#)
 __P_lm2
 __gnu_cxx::__legendre_p_t, [517](#)
 __P_lm2m

- __gnu_cxx::__assoc_legendre_p_t, 469
- __P_n
 - __gnu_cxx::__jacobi_t, 512
- __P_nm1
 - __gnu_cxx::__jacobi_t, 512
- __P_nm2
 - __gnu_cxx::__jacobi_t, 512
- __Q_l
 - __gnu_cxx::__legendre_q_t, 518
- __Q_lm
 - __gnu_cxx::__assoc_legendre_q_t, 471
- __Q_lm1
 - __gnu_cxx::__legendre_q_t, 519
- __Q_lm2
 - __gnu_cxx::__legendre_q_t, 519
- __Q_lmm1
 - __gnu_cxx::__assoc_legendre_q_t, 472
- __Q_lmm2
 - __gnu_cxx::__assoc_legendre_q_t, 472
- __STDCPP_MATH_SPEC_FUNCS__
 - specfun.h, 687
- __T_n
 - __gnu_cxx::__chebyshev_t_t, 474
- __T_nm1
 - __gnu_cxx::__chebyshev_t_t, 474
- __T_nm2
 - __gnu_cxx::__chebyshev_t_t, 474
- __U_n
 - __gnu_cxx::__chebyshev_u_t, 476
- __U_nm1
 - __gnu_cxx::__chebyshev_u_t, 476
- __U_nm2
 - __gnu_cxx::__chebyshev_u_t, 476
- __V_n
 - __gnu_cxx::__chebyshev_v_t, 478
- __V_nm1
 - __gnu_cxx::__chebyshev_v_t, 478
- __V_nm2
 - __gnu_cxx::__chebyshev_v_t, 478
- __W_n
 - __gnu_cxx::__chebyshev_w_t, 480
- __W_nm1
 - __gnu_cxx::__chebyshev_w_t, 480
- __W_nm2
 - __gnu_cxx::__chebyshev_w_t, 480
- __Wronskian
 - __gnu_cxx::__airy_t, 466
 - __gnu_cxx::__coulomb_t, 482
 - __gnu_cxx::__cyl_bessel_t, 484
 - __gnu_cxx::__cyl_hankel_t, 487
 - __gnu_cxx::__cyl_mod_bessel_t, 489
 - __gnu_cxx::__fock_airy_t, 492
 - __gnu_cxx::__sph_bessel_t, 524
 - __gnu_cxx::__sph_hankel_t, 527
 - __gnu_cxx::__sph_mod_bessel_t, 529
- __airy
 - std::__detail, 271
- __airy_ai
 - std::__detail, 272
- __airy_arg
 - std::__detail, 272
- __airy_bi
 - std::__detail, 273
- __alpha1
 - __gnu_cxx::__jacobi_t, 511
 - __gnu_cxx::__laguerre_t, 513
- __am
 - __gnu_cxx::__jacobi_ellint_t, 507
- __assoc_laguerre
 - std::__detail, 273
- __assoc_legendre_p
 - std::__detail, 274
- __assoc_legendre_q
 - std::__detail, 274
- __bell
 - std::__detail, 275
- __bell_series
 - std::__detail, 275
- __bernoulli
 - std::__detail, 276
- __bernoulli_2n
 - std::__detail, 277
- __bernoulli_series
 - std::__detail, 278
- __beta
 - std::__detail, 278
- __beta1
 - __gnu_cxx::__jacobi_t, 511
- __beta_gamma
 - std::__detail, 279
- __beta_inc
 - std::__detail, 279
- __beta_lgamma
 - std::__detail, 280
- __beta_p
 - std::__detail, 281
- __beta_product
 - std::__detail, 281
- __binomial
 - std::__detail, 282
- __binomial_p
 - std::__detail, 283
- __binomial_pdf
 - std::__detail, 284
- __binomial_q
 - std::__detail, 284
- __bose_einstein
 - std::__detail, 285

- `__cauchy_p`
 - `std::__detail`, [286](#)
- `__cd`
 - `__gnu_cxx::__jacobi_ellint_t`, [507](#)
- `__chebyshev_recur`
 - `std::__detail`, [286](#)
- `__chebyshev_t`
 - `std::__detail`, [287](#)
- `__chebyshev_u`
 - `std::__detail`, [287](#)
- `__chebyshev_v`
 - `std::__detail`, [288](#)
- `__chebyshev_w`
 - `std::__detail`, [288](#)
- `__chi_squared_pdf`
 - `std::__detail`, [289](#)
- `__chi_squared_pdfc`
 - `std::__detail`, [289](#)
- `__chshint`
 - `std::__detail`, [290](#)
- `__chshint_cont_frac`
 - `std::__detail`, [290](#)
- `__chshint_series`
 - `std::__detail`, [291](#)
- `__clamp_0_m2pi`
 - `std::__detail`, [291](#)
- `__clamp_pi`
 - `std::__detail`, [291](#)
- `__clausen`
 - `std::__detail`, [292](#)
- `__clausen_cl`
 - `std::__detail`, [293](#)
- `__clausen_sl`
 - `std::__detail`, [294](#), [295](#)
- `__cn_deriv`
 - `__gnu_cxx::__jacobi_ellint_t`, [507](#)
- `__cn_value`
 - `__gnu_cxx::__jacobi_ellint_t`, [509](#)
- `__comp_ellint_1`
 - `std::__detail`, [295](#)
- `__comp_ellint_2`
 - `std::__detail`, [297](#)
- `__comp_ellint_3`
 - `std::__detail`, [297](#)
- `__comp_ellint_d`
 - `std::__detail`, [298](#)
- `__comp_ellint_rf`
 - `std::__detail`, [298](#)
- `__comp_ellint_rg`
 - `std::__detail`, [299](#)
- `__conf_hyperg`
 - `std::__detail`, [299](#)
- `__conf_hyperg_lim`
 - `std::__detail`, [300](#)
- `__conf_hyperg_lim_series`
 - `std::__detail`, [300](#)
- `__conf_hyperg_luke`
 - `std::__detail`, [301](#)
- `__conf_hyperg_series`
 - `std::__detail`, [301](#)
- `__cos_pi`
 - `std::__detail`, [302](#)
- `__cos_v`
 - `__gnu_cxx::__sincos_t`, [523](#)
- `__cosh_pi`
 - `std::__detail`, [302](#), [303](#)
- `__coshint`
 - `std::__detail`, [303](#)
- `__coulomb_CF1`
 - `std::__detail`, [304](#)
- `__coulomb_CF2`
 - `std::__detail`, [304](#)
- `__coulomb_f_recur`
 - `std::__detail`, [304](#)
- `__coulomb_g_recur`
 - `std::__detail`, [305](#)
- `__coulomb_norm`
 - `std::__detail`, [305](#)
- `__cpp_lib_math_special_functions`
 - `specfun.h`, [687](#)
- `__cs`
 - `__gnu_cxx::__jacobi_ellint_t`, [507](#)
- `__cyl_bessel`
 - `std::__detail`, [305](#)
- `__cyl_bessel_i`
 - `std::__detail`, [306](#)
- `__cyl_bessel_ij_series`
 - `std::__detail`, [307](#)
- `__cyl_bessel_ik`
 - `std::__detail`, [307](#)
- `__cyl_bessel_ik_asymp`
 - `std::__detail`, [308](#)
- `__cyl_bessel_ik_steep`
 - `std::__detail`, [309](#)
- `__cyl_bessel_j`
 - `std::__detail`, [309](#)
- `__cyl_bessel_jn`
 - `std::__detail`, [310](#)
- `__cyl_bessel_jn_asymp`
 - `std::__detail`, [310](#)
- `__cyl_bessel_jn_neg_arg`
 - `std::__detail`, [311](#)
- `__cyl_bessel_jn_steep`
 - `std::__detail`, [311](#)
- `__cyl_bessel_k`
 - `std::__detail`, [312](#)
- `__cyl_hankel_1`
 - `std::__detail`, [312](#), [313](#)

- `__cyl_hankel_2`
 - `std::__detail`, 314
- `__cyl_neumann`
 - `std::__detail`, 315
- `__cyl_neumann_n`
 - `std::__detail`, 315
- `__dawson`
 - `std::__detail`, 316
- `__dawson_cont_frac`
 - `std::__detail`, 316
- `__dawson_series`
 - `std::__detail`, 317
- `__dc`
 - `__gnu_cxx::__jacobi_ellint_t`, 507
- `__debye`
 - `std::__detail`, 317
- `__debye_region`
 - `std::__detail`, 318
- `__delta`
 - `std::__detail::__weierstrass_invariants_t`, 561
 - `std::__detail::__weierstrass_roots_t`, 564
- `__digamma`
 - `std::__detail`, 318
- `__digamma_asymp`
 - `std::__detail`, 319
- `__digamma_series`
 - `std::__detail`, 319
- `__dilog`
 - `std::__detail`, 320
- `__dirichlet_beta`
 - `std::__detail`, 320, 321
- `__dirichlet_eta`
 - `std::__detail`, 321, 322
- `__dirichlet_lambda`
 - `std::__detail`, 323
- `__dn_value`
 - `__gnu_cxx::__jacobi_ellint_t`, 509
- `__double_factorial`
 - `std::__detail`, 323
- `__ds`
 - `__gnu_cxx::__jacobi_ellint_t`, 508
- `__e1`
 - `std::__detail::__weierstrass_roots_t`, 565
- `__e2`
 - `std::__detail::__weierstrass_roots_t`, 565
- `__e3`
 - `std::__detail::__weierstrass_roots_t`, 565
- `__ellint_1`
 - `std::__detail`, 324
- `__ellint_2`
 - `std::__detail`, 324
- `__ellint_3`
 - `std::__detail`, 325
- `__ellint_cel`
 - `std::__detail`, 326
- `__ellint_d`
 - `std::__detail`, 326
- `__ellint_el1`
 - `std::__detail`, 326
- `__ellint_el2`
 - `std::__detail`, 326
- `__ellint_el3`
 - `std::__detail`, 327
- `__ellint_rc`
 - `std::__detail`, 327
- `__ellint_rd`
 - `std::__detail`, 328
- `__ellint_rf`
 - `std::__detail`, 329
- `__ellint_rg`
 - `std::__detail`, 329
- `__ellint_rj`
 - `std::__detail`, 330
- `__ellnome`
 - `std::__detail`, 331
 - `std::__detail::__jacobi_lattice_t`, 548
- `__ellnome_k`
 - `std::__detail`, 332
- `__ellnome_series`
 - `std::__detail`, 332
- `__eta_arg`
 - `__gnu_cxx::__coulomb_t`, 482
- `__euler`
 - `std::__detail`, 332, 333
- `__euler_series`
 - `std::__detail`, 333
- `__eulerian_1`
 - `std::__detail`, 333, 334
- `__eulerian_1_recur`
 - `std::__detail`, 334
- `__eulerian_2`
 - `std::__detail`, 335
- `__eulerian_2_recur`
 - `std::__detail`, 335, 336
- `__exp2`
 - `std::__detail`, 336
- `__expint`
 - `std::__detail`, 336, 337
- `__expint_E1`
 - `std::__detail`, 338
- `__expint_E1_asymp`
 - `std::__detail`, 338
- `__expint_E1_series`
 - `std::__detail`, 339
- `__expint_Ei`
 - `std::__detail`, 339
- `__expint_Ei_asymp`
 - `std::__detail`, 340

- __expint_Ei_series
 - std::__detail, 340
- __expint_En_asyp
 - std::__detail, 341
- __expint_En_cont_frac
 - std::__detail, 342
- __expint_En_large_n
 - std::__detail, 342
- __expint_En_recursion
 - std::__detail, 343
- __expint_En_series
 - std::__detail, 343
- __exponential_p
 - std::__detail, 344
- __exponential_pdf
 - std::__detail, 344
- __exponential_q
 - std::__detail, 345
- __factorial
 - std::__detail, 345
 - std::__detail::__Factorial_table, 597
- __fai_deriv
 - std::__detail::__AiryAuxilliaryState, 590
- __fai_value
 - std::__detail::__AiryAuxilliaryState, 591
- __falling_factorial
 - std::__detail, 345, 346
- __fermi_dirac
 - std::__detail, 346
- __fisher_f_p
 - std::__detail, 347
- __fisher_f_pdf
 - std::__detail, 347
- __fisher_f_q
 - std::__detail, 348
- __fock_airy
 - std::__detail, 349
- __fp_is_equal
 - __gnu_cxx, 240
- __fp_is_even_integer
 - __gnu_cxx, 240
- __fp_is_half_integer
 - __gnu_cxx, 241
- __fp_is_half_odd_integer
 - __gnu_cxx, 241
- __fp_is_integer
 - __gnu_cxx, 242
- __fp_is_odd_integer
 - __gnu_cxx, 243
- __fp_is_zero
 - __gnu_cxx, 243
- __fp_max_abs
 - __gnu_cxx, 244
- __fresnel
 - std::__detail, 349
- __fresnel_cont_frac
 - std::__detail, 350
- __fresnel_series
 - std::__detail, 350
- __g_2
 - std::__detail::__weierstrass_invariants_t, 561
- __g_3
 - std::__detail::__weierstrass_invariants_t, 561
- __gai_deriv
 - std::__detail::__AiryAuxilliaryState, 591
- __gai_value
 - std::__detail::__AiryAuxilliaryState, 591
- __gamma
 - std::__detail, 350, 351
- __gamma_1_value
 - __gnu_cxx::__gamma_temme_t, 497
- __gamma_2_value
 - __gnu_cxx::__gamma_temme_t, 497
- __gamma_cont_frac
 - std::__detail, 351
- __gamma_minus_value
 - __gnu_cxx::__gamma_temme_t, 497
- __gamma_p
 - std::__detail, 351, 352
- __gamma_pdf
 - std::__detail, 352
- __gamma_plus_value
 - __gnu_cxx::__gamma_temme_t, 498
- __gamma_q
 - std::__detail, 353
- __gamma_reciprocal
 - std::__detail, 354
- __gamma_reciprocal_series
 - std::__detail, 354
- __gamma_series
 - std::__detail, 355
- __gamma_temme
 - std::__detail, 355
- __gappa_p_value
 - __gnu_cxx::__gappa_pq_t, 499
- __gappa_q_value
 - __gnu_cxx::__gappa_pq_t, 499
- __gauss
 - std::__detail, 356
- __gegenbauer_recur
 - std::__detail, 356
- __gegenbauer_zeros
 - std::__detail, 357
- __gnu_cxx, 225
 - __fp_is_equal, 240
 - __fp_is_even_integer, 240
 - __fp_is_half_integer, 241
 - __fp_is_half_odd_integer, 241

- __fp_is_integer, 242
- __fp_is_odd_integer, 243
- __fp_is_zero, 243
- __fp_max_abs, 244
- __parity, 244
- gauss_quad_type, 239
- __gnu_cxx::__airy_t
 - __Ai_deriv, 466
 - __Ai_value, 466
 - __Bi_deriv, 466
 - __Bi_value, 466
 - __Wronskian, 466
 - __x_arg, 467
- __gnu_cxx::__airy_t<_Tx, _Tp>, 465
- __gnu_cxx::__assoc_legendre_p_t
 - __P_lm, 468
 - __P_lm1m, 469
 - __P_lm2m, 469
 - __l, 468
 - __m, 468
 - __phase, 469
 - __x, 469
- deriv, 468
- __gnu_cxx::__assoc_legendre_p_t<_Tp>, 467
- __gnu_cxx::__assoc_legendre_q_t
 - __Q_lm, 471
 - __Q_lmm1, 472
 - __Q_lmm2, 472
 - __l, 471
 - __m, 471
 - __phase, 471
 - __x, 472
- deriv, 471
- __gnu_cxx::__assoc_legendre_q_t<_Tp>, 470
- __gnu_cxx::__chebyshev_t_t
 - __T_n, 474
 - __T_nm1, 474
 - __T_nm2, 474
 - __n, 474
 - __x, 474
- deriv, 473
- deriv2, 473
- __gnu_cxx::__chebyshev_t_t<_Tp>, 473
- __gnu_cxx::__chebyshev_u_t
 - __U_n, 476
 - __U_nm1, 476
 - __U_nm2, 476
 - __n, 476
 - __x, 476
- deriv, 475
- __gnu_cxx::__chebyshev_u_t<_Tp>, 475
- __gnu_cxx::__chebyshev_v_t
 - __V_n, 478
 - __V_nm1, 478
 - __V_nm2, 478
 - __n, 478
 - __x, 478
- deriv, 477
- __gnu_cxx::__chebyshev_v_t<_Tp>, 477
- __gnu_cxx::__chebyshev_w_t
 - __W_n, 480
 - __W_nm1, 480
 - __W_nm2, 480
 - __n, 480
 - __x, 480
- deriv, 479
- __gnu_cxx::__chebyshev_w_t<_Tp>, 479
- __gnu_cxx::__coulomb_t
 - __F_deriv, 482
 - __F_value, 482
 - __G_deriv, 482
 - __G_value, 483
 - __Wronskian, 482
 - __eta_arg, 482
 - __l, 483
 - __rho_arg, 483
- __gnu_cxx::__coulomb_t<_Teta, _Trho, _Tp>, 481
- __gnu_cxx::__cyl_bessel_t
 - __J_deriv, 485
 - __J_value, 485
 - __N_deriv, 485
 - __N_value, 485
 - __Wronskian, 484
 - __nu_arg, 485
 - __x_arg, 486
- __gnu_cxx::__cyl_bessel_t<_Tnu, _Tx, _Tp>, 484
- __gnu_cxx::__cyl_hankel_t
 - __H1_deriv, 487
 - __H1_value, 487
 - __H2_deriv, 487
 - __H2_value, 488
 - __Wronskian, 487
 - __nu_arg, 488
 - __x_arg, 488
- __gnu_cxx::__cyl_hankel_t<_Tnu, _Tx, _Tp>, 486
- __gnu_cxx::__cyl_mod_bessel_t
 - __I_deriv, 490
 - __I_value, 490
 - __K_deriv, 490
 - __K_value, 490
 - __Wronskian, 489
 - __nu_arg, 490
 - __x_arg, 491
- __gnu_cxx::__cyl_mod_bessel_t<_Tnu, _Tx, _Tp>, 489
- __gnu_cxx::__fock_airy_t
 - __Wronskian, 492
 - __w1_deriv, 492
 - __w1_value, 492

- __w2_deriv, [492](#)
- __w2_value, [493](#)
- __x_arg, [493](#)
- __gnu_cxx::__fock_airy_t<_Tx, _Tp>, [491](#)
- __gnu_cxx::__fp_is_integer_t, [493](#)
- __is_integral, [494](#)
- __value, [494](#)
- operator bool, [494](#)
- operator(), [494](#)
- __gnu_cxx::__gamma_inc_t
 - __lgamma_value, [495](#)
 - __tgamma_value, [495](#)
- __gnu_cxx::__gamma_inc_t<_Tp>, [495](#)
- __gnu_cxx::__gamma_temme_t
 - __gamma_1_value, [497](#)
 - __gamma_2_value, [497](#)
 - __gamma_minus_value, [497](#)
 - __gamma_plus_value, [498](#)
 - __mu_arg, [498](#)
- __gnu_cxx::__gamma_temme_t<_Tp>, [496](#)
- __gnu_cxx::__gappa_pq_t
 - __gappa_p_value, [499](#)
 - __gappa_q_value, [499](#)
- __gnu_cxx::__gappa_pq_t<_Tp>, [498](#)
- __gnu_cxx::__gegenbauer_t
 - __C_n, [500](#)
 - __C_nm1, [500](#)
 - __C_nm2, [501](#)
 - __lambda, [501](#)
 - __n, [501](#)
 - __x, [501](#)
 - deriv, [500](#)
- __gnu_cxx::__gegenbauer_t<_Tp>, [499](#)
- __gnu_cxx::__hermite_he_t
 - __He_n, [503](#)
 - __He_nm1, [503](#)
 - __He_nm2, [503](#)
 - __n, [503](#)
 - __x, [503](#)
 - deriv, [502](#)
 - deriv2, [502](#)
- __gnu_cxx::__hermite_he_t<_Tp>, [502](#)
- __gnu_cxx::__hermite_t
 - __H_n, [505](#)
 - __H_nm1, [505](#)
 - __H_nm2, [505](#)
 - __n, [505](#)
 - __x, [506](#)
 - deriv, [504](#)
 - deriv2, [505](#)
- __gnu_cxx::__hermite_t<_Tp>, [504](#)
- __gnu_cxx::__jacobi_ellint_t
 - __am, [507](#)
 - __cd, [507](#)
 - __cn_deriv, [507](#)
 - __cn_value, [509](#)
 - __cs, [507](#)
 - __dc, [507](#)
 - __dn_value, [509](#)
 - __ds, [508](#)
 - __nc, [508](#)
 - __nd, [508](#)
 - __ns, [508](#)
 - __sc, [508](#)
 - __sd, [509](#)
 - __sn_deriv, [509](#)
 - __sn_value, [510](#)
- __gnu_cxx::__jacobi_ellint_t<_Tp>, [506](#)
- __gnu_cxx::__jacobi_t
 - __P_n, [512](#)
 - __P_nm1, [512](#)
 - __P_nm2, [512](#)
 - __alpha1, [511](#)
 - __beta1, [511](#)
 - __n, [511](#)
 - __x, [512](#)
 - deriv, [511](#)
- __gnu_cxx::__jacobi_t<_Tp>, [510](#)
- __gnu_cxx::__laguerre_t
 - __L_n, [514](#)
 - __L_nm1, [514](#)
 - __L_nm2, [514](#)
 - __alpha1, [513](#)
 - __n, [514](#)
 - __x, [514](#)
 - deriv, [513](#)
- __gnu_cxx::__laguerre_t<_Tpa, _Tp>, [513](#)
- __gnu_cxx::__legendre_p_t
 - __P_l, [516](#)
 - __P_lm1, [516](#)
 - __P_lm2, [517](#)
 - __l, [516](#)
 - __x, [517](#)
 - deriv, [516](#)
 - lobatto, [516](#)
- __gnu_cxx::__legendre_p_t<_Tp>, [515](#)
- __gnu_cxx::__legendre_q_t
 - __Q_l, [518](#)
 - __Q_lm1, [519](#)
 - __Q_lm2, [519](#)
 - __l, [518](#)
 - __x, [519](#)
 - deriv, [518](#)
- __gnu_cxx::__legendre_q_t<_Tp>, [517](#)
- __gnu_cxx::__lgamma_t
 - __lgamma_sign, [520](#)
 - __lgamma_value, [520](#)
- __gnu_cxx::__lgamma_t<_Tp>, [519](#)

- __gnu_cxx::__quadrature_point_t
 - __point, 522
 - __quadrature_point_t, 521
 - __weight, 522
- __gnu_cxx::__quadrature_point_t<_Tp>, 521
- __gnu_cxx::__sincos_t
 - __cos_v, 523
 - __sin_v, 523
- __gnu_cxx::__sincos_t<_Tp>, 522
- __gnu_cxx::__sph_bessel_t
 - __Wronskian, 524
 - __j_deriv, 524
 - __j_value, 525
 - __n_arg, 525
 - __n_deriv, 525
 - __n_value, 525
 - __x_arg, 526
- __gnu_cxx::__sph_bessel_t<_Tn, _Tx, _Tp>, 523
- __gnu_cxx::__sph_hankel_t
 - __Wronskian, 527
 - __h1_deriv, 527
 - __h1_value, 527
 - __h2_deriv, 527
 - __h2_value, 528
 - __n_arg, 528
 - __x_arg, 528
- __gnu_cxx::__sph_hankel_t<_Tn, _Tx, _Tp>, 526
- __gnu_cxx::__sph_mod_bessel_t
 - __Wronskian, 529
 - __i_deriv, 530
 - __i_value, 530
 - __k_deriv, 530
 - __k_value, 530
 - __n_arg, 530
 - __x_arg, 531
- __gnu_cxx::__sph_mod_bessel_t<_Tn, _Tx, _Tp>, 529
- __gnu_cxx::__stirling_1_t
 - __sigma, 534
 - begin, 532
 - const_iterator, 532
 - degree, 533
 - end, 533
 - iterator, 532
 - operator(), 533
 - operator[], 533
- __gnu_cxx::__stirling_1_t<_Tp>, 531
- __gnu_cxx::__stirling_2_t
 - __S, 537
 - begin, 535
 - const_iterator, 535
 - degree, 536
 - end, 536
 - iterator, 535
 - operator(), 536
 - operator[], 536
- __gnu_cxx::__stirling_2_t<_Tp>, 534
- __h1_deriv
 - __gnu_cxx::__sph_hankel_t, 527
- __h1_value
 - __gnu_cxx::__sph_hankel_t, 527
- __h2_deriv
 - __gnu_cxx::__sph_hankel_t, 527
- __h2_value
 - __gnu_cxx::__sph_hankel_t, 528
- __hai_deriv
 - std::__detail::__AiryAuxilliaryState, 591
- __hai_value
 - std::__detail::__AiryAuxilliaryState, 591
- __hankel
 - std::__detail, 358
- __hankel_debye
 - std::__detail, 358
- __hankel_params
 - std::__detail, 359
- __hankel_uniform
 - std::__detail, 359
- __hankel_uniform_olver
 - std::__detail, 360
- __hankel_uniform_outer
 - std::__detail, 361
- __hankel_uniform_sum
 - std::__detail, 361
- __harmonic_number
 - std::__detail, 362
- __hermite
 - std::__detail, 363
- __hermite_asymp
 - std::__detail, 363
- __hermite_recur
 - std::__detail, 364
- __hermite_zeros
 - std::__detail, 365
- __heuman_lambda
 - std::__detail, 365
- __hurwitz_zeta
 - std::__detail, 366
- __hurwitz_zeta_euler_maclaurin
 - std::__detail, 366
- __hurwitz_zeta_polylog
 - std::__detail, 367
- __hydrogen
 - std::__detail, 368
- __hyperg
 - std::__detail, 368
- __hyperg_luke
 - std::__detail, 369
- __hyperg_recur
 - std::__detail, 369

- `__hyperg_reflect`
 - `std::__detail`, 370
- `__hyperg_series`
 - `std::__detail`, 370
- `__i_deriv`
 - `__gnu_cxx::__sph_mod_bessel_t`, 530
- `__i_value`
 - `__gnu_cxx::__sph_mod_bessel_t`, 530
- `__ibeta_cont_frac`
 - `std::__detail`, 371
- `__is_integral`
 - `__gnu_cxx::__fp_is_integer_t`, 494
- `__j_deriv`
 - `__gnu_cxx::__sph_bessel_t`, 524
- `__j_value`
 - `__gnu_cxx::__sph_bessel_t`, 525
- `__jacobi_ellint`
 - `std::__detail`, 372
- `__jacobi_lattice_t`
 - `std::__detail::__jacobi_lattice_t`, 547
- `__jacobi_recur`
 - `std::__detail`, 372
- `__jacobi_theta_0_t`
 - `std::__detail::__jacobi_theta_0_t`, 555
- `__jacobi_theta_1`
 - `std::__detail`, 373, 374
- `__jacobi_theta_1_prod`
 - `std::__detail`, 374
- `__jacobi_theta_1_sum`
 - `std::__detail`, 375
- `__jacobi_theta_2`
 - `std::__detail`, 376
- `__jacobi_theta_2_prod`
 - `std::__detail`, 377
- `__jacobi_theta_2_sum`
 - `std::__detail`, 378
- `__jacobi_theta_3`
 - `std::__detail`, 378, 379
- `__jacobi_theta_3_prod`
 - `std::__detail`, 380
- `__jacobi_theta_3_sum`
 - `std::__detail`, 380
- `__jacobi_theta_4`
 - `std::__detail`, 381, 382
- `__jacobi_theta_4_prod`
 - `std::__detail`, 382
- `__jacobi_theta_4_sum`
 - `std::__detail`, 383
- `__jacobi_zeros`
 - `std::__detail`, 383
- `__jacobi_zeta`
 - `std::__detail`, 384
- `__k_deriv`
 - `__gnu_cxx::__sph_mod_bessel_t`, 530
- `__k_value`
 - `__gnu_cxx::__sph_mod_bessel_t`, 530
- `__klein_j`
 - `std::__detail::__weierstrass_invariants_t`, 561
- `__kolmogorov_p`
 - `std::__detail`, 384
- `__l`
 - `__gnu_cxx::__assoc_legendre_p_t`, 468
 - `__gnu_cxx::__assoc_legendre_q_t`, 471
 - `__gnu_cxx::__coulomb_t`, 483
 - `__gnu_cxx::__legendre_p_t`, 516
 - `__gnu_cxx::__legendre_q_t`, 518
- `__laguerre`
 - `std::__detail`, 384, 385
- `__laguerre_hyperg`
 - `std::__detail`, 386
- `__laguerre_large_n`
 - `std::__detail`, 387
- `__laguerre_recur`
 - `std::__detail`, 388
- `__laguerre_zeros`
 - `std::__detail`, 389
- `__lah`
 - `std::__detail`, 389
- `__lah_recur`
 - `std::__detail`, 389, 390
- `__lambda`
 - `__gnu_cxx::__gegenbauer_t`, 501
- `__lanczos_binet1p`
 - `std::__detail`, 390
- `__lanczos_log_gamma1p`
 - `std::__detail`, 391
- `__legendre_p`
 - `std::__detail`, 391
- `__legendre_q`
 - `std::__detail`, 392
- `__legendre_q_series`
 - `std::__detail`, 393
- `__legendre_zeros`
 - `std::__detail`, 393
- `__lgamma_sign`
 - `__gnu_cxx::__lgamma_t`, 520
- `__lgamma_value`
 - `__gnu_cxx::__gamma_inc_t`, 495
 - `__gnu_cxx::__lgamma_t`, 520
- `__log_binomial`
 - `std::__detail`, 393, 394
- `__log_binomial_sign`
 - `std::__detail`, 394, 395
- `__log_double_factorial`
 - `std::__detail`, 395, 396
- `__log_factorial`
 - `std::__detail`, 396
 - `std::__detail::__Factorial_table`, 597

- `__log_falling_factorial`
 - `std::__detail`, 397
- `__log_gamma`
 - `std::__detail`, 397, 398
- `__log_gamma_bernoulli`
 - `std::__detail`, 398
- `__log_gamma_sign`
 - `std::__detail`, 399
- `__log_rising_factorial`
 - `std::__detail`, 400
- `__log_stirling_1`
 - `std::__detail`, 400
- `__log_stirling_1_sign`
 - `std::__detail`, 400
- `__log_stirling_2`
 - `std::__detail`, 401
- `__logint`
 - `std::__detail`, 401
- `__logistic_p`
 - `std::__detail`, 402
- `__logistic_pdf`
 - `std::__detail`, 402
- `__lognormal_p`
 - `std::__detail`, 402
- `__lognormal_pdf`
 - `std::__detail`, 403
- `__m`
 - `__gnu_cxx::__assoc_legendre_p_t`, 468
 - `__gnu_cxx::__assoc_legendre_q_t`, 471
 - `std::__detail::__jacobi_lattice_t::__arg_t`, 551
- `__max_FGH`
 - `std::__detail`, 458
- `__max_FGH< double >`
 - `std::__detail`, 458
- `__max_FGH< float >`
 - `std::__detail`, 458
- `__mu_arg`
 - `__gnu_cxx::__gamma_temme_t`, 498
- `__n`
 - `__gnu_cxx::__chebyshev_t_t`, 474
 - `__gnu_cxx::__chebyshev_u_t`, 476
 - `__gnu_cxx::__chebyshev_v_t`, 478
 - `__gnu_cxx::__chebyshev_w_t`, 480
 - `__gnu_cxx::__gegenbauer_t`, 501
 - `__gnu_cxx::__hermite_he_t`, 503
 - `__gnu_cxx::__hermite_t`, 505
 - `__gnu_cxx::__jacobi_t`, 511
 - `__gnu_cxx::__laguerre_t`, 514
 - `std::__detail::__jacobi_lattice_t::__arg_t`, 552
 - `std::__detail::__Factorial_table`, 597
- `__n_arg`
 - `__gnu_cxx::__sph_bessel_t`, 525
 - `__gnu_cxx::__sph_hankel_t`, 528
 - `__gnu_cxx::__sph_mod_bessel_t`, 530
- `__n_deriv`
 - `__gnu_cxx::__sph_bessel_t`, 525
- `__n_value`
 - `__gnu_cxx::__sph_bessel_t`, 525
- `__nc`
 - `__gnu_cxx::__jacobi_ellint_t`, 508
- `__nd`
 - `__gnu_cxx::__jacobi_ellint_t`, 508
- `__normal_p`
 - `std::__detail`, 403
- `__normal_pdf`
 - `std::__detail`, 403
- `__ns`
 - `__gnu_cxx::__jacobi_ellint_t`, 508
- `__nu_arg`
 - `__gnu_cxx::__cyl_bessel_t`, 485
 - `__gnu_cxx::__cyl_hankel_t`, 488
 - `__gnu_cxx::__cyl_mod_bessel_t`, 490
- `__omega_1`
 - `std::__detail::__jacobi_lattice_t`, 548
- `__omega_2`
 - `std::__detail::__jacobi_lattice_t`, 548
- `__omega_3`
 - `std::__detail::__jacobi_lattice_t`, 549
- `__owens_t`
 - `std::__detail`, 404
- `__parity`
 - `__gnu_cxx`, 244
- `__periodic_zeta`
 - `std::__detail`, 405
- `__phase`
 - `__gnu_cxx::__assoc_legendre_p_t`, 469
 - `__gnu_cxx::__assoc_legendre_q_t`, 471
- `__point`
 - `__gnu_cxx::__quadrature_point_t`, 522
- `__polar_pi`
 - `std::__detail`, 405
- `__polygamma`
 - `std::__detail`, 406
- `__polylog`
 - `std::__detail`, 406, 407
- `__polylog_exp`
 - `std::__detail`, 407
- `__polylog_exp_asymp`
 - `std::__detail`, 408
- `__polylog_exp_neg`
 - `std::__detail`, 409, 410
- `__polylog_exp_neg_int`
 - `std::__detail`, 410, 411
- `__polylog_exp_neg_real`
 - `std::__detail`, 411, 412
- `__polylog_exp_pos`
 - `std::__detail`, 413, 414
- `__polylog_exp_pos_int`

- std::__detail, 415
- __polylog_exp_pos_real
 - std::__detail, 416, 417
- __polylog_exp_sum
 - std::__detail, 417
- __prob_hermite_recur
 - std::__detail, 418
- __quadrature_point_t
 - __gnu_cxx::__quadrature_point_t, 521
- __radial_jacobi
 - std::__detail, 419
- __radial_jacobi_zeros
 - std::__detail, 420
- __reduce
 - std::__detail::__jacobi_lattice_t, 549
- __rho_arg
 - __gnu_cxx::__coulomb_t, 483
- __rice_pdf
 - std::__detail, 420
- __riemann_zeta
 - std::__detail, 421
- __riemann_zeta_euler_maclaurin
 - std::__detail, 422
- __riemann_zeta_glob
 - std::__detail, 422
- __riemann_zeta_laurent
 - std::__detail, 422
- __riemann_zeta_m_1
 - std::__detail, 422
- __riemann_zeta_m_1_glob
 - std::__detail, 423
- __riemann_zeta_product
 - std::__detail, 423
- __riemann_zeta_sum
 - std::__detail, 424
- __rising_factorial
 - std::__detail, 424, 425
- __sc
 - __gnu_cxx::__jacobi_ellint_t, 508
- __sd
 - __gnu_cxx::__jacobi_ellint_t, 509
- __sigma
 - __gnu_cxx::__stirling_1_t, 534
- __sin_pi
 - std::__detail, 425, 426
- __sin_v
 - __gnu_cxx::__sincos_t, 523
- __sinc
 - std::__detail, 426
- __sinc_pi
 - std::__detail, 426
- __sincos
 - std::__detail, 427
- __sincos_pi
 - std::__detail, 428
- __sincosint
 - std::__detail, 428
- __sincosint_asymp
 - std::__detail, 428
- __sincosint_cont_frac
 - std::__detail, 429
- __sincosint_series
 - std::__detail, 429
- __sinh_pi
 - std::__detail, 429, 430
- __sinhc
 - std::__detail, 430
- __sinhc_pi
 - std::__detail, 430
- __sinhint
 - std::__detail, 431
- __sn_deriv
 - __gnu_cxx::__jacobi_ellint_t, 509
- __sn_value
 - __gnu_cxx::__jacobi_ellint_t, 510
- __sph_bessel
 - std::__detail, 431, 432
- __sph_bessel_ik
 - std::__detail, 433
- __sph_bessel_jn
 - std::__detail, 433
- __sph_bessel_jn_neg_arg
 - std::__detail, 434
- __sph_hankel
 - std::__detail, 434
- __sph_hankel_1
 - std::__detail, 435
- __sph_hankel_2
 - std::__detail, 436, 437
- __sph_harmonic
 - std::__detail, 437
- __sph_legendre
 - std::__detail, 438
- __sph_neumann
 - std::__detail, 439
- __spouge_binet1p
 - std::__detail, 440
- __spouge_log_gamma1p
 - std::__detail, 440
- __stirling_1
 - std::__detail, 441, 442
- __stirling_1_recur
 - std::__detail, 442, 443
- __stirling_2
 - std::__detail, 443
- __stirling_2_recur
 - std::__detail, 444
- __stirling_2_series

- std::__detail, 445
- __student_t_p
 - std::__detail, 445
- __student_t_pdf
 - std::__detail, 446
- __student_t_q
 - std::__detail, 446
- __tan_pi
 - std::__detail, 447
- __tanh_pi
 - std::__detail, 447, 448
- __tau
 - std::__detail::__jacobi_lattice_t, 550
- __tau_t
 - std::__detail::__jacobi_lattice_t::__tau_t, 553
- __tgamma
 - std::__detail, 448
- __tgamma_lower
 - std::__detail, 448
- __tgamma_value
 - __gnu_cxx::__gamma_inc_t, 495
- __theta_1
 - std::__detail, 449
- __theta_2
 - std::__detail, 449
- __theta_2_asymp
 - std::__detail, 450
- __theta_2_sum
 - std::__detail, 450
- __theta_3
 - std::__detail, 451
- __theta_3_asymp
 - std::__detail, 451
- __theta_3_sum
 - std::__detail, 452
- __theta_4
 - std::__detail, 452
- __theta_c
 - std::__detail, 453
- __theta_d
 - std::__detail, 453
- __theta_n
 - std::__detail, 453
- __theta_s
 - std::__detail, 454
- __tricoli_u
 - std::__detail, 454
- __tricoli_u_naive
 - std::__detail, 455
- __val
 - std::__detail::__jacobi_lattice_t::__tau_t, 553
- __value
 - __gnu_cxx::__fp_is_integer_t, 494
- __w1_deriv
 - __gnu_cxx::__fock_airy_t, 492
- __w1_value
 - __gnu_cxx::__fock_airy_t, 492
- __w2_deriv
 - __gnu_cxx::__fock_airy_t, 492
- __w2_value
 - __gnu_cxx::__fock_airy_t, 493
- __weibull_p
 - std::__detail, 456
- __weibull_pdf
 - std::__detail, 456
- __weierstrass_invariants_t
 - std::__detail::__weierstrass_invariants_t, 560
- __weierstrass_roots_t
 - std::__detail::__weierstrass_roots_t, 564
- __weight
 - __gnu_cxx::__quadrature_point_t, 522
- __x
 - __gnu_cxx::__assoc_legendre_p_t, 469
 - __gnu_cxx::__assoc_legendre_q_t, 472
 - __gnu_cxx::__chebyshev_t_t, 474
 - __gnu_cxx::__chebyshev_u_t, 476
 - __gnu_cxx::__chebyshev_v_t, 478
 - __gnu_cxx::__chebyshev_w_t, 480
 - __gnu_cxx::__gegenbauer_t, 501
 - __gnu_cxx::__hermite_he_t, 503
 - __gnu_cxx::__hermite_t, 506
 - __gnu_cxx::__jacobi_t, 512
 - __gnu_cxx::__laguerre_t, 514
 - __gnu_cxx::__legendre_p_t, 517
 - __gnu_cxx::__legendre_q_t, 519
- __x_arg
 - __gnu_cxx::__airy_t, 467
 - __gnu_cxx::__cyl_bessel_t, 486
 - __gnu_cxx::__cyl_hankel_t, 488
 - __gnu_cxx::__cyl_mod_bessel_t, 491
 - __gnu_cxx::__fock_airy_t, 493
 - __gnu_cxx::__sph_bessel_t, 526
 - __gnu_cxx::__sph_hankel_t, 528
 - __gnu_cxx::__sph_mod_bessel_t, 531
- __z
 - std::__detail::__jacobi_lattice_t::__arg_t, 552
 - std::__detail::__AiryAuxilliaryState, 592
 - std::__detail::__AiryState, 594
- __zernike
 - std::__detail, 456
- __znorm1
 - std::__detail, 457
- __znorm2
 - std::__detail, 458
- airy_ai
 - GNU Extended Mathematical Special Functions, 60, 61

- airy_aif
 - GNU Extended Mathematical Special Functions, [61](#)
- airy_ail
 - GNU Extended Mathematical Special Functions, [61](#)
- airy_bi
 - GNU Extended Mathematical Special Functions, [62](#)
- airy_bif
 - GNU Extended Mathematical Special Functions, [63](#)
- airy_bil
 - GNU Extended Mathematical Special Functions, [63](#)
- assoc_laguerre
 - C++17/IS29124 Mathematical Special Functions, [15](#), [16](#)
- assoc_laguerref
 - C++17/IS29124 Mathematical Special Functions, [18](#)
- assoc_laguerrel
 - C++17/IS29124 Mathematical Special Functions, [18](#)
- assoc_legendre
 - C++17/IS29124 Mathematical Special Functions, [18](#)
- assoc_legendre_q
 - GNU Extended Mathematical Special Functions, [63](#)
- assoc_legendre_qf
 - GNU Extended Mathematical Special Functions, [64](#)
- assoc_legendre_ql
 - GNU Extended Mathematical Special Functions, [65](#)
- assoc_legendref
 - C++17/IS29124 Mathematical Special Functions, [19](#)
- assoc_legendrel
 - C++17/IS29124 Mathematical Special Functions, [20](#)
- begin
 - `__gnu_cxx::__stirling_1_t`, [532](#)
 - `__gnu_cxx::__stirling_2_t`, [535](#)
- bell
 - GNU Extended Mathematical Special Functions, [65](#)
- bernoulli
 - GNU Extended Mathematical Special Functions, [66](#)
- bernoullif
 - GNU Extended Mathematical Special Functions, [67](#)
- bernoullil
 - GNU Extended Mathematical Special Functions, [67](#)
- beta
 - C++17/IS29124 Mathematical Special Functions, [20](#)
- betaf
 - C++17/IS29124 Mathematical Special Functions, [21](#)
- betal
 - C++17/IS29124 Mathematical Special Functions, [21](#)
- binomial
 - GNU Extended Mathematical Special Functions, [67](#)
- binomial_p
 - GNU Extended Mathematical Special Functions, [68](#)
- binomial_pdf
 - GNU Extended Mathematical Special Functions, [69](#)
- binomialf
 - GNU Extended Mathematical Special Functions, [69](#)
- binomiall
 - GNU Extended Mathematical Special Functions, [69](#)
- bose_einstein
 - GNU Extended Mathematical Special Functions, [70](#)
- bose_einsteinf
 - GNU Extended Mathematical Special Functions, [71](#)
- bose_einsteinl
 - GNU Extended Mathematical Special Functions, [71](#)
- C++17/IS29124 Mathematical Special Functions, [13](#)
 - assoc_laguerre, [15](#), [16](#)
 - assoc_laguerref, [18](#)
 - assoc_laguerrel, [18](#)
 - assoc_legendre, [18](#)
 - assoc_legendref, [19](#)
 - assoc_legendrel, [20](#)
 - beta, [20](#)
 - betaf, [21](#)
 - betal, [21](#)
 - comp_ellint_1, [21](#)
 - comp_ellint_1f, [22](#)
 - comp_ellint_1l, [22](#)
 - comp_ellint_2, [23](#)
 - comp_ellint_2f, [24](#)
 - comp_ellint_2l, [24](#)
 - comp_ellint_3, [24](#)
 - comp_ellint_3f, [25](#)
 - comp_ellint_3l, [25](#)
 - cyl_bessel_i, [25](#)
 - cyl_bessel_if, [26](#)
 - cyl_bessel_il, [26](#)
 - cyl_bessel_j, [27](#)
 - cyl_bessel_jf, [27](#)
 - cyl_bessel_jl, [28](#)
 - cyl_bessel_k, [28](#)
 - cyl_bessel_kf, [29](#)
 - cyl_bessel_kl, [29](#)
 - cyl_neumann, [29](#)
 - cyl_neumannf, [30](#)
 - cyl_neumannl, [30](#)
 - ellint_1, [31](#)
 - ellint_1f, [32](#)
 - ellint_1l, [32](#)
 - ellint_2, [32](#)
 - ellint_2f, [33](#)
 - ellint_2l, [33](#)
 - ellint_3, [33](#)
 - ellint_3f, [34](#)
 - ellint_3l, [35](#)
 - expint, [35](#)
 - expintf, [36](#)
 - expintl, [36](#)
 - hermite, [36](#)

- hermitef, [37](#)
- hermitel, [37](#)
- laguerre, [38](#)
- laguerref, [38](#)
- laguerrel, [39](#)
- legendre, [39](#)
- legendref, [40](#)
- legendrel, [40](#)
- riemann_zeta, [40](#)
- riemann_zetaf, [41](#)
- riemann_zetal, [41](#)
- sph_bessel, [42](#)
- sph_besself, [42](#)
- sph_bessell, [43](#)
- sph_legendre, [43](#)
- sph_legendref, [44](#)
- sph_legendrel, [44](#)
- sph_neumann, [44](#)
- sph_neumannf, [45](#)
- sph_neumannl, [45](#)
- chebyshev_t
 - GNU Extended Mathematical Special Functions, [71](#)
- chebyshev_tf
 - GNU Extended Mathematical Special Functions, [72](#)
- chebyshev_tl
 - GNU Extended Mathematical Special Functions, [72](#)
- chebyshev_u
 - GNU Extended Mathematical Special Functions, [72](#)
- chebyshev_uf
 - GNU Extended Mathematical Special Functions, [73](#)
- chebyshev_ul
 - GNU Extended Mathematical Special Functions, [73](#)
- chebyshev_v
 - GNU Extended Mathematical Special Functions, [74](#)
- chebyshev_vf
 - GNU Extended Mathematical Special Functions, [74](#)
- chebyshev_vl
 - GNU Extended Mathematical Special Functions, [75](#)
- chebyshev_w
 - GNU Extended Mathematical Special Functions, [75](#)
- chebyshev_wf
 - GNU Extended Mathematical Special Functions, [76](#)
- chebyshev_wl
 - GNU Extended Mathematical Special Functions, [76](#)
- clausen
 - GNU Extended Mathematical Special Functions, [76](#), [77](#)
- clausen_cl
 - GNU Extended Mathematical Special Functions, [78](#)
- clausen_clf
 - GNU Extended Mathematical Special Functions, [78](#)
- clausen_cll
 - GNU Extended Mathematical Special Functions, [79](#)
- clausen_sl
 - GNU Extended Mathematical Special Functions, [79](#)
- clausen_slf
 - GNU Extended Mathematical Special Functions, [80](#)
- clausen_sll
 - GNU Extended Mathematical Special Functions, [80](#)
- clausenf
 - GNU Extended Mathematical Special Functions, [80](#)
- clausenl
 - GNU Extended Mathematical Special Functions, [81](#)
- comp_ellint_1
 - C++17/IS29124 Mathematical Special Functions, [21](#)
- comp_ellint_1f
 - C++17/IS29124 Mathematical Special Functions, [22](#)
- comp_ellint_1l
 - C++17/IS29124 Mathematical Special Functions, [22](#)
- comp_ellint_2
 - C++17/IS29124 Mathematical Special Functions, [23](#)
- comp_ellint_2f
 - C++17/IS29124 Mathematical Special Functions, [24](#)
- comp_ellint_2l
 - C++17/IS29124 Mathematical Special Functions, [24](#)
- comp_ellint_3
 - C++17/IS29124 Mathematical Special Functions, [24](#)
- comp_ellint_3f
 - C++17/IS29124 Mathematical Special Functions, [25](#)
- comp_ellint_3l
 - C++17/IS29124 Mathematical Special Functions, [25](#)
- comp_ellint_d
 - GNU Extended Mathematical Special Functions, [81](#)
- comp_ellint_df
 - GNU Extended Mathematical Special Functions, [82](#)
- comp_ellint_dl
 - GNU Extended Mathematical Special Functions, [82](#)
- comp_ellint_rf
 - GNU Extended Mathematical Special Functions, [82](#), [83](#)
- comp_ellint_rg
 - GNU Extended Mathematical Special Functions, [84](#)
- conf_hyperg
 - GNU Extended Mathematical Special Functions, [85](#)
- conf_hyperg_lim
 - GNU Extended Mathematical Special Functions, [85](#)
- conf_hyperg_limf
 - GNU Extended Mathematical Special Functions, [86](#)
- conf_hyperg_liml
 - GNU Extended Mathematical Special Functions, [86](#)
- conf_hypergf
 - GNU Extended Mathematical Special Functions, [86](#)
- conf_hypergl
 - GNU Extended Mathematical Special Functions, [87](#)
- const_iterator
 - __gnu_cxx::__stirling_1_t, [532](#)
 - __gnu_cxx::__stirling_2_t, [535](#)
- cos_pi

- GNU Extended Mathematical Special Functions, [87](#)
- `cos_pif`
 - GNU Extended Mathematical Special Functions, [88](#)
- `cos_pil`
 - GNU Extended Mathematical Special Functions, [88](#)
- `cosh_pi`
 - GNU Extended Mathematical Special Functions, [88](#)
- `cosh_pif`
 - GNU Extended Mathematical Special Functions, [89](#)
- `cosh_pil`
 - GNU Extended Mathematical Special Functions, [89](#)
- `coshint`
 - GNU Extended Mathematical Special Functions, [89](#)
- `coshintf`
 - GNU Extended Mathematical Special Functions, [90](#)
- `coshintl`
 - GNU Extended Mathematical Special Functions, [90](#)
- `cosint`
 - GNU Extended Mathematical Special Functions, [90](#)
- `cosintf`
 - GNU Extended Mathematical Special Functions, [91](#)
- `cosintl`
 - GNU Extended Mathematical Special Functions, [91](#)
- `cxx_fp_utils/include/ext/math_util.h`, [601](#)
- `cyl_bessel_i`
 - C++17/IS29124 Mathematical Special Functions, [25](#)
- `cyl_bessel_if`
 - C++17/IS29124 Mathematical Special Functions, [26](#)
- `cyl_bessel_il`
 - C++17/IS29124 Mathematical Special Functions, [26](#)
- `cyl_bessel_j`
 - C++17/IS29124 Mathematical Special Functions, [27](#)
- `cyl_bessel_jf`
 - C++17/IS29124 Mathematical Special Functions, [27](#)
- `cyl_bessel_jl`
 - C++17/IS29124 Mathematical Special Functions, [28](#)
- `cyl_bessel_k`
 - C++17/IS29124 Mathematical Special Functions, [28](#)
- `cyl_bessel_kf`
 - C++17/IS29124 Mathematical Special Functions, [29](#)
- `cyl_bessel_kl`
 - C++17/IS29124 Mathematical Special Functions, [29](#)
- `cyl_hankel_1`
 - GNU Extended Mathematical Special Functions, [91](#), [92](#)
- `cyl_hankel_1f`
 - GNU Extended Mathematical Special Functions, [92](#), [93](#)
- `cyl_hankel_1l`
 - GNU Extended Mathematical Special Functions, [93](#)
- `cyl_hankel_2`
 - GNU Extended Mathematical Special Functions, [94](#)
- `cyl_hankel_2f`
 - GNU Extended Mathematical Special Functions, [95](#)
- `cyl_hankel_2l`
 - GNU Extended Mathematical Special Functions, [96](#)
- `cyl_neumann`
 - C++17/IS29124 Mathematical Special Functions, [29](#)
- `cyl_neumannf`
 - C++17/IS29124 Mathematical Special Functions, [30](#)
- `cyl_neumannl`
 - C++17/IS29124 Mathematical Special Functions, [30](#)
- `dawson`
 - GNU Extended Mathematical Special Functions, [96](#)
- `dawsonf`
 - GNU Extended Mathematical Special Functions, [97](#)
- `dawsonl`
 - GNU Extended Mathematical Special Functions, [97](#)
- `debye`
 - GNU Extended Mathematical Special Functions, [97](#)
- `debyef`
 - GNU Extended Mathematical Special Functions, [98](#)
- `debyel`
 - GNU Extended Mathematical Special Functions, [98](#)
- `dedekind_eta`
 - `std::__detail::__jacobi_theta_0_t`, [556](#)
- `degree`
 - `__gnu_cxx::__stirling_1_t`, [533](#)
 - `__gnu_cxx::__stirling_2_t`, [536](#)
- `deriv`
 - `__gnu_cxx::__assoc_legendre_p_t`, [468](#)
 - `__gnu_cxx::__assoc_legendre_q_t`, [471](#)
 - `__gnu_cxx::__chebyshev_t_t`, [473](#)
 - `__gnu_cxx::__chebyshev_u_t`, [475](#)
 - `__gnu_cxx::__chebyshev_v_t`, [477](#)
 - `__gnu_cxx::__chebyshev_w_t`, [479](#)
 - `__gnu_cxx::__gegenbauer_t`, [500](#)
 - `__gnu_cxx::__hermite_he_t`, [502](#)
 - `__gnu_cxx::__hermite_t`, [504](#)
 - `__gnu_cxx::__jacobi_t`, [511](#)
 - `__gnu_cxx::__laguerre_t`, [513](#)
 - `__gnu_cxx::__legendre_p_t`, [516](#)
 - `__gnu_cxx::__legendre_q_t`, [518](#)
- `deriv2`
 - `__gnu_cxx::__chebyshev_t_t`, [473](#)
 - `__gnu_cxx::__hermite_he_t`, [502](#)
 - `__gnu_cxx::__hermite_t`, [505](#)
- `digamma`
 - GNU Extended Mathematical Special Functions, [99](#)
- `digammaf`
 - GNU Extended Mathematical Special Functions, [99](#)
- `digammal`
 - GNU Extended Mathematical Special Functions, [100](#)
- `dilog`
 - GNU Extended Mathematical Special Functions, [100](#)
- `dilogf`
 - GNU Extended Mathematical Special Functions, [100](#)

- dilogl
 - GNU Extended Mathematical Special Functions, [101](#)
- dirichlet_beta
 - GNU Extended Mathematical Special Functions, [101](#)
- dirichlet_betaf
 - GNU Extended Mathematical Special Functions, [102](#)
- dirichlet_betal
 - GNU Extended Mathematical Special Functions, [102](#)
- dirichlet_eta
 - GNU Extended Mathematical Special Functions, [102](#)
- dirichlet_etaf
 - GNU Extended Mathematical Special Functions, [103](#)
- dirichlet_etald
 - GNU Extended Mathematical Special Functions, [103](#)
- dirichlet_lambda
 - GNU Extended Mathematical Special Functions, [103](#)
- dirichlet_lambdaf
 - GNU Extended Mathematical Special Functions, [104](#)
- dirichlet_lambdal
 - GNU Extended Mathematical Special Functions, [104](#)
- double_factorial
 - GNU Extended Mathematical Special Functions, [104](#)
- double_factorialf
 - GNU Extended Mathematical Special Functions, [105](#)
- double_factoriall
 - GNU Extended Mathematical Special Functions, [105](#)
- ellint_1
 - C++17/IS29124 Mathematical Special Functions, [31](#)
- ellint_1f
 - C++17/IS29124 Mathematical Special Functions, [32](#)
- ellint_1l
 - C++17/IS29124 Mathematical Special Functions, [32](#)
- ellint_2
 - C++17/IS29124 Mathematical Special Functions, [32](#)
- ellint_2f
 - C++17/IS29124 Mathematical Special Functions, [33](#)
- ellint_2l
 - C++17/IS29124 Mathematical Special Functions, [33](#)
- ellint_3
 - C++17/IS29124 Mathematical Special Functions, [33](#)
- ellint_3f
 - C++17/IS29124 Mathematical Special Functions, [34](#)
- ellint_3l
 - C++17/IS29124 Mathematical Special Functions, [35](#)
- ellint_cel
 - GNU Extended Mathematical Special Functions, [105](#)
- ellint_celf
 - GNU Extended Mathematical Special Functions, [106](#)
- ellint_cell
 - GNU Extended Mathematical Special Functions, [106](#)
- ellint_d
 - GNU Extended Mathematical Special Functions, [107](#)
- ellint_df
 - GNU Extended Mathematical Special Functions, [107](#)
- ellint_dl
 - GNU Extended Mathematical Special Functions, [108](#)
- ellint_el1
 - GNU Extended Mathematical Special Functions, [108](#)
- ellint_el1f
 - GNU Extended Mathematical Special Functions, [108](#)
- ellint_el1l
 - GNU Extended Mathematical Special Functions, [109](#)
- ellint_el2
 - GNU Extended Mathematical Special Functions, [109](#)
- ellint_el2f
 - GNU Extended Mathematical Special Functions, [110](#)
- ellint_el2l
 - GNU Extended Mathematical Special Functions, [110](#)
- ellint_el3
 - GNU Extended Mathematical Special Functions, [110](#)
- ellint_el3f
 - GNU Extended Mathematical Special Functions, [111](#)
- ellint_el3l
 - GNU Extended Mathematical Special Functions, [111](#)
- ellint_rc
 - GNU Extended Mathematical Special Functions, [112](#)
- ellint_rcf
 - GNU Extended Mathematical Special Functions, [113](#)
- ellint_rcl
 - GNU Extended Mathematical Special Functions, [113](#)
- ellint_rd
 - GNU Extended Mathematical Special Functions, [113](#)
- ellint_rdf
 - GNU Extended Mathematical Special Functions, [114](#)
- ellint_rdl
 - GNU Extended Mathematical Special Functions, [114](#)
- ellint_rf
 - GNU Extended Mathematical Special Functions, [114](#)
- ellint_rff
 - GNU Extended Mathematical Special Functions, [115](#)
- ellint_rfl
 - GNU Extended Mathematical Special Functions, [115](#)
- ellint_rg
 - GNU Extended Mathematical Special Functions, [116](#)
- ellint_rgf
 - GNU Extended Mathematical Special Functions, [117](#)
- ellint_rgl
 - GNU Extended Mathematical Special Functions, [117](#)
- ellint_rj
 - GNU Extended Mathematical Special Functions, [117](#)
- ellint_rjf
 - GNU Extended Mathematical Special Functions, [119](#)
- ellint_rjl
 - GNU Extended Mathematical Special Functions, [119](#)
- ellnome
 - GNU Extended Mathematical Special Functions, [119](#)
- ellnomef
 - GNU Extended Mathematical Special Functions, [119](#)

- GNU Extended Mathematical Special Functions, [120](#)
- ellnomel
 - GNU Extended Mathematical Special Functions, [120](#)
- end
 - [__gnu_cxx::__stirling_1_t](#), [533](#)
 - [__gnu_cxx::__stirling_2_t](#), [536](#)
- eta_1
 - [std::__detail::__jacobi_theta_0_t](#), [556](#)
- eta_2
 - [std::__detail::__jacobi_theta_0_t](#), [556](#)
- eta_3
 - [std::__detail::__jacobi_theta_0_t](#), [557](#)
- euler
 - GNU Extended Mathematical Special Functions, [121](#)
- eulerian_1
 - GNU Extended Mathematical Special Functions, [121](#)
- eulerian_2
 - GNU Extended Mathematical Special Functions, [122](#)
- expint
 - C++17/IS29124 Mathematical Special Functions, [35](#)
 - GNU Extended Mathematical Special Functions, [122](#)
- expintf
 - C++17/IS29124 Mathematical Special Functions, [36](#)
 - GNU Extended Mathematical Special Functions, [123](#)
- expintl
 - C++17/IS29124 Mathematical Special Functions, [36](#)
 - GNU Extended Mathematical Special Functions, [123](#)
- exponential_p
 - GNU Extended Mathematical Special Functions, [123](#)
- exponential_pdf
 - GNU Extended Mathematical Special Functions, [124](#)
- factorial
 - GNU Extended Mathematical Special Functions, [124](#)
- factorialf
 - GNU Extended Mathematical Special Functions, [124](#)
- factoriall
 - GNU Extended Mathematical Special Functions, [125](#)
- falling_factorial
 - GNU Extended Mathematical Special Functions, [125](#)
- falling_factorialf
 - GNU Extended Mathematical Special Functions, [125](#)
- falling_factoriall
 - GNU Extended Mathematical Special Functions, [126](#)
- fermi_dirac
 - GNU Extended Mathematical Special Functions, [126](#)
- fermi_diracf
 - GNU Extended Mathematical Special Functions, [127](#)
- fermi_diracl
 - GNU Extended Mathematical Special Functions, [127](#)
- fisher_f_p
 - GNU Extended Mathematical Special Functions, [127](#)
- fisher_f_pdf
 - GNU Extended Mathematical Special Functions, [128](#)
- fresnel_c
 - GNU Extended Mathematical Special Functions, [129](#)
- fresnel_cf
 - GNU Extended Mathematical Special Functions, [129](#)
- fresnel_cl
 - GNU Extended Mathematical Special Functions, [129](#)
- fresnel_s
 - GNU Extended Mathematical Special Functions, [129](#)
- fresnel_sf
 - GNU Extended Mathematical Special Functions, [131](#)
- fresnel_sl
 - GNU Extended Mathematical Special Functions, [131](#)
- GNU Extended Mathematical Special Functions, [47](#)
 - [airy_ai](#), [60](#), [61](#)
 - [airy_aif](#), [61](#)
 - [airy_ail](#), [61](#)
 - [airy_bi](#), [62](#)
 - [airy_bif](#), [63](#)
 - [airy_bil](#), [63](#)
 - [assoc_legendre_q](#), [63](#)
 - [assoc_legendre_qf](#), [64](#)
 - [assoc_legendre_ql](#), [65](#)
 - [bell](#), [65](#)
 - [bernoulli](#), [66](#)
 - [bernoullif](#), [67](#)
 - [bernoullil](#), [67](#)
 - [binomial](#), [67](#)
 - [binomial_p](#), [68](#)
 - [binomial_pdf](#), [69](#)
 - [binomialf](#), [69](#)
 - [binomiall](#), [70](#)
 - [bose_einstein](#), [70](#)
 - [bose_einsteinf](#), [71](#)
 - [bose_einsteinl](#), [71](#)
 - [chebyshev_t](#), [71](#)
 - [chebyshev_tf](#), [72](#)
 - [chebyshev_tl](#), [72](#)
 - [chebyshev_u](#), [72](#)
 - [chebyshev_uf](#), [73](#)
 - [chebyshev_ul](#), [73](#)
 - [chebyshev_v](#), [74](#)
 - [chebyshev_vf](#), [74](#)
 - [chebyshev_vl](#), [75](#)
 - [chebyshev_w](#), [75](#)
 - [chebyshev_wf](#), [76](#)
 - [chebyshev_wl](#), [76](#)
 - [clausen](#), [76](#), [77](#)
 - [clausen_cl](#), [78](#)
 - [clausen_clf](#), [78](#)
 - [clausen_cll](#), [79](#)
 - [clausen_sl](#), [79](#)
 - [clausen_slf](#), [80](#)
 - [clausen_sll](#), [80](#)

clausenf, 80
 clausenl, 81
 comp_ellint_d, 81
 comp_ellint_df, 82
 comp_ellint_dl, 82
 comp_ellint_rf, 82, 83
 comp_ellint_rg, 84
 conf_hyperg, 85
 conf_hyperg_lim, 85
 conf_hyperg_limf, 86
 conf_hyperg_liml, 86
 conf_hypergf, 86
 conf_hypergl, 87
 cos_pi, 87
 cos_pif, 88
 cos_pil, 88
 cosh_pi, 88
 cosh_pif, 89
 cosh_pil, 89
 coshint, 89
 coshintf, 90
 coshintl, 90
 cosint, 90
 cosintf, 91
 cosintl, 91
 cyl_hankel_1, 91, 92
 cyl_hankel_1f, 92, 93
 cyl_hankel_1l, 93
 cyl_hankel_2, 94
 cyl_hankel_2f, 95
 cyl_hankel_2l, 96
 dawson, 96
 dawsonf, 97
 dawsonl, 97
 debye, 97
 debyef, 98
 debyel, 98
 digamma, 99
 digammaf, 99
 digammal, 100
 dilog, 100
 dilogf, 100
 dilogl, 101
 dirichlet_beta, 101
 dirichlet_betaf, 102
 dirichlet_betl, 102
 dirichlet_eta, 102
 dirichlet_etaf, 103
 dirichlet_etl, 103
 dirichlet_lambda, 103
 dirichlet_lambdaf, 104
 dirichlet_lambdal, 104
 double_factorial, 104
 double_factorialf, 105
 double_factoriall, 105
 ellint_cel, 105
 ellint_celf, 106
 ellint_cell, 106
 ellint_d, 107
 ellint_df, 107
 ellint_dl, 108
 ellint_el1, 108
 ellint_el1f, 108
 ellint_el1l, 109
 ellint_el2, 109
 ellint_el2f, 110
 ellint_el2l, 110
 ellint_el3, 110
 ellint_el3f, 111
 ellint_el3l, 111
 ellint_rc, 112
 ellint_rcf, 113
 ellint_rcl, 113
 ellint_rd, 113
 ellint_rdf, 114
 ellint_rdl, 114
 ellint_rf, 114
 ellint_rff, 115
 ellint_rfl, 115
 ellint_rg, 116
 ellint_rgf, 117
 ellint_rgl, 117
 ellint_rj, 117
 ellint_rjf, 119
 ellint_rjl, 119
 ellnome, 119
 ellnomef, 120
 ellnomel, 120
 euler, 121
 eulerian_1, 121
 eulerian_2, 122
 expint, 122
 expintf, 123
 expintl, 123
 exponential_p, 123
 exponential_pdf, 124
 factorial, 124
 factorialf, 124
 factoriall, 125
 falling_factorial, 125
 falling_factorialf, 125
 falling_factoriall, 126
 fermi_dirac, 126
 fermi_diracf, 127
 fermi_diracl, 127
 fisher_f_p, 127
 fisher_f_pdf, 128
 fresnel_c, 129

fresnel_cf, 129
fresnel_cl, 129
fresnel_s, 129
fresnel_sf, 131
fresnel_sl, 131
gamma_p, 131
gamma_pdf, 131
gamma_pf, 132
gamma_pl, 132
gamma_q, 132
gamma_qf, 133
gamma_ql, 133
gamma_reciprocal, 133
gamma_reciprocalf, 133
gamma_reciprocall, 134
gegenbauer, 134
gegenbauerf, 135
gegenbauerl, 135
harmonic, 135
heuman_lambda, 136
heuman_lambdaf, 137
heuman_lambdal, 137
hurwitz_zeta, 137
hurwitz_zetaf, 138
hurwitz_zetal, 138
hyperg, 138
hypergf, 139
hypergl, 139
ibeta, 140
ibetac, 141
ibetacf, 141
ibetacl, 141
ibetaf, 141
ibetal, 142
jacobi, 142
jacobi_cn, 143
jacobi_cnf, 144
jacobi_cnl, 144
jacobi_dn, 144
jacobi_dnf, 145
jacobi_dnl, 145
jacobi_sn, 145
jacobi_snf, 146
jacobi_snl, 146
jacobi_theta_1, 147
jacobi_theta_1f, 147
jacobi_theta_1l, 148
jacobi_theta_2, 148
jacobi_theta_2f, 149
jacobi_theta_2l, 149
jacobi_theta_3, 149
jacobi_theta_3f, 150
jacobi_theta_3l, 150
jacobi_theta_4, 150
jacobi_theta_4f, 151
jacobi_theta_4l, 151
jacobi_zeta, 151
jacobi_zetaf, 152
jacobi_zetal, 152
jacobif, 152
jacobil, 152
lah, 153
lbinomial, 153
lbinomialf, 154
lbinomiall, 154
ldouble_factorial, 155
ldouble_factorialf, 155
ldouble_factoriall, 155
legendre_q, 156
legendre_qf, 157
legendre_ql, 157
lerch_phi, 157
lerch_phif, 158
lerch_phil, 158
lfactorial, 158
lfactorialf, 159
lfactoriall, 159
lfalling_factorial, 159
lfalling_factorialf, 160
lfalling_factoriall, 160
lgamma, 160, 161
lgammaf, 161
lgammal, 162
logint, 162
logintf, 163
logintl, 163
logistic_p, 163
logistic_pdf, 164
lognormal_p, 164
lognormal_pdf, 164
lrising_factorial, 165
lrising_factorialf, 165
lrising_factoriall, 165
mittag_leffler, 166
normal_p, 166
normal_pdf, 166
owens_t, 167
owens_tf, 168
owens_tl, 168
periodic_zeta, 168, 169
periodic_zetaf, 169
periodic_zetal, 169
polygamma, 170
polygammaf, 170
polygammal, 171
polylog, 171
polylogf, 172
polylogl, 172, 173

- radpoly, 173
- radpolyf, 174
- radpolyl, 174
- rising_factorial, 174
- rising_factorialf, 175
- rising_factoriall, 175
- sin_pi, 175
- sin_pif, 176
- sin_pil, 176
- sinc, 176
- sinc_pi, 177
- sinc_pif, 177
- sinc_pil, 178
- sincf, 178
- sincl, 178
- sincos, 179
- sincos_pi, 179
- sincos_pif, 180
- sincos_pil, 180
- sincosf, 180
- sincosl, 181
- sinh_pi, 181
- sinh_pif, 182
- sinh_pil, 182
- sinhc, 182
- sinhc_pi, 183
- sinhc_pif, 183
- sinhc_pil, 184
- sinhcf, 184
- sinhcl, 184
- sinhint, 184
- sinhintf, 185
- sinhintl, 185
- sinint, 185
- sinintf, 186
- sinintl, 186
- sph_bessel_i, 186
- sph_bessel_if, 187
- sph_bessel_il, 187
- sph_bessel_k, 188
- sph_bessel_kf, 189
- sph_bessel_kl, 189
- sph_hankel_1, 189, 190
- sph_hankel_1f, 190, 191
- sph_hankel_1l, 191
- sph_hankel_2, 192, 193
- sph_hankel_2f, 193
- sph_hankel_2l, 194
- sph_harmonic, 194
- sph_harmonicf, 195
- sph_harmonicl, 195
- stirling_1, 196
- stirling_2, 197
- student_t_p, 197
- student_t_pdf, 198
- tan_pi, 198
- tan_pif, 199
- tan_pil, 199
- tanh_pi, 199
- tanh_pif, 200
- tanh_pil, 200
- tgamma, 200, 201
- tgamma_lower, 201
- tgamma_lowerf, 202
- tgamma_lowerl, 202
- tgammaf, 202, 203
- tgammal, 203, 204
- theta_1, 204
- theta_1f, 205
- theta_1l, 205
- theta_2, 205
- theta_2f, 206
- theta_2l, 206
- theta_3, 206
- theta_3f, 207
- theta_3l, 207
- theta_4, 207
- theta_4f, 208
- theta_4l, 208
- theta_c, 208
- theta_cf, 209
- theta_cl, 209
- theta_d, 210
- theta_df, 210
- theta_dl, 211
- theta_n, 211
- theta_nf, 212
- theta_nl, 212
- theta_s, 212
- theta_sf, 213
- theta_sl, 213
- tricoli_u, 214
- tricoli_uf, 215
- tricoli_ul, 215
- weibull_p, 215
- weibull_pdf, 215
- zernike, 216
- zernikef, 217
- zernikel, 217
- gamma_p
 - GNU Extended Mathematical Special Functions, 131
- gamma_pdf
 - GNU Extended Mathematical Special Functions, 131
- gamma_pf
 - GNU Extended Mathematical Special Functions, 132
- gamma_pl
 - GNU Extended Mathematical Special Functions, 132
- gamma_q

- GNU Extended Mathematical Special Functions, [132](#)
- gamma_qf
 - GNU Extended Mathematical Special Functions, [133](#)
- gamma_ql
 - GNU Extended Mathematical Special Functions, [133](#)
- gamma_reciprocal
 - GNU Extended Mathematical Special Functions, [133](#)
- gamma_reciprocalf
 - GNU Extended Mathematical Special Functions, [133](#)
- gamma_reciprocall
 - GNU Extended Mathematical Special Functions, [134](#)
- gauss_quad_type
 - __gnu_cxx, [239](#)
- gegenbauer
 - GNU Extended Mathematical Special Functions, [134](#)
- gegenbauerf
 - GNU Extended Mathematical Special Functions, [135](#)
- gegenbauerl
 - GNU Extended Mathematical Special Functions, [135](#)
- harmonic
 - GNU Extended Mathematical Special Functions, [135](#)
- hermite
 - C++17/IS29124 Mathematical Special Functions, [36](#)
- hermitef
 - C++17/IS29124 Mathematical Special Functions, [37](#)
- hermitel
 - C++17/IS29124 Mathematical Special Functions, [37](#)
- heuman_lambda
 - GNU Extended Mathematical Special Functions, [136](#)
- heuman_lambdaf
 - GNU Extended Mathematical Special Functions, [137](#)
- heuman_lambdal
 - GNU Extended Mathematical Special Functions, [137](#)
- hurwitz_zeta
 - GNU Extended Mathematical Special Functions, [137](#)
- hurwitz_zetaf
 - GNU Extended Mathematical Special Functions, [138](#)
- hurwitz_zetal
 - GNU Extended Mathematical Special Functions, [138](#)
- hyperg
 - GNU Extended Mathematical Special Functions, [138](#)
- hypergf
 - GNU Extended Mathematical Special Functions, [139](#)
- hypergl
 - GNU Extended Mathematical Special Functions, [139](#)
- ibeta
 - GNU Extended Mathematical Special Functions, [140](#)
- ibetac
 - GNU Extended Mathematical Special Functions, [141](#)
- ibetacf
 - GNU Extended Mathematical Special Functions, [141](#)
- ibetacl
 - GNU Extended Mathematical Special Functions, [141](#)
- ibetaf
 - GNU Extended Mathematical Special Functions, [141](#)
- ibetal
 - GNU Extended Mathematical Special Functions, [142](#)
- include/bits/sf_airy.tcc, [603](#)
- include/bits/sf_bernoulli.tcc, [605](#)
- include/bits/sf_bessel.tcc, [606](#)
- include/bits/sf_beta.tcc, [609](#)
- include/bits/sf_cardinal.tcc, [611](#)
- include/bits/sf_chebyshev.tcc, [613](#)
- include/bits/sf_coulomb.tcc, [614](#)
- include/bits/sf_dawson.tcc, [616](#)
- include/bits/sf_distributions.tcc, [618](#)
- include/bits/sf_ellint.tcc, [621](#)
- include/bits/sf_euler.tcc, [624](#)
- include/bits/sf_expint.tcc, [626](#)
- include/bits/sf_fresnel.tcc, [628](#)
- include/bits/sf_gamma.tcc, [630](#)
- include/bits/sf_gegenbauer.tcc, [638](#)
- include/bits/sf_hankel.tcc, [639](#)
- include/bits/sf_hermite.tcc, [642](#)
- include/bits/sf_hyperg.tcc, [644](#)
- include/bits/sf_hypint.tcc, [646](#)
- include/bits/sf_jacobi.tcc, [648](#)
- include/bits/sf_laguerre.tcc, [650](#)
- include/bits/sf_legendre.tcc, [652](#)
- include/bits/sf_mod_bessel.tcc, [654](#)
- include/bits/sf_owens_t.tcc, [656](#)
- include/bits/sf_polylog.tcc, [657](#)
- include/bits/sf_stirling.tcc, [660](#)
- include/bits/sf_theta.tcc, [662](#)
- include/bits/sf_trig.tcc, [665](#)
- include/bits/sf_trigint.tcc, [667](#)
- include/bits/sf_zeta.tcc, [669](#)
- include/bits/specfun.h, [671](#)
- include/bits/specfun_state.h, [688](#)
- inner_radius
 - std::__detail::__Airy, [568](#)
 - std::__detail::__Airy_default_radii< double >, [579](#)
 - std::__detail::__Airy_default_radii< float >, [580](#)
 - std::__detail::__Airy_default_radii< long double >, [581](#)
- iterator
 - __gnu_cxx::__stirling_1_t, [532](#)
 - __gnu_cxx::__stirling_2_t, [535](#)
- jacobi
 - GNU Extended Mathematical Special Functions, [142](#)
- jacobi_cn
 - GNU Extended Mathematical Special Functions, [143](#)
- jacobi_cnf
 - GNU Extended Mathematical Special Functions, [144](#)
- jacobi_cnl
 - GNU Extended Mathematical Special Functions, [144](#)

- jacobi_dn
 - GNU Extended Mathematical Special Functions, [144](#)
- jacobi_dnf
 - GNU Extended Mathematical Special Functions, [145](#)
- jacobi_dnl
 - GNU Extended Mathematical Special Functions, [145](#)
- jacobi_sn
 - GNU Extended Mathematical Special Functions, [145](#)
- jacobi_snf
 - GNU Extended Mathematical Special Functions, [146](#)
- jacobi_snl
 - GNU Extended Mathematical Special Functions, [146](#)
- jacobi_theta_1
 - GNU Extended Mathematical Special Functions, [147](#)
- jacobi_theta_1f
 - GNU Extended Mathematical Special Functions, [147](#)
- jacobi_theta_1l
 - GNU Extended Mathematical Special Functions, [148](#)
- jacobi_theta_2
 - GNU Extended Mathematical Special Functions, [148](#)
- jacobi_theta_2f
 - GNU Extended Mathematical Special Functions, [149](#)
- jacobi_theta_2l
 - GNU Extended Mathematical Special Functions, [149](#)
- jacobi_theta_3
 - GNU Extended Mathematical Special Functions, [149](#)
- jacobi_theta_3f
 - GNU Extended Mathematical Special Functions, [150](#)
- jacobi_theta_3l
 - GNU Extended Mathematical Special Functions, [150](#)
- jacobi_theta_4
 - GNU Extended Mathematical Special Functions, [150](#)
- jacobi_theta_4f
 - GNU Extended Mathematical Special Functions, [151](#)
- jacobi_theta_4l
 - GNU Extended Mathematical Special Functions, [151](#)
- jacobi_zeta
 - GNU Extended Mathematical Special Functions, [151](#)
- jacobi_zetaf
 - GNU Extended Mathematical Special Functions, [152](#)
- jacobi_zetal
 - GNU Extended Mathematical Special Functions, [152](#)
- jacobif
 - GNU Extended Mathematical Special Functions, [152](#)
- jacobil
 - GNU Extended Mathematical Special Functions, [152](#)
- laguerre
 - C++17/IS29124 Mathematical Special Functions, [38](#)
- laguerref
 - C++17/IS29124 Mathematical Special Functions, [38](#)
- laguerrel
 - C++17/IS29124 Mathematical Special Functions, [39](#)
- lah
 - GNU Extended Mathematical Special Functions, [153](#)
- lbinomial
 - GNU Extended Mathematical Special Functions, [153](#)
- lbinomialf
 - GNU Extended Mathematical Special Functions, [154](#)
- lbinomiall
 - GNU Extended Mathematical Special Functions, [154](#)
- ldouble_factorial
 - GNU Extended Mathematical Special Functions, [155](#)
- ldouble_factorialf
 - GNU Extended Mathematical Special Functions, [155](#)
- ldouble_factoriall
 - GNU Extended Mathematical Special Functions, [155](#)
- legendre
 - C++17/IS29124 Mathematical Special Functions, [39](#)
- legendre_q
 - GNU Extended Mathematical Special Functions, [156](#)
- legendre_qf
 - GNU Extended Mathematical Special Functions, [157](#)
- legendre_ql
 - GNU Extended Mathematical Special Functions, [157](#)
- legendref
 - C++17/IS29124 Mathematical Special Functions, [40](#)
- legendrel
 - C++17/IS29124 Mathematical Special Functions, [40](#)
- lerch_phi
 - GNU Extended Mathematical Special Functions, [157](#)
- lerch_phif
 - GNU Extended Mathematical Special Functions, [158](#)
- lerch_phil
 - GNU Extended Mathematical Special Functions, [158](#)
- lfactorial
 - GNU Extended Mathematical Special Functions, [158](#)
- lfactorialf
 - GNU Extended Mathematical Special Functions, [159](#)
- lfactoriall
 - GNU Extended Mathematical Special Functions, [159](#)
- lfalling_factorial
 - GNU Extended Mathematical Special Functions, [159](#)
- lfalling_factorialf
 - GNU Extended Mathematical Special Functions, [160](#)
- lfalling_factoriall
 - GNU Extended Mathematical Special Functions, [160](#)
- lgamma
 - GNU Extended Mathematical Special Functions, [160](#), [161](#)
- lgammaf
 - GNU Extended Mathematical Special Functions, [161](#)
- lgammal
 - GNU Extended Mathematical Special Functions, [162](#)
- lobatto
 - __gnu_cxx::__legendre_p_t, [516](#)
- logint
 - GNU Extended Mathematical Special Functions, [162](#)

- logintf
 - GNU Extended Mathematical Special Functions, [163](#)
- logintl
 - GNU Extended Mathematical Special Functions, [163](#)
- logistic_p
 - GNU Extended Mathematical Special Functions, [163](#)
- logistic_pdf
 - GNU Extended Mathematical Special Functions, [164](#)
- lognormal_p
 - GNU Extended Mathematical Special Functions, [164](#)
- lognormal_pdf
 - GNU Extended Mathematical Special Functions, [164](#)
- lrising_factorial
 - GNU Extended Mathematical Special Functions, [165](#)
- lrising_factorialf
 - GNU Extended Mathematical Special Functions, [165](#)
- lrising_factoriall
 - GNU Extended Mathematical Special Functions, [165](#)
- Mathematical Special Functions, [218](#)
- mittag_leffler
 - GNU Extended Mathematical Special Functions, [166](#)
- normal_p
 - GNU Extended Mathematical Special Functions, [166](#)
- normal_pdf
 - GNU Extended Mathematical Special Functions, [166](#)
- num_terms
 - std::__detail::__AsympTerminator, [596](#)
 - std::__detail::__Terminator, [599](#)
- operator bool
 - __gnu_cxx::__fp_is_integer_t, [494](#)
- operator<<
 - std::__detail::__AsympTerminator, [596](#)
- operator()
 - __gnu_cxx::__fp_is_integer_t, [494](#)
 - __gnu_cxx::__stirling_1_t, [533](#)
 - __gnu_cxx::__stirling_2_t, [536](#)
 - std::__detail::__Airy, [567](#)
 - std::__detail::__Airy_asymp, [571](#)
 - std::__detail::__Airy_asymp_series, [578](#)
 - std::__detail::__AsympTerminator, [596](#)
 - std::__detail::__Terminator, [599](#)
- operator[]
 - __gnu_cxx::__stirling_1_t, [533](#)
 - __gnu_cxx::__stirling_2_t, [536](#)
- outer_radius
 - std::__detail::__Airy, [568](#)
 - std::__detail::__Airy_default_radii< double >, [579](#)
 - std::__detail::__Airy_default_radii< float >, [580](#)
 - std::__detail::__Airy_default_radii< long double >, [581](#)
- owens_t
 - GNU Extended Mathematical Special Functions, [167](#)
- owens_tf
 - GNU Extended Mathematical Special Functions, [168](#)
- owens_tl
 - GNU Extended Mathematical Special Functions, [168](#)
- periodic_zeta
 - GNU Extended Mathematical Special Functions, [168](#), [169](#)
- periodic_zetaf
 - GNU Extended Mathematical Special Functions, [169](#)
- periodic_zetal
 - GNU Extended Mathematical Special Functions, [169](#)
- polygamma
 - GNU Extended Mathematical Special Functions, [170](#)
- polygammaf
 - GNU Extended Mathematical Special Functions, [170](#)
- polygamma1
 - GNU Extended Mathematical Special Functions, [171](#)
- polylog
 - GNU Extended Mathematical Special Functions, [171](#)
- polylogf
 - GNU Extended Mathematical Special Functions, [172](#)
- polylogl
 - GNU Extended Mathematical Special Functions, [172](#), [173](#)
- radpoly
 - GNU Extended Mathematical Special Functions, [173](#)
- radpolyf
 - GNU Extended Mathematical Special Functions, [174](#)
- radpolyl
 - GNU Extended Mathematical Special Functions, [174](#)
- riemann_zeta
 - C++17/IS29124 Mathematical Special Functions, [40](#)
- riemann_zetaf
 - C++17/IS29124 Mathematical Special Functions, [41](#)
- riemann_zetal
 - C++17/IS29124 Mathematical Special Functions, [41](#)
- rising_factorial
 - GNU Extended Mathematical Special Functions, [174](#)
- rising_factorialf
 - GNU Extended Mathematical Special Functions, [175](#)
- rising_factoriall
 - GNU Extended Mathematical Special Functions, [175](#)
- scalar_type
 - std::__detail::__Airy, [566](#)
 - std::__detail::__Airy_asymp_series, [577](#)
- sf_airy.tcc
 - _GLIBCXX_BITS_SF_AIRY_TCC, [604](#)
- sf_bernoulli.tcc
 - _GLIBCXX_BITS_SF_BERNOULLI_TCC, [606](#)
- sf_bessel.tcc
 - _GLIBCXX_BITS_SF_BESSEL_TCC, [608](#)
- sf_beta.tcc

[_GLIBCXX_BITS_SF_BETA_TCC](#), 610
[sf_cardinal.tcc](#)
[_GLIBCXX_BITS_SF_CARDINAL_TCC](#), 612
[sf_chebyshev.tcc](#)
[_GLIBCXX_BITS_SF_CHEBYSHEV_TCC](#), 614
[sf_coulomb.tcc](#)
[_GLIBCXX_BITS_SF_COULOMB_TCC](#), 616
[sf_dawson.tcc](#)
[_GLIBCXX_BITS_SF_DAWSON_TCC](#), 617
[sf_distributions.tcc](#)
[_GLIBCXX_BITS_SF_DISTRIBUTIONS_TCC](#), 621
[sf_ellint.tcc](#)
[_GLIBCXX_BITS_SF_ELLINT_TCC](#), 623
[sf_euler.tcc](#)
[_GLIBCXX_BITS_SF_EULER_TCC](#), 625
[sf_expint.tcc](#)
[_GLIBCXX_BITS_SF_EXPINT_TCC](#), 628
[sf_fresnel.tcc](#)
[_GLIBCXX_BITS_SF_FRESNEL_TCC](#), 629
[sf_gamma.tcc](#)
[_GLIBCXX_BITS_SF_GAMMA_TCC](#), 637
[sf_gegenbauer.tcc](#)
[_GLIBCXX_BITS_SF_GEGENBAUER_TCC](#), 639
[sf_hankel.tcc](#)
[_GLIBCXX_BITS_SF_HANKEL_TCC](#), 642
[sf_hermite.tcc](#)
[_GLIBCXX_BITS_SF_HERMITE_TCC](#), 644
[sf_hyperg.tcc](#)
[_GLIBCXX_BITS_SF_HYPERG_TCC](#), 646
[sf_hypint.tcc](#)
[_GLIBCXX_BITS_SF_HYPINT_TCC](#), 648
[sf_jacobi.tcc](#)
[_GLIBCXX_BITS_SF_JACOBI_TCC](#), 649
[sf_laguerre.tcc](#)
[_GLIBCXX_BITS_SF_LAGUERRE_TCC](#), 651
[sf_legendre.tcc](#)
[_GLIBCXX_BITS_SF_LEGENDRE_TCC](#), 653
[sf_mod_bessel.tcc](#)
[_GLIBCXX_BITS_SF_MOD_BESSEL_TCC](#), 655
[sf_owens.t.tcc](#)
[_GLIBCXX_BITS_SF_OWENS_T_TCC](#), 657
[sf_polylog.tcc](#)
[_GLIBCXX_BITS_SF_POLYLOG_TCC](#), 659
[sf_stirling.tcc](#)
[_GLIBCXX_BITS_SF_STIRLING_TCC](#), 662
[sf_theta.tcc](#)
[_GLIBCXX_BITS_SF_THETA_TCC](#), 664
[sf_trig.tcc](#)
[_GLIBCXX_BITS_SF_TRIG_TCC](#), 667
[sf_trigint.tcc](#)
[_GLIBCXX_BITS_SF_TRIGINT_TCC](#), 668
[sf_zeta.tcc](#)
[_GLIBCXX_BITS_SF_ZETA_TCC](#), 671
[sin_pi](#)

[GNU Extended Mathematical Special Functions](#), 175
[sin_pif](#)
[GNU Extended Mathematical Special Functions](#), 176
[sin_pil](#)
[GNU Extended Mathematical Special Functions](#), 176
[sinc](#)
[GNU Extended Mathematical Special Functions](#), 176
[sinc_pi](#)
[GNU Extended Mathematical Special Functions](#), 177
[sinc_pif](#)
[GNU Extended Mathematical Special Functions](#), 177
[sinc_pil](#)
[GNU Extended Mathematical Special Functions](#), 178
[sincof](#)
[GNU Extended Mathematical Special Functions](#), 178
[sincl](#)
[GNU Extended Mathematical Special Functions](#), 178
[sincos](#)
[GNU Extended Mathematical Special Functions](#), 179
[sincos_pi](#)
[GNU Extended Mathematical Special Functions](#), 179
[sincos_pif](#)
[GNU Extended Mathematical Special Functions](#), 180
[sincos_pil](#)
[GNU Extended Mathematical Special Functions](#), 180
[sincosf](#)
[GNU Extended Mathematical Special Functions](#), 180
[sincosl](#)
[GNU Extended Mathematical Special Functions](#), 181
[sinh_pi](#)
[GNU Extended Mathematical Special Functions](#), 181
[sinh_pif](#)
[GNU Extended Mathematical Special Functions](#), 182
[sinh_pil](#)
[GNU Extended Mathematical Special Functions](#), 182
[sinhc](#)
[GNU Extended Mathematical Special Functions](#), 182
[sinhc_pi](#)
[GNU Extended Mathematical Special Functions](#), 183
[sinhc_pif](#)
[GNU Extended Mathematical Special Functions](#), 183
[sinhc_pil](#)
[GNU Extended Mathematical Special Functions](#), 184
[sinhcf](#)
[GNU Extended Mathematical Special Functions](#), 184
[sinhcl](#)
[GNU Extended Mathematical Special Functions](#), 184
[sinhint](#)
[GNU Extended Mathematical Special Functions](#), 184
[sinhintf](#)
[GNU Extended Mathematical Special Functions](#), 185
[sinhintl](#)
[GNU Extended Mathematical Special Functions](#), 185
[sinint](#)

- GNU Extended Mathematical Special Functions, [185](#)
- `sinintf`
 - GNU Extended Mathematical Special Functions, [186](#)
- `sinintl`
 - GNU Extended Mathematical Special Functions, [186](#)
- `specfun.h`
 - `__STDCPP_MATH_SPEC_FUNCS__`, [687](#)
 - `__cpp_lib_math_special_functions`, [687](#)
- `sph_bessel`
 - C++17/IS29124 Mathematical Special Functions, [42](#)
- `sph_bessel_i`
 - GNU Extended Mathematical Special Functions, [186](#)
- `sph_bessel_if`
 - GNU Extended Mathematical Special Functions, [187](#)
- `sph_bessel_il`
 - GNU Extended Mathematical Special Functions, [187](#)
- `sph_bessel_k`
 - GNU Extended Mathematical Special Functions, [188](#)
- `sph_bessel_kf`
 - GNU Extended Mathematical Special Functions, [189](#)
- `sph_bessel_kl`
 - GNU Extended Mathematical Special Functions, [189](#)
- `sph_besself`
 - C++17/IS29124 Mathematical Special Functions, [42](#)
- `sph_bessell`
 - C++17/IS29124 Mathematical Special Functions, [43](#)
- `sph_hankel_1`
 - GNU Extended Mathematical Special Functions, [189](#), [190](#)
- `sph_hankel_1f`
 - GNU Extended Mathematical Special Functions, [190](#), [191](#)
- `sph_hankel_1l`
 - GNU Extended Mathematical Special Functions, [191](#)
- `sph_hankel_2`
 - GNU Extended Mathematical Special Functions, [192](#), [193](#)
- `sph_hankel_2f`
 - GNU Extended Mathematical Special Functions, [193](#)
- `sph_hankel_2l`
 - GNU Extended Mathematical Special Functions, [194](#)
- `sph_harmonic`
 - GNU Extended Mathematical Special Functions, [194](#)
- `sph_harmonicf`
 - GNU Extended Mathematical Special Functions, [195](#)
- `sph_harmonicl`
 - GNU Extended Mathematical Special Functions, [195](#)
- `sph_legendre`
 - C++17/IS29124 Mathematical Special Functions, [43](#)
- `sph_legendref`
 - C++17/IS29124 Mathematical Special Functions, [44](#)
- `sph_legendrel`
 - C++17/IS29124 Mathematical Special Functions, [44](#)
- `sph_neumann`
 - C++17/IS29124 Mathematical Special Functions, [44](#)
- `sph_neumannf`
 - C++17/IS29124 Mathematical Special Functions, [45](#)
- `sph_neumannl`
 - C++17/IS29124 Mathematical Special Functions, [45](#)
- `std`, [244](#)
- `std::`
 - `__detail`, [246](#)
 - `_Num_Euler_Maclaurin_zeta`, [458](#)
 - `_Num_Stieltjes`, [459](#)
 - `_S_Euler_Maclaurin_zeta`, [459](#)
 - `_S_Stieltjes`, [463](#)
 - `_S_double_factorial_table`, [459](#)
 - `_S_factorial_table`, [459](#)
 - `_S_harmonic_denom`, [460](#)
 - `_S_harmonic_numer`, [460](#)
 - `_S_neg_double_factorial_table`, [460](#)
 - `_S_num_double_factorials`, [460](#)
 - `_S_num_double_factorials< double >`, [460](#)
 - `_S_num_double_factorials< float >`, [461](#)
 - `_S_num_double_factorials< long double >`, [461](#)
 - `_S_num_factorials`, [461](#)
 - `_S_num_factorials< double >`, [461](#)
 - `_S_num_factorials< float >`, [461](#)
 - `_S_num_factorials< long double >`, [462](#)
 - `_S_num_harmonic_numer`, [462](#)
 - `_S_num_neg_double_factorials`, [462](#)
 - `_S_num_neg_double_factorials< double >`, [462](#)
 - `_S_num_neg_double_factorials< float >`, [462](#)
 - `_S_num_neg_double_factorials< long double >`, [463](#)
 - `_S_num_zetam1`, [463](#)
 - `_S_zetam1`, [463](#)
 - `__airy`, [271](#)
 - `__airy_ai`, [272](#)
 - `__airy_arg`, [272](#)
 - `__airy_bi`, [273](#)
 - `__assoc_laguerre`, [273](#)
 - `__assoc_legendre_p`, [274](#)
 - `__assoc_legendre_q`, [274](#)
 - `__bell`, [275](#)
 - `__bell_series`, [275](#)
 - `__bernoulli`, [276](#)
 - `__bernoulli_2n`, [277](#)
 - `__bernoulli_series`, [278](#)
 - `__beta`, [278](#)
 - `__beta_gamma`, [279](#)
 - `__beta_inc`, [279](#)
 - `__beta_lgamma`, [280](#)
 - `__beta_p`, [281](#)
 - `__beta_product`, [281](#)
 - `__binomial`, [282](#)
 - `__binomial_p`, [283](#)
 - `__binomial_pdf`, [284](#)
 - `__binomial_q`, [284](#)
 - `__bose_einstein`, [285](#)

- __cauchy_p, 286
- __chebyshev_recur, 286
- __chebyshev_t, 287
- __chebyshev_u, 287
- __chebyshev_v, 288
- __chebyshev_w, 288
- __chi_squared_pdf, 289
- __chi_squared_pdfc, 289
- __chshint, 290
- __chshint_cont_frac, 290
- __chshint_series, 291
- __clamp_0_m2pi, 291
- __clamp_pi, 291
- __clausen, 292
- __clausen_cl, 293
- __clausen_sl, 294, 295
- __comp_ellint_1, 295
- __comp_ellint_2, 297
- __comp_ellint_3, 297
- __comp_ellint_d, 298
- __comp_ellint_rf, 298
- __comp_ellint_rg, 299
- __conf_hyperg, 299
- __conf_hyperg_lim, 300
- __conf_hyperg_lim_series, 300
- __conf_hyperg_luke, 301
- __conf_hyperg_series, 301
- __cos_pi, 302
- __cosh_pi, 302, 303
- __coshint, 303
- __coulomb_CF1, 304
- __coulomb_CF2, 304
- __coulomb_f_recur, 304
- __coulomb_g_recur, 305
- __coulomb_norm, 305
- __cyl_bessel, 305
- __cyl_bessel_i, 306
- __cyl_bessel_ij_series, 307
- __cyl_bessel_ik, 307
- __cyl_bessel_ik_asymp, 308
- __cyl_bessel_ik_steel, 309
- __cyl_bessel_j, 309
- __cyl_bessel_jn, 310
- __cyl_bessel_jn_asymp, 310
- __cyl_bessel_jn_neg_arg, 311
- __cyl_bessel_jn_steel, 311
- __cyl_bessel_k, 312
- __cyl_hankel_1, 312, 313
- __cyl_hankel_2, 314
- __cyl_neumann, 315
- __cyl_neumann_n, 315
- __dawson, 316
- __dawson_cont_frac, 316
- __dawson_series, 317
- __debye, 317
- __debye_region, 318
- __digamma, 318
- __digamma_asymp, 319
- __digamma_series, 319
- __dilog, 320
- __dirichlet_beta, 320, 321
- __dirichlet_eta, 321, 322
- __dirichlet_lambda, 323
- __double_factorial, 323
- __ellint_1, 324
- __ellint_2, 324
- __ellint_3, 325
- __ellint_cel, 326
- __ellint_d, 326
- __ellint_el1, 326
- __ellint_el2, 326
- __ellint_el3, 327
- __ellint_rc, 327
- __ellint_rd, 328
- __ellint_rf, 329
- __ellint_rg, 329
- __ellint_rj, 330
- __ellnome, 331
- __ellnome_k, 332
- __ellnome_series, 332
- __euler, 332, 333
- __euler_series, 333
- __eulerian_1, 333, 334
- __eulerian_1_recur, 334
- __eulerian_2, 335
- __eulerian_2_recur, 335, 336
- __exp2, 336
- __expint, 336, 337
- __expint_E1, 338
- __expint_E1_asymp, 338
- __expint_E1_series, 339
- __expint_Ei, 339
- __expint_Ei_asymp, 340
- __expint_Ei_series, 340
- __expint_En_asymp, 341
- __expint_En_cont_frac, 342
- __expint_En_large_n, 342
- __expint_En_recursion, 343
- __expint_En_series, 343
- __exponential_p, 344
- __exponential_pdf, 344
- __exponential_q, 345
- __factorial, 345
- __falling_factorial, 345, 346
- __fermi_dirac, 346
- __fisher_f_p, 347
- __fisher_f_pdf, 347
- __fisher_f_q, 348

- [__fock_airy, 349](#)
- [__fresnel, 349](#)
- [__fresnel_cont_frac, 350](#)
- [__fresnel_series, 350](#)
- [__gamma, 350, 351](#)
- [__gamma_cont_frac, 351](#)
- [__gamma_p, 351, 352](#)
- [__gamma_pdf, 352](#)
- [__gamma_q, 353](#)
- [__gamma_reciprocal, 354](#)
- [__gamma_reciprocal_series, 354](#)
- [__gamma_series, 355](#)
- [__gamma_temme, 355](#)
- [__gauss, 356](#)
- [__gegenbauer_recur, 356](#)
- [__gegenbauer_zeros, 357](#)
- [__hankel, 358](#)
- [__hankel_debye, 358](#)
- [__hankel_params, 359](#)
- [__hankel_uniform, 359](#)
- [__hankel_uniform_olver, 360](#)
- [__hankel_uniform_outer, 361](#)
- [__hankel_uniform_sum, 361](#)
- [__harmonic_number, 362](#)
- [__hermite, 363](#)
- [__hermite_asymp, 363](#)
- [__hermite_recur, 364](#)
- [__hermite_zeros, 365](#)
- [__heuman_lambda, 365](#)
- [__hurwitz_zeta, 366](#)
- [__hurwitz_zeta_euler_maclaurin, 366](#)
- [__hurwitz_zeta_polylog, 367](#)
- [__hydrogen, 368](#)
- [__hyperg, 368](#)
- [__hyperg_luke, 369](#)
- [__hyperg_recur, 369](#)
- [__hyperg_reflect, 370](#)
- [__hyperg_series, 370](#)
- [__ibeta_cont_frac, 371](#)
- [__jacobi_ellint, 372](#)
- [__jacobi_recur, 372](#)
- [__jacobi_theta_1, 373, 374](#)
- [__jacobi_theta_1_prod, 374](#)
- [__jacobi_theta_1_sum, 375](#)
- [__jacobi_theta_2, 376](#)
- [__jacobi_theta_2_prod, 377](#)
- [__jacobi_theta_2_sum, 378](#)
- [__jacobi_theta_3, 378, 379](#)
- [__jacobi_theta_3_prod, 380](#)
- [__jacobi_theta_3_sum, 380](#)
- [__jacobi_theta_4, 381, 382](#)
- [__jacobi_theta_4_prod, 382](#)
- [__jacobi_theta_4_sum, 383](#)
- [__jacobi_zeros, 383](#)
- [__jacobi_zeta, 384](#)
- [__kolmogorov_p, 384](#)
- [__laguerre, 384, 385](#)
- [__laguerre_hyperg, 386](#)
- [__laguerre_large_n, 387](#)
- [__laguerre_recur, 388](#)
- [__laguerre_zeros, 389](#)
- [__lah, 389](#)
- [__lah_recur, 389, 390](#)
- [__lanczos_binet1p, 390](#)
- [__lanczos_log_gamma1p, 391](#)
- [__legendre_p, 391](#)
- [__legendre_q, 392](#)
- [__legendre_q_series, 393](#)
- [__legendre_zeros, 393](#)
- [__log_binomial, 393, 394](#)
- [__log_binomial_sign, 394, 395](#)
- [__log_double_factorial, 395, 396](#)
- [__log_factorial, 396](#)
- [__log_falling_factorial, 397](#)
- [__log_gamma, 397, 398](#)
- [__log_gamma_bernoulli, 398](#)
- [__log_gamma_sign, 399](#)
- [__log_rising_factorial, 400](#)
- [__log_stirling_1, 400](#)
- [__log_stirling_1_sign, 400](#)
- [__log_stirling_2, 401](#)
- [__logint, 401](#)
- [__logistic_p, 402](#)
- [__logistic_pdf, 402](#)
- [__lognormal_p, 402](#)
- [__lognormal_pdf, 403](#)
- [__max_FGH, 458](#)
- [__max_FGH< double >, 458](#)
- [__max_FGH< float >, 458](#)
- [__normal_p, 403](#)
- [__normal_pdf, 403](#)
- [__owens_t, 404](#)
- [__periodic_zeta, 405](#)
- [__polar_pi, 405](#)
- [__polygamma, 406](#)
- [__polylog, 406, 407](#)
- [__polylog_exp, 407](#)
- [__polylog_exp_asymp, 408](#)
- [__polylog_exp_neg, 409, 410](#)
- [__polylog_exp_neg_int, 410, 411](#)
- [__polylog_exp_neg_real, 411, 412](#)
- [__polylog_exp_pos, 413, 414](#)
- [__polylog_exp_pos_int, 415](#)
- [__polylog_exp_pos_real, 416, 417](#)
- [__polylog_exp_sum, 417](#)
- [__prob_hermite_recur, 418](#)
- [__radial_jacobi, 419](#)
- [__radial_jacobi_zeros, 420](#)

- [__rice_pdf](#), 420
- [__riemann_zeta](#), 421
- [__riemann_zeta_euler_maclaurin](#), 422
- [__riemann_zeta_glob](#), 422
- [__riemann_zeta_laurent](#), 422
- [__riemann_zeta_m_1](#), 422
- [__riemann_zeta_m_1_glob](#), 423
- [__riemann_zeta_product](#), 423
- [__riemann_zeta_sum](#), 424
- [__rising_factorial](#), 424, 425
- [__sin_pi](#), 425, 426
- [__sinc](#), 426
- [__sinc_pi](#), 426
- [__sincos](#), 427
- [__sincos_pi](#), 428
- [__sincosint](#), 428
- [__sincosint_asymp](#), 428
- [__sincosint_cont_frac](#), 429
- [__sincosint_series](#), 429
- [__sinh_pi](#), 429, 430
- [__sinhc](#), 430
- [__sinhc_pi](#), 430
- [__sinhint](#), 431
- [__sph_bessel](#), 431, 432
- [__sph_bessel_ik](#), 433
- [__sph_bessel_jn](#), 433
- [__sph_bessel_jn_neg_arg](#), 434
- [__sph_hankel](#), 434
- [__sph_hankel_1](#), 435
- [__sph_hankel_2](#), 436, 437
- [__sph_harmonic](#), 437
- [__sph_legendre](#), 438
- [__sph_neumann](#), 439
- [__spouge_binet1p](#), 440
- [__spouge_log_gamma1p](#), 440
- [__stirling_1](#), 441, 442
- [__stirling_1_recur](#), 442, 443
- [__stirling_2](#), 443
- [__stirling_2_recur](#), 444
- [__stirling_2_series](#), 445
- [__student_t_p](#), 445
- [__student_t_pdf](#), 446
- [__student_t_q](#), 446
- [__tan_pi](#), 447
- [__tanh_pi](#), 447, 448
- [__tgamma](#), 448
- [__tgamma_lower](#), 448
- [__theta_1](#), 449
- [__theta_2](#), 449
- [__theta_2_asymp](#), 450
- [__theta_2_sum](#), 450
- [__theta_3](#), 451
- [__theta_3_asymp](#), 451
- [__theta_3_sum](#), 452
- [__theta_4](#), 452
- [__theta_c](#), 453
- [__theta_d](#), 453
- [__theta_n](#), 453
- [__theta_s](#), 454
- [__tricomi_u](#), 454
- [__tricomi_u_naive](#), 455
- [__weibull_p](#), 456
- [__weibull_pdf](#), 456
- [__zernike](#), 456
- [__znorm1](#), 457
- [__znorm2](#), 458
- [std::__detail::__gamma_lanczos_data< _Tp >](#), 537
- [std::__detail::__gamma_lanczos_data< double >](#), 537
 - [_S_cheby](#), 538
 - [_S_g](#), 538
- [std::__detail::__gamma_lanczos_data< float >](#), 538
 - [_S_cheby](#), 539
 - [_S_g](#), 539
- [std::__detail::__gamma_lanczos_data< long double >](#), 539
 - [_S_cheby](#), 540
 - [_S_g](#), 540
- [std::__detail::__gamma_spouge_data< _Tp >](#), 541
- [std::__detail::__gamma_spouge_data< double >](#), 541
 - [_S_cheby](#), 541
- [std::__detail::__gamma_spouge_data< float >](#), 542
 - [_S_cheby](#), 542
- [std::__detail::__gamma_spouge_data< long double >](#), 543
 - [_S_cheby](#), 543
- [std::__detail::__jacobi_lattice_t](#)
 - [_Cmplx](#), 546
 - [_M_omega_1](#), 550
 - [_M_omega_3](#), 550
 - [_Real](#), 546
 - [_Real_Omega1](#), 546
 - [_Real_Omega3](#), 546
 - [_S_pi](#), 550
 - [_Tp_Nome](#), 547
 - [_ellnome](#), 548
 - [_jacobi_lattice_t](#), 547
 - [_omega_1](#), 548
 - [_omega_2](#), 548
 - [_omega_3](#), 549
 - [_reduce](#), 549
 - [_tau](#), 550
- [std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_←_Omega3 >](#), 544
- [std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_←_Omega3 >::__arg_t](#), 551
- [std::__detail::__jacobi_lattice_t< _Tp_Omega1, _Tp_←_Omega3 >::__tau_t](#), 552
- [std::__detail::__jacobi_lattice_t::__arg_t](#)

- [__m, 551](#)
- [__n, 552](#)
- [__z, 552](#)
- std::__detail::__jacobi_lattice_t::__tau_t
 - [__tau_t, 553](#)
 - [__val, 553](#)
- std::__detail::__jacobi_theta_0_t
 - [_Cmplx, 555](#)
 - [_Real, 555](#)
 - [_Type, 555](#)
 - [__jacobi_theta_0_t, 555](#)
 - [dedekind_eta, 556](#)
 - [eta_1, 556](#)
 - [eta_2, 556](#)
 - [eta_3, 557](#)
 - [th1p, 557](#)
 - [th1ppp, 557](#)
 - [th2, 557](#)
 - [th2pp, 557](#)
 - [th3, 558](#)
 - [th3pp, 558](#)
 - [th4, 558](#)
 - [th4pp, 558](#)
- std::__detail::__jacobi_theta_0_t< _Tp1, _Tp3 >, [554](#)
- std::__detail::__weierstrass_invariants_t
 - [_Cmplx, 560](#)
 - [_Real, 560](#)
 - [_Type, 560](#)
 - [__delta, 561](#)
 - [__g_2, 561](#)
 - [__g_3, 561](#)
 - [__klein_j, 561](#)
 - [__weierstrass_invariants_t, 560](#)
- std::__detail::__weierstrass_invariants_t< _Tp1, _Tp3 >, [559](#)
- std::__detail::__weierstrass_roots_t
 - [_Cmplx, 563](#)
 - [_Real, 563](#)
 - [_Type, 563](#)
 - [__delta, 564](#)
 - [__e1, 565](#)
 - [__e2, 565](#)
 - [__e3, 565](#)
 - [__weierstrass_roots_t, 564](#)
- std::__detail::__weierstrass_roots_t< _Tp1, _Tp3 >, [562](#)
- std::__detail::__Airy
 - [_Airy, 567](#)
 - [inner_radius, 568](#)
 - [operator\(\), 567](#)
 - [outer_radius, 568](#)
 - [scalar_type, 566](#)
 - [value_type, 566](#)
- std::__detail::__Airy< _Tp >, [566](#)
- std::__detail::__Airy_asymp
 - [_Airy_asymp, 570](#)
 - [_Cmplx, 570](#)
 - [_S_absarg_ge_pio3, 570](#)
 - [_S_absarg_lt_pio3, 571](#)
 - [operator\(\), 571](#)
- std::__detail::__Airy_asymp< _Tp >, [568](#)
- std::__detail::__Airy_asymp_data< _Tp >, [572](#)
- std::__detail::__Airy_asymp_data< double >, [573](#)
 - [_S_c, 573](#)
 - [_S_d, 573](#)
 - [_S_max_cd, 573](#)
- std::__detail::__Airy_asymp_data< float >, [574](#)
 - [_S_c, 574](#)
 - [_S_d, 574](#)
 - [_S_max_cd, 574](#)
- std::__detail::__Airy_asymp_data< long double >, [575](#)
 - [_S_c, 575](#)
 - [_S_d, 575](#)
 - [_S_max_cd, 575](#)
- std::__detail::__Airy_asymp_series
 - [_Airy_asymp_series, 577](#)
 - [_S_sqrt_pi, 578](#)
 - [operator\(\), 578](#)
 - [scalar_type, 577](#)
 - [value_type, 577](#)
- std::__detail::__Airy_asymp_series< _Sum >, [576](#)
- std::__detail::__Airy_default_radii< _Tp >, [579](#)
- std::__detail::__Airy_default_radii< double >, [579](#)
 - [inner_radius, 579](#)
 - [outer_radius, 579](#)
- std::__detail::__Airy_default_radii< float >, [580](#)
 - [inner_radius, 580](#)
 - [outer_radius, 580](#)
- std::__detail::__Airy_default_radii< long double >, [581](#)
 - [inner_radius, 581](#)
 - [outer_radius, 581](#)
- std::__detail::__Airy_series
 - [_Cmplx, 582](#)
 - [_N_FGH, 587](#)
 - [_S_Ai, 583](#)
 - [_S_Ai0, 587](#)
 - [_S_Aip0, 587](#)
 - [_S_Airy, 583](#)
 - [_S_Bi, 584](#)
 - [_S_Bi0, 587](#)
 - [_S_Bip0, 588](#)
 - [_S_FGH, 584](#)
 - [_S_Fock, 585](#)
 - [_S_Gi0, 588](#)
 - [_S_Gip0, 588](#)
 - [_S_Hi0, 588](#)
 - [_S_Hip0, 589](#)
 - [_S_Scorer, 585](#)
 - [_S_Scorer2, 586](#)

- [_S_eps, 588](#)
- [_S_i, 589](#)
- [_S_pi, 589](#)
- [_S_sqrt_pi, 589](#)
- [std::__detail::__Airy_series<_Tp>, 581](#)
- [std::__detail::__AiryAuxilliaryState](#)
 - [_Val, 590](#)
 - [__fai_deriv, 590](#)
 - [__fai_value, 591](#)
 - [__gai_deriv, 591](#)
 - [__gai_value, 591](#)
 - [__hai_deriv, 591](#)
 - [__hai_value, 591](#)
 - [__z, 592](#)
- [std::__detail::__AiryAuxilliaryState<_Tp>, 590](#)
- [std::__detail::__AiryState](#)
 - [_Real, 593](#)
 - [__Ai_deriv, 594](#)
 - [__Ai_value, 594](#)
 - [__Bi_deriv, 594](#)
 - [__Bi_value, 594](#)
 - [__z, 594](#)
- [true_Wronskian, 593](#)
- [Wronskian, 593](#)
- [std::__detail::__AiryState<_Tp>, 592](#)
- [std::__detail::__AsympTerminator](#)
 - [_AsympTerminator, 595](#)
 - [num_terms, 596](#)
 - [operator<<, 596](#)
 - [operator\(\), 596](#)
- [std::__detail::__AsympTerminator<_Tp>, 595](#)
- [std::__detail::__Factorial_table](#)
 - [__factorial, 597](#)
 - [__log_factorial, 597](#)
 - [__n, 597](#)
- [std::__detail::__Factorial_table<_Tp>, 597](#)
- [std::__detail::__Terminator](#)
 - [_Terminator, 598](#)
 - [num_terms, 599](#)
 - [operator\(\), 599](#)
- [std::__detail::__Terminator<_Tp>, 598](#)
- [stirling_1](#)
 - [GNU Extended Mathematical Special Functions, 196](#)
- [stirling_2](#)
 - [GNU Extended Mathematical Special Functions, 197](#)
- [student_t_p](#)
 - [GNU Extended Mathematical Special Functions, 197](#)
- [student_t_pdf](#)
 - [GNU Extended Mathematical Special Functions, 198](#)
- [tan_pi](#)
 - [GNU Extended Mathematical Special Functions, 198](#)
- [tan_pif](#)
 - [GNU Extended Mathematical Special Functions, 199](#)
- [tan_pil](#)
 - [GNU Extended Mathematical Special Functions, 199](#)
- [tanh_pi](#)
 - [GNU Extended Mathematical Special Functions, 199](#)
- [tanh_pif](#)
 - [GNU Extended Mathematical Special Functions, 200](#)
- [tanh_pil](#)
 - [GNU Extended Mathematical Special Functions, 200](#)
- [tgamma](#)
 - [GNU Extended Mathematical Special Functions, 200, 201](#)
- [tgamma_lower](#)
 - [GNU Extended Mathematical Special Functions, 201](#)
- [tgamma_lowerf](#)
 - [GNU Extended Mathematical Special Functions, 202](#)
- [tgamma_lowerl](#)
 - [GNU Extended Mathematical Special Functions, 202](#)
- [tgammaf](#)
 - [GNU Extended Mathematical Special Functions, 202, 203](#)
- [tgammaf](#)
 - [GNU Extended Mathematical Special Functions, 203, 204](#)
- [th1p](#)
 - [std::__detail::__jacobi_theta_0_t, 557](#)
- [th1ppp](#)
 - [std::__detail::__jacobi_theta_0_t, 557](#)
- [th2](#)
 - [std::__detail::__jacobi_theta_0_t, 557](#)
- [th2pp](#)
 - [std::__detail::__jacobi_theta_0_t, 557](#)
- [th3](#)
 - [std::__detail::__jacobi_theta_0_t, 558](#)
- [th3pp](#)
 - [std::__detail::__jacobi_theta_0_t, 558](#)
- [th4](#)
 - [std::__detail::__jacobi_theta_0_t, 558](#)
- [th4pp](#)
 - [std::__detail::__jacobi_theta_0_t, 558](#)
- [theta_1](#)
 - [GNU Extended Mathematical Special Functions, 204](#)
- [theta_1f](#)
 - [GNU Extended Mathematical Special Functions, 205](#)
- [theta_1l](#)
 - [GNU Extended Mathematical Special Functions, 205](#)
- [theta_2](#)
 - [GNU Extended Mathematical Special Functions, 205](#)
- [theta_2f](#)
 - [GNU Extended Mathematical Special Functions, 206](#)
- [theta_2l](#)
 - [GNU Extended Mathematical Special Functions, 206](#)
- [theta_3](#)
 - [GNU Extended Mathematical Special Functions, 206](#)
- [theta_3f](#)

- GNU Extended Mathematical Special Functions, [207](#)
- theta_3l
 - GNU Extended Mathematical Special Functions, [207](#)
- theta_4
 - GNU Extended Mathematical Special Functions, [207](#)
- theta_4f
 - GNU Extended Mathematical Special Functions, [208](#)
- theta_4l
 - GNU Extended Mathematical Special Functions, [208](#)
- theta_c
 - GNU Extended Mathematical Special Functions, [208](#)
- theta_cf
 - GNU Extended Mathematical Special Functions, [209](#)
- theta_cl
 - GNU Extended Mathematical Special Functions, [209](#)
- theta_d
 - GNU Extended Mathematical Special Functions, [210](#)
- theta_df
 - GNU Extended Mathematical Special Functions, [210](#)
- theta_dl
 - GNU Extended Mathematical Special Functions, [211](#)
- theta_n
 - GNU Extended Mathematical Special Functions, [211](#)
- theta_nf
 - GNU Extended Mathematical Special Functions, [212](#)
- theta_nl
 - GNU Extended Mathematical Special Functions, [212](#)
- theta_s
 - GNU Extended Mathematical Special Functions, [212](#)
- theta_sf
 - GNU Extended Mathematical Special Functions, [213](#)
- theta_sl
 - GNU Extended Mathematical Special Functions, [213](#)
- tricomi_u
 - GNU Extended Mathematical Special Functions, [214](#)
- tricomi_uf
 - GNU Extended Mathematical Special Functions, [215](#)
- tricomi_ul
 - GNU Extended Mathematical Special Functions, [215](#)
- true_Wronskian
 - std::__detail::_AiryState, [593](#)
- value_type
 - std::__detail::_Airy, [566](#)
 - std::__detail::_Airy_asymp_series, [577](#)
- weibull_p
 - GNU Extended Mathematical Special Functions, [215](#)
- weibull_pdf
 - GNU Extended Mathematical Special Functions, [215](#)
- Wronskian
 - std::__detail::_AiryState, [593](#)
- zernike
 - GNU Extended Mathematical Special Functions, [216](#)
- zernikef
 - GNU Extended Mathematical Special Functions, [217](#)
- zernikel
 - GNU Extended Mathematical Special Functions, [217](#)