TR29124 C++ Special Math Functions 2.0

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HINGS.

## **Mathematical Special Functions**

### 1.1 Introduction and History

The first significant library upgrade on the road to C++2011, TR1, included a set of 23 mathematical functions that significantly extended the standard transcendental functions inherited from C and declared in <cmath>.

Although most components from TR1 were eventually adopted for C++11 these math functions were left behind out of concern for implementability. The math functions were published as a separate international standard IS 29124 - Extensions to the C++ Library to Support Mathematical Special Functions.

For C++17 these functions were incorporated into the main standard.

#### 1.2 Contents

The following functions are implemented in namespace std:

- · assoc\_laguerre Associated Laguerre functions
- assoc\_legendre Associated Legendre functions
- beta Beta functions
- comp\_ellint\_1 Complete elliptic functions of the first kind
- · comp ellint 2 Complete elliptic functions of the second kind
- comp\_ellint\_3 Complete elliptic functions of the third kind
- cyl\_bessel\_i Regular modified cylindrical Bessel functions
- cyl\_bessel\_j Cylindrical Bessel functions of the first kind
- · cyl\_bessel\_k Irregular modified cylindrical Bessel functions
- cyl\_neumann Cylindrical Neumann functions or Cylindrical Bessel functions of the second kind
- · ellint 1 Incomplete elliptic functions of the first kind

- ellint\_2 Incomplete elliptic functions of the second kind
- · ellint\_3 Incomplete elliptic functions of the third kind
- expint The exponential integral
- · hermite Hermite polynomials
- · laguerre Laguerre functions
- · legendre Legendre polynomials
- · riemann zeta The Riemann zeta function
- · sph bessel Spherical Bessel functions
- sph legendre Spherical Legendre functions
- · sph\_neumann Spherical Neumann functions

The hypergeometric functions were stricken from the TR29124 and C++17 versions of this math library because of implementation concerns. However, since they were in the TR1 version and since they are popular we kept them as an extension in namespace \_\_gnu\_cxx:

- · conf\_hyperg Confluent hypergeometric functions
- · hyperg Hypergeometric functions

In addition a large number of new functions are added as extensions:

- airy\_ai Airy functions of the first kind
- · airy bi Airy functions of the second kind
- · bincoef Binomial coefficients
- chebyshev\_t Chebyshev polynomials of the first kind
- chebyshev u Chebyshev polynomials of the second kind
- chebyshev\_v Chebyshev polynomials of the third kind
- chebyshev\_w Chebyshev polynomials of the fourth kind
- clausen Clausen integrals
- · clausen\_c Clausen cosine integrals
- · clausen\_s Clausen sine integrals
- comp\_ellint\_d Incomplete Legendre D elliptic integral
- conf\_hyperg\_lim Confluent hypergeometric limit functions
- · coshint Hyperbolic cosine integral
- · cosint Cosine integral
- cyl\_hankel\_1 Cylindrical Hankel functions of the first kind
- · cyl hankel 2 Cylindrical Hankel functions of the second kind

1.2 Contents 3

- · dawson Dawson integrals
- · dilog Dilogarithm functions
- · dirichlet\_beta Dirichlet beta function
- dirichlet\_eta Dirichlet beta function
- · dirichlet lambda Dirichlet lambda function
- · double\_factorial -
- ellint\_d Legendre D elliptic integrals
- ellint\_rc Carlson elliptic functions R\_C
- · ellint\_rd Carlson elliptic functions R\_D
- ellint\_rf Carlson elliptic functions R\_F
- · ellint rg Carlson elliptic functions R G
- · ellint\_rj Carlson elliptic functions R\_J
- · expint Exponential integrals
- · factorial Factorials
- fresnel\_c Fresnel cosine integrals
- fresnel\_s Fresnel sine integrals
- gamma\_I Lower incomplete gamma functions
- pgamma Regularized lower incomplete gamma functions
- qgamma Regularized upper incomplete gamma functions
- · gamma u upper incomplete gamma functions
- gegenbauer Gegenbauer polynomials
- heuman\_lambda Heuman lambda functions
- · hurwitz zeta Hurwitz zeta functions
- · ibeta Regularized incomplete beta functions
- jacobi Jacobi polynomials
- jacobi\_sn Jacobi sine amplitude functions
- jacobi\_cn Jacobi cosine amplitude functions
- · jacobi\_dn Jacobi delta amplitude functions
- · jacobi\_zeta Jacobi zeta functions
- · Ibincoef Log binomial coefficients
- · Idouble factorial Log double factorials
- legendre\_q Legendre functions of the second kind
- · Ifactorial Log factorials

- · lpochhammer\_I Log lower Pochhammer functions
- · Ipochhammer\_u Log upper Pochhammer functions
- owens t Owens T functions
- pochhammer I Lower Pochhammer functions
- pochhammer\_u Upper Pochhammer functions
- psi Psi of digamma function
- · radpoly Radial polynomials
- sinhc Hyperbolic sinus cardinal function
- sinhc\_pi -
- · sinc Normalized sinus cardinal function
- · sinc pi Sinus cardinal function
- · sinhint Hyperbolic sine integral
- · sinint Sine integral
- sph\_bessel\_i Spherical regular modified Bessel functions
- sph\_bessel\_k Spherical iregular modified Bessel functions
- · sph\_hankel\_1 Spherical Hankel functions of the first kind
- · sph hankel 2 Spherical Hankel functions of the first kind
- sph\_harmonic Spherical
- · zernike Zernike polynomials

#### 1.3 General Features

#### 1.3.1 Argument Promotion

The arguments suppled to the non-suffixed functions will be promoted according to the following rules:

- 1. If any argument intended to be floating point is given an integral value That integral value is promoted to double.
- 2. All floating point arguments are promoted up to the largest floating point precision among them.

#### 1.3.2 NaN Arguments

If any of the floating point arguments supplied to these functions is invalid or NaN (std::numeric\_limits<Tp>::quiet\_← NaN), the value NaN is returned.

1.4 Implementation 5

### 1.4 Implementation

We strive to implement the underlying math with type generic algorithms to the greatest extent possible. In practice, the functions are thin wrappers that dispatch to function templates. Type dependence is controlled with std::numeric\_limits and functions thereof.

We don't promote float to double or double to long double reflexively. The goal is for float functions to operate more quickly, at the cost of float accuracy and possibly a smaller domain of validity. Similarly, long double should give you more dynamic range and slightly more pecision than double on many systems.

### 1.5 Testing

These functions have been tested against equivalent implementations from the Gnu Scientific Library, GSL and <a href="http://www.boost.org/doc/libs/1\_60\_0/libs/math/doc/html/index. $\leftarrow$ html>Boost and the ratio

 $\frac{f - f_{test}|}{|f_{test}|}$ 

is generally found to be within 10<sup>^</sup>-15 for 64-bit double on linux-x86\_64 systems over most of the ranges of validity.

**Todo** Provide accuracy comparisons on a per-function basis for a small number of targets.

### 1.6 General Bibliography

See also

Abramowitz and Stegun: Handbook of Mathematical Functions, with Formulas, Graphs, and Mathematical Tables Edited by Milton Abramowitz and Irene A. Stegun, National Bureau of Standards Applied Mathematics Series - 55 Issued June 1964, Tenth Printing, December 1972, with corrections Electronic versions of A&S abound including both pdf and navigable html.

for example http://people.math.sfu.ca/~cbm/aands/

The old A&S has been redone as the NIST Digital Library of Mathematical Functions: http://dlmf.nist. cov/ This version is far more navigable and includes more recent work.

An Atlas of Functions: with Equator, the Atlas Function Calculator 2nd Edition, by Oldham, Keith B., Myland, Jan, Spanier, Jerome

Asymptotics and Special Functions by Frank W. J. Olver, Academic Press, 1974

Numerical Recipes in C, The Art of Scientific Computing, by William H. Press, Second Ed., Saul A. Teukolsky, William T. Vetterling, and Brian P. Flannery, Cambridge University Press, 1992

The Special Functions and Their Approximations: Volumes 1 and 2, by Yudell L. Luke, Academic Press, 1969

## **Todo List**

```
page Mathematical Special Functions
   Provide accuracy comparisons on a per-function basis for a small number of targets.

Member std::__detail::__dawson_cont_frac (_Tp __x)
   this needs some compile-time construction!

Member std::__detail::__expint_E1 (_Tp __x)
   Find a good asymptotic switch point in E_1(x).

Member std::__detail::__expint_En_recursion (unsigned int __n, _Tp __x)
   Find a principled starting number for the E_n(x) downward recursion.

Member std::__detail::__hurwitz_zeta (_Tp __s, std::complex < _Tp > __a)
   This __hurwitz_zeta prefactor is prone to overflow. positive integer orders s?

Member std::__detail::_Airy_asymp < _Tp >::_S_absarg_lt_pio3 (std::complex < _Tp > __z) const
   Revisit these numbers of terms for the Airy asymptotic expansions.

Member std::__detail::_Airy_series < _Tp >::_S_Scorer (std::complex < _Tp > __t)
   Find out what is wrong with the Hi = fai + gai + hai scorer function.
```

8 Todo List

# **Module Index**

### 3.1 Modules

Here is a list of all modules:

C++ Mathematical Special Functions	19
C++17/IS29124 Mathematical Special Functions	20
GNU Extended Mathematical Special Functions	44

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# Namespace Index

## 4.1 Namespace List

Here is a list of all namespaces with brief descriptions:

gnt	n_cxx					 					 											 			12	26
std .						 					 											 			13	37
std::	detail					 					 							 				 			13	39

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## **Hierarchical Index**

## 5.1 Class Hierarchy

This inheritance list is sorted roughly, but not completely, alphabetically:

std::detail::_Airy< _Tp >	59
std::detail::_Airy_asymp_data< _Tp >	
std::detail::_Airy_asymp< _Tp >	32
std::detail::_Airy_asymp_data <float128></float128>	36
std::detail::_Airy_asymp_data< double >	37
std::detail::_Airy_asymp_data< float >	38
std::detail::_Airy_asymp_data< long double >	39
std::detail::_Airy_asymp_series< _Sum >	70
std::detail::_Airy_default_radii<_Tp>	72
std::detail::_Airy_default_radii< double >	72
std::detail::_Airy_default_radii< float >	73
std::detail::_Airy_default_radii< long double >	73
std::detail::_Airy_series< _Tp >	74
std::detail::_AiryAuxilliaryState< _Tp >	30
std::detail::_AiryState< _Tp >	32
std:: detail:: Factorial table< Tp >	34

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## **Class Index**

### 6.1 Class List

Here are the classes, structs, unions and interfaces with brief descriptions:

std::_	_detail::_Airy< _Tp >	59
std::_	_detail::_Airy_asymp< _Tp >	32
std::_	_detail::_Airy_asymp_data< _Tp >26	36
std::_	_detail::_Airy_asymp_data <float128></float128>	36
std::_	_detail::_Airy_asymp_data< double >	37
std::_	_detail::_Airy_asymp_data< float >	36
std::_	_detail::_Airy_asymp_data $<$ long double $>$ $\dots\dots\dots\dots$ 20	39
std::_	_detail::_Airy_asymp_series< _Sum >	70
std::_	_detail::_Airy_default_radii< _Tp >	72
std::_	_detail::_Airy_default_radii $<$ double $>$ $\dots$	72
std::_	_detail::_Airy_default_radii $<$ float $>$ $\dots$	73
std::_	_detail::_Airy_default_radii $<$ long double $>$ $\dots\dots\dots\dots$ 27	73
std::_	_detail::_Airy_series< _Tp >	74
std::_	_detail::_AiryAuxilliaryState $<$ _Tp $>$ $\dots$	30
std::_	_detail::_AiryState< _Tp >	32
std::	_detail::_Factorial_table< _Tp >	34

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# File Index

### 7.1 File List

Here is a list of all files with brief descriptions:

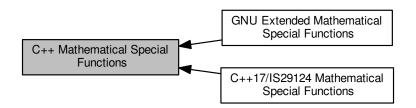
bits/sf_airy.tcc	85
bits/sf_bessel.tcc	87
bits/sf_beta.tcc	89
bits/sf_cardinal.tcc	91
bits/sf_chebyshev.tcc	93
bits/sf_dawson.tcc	94
bits/sf_ellint.tcc	96
bits/sf_expint.tcc	98
bits/sf_fresnel.tcc	01
bits/sf_gamma.tcc	
bits/sf_gegenbauer.tcc	30
bits/sf_hankel.tcc	09
bits/sf_hermite.tcc	
bits/sf_hydrogen.tcc	
bits/sf_hyperg.tcc	
bits/sf_hypint.tcc	
bits/sf_jacobi.tcc	
bits/sf_laguerre.tcc	
bits/sf_legendre.tcc	
bits/sf_mod_bessel.tcc	
bits/sf_owens_t.tcc	
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## **Module Documentation**

### 8.1 C++ Mathematical Special Functions

Collaboration diagram for C++ Mathematical Special Functions:



#### **Modules**

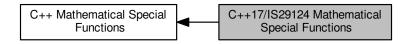
- C++17/IS29124 Mathematical Special Functions
- GNU Extended Mathematical Special Functions

### 8.1.1 Detailed Description

A collection of advanced mathematical special functions.

### 8.2 C++17/IS29124 Mathematical Special Functions

Collaboration diagram for C++17/IS29124 Mathematical Special Functions:



#### **Functions**

```
template<typename</li>Tp >
   __gnu_cxx::__promote< _Tp >::__type std::assoc_laguerre (unsigned int __n, unsigned int __m, _Tp __x)

    float std::assoc_laguerref (unsigned int __n, unsigned int __m, float __x)

    long double std::assoc_laguerrel (unsigned int __n, unsigned int __m, long double __x)

    template<typename</li>
    Tp >

    _gnu_cxx::__promote< _Tp >::__type std::assoc_legendre (unsigned int __I, unsigned int __m, _Tp __x)

    float std::assoc_legendref (unsigned int __l, unsigned int __m, float __x)

• long double std::assoc legendrel (unsigned int I, unsigned int m, long double x)

    template<typename _Tpa , typename _Tpb >

    _gnu_cxx::__promote_2< _Tpa, _Tpb >::__type std::beta (_Tpa __a, _Tpb __b)

    float std::betaf (float __a, float __b)

    long double std::betal (long double __a, long double __b)

• template<typename _Tp >
    _gnu_cxx::__promote< _Tp >::__type std::comp_ellint_1 (_Tp __k)

    float std::comp ellint 1f (float k)

    long double std::comp ellint 1l (long double k)

• template<typename _{\mathrm{Tp}} >
    _gnu_cxx::__promote< _Tp >::__type std::comp_ellint_2 (_Tp __k)

    float std::comp ellint 2f (float k)

    long double std::comp ellint 2l (long double k)

• template<typename _Tp , typename _Tpn >
    gnu cxx:: promote 2< Tp, Tpn >:: type std::comp ellint 3 ( Tp k, Tpn nu)

    float std::comp ellint 3f (float k, float nu)

      Return the complete elliptic integral of the third kind \Pi(k,\nu) for float modulus k.

    long double std::comp_ellint_3l (long double __k, long double __nu)

      Return the complete elliptic integral of the third kind \Pi(k,\nu) for long double modulus k.

    template<typename _Tpnu , typename _Tp >

    _gnu_cxx::__promote_2< _Tpnu, _Tp >::__type std::cyl_bessel_i (_Tpnu __nu, _Tp __x)

    float std::cyl_bessel_if (float __nu, float __x)

    long double std::cyl bessel il (long double nu, long double x)

    template<typename _Tpnu , typename _Tp >

   _gnu_cxx::__promote_2< _Tpnu, _Tp >::__type std::cyl_bessel_j (_Tpnu __nu, _Tp __x)

    float std::cyl bessel jf (float nu, float x)

• long double std::cyl_bessel_jl (long double __nu, long double __x)
```

```
• template<typename _Tpnu , typename _Tp >
    _gnu_cxx::__promote_2< _Tpnu, _Tp >::__type std::cyl_bessel_k (_Tpnu __nu, _Tp __x)

    float std::cyl bessel kf (float nu, float x)

    long double std::cyl_bessel_kl (long double __nu, long double __x)

• template<typename Tpnu, typename Tp >
    _gnu_cxx::__promote_2< _Tpnu, _Tp >::__type std::cyl_neumann (_Tpnu __nu, _Tp __x)

    float std::cyl neumannf (float nu, float x)

    long double std::cyl_neumannl (long double __nu, long double __x)

    template<typename</li>
    Tp , typename
    Tpp >

   _gnu_cxx::__promote_2< _Tp, _Tpp >::__type std::ellint_1 (_Tp __k, _Tpp __phi)

    float std::ellint_1f (float __k, float __phi)

    long double std::ellint 11 (long double k, long double phi)

    template<typename _Tp , typename _Tpp >

    _gnu_cxx::__promote_2< _Tp, _Tpp >::__type std::ellint_2 (_Tp __k, _Tpp __phi)

    float std::ellint 2f (float k, float phi)

      Return the incomplete elliptic integral of the second kind E(k,\phi) for float argument.

    long double std::ellint_2l (long double __k, long double __phi)

      Return the incomplete elliptic integral of the second kind E(k, \phi).

    template<typename _Tp , typename _Tpn , typename _Tpp >

   _gnu_cxx::__promote_3< _Tp, _Tpn, _Tpp >::__type std::ellint_3 (_Tp __k, _Tpn __nu, _Tpp __phi)
      Return the incomplete elliptic integral of the third kind \Pi(k, \nu, \phi).

    float std::ellint_3f (float __k, float __nu, float __phi)

      Return the incomplete elliptic integral of the third kind \Pi(k, \nu, \phi) for float argument.

    long double std::ellint 3l (long double k, long double nu, long double phi)

      Return the incomplete elliptic integral of the third kind \Pi(k, \nu, \phi).

    template<typename</li>
    Tp >

   _gnu_cxx::__promote< _Tp >::__type std::expint (_Tp __x)

    float std::expintf (float __x)

    long double std::expintl (long double x)

    template<typename</li>
    Tp >

   _gnu_cxx::__promote< _Tp >::__type std::hermite (unsigned int __n, _Tp __x)

    float std::hermitef (unsigned int __n, float __x)

    long double std::hermitel (unsigned int n, long double x)

template<typename _Tp >
    _gnu_cxx::__promote< _Tp >::__type std::laguerre (unsigned int __n, _Tp __x)

    float std::laguerref (unsigned int n, float x)

    long double std::laguerrel (unsigned int __n, long double __x)

• template<typename _Tp >
    _gnu_cxx::__promote< _Tp >::__type std::legendre (unsigned int __l, _Tp __x)

    float std::legendref (unsigned int I, float x)

    long double std::legendrel (unsigned int __I, long double __x)

template<typename _Tp >
    gnu cxx:: promote < Tp >:: type std::riemann zeta ( Tp s)

    float std::riemann_zetaf (float __s)

    long double std::riemann zetal (long double s)

template<typename _Tp >
    gnu cxx:: promote < Tp >:: type std::sph bessel (unsigned int n, Tp x)

    float std::sph besself (unsigned int n, float x)

    long double std::sph_bessell (unsigned int __n, long double __x)

template<typename _Tp >
    gnu cxx:: promote < Tp >:: type std::sph legendre (unsigned int I, unsigned int m, Tp theta)
```

- float std::sph\_legendref (unsigned int \_\_l, unsigned int \_\_m, float \_\_theta)
- long double std::sph legendrel (unsigned int I, unsigned int m, long double theta)
- template<typename \_Tp >
   \_\_gnu\_cxx::\_\_promote< \_Tp >::\_\_type std::sph\_neumann (unsigned int \_\_n, \_Tp \_\_x)
- float std::sph neumannf (unsigned int n, float x)
- long double std::sph\_neumannl (unsigned int \_\_n, long double \_\_x)

#### 8.2.1 Detailed Description

A collection of advanced mathematical special functions for C++17 and IS29124.

#### 8.2.2 Function Documentation

8.2.2.1 template<typename \_Tp > \_\_gnu\_cxx::\_\_promote<\_Tp>::\_\_type std::assoc\_laguerre ( unsigned int \_\_n, unsigned int \_\_n, \_Tp \_\_x ) [inline]

Return the associated Laguerre polynomial  $L_n^m(x)$  of nonnegative order n, nonnegative degree m and real argument x.

The associated Laguerre function of real degree  $\alpha$ ,  $L_n^{\alpha}(x)$ , is defined by

$$L_n^{\alpha}(x) = \frac{(\alpha+1)_n}{n!} {}_1F_1(-n;\alpha+1;x)$$

where  $(\alpha)_n$  is the Pochhammer symbol and  ${}_1F_1(a;c;x)$  is the confluent hypergeometric function.

The associated Laguerre polynomial is defined for integral degree  $\alpha=m$  by:

$$L_n^m(x) = (-1)^m \frac{d^m}{dx^m} L_{n+m}(x)$$

where the Laguerre polynomial is defined by:

$$L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n} (x^n e^{-x})$$

and x >= 0.

See also

laguerre for details of the Laguerre function of degree n

**Template Parameters** 

\_Tp | The floating-point type of the argument \_\_\_x.

#### **Parameters**

_~	The order of the Laguerre function, $\underline{\hspace{0.2cm}}$ n $>= 0$ .
_n	
_~	The degree of the Laguerre function,m >= 0.
_m	
_~	The argument of the Laguerre function, $\underline{} x >= 0$ .
_X	

#### **Exceptions**

std::domain_error	$   if \underline{} x < 0. $	
-------------------	------------------------------	--

Definition at line 372 of file specfun.h.

**8.2.2.2** float std::assoc\_laguerref ( unsigned int \_\_n, unsigned int \_\_m, float \_\_x ) [inline]

Return the associated Laguerre polynomial  $L_n^m(x)$  of order n, degree m, and float argument x.

#### See also

assoc laguerre for more details.

Definition at line 324 of file specfun.h.

**8.2.2.3** long double std::assoc\_laguerrel ( unsigned int \_\_n, unsigned int \_\_m, long double \_\_x ) [inline]

Return the associated Laguerre polynomial  $L_n^m(x)$  of order n, degree m and long double argument x.

#### See also

assoc laguerre for more details.

Definition at line 335 of file specfun.h.

8.2.2.4 template<typename \_Tp > \_\_gnu\_cxx::\_\_promote<\_Tp>::\_\_type std::assoc\_legendre ( unsigned int \_\_I, unsigned int \_

Return the associated Legendre function  $P_l^m(x)$  of degree l, order m, and real argument x.

The associated Legendre function is derived from the Legendre function  $P_l(x)$  by the Rodrigues formula:

$$P_l^m(x) = (1 - x^2)^{m/2} \frac{d^m}{dx^m} P_l(x)$$

#### See also

legendre for details of the Legendre function of degree 1

#### **Template Parameters**

_Тр	The floating-point type of the argument _	_x.
-----	---	-----

#### **Parameters**

_ <del>←</del>	The degree $\1 >= 0$ .
_ <del>←</del>	The order $\underline{}$ m $<=$ 1.
_← _X	The argument, abs (x) <= 1.

#### **Exceptions**

std::domain_error	if abs (x) > 1.
-------------------	-----------------

Definition at line 420 of file specfun.h.

8.2.2.5 float std::assoc\_legendref ( unsigned int \_\_l, unsigned int \_\_m, float \_\_x ) [inline]

Return the associated Legendre function  $P_l^m(x)$  of degree l, order m, and float argument x.

#### See also

assoc legendre for more details.

Definition at line 387 of file specfun.h.

8.2.2.6 long double std::assoc\_legendrel ( unsigned int \_\_l, unsigned int \_\_m, long double \_\_x ) [inline]

Return the associated Legendre function  $P_l^m(x)$  of degree l, order m, and long double argument x.

#### See also

assoc\_legendre for more details.

Definition at line 398 of file specfun.h.

8.2.2.7 template<typename \_Tpa , typename \_Tpb > \_\_gnu\_cxx::\_\_promote\_2<\_Tpa, \_Tpb>::\_\_type std::beta ( \_Tpa \_\_a, \_Tpb \_\_b ) [inline]

Return the beta function, B(a, b), for real parameters a, b.

The beta function is defined by

$$B(a,b) = \int_0^1 t^{a-1} (1-t)^{b-1} dt = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

where a > 0 and b > 0

#### **Template Parameters**

_Тра	The floating-point type of the parametera.
_Tpb	The floating-point type of the parameterb.

#### **Parameters**

_~	The first argument of the beta function, $\a > 0$ .
_a	
_~	The second argument of the beta function, $\_$ b $>$ 0 .
_b	

#### **Exceptions**

$$|$$
 std::domain\_error  $|$  if  $_a < 0$  or  $_b < 0$  .

Definition at line 465 of file specfun.h.

Return the beta function, B(a,b), for float parameters a, b.

See also

beta for more details.

Definition at line 434 of file specfun.h.

Return the beta function, B(a, b), for long double parameters a, b.

See also

beta for more details.

Definition at line 444 of file specfun.h.

Return the complete elliptic integral of the first kind K(k) for real modulus k.

The complete elliptic integral of the first kind is defined as

$$K(k) = F(k,\pi/2) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - k^2 sin^2 \theta}}$$

where  $F(k,\phi)$  is the incomplete elliptic integral of the first kind and the modulus |k|<=1.

See also

ellint 1 for details of the incomplete elliptic function of the first kind.

#### **Template Parameters**

_Тр	The floating-point type of the modulus	k.
-----	--	----

#### **Parameters**

```
\begin{array}{c|c} \_ \leftarrow & \text{The modulus, abs } (\_\_k) <= 1 \\ \_k & \end{array}
```

#### **Exceptions**

```
| std::domain_error | if abs (__k) > 1 .
```

Definition at line 513 of file specfun.h.

```
8.2.2.11 float std::comp_ellint_1f(float __k) [inline]
```

Return the complete elliptic integral of the first kind E(k) for float modulus k.

See also

comp\_ellint\_1 for details.

Definition at line 480 of file specfun.h.

```
8.2.2.12 long double std::comp_ellint_1I( long double __k ) [inline]
```

Return the complete elliptic integral of the first kind E(k) for long double modulus k.

See also

comp\_ellint\_1 for details.

Definition at line 490 of file specfun.h.

```
\textbf{8.2.2.13} \quad template < typename \_Tp > \_\_gnu\_cxx::\_promote < \_Tp > ::\_type \ std::comp\_ellint\_2 \ ( \_Tp \_k \ ) \quad [inline]
```

Return the complete elliptic integral of the second kind E(k) for real modulus k.

The complete elliptic integral of the second kind is defined as

$$E(k) = E(k, \pi/2) = \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 \theta}$$

where  $E(k,\phi)$  is the incomplete elliptic integral of the second kind and the modulus |k| <= 1.

See also

ellint 2 for details of the incomplete elliptic function of the second kind.

#### **Template Parameters**

_Тр	The floating-point type of the modulus _	k.
-----	--	----

#### **Parameters**

$$\begin{array}{c|c} - \leftarrow & \text{The modulus, abs } (\underline{\phantom{a}} k) <= 1 \\ \underline{\phantom{a}} k & \end{array}$$

#### **Exceptions**

```
std::domain\_error \mid if abs(\__k) > 1.
```

Definition at line 560 of file specfun.h.

Return the complete elliptic integral of the second kind E(k) for float modulus k.

#### See also

comp\_ellint\_2 for details.

Definition at line 528 of file specfun.h.

Return the complete elliptic integral of the second kind E(k) for long double modulus k.

#### See also

comp\_ellint\_2 for details.

Definition at line 538 of file specfun.h.

Return the complete elliptic integral of the third kind  $\Pi(k,\nu)=\Pi(k,\nu,\pi/2)$  for real modulus k.

The complete elliptic integral of the third kind is defined as

$$\Pi(k,\nu) = \Pi(k,\nu,\pi/2) = \int_0^{\pi/2} \frac{d\theta}{(1-\nu\sin^2\theta)\sqrt{1-k^2\sin^2\theta}}$$

where  $\Pi(k,\nu,\phi)$  is the incomplete elliptic integral of the second kind and the modulus |k| <= 1.

#### See also

ellint 3 for details of the incomplete elliptic function of the third kind.

#### **Template Parameters**

_Тр	The floating-point type of the modulusk.
_Tpn	The floating-point type of the argumentnu.

#### **Parameters**

k	The modulus, abs $(\underline{}$ k) <= 1
nu	The argument

#### **Exceptions**

std::domain_error	if abs (k) > 1.
-------------------	-----------------

Definition at line 611 of file specfun.h.

8.2.2.17 float std::comp\_ellint\_3f ( float \_\_k, float \_\_nu ) [inline]

Return the complete elliptic integral of the third kind  $\Pi(k,\nu)$  for float modulus k.

#### See also

comp\_ellint\_3 for details.

Definition at line 575 of file specfun.h.

8.2.2.18 long double std::comp\_ellint\_3l ( long double \_\_k, long double \_\_nu ) [inline]

Return the complete elliptic integral of the third kind  $\Pi(k,\nu)$  for long double modulus k.

#### See also

comp ellint 3 for details.

Definition at line 585 of file specfun.h.

8.2.2.19 template<typename \_Tpnu , typename \_Tp > \_\_gnu\_cxx::\_\_promote\_2<\_Tpnu, \_Tp>::\_\_type std::cyl\_bessel\_i ( \_Tpnu \_\_nu, \_Tp \_\_x ) [inline]

Return the regular modified Bessel function  $I_{\nu}(x)$  for real order  $\nu$  and argument x >= 0.

The regular modified cylindrical Bessel function is:

$$I_{\nu}(x) = i^{-\nu} J_{\nu}(ix) = \sum_{k=0}^{\infty} \frac{(x/2)^{\nu+2k}}{k!\Gamma(\nu+k+1)}$$

#### **Template Parameters**

_Tpnu	The floating-point type of the ordernu.
_Тр	The floating-point type of the argumentx.

#### **Parameters**

nu	The order
x	The argument, $\underline{}$ x $>= 0$

#### **Exceptions**

std::domain_error	ifx < 0 .
-------------------	-----------

Definition at line 657 of file specfun.h.

8.2.2.20 float std::cyl\_bessel\_if ( float \_\_nu, float \_\_x ) [inline]

Return the regular modified Bessel function  $I_{\nu}(x)$  for float order  $\nu$  and argument x>=0.

#### See also

cyl bessel i for setails.

Definition at line 626 of file specfun.h.

8.2.2.21 long double std::cyl\_bessel\_il ( long double \_\_nu, long double \_\_x ) [inline]

Return the regular modified Bessel function  $I_{\nu}(x)$  for long double order  $\nu$  and argument x>=0.

#### See also

cyl\_bessel\_i for setails.

Definition at line 636 of file specfun.h.

Return the Bessel function  $J_{\nu}(x)$  of real order  $\nu$  and argument x >= 0.

The cylindrical Bessel function is:

$$J_{\nu}(x) = \sum_{k=0}^{\infty} \frac{(-1)^k (x/2)^{\nu+2k}}{k! \Gamma(\nu+k+1)}$$

#### **Template Parameters**

_Tpnu	The floating-point type of the ordernu.
_Тр	The floating-point type of the argumentx.

#### **Parameters**

nu	The order
x	The argument, $\underline{}$ x $>= 0$

#### **Exceptions**

std::domain_error	if _	X	<	0		
-------------------	------	---	---	---	--	--

Definition at line 703 of file specfun.h.

**8.2.2.23** float std::cyl\_bessel\_jf ( float \_\_nu, float \_\_x ) [inline]

Return the Bessel function of the first kind  $J_{\nu}(x)$  for float order  $\nu$  and argument x>=0.

#### See also

cyl bessel i for setails.

Definition at line 672 of file specfun.h.

8.2.2.24 long double std::cyl\_bessel\_jl( long double \_\_nu, long double \_\_x ) [inline]

Return the Bessel function of the first kind  $J_{\nu}(x)$  for long double order  $\nu$  and argument x>=0.

#### See also

cyl\_bessel\_j for setails.

Definition at line 682 of file specfun.h.

8.2.2.25 template<typename \_Tpnu , typename \_Tp > \_\_gnu\_cxx::\_\_promote\_2<\_Tpnu, \_Tp>::\_\_type std::cyl\_bessel\_k ( \_Tpnu \_\_nu, \_Tp \_\_x ) [inline]

Return the irregular modified Bessel function  $K_{\nu}(x)$  of real order  $\nu$  and argument x.

The irregular modified Bessel function is defined by:

$$K_{\nu}(x) = \frac{\pi}{2} \frac{I_{-\nu}(x) - I_{\nu}(x)}{\sin \nu \pi}$$

where for integral  $\nu=n$  a limit is taken:  $lim_{\nu\to n}$ . For negative argument we have simply:

$$K_{-\nu}(x) = K_{\nu}(x)$$

#### **Template Parameters**

_Tpnu	The floating-point type of the ordernu.
_Тр	The floating-point type of the argumentx.

#### **Parameters**

nu	The order
x	The argument, $\underline{}$ x $>= 0$

#### **Exceptions**

std::domain_error	ifx < 0 .
-------------------	-----------

Definition at line 755 of file specfun.h.

8.2.2.26 float std::cyl\_bessel\_kf ( float \_\_nu, float \_\_x ) [inline]

Return the irregular modified Bessel function  $K_{\nu}(x)$  for float order  $\nu$  and argument x>=0.

#### See also

cyl\_bessel\_k for setails.

Definition at line 718 of file specfun.h.

8.2.2.27 long double std::cyl\_bessel\_kl ( long double \_\_nu, long double \_\_x ) [inline]

Return the irregular modified Bessel function  $K_{\nu}(x)$  for long double order  $\nu$  and argument x>=0.

#### See also

cyl\_bessel\_k for setails.

Definition at line 728 of file specfun.h.

8.2.2.28 template<typename \_Tpnu , typename \_Tp > \_\_gnu\_cxx::\_\_promote\_2<\_Tpnu, \_Tp>::\_\_type std::cyl\_neumann ( \_Tpnu \_\_nu, \_Tp \_\_x ) [inline]

Return the Neumann function  $N_{\nu}(x)$  of real order  $\nu$  and argument x>=0.

The Neumann function is defined by:

$$N_{\nu}(x) = \frac{J_{\nu}(x)\cos\nu\pi - J_{-\nu}(x)}{\sin\nu\pi}$$

where x >= 0 and for integral order  $\nu = n$  a limit is taken:  $\lim_{\nu \to n} u$ 

#### **Template Parameters**

_Tpnu	The floating-point type of the ordernu.
_Тр	The floating-point type of the argumentx.

#### **Parameters**

nu	The order
x	The argument, $\underline{}$ x $>= 0$

#### **Exceptions**

std::domain_error	if _	X	<	0		
-------------------	------	---	---	---	--	--

Definition at line 803 of file specfun.h.

8.2.2.29 float std::cyl\_neumannf ( float \_\_nu, float \_\_x ) [inline]

Return the Neumann function  $N_{
u}(x)$  of float order u and argument x.

See also

cyl\_neumann for setails.

Definition at line 770 of file specfun.h.

8.2.2.30 long double std::cyl\_neumannl ( long double \_\_nu, long double \_\_x ) [inline]

Return the Neumann function  $N_{\nu}(x)$  of long double order  $\nu$  and argument x.

See also

cyl\_neumann for setails.

Definition at line 780 of file specfun.h.

8.2.2.31 template<typename \_Tp , typename \_Tpp > \_\_gnu\_cxx::\_\_promote\_2<\_Tp, \_Tpp>::\_\_type std::ellint\_1 ( \_Tp \_\_k, \_Tpp \_\_phi ) [inline]

Return the incomplete elliptic integral of the first kind  $F(k,\phi)$  for real modulus k and angle  $\phi$ .

The incomplete elliptic integral of the first kind is defined as

$$F(k,\phi) = \int_0^{\phi} \frac{d\theta}{\sqrt{1 - k^2 sin^2 \theta}}$$

For  $\phi=\pi/2$  this becomes the complete elliptic integral of the first kind, K(k).

See also

comp\_ellint\_1.

#### **Template Parameters**

_Тр	The floating-point type of the modulus $\underline{}$ k.
_Трр	The floating-point type of the anglephi.

#### **Parameters**

k	The modulus, abs (k) <= 1
phi	The integral limit argument in radians

#### **Exceptions**

std::domain_error	if $abs(\underline{k}) > 1$ .
-------------------	-------------------------------

Definition at line 851 of file specfun.h.

Return the incomplete elliptic integral of the first kind  $E(k,\phi)$  for float modulus k and angle  $\phi$ .

#### See also

ellint\_1 for details.

Definition at line 818 of file specfun.h.

Return the incomplete elliptic integral of the first kind  $E(k,\phi)$  for long double modulus k and angle  $\phi$ .

#### See also

ellint\_1 for details.

Definition at line 828 of file specfun.h.

Return the incomplete elliptic integral of the second kind  $E(k, \phi)$ .

The incomplete elliptic integral of the second kind is defined as

$$E(k,\phi) = \int_0^{\phi} \sqrt{1 - k^2 sin^2 \theta}$$

For  $\phi = \pi/2$  this becomes the complete elliptic integral of the second kind, E(k).

#### See also

comp\_ellint\_2.

### **Template Parameters**

_Тр	The floating-point type of the modulusk.
_Трр	The floating-point type of the anglephi.

#### **Parameters**

k	The modulus, abs (k) <= 1
phi	The integral limit argument in radians

#### Returns

The elliptic function of the second kind.

### **Exceptions**

```
|std::domain\_error| if abs (\__k) > 1.
```

Definition at line 899 of file specfun.h.

```
8.2.2.35 float std::ellint_2f (float __k, float __phi ) [inline]
```

Return the incomplete elliptic integral of the second kind  $E(k,\phi)$  for float argument.

#### See also

ellint\_2 for details.

Definition at line 866 of file specfun.h.

8.2.2.36 long double std::ellint\_2l ( long double \_\_k, long double \_\_phi ) [inline]

Return the incomplete elliptic integral of the second kind  $E(k,\phi)$ .

#### See also

ellint\_2 for details.

Definition at line 876 of file specfun.h.

Return the incomplete elliptic integral of the third kind  $\Pi(k, \nu, \phi)$ .

The incomplete elliptic integral of the third kind is defined by:

$$\Pi(k,\nu,\phi) = \int_0^\phi \frac{d\theta}{(1-\nu\sin^2\theta)\sqrt{1-k^2\sin^2\theta}}$$

For  $\phi=\pi/2$  this becomes the complete elliptic integral of the third kind,  $\Pi(k,\nu)$ .

#### See also

comp\_ellint\_3.

#### **Template Parameters**

_Тр	The floating-point type of the modulusk.
_Tpn	The floating-point type of the argumentnu.
_Трр	The floating-point type of the anglephi.

#### **Parameters**

k	The modulus, abs $(\underline{}$ k) <= 1
nu	The second argument
phi	The integral limit argument in radians

#### Returns

The elliptic function of the third kind.

#### **Exceptions**

$$std::domain\_error \mid if abs(\__k) > 1$$
.

Definition at line 952 of file specfun.h.

Return the incomplete elliptic integral of the third kind  $\Pi(k,\nu,\phi)$  for float argument.

#### See also

ellint\_3 for details.

Definition at line 914 of file specfun.h.

8.2.2.39 long double std::ellint\_3I ( long double \_\_k, long double \_\_nu, long double \_\_phi ) [inline]

Return the incomplete elliptic integral of the third kind  $\Pi(k,\nu,\phi)$ .

#### See also

ellint\_3 for details.

Definition at line 924 of file specfun.h.

8.2.2.40 template<typename\_Tp>\_\_gnu\_cxx::\_\_promote<\_Tp>::\_\_type std::expint( \_Tp \_\_x ) [inline]

Return the exponential integral Ei(x) for real argument x.

The exponential integral is given by

$$Ei(x) = -\int_{-x}^{\infty} \frac{e^t}{t} dt$$

#### **Template Parameters**

Tp The floating-point type of the argument x.

#### **Parameters**

\_ ← The argument of the exponential integral function.

Definition at line 992 of file specfun.h.

**8.2.2.41** float std::expintf (float \_x ) [inline]

Return the exponential integral Ei(x) for float argument  ${\bf x}.$ 

See also

expint for details.

Definition at line 966 of file specfun.h.

**8.2.2.42** long double std::expintl (long double \_x ) [inline]

Return the exponential integral Ei(x) for long double argument x.

See also

expint for details.

Definition at line 976 of file specfun.h.

Return the Hermite polynomial  $H_n(x)$  of order n and real argument x.

The Hermite polynomial is defined by:

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2}$$

The Hermite polynomial obeys a reflection formula:

$$H_n(-x) = (-1)^n H_n(x)$$

# **Template Parameters**

$\_\mathit{Tp} \mid$ The floating-point type of the argument $\_$	_x.
---	-----

# **Parameters**

_~	The order
_n	
_←	The argument
_X	

Definition at line 1040 of file specfun.h.

8.2.2.44 float std::hermitef (unsigned int \_\_n, float \_\_x) [inline]

Return the Hermite polynomial  $H_n(x)$  of nonnegative order n and float argument x.

# See also

hermite for details.

Definition at line 1007 of file specfun.h.

8.2.2.45 long double std::hermitel ( unsigned int \_\_n, long double \_\_x ) [inline]

Return the Hermite polynomial  $H_n(x)$  of nonnegative order n and long double argument x.

## See also

hermite for details.

Definition at line 1017 of file specfun.h.

Returns the Laguerre polynomial  $L_n(x)$  of nonnegative degree n and real argument x >= 0.

The Laguerre polynomial is defined by:

$$L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n} (x^n e^{-x})$$

# **Template Parameters**

_Tp   The floating-point type of the argument _	x.
---	----

### **Parameters**

_~	The nonnegative order	
_n		
_←	The argument $\underline{}$ $x >= 0$	
_X		

# **Exceptions**

std::domain_error	$if _{x} < 0$ .
-------------------	-----------------

Definition at line 1084 of file specfun.h.

**8.2.2.47** float std::laguerref (unsigned int \_\_n, float \_\_x ) [inline]

Returns the Laguerre polynomial  $L_n(x)$  of nonnegative degree n and float argument x>=0.

### See also

laguerre for more details.

Definition at line 1055 of file specfun.h.

**8.2.2.48** long double std::laguerrel ( unsigned int \_\_n, long double \_\_x ) [inline]

Returns the Laguerre polynomial  $L_n(x)$  of nonnegative degree n and long double argument x>=0.

## See also

laguerre for more details.

Definition at line 1065 of file specfun.h.

Return the Legendre polynomial  $P_l(x)$  of nonnegative degree l and real argument |x| <= 0.

The Legendre function of order l and argument x,  $P_l(x)$ , is defined by:

$$P_l(x) = \frac{1}{2^l l!} \frac{d^l}{dx^l} (x^2 - 1)^l$$

# **Template Parameters**

_Tp   The floating-point type of the argument _	X.
---	----

### **Parameters**

_쓴	The degree $l>=0$
_/	
_←	The argument abs (x) <= 1
_X	

# **Exceptions**

std::domain_error	if abs (x) > 1
-------------------	----------------

Definition at line 1129 of file specfun.h.

Return the Legendre polynomial  $P_l(x)$  of nonnegative degree l and float argument |x| <= 0.

### See also

legendre for more details.

Definition at line 1099 of file specfun.h.

Return the Legendre polynomial  $P_l(x)$  of nonnegative degree l and long double argument |x| <= 0.

# See also

legendre for more details.

Definition at line 1109 of file specfun.h.

8.2.2.52 template<typename\_Tp > \_\_gnu\_cxx::\_\_promote<\_Tp>::\_\_type std::riemann\_zeta(\_Tp \_\_s) [inline]

Return the Riemann zeta function  $\zeta(s)$  for real argument s.

The Riemann zeta function is defined by:

$$\zeta(s) = \sum_{k=1}^{\infty} k^{-s} \text{ for } s > 1$$

and

$$\zeta(s) = \frac{1}{1-2^{1-s}} \sum_{k=1}^{\infty} (-1)^{k-1} k^{-s} \text{ for } 0 <= s <= 1$$

For s < 1 use the reflection formula:

$$\zeta(s) = 2^s \pi^{s-1} \sin(\frac{\pi s}{2}) \Gamma(1-s) \zeta(1-s)$$

# **Template Parameters**

\_*Tp* The floating-point type of the argument \_\_\_s.

# **Parameters**

_~	The argument s	! =	1
_s			

Definition at line 1180 of file specfun.h.

8.2.2.53 float std::riemann\_zetaf (float \_\_s ) [inline]

Return the Riemann zeta function  $\zeta(s)$  for float argument s.

### See also

riemann zeta for more details.

Definition at line 1144 of file specfun.h.

**8.2.2.54** long double std::riemann\_zetal ( long double \_\_s ) [inline]

Return the Riemann zeta function  $\zeta(s)$  for long double argument s.

## See also

riemann\_zeta for more details.

Definition at line 1154 of file specfun.h.

Return the spherical Bessel function  $j_n(x)$  of nonnegative order n and real argument x >= 0.

The spherical Bessel function is defined by:

$$j_n(x) = \left(\frac{\pi}{2x}\right)^{1/2} J_{n+1/2}(x)$$

# **Template Parameters**

_Tp	The floating-point type of the argument	_x.
-----	---	-----

### **Parameters**

_~	The integral order n >= 0
_n	
_~	The real argument $x >= 0$
_X	

# **Exceptions**

std::domain_error	if $_{}x < 0$ .
-------------------	-----------------

Definition at line 1224 of file specfun.h.

Return the spherical Bessel function  $j_n(x)$  of nonnegative order n and float argument x >= 0.

# See also

sph\_bessel for more details.

Definition at line 1195 of file specfun.h.

Return the spherical Bessel function  $j_n(x)$  of nonnegative order n and long double argument x>=0.

# See also

sph bessel for more details.

Definition at line 1205 of file specfun.h.

8.2.2.58 template<typename\_Tp > \_\_gnu\_cxx::\_\_promote<\_Tp>::\_\_type std::sph\_legendre ( unsigned int \_\_I, unsigned int \_\_m, \_\_Tp \_\_theta ) [inline]

Return the spherical Legendre function of nonnegative integral degree 1 and order m and real angle  $\theta$  in radians.

The spherical Legendre function is defined by

$$Y_l^m(\theta,\phi) = (-1)^m \left[ \frac{(2l+1)}{4\pi} \frac{(l-m)!}{(l+m)!} \right] P_l^m(\cos\theta) \exp^{im\phi}$$

# **Template Parameters**

_T	)	The floating-point type of the angle	_theta.
----	---	--------------------------------------	---------

### **Parameters**

/	The order $_{1} >= 0$
m	The degree $\underline{}$ $>= 0$ and $\underline{}$ $<=$
	1
theta	The radian polar angle argument

Definition at line 1271 of file specfun.h.

8.2.2.59 float std::sph\_legendref ( unsigned int \_\_l, unsigned int \_\_m, float \_\_theta ) [inline]

Return the spherical Legendre function of nonnegative integral degree 1 and order m and float angle  $\theta$  in radians.

# See also

sph\_legendre for details.

Definition at line 1239 of file specfun.h.

8.2.2.60 long double std::sph\_legendrel ( unsigned int \_\_l, unsigned int \_\_m, long double \_\_theta ) [inline]

Return the spherical Legendre function of nonnegative integral degree 1 and order m and long double angle  $\theta$  in radians.

# See also

sph\_legendre for details.

Definition at line 1250 of file specfun.h.

Return the spherical Neumann function of integral order n >= 0 and real argument x >= 0.

The spherical Neumann function is defined by

$$n_n(x) = \left(\frac{\pi}{2x}\right)^{1/2} N_{n+1/2}(x)$$

# **Template Parameters**

_Tp	The floating-point type of the argument	_x.
-----	---	-----

### **Parameters**

_~	The integral order n >= 0
_n	
_~	The real argumentx >= 0
_X	

# **Exceptions**

std::domain_error	$   if \underline{} x < 0 . $	
-------------------	-------------------------------	--

Definition at line 1315 of file specfun.h.

8.2.2.62 float std::sph\_neumannf ( unsigned int \_\_n, float \_\_x ) [inline]

Return the spherical Neumann function of integral order n>=0 and float argument x>=0.

# See also

sph neumann for details.

Definition at line 1286 of file specfun.h.

8.2.2.63 long double std::sph\_neumannl ( unsigned int \_\_n, long double \_\_x ) [inline]

Return the spherical Neumann function of integral order n >= 0 and long double x >= 0.

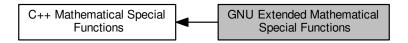
## See also

sph neumann for details.

Definition at line 1296 of file specfun.h.

# 8.3 GNU Extended Mathematical Special Functions

Collaboration diagram for GNU Extended Mathematical Special Functions:



# **Enumerations**

# **Functions**

```
template<typename _Tp >
   __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::airy_ai (_Tp __x)

    float gnu cxx::airy aif (float x)

    long double <u>__gnu_cxx::airy_ail</u> (long double <u>__x</u>)

template<typename_Tp>
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::airy_bi (_Tp __x)

    float gnu cxx::airy bif (float x)

    long double <u>__gnu_cxx::airy_bil</u> (long double <u>__x)</u>

• template<typename _Tp >
   gnu cxx::__promote_num_t< _Tp > __gnu_cxx::bernoulli (unsigned int __n)

    float gnu cxx::bernoullif (unsigned int n)

    long double __gnu_cxx::bernoullil (unsigned int __n)

template<typename_Tp>
  gnu cxx:: promote num t < Tp > gnu cxx::bincoef (unsigned int n, unsigned int k)
• float gnu cxx::bincoeff (unsigned int n, unsigned int k)

    long double <u>__gnu_cxx::bincoefl</u> (unsigned int __n, unsigned int __k)

template<typename _Tp >
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::chebyshev_t (unsigned int __n, _Tp __x)

    float gnu cxx::chebyshev tf (unsigned int n, float x)

    long double gnu cxx::chebyshev tl (unsigned int n, long double x)

    template<typename</li>
    Tp >

    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::chebyshev_u (unsigned int __n, _Tp __x)

    float __gnu_cxx::chebyshev_uf (unsigned int __n, float __x)

    long double gnu cxx::chebyshev ul (unsigned int n, long double x)

template<typename_Tp>
  __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::chebyshev_v (unsigned int __n, _Tp __x)

    float gnu cxx::chebyshev vf (unsigned int n, float x)

    long double gnu cxx::chebyshev vl (unsigned int n, long double x)
```

```
template<typename _Tp >
   _gnu_cxx::_promote_num_t< _Tp > __gnu_cxx::chebyshev_w (unsigned int __n, _Tp __x)

    float gnu cxx::chebyshev wf (unsigned int n, float x)

    long double __gnu_cxx::chebyshev_wl (unsigned int __n, long double __x)

template<typename</li>Tp >
   _gnu_cxx::_promote_num_t< _Tp > __gnu_cxx::clausen (unsigned int __m, _Tp __w)

    template<typename</li>
    Tp >

  std::complex< __gnu_cxx::__promote_num_t< _Tp >> __gnu_cxx::clausen (unsigned int __m, std::complex<
  _{\mathsf{Tp}} > _{\mathsf{w}}

    template<typename</li>
    Tp >

   __gnu_cxx::__promote_num_t<_Tp > __gnu_cxx::clausen_c (unsigned int __m, _Tp __w)

    float <u>gnu_cxx::clausen_cf</u> (unsigned int <u>m</u>, float <u>w</u>)

• long double gnu cxx::clausen cl (unsigned int m, long double w)

    template<typename</li>
    Tp >

    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::clausen_s (unsigned int __m, _Tp __w)

    float <u>gnu_cxx::clausen_sf</u> (unsigned int <u>m</u>, float <u>w</u>)

    long double gnu cxx::clausen sl (unsigned int m, long double w)

    float __gnu_cxx::clausenf (unsigned int __m, float __w)

• std::complex < float > gnu cxx::clausenf (unsigned int m, std::complex < float > w)

    long double gnu cxx::clausenl (unsigned int m, long double w)

• std::complex < long double > gnu cxx::clausenl (unsigned int m, std::complex < long double > w)
template<typename _Tk >
   gnu cxx:: promote num t < Tk > gnu cxx::comp ellint d (Tk k)

    float gnu cxx::comp ellint df (float k)

    long double __gnu_cxx::comp_ellint_dl (long double __k)

• float gnu cxx::comp ellint rf (float x, float y)

    long double __gnu_cxx::comp_ellint_rf (long double __x, long double __y)

• template<typename _Tx , typename _Ty >
    _gnu_cxx::__promote_num_t< _Tx, _Ty > __gnu_cxx::comp_ellint_rf (_Tx __x, _Ty __y)

    float __gnu_cxx::comp_ellint_rg (float __x, float __y)

    long double __gnu_cxx::comp_ellint_rg (long double __x, long double __y)

• template<typename _Tx , typename _Ty >
   gnu cxx:: promote num t < Tx, Ty > gnu cxx::comp ellint rg ( Tx x, Ty y)

    template<typename _Tpa , typename _Tpc , typename _Tp >

   _gnu_cxx::__promote_3< _Tpa, _Tpc, _Tp >::__type <u>__gnu_cxx::conf_hyperg</u> (_Tpa __a, _Tpc __c, _Tp __x)
• template<typename _Tpc , typename _Tp >
  __gnu_cxx::_promote_2< _Tpc, _Tp >::_type __gnu_cxx::conf_hyperg_lim (_Tpc __c, _Tp __x)

    float __gnu_cxx::conf_hyperg_limf (float __c, float __x)

    long double __gnu_cxx::conf_hyperg_liml (long double __c, long double __x)

    float gnu cxx::conf hypergf (float a, float c, float x)

    long double gnu cxx::conf hypergl (long double a, long double c, long double x)

template<typename _Tp >
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::coshint (_Tp __x)

    float gnu cxx::coshintf (float x)

    long double gnu cxx::coshintl (long double x)

template<typename _Tp >
   gnu cxx:: promote num t< Tp> gnu cxx::cosint (Tpx)

    float gnu cxx::cosintf (float x)

    long double <u>gnu_cxx::cosintl</u> (long double <u>x</u>)

template<typename _Tpnu , typename _Tp >
  std::complex< __gnu_cxx::_promote_num_t< _Tpnu, _Tp >> __gnu_cxx::cyl_hankel_1 (_Tpnu __nu, _Tp
  __z)
```

```
• template<typename _Tpnu , typename _Tp >
     std::complex< gnu cxx:: promote num t< Tpnu, Tp>> gnu cxx::cyl hankel 1 (std::complex< ←
     Tpnu > nu, std::complex < Tp > x

    std::complex< float > __gnu_cxx::cyl_hankel_1f (float __nu, float __z)

    std::complex < float > __gnu_cxx::cyl_hankel_1f (std::complex < float > __nu, std::complex < float > __x)

• std::complex < long double > gnu cxx::cyl hankel 1l (long double nu, long double z)
• std::complex < long double > gnu cxx::cyl hankel 1l (std::complex < long double > nu, std::complex < long
     double > __x)
template<typename _Tpnu , typename _Tp >
     std::complex< \underline{\quad} gnu\_cxx::\underline{\quad} promote\_num\_t< \underline{\quad} Tpnu, \underline{\quad} Tp>> \underline{\quad} gnu\_cxx::cyl\_hankel\_2 \ (\underline{\quad} Tpnu \underline{\quad} nu, \underline{\quad} Tpnu, \underline
• template<typename _Tpnu , typename _Tp >
     std::complex< gnu cxx:: promote num t< Tpnu, Tp>> gnu cxx::cyl hankel 2 (std::complex< ←
     Tpnu > __nu, std::complex< _Tp > __x)

    std::complex< float > __gnu_cxx::cyl_hankel_2f (float __nu, float __z)

    std::complex < float > __gnu_cxx::cyl_hankel_2f (std::complex < float > __nu, std::complex < float > __x)

    std::complex < long double > gnu cxx::cyl hankel 2l (long double nu, long double z)

• std::complex < long double > gnu cxx::cyl hankel 2l (std::complex < long double > nu, std::complex < long
     double > \underline{\hspace{1cm}} x)

    template<typename</li>
    Tp >

          _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::dawson (_Tp __x)

    float __gnu_cxx::dawsonf (float __x)

    long double <u>gnu_cxx::dawsonl</u> (long double <u>x</u>)

    template<typename</li>
    Tp >

          _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::digamma (_Tp __z)

    float gnu cxx::digammaf (float z)

    long double gnu cxx::digammal (long double z)

template<typename_Tp>
       __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::dilog (_Tp __x)

    float gnu cxx::dilogf (float x)

    long double gnu cxx::dilogl (long double x)

template<typename</li>Tp >
     _Tp __gnu_cxx::dirichlet_beta (_Tp __s)
• float __gnu_cxx::dirichlet betaf (float s)

    long double gnu cxx::dirichlet betal (long double s)

template<typename _Tp >
     _Tp __gnu_cxx::dirichlet_eta (_Tp __s)

    float gnu cxx::dirichlet etaf (float s)

    long double gnu cxx::dirichlet etal (long double s)

template<typename _Tp >
          gnu cxx:: promote num t < Tp > gnu cxx::double factorial (int n)

    float gnu cxx::double factorialf (int n)

    long double __gnu_cxx::double_factoriall (int __n)

• template<typename Tk, typename Tp, typename Ta, typename Tb>
          gnu_cxx::_promote_num_t<_Tk,_Tp,_Ta,_Tb>__gnu_cxx::ellint_cel(_Tk__k_c,_Tp__p,_Ta__a,_Tb
          b)

    float __gnu_cxx::ellint_celf (float __k_c, float __p, float __a, float __b)

    long double gnu cxx::ellint cell (long double k c, long double p, long double a, long double b)

• template<typename _Tk , typename _Tphi >
       _gnu_cxx::__promote_num_t< _Tk, _Tphi > __gnu_cxx::ellint_d (_Tk __k, _Tphi __phi)

    float gnu cxx::ellint df (float k, float phi)

    long double gnu cxx::ellint dl (long double k, long double phi)
```

```
    template<typename _Tp , typename _Tk >

   _gnu_cxx::__promote_num_t< _Tp, _Tk > __gnu_cxx::ellint_el1 (_Tp __x, _Tk __k_c)
• float gnu cxx::ellint el1f (float x, float k c)

    long double __gnu_cxx::ellint_el1l (long double __x, long double __k_c)

• template<typename Tp, typename Tk, typename Ta, typename Tb>
    gnu_cxx::_promote_num_t< _Tp, _Tk, _Ta, _Tb > __gnu_cxx::ellint_el2 (_Tp __x, _Tk __k_c, _Ta __a, _Tb
   b)

    float __gnu_cxx::ellint_el2f (float __x, float __k_c, float __a, float __b)

    long double __gnu_cxx::ellint_el2l (long double __x, long double __k_c, long double __a, long double __b)

• template<typename Tx, typename Tk, typename Tp>
  __gnu_cxx::__promote_num_t<_Tx,_Tk,_Tp > __gnu_cxx::ellint_el3 (_Tx __x, _Tk __k_c, _Tp __p)

    float __gnu_cxx::ellint_el3f (float __x, float __k_c, float __p)

    long double __gnu_cxx::ellint_el3l (long double __x, long double __k_c, long double __p)

• template<typename _Tp , typename _Up >
    _gnu_cxx::__promote_num_t< _Tp, _Up > __gnu_cxx::ellint_rc (_Tp __x, _Up __y)
• float gnu cxx::ellint rcf (float x, float y)

    long double gnu cxx::ellint rcl (long double x, long double y)

- template<typename _Tp , typename _Up , typename _Vp >
    gnu cxx:: promote num t< Tp, Up, Vp > gnu cxx::ellint rd (Tp x, Up y, Vp z)

    float gnu cxx::ellint rdf (float x, float y, float z)

    long double gnu cxx::ellint rdl (long double x, long double y, long double z)

- template<typename _Tp , typename _Up , typename _Vp >
    _gnu_cxx::__promote_num_t< _Tp, _Up, _Vp > __gnu_cxx::ellint_rf (_Tp __x, _Up __y, _Vp __z)

    float gnu cxx::ellint rff (float x, float y, float z)

    long double __gnu_cxx::ellint_rfl (long double __x, long double __y, long double __z)

template<typename _Tp , typename _Up , typename _Vp >
    _gnu_cxx::__promote_num_t< _Tp, _Up, _Vp > __gnu_cxx::ellint_rg (_Tp __x, _Up __y, _Vp __z)

    float gnu cxx::ellint rgf (float x, float y, float z)

    long double __gnu_cxx::ellint_rgl (long double __x, long double __y, long double __z)

ullet template<typename _Tp , typename _Up , typename _Vp , typename _Wp >
  __gnu_cxx::_promote_num_t< _Tp, _Up, _Vp, _Wp > __gnu_cxx::ellint_rj (_Tp __x, _Up __y, _Vp __z, _Wp
  __p)

    float __gnu_cxx::ellint_rjf (float __x, float __y, float __z, float __p)

    long double __gnu_cxx::ellint_rjl (long double __x, long double __y, long double __z, long double __p)

• template<typename Tp >
  _Tp __gnu_cxx::ellnome (_Tp __k)

    float gnu cxx::ellnomef (float k)

    long double __gnu_cxx::ellnomel (long double __k)

• template<typename_Tp>
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::expint (unsigned int __n, _Tp __x)

    float gnu cxx::expintf (unsigned int n, float x)

    long double __gnu_cxx::expintl (unsigned int __n, long double __x)

template<typename _Tp >
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::factorial (unsigned int __n)

    float gnu cxx::factorialf (unsigned int n)

    long double <u>gnu_cxx::factoriall</u> (unsigned int <u>n</u>)

template<typename_Tp>
    gnu cxx:: promote num t< Tp> gnu cxx::fresnel c (Tpx)

    float gnu cxx::fresnel cf (float x)

    long double <u>__gnu_cxx::fresnel_cl</u> (long double <u>__x</u>)

template<typename _Tp >
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::fresnel_s (_Tp __x)
```

```
    float __gnu_cxx::fresnel_sf (float __x)

    long double __gnu_cxx::fresnel_sl (long double __x)

• template<typename _Tn , typename _Tp >
    _gnu_cxx::__promote_num_t< _Tn, _Tp > __gnu_cxx::gamma_l (_Tn __n, _Tp __x)
• float gnu cxx::gamma lf (float n, float x)
• long double __gnu_cxx::gamma_ll (long double __n, long double __x)
• template<typename _Tn , typename _Tp >
    _gnu_cxx::__promote_num_t< _Tn, _Tp > __gnu_cxx::gamma_u (_Tn __n, _Tp __x)

    float gnu cxx::gamma uf (float n, float x)

• long double gnu cxx::gamma ul (long double n, long double x)
• template<typename _Talpha , typename _Tp >
   gnu cxx:: promote num t< Talpha, Tp > gnu cxx::gegenbauer (unsigned int n, Talpha alpha,
• float __gnu_cxx::gegenbauerf (unsigned int __n, float __alpha, float __x)

    long double gnu cxx::gegenbauerl (unsigned int n, long double alpha, long double x)

• template<typename Tk, typename Tphi >
   _gnu_cxx::__promote_num_t< _Tk, _Tphi > __gnu_cxx::heuman_lambda (_Tk __k, _Tphi __phi)

    float __gnu_cxx::heuman_lambdaf (float __k, float __phi)

• long double gnu cxx::heuman lambdal (long double k, long double phi)
• template<typename _Tp , typename _Up >
   __gnu_cxx::__promote_num_t< _Tp, _Up > __gnu_cxx::hurwitz_zeta (_Tp __s, _Up __a)
• template<typename _Tp , typename _Up >
  std::complex< _Tp > __gnu_cxx::hurwitz_zeta (_Tp __s, std::complex< _Up > __a)

    float gnu cxx::hurwitz zetaf (float s, float a)

    long double __gnu_cxx::hurwitz_zetal (long double __s, long double __a)

template<typename _Tpa , typename _Tpb , typename _Tpc , typename _Tp >
    _gnu_cxx::__promote_4< _Tpa, _Tpb, _Tpc, _Tp >::__type __gnu_cxx::hyperg (_Tpa __a, _Tpb __b, _Tpc

    float __gnu_cxx::hypergf (float __a, float __b, float __c, float __x)

• long double gnu cxx::hypergl (long double a, long double b, long double c, long double x)
- template<typename _Ta , typename _Tb , typename _Tp >
   gnu cxx:: promote num t < Ta, Tb, Tp > gnu cxx::ibeta ( Ta a, Tb b, Tp x)

    template<typename _Ta , typename _Tb , typename _Tp >

   _gnu_cxx::__promote_num_t< _Ta, _Tb, _Tp > <u>__gnu_cxx::ibetac</u> (_Ta __a, _Tb __b, _Tp __x)

    float gnu cxx::ibetacf (float a, float b, float x)

    long double gnu cxx::ibetacl (long double a, long double b, long double x)

    float gnu cxx::ibetaf (float a, float b, float x)

    long double __gnu_cxx::ibetal (long double __a, long double __b, long double __x)

• template<typename _Talpha , typename _Tbeta , typename _Tp >
    gnu cxx:: promote num t< Talpha, Tbeta, Tp > gnu cxx::jacobi (unsigned n, Talpha alpha,
   Tbeta beta, Tp x)
• template<typename _Kp , typename _Up >
   _gnu_cxx::__promote_num_t< _Kp, _Up > __gnu_cxx::jacobi_cn (_Kp __k, _Up __u)
• float gnu cxx::jacobi cnf (float k, float u)

    long double __gnu_cxx::jacobi_cnl (long double __k, long double __u)

    template<typename _Kp , typename _Up >

    _gnu_cxx::__promote_num_t< _Kp, _Up > __gnu_cxx::jacobi_dn (_Kp __k, _Up __u)
• float gnu cxx::jacobi dnf (float k, float u)

    long double gnu cxx::jacobi dnl (long double k, long double u)

• template<typename _Kp , typename _Up >
    _gnu_cxx::__promote_num_t< _Kp, _Up > __gnu_cxx::jacobi_sn (_Kp __k, _Up __u)

    float gnu cxx::jacobi snf (float k, float u)
```

```
    long double __gnu_cxx::jacobi_snl (long double __k, long double __u)

• template<typename _Tk , typename _Tphi >
    gnu cxx:: promote num t < Tk, Tphi > gnu cxx::jacobi zeta (Tk k, Tphi phi)

    float gnu cxx::jacobi zetaf (float k, float phi)

    long double gnu cxx::jacobi zetal (long double k, long double phi)

• float gnu cxx::jacobif (unsigned n, float alpha, float beta, float x)

    long double gnu cxx::jacobil (unsigned n, long double alpha, long double beta, long double x)

template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::lbincoef (unsigned int __n, unsigned int __k)

    float <u>__gnu_cxx::lbincoeff</u> (unsigned int __n, unsigned int __k)

    long double gnu cxx::lbincoefl (unsigned int n, unsigned int k)

template<typename_Tp>
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::ldouble_factorial (int __n)

    float gnu cxx::ldouble factorialf (int n)

    long double gnu cxx::ldouble factoriall (int n)

template<typename</li>Tp >
    gnu cxx:: promote num t< Tp > gnu cxx::legendre q (unsigned int n, Tp x)

    float gnu cxx::legendre qf (unsigned int n, float x)

    long double gnu cxx::legendre ql (unsigned int n, long double x)

    template<typename</li>
    Tp >

   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::lfactorial (unsigned int __n)

    float <u>__gnu_cxx::lfactorialf</u> (unsigned int <u>__n)</u>

    long double __gnu_cxx::lfactoriall (unsigned int __n)

template<typename _Tp >
   __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::logint (_Tp __x)

    float gnu cxx::logintf (float x)

    long double <u>gnu_cxx::logintl</u> (long double <u>x</u>)

• template<typename _Tp , typename _Tn >
   _gnu_cxx::__promote_num_t< _Tp, _Tn > __gnu_cxx::lpochhammer_l (_Tp __a, _Tn __n)

    float __gnu_cxx::lpochhammer_lf (float __a, float __n)

    long double __gnu_cxx::lpochhammer_ll (long double __a, long double __n)

• template<typename Tp, typename Tn >
   __gnu_cxx::__promote_num_t< _Tp, _Tn > __gnu_cxx::lpochhammer_u (_Tp __a, _Tn __n)
• float __gnu_cxx::lpochhammer_uf (float __a, float __n)

    long double gnu cxx::lpochhammer ul (long double a, long double n)

• template<typename _Tph , typename _Tpa >
    _gnu_cxx::__promote_num_t< _Tph, _Tpa > __gnu_cxx::owens_t (_Tph __h, _Tpa a)

    float gnu cxx::owens tf (float h, float a)

    long double gnu cxx::owens tl (long double h, long double a)

• template<typename _{\rm Ta} , typename _{\rm Tp} >
    _gnu_cxx::__promote_num_t< _Ta, _Tp > __gnu_cxx::pgamma (_Ta __a, _Tp __x)
• float gnu cxx::pgammaf (float a, float x)

    long double __gnu_cxx::pgammal (long double __a, long double __x)

• template<typename _Tp , typename _Tn >
    gnu cxx:: promote num t < Tp, Tn > gnu cxx::pochhammer I (Tp a, Tn n)

    float __gnu_cxx::pochhammer_lf (float __a, float __n)

    long double gnu cxx::pochhammer II (long double a, long double n)

• template<typename _Tp , typename _Tn >
   __gnu_cxx::__promote_num_t< _Tp, _Tn > __gnu_cxx::pochhammer_u (_Tp __a, _Tn __n)

    float __gnu_cxx::pochhammer_uf (float __a, float __n)

• long double gnu cxx::pochhammer ul (long double a, long double n)
```

```
template<typename _Tp , typename _Wp >
   _gnu_cxx::__promote_num_t< _Tp, _Wp > __gnu_cxx::polylog (_Tp __s, _Wp __w)

    template<typename</li>
    Tp , typename
    Wp >

  std::complex< __gnu_cxx::__promote_num_t< _Tp, _Wp >> __gnu_cxx::polylog (_Tp __s, std::complex< _Tp

    float gnu cxx::polylogf (float s, float w)

    std::complex < float > gnu cxx::polylogf (float s, std::complex < float > w)

    long double __gnu_cxx::polylogl (long double __s, long double __w)

    std::complex < long double > __gnu_cxx::polylogl (long double __s, std::complex < long double > __w)

template<typename</li>Tp >
   __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::psi (_Tp __x)

    float __gnu_cxx::psif (float __x)

    long double __gnu_cxx::psil (long double __x)

• template<typename _Ta , typename _Tp >
    _gnu_cxx::__promote_num_t< _Ta, _Tp > __gnu_cxx::qgamma (_Ta __a, _Tp __x)
• float gnu cxx::ggammaf (float a, float x)

    long double __gnu_cxx::qgammal (long double __a, long double __x)

template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::radpoly (unsigned int __n, unsigned int __m, _Tp __rho)
• float gnu cxx::radpolyf (unsigned int n, unsigned int m, float rho)

    long double __gnu_cxx::radpolyl (unsigned int __n, unsigned int __m, long double __rho)

template<typename</li>Tp >
   __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::sinc (_Tp __x)
template<typename</li>Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::sinc_pi (_Tp __x)

    float gnu cxx::sinc pif (float x)

    long double gnu cxx::sinc pil (long double x)

    float gnu cxx::sincf (float x)

    long double <u>gnu_cxx::sincl</u> (long double <u>x</u>)

template<typename</li>Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::sinhc (_Tp __x)
template<typename _Tp >
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::sinhc_pi (_Tp __x)

    float gnu cxx::sinhc pif (float x)

    long double gnu cxx::sinhc pil (long double x)

    float __gnu_cxx::sinhcf (float __x)

    long double gnu cxx::sinhcl (long double x)

template<typename _Tp >
   __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::sinhint (_Tp __x)

    float gnu cxx::sinhintf (float x)

    long double gnu cxx::sinhintl (long double x)

template<typename _Tp >
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::sinint (_Tp __x)

    float gnu cxx::sinintf (float x)

    long double gnu cxx::sinintl (long double x)

template<typename _Tp >
   _gnu_cxx::_promote_num_t< _Tp > __gnu_cxx::sph_bessel_i (unsigned int __n, _Tp __x)

    float gnu cxx::sph bessel if (unsigned int n, float x)

    long double gnu cxx::sph bessel il (unsigned int n, long double x)

template<typename _Tp >
    gnu cxx:: promote num t< Tp > gnu cxx::sph bessel k (unsigned int n, Tp x)

    float gnu cxx::sph bessel kf (unsigned int n, float x)
```

```
    long double __gnu_cxx::sph_bessel_kl (unsigned int __n, long double __x)

template<typename _Tp >
  std::complex< __gnu_cxx::__promote_num_t< _Tp >> __gnu_cxx::sph_hankel_1 (unsigned int __n, _Tp __z)
template<typename Tp >
  std::complex< __gnu_cxx::_promote_num_t< _Tp >> __gnu_cxx::sph_hankel_1 (unsigned int __n, std--
  ::complex < Tp > x)

    std::complex< float > __gnu_cxx::sph_hankel_1f (unsigned int __n, float __z)

    std::complex < float > __gnu_cxx::sph_hankel_1f (unsigned int __n, std::complex < float > __x)

    std::complex < long double > __gnu_cxx::sph_hankel_1l (unsigned int __n, long double __z)

    std::complex < long double > __gnu_cxx::sph_hankel_1l (unsigned int __n, std::complex < long double > __x)

    template<typename</li>
    Tp >

  std::complex< gnu cxx:: promote num t< Tp>> gnu cxx::sph hankel 2 (unsigned int n, Tp z)
template<typename _Tp >
  std::complex< __gnu_cxx::_promote_num_t< _Tp >> __gnu_cxx::sph_hankel_2 (unsigned int __n, std↔
  ::complex < _Tp > __x)
• std::complex< float > gnu cxx::sph hankel 2f (unsigned int n, float z)
• std::complex < float > gnu cxx::sph hankel 2f (unsigned int n, std::complex < float > x)

    std::complex < long double > __gnu_cxx::sph_hankel_2l (unsigned int __n, long double __z)

    std::complex < long double > gnu cxx::sph hankel 2l (unsigned int n, std::complex < long double > x)

• template<typename Ttheta, typename Tphi >
  std::complex< __gnu_cxx::_promote_num_t< _Ttheta, _Tphi >> __gnu_cxx::sph_harmonic (unsigned int ←
   _l, int __m, _Ttheta __theta, _Tphi __phi)
• std::complex < float > gnu cxx::sph harmonicf (unsigned int I, int m, float theta, float phi)
• std::complex < long double > __gnu_cxx::sph_harmonicl (unsigned int __l, int __m, long double __theta, long
  double phi)
• template<typename _Tpnu , typename _Tp >
  gnu_cxx:: promote_num_t< _Tpnu, _Tp > __gnu_cxx::theta_1 (_Tpnu __nu, _Tp __x)

    float __gnu_cxx::theta_1f (float __nu, float __x)

    long double __gnu_cxx::theta_1l (long double __nu, long double __x)

• template<typename _{\rm Tpnu}, typename _{\rm Tp} >
    _gnu_cxx::__promote_num_t< _Tpnu, _Tp > __gnu_cxx::theta_2 (_Tpnu __nu, _Tp __x)

    float __gnu_cxx::theta_2f (float __nu, float __x)

    long double __gnu_cxx::theta_2l (long double __nu, long double __x)

• template<typename _Tpnu , typename _Tp >
    gnu cxx:: promote num t< Tpnu, Tp > gnu cxx::theta 3 ( Tpnu nu, Tp x)

    float gnu cxx::theta 3f (float nu, float x)

    long double gnu cxx::theta 3l (long double nu, long double x)

template<typename _Tpnu , typename _Tp >
   _gnu_cxx::__promote_num_t< _Tpnu, _Tp > __gnu_cxx::theta_4 (_Tpnu __nu, _Tp __x)

    float __gnu_cxx::theta_4f (float __nu, float __x)

    long double __gnu_cxx::theta_4l (long double __nu, long double __x)

• template<typename _{\rm Tpk}, typename _{\rm Tp} >
    _gnu_cxx::__promote_num_t< _Tpk, _Tp > __gnu_cxx::theta_c (_Tpk __k, _Tp __x)

    float gnu cxx::theta cf (float k, float x)

    long double __gnu_cxx::theta_cl (long double __k, long double __x)

ullet template<typename _Tpk , typename _Tp >
    _gnu_cxx::__promote_num_t< _Tpk, _Tp > __gnu_cxx::theta_d (_Tpk __k, _Tp __x)

    float gnu cxx::theta df (float k, float x)

    long double gnu cxx::theta dl (long double k, long double x)

• template<typename _Tpk , typename _Tp >
    _gnu_cxx::__promote_num_t< _Tpk, _Tp > __gnu_cxx::theta_n (_Tpk __k, _Tp __x)

    float __gnu_cxx::theta_nf (float __k, float __x)
```

```
• long double <u>gnu_cxx::theta_nl</u> (long double <u>k</u>, long double <u>x</u>)
```

```
• template<typename _Tpk , typename _Tp >
```

- float \_\_gnu\_cxx::theta\_sf (float \_\_k, float \_\_x)
- long double <u>gnu\_cxx::theta\_sl</u> (long double <u>k</u>, long double <u>x</u>)
- template<typename \_Trho , typename \_Tphi >

```
__gnu_cxx::__promote_num_t< _Trho, _Tphi > __gnu_cxx::zernike (unsigned int __n, int __m, _Trho __rho, Tphi __phi)
```

- float \_\_gnu\_cxx::zernikef (unsigned int \_\_n, int \_\_m, float \_\_rho, float \_\_phi)
- long double \_\_gnu\_cxx::zernikel (unsigned int \_\_n, int \_\_m, long double \_\_rho, long double \_\_phi)

# 8.3.1 Detailed Description

An extended collection of advanced mathematical special functions for GNU.

# 8.3.2 Enumeration Type Documentation

### 8.3.2.1 anonymous enum

Enumerator

\_GLIBCXX\_JACOBI\_SN

\_GLIBCXX\_JACOBI\_CN

\_GLIBCXX\_JACOBI\_DN

Definition at line 1763 of file specfun.h.

# 8.3.3 Function Documentation

Return the Airy function Ai(x) of real argument x.

The Airy function is defined by:

$$Ai(x) = \frac{1}{\pi} \int_0^\infty \cos\left(\frac{t^3}{3} + xt\right) dt$$

**Template Parameters** 

Tp The real type of the argument

### **Parameters**

_~	The argument
_x	

Definition at line 2705 of file specfun.h.

Return the Airy function Ai(x) for float argument x.

# See also

airy ai for details.

Definition at line 2670 of file specfun.h.

Return the Airy function Ai(x) for long double argument x.

# See also

airy\_ai for details.

Definition at line 2684 of file specfun.h.

Return the Airy function Bi(x) of real argument x.

The Airy function is defined by:

$$Bi(x) = \frac{1}{\pi} \int_0^\infty \left[ \exp\left(-\frac{t^3}{3} + xt\right) + \sin\left(\frac{t^3}{3} + xt\right) \right] dt$$

# **Template Parameters**

# **Parameters**

_~	The argument
_X	

Definition at line 2757 of file specfun.h.

```
8.3.3.5 float __gnu_cxx::airy_bif(float __x) [inline]
```

Return the Airy function Bi(x) for float argument x.

# See also

airy\_bi for details.

Definition at line 2721 of file specfun.h.

```
8.3.3.6 long double __gnu_cxx::airy_bil ( long double __x ) [inline]
```

Return the Airy function Bi(x) for long double argument x.

# See also

airy\_bi for details.

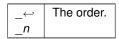
Definition at line 2735 of file specfun.h.

```
\textbf{8.3.3.7} \quad \textbf{template} < \textbf{typename\_Tp} > \underline{\quad} \textbf{gnu\_cxx::\_promote\_num\_t} < \underline{\quad} \textbf{Tp} > \underline{\quad} \textbf{gnu\_cxx::bernoulli (unsigned int\_n)} \quad [\texttt{inline}]
```

Return the Bernoulli number of integer order n.

The Bernoulli numbers are defined by

### **Parameters**



Definition at line 3744 of file specfun.h.

**8.3.3.8** float \_\_gnu\_cxx::bernoullif ( unsigned int \_\_n ) [inline]

Return the Bernoulli number of integer order n as a float.

See also

bernoulli for details.

Definition at line 3719 of file specfun.h.

**8.3.3.9** long double \_\_gnu\_cxx::bernoullil( unsigned int \_\_n ) [inline]

Return the Bernoulli number of integer order n as a long double.

See also

bernoulli for details.

Definition at line 3729 of file specfun.h.

8.3.3.10 template<typename\_Tp > \_\_gnu\_cxx::\_\_promote\_num\_t<\_Tp> \_\_gnu\_cxx::bincoef ( unsigned int \_\_n, unsigned int \_\_n,

Definition at line 3684 of file specfun.h.

8.3.3.11 float \_\_gnu\_cxx::bincoeff ( unsigned int \_\_n, unsigned int \_\_k ) [inline]

Definition at line 3672 of file specfun.h.

8.3.3.12 long double  $\_gnu\_cxx::bincoefl$  ( unsigned int  $\_n$ , unsigned int  $\_k$  ) [inline]

Definition at line 3676 of file specfun.h.

Return the Chebyshev polynomial of the first kind  $T_n(x)$  of non-negative order n and real argument x.

The Chebyshev polynomial of the first kind is defined by:

$$T_n(x) = \cos(n\theta)$$

# **Template Parameters**

_Тр	The real type of the argument
-----	-------------------------------

# **Parameters**

_~	The non-negative integral order
_n	
_~	The real argument $-1 \le x \le +1$
_X	

Definition at line 1964 of file specfun.h.

```
8.3.3.14 float __gnu_cxx::chebyshev_tf ( unsigned int __n, float __x ) [inline]
```

Return the Chebyshev polynomials of the first kind  $T_n(x)$  of non-negative order n and float argument x.

# See also

chebyshev\_t for details.

Definition at line 1935 of file specfun.h.

8.3.3.15 long double \_\_gnu\_cxx::chebyshev\_tl( unsigned int \_\_n, long double \_\_x ) [inline]

Return the Chebyshev polynomials of the first kind  $T_n(x)$  of non-negative order n and real argument x.

# See also

chebyshev\_t for details.

Definition at line 1945 of file specfun.h.

Return the Chebyshev polynomial of the second kind  $U_n(x)$  of non-negative order n and real argument x.

The Chebyshev polynomial of the second kind is defined by:

$$U_n(x) = \frac{\sin[(n+1)\theta]}{\sin(\theta)}$$

# **Template Parameters**

Tp The real type of the argument
----------------------------------

### **Parameters**

_~	The non-negative integral order
_n	
_~	The real argument $-1 \le x \le +1$
_X	

Definition at line 2008 of file specfun.h.

Return the Chebyshev polynomials of the second kind  $U_n(x)$  of non-negative order n and float argument x.

### See also

chebyshev u for details.

Definition at line 1979 of file specfun.h.

Return the Chebyshev polynomials of the second kind  $U_n(x)$  of non-negative order n and real argument x.

# See also

chebyshev\_u for details.

Definition at line 1989 of file specfun.h.

Return the Chebyshev polynomial of the third kind  $V_n(x)$  of non-negative order n and real argument x.

The Chebyshev polynomial of the third kind is defined by:

$$V_n(x) = \frac{\cos\left[\left(n + \frac{1}{2}\right)\theta\right]}{\cos\left(\frac{\theta}{2}\right)}$$

# **Template Parameters**

_Тр	The real type of the argument
-----	-------------------------------

# **Parameters**

_~	The non-negative integral order
_n	
_~	The real argument $-1 \le x \le +1$
_X	

Definition at line 2053 of file specfun.h.

```
8.3.3.20 float __gnu_cxx::chebyshev_vf ( unsigned int __n, float __x ) [inline]
```

Return the Chebyshev polynomials of the third kind  $V_n(x)$  of non-negative order n and  ${\tt float}$  argument x.

### See also

chebyshev v for details.

Definition at line 2023 of file specfun.h.

Return the Chebyshev polynomials of the third kind  $V_n(x)$  of non-negative order n and real argument x.

# See also

chebyshev\_v for details.

Definition at line 2033 of file specfun.h.

Return the Chebyshev polynomial of the fourth kind  $W_n(x)$  of non-negative order n and real argument x.

The Chebyshev polynomial of the fourth kind is defined by:

$$W_n(x) = \frac{\sin\left[\left(n + \frac{1}{2}\right)\theta\right]}{\sin\left(\frac{\theta}{2}\right)}$$

# **Template Parameters**

### **Parameters**

_~	The non-negative integral order
_n	
_~	The real argument $-1 \le x \le +1$
_x	

Definition at line 2098 of file specfun.h.

Return the Chebyshev polynomials of the fourth kind  $W_n(x)$  of non-negative order n and float argument x.

### See also

chebyshev\_w for details.

Definition at line 2068 of file specfun.h.

Return the Chebyshev polynomials of the fourth kind  $W_n(x)$  of non-negative order n and real argument x.

## See also

chebyshev\_w for details.

Definition at line 2078 of file specfun.h.

Return the Clausen function  $Cl_n(w)$  of integer order m and real argument w.

The Clausen function is defined by

$$Cl_n(w) = S_n(w) = \sum_{k=1}^{\infty} \frac{\sin(kx)}{k^n}$$
 for even  $m = C_n(w) = \sum_{k=1}^{\infty} \frac{\cos(kx)}{k^n}$  for odd  $m$ 

# **Template Parameters**

Tn	The real type of the argument
_'P	The real type of the argument

### **Parameters**

_~	The integral order
_m	
_~	The complex argument
_ <i>w</i>	

Definition at line 4700 of file specfun.h.

8.3.3.26 template<typename \_Tp > std::complex< \_gnu\_cxx::\_\_promote\_num\_t<\_Tp>> \_\_gnu\_cxx::clausen ( unsigned int \_\_m, std::complex< \_Tp > \_w ) [inline]

Return the Clausen function  $Cl_n(w)$  of integer order m and complex argument w.

The Clausen function is defined by

$$Cl_n(w) = S_n(w) = \sum_{k=1}^{\infty} \frac{\sin(kx)}{k^n}$$
 for even  $m = C_n(w) = \sum_{k=1}^{\infty} \frac{\cos(kx)}{k^n}$  for odd  $m$ 

### **Template Parameters**

Тр	The real type of the complex components

# **Parameters**

ſ	_~	The integral order
	_ <i>m</i>	
ſ	_~	The complex argument
	$_{w}$	

Definition at line 4744 of file specfun.h.

Return the Clausen cosine function  $C_n(w)$  of order m and real argument w.

The Clausen cosine function is defined by

$$C_n(w) = \sum_{k=1}^{\infty} \frac{\cos(kx)}{k^n}$$

# **Template Parameters**

_Тр	The real type of the argument
-----	-------------------------------

### **Parameters**

_~	The unsigned integer order
_m	
_←	The real argument
_ <i>w</i>	

Definition at line 4656 of file specfun.h.

Return the Clausen cosine function  $C_n(w)$  of order m and  ${\tt float}$  argument w.

# See also

clausen\_c for details.

Definition at line 4628 of file specfun.h.

Return the Clausen cosine function  $C_n(w)$  of order m and  $\log$  double argument w.

# See also

clausen\_c for details.

Definition at line 4638 of file specfun.h.

8.3.3.30 template 
$$\_$$
gnu\_cxx:: $\_$ promote\_num\_t<\_Tp>  $\_$ gnu\_cxx::clausen\_s ( unsigned int  $\_$ m,  $\_$ Tp  $\_$ w ) [inline]

Return the Clausen sine function  $S_n(\boldsymbol{w})$  of order m and real argument  $\boldsymbol{w}.$ 

The Clausen sine function is defined by

$$S_n(w) = \sum_{k=1}^{\infty} \frac{\sin(kx)}{k^n}$$

# **Template Parameters**

_Тр	The real type of the argument
-----	-------------------------------

### **Parameters**

_~	The unsigned integer order
_m	
_←	The real argument
_ <i>w</i>	

Definition at line 4613 of file specfun.h.

```
8.3.3.31 float __gnu_cxx::clausen_sf ( unsigned int __m, float __w ) [inline]
```

Return the Clausen sine function  $S_n(w)$  of order m and  ${\tt float}$  argument w.

### See also

clausen s for details.

Definition at line 4585 of file specfun.h.

```
8.3.3.32 long double __gnu_cxx::clausen_sl ( unsigned int __m, long double __w ) [inline]
```

Return the Clausen sine function  $S_n(w)$  of order m and long double argument w.

# See also

clausen\_s for details.

Definition at line 4595 of file specfun.h.

```
8.3.3.33 float __gnu_cxx::clausenf ( unsigned int __m, float __w ) [inline]
```

Return the Clausen function  $Cl_n(w)$  of integer order m and float argument w.

# See also

clausen for details.

Definition at line 4671 of file specfun.h.

8.3.3.34 std::complex<float> \_\_gnu\_cxx::clausenf ( unsigned int \_\_m, std::complex< float > \_\_w ) [inline]

Return the Clausen function  $Cl_n(w)$  of integer order m and std::complex < float > argument <math>w.

# See also

clausen for details.

Definition at line 4715 of file specfun.h.

8.3.3.35 long double \_\_gnu\_cxx::clausenl( unsigned int \_\_m, long double \_\_w ) [inline]

Return the Clausen function  $Cl_n(w)$  of integer order m and long double argument w.

### See also

clausen for details.

Definition at line 4681 of file specfun.h.

8.3.3.36 std::complex < long double > 
$$\_$$
gnu\_cxx::clausenl ( unsigned int  $\_$ m, std::complex < long double >  $\_$ w ) [inline]

Return the Clausen function  $Cl_n(w)$  of integer order m and std::complex<long double> argument <math>w.

# See also

clausen for details.

Definition at line 4725 of file specfun.h.

Return the complete Legendre elliptic integral D(k) of real modulus k.

The complete Legendre elliptic integral D is defined by

$$D(k) = \int_0^{\pi/2} \frac{\sin^2 \theta d\theta}{\sqrt{1 - k^2 sin 2\theta}}$$

# **Template Parameters**

\_*Tk* The type of the modulus k

### **Parameters**

Definition at line 3930 of file specfun.h.

```
8.3.3.38 float __gnu_cxx::comp_ellint_df(float __k) [inline]
```

Return the complete Legendre elliptic integral D(k) of float modulus k.

See also

```
comp_ellint_d for details.
```

Definition at line 3903 of file specfun.h.

```
8.3.3.39 long double __gnu_cxx::comp_ellint_dl( long double __k ) [inline]
```

Return the complete Legendre elliptic integral D(k) of long double modulus k.

See also

```
comp ellint d for details.
```

Definition at line 3913 of file specfun.h.

```
8.3.3.40 float _gnu_cxx::comp_ellint_rf( float _x, float _y ) [inline]
```

Return the complete Carlson elliptic function  $R_F(x,y,z)$  for float arguments.

See also

```
comp ellint rf for details.
```

Definition at line 2878 of file specfun.h.

```
8.3.3.41 long double __gnu_cxx::comp_ellint_rf( long double __x, long double __y) [inline]
```

Return the complete Carlson elliptic function  $R_F(x,y)$  for long double arguments.

See also

```
comp ellint rf for details.
```

Definition at line 2888 of file specfun.h.

Return the complete Carlson elliptic function  $R_F(x,y)$  for real arguments.

The complete Carlson elliptic function of the first kind is defined by:

$$R_F(x,y) = R_F(x,y,y) = \frac{1}{2} \int_0^\infty \frac{dt}{(t+x)^{1/2}(t+y)}$$

### **Parameters**

_~	The first argument.
_X	
_~	The second argument.
_y	

Definition at line 2906 of file specfun.h.

Return the Carlson complementary elliptic function  $R_G(x, y)$ .

See also

comp ellint rg for details.

Definition at line 3111 of file specfun.h.

Return the Carlson complementary elliptic function  $R_G(x,y)$ .

See also

comp\_ellint\_rg for details.

Definition at line 3120 of file specfun.h.

Return the complete Carlson elliptic function  $R_G(x,y)$  for real arguments.

The complete Carlson elliptic function is defined by:

$$R_G(x,y) = R_G(x,y,y) = \frac{1}{4} \int_0^\infty dt t(t+x)^{-1/2} (t+y)^{-1} (\frac{x}{t+x} + \frac{2y}{t+y})$$

## **Parameters**

_~	The first argument.
_X	
_~	The second argument.
V	

Definition at line 3139 of file specfun.h.

Return the confluent hypergeometric function  ${}_1F_1(a;c;x)$  of real numeratorial parameter a, denominatorial parameter c, and argument x.

The confluent hypergeometric function is defined by

$$_{1}F_{1}(a;c;x) = \sum_{n=0}^{\infty} \frac{(a)_{n}x^{n}}{(c)_{n}n!}$$

where the Pochhammer symbol is  $(x)_k = (x)(x+1)...(x+k-1)$ ,  $(x)_0 = 1$ 

### **Parameters**

_~	The numeratorial parameter
_a	
_←	The denominatorial parameter
_c	
_~	The argument
_x	

Definition at line 1378 of file specfun.h.

Return the confluent hypergeometric limit function  ${}_0F_1(;c;x)$  of real numeratorial parameter  ${}_{\mathbb{C}}$  and argument  ${}_{\mathbb{C}}$ .

The confluent hypergeometric limit function is defined by

$$_{0}F_{1}(;c;x) = \sum_{n=0}^{\infty} \frac{x^{n}}{(c)_{n}n!}$$

where the Pochhammer symbol is  $(x)_k = (x)(x+1)...(x+k-1), (x)_0 = 1$ 

### **Parameters**

_~	The denominatorial parameter
_c	
_~	The argument
_X	

Definition at line 1474 of file specfun.h.

8.3.3.48 float \_\_gnu\_cxx::conf\_hyperg\_limf(float \_\_c, float \_\_x) [inline]

Return the confluent hypergeometric limit function  ${}_0F_1(;c;x)$  of float numeratorial parameter c and argument x.

See also

conf\_hyperg\_lim for details.

Definition at line 1445 of file specfun.h.

8.3.3.49 long double \_\_gnu\_cxx::conf\_hyperg\_liml( long double \_\_c, long double \_\_x) [inline]

Return the confluent hypergeometric limit function  ${}_0F_1(;c;x)$  of long double numeratorial parameter c and argument x.

See also

conf\_hyperg\_lim for details.

Definition at line 1455 of file specfun.h.

8.3.3.50 float \_\_gnu\_cxx::conf\_hypergf ( float \_\_a, float \_\_c, float \_\_x ) [inline]

Return the confluent hypergeometric function  ${}_1F_1(a;c;x)$  of float numeratorial parameter a, denominatorial parameter c, and argument x.

See also

conf\_hyperg for details.

Definition at line 1346 of file specfun.h.

**8.3.3.51** long double \_\_gnu\_cxx::conf\_hypergl ( long double \_\_a, long double \_\_c, long double \_\_x ) [inline]

Return the confluent hypergeometric function  ${}_1F_1(a;c;x)$  of long double numeratorial parameter a, denominatorial parameter c, and argument x.

See also

conf hyperg for details.

Definition at line 1357 of file specfun.h.

8.3.3.52 template<typename  $_{Tp} > _{gnu\_cxx::\_promote\_num\_t < _{Tp} > _{gnu\_cxx::coshint}( _{Tp}_x )$  [inline]

Return the hyperbolic cosine integral Chi(x) of real argument x.

The hyperbolic cosine integral is defined by

$$Chi(x) = -\int_{x}^{\infty} \frac{\cosh(t)}{t} dt = \gamma_E + \ln(x) + \int_{0}^{x} \frac{\cosh(t) - 1}{t} dt$$

# **Template Parameters**

oe of the real argument	_Тр
-------------------------	-----

### **Parameters**

_~	The real argument
_x	

Definition at line 1756 of file specfun.h.

Return the hyperbolic cosine integral of float argument x.

### See also

coshint for details.

Definition at line 1728 of file specfun.h.

Return the hyperbolic cosine integral Chi(x) of long double argument x.

# See also

coshint for details.

Definition at line 1738 of file specfun.h.

$$\textbf{8.3.3.55} \quad template < typename \_Tp > \_gnu\_cxx::\_promote\_num\_t < \_Tp > \_gnu\_cxx::cosint(\_Tp \_x) \quad [inline]$$

Return the cosine integral Ci(x) of real argument x.

The cosine integral is defined by

$$Ci(x) = -\int_{x}^{\infty} \frac{\cos(t)}{t} dt = \gamma_E + \ln(x) + \int_{0}^{x} \frac{\cos(t) - 1}{t} dt$$

### **Parameters**

_~	The real upper integration limit
_X	

Definition at line 1673 of file specfun.h.

Return the cosine integral Ci(x) of float argument x.

### See also

cosint for details.

Definition at line 1647 of file specfun.h.

Return the cosine integral Ci(x) of long double argument x.

# See also

cosint for details.

Definition at line 1657 of file specfun.h.

Return the cylindrical Hankel function of the first kind  $H_n^{(1)}(x)$  of real order  $\nu$  and argument x>=0.

The cylindrical Hankel function of the first kind is defined by:

$$H_{\nu}^{(1)}(x) = \left(\frac{\pi}{2x}\right)^{1/2} \left[J_{n+1/2}(x) + iN_{n+1/2}(x)\right]$$

where  $J_{\nu}(x)$  and  $N_{\nu}(x)$  are the cylindrical Bessel and Neumann functions respectively (

# See also

cyl\_bessel and cyl\_neumann).

# **Template Parameters**

The real type of the argument	_Тр
-------------------------------	-----

### **Parameters**

nu	The real order
z	The real argument

Definition at line 2407 of file specfun.h.

Return the complex cylindrical Hankel function of the first kind  $H^{(1)}_{\nu}(x)$  of complex order  $\nu$  and argument x.

The cylindrical Hankel function of the first kind is defined by

$$H_{\nu}^{(1)}(x) = J_{\nu}(x) + iN_{\nu}(x)$$

# **Template Parameters**

_Tpnu	The complex type of the order
_Тр	The complex type of the argument

# **Parameters**

nu	The complex order
x	The complex argument

Definition at line 4207 of file specfun.h.

Return the cylindrical Hankel function of the first kind  $H_{\nu}^{(1)}(x)$  of float order  $\nu$  and argument x>=0.

### See also

cyl\_hankel\_1 for details.

Definition at line 2374 of file specfun.h.

8.3.3.61 std::complex < float >  $\_$ gnu\_cxx::cyl\_hankel\_1f ( std::complex < float >  $\_$ nu, std::complex < float >  $\_$ x ) [inline]

Return the complex cylindrical Hankel function of the first kind  $H_{\nu}^{(1)}(x)$  of std::complex<float> order  $\nu$  and argument x.

See also

cyl hankel 1 for more details.

Definition at line 4176 of file specfun.h.

8.3.3.62 std::complex < long double > \_\_gnu\_cxx::cyl\_hankel\_1I( long double \_\_nu, long double \_\_z) [inline]

Return the cylindrical Hankel function of the first kind  $H_{\nu}^{(1)}(x)$  of long double order  $\nu$  and argument x >= 0.

See also

cyl hankel 1 for details.

Definition at line 2385 of file specfun.h.

8.3.3.63 std::complex < long double >  $\_$ gnu\_cxx::cyl\_hankel\_1I ( std::complex < long double >  $\_$ nu, std::complex < long double >  $\_$ x ) [inline]

Return the complex cylindrical Hankel function of the first kind  $H^{(1)}_{\nu}(x)$  of std::complex<long double> order  $\nu$  and argument x.

See also

cyl hankel 1 for more details.

Definition at line 4187 of file specfun.h.

8.3.3.64 template<typename \_Tpnu , typename \_Tp > std::complex<\_\_gnu\_cxx::\_\_promote\_num\_t<\_Tpnu, \_Tp > \_\_gnu\_cxx::cyl\_hankel\_2( \_Tpnu \_\_nu, \_Tp \_\_z ) [inline]

Return the cylindrical Hankel function of the second kind  $H_n^{(2)}(x)$  of real order  $\nu$  and argument x >= 0.

The cylindrical Hankel function of the second kind is defined by:

$$H_{\nu}^{(2)}(x) = \left(\frac{\pi}{2r}\right)^{1/2} \left[J_{n+1/2}(x) - iN_{n+1/2}(x)\right]$$

where  $J_{\nu}(x)$  and  $N_{\nu}(x)$  are the cylindrical Bessel and Neumann functions respectively (

See also

cyl\_bessel and cyl\_neumann).

# **Template Parameters**

The real type of the argument	_Тр
-------------------------------	-----

### **Parameters**

nu	The real order
z	The real argument

Definition at line 2456 of file specfun.h.

Return the complex cylindrical Hankel function of the second kind  $H^{(2)}_{\nu}(x)$  of complex order  $\nu$  and argument x.

The cylindrical Hankel function of the second kind is defined by

$$H_{\nu}^{(2)}(x) = J_{\nu}(x) - iN_{\nu}(x)$$

# **Template Parameters**

_Tpnu	The complex type of the order
_Тр	The complex type of the argument

# **Parameters**

nu	The complex order
x	The complex argument

Definition at line 4254 of file specfun.h.

```
8.3.3.66 std::complex<float> __gnu_cxx::cyl_hankel_2f(float __nu, float __z) [inline]
```

Return the cylindrical Hankel function of the second kind  $H^{(2)}_{\nu}(x)$  of float order  $\nu$  and argument x>=0.

### See also

cyl\_hankel\_2 for details.

Definition at line 2423 of file specfun.h.

Return the complex cylindrical Hankel function of the second kind  $H^{(2)}_{\nu}(x)$  of std::complex<float> order  $\nu$  and argument x.

See also

cyl hankel 2 for more details.

Definition at line 4223 of file specfun.h.

8.3.3.68 std::complex<long double \_ gnu\_cxx::cyl\_hankel\_2l( long double \_ nu, long double \_ z ) [inline]

Return the cylindrical Hankel function of the second kind  $H_{\nu}^{(2)}(x)$  of long double order  $\nu$  and argument x >= 0.

See also

cyl hankel 2 for details.

Definition at line 2434 of file specfun.h.

8.3.3.69 std::complex < long double >  $\_$ gnu\_cxx::cyl\_hankel\_2I ( std::complex < long double >  $\_$ nu, std::complex < long double >  $\_$ x ) [inline]

Return the complex cylindrical Hankel function of the second kind  $H^{(2)}_{\nu}(x)$  of std::complex<long double> order  $\nu$  and argument x.

See also

cyl hankel 2 for more details.

Definition at line 4234 of file specfun.h.

8.3.3.70 template<typename\_Tp > \_\_gnu\_cxx::\_\_promote\_num\_t<\_Tp> \_\_gnu\_cxx::dawson( \_Tp \_\_x ) [inline]

Return the Dawson integral, F(x), for real argument x.

The Dawson integral is defined by:

$$F(x) = e^{-x^2} \int_0^x e^{y^2} dy$$

and it's derivative is:

$$F'(x) = 1 - 2xF(x)$$

### **Parameters**

Definition at line 3449 of file specfun.h.

```
8.3.3.71 float __gnu_cxx::dawsonf(float __x) [inline]
```

Return the Dawson integral, F(x), for float argument x.

See also

dawson for details.

Definition at line 3421 of file specfun.h.

```
8.3.3.72 long double __gnu_cxx::dawsonl(long double __x) [inline]
```

Return the Dawson integral, F(x), for long double argument x.

See also

dawson for details.

Definition at line 3430 of file specfun.h.

Definition at line 2822 of file specfun.h.

Definition at line 2810 of file specfun.h.

```
8.3.3.75 long double \_gnu\_cxx::digammal ( long double \_z ) [inline]
```

Definition at line 2814 of file specfun.h.

$$\textbf{8.3.3.76} \quad \textbf{template} < \textbf{typename\_Tp} > \underline{\textbf{gnu\_cxx::\_promote\_num\_t}} < \underline{\textbf{Tp}} > \underline{\textbf{gnu\_cxx::dilog}} \left( \underline{\textbf{Tp}\_\textbf{x}} \right) \quad [\texttt{inline}]$$

Return the dilogarithm function  $\psi(z)$  for real argument.

The dilogarithm is defined by:

$$Li_2(x) = \sum_{k=1}^{\infty} \frac{x^k}{k^2}$$

### **Parameters**

_~	The argument.
_x	

Definition at line 2863 of file specfun.h.

Return the dilogarithm function  $\psi(z)$  for float argument.

## See also

dilog for details.

Definition at line 2837 of file specfun.h.

Return the dilogarithm function  $\psi(z)$  for long double argument.

## See also

dilog for details.

Definition at line 2847 of file specfun.h.

Return the Dirichlet beta function of real argument s.

The Dirichlet beta function is defined by:

$$\beta(s) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)^s}$$

An important reflection formula is:

$$\beta(1-s) = \left(\frac{2}{\pi}\right)^s \sin(\frac{\pi s}{2})\Gamma(s)\beta(s)$$

Definition at line 4570 of file specfun.h.

```
8.3.3.80 float __gnu_cxx::dirichlet_betaf ( float __s ) [inline]
```

Return the Dirichlet beta function of real argument s.

See also

dirichlet beta for details.

Definition at line 4541 of file specfun.h.

```
8.3.3.81 long double __gnu_cxx::dirichlet_betal ( long double __s ) [inline]
```

Return the Dirichlet beta function of real argument s.

See also

dirichlet\_beta for details.

Definition at line 4550 of file specfun.h.

```
8.3.3.82 template<typename_Tp > _Tp __gnu_cxx::dirichlet_eta( _Tp __s ) [inline]
```

Return the Dirichlet eta function of real argument s.

The Dirichlet eta function is defined by

$$\eta(s) = \sum_{k=1}^{\infty} \frac{(-1)^k}{k^s} = (1 - 2^{1-s}) \zeta(s)$$

An important reflection formula is:

$$\eta(-s) = 2\frac{1 - 2^{-s-1}}{1 - 2^{-s}}\pi^{-s-1}s\sin(\frac{\pi s}{2})\Gamma(s)\eta(s+1)$$

**Parameters** 

Definition at line 4527 of file specfun.h.

8.3.3.83 float \_\_gnu\_cxx::dirichlet\_etaf ( float \_\_s ) [inline]

Return the Dirichlet eta function of real argument s.

See also

dirichlet eta for details.

Definition at line 4497 of file specfun.h.

**8.3.3.84** long double \_\_gnu\_cxx::dirichlet\_etal ( long double \_\_s ) [inline]

Return the Dirichlet eta function of real argument s.

See also

dirichlet\_eta for details.

Definition at line 4506 of file specfun.h.

8.3.3.85 template < typename \_Tp > \_\_gnu\_cxx::\_\_promote\_num\_t < \_Tp > \_\_gnu\_cxx::double\_factorial(int \_\_n) [inline]

Definition at line 3621 of file specfun.h.

8.3.3.86 float \_\_gnu\_cxx::double\_factorialf(int \_\_n) [inline]

Definition at line 3609 of file specfun.h.

**8.3.3.87** long double \_\_gnu\_cxx::double\_factoriall( int \_\_n ) [inline]

Definition at line 3613 of file specfun.h.

8.3.3.88 template<typename \_Tk , typename \_Tp , typename \_Ta , typename \_Tb > \_\_gnu\_cxx::\_\_promote\_num\_t<\_Tk, \_Tp, \_Ta, \_Tb> \_\_gnu\_cxx::ellint\_cel ( \_Tk \_k\_c, \_Tp \_\_p, \_Ta \_\_a, \_Tb \_\_b ) [inline]

Return the Bulirsch complete elliptic integral  $cel(k_c, p, a, b)$  of real complementary modulus  $k_c$ , and parameters p, a, and b.

The Bulirsch complete elliptic integral is defined by

$$cel(k_c, p, a, b) = \int_0^{\pi/2} \frac{a\cos^2\theta + b\sin^2\theta}{\cos^2\theta + p\sin^2\theta} \frac{d\theta}{\sqrt{\cos^2\theta + k_c^2\sin^2\theta}}$$

### **Parameters**

k⊷	The complementary modulus $k_c = \sqrt{1-k^2}$
_ <i>c</i>	
p	The parameter
a	The parameter
b	The parameter

Definition at line 4160 of file specfun.h.

```
8.3.3.89 float __gnu_cxx::ellint_celf ( float __k_c, float __p, float __a, float __b ) [inline]
```

Return the Bulirsch complete elliptic integral  $cel(k_c, p, a, b)$  of real complementary modulus  $k_c$ , and parameters p, a, and b.

## See also

ellint\_cel for details.

Definition at line 4128 of file specfun.h.

8.3.3.90 long double 
$$\_g$$
nu\_cxx::ellint\_cell ( long double  $\_k\_c$ , long double  $\_p$ , long double  $\_a$ , long double  $\_b$  ) [inline]

Return the Bulirsch complete elliptic integral  $cel(k_c, p, a, b)$ .

### See also

ellint cel for details.

Definition at line 4137 of file specfun.h.

Return the incomplete Legendre elliptic integral  $D(k,\phi)$  of real modulus k and angular limit  $\phi$ .

The Legendre elliptic integral D is defined by

$$D(k,\phi) = \int_0^\phi \frac{\sin^2 \theta d\theta}{\sqrt{1 - k^2 \sin^2 \theta}}$$

### **Parameters**

k	The modulus $-1 <= \underline{} k <= +1$
phi	The angle

Definition at line 3973 of file specfun.h.

Return the incomplete Legendre elliptic integral  $D(k,\phi)$  of float modulus k and angular limit  $\phi$ .

#### See also

ellint d for details.

Definition at line 3945 of file specfun.h.

Return the incomplete Legendre elliptic integral  $D(k,\phi)$  of long double modulus k and angular limit  $\phi$ .

### See also

ellint\_d for details.

Definition at line 3955 of file specfun.h.

Return the Bulirsch elliptic integral  $el1(x,k_c)$  of the first kind of real tangent limit x and complementary modulus  $k_c$ .

The Bulirsch elliptic integral of the first kind is defined by

$$el1(x, k_c) = el2(x, k_c, 1, 1) = \int_0^{\arctan x} \frac{1 + 1 \tan^2 \theta}{\sqrt{(1 + \tan^2 \theta)(1 + k_c^2 \tan^2 \theta)}} d\theta$$

x	The tangent of the angular integration limit
k⊷	The complementary modulus $k_c = \sqrt{1-k^2}$
_c	

Definition at line 4019 of file specfun.h.

```
8.3.3.95 float __gnu_cxx::ellint_el1f ( float __x, float __k_c ) [inline]
```

Return the Bulirsch elliptic integral  $el1(x,k_c)$  of the first kind of float tangent limit x and complementary modulus  $k_c$ .

### See also

ellint el1 for details.

Definition at line 3989 of file specfun.h.

Return the Bulirsch elliptic integral  $el1(x, k_c)$  of the first kind of real tangent limit x and complementary modulus  $k_c$ .

### See also

ellint\_el1 for details.

Definition at line 4000 of file specfun.h.

Return the Bulirsch elliptic integral of the second kind  $el2(x, k_c, a, b)$ .

The Bulirsch elliptic integral of the second kind is defined by

$$el2(x, k_c, a, b) = \int_0^{\arctan x} \frac{a + b \tan^2 \theta}{\sqrt{(1 + \tan^2 \theta)(1 + k_c^2 \tan^2 \theta)}} d\theta$$

# **Parameters**

x	The tangent of the angular integration limit	
k⊷	The complementary modulus $k_c = \sqrt{1-k^2}$	
_c		
a	The parameter	
b	The parameter	

Definition at line 4065 of file specfun.h.

Return the Bulirsch elliptic integral of the second kind  $el2(x, k_c, a, b)$ .

See also

ellint\_el2 for details.

Definition at line 4034 of file specfun.h.

Return the Bulirsch elliptic integral of the second kind  $el2(x, k_c, a, b)$ .

See also

ellint\_el2 for details.

Definition at line 4044 of file specfun.h.

8.3.3.100 template \_\_gnu\_cxx::\_\_promote\_num\_t<\_Tx, \_Tk, \_Tp> \_\_gnu\_cxx::ellint\_el3 ( \_Tx \_ x, \_Tk \_ 
$$k_c$$
, \_Tp \_  $p$  ) [inline]

Return the Bulirsch elliptic integral of the third kind  $el3(x,k_c,p)$  of real tangent limit x, complementary modulus  $k_c$ , and parameter p.

The Bulirsch elliptic integral of the third kind is defined by

$$el3(x, k_c, p) = \int_0^{\arctan x} \frac{d\theta}{(\cos^2 \theta + p \sin^2 \theta) \sqrt{\cos^2 \theta + k_c^2 \sin^2 \theta}}$$

# **Parameters**

x	The tangent of the angular integration limit
k↔	The complementary modulus $k_c = \sqrt{1-k^2}$
c	The paramenter

Definition at line 4112 of file specfun.h.

```
8.3.3.101 float __gnu_cxx::ellint_el3f ( float __x, float __k_c, float __p ) [inline]
```

Return the Bulirsch elliptic integral of the third kind  $el3(x,k_c,p)$  of float tangent limit x, complementary modulus  $k_c$ , and parameter p.

### See also

ellint el3 for details.

Definition at line 4081 of file specfun.h.

```
8.3.3.102 long double _gnu_cxx::ellint_el3l ( long double _x, long double _k_c, long double _p) [inline]
```

Return the Bulirsch elliptic integral of the third kind  $el3(x, k_c, p)$  of long double tangent limit x, complementary modulus  $k_c$ , and parameter p.

#### See also

ellint\_el3 for details.

Definition at line 4092 of file specfun.h.

Return the Carlson elliptic function  $R_C(x,y) = R_F(x,y,y)$  where  $R_F(x,y,z)$  is the Carlson elliptic function of the first kind.

The Carlson elliptic function is defined by:

$$R_C(x,y) = \frac{1}{2} \int_0^\infty \frac{dt}{(t+x)^{1/2}(t+y)}$$

Based on Carlson's algorithms:

- B. C. Carlson Numer. Math. 33, 1 (1979)
- B. C. Carlson, Special Functions of Applied Mathematics (1977)
- Numerical Recipes in C, 2nd ed, pp. 261-269, by Press, Teukolsky, Vetterling, Flannery (1992)

_←	The first argument.
_X	
_~	The second argument.
_y	

Definition at line 2998 of file specfun.h.

```
8.3.3.104 float __gnu_cxx::ellint_rcf(float __x, float __y) [inline]
```

Return the Carlson elliptic function  $R_C(x, y)$ .

### See also

ellint rc for details.

Definition at line 2964 of file specfun.h.

Return the Carlson elliptic function  $R_C(x, y)$ .

### See also

ellint\_rc for details.

Definition at line 2973 of file specfun.h.

Return the Carlson elliptic function of the second kind  $R_D(x,y,z) = R_J(x,y,z,z)$  where  $R_J(x,y,z,p)$  is the Carlson elliptic function of the third kind.

The Carlson elliptic function of the second kind is defined by:

$$R_D(x,y,z) = \frac{3}{2} \int_0^\infty \frac{dt}{(t+x)^{1/2}(t+y)^{1/2}(t+z)^{3/2}}$$

Based on Carlson's algorithms:

- B. C. Carlson Numer. Math. 33, 1 (1979)
- B. C. Carlson, Special Functions of Applied Mathematics (1977)
- Numerical Recipes in C, 2nd ed, pp. 261-269, by Press, Teukolsky, Vetterling, Flannery (1992)

_~	The first of two symmetric arguments.	
_x		
_←	The second of two symmetric arguments.	
Generate	d by Doxygen	
_←	The third argument.	
_z		

Definition at line 3097 of file specfun.h.

```
8.3.3.107 float __gnu_cxx::ellint_rdf(float __x, float __y, float __z) [inline]
```

Return the Carlson elliptic function  $R_D(x, y, z)$ .

### See also

ellint rd for details.

Definition at line 3061 of file specfun.h.

```
8.3.3.108 long double __gnu_cxx::ellint_rdl ( long double __x, long double __y, long double __z ) [inline]
```

Return the Carlson elliptic function  $R_D(x, y, z)$ .

## See also

ellint rd for details.

Definition at line 3070 of file specfun.h.

Return the Carlson elliptic function  $R_F(x,y,z)$  of the first kind for real arguments.

The Carlson elliptic function of the first kind is defined by:

$$R_F(x,y,z) = \frac{1}{2} \int_0^\infty \frac{dt}{(t+x)^{1/2}(t+y)^{1/2}(t+z)^{1/2}}$$

## Parameters

_~	The first of three symmetric arguments.
_X	
_~	The second of three symmetric arguments.
_y	
_~	The third of three symmetric arguments.
_z	

Definition at line 2950 of file specfun.h.

8.3.3.110 float \_\_gnu\_cxx::ellint\_rff(float \_\_x, float \_\_y, float \_\_z) [inline]

Return the Carlson elliptic function  $R_F(x,y,z)$  of the first kind for float arguments.

### See also

ellint\_rf for details.

Definition at line 2921 of file specfun.h.

8.3.3.111 long double \_\_gnu\_cxx::ellint\_fl( long double \_\_x, long double \_\_y, long double \_\_z) [inline]

Return the Carlson elliptic function  $R_F(x,y,z)$  of the first kind for long double arguments.

#### See also

ellint\_rf for details.

Definition at line 2931 of file specfun.h.

Return the symmetric Carlson elliptic function of the second kind  $R_G(x, y, z)$ .

The Carlson symmetric elliptic function of the second kind is defined by:

$$R_G(x,y,z) = \frac{1}{4} \int_0^\infty dt t [(t+x)(t+y)(t+z)]^{-1/2} \left(\frac{x}{t+x} + \frac{y}{t+y} + \frac{z}{t+z}\right)$$

Based on Carlson's algorithms:

- B. C. Carlson Numer. Math. 33, 1 (1979)
- B. C. Carlson, Special Functions of Applied Mathematics (1977)
- Numerical Recipes in C, 2nd ed, pp. 261-269, by Press, Teukolsky, Vetterling, Flannery (1992)

_←	The first of three symmetric arguments.
_X	
_←	The second of three symmetric arguments.
_ <b>y</b>	
_~	The third of three symmetric arguments.
_Z	

Definition at line 3188 of file specfun.h.

```
8.3.3.113 float __gnu_cxx::ellint_rgf(float __x, float __y, float __z) [inline]
```

Return the Carlson elliptic function  $R_G(x, y)$ .

### See also

ellint\_rg for details.

Definition at line 3153 of file specfun.h.

```
8.3.3.114 long double __gnu_cxx::ellint_rgl ( long double __x, long double __y, long double __z ) [inline]
```

Return the Carlson elliptic function  $R_G(x, y)$ .

### See also

ellint\_rg for details.

Definition at line 3162 of file specfun.h.

Return the Carlson elliptic function  $R_J(x,y,z,p)$  of the third kind.

The Carlson elliptic function of the third kind is defined by:

$$R_J(x,y,z,p) = \frac{3}{2} \int_0^\infty \frac{dt}{(t+x)^{1/2}(t+y)^{1/2}(t+z)^{1/2}(t+p)}$$

Based on Carlson's algorithms:

- B. C. Carlson Numer. Math. 33, 1 (1979)
- B. C. Carlson, Special Functions of Applied Mathematics (1977)
- Numerical Recipes in C, 2nd ed, pp. 261-269, by Press, Teukolsky, Vetterling, Flannery (1992)

_~	The first of three symmetric arguments.	
_x		
_~	The second of three symmetric arguments.	
y		
_~	The third of three symmetric arguments.	Generated by Doxyger
_z		
_~	The fourth argument.	
_p		

Definition at line 3047 of file specfun.h.

8.3.3.116 float \_\_gnu\_cxx::ellint\_rjf(float \_\_x, float \_\_y, float \_\_z, float \_\_p) [inline]

Return the Carlson elliptic function  $R_J(x, y, z, p)$ .

See also

ellint rj for details.

Definition at line 3012 of file specfun.h.

8.3.3.117 long double \_\_gnu\_cxx::ellint\_rjl( long double \_\_x, long double \_\_y, long double \_\_z, long double \_\_p) [inline]

Return the Carlson elliptic function  $R_J(x, y, z, p)$ .

See also

ellint rj for details.

Definition at line 3021 of file specfun.h.

8.3.3.118 template<typename\_Tp > \_Tp \_\_gnu\_cxx::ellnome( \_Tp \_\_k ) [inline]

Return the elliptic nome function q(k) of modulus k.

The elliptic nome function is defined by

$$q(k) = \exp\left(-\pi \frac{K(k)}{K(\sqrt{1-k^2})}\right)$$

where K(k) is the complete elliptic function of the first kind.

**Template Parameters** 

\_Tp | The real type of the modulus

## **Parameters**

$$\begin{array}{|c|c|c|} \hline \_ \leftarrow & \text{The modulus} \ -1 <= k <= +1 \\ \hline \_ k & \end{array}$$

Definition at line 4958 of file specfun.h.

8.3.3.119 float \_\_gnu\_cxx::ellnomef(float \_\_k) [inline]

Return the elliptic nome function q(k) of modulus k.

## See also

ellnome for details.

Definition at line 4931 of file specfun.h.

**8.3.3.120** long double \_\_gnu\_cxx::ellnomel( long double \_\_k ) [inline]

Return the elliptic nome function q(k) of long double modulus k.

#### See also

ellnome for details.

Definition at line 4941 of file specfun.h.

Return the exponential integral  $E_n(x)$  of integral order n and real argument x. The exponential integral is defined by:

$$E_n(x) = \int_1^\infty \frac{e^{-tx}}{t^n} dt$$

In particular

$$E_1(x) = \int_1^\infty \frac{e^{-tx}}{t} dt = -Ei(-x)$$

## **Template Parameters**

_Tp The real type of te argument	
----------------------------------	--

## **Parameters**

_←	The integral order
_n	
_~	The real argument
_X	

Definition at line 3495 of file specfun.h.

8.3.3.122 float \_\_gnu\_cxx::expintf ( unsigned int \_\_n, float \_\_x ) [inline]

Return the exponential integral  $E_n(x)$  for integral order n and float argument  ${\tt x}.$ 

See also

expint for details.

Definition at line 3464 of file specfun.h.

8.3.3.123 long double \_\_gnu\_cxx::expintl ( unsigned int \_\_n, long double \_\_x ) [inline]

Return the exponential integral  $E_n(x)$  for integral order n and long double argument x.

See also

expint for details.

Definition at line 3474 of file specfun.h.

Definition at line 3600 of file specfun.h.

**8.3.3.125** float \_\_gnu\_cxx::factorialf ( unsigned int \_\_n ) [inline]

Definition at line 3588 of file specfun.h.

**8.3.3.126** long double \_\_gnu\_cxx::factoriall ( unsigned int \_\_n ) [inline]

Definition at line 3592 of file specfun.h.

$$\textbf{8.3.3.127} \quad template < typename \_Tp > \_gnu\_cxx::\_promote\_num\_t < \_Tp > \_gnu\_cxx::fresnel\_c ( \_Tp \_x ) \quad \texttt{[inline]}$$

Return the Fresnel cosine integral of argument x.

The Fresnel cosine integral is defined by

$$C(x) = \int_0^x \cos(\frac{\pi}{2}t^2)dt$$

### **Parameters**

_~	The argument
_X	

Definition at line 3407 of file specfun.h.

```
8.3.3.128 float __gnu_cxx::fresnel_cf( float __x ) [inline]
```

Definition at line 3388 of file specfun.h.

```
8.3.3.129 long double __gnu_cxx::fresnel_cl( long double __x ) [inline]
```

Definition at line 3392 of file specfun.h.

Return the Fresnel sine integral of argument x.

The Fresnel sine integral is defined by

$$S(x) = \int_0^x \sin(\frac{\pi}{2}t^2)dt$$

### **Parameters**

_~	The argument
_X	

Definition at line 3379 of file specfun.h.

Definition at line 3360 of file specfun.h.

Definition at line 3364 of file specfun.h.

Definition at line 2801 of file specfun.h.

8.3.3.134 float \_\_gnu\_cxx::gamma\_lf(float \_\_n, float \_\_x) [inline]

Definition at line 2789 of file specfun.h.

8.3.3.135 long double \_\_gnu\_cxx::gamma\_ll( long double \_\_n, long double \_\_x) [inline]

Definition at line 2793 of file specfun.h.

8.3.3.136 template<typename \_Tn , typename \_Tp > \_\_gnu\_cxx::\_\_promote\_num\_t<\_Tn, \_Tp> \_\_gnu\_cxx::gamma\_u ( \_Tn \_\_n, \_Tp \_\_x ) [inline]

Definition at line 2780 of file specfun.h.

8.3.3.137 float \_\_gnu\_cxx::gamma\_uf(float \_\_n, float \_\_x) [inline]

Definition at line 2768 of file specfun.h.

8.3.3.138 long double \_\_gnu\_cxx::gamma\_ul ( long double \_\_n, long double \_\_x ) [inline]

Definition at line 2772 of file specfun.h.

8.3.3.139 template<typename \_Talpha , typename \_Tp > \_\_gnu\_cxx::\_\_promote\_num\_t<\_Talpha, \_Tp > \_\_gnu\_cxx::gegenbauer ( unsigned int \_\_n, \_Talpha \_\_alpha, \_Tp \_\_x ) [inline]

Return the Gegenbauer polynomial  $C_n^{\alpha}(x)$  of degree n and real order  $\alpha > -1/2, \alpha \neq 0$  and argument x.

The Gegenbauer polynomials are generated by a three-term recursion relation:

$$C_n^{\alpha}(x) = \frac{1}{n} \left[ 2x(n+\alpha-1)C_{n-1}^{\alpha}(x) - (n+2\alpha-2)C_{n-2}^{\alpha}(x) \right]$$

and  $C_0^{\alpha}(x) = 1$ ,  $C_1^{\alpha}(x) = 2\alpha x$ .

# **Template Parameters**

_Talpha	The real type of the order
_ <i>Tp</i>	The real type of the argument

n	The non-negative integral degree
alpha	The real order
X	The real argument

Definition at line 2206 of file specfun.h.

8.3.3.140 float \_\_gnu\_cxx::gegenbauerf ( unsigned int \_\_n, float \_\_alpha, float \_\_x ) [inline]

Return the Gegenbauer polynomial  $C_n^{\alpha}(x)$  of degree n and float order  $\alpha>-1/2, \alpha\neq 0$  and argument x.

## See also

gegenbauer for details.

Definition at line 2173 of file specfun.h.

8.3.3.141 long double \_\_gnu\_cxx::gegenbauerl( unsigned int \_\_n, long double \_\_alpha, long double \_\_x) [inline]

Return the Gegenbauer polynomial  $C_n^{\alpha}(x)$  of degree n and long double order  $\alpha > -1/2, \alpha \neq 0$  and argument x.

## See also

gegenbauer for details.

Definition at line 2184 of file specfun.h.

Return the Heuman lambda function  $\Lambda(k,\phi)$  of modulus k and angular limit  $\phi$ .

The complete Heuman lambda function is defined by

$$\Lambda(k,\phi) = \frac{F(1-m,\phi)}{K(1-m)} + \frac{2}{\pi}K(m)Z(1-m,\phi)$$

where  $m=k^2$ , K(k) is the complete elliptic function of the first kind, and Z(k,phi) is the Jacobi zeta function.

## **Template Parameters**

_Tk	the floating-point type of the modulus
_Tphi	the floating-point type of the angular limit argument

k	The modulus
phi	The angle

Definition at line 3888 of file specfun.h.

Definition at line 3862 of file specfun.h.

Definition at line 3866 of file specfun.h.

Return the Hurwitz zeta function of real argument s, and parameter a.

The the Hurwitz zeta function is defined by

$$\zeta(s,a) = \sum_{n=0}^{\infty} \frac{1}{(a+n)^s}$$

#### **Parameters**

_~	The argument
_s	
_~	The parameter
_a	

Definition at line 3229 of file specfun.h.

Return the Hurwitz zeta function of real argument s, and complex parameter a.

See also

hurwitz zeta for details.

Definition at line 3243 of file specfun.h.

Return the Hurwitz zeta function of float argument s, and parameter a.

See also

hurwitz zeta for details.

Definition at line 3203 of file specfun.h.

8.3.3.148 long double \_\_gnu\_cxx::hurwitz\_zetal ( long double \_\_s, long double \_\_a ) [inline]

Return the Hurwitz zeta function of long double argument s, and parameter a.

## See also

hurwitz\_zeta for details.

Definition at line 3213 of file specfun.h.

Return the hypergeometric function  ${}_2F_1(a,b;c;x)$  of real numeratorial parameters a and b, denominatorial parameter c, and argument x.

The hypergeometric function is defined by

$$_{2}F_{1}(a,b;c;x) = \sum_{n=0}^{\infty} \frac{(a)_{n}(b)_{n}x^{n}}{(c)_{n}n!}$$

where the Pochhammer symbol is  $(x)_k = (x)(x+1)...(x+k-1), (x)_0 = 1$ 

#### **Parameters**

_~	The first numeratorial parameter
_a	
_~	The second numeratorial parameter
_b	
_~	The denominatorial parameter
_c	
_~	The argument
_X	

Definition at line 1427 of file specfun.h.

```
8.3.3.150 float __gnu_cxx::hypergf ( float __a, float __b, float __c, float __x ) [inline]
```

Return the hypergeometric function  ${}_2F_1(a,b;c;x)$  of @ float numeratorial parameters a and b, denominatorial parameter c, and argument x.

## See also

hyperg for details.

Definition at line 1394 of file specfun.h.

8.3.3.151 long double \_\_gnu\_cxx::hypergl( long double \_\_a, long double \_\_b, long double \_\_c, long double \_\_x) [inline]

Return the hypergeometric function  ${}_2F_1(a,b;c;x)$  of long double numeratorial parameters a and b, denominatorial parameter c, and argument  ${\bf x}$ .

### See also

hyperg for details.

Definition at line 1405 of file specfun.h.

Return the regularized incomplete beta function of parameters a, b, and argument x.

The regularized incomplete beta function is defined by

$$I_x(a,b) = \frac{B_x(a,b)}{B(a,b)}$$

where

$$B_x(a,b) = \int_0^x t^{a-1} (1-t)^{b-1} dt$$

is the non-regularized beta function and B(a,b) is the usual beta function.

### **Parameters**

_←	The first parameter
_a	
_←	The second parameter
_b	
_~	The argument
_x	

Definition at line 3320 of file specfun.h.

Return the regularized complementary incomplete beta function of parameters a, b, and argument x.

The regularized complementary incomplete beta function is defined by

$$I_x(a,b) = I_x(a,b)$$

### **Parameters**

_~	The parameter
_a	
_←	The parameter
_b	
_~	The argument
_x	

Definition at line 3351 of file specfun.h.

```
8.3.3.154 float __gnu_cxx::ibetacf (float __a, float __b, float __x ) [inline]
```

Definition at line 3329 of file specfun.h.

References gnu cxx::ibetaf().

```
8.3.3.155 long double __gnu_cxx::ibetacl( long double __a, long double __b, long double __x) [inline]
```

Definition at line 3333 of file specfun.h.

References \_\_gnu\_cxx::ibetal().

```
8.3.3.156 float __gnu_cxx::ibetaf ( float __a, float __b, float __x ) [inline]
```

Return the regularized incomplete beta function of parameters a, b, and argument x.

See ibeta for details.

Definition at line 3286 of file specfun.h.

Referenced by \_\_gnu\_cxx::ibetacf().

```
8.3.3.157 long double __gnu_cxx::ibetal ( long double __a, long double __b, long double __x ) [inline]
```

Return the regularized incomplete beta function of parameters a, b, and argument x.

See ibeta for details.

Definition at line 3296 of file specfun.h.

Referenced by \_\_gnu\_cxx::ibetacl().

```
8.3.3.158 template<typename _Talpha , typename _Tp > __gnu_cxx::__promote_num_t<_Talpha, _Tbeta, _Tp > __gnu_cxx::__promote_num_t<_Talpha, _Tbeta, _Tp > __gnu_cxx::_promote_num_t<_Talpha, _Tbeta, _Tp = __x ) [inline]
```

Return the Jacobi polynomial  $P_n^{(\alpha,\beta)}(x)$  of degree n and float orders  $\alpha,\beta>-1$  and argument x.

The Jacobi polynomials are generated by a three-term recursion relation:

$$2n(\alpha+\beta+n)(\alpha+\beta+2n-2)P_{n}^{(\alpha,\beta)}(x) = (\alpha+\beta+2n-1)((\alpha^{2}-\beta^{2})+x(\alpha+\beta+2n-2)(\alpha+\beta+2n))P_{n-1}^{(\alpha,\beta)}(x) - 2(\alpha+n-1)(\beta+n-1)(\alpha+\beta+2n-2)(\alpha+2n-2)(\alpha+2n-2)(\alpha+2n-2)(\alpha+2n-2)(\alpha+2n-2)(\alpha+2n-2)(\alpha+2n-2$$

# **Template Parameters**

_Talpha	The real type of the order $\alpha$
_Tbeta	The real type of the order $eta$
_Тр	The real type of the argument

### **Parameters**

n	The non-negative integral degree	
alpha	The real order	
beta	The real order	
x	The real argument	

Definition at line 2158 of file specfun.h.

References std::\_\_detail::\_\_beta().

Return the Jacobi elliptic cn(k, u) integral of real modulus k and argument u.

The Jacobi elliptic cn integral is defined by

$$cos(\phi) = cn(k, F(k, \phi))$$

where  $F(k,\phi)$  is the elliptic integral of the first kind.

# **Template Parameters**

_Кр	The type of the real modulus
_Up	The type of the real argument

## **Parameters**

_~	The real modulus	
_k		
_~	The real argument	
_ <i>u</i>		

Definition at line 1867 of file specfun.h.

Return the Jacobi elliptic cn(k,u) integral of float modulus k and argument u.

#### See also

jacobi\_cn for details.

Definition at line 1831 of file specfun.h.

```
8.3.3.161 long double __gnu_cxx::jacobi_cnl( long double __k, long double __u) [inline]
```

Return the Jacobi elliptic cn(k,u) integral of long double modulus k and argument u.

### See also

jacobi\_cn for details.

Definition at line 1844 of file specfun.h.

Return the Jacobi elliptic dn(k,u) integral of real modulus k and argument u.

The Jacobi elliptic dn integral is defined by

$$\sqrt{1 - k^2 \sin(\phi)} = dn(k, F(k, \phi))$$

where  $F(k,\phi)$  is the elliptic integral of the first kind.

# **Template Parameters**

_Кр	The type of the real modulus
_Up	The type of the real argument

### **Parameters**

_← _k	The real modulus
_ <del>←</del>	The real argument

Definition at line 1919 of file specfun.h.

Return the Jacobi elliptic dn(k,u) integral of float modulus k and argument u.

#### See also

jacobi\_dn for details.

Definition at line 1883 of file specfun.h.

Return the Jacobi elliptic dn(k,u) integral of long double modulus k and argument u.

### See also

jacobi\_dn for details.

Definition at line 1896 of file specfun.h.

Return the Jacobi elliptic sn(k, u) integral of real modulus k and argument u.

The Jacobi elliptic sn integral is defined by

$$\sin(\phi) = sn(k, F(k, \phi))$$

where  $F(k, \phi)$  is the elliptic integral of the first kind.

# **Template Parameters**

_Kp	The type of the real modulus
_Up	The type of the real argument

### **Parameters**

_ <del>←</del> _k	The real modulus
_~	The real argument
и	

Definition at line 1815 of file specfun.h.

Return the Jacobi elliptic sn(k,u) integral of float modulus k and argument u.

#### See also

jacobi\_sn for details.

Definition at line 1779 of file specfun.h.

```
8.3.3.167 long double __gnu_cxx::jacobi_snl( long double __k, long double __u ) [inline]
```

Return the Jacobi elliptic sn(k,u) integral of long double modulus k and argument u.

### See also

jacobi\_sn for details.

Definition at line 1792 of file specfun.h.

Return the Jacobi zeta function of k and  $@c\phi$ .

The Jacobi zeta function is defined by

$$Z(m,\phi) = E(m,\phi) - \frac{E(m)F(m,\phi)}{K(m)}$$

where  $E(m,\phi)$  is the elliptic function of the second kind, E(m) is the complete ellitic function of the second kind, and  $F(m,\phi)$  is the elliptic function of the first kind.

# **Template Parameters**

_Tk	the real type of the modulus	
_Tphi the real type of the angle		

# **Parameters**

k	The modulus
phi	The angle

Definition at line 3853 of file specfun.h.

8.3.3.169 float \_gnu\_cxx::jacobi\_zetaf(float \_k, float \_phi) [inline]

Definition at line 3828 of file specfun.h.

```
8.3.3.170 long double __gnu_cxx::jacobi_zetal ( long double __k, long double __phi ) [inline]
```

Definition at line 3832 of file specfun.h.

```
8.3.3.171 float __gnu_cxx::jacobif ( unsigned __n, float __alpha, float __beta, float __x ) [inline]
```

Return the Jacobi polynomial  $P_n^{(\alpha,\beta)}(x)$  of degree n and float orders  $\alpha,\beta>-1$  and argument x.

See also

jacobi for details.

Definition at line 2114 of file specfun.h.

References std::\_\_detail::\_\_beta().

Return the Jacobi polynomial  $P_n^{(\alpha,\beta)}(x)$  of degree n and long double orders  $\alpha,\beta>-1$  and argument x.

See also

jacobi for details.

Definition at line 2125 of file specfun.h.

References std::\_\_detail::\_\_beta().

Definition at line 3705 of file specfun.h.

```
8.3.3.174 float __gnu_cxx::lbincoeff ( unsigned int __n, unsigned int __k ) [inline]
```

Definition at line 3693 of file specfun.h.

```
8.3.3.175 long double __gnu_cxx::lbincoefl ( unsigned int __n, unsigned int __k ) [inline]
```

Definition at line 3697 of file specfun.h.

Definition at line 3663 of file specfun.h.

```
8.3.3.177 float __gnu_cxx::ldouble_factorialf(int __n) [inline]
```

Definition at line 3651 of file specfun.h.

```
8.3.3.178 long double __gnu_cxx::ldouble_factoriall(int __n) [inline]
```

Definition at line 3655 of file specfun.h.

Definition at line 3777 of file specfun.h.

```
8.3.3.180 float __gnu_cxx::legendre_qf( unsigned int __n, float __x ) [inline]
```

Return the Legendre function of the second kind  $Q_l(x)$  for float argument.

See also

legendre\_q for details.

Definition at line 3759 of file specfun.h.

```
8.3.3.181 long double __gnu_cxx::legendre_ql( unsigned int __n, long double __x ) [inline]
```

Return the Legendre function of the second kind  $Q_l(x)$  for long double argument.

See also

legendre\_q for details.

Definition at line 3769 of file specfun.h.

Definition at line 3642 of file specfun.h.

**8.3.3.183** float \_\_gnu\_cxx::lfactorialf ( unsigned int \_\_n ) [inline]

Definition at line 3630 of file specfun.h.

8.3.3.184 long double \_\_gnu\_cxx::lfactoriall( unsigned int \_\_n ) [inline]

Definition at line 3634 of file specfun.h.

Return the logarithmic integral of argument x.

The logarithmic integral is defined by

$$li(x) = \int_0^x \frac{dt}{ln(t)}$$

### **Parameters**

_~	The real upper integration limit
_X	

Definition at line 1594 of file specfun.h.

8.3.3.186 float \_\_gnu\_cxx::logintf(float \_\_x) [inline]

Return the logarithmic integral of argument x.

## See also

logint for details.

Definition at line 1570 of file specfun.h.

**8.3.3.187** long double \_\_gnu\_cxx::logintl( long double \_\_x ) [inline]

Return the logarithmic integral of argument x.

## See also

logint for details.

Definition at line 1579 of file specfun.h.

Definition at line 3537 of file specfun.h.

8.3.3.189 float \_\_gnu\_cxx::lpochhammer\_lf ( float \_\_a, float \_\_n ) [inline]

Definition at line 3525 of file specfun.h.

8.3.3.190 long double \_\_gnu\_cxx::lpochhammer\_ll( long double \_\_a, long double \_\_n ) [inline]

Definition at line 3529 of file specfun.h.

Definition at line 3516 of file specfun.h.

8.3.3.192 float \_\_gnu\_cxx::lpochhammer\_uf(float \_\_a, float \_\_n) [inline]

Definition at line 3504 of file specfun.h.

8.3.3.193 long double \_\_gnu\_cxx::lpochhammer\_ul( long double \_\_a, long double \_\_n) [inline]

Definition at line 3508 of file specfun.h.

Return the Owens T function T(h,a) of shape factor h and integration limit a.

The Owens T function is defined by

$$T(h,a) = \frac{1}{2\pi} \int_0^a \frac{\exp\left[-\frac{1}{2}h^2(1+x^2)\right]}{1+x^2} dx$$

_~	The shape factor	
_h		
_←	The integration limit	
а		

Definition at line 5169 of file specfun.h.

```
8.3.3.195 float __gnu_cxx::owens_tf(float __h, float __a) [inline]
```

Return the Owens T function T(h, a) of shape factor h and integration limit a.

See also

owens\_t for details.

Definition at line 5141 of file specfun.h.

```
8.3.3.196 long double __gnu_cxx::owens_tl( long double __h, long double __a) [inline]
```

Return the Owens T function T(h,a) of long double shape factor h and integration limit a.

See also

owens t for details.

Definition at line 5151 of file specfun.h.

```
8.3.3.197 template<typename _Ta , typename _Tp > __gnu_cxx::__promote_num_t<_Ta, _Tp> __gnu_cxx::pgamma ( _Ta __a, __Tp __x ) [inline]
```

Definition at line 3798 of file specfun.h.

```
8.3.3.198 float __gnu_cxx::pgammaf(float __a, float __x) [inline]
```

Definition at line 3786 of file specfun.h.

```
8.3.3.199 long double __gnu_cxx::pgammal ( long double __a, long double __x ) [inline]
```

Definition at line 3790 of file specfun.h.

Definition at line 3579 of file specfun.h.

```
8.3.3.201 float __gnu_cxx::pochhammer_lf ( float __a, float __n ) [inline]
```

Definition at line 3567 of file specfun.h.

```
8.3.3.202 long double __gnu_cxx::pochhammer_II ( long double __a, long double __n ) [inline]
```

Definition at line 3571 of file specfun.h.

```
8.3.3.203 template<typename _Tp , typename _Tn > __gnu_cxx::__promote_num_t<_Tp, _Tn> __gnu_cxx::pochhammer_u ( _Tp __a, _Tn __n )  [inline]
```

Definition at line 3558 of file specfun.h.

```
8.3.3.204 float __gnu_cxx::pochhammer_uf(float __a, float __n) [inline]
```

Definition at line 3546 of file specfun.h.

```
8.3.3.205 long double __gnu_cxx::pochhammer_ul(long double __a, long double __n) [inline]
```

Definition at line 3550 of file specfun.h.

Return the complex polylogarithm function of real thing  $\,{\rm s}\,$  and complex argument  $\,{\rm w}.$ 

The polylogarithm function is defined by

# Parameters

_~	
_s	
_←	
W	

Definition at line 4443 of file specfun.h.

Return the complex polylogarithm function of real thing s and complex argument w.

The polylogarithm function is defined by

### **Parameters**

_~	
_s	
_←	
_ <i>w</i>	

Definition at line 4483 of file specfun.h.

```
8.3.3.208 float __gnu_cxx::polylogf(float __s, float __w) [inline]
```

Return the real polylogarithm function of real thing  $\ensuremath{\mathtt{s}}$  and real argument  $\ensuremath{\mathtt{w}}.$ 

## See also

polylog for details.

Definition at line 4416 of file specfun.h.

```
8.3.3.209 std::complex < float > \_gnu\_cxx::polylogf ( float \_s, std::complex < float > \_w ) [inline]
```

Return the complex polylogarithm function of real thing  ${\tt s}$  and complex argument  ${\tt w}.$ 

### See also

polylog for details.

Definition at line 4456 of file specfun.h.

```
\textbf{8.3.3.210} \quad \textbf{long double \_gnu\_cxx::polylogl( long double \_s, long double \_w )} \quad \texttt{[inline]}
```

Return the complex polylogarithm function of real thing s and complex argument w.

#### See also

polylog for details.

Definition at line 4426 of file specfun.h.

```
8.3.3.211 std::complex < long double > __gnu_cxx::polylogl ( long double __s, std::complex < long double > __w ) [inline]
```

Return the complex polylogarithm function of real thing  ${\tt s}$  and complex argument  ${\tt w}$ .

## See also

polylog for details.

Definition at line 4466 of file specfun.h.

Return the psi or digamma function of argument x.

The the psi or digamma function is defined by

$$\psi(x) =$$

#### **Parameters**

_~	The parameter
_X	

Definition at line 3271 of file specfun.h.

```
8.3.3.213 float __gnu_cxx::psif(float __x) [inline]
```

Definition at line 3252 of file specfun.h.

```
8.3.3.214 long double __gnu_cxx::psil( long double __x ) [inline]
```

Definition at line 3256 of file specfun.h.

```
8.3.3.215 template<typename _Ta , typename _Tp > __gnu_cxx::__promote_num_t<_Ta, _Tp> __gnu_cxx::qgamma ( _Ta __a, _Tp __x ) [inline]
```

Definition at line 3819 of file specfun.h.

```
8.3.3.216 float __gnu_cxx::qgammaf ( float __a, float __x ) [inline]
```

Definition at line 3807 of file specfun.h.

**8.3.3.217** long double \_\_gnu\_cxx::qgammal(long double \_\_a, long double \_\_x) [inline]

Definition at line 3811 of file specfun.h.

8.3.3.218 template<typename \_Tp > \_\_gnu\_cxx::\_\_promote\_num\_t<\_Tp> \_\_gnu\_cxx::radpoly ( unsigned int \_\_n, unsigned int \_\_n, unsigned int \_\_n, \_Tp \_\_rho ) [inline]

Return the radial polynomial  $R_n^m(\rho)$  for non-negative degree n, order m <= n, and real radial argument  $\rho$ .

The radial polynomials are defined by

$$R_n^m(\rho) = \sum_{k=0}^{\frac{n-m}{2}} \frac{(-1)^k (n-k)!}{k!(\frac{n+m}{2}-k)!(\frac{n-m}{2}-k)!} \rho^{n-2k}$$

for n-m even and identically 0 for n-m odd. The radial polynomials can be related to the Jacobi polynomials:

$$R_n^m(\rho) =$$

#### See also

jacobi for details on the Jacobi polynomials.

## **Template Parameters**

#### **Parameters**

n	The non-negative degree.
m	The non-negative azimuthal order
rho	The radial argument

Definition at line 2316 of file specfun.h.

8.3.3.219 float \_\_gnu\_cxx::radpolyf ( unsigned int \_\_n, unsigned int \_\_m, float \_\_rho ) [inline]

Return the radial polynomial  $R_n^m(\rho)$  for non-negative degree n, order m <= n, and float radial argument  $\rho$ .

#### See also

radpoly for details.

Definition at line 2277 of file specfun.h.

References std:: detail:: poly radial jacobi().

8.3.3.220 long double \_\_gnu\_cxx::radpolyl ( unsigned int \_\_n, unsigned int \_\_n, long double \_\_rho ) [inline]

Return the radial polynomial  $R_n^m(\rho)$  for non-negative degree n, order m <= n, and long double radial argument  $\rho$ .

#### See also

radpoly for details.

Definition at line 2288 of file specfun.h.

References std::\_\_detail::\_\_poly\_radial\_jacobi().

Return the normalized sinus cardinal function sinc(x) for real argument  $\underline{\phantom{a}}$ x. The normalized sinus cardinal function is defined by:

$$sinc(x) = \frac{sin(\pi x)}{\pi x}$$

# **Template Parameters**

real type of the argument	_Тр
---------------------------	-----

#### **Parameters**

_~	The argument
_X	

Definition at line 1556 of file specfun.h.

 $\textbf{8.3.3.222} \quad template < typename \_Tp > \_\_gnu\_cxx::\_promote\_num\_t < \_Tp > \_\_gnu\_cxx::sinc\_pi ( \_Tp \_\_x ) \quad \texttt{[inline]}$ 

Return the sinus cardinal function  $sinc_{\pi}(x)$  for real argument \_\_x. The sinus cardinal function is defined by:

$$sinc_{\pi}(x) = \frac{sin(x)}{x}$$

#### **Template Parameters**

#### **Parameters**

_←	The argument
_X	

Definition at line 1515 of file specfun.h.

```
8.3.3.223 float __gnu_cxx::sinc_pif(float __x) [inline]
```

Return the sinus cardinal function  $sinc_{\pi}(x)$  for float argument \_\_\_x.

See also

sinc\_pi for details.

Definition at line 1489 of file specfun.h.

```
8.3.3.224 long double __gnu_cxx::sinc_pil ( long double __x ) [inline]
```

Return the sinus cardinal function  $sinc_{\pi}(x)$  for long double argument \_\_\_x.

See also

sinc pi for details.

Definition at line 1499 of file specfun.h.

```
8.3.3.225 float __gnu_cxx::sincf(float __x) [inline]
```

Return the normalized sinus cardinal function sinc(x) for float argument  $\underline{\hspace{1cm}}$  x.

See also

sinc for details.

Definition at line 1530 of file specfun.h.

```
8.3.3.226 long double __gnu_cxx::sincl( long double __x ) [inline]
```

Return the normalized sinus cardinal function sinc(x) for long double argument \_\_\_x.

See also

sinc for details.

Definition at line 1540 of file specfun.h.

Definition at line 2358 of file specfun.h.

8.3.3.228 template<typename\_Tp > \_\_gnu\_cxx::\_\_promote\_num\_t<\_Tp> \_\_gnu\_cxx::sinhc\_pi(\_Tp \_\_x) [inline]

Definition at line 2337 of file specfun.h.

8.3.3.229 float \_\_gnu\_cxx::sinhc\_pif( float \_\_x ) [inline]

Definition at line 2325 of file specfun.h.

8.3.3.230 long double \_\_gnu\_cxx::sinhc\_pil( long double \_\_x ) [inline]

Definition at line 2329 of file specfun.h.

8.3.3.231 float \_\_gnu\_cxx::sinhcf(float \_\_x) [inline]

Definition at line 2346 of file specfun.h.

**8.3.3.232** long double \_\_gnu\_cxx::sinhcl( long double \_\_x ) [inline]

Definition at line 2350 of file specfun.h.

 $\textbf{8.3.3.233} \quad template < typename \_Tp > \_\_gnu\_cxx::\_promote\_num\_t < \_Tp > \_\_gnu\_cxx::sinhint ( \_Tp \_\_x ) \quad [\verb"inline"]$ 

Return the hyperbolic sine integral Shi(x) of real argument x.

The hyperbolic sine integral is defined by

$$Shi(x) = \int_0^x \frac{sinh(t)}{t} dt$$

**Template Parameters** 

\_Tp The type of the real argument

# **Parameters**

_~	The argument
_X	

Definition at line 1714 of file specfun.h.

```
8.3.3.234 float __gnu_cxx::sinhintf(float __x) [inline]
```

Return the hyperbolic sine integral of float argument x.

#### See also

sinhint for details.

Definition at line 1687 of file specfun.h.

```
8.3.3.235 long double __gnu_cxx::sinhintl( long double __x ) [inline]
```

Return the hyperbolic sine integral Shi(x) of long double argument x.

#### See also

sinhint for details.

Definition at line 1697 of file specfun.h.

$$\textbf{8.3.3.236} \quad template < typename \_Tp > \_\_gnu\_cxx::\_promote\_num\_t < \_Tp > \_\_gnu\_cxx::sinint ( \_Tp \_x ) \quad \texttt{[inline]}$$

Return the sine integral Si(x) of real argument x.

The sine integral is defined by

$$Si(x) = \int_0^x \frac{\sin(t)}{t} dt$$

#### **Parameters**

_~	The real upper integration limit
_X	

Definition at line 1633 of file specfun.h.

Return the sine integral Si(x) of float argument x.

#### See also

sinint for details.

Definition at line 1608 of file specfun.h.

**8.3.3.238** long double \_\_gnu\_cxx::sinintl( long double \_\_x ) [inline]

Return the sine integral Si(x) of long double argument x.

# See also

sinint for details.

Definition at line 1618 of file specfun.h.

8.3.3.239 template<typename\_Tp > \_\_gnu\_cxx::\_\_promote\_num\_t<\_Tp> \_\_gnu\_cxx::sph\_bessel\_i( unsigned int \_\_n, \_Tp \_\_x ) [inline]

Return the regular modified spherical Bessel function  $i_n(x)$  of nonnegative order n and real argument x >= 0.

The spherical Bessel function is defined by:

$$i_n(x) = \left(\frac{\pi}{2x}\right)^{1/2} I_{n+1/2}(x)$$

# **Template Parameters**

# **Parameters**

_~	The integral order $n >= 0$
_n	
_←	The real argument $x >= 0$
_X	

# **Exceptions**

std::domain_error	ifx < 0	
-------------------	---------	--

Definition at line 2596 of file specfun.h.

8.3.3.240 float \_\_gnu\_cxx::sph\_bessel\_if ( unsigned int \_\_n, float \_\_x ) [inline]

Return the regular modified spherical Bessel function  $i_n(x)$  of nonnegative order n and float argument x>=0.

# See also

sph\_bessel\_i for details.

Definition at line 2557 of file specfun.h.

8.3.3.241 long double \_\_gnu\_cxx::sph\_bessel\_il ( unsigned int \_\_n, long double \_\_x ) [inline]

Return the regular modified spherical Bessel function  $i_n(x)$  of nonnegative order n and long double argument x >= 0.

#### See also

sph\_bessel\_i for details.

Definition at line 2572 of file specfun.h.

8.3.3.242 template < typename \_Tp > \_\_gnu\_cxx::\_\_promote\_num\_t < \_Tp > \_\_gnu\_cxx::sph\_bessel\_k ( unsigned int \_\_n, \_Tp \_\_x ) [inline]

Return the irregular modified spherical Bessel function  $k_n(x)$  of nonnegative order n and real argument x >= 0.

The spherical Bessel function is defined by:

$$k_n(x) = \left(\frac{\pi}{2x}\right)^{1/2} K_{n+1/2}(x)$$

# **Template Parameters**

_Тр	The floating-point type of the argument _	_x.
-----	---	-----

# **Parameters**

_+	ے	The integral order $n >= 0$
_n		
_←	٥	The real argument $x >= 0$
_x		

# **Exceptions**

std::domain_error	if	_X	<	0	
-------------------	----	----	---	---	--

Definition at line 2653 of file specfun.h.

```
8.3.3.243 float __gnu_cxx::sph_bessel_kf( unsigned int __n, float __x ) [inline]
```

Return the irregular modified spherical Bessel function  $k_n(x)$  of nonnegative order n and float argument x>=0.

# See also

sph bessel k for more details.

Definition at line 2614 of file specfun.h.

```
8.3.3.244 long double __gnu_cxx::sph_bessel_kl ( unsigned int __n, long double __x ) [inline]
```

Return the irregular modified spherical Bessel function  $k_n(x)$  of nonnegative order n and long double argument x>=0.

#### See also

sph\_bessel\_k for more details.

Definition at line 2629 of file specfun.h.

Return the spherical Hankel function of the first kind  $h_n^{(1)}(x)$  of nonnegative order n and real argument x >= 0.

The spherical Hankel function of the first kind is defined by:

$$h_n^{(1)}(x) = \left(\frac{\pi}{2x}\right)^{1/2} H_{n+1/2}^{(1)}(x)$$

# **Template Parameters**

The real type of the argument	_Тр
-------------------------------	-----

#### **Parameters**

_~	The non-negative order
_n	
_←	The real argument
_Z	

Definition at line 2499 of file specfun.h.

```
8.3.3.246 template<typename _Tp > std::complex<__gnu_cxx::__promote_num_t<_Tp> > __gnu_cxx::sph_hankel_1 ( unsigned int __n, std::complex< _Tp > __x ) [inline]
```

Return the complex spherical Hankel function of the first kind  $h_n^{(1)}(x)$  of non-negative integral n and complex argument x

The spherical Hankel function of the first kind is defined by

$$h_n^{(1)}(x) = \left(\frac{\pi}{2x}\right)^{1/2} H_{n+1/2}^{(1)}(x) = j_n(x) + in_n(x)$$

where  $j_n(x)$  and  $n_n(x)$  are the spherical Bessel and Neumann functions respectively.

#### **Parameters**

_~	The integral order >= 0
_n	
_←	The complex argument
_x	

Definition at line 4302 of file specfun.h.

```
8.3.3.247 std::complex < float > __gnu_cxx::sph_hankel_1f ( unsigned int __n, float __z ) [inline]
```

Return the spherical Hankel function of the first kind  $h_n^{(1)}(x)$  of nonnegative order n and float argument x>=0.

#### See also

sph\_hankel\_1 for details.

Definition at line 2471 of file specfun.h.

```
8.3.3.248 std::complex < float > __gnu_cxx::sph_hankel_1f ( unsigned int __n, std::complex < float > __x ) [inline]
```

Return the complex spherical Hankel function of the first kind  $h_n^{(1)}(x)$  of non-negative integral n and  $std \leftarrow ::complex < float > argument <math>x$ .

# See also

sph\_hankel\_1 for more details.

Definition at line 4270 of file specfun.h.

8.3.3.249 std::complex < long double > \_\_gnu\_cxx::sph\_hankel\_II ( unsigned int \_\_n, long double \_\_z ) [inline]

Return the spherical Hankel function of the first kind  $h_n^{(1)}(x)$  of nonnegative order n and long double argument x>=0.

#### See also

sph hankel 1 for details.

Definition at line 2481 of file specfun.h.

8.3.3.250 std::complex < long double > 
$$\_$$
gnu\_cxx::sph\_hankel\_1I ( unsigned int  $\_$ n, std::complex < long double >  $\_$ x ) 
$$[inline]$$

Return the complex spherical Hankel function of the first kind  $h_n^{(1)}(x)$  of non-negative integral n and  $std \leftarrow ::complex < long double > argument <math>x$ .

# See also

sph hankel 1 for more details.

Definition at line 4281 of file specfun.h.

Return the spherical Hankel function of the second kind  $h_n^{(2)}(x)$  of nonnegative order n and real argument x >= 0.

The spherical Hankel function of the second kind is defined by:

$$h_n^{(2)}(x) = \left(\frac{\pi}{2x}\right)^{1/2} H_{n+1/2}^{(2)}(x)$$

# **Template Parameters**

_Тр	The real type of the argument
-----	-------------------------------

#### **Parameters**

_~	The non-negative order
_n	
_~	The real argument
_Z	

Definition at line 2542 of file specfun.h.

8.3.3.252 template<typename \_Tp > std::complex<\_\_gnu\_cxx::\_\_promote\_num\_t<\_Tp> > \_\_gnu\_cxx::sph\_hankel\_2 ( unsigned int \_\_n, std::complex< \_Tp > \_\_x ) [inline]

Return the complex spherical Hankel function of the second kind  $h_n^{(2)}(x)$  of nonnegative order n and complex argument x.

The spherical Hankel function of the second kind is defined by

$$h_n^{(2)}(x) = \left(\frac{\pi}{2x}\right)^{1/2} H_{n+1/2}^{(2)}(x) = j_n(x) - in_n(x)$$

where  $j_n(x)$  and  $n_n(x)$  are the spherical Bessel and Neumann functions respectively.

#### **Parameters**

_←	The integral order >= 0
_n	
_~	The complex argument
_X	

Definition at line 4350 of file specfun.h.

8.3.3.253 std::complex < float > \_\_gnu\_cxx::sph\_hankel\_2f( unsigned int \_\_n, float \_\_z ) [inline]

Return the spherical Hankel function of the second kind  $h_n^{(2)}(x)$  of nonnegative order n and float argument x>=0.

### See also

sph\_hankel\_2 for details.

Definition at line 2514 of file specfun.h.

8.3.3.254 std::complex<float> \_\_gnu\_cxx::sph\_hankel\_2f ( unsigned int \_\_n, std::complex< float > \_\_x ) [inline]

Return the complex spherical Hankel function of the second kind  $h_n^{(2)}(x)$  of non-negative integral n and  $std\leftarrow::complex<float>$  argument x.

#### See also

sph hankel 2 for more details.

Definition at line 4318 of file specfun.h.

8.3.3.255 std::complex < long double > \_\_gnu\_cxx::sph\_hankel\_2I ( unsigned int \_\_n, long double \_\_z ) [inline]

Return the spherical Hankel function of the second kind  $h_n^{(2)}(x)$  of nonnegative order n and long double argument x >= 0.

#### See also

sph hankel 2 for details.

Definition at line 2524 of file specfun.h.

8.3.3.256 std::complex < long double > 
$$\_$$
gnu\_cxx::sph\_hankel\_2I ( unsigned int  $\_$ n, std::complex < long double >  $\_$ x ) [inline]

Return the complex spherical Hankel function of the second kind  $h_n^{(2)}(x)$  of non-negative integral n and  $std \leftarrow ::complex < long double > argument <math>x$ .

#### See also

sph hankel 2 for more details.

Definition at line 4329 of file specfun.h.

Return the complex spherical harmonic function of degree 1, order m, and real zenith angle  $\theta$ , and azimuth angle  $\phi$ .

The spherical harmonic function is defined by:

$$Y_l^m(\theta,\phi) = (-1)^m \left[ \frac{(2l+1)}{4\pi} \frac{(l-m)!}{(l+m)!} \right] P_l^{|m|}(\cos\theta) \exp^{im\phi}$$

#### **Parameters**

/	The order
m	The degree
theta	The zenith angle in radians
phi	The azimuth angle in radians

Definition at line 4401 of file specfun.h.

8.3.3.258 std::complex<float> \_\_gnu\_cxx::sph\_harmonicf( unsigned int \_\_l, int \_\_m, float \_\_theta, float \_\_phi ) [inline]

Return the complex spherical harmonic function of degree 1, order m, and float zenith angle  $\theta$ , and azimuth angle  $\phi$ .

See also

sph harmonic for details.

Definition at line 4365 of file specfun.h.

Return the complex spherical harmonic function of degree 1, order m, and long double zenith angle  $\theta$ , and azimuth angle  $\phi$ .

See also

sph\_harmonic for details.

Definition at line 4377 of file specfun.h.

Return the exponential theta-1 function  $\theta_1(\nu,x)$  of period nu and argument x.

The Neville theta-1 function is defined by

$$\theta_1(\nu, x) = \frac{1}{\sqrt{\pi x}} \sum_{j=-\infty}^{+\infty} (-1)^j \exp\left(\frac{-(\nu + j - 1/2)^2}{x}\right)$$

#### **Parameters**

nu	The periodic (period = 2) argument
x	The argument

Definition at line 4787 of file specfun.h.

Return the exponential theta-1 function  $\theta_1(\nu,x)$  of period nu and argument x.

See also

theta\_1 for details.

Definition at line 4759 of file specfun.h.

```
8.3.3.262 long double __gnu_cxx::theta_1I ( long double __nu, long double __x ) [inline]
```

Return the exponential theta-1 function  $\theta_1(\nu,x)$  of period nu and argument x.

See also

theta\_1 for details.

Definition at line 4769 of file specfun.h.

Return the exponential theta-2 function  $\theta_2(\nu,x)$  of period nu and argument x.

The exponential theta-2 function is defined by

$$\theta_2(\nu, x) = \frac{1}{\sqrt{\pi x}} \sum_{j=-\infty}^{+\infty} (-1)^j \exp\left(\frac{-(\nu+j)^2}{x}\right)$$

#### **Parameters**

nu	The periodic (period = 2) argument
x	The argument

Definition at line 4830 of file specfun.h.

Return the exponential theta-2 function  $\theta_2(\nu,x)$  of period nu and argument x.

See also

theta 2 for details.

Definition at line 4802 of file specfun.h.

```
8.3.3.265 long double __gnu_cxx::theta_2l ( long double __nu, long double __x ) [inline]
```

Return the exponential theta-2 function  $\theta_2(\nu, x)$  of period nu and argument x.

See also

theta\_2 for details.

Definition at line 4812 of file specfun.h.

Return the exponential theta-3 function  $\theta_3(\nu, x)$  of period nu and argument x.

The exponential theta-3 function is defined by

$$\theta_3(\nu, x) = \frac{1}{\sqrt{\pi x}} \sum_{j=-\infty}^{+\infty} \exp\left(\frac{-(\nu+j)^2}{x}\right)$$

#### **Parameters**

nu	The periodic (period = 1) argument
x	The argument

Definition at line 4873 of file specfun.h.

8.3.3.267 float \_\_gnu\_cxx::theta\_3f (float \_\_nu, float \_\_x ) [inline]

Return the exponential theta-3 function  $\theta_3(\nu, x)$  of period nu and argument x.

#### See also

theta 3 for details.

Definition at line 4845 of file specfun.h.

8.3.3.268 long double \_\_gnu\_cxx::theta\_3l ( long double \_\_nu, long double \_\_x ) [inline]

Return the exponential theta-3 function  $\theta_3(\nu, x)$  of period nu and argument x.

#### See also

theta 3 for details.

Definition at line 4855 of file specfun.h.

Return the exponential theta-4 function  $\theta_4(\nu,x)$  of period nu and argument x.

The exponential theta-4 function is defined by

$$\theta_4(\nu, x) = \frac{1}{\sqrt{\pi x}} \sum_{i=-\infty}^{+\infty} \exp\left(\frac{-(\nu + j + 1/2)^2}{x}\right)$$

#### **Parameters**

nu	The periodic (period = 1) argument
x	The argument

Definition at line 4916 of file specfun.h.

```
8.3.3.270 float __gnu_cxx::theta_4f ( float __nu, float __x ) [inline]
```

Return the exponential theta-4 function  $\theta_4(\nu,x)$  of period nu and argument x.

See also

theta\_4 for details.

Definition at line 4888 of file specfun.h.

```
8.3.3.271 long double __gnu_cxx::theta_4l ( long double __nu, long double __x ) [inline]
```

Return the exponential theta-4 function  $\theta_4(\nu,x)$  of period nu and argument x.

See also

theta 4 for details.

Definition at line 4898 of file specfun.h.

Return the Neville theta-c function  $\theta_c(k,x)$  of modulus k and argument x.

The Neville theta-c function is defined by

### **Parameters**

$\leftarrow$	The modulus $-1 \le k \le +1$
_k	
_~	The argument
_x	

Definition at line 5042 of file specfun.h.

```
8.3.3.273 float __gnu_cxx::theta_cf(float __k, float __x) [inline]
```

Return the Neville theta-c function  $\theta_c(k,x)$  of modulus k and argument x.

# See also

theta\_c for details.

Definition at line 5015 of file specfun.h.

Return the Neville theta-c function  $\theta_c(k,x)$  of long double modulus k and argument x.

#### See also

theta\_c for details.

Definition at line 5025 of file specfun.h.

Return the Neville theta-d function  $\theta_d(k,x)$  of modulus k and argument x.

The Neville theta-d function is defined by

$$\theta_d(k,x) =$$

#### **Parameters**

_~	The modulus $-1 <= k <= +1$
_k	
_~	The argument
_X	

Definition at line 5084 of file specfun.h.

Return the Neville theta-d function  $\theta_d(k,x)$  of modulus k and argument x.

#### See also

theta d for details.

Definition at line 5057 of file specfun.h.

```
8.3.3.277 long double __gnu_cxx::theta_dl( long double __k, long double __x ) [inline]
```

Return the Neville theta-d function  $\theta_d(k,x)$  of long double modulus k and argument x.

See also

theta\_d for details.

Definition at line 5067 of file specfun.h.

Return the Neville theta-n function  $\theta_n(k,x)$  of modulus k and argument x.

The Neville theta-n function is defined by

$$\theta_n(k,x) =$$

#### **Parameters**

_~	The modulus $-1 <= k <= +1$
_k	
_←	The argument
_x	

Definition at line 5126 of file specfun.h.

```
8.3.3.279 float __gnu_cxx::theta_nf(float __k, float __x) [inline]
```

Return the Neville theta-n function  $\theta_n(k,x)$  of modulus k and argument x.

See also

theta n for details.

Definition at line 5099 of file specfun.h.

```
8.3.3.280 long double __gnu_cxx::theta_nl( long double __k, long double __x ) [inline]
```

Return the Neville theta-n function  $\theta_n(k,x)$  of long double modulus k and argument x.

See also

theta n for details.

Definition at line 5109 of file specfun.h.

8.3.3.281 template<typename \_Tpk , typename \_Tp > \_\_gnu\_cxx::\_\_promote\_num\_t<\_Tpk, \_Tp > \_\_gnu\_cxx::theta\_s ( \_Tpk \_\_k, \_Tp \_\_x ) [inline]

Return the Neville theta-s function  $\theta_s(k,x)$  of modulus k and argument x.

The Neville theta-s function is defined by

#### **Parameters**

_← _k	The modulus $-1 <= k <= +1$
_← _x	The argument

Definition at line 5000 of file specfun.h.

Return the Neville theta-s function  $\theta_s(k,x)$  of modulus k and argument x.

See also

theta\_s for details.

Definition at line 4973 of file specfun.h.

```
8.3.3.283 long double __gnu_cxx::theta_sl( long double __k, long double __x ) [inline]
```

Return the Neville theta-s function  $\theta_s(k,x)$  of long double modulus k and argument x.

See also

theta s for details.

Definition at line 4983 of file specfun.h.

Return the Zernicke polynomial  $Z_n^m(\rho,\phi)$  for non-negative degree n, signed order m, and real radial argument  $\rho$  and azimuthal angle  $\phi$ .

The even Zernicke polynomials are defined by:

$$Z_n^m(\rho,\phi) = R_n^m(\rho)\cos(m\phi)$$

and the odd Zernicke polynomials are defined by:

$$Z_n^{-m}(\rho,\phi) = R_n^m(\rho)\sin(m\phi)$$

for non-negative degree m and m <= n and where  $R_n^m(\rho)$  is the radial polynomial (

See also

radpoly).

# **Template Parameters**

_Trho	The real type of the radial coordinate
_Tphi	The real type of the azimuthal angle

#### **Parameters**

n	The non-negative degree.
m	The (signed) azimuthal order
rho	The radial coordinate
phi	The azimuthal angle

Definition at line 2261 of file specfun.h.

```
8.3.3.285 float _gnu_cxx::zernikef ( unsigned int _n, int _m, float _rho, float _phi ) [inline]
```

Return the Zernicke polynomial  $Z_n^m(\rho,\phi)$  for non-negative degree n, signed order m, and real radial argument  $\rho$  and azimuthal angle  $\phi$ .

# See also

zernike for details.

Definition at line 2222 of file specfun.h.

```
8.3.3.286 long double __gnu_cxx::zernikel ( unsigned int __n, int __m, long double __rho, long double __phi ) [inline]
```

Return the Zernicke polynomial  $Z_n^m(\rho,\phi)$  for non-negative degree n, signed order m, and real radial argument  $\rho$  and azimuthal angle  $\phi$ .

### See also

zernike for details.

Definition at line 2233 of file specfun.h.

# **Chapter 9**

# **Namespace Documentation**

# 9.1 \_\_gnu\_cxx Namespace Reference

# **Enumerations**

• enum { \_GLIBCXX\_JACOBI\_SN, \_GLIBCXX\_JACOBI\_CN, \_GLIBCXX\_JACOBI\_DN }

# **Functions**

```
template<typename _Tp >
  __gnu_cxx::__promote_num_t< _Tp > airy_ai (_Tp __x)

 float airy_aif (float __x)

    long double airy_ail (long double __x)

template<typename _Tp >
   \_gnu_cxx::\_promote_num_t< \_Tp > airy_bi (\_Tp \_\_x)

 float airy_bif (float __x)

    long double airy_bil (long double __x)

template<typename _Tp >
  __gnu_cxx::__promote_num_t< _Tp > bernoulli (unsigned int __n)
• float bernoullif (unsigned int __n)
• long double bernoullil (unsigned int __n)
template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > bincoef (unsigned int __n, unsigned int __k)

    float bincoeff (unsigned int __n, unsigned int __k)

• long double bincoefl (unsigned int __n, unsigned int __k)
template<typename_Tp>
    _gnu_cxx::__promote_num_t< _Tp > chebyshev_t (unsigned int __n, _Tp __x)

    float chebyshev_tf (unsigned int __n, float __x)

• long double <a href="mailto:chebyshev_tl">chebyshev_tl</a> (unsigned int __n, long double __x)
template<typename _Tp >
  __gnu_cxx::__promote_num_t< _Tp > chebyshev_u (unsigned int __n, _Tp __x)

    float chebyshev uf (unsigned int n, float x)

    long double chebyshev_ul (unsigned int __n, long double __x)
```

```
template<typename _Tp >
    gnu cxx:: promote num t< Tp> chebyshev v (unsigned int n, Tpx)

    float chebyshev vf (unsigned int n, float x)

    long double chebyshev vl (unsigned int n, long double x)

template<typename</li>Tp >
    _gnu_cxx::__promote_num_t< _Tp > chebyshev_w (unsigned int __n, _Tp __x)

    float chebyshev_wf (unsigned int __n, float __x)

• long double <a href="mailto:chebyshev_wl">chebyshev_wl</a> (unsigned int __n, long double __x)
template<typename_Tp>
    gnu cxx:: promote num t< Tp> clausen (unsigned int m, Tp w)

    template<typename</li>
    Tp >

  std::complex< __gnu_cxx::__promote_num_t< _Tp >> clausen (unsigned int __m, std::complex< _Tp > __w)
template<typename_Tp>
    _gnu_cxx::__promote_num_t< _Tp > clausen_c (unsigned int __m, _Tp __w)

    float clausen cf (unsigned int m, float w)

    long double clausen_cl (unsigned int __m, long double __w)

template<typename_Tp>
    _gnu_cxx::__promote_num_t< _Tp > clausen_s (unsigned int __m, _Tp __w)

    float clausen_sf (unsigned int __m, float __w)

    long double clausen sl (unsigned int m, long double w)

    float clausenf (unsigned int m, float w)

• std::complex< float > clausenf (unsigned int m, std::complex< float > w)

    long double clausenl (unsigned int __m, long double __w)

    std::complex < long double > clausenl (unsigned int m, std::complex < long double > w)

    template<typename Tk >

    _gnu_cxx::__promote_num_t< _Tk > comp_ellint_d (_Tk __k)

    float comp ellint df (float k)

    long double comp_ellint_dl (long double __k)

• float comp ellint rf (float x, float y)

    long double comp_ellint_rf (long double __x, long double __y)

• template<typename _Tx , typename _Ty >
    _gnu_cxx::__promote_num_t< _Tx, _Ty > comp_ellint_rf (_Tx __x, _Ty __y)

    float comp ellint rg (float x, float y)

    long double comp_ellint_rg (long double __x, long double __y)

    template<typename _Tx , typename _Ty >

    _gnu_cxx::__promote_num_t< _Tx, _Ty > comp_ellint_rg (_Tx __x, _Ty __y)
- template<typename _Tpa , typename _Tpc , typename _Tp >
   _gnu_cxx::__promote_3< _Tpa, _Tpc, _Tp >::__type conf_hyperg (_Tpa __a, _Tpc __c, _Tp __x)

    template<typename _Tpc , typename _Tp >

   _gnu_cxx::__promote_2< _Tpc, _Tp >::__type conf_hyperg_lim (_Tpc __c, _Tp __x)

    float conf_hyperg_limf (float __c, float __x)

    long double conf_hyperg_liml (long double __c, long double __x)

    float conf_hypergf (float __a, float __c, float __x)

    long double conf_hypergl (long double __a, long double __c, long double __x)

    template<typename</li>
    Tp >

    _gnu_cxx::__promote_num_t< _Tp > coshint (_Tp __x)

    float coshintf (float x)

    long double coshintl (long double x)

template<typename _Tp >
    gnu_cxx::__promote_num_t< _Tp > cosint (_Tp __x)

    float cosintf (float x)
```

```
    long double cosintl (long double __x)

• template<typename _Tpnu , typename _Tp >
  std::complex< __gnu_cxx::__promote_num_t< _Tpnu, _Tp >> cyl_hankel_1 (_Tpnu __nu, _Tp __z)
• template<typename _Tpnu , typename _Tp >
  std::complex< __gnu_cxx::__promote_num_t< _Tpnu, _Tp >> cyl_hankel_1 (std::complex< _Tpnu > __nu,
  std::complex < Tp > x)

    std::complex< float > cyl_hankel_1f (float __nu, float __z)

    std::complex < float > cyl_hankel_1f (std::complex < float > __nu, std::complex < float > __x)

    std::complex < long double > cyl_hankel_11 (long double __nu, long double __z)

    std::complex < long double > cyl_hankel_1l (std::complex < long double > __nu, std::complex < long double >

   __x)

    template<typename _Tpnu , typename _Tp >

  std::complex < __gnu_cxx::__promote_num_t < _Tpnu, _Tp >> cyl_hankel_2 (_Tpnu __nu, _Tp __z)
• template<typename _Tpnu , typename _Tp >
  std::complex< __gnu_cxx::__promote_num_t< _Tpnu, _Tp >> cyl_hankel_2 (std::complex< _Tpnu > __nu,
  std::complex < _Tp > __x)

    std::complex< float > cyl hankel 2f (float nu, float z)

    std::complex < float > cyl hankel 2f (std::complex < float > nu, std::complex < float > x)

    std::complex < long double > cyl_hankel_2l (long double __nu, long double __z)

    std::complex < long double > cyl hankel 2l (std::complex < long double > nu, std::complex < long double >

   x)
template<typename _Tp >
   _gnu_cxx::__promote_num_t< _Tp > dawson (_Tp __x)

    float dawsonf (float x)

    long double dawsonl (long double __x)

template<typename</li>Tp >
   __gnu_cxx::__promote_num_t< _Tp > digamma (_Tp __z)

    float digammaf (float z)

    long double digammal (long double __z)

template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > dilog (_Tp __x)

 float dilogf (float __x)

    long double dilogl (long double __x)

template<typename _Tp >
  Tp dirichlet beta (Tp s)

    float dirichlet betaf (float s)

    long double dirichlet betal (long double s)

template<typename _Tp >
  Tp dirichlet eta (Tp s)

    float dirichlet_etaf (float __s)

    long double dirichlet etal (long double s)

template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > double_factorial (int n)

    float double factorialf (int n)

    long double double factoriall (int n)

ullet template<typename _Tk , typename _Tp , typename _Ta , typename _Tb >
    _gnu_cxx::__promote_num_t< _Tk, _Tp, _Ta, _Tb > ellint_cel (_Tk __k_c, _Tp __p, _Ta __a, _Tb __b)

    float ellint celf (float k c, float p, float a, float b)

    long double ellint cell (long double k c, long double p, long double a, long double b)

• template<typename _Tk , typename _Tphi >
    _gnu_cxx::__promote_num_t< _Tk, _Tphi > ellint_d (_Tk __k, _Tphi __phi)

    float ellint df (float k, float phi)
```

```
    long double ellint_dl (long double ___k, long double ___phi)

• template<typename _{\rm Tp} , typename _{\rm Tk} >
    _gnu_cxx::__promote_num_t< _Tp, _Tk > ellint_el1 (_Tp __x, _Tk __k_c)

    float ellint el1f (float x, float k c)

    long double ellint el11 (long double x, long double k c)

    template<typename _Tp , typename _Tk , typename _Ta , typename _Tb >

    _gnu_cxx::__promote_num_t< _Tp, _Tk, _Ta, _Tb > ellint_el2 (_Tp __x, _Tk __k_c, _Ta __a, _Tb __b)

    float ellint_el2f (float __x, float __k_c, float __a, float __b)

    long double ellint_el2l (long double __x, long double __k_c, long double __a, long double __b)

• template<typename _Tx , typename _Tk , typename _Tp >
    _gnu_cxx::__promote_num_t< _Tx, _Tk, _Tp > ellint_el3 (_Tx __x, _Tk __k_c, _Tp __p)

    float ellint el3f (float x, float k c, float p)

    long double ellint_el3l (long double __x, long double __k_c, long double __p)

    template<typename _Tp , typename _Up >

    _gnu_cxx::__promote_num_t< _Tp, _Up > ellint_rc (_Tp __x, _Up __y)

    float ellint_rcf (float __x, float __y)

    long double ellint rcl (long double x, long double y)

• template<typename _Tp , typename _Up , typename _Vp >
    _gnu_cxx::__promote_num_t< _Tp, _Up, _Vp > ellint_rd (_Tp __x, _Up __y, _Vp __z)
• float ellint rdf (float x, float y, float z)

    long double ellint_rdl (long double __x, long double __y, long double __z)

ullet template<typename _Tp , typename _Up , typename _Vp >
    _gnu_cxx::__promote_num_t< _Tp, _Up, _Vp > ellint_rf (_Tp __x, _Up __y, _Vp __z)

    float ellint_rff (float __x, float __y, float __z)

    long double ellint_rfl (long double __x, long double __y, long double __z)

template<typename _Tp , typename _Up , typename _Vp >
    _gnu_cxx::__promote_num_t< _Tp, _Up, _Vp > ellint_rg (_Tp __x, _Up __y, _Vp __z)

    float ellint_rgf (float __x, float __y, float __z)

• long double ellint rgl (long double x, long double y, long double z)
ullet template<typename _Tp , typename _Up , typename _Vp , typename _Wp >
    _gnu_cxx::__promote_num_t< _Tp, _Up, _Vp, _Wp > ellint_rj (_Tp __x, _Up __y, _Vp __z, _Wp __p)

    float ellint_rjf (float __x, float __y, float __z, float __p)

    long double ellint rjl (long double x, long double y, long double z, long double p)

template<typename _Tp >
  _Tp ellnome (_Tp __k)

    float ellnomef (float k)

• long double ellnomel (long double __k)
template<typename _Tp >
    gnu cxx:: promote num t < Tp > expint (unsigned int n, Tp x)

    float expintf (unsigned int __n, float __x)

    long double expintl (unsigned int n, long double x)

template<typename</li>Tp >
    _gnu_cxx::__promote_num_t< _Tp > factorial (unsigned int __n)

    float factorialf (unsigned int n)

• long double factoriall (unsigned int __n)
template<typename</li>Tp >
   __gnu_cxx::__promote_num_t< _Tp > fresnel_c (_Tp __x)

    float fresnel cf (float x)

    long double fresnel_cl (long double __x)

template<typename _Tp >
  __gnu_cxx::__promote_num_t< _Tp > fresnel_s (_Tp __x)
```

```
 float fresnel_sf (float __x)

    long double fresnel sl (long double x)

• template<typename _{\rm Tn}, typename _{\rm Tp} >
    _gnu_cxx::__promote_num_t< _Tn, _Tp > gamma_l (_Tn __n, _Tp __x)

    float gamma If (float n, float x)

    long double gamma_II (long double __n, long double __x)

    template<typename Tn , typename Tp >

    _gnu_cxx::__promote_num_t< _Tn, _Tp > gamma_u (_Tn __n, _Tp __x)

    float gamma_uf (float __n, float __x)

    long double gamma ul (long double n, long double x)

• template<typename _Talpha , typename _Tp >
    gnu cxx:: promote num t< Talpha, Tp > gegenbauer (unsigned int n, Talpha alpha, Tp x)

    float gegenbauerf (unsigned int __n, float __alpha, float __x)

    long double gegenbauerl (unsigned int n, long double alpha, long double x)

    template<typename _Tk , typename _Tphi >

   _gnu_cxx::__promote_num_t< _Tk, _Tphi > heuman_lambda (_Tk __k, _Tphi __phi)

    float heuman_lambdaf (float __k, float __phi)

    long double heuman_lambdal (long double __k, long double __phi)

• template<typename _Tp , typename _Up >
    _gnu_cxx::__promote_num_t< _Tp, _Up > hurwitz_zeta (_Tp __s, _Up __a)
• template<typename _Tp , typename _Up >
  std::complex< _Tp > hurwitz_zeta (_Tp __s, std::complex< _Up > __a)

    float hurwitz zetaf (float s, float a)

    long double hurwitz_zetal (long double __s, long double __a)

    template<typename _Tpa , typename _Tpb , typename _Tpc , typename _Tp >

    _gnu_cxx::__promote_4< _Tpa, _Tpb, _Tpc, _Tp >::__type hyperg (_Tpa __a, _Tpb __b, _Tpc __c, _Tp __x)

    float hypergf (float __a, float __b, float __c, float __x)

    long double hypergl (long double __a, long double __b, long double __c, long double __x)

template<typename _Ta , typename _Tb , typename _Tp >
   _gnu_cxx::__promote_num_t< _Ta, _Tb, _Tp > ibeta (_Ta __a, _Tb __b, _Tp __x)
- template<typename _Ta , typename _Tb , typename _Tp >
    gnu cxx:: promote num t< Ta, Tb, Tp> ibetac ( Ta a, Tb b, Tp x)

    float ibetacf (float __a, float __b, float __x)

    long double ibetacl (long double __a, long double __b, long double __x)

 float ibetaf (float __a, float __b, float __x)

    long double ibetal (long double __a, long double __b, long double __x)

    template<typename _Talpha , typename _Tbeta , typename _Tp >

    _gnu_cxx::__promote_num_t< _Talpha, _Tbeta, _Tp > jacobi (unsigned __n, _Talpha __alpha, _Tbeta __beta,
  _Tp __x)
• template<typename _Kp , typename _Up >
    _gnu_cxx::__promote_num_t< _Kp, _Up > jacobi_cn (_Kp __k, _Up __u)

    float jacobi_cnf (float __k, float __u)

    long double jacobi cnl (long double k, long double u)

    template<typename Kp, typename Up >

    _gnu_cxx::__promote_num_t< _Kp, _Up > jacobi_dn (_Kp __k, _Up __u)
• float jacobi dnf (float k, float u)

    long double jacobi dnl (long double k, long double u)

    template<typename _Kp , typename _Up >

   _gnu_cxx::__promote_num_t< _Kp, _Up > jacobi_sn (_Kp __k, _Up __u)

    float jacobi_snf (float __k, float __u)

    long double jacobi snl (long double k, long double u)
```

```
    template<typename _Tk , typename _Tphi >

    gnu cxx:: promote num t< Tk, Tphi > jacobi zeta (Tk k, Tphi phi)

    float jacobi zetaf (float k, float phi)

    long double jacobi_zetal (long double __k, long double __phi)

    float jacobif (unsigned n, float alpha, float beta, float x)

    long double jacobil (unsigned __n, long double __alpha, long double __beta, long double __x)

template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > lbincoef (unsigned int __n, unsigned int __k)

    float lbincoeff (unsigned int n, unsigned int k)

    long double lbincoefl (unsigned int n, unsigned int k)

template<typename _Tp >
    gnu cxx:: promote num t < Tp > ldouble factorial (int n)

    float Idouble factorialf (int n)

• long double Idouble factoriall (int n)
template<typename _Tp >
    gnu cxx:: promote num t< Tp> legendre q (unsigned int n, Tp x)
• float legendre qf (unsigned int n, float x)

    long double legendre_ql (unsigned int __n, long double __x)

template<typename _Tp >
    gnu cxx:: promote num t< Tp> Ifactorial (unsigned int n)

    float Ifactorialf (unsigned int n)

    long double lfactoriall (unsigned int __n)

template<typename _Tp >
    gnu cxx:: promote num t < Tp > logint (Tp x)

    float logintf (float x)

    long double logintl (long double x)

• template<typename _Tp , typename _Tn >
    gnu cxx:: promote num t< Tp, Tn> lpochhammer I (Tp a, Tn n)

    float lpochhammer_lf (float __a, float __n)

    long double lpochhammer II (long double a, long double n)

• template<typename _Tp , typename _Tn >
    gnu cxx:: promote num t< Tp, Tn> lpochhammer u (Tp a, Tn n)

    float lpochhammer uf (float a, float n)

    long double lpochhammer ul (long double a, long double n)

• template<typename _Tph , typename _Tpa >
    _gnu_cxx::__promote_num_t< _Tph, _Tpa > owens_t (_Tph __h, _Tpa __a)

    float owens tf (float h, float a)

    long double owens tl (long double h, long double a)

• template<typename _Ta , typename _Tp >
    gnu cxx:: promote num t< Ta, Tp> pgamma ( Ta a, Tp x)

    float pgammaf (float a, float x)

• long double pgammal (long double a, long double x)
• template<typename _Tp , typename _Tn >
    _gnu_cxx::__promote_num_t< _Tp, _Tn > pochhammer_l (_Tp __a, _Tn __n)

    float pochhammer If (float a, float n)

• long double pochhammer_ll (long double __a, long double __n)

    template<typename _Tp , typename _Tn >

    gnu cxx:: promote num t< Tp, Tn> pochhammer u (Tp a, Tn n)

    float pochhammer uf (float a, float n)

    long double pochhammer_ul (long double __a, long double __n)

template<typename _Tp , typename _Wp >
   __gnu_cxx::__promote_num_t< _Tp, _Wp > polylog (_Tp __s, _Wp __w)
```

```
template<typename _Tp , typename _Wp >
  std::complex< \underline{\quad} gnu\_cxx::\underline{\quad} promote\_num\_t<\underline{\quad} Tp,\underline{\quad} Wp>>\underline{\quad} polylog\;(\underline{\quad} Tp\;\underline{\quad} s,\;std::complex<\underline{\quad} Tp>\underline{\quad} w)

    float polylogf (float s, float w)

    std::complex< float > polylogf (float __s, std::complex< float > __w)

    long double polylogi (long double s, long double w)

    std::complex < long double > polylogl (long double ___s, std::complex < long double > __w)

• template<typename _{\rm Tp}>
    _gnu_cxx::__promote_num_t< _Tp > psi (_Tp __x)

    float psif (float x)

    long double psil (long double __x)

• template<typename _Ta , typename _Tp >

    float qgammaf (float a, float x)

    long double qgammal (long double __a, long double __x)

template<typename _Tp >
    gnu cxx:: promote num t< Tp> radpoly (unsigned int n, unsigned int m, Tp rho)
• float radpolyf (unsigned int __n, unsigned int __m, float __rho)

    long double radpolyl (unsigned int __n, unsigned int __m, long double __rho)

template<typename_Tp>
    _gnu_cxx::__promote_num_t< _Tp > sinc (_Tp __x)
template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > sinc_pi (_Tp __x)

    float sinc pif (float x)

    long double sinc pil (long double x)

 float sincf (float __x)

    long double sincl (long double x)

template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > sinhc (_Tp __x)
template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > sinhc_pi (_Tp __x)

    float sinhc pif (float x)

    long double sinhc pil (long double x)

    float sinhcf (float x)

    long double sinhcl (long double x)

template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > sinhint (_Tp __x)

    float sinhintf (float x)

    long double sinhintl (long double x)

template<typename _Tp >
    gnu cxx:: promote num t < Tp > sinint (Tp x)

    float sinintf (float x)

    long double sinintl (long double __x)

template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > sph_bessel_i (unsigned int __n, _Tp __x)

    float sph bessel if (unsigned int n, float x)

    long double sph_bessel_il (unsigned int __n, long double __x)

template<typename _Tp >
    gnu cxx:: promote num t< Tp>sph bessel k (unsigned int n, Tpx)

    float sph bessel kf (unsigned int n, float x)

    long double sph_bessel_kl (unsigned int __n, long double __x)

template<typename _Tp >
  std::complex< __gnu_cxx::__promote_num_t< _Tp >> sph_hankel_1 (unsigned int __n, _Tp __z)
```

```
template<typename _Tp >
  std::complex < gnu cxx:: promote num t < Tp > sph hankel 1 (unsigned int n, std::complex < Tp > sph

    std::complex< float > sph_hankel_1f (unsigned int __n, float __z)

• std::complex < float > sph hankel 1f (unsigned int n, std::complex < float > x)

    std::complex < long double > sph hankel 1l (unsigned int n, long double z)

    std::complex < long double > sph hankel 1l (unsigned int n, std::complex < long double > x)

template<typename</li>Tp >
  std::complex< __gnu_cxx::__promote_num_t< _Tp >> sph_hankel_2 (unsigned int __n, _Tp __z)
template<typename Tp >
  std::complex< __gnu_cxx::__promote_num_t< _Tp >> sph_hankel_2 (unsigned int __n, std::complex< _Tp >
   __x)
• std::complex< float > sph_hankel_2f (unsigned int __n, float __z)

    std::complex < float > sph hankel 2f (unsigned int n, std::complex < float > x)

    std::complex < long double > sph_hankel_2l (unsigned int __n, long double __z)

    std::complex < long double > sph hankel 2l (unsigned int n, std::complex < long double > x)

• template<typename _Ttheta , typename _Tphi >
  std::complex < \underline{\quad} gnu\_cxx::\underline{\quad} promote\_num\_t < \underline{\quad} Ttheta, \underline{\quad} Tphi >> sph\_harmonic \ (unsigned \ int \underline{\quad} I, \ int \underline{\quad} m, \\
  _Ttheta __theta, _Tphi __phi)

    std::complex < float > sph_harmonicf (unsigned int __l, int __m, float __theta, float __phi)

• std::complex < long double > sph_harmonicl (unsigned int __l, int __m, long double __theta, long double __phi)
• template<typename _Tpnu , typename _Tp >
    gnu cxx:: promote num t < Tpnu, Tp > theta 1 (Tpnu nu, Tp x)
• float theta 1f (float nu, float x)

    long double theta_1l (long double __nu, long double __x)

• template<typename _Tpnu , typename _Tp >
    gnu cxx:: promote num t< Tpnu, Tp > theta 2 (Tpnu nu, Tp x)
• float theta 2f (float nu, float x)

    long double theta_2l (long double __nu, long double __x)

\bullet \;\; {\sf template}{<} {\sf typename} \; {\sf \_Tpnu} \; , \\ {\sf typename} \; {\sf \_Tp} > \\
    gnu cxx:: promote num t< Tpnu, Tp > theta 3 ( Tpnu nu, Tp x)
• float theta 3f (float nu, float x)

    long double theta_3l (long double __nu, long double __x)

• template<typename _Tpnu , typename _Tp >
    _gnu_cxx::__promote_num_t< _Tpnu, _Tp > theta_4 (_Tpnu __nu, _Tp __x)

 float theta_4f (float __nu, float __x)

    long double theta 4l (long double nu, long double x)

• template<typename _{\rm Tpk}, typename _{\rm Tp} >
    _gnu_cxx::__promote_num_t< _Tpk, _Tp > theta_c (_Tpk __k, _Tp __x)

    float theta_cf (float __k, float __x)

• long double theta cl (long double k, long double x)
• template<typename _{\rm Tpk}, typename _{\rm Tp} >
    _gnu_cxx::__promote_num_t< _Tpk, _Tp > theta_d (_Tpk __k, _Tp __x)

    float theta df (float k, float x)

    long double theta dl (long double k, long double x)

• template<typename _Tpk , typename _Tp >
    _gnu_cxx::__promote_num_t< _Tpk, _Tp > theta_n (_Tpk __k, _Tp __x)

    float theta nf (float k, float x)

    long double theta nl (long double k, long double x)

• template<typename _Tpk , typename _Tp >
    _gnu_cxx::__promote_num_t< _Tpk, _Tp > theta_s (_Tpk __k, _Tp __x)

    float theta sf (float k, float x)
```

```
long double theta_sl (long double __k, long double __x)
template<typename _Trho , typename _Tphi > __gnu_cxx::_promote_num_t< _Trho, _Tphi > zernike (unsigned int __n, int __m, _Trho __rho, _Tphi __phi)
float zernikef (unsigned int __n, int __m, float __rho, float __phi)
long double zernikel (unsigned int __n, int __m, long double __rho, long double __phi)
```

# 9.2 std Namespace Reference

# **Namespaces**

detail

#### **Functions**

```
template<typename _Tp >
   _gnu_cxx::__promote< _Tp >::__type assoc_laguerre (unsigned int __n, unsigned int __m, _Tp __x)

    float assoc_laguerref (unsigned int __n, unsigned int __m, float __x)

    long double assoc_laguerrel (unsigned int __n, unsigned int __m, long double __x)

    template<typename</li>
    Tp >

    _gnu_cxx::__promote< _Tp >::__type assoc_legendre (unsigned int __I, unsigned int __ m, Tp _ x)

    float assoc_legendref (unsigned int __l, unsigned int __m, float __x)

    long double assoc_legendrel (unsigned int __l, unsigned int __m, long double __x)

• template<typename Tpa, typename Tpb>
    _gnu_cxx::__promote_2< _Tpa, _Tpb >::__type beta (_Tpa __a, _Tpb __b)

    float betaf (float __a, float __b)

    long double betal (long double __a, long double __b)

ullet template<typename _Tp >
    _gnu_cxx::__promote< _Tp >::__type comp_ellint_1 (_Tp __k)

    float comp ellint 1f (float k)

    long double comp ellint 1l (long double k)

• template<typename _Tp >
    _gnu_cxx::__promote< _Tp >::__type comp_ellint_2 (_Tp __k)

    float comp ellint 2f (float k)

    long double comp_ellint_2l (long double __k)

• template<typename _Tp , typename _Tpn >
    gnu cxx:: promote 2< Tp, Tpn >:: type comp ellint 3 (Tp k, Tpn nu)

    float comp ellint 3f (float k, float nu)

      Return the complete elliptic integral of the third kind \Pi(k,\nu) for float modulus k.

    long double comp_ellint_3l (long double ___k, long double ___nu)

      Return the complete elliptic integral of the third kind \Pi(k,\nu) for long double modulus k.

    template<typename _Tpnu , typename _Tp >

    _gnu_cxx::__promote_2< _Tpnu, _Tp >::__type cyl_bessel_i (_Tpnu __nu, _Tp __x)

    float cyl_bessel_if (float __nu, float __x)

    long double cyl bessel il (long double nu, long double x)

• template<typename _Tpnu , typename _Tp >
   _gnu_cxx::__promote_2< _Tpnu, _Tp >::__type cyl_bessel_j (_Tpnu __nu, _Tp __x)

    float cyl bessel jf (float nu, float x)

• long double cyl_bessel_jl (long double __nu, long double __x)
```

```
    template<typename _Tpnu , typename _Tp >

    _gnu_cxx::__promote_2< _Tpnu, _Tp >::__type cyl_bessel_k (_Tpnu __nu, _Tp __x)

    float cyl bessel kf (float nu, float x)

    long double cyl_bessel_kl (long double __nu, long double __x)

• template<typename Tpnu, typename Tp >
    _gnu_cxx::__promote_2< _Tpnu, _Tp >::__type cyl_neumann (_Tpnu __nu, _Tp __x)

    float cyl_neumannf (float __nu, float __x)

    long double cyl_neumannl (long double __nu, long double __x)

• template<typename Tp, typename Tpp>

    float ellint_1f (float __k, float __phi)

    long double ellint 11 (long double k, long double phi)

template<typename _Tp , typename _Tpp >
    _gnu_cxx::__promote_2< _Tp, _Tpp >::__type ellint_2 (_Tp __k, _Tpp __phi)

    float ellint 2f (float k, float phi)

      Return the incomplete elliptic integral of the second kind E(k, \phi) for float argument.

    long double ellint_2l (long double ___k, long double ___phi)

      Return the incomplete elliptic integral of the second kind E(k, \phi).

    template<typename _Tp , typename _Tpn , typename _Tpp >

   _gnu_cxx::__promote_3< _Tp, _Tpn, _Tpp >::__type ellint_3 (_Tp __k, _Tpn __nu, _Tpp __phi)
      Return the incomplete elliptic integral of the third kind \Pi(k, \nu, \phi).

    float ellint 3f (float k, float nu, float phi)

      Return the incomplete elliptic integral of the third kind \Pi(k, \nu, \phi) for float argument.
• long double ellint 3I (long double k, long double nu, long double phi)
      Return the incomplete elliptic integral of the third kind \Pi(k, \nu, \phi).

    template<typename</li>
    Tp >

    _gnu_cxx::__promote< _Tp >::__type expint (_Tp __x)

 float expintf (float __x)

    long double expintl (long double __x)

template<typename</li>Tp >
   _gnu_cxx::__promote< _Tp >::__type hermite (unsigned int __n, _Tp __x)
• float hermitef (unsigned int __n, float __x)

    long double hermitel (unsigned int n, long double x)

template<typename _Tp >
    _gnu_cxx::__promote< _Tp >::__type laguerre (unsigned int __n, _Tp __x)

    float laguerref (unsigned int n, float x)

    long double laguerrel (unsigned int __n, long double __x)

• template<typename _Tp >
    _gnu_cxx::__promote< _Tp >::__type legendre (unsigned int __I, _Tp __x)

    float legendref (unsigned int I, float x)

    long double legendrel (unsigned int __l, long double __x)

template<typename _Tp >
    gnu cxx:: promote < Tp >:: type riemann zeta (Tp s)

    float riemann_zetaf (float __s)

    long double riemann zetal (long double s)

template<typename _Tp >
    gnu cxx:: promote < Tp >:: type sph bessel (unsigned int n, Tp x)

    float sph besself (unsigned int n, float x)

    long double sph_bessell (unsigned int __n, long double __x)

template<typename _Tp >
  gnu cxx:: promote < Tp >:: type sph legendre (unsigned int I, unsigned int m, Tp theta)
```

float sph\_legendref (unsigned int \_\_l, unsigned int \_\_m, float \_\_theta)
long double sph\_legendrel (unsigned int \_\_l, unsigned int \_\_m, long double \_\_theta)
template<typename \_Tp > \_\_gnu\_cxx::\_promote< \_Tp >::\_type sph\_neumann (unsigned int \_\_n, \_Tp \_\_x)
float sph\_neumannf (unsigned int \_\_n, float \_\_x)

long double sph neumannl (unsigned int n, long double x)

# 9.3 std::\_\_detail Namespace Reference

#### Classes

```
class _Airy
class _Airy_asymp
struct _Airy_asymp_data
struct _Airy_asymp_data<</li>
struct _Airy_asymp_data
double >
struct _Airy_asymp_data
float >
struct _Airy_asymp_data< long double >
class _Airy_asymp_series
struct _Airy_default_radii
struct _Airy_default_radii
struct _Airy_default_radii
struct _Airy_default_radii
struct _Airy_default_radii
long double >
class Airy series
```

- struct \_AiryAuxilliaryState
- struct AiryState
- struct \_Factorial\_table

### **Enumerations**

enum { SININT, COSINT }

# **Functions**

```
    template<typename _Tp >
        void __airy (_Tp __z, _Tp &_Ai, _Tp &_Bi, _Tp &_Aip, _Tp &_Bip)
        Compute the Airy functions Ai(x) and Bi(x) and their first derivatives Ai'(x) and Bi(x) respectively.
    template<typename _Tp >
        std::complex< _Tp > __airy_ai (std::complex< _Tp > __z)
        Return the complex Airy Ai function.
    template<typename _Tp >
        void __airy_arg (std::complex< _Tp > __num2d3, std::complex< _Tp > __zeta, std::complex< _Tp > &__argp, std::complex< _Tp > &__argm)
```

Compute the arguments for the Airy function evaluations carefully to prevent premature overflow. Note that the major work here is in safe\_div. A faster, but less safe implementation can be obtained without use of safe\_div.

```
template<typename _Tp >
  std::complex< Tp > airy bi (std::complex< Tp > z)
      Return the complex Airy Bi function.
template<typename _Tp >
  Tp assoc laguerre (unsigned int n, unsigned int m, Tp x)
      This routine returns the associated Laguerre polynomial of order n, degree m: L_n^m(x).
template<typename _Tp >
  _Tp __assoc_legendre_p (unsigned int __I, unsigned int __m, Tp _x)
      Return the associated Legendre function by recursion on l and downward recursion on m.
template<typename_Tp>
  GLIBCXX14 CONSTEXPR Tp bernoulli (int n)
      This returns Bernoulli number B_n.
template<typename _Tp >
  _GLIBCXX14_CONSTEXPR _Tp __bernoulli_2n (int __n)
      This returns Bernoulli number B_n.
template<typename _Tp >
  GLIBCXX14 CONSTEXPR Tp bernoulli series (unsigned int n)
      This returns Bernoulli numbers from a table or by summation for larger values.

    template<typename</li>
    Tp >

  Return the beta function B(a, b).

    template<typename</li>
    Tp >

  Return the beta function: B(a,b).
template<typename</li>Tp >
  _Tp __beta_inc (_Tp __a, _Tp __b, _Tp __x)
ullet template<typename_Tp>
  _Tp <u>__beta_inc_cont_fra</u>c (_Tp <u>__a, _</u>Tp <u>__b, _</u>Tp <u>__x</u>)

    template<typename _Tp >

  _Tp __beta_lgamma (_Tp __a, _Tp __b)
      Return the beta function B(a,b) using the log gamma functions.
template<typename _Tp >
  _Tp __beta_product (_Tp __a, _Tp __b)
      Return the beta function B(x, y) using the product form.
template<typename _Tp >
  _Tp __bincoef (unsigned int __n, unsigned int __k)
      Return the binomial coefficient. The binomial coefficient is given by:
                                                  \binom{n}{k} = \frac{n!}{(n-k)!k!}
• template<typename _Tp >
  _Tp __bose_einstein (_Tp __s, _Tp __x)
template<typename _Tp >
  _Tp __chebyshev_recur (unsigned int __n, _Tp __x, _Tp _C0, _Tp _C1)
template<typename _Tp >
  _Tp __chebyshev_t (unsigned int __n, _Tp __x)
• template<typename _{\mathrm{Tp}} >
  _Tp __chebyshev_u (unsigned int __n, _Tp __x)
template<typename _Tp >
  _Tp __chebyshev_v (unsigned int __n, _Tp __x)
```

```
template<typename _Tp >
  Tp chebyshev w (unsigned int n, Tp x)
template<typename_Tp>
  std::pair< _Tp, _Tp > __chshint (_Tp __x, _Tp &_Chi, _Tp &_Shi)
      This function returns the hyperbolic cosine Ci(x) and hyperbolic sine Si(x) integrals as a pair.
template<typename _Tp >
  void chshint cont frac (Tp t, Tp & Chi, Tp & Shi)
      This function computes the hyperbolic cosine Chi(x) and hyperbolic sine Shi(x) integrals by continued fraction for
      positive argument.
template<typename _Tp >
  void __chshint_series (_Tp __t, _Tp &_Chi, _Tp &_Shi)
      This function computes the hyperbolic cosine Chi(x) and hyperbolic sine Shi(x) integrals by series summation for
      positive argument.
template<typename _Tp >
  std::complex< _Tp > __clamp_0_m2pi (std::complex< _Tp > __w)
template<typename _Tp >
  std::complex< _Tp > __clamp_pi (std::complex< _Tp > __w)
template<typename _Tp >
  std::complex< _Tp > __clausen (unsigned int __m, std::complex< _Tp > __w)
template<typename _Tp >
  _Tp __clausen (unsigned int __m, _Tp __w)

    template<typename</li>
    Tp >

  _Tp <u>__clausen_c</u> (unsigned int __m, std::complex< _Tp > __w)
template<typename _Tp >
  Tp clausen c (unsigned int m, Tp w)
template<typename_Tp>
  _Tp __clausen_s (unsigned int __m, std::complex< _Tp > __w)
template<typename</li>Tp >
  _Tp <u>__clausen_s</u> (unsigned int __m, _Tp __w)
template<typename _Tp >
  _Tp __comp_ellint_1 (_Tp __k)
      Return the complete elliptic integral of the first kind K(k) using the Carlson formulation.
template<typename _Tp >
  _Tp __comp_ellint_2 (_Tp __k)
      Return the complete elliptic integral of the second kind E(k) using the Carlson formulation.
template<typename _Tp >
  _Tp __comp_ellint_3 (_Tp __k, _Tp __nu)
      Return the complete elliptic integral of the third kind \Pi(k,\nu) = \Pi(k,\nu,\pi/2) using the Carlson formulation.
template<typename _Tp >
  _Tp __comp_ellint_d (_Tp __k)
• template<typename _{\mathrm{Tp}}>
  _Tp __comp_ellint_rf (_Tp __x, _Tp __y)

    template<typename _Tp >

  _Tp __comp_ellint_rg (_Tp __x, _Tp __y)
template<typename _Tp >
  _Tp <u>__conf_hyperg</u> (_Tp <u>__a, _</u>Tp <u>__c, _</u>Tp <u>__x</u>)
      Return the confluent hypergeometric function {}_1F_1(a;c;x).
template<typename _Tp >
  _Tp __conf_hyperg_lim (_Tp __c, _Tp __x)
```

Return the confluent hypergeometric limit function  ${}_{0}F_{1}(-;c;x)$ .

```
template<typename _Tp >
  Tp conf hyperg lim series (Tp c, Tp x)
      This routine returns the confluent hypergeometric limit function by series expansion.

    template<typename</li>
    Tp >

  _Tp __conf_hyperg_luke (_Tp __a, _Tp __c, _Tp __xin)
      Return the hypergeometric function _1F_1(a;c;x) by an iterative procedure described in Luke, Algorithms for the Compu-
      tation of Mathematical Functions.
template<typename _Tp >
  _Tp __conf_hyperg_series (_Tp __a, _Tp __c, _Tp __x)
      This routine returns the confluent hypergeometric function by series expansion.
template<typename_Tp>
  _Tp __coshint (const _Tp __x)
      Return the hyperbolic cosine integral li(x).
template<typename _Tp >
  std::complex< Tp > cyl bessel (std::complex< Tp > nu, std::complex< Tp > z)
      Return the complex cylindrical Bessel function.

    template<typename</li>
    Tp >

  _Tp __cyl_bessel_i (_Tp __nu, _Tp __x)
      Return the regular modified Bessel function of order \nu: I_{\nu}(x).

    template<typename</li>
    Tp >

  _Tp __cyl_bessel_ij_series (_Tp __nu, _Tp __x, _Tp __sgn, unsigned int __max_iter)
      This routine returns the cylindrical Bessel functions of order \nu: J_{\nu} or I_{\nu} by series expansion.
template<typename_Tp>
  void <u>cyl_bessel_ik</u> (_Tp __nu, _Tp __x, _Tp &_lnu, _Tp &_Knu, _Tp &_lpnu, _Tp &_Kpnu)
      Return the modified cylindrical Bessel functions and their derivatives of order \nu by various means.
template<typename_Tp>
  void cyl bessel ik asymp (Tp nu, Tp x, Tp & Inu, Tp & Knu, Tp & Ipnu, Tp & Kpnu)
      This routine computes the asymptotic modified cylindrical Bessel and functions of order nu: I_{\nu}(x), N_{\nu}(x). Use this for
      x >> nu^2 + 1.
template<typename _Tp >
  void cyl bessel ik steed (Tp nu, Tp x, Tp & Inu, Tp & Knu, Tp & Ipnu, Tp & Kpnu)
      Compute the modified Bessel functions I_{\nu}(x) and K_{\nu}(x) and their first derivatives I'_{\nu}(x) and K'_{\nu}(x) respectively. These
      four functions are computed together for numerical stability.
template<typename _Tp >
  _Tp __cyl_bessel_j (_Tp __nu, _Tp __x)
      Return the Bessel function of order \nu: J_{\nu}(x).
template<typename_Tp>
  void <u>__cyl_bessel_jn</u> (_Tp __nu, _Tp __x, _Tp &_Jnu, _Tp &_Nnu, _Tp &_Jpnu, _Tp &_Npnu)
      Return the cylindrical Bessel functions and their derivatives of order \nu by various means.
template<typename _Tp >
```

```
void <u>__cyl_bessel_jn_asymp</u> (_Tp __nu, _Tp __x, _Tp &_Jnu, _Tp &_Nnu, _Tp &_Jpnu, _Tp &_Npnu)
```

This routine computes the asymptotic cylindrical Bessel and Neumann functions of order nu:  $J_{\nu}(x)$ ,  $N_{\nu}(x)$ . Use this for  $x >> nu^2 + 1$ .

• template<typename  $_{\rm Tp}>$ 

Compute the Bessel  $J_{\nu}(x)$  and Neumann  $N_{\nu}(x)$  functions and their first derivatives  $J'_{\nu}(x)$  and  $N'_{\nu}(x)$  respectively. These four functions are computed together for numerical stability.

template<typename \_Tp >

```
_Tp __cyl_bessel_k (_Tp __nu, _Tp __x)
```

Return the irregular modified Bessel function  $K_{\nu}(x)$  of order  $\nu$ .

```
template<typename _Tp >
  std::complex < Tp > cyl hankel 1 (Tp nu, Tp x)
      Return the cylindrical Hankel function of the first kind H_{\nu}^{(1)}(x).
template<typename _Tp >
  std::complex < _Tp > __cyl_hankel_1 (std::complex < _Tp > __nu, std::complex < _Tp > __z)
      Return the complex cylindrical Hankel function of the first kind.

    template<typename</li>
    Tp >

  std::complex< _Tp > __cyl_hankel_2 (_Tp __nu, _Tp __x)
      Return the cylindrical Hankel function of the second kind H_n^{(2)}u(x).
template<typename _Tp >
  std::complex< Tp > cyl hankel 2 (std::complex< Tp > nu, std::complex< Tp > z)
      Return the complex cylindrical Hankel function of the second kind.
template<typename _Tp >
  std::complex < _Tp > __cyl_neumann (std::complex < _Tp > __nu, std::complex < _Tp > __z)
      Return the complex cylindrical Neumann function.
template<typename</li>Tp >
  _Tp <u>__cyl_neumann_</u>n (_Tp __nu, _Tp __x)
      Return the Neumann function of order \nu: N_{\nu}(x).
template<typename</li>Tp >
  _Tp __dawson (_Tp __x)
      Return the Dawson integral, F(x), for real argument x.
template<typename _Tp >
  _Tp __dawson_cont_frac (_Tp __x)
      Compute the Dawson integral using a sampling theorem representation.

    template<typename</li>
    Tp >

  _Tp __dawson_series (_Tp __x)
      Compute the Dawson integral using the series expansion.
template<typename_Tp>
  void debye region (std::complex < Tp > alpha, int & indexr, char & aorb)
template<typename_Tp>
  _Tp __dilog (_Tp __x)
      Compute the dilogarithm function Li_2(x) by summation for x \le 1.
template<typename</li>Tp >
  _Tp __dirichlet_beta (std::complex< _Tp > __w)
template<typename _Tp >
  _Tp __dirichlet_beta (_Tp __w)
template<typename_Tp>
  std::complex< Tp > dirichlet eta (std::complex< Tp > w)
template<typename _Tp >
  _Tp __dirichlet_eta (_Tp __w)
• template<typename _{\rm Tp}>
  GLIBCXX14 CONSTEXPR Tp double factorial (int n)
      Return the double factorial of the integer n.
template<typename_Tp>
  _Tp __ellint_1 (_Tp __k, _Tp __phi)
      Return the incomplete elliptic integral of the first kind F(k,\phi) using the Carlson formulation.
template<typename_Tp>
  _Tp __ellint_2 (_Tp __k, _Tp __phi)
      Return the incomplete elliptic integral of the second kind E(k,\phi) using the Carlson formulation.
```

```
template<typename _Tp >
  _Tp <u>__ellint_3</u> (_Tp __k, _Tp __nu, _Tp __phi)
      Return the incomplete elliptic integral of the third kind \Pi(k, \nu, \phi) using the Carlson formulation.
template<typename _Tp >
  _Tp __ellint_cel (_Tp __k_c, _Tp __p, _Tp __a, _Tp __b)
• template<typename _{\rm Tp}>

    template<typename</li>
    Tp >

  _Tp __ellint_el1 (_Tp __x, _Tp __k_c)
• template<typename _{\rm Tp}>
  _Tp __ellint_el2 (_Tp __x, _Tp __k_c, _Tp __a, _Tp __b)

    template<typename</li>
    Tp >

  _Tp __ellint_el3 (_Tp __x, _Tp __k_c, _Tp __p)
ullet template<typename _Tp >
  _Tp __ellint_rc (_Tp __x, _Tp __y)
      Return the Carlson elliptic function R_C(x,y) = R_F(x,y,y) where R_F(x,y,z) is the Carlson elliptic function of the first
template<typename _Tp >
  _Tp __ellint_rd (_Tp __x, _Tp __y, _Tp __z)
      Return the Carlson elliptic function of the second kind R_D(x,y,z) = R_J(x,y,z,z) where R_J(x,y,z,p) is the Carlson
      elliptic function of the third kind.
template<typename _Tp >
  _Tp __ellint_rf (_Tp __x, _Tp __y, _Tp __z)
      Return the Carlson elliptic function R_F(x, y, z) of the first kind.

    template<typename</li>
    Tp >

  _Tp <u>__ellint_rg</u> (_Tp __x, _Tp __y, _Tp __z)
      Return the symmetric Carlson elliptic function of the second kind R_G(x, y, z).
• template<typename _Tp >
  _Tp <u>__ellint_rj</u> (_Tp __x, _Tp __y, _Tp __z, _Tp __p)
      Return the Carlson elliptic function R_J(x, y, z, p) of the third kind.
template<typename</li>Tp >
  _Tp __ellnome (_Tp __k)
template<typename_Tp>
  _Tp __ellnome_k (_Tp __k)
• template<typename _{\rm Tp}>
  _Tp __ellnome_series (_Tp __k)
• template<typename _{\mathrm{Tp}} >
  _Tp __expint (unsigned int __n, _Tp __x)
      Return the exponential integral E_n(x).
template<typename_Tp>
  _Tp __expint (_Tp __x)
      Return the exponential integral Ei(x).
template<typename_Tp>
  _Tp __expint_asymp (unsigned int __n, _Tp __x)
      Return the exponential integral E_n(x) for large argument.
template<typename _Tp >
  Tp expint E1 (Tp x)
      Return the exponential integral E_1(x).
template<typename _Tp >
  _Tp __expint_E1_asymp (_Tp __x)
```

```
Return the exponential integral E_1(x) by asymptotic expansion.
template<typename _Tp >
  _Tp __expint_E1_series (_Tp __x)
      Return the exponential integral E_1(x) by series summation. This should be good for x < 1.
template<typename</li>Tp >
  _Tp __expint_Ei (_Tp __x)
      Return the exponential integral Ei(x).
template<typename_Tp>
  _Tp __expint_Ei_asymp (_Tp __x)
      Return the exponential integral Ei(x) by asymptotic expansion.

    template<typename</li>
    Tp >

  _Tp __expint_Ei_series (_Tp __x)
      Return the exponential integral Ei(x) by series summation.
template<typename _Tp >
  Tp expint En cont frac (unsigned int n, Tp x)
      Return the exponential integral E_n(x) by continued fractions.
template<typename</li>Tp >
  _Tp __expint_En_recursion (unsigned int __n, _Tp __x)
      Return the exponential integral E_n(x) by recursion. Use upward recursion for x < n and downward recursion (Miller's
      algorithm) otherwise.

    template<typename</li>
    Tp >

  _Tp __expint_En_series (unsigned int __n, _Tp __x)
      Return the exponential integral E_n(x) by series summation.
template<typename _Tp >
  _Tp __expint_large_n (unsigned int __n, _Tp __x)
      Return the exponential integral E_n(x) for large order.
template<typename</li>Tp >
  _GLIBCXX14_CONSTEXPR _Tp __factorial (unsigned int __n)
      Return the factorial of the integer n.
template<typename _Tp >
  _Tp __fermi_dirac (_Tp __s, _Tp __x)

    template<typename</li>
    Tp >

  void __fock_airy (_Tp __x, std::complex< _Tp > &__w1, std::complex< _Tp > &__w2, std::complex< _Tp >
  _{\text{w1p, std::complex}} < _{\text{Tp}} > _{\text{w2p}}
      Compute the Fock-type Airy functions w_1(x) and w_2(x) and their first derivatives w_1'(x) and w_2'(x) respectively.
                                               w_1(x) = \sqrt{\pi}(Ai(x) + iBi(x))
                                               w_2(x) = \sqrt{\pi}(Ai(x) - iBi(x))
template<typename _Tp >
  bool __fpequal (const _Tp &__a, const _Tp &__b)
template<typename _Tp >
  bool <u>__fpimag</u> (const std::complex < _Tp > &__w)
template<typename _Tp >
  bool __fpimag (const _Tp)
template<typename Tp >
  bool __fpreal (const std::complex< _Tp > &__w)
• template<typename _Tp >
  bool __fpreal (const _Tp)
template<typename _Tp >
```

std::complex< \_Tp > \_\_fresnel (const \_Tp \_\_x)

Return the Fresnel cosine and sine integrals as a complex number f(C(x) + iS(x))

template<typename</li>Tp >

This function computes the Fresnel cosine and sine integrals by continued fractions for positive argument.

template<typename \_Tp >

This function returns the Fresnel cosine and sine integrals as a pair by series expansion for positive argument.

• template<typename\_Tp>

Return  $\Gamma(x)$ .

• template<typename \_Tp >

 $\bullet \ \ \mathsf{template} \mathord{<} \mathsf{typename} \ \mathsf{\_Tp} \mathord{>}$ 

Return the lower incomplete gamma function. The lower incomplete gamma function is defined by

$$\gamma(a, x) = \int_0^x e^{-t} t^{a-1} dt (a > 0)$$

.

• template<typename  $_{\rm Tp}>$ 

• template<typename  $_{\rm Tp}>$ 

Compute the gamma functions required by the Temme series expansions of  $N_{\nu}(x)$  and  $K_{\nu}(x)$ .

$$\Gamma_1 = \frac{1}{2\mu} \left[ \frac{1}{\Gamma(1-\mu)} - \frac{1}{\Gamma(1+\mu)} \right]$$

and

$$\Gamma_2 = \frac{1}{2} \left[ \frac{1}{\Gamma(1-\mu)} + \frac{1}{\Gamma(1+\mu)} \right]$$

where  $-1/2 <= \mu <= 1/2$  is  $\mu = \nu - N$  and N. is the nearest integer to  $\nu$ . The values of  $\Gamma(1+\mu)$  and  $\Gamma(1-\mu)$  are returned as well.

template<typename</li>
 Tp >

Return the upper incomplete gamma function. The lower incomplete gamma function is defined by

$$\Gamma(a,x) = \int_{x}^{\infty} e^{-t} t^{a-1} dt (a > 0)$$

.

• template<typename  $_{\rm Tp}>$ 

• template<typename  $_{\mathrm{Tp}}>$ 

template<typename \_Tp >

void \_\_hankel (std::complex< \_Tp > \_\_nu, std::complex< \_Tp > \_\_z, std::complex< \_Tp > &\_H1, std $\leftarrow$ ::complex< \_Tp > &\_H2, std::complex< \_Tp > &\_H2p)

• template<typename\_Tp>

template<typename \_Tp >
 void \_\_hankel\_params (std::complex< \_Tp > \_\_nu, std::complex

Compute parameters depending on z and nu that appear in the uniform asymptotic expansions of the Hankel functions and their derivatives, except the arguments to the Airy functions.

template<typename \_Tp >

```
void __hankel_uniform (std::complex< _Tp > __nu, std::complex< _Tp > __z, std::complex< _Tp > &_H1, std::complex< Tp > & H2, std::complex< Tp > & H2p)
```

This routine computes the uniform asymptotic approximations of the Hankel functions and their derivatives including a patch for the case when the order equals or nearly equals the argument. At such points, Olver's expressions have zero denominators (and numerators) resulting in numerical problems. This routine averages results from four surrounding points in the complex plane to obtain the result in such cases.

template<typename \_Tp >

Compute approximate values for the Hankel functions of the first and second kinds using Olver's uniform asymptotic expansion to of order nu along with their derivatives.

template<typename \_Tp >

```
\label{lem:complex} $$\operatorname{\begin{tikzpicture}{0.5\textwidth} $\operatorname{\begin{tikzpicture}{0.5\textwidth} $\operatorname{\begin{tikzpicture} $\operatorname{\begin{tikzpictu
```

Compute outer factors and associated functions of z and nu appearing in Olver's uniform asymptotic expansions of the Hankel functions of the first and second kinds and their derivatives. The various functions of z and nu returned by hankel\_uniform\_outer are available for use in computing further terms in the expansions.

template<typename</li>Tp >

```
void __hankel_uniform_sum (std::complex < _Tp > __p, std::complex < _Tp > __p2, std::complex < _Tp > __ num2, std::complex < _Tp > __zetam3hf, std::complex < _Tp > __alip, std::complex < _Tp > __o4dp, std \leftrightarrow ::complex < _Tp > __alim, std::complex < _Tp > __o4dm, std::complex < _Tp > __o4dp, std::complex
```

Compute the sums in appropriate linear combinations appearing in Olver's uniform asymptotic expansions for the Hankel functions of the first and second kinds and their derivatives, using up to nterms (less than 5) to achieve relative error eps.

template<typename \_Tp >

```
_Tp __heuman_lambda (_Tp __k, _Tp __phi)
```

template<typename \_Tp >

Return the Hurwitz zeta function  $\zeta(s,a)$  for all s = 1 and a > -1.

template<typename \_Tp >

```
std::complex< _Tp > __hurwitz_zeta (_Tp __s, std::complex< _Tp > __a)
```

template<typename \_Tp >

Return the Hurwitz zeta function  $\zeta(s, a)$  for all s = 1 and a > -1.

• template<typename Tp >

```
std::complex < _Tp > __hydrogen (const unsigned int __n, const unsigned int __l, const unsigned int __m, const _Tp _Z, const _Tp __r, const _Tp __theta, const _Tp __phi)
```

template<typename\_Tp>

```
_Tp __hyperg (_Tp __a, _Tp __b, _Tp __c, _Tp __x)
```

Return the hypergeometric function  ${}_{2}F_{1}(a,b;c;x)$ .

template<typename \_Tp >

Return the hypergeometric function  $_2F_1(a,b;c;x)$  by an iterative procedure described in Luke, Algorithms for the Computation of Mathematical Functions.

template<typename\_Tp>

Return the hypergeometric function  ${}_2F_1(a,b;c;x)$  by the reflection formulae in Abramowitz & Stegun formula 15.3.6 for d=c-a b not integral and formula 15.3.11 for d=c-a b integral. This assumes a,b,c!= negative integer.

template<typename \_Tp >

Return the hypergeometric function  ${}_2F_1(a,b;c;x)$  by series expansion.

template<typename \_Tp >

template<typename \_Tp >

template<typename\_Tp>

This routine returns the Laguerre polynomial of order n:  $L_n(x)$ .

template<typename \_Tp >

Return the Legendre function of the second kind by upward recursion on order l.

template<typename \_Tp >

Return the logarithm of the binomial coefficient. The binomial coefficient is given by:

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

template<typename \_Tp >

template<typename\_Tp>

Return the logarithm of the double factorial of the integer n.

template<typename</li>Tp >

Return the logarithm of the factorial of the integer n.

template<typename\_Tp>

Return  $log(|\Gamma(x)|)$ . This will return values even for x < 0. To recover the sign of  $\Gamma(x)$  for any argument use  $\_log\_ \hookleftarrow gamma\_sign$ .

template<typename \_Tp >

Return  $log(\Gamma(x))$  by asymptotic expansion with Bernoulli number coefficients. This is like Sterling's approximation.

template<typename</li>Tp >

Return  $log(\Gamma(x))$  by the Lanczos method. This method dominates all others on the positive axis I think.

• template<typename  $_{\rm Tp}>$ 

Return the sign of  $\Gamma(x)$ . At nonpositive integers zero is returned.

template<typename \_Tp >

\_GLIBCXX14\_CONSTEXPR \_Tp \_\_log\_gamma\_spouge (\_Tp \_\_z)

Return  $\Gamma(z)$  by the Spouge algorithm:

$$\Gamma(z+1) = (z+a)^{z+1/2} e^{-z-a} \left[ \sqrt{2\pi} \sum_{k=1}^{\lceil a \rceil + 1} \frac{c_k(a)}{z+k} \right]$$

where

$$c_k(a) = \frac{(-1)^{k-1}}{(k-1)!} (a-k)^{k-1/2} e^{a-k}$$

and the error is bounded by

$$\epsilon(a) < a^{-1/2} (2\pi)^{-a-1/2}$$

.

ullet template<typename\_Tp>

Return the logarithm of the lower Pochhammer symbol or the falling factorial function. The lower Pochammer symbol is defined by

$$(a)_n = \prod_{k=0}^{n-1} (a-k), (a)_0 = 1 = \Gamma(a+1)/\Gamma(a-n+1)$$

In particular, f(n) = n! f. Thus this function returns

$$ln[(a)_n] = \Gamma(a+1) - \Gamma(a-n+1), ln[(a)_0] = 0$$

Many notations exist:

 $a^{\underline{n}}$ 

,

$$\left\{\begin{array}{c} a \\ n \end{array}\right\}$$

, and others.

ullet template<typename\_Tp>

Return the logarithm of the (upper) Pochhammer symbol or the rising factorial function. The Pochammer symbol is defined by

$$(a)_n = \prod_{k=0}^{n-1} (a+k), (a)_0 = 1 = \Gamma(a+n)/\Gamma(n)$$

Thus this function returns

$$ln[(a)_n] = \Gamma(a+n) - \Gamma(n), ln[(a)_0] = 0$$

Many notations exist:

$$a^n$$

,

$$\begin{bmatrix} a \\ n \end{bmatrix}$$

, and others.

 $\bullet \ \ \text{template} {<} \text{typename} \ \_{\text{Tp}} >$ 

Return the logarithmic integral li(x).

template<typename \_Tp >

• template<typename  $_{\rm Tp}>$ 

Return the regularized lower incomplete gamma function. The regularized lower incomplete gamma function is defined by

$$P(a,x) = \frac{\gamma(a,x)}{\Gamma(a)}$$

where  $\Gamma(a)$  is the gamma function and

$$\gamma(a, x) = \int_0^x e^{-t} t^{a-1} dt (a > 0)$$

is the lower incomplete gamma function.

• template<typename  $_{\rm Tp}>$ 

Return the logarithm of the lower Pochhammer symbol or the falling factorial function. The lower Pochammer symbol is defined by

$$(a)_n = \prod_{k=0}^{n-1} (a-k), (a)_0 = 1 = \Gamma(a+1)/\Gamma(a-n+1)$$

In particular,  $f(n)_n = n! f(n)$ 

template<typename \_Tp >

Return the (upper) Pochhammer function or the rising factorial function. The Pochammer symbol is defined by

$$(a)_n = \prod_{k=0}^{n-1} (a+k), (a)_0 = 1 = \Gamma(a+n)/\Gamma(n)$$

Many notations exist:

 $a^{\bar{\imath}}$ 

 $\begin{bmatrix} a \\ n \end{bmatrix}$ 

, and others.

template<typename \_Tp >

This routine returns the Hermite polynomial of order n:  $H_n(x)$ .

• template<typename  $_{\rm Tp}>$ 

This routine returns the Hermite polynomial of large order n:  $H_n(x)$ . We assume here that x >= 0.

• template<typename\_Tp>

This routine returns the Hermite polynomial of order n:  $H_n(x)$  by recursion on n.

template<typename</li>
 Tp >

• template<typename \_Tpa , typename \_Tp >

This routine returns the associated Laguerre polynomial of order n, degree  $\alpha$ :  $L_n^a lpha(x)$ .

• template<typename \_Tpa , typename \_Tp >

Evaluate the polynomial based on the confluent hypergeometric function in a safe way, with no restriction on the arguments.

• template<typename \_Tpa , typename \_Tp >

This routine returns the associated Laguerre polynomial of order n, degree  $\alpha > -1$  for large n. Abramowitz & Stegun, 13.5.21.

• template<typename \_Tpa , typename \_Tp >

```
This routine returns the associated Laguerre polynomial of order n, degree \alpha: L_n^n(x) by recursion.
template<typename _Tp >
  _Tp __poly_legendre_p (unsigned int __l, _Tp __x)
      Return the Legendre polynomial by upward recursion on order l.
template<typename_Tp>
  _Tp __poly_radial_jacobi (unsigned int __n, unsigned int __m, _Tp __rho)
template<typename _Tp >
  _Tp __polylog (_Tp __s, _Tp __x)
template<typename Tp >
  std::complex< _Tp > __polylog (_Tp __s, std::complex< _Tp > __w)

    template<typename _Tp , typename ArgType >

   _gnu_cxx::__promote_num_t< std::complex< _Tp >, ArgType > __polylog_exp (_Tp __s, ArgType __w)

    template<typename</li>
    Tp >

  std::complex < Tp > polylog exp asymp ( Tp s, std::complex < Tp > w)
template<typename</li>Tp >
  std::complex< _Tp > __polylog_exp_int_neg (int __s, std::complex< _Tp > __w)
template<typename Tp >
  std::complex< _Tp > __polylog_exp_int_neg (const int __s, _Tp __w)
template<typename _Tp >
  std::complex< Tp > polylog exp int pos (unsigned int s, std::complex< Tp > w)
template<typename _Tp >
  std::complex< _Tp > __polylog_exp_int_pos (unsigned int __s, _Tp __w)
template<typename Tp >
  std::complex< _Tp > __polylog_exp_neg (_Tp __s, std::complex< _Tp > __w)
template<typename _Tp >
  std::complex< _Tp > __polylog_exp_neg (int __s, std::complex< _Tp > __w)
• template<typename Tp , int sigma>
  std::complex < _Tp > __polylog_exp_neg_even (unsigned int __n, std::complex < _Tp > __w)
• template<typename _Tp , int __sigma>
  std::complex< _Tp > __polylog_exp_neg_odd (unsigned int __n, std::complex< _Tp > __w)
• template<typename \_PowTp , typename \_Tp >
  _Tp __polylog_exp_negative_real_part (_PowTp __s, _Tp __w)

    template<typename</li>
    Tp >

  std::complex < _Tp > __polylog_exp_pos (unsigned int __s, std::complex < _Tp > __w)
template<typename _Tp >
  std::complex < _Tp > __polylog_exp_pos (unsigned int __s, _Tp w)

    template<typename</li>
    Tp >

  std::complex< _Tp > __polylog_exp_pos (_Tp __s, std::complex< _Tp > __w)
template<typename _Tp >
  std::complex< _Tp > __polylog_exp_real_neg (_Tp __s, std::complex< _Tp > __w)

    template<typename</li>
    Tp >

  std::complex< _Tp > __polylog_exp_real_neg (_Tp __s, _Tp __w)
template<typename _Tp >
  std::complex< _Tp > __polylog_exp_real_pos (_Tp __s, std::complex< _Tp > __w)
template<typename _Tp >
  std::complex< _Tp > __polylog_exp_real_pos (_Tp __s, _Tp __w)

    template<typename</li>
    Tp >

  _Tp __psi (_Tp __x)
      Return the digamma function. The digamma or \psi(x) function is defined by
                                                    \psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}
```

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For negative argument the reflection formula is used:

$$\psi(x) = \psi(1-x) - \pi \cot(\pi x)$$

.

 $\bullet \ \ template\!<\!typename\,\_Tp>$ 

Return the polygamma function  $\psi^{(n)}(x)$ .

template<typename \_Tp >

Return the digamma function for large argument. The digamma or  $\psi(x)$  function is defined by

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

.

 $\bullet \ \ template {<} typename \ \_Tp >$ 

Return the digamma function by series expansion. The digamma or  $\psi(x)$  function is defined by

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

.

template<typename \_Tp >

Return the regularized upper incomplete gamma function. The regularized upper incomplete gamma function is defined by

$$Q(a,x) = \frac{\Gamma(a,x)}{\Gamma(a)}$$

where  $\Gamma(a)$  is the gamma function and

$$\Gamma(a,x) = \int_{x}^{\infty} e^{-t} t^{a-1} dt (a > 0)$$

is the upper incomplete gamma function.

template<typename \_Tp >

Return the Riemann zeta function  $\zeta(s)$ .

template<typename \_Tp >

Evaluate the Riemann zeta function  $\zeta(s)$  by an alternate series for s > 0.

template<typename \_Tp >

Evaluate the Riemann zeta function  $\zeta(s)$  by an alternate series for s > 0.

• template<typename \_Tp >

Evaluate the Riemann zeta function by series for all  $s \neq 1$ . Convergence is great until largish negative numbers. Then the convergence of the > 0 sum gets better.

template<typename\_Tp>

Return the Riemann zeta function  $\zeta(s) - 1$ .

• template<typename  $_{\mathrm{Tp}}>$ 

Return the Riemann zeta function  $\zeta(s)-1$  by summation for s>1. This is a small remainder for large s.

template<typename \_Tp >
 \_riemann\_zeta\_product (\_Tp \_\_s)

Compute the Riemann zeta function  $\zeta(s)$  using the product over prime factors.

template<typename \_Tp >

Compute the Riemann zeta function  $\zeta(s)$  by summation for s > 1.

template<typename \_Tp >

$$\_\_gnu\_cxx::\_promote\_num\_t < \_Tp > \_\_sinc (\_Tp \_\_a, \_Tp \_\_x)$$

Return the generalized sinus cardinal function

$$sinc_a(x) = \frac{\sin(\pi x/a)}{(\pi x/a)}$$

.

• template<typename\_Tp>

Return the normalized sinus cardinal function

$$sinc(x) = \frac{\sin(\pi x)}{\pi x}$$

•

template<typename</li>
 Tp >

Return the unnormalized sinus cardinal function

$$sinc_{\pi}(x) = \frac{\sin(x)}{x}$$

.

• template<typename\_Tp>

$$std::pair < Tp, Tp > \underline{sincosint} (Tp \underline{x})$$

This function returns the sine Si(x) and cosine Ci(x) integrals as a pair.

template<typename</li>Tp >

This function computes the sine Si(x) and cosine Ci(x) integrals by asymptotic series summation for positive argument.

template<typename</li>
 Tp >

This function computes the sine Si(x) and cosine Ci(x) integrals by continued fraction for positive argument.

template<typename \_Tp >

This function computes the sine Si(x) and cosine Ci(x) integrals by series summation for positive argument.

template<typename \_Tp >

Return the generalized hyperbolic sinus cardinal function

$$sinhc_a(x) = \frac{\sinh(\pi x/a)}{\pi x/a}$$

.

template<typename \_Tp >

$$_{\rm gnu\_cxx::\_promote\_num\_t < _Tp > \__sinhc (_Tp \__x)}$$

Return the normalized hyperbolic sinus cardinal function

$$sinhc(x) = \frac{\sinh(\pi x)}{\pi x}$$

.

template<typename\_Tp>

Return the unnormalized hyperbolic sinus cardinal function

$$sinhc_{\pi}(x) = \frac{\sinh(x)}{x}$$

```
template<typename_Tp >
    _Tp __sinhint (const _Tp __x)
```

Return the hyperbolic sine integral li(x).

template<typename \_Tp >

```
_Tp __sph_bessel (unsigned int __n, Tp x)
```

Return the spherical Bessel function  $j_n(x)$  of order n and non-negative real argument x.

template<typename</li>Tp >

```
std::complex< _Tp > __sph_bessel (unsigned int __n, std::complex< _Tp > __z)
```

Return the complex spherical Bessel function.

template<typename\_Tp>

```
void sph_bessel_ik (unsigned int __n, _Tp __x, _Tp &_ i_n, _Tp &_ k_n, _Tp &_ ip_n, _Tp &_ kp_n)
```

Compute the spherical modified Bessel functions  $i_n(x)$  and  $k_n(x)$  and their first derivatives  $i'_n(x)$  and  $k'_n(x)$  respectively.

template<typename\_Tp>

```
void <u>sph_bessel_jn</u> (unsigned int __n, _Tp __x, _Tp &__j_n, _Tp &__n, _Tp &__jp_n, _Tp &__np_n)
```

Compute the spherical Bessel  $j_n(x)$  and Neumann  $n_n(x)$  functions and their first derivatives  $j_n(x)$  and  $n'_n(x)$  respectively.

template<typename\_Tp>

```
void \_sph_hankel (unsigned int \_n, std::complex< \_Tp > \_z, std::complex< \_Tp > &\_H1, std::complex< \_Tp > &\_H2, std::complex< \_Tp > &\_H2p)
```

Helper to compute complex spherical Hankel functions and their derivatives.

template<typename \_Tp >

```
std::complex< Tp > sph hankel 1 (unsigned int n, Tp x)
```

Return the spherical Hankel function of the first kind  $h_n^{(1)}(x)$ .

template<typename</li>
 Tp >

Return the complex spherical Hankel function of the first kind.

template<typename \_Tp >

Return the spherical Hankel function of the second kind  $h_n^{(2)}(x)$ .

ullet template<typename\_Tp>

Return the complex spherical Hankel function of the second kind.

template<typename</li>Tp >

Return the spherical harmonic function.

• template<typename  $_{\mathrm{Tp}}>$ 

Return the spherical associated Legendre function.

• template<typename  $_{\rm Tp}>$ 

Return the spherical Neumann function  $n_n(x)$  of order n and non-negative real argument x.

template<typename</li>
 Tp >

```
std::complex<\_Tp>\_\_sph\_neumann \ (unsigned \ int \_\_n, \ std::complex<\_Tp>\_\_z)
```

Return the complex spherical Neumann function.

```
template<typename _Tp >
  _Tp <u>__theta_</u>1 (_Tp __nu, _Tp __x)
template<typename_Tp>
  _Tp <u>__theta_</u>2 (_Tp __nu, _Tp __x)
template<typename _Tp >
  _Tp __theta_2_asymp (_Tp __nu, _Tp __x)
• template<typename _{\rm Tp}>
  _Tp <u>__theta_2_sum</u> (_Tp __nu, _Tp __x)
template<typename _Tp >
  _Tp <u>__theta_3</u> (_Tp __nu, _Tp __x)
• template<typename _{\rm Tp}>
  _Tp __theta_3_asymp (_Tp __nu, _Tp __x)
template<typename_Tp>
  _Tp __theta_3_sum (_Tp __nu, _Tp __x)
template<typename _Tp >
  _Tp <u>__theta_4</u> (_Tp __nu, _Tp __x)

    template<typename</li>
    Tp >

  _Tp __theta_c (_Tp __k, _Tp __x)
template<typename _Tp >
  _Tp <u>__theta_d</u> (_Tp __k, _Tp __x)
template<typename _Tp >
  _Tp <u>theta_n (_Tp __k, _Tp __</u>x)
template<typename _Tp >
  _Tp <u>__theta_s</u> (_Tp __k, _Tp __x)
template<typename _Tp >
   gnu cxx:: promote num t < Tp > zernike (unsigned int n, int m, Tp rho, Tp phi)
template<typename _Tp >
  _Tp __znorm1 (_Tp __x)
template<typename _Tp >
  _Tp <u>__znorm2</u> (_Tp __x)
• template<typename _Tp = double>
  _Tp evenzeta (unsigned int __k)
```

## **Variables**

```
template<typename_Tp > constexpr int __max_FGH = _Airy_series<_Tp>::_N_FGH
template<> constexpr int __max_FGH< double > = 79
template<> constexpr int __max_FGH< float > = 15
constexpr size_t_Num_Euler_Maclaurin_zeta = 100
constexpr _Factorial_table< long double > _S_double_factorial_table [301]
constexpr long double _S_Euler_Maclaurin_zeta [_Num_Euler_Maclaurin_zeta]
constexpr _Factorial_table< long double > _S_factorial_table [171]
constexpr _Factorial_table< long double > _S_neg_double_factorial_table [999]
template<typename_Tp > constexpr std::size_t_S_num_double_factorials< double > = 301
```

```
• template<>
  constexpr std::size t S num double factorials < float > = 57
  constexpr std::size_t _S_num_double_factorials< long double > = 301
template<typename _Tp >
  constexpr std::size t S num factorials = 0
• template<>
  constexpr std::size_t _S_num_factorials< double > = 171
• template<>
  constexpr std::size t S num factorials < float > = 35
template<>
  constexpr std::size_t _S_num_factorials< long double > = 171
template<typename</li>Tp >
  constexpr std::size_t _S_num_neg_double_factorials = 0
• template<>
  constexpr std::size_t _S_num_neg_double_factorials< double > = 150
template<>
  constexpr std::size_t _S_num_neg_double_factorials< float > = 27
template<>
  constexpr std::size_t _S_num_neg_double_factorials< long double > = 999
• constexpr size t S num zetam1 = 33

    constexpr long double _S_zetam1 [_S_num_zetam1]
```

# 9.3.1 Enumeration Type Documentation

## 9.3.1.1 anonymous enum

Enumerator

**SININT** 

**COSINT** 

Definition at line 43 of file sf\_trigint.tcc.

## 9.3.2 Function Documentation

9.3.2.1 template < typename \_Tp > void std::\_\_detail::\_\_airy ( \_Tp \_z, \_Tp & \_Ai, \_Tp & \_Bi, \_Tp & \_Aip, \_Tp & \_Bip )

Compute the Airy functions Ai(x) and Bi(x) and their first derivatives Ai'(x) and Bi(x) respectively.

#### **Parameters**

_~	The argument of the Airy functions.
_Z	
_Ai	The output Airy function of the first kind.
_Bi	The output Airy function of the second kind.
_Aip	The output derivative of the Airy function of the first kind.
_Bip	The output derivative of the Airy function of the second kind.

Definition at line 498 of file sf\_mod\_bessel.tcc.

References \_\_cyl\_bessel\_ik(), and \_\_cyl\_bessel\_in().

Referenced by \_\_poly\_hermite\_asymp().

9.3.2.2 template<typename \_Tp > std::complex<\_Tp> std::\_\_detail::\_\_airy\_ai ( std::complex< \_Tp > \_\_z )

Return the complex Airy Ai function.

Definition at line 3872 of file sf\_airy.tcc.

9.3.2.3 template<typename \_Tp > void std::\_\_detail::\_\_airy\_arg ( std::complex< \_Tp > \_\_num2d3, std::complex< \_Tp > \_\_zeta, std::complex< \_Tp > & \_\_argp, std::complex< \_Tp > & \_\_argm )

Compute the arguments for the Airy function evaluations carefully to prevent premature overflow. Note that the major work here is in safe\_div. A faster, but less safe implementation can be obtained without use of safe\_div.

#### **Parameters**

in	num2d3	$ u^{-2/3}$ - output from hankel_params
in	zeta	zeta in the uniform asymptotic expansions - output from hankel_params
out	argp	$e^{+i2\pi/3} u^{2/3}\zeta$
out	argm	$e^{-i2\pi/3} u^{2/3}\zeta$

## **Exceptions**

std::runtime_error	if unable to compute Airy function arguments

Definition at line 217 of file sf\_hankel.tcc.

Referenced by \_\_hankel\_uniform\_outer().

 $9.3.2.4 \quad template < typename \_Tp > std::\_detail::\_airy\_bi \ ( \ std::complex < \_Tp > \_\_z \ )$ 

Return the complex Airy Bi function.

Definition at line 3884 of file sf airy.tcc.

9.3.2.5 template < typename \_Tp > \_Tp std::\_\_detail::\_assoc\_laguerre ( unsigned int \_\_n, unsigned int \_\_m, \_Tp \_\_x )

This routine returns the associated Laguerre polynomial of order n, degree m:  $L_n^m(x)$ .

The associated Laguerre polynomial is defined for integral  $\alpha=m$  by:

$$L_n^m(x) = (-1)^m \frac{d^m}{dx^m} L_{n+m}(x)$$

where the Laguerre polynomial is defined by:

$$L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n} (x^n e^{-x})$$

## **Template Parameters**

### **Parameters**

_~	The order
_n	
_←	The degree
_m	
_~	The argument
_X	

## Returns

The value of the associated Laguerre polynomial of order n, degree m, and argument x.

Definition at line 301 of file sf\_laguerre.tcc.

Referenced by \_\_hydrogen().

 $9.3.2.6 \quad template < typename \_Tp > \_Tp \ std::\_detail::\_assoc\_legendre\_p \ ( \ unsigned \ int \_\_I, \ unsigned \ int \_\_m, \ \_Tp \_\_x \ )$ 

Return the associated Legendre function by recursion on l and downward recursion on m.

The associated Legendre function is derived from the Legendre function  $P_l(x)$  by the Rodrigues formula:

$$P_l^m(x) = (1 - x^2)^{m/2} \frac{d^m}{dx^m} P_l(x)$$

# **Parameters**

_ <del>←</del>	The order of the associated Legendre function. $l>=0. \label{eq:local_local_local_local}$	
_ <del>_</del>	The order of the associated Legendre function. $m <= l$ .	
_m		Generated by Doxygen
_←	The argument of the associated Legendre function. $\lvert x \rvert <= 1$ .	
_X		

Definition at line 176 of file sf\_legendre.tcc.

```
References __poly_legendre_p().
```

```
9.3.2.7 template<typename _Tp > _GLIBCXX14_CONSTEXPR _Tp std::__detail::__bernoulli ( int __n )
```

This returns Bernoulli number  $B_n$ .

### **Parameters**

```
_ ← the order n of the Bernoulli number.
```

## Returns

The Bernoulli number of order n.

Definition at line 1673 of file sf gamma.tcc.

```
References\ std::\_detail::\_Factorial\_table < \_Tp > ::\__n.
```

```
9.3.2.8 template < typename _Tp > _GLIBCXX14_CONSTEXPR _Tp std::__detail::__bernoulli_2n ( int __n )
```

This returns Bernoulli number  $B_n$ .

## **Parameters**

```
_ ← the order n of the Bernoulli number.
```

## Returns

The Bernoulli number of order n.

Definition at line 1685 of file sf gamma.tcc.

```
References\ std::\_detail::\_Factorial\_table < \_Tp > ::\__n.
```

```
9.3.2.9 template < typename _Tp > _GLIBCXX14_CONSTEXPR _Tp std::__detail::__bernoulli_series ( unsigned int __n )
```

This returns Bernoulli numbers from a table or by summation for larger values.

Upward recursion is unstable.

_←	the order n of the Bernoulli number.
_n	

### Returns

The Bernoulli number of order n.

Definition at line 1608 of file sf\_gamma.tcc.

References std::\_\_detail::\_Factorial\_table< \_Tp >::\_\_n.

9.3.2.10 template<typename \_Tp > \_Tp std::\_\_detail::\_\_beta ( \_Tp \_\_a, \_Tp \_\_b )

Return the beta function B(a, b).

The beta function is defined by

$$B(a,b) = \int_0^1 t^{a-1} (1-t)^{b-1} dt = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

#### **Parameters**

_~	The first argument of the beta function.
_a	
_~	The second argument of the beta function.
_b	

## Returns

The beta function.

Definition at line 173 of file sf beta.tcc.

References beta Igamma().

Referenced by  $\_$ poly\_jacobi(),  $\_$ gnu\_cxx::jacobi(),  $\_$ gnu\_cxx::jacobif(),  $\_$ gnu\_cxx::jacobil(), and std:: $\_$ detail:: $\_$  $\leftarrow$  Airy<  $\_$ Tp >::operator()().

9.3.2.11 template < typename \_Tp > \_Tp std::\_\_detail::\_\_beta\_gamma ( \_Tp \_\_a, \_Tp \_\_b )

Return the beta function: B(a, b).

The beta function is defined by

$$B(a,b) = \int_0^1 t^{a-1} (1-t)^{b-1} dt = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

_~	The first argument of the beta function.
_a	
_~	The second argument of the beta function.
_b	

#### Returns

The beta function.

Definition at line 75 of file sf beta.tcc.

References \_\_gamma().

Return the regularized incomplete beta function,  $I_x(a,b)$ , of arguments a, b, and x.

The regularized incomplete beta function is defined by:

$$I_x(a,b) = \frac{B_x(a,b)}{B(a,b)}$$

where

$$B_x(a,b) = \int_0^x t^{a-1} (1-t)^{b-1} dt$$

is the non-regularized beta function and B(a,b) is the usual beta function.

## **Parameters**

_←	The first parameter
_a	
_~	The second parameter
_b	
_~	The argument
_X	

Definition at line 262 of file sf\_beta.tcc.

References \_\_beta\_inc\_cont\_frac().

9.3.2.13 template < typename \_Tp > \_Tp std::\_\_detail::\_\_beta\_inc\_cont\_frac ( \_Tp 
$$\_a$$
, \_Tp  $\_b$ , \_Tp  $\_x$  )

Return the regularized incomplete beta function,  $I_x(a,b)$ , of arguments a,b, and x.

_~	The first parameter
_a	
_~	The second parameter
_b	
_~	The argument
_x	

Definition at line 193 of file sf beta.tcc.

Referenced by \_\_beta\_inc().

9.3.2.14 template < typename \_Tp > \_Tp std::\_\_detail::\_\_beta\_lgamma ( \_Tp  $\_a$ , \_Tp  $\_b$  )

Return the beta function B(a,b) using the log gamma functions.

The beta function is defined by

$$B(a,b) = \int_0^1 t^{a-1} (1-t)^{b-1} dt = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

#### **Parameters**

_~	The first argument of the beta function.
_a	
_←	The second argument of the beta function.
_b	

## Returns

The beta function.

Definition at line 109 of file sf\_beta.tcc.

References \_\_log\_gamma().

Referenced by \_\_beta().

9.3.2.15 template < typename \_Tp > \_Tp std::\_\_detail::\_\_beta\_product ( \_Tp  $\_a$ , \_Tp  $\_b$  )

Return the beta function B(x, y) using the product form.

The beta function is defined by

$$B(a,b) = \int_0^1 t^{a-1} (1-t)^{b-1} dt = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

Here, we employ the product form:

$$B(a,b) = \frac{a+b}{ab} \prod_{k=1}^{\infty} \frac{1 + (a+b)/k}{(1+a/k)(1+b/k)}$$

_~	The first argument of the beta function.
_a	
_~	The second argument of the beta function.
_b	

### **Returns**

The beta function.

Definition at line 140 of file sf\_beta.tcc.

9.3.2.16 template < typename  $_{\rm Tp}$  >  $_{\rm Tp}$  std::\_\_detail::\_\_bincoef ( unsigned int  $_{\rm n}$ , unsigned int  $_{\rm k}$  )

Return the binomial coefficient. The binomial coefficient is given by:

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

.

## **Parameters**

_~	The first argument of the binomial coefficient.
_n	
_~	The second argument of the binomial coefficient.
_k	

## Returns

The binomial coefficient.

Definition at line 1888 of file sf\_gamma.tcc.

 $References\ std::\_detail::\_Factorial\_table < \_Tp > ::\__n.$ 

9.3.2.17 template < typename \_Tp > \_Tp std::\_\_detail::\_\_bose\_einstein ( \_Tp  $\_s$ , \_Tp  $\_x$  )

Return the Bose-Einstein integral of real order s and real argument x.

# See also

```
https://en.wikipedia.org/wiki/Clausen_function
http://dlmf.nist.gov/25.12#iii
```

_~	The order $s >= 0$ .
_s	
_~	The real argument.
_X	

#### Returns

The real Fermi-Dirac cosine sum  $G_s(x)$ ,

Definition at line 1402 of file sf\_polylog.tcc.

References \_\_polylog\_exp().

9.3.2.18 template<typename \_Tp > \_Tp std::\_\_detail::\_\_chebyshev\_recur ( unsigned int \_\_n, \_Tp \_\_x, \_Tp \_C0, \_Tp \_C1 )

Return a Chebyshev polynomial of non-negative order n and real argument x by the recursion

$$C_n(x) = 2xC_{n-1} - C_{n-2}$$

# **Template Parameters**

_Tp   The real type of the argumen
------------------------------------

## **Parameters**

_~	The non-negative integral order
_n	
_←	The real argument $-1 \le x \le +1$
_X	
_C0	The value of the zeroth-order Chebyshev polynomial at $\boldsymbol{x}$
_C1	The value of the first-order Chebyshev polynomial at $\boldsymbol{x}$

Definition at line 57 of file sf\_chebyshev.tcc.

Referenced by \_\_chebyshev\_t(), \_\_chebyshev\_u(), \_\_chebyshev\_v(), and \_\_chebyshev\_w().

9.3.2.19 template < typename \_Tp > \_Tp std::\_\_detail::\_\_chebyshev\_t ( unsigned int \_\_n, \_Tp \_\_x )

Return the Chebyshev polynomial of the first kind  $T_n(x)$  of non-negative order n and real argument x.

The Chebyshev polynomial of the first kind is defined by:

$$T_n(x) = \cos(n\theta)$$

where  $\theta = \arccos(x)$ ,  $-1 \le x \le +1$ .

## **Template Parameters**

### **Parameters**

_~	The non-negative integral order
_n	
_~	The real argument $-1 \le x \le +1$
_X	

Definition at line 85 of file sf\_chebyshev.tcc.

References \_\_chebyshev\_recur().

Return the Chebyshev polynomial of the second kind  $U_n(x)$  of non-negative order n and real argument x.

The Chebyshev polynomial of the second kind is defined by:

$$U_n(x) = \frac{\sin[(n+1)\theta]}{\sin(\theta)}$$

where  $\theta = \arccos(x)$ ,  $-1 \le x \le +1$ .

## **Template Parameters**

_Тр	The real type of the argument

# **Parameters**

_~	The non-negative integral order
_n	
_~	The real argument $-1 \le x \le +1$
_x	

Definition at line 114 of file sf chebyshev.tcc.

References \_\_chebyshev\_recur().

Return the Chebyshev polynomial of the third kind  $V_n(x)$  of non-negative order n and real argument x.

The Chebyshev polynomial of the third kind is defined by:

$$V_n(x) = \frac{\cos\left[\left(n + \frac{1}{2}\right)\theta\right]}{\cos\left(\frac{\theta}{2}\right)}$$

where  $\theta = \arccos(x)$ ,  $-1 \le x \le +1$ .

# **Template Parameters**

_Тр	The real type of the argument
-----	-------------------------------

### **Parameters**

_~	The non-negative integral order
_n	
_←	The real argument $-1 \le x \le +1$
_X	

Definition at line 144 of file sf\_chebyshev.tcc.

References \_\_chebyshev\_recur().

9.3.2.22 template < typename \_Tp > \_Tp std::\_\_detail::\_\_chebyshev\_w ( unsigned int \_\_n, \_Tp \_\_x )

Return the Chebyshev polynomial of the fourth kind  $W_n(x)$  of non-negative order n and real argument x.

The Chebyshev polynomial of the fourth kind is defined by:

$$W_n(x) = \frac{\sin\left[\left(n + \frac{1}{2}\right)\theta\right]}{\sin\left(\frac{\theta}{2}\right)}$$

where  $\theta = \arccos(x)$ ,  $-1 \le x \le +1$ .

# **Template Parameters**

_Tp   The real type of the argumer
------------------------------------

## **Parameters**

_~	The non-negative integral order
_n	
_~	The real argument $-1 \le x \le +1$
_x	

Definition at line 174 of file sf chebyshev.tcc.

References \_\_chebyshev\_recur().

9.3.2.23 template < typename \_Tp > std::pair < \_Tp, \_Tp> std::\_\_chshint ( \_Tp \_\_x, \_Tp & \_Chi, \_Tp & \_Shi )

This function returns the hyperbolic cosine Ci(x) and hyperbolic sine Si(x) integrals as a pair.

The hyperbolic cosine integral is defined by:

$$Chi(x) = \gamma_E + \log(x) + \int_0^x dt \frac{\cosh(t) - 1}{t}$$

The hyperbolic sine integral is defined by:

$$Shi(x) = \int_0^x dt \frac{\sinh(t)}{t}$$

Definition at line 164 of file sf\_hypint.tcc.

References chshint cont frac(), and chshint series().

9.3.2.24 template < typename \_Tp > void std::\_\_detail::\_\_chshint\_cont\_frac ( \_Tp \_\_t, \_Tp & \_Chi, \_Tp & \_Shi )

This function computes the hyperbolic cosine Chi(x) and hyperbolic sine Shi(x) integrals by continued fraction for positive argument.

Definition at line 51 of file sf\_hypint.tcc.

Referenced by chshint().

9.3.2.25 template<typename\_Tp > void std::\_\_chshint\_series ( \_Tp \_\_t, \_Tp & \_Chi, \_Tp & \_Shi )

This function computes the hyperbolic cosine Chi(x) and hyperbolic sine Shi(x) integrals by series summation for positive argument.

Definition at line 94 of file sf hypint.tcc.

Referenced by chshint().

 $9.3.2.26 \quad template < typename \_Tp > std::complex < \_Tp > std::\_detail::\_clamp\_0\_m2pi ( \ std::complex < \_Tp > \_w )$ 

Definition at line 137 of file sf polylog.tcc.

Referenced by  $\_$ polylog\_exp\_int\_neg(),  $\_$ polylog\_exp\_int\_pos(),  $\_$ polylog\_exp\_real\_neg(), and  $\_$ polylog\_exp\_ $\leftarrow$ real\_pos().

9.3.2.27 template < typename  $_{Tp} >$  std::complex  $<_{Tp} >$  std::\_\_detail::\_\_clamp\_pi ( std::complex  $<_{Tp} >$   $_{w}$  )

Definition at line 124 of file sf\_polylog.tcc.

Referenced by  $\_$ polylog\_exp\_int\_neg(),  $\_$ polylog\_exp\_int\_pos(),  $\_$ polylog\_exp\_real\_neg(), and  $\_$ polylog\_exp\_ $\leftarrow$  real\_pos().

9.3.2.28 template < typename \_Tp > std::complex < \_Tp > std::\_\_detail::\_\_clausen ( unsigned int \_\_m, std::complex < \_Tp > \_\_w )

Return Clausen's function of integer order m and complex argument w. The notation and connection to polylog is from Wikipedia

_~	The non-negative integral order.
_m	
_←	The complex argument.
_ <i>w</i>	

## Returns

The complex Clausen function.

Definition at line 1231 of file sf\_polylog.tcc.

References \_\_polylog\_exp().

Return Clausen's function of integer order m and real argument w. The notation and connection to polylog is from Wikipedia

## **Parameters**

_~	The integer order $m \ge 1$ .
_m	
_←	The real argument.
_w	

### Returns

The Clausen function.

Definition at line 1255 of file sf polylog.tcc.

References \_\_polylog\_exp().

9.3.2.30 template < typename \_Tp > \_Tp std::\_\_detail::\_\_clausen\_c ( unsigned int \_\_m, std::complex < \_Tp > \_\_w )

Return Clausen's cosine sum Cl\_m for positive integer order m and complex argument w.

## See also

https://en.wikipedia.org/wiki/Clausen\_function

_~	The integer order $m >= 1$ .
_m	
_~	The real argument.
_ <i>w</i>	

#### Returns

The Clausen cosine sum Cl\_m(w),

Definition at line 1330 of file sf\_polylog.tcc.

References \_\_polylog\_exp().

9.3.2.31 template<typename \_Tp > \_Tp std::\_\_detail::\_\_clausen\_c ( unsigned int \_\_m, \_Tp \_\_w )

Return Clausen's cosine sum Cl\_m for positive integer order m and real argument w.

### See also

https://en.wikipedia.org/wiki/Clausen\_function

# **Parameters**

_~	The integer order $m >= 1$ .
_m	
_~	The real argument.
W	

## Returns

The real Clausen cosine sum Cl\_m(w),

Definition at line 1355 of file sf\_polylog.tcc.

References \_\_polylog\_exp().

9.3.2.32 template<typename\_Tp > \_Tp std::\_\_detail::\_\_clausen\_s ( unsigned int \_\_m, std::complex < \_Tp > \_\_w )

Return Clausen's sine sum SI\_m for positive integer order m and complex argument w.

# See also

https://en.wikipedia.org/wiki/Clausen\_function

_~	The integer order $m >= 1$ .
_m	
_~	The complex argument.
_ <i>w</i>	

## Returns

The Clausen sine sum SI\_m(w),

Definition at line 1280 of file sf\_polylog.tcc.

References \_\_polylog\_exp().

Return Clausen's sine sum SI\_m for positive integer order m and real argument w.

### See also

https://en.wikipedia.org/wiki/Clausen\_function

## **Parameters**

_←	The integer order $m >= 1$ .
_m	
_~	The complex argument.
_ <i>w</i>	

## Returns

The Clausen sine sum SI\_m(w),

Definition at line 1305 of file sf polylog.tcc.

References \_\_polylog\_exp().

9.3.2.34 template> \_Tp std::\_\_detail::\_\_comp\_ellint\_1 ( \_Tp 
$$\_k$$
 )

Return the complete elliptic integral of the first kind K(k) using the Carlson formulation.

The complete elliptic integral of the first kind is defined as

$$K(k) = F(k, \pi/2) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - k^2 sin^2 \theta}}$$

where  $F(k,\phi)$  is the incomplete elliptic integral of the first kind.

_~	The modulus of the complete elliptic function.
_k	

### Returns

The complete elliptic function of the first kind.

Definition at line 566 of file sf\_ellint.tcc.

References \_\_comp\_ellint\_rf().

 $Referenced \ by \underline{\hspace{0.4cm}} ellint\underline{\hspace{0.4cm}} 1(), \underline{\hspace{0.4cm}} ellnome\underline{\hspace{0.4cm}} k(), \underline{\hspace{0.4cm}} jacobi\underline{\hspace{0.4cm}} zeta(), \underline{\hspace{0.4cm}} theta\underline{\hspace{0.4cm}} c(), \underline{\hspace{0.4cm}} theta\underline{\hspace{0.4cm}} d(), \underline{\hspace{0.4cm}} theta\underline{\hspace{0.4cm}} n(), and \underline{\hspace{0.4cm}} theta\underline{\hspace{0.4cm}} s().$ 

9.3.2.35 template < typename  $_{\rm Tp} > _{\rm Tp}$  std::\_\_detail::\_\_comp\_ellint\_2 (  $_{\rm Tp}$  \_\_k )

Return the complete elliptic integral of the second kind E(k) using the Carlson formulation.

The complete elliptic integral of the second kind is defined as

$$E(k, \pi/2) = \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 \theta}$$

### **Parameters**

## Returns

The complete elliptic function of the second kind.

Definition at line 639 of file sf\_ellint.tcc.

References \_\_ellint\_rd(), and \_\_ellint\_rf().

Referenced by \_\_ellint\_2().

9.3.2.36 template < typename \_Tp > \_Tp std::\_\_detail::\_\_comp\_ellint\_3 ( \_Tp  $\_k$ , \_Tp  $\_nu$  )

Return the complete elliptic integral of the third kind  $\Pi(k,\nu)=\Pi(k,\nu,\pi/2)$  using the Carlson formulation.

The complete elliptic integral of the third kind is defined as

$$\Pi(k,\nu) = \int_0^{\pi/2} \frac{d\theta}{(1 - \nu \sin^2 \theta) \sqrt{1 - k^2 \sin^2 \theta}}$$

k	The argument of the elliptic function.
nu	The second argument of the elliptic function.

## Returns

The complete elliptic function of the third kind.

Definition at line 728 of file sf\_ellint.tcc.

References \_\_ellint\_rf(), and \_\_ellint\_rj().

Referenced by \_\_ellint\_3().

9.3.2.37 template> \_Tp std::\_\_detail::\_\_comp\_ellint\_d ( \_Tp 
$$\_k$$
 )

Return the complete Legendre elliptic integral D.

Definition at line 833 of file sf\_ellint.tcc.

References \_\_ellint\_rd().

Definition at line 236 of file sf ellint.tcc.

Referenced by \_\_comp\_ellint\_1(), and \_\_ellint\_rf().

Definition at line 347 of file sf\_ellint.tcc.

Referenced by \_\_ellint\_rg().

Return the confluent hypergeometric function  ${}_1F_1(a;c;x)$ .

## **Parameters**

_←	The <i>numerator</i> parameter.	
_a		
_~	The denominator parameter.	
c_		
_~	The argument of the confluent hypergeometric function.	Generated by Doxyge
_x		

#### Returns

The confluent hypergeometric function.

Definition at line 281 of file sf\_hyperg.tcc.

References \_\_conf\_hyperg\_luke(), and \_\_conf\_hyperg\_series().

9.3.2.41 template < typename \_Tp > \_Tp std::\_\_detail::\_\_conf\_hyperg\_lim ( \_Tp 
$$\_c$$
, \_Tp  $\_x$  )

Return the confluent hypergeometric limit function  ${}_0F_1(-;c;x)$ .

## **Parameters**

_~	The denominator parameter.
_c	
_~	The argument of the confluent hypergeometric limit function.
_X	

### Returns

The confluent limit hypergeometric function.

Definition at line 109 of file sf hyperg.tcc.

References \_\_conf\_hyperg\_lim\_series().

This routine returns the confluent hypergeometric limit function by series expansion.

$$_{0}F_{1}(-;c;x) = \Gamma(c) \sum_{n=0}^{\infty} \frac{1}{\Gamma(c+n)} \frac{x^{n}}{n!}$$

If a and b are integers and a < 0 and either b > 0 or b < a then the series is a polynomial with a finite number of terms.

## **Parameters**

_~	The "denominator" parameter.
_c	
_~	The argument of the confluent hypergeometric limit function.
_x	

## Returns

The confluent hypergeometric limit function.

Definition at line 76 of file sf\_hyperg.tcc.

Referenced by \_\_conf\_hyperg\_lim().

Return the hypergeometric function  ${}_1F_1(a;c;x)$  by an iterative procedure described in Luke, Algorithms for the Computation of Mathematical Functions.

Like the case of the 2F1 rational approximations, these are probably guaranteed to converge for x < 0, barring gross numerical instability in the pre-asymptotic regime.

Definition at line 176 of file sf\_hyperg.tcc.

Referenced by \_\_conf\_hyperg().

9.3.2.44 template < typename \_Tp > \_Tp std::\_\_detail::\_\_conf\_hyperg\_series ( \_Tp 
$$\_a$$
, \_Tp  $\_c$ , \_Tp  $\_x$  )

This routine returns the confluent hypergeometric function by series expansion.

$$_{1}F_{1}(a;c;x) = \frac{\Gamma(c)}{\Gamma(a)} \sum_{n=0}^{\infty} \frac{\Gamma(a+n)}{\Gamma(c+n)} \frac{x^{n}}{n!}$$

## **Parameters**

_~	The "numerator" parameter.
_a	
_←	The "denominator" parameter.
_c	
_~	The argument of the confluent hypergeometric function.
_x	

## Returns

The confluent hypergeometric function.

Definition at line 141 of file sf hyperg.tcc.

Referenced by \_\_conf\_hyperg().

9.3.2.45 template<typename \_Tp > \_Tp std::\_\_detail::\_\_coshint ( const \_Tp \_\_x )

Return the hyperbolic cosine integral li(x).

The hyperbolic cosine integral is given by

$$Chi(x) = (Ei(x) - E_1(x))/2$$

#### **Parameters**

_~	The argument of the hyperbolic cosine integral function.
_X	

### Returns

The hyperbolic cosine integral.

Definition at line 554 of file sf\_expint.tcc.

References \_\_expint\_E1(), and \_\_expint\_Ei().

9.3.2.46 template<typename\_Tp > std::complex<\_Tp> std::\_\_detail::\_\_cyl\_bessel ( std::complex<\_Tp > \_\_nu, std::complex< \_Tp > \_\_z )

Return the complex cylindrical Bessel function.

## **Parameters**

in	nu	The order for which the cylindrical Bessel function is evaluated.
in	z	The argument at which the cylindrical Bessel function is evaluated.

## Returns

The complex cylindrical Bessel function.

Definition at line 1178 of file sf\_hankel.tcc.

References \_\_hankel().

9.3.2.47 template < typename \_Tp > \_Tp std::\_\_detail::\_\_cyl\_bessel\_i ( \_Tp  $\_nu$ , \_Tp  $\_x$  )

Return the regular modified Bessel function of order  $\nu$ :  $I_{\nu}(x)$ .

The regular modified cylindrical Bessel function is:

$$I_{\nu}(x) = \sum_{k=0}^{\infty} \frac{(x/2)^{\nu+2k}}{k!\Gamma(\nu+k+1)}$$

nu	The order of the regular modified Bessel function.
x	The argument of the regular modified Bessel function.

### Returns

The output regular modified Bessel function.

Definition at line 387 of file sf\_mod\_bessel.tcc.

References \_\_cyl\_bessel\_ij\_series(), and \_\_cyl\_bessel\_ik().

This routine returns the cylindrical Bessel functions of order  $\nu$ :  $J_{\nu}$  or  $I_{\nu}$  by series expansion.

The modified cylindrical Bessel function is:

$$Z_{\nu}(x) = \sum_{k=0}^{\infty} \frac{\sigma^{k}(x/2)^{\nu+2k}}{k!\Gamma(\nu+k+1)}$$

where  $\sigma = +1$  or -1 for Z = I or J respectively.

See Abramowitz & Stegun, 9.1.10 Abramowitz & Stegun, 9.6.7 (1) Handbook of Mathematical Functions, ed. Milton Abramowitz and Irene A. Stegun, Dover Publications, Equation 9.1.10 p. 360 and Equation 9.6.10 p. 375

## **Parameters**

nu	The order of the Bessel function.	
x	The argument of the Bessel function.	
sgn	The sign of the alternate terms -1 for the Bessel function of the first kind. +1 for the modified Bessel function of the first kind.	
max_iter	The maximum number of iterations for sum.	

#### Returns

The output Bessel function.

Definition at line 414 of file sf bessel.tcc.

References \_\_log\_gamma().

Referenced by \_\_cyl\_bessel\_i(), and \_\_cyl\_bessel\_j().

9.3.2.49 template<typename\_Tp > void std::\_\_detail::\_\_cyl\_bessel\_ik ( \_Tp \_\_nu, \_Tp \_\_x, \_Tp & \_Inu, \_Tp & \_Knu, \_Tp & \_Ipnu, \_Tp & \_Kpnu )

Return the modified cylindrical Bessel functions and their derivatives of order  $\nu$  by various means.

nu	The order of the Bessel functions.	
x	The argument of the Bessel functions.	
_Inu	The output regular modified Bessel function.	
_Knu	The output irregular modified Bessel function.	
_lpnu	The output derivative of the regular modified Bessel function.	
_Kpnu    The output derivative of the irregular modified Bessel fu		

Definition at line 317 of file sf mod bessel.tcc.

References \_\_cyl\_bessel\_ik\_asymp(), and \_\_cyl\_bessel\_ik\_steed().

Referenced by \_\_airy(), \_\_cyl\_bessel\_i(), \_\_cyl\_bessel\_k(), and \_\_sph\_bessel\_ik().

9.3.2.50 template<typename\_Tp > void std::\_\_cyl\_bessel\_ik\_asymp ( \_Tp \_\_nu, \_Tp \_\_x, \_Tp & \_Inu, \_Tp & \_Knu, \_Tp & \_Inu, \_Tp & \_Knu, \_Tp &

This routine computes the asymptotic modified cylindrical Bessel and functions of order nu:  $I_{\nu}(x)$ ,  $N_{\nu}(x)$ . Use this for  $x >> nu^2 + 1$ .

References: (1) Handbook of Mathematical Functions, ed. Milton Abramowitz and Irene A. Stegun, Dover Publications, Section 9 p. 364, Equations 9.2.5-9.2.10

## **Parameters**

nu	The order of the Bessel functions.		
x	The argument of the Bessel functions.		
_Inu	The output regular modified Bessel function.		
_Knu	The output irregular modified Bessel function.		
_lpnu			
_Kpnu   The output derivative of the irregular modified Bessel fund			

Definition at line 82 of file sf\_mod\_bessel.tcc.

Referenced by \_\_cyl\_bessel\_ik(), and \_\_cyl\_bessel\_ik\_steed().

9.3.2.51 template<typename\_Tp > void std::\_\_detail::\_\_cyl\_bessel\_ik\_steed ( \_Tp \_\_nu, \_Tp \_\_x, \_Tp & \_Inu, \_Tp & \_Knu, \_Tp & \_Ipnu, \_Tp & \_Kpnu )

Compute the modified Bessel functions  $I_{\nu}(x)$  and  $K_{\nu}(x)$  and their first derivatives  $I'_{\nu}(x)$  and  $K'_{\nu}(x)$  respectively. These four functions are computed together for numerical stability.

### **Parameters**

nu	The order of the Bessel functions.
----	------------------------------------

x	The argument of the Bessel functions.	
_Inu	_Inu The output regular modified Bessel function.	
_Knu	The output irregular modified Bessel function.	
_lpnu	The output derivative of the regular modified Bessel function.	
_Kpnu	The output derivative of the irregular modified Bessel function.	

Definition at line 153 of file sf\_mod\_bessel.tcc.

References \_\_cyl\_bessel\_ik\_asymp(), and \_\_gamma\_temme().

Referenced by \_\_cyl\_bessel\_ik().

9.3.2.52 template < typename \_Tp > \_Tp std::\_\_detail::\_\_cyl\_bessel\_j ( \_Tp \_\_nu, \_Tp \_\_x )

Return the Bessel function of order  $\nu$ :  $J_{\nu}(x)$ .

The cylindrical Bessel function is:

$$J_{\nu}(x) = \sum_{k=0}^{\infty} \frac{(-1)^k (x/2)^{\nu+2k}}{k! \Gamma(\nu+k+1)}$$

## **Parameters**

nu	The order of the Bessel function.
x	The argument of the Bessel function.

# Returns

The output Bessel function.

Definition at line 534 of file sf\_bessel.tcc.

References \_\_cyl\_bessel\_ij\_series(), and \_\_cyl\_bessel\_in().

Return the cylindrical Bessel functions and their derivatives of order  $\nu$  by various means.

Definition at line 453 of file sf\_bessel.tcc.

References cyl bessel in asymp(), and cyl bessel in steed().

Referenced by  $\_airy()$ ,  $\_cyl\_bessel\_j()$ ,  $\_cyl\_hankel\_1()$ ,  $\_cyl\_hankel\_2()$ ,  $\_cyl\_neumann\_n()$ , and  $\_sph\_\leftrightarrow bessel\_jn()$ .

9.3.2.54 template < typename \_Tp > void std::\_\_detail::\_\_cyl\_bessel\_jn\_asymp ( \_Tp \_\_nu, \_Tp \_\_x, \_Tp & \_Jnu, \_Tp & \_Nnu, \_Tp & \_Jnu, \_Tp & \_Nnu, \_Tp

This routine computes the asymptotic cylindrical Bessel and Neumann functions of order nu:  $J_{\nu}(x)$ ,  $N_{\nu}(x)$ . Use this for  $x >> nu^2 + 1$ .

References: (1) Handbook of Mathematical Functions, ed. Milton Abramowitz and Irene A. Stegun, Dover Publications, Section 9 p. 364, Equations 9.2.5-9.2.10

#### **Parameters**

	nu	The order of the Bessel functions.
	x	The argument of the Bessel functions.
out	_Jnu	The Bessel function of the first kind.
out	_Nnu	The Neumann function (Bessel function of the second kind).
out	_Jpnu	The Bessel function of the first kind.
out	_Npnu	The Neumann function (Bessel function of the second kind).

Definition at line 80 of file sf\_bessel.tcc.

Referenced by \_\_cyl\_bessel\_jn(), and \_\_cyl\_bessel\_jn\_steed().

Compute the Bessel  $J_{\nu}(x)$  and Neumann  $N_{\nu}(x)$  functions and their first derivatives  $J'_{\nu}(x)$  and  $N'_{\nu}(x)$  respectively. These four functions are computed together for numerical stability.

## **Parameters**

	nu	The order of the Bessel functions.
	x	The argument of the Bessel functions.
out	_Jnu	The output Bessel function of the first kind.
out	_Nnu	The output Neumann function (Bessel function of the second kind).
out	_Jpnu	The output derivative of the Bessel function of the first kind.
out	_Npnu	The output derivative of the Neumann function.

Definition at line 198 of file sf\_bessel.tcc.

References cyl bessel in asymp(), and gamma temme().

Referenced by \_\_cyl\_bessel\_jn().

9.3.2.56 template<typename \_Tp > \_Tp std::\_\_detail::\_\_cyl\_bessel\_k ( \_Tp \_\_nu, \_Tp \_\_x )

Return the irregular modified Bessel function  $K_{\nu}(x)$  of order  $\nu$ .

The irregular modified Bessel function is defined by:

$$K_{\nu}(x) = \frac{\pi}{2} \frac{I_{-\nu}(x) - I_{\nu}(x)}{\sin \nu \pi}$$

where for integral  $\nu=n$  a limit is taken:  $lim_{\nu\to n}$ . For negative argument we have simply:

$$K_{-\nu}(x) = K_{\nu}(x)$$

#### **Parameters**

nu	The order of the irregular modified Bessel function.
x	The argument of the irregular modified Bessel function.

#### Returns

The output irregular modified Bessel function.

Definition at line 425 of file sf\_mod\_bessel.tcc.

References \_\_cyl\_bessel\_ik().

9.3.2.57 template<typename\_Tp > std::complex<\_Tp> std::\_\_detail::\_\_cyl\_hankel\_1 ( \_Tp \_\_nu, \_Tp \_\_x )

Return the cylindrical Hankel function of the first kind  $H^{(1)}_{\nu}(x)$ .

The cylindrical Hankel function of the first kind is defined by:

$$H_{\nu}^{(1)}(x) = J_{\nu}(x) + iN_{\nu}(x)$$

# **Parameters**

nu	The order of the spherical Neumann function.
x	The argument of the spherical Neumann function.

# Returns

The output spherical Neumann function.

Definition at line 599 of file sf\_bessel.tcc.

References \_\_cyl\_bessel\_jn().

9.3.2.58 template < typename \_Tp > std::complex < \_Tp > std::\_\_detail::\_\_cyl\_hankel\_1 ( std::complex < \_Tp > \_\_nu, std::complex < \_Tp > \_\_z )

Return the complex cylindrical Hankel function of the first kind.

in	nu	The order for which the cylindrical Hankel function of the first kind is evaluated.
in	z	The argument at which the cylindrical Hankel function of the first kind is evaluated.

# Returns

The complex cylindrical Hankel function of the first kind.

Definition at line 1146 of file sf hankel.tcc.

References \_\_hankel().

9.3.2.59 template<typename\_Tp > std::complex<\_Tp> std::\_\_detail::\_\_cyl\_hankel\_2 ( \_Tp \_\_nu, \_Tp \_\_x )

Return the cylindrical Hankel function of the second kind  $H_n^{(2)}u(x)$ .

The cylindrical Hankel function of the second kind is defined by:

$$H_{\nu}^{(2)}(x) = J_{\nu}(x) - iN_{\nu}(x)$$

# **Parameters**

nu	The order of the spherical Neumann function.
x	The argument of the spherical Neumann function.

# Returns

The output spherical Neumann function.

Definition at line 634 of file sf\_bessel.tcc.

References \_\_cyl\_bessel\_jn().

9.3.2.60 template<typename \_Tp > std::complex<\_Tp> std::\_\_detail::\_\_cyl\_hankel\_2 ( std::complex< \_Tp > \_\_nu, std::complex< \_Tp > \_\_z )

Return the complex cylindrical Hankel function of the second kind.

in	nu	The order for which the cylindrical Hankel function of the second kind is evaluated.
in	z	The argument at which the cylindrical Hankel function of the second kind is evaluated.

#### Returns

The complex cylindrical Hankel function of the second kind.

Definition at line 1162 of file sf\_hankel.tcc.

References \_\_hankel().

9.3.2.61 template < typename \_Tp > std::complex < \_Tp > std::\_\_detail::\_\_cyl\_neumann ( std::complex < \_Tp > \_\_nu, std::complex < \_Tp > \_\_z )

Return the complex cylindrical Neumann function.

# **Parameters**

in	nu	The order for which the cylindrical Neumann function is evaluated.
in	z	The argument at which the cylindrical Neumann function is evaluated.

# Returns

The complex cylindrical Neumann function.

Definition at line 1194 of file sf\_hankel.tcc.

References \_\_hankel().

9.3.2.62 template<typename\_Tp > \_Tp std::\_\_detail::\_\_cyl\_neumann\_n ( \_Tp \_\_nu, \_Tp \_\_x )

Return the Neumann function of order  $\nu$ :  $N_{\nu}(x)$ .

The Neumann function is defined by:

$$N_{\nu}(x) = \frac{J_{\nu}(x)\cos\nu\pi - J_{-\nu}(x)}{\sin\nu\pi}$$

where for integral  $\nu = n$  a limit is taken:  $\lim_{\nu \to n}$ .

# **Parameters**

nu	The order of the Neumann function.
X	The argument of the Neumann function.

# Returns

The output Neumann function.

Definition at line 569 of file sf\_bessel.tcc.

References \_\_cyl\_bessel\_jn().

9.3.2.63 template<typename \_Tp > \_Tp std::\_\_dawson ( \_Tp \_\_x )

Return the Dawson integral, F(x), for real argument x.

The Dawson integral is defined by:

$$F(x) = e^{-x^2} \int_0^x e^{y^2} dy$$

and it's derivative is:

$$F'(x) = 1 - 2xF(x)$$

# **Parameters**

$$\begin{array}{|c|c|c|c|} \hline \_ \leftarrow & \text{The argument } -inf < x < inf. \\ \_ \textbf{\textit{X}} & \end{array}$$

Definition at line 233 of file sf\_dawson.tcc.

References \_\_dawson\_cont\_frac(), and \_\_dawson\_series().

9.3.2.64 template<typename \_Tp > \_Tp std::\_\_detail::\_\_dawson\_cont\_frac ( \_Tp \_\_x )

Compute the Dawson integral using a sampling theorem representation.

Todo this needs some compile-time construction!

Definition at line 71 of file sf\_dawson.tcc.

Referenced by \_\_dawson().

9.3.2.65 template<typename \_Tp > \_Tp std::\_\_detail::\_\_dawson\_series ( \_Tp \_\_x )

Compute the Dawson integral using the series expansion.

Definition at line 47 of file sf\_dawson.tcc.

Referenced by \_\_dawson().

9.3.2.66 template < typename \_Tp > void std::\_\_detail::\_\_debye\_region ( std::complex < \_Tp > \_\_alpha, int & \_\_indexr, char & \_\_aorb )

Compute the Debye region in te complex plane.

Definition at line 56 of file sf hankel.tcc.

Referenced by \_\_hankel().

9.3.2.67 template<typename \_Tp > \_Tp std::\_\_detail::\_\_dilog ( \_Tp \_\_x )

Compute the dilogarithm function  $Li_2(x)$  by summation for x <= 1.

The Riemann zeta function is defined by:

$$Li_2(x) = \sum_{k=1}^{\infty} \frac{1}{k^s} fors > 1$$

For |x| near 1 use the reflection formulae:

$$Li_2(-x) + Li_2(1-x) = \frac{\pi^2}{6} - \ln(x)\ln(1-x)$$

$$Li_2(-x) - Li_2(1-x) - \frac{1}{2}Li_2(1-x^2) = -\frac{\pi^2}{12} - \ln(x)\ln(1-x)$$

For x < 1 use the reflection formula:

$$Li_2(1-x) - Li_2(1-\frac{1}{1-x}) - \frac{1}{2}(\ln(x))^2$$

Definition at line 194 of file sf\_zeta.tcc.

9.3.2.68 template < typename \_Tp > \_Tp std::\_\_detail::\_\_dirichlet\_beta ( std::complex < \_Tp > \_\_w )

Return the Dirichlet beta function. Currently, w must be real (complex type but negligible imaginary part.) Otherwise std::domain error is thrown.

#### **Parameters**

_~	The complex (but on-real-axis) argument.
_ <i>w</i>	

# Returns

The Dirichlet Beta function of real argument.

# **Exceptions**

main_error if the argument has a significant imaginary part.	std::domain_error
--	-------------------

Definition at line 1193 of file sf polylog.tcc.

References \_\_fpequal(), and \_\_polylog().

9.3.2.69 template < typename \_Tp > \_Tp std::\_\_detail::\_\_dirichlet\_beta ( \_Tp  $\_w$  )

Return the Dirichlet beta function for real argument.

_~	The real argument.
_ <i>w</i>	

# Returns

The Dirichlet Beta function of real argument.

Definition at line 1212 of file sf\_polylog.tcc.

References \_\_polylog().

Return the Dirichlet eta function. Currently, w must be real (complex type but negligible imaginary part.) Otherwise std::domain\_error is thrown.

#### **Parameters**

```
_ ← The complex (but on-real-axis) argument.
```

# Returns

The complex Dirichlet eta function.

# **Exceptions**

	14.1
std::domain_error	if the argument has a significant imaginary part.

Definition at line 1156 of file sf\_polylog.tcc.

References \_\_fpequal(), and \_\_polylog().

9.3.2.71 template < typename \_Tp > \_Tp std::\_\_detail::\_\_dirichlet\_eta ( \_Tp  $\_w$  )

Return the Dirichlet eta function for real argument.

_~	The real argument.
_ <i>w</i>	

Returns

The Dirichlet eta function.

Definition at line 1174 of file sf polylog.tcc.

References \_\_polylog().

9.3.2.72 template < typename \_Tp > \_GLIBCXX14\_CONSTEXPR \_Tp std::\_\_detail::\_\_double\_factorial ( int \_\_n )

Return the double factorial of the integer n.

The double factorial is defined for integral n by:

$$n!! = 135...(n-2)n, noddn!! = 246...(n-2)n, neven - 1!! = 10!! = 1$$

The double factorial is defined for odd negative integers in the obvious way:

$$(-2m-1)!! = 1/(1(-1)(-3)...(-2m+1)(-2m-1)) = \frac{(-1)^m}{(2m-1)!!}$$

for f[ n = -2m - 1 f].

Definition at line 2482 of file sf gamma.tcc.

References std::\_\_detail::\_Factorial\_table< \_Tp >::\_\_factorial, \_\_log\_double\_factorial(), std::\_\_detail::\_Factorial\_ $\leftarrow$  table< \_Tp >::\_\_n, \_S\_\_double\_factorial\_table, and \_S\_neg\_\_double\_factorial\_table.

9.3.2.73 template<typename \_Tp > \_Tp std::\_\_detail::\_\_ellint\_1 ( \_Tp \_\_k, \_Tp \_\_phi )

Return the incomplete elliptic integral of the first kind  $F(k,\phi)$  using the Carlson formulation.

The incomplete elliptic integral of the first kind is defined as

$$F(k,\phi) = \int_0^\phi \frac{d\theta}{\sqrt{1 - k^2 sin^2 \theta}}$$

# **Parameters**

k	The argument of the elliptic function.
phi	The integral limit argument of the elliptic function.

# Returns

The elliptic function of the first kind.

Definition at line 595 of file sf\_ellint.tcc.

References comp ellint 1(), and ellint rf().

9.3.2.74 template<typename \_Tp > \_Tp std::\_\_detail::\_\_ellint\_2 ( \_Tp \_\_k, \_Tp \_\_phi )

Return the incomplete elliptic integral of the second kind  $E(k,\phi)$  using the Carlson formulation.

The incomplete elliptic integral of the second kind is defined as

$$E(k,\phi) = \int_0^{\phi} \sqrt{1 - k^2 sin^2 \theta}$$

# **Parameters**

k	The argument of the elliptic function.
phi	The integral limit argument of the elliptic function.

# Returns

The elliptic function of the second kind.

Definition at line 674 of file sf ellint.tcc.

References \_\_comp\_ellint\_2(), \_\_ellint\_rd(), and \_\_ellint\_rf().

9.3.2.75 template < typename \_Tp > \_Tp std::\_\_detail::\_\_ellint\_3 ( \_Tp  $\_k$ , \_Tp  $\_nu$ , \_Tp  $\_phi$  )

Return the incomplete elliptic integral of the third kind  $\Pi(k,\nu,\phi)$  using the Carlson formulation.

The incomplete elliptic integral of the third kind is defined as

$$\Pi(k,\nu,\phi) = \int_0^\phi \frac{d\theta}{(1-\nu\sin^2\theta)\sqrt{1-k^2\sin^2\theta}}$$

#### **Parameters**

k	The argument of the elliptic function.
nu	The second argument of the elliptic function.
phi	The integral limit argument of the elliptic function.

# Returns

The elliptic function of the third kind.

Definition at line 769 of file sf\_ellint.tcc.

References \_\_comp\_ellint\_3(), \_\_ellint\_rf(), and \_\_ellint\_rj().

9.3.2.76 template < typename  $_{\rm Tp} > _{\rm Tp}$  std::\_\_ellint\_cel (  $_{\rm Tp} \_k\_c, _{\rm Tp} \_p, _{\rm Tp} \_a, _{\rm Tp} \_b$  )

Return the Bulirsch complete elliptic integrals.

Definition at line 921 of file sf\_ellint.tcc.

References \_\_ellint\_rf(), and \_\_ellint\_rj().

9.3.2.77 template<typename \_Tp > \_Tp std::\_\_detail::\_\_ellint\_d ( \_Tp \_\_k, \_Tp \_\_phi )

Return the Legendre elliptic integral D.

Definition at line 810 of file sf\_ellint.tcc.

References \_\_ellint\_rd().

9.3.2.78 template < typename \_Tp > \_Tp std::\_\_detail::\_\_ellint\_el1 ( \_Tp  $\_x$ , \_Tp  $\_k\_c$  )

Return the Bulirsch elliptic integrals of the first kind.

Definition at line 849 of file sf ellint.tcc.

References \_\_ellint\_rf().

9.3.2.79 template<typename\_Tp > \_Tp std::\_\_detail::\_\_ellint\_el2 ( \_Tp \_\_x, \_Tp \_\_k\_c, \_Tp \_\_a, \_Tp \_\_b )

Return the Bulirsch elliptic integrals of the second kind.

Definition at line 870 of file sf ellint.tcc.

References \_\_ellint\_rd(), and \_\_ellint\_rf().

9.3.2.80 template < typename \_Tp > \_Tp std::\_\_detail::\_\_ellint\_el3 ( \_Tp \_\_x, \_Tp \_\_k\_c, \_Tp \_\_p )

Return the Bulirsch elliptic integrals of the third kind.

Definition at line 895 of file sf ellint.tcc.

References \_\_ellint\_rf(), and \_\_ellint\_rj().

9.3.2.81 template < typename  $_{\rm Tp} > _{\rm Tp}$  std::\_\_ellint\_rc (  $_{\rm Tp}$  \_\_x,  $_{\rm Tp}$  \_\_y )

Return the Carlson elliptic function  $R_C(x,y) = R_F(x,y,y)$  where  $R_F(x,y,z)$  is the Carlson elliptic function of the first kind.

The Carlson elliptic function is defined by:

$$R_C(x,y) = \frac{1}{2} \int_0^\infty \frac{dt}{(t+x)^{1/2}(t+y)}$$

Based on Carlson's algorithms:

- B. C. Carlson Numer. Math. 33, 1 (1979)
- B. C. Carlson, Special Functions of Applied Mathematics (1977)
- Numerical Recipes in C, 2nd ed, pp. 261-269, by Press, Teukolsky, Vetterling, Flannery (1992)

_~	The first argument.
_X	
_~	The second argument.
y	

# Returns

The Carlson elliptic function.

Definition at line 82 of file sf\_ellint.tcc.

Referenced by \_\_ellint\_rf(), and \_\_ellint\_rj().

9.3.2.82 template<typename\_Tp > \_Tp std::\_\_detail::\_\_ellint\_rd ( \_Tp \_\_x, \_Tp \_\_y, \_Tp \_\_z )

Return the Carlson elliptic function of the second kind  $R_D(x,y,z) = R_J(x,y,z,z)$  where  $R_J(x,y,z,p)$  is the Carlson elliptic function of the third kind.

The Carlson elliptic function of the second kind is defined by:

$$R_D(x,y,z) = \frac{3}{2} \int_0^\infty \frac{dt}{(t+x)^{1/2}(t+y)^{1/2}(t+z)^{3/2}}$$

Based on Carlson's algorithms:

- B. C. Carlson Numer. Math. 33, 1 (1979)
- B. C. Carlson, Special Functions of Applied Mathematics (1977)
- Numerical Recipes in C, 2nd ed, pp. 261-269, by Press, Teukolsky, Vetterling, Flannery (1992)

#### **Parameters**

_~	The first of two symmetric arguments.
_X	
_~	The second of two symmetric arguments.
_y	
_~	The third argument.
_Z	

# Returns

The Carlson elliptic function of the second kind.

Definition at line 164 of file sf ellint.tcc.

Referenced by  $\_$ comp $\_$ ellint $\_$ 2(),  $\_$ ellint $\_$ d(),  $\_$ ellint $\_$ d(),  $\_$ ellint $\_$ d(),  $\_$ ellint $\_$ ellint $\_$ rg(), and  $\_$ ellint $\_$ rj().

9.3.2.83 template<typename \_Tp > \_Tp std::\_\_detail::\_\_ellint\_rf ( \_Tp \_\_x, \_Tp \_\_y, \_Tp \_\_z )

Return the Carlson elliptic function  $R_F(x,y,z)$  of the first kind.

The Carlson elliptic function of the first kind is defined by:

$$R_F(x,y,z) = \frac{1}{2} \int_0^\infty \frac{dt}{(t+x)^{1/2}(t+y)^{1/2}(t+z)^{1/2}}$$

#### **Parameters**

_~	The first of three symmetric arguments.
_X	
_~	The second of three symmetric arguments.
_y	
_~	The third of three symmetric arguments.
_Z	

#### Returns

The Carlson elliptic function of the first kind.

Definition at line 278 of file sf ellint.tcc.

References comp ellint rf(), and ellint rc().

Referenced by \_\_comp\_ellint\_2(), \_\_comp\_ellint\_3(), \_\_ellint\_1(), \_\_ellint\_2(), \_\_ellint\_3(), \_\_ellint\_cel(), \_\_ellint\_el1(), \_\_ellint\_el2(), \_\_ellint\_el3(), and \_\_heuman\_lambda().

9.3.2.84 template<typename\_Tp > \_Tp std::\_\_detail::\_\_ellint\_rg ( \_Tp \_\_x, \_Tp \_\_y, \_Tp \_\_z )

Return the symmetric Carlson elliptic function of the second kind  $R_G(x, y, z)$ .

The Carlson symmetric elliptic function of the second kind is defined by:

$$R_G(x,y,z) = \frac{1}{4} \int_0^\infty dt t [(t+x)(t+y)(t+z)]^{-1/2} \left(\frac{x}{t+x} + \frac{y}{t+y} + \frac{z}{t+z}\right)$$

Based on Carlson's algorithms:

- B. C. Carlson Numer. Math. 33, 1 (1979)
- B. C. Carlson, Special Functions of Applied Mathematics (1977)
- Numerical Recipes in C, 2nd ed, pp. 261-269, by Press, Teukolsky, Vetterling, Flannery (1992)

_~	The first of three symmetric arguments.
_X	
_~	The second of three symmetric arguments.
_y	
_~	The third of three symmetric arguments.
_Z	

# Returns

The Carlson symmetric elliptic function of the second kind.

Definition at line 409 of file sf\_ellint.tcc.

References \_\_comp\_ellint\_rg(), and \_\_ellint\_rd().

Return the Carlson elliptic function  $R_J(x, y, z, p)$  of the third kind.

The Carlson elliptic function of the third kind is defined by:

$$R_J(x,y,z,p) = \frac{3}{2} \int_0^\infty \frac{dt}{(t+x)^{1/2}(t+y)^{1/2}(t+z)^{1/2}(t+p)}$$

Based on Carlson's algorithms:

- B. C. Carlson Numer. Math. 33, 1 (1979)
- B. C. Carlson, Special Functions of Applied Mathematics (1977)
- Numerical Recipes in C, 2nd ed, pp. 261-269, by Press, Teukolsky, Vetterling, Flannery (1992)

_~	The first of three symmetric arguments.
_x	
_←	The second of three symmetric arguments.
_y	
_~	The third of three symmetric arguments.
_Z	
_~	The fourth argument.
_p	

Returns

The Carlson elliptic function of the fourth kind.

Definition at line 457 of file sf\_ellint.tcc.

References ellint rc(), and ellint rd().

Referenced by \_\_comp\_ellint\_3(), \_\_ellint\_cel(), \_\_ellint\_el3(), \_\_heuman\_lambda(), and \_\_jacobi\_zeta().

9.3.2.86 template < typename \_Tp > \_Tp std::\_\_detail::\_\_ellnome ( \_Tp  $\underline{\hspace{0.1cm}}$  )

Return the elliptic nome given the modulus k.

Definition at line 292 of file sf\_theta.tcc.

References \_\_ellnome\_k(), and \_\_ellnome\_series().

Referenced by \_\_theta\_c(), \_\_theta\_d(), \_\_theta\_n(), and \_\_theta\_s().

9.3.2.87 template < typename  $_{\rm Tp} > _{\rm Tp}$  std::\_\_detail::\_\_ellnome\_k (  $_{\rm Tp}$  \_\_k )

Use the arithmetic-geometric mean to calculate the elliptic nome given the , k.

Definition at line 278 of file sf theta.tcc.

References \_\_comp\_ellint\_1().

Referenced by \_\_ellnome().

9.3.2.88 template < typename \_Tp > \_Tp std::\_\_detail::\_\_ellnome\_series ( \_Tp  $\_k$  )

Use MacLaurin series to calculate the elliptic nome given the , k.

Definition at line 262 of file sf\_theta.tcc.

Referenced by \_\_ellnome().

9.3.2.89 template < typename  $_{\rm Tp}$  >  $_{\rm Tp}$  std::\_\_expint ( unsigned int  $_{\rm n}$ ,  $_{\rm Tp}$  \_\_x )

Return the exponential integral  $E_n(x)$ .

$$E_n(x) = \int_1^\infty \frac{e^{-xt}}{t^n} dt$$

_~	The order of the exponential integral function.
_n	
_←	The argument of the exponential integral function.
_x	

# Returns

The exponential integral.

Definition at line 470 of file sf\_expint.tcc.

References \_\_expint\_E1(), and \_\_expint\_En\_recursion().

Referenced by \_\_logint().

9.3.2.90 template < typename  $_{\rm Tp}$  >  $_{\rm Tp}$  std::\_\_detail::\_\_expint (  $_{\rm Tp}$  \_\_x )

Return the exponential integral Ei(x).

The exponential integral is given by

$$Ei(x) = -\int_{-x}^{\infty} \frac{e^t}{t} dt$$

# Parameters

_~	The argument of the exponential integral function.
_X	

# Returns

The exponential integral.

Definition at line 510 of file sf\_expint.tcc.

References \_\_expint\_Ei().

9.3.2.91 template<typename\_Tp > \_Tp std::\_\_detail::\_\_expint\_asymp ( unsigned int \_\_n, \_Tp \_\_x )

Return the exponential integral  $E_n(x)$  for large argument.

The exponential integral is given by

$$E_n(x) = \int_1^\infty \frac{e^{-xt}}{t^n} dt$$

This is something of an extension.

_~	The order of the exponential integral function.
_n	
_~	The argument of the exponential integral function.
_X	

# Returns

The exponential integral.

Definition at line 403 of file sf expint.tcc.

9.3.2.92 template < typename \_Tp > \_Tp std::\_\_detail::\_\_expint\_E1 ( \_Tp \_\_x )

Return the exponential integral  $E_1(x)$ .

The exponential integral is given by

$$E_1(x) = \int_1^\infty \frac{e^{-xt}}{t} dt$$

#### **Parameters**

_~	The argument of the exponential integral function.
_X	

# Returns

The exponential integral.

**Todo** Find a good asymptotic switch point in  $E_1(x)$ .

**Todo** Find a good asymptotic switch point in  $E_1(x)$ .

Definition at line 372 of file sf\_expint.tcc.

References \_\_expint\_E1\_asymp(), \_\_expint\_E1\_series(), \_\_expint\_Ei(), and \_\_expint\_En\_cont\_frac().

Referenced by \_\_coshint(), \_\_expint(), \_\_expint\_Ei(), \_\_expint\_En\_recursion(), and \_\_sinhint().

9.3.2.93 template < typename  $_{\rm Tp} > _{\rm Tp}$  std::\_\_expint\_E1\_asymp (  $_{\rm Tp} _{\rm x}$  )

Return the exponential integral  $E_1(x)$  by asymptotic expansion.

$$E_1(x) = \int_1^\infty \frac{e^{-xt}}{t} dt$$

_~	The argument of the exponential integral function.
_X	

# Returns

The exponential integral.

Definition at line 111 of file sf\_expint.tcc.

Referenced by \_\_expint\_E1().

Return the exponential integral  $E_1(x)$  by series summation. This should be good for x < 1.

The exponential integral is given by

$$E_1(x) = \int_1^\infty \frac{e^{-xt}}{t} dt$$

# **Parameters**

_~	The argument of the exponential integral function.
_X	

# Returns

The exponential integral.

Definition at line 74 of file sf\_expint.tcc.

Referenced by \_\_expint\_E1().

Return the exponential integral Ei(x).

$$Ei(x) = -\int_{-x}^{\infty} \frac{e^t}{t} dt$$

_~	The argument of the exponential integral function.
_X	

# Returns

The exponential integral.

Definition at line 348 of file sf\_expint.tcc.

References \_\_expint\_E1(), \_\_expint\_Ei\_asymp(), and \_\_expint\_Ei\_series().

Referenced by \_\_coshint(), \_\_expint(), \_\_expint\_E1(), and \_\_sinhint().

9.3.2.96 template<typename \_Tp > \_Tp std::\_\_detail::\_\_expint\_Ei\_asymp ( \_Tp \_\_x )

Return the exponential integral Ei(x) by asymptotic expansion.

The exponential integral is given by

$$Ei(x) = -\int_{-x}^{\infty} \frac{e^t}{t} dt$$

# **Parameters**

_~	The argument of the exponential integral function.
_X	

# Returns

The exponential integral.

Definition at line 315 of file sf\_expint.tcc.

Referenced by \_\_expint\_Ei().

9.3.2.97 template<typename \_Tp > \_Tp std::\_\_detail::\_\_expint\_Ei\_series ( \_Tp \_\_x )

Return the exponential integral Ei(x) by series summation.

$$Ei(x) = -\int_{-x}^{\infty} \frac{e^t}{t} dt$$

_~	The argument of the exponential integral function.
_X	

# Returns

The exponential integral.

Definition at line 283 of file sf\_expint.tcc.

Referenced by \_\_expint\_Ei().

9.3.2.98 template < typename \_Tp > \_Tp std::\_\_detail::\_\_expint\_En\_cont\_frac ( unsigned int \_\_n, \_Tp \_\_x )

Return the exponential integral  $E_n(x)$  by continued fractions.

The exponential integral is given by

$$E_n(x) = \int_1^\infty \frac{e^{-xt}}{t^n} dt$$

# **Parameters**

_~	The order of the exponential integral function.
_n	
_~	The argument of the exponential integral function.
_X	

# Returns

The exponential integral.

Definition at line 193 of file sf\_expint.tcc.

Referenced by \_\_expint\_E1().

9.3.2.99 template < typename  $_{\rm Tp}$  >  $_{\rm Tp}$  std::\_\_expint\_En\_recursion ( unsigned int  $_{\rm n}$ ,  $_{\rm Tp}$   $_{\rm x}$  )

Return the exponential integral  $E_n(x)$  by recursion. Use upward recursion for x < n and downward recursion (Miller's algorithm) otherwise.

$$E_n(x) = \int_1^\infty \frac{e^{-xt}}{t^n} dt$$

_~	The order of the exponential integral function.
_n	
_~	The argument of the exponential integral function.
_X	

# Returns

The exponential integral.

**Todo** Find a principled starting number for the  $E_n(x)$  downward recursion.

Definition at line 238 of file sf\_expint.tcc.

References \_\_expint\_E1().

Referenced by \_\_expint().

9.3.2.100 template<typename \_Tp > \_Tp std::\_\_expint\_En\_series ( unsigned int \_\_n, \_Tp \_\_x )

Return the exponential integral  $E_n(x)$  by series summation.

The exponential integral is given by

$$E_n(x) = \int_1^\infty \frac{e^{-xt}}{t^n} dt$$

# **Parameters**

_~	The order of the exponential integral function.
_n	
_←	The argument of the exponential integral function.
_X	

# Returns

The exponential integral.

Definition at line 147 of file sf\_expint.tcc.

References \_\_psi().

9.3.2.101 template<typename \_Tp > \_Tp std::\_\_expint\_large\_n ( unsigned int \_\_n, \_Tp \_\_x )

Return the exponential integral  $E_n(x)$  for large order.

The exponential integral is given by

$$E_n(x) = \int_1^\infty \frac{e^{-xt}}{t^n} dt$$

This is something of an extension.

# **Parameters**

_~	The order of the exponential integral function.
_n	
_~	The argument of the exponential integral function.
_X	

# Returns

The exponential integral.

Definition at line 437 of file sf\_expint.tcc.

9.3.2.102 template < typename \_Tp > \_GLIBCXX14\_CONSTEXPR \_Tp std::\_\_detail::\_\_factorial ( unsigned int \_\_n )

Return the factorial of the integer n.

The factorial is:

$$n! = 12...(n-1)n, 0! = 1$$

Definition at line 2424 of file sf\_gamma.tcc.

References std::\_\_detail::\_Factorial\_table< \_Tp >::\_\_n, and \_S\_factorial\_table.

9.3.2.103 template<typename \_Tp > \_Tp std::\_\_detail::\_\_fermi\_dirac ( \_Tp \_\_s, \_Tp \_\_x )

Return the Fermi-Dirac integral of real order s and real argument  $\boldsymbol{x}$ .

# See also

```
https://en.wikipedia.org/wiki/Clausen_function
http://dlmf.nist.gov/25.12#iii
```

_←	The order $s >= 0$ .
_s	
_~	The real argument.
_x	

#### Returns

The real Fermi-Dirac cosine sum  $F_s(x)$ ,

Definition at line 1380 of file sf\_polylog.tcc.

References \_\_polylog\_exp().

Compute the Fock-type Airy functions  $w_1(x)$  and  $w_2(x)$  and their first derivatives  $w_1'(x)$  and  $w_2'(x)$  respectively.

$$w_1(x) = \sqrt{\pi}(Ai(x) + iBi(x))$$

$$w_2(x) = \sqrt{\pi}(Ai(x) - iBi(x))$$

# Parameters

x	The argument of the Airy functions.	
w1	w1 The output Fock-type Airy function of the first kind.	
w2	_w2 The output Fock-type Airy function of the second kind.	
w1p	The output derivative of the Fock-type Airy function of the first kind.	
w2p	The output derivative of the Fock-type Airy function of the second kind.	

Definition at line 581 of file sf mod bessel.tcc.

9.3.2.105 template<typename \_Tp > bool std::\_\_detail::\_\_fpequal ( const \_Tp & \_\_a, const \_Tp & \_\_b )

A function to reliably compare two floating point numbers.

# **Parameters**

_~	the left hand side.	
_a		
_~	the right hand side	
_b		

# Returns

returns true if a and b are equal to zero or differ only by max(a,b)\*5\*eps

Definition at line 63 of file sf\_polylog.tcc.

```
9.3.2.106 template<typename _Tp > bool std::__detail::__fpimag ( const std::complex < _Tp > & __w )
```

A function to reliably test a complex number for imaginaryness [?].

# **Parameters**

```
_ ← The complex argument.
```

# Returns

```
true if Re(w) is zero within 5*epsilon, false otherwize.
```

Definition at line 108 of file sf\_polylog.tcc.

```
References fpequal().
```

```
9.3.2.107 template<typename _Tp > bool std::__detail::__fpimag ( const _Tp )
```

Definition at line 118 of file sf\_polylog.tcc.

```
9.3.2.108 template<typename_Tp > bool std::__detail::__fpreal ( const std::complex< _Tp > & __w )
```

A function to reliably test a complex number for realness.

# **Parameters**

```
_ ← The complex argument.
```

#### Returns

```
true if Im(w) is zero within 5*epsilon, false otherwize.
```

Definition at line 85 of file sf\_polylog.tcc.

References \_\_fpequal().

Referenced by \_\_polylog\_exp\_int\_pos(), and \_\_polylog\_exp\_real\_pos().

9.3.2.109 template<typename \_Tp > bool std::\_\_detail::\_\_fpreal ( const \_Tp )

Definition at line 95 of file sf polylog.tcc.

9.3.2.110 template<typename \_Tp > std::complex<\_Tp> std::\_\_detail::\_\_fresnel ( const \_Tp \_\_x )

Return the Fresnel cosine and sine integrals as a complex number f[C(x) + iS(x)].

The Fresnel cosine integral is defined by:

$$C(x) = \int_0^x \cos(\frac{\pi}{2}t^2)dt$$

The Fresnel sine integral is defined by:

$$S(x) = \int_0^x \sin(\frac{\pi}{2}t^2)dt$$

# **Parameters**

_~	The argument
_X	

Definition at line 168 of file sf\_fresnel.tcc.

References \_\_fresnel\_cont\_frac(), and \_\_fresnel\_series().

9.3.2.111 template < typename \_Tp > void std::\_\_detail::\_\_fresnel\_cont\_frac ( const \_Tp \_\_ax, \_Tp & \_Cf, \_Tp & \_Sf )

This function computes the Fresnel cosine and sine integrals by continued fractions for positive argument.

Definition at line 107 of file sf\_fresnel.tcc.

Referenced by \_\_fresnel().

9.3.2.112 template < typename \_Tp > void std::\_\_detail::\_\_fresnel\_series ( const \_Tp \_\_ax, \_Tp & \_Cf, \_Tp & \_Sf )

This function returns the Fresnel cosine and sine integrals as a pair by series expansion for positive argument.

Definition at line 49 of file sf fresnel.tcc.

Referenced by fresnel().

9.3.2.113 template < typename  $_{\rm Tp} > _{\rm Tp}$  std::\_\_detail::\_\_gamma (  $_{\rm Tp} _{\rm \_x}$  )

Return  $\Gamma(x)$ .

# **Parameters**

Returns

The gamma function.

Definition at line 1918 of file sf\_gamma.tcc.

References \_\_log\_gamma().

Referenced by \_\_beta\_gamma(), and \_\_riemann\_zeta().

9.3.2.114 template<typename\_Tp > std::pair<\_Tp, \_Tp> std::\_\_detail::\_\_gamma\_cont\_frac ( \_Tp \_\_a, \_Tp \_\_x )

Definition at line 1965 of file sf\_gamma.tcc.

References std:: detail:: Factorial table< Tp >:: n.

Referenced by \_\_gamma\_I(), \_\_gamma\_u(), \_\_pgamma(), and \_\_qgamma().

9.3.2.115 template<typename\_Tp > \_Tp std::\_\_detail::\_\_gamma\_I ( \_Tp \_\_a, \_Tp \_\_x )

Return the lower incomplete gamma function. The lower incomplete gamma function is defined by

$$\gamma(a, x) = \int_0^x e^{-t} t^{a-1} dt (a > 0)$$

.

Definition at line 2072 of file sf gamma.tcc.

References \_\_gamma\_cont\_frac(), and \_\_gamma\_series().

9.3.2.116 template<typename\_Tp > std::pair<\_Tp,\_Tp> std::\_\_detail::\_\_gamma\_series( \_Tp \_\_a, \_Tp \_\_x )

Definition at line 1930 of file sf\_gamma.tcc.

References std::\_\_detail::\_Factorial\_table< \_Tp >::\_\_n.

Referenced by \_\_gamma\_I(), \_\_gamma\_u(), \_\_pgamma(), and \_\_qgamma().

9.3.2.117 template<typename \_Tp > void std::\_\_detail::\_\_gamma\_temme ( \_Tp \_\_mu, \_Tp & \_\_gam1, \_Tp & \_\_gam2, \_Tp & \_\_gammi )

Compute the gamma functions required by the Temme series expansions of  $N_{\nu}(x)$  and  $K_{\nu}(x)$ .

$$\Gamma_1 = \frac{1}{2\mu} \left[ \frac{1}{\Gamma(1-\mu)} - \frac{1}{\Gamma(1+\mu)} \right]$$

and

$$\Gamma_2 = \frac{1}{2} \left[ \frac{1}{\Gamma(1-\mu)} + \frac{1}{\Gamma(1+\mu)} \right]$$

where  $-1/2 <= \mu <= 1/2$  is  $\mu = \nu - N$  and N. is the nearest integer to  $\nu$ . The values of  $\Gamma(1+\mu)$  and  $\Gamma(1-\mu)$  are returned as well.

The accuracy requirements on this are exquisite.

	mu	The input parameter of the gamma functions.
out	gam1	The output function $\Gamma_1(\mu)$
out	gam2	The output function $\Gamma_2(\mu)$
out	gampl	The output function $\Gamma(1+\mu)$
out	gammi	The output function $\Gamma(1-\mu)$

Definition at line 164 of file sf bessel.tcc.

Referenced by \_\_cyl\_bessel\_ik\_steed(), and \_\_cyl\_bessel\_jn\_steed().

9.3.2.118 template < typename \_Tp > \_Tp std::\_\_detail::\_\_gamma\_u ( \_Tp \_\_a, \_Tp \_\_x )

Return the upper incomplete gamma function. The lower incomplete gamma function is defined by

$$\Gamma(a,x) = \int_{x}^{\infty} e^{-t} t^{a-1} dt (a > 0)$$

.

Definition at line 2104 of file sf gamma.tcc.

References \_\_gamma\_cont\_frac(), and \_\_gamma\_series().

9.3.2.119 template<typename \_Tp > \_Tp std::\_\_detail::\_\_gauss ( \_Tp \_\_x )

The CDF of the normal distribution. i.e. the integrated lower tail of the normal PDF.

Definition at line 70 of file sf\_owens\_t.tcc.

9.3.2.120 template<typename \_Tp > \_Tp std::\_\_gegenbauer\_poly ( unsigned int \_\_n, \_Tp \_\_alpha, \_Tp \_\_x )

Return the Gegenbauer polynomial  $C_n^{\alpha}(x)$  of degree n and real order  $\alpha$  and argument x.

The Gegenbauer polynomials are generated by a three-term recursion relation:

$$C_{n}^{\alpha}(x) = \frac{1}{n} \left[ 2x(n+\alpha-1)C_{n-1}^{\alpha}(x) - (n+2\alpha-2)C_{n-2}^{\alpha}(x) \right]$$

and  $C_0^{\alpha}(x) = 1$ ,  $C_1^{\alpha}(x) = 2\alpha x$ .

# **Template Parameters**

_Talpha	The real type of the order
_ <i>Tp</i>	The real type of the argument

n	The non-negative integral degree
alpha	The real order
x	The real argument

Definition at line 61 of file sf gegenbauer.tcc.

```
9.3.2.121 template < typename _Tp > void std::__detail::__hankel ( std::complex < _Tp > __nu, std::complex < _Tp > __z, std::complex < _Tp > & _H1, std::complex < _Tp > & _H2, std::complex < _Tp > & _H1p, std::complex < _Tp > & _H2p )
```

# **Parameters**

in	nu	The order for which the Hankel functions are evaluated.
in	z	The argument at which the Hankel functions are evaluated.
out	_H1	The Hankel function of the first kind.
out	_H1p	The derivative of the Hankel function of the first kind.
out	_H2	The Hankel function of the second kind.
out	_H2p	The derivative of the Hankel function of the second kind.

Definition at line 1083 of file sf\_hankel.tcc.

References \_\_debye\_region(), \_\_hankel\_debye(), and \_\_hankel\_uniform().

Referenced by \_\_cyl\_bessel(), \_\_cyl\_hankel\_1(), \_\_cyl\_hankel\_2(), \_\_cyl\_neumann(), and \_\_sph\_hankel().

9.3.2.122 template < typename \_Tp > void std::\_\_detail::\_\_hankel\_debye ( std::complex < \_Tp > \_\_nu, std::complex < \_Tp > \_\_z, std::complex < \_Tp > \_\_alpha, int \_\_indexr, char & \_\_aorb, int & \_\_morn, std::complex < \_Tp > & \_H1, std::complex < \_Tp > & \_H2, std::complex < \_Tp > & \_H2p, std::complex < \_Tp > & \_H2p )

in	nu	The order for which the Hankel functions are evaluated.
in	z	The argument at which the Hankel functions are evaluated.
in	alpha	
in	indexr	
out	aorb	
out	morn	
out	_H1	The Hankel function of the first kind.
out	_H1p	The derivative of the Hankel function of the first kind.
out	_H2	The Hankel function of the second kind.
out	_H2p	The derivative of the Hankel function of the second kind.

Definition at line 915 of file sf\_hankel.tcc.

Referenced by hankel().

```
9.3.2.123 template<typename_Tp > void std::__detail::__hankel_params ( std::complex<_Tp > __nu, std::complex<_Tp > __zhat, std::complex<_Tp > & __p, std::complex<_Tp > & __p2, std::complex<_Tp > & __nup2, std::complex<_Tp > & __nup2, std::complex<_Tp > & __num2d3, std::complex<_Tp > & __num2d3, std::complex<_Tp > & __num2d3, std::complex<_Tp > & __zetanhf, std::complex<_Tp > & __zetamhf, std::complex<_Tp > & __zetam3hf, std::complex<_Tp > & __zetatat )
```

Compute parameters depending on z and nu that appear in the uniform asymptotic expansions of the Hankel functions and their derivatives, except the arguments to the Airy functions.

Definition at line 112 of file sf hankel.tcc.

Referenced by \_\_hankel\_uniform\_outer().

```
9.3.2.124 template < typename _Tp > void std::__detail::__hankel_uniform ( std::complex < _Tp > __nu, std::complex < _Tp > __nu, std::complex < _Tp > & _H1p, std::complex < _Tp > & _H2p )
```

This routine computes the uniform asymptotic approximations of the Hankel functions and their derivatives including a patch for the case when the order equals or nearly equals the argument. At such points, Olver's expressions have zero denominators (and numerators) resulting in numerical problems. This routine averages results from four surrounding points in the complex plane to obtain the result in such cases.

# **Parameters**

in	nu	The order for which the Hankel functions are evaluated.
in	z	The argument at which the Hankel functions are evaluated.
out	_H1	The Hankel function of the first kind.
out	_H1p	The derivative of the Hankel function of the first kind.
out	_H2	The Hankel function of the second kind.
out	_H2p	The derivative of the Hankel function of the second kind.

Definition at line 860 of file sf\_hankel.tcc.

References hankel uniform olver().

Referenced by \_\_hankel().

```
9.3.2.125 template < typename _Tp > void std::__detail::__hankel_uniform_olver ( std::complex < _Tp > __nu, std::complex < _Tp > __z, std::complex < _Tp > & _H1, std::complex < _Tp > & _H2, std::complex < _Tp > & _H1p, std::complex < _Tp > & _H2p )
```

Compute approximate values for the Hankel functions of the first and second kinds using Olver's uniform asymptotic expansion to of order nu along with their derivatives.

in	nu	The order for which the Hankel functions are evaluated.
in	z	The argument at which the Hankel functions are evaluated.
out	_H1	The Hankel function of the first kind.
out	_H1p	The derivative of the Hankel function of the first kind.
out	_H2	The Hankel function of the second kind.
out	_H2p	The derivative of the Hankel function of the second kind.

Definition at line 774 of file sf hankel.tcc.

References hankel uniform outer(), and hankel uniform sum().

Referenced by \_\_hankel\_uniform().

9.3.2.126 template < typename \_Tp > void std::\_\_detail::\_\_hankel\_uniform\_outer ( std::complex < \_Tp > \_\_nu, std::complex < \_Tp > \_\_z, \_Tp \_\_eps, std::complex < \_Tp > & \_\_zhat, std::complex < \_Tp > & \_\_num1d3, std::complex < \_Tp > & \_\_num2d3, std::complex < \_Tp > & \_\_p, std::complex < \_Tp > & \_\_p2, std::complex < \_Tp > & \_\_etrat, std::complex < \_Tp > & \_\_aip, std::complex < \_Tp > & \_\_aip, std::complex < \_Tp > & \_\_o4dp, std::complex < \_Tp > & \_\_o4dm, std::complex < \_Tp > & \_\_o4ddm) }

Compute outer factors and associated functions of z and nu appearing in Olver's uniform asymptotic expansions of the Hankel functions of the first and second kinds and their derivatives. The various functions of z and nu returned by nu form\_outer are available for use in computing further terms in the expansions.

Definition at line 249 of file sf hankel.tcc.

References airy arg(), and hankel params().

Referenced by \_\_hankel\_uniform\_olver().

9.3.2.127 template < typename \_Tp > void std::\_\_detail::\_\_hankel\_uniform\_sum ( std::complex < \_Tp > \_\_p, std::complex < \_Tp > \_\_p, std::complex < \_Tp > \_\_p, std::complex < \_Tp > \_\_aip, std::complex < \_Tp > \_\_o4dp, std::complex < \_Tp > \_\_o4dm, \_Tp \_\_eps, std::complex < \_Tp > \_\_o4dm, std::complex < \_Tp > \_\_o4dm, std::complex < \_Tp > \_\_o4dm, \_Tp \_\_eps, std::complex < \_Tp > & \_\_H1sum, std::complex < \_Tp > & \_\_H2sum, std::complex < \_Tp > & \_\_H2sum)

Compute the sums in appropriate linear combinations appearing in Olver's uniform asymptotic expansions for the Hankel functions of the first and second kinds and their derivatives, using up to nterms (less than 5) to achieve relative error eps.

in	p	
in	p2	
in	num2	

in	zetam3hf	
in	_Aip	The Airy function value $Ai()$ .
in	o4dp	
in	_Aim	The Airy function value $Ai()$ .
in	o4dm	
in	od2p	
in	od0dp	
in	od2m	
in	od0dm	
in	eps	The error tolerance
out	_H1sum	The Hankel function of the first kind.
out	_H1psum	The derivative of the Hankel function of the first kind.
out	_H2sum	The Hankel function of the second kind.
out	_H2psum	The derivative of the Hankel function of the second kind.

Definition at line 325 of file sf\_hankel.tcc.

Referenced by \_\_hankel\_uniform\_olver().

9.3.2.128 template> \_Tp std::\_\_detail::\_\_heuman\_lambda ( \_Tp 
$$\_$$
k, \_Tp  $\_$ phi )

Return the Heuman lambda function.

Definition at line 942 of file sf\_ellint.tcc.

References \_\_ellint\_rf(), and \_\_ellint\_rj().

9.3.2.129 template> \_Tp std::\_\_detail::\_\_hurwitz\_zeta ( \_Tp 
$$\_s$$
, \_Tp  $\_a$  )

Return the Hurwitz zeta function  $\zeta(s,a)$  for all s != 1 and a > -1.

The Hurwitz zeta function is defined by:

$$\zeta(s,a) = \sum_{n=0}^{\infty} \frac{1}{(n+a)^s}$$

The Riemann zeta function is a special case:

$$\zeta(s) = \zeta(s, 1)$$

_~	The argument $s! = 1$
_s	
_←	The scale parameter $a>-1$
_a	

Definition at line 703 of file sf\_zeta.tcc.

References \_\_hurwitz\_zeta\_euler\_maclaurin().

 $9.3.2.130 \quad template < typename \_Tp > std::complex < \_Tp > std::\_detail::\_hurwitz\_zeta \ ( \ \_Tp \_\_s, \ std::complex < \_Tp > \_\_a \ )$ 

Return the Hurwitz Zeta function for real s and complex a.

# **Parameters**

_~	The real argument
_s	
_~	The complex parameter
_a	

Todo This \_\_hurwitz\_zeta prefactor is prone to overflow. positive integer orders s?

Definition at line 1120 of file sf\_polylog.tcc.

References polylog exp().

Referenced by \_\_psi().

9.3.2.131 template<typename\_Tp > \_Tp std::\_\_detail::\_\_hurwitz\_zeta\_euler\_maclaurin( \_Tp \_\_s, \_Tp \_\_a )

Return the Hurwitz zeta function  $\zeta(s,a)$  for all s != 1 and a > -1.

# See also

An efficient algorithm for accelerating the convergence of oscillatory series, useful for computing the polylogarithm and Hurwitz zeta functions, Linas Vep

# **Parameters**

_~	The argument $s! = 1$
_s	
_←	The scale parameter $a>-1$
_a	

Definition at line 561 of file sf\_zeta.tcc.

References \_S\_Euler\_Maclaurin\_zeta.

Referenced by \_\_hurwitz\_zeta().

9.3.2.132 template<typename \_Tp > std::complex<\_Tp> std::\_\_detail::\_\_hydrogen ( const unsigned int \_\_n, const unsigned int \_\_n, const unsigned int \_\_n, const \_Tp \_\_r, const \_Tp \_\_theta, const \_Tp \_\_phi )

Definition at line 46 of file sf\_hydrogen.tcc.

References \_\_assoc\_laguerre(), \_\_psi(), and \_\_sph\_legendre().

9.3.2.133 template<typename \_Tp > \_Tp std::\_\_detail::\_\_hyperg ( \_Tp \_\_a, \_Tp \_\_b, \_Tp \_\_c, \_Tp \_\_x )

Return the hypergeometric function  ${}_{2}F_{1}(a,b;c;x)$ .

The hypergeometric function is defined by

$$_{2}F_{1}(a,b;c;x) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)} \sum_{n=0}^{\infty} \frac{\Gamma(a+n)\Gamma(b+n)}{\Gamma(c+n)} \frac{x^{n}}{n!}$$

#### **Parameters**

_~	The first <i>numerator</i> parameter.
_a	
_←	The second <i>numerator</i> parameter.
_b	
_~	The denominator parameter.
_c	
_~	The argument of the confluent hypergeometric function.
_x	

# Returns

The confluent hypergeometric function.

Definition at line 776 of file sf\_hyperg.tcc.

References \_\_hyperg\_luke(), \_\_hyperg\_reflect(), \_\_hyperg\_series(), \_\_log\_gamma(), and \_\_log\_gamma\_sign().

Return the hypergeometric function  ${}_2F_1(a,b;c;x)$  by an iterative procedure described in Luke, Algorithms for the Computation of Mathematical Functions.

Definition at line 352 of file sf\_hyperg.tcc.

Referenced by \_\_hyperg().

9.3.2.135 template<typename\_Tp > \_Tp std::\_\_detail::\_\_hyperg\_reflect ( \_Tp \_\_a, \_Tp \_\_b, \_Tp \_\_c, \_Tp \_\_x )

Return the hypergeometric function  ${}_2F_1(a,b;c;x)$  by the reflection formulae in Abramowitz & Stegun formula 15.3.6 for d = c - a - b not integral and formula 15.3.11 for d = c - a - b integral. This assumes a, b, c != negative integer.

The hypergeometric function is defined by

$$_{2}F_{1}(a,b;c;x) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)} \sum_{n=0}^{\infty} \frac{\Gamma(a+n)\Gamma(b+n)}{\Gamma(c+n)} \frac{x^{n}}{n!}$$

The reflection formula for nonintegral d=c-a-b is:

$${}_{2}F_{1}(a,b;c;x) = \frac{\Gamma(c)\Gamma(d)}{\Gamma(c-a)\Gamma(c-b)} {}_{2}F_{1}(a,b;1-d;1-x) + \frac{\Gamma(c)\Gamma(-d)}{\Gamma(a)\Gamma(b)} {}_{2}F_{1}(c-a,c-b;1+d;1-x)$$

The reflection formula for integral m=c-a-b is:

$$_{2}F_{1}(a,b;a+b+m;x) = \frac{\Gamma(m)\Gamma(a+b+m)}{\Gamma(a+m)\Gamma(b+m)} \sum_{k=0}^{m-1} \frac{(m+a)_{k}(m+b)_{k}}{k!(1-m)_{k}} - \frac{(m+a)_{k}(m+b)_{k}}{k!(1-m)_{k}}$$

Definition at line 486 of file sf hyperg.tcc.

References \_\_hyperg\_series(), \_\_log\_gamma(), \_\_log\_gamma\_sign(), and \_\_psi().

Referenced by \_\_hyperg().

Return the hypergeometric function  ${}_{2}F_{1}(a,b;c;x)$  by series expansion.

The hypergeometric function is defined by

$$_{2}F_{1}(a,b;c;x) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)} \sum_{n=0}^{\infty} \frac{\Gamma(a+n)\Gamma(b+n)}{\Gamma(c+n)} \frac{x^{n}}{n!}$$

This works and it's pretty fast.

_~	The first <i>numerator</i> parameter.
_a	
_←	The second <i>numerator</i> parameter.
_b	
_~	The denominator parameter.
_c	
_~	The argument of the confluent hypergeometric function.
_x	

#### Returns

The confluent hypergeometric function.

Definition at line 321 of file sf hyperg.tcc.

Referenced by \_\_hyperg(), and \_\_hyperg\_reflect().

9.3.2.137 template<typename \_Tp > std::tuple<\_Tp, \_Tp> std::\_\_detail::\_\_jacobi\_sncndn ( \_Tp  $\underline{\hspace{0.4cm}}$ , \_Tp  $\underline{\hspace{0.4cm}}$ u )

Return a tuple of the three primary Jacobi elliptic functions: sn(k, u), cn(k, u), dn(k, u).

Definition at line 414 of file sf theta.tcc.

9.3.2.138 template<typename \_Tp > \_Tp std::\_\_detail::\_\_jacobi\_zeta ( \_Tp \_\_k, \_Tp \_\_phi )

Return the Jacobi zeta function.

Definition at line 972 of file sf\_ellint.tcc.

References \_\_comp\_ellint\_1(), and \_\_ellint\_rj().

9.3.2.139 template<typename \_Tp > \_Tp std::\_\_detail::\_\_laguerre ( unsigned int \_\_n, \_Tp \_\_x )

This routine returns the Laguerre polynomial of order n:  $L_n(x)$ .

The Laguerre polynomial is defined by:

$$L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n} (x^n e^{-x})$$

# **Parameters**

_~	The order of the Laguerre polynomial.
_n	
_~	The argument of the Laguerre polynomial.
_X	

# Returns

The value of the Laguerre polynomial of order n and argument x.

Definition at line 321 of file sf laguerre.tcc.

9.3.2.140 template<typename \_Tp > \_Tp std::\_\_detail::\_\_legendre\_q ( unsigned int \_\_l, \_Tp \_\_x )

Return the Legendre function of the second kind by upward recursion on order l.

The Legendre function of the second kind of order l and argument x,  $Q_l(x)$ , is defined by:

$$Q_{l}(x) = \frac{1}{2^{l} l!} \frac{d^{l}}{dx^{l}} (x^{2} - 1)^{l}$$

#### **Parameters**

_~	The order of the Legendre function. $l>=0$ .
_/	
_~	The argument of the Legendre function. $ x  <= 1$ .
_x	

Definition at line 124 of file sf\_legendre.tcc.

9.3.2.141 template < typename \_Tp > \_Tp std::\_\_detail::\_\_log\_bincoef ( unsigned int \_\_n, unsigned int \_\_k )

Return the logarithm of the binomial coefficient. The binomial coefficient is given by:

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

# **Parameters**

_~	The first argument of the binomial coefficient.
_n	
_~	The second argument of the binomial coefficient.
_k	

# Returns

The logarithm of the binomial coefficient.

Definition at line 1862 of file sf\_gamma.tcc.

9.3.2.142 template < typename \_Tp > \_GLIBCXX14\_CONSTEXPR \_Tp std::\_\_detail::\_\_log\_double\_factorial ( \_Tp \_\_x )

Definition at line 2452 of file sf\_gamma.tcc.

References \_\_log\_gamma().

Referenced by \_\_double\_factorial(), and \_\_log\_double\_factorial().

9.3.2.143 template < typename \_Tp > \_GLIBCXX14\_CONSTEXPR \_Tp std::\_\_detail::\_\_log\_double\_factorial ( int \_\_n )

Return the logarithm of the double factorial of the integer n.

The double factorial is defined for integral n by:

$$n!! = 135...(n-2)n, noddn!! = 246...(n-2)n, neven - 1!! = 10!! = 1$$

The double factorial is defined for odd negative integers in the obvious way:

$$(-2m-1)!! = 1/(1(-1)(-3)...(-2m+1)(-2m-1)) = \frac{(-1)^m}{(2m-1)!!}$$

for f[ n = -2m - 1 f].

Definition at line 2518 of file sf\_gamma.tcc.

 $\label{log_double_factorial} References \_\_log\_double\_factorial(), std::\__detail::\_Factorial\_table < \_Tp >::\_\_log\_factorial, std::\_\_detail::\_Factorial \leftarrow \_table < \_Tp >::\_\_n, \_S\_double\_factorial\_table, and \_S\_neg\_double\_factorial\_table.$ 

9.3.2.144 template < typename \_Tp > \_GLIBCXX14\_CONSTEXPR \_Tp std::\_\_detail::\_\_log\_factorial ( unsigned int \_\_n )

Return the logarithm of the factorial of the integer n.

The factorial is:

$$n! = 12...(n-1)n, 0! = 1$$

Definition at line 2442 of file sf\_gamma.tcc.

References log gamma(), std:: detail:: Factorial table< Tp >:: n, and S factorial table.

9.3.2.145 template<typename \_Tp > \_Tp std::\_\_detail::\_\_log\_gamma ( \_Tp \_\_x )

Return  $log(|\Gamma(x)|)$ . This will return values even for x < 0. To recover the sign of  $\Gamma(x)$  for any argument use  $\_\_log\_ \leftarrow gamma\_sign$ .

# **Parameters**

# Returns

The logarithm of the gamma function.

Definition at line 1800 of file sf gamma.tcc.

References log gamma lanczos().

```
Referenced by __beta_lgamma(), __cyl_bessel_ij_series(), __gamma(), __hyperg(), __hyperg_reflect(), __log_\leftarrow double_factorial(), __log_factorial(), __log_pochhammer_u(), __poly_laguerre_large_n(), __psi(), __riemann_zeta(), \leftarrow __riemann_zeta_glob(), and __sph_legendre().
```

```
9.3.2.146 template<typename_Tp > _GLIBCXX14_CONSTEXPR _Tp std::__detail::__log_gamma_bernoulli ( _Tp __x )
```

Return  $log(\Gamma(x))$  by asymptotic expansion with Bernoulli number coefficients. This is like Sterling's approximation.

# **Parameters**

```
_← The argument of the log of the gamma function.
```

# Returns

The logarithm of the gamma function.

Definition at line 1699 of file sf gamma.tcc.

```
9.3.2.147 template < typename _Tp > _GLIBCXX14_CONSTEXPR _Tp std::__detail::__log_gamma_lanczos ( _Tp __x )
```

Return  $log(\Gamma(x))$  by the Lanczos method. This method dominates all others on the positive axis I think.

# **Parameters**

```
_ ← The argument of the log of the gamma function.
```

# Returns

The logarithm of the gamma function.

Definition at line 1755 of file sf gamma.tcc.

Referenced by log gamma().

```
9.3.2.148 template<typename_Tp > _Tp std::__detail::__log_gamma_sign ( _Tp __x )
```

Return the sign of  $\Gamma(x)$ . At nonpositive integers zero is returned.

_~	The argument of the gamma function.
_X	

The sign of the gamma function.

Definition at line 1831 of file sf\_gamma.tcc.

Referenced by \_\_hyperg(), \_\_hyperg\_reflect(), and \_\_pochhammer\_l().

9.3.2.149 template<typename\_Tp > \_GLIBCXX14\_CONSTEXPR\_Tp std::\_\_detail::\_\_log\_gamma\_spouge( \_Tp \_\_z )

Return  $\Gamma(z)$  by the Spouge algorithm:

$$\Gamma(z+1) = (z+a)^{z+1/2} e^{-z-a} \left[ \sqrt{2\pi} \sum_{k=1}^{\lceil a \rceil + 1} \frac{c_k(a)}{z+k} \right]$$

where

$$c_k(a) = \frac{(-1)^{k-1}}{(k-1)!} (a-k)^{k-1/2} e^{a-k}$$

and the error is bounded by

$$\epsilon(a) < a^{-1/2} (2\pi)^{-a-1/2}$$

.

See also

Spouge, J.L., Computation of the gamma, digamma, and trigamma functions. SIAM Journal on Numerical Analysis 31, 3 (1994), pp. 931-944

## **Parameters**

_~	The argument of the gamma function.
Z	

# Returns

The the gamma function.

Definition at line 1739 of file sf gamma.tcc.

9.3.2.150 template<typename\_Tp > \_Tp std::\_\_detail::\_\_log\_pochhammer\_I ( \_Tp \_\_a, \_Tp \_\_n )

Return the logarithm of the lower Pochhammer symbol or the falling factorial function. The lower Pochammer symbol is defined by

$$(a)_n = \prod_{k=0}^{n-1} (a-k), (a)_0 = 1 = \Gamma(a+1)/\Gamma(a-n+1)$$

In particular,  $f(n)_n = n! f$ . Thus this function returns

$$ln[(a)_n] = \Gamma(a+1) - \Gamma(a-n+1), ln[(a)_0] = 0$$

Many notations exist:

 $a^{\underline{n}}$ 

,

$$\left\{\begin{array}{c} a \\ n \end{array}\right\}$$

, and others.

Definition at line 2211 of file sf\_gamma.tcc.

9.3.2.151 template \_Tp std::\_\_detail::\_\_log\_pochhammer\_u ( \_Tp 
$$\_a$$
, \_Tp  $\_n$  )

Return the logarithm of the (upper) Pochhammer symbol or the rising factorial function. The Pochammer symbol is defined by

$$(a)_n = \prod_{k=0}^{n-1} (a+k), (a)_0 = 1 = \Gamma(a+n)/\Gamma(n)$$

Thus this function returns

$$ln[(a)_n] = \Gamma(a+n) - \Gamma(n), ln[(a)_0] = 0$$

Many notations exist:

 $a^{\overline{n}}$ 

,

$$\begin{bmatrix} a \\ n \end{bmatrix}$$

, and others.

Definition at line 2146 of file sf\_gamma.tcc.

References log gamma().

Return the logarithmic integral li(x).

The logarithmic integral is given by

$$li(x) = Ei(\log(x))$$

	The argument of the logarithmic integral function.
_X	

The logarithmic integral.

Definition at line 531 of file sf expint.tcc.

References \_\_expint().

9.3.2.153 template<typename \_Tp > \_Tp std::\_\_detail::\_\_owens\_t ( \_Tp \_\_h, \_Tp \_\_a )

Return the Owens T function:

$$T(h,a) = \frac{1}{2\pi} \int_0^a \frac{\exp[-\frac{1}{2}h^2(1+x^2)]}{1+x^2} dx$$

This implementation is a translation of the Fortran implementation in

## See also

Patefield, M. and Tandy, D. "Fast and accurate Calculation of Owen's T-Function", Journal of Statistical Software, 5 (5), 1 - 25 (2000)

#### **Parameters**

in	_~	The scale parameter.
	_h	
in	_~	The integration limit.
	а	

## Returns

The owens T function.

Definition at line 92 of file sf owens t.tcc.

References \_\_znorm1(), and \_\_znorm2().

9.3.2.154 template<typename \_Tp > \_Tp std::\_\_detail::\_\_pgamma ( \_Tp \_\_a, \_Tp \_\_x )

Return the regularized lower incomplete gamma function. The regularized lower incomplete gamma function is defined by

$$P(a,x) = \frac{\gamma(a,x)}{\Gamma(a)}$$

where  $\Gamma(a)$  is the gamma function and

$$\gamma(a,x) = \int_0^x e^{-t} t^{a-1} dt (a > 0)$$

is the lower incomplete gamma function.

Definition at line 2015 of file sf gamma.tcc.

References \_\_gamma\_cont\_frac(), and \_\_gamma\_series().

9.3.2.155 template < typename \_Tp > \_Tp std::\_\_detail::\_\_pochhammer\_I ( \_Tp \_\_a, \_Tp \_\_n )

Return the logarithm of the lower Pochhammer symbol or the falling factorial function. The lower Pochammer symbol is defined by

$$(a)_n = \prod_{k=0}^{n-1} (a-k), (a)_0 = 1 = \Gamma(a+1)/\Gamma(a-n+1)$$

In particular,  $f[(n)_n = n! f]$ .

Definition at line 2234 of file sf\_gamma.tcc.

References \_\_log\_gamma\_sign().

9.3.2.156 template<typename \_Tp > \_Tp std::\_\_detail::\_\_pochhammer\_u ( \_Tp \_\_a, \_Tp \_\_n )

Return the (upper) Pochhammer function or the rising factorial function. The Pochammer symbol is defined by

$$(a)_n = \prod_{k=0}^{n-1} (a+k), (a)_0 = 1 = \Gamma(a+n)/\Gamma(n)$$

Many notations exist:

 $a^{\overline{n}}$ 

,

$$\left[\begin{array}{c} a \\ n \end{array}\right]$$

, and others.

Definition at line 2172 of file sf gamma.tcc.

9.3.2.157 template<typename \_Tp > \_Tp std::\_\_detail::\_\_poly\_hermite ( unsigned int \_\_n, \_Tp \_\_x )

This routine returns the Hermite polynomial of order n:  $H_n(x)$ .

The Hermite polynomial is defined by:

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2}$$

The Hermite polynomial obeys a reflection formula:

$$H_n(-x) = (-1)^n H_n(x)$$

1	The order of the Hermite polynomial.
_n	
1	The argument of the Hermite polynomial.
_X	

The value of the Hermite polynomial of order n and argument x.

Definition at line 179 of file sf\_hermite.tcc.

References \_\_poly\_hermite\_asymp(), and \_\_poly\_hermite\_recursion().

9.3.2.158 template<typename\_Tp > \_Tp std::\_\_detail::\_\_poly\_hermite\_asymp ( unsigned int \_\_n, \_Tp \_\_x )

This routine returns the Hermite polynomial of large order n:  $H_n(x)$ . We assume here that  $x \ge 0$ .

The Hermite polynomial is defined by:

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2}$$

see "Asymptotic analysis of the Hermite polynomials from their differential-difference equation", Diego Dominici, arXiv↔ :math/0601078v1 [math.CA] 4 Jan 2006

#### **Parameters**

_~	The order of the Hermite polynomial.	
_n		
_~	The argument of the Hermite polynomial.	
_X		

# Returns

The value of the Hermite polynomial of order n and argument x.

Definition at line 113 of file sf\_hermite.tcc.

References \_\_airy().

Referenced by \_\_poly\_hermite().

9.3.2.159 template<typename\_Tp > \_Tp std::\_\_detail::\_\_poly\_hermite\_recursion ( unsigned int \_\_n, \_Tp \_\_x )

This routine returns the Hermite polynomial of order n:  $H_n(x)$  by recursion on n.

The Hermite polynomial is defined by:

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2}$$

_~	The order of the Hermite polynomial.
_n	
← Generate	The argument of the Hermite polynomial. d by Doxygen

The value of the Hermite polynomial of order n and argument x.

Definition at line 69 of file sf hermite.tcc.

Referenced by \_\_poly\_hermite().

9.3.2.160 template<typename \_Tp > \_Tp std::\_\_detail::\_\_poly\_jacobi ( unsigned int \_\_n, \_Tp \_\_alpha, \_Tp \_\_beta, \_Tp \_\_x )

Compute the Jacobi polynomial by recursion on n:

$$2n(\alpha+\beta+n)(\alpha+\beta+2n-2)P_n^{(\alpha,\beta)}(x) = (\alpha+\beta+2n-1)((\alpha^2-\beta^2)+x(\alpha+\beta+2n-2)(\alpha+\beta+2n))P_{n-1}^{(\alpha,\beta)}(x) - 2(\alpha+n-1)(\beta+n-1)(\alpha+\beta+2n-2)P_n^{(\alpha,\beta)}(x) = (\alpha+\beta+2n-1)((\alpha^2-\beta^2)+x(\alpha+\beta+2n-2)(\alpha+\beta+2n))P_{n-1}^{(\alpha,\beta)}(x) - 2(\alpha+n-1)(\beta+n-1)(\alpha+\beta+2n-2)(\alpha+2n-2$$

Definition at line 57 of file sf jacobi.tcc.

References \_\_beta().

Referenced by \_\_poly\_radial\_jacobi().

This routine returns the associated Laguerre polynomial of order n, degree  $\alpha$ :  $L_n^a lpha(x)$ .

The associated Laguerre function is defined by

$$L_n^{\alpha}(x) = \frac{(\alpha+1)_n}{n!} {}_1F_1(-n; \alpha+1; x)$$

where  $(\alpha)_n$  is the Pochhammer symbol and  ${}_1F_1(a;c;x)$  is the confluent hypergeometric function.

The associated Laguerre polynomial is defined for integral  $\alpha=m$  by:

$$L_n^m(x) = (-1)^m \frac{d^m}{dx^m} L_{n+m}(x)$$

where the Laguerre polynomial is defined by:

$$L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n} (x^n e^{-x})$$

# **Template Parameters**

_Тра	The type of the degree.
_Tp	The type of the parameter.

n	The order of the Laguerre function.
alpha1	The degree of the Laguerre function.
X	The argument of the Laguerre function.

## Returns

The value of the Laguerre function of order n, degree  $\alpha$ , and argument x.

Definition at line 248 of file sf\_laguerre.tcc.

References \_\_poly\_laguerre\_hyperg(), \_\_poly\_laguerre\_large\_n(), and \_\_poly\_laguerre\_recursion().

Evaluate the polynomial based on the confluent hypergeometric function in a safe way, with no restriction on the arguments.

The associated Laguerre function is defined by

$$L_n^{\alpha}(x) = \frac{(\alpha+1)_n}{n!} F_1(-n; \alpha+1; x)$$

where  $(\alpha)_n$  is the Pochhammer symbol and  ${}_1F_1(a;c;x)$  is the confluent hypergeometric function.

This function assumes x = 0.

This is from the GNU Scientific Library.

## **Template Parameters**

_Тра	The type of the degree.
_Тр	The type of the parameter.

## **Parameters**

n	The order of the Laguerre function.
alpha1	The degree of the Laguerre function.
X	The argument of the Laguerre function.

# Returns

The value of the Laguerre function of order n, degree  $\alpha$ , and argument x.

Definition at line 129 of file sf laguerre.tcc.

Referenced by \_\_poly\_laguerre().

9.3.2.163 template<typename \_Tpa , typename \_Tp > \_Tp std::\_\_detail::\_\_poly\_laguerre\_large\_n ( unsigned \_\_n, \_Tpa \_\_alpha1, \_\_Tp \_\_x )

This routine returns the associated Laguerre polynomial of order n, degree  $\alpha > -1$  for large n. Abramowitz & Stegun, 13.5.21.

## **Template Parameters**

_Тра	The type of the degree.
_Тр	The type of the parameter.

#### **Parameters**

n	The order of the Laguerre function.
alpha1	The degree of the Laguerre function.
X	The argument of the Laguerre function.

# Returns

The value of the Laguerre function of order n, degree  $\alpha$ , and argument x.

This is from the GNU Scientific Library.

Definition at line 72 of file sf laguerre.tcc.

References \_\_log\_gamma().

Referenced by \_\_poly\_laguerre().

9.3.2.164 template<typename \_Tpa , typename \_Tp > \_Tp std::\_\_detail::\_\_poly\_laguerre\_recursion ( unsigned int \_\_n, \_Tpa \_\_alpha1, \_Tp \_\_x )

This routine returns the associated Laguerre polynomial of order n, degree  $\alpha$ :  $L_n^{\alpha}(x)$  by recursion.

The associated Laguerre function is defined by

$$L_n^{\alpha}(x) = \frac{(\alpha+1)_n}{n!} {}_1F_1(-n; \alpha+1; x)$$

where  $(\alpha)_n$  is the Pochhammer symbol and  ${}_1F_1(a;c;x)$  is the confluent hypergeometric function.

The associated Laguerre polynomial is defined for integral  $\alpha=m$  by:

$$L_n^m(x) = (-1)^m \frac{d^m}{dx^m} L_{n+m}(x)$$

where the Laguerre polynomial is defined by:

$$L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n} (x^n e^{-x})$$

# **Template Parameters**

_Тра	The type of the degree.
_Tp	The type of the parameter.

#### **Parameters**

n	The order of the Laguerre function.
alpha1	The degree of the Laguerre function.
X	The argument of the Laguerre function.

#### Returns

The value of the Laguerre function of order n, degree  $\alpha$ , and argument x.

Definition at line 187 of file sf\_laguerre.tcc.

Referenced by \_\_poly\_laguerre().

 $9.3.2.165 \quad template < typename \_Tp > \_Tp \ std::\_\_detail::\_\_poly\_legendre\_p \ ( \ unsigned \ int \_\_I, \ \_Tp \_\_x \ )$ 

Return the Legendre polynomial by upward recursion on order l.

The Legendre function of order l and argument x,  $P_l(x)$ , is defined by:

$$P_l(x) = \frac{1}{2^l l!} \frac{d^l}{dx^l} (x^2 - 1)^l$$

## **Parameters**

_~	The order of the Legendre polynomial. $l>=0$ .
_/	
_←	The argument of the Legendre polynomial. $ x  <= 1$ .
_X	

Definition at line 74 of file sf\_legendre.tcc.

Referenced by \_\_assoc\_legendre\_p(), and \_\_sph\_legendre().

 $9.3.2.166 \quad template < typename \_Tp > \_Tp \ std::\_detail::\_poly\_radial\_jacobi \ (\ unsigned \ int \_n, \ unsigned \ int \_m, \_Tp \_rho \ )$ 

Return the radial polynomial  $R_n^m(\rho)$  for non-negative degree n, order m <= n, and real radial argument  $\rho$ .

The radial polynomials are defined by

$$R_n^m(\rho) = \sum_{k=0}^{\frac{n-m}{2}} \frac{(-1)^k (n-k)!}{k!(\frac{n+m}{2}-k)!(\frac{n-m}{2}-k)!} \rho^{n-2k}$$

for n-m even and identically 0 for n-m odd. The radial polynomials can be related to the Zernike polynomials:

$$Z_n^m(\rho,\phi) = R_n^m(\rho)\cos(m\phi)$$

$$Z_n^{-m}(\rho,\phi) = R_n^m(\rho)\sin(m\phi)$$

for non-negative m, n.

# See also

zernike for details on the Zernike polynomials.

# **Template Parameters**

_Тр	The real type of the radial coordinate
-----	--

## **Parameters**

n	The non-negative degree.
m	The non-negative azimuthal order
rho	The radial argument

Definition at line 139 of file sf\_jacobi.tcc.

References \_\_poly\_jacobi().

Referenced by \_\_zernike(), \_\_gnu\_cxx::radpolyf(), and \_\_gnu\_cxx::radpolyl().

9.3.2.167 template<typename \_Tp > \_Tp std::\_\_detail::\_\_polylog ( \_Tp \_\_s, \_Tp \_\_x )

Return the polylog Li s(x) for two real arguments.

# **Parameters**

_~	The real index.
_s	
_~	The real argument.
_X	

# Returns

The complex value of the polylogarithm.

Definition at line 1073 of file sf\_polylog.tcc.

References \_\_fpequal(), and \_\_polylog\_exp().

Referenced by \_\_dirichlet\_beta(), \_\_dirichlet\_eta(), and \_\_polylog().

9.3.2.168 template<typename\_Tp > std::complex<\_Tp > std::\_\_detail::\_\_polylog ( \_Tp \_\_s, std::complex<\_Tp > \_\_w )

Return the polylog in those cases where we can calculate it.

#### **Parameters**

_~	The real index.
_s	
_~	The complex argument.
_ <i>w</i>	

# Returns

The complex value of the polylogarithm.

Definition at line 1103 of file sf\_polylog.tcc.

References \_\_fpequal(), \_\_polylog(), and \_\_polylog\_exp().

9.3.2.169 template<typename \_Tp , typename ArgType > \_\_gnu\_cxx::\_promote\_num\_t < std::complex < \_Tp >, ArgType > std::\_\_detail::\_polylog\_exp ( \_Tp \_\_s, ArgType \_\_w )

This is the frontend function which calculates  $Li_s(e^w)$  First we branch into different parts depending on the properties of s. This function is the same irrespective of a real or complex w, hence the template parameter ArgType.

#### Note

: I really wish we could return a variant<Tp, std::complex<Tp>>.

## **Parameters**

_~	The real order.
_s	
_~	The real or complex argument.
_ <i>w</i>	

## Returns

The real or complex value of Li\_s( $e^{\wedge}$ w).

Definition at line 1040 of file sf\_polylog.tcc.

References \_\_polylog\_exp\_int\_neg(), \_\_polylog\_exp\_int\_pos(), \_\_polylog\_exp\_negative\_real\_part(), \_\_polylog\_exp← \_real\_neg(), and \_\_polylog\_exp\_real\_pos().

Referenced by \_\_bose\_einstein(), \_\_clausen(), \_\_clausen\_c(), \_\_clausen\_s(), \_\_fermi\_dirac(), \_\_hurwitz\_zeta(), and \_\_polylog().

9.3.2.170 template<typename \_Tp > std::complex<\_Tp> std::\_\_detail::\_\_polylog\_exp\_asymp ( \_Tp \_\_s, std::complex< \_Tp > \_\_w )

This function implements the asymptotic series for the polylog. It is given by

$$2\sum_{k=0}^{\infty} \zeta(2k)w^{s-2k}/\Gamma(s-2k+1) - i\pi w^{(s-1)}/\Gamma(s)$$

for Re(w) >> 1

Don't check this against Mathematica 8. For real u the imaginary part of the polylog is given by  $Im(Li_s(e^u)) = -\pi u^{s-1}/\Gamma(s)$  Check this relation for any benchmark that you use. The use of evenzeta leads to a speedup of about 1000.

#### **Parameters**

_~	the real index s.
_s	
_~	the large complex argument w.
_ <i>w</i>	

# Returns

the value of the polylogarithm.

Definition at line 687 of file sf\_polylog.tcc.

References fpequal().

Referenced by \_\_polylog\_exp\_int\_neg(), \_\_polylog\_exp\_int\_pos(), \_\_polylog\_exp\_real\_neg(), and \_\_polylog\_exp\_\times real\_pos().

9.3.2.171 template<typename \_Tp > std::complex<\_Tp> std::\_\_detail::\_\_polylog\_exp\_int\_neg ( int \_\_s, std::complex< \_Tp > \_\_w )

This treats the case where s is a negative integer.

_←	a negative integer.
_s	
_~	an arbitrary complex number
W	

## Returns

the value of the polylogarith,.

Definition at line 857 of file sf\_polylog.tcc.

 $References \ \_clamp\_0\_m2pi(), \ \_\_clamp\_pi(), \ \_\_polylog\_exp\_asymp(), \ \_\_polylog\_exp\_neg(), \ and \ \_\_colylog\_exp\_negative\_real\_part().$ 

Referenced by \_\_polylog\_exp().

9.3.2.172 template<typename\_Tp > std::complex<\_Tp> std::\_\_detail::\_\_polylog\_exp\_int\_neg ( const int \_\_s, \_Tp \_\_w )

This treats the case where s is a negative integer and w is a real.

## **Parameters**

_~	a negative integer.
_s	
_←	the argument.
_ <i>w</i>	

## Returns

the value of the polylogarithm.

Definition at line 899 of file sf\_polylog.tcc.

References \_\_fpequal(), \_\_polylog\_exp\_asymp(), \_\_polylog\_exp\_neg(), and \_\_polylog\_exp\_negative\_real\_part().

9.3.2.173 template<typename \_Tp > std::complex<\_Tp> std::\_\_detail::\_\_polylog\_exp\_int\_pos ( unsigned int \_\_s, std::complex< \_Tp > \_\_w )

Here s is a positive integer and the function descends into the different kernels depending on w.

_~	a positive integer.	
_s		
←	an arbitrary complex number.	
Generate	Generated by Doxygen	

The value of the polylogarithm.

Definition at line 768 of file sf polylog.tcc.

Referenced by \_\_polylog\_exp().

 $9.3.2.174 \quad template < typename \_Tp > std::\_detail::\_polylog\_exp\_int\_pos \ ( \ unsigned \ int \_s, \ \_Tp \_w \ )$ 

Here s is a positive integer and the function descends into the different kernels depending on w.

#### **Parameters**

_~	a positive integer
_s	
_~	an arbitrary real argument w
_ <i>w</i>	

## Returns

the value of the polylogarithm.

Definition at line 816 of file sf\_polylog.tcc.

 $References \_\_fpequal(), \_\_polylog\_exp\_asymp(), \_\_polylog\_exp\_negative\_real\_part(), \_\_polylog\_exp\_pos(), and \_\_\leftarrow riemann\_zeta().$ 

9.3.2.175 template<typename \_Tp > std::complex<\_Tp> std::\_\_detail::\_\_polylog\_exp\_neg ( \_Tp \_\_s, std::complex< \_Tp > \_\_w )

This function treats the cases of negative real index s. Theoretical convergence is present for  $|w| < 2\pi$ . We use an optimized version of

$$Li_s(e^w) = \Gamma(1-s)(-w)^{(s-1)} + (2\pi)^{(s-1)} + ($$

_←	The real index
_s	
_~	The complex argument
_ <i>w</i>	

The value of the polylogarithm.

Definition at line 347 of file sf polylog.tcc.

References \_\_fpequal(), \_\_riemann\_zeta(), and \_\_riemann\_zeta\_m\_1().

Referenced by polylog exp int neg(), and polylog exp real neg().

9.3.2.176 template<typename\_Tp > std::complex<\_Tp> std::\_detail::\_polylog\_exp\_neg ( int \_\_s, std::complex<\_Tp > \_\_w )

This function treats the cases of negative integer index s and branches accordingly

#### **Parameters**

_~	the integer index s.
_s	
_~	The Argument w
_ <i>w</i>	

#### Returns

The value of the Polylogarithm evaluated by a suitable function.

Definition at line 565 of file sf polylog.tcc.

References polylog exp neg even(), and polylog exp neg odd().

9.3.2.177 template<typename \_Tp , int \_\_sigma> std::complex<\_Tp> std::\_\_detail::\_\_polylog\_exp\_neg\_even ( unsigned int \_\_n, std::complex< \_Tp > \_\_w )

This function treats the cases of negative integer index s which are multiples of two.

In that case the sine occurring in the expansion occasionally takes on the value zero. We use that to provide an optimized series for p = 2n:

In the template parameter sigma we transport whether p = 4k(sigma = 1) or p = 4k + 2(sigma = -1)

$$Li_{p}(e^{w}) = Gamma(1-p)(-w)^{p-1} - A_{p}(w) - \sigma * B_{p}(w)$$

with

$$A_p(w) = 2(2\pi)^{(p-1)(-p)!/(2\pi)^{(-p/2)(1+w^2/(4pi^2))^{-1/2+p/2}}\cos((1-p)ArcTan(2pi/w))$$

and

$$B_p(w) = -2(2\pi)^{\ell}(p-1)\sum_{k=0}^{\infty} \Gamma(2+2k-p)/(2k+1)!(-1)^k(w/2\pi)^{\ell}(2k+1)(\zeta(2+2k-p)-1)$$

This is suitable for  $|w| < 2\pi$  The original series is (This might be worthwhile if we use the already present table of the Bernoullis)

$$Li_p(e^w) = \Gamma(1-p)(-w)^{p-1} - \sigma(2\pi)^p/\pi \sum_{k=0}^{\infty} \Gamma(2+2k-p)/(2k+1)!(-1)^k (w/2\pi)^{(2k+1)}\zeta(2+2k-p)$$

_~	the integral index $n=4k$ .
_n	
_~	The complex argument w
_w	

#### Returns

the value of the Polylogarithm.

Definition at line 451 of file sf polylog.tcc.

References \_\_fpequal().

Referenced by polylog exp neg().

9.3.2.178 template<typename \_Tp , int \_\_sigma> std::complex<\_Tp> std::\_\_detail::\_\_polylog\_exp\_neg\_odd ( unsigned int \_\_n, std::complex< \_Tp > \_\_w )

This function treats the cases of negative integer index s which are odd.

In that case the sine occurring in the expansion occasionally vanishes. We use that to provide an optimized series for p=1+2k: In the template parameter sigma we transport whether  $p=1+4k(\sigma=1)$  or  $p=3+4k(\sigma=-1)$ 

$$Li_p(e^w) = \Gamma(1-p)(-w)^{p-1} + \sigma * A_p(w) - \sigma * B_p(w)$$

with

$$A_p(w) = 2(2\pi)^{(p-1)}\Gamma(1-p)(1+w^2/(4\pi^2))^{-1/2+p/2}\cos((1-p)ArcTan(2pi/w))$$

and

$$B_p(w) = 2(2pi)^{\ell}(p-1)\sum_{k=0}^{\infty} \Gamma(1+2k-p)/(2k)!(-w^2/4/\pi^2)^k(\zeta(1+2k-p)-T)$$

This is suitable for  $|w| < 2\pi$ . The use of evenzeta gives a speedup of about 50 The original series is (This might be worthwhile if we use the already present table of the Bernoullis)

$$Li_p(e^w) = \Gamma(1-p) * (-w)^{p-1} - \sigma 2(2\pi)^{(p-1)} * \sum_{k=0}^{\infty} \Gamma(1+2k-p)/(2k)!(-1)^k (w/2/\pi)^{(2k)} \zeta(1+2k-p)$$

_~	the integral index $n = 4k$ .
_n	
_←	The complex argument w.
_ <i>w</i>	

The value of the Polylogarithm.

Definition at line 518 of file sf polylog.tcc.

References \_\_fpequal().

Referenced by polylog exp neg().

9.3.2.179 template<typename \_PowTp , typename \_Tp > \_Tp std::\_\_detail::\_\_polylog\_exp\_negative\_real\_part ( \_PowTp \_\_s, \_Tp \_\_w )

Theoretical convergence for Re(w) < 0.

Seems to beat the other expansions for  $Re(w) < -\pi/2 - \pi/5$ . Note that this is an implementation of the basic series:

$$Li_s(e^z) = \sum_{k=1} e^{(k*z)*k^{(-s)}}$$

## **Parameters**

_~	is an arbitrary type, integral or float.
_s	
_~	something with a negative real part.
_ <i>w</i>	

# Returns

the value of the polylogarithm.

Definition at line 738 of file sf\_polylog.tcc.

References fpequal().

Referenced by  $\_$ polylog\_exp(),  $\_$ polylog\_exp\_int\_neg(),  $\_$ polylog\_exp\_int\_pos(),  $\_$ polylog\_exp\_real\_neg(), and  $\hookleftarrow$   $\_$ polylog\_exp\_real\_pos().

9.3.2.180 template<typename \_Tp > std::complex<\_Tp> std::\_\_detail::\_\_polylog\_exp\_pos ( unsigned int \_\_s, std::complex< \_Tp > \_w )

This function treats the cases of positive integer index s.

$$Li_s(e^w) = \sum_{k=0}^{\infty} \frac{\zeta(s-k)w^k}{k!} + (H_{s-1} - \log(-w))w^(s-1)/(s-1)!$$

The radius of convergence is |w| < 2pi. Note that this series involves a  $\log(-x)$ . gcc and Mathematica differ in their implementation of  $\log(e^(i\pi))$ : gcc:  $\log(e^(+-i*\pi)) = +-i\pi$  whereas Mathematica doesn't preserve the sign in this case:  $\log(e^(+-i\pi)) = +i\pi$ 

_~	the index s.
_s	
_~	the argument w.
W	

#### Returns

the value of the polylogarithm.

Definition at line 207 of file sf\_polylog.tcc.

References \_\_fpequal(), and \_\_riemann\_zeta().

Referenced by \_\_polylog\_exp\_int\_pos(), and \_\_polylog\_exp\_real\_pos().

 $9.3.2.181 \quad template < typename \_Tp > std::\_detail::\_polylog\_exp\_pos \ ( \ unsigned \ int \_s, \ \_Tp \_w \ )$ 

This function treats the cases of positive integer index s for real w.

This specialization is worthwhile to catch the differing behaviour of log(x).

$$Li_s(e^w) = \sum_{k=0, k!=s-1} \zeta(s-k)w^k/k! + (H_{s-1} - \log(-w))w^(s-1)/(s-1)!$$

The radius of convergence is  $|w| < 2\pi$ . Note that this series involves a  $\log(-x)$ . The use of evenzeta yields a speedup of about 2.5. gcc and Mathematica differ in their implementation of  $\log(e^{(i\pi)})$ : gcc:  $\log(e^{(i\pi)}) = -i\pi$  whereas Mathematica doesn't preserve the sign in this case:  $\log(e^{(i\pi)}) = +i\pi$ 

# **Parameters**

_~	the index.
_s	
_~	the argument
_w	

# Returns

the value of the Polylogarithm

Definition at line 280 of file sf\_polylog.tcc.

References \_\_fpequal(), and \_\_riemann\_zeta().

9.3.2.182 template < typename \_Tp > std::complex < \_Tp > std::\_\_detail::\_\_polylog\_exp\_pos ( \_Tp \_\_s, std::complex < \_Tp > \_\_w )

This function treats the cases of positive real index s.

The defining series is

$$Li_s(e^w) = A_s(w) + B_s(w) + \Gamma(1-s)(-w)(s-1)$$

with

$$A_s(w) = \sum_{k=0}^{m} \zeta(s-k)w^k/k!$$

$$B_s(w) = \sum_{k=m+1}^{\infty} \sin(\pi/2(s-k))\Gamma(1-s+k)\zeta(1-s+k)(w/2/\pi)^k/k!$$

#### **Parameters**

_←	the positive real index s.
_s	
_←	The complex argument w.
_ <i>w</i>	

# Returns

the value of the polylogarithm.

Definition at line 604 of file sf\_polylog.tcc.

References \_\_fpequal(), and \_\_riemann\_zeta().

Return the polylog where s is a negative real value and for complex argument. Now we branch depending on the properties of w in the specific functions

## **Parameters**

_~	A negative real value that does not reduce to a negative integer.
_s	
_~	The complex argument.
_w	

## Returns

The value of the polylogarithm.

Definition at line 986 of file sf polylog.tcc.

Referenced by \_\_polylog\_exp().

```
9.3.2.184 template<typename_Tp > std::complex<_Tp> std::__detail::__polylog_exp_real_neg( _Tp __s, _Tp __w )
```

Return the polylog where s is a negative real value and for real argument. Now we branch depending on the properties of w in the specific functions.

#### **Parameters**

_←	A negative real value.
_s	
_~	A real argument.
w	

# Returns

The value of the polylogarithm.

Definition at line 1014 of file sf polylog.tcc.

 $References \underline{\hspace{0.5cm}} polylog\underline{\hspace{0.5cm}} exp\underline{\hspace{0.5cm}} asymp(), \underline{\hspace{0.5cm}} polylog\underline{\hspace{0.5cm}} exp\underline{\hspace{0.5cm}} neg(), and \underline{\hspace{0.5cm}} polylog\underline{\hspace{0.5cm}} exp\underline{\hspace{0.5cm}} negative\underline{\hspace{0.5cm}} real\underline{\hspace{0.5cm}} part().$ 

Return the polylog where s is a positive real value and for complex argument.

# Parameters

_~	A positive real number.
_s	
_~	the complex argument.
_ <i>w</i>	

# Returns

The value of the polylogarithm.

Definition at line 923 of file sf\_polylog.tcc.

```
References \_clamp\_0\_m2pi(), \_clamp\_pi(), \_fpreal(), \_polylog\_exp\_asymp(), \_polylog\_exp\_casymp(), \_polylog\_exp\_pos(), and \_riemann\_zeta().
```

Referenced by \_\_polylog\_exp().

9.3.2.186 template<typename \_Tp > std::complex<\_Tp> std::\_\_detail::\_\_polylog\_exp\_real\_pos( \_Tp \_\_s, \_Tp \_\_w )

Return the polylog where s is a positive real value and the argument is real.

#### **Parameters**

_~	A positive real number tht does not reduce to an integer.
_s	
_←	The real argument w.
_ <i>w</i>	

# Returns

The value of the polylogarithm.

Definition at line 957 of file sf\_polylog.tcc.

References  $\_$ fpequal(),  $\_$ polylog\_exp\_asymp(),  $\_$ polylog\_exp\_negative\_real\_part(),  $\_$ polylog\_exp\_pos(), and  $\_$  $\leftarrow$ riemann\_zeta().

9.3.2.187 template<typename  $_{Tp} > _{Tp}$  std::\_\_detail::\_\_psi (  $_{Tp} _{x}$  )

Return the digamma function. The digamma or  $\psi(x)$  function is defined by

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

For negative argument the reflection formula is used:

$$\psi(x) = \psi(1-x) - \pi \cot(\pi x)$$

.

Definition at line 2332 of file sf\_gamma.tcc.

References std::\_\_detail::\_Factorial\_table< \_Tp >::\_\_n, and \_\_psi\_asymp().

Referenced by \_\_expint\_En\_series(), \_\_hydrogen(), \_\_hyperg\_reflect(), and \_\_psi().

9.3.2.188 template < typename  $_{\rm Tp}$  >  $_{\rm Tp}$  std::\_\_psi ( unsigned int  $_{\rm n}$ ,  $_{\rm Tp}$   $_{\rm x}$  )

Return the polygamma function  $\psi^{(n)}(x)$ .

The polygamma function is related to the Hurwitz zeta function:

$$\psi^{(n)}(x) = (-1)^{n+1} m! \zeta(m+1, x)$$

Definition at line 2397 of file sf\_gamma.tcc.

References \_\_hurwitz\_zeta(), \_\_log\_gamma(), and \_\_psi().

9.3.2.189 template<typename \_Tp > \_Tp std::\_\_detail::\_\_psi\_asymp ( \_Tp \_\_x )

Return the digamma function for large argument. The digamma or  $\psi(x)$  function is defined by

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

The asymptotic series is given by:

$$\psi(x) = \ln(x) - \frac{1}{2x} - \sum_{n=1}^{\infty} \frac{B_{2n}}{2nx^{2n}}$$

Definition at line 2301 of file sf gamma.tcc.

Referenced by psi().

9.3.2.190 template<typename \_Tp > \_Tp std::\_\_detail::\_\_psi\_series ( \_Tp \_\_x )

Return the digamma function by series expansion. The digamma or  $\psi(x)$  function is defined by

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

The series is given by:

$$\psi(x) = -\gamma_E - \frac{1}{x} \sum_{k=1}^{\infty} \frac{x-1}{(k+1)(x+k)}$$

Definition at line 2270 of file sf\_gamma.tcc.

9.3.2.191 template<typename \_Tp > \_Tp std::\_\_qgamma ( \_Tp \_\_a, \_Tp \_\_x )

Return the regularized upper incomplete gamma function. The regularized upper incomplete gamma function is defined by

$$Q(a,x) = \frac{\Gamma(a,x)}{\Gamma(a)}$$

where  $\Gamma(a)$  is the gamma function and

$$\Gamma(a,x) = \int_x^\infty e^{-t} t^{a-1} dt (a > 0)$$

is the upper incomplete gamma function.

Definition at line 2046 of file sf\_gamma.tcc.

References \_\_gamma\_cont\_frac(), and \_\_gamma\_series().

9.3.2.192 template<typename \_Tp > \_Tp std::\_\_detail::\_\_riemann\_zeta ( \_Tp \_\_s )

Return the Riemann zeta function  $\zeta(s)$ .

The Riemann zeta function is defined by:

$$\zeta(s) = \sum_{k=1}^\infty k^{-s} \text{ for } s > 1 \frac{(2\pi)^s}{\pi} \sin(\frac{\pi s}{2}) \Gamma(1-s) \zeta(1-s) \text{ for } s < 1$$

For s < 1 use the reflection formula:

$$\zeta(s) = 2^s \pi^{s-1} \Gamma(1-s) \zeta(1-s)$$

_~	The argument
_s	

Definition at line 506 of file sf zeta.tcc.

References  $\_$ gamma(),  $\_$ log $\_$ gamma(),  $\_$ riemann $\_$ zeta $\_$ glob(),  $\_$ riemann $\_$ zeta $\_$ product(), and  $\_$ riemann $\_$ zeta $\_$ cum().

Referenced by \_\_polylog\_exp\_int\_pos(), \_\_polylog\_exp\_neg(), \_\_polylog\_exp\_pos(), \_\_polylog\_exp\_real\_pos(), and evenzeta().

9.3.2.193 template<typename \_Tp > \_Tp std::\_\_detail::\_\_riemann\_zeta\_alt ( \_Tp \_\_s )

Evaluate the Riemann zeta function  $\zeta(s)$  by an alternate series for s > 0.

The Riemann zeta function is defined by:

$$\zeta(s) = \sum_{k=1}^{\infty} \frac{1}{k^s} fors > 1$$

For s < 1 use the reflection formula:

$$\zeta(s) = 2^s \pi^{s-1} \Gamma(1-s) \zeta(1-s)$$

Definition at line 330 of file sf\_zeta.tcc.

9.3.2.194 template < typename \_Tp > \_Tp std::\_\_detail::\_\_riemann\_zeta\_euler\_maclaurin ( \_Tp  $\_s$  )

Evaluate the Riemann zeta function  $\zeta(s)$  by an alternate series for s > 0.

This is a specialization of the code for the Hurwitz zeta function.

Definition at line 283 of file sf\_zeta.tcc.

References \_S\_Euler\_Maclaurin\_zeta.

9.3.2.195 template<typename \_Tp > \_Tp std::\_\_detail::\_\_riemann\_zeta\_glob ( \_Tp  $\_s$  )

Evaluate the Riemann zeta function by series for all s != 1. Convergence is great until largish negative numbers. Then the convergence of the > 0 sum gets better.

The series is:

$$\zeta(s) = \frac{1}{1 - 2^{1 - s}} \sum_{n = 0}^{\infty} \frac{1}{2^{n + 1}} \sum_{k = 0}^{n} (-1)^k \frac{n!}{(n - k)! k!} (k + 1)^{-s}$$

Havil 2003, p. 206.

The Riemann zeta function is defined by:

$$\zeta(s) = \sum_{k=1}^{\infty} \frac{1}{k^s} fors > 1$$

For s < 1 use the reflection formula:

$$\zeta(s) = 2^s \pi^{s-1} \Gamma(1-s) \zeta(1-s)$$

Definition at line 375 of file sf\_zeta.tcc.

References \_\_log\_gamma().

Referenced by \_\_riemann\_zeta().

9.3.2.196 template<typename\_Tp > \_Tp std::\_\_detail::\_\_riemann\_zeta\_m\_1 ( \_Tp \_\_s )

Return the Riemann zeta function  $\zeta(s)-1$ .

## **Parameters**

Definition at line 673 of file sf zeta.tcc.

 $References \underline{\hspace{0.3cm}} riemann\_zeta\_m\_1\_sum(), \underline{\hspace{0.3cm}} S\_num\_zetam1, and \underline{\hspace{0.3cm}} S\_zetam1.$ 

Referenced by \_\_polylog\_exp\_neg().

9.3.2.197 template<typename \_Tp > \_Tp std::\_\_detail::\_\_riemann\_zeta\_m\_1\_sum ( \_Tp \_\_s )

Return the Riemann zeta function  $\zeta(s)-1$  by summation for s>1. This is a small remainder for large s.

The Riemann zeta function is defined by:

$$\zeta(s) = \sum_{k=1}^{\infty} \frac{1}{k^s} fors > 1$$

# **Parameters**

$$\begin{array}{|c|c|c|c|} \hline \_ \leftarrow & \text{The argument } s! = 1 \\ \hline \_ \textbf{s} & \end{array}$$

Definition at line 646 of file sf zeta.tcc.

Referenced by \_\_riemann\_zeta\_m\_1().

9.3.2.198 template<typename \_Tp > \_Tp std::\_\_detail::\_\_riemann\_zeta\_product ( \_Tp  $\_s$  )

Compute the Riemann zeta function  $\zeta(s)$  using the product over prime factors.

$$\zeta(s) = \prod_{i=1}^{\infty} \frac{1}{1 - p_i^{-s}}$$

where  $p_i$  are the prime numbers.

The Riemann zeta function is defined by:

$$\zeta(s) = \sum_{k=1}^{\infty} \frac{1}{k^s} fors > 1$$

For s < 1 use the reflection formula:

$$\zeta(s) = 2^s \pi^{s-1} \Gamma(1-s) \zeta(1-s)$$

_~	The argument
_s	

Definition at line 464 of file sf\_zeta.tcc.

Referenced by \_\_riemann\_zeta().

9.3.2.199 template<typename  $_{\rm Tp}$  >  $_{\rm Tp}$  std::\_\_detail::\_\_riemann\_zeta\_sum (  $_{\rm Tp}$  \_\_s )

Compute the Riemann zeta function  $\zeta(s)$  by summation for s > 1.

The Riemann zeta function is defined by:

$$\zeta(s) = \sum_{k=1}^{\infty} \frac{1}{k^s} fors > 1$$

For s < 1 use the reflection formula:

$$\zeta(s) = 2^s \pi^{s-1} \Gamma(1-s) \zeta(1-s)$$

Definition at line 254 of file sf zeta.tcc.

Referenced by \_\_riemann\_zeta().

9.3.2.200 template<typename \_Tp > \_\_gnu\_cxx::\_\_promote\_num\_t<\_Tp> std::\_\_detail::\_\_sinc ( \_Tp \_\_a, \_Tp \_\_x )

Return the generalized sinus cardinal function

$$sinc_a(x) = \frac{\sin(\pi x/a)}{(\pi x/a)}$$

.

Definition at line 51 of file sf\_cardinal.tcc.

9.3.2.201 template<typename \_Tp > \_\_gnu\_cxx::\_\_promote\_num\_t<\_Tp> std::\_\_detail::\_\_sinc ( \_Tp \_\_x )

Return the normalized sinus cardinal function

$$sinc(x) = \frac{\sin(\pi x)}{\pi x}$$

.

Definition at line 98 of file sf cardinal.tcc.

9.3.2.202 template<typename\_Tp > \_\_gnu\_cxx::\_\_promote\_num\_t<\_Tp> std::\_\_detail::\_\_sinc\_pi ( \_Tp \_\_x )

Return the unnormalized sinus cardinal function

$$sinc_{\pi}(x) = \frac{\sin(x)}{x}$$

.

Definition at line 78 of file sf\_cardinal.tcc.

9.3.2.203 template<typename \_Tp > std::pair<\_Tp, \_Tp> std::\_\_detail::\_\_sincosint ( \_Tp \_\_x )

This function returns the sine Si(x) and cosine Ci(x) integrals as a pair.

The sine integral is defined by:

$$Si(x) = \int_0^x dt \frac{\sin(t)}{t}$$

The cosine integral is defined by:

$$Ci(x) = \gamma_E + \log(x) + \int_0^x dt \frac{\cos(t) - 1}{t}$$

Definition at line 229 of file sf trigint.tcc.

References \_\_sincosint\_asymp(), \_\_sincosint\_cont\_frac(), and \_\_sincosint\_series().

9.3.2.204 template<typename\_Tp > void std::\_\_detail::\_\_sincosint\_asymp ( \_Tp \_\_t, \_Tp & \_Si, \_Tp & \_Ci )

This function computes the sine Si(x) and cosine Ci(x) integrals by asymptotic series summation for positive argument.

The asymptotic series is very good for x > 50.

Definition at line 164 of file sf\_trigint.tcc.

Referenced by \_\_sincosint().

9.3.2.205 template<typename\_Tp > void std::\_\_detail::\_\_sincosint\_cont\_frac ( \_Tp \_\_t, \_Tp & \_Si, \_Tp & \_Ci )

This function computes the sine Si(x) and cosine Ci(x) integrals by continued fraction for positive argument.

Definition at line 56 of file sf trigint.tcc.

Referenced by sincosint().

9.3.2.206 template<typename\_Tp > void std::\_\_detail::\_\_sincosint\_series ( \_Tp \_\_t, \_Tp & \_Si, \_Tp & \_Ci )

This function computes the sine Si(x) and cosine Ci(x) integrals by series summation for positive argument.

Definition at line 99 of file sf trigint.tcc.

Referenced by \_\_sincosint().

 $9.3.2.207 \quad template < typename \_Tp > \_gnu\_cxx::\_promote\_num\_t < \_Tp > std::\_detail::\_sinhc ( \_Tp \_a, \_Tp \_x )$ 

Return the generalized hyperbolic sinus cardinal function

$$sinhc_a(x) = \frac{\sinh(\pi x/a)}{\pi x/a}$$

.

Definition at line 124 of file sf\_cardinal.tcc.

9.3.2.208 template<typename\_Tp > \_\_gnu\_cxx::\_\_promote\_num\_t<\_Tp> std::\_\_detail::\_\_sinhc ( \_Tp \_\_x )

Return the normalized hyperbolic sinus cardinal function

$$sinhc(x) = \frac{\sinh(\pi x)}{\pi x}$$

.

Definition at line 167 of file sf\_cardinal.tcc.

9.3.2.209 template<typename\_Tp > \_\_gnu\_cxx::\_\_promote\_num\_t<\_Tp> std::\_\_detail::\_\_sinhc\_pi ( \_Tp \_\_x )

Return the unnormalized hyperbolic sinus cardinal function

$$sinhc_{\pi}(x) = \frac{\sinh(x)}{x}$$

.

Definition at line 149 of file sf\_cardinal.tcc.

9.3.2.210 template < typename  $_{\rm Tp} > _{\rm Tp}$  std::\_\_detail::\_\_sinhint ( const  $_{\rm Tp}$  \_\_x )

Return the hyperbolic sine integral li(x).

The hyperbolic sine integral is given by

$$Shi(x) = (Ei(x) - E_1(x))/2$$

_~	The argument of the hyperbolic sine integral function.
_X	

# Returns

The hyperbolic sine integral.

Definition at line 577 of file sf\_expint.tcc.

References \_\_expint\_E1(), and \_\_expint\_Ei().

9.3.2.211 template<typename \_Tp > \_Tp std::\_\_detail::\_\_sph\_bessel ( unsigned int \_\_n, \_Tp \_\_x )

Return the spherical Bessel function  $j_n(x)$  of order n and non-negative real argument x.

The spherical Bessel function is defined by:

$$j_n(x) = \left(\frac{\pi}{2x}\right)^{1/2} J_{n+1/2}(x)$$

# **Parameters**

_←	The non-negative integral order
_n	
_~	The non-negative real argument
_X	

# Returns

The output spherical Bessel function.

Definition at line 704 of file sf\_bessel.tcc.

References \_\_sph\_bessel\_in().

9.3.2.212 template<typename \_Tp > std::complex< \_Tp> std::\_\_detail::\_\_sph\_bessel ( unsigned int \_\_n, std::complex< \_Tp > \_\_z )

Return the complex spherical Bessel function.

in	_~	The order for which the spherical Bessel function is evaluated.
	_n	
in	_ <del></del>	The argument at which the spherical Bessel function is evaluated.
	Z	

The complex spherical Bessel function.

Definition at line 1270 of file sf\_hankel.tcc.

References \_\_sph\_hankel().

Compute the spherical modified Bessel functions  $i_n(x)$  and  $k_n(x)$  and their first derivatives  $i'_n(x)$  and  $k'_n(x)$  respectively.

# **Parameters**

n	The order of the modified spherical Bessel function.	
x	The argument of the modified spherical Bessel function.	
i_n	The output regular modified spherical Bessel function.	
k_n	The output irregular modified spherical Bessel function.	
ip⊷	← The output derivative of the regular modified spherical Bessel function.	
_n		
kp↔	The output derivative of the irregular modified spherical Bessel function.	
_n		

Definition at line 457 of file sf\_mod\_bessel.tcc.

References \_\_cyl\_bessel\_ik().

Compute the spherical Bessel  $j_n(x)$  and Neumann  $n_n(x)$  functions and their first derivatives  $j_n(x)$  and  $n'_n(x)$  respectively.

	n	The order of the spherical Bessel function.
	x	The argument of the spherical Bessel function.
out	j_n	The output spherical Bessel function.
out	n_n	The output spherical Neumann function.
out	<i>jp</i> ←	The output derivative of the spherical Bessel function.
	_n	
out	np⊷	The output derivative of the spherical Neumann function.
	_n	

Definition at line 669 of file sf\_bessel.tcc.

References \_\_cyl\_bessel\_jn().

Referenced by \_\_sph\_bessel(), \_\_sph\_hankel\_1(), \_\_sph\_hankel\_2(), and \_\_sph\_neumann().

```
9.3.2.215 template < typename _Tp > void std::__detail::__sph_hankel ( unsigned int __n, std::complex < _Tp > __z, std::complex < _Tp > & _H1, std::complex < _Tp > & _H2, std::complex < _Tp > & _H2, std::complex < _Tp > & _H2p )
```

Helper to compute complex spherical Hankel functions and their derivatives.

## **Parameters**

in	n	The order for which the spherical Hankel functions are evaluated.
in	z	The argument at which the spherical Hankel functions are evaluated.
out	_H1	The spherical Hankel function of the first kind.
out	_H1p	The derivative of the spherical Hankel function of the first kind.
out	_H2	The spherical Hankel function of the second kind.
out	_H2p	The derivative of the spherical Hankel function of the second kind.

Definition at line 1214 of file sf\_hankel.tcc.

References \_\_hankel().

Referenced by \_\_sph\_bessel(), \_\_sph\_hankel\_1(), \_\_sph\_hankel\_2(), and \_\_sph\_neumann().

9.3.2.216 template<typename\_Tp > std::complex<\_Tp> std::\_\_detail::\_\_sph\_hankel\_1 ( unsigned int \_\_n, \_Tp \_\_x )

Return the spherical Hankel function of the first kind  $h_n^{(1)}(x)$ .

The spherical Hankel function of the first kind is defined by:

$$h_n^{(1)}(x) = j_n(x) + i n_n(x)$$

# Parameters

_←	The order of the spherical Neumann function.
_n	
_~	The argument of the spherical Neumann function.
X	

## Returns

The output spherical Neumann function.

Definition at line 773 of file sf\_bessel.tcc.

References \_\_sph\_bessel\_jn().

9.3.2.217 template<typename \_Tp > std::complex<\_Tp> std::\_\_detail::\_\_sph\_hankel\_1 ( unsigned int \_\_n, std::complex< \_Tp > \_\_z )

Return the complex spherical Hankel function of the first kind.

# **Parameters**

in	_~	The order for which the spherical Hankel function of the first kind is evaluated.
	_n	
in	_~	The argument at which the spherical Hankel function of the first kind is evaluated.
	_Z	

# Returns

The complex spherical Hankel function of the first kind.

Definition at line 1238 of file sf\_hankel.tcc.

References \_\_sph\_hankel().

9.3.2.218 template<typename\_Tp > std::complex<\_Tp> std::\_\_detail::\_\_sph\_hankel\_2 ( unsigned int \_\_n, \_Tp \_\_x )

Return the spherical Hankel function of the second kind  $h_n^{(2)}(\boldsymbol{x}).$ 

The spherical Hankel function of the second kind is defined by:

$$h_n^{(2)}(x) = j_n(x) - in_n(x)$$

## **Parameters**

_~	The non-negative integral order
_n	
_~	The non-negative real argument
_X	

# Returns

The output spherical Neumann function.

Definition at line 805 of file sf\_bessel.tcc.

References \_\_sph\_bessel\_jn().

9.3.2.219 template < typename \_Tp > std::complex < \_Tp > std::\_\_detail::\_\_sph\_hankel\_2 ( unsigned int \_\_n, std::complex < \_Tp > z )

Return the complex spherical Hankel function of the second kind.

## **Parameters**

in	_~	The order for which the spherical Hankel function of the second kind is evaluated.
	_n	
in	_~	The argument at which the spherical Hankel function of the second kind is evaluated.
	_Z	

## **Returns**

The complex spherical Hankel function of the second kind.

Definition at line 1254 of file sf hankel.tcc.

References \_\_sph\_hankel().

9.3.2.220 template<typename \_Tp > std::complex<\_Tp> std::\_\_detail::\_\_sph\_harmonic ( unsigned int \_\_l, int \_\_m, \_Tp \_\_theta, \_\_Tp \_\_phi )

Return the spherical harmonic function.

The spherical harmonic function of l, m, and  $\theta, \phi$  is defined by:

$$Y_l^m(\theta,\phi) = (-1)^m \left[ \frac{(2l+1)}{4\pi} \frac{(l-m)!}{(l+m)!} \right] P_l^{|m|}(\cos\theta) \exp^{im\phi}$$

# **Parameters**

/	The order of the spherical harmonic function. $l>=0$ .
m	The order of the spherical harmonic function. $m <= l$ .
theta	The radian polar angle argument of the spherical harmonic function.
phi	The radian azimuthal angle argument of the spherical harmonic function.

Definition at line 351 of file sf\_legendre.tcc.

References \_\_sph\_legendre().

 $9.3.2.221 \quad template < typename \_Tp > \_Tp \ std::\_\_detail::\_\_sph\_legendre \ ( \ unsigned \ int \_\_I, \ unsigned \ int \_\_m, \ \_Tp \_\_theta \ )$ 

Return the spherical associated Legendre function.

The spherical associated Legendre function of l, m, and  $\theta$  is defined as  $Y_l^m(\theta, 0)$  where

$$Y_{l}^{m}(\theta,\phi) = (-1)^{m} \left[ \frac{(2l+1)}{4\pi} \frac{(l-m)!}{(l+m)!} \right] P_{l}^{m}(\cos\theta) \exp^{im\phi}$$

is the spherical harmonic function and  $P_l^m(x)$  is the associated Legendre function.

This function differs from the associated Legendre function by argument ( $x = \cos(\theta)$ ) and by a normalization factor but this factor is rather large for large l and m and so this function is stable for larger differences of l and m.

#### **Parameters**

/	The order of the spherical associated Legendre function. $l>=0$ .
m	The order of the spherical associated Legendre function. $m <= l$ .
theta	The radian polar angle argument of the spherical associated Legendre function.

Definition at line 254 of file sf\_legendre.tcc.

References \_\_log\_gamma(), and \_\_poly\_legendre\_p().

Referenced by \_\_hydrogen(), and \_\_sph\_harmonic().

9.3.2.222 template < typename \_Tp > \_Tp std::\_\_detail::\_\_sph\_neumann ( unsigned int \_\_n, \_Tp \_\_x )

Return the spherical Neumann function  $n_n(x)$  of order n and non-negative real argument x.

The spherical Neumann function is defined by:

$$n_n(x) = \left(\frac{\pi}{2x}\right)^{1/2} N_{n+1/2}(x)$$

# Parameters

_~	The order of the spherical Neumann function.
_n	
_~	The argument of the spherical Neumann function.
_X	

## Returns

The output spherical Neumann function.

Definition at line 741 of file sf bessel.tcc.

References sph bessel in().

9.3.2.223 template < typename \_Tp > std::complex < \_Tp > std::\_\_detail::\_\_sph\_neumann ( unsigned int \_\_n, std::complex < \_Tp > \_\_z )

Return the complex spherical Neumann function.

in	_←	The order for which the spherical Neumann function is evaluated.
	_n	
in	_~	The argument at which the spherical Neumann function is evaluated.
	_Z	

## Returns

The complex spherical Neumann function.

Definition at line 1286 of file sf hankel.tcc.

References \_\_sph\_hankel().

Return the exponential theta-1 function of period nu and argument x.

The Neville theta-1 function is defined by

$$\theta_1(\nu, x) = \frac{1}{\sqrt{\pi x}} \sum_{j=-\infty}^{+\infty} (-1)^j \exp\left(\frac{-(\nu + j - 1/2)^2}{x}\right)$$

# **Parameters**

nu	The periodic (period = 2) argument
x	The argument

Definition at line 190 of file sf theta.tcc.

References \_\_theta\_2().

Referenced by \_\_theta\_s().

9.3.2.225 template> \_Tp std::\_\_detail::\_\_theta\_2 ( \_Tp 
$$\_nu$$
, \_Tp  $\_x$  )

Return the exponential theta-2 function of period nu and argument x.

The exponential theta-2 function is defined by

$$\theta_2(\nu, x) = \frac{1}{\sqrt{\pi x}} \sum_{j=-\infty}^{+\infty} (-1)^j \exp\left(\frac{-(\nu+j)^2}{x}\right)$$

nu	The periodic (period = 2) argument
x	The argument

Definition at line 162 of file sf\_theta.tcc.

References \_\_theta\_2\_asymp(), and \_\_theta\_2\_sum().

Referenced by \_\_theta\_1(), and \_\_theta\_c().

9.3.2.226 template<typename \_Tp > \_Tp std::\_\_detail::\_\_theta\_2\_asymp ( \_Tp \_\_nu, \_Tp \_\_x )

Compute and return the  $\theta_2$  function by series expansion.

Definition at line 103 of file sf theta.tcc.

Referenced by \_\_theta\_2().

9.3.2.227 template<typename \_Tp > \_Tp std::\_\_detail::\_\_theta\_2\_sum ( \_Tp  $\_nu$ , \_Tp  $\_x$  )

Compute and return the  $\theta_1$  function by series expansion.

Definition at line 49 of file sf\_theta.tcc.

Referenced by \_\_theta\_2().

9.3.2.228 template<typename\_Tp > \_Tp std::\_\_detail::\_\_theta\_3 ( \_Tp \_\_nu, \_Tp \_\_x )

Return the exponential theta-3 function of period nu and argument x.

The exponential theta-3 function is defined by

$$\theta_3(\nu, x) = \frac{1}{\sqrt{\pi x}} \sum_{j=-\infty}^{+\infty} \exp\left(\frac{-(\nu+j)^2}{x}\right)$$

## **Parameters**

nu	The periodic (period = 1) argument
X	The argument

Definition at line 216 of file sf\_theta.tcc.

References \_\_theta\_3\_asymp(), and \_\_theta\_3\_sum().

Referenced by \_\_theta\_4(), and \_\_theta\_d().

Compute and return the  $\theta_3$  function by asymptotic series expansion.

Definition at line 128 of file sf\_theta.tcc.

Referenced by \_\_theta\_3().

9.3.2.230 template < typename 
$$_{\rm Tp} > _{\rm Tp}$$
 std::\_\_detail::\_\_theta\_3\_sum (  $_{\rm Tp}$  \_\_nu,  $_{\rm Tp}$  \_\_x )

Compute and return the  $\theta_3$  function by series expansion.

Definition at line 77 of file sf\_theta.tcc.

Referenced by \_\_theta\_3().

Return the exponential theta-2 function of period nu and argument x.

The exponential theta-2 function is defined by

$$\theta_2(\nu, x) = \frac{1}{\sqrt{\pi x}} \sum_{j=-\infty}^{+\infty} (-1)^j \exp\left(\frac{-(\nu+j)^2}{x}\right)$$

#### **Parameters**

nu	The periodic (period = 2) argument
x	The argument

Definition at line 244 of file sf\_theta.tcc.

References \_\_theta\_3().

Referenced by \_\_theta\_n().

9.3.2.232 template < typename \_Tp > \_Tp std::\_\_detail::\_\_theta\_c ( \_Tp 
$$\underline{\hspace{0.1cm}}$$
  $k$ , \_Tp  $\underline{\hspace{0.1cm}}$   $x$  )

Return the Neville  $\theta_c$  function

Definition at line 337 of file sf theta.tcc.

References \_\_comp\_ellint\_1(), \_\_ellnome(), and \_\_theta\_2().

9.3.2.233 template<typename \_Tp > \_Tp std::\_\_detail::\_\_theta\_d ( \_Tp  $\_$ k, \_Tp  $\_$ x )

Return the Neville  $\theta_d$  function

Definition at line 362 of file sf theta.tcc.

References \_\_comp\_ellint\_1(), \_\_ellnome(), and \_\_theta\_3().

9.3.2.234 template < typename  $_{\rm Tp} > _{\rm Tp}$  std::\_\_detail::\_\_theta\_n (  $_{\rm Tp}$  \_\_k,  $_{\rm Tp}$  \_\_x )

Return the Neville  $\theta_n$  function

Definition at line 387 of file sf\_theta.tcc.

References \_\_comp\_ellint\_1(), \_\_ellnome(), and \_\_theta\_4().

9.3.2.235 template<typename \_Tp > \_Tp std::\_\_detail::\_\_theta\_s ( \_Tp  $\_k$ , \_Tp  $\_x$  )

Return the Neville  $\theta_s$  function

Definition at line 311 of file sf theta.tcc.

References \_\_comp\_ellint\_1(), \_\_ellnome(), and \_\_theta\_1().

9.3.2.236 template<typename \_Tp > \_\_gnu\_cxx::\_\_promote\_num\_t<\_Tp> std::\_\_detail::\_\_zernike ( unsigned int \_\_n, int \_\_m, \_\_Tp \_\_rho, \_Tp \_\_phi )

Return the Zernicke polynomial  $Z_n^m(\rho,\phi)$  for non-negative degree n, signed order m, and real radial argument  $\rho$  and azimuthal angle  $\phi$ .

The even Zernicke polynomials are defined by:

$$Z_n^m(\rho,\phi) = R_n^m(\rho)\cos(m\phi)$$

and the odd Zernicke polynomials are defined by:

$$Z_n^{-m}(\rho,\phi) = R_n^m(\rho)\sin(m\phi)$$

for non-negative degree m and m <= n and where  $R_n^m(\rho)$  is the radial polynomial (

See also

\_poly\_radial\_jacobi).

**Template Parameters** 

\_Tp | The real type of the radial coordinate and azimuthal angle

#### **Parameters**

n	The non-negative degree.
m	The azimuthal order
rho	The radial coordinate
phi	The azimuthal angle

Definition at line 184 of file sf\_jacobi.tcc.

References \_\_poly\_radial\_jacobi().

9.3.2.237 template < typename  $_{Tp} > _{Tp}$  std::\_\_detail::\_\_znorm1 (  $_{Tp} _{x}$  )

Definition at line 58 of file sf\_owens\_t.tcc.

Referenced by \_\_owens\_t().

9.3.2.238 template<typename \_Tp > \_Tp std::\_\_detail::\_\_znorm2 ( \_Tp  $\_x$  )

Definition at line 47 of file sf\_owens\_t.tcc.

Referenced by \_\_owens\_t().

9.3.2.239 template<typename \_Tp = double> \_Tp std::\_\_detail::evenzeta ( unsigned int \_\_k )

A function to calculate the values of zeta at even positive integers. For values smaller than thirty a table is used.

### **Parameters**

_←	an integer at which we evaluate the Riemann zeta function.
_k	

### Returns

zeta(k)

Definition at line 157 of file sf\_polylog.tcc.

References \_\_riemann\_zeta().

### 9.3.3 Variable Documentation

 $9.3.3.1 \quad template < typename \_Tp > constexpr \ int \ std::\_detail::\_max\_FGH = \_Airy\_series < \_Tp > ::\_N\_FGH$ 

Definition at line 1427 of file sf\_airy.tcc.

```
9.3.3.2 template<> constexpr int std::__detail::__max_FGH< double > = 79
Definition at line 1433 of file sf_airy.tcc.
9.3.3.3 template <> constexpr int std:: detail:: max FGH < float > = 15
Definition at line 1430 of file sf_airy.tcc.
9.3.3.4 constexpr size_t std::__detail::_Num_Euler_Maclaurin_zeta = 100
Coefficients for Euler-Maclaurin summation of zeta functions.
                                                        B_{2j}/(2j)!
where B_k are the Bernoulli numbers.
Definition at line 65 of file sf_zeta.tcc.
9.3.3.5 constexpr Factorial_table < long double > std::__detail::_S_double_factorial_table[301]
Definition at line 274 of file sf gamma.tcc.
Referenced by __double_factorial(), and __log_double_factorial().
9.3.3.6 constexpr long double std:: detail:: S Euler Maclaurin zeta[ Num Euler Maclaurin zeta]
Definition at line 68 of file sf zeta.tcc.
Referenced by __hurwitz_zeta_euler_maclaurin(), and __riemann_zeta_euler_maclaurin().
9.3.3.7 constexpr Factorial table<long double> std:: detail:: S factorial table[171]
Definition at line 84 of file sf_gamma.tcc.
Referenced by factorial(), and log factorial().
```

9.3.3.8 constexpr\_Factorial\_table<long double> std::\_\_detail::\_S\_neg\_double\_factorial\_table[999]

Referenced by \_\_double\_factorial(), and \_\_log\_double\_factorial().

Definition at line 595 of file sf\_gamma.tcc.

9.3.3.9 template<typename \_Tp > constexpr std::size\_t std::\_\_detail::\_S\_num\_double\_factorials = 0

Definition at line 259 of file sf gamma.tcc.

9.3.3.10 template<> constexpr std::size\_t std::\_\_detail::\_S\_num\_double\_factorials< double > = 301

Definition at line 264 of file sf\_gamma.tcc.

9.3.3.11 template <> constexpr std::size\_t std:: detail:: S num double factorials < float > = 57

Definition at line 262 of file sf gamma.tcc.

 $9.3.3.12 \quad template <> constexpr \ std::size\_t \ std::\_detail::\_S\_num\_double\_factorials < long \ double> = 301$ 

Definition at line 266 of file sf gamma.tcc.

9.3.3.13 template<typename \_Tp > constexpr std::size\_t std::\_\_detail::\_S\_num\_factorials = 0

Definition at line 69 of file sf\_gamma.tcc.

9.3.3.14 template<> constexpr std::size\_t std::\_\_detail::\_S\_num\_factorials< double > = 171

Definition at line 74 of file sf\_gamma.tcc.

9.3.3.15 template <> constexpr std::size\_t std::\_\_detail::\_S\_num\_factorials < float > = 35

Definition at line 72 of file sf gamma.tcc.

9.3.3.16 template <> constexpr std::size\_t std::\_\_detail::\_S\_num\_factorials < long double > = 171

Definition at line 76 of file sf\_gamma.tcc.

 $9.3.3.17 \quad template < typename \_Tp > constexpr \ std:: \_detail:: \_S_num\_neg\_double\_factorials = 0$ 

Definition at line 579 of file sf gamma.tcc.

9.3.3.18 template <> constexpr std::size\_t std:: detail:: S num neg double factorials < double > = 150

Definition at line 584 of file sf gamma.tcc.

Referenced by \_\_\_riemann\_zeta\_m\_1().

```
9.3.3.19 template<> constexpr std::size_t std::__detail::_S_num_neg_double_factorials< float > = 27

Definition at line 582 of file sf_gamma.tcc.

9.3.3.20 template<> constexpr std::size_t std::__detail::_S_num_neg_double_factorials< long double > = 999

Definition at line 586 of file sf_gamma.tcc.

9.3.3.21 constexpr size_t std::__detail::_S_num_zetam1 = 33

Table of zeta(n) - 1 from 2 - 32. MPFR - 128 bits.

Definition at line 593 of file sf_zeta.tcc.

Referenced by __riemann_zeta_m_1().

9.3.3.22 constexpr long double std::__detail::_S_zetam1[_S_num_zetam1]

Definition at line 597 of file sf_zeta.tcc.
```

# **Chapter 10**

# **Class Documentation**

```
10.1 std::__detail::_Airy< _Tp > Class Template Reference
```

### **Public Types**

- using scalar\_type = std::\_\_detail::\_\_num\_traits\_t< value\_type >
- using value\_type = \_Tp

#### **Public Member Functions**

- constexpr\_Airy ()=default
- \_Airy (const \_Airy &)=default
- \_Airy (\_Airy &&)=default
- constexpr\_AiryState< value\_type > operator() (value\_type \_\_y) const

### **Public Attributes**

- scalar\_type inner\_radius { \_Airy\_default\_radii<scalar\_type>::inner\_radius}
- scalar\_type outer\_radius {\_Airy\_default\_radii<scalar\_type>::outer\_radius}

### **Static Public Attributes**

- static constexpr scalar\_type \_S\_2pi\_3 = scalar\_type{2} \* \_S\_pi\_3
- static constexpr scalar type S 5pi 6 = scalar type{5} \* S pi 6
- static constexpr auto \_S\_cNaN = value\_type(\_S\_NaN, \_S\_NaN)
- static constexpr value\_type \_S\_i = value\_type{0, 1}
- static constexpr auto \_S\_NaN = \_\_gnu\_cxx::\_\_quiet\_NaN<scalar\_type>()
- static constexpr scalar\_type \_S\_pi = \_\_gnu\_cxx::\_\_math\_constants<scalar\_type>::\_\_pi
- static constexpr scalar\_type \_S\_pi\_3 = \_\_gnu\_cxx::\_\_math\_constants<scalar\_type>::\_\_pi\_third
- static constexpr scalar\_type \_S\_pi\_6 = \_S\_pi\_3 / scalar\_type{2}
- static constexpr scalar\_type \_S\_sqrt\_pi = \_\_gnu\_cxx::\_\_math\_constants<scalar\_type>::\_\_root\_pi

### 10.1.1 Detailed Description

```
template<typename _Tp> class std::__detail::_Airy< _Tp >
```

Class to manage the asymptotic expansions for Airy functions. The parameters describing the various regions are adjustable.

Definition at line 3729 of file sf\_airy.tcc.

### 10.1.2 Member Typedef Documentation

10.1.2.1 template < typename \_Tp > using std::\_\_detail::\_Airy < \_Tp >::scalar\_type = std::\_\_detail::\_\_num\_traits\_t < value ← \_type >

Definition at line 3734 of file sf\_airy.tcc.

10.1.2.2 template<typename \_Tp> using std::\_\_detail::\_Airy< \_Tp >::value\_type = \_Tp

Definition at line 3733 of file sf\_airy.tcc.

#### 10.1.3 Constructor & Destructor Documentation

```
10.1.3.1 template<typename_Tp> constexpr std::__detail::_Airy<_Tp>::_Airy( ) [default]
```

 $10.1.3.2 \quad template < typename \_Tp > std::\__detail::\_Airy < \_Tp > ::\_Airy (\ const\_Airy < \_Tp > \&\ ) \quad \texttt{[default]}$ 

10.1.3.3 template<typename\_Tp> std::\_\_detail::\_Airy< \_Tp>::\_Airy( \_Airy< \_Tp > && ) [default]

#### 10.1.4 Member Function Documentation

10.1.4.1 template < typename \_Tp > constexpr \_AiryState < \_Tp > std::\_\_detail::\_Airy < \_Tp >::operator() ( value\_type \_\_y ) const

Return the Airy functions for complex argument.

Definition at line 3781 of file sf\_airy.tcc.

References std::\_\_detail::\_\_beta(), std::\_\_detail::\_Airy\_series< \_Tp >::\_S\_Ai(), and std::\_\_detail::\_Airy\_series< \_Tp >::\_S\_Bi().

#### 10.1.5 Member Data Documentation

Definition at line 3741 of file sf airy.tcc.

Definition at line 3743 of file sf\_airy.tcc.

Definition at line 3747 of file sf\_airy.tcc.

```
10.1.5.4 template<typename _Tp> constexpr _Airy< _Tp>::value_type std::__detail::_Airy< _Tp>::_S_i = value_type{0, 1} [static]
```

Definition at line 3744 of file sf\_airy.tcc.

```
10.1.5.5 template<typename _Tp> constexpr auto std::__detail::_Airy< _Tp >::_S_NaN = __gnu_cxx::__quiet_NaN<scalar_type>() [static]
```

Definition at line 3746 of file sf airy.tcc.

```
10.1.5.6 template<typename _Tp> constexpr scalar_type std::__detail::_Airy< _Tp >::_S_pi = __gnu_cxx::__math_constants<scalar_type>::__pi  [static]
```

Definition at line 3736 of file sf\_airy.tcc.

```
10.1.5.7 template<typename _Tp> constexpr _Airy< _Tp>::scalar_type std::__detail::_Airy< _Tp>::_S_pi_3 = __gnu_cxx::__math_constants<scalar_type>::_pi_third [static]
```

Definition at line 3740 of file sf\_airy.tcc.

```
10.1.5.8 template<typename_Tp> constexpr_Airy< _Tp>:::scalar_type std::__detail::_Airy< _Tp>:::_S_pi_6 = _S_pi_3 / scalar_type{2} [static]
```

Definition at line 3742 of file sf airy.tcc.

Definition at line 3738 of file sf\_airy.tcc.

Definition at line 3756 of file sf\_airy.tcc.

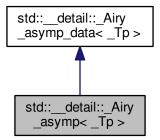
Definition at line 3757 of file sf airy.tcc.

The documentation for this class was generated from the following file:

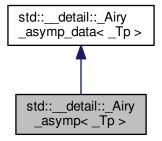
• bits/sf\_airy.tcc

# 10.2 std::\_\_detail::\_Airy\_asymp< \_Tp > Class Template Reference

Inheritance diagram for std::\_\_detail::\_Airy\_asymp< \_Tp >:



Collaboration diagram for std::\_\_detail::\_Airy\_asymp< \_Tp >:



### **Public Types**

using <u>\_\_cmplx</u> = std::complex< \_Tp >

#### **Public Member Functions**

- constexpr \_Airy\_asymp ()=default
- \_AiryState< std::complex< \_Tp >> \_S\_absarg\_ge\_pio3 (std::complex< \_Tp > \_\_z) const This function evaluates Ai(z), Ai'(z) and Bi(z), Bi'(z) from their asymptotic expansions for  $|arg(z)| < 2 * \pi/3$  i.e. roughly along the negative real axis.
- \_AiryState< std::complex< \_Tp >> \_S\_absarg\_lt\_pio3 (std::complex< \_Tp > \_\_z) const This function evaluates Ai(z) and Ai'(z) from their asymptotic expansions for  $|arg(-z)| < \pi/3$  i.e. roughly along the negative real axis.
- \_AiryState< std::complex< \_Tp >> operator() (std::complex< \_Tp > \_\_t, bool \_\_return\_fock\_airy=false) const

### 10.2.1 Detailed Description

```
\label{template} \begin{tabular}{ll} template < typename $\_Tp$ > \\ class std::$\_detail::$\_Airy$\_asymp < $\_Tp$ > \\ \end{tabular}
```

A class encapsulating the asymptotic expansions of Airy functions and thier derivatives.

### **Template Parameters**



Definition at line 3231 of file sf airy.tcc.

### 10.2.2 Member Typedef Documentation

10.2.2.1 template < typename \_Tp > using std::\_\_detail::\_\_diry\_asymp < \_Tp >::\_\_cmplx = std::complex < \_Tp >

Definition at line 3236 of file sf airy.tcc.

#### 10.2.3 Constructor & Destructor Documentation

10.2.3.1 template < typename \_Tp > constexpr std:: \_detail:: Airy\_asymp < \_Tp >:: Airy\_asymp ( ) [default]

#### 10.2.4 Member Function Documentation

This function evaluates Ai(z), Ai'(z) and Bi(z), Bi'(z) from their asymptotic expansions for  $|arg(z)| < 2 * \pi/3$  i.e. roughly along the negative real axis.

#### **Template Parameters**

### **Parameters**

in	_~	Complex argument at which Ai(z) and Bi(z) and their derivative are evaluated. This function assumes
	_Z	$ z >15$ and $ (arg(z) <2\pi/3.$

#### Returns

A struct containing z, Ai(z), Ai'(z), Bi(z), Bi'(z).

Definition at line 3503 of file sf\_airy.tcc.

This function evaluates Ai(z) and Ai'(z) from their asymptotic expansions for  $|arg(-z)| < \pi/3$  i.e. roughly along the negative real axis.

For speed, the number of terms needed to achieve about 16 decimals accuracy is tabled and determined for |z|. This function assumes |z| > 15 and  $|arg(-z)| < \pi/3$ .

Note that for speed and since this function is called by another, checks for valid arguments are not made. Hence, an error return is not needed.

### **Template Parameters**

_Тр	A real type
-----	-------------

#### **Parameters**

in	_~	The value at which the Airy function and their derivatives are evaluated.
	_Z	

#### Returns

A struct containing z, Ai(z), Ai'(z), Bi(z), Bi'(z).

**Todo** Revisit these numbers of terms for the Airy asymptotic expansions.

Definition at line 3533 of file sf\_airy.tcc.

Return the Airy functions for a given argument using asymptotic series.

### **Template Parameters**

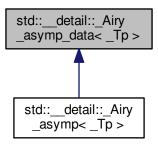
Definition at line 3262 of file sf\_airy.tcc.

The documentation for this class was generated from the following file:

• bits/sf\_airy.tcc

# 10.3 std::\_\_detail::\_Airy\_asymp\_data< \_Tp > Struct Template Reference

Inheritance diagram for std::\_\_detail::\_Airy\_asymp\_data< \_Tp >:



### 10.3.1 Detailed Description

```
template<typename _Tp> struct std::__detail::_Airy_asymp_data< _Tp >
```

A class encapsulating data for the asymptotic expansions of Airy functions and thier derivatives.

### **Template Parameters**

Definition at line 1867 of file sf\_airy.tcc.

The documentation for this struct was generated from the following file:

• bits/sf\_airy.tcc

# 10.4 std::\_\_detail::\_Airy\_asymp\_data < \_\_float128 > Struct Template Reference

### **Static Public Attributes**

- static constexpr \_\_float128 \_S\_c [\_S\_max\_cd]
- static constexpr \_\_float128 \_S\_d [\_S\_max\_cd]
- static constexpr int \_S\_max\_cd = 201

### 10.4.1 Detailed Description

```
template<> struct std::__detail::_Airy_asymp_data< __float128 >
```

Definition at line 2806 of file sf\_airy.tcc.

#### 10.4.2 Member Data Documentation

```
10.4.2.1 constexpr_float128 std::__detail::_Airy_asymp_data< _float128 >::_S_c[_S_max_cd] [static]
```

Definition at line 2812 of file sf\_airy.tcc.

```
10.4.2.2 constexpr_float128 std::__detail::_Airy_asymp_data< _float128 >::_S_d[_S_max_cd] [static]
```

Definition at line 3018 of file sf\_airy.tcc.

```
10.4.2.3 constexpr int std::__detail::_Airy_asymp_data< __float128 >::_S_max_cd = 201 [static]
```

Definition at line 2808 of file sf\_airy.tcc.

The documentation for this struct was generated from the following file:

• bits/sf\_airy.tcc

## 10.5 std::\_\_detail::\_Airy\_asymp\_data< double > Struct Template Reference

#### **Static Public Attributes**

- static constexpr double \_S\_c [\_S\_max\_cd]
- static constexpr double \_S\_d [\_S\_max\_cd]
- static constexpr int \_S\_max\_cd = 198

### 10.5.1 Detailed Description

```
\label{lem:lemplate} \mbox{template} <> \\ \mbox{struct std::\_detail::\_Airy\_asymp\_data} < \mbox{double} >
```

Definition at line 1974 of file sf\_airy.tcc.

### 10.5.2 Member Data Documentation

```
10.5.2.1 constexpr double std::__detail::_Airy_asymp_data< double >::_S_c[_S_max_cd] [static]
```

Definition at line 1980 of file sf\_airy.tcc.

```
10.5.2.2 constexpr double std::__detail::_Airy_asymp_data< double >::_S_d[_S_max_cd] [static]
```

Definition at line 2183 of file sf\_airy.tcc.

```
10.5.2.3 constexpr int std::__detail::_Airy_asymp_data< double >::_S_max_cd = 198 [static]
```

Definition at line 1976 of file sf\_airy.tcc.

The documentation for this struct was generated from the following file:

· bits/sf\_airy.tcc

# 10.6 std::\_\_detail::\_Airy\_asymp\_data< float > Struct Template Reference

### **Static Public Attributes**

- static constexpr float \_S\_c [\_S\_max\_cd]
- static constexpr float \_S\_d [\_S\_max\_cd]
- static constexpr int \_S\_max\_cd = 43

#### 10.6.1 Detailed Description

```
\label{lem:lemplate} \begin{tabular}{ll} template<> \\ struct std::\_detail::\_Airy\_asymp\_data< float> \\ \end{tabular}
```

Definition at line 1871 of file sf\_airy.tcc.

### 10.6.2 Member Data Documentation

10.6.2.1 constexpr float std::\_\_detail::\_Airy\_asymp\_data< float >::\_S\_c[\_S\_max\_cd] [static]

Definition at line 1877 of file sf\_airy.tcc.

```
10.6.2.2 constexpr float std::__detail::_Airy_asymp_data< float >::_S_d[_S_max_cd] [static]
```

Definition at line 1925 of file sf\_airy.tcc.

```
10.6.2.3 constexpr int std:__detail::_Airy_asymp_data< float >::_S_max_cd = 43 [static]
```

Definition at line 1873 of file sf\_airy.tcc.

The documentation for this struct was generated from the following file:

• bits/sf\_airy.tcc

## 10.7 std::\_\_detail::\_Airy\_asymp\_data < long double > Struct Template Reference

#### Static Public Attributes

- static constexpr long double \_S\_c [\_S\_max\_cd]
- static constexpr long double \_S\_d [\_S\_max\_cd]
- static constexpr int \_S\_max\_cd = 201

### 10.7.1 Detailed Description

```
\label{lem:lemplate} \mbox{template} <> \\ \mbox{struct std::\_detail::\_Airy\_asymp\_data} < \mbox{long double} >
```

Definition at line 2387 of file sf\_airy.tcc.

#### 10.7.2 Member Data Documentation

```
10.7.2.1 constexpr long double std::__detail::_Airy_asymp_data< long double >::_S_c[_S_max_cd] [static]
```

Definition at line 2393 of file sf\_airy.tcc.

10.7.2.2 constexpr long double std:: detail:: Airy asymp data < long double >::\_S\_d[ S max cd] [static]

Definition at line 2599 of file sf\_airy.tcc.

```
10.7.2.3 constexpr int std::__detail::_Airy_asymp_data< long double >::_S_max_cd = 201 [static]
```

Definition at line 2389 of file sf\_airy.tcc.

The documentation for this struct was generated from the following file:

· bits/sf\_airy.tcc

## 10.8 std::\_\_detail::\_Airy\_asymp\_series < \_Sum > Class Template Reference

### **Public Types**

- using scalar type = std:: detail:: num traits t < value type >
- using value\_type = typename \_Sum::value\_type

### **Public Member Functions**

- \_Airy\_asymp\_series (\_Sum \_\_proto)
- \_Airy\_asymp\_series (const \_Airy\_asymp\_series &)=default
- \_Airy\_asymp\_series (\_Airy\_asymp\_series &&)=default
- \_AiryState< value\_type > operator() (value\_type \_\_\_y)

#### **Static Public Attributes**

• static constexpr scalar\_type \_S\_sqrt\_pi = \_\_gnu\_cxx::\_\_math\_constants<scalar\_type>::\_\_root\_pi

#### 10.8.1 Detailed Description

```
template<typename _Sum> class std::__detail::_Airy_asymp_series< _Sum >
```

Class to manage the asymptotic series for Airy functions.

#### **Template Parameters**

```
_Sum | A sum type
```

Definition at line 3596 of file sf airy.tcc.

### 10.8.2 Member Typedef Documentation

10.8.2.1 template<typename \_Sum> using std::\_\_detail::\_Airy\_asymp\_series< \_Sum >::scalar\_type = std:: detail:: num traits t<value type>

Definition at line 3601 of file sf\_airy.tcc.

10.8.2.2 template < typename \_Sum > using std::\_\_detail::\_Airy\_asymp\_series < \_Sum > ::value\_type = typename \_Sum::value\_type

Definition at line 3600 of file sf airy.tcc.

#### 10.8.3 Constructor & Destructor Documentation

```
10.8.3.1 template < typename _Sum > std::__detail::_Airy_asymp_series < _Sum >::_Airy_asymp_series ( _Sum __proto ) [inline]
```

Definition at line 3605 of file sf\_airy.tcc.

```
10.8.3.2 template<typename_Sum> std::__detail::_Airy_asymp_series< _Sum >::_Airy_asymp_series ( const __Airy_asymp_series< _Sum > & ) [default]
```

#### 10.8.4 Member Function Documentation

```
10.8.4.1 template < typename _Sum > _AiryState < typename _Airy_asymp_series < _Sum >::value_type > std::__detail::_Airy_asymp_series < _Sum >::operator() ( value_type __y )
```

Return an \_AiryState containing, not actual Airy functions, but four asymptotic Airy components:

#### **Template Parameters**

```
_Sum | A sum type
```

Definition at line 3650 of file sf\_airy.tcc.

### 10.8.5 Member Data Documentation

```
10.8.5.1 template < typename _Sum > constexpr _Airy_asymp_series < _Sum >::scalar_type std::__detail:: ←
    __Airy_asymp_series < _Sum >::_S_sqrt_pi = __gnu_cxx::__math_constants < scalar_type >::__root_pi
    [static]
```

Definition at line 3603 of file sf airy.tcc.

The documentation for this class was generated from the following file:

bits/sf\_airy.tcc

# 10.9 std::\_\_detail::\_Airy\_default\_radii< \_Tp > Struct Template Reference

### 10.9.1 Detailed Description

```
template<typename _Tp>
struct std::__detail::_Airy_default_radii< _Tp>
```

Definition at line 3700 of file sf\_airy.tcc.

The documentation for this struct was generated from the following file:

• bits/sf\_airy.tcc

# 10.10 std::\_\_detail::\_Airy\_default\_radii< double > Struct Template Reference

#### **Static Public Attributes**

- static constexpr double inner radius {4.0}
- static constexpr double outer\_radius {12.0}

### 10.10.1 Detailed Description

```
template<> struct std::__detail::_Airy_default_radii< double >
```

Definition at line 3711 of file sf\_airy.tcc.

### 10.10.2 Member Data Documentation

```
10.10.2.1 constexpr double std::__detail::_Airy_default_radii < double >::inner_radius {4.0} [static]
```

Definition at line 3713 of file sf airy.tcc.

10.10.2.2 constexpr double std::\_\_detail::\_\_default\_\_radii< double >::outer\_\_radius {12.0} [static]

Definition at line 3714 of file sf\_airy.tcc.

The documentation for this struct was generated from the following file:

• bits/sf\_airy.tcc

### 10.11 std::\_\_detail::\_Airy\_default\_radii< float > Struct Template Reference

#### Static Public Attributes

- static constexpr float inner\_radius {2.0F}
- static constexpr float outer\_radius {6.0F}

### 10.11.1 Detailed Description

```
template<> struct std::__detail::_Airy_default_radii< float >
```

Definition at line 3704 of file sf\_airy.tcc.

#### 10.11.2 Member Data Documentation

```
10.11.2.1 constexpr float std::__detail::_Airy_default_radii < float >::inner_radius {2.0F} [static]
```

Definition at line 3706 of file sf\_airy.tcc.

```
10.11.2.2 constexpr float std::__detail::_Airy_default_radii< float >::outer_radius {6.0F} [static]
```

Definition at line 3707 of file sf airy.tcc.

The documentation for this struct was generated from the following file:

bits/sf\_airy.tcc

### 10.12 std::\_\_detail::\_Airy\_default\_radii< long double > Struct Template Reference

### **Static Public Attributes**

- static constexpr long double inner radius {5.0L}
- static constexpr long double outer radius {15.0L}

### 10.12.1 Detailed Description

```
\label{lem:lemplate} \mbox{template} <> \\ \mbox{struct std::\_detail::\_Airy\_default\_radii} < \mbox{long double} >
```

Definition at line 3718 of file sf airy.tcc.

#### 10.12.2 Member Data Documentation

```
10.12.2.1 constexpr long double std::__detail::_Airy_default_radii < long double >::inner_radius {5.0L} [static]
```

Definition at line 3720 of file sf\_airy.tcc.

```
10.12.2.2 constexpr long double std:: detail:: Airy default radii< long double >::outer radius {15.0L} [static]
```

Definition at line 3721 of file sf\_airy.tcc.

The documentation for this struct was generated from the following file:

· bits/sf\_airy.tcc

### 10.13 std::\_\_detail::\_Airy\_series< \_Tp > Class Template Reference

#### **Static Public Member Functions**

```
• static _AiryState< std::complex< _Tp >> _S_Airy (std::complex< _Tp > __t)
```

- static std::pair< std::complex< \_Tp >, std::complex< \_Tp >> \_S\_Bi (std::complex< \_Tp > \_\_t)
- $\bullet \ \ static \ \_AiryAuxilliaryState < std::complex < \ \_Tp >> \ \_S\_FGH \ (std::complex < \ \_Tp > \ \_t) \\$
- $\bullet \ \ \text{static} \ \_\text{AiryState} < \ \text{std}:: complex < \_\text{Tp} >> \_S\_\text{Fock} \ (\text{std}:: complex < \_\text{Tp} > \_\_t) \\$
- $\bullet \ \ \mathsf{static} \ \_\mathsf{AiryState} < \ \mathsf{std} :: \mathsf{complex} < \ \_\mathsf{Tp} >> \ \_\mathsf{S} \_\mathsf{Scorer} \ (\mathsf{std} :: \mathsf{complex} < \ \_\mathsf{Tp} > \ \_\mathsf{t})$
- static \_AiryState< std::complex< \_Tp >> \_S\_Scorer2 (std::complex< \_Tp > \_\_t)

### Static Public Attributes

```
• static constexpr int N FGH = 200
```

- static constexpr \_Tp \_S\_Ai0 = \_Tp{3.550280538878172392600631860041831763980e-1Q}
- static constexpr \_Tp \_S\_Aip0 = \_Tp{-2.588194037928067984051835601892039634793e-1Q}
- static constexpr Tp S Bi0 = Tp{6.149266274460007351509223690936135535960e-1Q}
- static constexpr \_Tp \_S\_Bip0 = \_Tp{4.482883573538263579148237103988283908668e-1Q}
- static constexpr \_Tp \_S\_eps = \_\_gnu\_cxx::\_\_epsilon(\_Tp{})
- static constexpr \_Tp \_S\_Gi0 = \_Tp{2.049755424820002450503074563645378511979e-1Q}
- static constexpr \_Tp \_S\_Gip0 = \_Tp{1.494294524512754526382745701329427969551e-1Q}
- static constexpr Tp S Hi0 = Tp{4.099510849640004901006149127290757023959e-1Q}
- static constexpr \_Tp \_S\_Hip0 = \_Tp{2.988589049025509052765491402658855939102e-1Q}
- static constexpr \_\_cmplx \_S\_i {\_Tp{0}, \_Tp{1}}
- static constexpr \_Tp \_S\_log10min = \_\_gnu\_cxx::\_\_log10\_min(\_Tp{})
- static constexpr \_Tp \_S\_pi = \_\_gnu\_cxx::\_\_math\_constants<\_Tp>::\_\_pi
- static constexpr \_Tp \_S\_sqrt\_pi = \_\_gnu\_cxx::\_\_math\_constants<\_Tp>::\_\_root\_pi

### 10.13.1 Detailed Description

template<typename \_Tp> class std::\_\_detail::\_Airy\_series< \_Tp >

This class orgianizes series solutions of the Airy function.

$$fai(x) = \sum_{k=0}^{\infty} \frac{(2k+1)!!!x^{3k}}{(2k+1)!}$$

$$gai(x) = \sum_{k=0}^{\infty} \frac{(2k+2)!!!x^{3k+1}}{(2k+2)!}$$

$$hai(x) = \sum_{k=0}^{\infty} \frac{(2k+3)!!!x^{3k+2}}{(2k+3)!}$$

This class contains tabulations of the factors appearing in the sums above.

Definition at line 108 of file sf airy.tcc.

#### 10.13.2 Member Function Documentation

10.13.2.1 template<typename \_Tp > std::pair< std::complex< \_Tp >, std::complex< \_Tp >> std::\_\_detail::\_Airy\_series< \_Tp >::\_S\_Ai( std::complex< \_Tp > \_\_t) [static]

Return the Airy function of the first kind and its derivative by using the series expansions of the auxilliary Airy functions:

$$fai(x) = \sum_{k=0}^{\infty} \frac{(2k+1)!!!x^{3k}}{(2k+1)!}$$

$$gai(x) = \sum_{k=0}^{\infty} \frac{(2k+2)!!!x^{3k+1}}{(2k+2)!}$$

The Airy function of the first kind is then defined by:

$$Ai(x) = Ai(0) fai(x) + Ai'(0) qai(x)$$

where 
$$Ai(0)=3^{-2/3}/\Gamma(2/3), Ai'(0)=-3-1/2Bi'(0)$$
 and  $Bi(0)=3^{1/2}Ai(0), Bi'(0)=3^{1/6}/\Gamma(1/3)$ 

**Template Parameters** 

Definition at line 1590 of file sf\_airy.tcc.

Referenced by std::\_\_detail::\_Airy< \_Tp >::operator()().

10.13.2.2 template<typename \_Tp > \_AiryState< std::complex< \_Tp >> std::\_\_detail::\_Airy\_series< \_Tp >::\_S\_Airy ( std::complex< \_Tp > \_\_t ) [static]

Return the Fock-type Airy functions Ai(t), and Bi(t) and their derivatives of complex argument.

#### **Template Parameters**

_Тр	A real type
-----	-------------

#### **Parameters**

$\leftarrow$	The complex argument
_←	
$\leftarrow$	
_←	
t	

Definition at line 1844 of file sf\_airy.tcc.

10.13.2.3 template<typename \_Tp > std::pair< std::complex< \_Tp >, std::complex< \_Tp >> std::\_\_detail::\_Airy\_series< \_Tp >::\_S\_Bi( std::complex< \_Tp > \_\_t) [static]

Return the Airy function of the second kind and its derivative by using the series expansions of the auxilliary Airy functions:

$$fai(x) = \sum_{k=0}^{\infty} \frac{(2k+1)!!!x^{3k}}{(2k+1)!}$$

$$gai(x) = \sum_{k=0}^{\infty} \frac{(2k+2)!!!x^{3k+1}}{(2k+2)!}$$

The Airy function of the second kind is then defined by:

$$Bi(x) = Bi(0)fai(x) + Bi'(0)gai(x)$$

where  $Ai(0) = 3^{-2/3}/\Gamma(2/3)$ , Ai'(0) = -3 - 1/2Bi'(0) and  $Bi(0) = 3^{1/2}Ai(0)$ ,  $Bi'(0) = 3^{1/6}/\Gamma(1/3)$ 

#### **Template Parameters**

Definition at line 1613 of file sf airy.tcc.

Referenced by std::\_\_detail::\_Airy< \_Tp >::operator()().

Return the auxilliary Airy functions:

$$fai(x) = \sum_{k=0}^{\infty} \frac{(2k+1)!!!x^{3k}}{(2k+1)!}$$

$$gai(x) = \sum_{k=0}^{\infty} \frac{(2k+2)!!!x^{3k+1}}{(2k+2)!}$$

$$hai(x) = \sum_{k=0}^{\infty} \frac{(2k+3)!!!x^{3k+2}}{(2k+3)!}$$

#### **Template Parameters**

_ <i>Tp</i>	A real type
-------------	-------------

Definition at line 1632 of file sf\_airy.tcc.

Return the Fock-type Airy functions  $w_1(t)$ , and  $w_2(t)$  and their derivatives of complex argument.

### **Template Parameters**

### **Parameters**

$\leftarrow$	The complex argument
_←	
$\leftarrow$	
_←	
t	

Definition at line 1856 of file sf airy.tcc.

10.13.2.6 template<typename\_Tp > \_AiryState< std::complex< \_Tp >> std::\_\_detail::\_Airy\_series< \_Tp >::\_S\_Scorer ( std::complex< \_Tp > \_\_t ) [static]

Return the Scorer functions by using the series expansions of the auxilliary Airy functions:

$$fai(x) = \sum_{k=0}^{\infty} \frac{(2k+1)!!!x^{3k}}{(2k+1)!}$$

$$gai(x) = \sum_{k=0}^{\infty} \frac{(2k+2)!!!x^{3k+1}}{(2k+2)!}$$

$$hai(x) = \sum_{k=0}^{\infty} \frac{(2k+3)!!!x^{3k+2}}{(2k+3)!}$$

The Scorer function is then defined by:

$$Hi(x) = Hi(0) \left( fai(x) + gai(x) + hai(x) \right)$$

where  $Hi(0)=2/(3^{7/6}\Gamma(2/3))$  and  $Hi'(0)=2/(3^{5/6}\Gamma(1/3))$ . The other Scorer function is found from the identity

$$Gi(x) + Hi(x) = Bi(x)$$

**Todo** Find out what is wrong with the Hi = fai + gai + hai scorer function.

### **Template Parameters**

Definition at line 1706 of file sf\_airy.tcc.

Return the Scorer functions by using the series expansions:

$$Hi(x) = \frac{3^{-2/3}}{\pi} \sum_{k=0}^{\infty} \Gamma\left(\frac{k+1}{3}\right) \frac{3^{1/3}x}{k!}$$

$$Hi'(x) = \frac{3^{-1/3}}{\pi} \sum_{k=0}^{\infty} \Gamma\left(\frac{k+2}{3}\right) \frac{3^{1/3}x}{k!}$$

$$Gi(x) = \frac{3^{-2/3}}{\pi} \sum_{k=0}^{\infty} \cos\left(\frac{2k-1}{3}\pi\right) \Gamma\left(\frac{k+1}{3}\right) \frac{3^{1/3}x}{k!}$$

$$Gi'(x) = \frac{3^{-1/3}}{\pi} \sum_{k=0}^{\infty} \cos\left(\frac{2k+1}{3}\pi\right) \Gamma\left(\frac{k+2}{3}\right) \frac{3^{1/3}x}{k!}$$

Definition at line 1743 of file sf airy.tcc.

#### 10.13.3 Member Data Documentation

10.13.3.1 template<typename\_Tp > constexpr int std::\_\_detail::\_Airy\_series< \_Tp >::\_N\_FGH = 200 [static]

Definition at line 113 of file sf airy.tcc.

```
10.13.3.2 template<typename _Tp > constexpr _Tp std::__detail::_Airy_series< _Tp >::_S_Ai0 = _Tp{3.550280538878172392600631860041831763980e-1Q} [static]
```

Definition at line 1353 of file sf airy.tcc.

```
10.13.3.3 template<typename _Tp > constexpr _Tp std::__detail::_Airy_series< _Tp >::_S_Aip0 = _Tp{-2.588194037928067984051835601892039634793e-1Q} [static]
```

Definition at line 1355 of file sf\_airy.tcc.

```
10.13.3.4 template<typename _Tp > constexpr _Tp std::__detail::_Airy_series< _Tp >::_S_Bi0 = _Tp{6.149266274460007351509223690936135535960e-1Q} [static]
```

Definition at line 1357 of file sf\_airy.tcc.

```
10.13.3.5 template<typename _Tp > constexpr _Tp std::__detail::_Airy_series< _Tp >::_S_Bip0 = 
    _Tp{4.482883573538263579148237103988283908668e-1Q} [static]
```

Definition at line 1359 of file sf airy.tcc.

```
10.13.3.6 template<typename _Tp > constexpr _Tp std::__detail::_Airy_series< _Tp >::_S_eps = __gnu_cxx::_epsilon(_Tp{}) [static]
```

Definition at line 1348 of file sf\_airy.tcc.

```
10.13.3.7 template<typename _Tp > constexpr _Tp std::__detail::_Airy_series< _Tp >::_S_Gi0 = _Tp{2.049755424820002450503074563645378511979e-1Q} [static]
```

Definition at line 1365 of file sf\_airy.tcc.

```
10.13.3.8 template<typename _Tp > constexpr _Tp std::__detail::_Airy_series< _Tp >::_S_Gip0 = _Tp{1.494294524512754526382745701329427969551e-1Q} [static]
```

Definition at line 1367 of file sf\_airy.tcc.

```
10.13.3.9 template<typename _Tp > constexpr _Tp std::__detail::_Airy_series< _Tp >::_S_Hi0 = _Tp{4.099510849640004901006149127290757023959e-1Q} [static]
```

Definition at line 1361 of file sf airy.tcc.

Definition at line 1363 of file sf airy.tcc.

```
10.13.3.11 template < typename _Tp > constexpr std::complex < _Tp > std::__detail::_Airy_series < _Tp >::_S_i {_Tp{0}, _Tp{1}} [static]
```

Definition at line 1368 of file sf\_airy.tcc.

```
10.13.3.12 template<typename _Tp > constexpr _Tp std::__detail::_Airy_series< _Tp >::_S_log10min = __gnu_cxx::__log10_min(_Tp{}) [static]
```

Definition at line 1369 of file sf\_airy.tcc.

```
10.13.3.13 template<typename _Tp > constexpr _Tp std::__detail::_Airy_series< _Tp >::_S_pi = __gnu_cxx::__math_constants<_Tp>::_pi [static]
```

Definition at line 1349 of file sf\_airy.tcc.

```
10.13.3.14 template<typename _Tp > constexpr _Tp std::__detail::_Airy_series< _Tp >::_S_sqrt_pi = __gnu_cxx::__math_constants<_Tp>::__root_pi  [static]
```

Definition at line 1351 of file sf airy.tcc.

The documentation for this class was generated from the following file:

· bits/sf airy.tcc

# 10.14 std::\_\_detail::\_AiryAuxilliaryState< \_Tp > Struct Template Reference

### **Public Types**

```
using _Val = std::__detail::__num_traits_t< _Tp >
```

#### **Public Attributes**

- \_Tp fai
- \_Tp faip
- \_Tp gai
- \_Tp gaip
- \_Tp hai
- \_Tp haip
- \_Tp z

### 10.14.1 Detailed Description

```
template<typename _Tp> struct std::__detail::_AiryAuxilliaryState< _Tp >
```

A structure containing three auxilliary Airy functions and their derivatives.

Definition at line 80 of file sf airy.tcc.

### 10.14.2 Member Typedef Documentation

```
10.14.2.1 template<typename _Tp> using std::__detail::_AiryAuxilliaryState< _Tp >::_Val = std::__detail::_num_traits_t<_Tp>
```

Definition at line 82 of file sf\_airy.tcc.

#### 10.14.3 Member Data Documentation

10.14.3.1 template<typename \_Tp> \_Tp std::\_\_detail::\_AiryAuxilliaryState< \_Tp >::fai

Definition at line 85 of file sf\_airy.tcc.

10.14.3.2 template<typename \_Tp> \_Tp std::\_\_detail::\_AiryAuxilliaryState< \_Tp >::faip

Definition at line 86 of file sf\_airy.tcc.

10.14.3.3 template<typename\_Tp>\_Tp std::\_\_detail::\_AiryAuxilliaryState< \_Tp >::gai

Definition at line 87 of file sf\_airy.tcc.

10.14.3.4 template<typename \_Tp> \_Tp std::\_\_detail::\_AiryAuxilliaryState< \_Tp >::gaip

Definition at line 88 of file sf\_airy.tcc.

10.14.3.5 template<typename \_Tp> \_Tp std::\_\_detail::\_AiryAuxilliaryState< \_Tp >::hai

Definition at line 89 of file sf airy.tcc.

```
10.14.3.6 template<typename _Tp> _Tp std::__detail::_AiryAuxilliaryState< _Tp >::haip
```

Definition at line 90 of file sf\_airy.tcc.

```
10.14.3.7 template<typename _Tp> _Tp std:: _detail:: AiryAuxilliaryState< _Tp >::z
```

Definition at line 84 of file sf\_airy.tcc.

The documentation for this struct was generated from the following file:

• bits/sf\_airy.tcc

# 10.15 std::\_\_detail::\_AiryState< \_Tp > Struct Template Reference

### **Public Types**

```
using _Val = std::__detail::__num_traits_t< _Tp >
```

#### **Public Member Functions**

• constexpr \_Tp Wronskian () const

### **Static Public Member Functions**

• static constexpr \_Val true\_Wronskian ()

### **Public Attributes**

- \_Tp Ai
- \_Tp Aip
- \_Tp Bi
- \_Tp Bip
- \_Tp z

### 10.15.1 Detailed Description

```
template<typename _Tp> struct std::__detail::_AiryState< _Tp >
```

This struct defines the Airy function state with two presumably numerically useful Airy functions and their derivatives. The data mambers are directly accessible. The lone method computes the Wronskian from the stord functions. A static method returns the correct Wronskian.

Definition at line 55 of file sf airy.tcc.

### 10.15.2 Member Typedef Documentation

10.15.2.1 template<typename\_Tp> using std::\_\_detail::\_AiryState<\_Tp>::\_Val = std::\_\_detail::\_num\_traits\_t<\_Tp>

Definition at line 57 of file sf\_airy.tcc.

#### 10.15.3 Member Function Documentation

10.15.3.1 template<typename\_Tp> static constexpr\_Val std::\_\_detail::\_AiryState< \_Tp >::true\_Wronskian ( ) [inline], [static]

Definition at line 70 of file sf airy.tcc.

10.15.3.2 template<typename\_Tp> constexpr\_Tp std::\_\_detail::\_AiryState< \_Tp >::Wronskian( ) const [inline]

Definition at line 66 of file sf airy.tcc.

References std::\_\_detail::\_AiryState< \_Tp >::Aip.

#### 10.15.4 Member Data Documentation

10.15.4.1 template<typename \_Tp> \_Tp std::\_\_detail::\_AiryState< \_Tp >::Ai

Definition at line 60 of file sf\_airy.tcc.

10.15.4.2 template<typename \_Tp> \_Tp std::\_\_detail::\_AiryState< \_Tp >::Aip

Definition at line 61 of file sf\_airy.tcc.

Referenced by std::\_\_detail::\_AiryState< \_Tp >::Wronskian().

10.15.4.3 template<typename \_Tp> \_Tp std::\_\_detail::\_AiryState< \_Tp >::Bi

Definition at line 62 of file sf\_airy.tcc.

10.15.4.4 template<typename \_Tp> \_Tp std::\_\_detail::\_AiryState< \_Tp >::Bip

Definition at line 63 of file sf airy.tcc.

```
10.15.4.5 \quad template < typename \_Tp > \_Tp \ std:: \__detail:: \_AiryState < \_Tp >::z
```

Definition at line 59 of file sf\_airy.tcc.

The documentation for this struct was generated from the following file:

· bits/sf airy.tcc

# 10.16 std::\_\_detail::\_Factorial\_table< \_Tp > Struct Template Reference

### **Public Attributes**

```
    _Tp __factorial
```

- · \_Tp \_\_log\_factorial
- unsigned int \_\_n

### 10.16.1 Detailed Description

```
template<typename _Tp>
struct std::__detail::_Factorial_table< _Tp >
```

Definition at line 61 of file sf gamma.tcc.

#### 10.16.2 Member Data Documentation

```
10.16.2.1 template < typename _Tp > _Tp std::__detail::_Factorial_table < _Tp >::__factorial
```

Definition at line 64 of file sf gamma.tcc.

Referenced by std::\_\_detail::\_\_double\_factorial().

```
10.16.2.2 \quad template < typename \_Tp > \_Tp \ std:: \_\_detail:: \_Factorial\_table < \_Tp >:: \_log\_factorial\_table < \_Tp >:: \_log_factorial\_table < \_Tp >: \_tp >: \_log_factorial\_table < \_Tp >: \_tp
```

Definition at line 65 of file sf\_gamma.tcc.

Referenced by std:: detail:: log double factorial().

```
10.16.2.3 template<typename _Tp > unsigned int std::__detail::_Factorial_table< _Tp >::__n
```

Definition at line 63 of file sf\_gamma.tcc.

Referenced by std::\_\_detail::\_\_bernoulli(), std::\_\_detail::\_\_bernoulli\_2n(), std::\_\_detail::\_\_bernoulli\_series(), std::\_\_detail::\_\_bernoulli\_series(), std::\_\_detail::\_\_bernoulli\_series(), std::\_\_detail::\_\_gamma\_cont\_frac(), std::\_\_detail::\_\_gamma\_series(), std::\_\_detail::\_\_log\_double\_factorial(), std::\_\_detail::\_\_log\_factorial(), and std::\_\_detail::\_\_psi().

The documentation for this struct was generated from the following file:

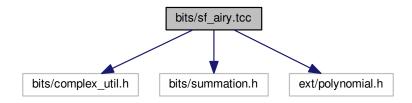
• bits/sf\_gamma.tcc

# **Chapter 11**

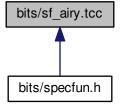
# **File Documentation**

# 11.1 bits/sf\_airy.tcc File Reference

```
#include <bits/complex_util.h>
#include <bits/summation.h>
#include <ext/polynomial.h>
Include dependency graph for sf_airy.tcc:
```



This graph shows which files directly or indirectly include this file:



286 File Documentation

#### Classes

```
class std::__detail::_Airy<_Tp>
class std::__detail::_Airy_asymp<_Tp>
struct std::__detail::_Airy_asymp_data<_Tp>
struct std::__detail::_Airy_asymp_data< __float128 >
struct std::__detail::_Airy_asymp_data< double >
struct std::__detail::_Airy_asymp_data< float >
struct std::__detail::_Airy_asymp_data< long double >
class std::__detail::_Airy_asymp_series<_Sum >
struct std::__detail::_Airy_default_radii<_Tp >
struct std::__detail::_Airy_default_radii< double >
struct std::__detail::_Airy_default_radii< long double >
struct std::__detail::_Airy_default_radii< long double >
class std::__detail::_Airy_series<_Tp >
struct std::__detail::_AiryAuxilliaryState<_Tp >
struct std::__detail::_AiryState<_Tp >
```

### **Namespaces**

- std
- std::\_\_detail

#### **Macros**

#define \_GLIBCXX\_BITS\_SF\_AIRY\_TCC 1

#### **Functions**

```
    template<typename _Tp >
        std::complex< _Tp > std::__detail::__airy_ai (std::complex< _Tp > __z)
        Return the complex Airy Ai function.
    template<typename _Tp >
        std::complex< _Tp > std::__detail::__airy_bi (std::complex< _Tp > __z)
        Return the complex Airy Bi function.
```

#### **Variables**

```
    template<typename _Tp >
        constexpr int std::__detail::__max_FGH = _Airy_series<_Tp>::_N_FGH
    template<>
        constexpr int std::__detail::__max_FGH< double > = 79
    template<>
        constexpr int std::__detail::__max_FGH< float > = 15
```

### 11.1.1 Detailed Description

This is an internal header file, included by other library headers. You should not attempt to use it directly.

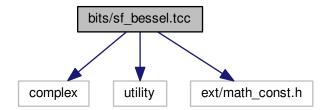
#### 11.1.2 Macro Definition Documentation

11.1.2.1 #define \_GLIBCXX\_BITS\_SF\_AIRY\_TCC 1

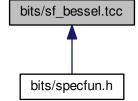
Definition at line 31 of file sf\_airy.tcc.

### 11.2 bits/sf\_bessel.tcc File Reference

```
#include <complex>
#include <utility>
#include <ext/math_const.h>
Include dependency graph for sf_bessel.tcc:
```



This graph shows which files directly or indirectly include this file:



288 File Documentation

### **Namespaces**

- std
- std:: detail

#### **Macros**

• #define GLIBCXX BITS SF BESSEL TCC 1

#### **Functions**

template<typename \_Tp >
 \_Tp std::\_\_detail::\_\_cyl\_bessel\_ij\_series (\_Tp \_\_nu, \_Tp \_\_x, \_Tp \_\_sgn, unsigned int \_\_max\_iter)

This routine returns the cylindrical Bessel functions of order  $\nu$ :  $J_{\nu}$  or  $I_{\nu}$  by series expansion.

ullet template<typename \_Tp >

```
_Tp std:: __detail:: __cyl_bessel_j (_Tp __nu, _Tp __x)
```

Return the Bessel function of order  $\nu$ :  $J_{\nu}(x)$ .

template<typename \_Tp >

```
void std::__detail::__cyl_bessel_jn (_Tp __nu, _Tp __x, _Tp &_Jnu, _Tp &_Nnu, _Tp &_Jpnu, _Tp &_Npnu)
```

Return the cylindrical Bessel functions and their derivatives of order  $\nu$  by various means.

template<typename</li>
 Tp >

void std::\_\_detail::\_\_cyl\_bessel\_jn\_asymp (\_Tp \_\_nu, \_Tp \_\_x, \_Tp &\_Jnu, \_Tp &\_Nnu, \_Tp &\_Jpnu, \_Tp &\_↔ Npnu)

This routine computes the asymptotic cylindrical Bessel and Neumann functions of order nu:  $J_{\nu}(x)$ ,  $N_{\nu}(x)$ . Use this for  $x >> nu^2 + 1$ .

template<typename\_Tp>

```
void std::__detail::__cyl_bessel_jn_steed (_Tp __nu, _Tp __x, _Tp &_Jnu, _Tp &_Nnu, _Tp &_Jpnu, _Tp &_↔ Npnu)
```

Compute the Bessel  $J_{\nu}(x)$  and Neumann  $N_{\nu}(x)$  functions and their first derivatives  $J'_{\nu}(x)$  and  $N'_{\nu}(x)$  respectively. These four functions are computed together for numerical stability.

template<typename \_Tp >

Return the cylindrical Hankel function of the first kind  $H_{\nu}^{(1)}(x)$ .

template<typename\_Tp>

Return the cylindrical Hankel function of the second kind  $H_n^{(2)}u(x)$ .

template<typename</li>
 Tp >

Return the Neumann function of order  $\nu$ :  $N_{\nu}(x)$ .

• template<typename  $_{\rm Tp}>$ 

 $void \ std:: \underline{\quad \ } gamma\_temme \ (\underline{\quad \ } Tp \ \underline{\quad \ } gam1, \underline{\quad \ } Tp \ \& \underline{\quad \ } gam2, \underline{\quad \ } Tp \ \& \underline{\quad \ } gampl, \underline{\quad \ } Tp \ \& \underline{\quad \ } gammi)$ 

Compute the gamma functions required by the Temme series expansions of  $N_{\nu}(x)$  and  $K_{\nu}(x)$ .

$$\Gamma_1 = \frac{1}{2\mu} \left[ \frac{1}{\Gamma(1-\mu)} - \frac{1}{\Gamma(1+\mu)} \right]$$

and

$$\Gamma_2 = \frac{1}{2} \left[ \frac{1}{\Gamma(1-\mu)} + \frac{1}{\Gamma(1+\mu)} \right]$$

where  $-1/2 <= \mu <= 1/2$  is  $\mu = \nu - N$  and N. is the nearest integer to  $\nu$ . The values of  $\Gamma(1+\mu)$  and  $\Gamma(1-\mu)$  are returned as well.

```
template<typename _Tp >
  Tp std:: detail:: sph bessel (unsigned int n, Tp x)
      Return the spherical Bessel function j_n(x) of order n and non-negative real argument x.
template<typename _Tp >
  void std:: detail:: sph bessel jn (unsigned int n, Tp x, Tp & j n, Tp & n n, Tp & jp n, Tp
  &__np_n)
      Compute the spherical Bessel j_n(x) and Neumann n_n(x) functions and their first derivatives j_n(x) and n'_n(x) respec-
     tively.
template<typename _Tp >
  std::complex< _Tp > std::__detail::__sph_hankel_1 (unsigned int __n, _Tp __x)
      Return the spherical Hankel function of the first kind h_n^{(1)}(x).
template<typename _Tp >
  std::complex < _Tp > std::__detail::__sph_hankel_2 (unsigned int __n, _Tp __x)
     Return the spherical Hankel function of the second kind h_n^{(2)}(x).
template<typename_Tp>
  Tp std:: detail:: sph neumann (unsigned int n, Tp x)
      Return the spherical Neumann function n_n(x) of order n and non-negative real argument x.
```

## 11.2.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

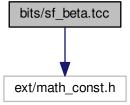
#### 11.2.2 Macro Definition Documentation

11.2.2.1 #define GLIBCXX BITS SF BESSEL TCC 1

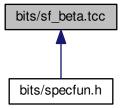
Definition at line 47 of file sf\_bessel.tcc.

# 11.3 bits/sf beta.tcc File Reference

#include <ext/math\_const.h>
Include dependency graph for sf beta.tcc:



This graph shows which files directly or indirectly include this file:



## **Namespaces**

- std
- std::\_\_detail

#### **Macros**

• #define \_GLIBCXX\_BITS\_SF\_BETA\_TCC 1

#### **Functions**

```
template<typename _Tp >
  _Tp std::__detail::__beta (_Tp __a, _Tp __b)
      Return the beta function B(a,b).
template<typename _Tp >
  _Tp std::__detail::__beta_gamma (_Tp __a, _Tp __b)
      Return the beta function: B(a, b).
template<typename _Tp >
  _Tp std::__detail::__beta_inc (_Tp __a, _Tp __b, _Tp __x)
• template<typename _{\mathrm{Tp}} >
  _Tp std::__detail::__beta_inc_cont_frac (_Tp __a, _Tp __b, _Tp __x)
ullet template<typename _Tp >
  _Tp std::__detail::__beta_lgamma (_Tp __a, _Tp __b)
      Return the beta function B(a,b) using the log gamma functions.
template<typename _Tp >
  _Tp std::__detail::__beta_product (_Tp __a, _Tp __b)
      Return the beta function B(x, y) using the product form.
```

## 11.3.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

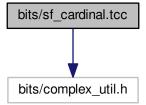
# 11.3.2 Macro Definition Documentation

11.3.2.1 #define \_GLIBCXX\_BITS\_SF\_BETA\_TCC 1

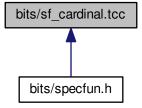
Definition at line 49 of file sf\_beta.tcc.

# 11.4 bits/sf\_cardinal.tcc File Reference

#include <bits/complex\_util.h>
Include dependency graph for sf\_cardinal.tcc:



This graph shows which files directly or indirectly include this file:



# **Namespaces**

- std
- std::\_\_detail

#### **Macros**

• #define GLIBCXX BITS SF CARDINAL TCC 1

#### **Functions**

template < typename \_Tp >
 \_\_gnu\_cxx::\_\_promote\_num\_t < \_Tp > std::\_\_detail::\_\_sinc (\_Tp \_\_a, \_Tp \_\_x)

Return the generalized sinus cardinal function

$$sinc_a(x) = \frac{\sin(\pi x/a)}{(\pi x/a)}$$

template<typename \_Tp >

 $\_\_gnu\_cxx::\_promote\_num\_t < \_Tp > std::\__detail::\__sinc (\_Tp \__x)$ 

Return the normalized sinus cardinal function

$$sinc(x) = \frac{\sin(\pi x)}{\pi x}$$

template<typename\_Tp>

\_\_gnu\_cxx::\_\_promote\_num\_t< \_Tp > std::\_\_detail::\_\_sinc\_pi (\_Tp \_\_x)

Return the unnormalized sinus cardinal function

$$sinc_{\pi}(x) = \frac{\sin(x)}{x}$$

.

template<typename \_Tp >

\_\_gnu\_cxx::\_\_promote\_num\_t< \_Tp > std::\_\_detail::\_\_sinhc (\_Tp \_\_a, \_Tp \_\_x)

Return the generalized hyperbolic sinus cardinal function

$$sinhc_a(x) = \frac{\sinh(\pi x/a)}{\pi x/a}$$

template<typename \_Tp >

Return the normalized hyperbolic sinus cardinal function

$$sinhc(x) = \frac{\sinh(\pi x)}{\pi x}$$

template<typename</li>
 Tp >

Return the unnormalized hyperbolic sinus cardinal function

$$sinhc_{\pi}(x) = \frac{\sinh(x)}{x}$$

.

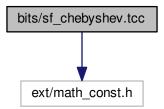
#### 11.4.1 Macro Definition Documentation

11.4.1.1 #define \_GLIBCXX\_BITS\_SF\_CARDINAL\_TCC 1

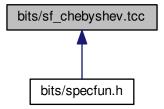
Definition at line 30 of file sf cardinal.tcc.

# 11.5 bits/sf\_chebyshev.tcc File Reference

#include <ext/math\_const.h>
Include dependency graph for sf\_chebyshev.tcc:



This graph shows which files directly or indirectly include this file:



# **Namespaces**

- std
- std::\_\_detail

## **Macros**

#define \_GLIBCXX\_BITS\_SF\_CHEBYSHEV\_TCC 1

#### **Functions**

```
template<typename _Tp >
    _Tp std::__detail::__chebyshev_recur (unsigned int __n, _Tp __x, _Tp _C0, _Tp _C1)
template<typename _Tp >
    _Tp std::__detail::__chebyshev_t (unsigned int __n, _Tp __x)
template<typename _Tp >
    _Tp std::__detail::__chebyshev_u (unsigned int __n, _Tp __x)
template<typename _Tp >
    _Tp std::__detail::__chebyshev_v (unsigned int __n, _Tp __x)
template<typename _Tp >
    _Tp std::__detail::__chebyshev_w (unsigned int __n, _Tp __x)
```

## 11.5.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

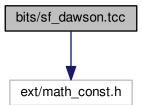
#### 11.5.2 Macro Definition Documentation

11.5.2.1 #define \_GLIBCXX\_BITS\_SF\_CHEBYSHEV\_TCC 1

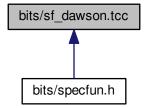
Definition at line 31 of file sf\_chebyshev.tcc.

# 11.6 bits/sf\_dawson.tcc File Reference

```
#include <ext/math_const.h>
Include dependency graph for sf_dawson.tcc:
```



This graph shows which files directly or indirectly include this file:



## **Namespaces**

- std
- std::\_\_detail

#### **Macros**

#define \_GLIBCXX\_BITS\_SF\_DAWSON\_TCC 1

### **Functions**

```
    template < typename _Tp >
        _Tp std::__detail::__dawson (_Tp __x)
        Return the Dawson integral, F(x), for real argument x.
    template < typename _Tp >
        _Tp std::__detail::__dawson_cont_frac (_Tp __x)
        Compute the Dawson integral using a sampling theorem representation.
    template < typename _Tp >
        _Tp std::__detail::__dawson_series (_Tp __x)
        Compute the Dawson integral using the series expansion.
```

## 11.6.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

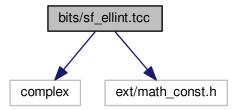
### 11.6.2 Macro Definition Documentation

11.6.2.1 #define \_GLIBCXX\_BITS\_SF\_DAWSON\_TCC 1

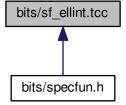
Definition at line 31 of file sf dawson.tcc.

# 11.7 bits/sf\_ellint.tcc File Reference

```
#include <complex>
#include <ext/math_const.h>
Include dependency graph for sf_ellint.tcc:
```



This graph shows which files directly or indirectly include this file:



# **Namespaces**

- std
- std::\_\_detail

## **Macros**

#define \_GLIBCXX\_BITS\_SF\_ELLINT\_TCC 1

#### **Functions**

```
template<typename _Tp >
  _Tp std::__detail::__comp_ellint_1 (_Tp __k)
      Return the complete elliptic integral of the first kind K(k) using the Carlson formulation.
template<typename _Tp >
  Tp std:: detail:: comp ellint 2 (Tp k)
      Return the complete elliptic integral of the second kind E(k) using the Carlson formulation.
  _Tp std::__detail::__comp_ellint_3 (_Tp __k, _Tp __nu)
      Return the complete elliptic integral of the third kind \Pi(k,\nu) = \Pi(k,\nu,\pi/2) using the Carlson formulation.

    template<typename</li>
    Tp >

  _Tp std::__detail::__comp_ellint_d (_Tp __k)
template<typename _Tp >
  _Tp std::__detail::__comp_ellint_rf (_Tp __x, _Tp __y)

    template<typename</li>
    Tp >

  _Tp std::__detail::__comp_ellint_rg (_Tp __x, _Tp __y)
template<typename _Tp >
  _Tp std::__detail::__ellint_1 (_Tp __k, _Tp __phi)
      Return the incomplete elliptic integral of the first kind F(k,\phi) using the Carlson formulation.
template<typename_Tp>
  _Tp std::__detail::__ellint_2 (_Tp __k, _Tp __phi)
      Return the incomplete elliptic integral of the second kind E(k,\phi) using the Carlson formulation.

    template<typename</li>
    Tp >

  _Tp std::__detail::__ellint_3 (_Tp __k, _Tp __nu, _Tp __phi)
      Return the incomplete elliptic integral of the third kind \Pi(k, \nu, \phi) using the Carlson formulation.
template<typename_Tp>
  _Tp std::__detail::__ellint_cel (_Tp __k_c, _Tp __p, _Tp __a, _Tp __b)
template<typename _Tp >
  _Tp std::__detail::__ellint_d (_Tp __k, _Tp __phi)
template<typename _Tp >
  _Tp std::__detail::__ellint_el1 (_Tp __x, _Tp __k_c)
template<typename _Tp >
  _Tp std::__detail::__ellint_el2 (_Tp __x, _Tp __k_c, _Tp __a, _Tp __b)
template<typename _Tp >
  _Tp std::__detail::__ellint_el3 (_Tp __x, _Tp __k_c, _Tp __p)

    template<typename</li>
    Tp >

  _Tp std::__detail::__ellint_rc (_Tp __x, _Tp __y)
      Return the Carlson elliptic function R_C(x,y) = R_F(x,y,y) where R_F(x,y,z) is the Carlson elliptic function of the first
      kind.
template<typename _Tp >
  _Tp std::__detail::__ellint_rd (_Tp __x, _Tp __y, _Tp __z)
      Return the Carlson elliptic function of the second kind R_D(x,y,z) = R_J(x,y,z,z) where R_J(x,y,z,p) is the Carlson
      elliptic function of the third kind.
template<typename _Tp >
  _Tp std::__detail::__ellint_rf (_Tp __x, _Tp __y, _Tp __z)
      Return the Carlson elliptic function R_F(x, y, z) of the first kind.
template<typename_Tp>
  _Tp std::__detail::__ellint_rg (_Tp __x, _Tp __y, _Tp __z)
      Return the symmetric Carlson elliptic function of the second kind R_G(x, y, z).
```

```
    template<typename_Tp >
        _Tp std::__detail::__ellint_rj (_Tp __x, _Tp __y, _Tp __z, _Tp __p)
        Return the Carlson elliptic function R<sub>J</sub>(x, y, z, p) of the third kind.
    template<typename_Tp >
        _Tp std::__detail::__heuman_lambda (_Tp __k, _Tp __phi)
    template<typename_Tp >
        _Tp std::__detail::__jacobi_zeta (_Tp __k, _Tp __phi)
```

# 11.7.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

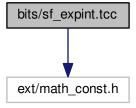
## 11.7.2 Macro Definition Documentation

11.7.2.1 #define \_GLIBCXX\_BITS\_SF\_ELLINT\_TCC 1

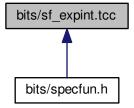
Definition at line 47 of file sf\_ellint.tcc.

# 11.8 bits/sf\_expint.tcc File Reference

#include <ext/math\_const.h>
Include dependency graph for sf\_expint.tcc:



This graph shows which files directly or indirectly include this file:



## **Namespaces**

- std
- std:: detail

#### **Macros**

#define \_GLIBCXX\_BITS\_SF\_EXPINT\_TCC 1

#### **Functions**

```
ullet template<typename _Tp >
  _Tp std::__detail::__coshint (const _Tp __x)
      Return the hyperbolic cosine integral li(x).
template<typename _Tp >
  _Tp std::__detail::__expint (unsigned int __n, _Tp __x)
      Return the exponential integral E_n(x).
• template<typename _{\mathrm{Tp}} >
  _Tp std::__detail::__expint (_Tp __x)
      Return the exponential integral Ei(x).
• template<typename _{\mathrm{Tp}} >
  _Tp std::__detail::__expint_asymp (unsigned int __n, _Tp __x)
      Return the exponential integral E_n(x) for large argument.
• template<typename _{\mathrm{Tp}} >
  _Tp std::__detail::__expint_E1 (_Tp __x)
      Return the exponential integral E_1(x).
template<typename _Tp >
  _Tp std::__detail::__expint_E1_asymp (_Tp __x)
      Return the exponential integral E_1(x) by asymptotic expansion.
• template<typename _{\mathrm{Tp}} >
  _Tp std::__detail::__expint_E1_series (_Tp __x)
```

```
Return the exponential integral E_1(x) by series summation. This should be good for x < 1.
template<typename_Tp>
  _Tp std::__detail::__expint_Ei (_Tp __x)
      Return the exponential integral Ei(x).
template<typename _Tp >
  _Tp std::__detail::__expint_Ei_asymp (_Tp __x)
      Return the exponential integral Ei(x) by asymptotic expansion.

    template<typename</li>
    Tp >

  _Tp std::__detail::__expint_Ei_series (_Tp __x)
      Return the exponential integral Ei(x) by series summation.
• template<typename _Tp >
  _Tp std::__detail::__expint_En_cont_frac (unsigned int __n, _Tp __x)
      Return the exponential integral E_n(x) by continued fractions.
template<typename _Tp >
  _Tp std::__detail::__expint_En_recursion (unsigned int __n, _Tp __x)
      Return the exponential integral E_n(x) by recursion. Use upward recursion for x < n and downward recursion (Miller's
      algorithm) otherwise.
template<typename _Tp >
  Tp std:: detail:: expint En series (unsigned int n, Tp x)
      Return the exponential integral E_n(x) by series summation.
template<typename _Tp >
  _Tp std::__detail::__expint_large_n (unsigned int __n, _Tp __x)
      Return the exponential integral E_n(x) for large order.
template<typename _Tp >
  _Tp std::__detail::__logint (const _Tp __x)
      Return the logarithmic integral li(x).
template<typename _Tp >
  _Tp std::__detail::__sinhint (const _Tp __x)
      Return the hyperbolic sine integral li(x).
```

### 11.8.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <cmath>.

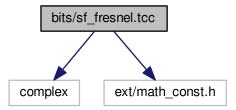
### 11.8.2 Macro Definition Documentation

11.8.2.1 #define GLIBCXX BITS SF EXPINT TCC 1

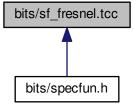
Definition at line 47 of file sf expint.tcc.

# 11.9 bits/sf\_fresnel.tcc File Reference

```
#include <complex>
#include <ext/math_const.h>
Include dependency graph for sf_fresnel.tcc:
```



This graph shows which files directly or indirectly include this file:



# **Namespaces**

- std
- std::\_\_detail

## **Macros**

#define \_GLIBCXX\_BITS\_SF\_FRESNEL\_TCC 1

#### **Functions**

```
    template<typename _Tp >
        std::complex< _Tp > std::__detail::__fresnel (const _Tp __x)
```

Return the Fresnel cosine and sine integrals as a complex number f(C(x) + iS(x))

```
    template<typename _Tp >
        void std::__detail::__fresnel_cont_frac (const _Tp __ax, _Tp &_Cf, _Tp &_Sf)
```

This function computes the Fresnel cosine and sine integrals by continued fractions for positive argument.

```
    template<typename _Tp >
        void std::__detail::__fresnel_series (const _Tp __ax, _Tp &_Cf, _Tp &_Sf)
```

This function returns the Fresnel cosine and sine integrals as a pair by series expansion for positive argument.

# 11.9.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

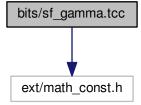
#### 11.9.2 Macro Definition Documentation

11.9.2.1 #define \_GLIBCXX\_BITS\_SF\_FRESNEL\_TCC 1

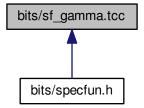
Definition at line 31 of file sf\_fresnel.tcc.

# 11.10 bits/sf\_gamma.tcc File Reference

```
#include <ext/math_const.h>
Include dependency graph for sf_gamma.tcc:
```



This graph shows which files directly or indirectly include this file:



# **Classes**

struct std::\_\_detail::\_Factorial\_table< \_Tp >

## **Namespaces**

- std
- std::\_\_detail

#### **Macros**

#define \_GLIBCXX\_BITS\_SF\_GAMMA\_TCC 1

#### **Functions**

```
template<typename _Tp >
  GLIBCXX14 CONSTEXPR Tp std:: detail:: double factorial (int n)
      Return the double factorial of the integer n.
template<typename _Tp >
  GLIBCXX14 CONSTEXPR Tp std:: detail:: factorial (unsigned int n)
      Return the factorial of the integer n.
template<typename _Tp >
  _Tp std::__detail::__gamma (_Tp __x)
      Return \Gamma(x).
template<typename _Tp >
  std::pair< _Tp, _Tp > std::__detail::__gamma_cont_frac (_Tp __a, _Tp __x)

    template<typename</li>
    Tp >

  _Tp std::__detail::__gamma_l (_Tp __a, _Tp __x)
      Return the lower incomplete gamma function. The lower incomplete gamma function is defined by
                                                \gamma(a,x) = \int_{0}^{x} e^{-t} t^{a-1} dt (a > 0)
template<typename _Tp >
  std::pair< _Tp, _Tp > std::__detail::__gamma_series (_Tp __a, _Tp __x)

    template<typename</li>
    Tp >

  _Tp std::__detail::__gamma_u (_Tp __a, _Tp __x)
      Return the upper incomplete gamma function. The lower incomplete gamma function is defined by
                                               \Gamma(a,x) = \int_{-\infty}^{\infty} e^{-t} t^{a-1} dt (a > 0)
template<typename _Tp >
  Tp std:: detail:: log bincoef (unsigned int n, unsigned int k)
      Return the logarithm of the binomial coefficient. The binomial coefficient is given by:
                                                       \binom{n}{k} = \frac{n!}{(n-k)!k!}
template<typename _Tp >
  _GLIBCXX14_CONSTEXPR _Tp std::__detail::__log_double_factorial (_Tp __x)

    template<typename</li>
    Tp >

  GLIBCXX14 CONSTEXPR Tp std:: detail:: log double factorial (int n)
      Return the logarithm of the double factorial of the integer n.

    template<typename</li>
    Tp >

  _GLIBCXX14_CONSTEXPR _Tp std::__detail::__log_factorial (unsigned int __n)
      Return the logarithm of the factorial of the integer n.

    template<typename</li>
    Tp >

  _Tp std::__detail::__log_gamma (_Tp __x)
      Return log(|\Gamma(x)|). This will return values even for x < 0. To recover the sign of \Gamma(x) for any argument use \underline{\quad} log \hookleftarrow
      gamma_sign.
template<typename</li>Tp >
  _GLIBCXX14_CONSTEXPR _Tp std::__detail::__log_gamma_bernoulli (_Tp __x)
      Return log(\Gamma(x)) by asymptotic expansion with Bernoulli number coefficients. This is like Sterling's approximation.

    template<typename</li>
    Tp >

  _GLIBCXX14_CONSTEXPR _Tp std::__detail::__log_gamma_lanczos (_Tp __x)
```

Return  $log(\Gamma(x))$  by the Lanczos method. This method dominates all others on the positive axis I think.

template<typename \_Tp >
 \_Tp std::\_\_detail::\_\_log\_gamma\_sign (\_Tp \_\_x)

Return the sign of  $\Gamma(x)$ . At nonpositive integers zero is returned.

template<typename \_Tp >

Return  $\Gamma(z)$  by the Spouge algorithm:

$$\Gamma(z+1) = (z+a)^{z+1/2} e^{-z-a} \left[ \sqrt{2\pi} \sum_{k=1}^{\lceil a \rceil + 1} \frac{c_k(a)}{z+k} \right]$$

where

$$c_k(a) = \frac{(-1)^{k-1}}{(k-1)!} (a-k)^{k-1/2} e^{a-k}$$

and the error is bounded by

$$\epsilon(a) < a^{-1/2} (2\pi)^{-a-1/2}$$

.

template<typename</li>
 Tp >

Return the logarithm of the lower Pochhammer symbol or the falling factorial function. The lower Pochammer symbol is defined by

$$(a)_n = \prod_{k=0}^{n-1} (a-k), (a)_0 = 1 = \Gamma(a+1)/\Gamma(a-n+1)$$

In particular, f(n) = n! f. Thus this function returns

$$ln[(a)_n] = \Gamma(a+1) - \Gamma(a-n+1), ln[(a)_0] = 0$$

Many notations exist:

 $a^{\underline{n}}$ 

,

$$\left\{\begin{array}{c} a \\ n \end{array}\right.$$

, and others.

template<typename \_Tp >

Return the logarithm of the (upper) Pochhammer symbol or the rising factorial function. The Pochammer symbol is defined by

$$(a)_n = \prod_{k=0}^{n-1} (a+k), (a)_0 = 1 = \Gamma(a+n)/\Gamma(n)$$

Thus this function returns

$$ln[(a)_n] = \Gamma(a+n) - \Gamma(n), ln[(a)_0] = 0$$

Many notations exist:

 $a^{\overline{n}}$ 

,

$$\begin{bmatrix} a \\ n \end{bmatrix}$$

, and others.

template<typename \_Tp >

Return the regularized lower incomplete gamma function. The regularized lower incomplete gamma function is defined by

$$P(a,x) = \frac{\gamma(a,x)}{\Gamma(a)}$$

where  $\Gamma(a)$  is the gamma function and

$$\gamma(a, x) = \int_0^x e^{-t} t^{a-1} dt (a > 0)$$

is the lower incomplete gamma function.

template<typename</li>
 Tp >

Return the logarithm of the lower Pochhammer symbol or the falling factorial function. The lower Pochammer symbol is defined by

$$(a)_n = \prod_{k=0}^{n-1} (a-k), (a)_0 = 1 = \Gamma(a+1)/\Gamma(a-n+1)$$

In particular,  $f(n)_n = n! f$ .

template<typename \_Tp >

Return the (upper) Pochhammer function or the rising factorial function. The Pochammer symbol is defined by

$$(a)_n = \prod_{k=0}^{n-1} (a+k), (a)_0 = 1 = \Gamma(a+n)/\Gamma(n)$$

Many notations exist:

 $a^{\overline{n}}$ 

 $\begin{bmatrix} a \\ n \end{bmatrix}$ 

, and others.

• template<typename \_Tp >

Return the digamma function. The digamma or  $\psi(x)$  function is defined by

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

For negative argument the reflection formula is used:

$$\psi(x) = \psi(1-x) - \pi \cot(\pi x)$$

.

template<typename\_Tp>

Return the polygamma function  $\psi^{(n)}(x)$ .

 $\bullet \ \ template {<} typename\ \_Tp >$ 

Return the digamma function for large argument. The digamma or  $\psi(x)$  function is defined by

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

.

template<typename \_Tp >

Return the digamma function by series expansion. The digamma or  $\psi(x)$  function is defined by

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

.

template < typename \_Tp >
 \_Tp std::\_\_detail::\_\_qgamma (\_Tp \_\_a, \_Tp \_\_x)

Return the regularized upper incomplete gamma function. The regularized upper incomplete gamma function is defined by

$$Q(a,x) = \frac{\Gamma(a,x)}{\Gamma(a)}$$

where  $\Gamma(a)$  is the gamma function and

$$\Gamma(a,x) = \int_{x}^{\infty} e^{-t} t^{a-1} dt (a > 0)$$

is the upper incomplete gamma function.

#### **Variables**

```
    constexpr Factorial table < long double > std:: detail:: S double factorial table [301]

    constexpr _Factorial_table < long double > std::__detail::_S_factorial_table [171]

    constexpr Factorial table < long double > std:: detail:: S neg double factorial table [999]

template<typename _Tp >
  constexpr std::size_t std::__detail::_S_num_double_factorials = 0
template<>
  constexpr std::size t std:: detail:: S num double factorials < double > = 301
template<>
  constexpr std::size t std:: detail:: S num double factorials < float > = 57
• template<>
  constexpr std::size t std:: detail:: S num double factorials < long double > = 301
template<typename _Tp >
  constexpr std::size_t std::__detail::_S_num_factorials = 0
template<>
  constexpr std::size_t std::__detail::_S_num_factorials< double > = 171
  constexpr std::size t std:: detail:: S num factorials < float > = 35
template<>
  constexpr std::size_t std::__detail::_S_num_factorials< long double > = 171
template<typename _Tp >
  constexpr std::size t std:: detail:: S num neg double factorials = 0
template<>
  constexpr std::size_t std::__detail::_S_num_neg_double_factorials< double > = 150
  constexpr std::size t std:: detail:: S num neg double factorials< float > = 27
  constexpr std::size_t std::__detail::_S_num_neg_double_factorials< long double > = 999
```

# 11.10.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <cmath>.

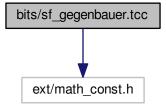
## 11.10.2 Macro Definition Documentation

11.10.2.1 #define \_GLIBCXX\_BITS\_SF\_GAMMA\_TCC 1

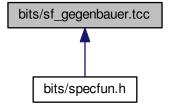
Definition at line 49 of file sf\_gamma.tcc.

# 11.11 bits/sf\_gegenbauer.tcc File Reference

#include <ext/math\_const.h>
Include dependency graph for sf\_gegenbauer.tcc:



This graph shows which files directly or indirectly include this file:



# **Namespaces**

- std
- std::\_\_detail

#### **Macros**

#define \_GLIBCXX\_BITS\_SF\_GEGENBAUER\_TCC 1

#### **Functions**

```
    template<typename _Tp >
        _Tp std::__detail::__gegenbauer_poly (unsigned int __n, _Tp __alpha, _Tp __x)
```

### 11.11.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

## 11.11.2 Macro Definition Documentation

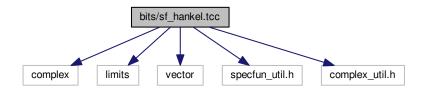
```
11.11.2.1 #define _GLIBCXX_BITS_SF_GEGENBAUER_TCC 1
```

Definition at line 31 of file sf\_gegenbauer.tcc.

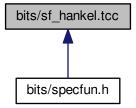
# 11.12 bits/sf\_hankel.tcc File Reference

```
#include <complex>
#include <limits>
#include <vector>
#include "specfun_util.h"
#include "complex_util.h"
```

Include dependency graph for sf\_hankel.tcc:



This graph shows which files directly or indirectly include this file:



## **Namespaces**

- std
- std::\_\_detail

#### **Macros**

• #define GLIBCXX BITS SF HANKEL TCC 1

#### **Functions**

```
    template<typename _Tp >
        void std::__detail::__airy_arg (std::complex< _Tp > __num2d3, std::complex< _Tp > __zeta, std::complex<
        _Tp > &__argp, std::complex< _Tp > &__argm)
```

Compute the arguments for the Airy function evaluations carefully to prevent premature overflow. Note that the major work here is in  $safe\_div$ . A faster, but less safe implementation can be obtained without use of safe\\_div.

- template<typename\_Tp >
   std::complex< \_Tp > std::\_\_detail::\_\_cyl\_bessel (std::complex< \_Tp > \_\_nu, std::complex< \_Tp > \_\_z)

   Return the complex cylindrical Bessel function.
- template<typename \_Tp >
   std::complex< \_Tp > std::\_\_cyl\_hankel\_1 (std::complex< \_Tp > \_\_nu, std::complex< \_Tp > \_\_z)

   Return the complex cylindrical Hankel function of the first kind.
- template<typename\_Tp >
   std::complex< \_Tp > std::\_\_detail::\_\_cyl\_hankel\_2 (std::complex< \_Tp > \_\_nu, std::complex< \_Tp > \_\_z)

   Return the complex cylindrical Hankel function of the second kind.
- template<typename\_Tp >
   std::complex< \_Tp > std::\_\_detail::\_\_cyl\_neumann (std::complex< \_Tp > \_\_nu, std::complex< \_Tp > \_\_z)
   Return the complex cylindrical Neumann function.
- template < typename \_Tp >
   void std::\_\_detail::\_\_debye\_region (std::complex < \_Tp > \_\_alpha, int &\_\_indexr, char &\_\_aorb)

- template<typename \_Tp >
   void std::\_\_detail::\_\_hankel (std::complex < \_Tp > \_\_nu, std::complex < \_Tp > \_\_z, std::complex < \_Tp > &\_H1, std::complex < \_Tp > & H2, std::complex < \_Tp > & H2p)
- template<typename\_Tp >
   void std::\_\_detail::\_\_hankel\_debye (std::complex< \_Tp > \_\_nu, std::complex< \_Tp > \_\_z, std::complex< \_Tp >
   \_alpha, int \_\_indexr, char &\_\_aorb, int &\_\_morn, std::complex< \_Tp > &\_H1, std::complex< \_Tp > &\_H2, std::complex< \_Tp > &\_H1p, std::complex< \_Tp > &\_H2p)
- template<typename \_Tp >
   void std::\_\_detail::\_\_hankel\_params (std::complex< \_Tp > \_\_nu, std::complex< \_Tp > \_\_zhat, std::complex<
   \_Tp > &\_\_p, std::complex< \_Tp > &\_\_nup2, std::complex< \_Tp > &\_\_num2, std::complex< \_Tp > &\_\_num1d3, std::complex< \_Tp > &\_\_num2d3, std::complex< \_Tp > &\_\_num4d3, std::complex< \_Tp > &\_\_num4d3, std::complex< \_Tp > &\_\_zetanhf, std::complex< \_Tp > &\_\_zetanhf, std::complex< \_Tp > &\_\_zetanhf, std::complex<</li>
   Tp > &\_\_zetanhf, std::complex
   Tp > &\_\_zetanhf, std::complex
   Tp > &\_\_zetanhf, std::complex

Compute parameters depending on z and nu that appear in the uniform asymptotic expansions of the Hankel functions and their derivatives, except the arguments to the Airy functions.

template<typename \_Tp >
 void std::\_\_detail::\_\_hankel\_uniform (std::complex< \_Tp > \_\_nu, std::complex< \_Tp > \_\_z, std::complex< \_Tp > &\_H1, std::complex< \_Tp > &\_H2, std::complex< \_Tp > &\_H1p, std::complex< \_Tp > &\_H2p)

This routine computes the uniform asymptotic approximations of the Hankel functions and their derivatives including a patch for the case when the order equals or nearly equals the argument. At such points, Olver's expressions have zero denominators (and numerators) resulting in numerical problems. This routine averages results from four surrounding points in the complex plane to obtain the result in such cases.

template<typename\_Tp >
 void std::\_\_detail::\_\_hankel\_uniform\_olver (std::complex< \_Tp > \_\_nu, std::complex< \_Tp > \_\_z, std
 ::complex< \_Tp > &\_H1, std::complex< \_Tp > &\_H2, std::complex< \_Tp > &\_H1p, std::complex< \_Tp >
 & H2p)

Compute approximate values for the Hankel functions of the first and second kinds using Olver's uniform asymptotic expansion to of order nu along with their derivatives.

template<typename \_Tp >
 void std::\_\_detail::\_\_hankel\_uniform\_outer (std::complex< \_Tp > \_\_nu, std::complex< \_Tp > \_\_z, \_Tp \_\_ 
 eps, std::complex< \_Tp > &\_\_num1d3, std
 ::complex< \_Tp > &\_\_num2d3, std::complex< \_Tp > &\_\_p, std::complex< \_Tp > &\_\_p2, std::complex< \_Tp >
 &\_\_etm3h, std::complex< \_Tp > &\_\_etrat, std::complex< \_Tp > &\_\_o4p, std::complex< \_Tp > &

Compute outer factors and associated functions of z and nu appearing in Olver's uniform asymptotic expansions of the Hankel functions of the first and second kinds and their derivatives. The various functions of z and nu returned by  $hankel\_uniform\_outer$  are available for use in computing further terms in the expansions.

template<typename \_Tp >
 void std::\_\_detail::\_\_hankel\_uniform\_sum (std::complex< \_Tp > \_\_p, std::complex< \_Tp > \_\_p2, std::complex<
 \_Tp > \_\_num2, std::complex< \_Tp > \_\_o4dp, std::complex< \_Tp > \_\_o4

Compute the sums in appropriate linear combinations appearing in Olver's uniform asymptotic expansions for the Hankel functions of the first and second kinds and their derivatives, using up to nterms (less than 5) to achieve relative error eps.

- template<typename \_Tp >
   std::complex< \_Tp > std::\_\_detail::\_\_sph\_bessel (unsigned int \_\_n, std::complex< \_Tp > \_\_z)

   Return the complex spherical Bessel function.
- template<typename \_Tp > void std:: \_\_detail:: \_\_sph\_hankel (unsigned int \_\_n, std::complex< \_Tp > \_\_z, std::complex< \_Tp > &\_H1, std  $\hookleftarrow$  ::complex< \_Tp > &\_H1p, std::complex< \_Tp > &\_H2p)

Helper to compute complex spherical Hankel functions and their derivatives.

```
    template<typename _Tp >
        std::complex< _Tp > std::__detail::__sph_hankel_1 (unsigned int __n, std::complex< _Tp > __z)

    Return the complex spherical Hankel function of the first kind.
```

```
    template<typename _Tp >
        std::complex< _Tp > std::__detail::__sph_hankel_2 (unsigned int __n, std::complex< _Tp > __z)

    Return the complex spherical Hankel function of the second kind.
```

template<typename \_Tp >
 std::complex< \_Tp > std::\_\_detail::\_\_sph\_neumann (unsigned int \_\_n, std::complex< \_Tp > \_\_z)

 Return the complex spherical Neumann function.

## 11.12.1 Detailed Description

This is an internal header file, included by other library headers. You should not attempt to use it directly.

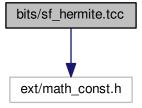
#### 11.12.2 Macro Definition Documentation

11.12.2.1 #define GLIBCXX\_BITS\_SF\_HANKEL\_TCC 1

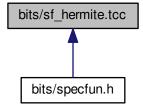
Definition at line 31 of file sf\_hankel.tcc.

# 11.13 bits/sf hermite.tcc File Reference

#include <ext/math\_const.h>
Include dependency graph for sf hermite.tcc:



This graph shows which files directly or indirectly include this file:



## **Namespaces**

- std
- std::\_\_detail

#### **Macros**

#define \_GLIBCXX\_BITS\_SF\_HERMITE\_TCC 1

### **Functions**

```
    template<typename _Tp >
        _Tp std::__detail::__poly_hermite (unsigned int __n, _Tp __x)
        This routine returns the Hermite polynomial of order n: H<sub>n</sub>(x).
    template<typename _Tp >
        _Tp std::__detail::__poly_hermite_asymp (unsigned int __n, _Tp __x)
        This routine returns the Hermite polynomial of large order n: H<sub>n</sub>(x). We assume here that x >= 0.
    template<typename _Tp >
        _Tp std::__detail::__poly_hermite_recursion (unsigned int __n, _Tp __x)
        This routine returns the Hermite polynomial of order n: H<sub>n</sub>(x) by recursion on n.
```

#### 11.13.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

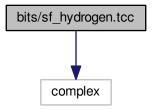
#### 11.13.2 Macro Definition Documentation

11.13.2.1 #define \_GLIBCXX\_BITS\_SF\_HERMITE\_TCC 1

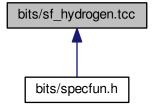
Definition at line 42 of file sf hermite.tcc.

# 11.14 bits/sf\_hydrogen.tcc File Reference

#include <complex>
Include dependency graph for sf\_hydrogen.tcc:



This graph shows which files directly or indirectly include this file:



# **Namespaces**

- std
- std::\_\_detail

# **Macros**

#define \_GLIBCXX\_BITS\_SF\_HYDROGEN\_TCC 1

# **Functions**

template<typename \_Tp >
 std::complex< \_Tp > std::\_\_detail::\_\_hydrogen (const unsigned int \_\_n, const unsigned int \_\_n, const unsigned int \_\_n, const \_Tp \_Z, const \_Tp \_\_r, const \_Tp \_\_theta, const \_Tp \_\_phi)

# 11.14.1 Detailed Description

This is an internal header file, included by other library headers. You should not attempt to use it directly.

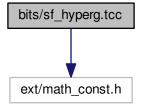
# 11.14.2 Macro Definition Documentation

11.14.2.1 #define \_GLIBCXX\_BITS\_SF\_HYDROGEN\_TCC 1

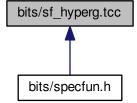
Definition at line 31 of file sf\_hydrogen.tcc.

# 11.15 bits/sf\_hyperg.tcc File Reference

#include <ext/math\_const.h>
Include dependency graph for sf\_hyperg.tcc:



This graph shows which files directly or indirectly include this file:



## **Namespaces**

- std
- std:: detail

#### **Macros**

• #define GLIBCXX BITS SF HYPERG TCC 1

#### **Functions**

```
 \begin{array}{lll} \bullet & \mathsf{template} < \mathsf{typename\_Tp} > \\ & \_\mathsf{Tp} \; \mathsf{std} :: \_ \; \mathsf{detail} :: \_ \; \mathsf{conf\_hyperg} \; (\_\mathsf{Tp} \_ a, \_\mathsf{Tp} \_ c, \_\mathsf{Tp} \_ x) \\ & & \mathit{Return} \; \mathsf{the} \; \mathsf{confluent} \; \mathsf{hypergeometric} \; \mathsf{function} \; {}_1F_1(a;c;x). \\ \bullet & \mathsf{template} < \mathsf{typename} \_\mathsf{Tp} > \\ & \_\mathsf{Tp} \; \mathsf{std} :: \_ \; \mathsf{detail} :: \_ \; \mathsf{conf\_hyperg\_lim} \; (\_\mathsf{Tp} \_ c, \_\mathsf{Tp} \_ x) \\ & \mathit{Return} \; \mathsf{the} \; \mathsf{confluent} \; \mathsf{hypergeometric} \; \mathsf{limit} \; \mathsf{function} \; {}_0F_1(-;c;x). \\ \end{array}
```

template<typename</li>Tp >

```
_Tp std::__detail::__conf_hyperg_lim_series (_Tp __c, _Tp __x)
```

This routine returns the confluent hypergeometric limit function by series expansion.

template<typename\_Tp>

```
_Tp std::__detail::__conf_hyperg_luke (_Tp __a, _Tp __c, _Tp __xin)
```

Return the hypergeometric function  $_1F_1(a;c;x)$  by an iterative procedure described in Luke, Algorithms for the Computation of Mathematical Functions.

template<typename\_Tp>

```
_Tp std::__detail::__conf_hyperg_series (_Tp __a, _Tp __c, _Tp __x)
```

This routine returns the confluent hypergeometric function by series expansion.

template<typename\_Tp>

```
_Tp std::__detail::__hyperg (_Tp __a, _Tp __b, _Tp __c, _Tp __x)
```

Return the hypergeometric function  $_2F_1(a,b;c;x)$ .

• template<typename\_Tp>

```
_Tp std::__detail::__hyperg_luke (_Tp __a, _Tp __b, _Tp __c, _Tp __xin)
```

Return the hypergeometric function  $_2F_1(a,b;c;x)$  by an iterative procedure described in Luke, Algorithms for the Computation of Mathematical Functions.

template<typename \_Tp >

```
_Tp std::__detail::__hyperg_reflect (_Tp __a, _Tp __b, _Tp __c, _Tp __x)
```

Return the hypergeometric function  ${}_2F_1(a,b;c;x)$  by the reflection formulae in Abramowitz & Stegun formula 15.3.6 for d=c-a - b not integral and formula 15.3.11 for d=c - a - b integral. This assumes a, b, c != negative integer.

template<typename</li>
 Tp >

```
_Tp std::__detail::__hyperg_series (_Tp __a, _Tp __b, _Tp __c, _Tp __x)
```

Return the hypergeometric function  ${}_2F_1(a,b;c;x)$  by series expansion.

## 11.15.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <cmath>.

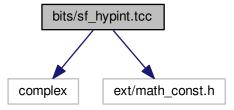
# 11.15.2 Macro Definition Documentation

11.15.2.1 #define \_GLIBCXX\_BITS\_SF\_HYPERG\_TCC 1

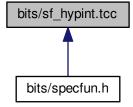
Definition at line 44 of file sf\_hyperg.tcc.

# 11.16 bits/sf\_hypint.tcc File Reference

```
#include <complex>
#include <ext/math_const.h>
Include dependency graph for sf_hypint.tcc:
```



This graph shows which files directly or indirectly include this file:



# **Namespaces**

- std
- std::\_\_detail

#### **Macros**

#define \_GLIBCXX\_BITS\_SF\_HYPINT\_TCC 1

#### **Functions**

```
    template<typename _Tp >
        std::pair< _Tp, _Tp > std::__detail::__chshint (_Tp __x, _Tp &_Chi, _Tp & Shi)
```

This function returns the hyperbolic cosine Ci(x) and hyperbolic sine Si(x) integrals as a pair.

 $\bullet \ \ template\!<\!typename\,\_Tp>$ 

```
void std::__detail::__chshint_cont_frac (_Tp __t, _Tp &_Chi, _Tp &_Shi)
```

This function computes the hyperbolic cosine Chi(x) and hyperbolic sine Shi(x) integrals by continued fraction for positive argument.

 $\bullet \ \ template {<} typename \ \_Tp >$ 

```
void std::__detail::__chshint_series (_Tp __t, _Tp &_Chi, _Tp &_Shi)
```

This function computes the hyperbolic cosine Chi(x) and hyperbolic sine Shi(x) integrals by series summation for positive argument.

## 11.16.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

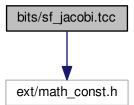
#### 11.16.2 Macro Definition Documentation

11.16.2.1 #define \_GLIBCXX\_BITS\_SF\_HYPINT\_TCC 1

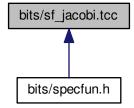
Definition at line 31 of file sf hypint.tcc.

# 11.17 bits/sf\_jacobi.tcc File Reference

```
#include <ext/math_const.h>
Include dependency graph for sf_jacobi.tcc:
```



This graph shows which files directly or indirectly include this file:



## **Namespaces**

- std
- std::\_\_detail

#### **Macros**

#define \_GLIBCXX\_BITS\_SF\_JACOBI\_TCC 1

#### **Functions**

```
    template < typename _Tp >
        _Tp std::__detail::__poly_jacobi (unsigned int __n, _Tp __alpha, _Tp __beta, _Tp __x)
    template < typename _Tp >
        _Tp std::__detail::__poly_radial_jacobi (unsigned int __n, unsigned int __m, _Tp __rho)
    template < typename _Tp >
        __gnu_cxx::__promote_num_t < _Tp > std::__detail::__zernike (unsigned int __n, int __m, _Tp __rho, _Tp __phi)
```

### 11.17.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

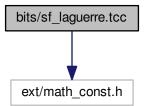
## 11.17.2 Macro Definition Documentation

11.17.2.1 #define \_GLIBCXX\_BITS\_SF\_JACOBI\_TCC 1

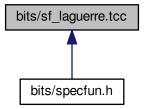
Definition at line 31 of file sf jacobi.tcc.

# 11.18 bits/sf\_laguerre.tcc File Reference

#include <ext/math\_const.h>
Include dependency graph for sf\_laguerre.tcc:



This graph shows which files directly or indirectly include this file:



# **Namespaces**

- std
- std::\_\_detail

## **Macros**

#define \_GLIBCXX\_BITS\_SF\_LAGUERRE\_TCC 1

template<typename \_Tp >

#### **Functions**

```
_Tp std::__detail::__assoc_laguerre (unsigned int __n, unsigned int __m, _Tp __x)
      This routine returns the associated Laguerre polynomial of order n, degree m: L_n^m(x).
template<typename</li>Tp >
  _Tp std::__detail::__laguerre (unsigned int __n, _Tp __x)
      This routine returns the Laguerre polynomial of order n: L_n(x).
• template<typename _Tpa , typename _Tp >
  _Tp std::__detail::__poly_laguerre (unsigned int __n, _Tpa __alpha1, _Tp __x)
      This routine returns the associated Laguerre polynomial of order n, degree \alpha: L_n^a lpha(x).

    template<typename _Tpa , typename _Tp >

  _Tp std::__detail::__poly_laguerre_hyperg (unsigned int __n, _Tpa __alpha1, _Tp __x)
      Evaluate the polynomial based on the confluent hypergeometric function in a safe way, with no restriction on the arguments.
• template<typename _{\rm Tpa}, typename _{\rm Tp} >
  _Tp std::__detail::__poly_laguerre_large_n (unsigned __n, _Tpa __alpha1, _Tp __x)
      This routine returns the associated Laguerre polynomial of order n, degree \alpha > -1 for large n. Abramowitz & Stegun,
      13.5.21.

    template<typename _Tpa , typename _Tp >

  _Tp std::__detail::__poly_laguerre_recursion (unsigned int __n, _Tpa __alpha1, _Tp __x)
      This routine returns the associated Laguerre polynomial of order n, degree \alpha: L_n^{\alpha}(x) by recursion.
```

## 11.18.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <cmath>.

#### 11.18.2 Macro Definition Documentation

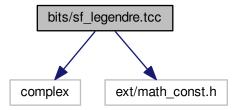
```
11.18.2.1 #define GLIBCXX BITS SF LAGUERRE TCC 1
```

Definition at line 44 of file sf laguerre.tcc.

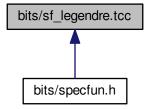
# 11.19 bits/sf\_legendre.tcc File Reference

```
#include <complex>
#include <ext/math_const.h>
```

Include dependency graph for sf\_legendre.tcc:



This graph shows which files directly or indirectly include this file:



## **Namespaces**

- std
- std::\_\_detail

## **Macros**

• #define \_GLIBCXX\_BITS\_SF\_LEGENDRE\_TCC 1

## **Functions**

```
    template<typename_Tp >
        _Tp std::__detail::__assoc_legendre_p (unsigned int __I, unsigned int __m, _Tp __x)
        Return the associated Legendre function by recursion on l and downward recursion on m.
```

```
    template < typename _Tp >
        _Tp std:: __detail:: __legendre_q (unsigned int __l, _Tp __x)
        Return the Legendre function of the second kind by upward recursion on order l.
    template < typename _Tp >
        _Tp std:: __detail:: __poly_legendre_p (unsigned int __l, _Tp __x)
        Return the Legendre polynomial by upward recursion on order l.
    template < typename _Tp >
        std::complex < _Tp > std:: __detail:: __sph_harmonic (unsigned int __l, int __m, _Tp __theta, _Tp __phi)
        Return the spherical harmonic function.
    template < typename _Tp >
        _Tp std:: __detail:: _sph_legendre (unsigned int __l, unsigned int __m, _Tp __theta)
        Return the spherical associated Legendre function.
```

### 11.19.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

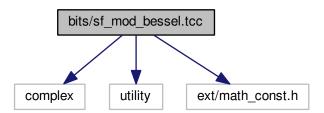
#### 11.19.2 Macro Definition Documentation

```
11.19.2.1 #define GLIBCXX BITS SF_LEGENDRE_TCC 1
```

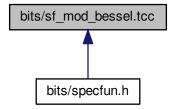
Definition at line 47 of file sf\_legendre.tcc.

# 11.20 bits/sf\_mod\_bessel.tcc File Reference

```
#include <complex>
#include <utility>
#include <ext/math_const.h>
Include dependency graph for sf_mod_bessel.tcc:
```



This graph shows which files directly or indirectly include this file:



## **Namespaces**

- std
- std:: detail

#### **Macros**

#define \_GLIBCXX\_BITS\_SF\_MOD\_BESSEL\_TCC 1

four functions are computed together for numerical stability.

\_Tp std::\_\_detail::\_\_cyl\_bessel\_k (\_Tp \_\_nu, \_Tp \_\_x)

template<typename \_Tp >

#### **Functions**

void std::\_\_detail::\_\_cyl\_bessel\_ik\_steed (\_Tp \_\_nu, \_Tp \_\_x, \_Tp &\_Inu, \_Tp &\_Knu, \_Tp &\_Ipnu, \_Tp &\_Kpnu) Compute the modified Bessel functions  $I_{\nu}(x)$  and  $K_{\nu}(x)$  and their first derivatives  $I'_{\nu}(x)$  and  $K'_{\nu}(x)$  respectively. These Return the irregular modified Bessel function  $K_{\nu}(x)$  of order  $\nu$ .

• template<typename\_Tp>

void std::\_\_detail::\_\_fock\_airy (\_Tp \_\_x, std::complex< \_Tp > &\_\_w1, std::complex< \_Tp > &\_\_w2, std 
$$\leftrightarrow$$
 ::complex< \_Tp > &\_\_w1p, std::complex< \_Tp > &\_\_w2p)

Compute the Fock-type Airy functions  $w_1(x)$  and  $w_2(x)$  and their first derivatives  $w_1'(x)$  and  $w_2'(x)$  respectively.

$$w_1(x) = \sqrt{\pi}(Ai(x) + iBi(x))$$

$$w_2(x) = \sqrt{\pi}(Ai(x) - iBi(x))$$

template < typename \_Tp >
 void std::\_\_detail::\_\_sph\_bessel\_ik (unsigned int \_\_n, \_Tp \_\_x, \_Tp &\_\_i\_n, \_Tp &\_\_k\_n, \_Tp &\_\_ip\_n, \_Tp &\_\_kp\_n)

Compute the spherical modified Bessel functions  $i_n(x)$  and  $k_n(x)$  and their first derivatives  $i_n'(x)$  and  $k_n'(x)$  respectively.

## 11.20.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

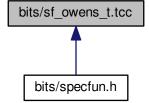
#### 11.20.2 Macro Definition Documentation

11.20.2.1 #define \_GLIBCXX\_BITS\_SF\_MOD\_BESSEL\_TCC 1

Definition at line 47 of file sf\_mod\_bessel.tcc.

## 11.21 bits/sf owens t.tcc File Reference

This graph shows which files directly or indirectly include this file:



## **Namespaces**

- std
- std:: detail

#### **Macros**

• #define \_GLIBCXX\_BITS\_SF\_OWENS\_T\_TCC 1

#### **Functions**

```
template<typename _Tp >
    _Tp std::__detail::__gauss (_Tp __x)
template<typename _Tp >
    _Tp std::__detail::__owens_t (_Tp __h, _Tp __a)
template<typename _Tp >
    _Tp std::__detail::__znorm1 (_Tp __x)
template<typename _Tp >
    _Tp std::__detail::__znorm2 (_Tp __x)
```

## 11.21.1 Detailed Description

This is an internal header file, included by other library headers. You should not attempt to use it directly.

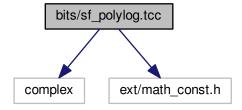
## 11.21.2 Macro Definition Documentation

```
11.21.2.1 #define _GLIBCXX_BITS_SF_OWENS_T_TCC 1
```

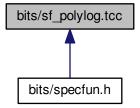
Definition at line 31 of file sf\_owens\_t.tcc.

# 11.22 bits/sf\_polylog.tcc File Reference

```
#include <complex>
#include <ext/math_const.h>
Include dependency graph for sf_polylog.tcc:
```



This graph shows which files directly or indirectly include this file:



## **Namespaces**

- std
- std:: detail

### **Macros**

• #define \_GLIBCXX\_BITS\_SF\_POLYLOG\_TCC 1

## **Functions**

```
template<typename Tp >
  _Tp std::__detail::__bose_einstein (_Tp __s, _Tp __x)
• template<typename _{\rm Tp}>
  std::complex< _Tp > std::__detail::__clamp_0_m2pi (std::complex< _Tp > __w)
template<typename Tp >
  std::complex< _Tp > std::__detail::__clamp_pi (std::complex< _Tp > __w)
• template<typename _Tp >
  std::complex < _Tp > std::__detail::__clausen (unsigned int __m, std::complex < _Tp > __w)
template<typename Tp >
  _Tp std::__detail::__clausen (unsigned int __m, _Tp __w)
template<typename _Tp >
  _Tp std::__detail::__clausen_c (unsigned int __m, std::complex< _Tp > __w)
template<typename _Tp >
  _Tp std::__detail::__clausen_c (unsigned int __m, _Tp __w)
template<typename _Tp >
  _Tp std::\_detail::\_clausen\_s (unsigned int \_m, std::complex< \_Tp > \_w)
template<typename _Tp >
  _Tp std::__detail::__clausen_s (unsigned int __m, _Tp __w)
template<typename _Tp >
  _Tp std::__detail::__dirichlet_beta (std::complex < _Tp > __w)
```

```
template<typename _Tp >
  Tp std:: detail:: dirichlet beta (Tp w)
template<typename _Tp >
  std::complex < _Tp > std::__detail::__dirichlet_eta (std::complex < _Tp > __w)
template<typename _Tp >
  Tp std:: detail:: dirichlet eta (Tp w)
template<typename _Tp >
  _Tp std::__detail::__fermi_dirac (_Tp __s, _Tp __x)
template<typename _Tp >
  bool std::__detail::__fpequal (const _Tp &__a, const _Tp &__b)
template<typename</li>Tp >
  bool std::__detail::__fpimag (const std::complex < _Tp > &__w)
template<typename _Tp >
  bool std:: detail:: fpimag (const Tp)

    template<typename</li>
    Tp >

  bool std::__detail::__fpreal (const std::complex < _Tp > &__w)
• template<typename _Tp >
  bool std:: detail:: fpreal (const Tp)
template<typename _Tp >
  std::complex< _Tp > std::__detail::__hurwitz_zeta (_Tp __s, std::complex< _Tp > __a)
template<typename _Tp >
  Tp std:: detail:: polylog (Tp s, Tp x)
template<typename _Tp >
  std::complex< Tp > std:: detail:: polylog ( Tp s, std::complex< Tp > w)

    template<typename _Tp , typename ArgType >

    _gnu_cxx::__promote_num_t< std::complex< _Tp >, ArgType > std::__detail::__polylog_exp (_Tp __s, Arg ↔
  Type w)
template<typename _Tp >
  std::complex< _Tp > std::__detail::__polylog_exp_asymp (_Tp __s, std::complex< _Tp > __w)
template<typename _Tp >
  std::complex < _Tp > std::__detail::__polylog_exp_int_neg (int __s, std::complex < _Tp > __w)
template<typename</li>Tp >
  std::complex < _Tp > std:: _detail:: _polylog_exp_int_neg (const int __s, _Tp __w)
template<typename _Tp >
  std::complex< Tp > std:: detail:: polylog exp int pos (unsigned int s, std::complex< Tp > w)
template<typename _Tp >
  std::complex < _Tp > std::__detail::__polylog_exp_int_pos (unsigned int __s, _Tp __w)
template<typename_Tp>
  std::complex < Tp > std:: detail:: polylog exp neg ( Tp s, std::complex < Tp > w)
template<typename _Tp >
  std::complex< _Tp > std::__detail::__polylog_exp_neg (int __s, std::complex< _Tp > __w)
• template<typename _Tp , int __sigma>
  std::complex< _Tp > std::__detail::__polylog_exp_neg_even (unsigned int __n, std::complex< _Tp > __w)
• template<typename Tp , int sigma>
  std::complex< _Tp > std::__detail::__polylog_exp_neg_odd (unsigned int __n, std::complex< _Tp > __w)

    template<typename _PowTp , typename _Tp >

  _Tp std::__detail::__polylog_exp_negative_real_part (_PowTp __s, _Tp __w)
template<typename</li>Tp >
  std::complex< _Tp > std::__detail::__polylog_exp_pos (unsigned int __s, std::complex< _Tp > __w)
template<typename_Tp>
  std::complex < _Tp > std::__detail::__polylog_exp_pos (unsigned int __s, _Tp __w)
template<typename _Tp >
  std::complex< Tp > std:: detail:: polylog exp pos (Tp s, std::complex< Tp > w)
```

```
template<typename _Tp > std::__detail::__polylog_exp_real_neg (_Tp __s, std::complex< _Tp > __w)
template<typename _Tp > std::_detail::__polylog_exp_real_neg (_Tp __s, _Tp __w)
template<typename _Tp > std::_detail::__polylog_exp_real_neg (_Tp __s, _Tp __w)
template<typename _Tp > std::_detail::__polylog_exp_real_pos (_Tp __s, std::complex< _Tp > __w)
template<typename _Tp > std::_detail::__polylog_exp_real_pos (_Tp __s, _Tp __w)
template<typename _Tp > std::__detail::_polylog_exp_real_pos (_Tp __s, _Tp __w)
template<typename _Tp = double> __Tp std::__detail::evenzeta (unsigned int __k)
```

## 11.22.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

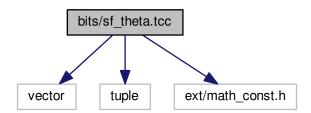
### 11.22.2 Macro Definition Documentation

11.22.2.1 #define \_GLIBCXX\_BITS\_SF\_POLYLOG\_TCC 1

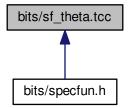
Definition at line 41 of file sf polylog.tcc.

## 11.23 bits/sf theta.tcc File Reference

```
#include <vector>
#include <tuple>
#include <ext/math_const.h>
Include dependency graph for sf_theta.tcc:
```



This graph shows which files directly or indirectly include this file:



## **Namespaces**

- std
- std::\_\_detail

### **Macros**

#define \_GLIBCXX\_BITS\_SF\_THETA\_TCC 1

## **Functions**

```
template<typename</li>Tp >
  _Tp std::__detail::__ellnome (_Tp __k)
\bullet \ \ \mathsf{template} \!<\! \mathsf{typename} \ \_\mathsf{Tp} >
  _Tp std::__detail::__ellnome_k (_Tp __k)
template<typename</li>Tp >
  _Tp std::__detail::__ellnome_series (_Tp __k)
\bullet \ \ template\!<\!typename\,\_Tp>
  std::tuple < _Tp, _Tp, _Tp > std::__detail::__jacobi_sncndn (_Tp __k, _Tp __u)
template<typename Tp >
  _Tp std::__detail::__theta_1 (_Tp __nu, _Tp __x)
• template<typename _{\mathrm{Tp}} >
  _Tp std::__detail::__theta_2 (_Tp __nu, _Tp __x)
template<typename _Tp >
  _Tp std::__detail::__theta_2_asymp (_Tp __nu, _Tp __x)
• template<typename _{\rm Tp}>
  _Tp std::__detail::__theta_2_sum (_Tp __nu, _Tp __x)
template<typename _Tp >
  _Tp std::__detail::__theta_3 (_Tp __nu, _Tp __x)
• template<typename _{\mathrm{Tp}} >
  _Tp std::__detail::__theta_3_asymp (_Tp __nu, _Tp __x)
```

```
template<typename _Tp >
    _Tp std::__detail::__theta_3_sum (_Tp __nu, _Tp __x)
template<typename _Tp >
    _Tp std::__detail::__theta_4 (_Tp __nu, _Tp __x)
template<typename _Tp >
    _Tp std::__detail::__theta_c (_Tp __k, _Tp __x)
template<typename _Tp >
    _Tp std::__detail::__theta_d (_Tp __k, _Tp __x)
template<typename _Tp >
    _Tp std::__detail::__theta_n (_Tp __k, _Tp __x)
template<typename _Tp >
    _Tp std::__detail::__theta_n (_Tp __k, _Tp __x)
template<typename _Tp >
    _Tp std::__detail::__theta_s (_Tp __k, _Tp __x)
```

## 11.23.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

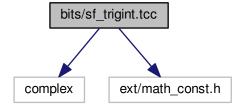
### 11.23.2 Macro Definition Documentation

11.23.2.1 #define \_GLIBCXX\_BITS\_SF\_THETA\_TCC 1

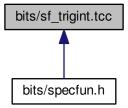
Definition at line 31 of file sf theta.tcc.

# 11.24 bits/sf\_trigint.tcc File Reference

```
#include <complex>
#include <ext/math_const.h>
Include dependency graph for sf_trigint.tcc:
```



This graph shows which files directly or indirectly include this file:



## **Namespaces**

- std
- std:: detail

## **Macros**

• #define \_GLIBCXX\_BITS\_SF\_TRIGINT\_TCC 1

#### **Enumerations**

enum { std::\_\_detail::SININT, std::\_\_detail::COSINT }

### **Functions**

```
    template<typename _Tp > std::__sincosint (_Tp __x)  
        This function returns the sine Si(x) and cosine Ci(x) integrals as a pair.
    template<typename _Tp >
```

void std::\_\_detail::\_\_sincosint\_asymp (\_Tp \_\_t, \_Tp &\_Si, \_Tp &\_Ci)

This function computes the sine Si(x) and cosine Ci(x) integrals by asymptotic series summation for positive argument.

```
    template < typename _Tp >
    void std:: __detail:: __sincosint _cont_frac (_Tp __t, _Tp &_Si, _Tp &_Ci)
```

This function computes the sine Si(x) and cosine Ci(x) integrals by continued fraction for positive argument.

```
    template<typename _Tp >
        void std::__detail::__sincosint_series (_Tp __t, _Tp &_Si, _Tp &_Ci)
```

This function computes the sine Si(x) and cosine Ci(x) integrals by series summation for positive argument.

## 11.24.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

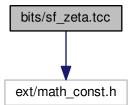
## 11.24.2 Macro Definition Documentation

11.24.2.1 #define \_GLIBCXX\_BITS\_SF\_TRIGINT\_TCC 1

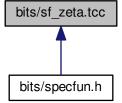
Definition at line 31 of file sf\_trigint.tcc.

# 11.25 bits/sf\_zeta.tcc File Reference

#include <ext/math\_const.h>
Include dependency graph for sf\_zeta.tcc:



This graph shows which files directly or indirectly include this file:



## **Namespaces**

```
std
```

• std:: detail

### **Macros**

• #define GLIBCXX BITS SF ZETA TCC 1

#### **Functions**

```
template<typename _Tp >
  _Tp std::__detail::__dilog (_Tp __x)
      Compute the dilogarithm function Li_2(x) by summation for x \le 1.

    template<typename</li>
    Tp >

  _Tp std::__detail::__hurwitz_zeta (_Tp __s, _Tp __a)
      Return the Hurwitz zeta function \zeta(s,a) for all s = 1 and a > -1.
template<typename _Tp >
  _Tp std::__detail::__hurwitz_zeta_euler_maclaurin (_Tp __s, _Tp __a)
      Return the Hurwitz zeta function \zeta(s,a) for all s = 1 and a > -1.
template<typename _Tp >
  Tp std:: detail:: riemann zeta (Tp s)
      Return the Riemann zeta function \zeta(s).
template<typename_Tp>
  _Tp std::__detail::__riemann_zeta_alt (_Tp __s)
      Evaluate the Riemann zeta function \zeta(s) by an alternate series for s>0.
template<typename _Tp >
  _Tp std::__detail::__riemann_zeta_euler_maclaurin (_Tp __s)
      Evaluate the Riemann zeta function \zeta(s) by an alternate series for s > 0.
template<typename _Tp >
  _Tp std::__detail::__riemann_zeta_glob (_Tp __s)
      Evaluate the Riemann zeta function by series for all s != 1. Convergence is great until largish negative numbers. Then the
      convergence of the > 0 sum gets better.
template<typename _Tp >
  _Tp std::__detail::__riemann_zeta_m_1 (_Tp __s)
      Return the Riemann zeta function \zeta(s) - 1.
template<typename _Tp >
  _Tp std::__detail::__riemann_zeta_m_1_sum (_Tp __s)
      Return the Riemann zeta function \zeta(s)-1 by summation for s>1. This is a small remainder for large s.
template<typename _Tp >
  _Tp std::__detail::__riemann_zeta_product (_Tp __s)
      Compute the Riemann zeta function \zeta(s) using the product over prime factors.
template<typename _Tp >
  _Tp std::__detail::__riemann_zeta_sum (_Tp __s)
      Compute the Riemann zeta function \zeta(s) by summation for s > 1.
```

# **Variables**

<ul> <li>constexpr size_t std::detail::_Num_Euler_Maclaurin_zeta = 100</li> <li>constexpr long double std::detail::_S_Euler_Maclaurin_zeta [_Num_Euler_Maclaurin_zeta]</li> <li>constexpr size_t std::detail::_S_num_zetam1 = 33</li> <li>constexpr long double std::detail::_S_zetam1 [_S_num_zetam1]</li> </ul>
11.25.1 Detailed Description
This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include $.$
11.25.2 Macro Definition Documentation
11.25.2.1 #define _GLIBCXX_BITS_SF_ZETA_TCC 1

Definition at line 46 of file sf\_zeta.tcc.

# 11.26 bits/specfun.h File Reference

```
#include <bits/c++config.h>
#include <limits>
#include <bits/stl algobase.h>
#include <bits/specfun_util.h>
#include <type_traits>
#include <bits/numeric_limits.h>
#include <bits/complex_util.h>
#include <bits/sf_gamma.tcc>
#include <bits/sf bessel.tcc>
#include <bits/sf_beta.tcc>
#include <bits/sf_cardinal.tcc>
#include <bits/sf_chebyshev.tcc>
#include <bits/sf_dawson.tcc>
#include <bits/sf_ellint.tcc>
#include <bits/sf expint.tcc>
#include <bits/sf_fresnel.tcc>
#include <bits/sf_gegenbauer.tcc>
#include <bits/sf_hyperg.tcc>
#include <bits/sf_hypint.tcc>
#include <bits/sf_jacobi.tcc>
#include <bits/sf_laguerre.tcc>
#include <bits/sf_legendre.tcc>
#include <bits/sf_hydrogen.tcc>
#include <bits/sf_mod_bessel.tcc>
#include <bits/sf_hermite.tcc>
#include <bits/sf_theta.tcc>
#include <bits/sf_trigint.tcc>
#include <bits/sf_zeta.tcc>
#include <bits/sf_owens_t.tcc>
#include <bits/sf_polylog.tcc>
#include <bits/sf_airy.tcc>
#include <bits/sf_hankel.tcc>
#include <bits/sf_distributions.tcc>
Include dependency graph for specfun.h:
```



### **Namespaces**

- \_\_gnu\_cxx
- std

#### **Macros**

- #define \_\_cpp\_lib\_math\_special\_functions 201603L
- #define STDCPP MATH SPEC FUNCS 201003L

### **Enumerations**

#### **Functions**

```
template<typename _Tp >
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::airy_ai (_Tp __x)

    float gnu cxx::airy aif (float x)

    long double <u>gnu_cxx::airy_ail</u> (long double <u>x</u>)

template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::airy_bi (_Tp __x)

    float __gnu_cxx::airy_bif (float __x)

    long double gnu cxx::airy bil (long double x)

template<typename _Tp >
   gnu cxx:: promote < Tp >:: type std::assoc laguerre (unsigned int n, unsigned int m, Tp x)
• float std::assoc_laguerref (unsigned int __n, unsigned int __m, float __x)
• long double std::assoc laguerrel (unsigned int n, unsigned int m, long double x)
template<typename _Tp >
   gnu cxx:: promote < Tp >:: type std::assoc legendre (unsigned int I, unsigned int m, Tp x)

    float std::assoc legendref (unsigned int I, unsigned int m, float x)

    long double std::assoc legendrel (unsigned int I, unsigned int m, long double x)

template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::bernoulli (unsigned int __n)

    float gnu cxx::bernoullif (unsigned int n)

    long double gnu cxx::bernoullil (unsigned int n)

template<typename _Tpa , typename _Tpb >
    _gnu_cxx::__promote_2< _Tpa, _Tpb >::__type std::beta (_Tpa __a, _Tpb __b)

    float std::betaf (float a, float b)

    long double std::betal (long double __a, long double __b)

template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::bincoef (unsigned int __n, unsigned int __k)

    float gnu cxx::bincoeff (unsigned int n, unsigned int k)

    long double __gnu_cxx::bincoefl (unsigned int __n, unsigned int __k)

template<typename</li>Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::chebyshev_t (unsigned int __n, _Tp __x)

    float gnu cxx::chebyshev tf (unsigned int n, float x)

    long double __gnu_cxx::chebyshev_tl (unsigned int __n, long double __x)

template<typename</li>Tp >
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::chebyshev_u (unsigned int __n, _Tp __x)

    float gnu cxx::chebyshev uf (unsigned int n, float x)

    long double gnu cxx::chebyshev ul (unsigned int n, long double x)

template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::chebyshev_v (unsigned int __n, _Tp __x)

    float __gnu_cxx::chebyshev_vf (unsigned int __n, float __x)

• long double __gnu_cxx::chebyshev_vl (unsigned int __n, long double __x)
template<typename_Tp>
   _gnu_cxx::_promote_num_t< _Tp > __gnu_cxx::chebyshev_w (unsigned int __n, _Tp __x)

    float gnu cxx::chebyshev wf (unsigned int n, float x)

    long double <u>__gnu_cxx::chebyshev_wl</u> (unsigned int __n, long double __x)
```

```
template<typename _Tp >
   gnu cxx:: promote num t < Tp > gnu cxx::clausen (unsigned int m, Tp w)

    template<typename</li>
    Tp >

  std::complex< __gnu_cxx::__promote_num_t< _Tp >> __gnu_cxx::clausen (unsigned int __m, std::complex<
  _{\mathsf{Tp}} > _{\mathsf{w}}

    template<typename</li>
    Tp >

   _gnu_cxx::_ promote_num_t< _Tp > __gnu_cxx::clausen_c (unsigned int __m, _Tp __w)
• float <u>gnu_cxx::clausen_cf</u> (unsigned int <u>m</u>, float <u>w</u>)
• long double gnu cxx::clausen cl (unsigned int m, long double w)
template<typename</li>Tp >
   __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::clausen_s (unsigned int __m, _Tp __w)

    float gnu cxx::clausen sf (unsigned int m, float w)

    long double gnu cxx::clausen sl (unsigned int m, long double w)

    float __gnu_cxx::clausenf (unsigned int __m, float __w)

• std::complex < float > gnu cxx::clausenf (unsigned int m, std::complex < float > w)

    long double __gnu_cxx::clausenl (unsigned int __m, long double __w)

    std::complex < long double > __gnu_cxx::clausenl (unsigned int __m, std::complex < long double > __w)

template<typename _Tp >
    gnu cxx:: promote < Tp >:: type std::comp ellint 1 (Tp k)

    float std::comp ellint 1f (float k)

    long double std::comp ellint 11 (long double k)

template<typename_Tp>
    _gnu_cxx::__promote< _Tp >::__type std::comp_ellint_2 (_Tp __k)

    float std::comp ellint 2f (float k)

    long double std::comp ellint 2l (long double k)

template<typename _Tp , typename _Tpn >
   _gnu_cxx::__promote_2< _Tp, _Tpn >::__type std::comp_ellint_3 (_Tp __k, _Tpn __nu)

    float std::comp ellint 3f (float k, float nu)

      Return the complete elliptic integral of the third kind \Pi(k,\nu) for float modulus k.

    long double std::comp ellint 3l (long double k, long double nu)

      Return the complete elliptic integral of the third kind \Pi(k,\nu) for long double modulus k.

    template<typename Tk >

    gnu cxx:: promote num t < Tk > gnu cxx::comp ellint d (Tk k)

    float gnu cxx::comp ellint df (float k)

    long double gnu cxx::comp ellint dl (long double k)

    float __gnu_cxx::comp_ellint_rf (float __x, float __y)

• long double gnu cxx::comp ellint rf (long double x, long double y)
• template<typename _{\rm Tx}, typename _{\rm Ty} >
   _gnu_cxx::__promote_num_t< _Tx, _Ty > __gnu_cxx::comp_ellint_rf (_Tx __x, _Ty __y)

    float __gnu_cxx::comp_ellint_rg (float __x, float __y)

    long double gnu cxx::comp ellint rg (long double x, long double y)

    template<typename _Tx , typename _Ty >

    _gnu_cxx::__promote_num_t< _Tx, _Ty > __gnu_cxx::comp_ellint_rg (_Tx __x, _Ty __y)
template<typename _Tpa , typename _Tpc , typename _Tp >
   _gnu_cxx::__promote_3< _Tpa, _Tpc, _Tp >::__type __gnu_cxx::conf_hyperg (_Tpa __a, _Tpc __c, _Tp __x)

    template<typename _Tpc , typename _Tp >

  __gnu_cxx::_promote_2< _Tpc, _Tp >::_type __gnu_cxx::conf_hyperg_lim (_Tpc __c, _Tp __x)

    float gnu cxx::conf hyperg limf (float c, float x)

    long double __gnu_cxx::conf_hyperg_liml (long double __c, long double __x)

    float gnu cxx::conf hypergf (float a, float c, float x)

    long double gnu cxx::conf hypergl (long double a, long double c, long double x)
```

```
template<typename _Tp >
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::coshint (_Tp __x)

    float gnu cxx::coshintf (float x)

    long double <u>gnu_cxx::coshintl</u> (long double <u>x</u>)

template<typename</li>Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::cosint (_Tp __x)

    float gnu cxx::cosintf (float x)

    long double gnu cxx::cosintl (long double x)

    template<typename _Tpnu , typename _Tp >

   _gnu_cxx::__promote_2< _Tpnu, _Tp >::__type std::cyl_bessel_i (_Tpnu __nu, _Tp __x)

    float std::cyl_bessel_if (float __nu, float __x)

    long double std::cyl bessel il (long double nu, long double x)

• template<typename Tpnu, typename Tp>
    _gnu_cxx::__promote_2< _Tpnu, _Tp >::__type std::cyl_bessel_j (_Tpnu __nu, _Tp __x)

    float std::cyl bessel if (float nu, float x)

• long double std::cyl_bessel_jl (long double __nu, long double __x)
• template<typename _Tpnu , typename _Tp >
    _gnu_cxx::__promote_2< _Tpnu, _Tp >::__type std::cyl_bessel_k (_Tpnu __nu, _Tp __x)

    float std::cyl bessel kf (float nu, float x)

    long double std::cyl_bessel_kl (long double __nu, long double __x)

• template<typename _Tpnu , typename _Tp >
  std::complex< __gnu_cxx::_promote_num_t< _Tpnu, _Tp >> __gnu_cxx::cyl_hankel_1 (_Tpnu __nu, _Tp
  __z)
• template<typename _{\rm Tpnu}, typename _{\rm Tp} >
  std::complex < \underline{gnu\_cxx::\_promote\_num\_t} < \underline{Tpnu, \_Tp} > \underline{gnu\_cxx::cyl\_hankel\_1} (std::complex < \underline{\leftarrow}
  Tpnu > __nu, std::complex< _Tp > __x)

    std::complex< float > __gnu_cxx::cyl_hankel_1f (float __nu, float __z)

    std::complex < float > __gnu_cxx::cyl_hankel_1f (std::complex < float > __nu, std::complex < float > __x)

• std::complex < long double > gnu cxx::cyl hankel 1l (long double nu, long double z)

    std::complex < long double > gnu cxx::cyl hankel 1l (std::complex < long double > nu, std::complex < long</li>

  double > x)
\bullet \;\; {\sf template}{<} {\sf typename} \; {\sf \_Tpnu} \; , \; {\sf typename} \; {\sf \_Tp} >
  std::complex< __gnu_cxx::__promote_num_t< _Tpnu, _Tp >> __gnu_cxx::cyl_hankel_2 (_Tpnu __nu, _Tp
  __z)
• template<typename _{\rm Tpnu}, typename _{\rm Tp} >
  std::complex < \underline{gnu\_cxx::\_promote\_num\_t < \underline{Tpnu}, \underline{Tp} > \underline{gnu\_cxx::cyl\_hankel\_2} (std::complex < \underline{\leftarrow}
  Tpnu > __nu, std::complex< _Tp > __x)

    std::complex< float > __gnu_cxx::cyl_hankel_2f (float __nu, float __z)

    std::complex < float > __nu, std::complex < float > __nu, std::complex < float > __x)

• std::complex < long double > gnu cxx::cyl hankel 2l (long double nu, long double z)

    std::complex < long double > gnu cxx::cyl hankel 2l (std::complex < long double > nu, std::complex < long</li>

  double > x)
• template<typename _Tpnu , typename _Tp >
   gnu cxx:: promote 2< Tpnu, Tp >:: type std::cyl neumann (Tpnu nu, Tp x)

    float std::cyl neumannf (float nu, float x)

    long double std::cyl_neumannl (long double __nu, long double __x)

template<typename _Tp >
   gnu cxx:: promote num t< Tp> gnu cxx::dawson (Tpx)

    float __gnu_cxx::dawsonf (float __x)

    long double <u>__gnu_cxx::dawsonl</u> (long double <u>__x</u>)

template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::digamma (_Tp __z)
```

```
    float __gnu_cxx::digammaf (float __z)

    long double __gnu_cxx::digammal (long double __z)

template<typename_Tp>
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::dilog (_Tp __x)

    float gnu cxx::dilogf (float x)

    long double __gnu_cxx::dilogl (long double __x)

template<typename _Tp >
  _Tp __gnu_cxx::dirichlet_beta (_Tp __s)

    float gnu cxx::dirichlet betaf (float s)

    long double gnu cxx::dirichlet betal (long double s)

template<typename _Tp >
  Tp gnu cxx::dirichlet eta (Tp s)

    float gnu cxx::dirichlet etaf (float s)

    long double <u>__gnu_cxx::dirichlet_etal</u> (long double <u>__s)</u>

template<typename</li>Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::double_factorial (int __n)

    float gnu cxx::double factorialf (int n)

    long double __gnu_cxx::double_factoriall (int __n)

• template<typename _Tp , typename _Tpp >
   _gnu_cxx::__promote_2< _Tp, _Tpp >::__type std::ellint_1 (_Tp __k, _Tpp __phi)

    float std::ellint_1f (float __k, float __phi)

    long double std::ellint 11 (long double k, long double phi)

template<typename _Tp , typename _Tpp >
    _gnu_cxx::__promote_2< _Tp, _Tpp >::__type std::ellint_2 (_Tp __k, _Tpp __phi)

    float std::ellint 2f (float k, float phi)

      Return the incomplete elliptic integral of the second kind E(k, \phi) for float argument.

    long double std::ellint 2l (long double k, long double phi)

      Return the incomplete elliptic integral of the second kind E(k, \phi).

    template<typename Tp, typename Tpn, typename Tpp>

   _gnu_cxx::__promote_3< _Tp, _Tpn, _Tpp >::__type std::ellint_3 (_Tp __k, _Tpn __nu, _Tpp __phi)
      Return the incomplete elliptic integral of the third kind \Pi(k, \nu, \phi).

    float std::ellint 3f (float k, float nu, float phi)

      Return the incomplete elliptic integral of the third kind \Pi(k, \nu, \phi) for float argument.

    long double std::ellint 3l (long double k, long double nu, long double phi)

      Return the incomplete elliptic integral of the third kind \Pi(k, \nu, \phi).

    template<typename _Tk , typename _Tp , typename _Ta , typename _Tb >

    _gnu_cxx::__promote_num_t< _Tk, _Tp, _Ta, _Tb > <u>__gnu_cxx::ellint_cel</u> (_Tk <u>__k_c, _</u>Tp <u>__p, _</u>Ta <u>__a, _</u>Tb
   b)

    float __gnu_cxx::ellint_celf (float __k_c, float __p, float _ a, float _ b)

    long double gnu cxx::ellint cell (long double k c, long double p, long double a, long double b)

• template<typename _Tk , typename _Tphi >
    _gnu_cxx::__promote_num_t< _Tk, _Tphi > __gnu_cxx::ellint_d (_Tk __k, _Tphi _ phi)

    float __gnu_cxx::ellint_df (float __k, float __phi)

    long double gnu cxx::ellint dl (long double k, long double phi)

• template<typename _Tp , typename _Tk >
    _gnu_cxx::__promote_num_t< _Tp, _Tk > __gnu_cxx::ellint_el1 (_Tp __x, _Tk __k_c)

    float __gnu_cxx::ellint_el1f (float __x, float __k_c)

    long double __gnu_cxx::ellint_el1l (long double __x, long double __k_c)

    template<typename _Tp , typename _Tk , typename _Ta , typename _Tb >

    _gnu_cxx::__promote_num_t< _Tp, _Tk, _Ta, _Tb > __gnu_cxx::ellint_el2 (_Tp __x, _Tk __k_c, _Ta __a, _Tb
  ___b)
```

```
    float __gnu_cxx::ellint_el2f (float __x, float __k_c, float __a, float __b)

    long double __gnu_cxx::ellint_el2l (long double __x, long double __k_c, long double __a, long double __b)

• template<typename \_Tx, typename \_Tk, typename \_Tp>
    _gnu_cxx::__promote_num_t< _Tx, _Tk, _Tp > __gnu_cxx::ellint_el3 (_Tx __x, _Tk __k_c, _Tp __p)
• float gnu cxx::ellint el3f (float x, float k c, float p)
• long double gnu cxx::ellint el3l (long double x, long double k c, long double p)
• template<typename Tp, typename Up>
    _gnu_cxx::__promote_num_t< _Tp, _Up > __gnu_cxx::ellint_rc (_Tp __x, _Up __y)

    float __gnu_cxx::ellint_rcf (float __x, float __y)

    long double __gnu_cxx::ellint_rcl (long double __x, long double __y)

- template<typename _Tp , typename _Up , typename _Vp >
    gnu cxx:: promote num t< Tp, Up, Vp > gnu cxx::ellint rd (Tp x, Up y, Vp z)

    float __gnu_cxx::ellint_rdf (float __x, float __y, float __z)

    long double gnu cxx::ellint rdl (long double x, long double y, long double z)

template<typename _Tp , typename _Up , typename _Vp >
   _gnu_cxx::_promote_num_t< _Tp, _Up, _Vp > __gnu_cxx::ellint_rf (_Tp __x, _Up __y, _Vp __z)

    float __gnu_cxx::ellint_rff (float __x, float __y, float __z)

    long double gnu cxx::ellint rfl (long double x, long double y, long double z)

• template<typename _Tp , typename _Up , typename _Vp >
    _gnu_cxx::__promote_num_t< _Tp, _Up, _Vp > __gnu_cxx::ellint_rg (_Tp __x, _Up __y, _Vp __z)

    float __gnu_cxx::ellint_rgf (float __x, float __y, float __z)

    long double __gnu_cxx::ellint_rgl (long double __x, long double __y, long double __z)

template<typename _Tp , typename _Up , typename _Vp , typename _Wp >
   _gnu_cxx::__promote_num_t< _Tp, _Up, _Vp, _Wp > __gnu_cxx::ellint_rj (_Tp __x, _Up __y, _Vp __z, _Wp
  __p)
• float gnu cxx::ellint rif (float x, float y, float z, float p)

    long double __gnu_cxx::ellint_rjl (long double __x, long double __y, long double __z, long double __p)

template<typename _Tp >
  Tp gnu cxx::ellnome (Tp k)

    float gnu cxx::ellnomef (float k)

    long double gnu cxx::ellnomel (long double k)

template<typename</li>Tp >
   __gnu_cxx::__promote< _Tp >::__type std::expint (_Tp __x)
template<typename _Tp >
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::expint (unsigned int __n, _Tp __x)

    float std::expintf (float x)

    float gnu cxx::expintf (unsigned int n, float x)

    long double std::expintl (long double x)

    long double gnu cxx::expintl (unsigned int n, long double x)

template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::factorial (unsigned int __n)

    float gnu cxx::factorialf (unsigned int n)

    long double __gnu_cxx::factoriall (unsigned int __n)

template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::fresnel_c (_Tp __x)

    float gnu cxx::fresnel cf (float x)

    long double gnu cxx::fresnel cl (long double x)

template<typename _Tp >
   __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::fresnel_s (_Tp __x)

    float gnu_cxx::fresnel_sf (float __x)

    long double gnu cxx::fresnel sl (long double x)
```

```
template<typename _Tn , typename _Tp >
   _gnu_cxx::__promote_num_t< _Tn, _Tp > __gnu_cxx::gamma_l (_Tn __n, _Tp __x)

    float gnu cxx::gamma If (float n, float x)

    long double __gnu_cxx::gamma_ll (long double __n, long double __x)

• template<typename Tn , typename Tp >
   _gnu_cxx::__promote_num_t< _Tn, _Tp > __gnu_cxx::gamma_u (_Tn __n, _Tp __x)

    float __gnu_cxx::gamma_uf (float __n, float __x)

• long double gnu cxx::gamma ul (long double n, long double x)
• template<typename _Talpha , typename _Tp >
    _gnu_cxx::__promote_num_t< _Talpha, _Tp > __gnu_cxx::gegenbauer (unsigned int __n, _Talpha __alpha,
  Tp x)
• float gnu cxx::gegenbauerf (unsigned int n, float alpha, float x)

    long double __gnu_cxx::gegenbauerl (unsigned int __n, long double __alpha, long double __x)

template<typename</li>Tp >
    _gnu_cxx::__promote< _Tp >::__type std::hermite (unsigned int __n, _Tp __x)

    float std::hermitef (unsigned int n, float x)

    long double std::hermitel (unsigned int n, long double x)

• template<typename _Tk , typename _Tphi >
   _gnu_cxx::__promote_num_t< _Tk, _Tphi > __gnu_cxx::heuman_lambda (_Tk __k, _Tphi __phi)

    float gnu cxx::heuman lambdaf (float k, float phi)

    long double __gnu_cxx::heuman_lambdal (long double __k, long double __phi)

    template<typename _Tp , typename _Up >

   _gnu_cxx::__promote_num_t< _Tp, _Up > __gnu_cxx::hurwitz_zeta (_Tp __s, _Up __a)
• template<typename _Tp , typename _Up >
  std::complex< _Tp > __gnu_cxx::hurwitz_zeta (_Tp __s, std::complex< _Up > __a)

    float gnu cxx::hurwitz zetaf (float s, float a)

    long double gnu cxx::hurwitz zetal (long double s, long double a)

ullet template<typename _Tpa , typename _Tpb , typename _Tpc , typename _Tp >
   _gnu_cxx::_promote_4< _Tpa, _Tpb, _Tpc, _Tp >::_type __gnu_cxx::hyperg (_Tpa __a, _Tpb __b, _Tpc
   __c, _Tp ___x)

    float gnu cxx::hypergf (float a, float b, float c, float x)

• long double gnu cxx::hypergl (long double a, long double b, long double c, long double x)
• template<typename _Ta , typename _Tb , typename _Tp >
   _gnu_cxx::__promote_num_t< _Ta, _Tb, _Tp > __gnu_cxx::ibeta (_Ta __a, _Tb __b, _Tp __x)
ullet template<typename _Ta , typename _Tb , typename _Tp >
   gnu cxx:: promote num t < Ta, Tb, Tp > gnu cxx::ibetac ( Ta a, Tb b, Tp x)

    float gnu cxx::ibetacf (float a, float b, float x)

• long double gnu cxx::ibetacl (long double a, long double b, long double x)

    float gnu cxx::ibetaf (float a, float b, float x)

    long double gnu cxx::ibetal (long double a, long double b, long double x)

• template<typename Talpha, typename Tbeta, typename Tp >
    _gnu_cxx::__promote_num_t< _Talpha, _Tbeta, _Tp > __gnu_cxx::jacobi (unsigned __n, _Talpha __alpha,
  _Tbeta __beta, _Tp __x)
template<typename _Kp , typename _Up >
   gnu cxx:: promote num t < Kp, Up > gnu cxx::jacobi cn ( Kp k, Up u)

    float __gnu_cxx::jacobi_cnf (float __k, float __u)

• long double gnu cxx::jacobi cnl (long double k, long double u)
template<typename _Kp , typename _Up >
   __gnu_cxx::__promote_num_t< _Kp, _Up > __gnu_cxx::jacobi_dn (_Kp __k, _Up __u)

    float __gnu_cxx::jacobi_dnf (float __k, float __u)

    long double gnu cxx::jacobi dnl (long double k, long double u)
```

```
    template<typename _Kp , typename _Up >

    gnu cxx:: promote num t< Kp, Up > gnu cxx::jacobi sn ( Kp k, Up u)

    float gnu cxx::jacobi snf (float k, float u)

    long double __gnu_cxx::jacobi_snl (long double __k, long double __u)

• template<typename Tk, typename Tphi >
    gnu cxx:: promote num t < Tk, Tphi > gnu cxx::jacobi zeta (Tk k, Tphi phi)

    float gnu cxx::jacobi zetaf (float k, float phi)

    long double __gnu_cxx::jacobi_zetal (long double __k, long double __phi)

    float gnu cxx::jacobif (unsigned n, float alpha, float beta, float x)

    long double __gnu_cxx::jacobil (unsigned __n, long double __alpha, long double __beta, long double __x)

template<typename _Tp >
    gnu cxx:: promote < Tp >:: type std::laguerre (unsigned int n, Tp x)

    float std::laguerref (unsigned int n, float x)

    long double std::laguerrel (unsigned int __n, long double __x)

template<typename _Tp >
    gnu cxx:: promote num t< Tp > gnu cxx::lbincoef (unsigned int n, unsigned int k)
• float gnu cxx::lbincoeff (unsigned int n, unsigned int k)

    long double <u>gnu_cxx::lbincoefl</u> (unsigned int _n, unsigned int _k)

template<typename _Tp >
    gnu cxx:: promote num t < Tp > gnu cxx::ldouble factorial (int n)

    float gnu cxx::ldouble factorialf (int n)

    long double __gnu_cxx::ldouble_factoriall (int __n)

template<typename _Tp >
    gnu cxx:: promote< Tp >:: type std::legendre (unsigned int I, Tp x)
template<typename _Tp >
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::legendre_q (unsigned int __n, _Tp __x)
• float __gnu _cxx::legendre_qf (unsigned int __n, float __x)

    long double gnu cxx::legendre ql (unsigned int n, long double x)

    float std::legendref (unsigned int I, float x)

    long double std::legendrel (unsigned int I, long double x)

template<typename _Tp >
    gnu cxx:: promote num t < Tp > gnu cxx::lfactorial (unsigned int n)

    float __gnu_cxx::lfactorialf (unsigned int __n)

    long double gnu cxx::lfactoriall (unsigned int n)

template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::logint (_Tp __x)

    float gnu cxx::logintf (float x)

    long double gnu cxx::logintl (long double x)

• template<typename _Tp , typename _Tn >
    gnu cxx:: promote num t< Tp, Tn > gnu cxx::lpochhammer I ( Tp a, Tn n)

    float gnu cxx::lpochhammer lf (float a, float n)

• long double __gnu_cxx::lpochhammer_ll (long double __a, long double __n)
• template<typename _Tp , typename _Tn >
    _gnu_cxx::__promote_num_t< _Tp, _Tn > __gnu_cxx::lpochhammer_u (_Tp __a, _Tn __n)

    float gnu cxx::lpochhammer uf (float a, float n)

    long double __gnu_cxx::lpochhammer_ul (long double __a, long double __n)

    template<typename _Tph , typename _Tpa >

    _gnu_cxx::__promote_num_t< _Tph, _Tpa > __gnu_cxx::owens_t (_Tph __h, _Tpa __a)

    float gnu cxx::owens tf (float h, float a)

    long double __gnu_cxx::owens_tl (long double __h, long double __a)

template<typename _Ta , typename _Tp >
    _gnu_cxx::__promote_num_t< _Ta, _Tp > __gnu_cxx::pgamma (_Ta __a, _Tp __x)
```

```
    float __gnu_cxx::pgammaf (float __a, float __x)

    long double __gnu_cxx::pgammal (long double __a, long double __x)

• template<typename _Tp , typename _Tn >
    _gnu_cxx::__promote_num_t< _Tp, _Tn > __gnu_cxx::pochhammer_l (_Tp __a, _Tn __n)

    float gnu cxx::pochhammer lf (float a, float n)

    long double gnu cxx::pochhammer II (long double a, long double n)

• template<typename Tp, typename Tn>
    _gnu_cxx::__promote_num_t< _Tp, _Tn > __gnu_cxx::pochhammer_u (_Tp __a, _Tn __n)

    float __gnu_cxx::pochhammer_uf (float __a, float __n)

    long double __gnu_cxx::pochhammer_ul (long double __a, long double __n)

template<typename _Tp , typename _Wp >
   _gnu_cxx::__promote_num_t< _Tp, _Wp > __gnu_cxx::polylog (_Tp __s, _Wp __w)
• template<typename _{\rm Tp}, typename _{\rm Wp} >
  std::complex< __gnu_cxx::__promote_num_t< _Tp, _Wp >> __gnu_cxx::polylog (_Tp __s, std::complex< _Tp
  > w)

    float __gnu_cxx::polylogf (float __s, float __w)

    std::complex< float > gnu cxx::polylogf (float s, std::complex< float > w)

    long double __gnu_cxx::polylogl (long double __s, long double __w)

    std::complex < long double > __gnu_cxx::polylogl (long double __s, std::complex < long double > __w)

    template<typename</li>
    Tp >

   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::psi (_Tp __x)

    float gnu cxx::psif (float x)

    long double gnu cxx::psil (long double x)

    template<typename</li>
    Ta , typename
    Tp >

   _gnu_cxx::__promote_num_t< _Ta, _Tp > __gnu_cxx::qgamma (_Ta __a, _Tp __x)
• float gnu cxx::ggammaf (float a, float x)

    long double gnu cxx::ggammal (long double a, long double x)

template<typename _Tp >
    gnu cxx:: promote num t < Tp > gnu cxx::radpoly (unsigned int n, unsigned int m, Tp rho)

    float gnu cxx::radpolyf (unsigned int n, unsigned int m, float rho)

    long double gnu cxx::radpolyl (unsigned int n, unsigned int m, long double rho)

    template<typename</li>
    Tp >

   _gnu_cxx::__promote< _Tp >::__type std::riemann_zeta (_Tp __s)

    float std::riemann zetaf (float s)

• long double std::riemann zetal (long double s)
template<typename_Tp>
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::sinc (_Tp __x)

    template<typename</li>
    Tp >

   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::sinc_pi (_Tp __x)
float __gnu_cxx::sinc_pif (float __x)

    long double __gnu_cxx::sinc_pil (long double __x)

    float gnu cxx::sincf (float x)

    long double <u>gnu_cxx::sincl</u> (long double <u>x</u>)

template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::sinhc (_Tp __x)
template<typename _Tp >
   __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::sinhc_pi (_Tp __x)

    float gnu cxx::sinhc pif (float x)

    long double gnu cxx::sinhc pil (long double x)

    float gnu cxx::sinhcf (float x)

    long double gnu cxx::sinhcl (long double x)
```

```
template<typename _Tp >
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::sinhint (_Tp __x)

    float gnu cxx::sinhintf (float x)

    long double <u>__gnu_cxx::sinhintl</u> (long double <u>__x)</u>

template<typename</li>Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::sinint (_Tp __x)

    float gnu cxx::sinintf (float x)

    long double <u>gnu_cxx::sinintl</u> (long double <u>x</u>)

template<typename _Tp >
   __gnu_cxx::__promote< _Tp >::__type std::sph_bessel (unsigned int __n, _Tp __x)
template<typename _Tp >
   gnu cxx:: promote num t< Tp > gnu cxx::sph bessel i (unsigned int n, Tp x)

    float gnu cxx::sph bessel if (unsigned int n, float x)

• long double <u>__gnu_cxx::sph_bessel_il</u> (unsigned int __n, long double __x)
template<typename _Tp >
   gnu cxx:: promote num t< Tp > gnu cxx::sph bessel k (unsigned int n, Tp x)

    float gnu cxx::sph bessel kf (unsigned int n, float x)

    long double __gnu_cxx::sph_bessel_kl (unsigned int __n, long double __x)

    float std::sph besself (unsigned int n, float x)

    long double std::sph bessell (unsigned int n, long double x)

    template<typename</li>
    Tp >

  std::complex< __gnu_cxx::__promote_num_t< _Tp >> __gnu_cxx::sph_hankel_1 (unsigned int __n, _Tp __z)
template<typename</li>Tp >
  std::complex< __gnu_cxx::_promote_num_t< _Tp >> __gnu_cxx::sph_hankel_1 (unsigned int __n, std↔
  ::complex < _Tp > __x)

    std::complex< float > __gnu_cxx::sph_hankel_1f (unsigned int __n, float __z)

    std::complex < float > __gnu_cxx::sph_hankel_1f (unsigned int __n, std::complex < float > __x)

    std::complex < long double > __gnu_cxx::sph_hankel_1I (unsigned int __n, long double __z)

    std::complex < long double > gnu cxx::sph hankel 1l (unsigned int n, std::complex < long double > x)

    template<typename</li>
    Tp >

  std::complex < __gnu_cxx::__promote_num_t < _Tp > > __gnu_cxx::sph_hankel_2 (unsigned int __n, _Tp __z)
• template<typename Tp >
  std::complex< gnu cxx:: promote num t< Tp >> gnu cxx::sph hankel 2 (unsigned int n, std←
  ::complex < _Tp > __x)

    std::complex< float > gnu cxx::sph hankel 2f (unsigned int n, float z)

    std::complex < float > gnu cxx::sph hankel 2f (unsigned int n, std::complex < float > x)

    std::complex < long double > gnu cxx::sph hankel 2l (unsigned int n, long double z)

    std::complex < long double > __gnu_cxx::sph_hankel_2l (unsigned int __n, std::complex < long double > __x)

• template<typename _Ttheta , typename _Tphi >
  std::complex< gnu cxx:: promote num t< Ttheta, Tphi >> gnu cxx::sph harmonic (unsigned int ←
   I, int m, Ttheta theta, Tphi phi)

    std::complex < float > __gnu_cxx::sph_harmonicf (unsigned int __l, int __m, float __theta, float __phi)

• std::complex < long double > __gnu_cxx::sph_harmonicl (unsigned int __l, int __m, long double __theta, long
  double phi)
template<typename</li>Tp >
    _gnu_cxx::__promote< _Tp >::__type std::sph_legendre (unsigned int __I, unsigned int __m, _Tp __theta)
• float std::sph_legendref (unsigned int __I, unsigned int __m, float __theta)
• long double std::sph legendrel (unsigned int __l, unsigned int __m, long double __theta)
template<typename_Tp>
   _gnu_cxx::__promote< _Tp >::__type std::sph_neumann (unsigned int __n, _Tp __x)

    float std::sph neumannf (unsigned int n, float x)

    long double std::sph neumannl (unsigned int n, long double x)
```

```
    template<typename _Tpnu , typename _Tp >

   _gnu_cxx::__promote_num_t< _Tpnu, _Tp > <u>__gnu_cxx::theta_</u>1 (_Tpnu __nu, _Tp __x)

    float gnu cxx::theta 1f (float nu, float x)

    long double __gnu_cxx::theta_1l (long double __nu, long double __x)

• template<typename _Tpnu , typename _Tp >
   _gnu_cxx::__promote_num_t< _Tpnu, _Tp > __gnu_cxx::theta_2 (_Tpnu __nu, _Tp __x)
• float gnu cxx::theta 2f (float nu, float x)

    long double __gnu_cxx::theta_2l (long double __nu, long double __x)

• template<typename Tpnu, typename Tp>
    _gnu_cxx::__promote_num_t< _Tpnu, _Tp > __gnu_cxx::theta_3 (_Tpnu __nu, _Tp __x)

    float gnu cxx::theta 3f (float nu, float x)

    long double __gnu_cxx::theta_3I (long double __nu, long double __x)

• template<typename Tpnu, typename Tp>
    _gnu_cxx::__promote_num_t< _Tpnu, _Tp > __gnu_cxx::theta_4 (_Tpnu __nu, _Tp __x)
float gnu_cxx::theta_4f (float __nu, float __x)

    long double __gnu_cxx::theta_4l (long double __nu, long double __x)

• template<typename _{\rm Tpk}, typename _{\rm Tp} >
    gnu cxx:: promote num t < Tpk, Tp > gnu cxx::theta c ( Tpk k, Tp x)

    float __gnu_cxx::theta_cf (float __k, float __x)

    long double __gnu_cxx::theta_cl (long double __k, long double __x)

• template<typename _{\rm Tpk}, typename _{\rm Tp} >
    _gnu_cxx::__promote_num_t< _Tpk, _Tp > __gnu_cxx::theta_d (_Tpk __k, _Tp __x)

    float __gnu_cxx::theta_df (float __k, float __x)

    long double gnu cxx::theta dl (long double k, long double x)

template<typename _Tpk , typename _Tp >
    _gnu_cxx::__promote_num_t< _Tpk, _Tp > __gnu_cxx::theta_n (_Tpk __k, _Tp __x)

    float gnu cxx::theta nf (float k, float x)

    long double gnu cxx::theta nl (long double k, long double x)

    template<typename</li>
    Tpk , typename
    Tp >

    _gnu_cxx::__promote_num_t< _Tpk, _Tp > __gnu_cxx::theta_s (_Tpk __k, _Tp __x)

    float __gnu_cxx::theta_sf (float __k, float __x)

    long double __gnu_cxx::theta_sl (long double __k, long double __x)

• template<typename _Trho , typename _Tphi >
   gnu cxx:: promote num t< Trho, Tphi > gnu cxx::zernike (unsigned int n, int m, Trho rho,
  Tphi phi)

    float gnu cxx::zernikef (unsigned int n, int m, float rho, float phi)

    long double __gnu_cxx::zernikel (unsigned int __n, int __m, long double __rho, long double __phi)
```

## 11.26.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <cmath>.

#### 11.26.2 Macro Definition Documentation

11.26.2.1 #define \_\_cpp\_lib\_math\_special\_functions 201603L

Definition at line 39 of file specfun.h.

11.26.2.2 #define \_\_STDCPP\_MATH\_SPEC\_FUNCS\_\_ 201003L

Definition at line 37 of file specfun.h.

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