TR29124 C++ Special Math Functions 2.0

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Contents

1	Math	hematical Special Functions	1
	1.1	Introduction and History	1
	1.2	Contents	1
	1.3	General Features	2
		1.3.1 Argument Promotion	2
		1.3.2 NaN Arguments	2
	1.4	Implementation	2
	1.5	Testing	3
	1.6	General Bibliography	3
2	Todo	o List	5
3	Mod	lule Index	7
	3.1	Modules	7
4	Nam	nespace Index	9
	4.1	Namespace List	9
5	Clas	es Index	11
	5.1	Class List	11
6			13
	6.1	File List	13
7	Mod	lule Documentation	15
	7.1	C++ Mathematical Special Functions	15
		7.1.1 Detailed Description	15
	7.2	C++17/IS29124 Mathematical Special Functions	16
		7.2.1 Detailed Description	18
		7.2.2 Function Documentation	18
		7.2.2.1 assoc_laguerre	18

iv CONTENTS

7.2.2.2	assoc_laguerref	. 19
7.2.2.3	assoc_laguerrel	. 19
7.2.2.4	assoc_legendre	. 19
7.2.2.5	assoc_legendref	. 20
7.2.2.6	assoc_legendrel	. 20
7.2.2.7	beta	. 20
7.2.2.8	betaf	. 20
7.2.2.9	betal	. 21
7.2.2.10	comp_ellint_1	. 21
7.2.2.11	comp_ellint_1f	. 21
7.2.2.12	comp_ellint_1I	. 22
7.2.2.13	comp_ellint_2	. 22
7.2.2.14	comp_ellint_2f	. 22
7.2.2.15	comp_ellint_2l	. 22
7.2.2.16	comp_ellint_3	. 23
7.2.2.17	comp_ellint_3f	. 23
7.2.2.18	comp_ellint_3l	. 23
7.2.2.19	cyl_bessel_i	. 24
7.2.2.20	cyl_bessel_if	. 24
7.2.2.21	cyl_bessel_il	. 24
7.2.2.22	cyl_bessel_j	. 24
7.2.2.23	cyl_bessel_jf	. 25
7.2.2.24	cyl_bessel_jl	. 25
7.2.2.25	cyl_bessel_k	. 25
7.2.2.26	cyl_bessel_kf	. 26
7.2.2.27	cyl_bessel_kl	. 26
7.2.2.28	cyl_neumann	. 26
7.2.2.29	cyl_neumannf	. 27
7.2.2.30	cyl_neumannl	. 27
7.2.2.31	ellint_1	. 27
7.2.2.32	ellint_1f	. 28
7.2.2.33	ellint_1I	. 28
7.2.2.34	ellint_2	. 28
7.2.2.35	ellint_2f	. 29
7.2.2.36	ellint_2l	. 29
7.2.2.37	ellint_3	. 29
7.2.2.38	ellint_3f	. 30

CONTENTS

		7.2.2.39	ellint_3I	 30
		7.2.2.40	expint	 30
		7.2.2.41	expintf	 31
		7.2.2.42	expintl	 31
		7.2.2.43	hermite	 31
		7.2.2.44	hermitef	 32
		7.2.2.45	hermitel	 32
		7.2.2.46	laguerre	 32
		7.2.2.47	laguerref	 32
		7.2.2.48	laguerrel	 33
		7.2.2.49	legendre	 33
		7.2.2.50	legendref	 33
		7.2.2.51	legendrel	 33
		7.2.2.52	riemann_zeta	 34
		7.2.2.53	riemann_zetaf	 34
		7.2.2.54	riemann_zetal	 34
		7.2.2.55	sph_bessel	 35
		7.2.2.56	sph_besself	 35
		7.2.2.57	sph_bessell	 35
		7.2.2.58	sph_legendre	 35
		7.2.2.59	sph_legendref	 36
		7.2.2.60	sph_legendrel	 36
		7.2.2.61	sph_neumann	 36
		7.2.2.62	sph_neumannf	 37
		7.2.2.63	sph_neumannl	 37
7.3	GNU E	xtended M	Mathematical Special Functions	 38
	7.3.1	Detailed I	Description	 46
	7.3.2	Enumerat	tion Type Documentation	 46
		7.3.2.1	anonymous enum	 46
	7.3.3	Function	Documentation	 46
		7.3.3.1	airy_ai	 46
		7.3.3.2	airy_aif	 46
		7.3.3.3	airy_ail	 46
		7.3.3.4	airy_bi	 46
		7.3.3.5	airy_bif	 47
		7.3.3.6	airy_bil	 47
		7.3.3.7	bernoulli	 47

vi CONTENTS

7.3.3.8	bernoullif
7.3.3.9	bernoullil
7.3.3.10	bincoef
7.3.3.11	bincoeff
7.3.3.12	bincoefl
7.3.3.13	chebyshev_t
7.3.3.14	chebyshev_tf
7.3.3.15	chebyshev_tl
7.3.3.16	chebyshev_u
7.3.3.17	chebyshev_uf
7.3.3.18	chebyshev_ul
7.3.3.19	chebyshev_v
7.3.3.20	chebyshev_vf
7.3.3.21	chebyshev_vl
7.3.3.22	chebyshev_w
7.3.3.23	chebyshev_wf
7.3.3.24	chebyshev_wl
7.3.3.25	clausen
7.3.3.26	clausen
7.3.3.27	clausen_c
7.3.3.28	clausen_cf
7.3.3.29	clausen_cl
7.3.3.30	clausen_s
7.3.3.31	clausen_sf
7.3.3.32	clausen_sl
7.3.3.33	clausenf
7.3.3.34	clausenf
7.3.3.35	clausenl
7.3.3.36	clausenl
7.3.3.37	comp_ellint_d
7.3.3.38	comp_ellint_df
7.3.3.39	comp_ellint_dl
7.3.3.40	comp_ellint_rf
7.3.3.41	comp_ellint_rf
7.3.3.42	comp_ellint_rf
7.3.3.43	comp_ellint_rg
7.3.3.44	comp_ellint_rg

CONTENTS vii

7.3.3.45	comp_ellint_rg	 54
7.3.3.46	conf_hyperg	 54
7.3.3.47	conf_hyperg_lim	 55
7.3.3.48	conf_hyperg_limf	 55
7.3.3.49	conf_hyperg_liml	 55
7.3.3.50	conf_hypergf	 55
7.3.3.51	conf_hypergl	 56
7.3.3.52	coshint	 56
7.3.3.53	coshintf	 56
7.3.3.54	coshintl	 56
7.3.3.55	cosint	 56
7.3.3.56	cosintf	 57
7.3.3.57	cosintl	 57
7.3.3.58	cyl_hankel_1	 57
7.3.3.59	cyl_hankel_1	 57
7.3.3.60	cyl_hankel_1f	 57
7.3.3.61	cyl_hankel_1f	 58
7.3.3.62	cyl_hankel_1l	 58
7.3.3.63	cyl_hankel_1l	 58
7.3.3.64	cyl_hankel_2	 58
7.3.3.65	cyl_hankel_2	 58
7.3.3.66	cyl_hankel_2f	 58
7.3.3.67	cyl_hankel_2f	 59
7.3.3.68	cyl_hankel_2l	 59
7.3.3.69	cyl_hankel_2l	 59
7.3.3.70	dawson	 59
7.3.3.71	dawsonf	 59
7.3.3.72	dawsonl	 60
7.3.3.73	digamma	 60
7.3.3.74	digammaf	 60
7.3.3.75	digammal	 60
7.3.3.76	dilog	 60
7.3.3.77	dilogf	 60
7.3.3.78	dilogl	 60
7.3.3.79	dirichlet_beta	 61
7.3.3.80	dirichlet_betaf	 61
7.3.3.81	dirichlet_betal	 61

viii CONTENTS

7.3.3.82	dirichlet_eta	
7.3.3.83	dirichlet_etaf	61
7.3.3.84	dirichlet_etal	62
7.3.3.85	double_factorial	62
7.3.3.86	double_factorialf	62
7.3.3.87	double_factoriall	62
7.3.3.88	ellint_cel	62
7.3.3.89	ellint_celf	62
7.3.3.90	ellint_cell	
7.3.3.91	ellint_d	63
7.3.3.92	ellint_df	63
7.3.3.93	ellint_dl	63
7.3.3.94	ellint_el1	63
7.3.3.95	ellint_el1f	63
7.3.3.96	ellint_el1I	63
7.3.3.97	ellint_el2	64
7.3.3.98	ellint_el2f	
7.3.3.99	ellint_el2l	64
	0 ellint_el3	
7.3.3.101	l ellint_el3f	64
	2 ellint_el3l	
7.3.3.103	B ellint_rc	65
7.3.3.104	4 ellint_rcf	65
7.3.3.105	5 ellint_rcl	65
7.3.3.106	6 ellint_rd	65
7.3.3.107	7 ellint_rdf	66
7.3.3.108	3 ellint_rdl	66
7.3.3.109	9 ellint_rf	66
7.3.3.110	Oellint_rff	67
7.3.3.111	l ellint_rfl	67
7.3.3.112	2 ellint_rg	67
	Bellint_rgf	68
7.3.3.114	4 ellint_rgl	68
	5 ellint_rj	68
	6 ellint_rjf	68
7.3.3.117	⁷ ellint_rjl	69
7.3.3.118	3 ellnome	69

CONTENTS ix

7.3.3.119 ellnomef
7.3.3.120 ellnomel
7.3.3.121 expint_e1
7.3.3.122 expint_e1f
7.3.3.123 expint_e1l
7.3.3.124 expint_en
7.3.3.125 expint_enf
7.3.3.126 expint_enl
7.3.3.127 factorial
7.3.3.128 factorialf
7.3.3.129 factoriall
7.3.3.130 fresnel_c
7.3.3.131 fresnel_cf
7.3.3.132 fresnel_cl
7.3.3.133 fresnel_s
7.3.3.134 fresnel_sf
7.3.3.135 fresnel_sl
7.3.3.136 gamma_l
7.3.3.137 gamma_lf
7.3.3.138 gamma_ll
7.3.3.139 gamma_p
7.3.3.140 gamma_pf
7.3.3.141 gamma_pl
7.3.3.142 gamma_q
7.3.3.143 gamma_qf
7.3.3.144 gamma_ql
7.3.3.145 gamma_u
7.3.3.146 gamma_uf
7.3.3.147 gamma_ul
7.3.3.148 gegenbauer
7.3.3.149 gegenbauerf
7.3.3.150 gegenbauerl
7.3.3.151 heuman_lambda
7.3.3.152 heuman_lambdaf
7.3.3.153 heuman_lambdal
7.3.3.154 hurwitz_zeta
7.3.3.155 hurwitz_zeta

CONTENTS

7.3.3.156 hurwitz_zetaf
7.3.3.157 hurwitz_zetal
7.3.3.158 hyperg
7.3.3.159 hypergf
7.3.3.160 hypergl
7.3.3.161 ibeta
7.3.3.162 ibetac
7.3.3.163 ibetacf
7.3.3.164 ibetacl
7.3.3.165 ibetaf
7.3.3.166 ibetal
7.3.3.167 jacobi
7.3.3.168 jacobi_cn
7.3.3.169 jacobi_cnf
7.3.3.170 jacobi_cnl
7.3.3.171 jacobi_dn
7.3.3.172 jacobi_dnf
7.3.3.173 jacobi_dnl
7.3.3.174 jacobi_sn
7.3.3.175 jacobi_snf
7.3.3.176 jacobi_snl
7.3.3.177 jacobi_zeta
7.3.3.178 jacobi_zetaf
7.3.3.179 jacobi_zetal
7.3.3.180 jacobif
7.3.3.181 jacobil
7.3.3.182 lbincoef
7.3.3.183 lbincoeff
7.3.3.184 lbincoefl
7.3.3.185 Idouble_factorial
7.3.3.186 Idouble_factorialf
7.3.3.187 Idouble_factoriall
7.3.3.188 legendre_q
7.3.3.189 legendre_qf
7.3.3.190 legendre_ql
7.3.3.191 factorial
7.3.3.192 factorialf

CONTENTS xi

7.3.3.193 factorial
7.3.3.194 logint
7.3.3.195 logintf
7.3.3.196 logintl
7.3.3.197 lpochhammer_I
7.3.3.198 lpochhammer_lf
7.3.3.199 lpochhammer_ll
7.3.3.200 lpochhammer_u
7.3.3.201 lpochhammer_uf
7.3.3.202 lpochhammer_ul
7.3.3.203 owens_t
7.3.3.204 owens_tf
7.3.3.205 owens_tl
7.3.3.206 pochhammer_I
7.3.3.207 pochhammer_lf
7.3.3.208 pochhammer_II
7.3.3.209 pochhammer_u
7.3.3.210 pochhammer_uf
7.3.3.211 pochhammer_ul
7.3.3.212 polylog
7.3.3.213 polylogf
7.3.3.214 polylogl
7.3.3.215 psi
7.3.3.216 psif
7.3.3.217 psil
7.3.3.218 radpoly
7.3.3.219 radpolyf
7.3.3.220 radpolyl
7.3.3.221 sinc
7.3.3.222 sinc_pi
7.3.3.223 sinc_pif
7.3.3.224 sinc_pil
7.3.3.225 sincf
7.3.3.226 sincl
7.3.3.227 sinhc
7.3.3.228 sinhc_pi
7.3.3.229 sinhc_pif

xii CONTENTS

7.3.3.230 sinhc_pil
7.3.3.231 sinhcf
7.3.3.232 sinhcl
7.3.3.233 sinhint
7.3.3.234 sinhintf
7.3.3.235 sinhintl
7.3.3.236 sinint
7.3.3.237 sinintf
7.3.3.238 sinintl
7.3.3.239 sph_bessel_i
7.3.3.240 sph_bessel_if
7.3.3.241 sph_bessel_il
7.3.3.242 sph_bessel_k
7.3.3.243 sph_bessel_kf
7.3.3.244 sph_bessel_kl
7.3.3.245 sph_hankel_1
7.3.3.246 sph_hankel_1
7.3.3.247 sph_hankel_1f
7.3.3.248 sph_hankel_1f
7.3.3.249 sph_hankel_1l
7.3.3.250 sph_hankel_1l
7.3.3.251 sph_hankel_2
7.3.3.252 sph_hankel_2
7.3.3.253 sph_hankel_2f
7.3.3.254 sph_hankel_2f
7.3.3.255 sph_hankel_2l
7.3.3.256 sph_hankel_2l
7.3.3.257 sph_harmonic
7.3.3.258 sph_harmonicf
7.3.3.259 sph_harmonicl
7.3.3.260 theta_1
7.3.3.261 theta_1f
7.3.3.262 theta_1I
7.3.3.263 theta_2
7.3.3.264 theta_2f
7.3.3.265 theta_2l
7.3.3.266 theta_3

CONTENTS xiii

			7.3.3.267 theta_3f
			7.3.3.268 theta_3I
			7.3.3.269 theta_4
			7.3.3.270 theta_4f
			7.3.3.271 theta_4l
			7.3.3.272 theta_c
			7.3.3.273 theta_cf
			7.3.3.274 theta_cl
			7.3.3.275 theta_d
			7.3.3.276 theta_df
			7.3.3.277 theta_dl
			7.3.3.278 theta_n
			7.3.3.279 theta_nf
			7.3.3.280 theta_nl
			7.3.3.281 theta_s
			7.3.3.282 theta_sf
			7.3.3.283 theta_sl
			7.3.3.284 zernike
			7.3.3.285 zernikef
			7.3.3.286 zernikel
8	Nam	nespace	Documentation 95
		_	_cxx Namespace Reference
	8.2		- mespace Reference
	8.3		•
			detail Namespace Reference
		8.3.1	detail Namespace Reference
		8.3.1	
		8.3.1	Enumeration Type Documentation
			Enumeration Type Documentation
			Enumeration Type Documentation
			Enumeration Type Documentation .122 8.3.1.1 anonymous enum .122 Function Documentation .123 8.3.2.1airy .123
			Enumeration Type Documentation .122 8.3.1.1 anonymous enum .122 Function Documentation .123 8.3.2.1airy .123 8.3.2.2airy .123
			Enumeration Type Documentation .122 8.3.1.1 anonymous enum .122 Function Documentation .123 8.3.2.1airy .123 8.3.2.2airy .123 8.3.2.3airy_ai .125
			Enumeration Type Documentation 122 8.3.1.1 anonymous enum 122 Function Documentation 123 8.3.2.1airy 123 8.3.2.2airy 123 8.3.2.3airy_ai 125 8.3.2.4airy_arg 125
			Enumeration Type Documentation .122 8.3.1.1 anonymous enum .122 Function Documentation .123 8.3.2.1airy .123 8.3.2.2airy .123 8.3.2.3airy_ai .125 8.3.2.4airy_arg .125 8.3.2.5airy_asymp_absarg_ge_pio3 .125
			Enumeration Type Documentation 122 8.3.1.1 anonymous enum 122 Function Documentation 123 8.3.2.1airy 123 8.3.2.2airy 123 8.3.2.3airy_ai 125 8.3.2.4airy_arg 125 8.3.2.5airy_asymp_absarg_ge_pio3 125 8.3.2.6airy_asymp_absarg_lt_pio3 126
			Enumeration Type Documentation .122 8.3.1.1 anonymous enum .122 Function Documentation .123 8.3.2.1airy .123 8.3.2.2airy .123 8.3.2.3airy_ai .125 8.3.2.4airy_arg .125 8.3.2.5airy_asymp_absarg_ge_pio3 .125 8.3.2.6airy_asymp_absarg_lt_pio3 .126 8.3.2.7airy_bessel_i .126

xiv CONTENTS

8.3.2.10airy_hyperg_rational
8.3.2.11assoc_laguerre
8.3.2.12assoc_legendre_p
8.3.2.13bernoulli
8.3.2.14bernoulli_2n
8.3.2.15bernoulli_series
8.3.2.16beta
8.3.2.17beta_gamma
8.3.2.18beta_inc
8.3.2.19beta_inc_cont_frac
8.3.2.20beta_lgamma
8.3.2.21beta_product
8.3.2.22bincoef
8.3.2.23binomial_cdf
8.3.2.24binomial_cdfc
8.3.2.25bose_einstein
8.3.2.26chebyshev_recur
8.3.2.27chebyshev_t
8.3.2.28chebyshev_u
8.3.2.29chebyshev_v
8.3.2.30chebyshev_w
8.3.2.31chi_squared_pdf
8.3.2.32chi_squared_pdfc
8.3.2.33chshint
8.3.2.34chshint_cont_frac
8.3.2.35chshint_series
8.3.2.36clamp_0_m2pi
8.3.2.37clamp_pi
8.3.2.38clausen
8.3.2.39clausen
8.3.2.40clausen_c
8.3.2.41clausen_c
8.3.2.42clausen_s
8.3.2.43clausen_s
8.3.2.44comp_ellint_1
8.3.2.45comp_ellint_2
8.3.2.46comp_ellint_3

CONTENTS xv

8.3.2.47comp_ellint_d
8.3.2.48comp_ellint_rf
8.3.2.49comp_ellint_rg
8.3.2.50conf_hyperg
8.3.2.51conf_hyperg_lim
8.3.2.52conf_hyperg_lim_series
8.3.2.53conf_hyperg_luke
8.3.2.54conf_hyperg_series
8.3.2.55coshint
8.3.2.56cyl_bessel
8.3.2.57cyl_bessel_i
8.3.2.58cyl_bessel_ij_series
8.3.2.59cyl_bessel_ik
8.3.2.60cyl_bessel_ik_asymp
8.3.2.61cyl_bessel_ik_steed
8.3.2.62cyl_bessel_j
8.3.2.63cyl_bessel_jn
8.3.2.64cyl_bessel_jn_asymp
8.3.2.65cyl_bessel_jn_steed
8.3.2.66cyl_bessel_k
8.3.2.67cyl_hankel_1
8.3.2.68cyl_hankel_1
8.3.2.69cyl_hankel_2
8.3.2.70cyl_hankel_2
8.3.2.71cyl_neumann
8.3.2.72cyl_neumann_n
8.3.2.73dawson
8.3.2.74dawson_const_frac
8.3.2.75dawson_series
8.3.2.76debye_region
8.3.2.77dilog
8.3.2.78dirichlet_beta
8.3.2.79dirichlet_beta
8.3.2.80dirichlet_eta
8.3.2.81dirichlet_eta
8.3.2.82double_factorial
8.3.2.83ellint_1

xvi CONTENTS

8.3.2.84ellint_2
8.3.2.85ellint_3
8.3.2.86ellint_cel
8.3.2.87ellint_d
8.3.2.88ellint_el1
8.3.2.89ellint_el2
8.3.2.90ellint_el3
8.3.2.91ellint_rc
8.3.2.92ellint_rd
8.3.2.93ellint_rf
8.3.2.94ellint_rg
8.3.2.95ellint_rj
8.3.2.96ellnome
8.3.2.97ellnome_k
8.3.2.98ellnome_series
8.3.2.99expint
8.3.2.100expint
8.3.2.101expint_asymp
8.3.2.102expint_E1
8.3.2.103expint_E1_asymp
8.3.2.104expint_E1_series
8.3.2.105expint_Ei
8.3.2.106expint_Ei_asymp
8.3.2.107expint_Ei_series
8.3.2.108expint_En_cont_frac
8.3.2.109expint_En_recursion
8.3.2.110expint_En_series
8.3.2.111expint_large_n
8.3.2.112factorial
8.3.2.113fermi_dirac
8.3.2.114fisher_f_cdf
8.3.2.115fisher_f_cdfc
8.3.2.116fock_airy
8.3.2.117fpequal
8.3.2.118fpimag
8.3.2.119fpimag
8.3.2.120fpreal

CONTENTS xvii

8.3.2.121fpreal
8.3.2.122fresnel
8.3.2.123fresnel_cont_frac
8.3.2.124fresnel_series
8.3.2.125 <u>gamma</u>
8.3.2.126gamma_cont_frac
8.3.2.127gamma_l
8.3.2.128gamma_p
8.3.2.129gamma_q
8.3.2.130gamma_series
8.3.2.131gamma_temme
8.3.2.132gamma_u
8.3.2.133gauss
8.3.2.134gegenbauer_poly
8.3.2.135hankel
8.3.2.136hankel_debye
8.3.2.137hankel_params
8.3.2.138hankel_uniform
8.3.2.139hankel_uniform_olver
8.3.2.140hankel_uniform_outer
8.3.2.141hankel_uniform_sum
8.3.2.142heuman_lambda
8.3.2.143hurwitz_zeta
8.3.2.144hurwitz_zeta
8.3.2.145hurwitz_zeta_euler_maclaurin
8.3.2.146hydrogen
8.3.2.147hyperg
8.3.2.148hyperg_luke
8.3.2.149hyperg_reflect
8.3.2.150hyperg_series
8.3.2.151jacobi_sncndn
8.3.2.152jacobi_zeta
8.3.2.153laguerre
8.3.2.154log_bincoef
8.3.2.155log_double_factorial
8.3.2.156log_double_factorial
8.3.2.157log_factorial

xviii CONTENTS

8.3.2.158 <u>log_gamma</u>	175
8.3.2.159log_gamma_bernoulli	175
8.3.2.160log_gamma_lanczos	175
8.3.2.161log_gamma_sign	176
8.3.2.162log_gamma_spouge	176
8.3.2.163log_pochhammer_l	177
8.3.2.164log_pochhammer_u	177
8.3.2.165logint	177
8.3.2.166owens_t	
8.3.2.167pochhammer_l	178
8.3.2.168pochhammer_u	
8.3.2.169poly_hermite	
8.3.2.170poly_hermite_asymp	
8.3.2.171poly_hermite_recursion	180
8.3.2.172poly_jacobi	
8.3.2.173poly_laguerre	
8.3.2.174poly_laguerre_hyperg	
8.3.2.175poly_laguerre_large_n	181
8.3.2.176poly_laguerre_recursion	182
8.3.2.177poly_legendre_p	182
8.3.2.178poly_legendre_q	183
8.3.2.179poly_radial_jacobi	183
8.3.2.180polylog	
8.3.2.181polylog	
8.3.2.182polylog_exp	
8.3.2.183polylog_exp_asymp	184
8.3.2.184polylog_exp_int_neg	185
8.3.2.185polylog_exp_int_neg	185
8.3.2.186polylog_exp_int_pos	
8.3.2.187polylog_exp_int_pos	
8.3.2.188polylog_exp_neg	186
8.3.2.189polylog_exp_neg	187
8.3.2.190polylog_exp_neg_even	187
8.3.2.191polylog_exp_neg_odd	188
8.3.2.192polylog_exp_negative_real_part	188
8.3.2.193polylog_exp_pos	189
8.3.2.194polylog_exp_pos	189

CONTENTS xix

8.3.2.195polylog_exp_pos
8.3.2.196polylog_exp_real_neg
8.3.2.197polylog_exp_real_neg
8.3.2.198polylog_exp_real_pos
8.3.2.199polylog_exp_real_pos
8.3.2.200psi
8.3.2.201psi
8.3.2.202psi_asymp
8.3.2.203psi_series
8.3.2.204riemann_zeta
8.3.2.205riemann_zeta_alt
8.3.2.206riemann_zeta_euler_maclaurin
8.3.2.207riemann_zeta_glob
8.3.2.208riemann_zeta_m_1
8.3.2.209riemann_zeta_m_1_sum
8.3.2.210riemann_zeta_product
8.3.2.211riemann_zeta_sum
8.3.2.212 <u>sinc</u>
8.3.2.213 <u>sinc</u>
8.3.2.214 <u>sinc</u> pi
8.3.2.215 <u>sincosint</u>
8.3.2.216sincosint_asymp
8.3.2.217sincosint_cont_frac
8.3.2.218sincosint_series
8.3.2.219 <u>sinhc</u>
8.3.2.220sinhc
8.3.2.221sinhc_pi
8.3.2.222 <u>sinhint</u>
8.3.2.223sph_bessel
8.3.2.224sph_bessel
8.3.2.225sph_bessel_ik
8.3.2.226sph_bessel_jn
8.3.2.227sph_hankel
8.3.2.228sph_hankel_1
8.3.2.229sph_hankel_1
8.3.2.230sph_hankel_2
8.3.2.231sph_hankel_2

XX CONTENTS

	8.3.2.232sph_harmonic
	8.3.2.233sph_legendre
	8.3.2.234sph_neumann
	8.3.2.235sph_neumann
	8.3.2.236student_t_cdf
	8.3.2.237student_t_cdfc
	8.3.2.238theta_1
	8.3.2.239theta_2
	8.3.2.240theta_2_asymp
	8.3.2.241theta_2_sum
	8.3.2.242theta_3
	8.3.2.243theta_3_asymp
	8.3.2.244theta_3_sum
	8.3.2.245theta_4
	8.3.2.246theta_c
	8.3.2.247theta_d
	8.3.2.248theta_n
	8.3.2.249theta_s
	8.3.2.250zernike
	8.3.2.251znorm1
	8.3.2.252znorm2
	8.3.2.253 evenzeta
8.3.3	Variable Documentation
	8.3.3.1 _Num_Euler_Maclaurin_zeta
	8.3.3.2 _S_double_factorial_table
	8.3.3.3 _S_Euler_Maclaurin_zeta
	8.3.3.4 _S_factorial_table
	8.3.3.5 _S_neg_double_factorial_table
	8.3.3.6 _S_num_double_factorials
	8.3.3.7 _S_num_double_factorials< double >
	8.3.3.8 _S_num_double_factorials< float >
	8.3.3.9 _S_num_double_factorials< long double >
	8.3.3.10 _S_num_factorials
	8.3.3.11 _S_num_factorials< double >
	8.3.3.12 _S_num_factorials< float >
	8.3.3.13 _S_num_factorials< long double >
	8.3.3.14 _S_num_neg_double_factorials

CONTENTS xxi

			8.3.3.15 _S_num_neg_double_factorials< double >	. 208
			8.3.3.16 _S_num_neg_double_factorials< float >	. 209
			8.3.3.17 _S_num_neg_double_factorials< long double >	. 209
			8.3.3.18 _S_num_zetam1	. 209
			8.3.3.19 _S_zetam1	. 209
9	Clas	e Docu	mentation	211
3	9.1		detail::_Factorial_table< _Tp > Struct Template Reference	
	0.1	9.1.1	Detailed Description	
		9.1.2	Member Data Documentation	
		3.1.2	9.1.2.1 factorial	
			9.1.2.2 log factorial	
			9.1.2.3 n	
			9.1.2.511	. 211
10	File I	Docum	entation	213
	10.1	bits/sf_	_airy.tcc File Reference	. 213
		10.1.1	Detailed Description	. 215
		10.1.2	Macro Definition Documentation	. 215
			10.1.2.1 _GLIBCXX_BITS_SF_AIRY_TCC	. 215
	10.2	bits/sf_	_bessel.tcc File Reference	. 215
		10.2.1	Detailed Description	. 217
		10.2.2	Macro Definition Documentation	. 217
			10.2.2.1 _GLIBCXX_BITS_SF_BESSEL_TCC	. 217
	10.3	bits/sf_	_beta.tcc File Reference	. 217
		10.3.1	Detailed Description	. 219
		10.3.2	Macro Definition Documentation	. 219
			10.3.2.1 _GLIBCXX_BITS_SF_BETA_TCC	. 219
	10.4	bits/sf_	_cardinal.tcc File Reference	. 219
		10.4.1	Macro Definition Documentation	. 221
			10.4.1.1 _GLIBCXX_BITS_SF_CARDINAL_TCC	. 221
	10.5	bits/sf_	_chebyshev.tcc File Reference	. 221
		10.5.1	Detailed Description	. 222
		10.5.2	Macro Definition Documentation	. 222
			10.5.2.1 _GLIBCXX_SF_CHEBYSHEV_TCC	. 222
	10.6	bits/sf_	_dawson.tcc File Reference	. 223
		10.6.1	Detailed Description	. 224
		10.6.2	Macro Definition Documentation	. 224
			10.6.2.1 _GLIBCXX_SF_DAWSON_TCC	. 224

xxii CONTENTS

10.7 bits/sf_ellint.tcc File Reference
10.7.1 Detailed Description
10.7.2 Macro Definition Documentation
10.7.2.1 _GLIBCXX_BITS_SF_ELLINT_TCC
10.8 bits/sf_expint.tcc File Reference
10.8.1 Detailed Description
10.8.2 Macro Definition Documentation
10.8.2.1 _GLIBCXX_BITS_SF_EXPINT_TCC
10.9 bits/sf_fresnel.tcc File Reference
10.9.1 Detailed Description
10.9.2 Macro Definition Documentation
10.9.2.1 _GLIBCXX_SF_FRESNEL_TCC
10.10bits/sf_gamma.tcc File Reference
10.10.1 Detailed Description
10.10.2 Macro Definition Documentation
10.10.2.1 _GLIBCXX_BITS_SF_GAMMA_TCC
10.11bits/sf_gegenbauer.tcc File Reference
10.11.1 Detailed Description
10.11.2 Macro Definition Documentation
10.11.2.1 _GLIBCXX_SF_GEGENBAUER_TCC
10.12bits/sf_hankel.tcc File Reference
10.12.1 Detailed Description
10.12.2 Macro Definition Documentation
10.12.2.1 _GLIBCXX_BITS_SF_HANKEL_TCC
10.13bits/sf_hankel_new.tcc File Reference
10.13.1 Macro Definition Documentation
10.13.1.1 _GLIBCXX_BITS_SF_HANKEL_NEW_TCC
10.14bits/sf_hermite.tcc File Reference
10.14.1 Detailed Description
10.14.2 Macro Definition Documentation
10.14.2.1 _GLIBCXX_BITS_SF_HERMITE_TCC
10.15bits/sf_hydrogen.tcc File Reference
10.15.1 Detailed Description
10.15.2 Macro Definition Documentation
10.15.2.1 _GLIBCXX_BITS_SF_HYDROGEN_TCC
10.16bits/sf_hyperg.tcc File Reference
10.16.1 Detailed Description

CONTENTS xxiii

10.16.2 Macro Definition Documentation	. 245
10.16.2.1 _GLIBCXX_BITS_SF_HYPERG_TCC	. 245
10.17bits/sf_hypint.tcc File Reference	. 245
10.17.1 Detailed Description	. 246
10.17.2 Macro Definition Documentation	. 246
10.17.2.1 _GLIBCXX_SF_HYPINT_TCC	. 246
10.18bits/sf_jacobi.tcc File Reference	. 247
10.18.1 Detailed Description	. 248
10.18.2 Macro Definition Documentation	. 248
10.18.2.1 _GLIBCXX_SF_JACOBI_TCC	. 248
10.19bits/sf_laguerre.tcc File Reference	. 248
10.19.1 Detailed Description	. 250
10.19.2 Macro Definition Documentation	. 250
10.19.2.1 _GLIBCXX_BITS_SF_LAGUERRE_TCC	. 250
10.20bits/sf_legendre.tcc File Reference	. 250
10.20.1 Detailed Description	. 251
10.20.2 Macro Definition Documentation	. 251
10.20.2.1 _GLIBCXX_BITS_SF_LEGENDRE_TCC	. 251
10.21bits/sf_mod_bessel.tcc File Reference	. 251
10.21.1 Detailed Description	. 253
10.21.2 Macro Definition Documentation	. 253
10.21.2.1 _GLIBCXX_BITS_SF_MOD_BESSEL_TCC	. 253
10.22bits/sf_owens_t.tcc File Reference	. 254
10.22.1 Detailed Description	. 254
10.22.2 Macro Definition Documentation	. 254
10.22.2.1 _GLIBCXX_BITS_SF_OWENS_T_TCC	. 254
10.23bits/sf_polylog.tcc File Reference	. 255
10.23.1 Detailed Description	. 257
10.23.2 Macro Definition Documentation	. 257
10.23.2.1 _GLIBCXX_BITS_SF_POLYLOG_TCC	. 257
10.24bits/sf_theta.tcc File Reference	. 257
10.24.1 Detailed Description	. 259
10.24.2 Macro Definition Documentation	. 259
10.24.2.1 _GLIBCXX_SF_THETA_TCC	. 259
10.25bits/sf_trigint.tcc File Reference	. 260
10.25.1 Detailed Description	. 261
10.25.2 Macro Definition Documentation	. 261

xxiv CONTENTS

Index	(275
	10.27.2.2STDCPP_MATH_SPEC_FUNCS	274
	10.27.2.1cpp_lib_math_special_functions	274
	10.27.2 Macro Definition Documentation	274
	10.27.1 Detailed Description	274
10	0.27bits/specfun.h File Reference	264
	10.26.2.1 _GLIBCXX_BITS_SF_ZETA_TCC	263
	10.26.2 Macro Definition Documentation	263
	10.26.1 Detailed Description	263
10	0.26bits/sf_zeta.tcc File Reference	261
	10.25.2.1 _GLIBCXX_SF_TRIGINT_TCC	261

Mathematical Special Functions

1.1 Introduction and History

The first significant library upgrade on the road to C++2011, TR1, included a set of 23 mathematical functions that significantly extended the standard trancendental functions inherited from C and declared in <cmath>.

Although most components from TR1 were eventually adopted for C++11 these math function were left behind out of concern for implementability. The math functions were published as a separate international standard IS 29124 - Extensions to the C++ Library to Support Mathematical Special Functions.

For C++17 these functions were incorporated into the main standard.

1.2 Contents

The following functions are implemented in namespace std:

- assoc_laguerre Associated Laguerre functions
- · assoc legendre Associated Legendre functions
- beta Beta functions
- comp_ellint_1 Complete elliptic functions of the first kind
- · comp ellint 2 Complete elliptic functions of the second kind
- comp_ellint_3 Complete elliptic functions of the third kind
- · cyl bessel i Regular modified cylindrical Bessel functions
- cyl_bessel_j Cylindrical Bessel functions of the first kind
- cyl_bessel_k Irregular modified cylindrical Bessel functions
- · cyl neumann Cylindrical Neumann functions or Cylindrical Bessel functions of the second kind
- ellint_1 Incomplete elliptic functions of the first kind
- ellint_2 Incomplete elliptic functions of the second kind
- · ellint 3 Incomplete elliptic functions of the third kind

- · expint The exponential integral
- · hermite Hermite polynomials
- · laguerre Laguerre functions
- · legendre Legendre polynomials
- · riemann zeta The Riemann zeta function
- · sph bessel Spherical Bessel functions
- · sph legendre Spherical Legendre functions
- · sph neumann Spherical Neumann functions

The hypergeometric functions were stricken from the TR29124 and C++17 versions of this math library because of implementation concerns. However, since they were in the TR1 version and since they are popular we kept them as an extension in namespace __gnu_cxx:

- conf_hyperg Confluent hypergeometric functions
- hyperg Hypergeometric functions

1.3 General Features

1.3.1 Argument Promotion

The arguments suppled to the non-suffixed functions will be promoted according to the following rules:

- 1. If any argument intended to be floating opint is given an integral value That integral value is promoted to double.
- 2. All floating point arguments are promoted up to the largest floating point precision among them.

1.3.2 NaN Arguments

If any of the floating point arguments supplied to these functions is invalid or NaN (std::numeric_limits<Tp>::quiet_ \(\to \) NaN), the value NaN is returned.

1.4 Implementation

We strive to implement the underlying math with type generic algorithms to the greatest extent possible. In practice, the function are thin wrappers that dispatch to function templates. Type dependence is controlled with std::numeric_limits and functions thereof.

We don't promote *c float to *c double or *c double to long double reflexively. The goal is for float functions to operate more quickly, at the cost of float accuracy and possibly a smaller domain of validity. Similarly, long double should give you more dynamic range and slightly more pecision than double on many systems.

1.5 Testing 3

1.5 Testing

These functions have been tested against equivalent implementations from the Gnu Scientific Library, GSL and Boost and the ratio

 $\frac{|f - f_{test}|}{|f_{test}|}$

is generally found to be within 10^-15 for 64-bit double on linux-x86_64 systems over most of the ranges of validity.

Todo Provide accuracy comparisons on a per-function basis for a small number of targets.

1.6 General Bibliography

See also

Abramowitz and Stegun: Handbook of Mathematical Functions, with Formulas, Graphs, and Mathematical Tables Edited by Milton Abramowitz and Irene A. Stegun, National Bureau of Standards Applied Mathematics Series - 55 Issued June 1964, Tenth Printing, December 1972, with corrections Electronic versions of A&S abound including both pdf and navigable html.

for example http://people.math.sfu.ca/~cbm/aands/

The old A&S has been redone as the NIST Digital Library of Mathematical Functions: http://dlmf.nist. ← gov/ This version is far more navigable and includes more recent work.

An Atlas of Functions: with Equator, the Atlas Function Calculator 2nd Edition, by Oldham, Keith B., Myland, Jan, Spanier, Jerome

Asymptotics and Special Functions by Frank W. J. Olver, Academic Press, 1974

Numerical Recipes in C, The Art of Scientific Computing, by William H. Press, Second Ed., Saul A. Teukolsky, William T. Vetterling, and Brian P. Flannery, Cambridge University Press, 1992

The Special Functions and Their Approximations: Volumes 1 and 2, by Yudell L. Luke, Academic Press, 1969

N	lathen	natical	Speci	ial Fur	nctions

Todo List

```
page Mathematical Special Functions

Provide accuracy comparisons on a per-function basis for a small number of targets.

Member std::__detail::__dawson_const_frac (_Tp __x)
    this needs some compile-time construction!

Member std::__detail::__expint_E1 (_Tp __x)
    Find a good asymptotic switch point in E_1(x).

Member std::__detail::__expint_En_recursion (unsigned int __n, _Tp __x)
    Find a principled starting number for the E_n(x) downward recursion.

Member std::__detail::__hurwitz_zeta (_Tp __s, std::complex < _Tp > __a)
    This __hurwitz_zeta prefactor is prone to overflow. positive integer orders s?
```

6	Todo List

Module Index

3.1 Modules

Here	19 2	a liet	ot a	ll mor	29111

C++ Mathematical Special Functions	 											15
C++17/IS29124 Mathematical Special Functions	 											16
GNU Extended Mathematical Special Functions	 											38

8	Module Index

Namespace Index

4.1	Namespace	List
-----	------------------	------

Here is a list of all namespaces with brief descriptions:

gn	u_cxx		 							 												 . 9)!
std .			 							 												 . 10);
std::	detail		 							 												 10)!

10	Namespace Index

Class Index

E 4	O I	1.2 - 4
5.1	Class	I ICT
J. I	Viass	LISL

Here are the classes, structs, unions and ir $% \left(1\right) =\left(1\right) \left(1\right) \left$	terfaces with brief descriptions:	
std::detail::_Factorial_table< _Tp >		21

12	Class Index

Chapter 6

File Index

6.1 File List

Here is a list of all files with brief descriptions:

bits/sf_airy.tcc
bits/sf_bessel.tcc
bits/sf_beta.tcc
bits/sf_cardinal.tcc
bits/sf_chebyshev.tcc
bits/sf_dawson.tcc
bits/sf_ellint.tcc
bits/sf_expint.tcc
bits/sf_fresnel.tcc
bits/sf_gamma.tcc
bits/sf_gegenbauer.tcc
bits/sf_hankel.tcc
bits/sf_hankel_new.tcc
bits/sf_hermite.tcc
bits/sf_hydrogen.tcc
bits/sf_hyperg.tcc
bits/sf_hypint.tcc
bits/sf_jacobi.tcc
bits/sf_laguerre.tcc
bits/sf_legendre.tcc
bits/sf_mod_bessel.tcc
bits/sf_owens_t.tcc
bits/sf_polylog.tcc
bits/sf_theta.tcc
bits/sf_trigint.tcc
bits/sf_zeta.tcc
bits/specfun.h

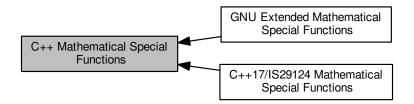
14	File Index

Chapter 7

Module Documentation

7.1 C++ Mathematical Special Functions

Collaboration diagram for C++ Mathematical Special Functions:



Modules

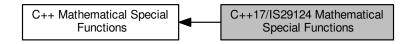
- C++17/IS29124 Mathematical Special Functions
- GNU Extended Mathematical Special Functions

7.1.1 Detailed Description

A collection of advanced mathematical special functions.

7.2 C++17/IS29124 Mathematical Special Functions

Collaboration diagram for C++17/IS29124 Mathematical Special Functions:



Functions

```
template<typename</li>Tp >
  __gnu_cxx::__promote< _Tp >::__type std::assoc_laguerre (unsigned int __n, unsigned int __m, _Tp __x)
• float std::assoc_laguerref (unsigned int __n, unsigned int __m, float __x)
• long double std::assoc laguerrel (unsigned int n, unsigned int m, long double x)
template<typename _Tp >
   _gnu_cxx::__promote< _Tp >::__type std::assoc_legendre (unsigned int __l, unsigned int __m, _Tp __x)

    float std::assoc legendref (unsigned int I, unsigned int m, float x)

    long double std::assoc_legendrel (unsigned int __l, unsigned int __m, long double __x)

template<typename _Tpa , typename _Tpb >
    gnu cxx:: promote 2< Tpa, Tpb >:: type std::beta ( Tpa a, Tpb b)

    float std::betaf (float a, float b)

    long double std::betal (long double __a, long double __b)

template<typename _Tp >
   __gnu_cxx::__promote< _Tp >::__type std::comp_ellint_1 (_Tp __k)

    float std::comp_ellint_1f (float __k)

    long double std::comp ellint 1l (long double k)

• template<typename _{\rm Tp}>
    _gnu_cxx::__promote< _Tp >::__type std::comp_ellint_2 (_Tp __k)

    float std::comp ellint 2f (float k)

    long double std::comp ellint 2l (long double k)

• template<typename Tp, typename Tpn >
    _gnu_cxx::__promote_2< _Tp, _Tpn >::__type std::comp_ellint_3 (_Tp __k, _Tpn __nu)

    float std::comp_ellint_3f (float __k, float __nu)

      Return the complete elliptic integral of the third kind \Pi(k,\nu) for float modulus k.

    long double std::comp_ellint_3l (long double __k, long double __nu)

      Return the complete elliptic integral of the third kind \Pi(k,\nu) for long double modulus k.
• template<typename _Tpnu , typename _Tp >
    _gnu_cxx::__promote_2< _Tpnu, _Tp >::__type std::cyl_bessel_i (_Tpnu __nu, _Tp __x)

    float std::cyl_bessel_if (float __nu, float __x)

    long double std::cyl bessel il (long double nu, long double x)

• template<typename _Tpnu , typename _Tp >
   __gnu_cxx::__promote_2< _Tpnu, _Tp >::__type std::cyl_bessel_j (_Tpnu __nu, _Tp __x)

    float std::cyl_bessel_if (float __nu, float __x)

• long double std::cyl_bessel_jl (long double __nu, long double __x)
```

```
• template<typename _Tpnu , typename _Tp >
   _gnu_cxx::__promote_2< _Tpnu, _Tp >::__type std::cyl_bessel_k (_Tpnu __nu, _Tp __x)

    float std::cyl bessel kf (float nu, float x)

    long double std::cyl_bessel_kl (long double __nu, long double __x)

• template<typename Tpnu, typename Tp >
    _gnu_cxx::__promote_2< _Tpnu, _Tp >::__type std::cyl_neumann (_Tpnu __nu, _Tp __x)

    float std::cyl neumannf (float nu, float x)

    long double std::cyl_neumannl (long double __nu, long double __x)

    template<typename</li>
    Tp , typename
    Tpp >

   _gnu_cxx::__promote_2< _Tp, _Tpp >::__type std::ellint_1 (_Tp __k, _Tpp __phi)

    float std::ellint_1f (float __k, float __phi)

    long double std::ellint 11 (long double k, long double phi)

    template<typename _Tp , typename _Tpp >

    _gnu_cxx::__promote_2< _Tp, _Tpp >::__type std::ellint_2 (_Tp __k, _Tpp __phi)

    float std::ellint 2f (float k, float phi)

      Return the incomplete elliptic integral of the second kind E(k,\phi) for float argument.

    long double std::ellint_2l (long double __k, long double __phi)

      Return the incomplete elliptic integral of the second kind E(k, \phi).

    template<typename _Tp , typename _Tpn , typename _Tpp >

   _gnu_cxx::_promote_3< _Tp, _Tpn, _Tpp >::_type std::ellint_3 (_Tp __k, _Tpn __nu, _Tpp __phi)
      Return the incomplete elliptic integral of the third kind \Pi(k, \nu, \phi).

    float std::ellint_3f (float __k, float __nu, float __phi)

      Return the incomplete elliptic integral of the third kind \Pi(k, \nu, \phi) for float argument.
• long double std::ellint 3l (long double k, long double nu, long double phi)
      Return the incomplete elliptic integral of the third kind \Pi(k, \nu, \phi).

    template<typename</li>
    Tp >

   _gnu_cxx::__promote< _Tp >::__type std::expint (_Tp __x)

    float std::expintf (float __x)

    long double std::expintl (long double x)

template<typename</li>Tp >
   _gnu_cxx::__promote< _Tp >::__type std::hermite (unsigned int __n, _Tp __x)

    float std::hermitef (unsigned int __n, float __x)

    long double std::hermitel (unsigned int n, long double x)

template<typename _Tp >
    _gnu_cxx::__promote< _Tp >::__type std::laguerre (unsigned int __n, _Tp __x)

    float std::laguerref (unsigned int n, float x)

    long double std::laguerrel (unsigned int __n, long double __x)

• template<typename _Tp >
    _gnu_cxx::__promote< _Tp >::__type std::legendre (unsigned int __l, _Tp __x)

    float std::legendref (unsigned int I, float x)

    long double std::legendrel (unsigned int __I, long double __x)

template<typename _Tp >
    gnu cxx:: promote < Tp >:: type std::riemann zeta ( Tp s)

    float std::riemann_zetaf (float __s)

    long double std::riemann zetal (long double s)

template<typename _Tp >
    gnu cxx:: promote < Tp >:: type std::sph bessel (unsigned int n, Tp x)

    float std::sph besself (unsigned int n, float x)

    long double std::sph_bessell (unsigned int __n, long double __x)

template<typename _Tp >
   gnu cxx:: promote < Tp >:: type std::sph legendre (unsigned int I, unsigned int m, Tp theta)
```

- float std::sph_legendref (unsigned int __l, unsigned int __m, float __theta)
- long double std::sph_legendrel (unsigned int __l, unsigned int __m, long double __theta)
- template<typename _Tp >
 __gnu_cxx::__promote< _Tp >::__type std::sph_neumann (unsigned int __n, _Tp __x)
- float std::sph neumannf (unsigned int n, float x)
- long double std::sph_neumannl (unsigned int __n, long double __x)

7.2.1 Detailed Description

A collection of advanced mathematical special functions for C++17 and IS29124.

7.2.2 Function Documentation

7.2.2.1 template<typename _Tp > __gnu_cxx::__promote<_Tp>::__type std::assoc_laguerre (unsigned int __n, unsigned int __n, _Tp __x) [inline]

Return the associated Laguerre polynomial of nonnegative order n, nonnegative degree m and real argument $x \leftarrow L_n^m(x)$.

The associated Laguerre function of real degree α , $L_n^{\alpha}(x)$, is defined by

$$L_n^{\alpha}(x) = \frac{(\alpha+1)_n}{n!} {}_1F_1(-n;\alpha+1;x)$$

where $(\alpha)_n$ is the Pochhammer symbol and ${}_1F_1(a;c;x)$ is the confluent hypergeometric function.

The associated Laguerre polynomial is defined for integral degree $\alpha=m$ by:

$$L_n^m(x) = (-1)^m \frac{d^m}{dx^m} L_{n+m}(x)$$

where the Laguerre polynomial is defined by:

$$L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n} (x^n e^{-x})$$

and x >= 0.

See also

laguerre for details of the Laguerre function of degree n

Template Parameters

_Tp	The floating-point type of the argumentx.

Parameters

n	The order of the Laguerre function, $\underline{\hspace{1cm}} n >= 0$.
m	The degree of the Laguerre function, $\underline{\hspace{1cm}}$ $m >= 0$.
X	The argument of the Laguerre function, $\underline{}x >= 0$.

Exceptions

Definition at line 299 of file specfun.h.

7.2.2.2 float std::assoc_laguerref (unsigned int __n, unsigned int __m, float __x) [inline]

Return the associated Laguerre polynomial of order n, degree ${\tt m}\colon L^m_n(x)$ for float argument.

See also

assoc_laguerre for more details.

Definition at line 253 of file specfun.h.

7.2.2.3 long double std::assoc_laguerrel (unsigned int _n, unsigned int _m, long double _x) [inline]

Return the associated Laguerre polynomial of order n, degree m: $L_n^m(x)$.

See also

assoc_laguerre for more details.

Definition at line 263 of file specfun.h.

7.2.2.4 template<typename _Tp > __gnu_cxx::__promote<_Tp>::__type std::assoc_legendre (unsigned int __I, unsigned int _

Return the associated Legendre function of degree 1 and order m.

The associated Legendre function is derived from the Legendre function $P_l(x)$ by the Rodrigues formula:

$$P_l^m(x) = (1 - x^2)^{m/2} \frac{d^m}{dx^m} P_l(x)$$

See also

legendre for details of the Legendre function of degree 1

Template Parameters

_Tp The floating-point type of the argument _	_X.
---	-----

Parameters

	1	The degree1 >= 0.
	m	The order $_{m} <= 1$.
Ī	X	The argument, abs (x) <= 1.

Exceptions

std::domain_error	if $abs(\underline{x}) > 1$.

Definition at line 344 of file specfun.h.

7.2.2.5 float std::assoc_legendref (unsigned int __l, unsigned int __m, float __x) [inline]

Return the associated Legendre function of degree 1 and order m for float argument.

See also

assoc_legendre for more details.

Definition at line 314 of file specfun.h.

7.2.2.6 long double std::assoc_legendrel (unsigned int __I, unsigned int __I, long double __x) [inline]

Return the associated Legendre function of degree $\ensuremath{\mathbb{1}}$ and order $\ensuremath{\mathbb{m}}$.

See also

assoc_legendre for more details.

Definition at line 323 of file specfun.h.

7.2.2.7 template<typename _Tpa , typename _Tpb > __gnu_cxx::__promote_2<_Tpa, _Tpb>::__type std::beta (_Tpa __a, _Tpb ___b) [inline]

Return the beta function, B(a, b), for real parameters a, b.

The beta function is defined by

$$B(a,b) = \int_0^1 t^{a-1} (1-t)^{b-1} dt = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

where x > 0 and y > 0

Template Parameters

_Тра	The floating-point type of the parametera.
_Tpb	The floating-point type of the parameterb.

Parameters

a	The first argument of the beta function, $\underline{} = 0$.
b	The second argument of the beta function, $_$ b > 0 .

Exceptions

Definition at line 389 of file specfun.h.

7.2.2.8 float std::betaf (float _a, float _b) [inline]

Return the beta function, B(a,b), for float parameters a, b.

See also

beta for more details.

Definition at line 358 of file specfun.h.

7.2.2.9 long double std::betal (long double _a, long double _b) [inline]

Return the beta function, B(a, b), for long double parameters a, b.

See also

beta for more details.

Definition at line 368 of file specfun.h.

7.2.2.10 template<typename_Tp>__gnu_cxx:__promote<_Tp>::_type std::comp_ellint_1(_Tp _k) [inline]

Return the complete elliptic integral of the first kind K(k) for real modulus ${\bf k}$.

The complete elliptic integral of the first kind is defined as

$$K(k) = F(k, \pi/2) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - k^2 sin^2 \theta}}$$

where $F(k,\phi)$ is the incomplete elliptic integral of the first kind and the modulus |k|<=1.

See also

ellint_1 for details of the incomplete elliptic function of the first kind.

Template Parameters

_ <i>Tp</i>	The floating-point type of the modulusk.

Parameters

$$\underline{\hspace{0.5cm}}$$
 The modulus, abs ($\underline{\hspace{0.5cm}}$ k) <= 1

Exceptions

Definition at line 437 of file specfun.h.

7.2.2.11 float std::comp_ellint_1f(float __k) [inline]

Return the complete elliptic integral of the first kind E(k) for float modulus k.

See also

comp_ellint_1 for details.

Definition at line 404 of file specfun.h.

7.2.2.12 long double std::comp_ellint_1I(long double __k) [inline]

Return the complete elliptic integral of the first kind E(k) for long double modulus k.

See also

comp ellint 1 for details.

Definition at line 414 of file specfun.h.

Return the complete elliptic integral of the second kind E(k) for real modulus k.

The complete elliptic integral of the second kind is defined as

$$E(k) = E(k, \pi/2) = \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 \theta}$$

where $E(k,\phi)$ is the incomplete elliptic integral of the second kind and the modulus |k| <= 1.

See also

ellint_2 for details of the incomplete elliptic function of the second kind.

Template Parameters

_ <i>Tp</i>	The floating-point type of the modulusk.

Parameters

```
\underline{\hspace{0.5cm}} The modulus, abs (\underline{\hspace{0.5cm}} k) <= 1
```

Exceptions

```
std::domain_error | if abs (__k) > 1.
```

Definition at line 484 of file specfun.h.

7.2.2.14 float std::comp_ellint_2f (float __k) [inline]

Return the complete elliptic integral of the second kind E(k) for float modulus k.

See also

comp_ellint_2 for details.

Definition at line 452 of file specfun.h.

7.2.2.15 long double std::comp_ellint_2l (long double __k) [inline]

Return the complete elliptic integral of the second kind E(k) for long double modulus k.

See also

comp ellint 2 for details.

Definition at line 462 of file specfun.h.

Return the complete elliptic integral of the third kind $\Pi(k,\nu)=\Pi(k,\nu,\pi/2)$ for real modulus k.

The complete elliptic integral of the third kind is defined as

$$\Pi(k,\nu) = \Pi(k,\nu,\pi/2) = \int_0^{\pi/2} \frac{d\theta}{(1-\nu\sin^2\theta)\sqrt{1-k^2\sin^2\theta}}$$

where $\Pi(k,\nu,\phi)$ is the incomplete elliptic integral of the second kind and the modulus |k|<=1.

See also

ellint_3 for details of the incomplete elliptic function of the third kind.

Template Parameters

_Tp	The floating-point type of the modulusk.
_Tpn	The floating-point type of the argumentnu.

Parameters

k	The modulus, abs $(\underline{}k) \le 1$
nu	The argument

Exceptions

Definition at line 535 of file specfun.h.

7.2.2.17 float std::comp_ellint_3f (float __k, float __nu) [inline]

Return the complete elliptic integral of the third kind $\Pi(k,\nu)$ for float modulus k.

See also

comp_ellint_3 for details.

Definition at line 499 of file specfun.h.

7.2.2.18 long double std::comp_ellint_3l(long double __k, long double __nu) [inline]

Return the complete elliptic integral of the third kind $\Pi(k,\nu)$ for long double modulus k.

See also

comp_ellint_3 for details.

Definition at line 509 of file specfun.h.

7.2.2.19 template<typename _Tpnu , typename _Tp > __gnu_cxx::__promote_2<_Tpnu, _Tp>::__type std::cyl_bessel_i (_Tpnu __nu, _Tp __x) [inline]

Return the regular modified Bessel function $I_{\nu}(x)$ for real order ν and argument x >= 0.

The regular modified cylindrical Bessel function is:

$$I_{\nu}(x) = i^{-\nu} J_{\nu}(ix) = \sum_{k=0}^{\infty} \frac{(x/2)^{\nu+2k}}{k! \Gamma(\nu+k+1)}$$

Template Parameters

_Tpnu	The floating-point type of the ordernu.
_ <i>Tp</i>	The floating-point type of the argumentx.

Parameters

nu	The order
x	The argument, $\underline{} x >= 0$

Exceptions

std::domain_error	$ \text{ if } \underline{x} < 0 .$

Definition at line 581 of file specfun.h.

7.2.2.20 float std::cyl_bessel_if (float __nu, float __x) [inline]

Return the regular modified Bessel function $I_{\nu}(x)$ for float order ν and argument x>=0.

See also

cyl_bessel_i for setails.

Definition at line 550 of file specfun.h.

7.2.2.21 long double std::cyl_bessel_il (long double __nu, long double __x) [inline]

Return the regular modified Bessel function $I_{\nu}(x)$ for long double order ν and argument x >= 0.

See also

cyl_bessel_i for setails.

Definition at line 560 of file specfun.h.

Return the Bessel function $J_{\nu}(x)$ of real order ν and argument x >= 0.

The cylindrical Bessel function is:

$$J_{\nu}(x) = \sum_{k=0}^{\infty} \frac{(-1)^k (x/2)^{\nu+2k}}{k! \Gamma(\nu+k+1)}$$

Template Parameters

_Tpnu	The floating-point type of the ordernu.
_ <i>Tp</i>	The floating-point type of the argumentx.

Parameters

nu	The order
X	The argument, $\underline{}x >= 0$

Exceptions

std::domain_error	if v < 0
stadomaii_enti	" <u></u>

Definition at line 627 of file specfun.h.

7.2.2.23 float std::cyl_bessel_jf (float __nu, float __x) [inline]

Return the Bessel function of the first kind $J_{\nu}(x)$ for float order ν and argument x>=0.

See also

cyl_bessel_j for setails.

Definition at line 596 of file specfun.h.

7.2.2.24 long double std::cyl_bessel_il (long double __nu, long double __x) [inline]

Return the Bessel function of the first kind $J_{\nu}(x)$ for long double order ν and argument x>=0.

See also

cyl_bessel_j for setails.

Definition at line 606 of file specfun.h.

7.2.2.25 template<typename _Tpnu , typename _Tp > __gnu_cxx::__promote_2<_Tpnu, _Tp>::__type std::cyl_bessel_k (_Tpnu __nu, _Tp __x) [inline]

Return the irregular modified Bessel function $K_{\nu}(x)$ of real order ν and argument x.

The irregular modified Bessel function is defined by:

$$K_{\nu}(x) = \frac{\pi}{2} \frac{I_{-\nu}(x) - I_{\nu}(x)}{\sin \nu \pi}$$

where for integral $\nu=n$ a limit is taken: $lim_{\nu\to n}$. For negative argument we have simply:

$$K_{-\nu}(x) = K_{\nu}(x)$$

Template Parameters

_Tpnu	The floating-point type of the ordernu.
_ <i>Tp</i>	The floating-point type of the argumentx.

Parameters

nu	The order
X	The argument, $\underline{} x >= 0$

Exceptions

std::domain_error	$if \underline{x} < 0.$

Definition at line 679 of file specfun.h.

7.2.2.26 float std::cyl_bessel_kf (float __nu, float __x) [inline]

Return the irregular modified Bessel function $K_{\nu}(x)$ for float order ν and argument x>=0.

See also

cyl_bessel_k for setails.

Definition at line 642 of file specfun.h.

7.2.2.27 long double std::cyl_bessel_kl (long double __nu, long double __x) [inline]

Return the irregular modified Bessel function $K_{\nu}(x)$ for long double order ν and argument x>=0.

See also

cyl_bessel_k for setails.

Definition at line 652 of file specfun.h.

7.2.2.28 template<typename _Tpnu , typename _Tp > __gnu_cxx::__promote_2<_Tpnu, _Tp>::__type std::cyl_neumann (_Tpnu __nu, _Tp __x) [inline]

Return the Neumann function $N_{\nu}(x)$ of real order ν and argument x >= 0.

The Neumann function is defined by:

$$N_{\nu}(x) = \frac{J_{\nu}(x)\cos\nu\pi - J_{-\nu}(x)}{\sin\nu\pi}$$

where x >= 0 and for integral order $\nu = n$ a limit is taken: $\lim_{\nu \to n} d\nu$

Template Parameters

_Tpnu	The floating-point type of the ordernu.
_ <i>Tp</i>	The floating-point type of the argumentx.

Parameters

nu	The order
X	The argument, $\underline{}x >= 0$

Exceptions

```
std::domain_error | if __x < 0 .
```

Definition at line 727 of file specfun.h.

7.2.2.29 float std::cyl_neumannf (float __nu, float __x) [inline]

Return the Neumann function $N_{\nu}(x)$ of float order ν and argument x.

See also

cyl_neumann for setails.

Definition at line 694 of file specfun.h.

7.2.2.30 long double std::cyl_neumannl(long double __nu, long double __x) [inline]

Return the Neumann function $N_{\nu}(x)$ of long double order ν and argument x.

See also

cyl_neumann for setails.

Definition at line 704 of file specfun.h.

7.2.2.31 template<typename_Tp , typename_Tpp > __gnu_cxx::__promote_2<_Tp, _Tpp>::__type std::ellint_1 (_Tp __k, _Tpp __phi) [inline]

Return the incomplete elliptic integral of the first kind $F(k,\phi)$ for real modulus k and angle ϕ .

The incomplete elliptic integral of the first kind is defined as

$$F(k,\phi) = \int_0^{\phi} \frac{d\theta}{\sqrt{1 - k^2 sin^2 \theta}}$$

For $\phi = \pi/2$ this becomes the complete elliptic integral of the first kind, K(k).

See also

comp ellint 1.

Template Parameters

_Tp	The floating-point type of the modulusk.

_ <i>Tpp</i>	The floating-point type of the anglephi.

Parameters

k	The modulus, abs (k) <= 1
phi	The integral limit argument in radians

Exceptions

```
std::domain_error | if abs (__k) > 1 .
```

Definition at line 775 of file specfun.h.

Return the incomplete elliptic integral of the first kind $E(k,\phi)$ for float modulus k and angle ϕ .

See also

ellint 1 for details.

Definition at line 742 of file specfun.h.

7.2.2.33 long double std::ellint_11 (long double __k, long double __phi) [inline]

Return the incomplete elliptic integral of the first kind $E(k,\phi)$ for long double modulus k and angle ϕ .

See also

ellint 1 for details.

Definition at line 752 of file specfun.h.

Return the incomplete elliptic integral of the second kind $E(k, \phi)$.

The incomplete elliptic integral of the second kind is defined as

$$E(k,\phi) = \int_0^\phi \sqrt{1 - k^2 sin^2 \theta}$$

For $\phi = \pi/2$ this becomes the complete elliptic integral of the second kind, E(k).

See also

comp_ellint_2.

Template Parameters

_ <i>Tp</i>	The floating-point type of the modulusk.
_Трр	The floating-point type of the anglephi.

Parameters

k	The modulus, abs (k) <= 1
phi	The integral limit argument in radians

Returns

The elliptic function of the second kind.

Exceptions

$$std::domain_error \mid if abs(__k) > 1$$
.

Definition at line 823 of file specfun.h.

Return the incomplete elliptic integral of the second kind $E(k,\phi)$ for float argument.

See also

ellint_2 for details.

Definition at line 790 of file specfun.h.

Return the incomplete elliptic integral of the second kind $E(k, \phi)$.

See also

ellint_2 for details.

Definition at line 800 of file specfun.h.

Return the incomplete elliptic integral of the third kind $\Pi(k, \nu, \phi)$.

The incomplete elliptic integral of the third kind is defined by:

$$\Pi(k,\nu,\phi) = \int_0^\phi \frac{d\theta}{(1-\nu\sin^2\theta)\sqrt{1-k^2\sin^2\theta}}$$

For $\phi=\pi/2$ this becomes the complete elliptic integral of the third kind, $\Pi(k,\nu)$.

See also

comp_ellint_3.

Template Parameters

_Tp	The floating-point type of the modulusk.
_Tpn	The floating-point type of the argumentnu.
_ <i>Tpp</i>	The floating-point type of the anglephi.

Parameters

k	The modulus, abs (k) <= 1
nu	The second argument
phi	The integral limit argument in radians

Returns

The elliptic function of the third kind.

Exceptions

std::domain_erro	if abs (k) > 1 .
------------------	------------------

Definition at line 876 of file specfun.h.

Return the incomplete elliptic integral of the third kind $\Pi(k,\nu,\phi)$ for float argument.

See also

ellint_3 for details.

Definition at line 838 of file specfun.h.

7.2.2.39 long double std::ellint_3I (long double __k, long double __nu, long double __phi) [inline]

Return the incomplete elliptic integral of the third kind $\Pi(k,\nu,\phi)$.

See also

ellint_3 for details.

Definition at line 848 of file specfun.h.

7.2.2.40 template<typename_Tp > __gnu_cxx::__promote<_Tp>::__type std::expint(_Tp__x) [inline]

Return the exponential integral Ei(x) for real argument x.

The exponential integral is given by

$$Ei(x) = -\int_{-x}^{\infty} \frac{e^t}{t} dt$$

Template Parameters

_ <i>Tp</i>	The floating-point type of the argumentx.

Parameters

__x The argument of the exponential integral function.

Definition at line 916 of file specfun.h.

7.2.2.41 float std::expintf (float _x) [inline]

Return the exponential integral Ei(x) for float argument x.

See also

expint for details.

Definition at line 890 of file specfun.h.

7.2.2.42 long double std::expintl (long double _x) [inline]

Return the exponential integral Ei(x) for long double argument x.

See also

expint for details.

Definition at line 900 of file specfun.h.

Return the Hermite polynomial $H_n(x)$ of order n and real argument x.

The Hermite polynomial is defined by:

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2}$$

The Hermite polynomial obeys a reflection formula:

$$H_n(-x) = (-1)^n H_n(x)$$

Template Parameters

_Tp	The floating-point type of the argumentx.

Parameters

n	The order

__x The argument

Definition at line 964 of file specfun.h.

7.2.2.44 float std::hermitef (unsigned int __n, float __x) [inline]

Return the Hermite polynomial $H_n(x)$ of nonnegative order n and float argument x.

See also

hermite for details.

Definition at line 931 of file specfun.h.

7.2.2.45 long double std::hermitel (unsigned int __n, long double __x) [inline]

Return the Hermite polynomial $H_n(x)$ of nonnegative order n and long double argument x.

See also

hermite for details.

Definition at line 941 of file specfun.h.

Returns the Laguerre polynomial $L_n(x)$ of nonnegative degree n and real argument x >= 0.

The Laguerre polynomial is defined by:

$$L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n} (x^n e^{-x})$$

Template Parameters

_ <i>Tp</i> T	The floating-point type of the argumentx.
-----------------	---

Parameters

n	The nonnegative order	
x	The argument $_{x} >= 0$	

Exceptions

Definition at line 1008 of file specfun.h.

7.2.2.47 float std::laguerref (unsigned int __n, float __x) [inline]

Returns the Laguerre polynomial $L_n(x)$ of nonnegative degree n and float argument x>=0.

See also

laguerre for more details.

Definition at line 979 of file specfun.h.

7.2.2.48 long double std::laguerrel (unsigned int __n, long double __x) [inline]

Returns the Laguerre polynomial $L_n(x)$ of nonnegative degree n and long double argument x >= 0.

See also

laguerre for more details.

Definition at line 989 of file specfun.h.

Return the Legendre polynomial $P_l(x)$ of nonnegative degree l and real argument |x| <= 0.

The Legendre function of order l and argument x, $P_l(x)$, is defined by:

$$P_l(x) = \frac{1}{2^l l!} \frac{d^l}{dx^l} (x^2 - 1)^l$$

Template Parameters

_ <i>Tp</i> The floating-point type of the argumentx.

Parameters

	The degree $l>=0$
X	The argument abs $(\underline{x}) \le 1$

Exceptions

Definition at line 1053 of file specfun.h.

7.2.2.50 float std::legendref (unsigned int __I, float __x) [inline]

Return the Legendre polynomial $P_l(x)$ of nonnegative degree l and float argument |x| <= 0.

See also

legendre for more details.

Definition at line 1023 of file specfun.h.

7.2.2.51 long double std::legendrel (unsigned int __I, long double __x) [inline]

Return the Legendre polynomial $P_l(x)$ of nonnegative degree l and long double argument |x| <= 0.

See also

legendre for more details.

Definition at line 1033 of file specfun.h.

 $\textbf{7.2.2.52} \quad template < typename _Tp > \underline{\quad} gnu_cxx::\underline{\quad} promote < \underline{\quad} Tp > ::\underline{\quad} type \ std::riemann_zeta \ (\ \underline{\quad} Tp \underline{\quad} s \) \quad \texttt{[inline]}$

Return the Riemann zeta function $\zeta(s)$ for real argument s.

The Riemann zeta function is defined by:

$$\zeta(s) = \sum_{k=1}^{\infty} k^{-s} \text{ for } s > 1$$

and

$$\zeta(s) = \frac{1}{1 - 2^{1 - s}} \sum_{k = 1}^{\infty} (-1)^{k - 1} k^{-s} \text{ for } 0 <= s <= 1$$

For s < 1 use the reflection formula:

$$\zeta(s) = 2^s \pi^{s-1} \sin(\frac{\pi s}{2}) \Gamma(1-s) \zeta(1-s)$$

Template Parameters

_Tp	The floating-point type of the arguments.
-----	---

Parameters

```
__s | The argument s != 1
```

Definition at line 1104 of file specfun.h.

7.2.2.53 float std::riemann_zetaf (float __s) [inline]

Return the Riemann zeta function $\zeta(s)$ for float argument s.

See also

riemann_zeta for more details.

Definition at line 1068 of file specfun.h.

7.2.2.54 long double std::riemann_zetal (long double __s) [inline]

Return the Riemann zeta function $\zeta(s)$ for long double argument s.

See also

riemann zeta for more details.

Definition at line 1078 of file specfun.h.

Return the spherical Bessel function $j_n(x)$ of nonnegative order n and real argument x >= 0.

The spherical Bessel function is defined by:

$$j_n(x) = \left(\frac{\pi}{2x}\right)^{1/2} J_{n+1/2}(x)$$

Template Parameters

To	The fleeting point type of the argument
_1p	The floating-point type of the argumentx.

Parameters

n	The integral order $n >= 0$
X	The real argument $x >= 0$

Exceptions

Definition at line 1148 of file specfun.h.

7.2.2.56 float std::sph_besself (unsigned int __n, float __x) [inline]

Return the spherical Bessel function $j_n(x)$ of nonnegative order n and float argument x >= 0.

See also

sph_bessel for more details.

Definition at line 1119 of file specfun.h.

7.2.2.57 long double std::sph_bessell (unsigned int _n, long double _x) [inline]

Return the spherical Bessel function $j_n(x)$ of nonnegative order n and long double argument x >= 0.

See also

sph bessel for more details.

Definition at line 1129 of file specfun.h.

7.2.2.58 template<typename_Tp > __gnu_cxx::__promote<_Tp>::__type std::sph_legendre(unsigned int __I, unsigned int __m, __Tp __theta) [inline]

Return the spherical Legendre function of nonnegative integral degree 1 and order m and real angle θ in radians.

The spherical Legendre function is defined by

$$Y_l^m(\theta,\phi) = (-1)^m \left[\frac{(2l+1)}{4\pi} \frac{(l-m)!}{(l+m)!} \right] P_l^m(\cos\theta) \exp^{im\phi}$$

Template Parameters

_ <i>Tp</i>	The floating-point type of the angletheta.

Parameters

1	The order $_1 >= 0$	
m	The degreem >= 0 andm <=1	
theta	The radian polar angle argument	

Definition at line 1195 of file specfun.h.

7.2.2.59 float std::sph_legendref (unsigned int __n, unsigned int __m, float __theta) [inline]

Return the spherical Legendre function of nonnegative integral degree 1 and order m and float angle θ in radians.

See also

sph legendre for details.

Definition at line 1163 of file specfun.h.

7.2.2.60 long double std::sph_legendrel (unsigned int __l, unsigned int __m, long double __theta) [inline]

Return the spherical Legendre function of nonnegative integral degree 1 and order m and long double angle θ in radians.

See also

sph_legendre for details.

Definition at line 1174 of file specfun.h.

Return the spherical Neumann function of integral order $n \ge 0$ and real argument $x \ge 0$.

The spherical Neumann function is defined by

$$n_n(x) = \left(\frac{\pi}{2x}\right)^{1/2} N_{n+1/2}(x)$$

Template Parameters

_ <i>Tp</i>	The floating-point type of the argumentx.

Parameters

n	The integral order $n >= 0$

```
__x | The real argument __x >= 0
```

Exceptions

```
std::domain_error | if __x < 0 .
```

Definition at line 1239 of file specfun.h.

```
7.2.2.62 float std::sph_neumannf ( unsigned int __n, float __x ) [inline]
```

Return the spherical Neumann function of integral order n>=0 and float argument x>=0.

See also

sph_neumann for details.

Definition at line 1210 of file specfun.h.

```
7.2.2.63 long double std::sph_neumannl ( unsigned int __n, long double __x ) [inline]
```

Return the spherical Neumann function of integral order n >= 0 and long double x >= 0.

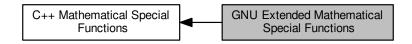
See also

sph_neumann for details.

Definition at line 1220 of file specfun.h.

7.3 GNU Extended Mathematical Special Functions

Collaboration diagram for GNU Extended Mathematical Special Functions:



Enumerations

enum { __gnu_cxx::_GLIBCXX_JACOBI_SN, __gnu_cxx::_GLIBCXX_JACOBI_CN, __gnu_cxx::_GLIBCXX_J
 ACOBI_DN }

Functions

```
template<typename _Tp >
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::airy_ai (_Tp __x)

    float __gnu_cxx::airy_aif (float __x)

    long double gnu cxx::airy ail (long double x)

template<typename _Tp >
   __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::airy_bi (_Tp __x)

    float gnu cxx::airy bif (float x)

    long double gnu cxx::airy bil (long double x)

template<typename _Tp >
    \_gnu\_cxx::\_promote\_num\_t < \_Tp > \_\_gnu\_cxx::bernoulli \ (unsigned \ int \ \_\_n)

    float gnu cxx::bernoullif (unsigned int n)

    long double gnu cxx::bernoullil (unsigned int n)

template<typename_Tp>
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::bincoef (unsigned int __n, unsigned int __k)

    float gnu cxx::bincoeff (unsigned int n, unsigned int k)

• long double gnu cxx::bincoefl (unsigned int n, unsigned int k)
template<typename _Tp >
    _gnu_cxx::__promote_num_t < _Tp > __gnu_cxx::chebyshev_t (unsigned int __n, Tp x)

    float gnu cxx::chebyshev tf (unsigned int n, float x)

    long double __gnu_cxx::chebyshev_tl (unsigned int __n, long double __x)

template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::chebyshev_u (unsigned int __n, _Tp __x)

    float gnu cxx::chebyshev uf (unsigned int n, float x)

    long double __gnu_cxx::chebyshev_ul (unsigned int __n, long double __x)

template<typename</li>Tp >
   gnu cxx:: promote num t< Tp > gnu cxx::chebyshev v (unsigned int n, Tp x)

    float gnu cxx::chebyshev vf (unsigned int n, float x)

    long double __gnu_cxx::chebyshev_vl (unsigned int __n, long double __x)

template<typename _Tp >
  gnu cxx:: promote num t< Tp > gnu cxx::chebyshev w (unsigned int n, Tp x)
```

```
    float <u>__gnu_cxx::chebyshev_wf</u> (unsigned int <u>__n, float __x)</u>

    long double __gnu_cxx::chebyshev_wl (unsigned int __n, long double __x)

template<typename _Tp >
   __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::clausen (unsigned int __m, _Tp __w)
template<typename _Tp >
  std::complex< __gnu_cxx::__promote_num_t< _Tp >> __gnu_cxx::clausen (unsigned int __m, std::complex<
  _{\mathsf{Tp}} > _{\mathsf{w}}
template<typename _Tp >
   __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::clausen_c (unsigned int __m, _Tp __w)

    float <u>__gnu_cxx::clausen_cf</u> (unsigned int <u>__</u>m, float <u>__</u>w)

    long double gnu cxx::clausen cl (unsigned int m, long double w)

template<typename _Tp >
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::clausen_s (unsigned int __m, _Tp __w)

    float gnu cxx::clausen sf (unsigned int m, float w)

    long double __gnu_cxx::clausen_sl (unsigned int __m, long double __w)

    float gnu cxx::clausenf (unsigned int m, float w)

    std::complex < float > __gnu_cxx::clausenf (unsigned int __m, std::complex < float > __w)

• long double gnu cxx::clausenl (unsigned int m, long double w)

    std::complex < long double > __gnu_cxx::clausenl (unsigned int __m, std::complex < long double > __w)

    template<typename Tk >

   _gnu_cxx::__promote_num_t< _Tk > __gnu_cxx::comp_ellint_d (_Tk __k)

    float __gnu_cxx::comp_ellint_df (float __k)

    long double gnu cxx::comp ellint dl (long double k)

float __gnu_cxx::comp_ellint_rf (float __x, float __y)

    long double gnu cxx::comp ellint rf (long double x, long double y)

    template<typename _Tx , typename _Ty >

   _gnu_cxx::__promote_num_t< _Tx, _Ty > __gnu_cxx::comp_ellint_rf (_Tx __x, _Ty __y)

    float <u>__gnu_cxx::comp_ellint_rg</u> (float <u>__x</u>, float <u>__y</u>)

    long double __gnu_cxx::comp_ellint_rg (long double __x, long double __y)

    template<typename _Tx , typename _Ty >

   _gnu_cxx::__promote_num_t< _Tx, _Ty > __gnu_cxx::comp_ellint_rg (_Tx __x, _Ty __y)
• template<typename _Tpa , typename _Tpc , typename _Tp >
   _gnu_cxx::__promote_3< _Tpa, _Tpc, _Tp >::__type <u>__gnu_cxx::conf_hyperg</u> (_Tpa __a, _Tpc __c, _Tp __x)
• template<typename _Tpc , typename _Tp >
   __gnu_cxx::__promote_2< _Tpc, _Tp >::__type __gnu_cxx::conf_hyperg_lim (_Tpc __c, _Tp __x)

    float gnu cxx::conf hyperg limf (float c, float x)

    long double gnu cxx::conf hyperg liml (long double c, long double x)

    float gnu cxx::conf hypergf (float a, float c, float x)

    long double gnu cxx::conf hypergl (long double a, long double c, long double x)

template<typename _Tp >
    gnu cxx:: promote num t< Tp> gnu cxx::coshint (Tpx)

    float gnu cxx::coshintf (float x)

    long double gnu cxx::coshintl (long double x)

template<typename_Tp>
   __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::cosint (_Tp __x)

    float gnu cxx::cosintf (float x)

    long double gnu cxx::cosintl (long double x)

    template<typename _Tpnu , typename _Tp >

  std::complex< __gnu_cxx::_promote_num_t< _Tpnu, _Tp >> __gnu_cxx::cyl_hankel_1 (_Tpnu __nu, _Tp
  __z)
```

```
• template<typename _Tpnu , typename _Tp >
     std::complex< gnu cxx:: promote num t< Tpnu, Tp>> gnu cxx::cyl hankel 1 (std::complex< ←
     Tpnu > nu, std::complex < Tp > x

    std::complex< float > __gnu_cxx::cyl_hankel_1f (float __nu, float __z)

    std::complex < float > __gnu_cxx::cyl_hankel_1f (std::complex < float > __nu, std::complex < float > __x)

• std::complex < long double > gnu cxx::cyl hankel 1l (long double nu, long double z)
• std::complex < long double > gnu cxx::cyl hankel 1l (std::complex < long double > nu, std::complex < long
     double > __x)

    template<typename _Tpnu , typename _Tp >

     std::complex< \underline{\quad} gnu\_cxx::\underline{\quad} promote\_num\_t< \underline{\quad} Tpnu, \underline{\quad} Tp>> \underline{\quad} gnu\_cxx::cyl\_hankel\_2 \ (\underline{\quad} Tpnu \ \underline{\quad} nu, \underline{\quad} Tp>> \underline{\quad} gnu\_cxx::cyl\_hankel\_2 \ (\underline{\quad} Tpnu \ \underline{\quad} nu, \underline{\quad} Tp>> \underline{\quad} gnu\_cxx::cyl\_hankel\_2 \ (\underline{\quad} Tpnu \ \underline{\quad} nu, \underline{\quad} Tp>> \underline{\quad} gnu\_cxx::cyl\_hankel\_2 \ (\underline{\quad} Tpnu \ \underline{\quad} nu, \underline{\quad} Tp>> \underline{\quad} gnu\_cxx::cyl\_hankel\_2 \ (\underline{\quad} Tpnu \ \underline{\quad} nu, \underline{\quad} Tp>> \underline{\quad} gnu\_cxx::cyl\_hankel\_2 \ (\underline{\quad} Tpnu \ \underline{\quad} nu, \underline{\quad} Tp>> \underline{\quad} gnu\_cxx::cyl\_hankel\_2 \ (\underline{\quad} Tpnu \ \underline{\quad} nu, \underline{\quad} Tp>> \underline{\quad} gnu\_cxx::cyl\_hankel\_2 \ (\underline{\quad} Tpnu \ \underline{\quad} nu, \underline{\quad} Tp>> \underline{\quad} gnu\_cxx::cyl\_hankel\_2 \ (\underline{\quad} Tpnu \ \underline{\quad} nu, \underline{\quad} Tp>> \underline{\quad} gnu\_cxx::cyl\_hankel\_2 \ (\underline{\quad} Tpnu \ \underline{\quad} nu, \underline{\quad} Tp>> \underline{\quad} gnu\_cxx::cyl\_hankel\_2 \ (\underline{\quad} Tpnu \ \underline{\quad} nu, \underline{\quad} Tp>> \underline{\quad} gnu\_cxx::cyl\_hankel\_2 \ (\underline{\quad} Tpnu \ \underline{\quad} nu, \underline{\quad} Tp>> \underline{\quad} gnu\_cxx::cyl\_hankel\_2 \ (\underline{\quad} Tpnu \ \underline{\quad} nu, \underline{\quad} \underline{\quad} nu
• template<typename _Tpnu , typename _Tp >
     std::complex< gnu cxx:: promote num t< Tpnu, Tp>> gnu cxx::cyl hankel 2 (std::complex< ←
     Tpnu > __nu, std::complex< _Tp > __x)

    std::complex< float > __gnu_cxx::cyl_hankel_2f (float __nu, float __z)

    std::complex < float > __gnu_cxx::cyl_hankel_2f (std::complex < float > __nu, std::complex < float > __x)

    std::complex < long double > gnu cxx::cyl hankel 2l (long double nu, long double z)

• std::complex < long double > gnu cxx::cyl hankel 2l (std::complex < long double > nu, std::complex < long
     double > \underline{\hspace{1cm}} x)
template<typename</li>Tp >
          _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::dawson (_Tp __x)

    float __gnu_cxx::dawsonf (float __x)

    long double <u>gnu_cxx::dawsonl</u> (long double <u>x</u>)

    template<typename</li>
    Tp >

          _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::digamma (_Tp __z)

    float gnu cxx::digammaf (float z)

    long double gnu cxx::digammal (long double z)

template<typename_Tp>
       __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::dilog (_Tp __x)

    float gnu cxx::dilogf (float x)

    long double gnu cxx::dilogl (long double x)

template<typename</li>Tp >
     _Tp __gnu_cxx::dirichlet_beta (_Tp __x)

    float gnu cxx::dirichlet betaf (float x)

    long double gnu cxx::dirichlet betal (long double x)

template<typename _Tp >
     _Tp __gnu_cxx::dirichlet_eta (_Tp __x)

    float gnu cxx::dirichlet etaf (float x)

    long double gnu cxx::dirichlet etal (long double x)

template<typename _Tp >
          gnu cxx:: promote num t < Tp > gnu cxx::double factorial (int n)

    float gnu cxx::double factorialf (int n)

    long double __gnu_cxx::double_factoriall (int __n)

• template<typename Tk, typename Tp, typename Ta, typename Tb>
          _gnu_cxx::__promote_num_t<_Tk,_Tp,_Ta,_Tb>__gnu_cxx::ellint_cel (_Tk__k_c,_Tp__p,_Ta__a,_Tb
          b)

    float __gnu_cxx::ellint_celf (float __k_c, float __p, float __a, float __b)

    long double gnu cxx::ellint cell (long double k c, long double p, long double a, long double b)

• template<typename _Tk , typename _Tphi >
       __gnu_cxx::__promote_num_t< _Tk, _Tphi > __gnu_cxx::ellint_d (_Tk __k, _Tphi __phi)

    float <u>__gnu_cxx::ellint_df</u> (float <u>__k</u>, float <u>__phi</u>)

• long double gnu cxx::ellint dl (long double k, long double phi)
```

```
• template<typename _{\rm Tp}, typename _{\rm Tk} >
   _gnu_cxx::__promote_num_t< _Tp, _Tk > __gnu_cxx::ellint_el1 (_Tp __x, _Tk __k_c)
• float gnu cxx::ellint el1f (float x, float k c)

    long double __gnu_cxx::ellint_el1l (long double __x, long double __k_c)

• template<typename Tp, typename Tk, typename Ta, typename Tb>
    gnu_cxx::_promote_num_t< _Tp, _Tk, _Ta, _Tb > __gnu_cxx::ellint_el2 (_Tp __x, _Tk __k_c, _Ta __a, _Tb
   b)

    float __gnu_cxx::ellint_el2f (float __x, float __k_c, float __a, float __b)

    long double __gnu_cxx::ellint_el2l (long double __x, long double __k_c, long double __a, long double __b)

• template<typename Tx, typename Tk, typename Tp>
  __gnu_cxx::__promote_num_t<_Tx,_Tk,_Tp > __gnu_cxx::ellint_el3 (_Tx __x, _Tk __k_c, _Tp __p)

    float __gnu_cxx::ellint_el3f (float __x, float __k_c, float __p)

    long double __gnu_cxx::ellint_el3l (long double __x, long double __k_c, long double __p)

• template<typename _Tp , typename _Up >
    _gnu_cxx::__promote_num_t< _Tp, _Up > __gnu_cxx::ellint_rc (_Tp __x, _Up __y)
• float gnu cxx::ellint rcf (float x, float y)

    long double gnu cxx::ellint rcl (long double x, long double y)

- template<typename _Tp , typename _Up , typename _Vp >
    gnu cxx:: promote num t< Tp, Up, Vp > gnu cxx::ellint rd (Tp x, Up y, Vp z)

    float gnu cxx::ellint rdf (float x, float y, float z)

    long double gnu cxx::ellint rdl (long double x, long double y, long double z)

- template<typename _Tp , typename _Up , typename _Vp >
    _gnu_cxx::__promote_num_t< _Tp, _Up, _Vp > __gnu_cxx::ellint_rf (_Tp __x, _Up __y, _Vp __z)

    float gnu cxx::ellint rff (float x, float y, float z)

• long double __gnu_cxx::ellint_rfl (long double __x, long double __y, long double __z)
template<typename _Tp , typename _Up , typename _Vp >
    _gnu_cxx::__promote_num_t< _Tp, _Up, _Vp > __gnu_cxx::ellint_rg (_Tp __x, _Up __y, _Vp __z)

    float gnu cxx::ellint rgf (float x, float y, float z)

    long double __gnu_cxx::ellint_rgl (long double __x, long double __y, long double __z)

ullet template<typename _Tp , typename _Up , typename _Vp , typename _Wp >
  __gnu_cxx::_promote_num_t< _Tp, _Up, _Vp, _Wp > __gnu_cxx::ellint_rj (_Tp __x, _Up __y, _Vp __z, _Wp
  __p)

    float __gnu_cxx::ellint_rjf (float __x, float __y, float __z, float __p)

    long double __gnu_cxx::ellint_rjl (long double __x, long double __y, long double __z, long double __p)

• template<typename Tp >
  _Tp __gnu_cxx::ellnome (_Tp __k)

    float gnu cxx::ellnomef (float k)

• long double __gnu_cxx::ellnomel (long double __k)
• template<typename_Tp>
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::expint_e1 (_Tp __x)

    float gnu cxx::expint e1f (float x)

    long double __gnu_cxx::expint_e1l (long double __x)

template<typename _Tp >
   __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::expint_en (unsigned int __n, _Tp __x)

    float gnu cxx::expint enf (unsigned int n, float x)

    long double __gnu_cxx::expint_enl (unsigned int __n, long double __x)

template<typename_Tp>
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::factorial (unsigned int __n)

    float gnu cxx::factorialf (unsigned int

    long double <u>__gnu_cxx::factoriall</u> (unsigned int __n)

template<typename _Tp >
   __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::fresnel_c (_Tp __x)
```

```
    float __gnu_cxx::fresnel_cf (float __x)

• long double __gnu_cxx::fresnel_cl (long double __x)
template<typename _Tp >
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::fresnel_s (_Tp __x)

    float __gnu_cxx::fresnel_sf (float __x)

    long double __gnu_cxx::fresnel_sl (long double __x)

ullet template<typename _Tn , typename _Tp >
   __gnu_cxx::__promote_num_t< _Tn, _Tp > __gnu_cxx::gamma_l (_Tn __n, _Tp __x)
• float __gnu_cxx::gamma_lf (float __n, float __x)

    long double __gnu_cxx::gamma_ll (long double __n, long double __x)

• template<typename _Ta , typename _Tp >
    _gnu_cxx::__promote_num_t< _Ta, _Tp > __gnu_cxx::gamma_p (_Ta __a, _Tp __x)

    float gnu cxx::gamma pf (float a, float x)

    long double __gnu_cxx::gamma_pl (long double __a, long double __x)

template<typename _Ta , typename _Tp >
    _gnu_cxx::__promote_num_t< _Ta, _Tp > __gnu_cxx::gamma_q (_Ta __a, _Tp __x)

    float gnu cxx::gamma gf (float a, float x)

    long double gnu cxx::gamma ql (long double a, long double x)

• template<typename _{\rm Tn}, typename _{\rm Tp} >
    gnu cxx:: promote num t< Tn, Tp > gnu cxx::gamma u (Tn n, Tp x)

    float gnu cxx::gamma uf (float n, float x)

    long double gnu cxx::gamma ul (long double n, long double x)

    template<typename _Talpha , typename _Tp >

    _gnu_cxx::__promote_num_t< _Talpha, _Tp > __gnu_cxx::gegenbauer (unsigned int __n, _Talpha __alpha,
  _Tp __x)
• float gnu cxx::gegenbauerf (unsigned int n, float alpha, float x)

    long double __gnu_cxx::gegenbauerl (unsigned int __n, long double __alpha, long double __x)

    template<typename _Tk , typename _Tphi >

   __gnu_cxx::__promote_num_t< _Tk, _Tphi > __gnu_cxx::heuman_lambda (_Tk __k, _Tphi __phi)

    float __gnu_cxx::heuman_lambdaf (float __k, float __phi)

    long double gnu cxx::heuman lambdal (long double k, long double phi)

• template<typename _Tp , typename _Up >
    _gnu_cxx::__promote_num_t< _Tp, _Up > __gnu_cxx::hurwitz_zeta (_Tp __s, _Up __a)

    template<typename _Tp , typename _Up >

  std::complex< _Tp > __gnu_cxx::hurwitz_zeta (_Tp __s, std::complex< _Up > __a)

    float gnu cxx::hurwitz zetaf (float s, float a)

    long double gnu cxx::hurwitz zetal (long double s, long double a)

    template<typename _Tpa , typename _Tpb , typename _Tpc , typename _Tp >

   _gnu_cxx::__promote_4< _Tpa, _Tpb, _Tpc, _Tp >::__type __gnu_cxx::hyperg (_Tpa __a, _Tpb __b, _Tpc
   __c, _Tp ___x)

    float __gnu_cxx::hypergf (float __a, float __b, float __c, float __x)

    long double __gnu_cxx::hypergl (long double __a, long double __b, long double __c, long double __x)

- template<typename _Ta , typename _Tb , typename _Tp >
    _gnu_cxx::__promote_num_t< _Ta, _Tb, _Tp > __gnu_cxx::ibeta (_Ta __a, _Tb __b, _Tp __x)
template<typename _Ta , typename _Tb , typename _Tp >
    _gnu_cxx::__promote_num_t< _Ta, _Tb, _Tp > __gnu_cxx::ibetac (_Ta __a, _Tb __b, _Tp __x)

    float gnu cxx::ibetacf (float a, float b, float x)

    long double gnu cxx::ibetacl (long double a, long double b, long double x)

    float gnu cxx::ibetaf (float a, float b, float x)

    long double gnu cxx::ibetal (long double a, long double b, long double x)
```

```
ullet template<typename _Talpha , typename _Tbeta , typename _Tp >
   _gnu_cxx::__promote_num_t< _Talpha, _Tbeta, _Tp > __gnu_cxx::jacobi (unsigned __n, _Talpha __alpha,
  Tbeta beta, Tp x)
• template<typename _Kp , typename _Up >
   _gnu_cxx::__promote_num_t< _Kp, _Up > __gnu_cxx::jacobi_cn (_Kp __k, _Up __u)
• float gnu cxx::jacobi cnf (float k, float u)
• long double gnu cxx::jacobi cnl (long double k, long double u)
• template<typename _Kp , typename _Up >
    _gnu_cxx::__promote_num_t< _Kp, _Up > __gnu_cxx::jacobi_dn (_Kp __k, _Up __u)
• float gnu cxx::jacobi dnf (float k, float u)

    long double __gnu_cxx::jacobi_dnl (long double __k, long double __u)

• template<typename _Kp , typename _Up >
    _gnu_cxx::__promote_num_t< _Kp, _Up > __gnu_cxx::jacobi_sn (_Kp __k, _Up __u)

    float gnu cxx::jacobi snf (float k, float u)

    long double __gnu_cxx::jacobi_snl (long double __k, long double __u)

• template<typename Tk, typename Tphi >
    gnu_cxx::__promote_num_t< _Tk, _Tphi > __gnu_cxx::jacobi_zeta ( Tk k, Tphi phi)

    float gnu cxx::jacobi zetaf (float k, float phi)

    long double __gnu_cxx::jacobi_zetal (long double __k, long double __phi)

    float gnu cxx::jacobif (unsigned n, float alpha, float beta, float x)

    long double __gnu_cxx::jacobil (unsigned __n, long double __alpha, long double __beta, long double __x)

template<typename _Tp >
   _gnu_cxx::_promote_num_t< _Tp > __gnu_cxx::lbincoef (unsigned int __n, unsigned int __k)

    float gnu cxx::lbincoeff (unsigned int n, unsigned int k)

• long double __gnu_cxx::lbincoefl (unsigned int __n, unsigned int __k)
template<typename _Tp >
   gnu cxx:: promote num t < Tp > gnu cxx::ldouble factorial (int n)

    float gnu cxx::ldouble factorialf (int n)

• long double __gnu_cxx::ldouble_factoriall (int __n)

    template<typename</li>
    Tp >

    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::legendre_q (unsigned int __n, _Tp __x)

    float __gnu_cxx::legendre_qf (unsigned int __n, float __x)

    long double __gnu_cxx::legendre_ql (unsigned int __n, long double __x)

template<typename_Tp>
    gnu cxx:: promote num t < Tp > gnu cxx::lfactorial (unsigned int n)

    float __gnu_cxx::lfactorialf (unsigned int __n)

    long double gnu cxx::lfactoriall (unsigned int n)

template<typename_Tp>
   __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::logint (_Tp __x)

    float gnu cxx::logintf (float x)

    long double gnu cxx::logintl (long double x)

    template<typename _Tp , typename _Tn >

    _gnu_cxx::__promote_num_t< _Tp, _Tn > __gnu_cxx::lpochhammer_l (_Tp __a, _Tn __n)

    float gnu cxx::lpochhammer lf (float a, float n)

    long double gnu cxx::lpochhammer II (long double a, long double n)

• template<typename _Tp , typename _Tn >
   _gnu_cxx::__promote_num_t< _Tp, _Tn > __gnu_cxx::lpochhammer_u (_Tp __a, _Tn __n)

    float gnu cxx::lpochhammer uf (float a, float n)

    long double gnu cxx::lpochhammer ul (long double a, long double n)

    template<typename _Tph , typename _Tpa >

    gnu cxx:: promote num t< Tph, Tpa > gnu cxx::owens t (Tph h, Tpa a)

    float gnu cxx::owens tf (float h, float a)
```

```
    long double __gnu_cxx::owens_tl (long double __h, long double __a)

• template<typename _Tp , typename _Tn >
    _gnu_cxx::__promote_num_t< _Tp, _Tn > __gnu_cxx::pochhammer_l (_Tp __a, _Tn __n)

    float __gnu_cxx::pochhammer_lf (float __a, float __n)

    long double gnu cxx::pochhammer II (long double a, long double n)

• template<typename _Tp , typename _Tn >
    _gnu_cxx::__promote_num_t< _Tp, _Tn > __gnu_cxx::pochhammer_u (_Tp __a, _Tn __n)

    float __gnu_cxx::pochhammer_uf (float __a, float __n)

• long double __gnu_cxx::pochhammer_ul (long double __a, long double __n)
template<typename _Tp >
  std::complex< __gnu_cxx::_promote_num_t< _Tp >> __gnu_cxx::polylog (_Tp __s, std::complex< _Tp >
  __w)
• std::complex< float > __gnu_cxx::polylogf (float __s, std::complex< float > __w)

    std::complex < long double > __gnu_cxx::polylogl (long double __s, std::complex < long double > __w)

template<typename</li>Tp >
  __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::psi (_Tp __x)

    float __gnu_cxx::psif (float __x)

    long double __gnu_cxx::psil (long double __x)

template<typename</li>Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::radpoly (unsigned int __n, unsigned int __m, _Tp __rho)
• float <u>gnu_cxx::radpolyf</u> (unsigned int __n, unsigned int __m, float __rho)

    long double __gnu_cxx::radpolyl (unsigned int __n, unsigned int __m, long double __rho)

template<typename</li>Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::sinc (_Tp __x)
template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::sinc_pi (_Tp __x)

    float gnu cxx::sinc pif (float x)

    long double __gnu_cxx::sinc_pil (long double __x)

    float __gnu_cxx::sincf (float __x)

    long double <u>gnu_cxx::sincl</u> (long double <u>x</u>)

    template<typename</li>
    Tp >

   __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::sinhc (_Tp __x)
template<typename _Tp >
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::sinhc_pi (_Tp __x)

    float gnu cxx::sinhc pif (float x)

    long double __gnu_cxx::sinhc_pil (long double __x)

    float gnu cxx::sinhcf (float x)

    long double gnu cxx::sinhcl (long double x)

• template<typename _Tp >
    gnu cxx:: promote num t< Tp> gnu cxx::sinhint (Tpx)

    float gnu cxx::sinhintf (float x)

    long double gnu cxx::sinhintl (long double x)

template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::sinint (_Tp __x)

    float gnu cxx::sinintf (float x)

    long double <u>__gnu_cxx::sinintl</u> (long double <u>__x)</u>

template<typename_Tp>
   gnu cxx:: promote num t< Tp > gnu cxx::sph bessel i (unsigned int n, Tp x)

    float gnu cxx::sph bessel if (unsigned int n, float x)

    long double <u>__gnu_cxx::sph_bessel_il</u> (unsigned int __n, long double __x)

template<typename_Tp>
  __gnu_cxx::_promote_num_t< _Tp > __gnu_cxx::sph_bessel_k (unsigned int __n, _Tp __x)
```

```
    float __gnu_cxx::sph_bessel_kf (unsigned int __n, float __x)

    long double __gnu_cxx::sph_bessel_kl (unsigned int __n, long double __x)

template<typename</li>Tp >
  std::complex< __gnu_cxx::__promote_num_t< _Tp >> __gnu_cxx::sph_hankel_1 (unsigned int __n, _Tp __z)
template<typename</li>Tp >
  std::complex< __gnu_cxx::_promote_num_t< _Tp >> __gnu_cxx::sph_hankel_1 (unsigned int __n, std↔
  ::complex < _Tp > __x)

    std::complex< float > __gnu_cxx::sph_hankel_1f (unsigned int __n, float __z)

• std::complex < float > gnu cxx::sph hankel 1f (unsigned int n, std::complex < float > x)

    std::complex < long double > __gnu_cxx::sph_hankel_1l (unsigned int __n, long double __z)

    std::complex < long double > __gnu_cxx::sph_hankel_1l (unsigned int __n, std::complex < long double > __x)

    template<typename</li>
    Tp >

  std::complex < __gnu_cxx::__promote_num_t < _Tp > > __gnu_cxx::sph_hankel_2 (unsigned int __n, _Tp __z)
template<typename _Tp >
  std::complex< __gnu_cxx::_promote_num_t< _Tp >> __gnu_cxx::sph_hankel_2 (unsigned int __n, std-
  ::complex < Tp > x)

    std::complex< float > __gnu_cxx::sph_hankel_2f (unsigned int __n, float __z)

    std::complex< float > __gnu_cxx::sph_hankel_2f (unsigned int __n, std::complex< float > __x)

• std::complex < long double > gnu cxx::sph hankel 2l (unsigned int n, long double z)

    std::complex < long double > gnu cxx::sph hankel 2l (unsigned int n, std::complex < long double > x)

• template<typename Ttheta, typename Tphi >
  std::complex< __gnu_cxx::_promote_num_t< _Ttheta, _Tphi >> __gnu_cxx::sph_harmonic (unsigned int ←
  I, int m, Ttheta theta, Tphi phi)
• std::complex< float > __gnu_cxx::sph_harmonicf (unsigned int __l, int __m, float __theta, float __phi)
• std::complex< long double > __gnu_cxx::sph_harmonicl (unsigned int __l, int __m, long double __theta, long
  double phi)
• template<typename Tpnu, typename Tp >
   _gnu_cxx::_promote_num_t< _Tpnu, _Tp > __gnu_cxx::theta_1 (_Tpnu __nu, _Tp __x)

    float __gnu_cxx::theta_1f (float __nu, float __x)

• long double __nu, long double __nu, long double __x)
• template<typename Tpnu, typename Tp >
  gnu_cxx::_promote_num_t< _Tpnu, _Tp > __gnu_cxx::theta_2 (_Tpnu __nu, _Tp __x)

    float __gnu_cxx::theta_2f (float __nu, float __x)

    long double gnu cxx::theta 2l (long double nu, long double x)

• template<typename _Tpnu , typename _Tp >
    _gnu_cxx::__promote_num_t< _Tpnu, _Tp > __gnu_cxx::theta_3 (_Tpnu __nu, _Tp __x)
• float gnu cxx::theta 3f (float nu, float x)

    long double gnu cxx::theta 3l (long double nu, long double x)

• template<typename _Tpnu , typename _Tp >
   _gnu_cxx::__promote_num_t< _Tpnu, _Tp > __gnu_cxx::theta_4 (_Tpnu __nu, _Tp __x)
• float gnu cxx::theta 4f (float nu, float x)

    long double __gnu_cxx::theta_4l (long double __nu, long double __x)

• template<typename _Tpk , typename _Tp >
   _gnu_cxx::__promote_num_t< _Tpk, _Tp > __gnu_cxx::theta_c (_Tpk __k, _Tp _ x)

    float __gnu_cxx::theta_cf (float __k, float __x)

    long double __gnu_cxx::theta_cl (long double __k, long double __x)

template<typename _Tpk , typename _Tp >
   gnu cxx:: promote num t < Tpk, Tp > gnu cxx::theta d ( Tpk k, Tp x)

    float gnu cxx::theta df (float k, float x)

    long double __gnu_cxx::theta_dl (long double __k, long double __x)

template<typename _Tpk , typename _Tp >
  \underline{\hspace{0.3cm}} gnu\_cxx::\underline{\hspace{0.3cm}} promote\_num\_t<\underline{\hspace{0.3cm}} Tpk,\underline{\hspace{0.3cm}} Tp>\underline{\hspace{0.3cm}} gnu\_cxx::theta\_n (\underline{\hspace{0.3cm}} Tpk\underline{\hspace{0.3cm}} k,\underline{\hspace{0.3cm}} Tp\underline{\hspace{0.3cm}} x)
```

```
float __gnu_cxx::theta_nf (float __k, float __x)
long double __gnu_cxx::theta_nl (long double __k, long double __x)
template<typename _Tpk , typename _Tp > __gnu_cxx::_promote_num_t< _Tpk, _Tp > __gnu_cxx::theta_s (_Tpk __k, _Tp __x)
float __gnu_cxx::theta_sf (float __k, float __x)
long double __gnu_cxx::theta_sl (long double __k, long double __x)
template<typename _Trho , typename _Tphi > __gnu_cxx::_promote_num_t< _Trho, _Tphi > __gnu_cxx::zernike (unsigned int __n, int __m, float __rho, float __phi)
float __gnu_cxx::zernikef (unsigned int __n, int __m, long double __rho, long double __phi)
```

Detailed Description

An extended collection of advanced mathematical special functions for GNU.

7.3.2 Enumeration Type Documentation

7.3.2.1 anonymous enum

Enumerator

7.3.1

```
_GLIBCXX_JACOBI_SN
_GLIBCXX_JACOBI_CN
_GLIBCXX_JACOBI_DN
```

Definition at line 1629 of file specfun.h.

7.3.3 Function Documentation

Definition at line 2243 of file specfun.h.

```
7.3.3.2 float __gnu_cxx::airy_aif(float __x) [inline]
```

Definition at line 2223 of file specfun.h.

```
7.3.3.3 long double __gnu_cxx::airy_ail ( long double __x ) [inline]
```

Definition at line 2231 of file specfun.h.

Definition at line 2274 of file specfun.h.

```
7.3.3.5 float __gnu_cxx::airy_bif(float __x) [inline]
```

Definition at line 2254 of file specfun.h.

```
7.3.3.6 long double __gnu_cxx::airy_bil ( long double __x ) [inline]
```

Definition at line 2262 of file specfun.h.

Return the Bernoulli number of integer order n.

The Bernoulli numbers are defined by

Parameters

```
__n The order.
```

Definition at line 3239 of file specfun.h.

```
7.3.3.8 float __gnu_cxx::bernoullif( unsigned int __n ) [inline]
```

Definition at line 3220 of file specfun.h.

```
7.3.3.9 long double __gnu_cxx::bernoullil( unsigned int __n ) [inline]
```

Definition at line 3224 of file specfun.h.

```
7.3.3.10 template<typename_Tp > __gnu_cxx::__promote_num_t<_Tp> __gnu_cxx::bincoef ( unsigned int __n, unsigned int __n
```

Definition at line 3190 of file specfun.h.

```
7.3.3.11 float __gnu_cxx::bincoeff ( unsigned int __n, unsigned int __k ) [inline]
```

Definition at line 3178 of file specfun.h.

```
7.3.3.12 long double __gnu_cxx::bincoefl ( unsigned int __n, unsigned int __k ) [inline]
```

Definition at line 3182 of file specfun.h.

Return the Chebyshev polynomials of the first kind of order n and argument x.

The Chebyshev polynomials of the first kind is defined by

Parameters

n	
x	

Definition at line 1810 of file specfun.h.

```
7.3.3.14 float __gnu_cxx::chebyshev_tf ( unsigned int __n, float __x ) [inline]
```

Return the Chebyshev polynomials of the first kind of order n and argument x.

See also

chebyshev_t for details.

Definition at line 1783 of file specfun.h.

```
7.3.3.15 long double __gnu_cxx::chebyshev_tl( unsigned int __n, long double __x ) [inline]
```

Return the Chebyshev polynomials of the first kind of order n and argument x.

See also

chebyshev_t for details.

Definition at line 1793 of file specfun.h.

Return the Chebyshev polynomials of the second kind of order n and argument x.

The Chebyshev polynomials of the second kind is defined by

Parameters

n	
X	

Definition at line 1852 of file specfun.h.

```
7.3.3.17 float __gnu_cxx::chebyshev_uf ( unsigned int __n, float __x ) [inline]
```

Return the Chebyshev polynomials of the second kind of order n and argument x.

See also

chebyshev_u for details.

Definition at line 1825 of file specfun.h.

7.3.3.18 long double __gnu_cxx::chebyshev_ul (unsigned int __n, long double __x) [inline]

Return the Chebyshev polynomials of the second kind of order n and argument x.

See also

chebyshev_u for details.

Definition at line 1835 of file specfun.h.

Return the Chebyshev polynomials of the third kind of order n and argument x.

The Chebyshev polynomials of the third kind is defined by

Parameters

n	
X	

Definition at line 1894 of file specfun.h.

```
7.3.3.20 float __gnu_cxx::chebyshev_vf( unsigned int __n, float __x ) [inline]
```

Return the Chebyshev polynomials of the third kind of order n and argument x.

See also

chebyshev_v for details.

Definition at line 1867 of file specfun.h.

```
7.3.3.21 long double __gnu_cxx::chebyshev_vI ( unsigned int __n, long double __x ) [inline]
```

Return the Chebyshev polynomials of the third kind of order n and argument x.

See also

chebyshev_v for details.

Definition at line 1877 of file specfun.h.

Return the Chebyshev polynomials of the fourth kind of order n and argument x.

The Chebyshev polynomials of the fourth kind is defined by

Parameters

n	
X	

Definition at line 1936 of file specfun.h.

```
7.3.3.23 float __gnu_cxx::chebyshev_wf( unsigned int __n, float __x) [inline]
```

Return the Chebyshev polynomials of the fourth kind of order n and argument x.

See also

```
chebyshev_w for details.
```

Definition at line 1909 of file specfun.h.

```
7.3.3.24 long double __gnu_cxx::chebyshev_wl ( unsigned int __n, long double __x ) [inline]
```

Return the Chebyshev polynomials of the fourth kind of order n and argument x.

See also

```
chebyshev_w for details.
```

Definition at line 1919 of file specfun.h.

Return the Clausen function of integer order $\ensuremath{\mathtt{m}}$ and complex argument $\ensuremath{\mathtt{w}}.$

The Clausen function is defined by

Parameters

m	
w	The complex argument

Definition at line 3959 of file specfun.h.

Definition at line 3980 of file specfun.h.

7.3.3.27 templateTp > \underline{gnu_cxx::_promote_num_t} < Tp > \underline{gnu_cxx::clausen_c} (unsigned int
$$\underline{m}$$
, $\underline{Tp}\underline{w}$) [inline]

Return the Clausen cosine function of order m and real argument x.

The Clausen cosine function is defined by

Parameters

m	
W	

Definition at line 3920 of file specfun.h.

```
7.3.3.28 float __gnu_cxx::clausen_cf ( unsigned int __m, float __w ) [inline]
```

Return the Clausen cosine function of order m and real argument x.

See also

clausen_c for details.

Definition at line 3895 of file specfun.h.

```
7.3.3.29 long double __gnu_cxx::clausen_cl ( unsigned int __m, long double __w ) [inline]
```

Return the Clausen cosine function of order m and real argument x.

See also

clausen c for details.

Definition at line 3904 of file specfun.h.

Return the Clausen sine function of order m and real argument x.

The Clausen sine function is defined by

Parameters

m	
w	

Definition at line 3881 of file specfun.h.

Return the Clausen sine function of order m and real argument x.

See also

clausen_s for details.

Definition at line 3856 of file specfun.h.

7.3.3.32 long double __gnu_cxx::clausen_sl (unsigned int __m, long double __w) [inline]

Return the Clausen sine function of order m and real argument x.

See also

clausen s for details.

Definition at line 3865 of file specfun.h.

7.3.3.33 float __gnu_cxx::clausenf (unsigned int __m, float __w) [inline]

Return the Clausen function of integer order m and complex argument w.

See also

clausen for details.

Definition at line 3934 of file specfun.h.

7.3.3.34 std::complex<float> __gnu_cxx::clausenf (unsigned int __m, std::complex< float > __w) [inline]

Definition at line 3968 of file specfun.h.

7.3.3.35 long double __gnu_cxx::clausenl (unsigned int __m, long double __w) [inline]

Return the Clausen function of integer order $\ensuremath{\mathtt{m}}$ and complex argument $\ensuremath{\mathtt{w}}.$

See also

clausen for details.

Definition at line 3943 of file specfun.h.

7.3.3.36 std::complex < long double > __gnu_cxx::clausenl (unsigned int __m, std::complex < long double > __w) [inline]

Definition at line 3972 of file specfun.h.

 $7.3.3.37 \quad template < typename _Tk > _gnu_cxx::_promote_num_t < _Tk > _gnu_cxx::comp_ellint_d (_Tk _k) \quad [inline]$

Return the complete Legendre elliptic integral D of k and ϕ .

The complete Legendre elliptic integral D is defined by

$$D(k) = \int_0^{\pi/2} \frac{\sin^2 \theta d\theta}{\sqrt{1 - k^2 \sin 2\theta}}$$

Parameters

$$__k \mid \mathsf{The} \ \mathsf{modulus} \ -1 <= k <= +1$$

Definition at line 3388 of file specfun.h.

Definition at line 3369 of file specfun.h.

Definition at line 3373 of file specfun.h.

Return the complete Carlson elliptic function $R_F(x,y,z)$ for float arguments.

See also

comp ellint rf for details.

Definition at line 2395 of file specfun.h.

Return the complete Carlson elliptic function $R_F(x,y)$ for long double arguments.

See also

comp_ellint_rf for details.

Definition at line 2405 of file specfun.h.

Return the complete Carlson elliptic function $R_F(x,y)$ for real arguments.

The complete Carlson elliptic function of the first kind is defined by:

$$R_F(x,y) = R_F(x,y,y) = \frac{1}{2} \int_0^\infty \frac{dt}{(t+x)^{1/2}(t+y)}$$

Parameters

x The first argument.

у	The second argument.
---	----------------------

Definition at line 2423 of file specfun.h.

7.3.3.43 float __gnu_cxx::comp_ellint_rg(float __x, float __y) [inline]

Return the Carlson complementary elliptic function $R_G(x, y)$.

See also

comp_ellint_rg for details.

Definition at line 2628 of file specfun.h.

7.3.3.44 long double __gnu_cxx::comp_ellint_rg (long double __x, long double __y) [inline]

Return the Carlson complementary elliptic function $R_G(x, y)$.

See also

comp_ellint_rg for details.

Definition at line 2637 of file specfun.h.

Return the complete Carlson elliptic function $R_G(x,y)$ for real arguments.

The complete Carlson elliptic function is defined by:

$$R_G(x,y) = R_G(x,y,y) = \frac{1}{4} \int_0^\infty dt t(t+x)^{-1/2} (t+y)^{-1} (\frac{x}{t+x} + \frac{2y}{t+y})$$

Parameters

X	The first argument.
у	The second argument.

Definition at line 2656 of file specfun.h.

Return the confluent hypergeometric function ${}_1F_1(a;c;x)$ of real numeratorial parameter a, denominatorial parameter c, and argument x.

The confluent hypergeometric function is defined by

$$_{1}F_{1}(a;c;x) = \sum_{n=0}^{\infty} \frac{(a)_{n}x^{n}}{(c)_{n}n!}$$

where the Pochhammer symbol is $(x)_k = (x)(x+1)...(x+k-1), (x)_0 = 1$

Parameters

a	The numeratorial parameter
c	The denominatorial parameter
X	The argument

Definition at line 1302 of file specfun.h.

Return the confluent hypergeometric limit function ${}_0F_1(;c;x)$ of real numeratorial parameter ${}_{\mathbb{C}}$ and argument ${}_{\mathbb{C}}$.

The confluent hypergeometric limit function is defined by

$$_{0}F_{1}(;c;x) = \sum_{n=0}^{\infty} \frac{x^{n}}{(c)_{n}n!}$$

where the Pochhammer symbol is $(x)_k = (x)(x+1)...(x+k-1), (x)_0 = 1$

Parameters

c	The denominatorial parameter
X	The argument

Definition at line 1398 of file specfun.h.

Return the confluent hypergeometric limit function ${}_0F_1(;c;x)$ of float numeratorial parameter c and argument x.

See also

conf_hyperg_lim for details.

Definition at line 1369 of file specfun.h.

Return the confluent hypergeometric limit function ${}_0F_1(;c;x)$ of long double numeratorial parameter c and argument x.

See also

conf_hyperg_lim for details.

Definition at line 1379 of file specfun.h.

Return the confluent hypergeometric function ${}_1F_1(a;c;x)$ of float numeratorial parameter a, denominatorial parameter c, and argument x.

See also

conf_hyperg for details.

Definition at line 1270 of file specfun.h.

```
7.3.3.51 long double __gnu_cxx::conf_hypergl ( long double __a, long double __c, long double __x ) [inline]
```

Return the confluent hypergeometric function ${}_1F_1(a;c;x)$ of long double numeratorial parameter a, denominatorial parameter c, and argument x.

See also

conf_hyperg for details.

Definition at line 1281 of file specfun.h.

```
7.3.3.52 template<typename_Tp > __gnu_cxx::__promote_num_t<_Tp> __gnu_cxx::coshint( _Tp __x ) [inline]
```

Return the hyperbolic cosine integral of argument x.

The hyperbolic cosine integral is defined by

Parameters

X	The argument

Definition at line 1622 of file specfun.h.

```
7.3.3.53 float __gnu_cxx::coshintf(float __x) [inline]
```

Definition at line 1598 of file specfun.h.

```
7.3.3.54 long double __gnu_cxx::coshintl( long double __x ) [inline]
```

Return the hyperbolic cosine integral of argument x.

See also

coshint for details.

Definition at line 1607 of file specfun.h.

Return the cosine integral of argument x.

The cosine integral is defined by

Parameters

.,	The average of
X	I The argument

Definition at line 1551 of file specfun.h.

7.3.3.56 float __gnu_cxx::cosintf(float __x) [inline]

Return the cosine integral of argument x.

See also

cosint for details.

Definition at line 1527 of file specfun.h.

7.3.3.57 long double __gnu_cxx::cosintl(long double __x) [inline]

Return the cosine integral of argument x.

See also

cosint for details.

Definition at line 1536 of file specfun.h.

Definition at line 2083 of file specfun.h.

Return the complex cylindrical Hankel function of the first kind of complex order ν and complex argument x.

The cylindrical Hankel function of the first kind is defined by

$$H_{\nu}^{(1)}(x) = J_{\nu}(x) + iN_{\nu}(x)$$

Parameters

nu	The complex order
x	The complex argument

Definition at line 3582 of file specfun.h.

7.3.3.60 std::complex<float> __gnu_cxx::cyl_hankel_1f(float __nu, float __z) [inline]

Definition at line 2071 of file specfun.h.

7.3.3.61 std::complex < float >
$$_$$
gnu_cxx::cyl_hankel_1f (std::complex < float > $_$ nu, std::complex < float > $_$ x) $[inline]$

Return the complex cylindrical Hankel function of the first kind of complex order ν and complex argument x.

See also

cyl_hankel_1 for more details.

Definition at line 3555 of file specfun.h.

Definition at line 2075 of file specfun.h.

7.3.3.63 std::complex < long double >
$$_$$
gnu_cxx::cyl_hankel_1I (std::complex < long double > $_$ nu, std::complex < long double > $_$ x) [inline]

Return the complex cylindrical Hankel function of the first kind of complex order ν and complex argument x.

See also

cyl hankel 1 for more details.

Definition at line 3565 of file specfun.h.

Definition at line 2104 of file specfun.h.

Return the complex cylindrical Hankel function of the second kind of complex order ν and complex argument x.

The cylindrical Hankel function of the second kind is defined by

$$H_{\nu}^{(2)}(x) = J_{\nu}(x) - iN_{\nu}(x)$$

Parameters

nu	The complex order
X	The complex argument

Definition at line 3624 of file specfun.h.

7.3.3.66 std::complex<float> __gnu_cxx::cyl_hankel_2f(float __nu, float __z) [inline]

Definition at line 2092 of file specfun.h.

Return the complex cylindrical Hankel function of the second kind of complex order ν and complex argument x.

See also

cyl hankel 2 for more details.

Definition at line 3597 of file specfun.h.

7.3.3.68 std::complex<long double> __gnu_cxx::cyl_hankel_2l(long double __nu, long double __z) [inline]

Definition at line 2096 of file specfun.h.

7.3.3.69 std::complex < long double > $_$ gnu_cxx::cyl_hankel_2I (std::complex < long double > $_$ nu, std::complex < long double > $_$ x) [inline]

Return the complex cylindrical Hankel function of the second kind of complex order ν and complex argument x.

See also

cyl hankel 2 for more details.

Definition at line 3607 of file specfun.h.

7.3.3.70 template<typename_Tp > __gnu_cxx::__promote_num_t<_Tp> __gnu_cxx::dawson(_Tp __x) [inline]

Return the Dawson integral, F(x), for real argument x.

The Dawson integral is defined by:

$$F(x) = e^{-x^2} \int_0^x e^{y^2} dy$$

and it's derivative is:

$$F'(x) = 1 - 2xF(x)$$

Parameters

 $\underline{\hspace{0.5cm}}$ The argument -inf < x < inf.

Definition at line 2966 of file specfun.h.

7.3.3.71 float __gnu_cxx::dawsonf(float __x) [inline]

Return the Dawson integral, F(x), for float argument x.

See also

dawson for details.

Definition at line 2938 of file specfun.h.

7.3.3.72 long double __gnu_cxx::dawsonl(long double __x) [inline]

Return the Dawson integral, F(x), for long double argument x.

See also

dawson for details.

Definition at line 2947 of file specfun.h.

Definition at line 2339 of file specfun.h.

7.3.3.74 float __gnu_cxx::digammaf(float __z) [inline]

Definition at line 2327 of file specfun.h.

7.3.3.75 long double __gnu_cxx::digammal(long double __z) [inline]

Definition at line 2331 of file specfun.h.

7.3.3.76 template<typename_Tp > __gnu_cxx::__promote_num_t<_Tp> __gnu_cxx::dilog(_Tp __x) [inline]

Return the dilogarithm function $\psi(z)$ for real argument.

The dilogarithm is defined by:

$$Li_2(x) = \sum_{k=1}^{\infty} \frac{x^k}{k^2}$$

Parameters

__x The argument.

Definition at line 2380 of file specfun.h.

7.3.3.77 float __gnu_cxx::dilogf(float __x) [inline]

Return the dilogarithm function $\psi(z)$ for float argument.

See also

dilog for details.

Definition at line 2354 of file specfun.h.

7.3.3.78 long double __gnu_cxx::dilogl(long double __x) [inline]

Return the dilogarithm function $\psi(z)$ for long double argument.

See also

dilog for details.

Definition at line 2364 of file specfun.h.

```
7.3.3.79 template<typename_Tp > _Tp __gnu_cxx::dirichlet_beta(_Tp __x) [inline]
```

Return the Dirichlet beta function of real argument x.

The Dirichlet beta function is defined by

Parameters

```
__x
```

Definition at line 3842 of file specfun.h.

```
7.3.3.80 float __gnu_cxx::dirichlet_betaf ( float __x ) [inline]
```

Definition at line 3823 of file specfun.h.

```
7.3.3.81 long double __gnu_cxx::dirichlet_betal ( long double __x ) [inline]
```

Definition at line 3827 of file specfun.h.

Return the Dirichlet eta function of real argument x.

The Dirichlet eta function is defined by

Parameters

```
__x
```

Definition at line 3814 of file specfun.h.

```
7.3.3.83 float __gnu_cxx::dirichlet_etaf ( float __x ) [inline]
```

Return the Dirichlet eta function of real argument x.

See also

dirichlet_eta for details.

Definition at line 3790 of file specfun.h.

```
7.3.3.84 long double __gnu_cxx::dirichlet_etal( long double __x ) [inline]
```

Return the Dirichlet eta function of real argument x.

See also

dirichlet eta for details.

Definition at line 3799 of file specfun.h.

```
7.3.3.85 template<typename_Tp>__gnu_cxx::__promote_num_t<_Tp>__gnu_cxx::double_factorial(int __n) [inline]
```

Definition at line 3127 of file specfun.h.

```
7.3.3.86 float __gnu_cxx::double_factorialf(int __n) [inline]
```

Definition at line 3115 of file specfun.h.

Definition at line 3119 of file specfun.h.

7.3.3.88 template < typename _Tb , typename _Tb , typename _Tb > __gnu_cxx::_promote_num_t < _Tk, _Tp, _Ta, _Tb > __gnu_cxx::ellint_cel(_Tk _
$$k$$
_c, _Tp _ p , _Ta _ a , _Tb _ b) [inline]

Return the Bulirsch complete elliptic integral of ...

The Bulirsch complete elliptic integral is defined by

Parameters

	k_c	The complementary modulus $k_c=\sqrt{(1-k^2)}$
_	p	The
_	a	The
_	b	The

Definition at line 3540 of file specfun.h.

```
7.3.3.89 float __gnu_cxx::ellint_celf ( float __k_c, float __p, float __a, float __b ) [inline]
```

Definition at line 3517 of file specfun.h.

```
7.3.3.90 long double __gnu_cxx::ellint_cell ( long double __k_c, long double __p, long double __a, long double __b ) [inline]
```

Definition at line 3521 of file specfun.h.

7.3.3.91 template<typename _Tk , typename _Tphi > __gnu_cxx::__promote_num_t<_Tk, _Tphi> __gnu_cxx::ellint_d (_Tk __k, _Tphi __phi) [inline]

Return the incomplete Legendre elliptic integral D of k and ϕ .

The Legendre elliptic integral D is defined by

$$D(k,\phi) = \int_0^{\phi} \frac{\sin^2 \theta d\theta}{\sqrt{1 - k^2 \sin 2\theta}}$$

Parameters

k	The modulus $-1 <= k <= +1$
phi	The angle

Definition at line 3417 of file specfun.h.

Definition at line 3397 of file specfun.h.

Definition at line 3401 of file specfun.h.

Return the Bulirsch elliptic integral of the first kind of ...

The Bulirsch elliptic integral of the first kind is defined by

Parameters

X	The argument
k_c	The complementary modulus $k_c=\sqrt(1-k^2)$

Definition at line 3446 of file specfun.h.

Definition at line 3426 of file specfun.h.

Definition at line 3430 of file specfun.h.

```
7.3.3.97 template < typename _Tp , typename _Tk , typename _Ta , typename _Tb > __gnu_cxx::__promote_num_t < _Tp, _Tk, _Ta, _Tb > __gnu_cxx::ellint_el2 ( _Tp _x, _Tk _k_c, _Ta _a, _Tb _b ) [inline]
```

Return the Bulirsch elliptic integral of the second kind of ...

The Bulirsch elliptic integral of the second kind is defined by

Parameters

X	The argument
k_c	The complementary modulus $k_c=\sqrt(1-k^2)$
a	The
b	The

Definition at line 3478 of file specfun.h.

```
7.3.3.98 float __gnu_cxx::ellint_el2f ( float __x, float __k_c, float __a, float __b ) [inline]
```

Definition at line 3455 of file specfun.h.

Definition at line 3459 of file specfun.h.

7.3.3.100 template __gnu_cxx::__promote_num_t<_Tx, _Tk, _Tp> __gnu_cxx::ellint_el3 (_Tx _ x, _Tk _
$$k_c$$
, _Tp _ p) [inline]

Return the Bulirsch elliptic integral of the third kind of ...

The Bulirsch elliptic integral of the third kind is defined by

Parameters

X	The
k_c	The complementary modulus $k_c=\sqrt{(1-k^2)}$
р	The

Definition at line 3508 of file specfun.h.

```
7.3.3.101 float __gnu_cxx::ellint_el3f ( float __x, float __k_c, float __p ) [inline]
```

Definition at line 3487 of file specfun.h.

7.3.3.102 long double $_gnu_cxx$::ellint_el3l (long double $_x$, long double $_k_c$, long double $_p$) [inline]

Definition at line 3491 of file specfun.h.

Return the Carlson elliptic function $R_C(x,y) = R_F(x,y,y)$ where $R_F(x,y,z)$ is the Carlson elliptic function of the first kind.

The Carlson elliptic function is defined by:

$$R_C(x,y) = \frac{1}{2} \int_0^\infty \frac{dt}{(t+x)^{1/2}(t+y)}$$

Based on Carlson's algorithms:

- B. C. Carlson Numer. Math. 33, 1 (1979)
- B. C. Carlson, Special Functions of Applied Mathematics (1977)
- Numerical Recipes in C, 2nd ed, pp. 261-269, by Press, Teukolsky, Vetterling, Flannery (1992)

Parameters

X	The first argument.
у	The second argument.

Definition at line 2515 of file specfun.h.

7.3.3.104 float __gnu_cxx::ellint_rcf(float __x, float __y) [inline]

Return the Carlson elliptic function $R_C(x, y)$.

See also

ellint_rc for details.

Definition at line 2481 of file specfun.h.

7.3.3.105 long double __gnu_cxx::ellint_rcl (long double __x, long double __y) [inline]

Return the Carlson elliptic function $R_C(x, y)$.

See also

ellint rc for details.

Definition at line 2490 of file specfun.h.

Return the Carlson elliptic function of the second kind $R_D(x,y,z) = R_J(x,y,z,z)$ where $R_J(x,y,z,p)$ is the Carlson elliptic function of the third kind.

The Carlson elliptic function of the second kind is defined by:

$$R_D(x, y, z) = \frac{3}{2} \int_0^\infty \frac{dt}{(t+x)^{1/2}(t+y)^{1/2}(t+z)^{3/2}}$$

Based on Carlson's algorithms:

- B. C. Carlson Numer. Math. 33, 1 (1979)
- B. C. Carlson, Special Functions of Applied Mathematics (1977)
- Numerical Recipes in C, 2nd ed, pp. 261-269, by Press, Teukolsky, Vetterling, Flannery (1992)

Parameters

	X	The first of two symmetric arguments.
	у	The second of two symmetric arguments.
ĺ	z	The third argument.

Definition at line 2614 of file specfun.h.

7.3.3.107 float __gnu_cxx::ellint_rdf (float __x, float __y, float __z) [inline]

Return the Carlson elliptic function $R_D(x, y, z)$.

See also

ellint rd for details.

Definition at line 2578 of file specfun.h.

7.3.3.108 long double __gnu_cxx::ellint_rdl (long double __x, long double __y, long double __z) [inline]

Return the Carlson elliptic function $R_D(x, y, z)$.

See also

ellint_rd for details.

Definition at line 2587 of file specfun.h.

Return the Carlson elliptic function $R_F(x,y,z)$ of the first kind for real arguments.

The Carlson elliptic function of the first kind is defined by:

$$R_F(x,y,z) = \frac{1}{2} \int_0^\infty \frac{dt}{(t+x)^{1/2}(t+y)^{1/2}(t+z)^{1/2}}$$

Parameters

X	The first of three symmetric arguments.

у	The second of three symmetric arguments.
Z	The third of three symmetric arguments.

Definition at line 2467 of file specfun.h.

Return the Carlson elliptic function $R_F(x,y,z)$ of the first kind for float arguments.

See also

ellint rf for details.

Definition at line 2438 of file specfun.h.

Return the Carlson elliptic function $R_F(x,y,z)$ of the first kind for long double arguments.

See also

ellint_rf for details.

Definition at line 2448 of file specfun.h.

Return the symmetric Carlson elliptic function of the second kind $R_G(x, y, z)$.

The Carlson symmetric elliptic function of the second kind is defined by:

$$R_G(x,y,z) = \frac{1}{4} \int_0^\infty dt t [(t+x)(t+y)(t+z)]^{-1/2} \left(\frac{x}{t+x} + \frac{y}{t+y} + \frac{z}{t+z}\right)$$

Based on Carlson's algorithms:

- B. C. Carlson Numer. Math. 33, 1 (1979)
- B. C. Carlson, Special Functions of Applied Mathematics (1977)
- Numerical Recipes in C, 2nd ed, pp. 261-269, by Press, Teukolsky, Vetterling, Flannery (1992)

Parameters

X	The first of three symmetric arguments.
y	The second of three symmetric arguments.
Z	The third of three symmetric arguments.

Definition at line 2705 of file specfun.h.

7.3.3.113 float __gnu_cxx::ellint_rgf (float __x, float __y, float __z) [inline]

Return the Carlson elliptic function $R_G(x, y)$.

See also

ellint_rg for details.

Definition at line 2670 of file specfun.h.

7.3.3.114 long double __gnu_cxx::ellint_rgl (long double __x, long double __y, long double __z) [inline]

Return the Carlson elliptic function $R_G(x, y)$.

See also

ellint rg for details.

Definition at line 2679 of file specfun.h.

Return the Carlson elliptic function $R_J(x, y, z, p)$ of the third kind.

The Carlson elliptic function of the third kind is defined by:

$$R_J(x,y,z,p) = \frac{3}{2} \int_0^\infty \frac{dt}{(t+x)^{1/2}(t+y)^{1/2}(t+z)^{1/2}(t+p)}$$

Based on Carlson's algorithms:

- B. C. Carlson Numer. Math. 33, 1 (1979)
- B. C. Carlson, Special Functions of Applied Mathematics (1977)
- Numerical Recipes in C, 2nd ed, pp. 261-269, by Press, Teukolsky, Vetterling, Flannery (1992)

Parameters

X	The first of three symmetric arguments.
у	The second of three symmetric arguments.
Z	The third of three symmetric arguments.
p	The fourth argument.

Definition at line 2564 of file specfun.h.

7.3.3.116 float _gnu_cxx::ellint_rjf(float _x, float _y, float _z, float _p) [inline]

Return the Carlson elliptic function $R_J(x, y, z, p)$.

See also

ellint_rj for details.

Definition at line 2529 of file specfun.h.

7.3.3.117 long double __gnu_cxx::ellint_rjl(long double __x, long double __y, long double __z, long double __p) [inline]

Return the Carlson elliptic function $R_J(x, y, z, p)$.

See also

ellint_rj for details.

Definition at line 2538 of file specfun.h.

```
7.3.3.118 template<typename_Tp > _Tp __gnu_cxx::ellnome( _Tp __k ) [inline]
```

Return the elliptic nome function of modulus k.

The elliptic nome function is defined by

Parameters

```
\underline{\hspace{0.5cm}} The modulus -1 <= k <= +1
```

Definition at line 4168 of file specfun.h.

```
7.3.3.119 float __gnu_cxx::ellnomef(float __k) [inline]
```

Definition at line 4149 of file specfun.h.

```
7.3.3.120 long double __gnu_cxx::ellnomel( long double __k ) [inline]
```

Definition at line 4153 of file specfun.h.

```
7.3.3.121 template<typename_Tp > __gnu_cxx::_promote_num_t<_Tp>__gnu_cxx::expint_e1( _Tp __x ) [inline]
```

Definition at line 2982 of file specfun.h.

```
7.3.3.122 float __gnu_cxx::expint_e1f(float __x) [inline]
```

Definition at line 2973 of file specfun.h.

```
7.3.3.123 long double __gnu_cxx::expint_e1l( long double __x ) [inline]
```

Definition at line 2977 of file specfun.h.

Definition at line 3001 of file specfun.h.

7.3.3.125 float __gnu_cxx::expint_enf(unsigned int __n, float __x) [inline]

Definition at line 2989 of file specfun.h.

7.3.3.126 long double __gnu_cxx::expint_enl (unsigned int __n, long double __x) [inline]

Definition at line 2993 of file specfun.h.

Definition at line 3106 of file specfun.h.

7.3.3.128 float __gnu_cxx::factorialf (unsigned int __n) [inline]

Definition at line 3094 of file specfun.h.

7.3.3.129 long double __gnu_cxx::factoriall (unsigned int __n) [inline]

Definition at line 3098 of file specfun.h.

7.3.3.130 template<typename_Tp > __gnu_cxx::_promote_num_t<_Tp> __gnu_cxx::fresnel_c (_Tp __x) [inline]

Return the Fresnel cosine integral of argument x.

The Fresnel cosine integral is defined by

$$C(x) = \int_0^x \cos(\frac{\pi}{2}t^2)dt$$

Parameters

X	The argument

Definition at line 2924 of file specfun.h.

7.3.3.131 float __gnu_cxx::fresnel_cf(float __x) [inline]

Definition at line 2905 of file specfun.h.

7.3.3.132 long double __gnu_cxx::fresnel_cl (long double __x) [inline]

Definition at line 2909 of file specfun.h.

Return the Fresnel sine integral of argument x.

The Fresnel sine integral is defined by

$$S(x) = \int_0^x \sin(\frac{\pi}{2}t^2)dt$$

Parameters

X	The argument

Definition at line 2896 of file specfun.h.

```
7.3.3.134 float __gnu_cxx::fresnel_sf( float __x ) [inline]
```

Definition at line 2877 of file specfun.h.

```
7.3.3.135 long double __gnu_cxx::fresnel_sl( long double __x ) [inline]
```

Definition at line 2881 of file specfun.h.

Definition at line 2318 of file specfun.h.

```
7.3.3.137 float __gnu_cxx::gamma_lf(float __n, float __x) [inline]
```

Definition at line 2306 of file specfun.h.

```
7.3.3.138 long double __gnu_cxx::gamma_ll( long double __n, long double __x) [inline]
```

Definition at line 2310 of file specfun.h.

```
7.3.3.139 template<typename _Ta , typename _Tp > __gnu_cxx::__promote_num_t<_Ta, _Tp> __gnu_cxx::gamma_p ( _Ta __a, _Tp __x ) [inline]
```

Definition at line 3281 of file specfun.h.

```
7.3.3.140 float __gnu_cxx::gamma_pf(float __a, float __x) [inline]
```

Definition at line 3269 of file specfun.h.

```
7.3.3.141 long double __gnu_cxx::gamma_pl( long double __a, long double __x) [inline]
```

Definition at line 3273 of file specfun.h.

```
7.3.3.142 template<typename_Ta , typename _Tp > __gnu_cxx::__promote_num_t<_Ta, _Tp> __gnu_cxx::gamma_q ( _Ta __a, __Tp __x ) [inline]
```

Definition at line 3302 of file specfun.h.

```
7.3.3.143 float __gnu_cxx::gamma_qf(float __a, float __x) [inline]
```

Definition at line 3290 of file specfun.h.

```
7.3.3.144 long double __gnu_cxx::gamma_ql( long double __a, long double __x) [inline]
```

Definition at line 3294 of file specfun.h.

```
7.3.3.145 template < typename _Tn , typename _Tp > __gnu_cxx::__promote_num_t < _Tn, _Tp > __gnu_cxx::gamma_u ( _Tn __n, _Tp __x ) [inline]
```

Definition at line 2297 of file specfun.h.

```
7.3.3.146 float __gnu_cxx::gamma_uf(float __n, float __x) [inline]
```

Definition at line 2285 of file specfun.h.

```
7.3.3.147 long double __gnu_cxx::gamma_ul ( long double __n, long double __x ) [inline]
```

Definition at line 2289 of file specfun.h.

```
7.3.3.148 template<typename _Talpha , typename _Tp > __gnu_cxx::__promote_num_t<_Talpha, _Tp > __gnu_cxx::gegenbauer ( unsigned int __n, _Talpha __alpha, _Tp __x ) [inline]
```

Definition at line 1978 of file specfun.h.

```
7.3.3.149 float _gnu cxx::gegenbauerf ( unsigned int _ n, float _ alpha, float _ x ) [inline]
```

Definition at line 1966 of file specfun.h.

```
7.3.3.150 long double __gnu_cxx::gegenbauerl ( unsigned int __n, long double __alpha, long double __x ) [inline]
```

Definition at line 1970 of file specfun.h.

```
7.3.3.151 template < typename _Tk , typename _Tphi > __gnu_cxx::__promote_num_t < _Tk, _Tphi > __gnu_cxx::heuman_lambda ( _Tk _ k, _Tphi _ phi ) [inline]
```

Return the Heuman lambda function of k and $@c\phi$.

The complete Heuman lambda function is defined by

Parameters

k	The modulus

iha	The angle

Definition at line 3360 of file specfun.h.

Definition at line 3340 of file specfun.h.

Definition at line 3344 of file specfun.h.

Return the Hurwitz zeta function of real argument s, and parameter a.

The the Hurwitz zeta function is defined by

$$\zeta(s,a) = \sum_{n=0}^{\infty} \frac{1}{(a+n)^s}$$

Parameters

s	The argument
a	The parameter

Definition at line 2746 of file specfun.h.

Return the Hurwitz zeta function of real argument s, and complex parameter a.

See also

hurwitz_zeta for details.

Definition at line 2760 of file specfun.h.

Return the Hurwitz zeta function of float argument s, and parameter a.

See also

hurwitz zeta for details.

Definition at line 2720 of file specfun.h.

7.3.3.157 long double __gnu_cxx::hurwitz_zetal (long double __s, long double __a) [inline]

Return the Hurwitz zeta function of long double argument s, and parameter a.

See also

hurwitz zeta for details.

Definition at line 2730 of file specfun.h.

Return the hypergeometric function ${}_2F_1(a,b;c;x)$ of real numeratorial parameters a and b, denominatorial parameter c, and argument x.

The hypergeometric function is defined by

$$_{2}F_{1}(a;c;x) = \sum_{n=0}^{\infty} \frac{(a)_{n}(b)_{n}x^{n}}{(c)_{n}n!}$$

where the Pochhammer symbol is $(x)_k = (x)(x+1)...(x+k-1), (x)_0 = 1$

Parameters

a	The first numeratorial parameter
b	The second numeratorial parameter
c	The denominatorial parameter
X	The argument

Definition at line 1351 of file specfun.h.

Return the hypergeometric function ${}_2F_1(a,b;c;x)$ of @ float numeratorial parameters a and b, denominatorial parameter c, and argument x.

See also

hyperg for details.

Definition at line 1318 of file specfun.h.

```
7.3.3.160 long double __gnu_cxx::hypergl( long double __a, long double __b, long double __c, long double __x) [inline]
```

Return the hypergeometric function ${}_2F_1(a,b;c;x)$ of long double numeratorial parameters a and b, denominatorial parameter c, and argument x.

See also

hyperg for details.

Definition at line 1329 of file specfun.h.

Return the regularized incomplete beta function of parameters a, b, and argument x.

The regularized incomplete beta function is defined by

$$I_x(a,b) = \frac{B_x(a,b)}{B(a,b)}$$

where

$$B_x(a,b) = \int_0^x t^{a-1} (1-t)^{b-1} dt$$

is the non-regularized beta function and B(a,b) is the usual beta function.

Parameters

a	The first parameter
b	The second parameter
X	The argument

Definition at line 2837 of file specfun.h.

Return the regularized complementary incomplete beta function of parameters a, b, and argument x.

The regularized complementary incomplete beta function is defined by

$$I_x(a,b) = I_x(a,b)$$

Parameters

a	The parameter
b	The parameter
X	The argument

Definition at line 2868 of file specfun.h.

Definition at line 2846 of file specfun.h.

References gnu cxx::ibetaf().

7.3.3.164 long double __gnu_cxx::ibetacl (long double __a, long double __b, long double __x) [inline]

Definition at line 2850 of file specfun.h.

References __gnu_cxx::ibetal().

Return the regularized incomplete beta function of parameters a, b, and argument x.

See ibeta for details.

Definition at line 2803 of file specfun.h.

Referenced by __gnu_cxx::ibetacf().

```
7.3.3.166 long double __gnu_cxx::ibetal ( long double __a, long double __b, long double __x ) [inline]
```

Return the regularized incomplete beta function of parameters a, b, and argument x.

See ibeta for details.

Definition at line 2813 of file specfun.h.

Referenced by __gnu_cxx::ibetacl().

```
7.3.3.167 template<typename _Talpha , typename _Tbeta , typename _Tp > __gnu_cxx::__promote_num_t<_Talpha, _Tbeta, _Tp > __gnu_cxx::jacobi ( unsigned __n, _Talpha __alpha, _Tbeta __beta, _Tp __x ) [inline]
```

Definition at line 1957 of file specfun.h.

References std::__detail::__beta().

Return the Jacobi elliptic cn integral of modulus k and argument u.

The Jacobi elliptic cn integral is defined by

Parameters

k	The modulus
u	The argument

Definition at line 1721 of file specfun.h.

```
7.3.3.169 float __gnu_cxx::jacobi_cnf(float __k, float __u) [inline]
```

Return the Jacobi elliptic cn integral of modulus k and argument u.

See also

jacobi_cn for details.

Definition at line 1690 of file specfun.h.

```
7.3.3.170 long double __gnu_cxx::jacobi_cnl( long double __k, long double __u ) [inline]
```

Return the Jacobi elliptic cn integral of modulus k and argument u.

See also

jacobi cn for details.

Definition at line 1702 of file specfun.h.

Return the Jacobi elliptic ${\tt dn}$ integral of modulus ${\tt k}$ and argument ${\tt u}.$

The Jacobi elliptic dn integral is defined by

Parameters

k	The modulus
u	The argument

Definition at line 1767 of file specfun.h.

```
7.3.3.172 float __gnu_cxx::jacobi_dnf(float __k, float __u) [inline]
```

Return the Jacobi elliptic dn integral of modulus k and argument u.

See also

jacobi_dn for details.

Definition at line 1736 of file specfun.h.

```
7.3.3.173 long double __gnu_cxx::jacobi_dnl( long double __k, long double __u) [inline]
```

Return the Jacobi elliptic $\mathtt{d} n$ integral of modulus \mathtt{k} and argument $\mathtt{u}.$

See also

jacobi_dn for details.

Definition at line 1748 of file specfun.h.

Return the Jacobi elliptic sn integral of modulus k and argument u.

The Jacobi elliptic sn integral is defined by

Parameters

k	The modulus
u	The argument

Definition at line 1675 of file specfun.h.

```
7.3.3.175 float __gnu_cxx::jacobi_snf(float __k, float __u) [inline]
```

Return the Jacobi elliptic sn integral of modulus k and argument u.

See also

jacobi_sn for details.

Definition at line 1644 of file specfun.h.

```
7.3.3.176 long double __gnu_cxx::jacobi_snl( long double __k, long double __u) [inline]
```

Return the Jacobi elliptic sn integral of modulus k and argument u.

See also

jacobi_sn for details.

Definition at line 1656 of file specfun.h.

```
7.3.3.177 template<typename _Tk , typename _Tphi > __gnu_cxx::__promote_num_t<_Tk, _Tphi> __gnu_cxx::jacobi_zeta ( _Tk __k, _Tphi __phi ) [inline]
```

Return the Jacobi zeta function of k and $@c\phi$.

The Jacobi zeta function is defined by

Parameters

k	The modulus
phi	The angle

Definition at line 3331 of file specfun.h.

```
7.3.3.178 float __gnu_cxx::jacobi_zetaf ( float __k, float __phi ) [inline]
```

Definition at line 3311 of file specfun.h.

```
7.3.3.179 long double __gnu_cxx::jacobi_zetal(long double __k, long double __phi) [inline]
```

Definition at line 3315 of file specfun.h.

```
7.3.3.180 float _gnu_cxx::jacobif ( unsigned _n, float _alpha, float _beta, float _x ) [inline]
```

Definition at line 1945 of file specfun.h.

References std::__detail::__beta().

Definition at line 1949 of file specfun.h.

References std:: detail:: beta().

```
7.3.3.182 template<typename_Tp > __gnu_cxx::_promote_num_t<_Tp> __gnu_cxx::lbincoef ( unsigned int __n, unsigned int
          k) [inline]
Definition at line 3211 of file specfun.h.
7.3.3.183 float __gnu_cxx::lbincoeff ( unsigned int __n, unsigned int __k ) [inline]
Definition at line 3199 of file specfun.h.
7.3.3.184 long double __gnu_cxx::lbincoefl ( unsigned int __n, unsigned int __k ) [inline]
Definition at line 3203 of file specfun.h.
7.3.3.185 template<typename_Tp > __gnu_cxx::__promote_num_t<_Tp > __gnu_cxx::Idouble_factorial ( int __n )
          [inline]
Definition at line 3169 of file specfun.h.
7.3.3.186 float __gnu_cxx::ldouble_factorialf(int __n) [inline]
Definition at line 3157 of file specfun.h.
7.3.3.187 long double __gnu_cxx::ldouble_factoriall(int __n) [inline]
Definition at line 3161 of file specfun.h.
7.3.3.188 template<typename_Tp > __gnu_cxx::_promote_num_t<_Tp> __gnu_cxx::legendre_q ( unsigned int __n, _Tp __x )
          [inline]
Definition at line 3260 of file specfun.h.
7.3.3.189 float __gnu_cxx::legendre_qf( unsigned int __n, float __x ) [inline]
Definition at line 3248 of file specfun.h.
7.3.3.190 long double __gnu_cxx::legendre_ql( unsigned int __n, long double __x ) [inline]
Definition at line 3252 of file specfun.h.
7.3.3.191 template<typename_Tp > __gnu_cxx::_promote_num_t<_Tp> __gnu_cxx::lfactorial ( unsigned int __n )
          [inline]
Definition at line 3148 of file specfun.h.
7.3.3.192 float __gnu_cxx::lfactorialf ( unsigned int __n ) [inline]
```

Definition at line 3136 of file specfun.h.

```
7.3.3.193 long double __gnu_cxx::lfactoriall ( unsigned int __n ) [inline]
```

Definition at line 3140 of file specfun.h.

Return the logarithmic integral of argument x.

The logarithmic integral is defined by

Parameters

```
__x
```

Definition at line 1475 of file specfun.h.

```
7.3.3.195 float __gnu_cxx::logintf(float __x) [inline]
```

Return the logarithmic integral of argument x.

See also

logint for details.

Definition at line 1451 of file specfun.h.

```
7.3.3.196 long double __gnu_cxx::logintl(long double __x) [inline]
```

Return the logarithmic integral of argument x.

See also

logint for details.

Definition at line 1460 of file specfun.h.

```
7.3.3.197 template<typename_Tp , typename_Tn > __gnu_cxx::__promote_num_t<_Tp, _Tn> __gnu_cxx::lpochhammer_I ( _Tp __a, _Tn __n ) [inline]
```

Definition at line 3043 of file specfun.h.

```
7.3.3.198 float __gnu_cxx::lpochhammer_lf(float __a, float __n) [inline]
```

Definition at line 3031 of file specfun.h.

7.3.3.199 long double __gnu_cxx::lpochhammer_ll(long double __a, long double __n) [inline]

Definition at line 3035 of file specfun.h.

7.3.3.200 template<typename_Tp , typename _Tn > __gnu_cxx::__promote_num_t<_Tp, _Tn> __gnu_cxx::|pochhammer_u (_Tp __a, _Tn __n) [inline]

Definition at line 3022 of file specfun.h.

7.3.3.201 float __gnu_cxx::lpochhammer_uf(float __a, float __n) [inline]

Definition at line 3010 of file specfun.h.

7.3.3.202 long double __gnu_cxx::lpochhammer_ul(long double __a, long double __n) [inline]

Definition at line 3014 of file specfun.h.

7.3.3.203 template<typename _Tph , typename _Tpa > __gnu_cxx::__promote_num_t<_Tph, _Tpa > __gnu_cxx::owens_t (_Tph __h, _Tpa __a) [inline]

Return the Owens T function of thing1 h and thing2 a.

The Owens T function is defined by

$$T(h,a) = \frac{1}{2\pi} \int_0^a \frac{\exp\left[-\frac{1}{2}h^2(1+x^2)\right]}{1+x^2} dx$$

Parameters

h	The shape factor
a	The integration lomit

Definition at line 4364 of file specfun.h.

7.3.3.204 float __gnu_cxx::owens_tf(float __h, float __a) [inline]

Return the Owens T function function of thing h and argument a.

See also

owens_t for details.

Definition at line 4338 of file specfun.h.

7.3.3.205 long double __gnu_cxx::owens_tl(long double __h, long double __a) [inline]

Return the Owens T function function of thing h and argument a.

See also

owens t for details.

Definition at line 4347 of file specfun.h.

```
7.3.3.206 template<typename_Tp , typename_Tn > __gnu_cxx::__promote_num_t<_Tp, _Tn> __gnu_cxx::pochhammer_I ( _Tp a. Tn n ) [inline]
```

Definition at line 3085 of file specfun.h.

```
7.3.3.207 float __gnu_cxx::pochhammer_lf ( float __a, float __n ) [inline]
```

Definition at line 3073 of file specfun.h.

```
7.3.3.208 long double __gnu_cxx::pochhammer_ll( long double __a, long double __n) [inline]
```

Definition at line 3077 of file specfun.h.

Definition at line 3064 of file specfun.h.

```
7.3.3.210 float __gnu_cxx::pochhammer_uf( float __a, float __n ) [inline]
```

Definition at line 3052 of file specfun.h.

```
7.3.3.211 long double __gnu_cxx::pochhammer_ul ( long double __a, long double __n ) [inline]
```

Definition at line 3056 of file specfun.h.

Return the complex polylogarithm function of real thing s and complex argument w.

The polylogarithm function is defined by

Parameters

s	
w	

Definition at line 3776 of file specfun.h.

```
7.3.3.213 std::complex < float > __gnu_cxx::polylogf ( float __s, std::complex < float > __w ) [inline]
```

Return the complex polylogarithm function of real thing s and complex argument w.

See also

polylog for details.

Definition at line 3749 of file specfun.h.

7.3.3.214 std::complex < long double > __gnu_cxx::polylogl (long double __s, std::complex < long double > __w) [inline]

Return the complex polylogarithm function of real thing s and complex argument w.

See also

polylog for details.

Definition at line 3759 of file specfun.h.

Return the psi or digamma function of argument x.

The the psi or digamma function is defined by

$$\psi(x) =$$

Parameters

X	The parameter

Definition at line 2788 of file specfun.h.

Definition at line 2769 of file specfun.h.

Definition at line 2773 of file specfun.h.

Definition at line 2020 of file specfun.h.

```
7.3.3.219 float __gnu_cxx::radpolyf ( unsigned int __n, unsigned int __m, float __rho ) [inline]
```

Definition at line 2008 of file specfun.h.

References std:: detail:: poly radial jacobi().

```
7.3.3.220 long double __gnu_cxx::radpolyl ( unsigned int __n, unsigned int __n, long double __rho ) [inline]
```

Definition at line 2012 of file specfun.h.

References std::__detail::__poly_radial_jacobi().

Definition at line 1437 of file specfun.h.

```
7.3.3.222 template<typename_Tp > __gnu_cxx::_promote_num_t<_Tp> __gnu_cxx::sinc_pi( _Tp __x ) [inline]
Definition at line 1419 of file specfun.h.
7.3.3.223 float __gnu_cxx::sinc_pif(float __x) [inline]
Definition at line 1407 of file specfun.h.
7.3.3.224 long double __gnu_cxx::sinc_pil( long double __x ) [inline]
Definition at line 1411 of file specfun.h.
7.3.3.225 float __gnu_cxx::sincf(float __x) [inline]
Definition at line 1428 of file specfun.h.
7.3.3.226 long double __gnu_cxx::sincl( long double __x ) [inline]
Definition at line 1432 of file specfun.h.
\textbf{7.3.3.227} \quad template < typename \_Tp > \_\_gnu\_cxx::\_promote\_num\_t < \_Tp > \_\_gnu\_cxx::sinhc ( \_Tp \_x ) \quad \texttt{[inline]}
Definition at line 2062 of file specfun.h.
Definition at line 2041 of file specfun.h.
7.3.3.229 float __gnu_cxx::sinhc_pif(float __x) [inline]
Definition at line 2029 of file specfun.h.
7.3.3.230 long double __gnu_cxx::sinhc_pil( long double __x ) [inline]
Definition at line 2033 of file specfun.h.
7.3.3.231 float __gnu_cxx::sinhcf(float __x) [inline]
Definition at line 2050 of file specfun.h.
7.3.3.232 long double __gnu_cxx::sinhcl( long double __x ) [inline]
Definition at line 2054 of file specfun.h.
```

```
7.3.3.233 template<typename_Tp > __gnu_cxx::_promote_num_t<_Tp> __gnu_cxx::sinhint( _Tp __x ) [inline]
```

Return the hyperbolic sine integral of argument x.

The sine hyperbolic integral is defined by

Parameters

```
__x The argument
```

Definition at line 1589 of file specfun.h.

```
7.3.3.234 float __gnu_cxx::sinhintf(float __x) [inline]
```

Return the hyperbolic sine integral of argument x.

See also

sinhint for details.

Definition at line 1565 of file specfun.h.

```
7.3.3.235 long double __gnu_cxx::sinhintl( long double __x ) [inline]
```

Return the hyperbolic sine integral of argument x.

See also

sinhint for details.

Definition at line 1574 of file specfun.h.

Return the sine integral of argument x.

The sine integral is defined by

Parameters

```
__x The argument
```

Definition at line 1513 of file specfun.h.

```
7.3.3.237 float __gnu_cxx::sinintf(float __x) [inline]
```

Return the sine integral of argument x.

See also

sinint for details.

Definition at line 1489 of file specfun.h.

```
7.3.3.238 long double __gnu_cxx::sinintl( long double __x ) [inline]
```

Return the sine integral of argument x.

See also

sinint for details.

Definition at line 1498 of file specfun.h.

```
7.3.3.239 template<typename_Tp > __gnu_cxx::__promote_num_t<_Tp> __gnu_cxx::sph_bessel_i( unsigned int __n, _Tp __x ) [inline]
```

Definition at line 2177 of file specfun.h.

```
7.3.3.240 float __gnu_cxx::sph_bessel_if ( unsigned int __n, float __x ) [inline]
```

Definition at line 2155 of file specfun.h.

Definition at line 2164 of file specfun.h.

Definition at line 2211 of file specfun.h.

```
7.3.3.243 float __gnu_cxx::sph_bessel_kf( unsigned int __n, float __x ) [inline]
```

Definition at line 2189 of file specfun.h.

```
7.3.3.244 long double _gnu cxx::sph bessel kl ( unsigned int _n, long double _x ) [inline]
```

Definition at line 2198 of file specfun.h.

Definition at line 2125 of file specfun.h.

Return the complex spherical Hankel function of the first kind of nonnegative order n and complex argument x.

The spherical Hankel function of the first kind is defined by

$$h_n^{(1)}(x) = j_n(x) + i n_n(x)$$

Parameters

n	The integral order >= 0
X	The complex argument

Definition at line 3654 of file specfun.h.

7.3.3.247 std::complex<float> __gnu_cxx::sph_hankel_1f(unsigned int __n, float __z) [inline]

Definition at line 2113 of file specfun.h.

7.3.3.248 std::complex < float > __gnu_cxx::sph_hankel_1f (unsigned int __n, std::complex < float > __x) [inline]

Definition at line 3633 of file specfun.h.

7.3.3.249 std::complex < long double > __gnu_cxx::sph_hankel_1I (unsigned int __n, long double __z) [inline]

Definition at line 2117 of file specfun.h.

7.3.3.250 std::complex < long double > __gnu_cxx::sph_hankel_1I (unsigned int __n, std::complex < long double > __x) [inline]

Definition at line 3637 of file specfun.h.

Definition at line 2146 of file specfun.h.

7.3.3.252 template<typename _Tp > std::complex<__gnu_cxx::__promote_num_t<_Tp> > __gnu_cxx::sph_hankel_2 (unsigned int __n, std::complex< _Tp > __x) [inline]

Return the complex spherical Hankel function of the second kind of nonnegative order n and complex argument x.

The spherical Hankel function of the second kind is defined by

$$h_n^{(2)}(x) = j_n(x) - in_n(x)$$

Parameters

n	The integral order >= 0
X	The complex argument

Definition at line 3684 of file specfun.h.

7.3.3.253 std::complex < float > __gnu_cxx::sph_hankel_2f (unsigned int __n, float __z) [inline]

Definition at line 2134 of file specfun.h.

7.3.3.254 std::complex < float > __gnu_cxx::sph_hankel_2f (unsigned int __n, std::complex < float > __x) [inline]

Definition at line 3663 of file specfun.h.

7.3.3.255 std::complex < long double > __gnu_cxx::sph_hankel_2I (unsigned int __n, long double __z) [inline]

Definition at line 2138 of file specfun.h.

7.3.3.256 std::complex < long double > $_$ gnu_cxx::sph_hankel_2l (unsigned int $_$ n, std::complex < long double > $_$ x) [inline]

Definition at line 3667 of file specfun.h.

Return the complex spherical harmonic function of degree 1, order m, and real zenith angle θ , and real azimuth angle ϕ .

The spherical harmonic function is defined by:

$$Y_l^m(\theta,\phi) = (-1)^m \left[\frac{(2l+1)}{4\pi} \frac{(l-m)!}{(l+m)!} \right] P_l^{|m|}(\cos\theta) \exp^{im\phi}$$

Parameters

	The order
m	The degree
theta	The zenith angle in radians
phi	The azimuth angle in radians

Definition at line 3734 of file specfun.h.

Return the complex spherical harmonic function of degree 1, order m, and real zenith angle θ , and real azimuth angle ϕ .

See also

sph_harmonic for details.

Definition at line 3699 of file specfun.h.

7.3.3.259 std::complex<long double> __gnu_cxx::sph_harmonicl(unsigned int __l, int __m, long double __theta, long double __phi) [inline]

Return the complex spherical harmonic function of degree 1, order m, and real zenith angle θ , and real azimuth angle ϕ .

See also

sph harmonic for details.

Definition at line 3710 of file specfun.h.

Return the exponential theta-1 function of period nu and argument x.

The Neville theta-1 function is defined by

$$\theta_1(\nu, x) = \frac{1}{\sqrt{\pi x}} \sum_{j=-\infty}^{+\infty} (-1)^j \exp\left(\frac{-(\nu + j - 1/2)^2}{x}\right)$$

Parameters

nu	The periodic (period = 2) argument
x	The argument

Definition at line 4020 of file specfun.h.

Return the exponential theta-1 function of period nu and argument x.

See also

theta_1 for details.

Definition at line 3994 of file specfun.h.

Return the exponential theta-1 function of period nu and argument x.

See also

theta_1 for details.

Definition at line 4003 of file specfun.h.

Return the exponential theta-2 function of period nu and argument x.

The exponential theta-2 function is defined by

$$\theta_2(\nu, x) = \frac{1}{\sqrt{\pi x}} \sum_{j=-\infty}^{+\infty} (-1)^j \exp\left(\frac{-(\nu+j)^2}{x}\right)$$

Parameters

nu	The periodic (period = 2) argument
X	The argument

Definition at line 4060 of file specfun.h.

```
7.3.3.264 float __gnu_cxx::theta_2f(float __nu, float __x) [inline]
```

Return the exponential theta-2 function of period nu and argument x.

See also

theta 2 for details.

Definition at line 4034 of file specfun.h.

```
7.3.3.265 long double __gnu_cxx::theta_2l ( long double __nu, long double __x ) [inline]
```

Return the exponential theta-2 function of period nu and argument x.

See also

theta 2 for details.

Definition at line 4043 of file specfun.h.

Return the exponential theta-3 function of period nu and argument x.

The exponential theta-3 function is defined by

$$\theta_3(\nu, x) = \frac{1}{\sqrt{\pi x}} \sum_{j=-\infty}^{+\infty} \exp\left(\frac{-(\nu+j)^2}{x}\right)$$

Parameters

nu	The periodic (period = 1) argument
X	The argument

Definition at line 4100 of file specfun.h.

Return the exponential theta-3 function of period nu and argument x.

See also

theta 3 for details.

Definition at line 4074 of file specfun.h.

7.3.3.268 long double __gnu_cxx::theta_3l (long double __nu, long double __x) [inline]

Return the exponential theta-3 function of period nu and argument x.

See also

theta_3 for details.

Definition at line 4083 of file specfun.h.

Return the exponential theta-4 function of period nu and argument x.

The exponential theta-4 function is defined by

$$\theta_4(\nu, x) = \frac{1}{\sqrt{\pi x}} \sum_{j=-\infty}^{+\infty} \exp\left(\frac{-(\nu + j + 1/2)^2}{x}\right)$$

Parameters

nu	The periodic (period = 1) argument
X	The argument

Definition at line 4140 of file specfun.h.

7.3.3.270 float __gnu_cxx::theta_4f(float __nu, float __x) [inline]

Return the exponential theta-4 function of period nu and argument x.

See also

theta 4 for details.

Definition at line 4114 of file specfun.h.

Return the exponential theta-4 function of period nu and argument x.

See also

theta_4 for details.

Definition at line 4123 of file specfun.h.

Return the Neville theta-c function of modulus ${\bf k}$ and argument ${\bf x}$.

The Neville theta-c function is defined by

Parameters

k	The modulus $-1 <= k <= +1$
X	The argument

Definition at line 4246 of file specfun.h.

```
7.3.3.273 float __gnu_cxx::theta_cf(float __k, float __x) [inline]
```

Return the Neville theta-c function of modulus ${\bf k}$ and argument ${\bf x}$.

See also

theta_c for details.

Definition at line 4221 of file specfun.h.

```
7.3.3.274 long double __gnu_cxx::theta_cl( long double __k, long double __x ) [inline]
```

Return the Neville theta-c function of modulus ${\bf k}$ and argument ${\bf x}$.

See also

theta c for details.

Definition at line 4230 of file specfun.h.

Return the Neville theta-d function of modulus ${\bf k}$ and argument ${\bf x}$.

The Neville theta-d function is defined by

Parameters

k	The modulus $-1 <= k <= +1$
X	The argument

Definition at line 4285 of file specfun.h.

```
7.3.3.276 float __gnu_cxx::theta_df ( float __k, float __x ) [inline]
```

Return the Neville theta-d function of modulus ${\bf k}$ and argument ${\bf x}$.

See also

theta d for details.

Definition at line 4260 of file specfun.h.

7.3.3.277 long double __gnu_cxx::theta_dl(long double __k, long double __x) [inline]

Return the Neville theta-d function of modulus k and argument x.

See also

theta d for details.

Definition at line 4269 of file specfun.h.

Return the Neville theta-n function of modulus ${\bf k}$ and argument ${\bf x}$.

The Neville theta-n function is defined by

Parameters

k	The modulus $-1 <= k <= +1$
X	The argument

Definition at line 4324 of file specfun.h.

Return the Neville theta-n function of modulus ${\bf k}$ and argument ${\bf x}.$

See also

theta n for details.

Definition at line 4299 of file specfun.h.

```
7.3.3.280 long double __gnu_cxx::theta_nl( long double __k, long double __x ) [inline]
```

Return the Neville theta-n function of modulus ${\bf k}$ and argument ${\bf x}.$

See also

theta n for details.

Definition at line 4308 of file specfun.h.

Return the Neville theta-s function of modulus ${\bf k}$ and argument ${\bf x}$.

The Neville theta-s function is defined by

Parameters

k	The modulus $-1 <= k <= +1$
X	The argument

Definition at line 4207 of file specfun.h.

```
7.3.3.282 float __gnu_cxx::theta_sf(float __k, float __x) [inline]
```

Return the Neville theta-s function of modulus ${\bf k}$ and argument ${\bf x}.$

See also

theta_s for details.

Definition at line 4182 of file specfun.h.

```
7.3.3.283 long double __gnu_cxx::theta_sl( long double __k, long double __x ) [inline]
```

Return the Neville theta-s function of modulus ${\bf k}$ and argument ${\bf x}.$

See also

theta_s for details.

Definition at line 4191 of file specfun.h.

```
7.3.3.284 template<typename _Trho , typename _Tphi > __gnu_cxx::__promote_num_t<_Trho, _Tphi> __gnu_cxx::zernike ( unsigned int __n, int __m, _Trho __rho, _Tphi __phi ) [inline]
```

Definition at line 1999 of file specfun.h.

```
7.3.3.285 float _gnu_cxx::zernikef ( unsigned int _n, int _m, float _rho, float _phi ) [inline]
```

Definition at line 1987 of file specfun.h.

```
7.3.3.286 long double __gnu_cxx::zernikel ( unsigned int __n, int __m, long double __rho, long double __phi ) [inline]
```

Definition at line 1991 of file specfun.h.

Chapter 8

Namespace Documentation

8.1 __gnu_cxx Namespace Reference

Enumerations

enum { _GLIBCXX_JACOBI_SN, _GLIBCXX_JACOBI_CN, _GLIBCXX_JACOBI_DN }

Functions

```
template<typename _Tp >
  __gnu_cxx::__promote_num_t< _Tp > airy_ai (_Tp __x)

 float airy_aif (float __x)

• long double airy ail (long double x)
• template<typename _{\mathrm{Tp}} >
    _gnu_cxx::__promote_num_t< _Tp > airy_bi (_Tp __x)

    float airy bif (float x)

    long double airy_bil (long double __x)

• template<typename _Tp >
   _gnu_cxx::__promote_num_t< _Tp > bernoulli (unsigned int __n)

    float bernoullif (unsigned int n)

    long double bernoullil (unsigned int __n)

template<typename</li>Tp >
   _gnu_cxx::__promote_num_t< _Tp > bincoef (unsigned int __n, unsigned int __k)
• float bincoeff (unsigned int __n, unsigned int __k)
• long double bincoefl (unsigned int n, unsigned int k)
template<typename</li>Tp >
  __gnu_cxx::__promote_num_t< _Tp > chebyshev_t (unsigned int __n, _Tp __x)

    float chebyshev_tf (unsigned int __n, float __x)

    long double chebyshev_tl (unsigned int __n, long double __x)

• template<typename _{\mathrm{Tp}} >
    _gnu_cxx::__promote_num_t< _Tp > chebyshev_u (unsigned int __n, _Tp __x)

    float chebyshev uf (unsigned int n, float x)

    long double chebyshev_ul (unsigned int __n, long double __x)

• template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > chebyshev_v (unsigned int __n, _Tp __x)

    float chebyshev_vf (unsigned int __n, float __x)
```

```
    long double chebyshev_vl (unsigned int __n, long double __x)

template<typename _Tp >
    gnu cxx:: promote num t < Tp > chebyshev w (unsigned int n, Tp x)

    float chebyshev wf (unsigned int n, float x)

    long double chebyshev wl (unsigned int n, long double x)

template<typename_Tp>
   _gnu_cxx::__promote_num_t< _Tp > clausen (unsigned int __m, _Tp __w)

    template<typename _Tp >

  std::complex < gnu cxx:: promote num t < Tp > > clausen (unsigned int m, std::complex < Tp > w)
template<typename _Tp >
   gnu cxx:: promote num t< Tp> clausen c (unsigned int m, Tp w)

    float clausen_cf (unsigned int __m, float __w)

• long double clausen_cl (unsigned int __m, long double __w)
template<typename</li>Tp >
    gnu cxx:: promote num t< Tp > clausen s (unsigned int m, Tp w)

    float clausen_sf (unsigned int __m, float __w)

    long double clausen_sl (unsigned int __m, long double __w)

    float clausenf (unsigned int m, float w)

• std::complex< float > clausenf (unsigned int m, std::complex< float > w)
• long double clausenl (unsigned int m, long double w)

    std::complex < long double > clausenl (unsigned int __m, std::complex < long double > __w)

• template<typename_Tk>
    _gnu_cxx::__promote_num_t< _Tk > comp_ellint_d (_Tk k)

    float comp ellint df (float k)

    long double comp_ellint_dl (long double __k)

    float comp ellint rf (float x, float y)

    long double comp_ellint_rf (long double __x, long double __y)

    template<typename _Tx , typename _Ty >

   _gnu_cxx::__promote_num_t< _Tx, _Ty > comp_ellint_rf (_Tx __x, _Ty __y)

    float comp_ellint_rg (float __x, float __y)

• long double comp_ellint_rg (long double __x, long double __y)

    template<typename _Tx , typename _Ty >

   gnu cxx:: promote num t< Tx, Ty> comp ellint rg (Txx, Tyy)

    template<typename _Tpa , typename _Tpc , typename _Tp >

   __gnu_cxx::__promote_3< _Tpa, _Tpc, _Tp >::__type conf_hyperg (_Tpa __a, _Tpc __c, _Tp __x)
• template<typename _Tpc , typename _Tp >
    _gnu_cxx::__promote_2< _Tpc, _Tp >::__type conf_hyperg_lim (_Tpc __c, _Tp __x)

    float conf hyperg limf (float c, float x)

    long double conf hyperg liml (long double c, long double x)

    float conf_hypergf (float __a, float __c, float __x)

    long double conf hypergl (long double a, long double c, long double x)

template<typename</li>Tp >
    _gnu_cxx::__promote_num_t< _Tp > coshint (_Tp __x)

    float coshintf (float x)

    long double coshintl (long double x)

template<typename</li>Tp >
   __gnu_cxx::__promote_num_t< _Tp > cosint (_Tp x)

    float cosintf (float x)

    long double cosintl (long double __x)

template<typename _Tpnu , typename _Tp >
  std::complex< __gnu_cxx::__promote_num_t< _Tpnu, _Tp >> cyl_hankel_1 (_Tpnu __nu, _Tp __z)
```

```
    template<typename _Tpnu , typename _Tp >

  std::complex< __gnu_cxx::__promote_num_t< _Tpnu, _Tp >> cyl_hankel_1 (std::complex< _Tpnu > __nu,
  std::complex < Tp > x)

    std::complex< float > cyl_hankel_1f (float __nu, float __z)

    std::complex < float > cyl_hankel_1f (std::complex < float > __nu, std::complex < float > __x)

• std::complex < long double > cyl hankel 1l (long double nu, long double z)
• std::complex < long double > cyl hankel 1l (std::complex < long double > nu, std::complex < long double >
   __x)

    template<typename _Tpnu , typename _Tp >

  std::complex< __gnu_cxx::__promote_num_t< _Tpnu, _Tp >> cyl_hankel_2 (_Tpnu __nu, _Tp __z)
• template<typename _{\rm Tpnu}, typename _{\rm Tp} >
  std::complex< __gnu_cxx::__promote_num_t< _Tpnu, _Tp >> cyl_hankel_2 (std::complex< _Tpnu > __nu,
  std::complex < _Tp > __x)

    std::complex< float > cyl_hankel_2f (float __nu, float __z)

    std::complex < float > cyl hankel 2f (std::complex < float > nu, std::complex < float > x)

    std::complex < long double > cyl hankel 2l (long double nu, long double z)

    std::complex < long double > cyl hankel 2l (std::complex < long double > nu, std::complex < long double >

   X)
template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > dawson (_Tp __x)

    float dawsonf (float x)

    long double dawsonl (long double x)

template<typename</li>Tp >
    _gnu_cxx::__promote_num_t< _Tp > digamma (_Tp __z)
• float digammaf (float z)

    long double digammal (long double __z)

template<typename</li>Tp >
   _gnu_cxx::__promote_num_t< _Tp > dilog (_Tp __x)

 float dilogf (float __x)

    long double dilogl (long double __x)

    template<typename</li>
    Tp >

  _Tp dirichlet_beta (_Tp __x)

    float dirichlet_betaf (float __x)

    long double dirichlet betal (long double x)

template<typename _Tp >
  _Tp dirichlet_eta (_Tp __x)

    float dirichlet etaf (float x)

    long double dirichlet etal (long double x)

template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > double_factorial (int __n)

    float double factorialf (int n)

    long double double_factoriall (int __n)

- template<typename _Tk , typename _Tp , typename _Ta , typename _Tb >
   _gnu_cxx::__promote_num_t< _Tk, _Tp, _Ta, _Tb > ellint_cel (_Tk __k_c, _Tp __p, _Ta _ a, Tb b)

    float ellint_celf (float __k_c, float __p, float __a, float __b)

    long double ellint_cell (long double __k_c, long double __p, long double __a, long double __b)

• template<typename _Tk , typename _Tphi >
    gnu cxx:: promote num t < Tk, Tphi > ellint d (Tk k, Tphi phi)

    float ellint df (float k, float phi)

    long double ellint_dl (long double ___k, long double ___phi)

    template<typename _Tp , typename _Tk >

   _gnu_cxx::__promote_num_t< _Tp, _Tk > ellint_el1 (_Tp __x, _Tk __k_c)
```

```
    float ellint_el1f (float __x, float __k_c)

• long double ellint_el1l (long double __x, long double __k_c)
ullet template<typename _Tp , typename _Tk , typename _Ta , typename _Tb >
    _gnu_cxx::__promote_num_t< _Tp, _Tk, _Ta, _Tb > ellint_el2 (_Tp __x, _Tk __k_c, _Ta __a, _Tb __b)
• float ellint el2f (float x, float k c, float a, float b)

    long double ellint_el2l (long double __x, long double __k_c, long double __a, long double __b)

• template<typename _Tx , typename _Tk , typename _Tp >
    _gnu_cxx::__promote_num_t< _Tx, _Tk, _Tp > ellint_el3 (_Tx __x, _Tk __k_c, _Tp __p)
• float ellint el3f (float x, float k c, float p)

    long double ellint_el3l (long double __x, long double __k_c, long double __p)

• template<typename _Tp , typename _Up >
    gnu cxx:: promote num t< Tp, Up > ellint rc (Tp x, Up y)

    float ellint rcf (float x, float y)

    long double ellint_rcl (long double __x, long double __y)

• template<typename _Tp , typename _Up , typename _Vp >
    gnu cxx:: promote num t< Tp, Up, Vp> ellint rd ( Tp x, Up y, Vp z)
• float ellint rdf (float x, float y, float z)

    long double ellint_rdl (long double __x, long double __y, long double __z)

    template<typename _Tp , typename _Up , typename _Vp >

    _gnu_cxx::__promote_num_t< _Tp, _Up, _Vp > ellint_rf (_Tp __x, _Up __y, _Vp __z)

    float ellint_rff (float __x, float __y, float __z)

    long double ellint_rfl (long double __x, long double __y, long double __z)

• template<typename _Tp , typename _Up , typename _Vp >
    _gnu_cxx::__promote_num_t< _Tp, _Up, _Vp > ellint_rg (_Tp __x, _Up __y, _Vp __z)

    float ellint_rgf (float __x, float __y, float __z)

    long double ellint rgl (long double x, long double y, long double z)

- template<typename _Tp , typename _Up , typename _Vp , typename _Wp >
    _gnu_cxx::__promote_num_t< _Tp, _Up, _Vp, _Wp > ellint_rj (_Tp __x, _Up __y, _Vp __z, _Wp __p)

    float ellint_rjf (float __x, float __y, float __z, float __p)

    long double ellint rjl (long double x, long double y, long double z, long double p)

template<typename_Tp>
  Tp ellnome (Tp k)

 float ellnomef (float __k)

    long double ellnomel (long double k)

template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > expint_e1 (_Tp __x)

    float expint e1f (float x)

    long double expint e1l (long double x)

• template<typename _Tp >
    gnu cxx:: promote num t < Tp > expint en (unsigned int n, Tp x)

    float expint enf (unsigned int n, float x)

    long double expint enl (unsigned int n, long double x)

template<typename _Tp >
    gnu cxx:: promote num t< Tp> factorial (unsigned int n)

    float factorialf (unsigned int n)

    long double factoriall (unsigned int n)

template<typename_Tp>
    gnu cxx:: promote num t < Tp > fresnel c (Tp x)

    float fresnel cf (float x)

    long double fresnel_cl (long double __x)

template<typename_Tp>
   gnu cxx:: promote num t < Tp > fresnel s (Tp x)
```

```
 float fresnel_sf (float __x)

    long double fresnel sl (long double x)

• template<typename _Tn , typename _Tp >
    _gnu_cxx::__promote_num_t< _Tn, _Tp > gamma_l (_Tn __n, _Tp __x)

    float gamma If (float n, float x)

    long double gamma_II (long double __n, long double __x)

    template<typename</li>
    Ta , typename
    Tp >

    _gnu_cxx::__promote_num_t< _Ta, _Tp > gamma_p (_Ta __a, _Tp __x)

 float gamma_pf (float __a, float __x)

    long double gamma pl (long double a, long double x)

• template<typename _Ta , typename _Tp >
    _gnu_cxx::__promote_num_t< _Ta, _Tp > gamma_q (_Ta __a, _Tp __x)

    float gamma_qf (float __a, float __x)

    long double gamma gl (long double a, long double x)

• template<typename _Tn , typename _Tp >
   _gnu_cxx::__promote_num_t< _Tn, _Tp > gamma_u (_Tn __n, _Tp __x)

    float gamma_uf (float __n, float __x)

    long double gamma_ul (long double ___n, long double ___x)

• template<typename _Talpha , typename _Tp >
    gnu cxx:: promote num t< Talpha, Tp > gegenbauer (unsigned int n, Talpha alpha, Tp x)

    float gegenbauerf (unsigned int __n, float __alpha, float __x)

    long double gegenbauerl (unsigned int __n, long double __alpha, long double __x)

• template<typename Tk, typename Tphi >
    _gnu_cxx::__promote_num_t< _Tk, _Tphi > heuman_lambda (_Tk __k, _Tphi __phi)

    float heuman lambdaf (float k, float phi)

    long double heuman lambdal (long double k, long double phi)

    template<typename _Tp , typename _Up >

    gnu cxx:: promote num t< Tp, Up > hurwitz zeta (Tp s, Up a)
template<typename _Tp , typename _Up >
  std::complex< _Tp > hurwitz_zeta (_Tp __s, std::complex< _Up > __a)

    float hurwitz_zetaf (float __s, float __a)

    long double hurwitz_zetal (long double __s, long double __a)

• template<typename _Tpa , typename _Tpb , typename _Tpc , typename _Tp >
   _gnu_cxx::__promote_4< _Tpa, _Tpb, _Tpc, _Tp >::__type hyperg (_Tpa __a, _Tpb __b, _Tpc __c, _Tp __x)

    float hypergf (float __a, float __b, float __c, float __x)

• long double hypergl (long double __a, long double __b, long double __c, long double __x)
template<typename _Ta , typename _Tb , typename _Tp >
    _gnu_cxx::__promote_num_t< _Ta, _Tb, _Tp > ibeta (_Ta __a, _Tb __b, _Tp __x)
- template<typename _Ta , typename _Tb , typename _Tp >
   _gnu_cxx::__promote_num_t< _Ta, _Tb, _Tp > ibetac (_Ta __a, _Tb __b, _Tp __x)
• float ibetacf (float a, float b, float x)

    long double ibetacl (long double a, long double b, long double x)

 float ibetaf (float __a, float __b, float __x)

    long double ibetal (long double a, long double b, long double x)

    template<typename _Talpha , typename _Tbeta , typename _Tp >

    _gnu_cxx::__promote_num_t< _Talpha, _Tbeta, _Tp > jacobi (unsigned __n, _Talpha __alpha, _Tbeta __beta,
  Tp x)
template<typename _Kp , typename _Up >
   __gnu_cxx::__promote_num_t< _Kp, _Up > jacobi_cn (_Kp __k, _Up __u)

    float jacobi_cnf (float __k, float __u)

    long double jacobi cnl (long double k, long double u)
```

```
    template<typename _Kp , typename _Up >

    gnu cxx:: promote num t< Kp, Up> jacobi dn ( Kp k, Up u)

    float jacobi dnf (float k, float u)

    long double jacobi_dnl (long double __k, long double __u)

    template<typename</li>
    Kp , typename
    Up >

    gnu cxx:: promote num t< Kp, Up> jacobi sn ( Kp k, Up u)

    float jacobi snf (float k, float u)

    long double jacobi_snl (long double __k, long double __u)

• template<typename _Tk , typename _Tphi >
    _gnu_cxx::__promote_num_t< _Tk, _Tphi > jacobi_zeta (_Tk __k, _Tphi __phi)

    float jacobi zetaf (float k, float phi)

• long double jacobi_zetal (long double __k, long double __phi)
• float jacobif (unsigned n, float alpha, float beta, float x)

    long double jacobil (unsigned __n, long double __alpha, long double __beta, long double __x)

template<typename _Tp >
    gnu cxx:: promote num t< Tp > lbincoef (unsigned int n, unsigned int k)

    float lbincoeff (unsigned int n, unsigned int k)

    long double lbincoefl (unsigned int __n, unsigned int __k)

template<typename _Tp >
    gnu cxx:: promote num t< Tp> Idouble factorial (int n)

    float Idouble factorialf (int n)

    long double ldouble_factoriall (int __n)

template<typename _Tp >
    gnu cxx:: promote num t< Tp> legendre q (unsigned int n, Tp x)

    float legendre_qf (unsigned int __n, float __x)

    long double legendre_ql (unsigned int __n, long double __x)

template<typename_Tp>
    gnu cxx:: promote num t< Tp> Ifactorial (unsigned int n)

    float Ifactorialf (unsigned int __n)

    long double lfactoriall (unsigned int n)

template<typename_Tp>
    gnu cxx:: promote num t < Tp > logint (Tp x)

    float logintf (float x)

    long double logintl (long double x)

• template<typename _Tp , typename _Tn >
    _gnu_cxx::__promote_num_t< _Tp, _Tn > lpochhammer_l (_Tp __a, _Tn __n)

    float lpochhammer If (float a, float n)

    long double lpochhammer_ll (long double __a, long double __n)

• template<typename _Tp , typename _Tn >
    gnu cxx:: promote num t< Tp, Tn> lpochhammer u (Tp a, Tn n)

    float lpochhammer uf (float a, float n)

    long double lpochhammer ul (long double a, long double n)

• template<typename _Tph , typename _Tpa >
    _gnu_cxx::__promote_num_t< _Tph, _Tpa > owens_t (_Tph __h, _Tpa __a)

    float owens tf (float h, float a)

    long double owens_tl (long double __h, long double __a)

• template<typename _Tp , typename _Tn >
    _gnu_cxx::__promote_num_t< _Tp, _Tn > pochhammer_l (_Tp __a, _Tn __n)

    float pochhammer If (float a, float n)

    long double pochhammer_II (long double __a, long double __n)

    template<typename _Tp , typename _Tn >

   __gnu_cxx::__promote_num_t< _Tp, _Tn > pochhammer_u (_Tp __a, _Tn __n)
```

```
    float pochhammer_uf (float __a, float __n)

    long double pochhammer_ul (long double __a, long double __n)

template<typename _Tp >
  std::complex < __gnu_cxx::__promote_num_t < _Tp >> polylog (_Tp __s, std::complex < _Tp > __w)

    std::complex< float > polylogf (float __s, std::complex< float > __w)

    std::complex < long double > polylogl (long double ___s, std::complex < long double > __w)

template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > psi (_Tp __x)

    float psif (float x)

    long double psil (long double __x)

template<typename _Tp >
    gnu cxx:: promote num t < Tp > radpoly (unsigned int n, unsigned int m, Tp rho)

    float radpolyf (unsigned int n, unsigned int m, float rho)

    long double radpolyl (unsigned int __n, unsigned int __m, long double __rho)

template<typename</li>Tp >
    gnu cxx:: promote num t < Tp > sinc ( Tp x)
template<typename _Tp >
   __gnu_cxx::__promote_num_t< _Tp > sinc_pi (_Tp __x)

    float sinc pif (float x)

    long double sinc pil (long double x)

 float sincf (float __x)

    long double sincl (long double __x)

template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > sinhc (_Tp __x)
template<typename _Tp >
   __gnu_cxx::__promote_num_t< _Tp > sinhc_pi (_Tp x)

    float sinhc pif (float x)

    long double sinhc pil (long double x)

 float sinhcf (float __x)

    long double sinhcl (long double x)

template<typename_Tp>
    _gnu_cxx::__promote_num_t< _Tp > sinhint (_Tp __x)

    float sinhintf (float x)

    long double sinhintl (long double x)

template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > sinint (_Tp __x)

    float sinintf (float x)

    long double sinintl (long double x)

template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > sph_bessel_i (unsigned int __n, _Tp __x)

    float sph_bessel_if (unsigned int __n, float __x)

    long double sph bessel il (unsigned int n, long double x)

template<typename_Tp>
    _gnu_cxx::__promote_num_t< _Tp > sph_bessel_k (unsigned int __n, _Tp __x)

    float sph bessel kf (unsigned int n, float x)

    long double sph bessel kl (unsigned int n, long double x)

template<typename_Tp>
  std::complex< __gnu_cxx::__promote_num_t< _Tp >> sph_hankel_1 (unsigned int __n, _Tp __z)
template<typename _Tp >
  std::complex< __gnu_cxx::__promote_num_t< _Tp >> sph_hankel_1 (unsigned int __n, std::complex< _Tp >

    std::complex< float > sph hankel 1f (unsigned int n, float z)
```

```
    std::complex < float > sph_hankel_1f (unsigned int __n, std::complex < float > __x)

    std::complex < long double > sph hankel 1l (unsigned int n, long double z)

    std::complex < long double > sph hankel 1! (unsigned int n, std::complex < long double > x)

template<typename _Tp >
  std::complex< __gnu_cxx::__promote_num_t< _Tp >> sph_hankel_2 (unsigned int __n, _Tp __z)

    template<typename</li>
    Tp >

  std::complex < __gnu_cxx::__promote_num_t < _Tp > > sph_hankel_2 (unsigned int __n, std::complex < _Tp >

    std::complex< float > sph hankel 2f (unsigned int n, float z)

    std::complex< float > sph_hankel_2f (unsigned int __n, std::complex< float > __x)

• std::complex < long double > sph hankel 2l (unsigned int n, long double z)

    std::complex < long double > sph hankel 2l (unsigned int n, std::complex < long double > x)

• template<typename _Ttheta , typename _Tphi >
  std::complex < \underline{\quad} gnu\_cxx::\underline{\quad} promote\_num\_t < \underline{\quad} Ttheta, \underline{\quad} Tphi >> sph\_harmonic \ (unsigned \ int \underline{\quad} I, \ int \underline{\quad} m, \\
  Ttheta theta, Tphi phi)

    std::complex < float > sph_harmonicf (unsigned int __l, int __m, float __theta, float __phi)

• std::complex < long double > sph_harmonicl (unsigned int __I, int __m, long double __theta, long double __phi)
template<typename _Tpnu , typename _Tp >
    gnu cxx:: promote num t< Tpnu, Tp> theta 1 (Tpnu nu, Tpx)

    float theta 1f (float nu, float x)

    long double theta_1l (long double __nu, long double __x)

• template<typename _Tpnu , typename _Tp >
    gnu cxx:: promote num t< Tpnu, Tp> theta 2 ( Tpnu nu, Tp> x)

 float theta_2f (float __nu, float __x)

    long double theta_2l (long double __nu, long double __x)

• template<typename _Tpnu , typename _Tp >
    _gnu_cxx::__promote_num_t< _Tpnu, _Tp > theta_3 (_Tpnu __nu, _Tp __x)

    float theta 3f (float nu, float x)

    long double theta 3l (long double nu, long double x)

• template<typename _Tpnu , typename _Tp >
    gnu cxx:: promote num t< Tpnu, Tp> theta 4 (Tpnu nu, Tpx)
• float theta_4f (float _ nu, float _ x)

    long double theta_4l (long double __nu, long double __x)

• template<typename _Tpk , typename _Tp >
    \_gnu\_cxx::\_promote\_num\_t < \_Tpk, \_Tp > theta\_c (\_Tpk \__k, \_Tp \__x)
• float theta cf (float k, float x)

    long double theta_cl (long double ___k, long double ___x)

• template<typename \_\mathsf{Tpk} , typename \_\mathsf{Tp}>
    _gnu_cxx::__promote_num_t< _Tpk, _Tp > theta_d (_Tpk __k, _Tp __x)
• float theta df (float k, float x)

    long double theta dl (long double k, long double x)

• template<typename _{\rm Tpk}, typename _{\rm Tp} >
    _gnu_cxx::__promote_num_t< _Tpk, _Tp > theta_n (_Tpk __k, _Tp __x)

    float theta nf (float k, float x)

    long double theta_nl (long double __k, long double __x)

• template<typename _Tpk , typename _Tp >
    _gnu_cxx::__promote_num_t< _Tpk, _Tp > theta_s (_Tpk __k, _Tp __x)
• float theta sf (float k, float x)

    long double theta_sl (long double __k, long double __x)

• template<typename _Trho , typename _Tphi >
    _gnu_cxx::__promote_num_t< _Trho, _Tphi > zernike (unsigned int __n, int __m, _Trho __rho, _Tphi __phi)

    float zernikef (unsigned int __n, int __m, float __rho, float __phi)

    long double zernikel (unsigned int n, int m, long double rho, long double phi)
```

8.2 std Namespace Reference

Namespaces

detail

Functions

```
template<typename</li>Tp >
   __gnu_cxx::__promote< _Tp >::__type assoc_laguerre (unsigned int __n, unsigned int __m, _Tp __x)

    float assoc_laguerref (unsigned int __n, unsigned int __m, float __x)

    long double assoc laguerrel (unsigned int n, unsigned int m, long double x)

template<typename _Tp >
   _gnu_cxx::_promote< _Tp >::_type assoc_legendre (unsigned int __l, unsigned int __m, _Tp __x)

    float assoc legendref (unsigned int I, unsigned int m, float x)

    long double assoc_legendrel (unsigned int __l, unsigned int __m, long double __x)

• template<typename _Tpa , typename _Tpb >
    gnu_cxx::__promote_2< _Tpa, _Tpb >::__type beta (_Tpa __a, _Tpb __b)

    float betaf (float a, float b)

    long double betal (long double __a, long double __b)

    template<typename</li>
    Tp >

   _gnu_cxx::__promote< _Tp >::__type comp_ellint_1 (_Tp __k)

    float comp_ellint_1f (float __k)

    long double comp_ellint_1l (long double __k)

template<typename _Tp >
    _gnu_cxx::__promote< _Tp >::__type comp_ellint_2 (_Tp __k)

    float comp ellint 2f (float k)

    long double comp ellint 2l (long double k)

• template<typename _Tp , typename _Tpn >
    _gnu_cxx::__promote_2< _Tp, _Tpn >::__type comp_ellint_3 (_Tp __k, _Tpn __nu)

    float comp_ellint_3f (float __k, float __nu)

      Return the complete elliptic integral of the third kind \Pi(k,\nu) for float modulus k.
• long double comp ellint 3l (long double k, long double nu)
      Return the complete elliptic integral of the third kind \Pi(k,\nu) for long double modulus k.
• template<typename _Tpnu , typename _Tp >
    _gnu_cxx::__promote_2< _Tpnu, _Tp >::__type cyl_bessel_i (_Tpnu __nu, _Tp __x)

    float cyl bessel if (float nu, float x)

    long double cyl bessel il (long double nu, long double x)

• template<typename _Tpnu , typename _Tp >
    _gnu_cxx::__promote_2< _Tpnu, _Tp >::__type cyl_bessel_j (_Tpnu __nu, _Tp __x)

    float cyl_bessel_jf (float __nu, float __x)

    long double cyl_bessel_jl (long double __nu, long double __x)

• template<typename _Tpnu , typename _Tp >
    gnu cxx:: promote 2< Tpnu, Tp >:: type cyl bessel k (Tpnu nu, Tp x)

    float cyl_bessel_kf (float __nu, float __x)

    long double cyl bessel kl (long double nu, long double x)

template<typename _Tpnu , typename _Tp >
   _gnu_cxx::__promote_2< _Tpnu, _Tp >::__type cyl_neumann (_Tpnu __nu, _Tp __x)

    float cyl_neumannf (float __nu, float __x)

• long double cyl_neumannl (long double __nu, long double __x)
```

```
template<typename _Tp , typename _Tpp >
   _gnu_cxx::__promote_2< _Tp, _Tpp >::__type ellint_1 (_Tp __k, _Tpp __phi)

    float ellint 1f (float k, float phi)

    long double ellint 11 (long double k, long double phi)

• template<typename _Tp , typename _Tpp >
    _gnu_cxx::__promote_2< _Tp, _Tpp >::__type ellint_2 (_Tp __k, _Tpp __phi)

    float ellint 2f (float k, float phi)

      Return the incomplete elliptic integral of the second kind E(k, \phi) for float argument.

    long double ellint 2l (long double k, long double phi)

      Return the incomplete elliptic integral of the second kind E(k, \phi).

    template<typename Tp , typename Tpn , typename Tpp >

   _gnu_cxx::__promote_3< _Tp, _Tpn, _Tpp >::__type ellint_3 (_Tp __k, _Tpn __nu, _Tpp __phi)
      Return the incomplete elliptic integral of the third kind \Pi(k,\nu,\phi).

    float ellint 3f (float k, float nu, float phi)

      Return the incomplete elliptic integral of the third kind \Pi(k, \nu, \phi) for float argument.

    long double ellint_3l (long double ___k, long double ___nu, long double ___phi)

      Return the incomplete elliptic integral of the third kind \Pi(k, \nu, \phi).
template<typename _Tp >
    _gnu_cxx::__promote< _Tp >::__type expint (_Tp __x)

    float expintf (float x)

    long double expintl (long double x)

template<typename_Tp>
    gnu cxx:: promote < Tp >:: type hermite (unsigned int n, Tp x)

    float hermitef (unsigned int __n, float __x)

    long double hermitel (unsigned int n, long double x)

template<typename _Tp >
    _gnu_cxx::__promote< _Tp >::__type laguerre (unsigned int __n, _Tp __x)

    float laguerref (unsigned int __n, float __x)

    long double laguerrel (unsigned int __n, long double __x)

template<typename</li>Tp >
   _gnu_cxx::__promote< _Tp >::__type legendre (unsigned int __I, _Tp __x)

    float legendref (unsigned int ___l, float ___x)

    long double legendrel (unsigned int I, long double x)

template<typename _Tp >
   _gnu_cxx::__promote< _Tp >::__type riemann_zeta (_Tp __s)

    float riemann_zetaf (float __s)

    long double riemann_zetal (long double __s)

template<typename</li>Tp >
   _gnu_cxx::__promote< _Tp >::__type sph_bessel (unsigned int __n, _Tp __x)

    float sph_besself (unsigned int __n, float __x)

    long double sph_bessell (unsigned int __n, long double __x)

template<typename _Tp >
    _gnu_cxx::__promote< _Tp >::__type sph_legendre (unsigned int __I, unsigned int __m, _Tp __theta)

    float sph legendref (unsigned int I, unsigned int m, float theta)

    long double sph_legendrel (unsigned int __l, unsigned int __m, long double __theta)

template<typename_Tp>
    _gnu_cxx::__promote< _Tp >::__type sph_neumann (unsigned int __n, _Tp __x)

    float sph neumannf (unsigned int n, float x)

    long double sph neumannl (unsigned int n, long double x)
```

8.3 std::__detail Namespace Reference

Classes

· struct Factorial table

Enumerations

enum { SININT, COSINT }

Functions

```
    template<typename _Tp >
        void __airy (_Tp __z, _Tp &_Ai, _Tp &_Bi, _Tp &_Aip, _Tp &_Bip)
```

Compute the Airy functions Ai(x) and Bi(x) and their first derivatives Ai'(x) and Bi(x) respectively.

template<typename _Tp >

```
void __airy (const std::complex< _Tp > &__z, _Tp __eps, std::complex< _Tp > &_Ai, std::complex< _Tp > &_Aip, std::complex< _Tp > &_Bip)
```

This function computes the Airy function Ai(z) and its first derivative in the complex z-plane.

template<typename _Tp >

```
std::complex< _Tp > __airy_ai (std::complex< _Tp > __z)
```

Return the complex Airy Ai function.

ullet template<typename_Tp>

```
\label{lem:complex} \mbox{void} \begin{subarray}{ll} \mbox{ airy\_arg (std::complex} < \mbox{ $Tp > $\_$num2d3, std::complex} < \mbox{ $Tp > $\_$argp, std::complex} < \mbox{ $Tp > $\&$ argm)} \end{subarray}
```

Compute the arguments for the Airy function evaluations carefully to prevent premature overflow. Note that the major work here is in safe_div. A faster, but less safe implementation can be obtained without use of safe div.

template<typename_Tp>

```
\label{local_complex} $$\operatorname{void}$ $\_\operatorname{airy}$ $_\operatorname{asymp}$ $_\operatorname{absarg}$ $_\operatorname{poisson}$ (std::complex < $_\operatorname{Tp} > $_z$, std::complex < $_\operatorname{Tp} > $_x$, std::co
```

This function evaluates Ai(z) and Ai'(z) from their asymptotic expansions for $|arg(z)| < 2 * \pi/3$. For speed, the number of terms needed to achieve about 16 decimals accuracy is tabled and determined from abs(z).

template<typenameTp >

This function evaluates Ai(z) and Ai'(z) from their asymptotic expansions for |arg(-z)| < pi/3. For speed, the number of terms needed to achieve about 16 decimals accuracy is tabled and determined from |z|.

 $\bullet \ \ template\!<\!typename\,_Tp>$

```
\label{local_post_std} $$\operatorname{\sc wold}$$ $\_\operatorname{\sc airy\_bessel\_i}$ (const std::complex< $\_\operatorname{Tp} > \&\_z$, $_\operatorname{Tp} $\_\operatorname{\sc eps}$, std::complex< $_\operatorname{Tp} > \&_\operatorname{\sc lp1d3}$, std::complex< $_\operatorname{Tp} > \&_\operatorname{\sc lp2d3}$) $$
```

• template<typename $_{\mathrm{Tp}}$ >

```
void __airy_bessel_k (const std::complex< _Tp > &__z, _Tp __eps, std::complex< _Tp > &_Kp1d3, std\leftrightarrow ::complex< _Tp > &_Kp2d3)
```

Compute approximations to the modified Bessel functions of the second kind of orders 1/3 and 2/3 for moderate arguments.

• template<typename_Tp>

```
std::complex < \_Tp > \underline{\quad} airy\_bi \ (std::complex < \_Tp > \underline{\quad} z)
```

Return the complex Airy Bi function.

template<typename_Tp>

```
\label{local_problem} $$\operatorname{void}$ $\_\operatorname{airy\_hyperg\_rational}$ (const std::complex < \_Tp > \&\_z, std::complex < \_Tp > \&\_Ai, std::complex < \_Tp > \&\_Aip, std::complex < \_Tp > \&\_Bip) $$
```

This function computes rational approximations to the hypergeometric functions related to the modified Bessel functions of orders $\nu = + -1/3$ and $\nu + -2/3$. That is, A(z)/B(z), Where A(z) and B(z) are cubic polynomials with real coefficients, approximates

$$\frac{\Gamma(\nu+1)}{(z/2)^n u} I_{\nu}(z) =_0 F_1(; \nu+1; z^2/4),$$

where the function on the right is a confluent hypergeometric limit function. For |z| <= 1/4 and |arg(z)| <= pi/2, the approximations are accurate to about 16 decimals.

- template<typename
 Tp >
 - _Tp __assoc_laguerre (unsigned int __n, unsigned int __m, _Tp __x)

This routine returns the associated Laguerre polynomial of order n, degree m: $L_n^m(x)$.

- template<typename
 Tp >
 - _Tp __assoc_legendre_p (unsigned int __I, unsigned int __m, _Tp __x)

Return the associated Legendre function by recursion on l and downward recursion on m.

template<typename_Tp>

This returns Bernoulli number B_n .

template<typename
 Tp >

This returns Bernoulli number B_n .

- template<typename_Tp>
 - _GLIBCXX14_CONSTEXPR _Tp __bernoulli_series (unsigned int __n)

This returns Bernoulli numbers from a table or by summation for larger values.

template<typename_Tp>

Return the beta function B(a, b).

template<typename _Tp >

Return the beta function: B(a,b).

template<typename _Tp >

• template<typename $_{\rm Tp}>$

template<typename _Tp >

Return the beta function B(a,b) using the log gamma functions.

template<typename_Tp>

Return the beta function B(x, y) using the product form.

- template<typename_Tp>
 - Tp bincoef (unsigned int n, unsigned int k)

Return the binomial coefficient. The binomial coefficient is given by:

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

- template<typename
 Tp >
 - GLIBCXX14 CONSTEXPR Tp binomial cdf (Tp p, unsigned int n, unsigned int k)

Return the binomial cumulative distribution function.

- template<typename _Tp >
 - _GLIBCXX14_CONSTEXPR _Tp <u>__binomial_cdfc</u> (_Tp __p, unsigned int __n, unsigned int __k)

Return the complementary binomial cumulative distribution function.

```
template<typename _Tp >
  _Tp __bose_einstein (_Tp __s, _Tp __x)

    template<typename</li>
    Tp >

  _Tp __chebyshev_recur (unsigned int __n, _Tp __x, _Tp _C0, _Tp _C1)
template<typename _Tp >
  Tp chebyshev t (unsigned int n, Tp x)

    template<typename</li>
    Tp >

  _Tp __chebyshev_u (unsigned int __n, _Tp __x)
template<typename _Tp >
  Tp chebyshev v (unsigned int n, Tp x)
template<typename _Tp >
  _Tp __chebyshev_w (unsigned int __ n, Tp x)
template<typename_Tp>
  _GLIBCXX14_CONSTEXPR _Tp __chi_squared_pdf (_Tp __chi2, unsigned int __nu)
     Return the chi-squared propability function. This returns the probability that the observed chi-squared for a correct model
     is less than the value \chi^2.

    template<typename</li>
    Tp >

  GLIBCXX14 CONSTEXPR Tp chi squared pdfc (Tp chi2, unsigned int nu)
     Return the complementary chi-squared propability function. This returns the probability that the observed chi-squared for
     a correct model is greater than the value \chi^2.
template<typename Tp >
  std::pair< _Tp, _Tp > __chshint (_Tp __x, _Tp &_Chi, _Tp &_Shi)
     This function returns the hyperbolic cosine Ci(x) and hyperbolic sine Si(x) integrals as a pair.
template<typename_Tp>
  void __chshint_cont_frac (_Tp __t, _Tp &_Chi, _Tp &_Shi)
     This function computes the hyperbolic cosine Chi(x) and hyperbolic sine Shi(x) integrals by continued fraction for
     positive argument.
template<typename _Tp >
  void __chshint_series (_Tp __t, _Tp &_Chi, _Tp &_Shi)
     This function computes the hyperbolic cosine Chi(x) and hyperbolic sine Shi(x) integrals by series summation for
     positive argument.
template<typename _Tp >
  std::complex< Tp > clamp 0 m2pi (std::complex< Tp > w)
template<typename_Tp>
  std::complex< _Tp > __clamp_pi (std::complex< _Tp > __w)
template<typename _Tp >
  std::complex< Tp > clausen (unsigned int m, std::complex< Tp > w)
template<typename_Tp>
  _Tp __clausen (unsigned int __m, _Tp __w)
template<typename_Tp>
  _Tp __clausen_c (unsigned int __m, std::complex< _Tp > __w)
template<typename _Tp >
  Tp clausen c (unsigned int m, Tp w)
template<typename_Tp>
  Tp clausen s (unsigned int m, std::complex < Tp > w)
template<typename_Tp>
  _Tp __clausen_s (unsigned int __m, _Tp __w)
template<typename</li>Tp >
  Tp comp ellint 1 (Tp k)
     Return the complete elliptic integral of the first kind K(k) using the Carlson formulation.
template<typename _Tp >
  _Tp __comp_ellint_2 (_Tp __k)
```

Return the complete elliptic integral of the second kind E(k) using the Carlson formulation.

template<typename_Tp>

```
_Tp __comp_ellint_3 (_Tp __k, _Tp __nu)
```

Return the complete elliptic integral of the third kind $\Pi(k,\nu) = \Pi(k,\nu,\pi/2)$ using the Carlson formulation.

template<typename _Tp >

```
_Tp __comp_ellint_d (_Tp __k)
```

template<typename _Tp >

template<typename_Tp>

template<typename _Tp >

Return the confluent hypergeometric function ${}_1F_1(a;c;x)$.

template<typename_Tp>

Return the confluent hypergeometric limit function ${}_0F_1(-;c;x)$.

template<typenameTp >

```
_Tp __conf_hyperg_lim_series (_Tp __c, _Tp __x)
```

This routine returns the confluent hypergeometric limit function by series expansion.

template<typename _Tp >

Return the hypergeometric function $_1F_1(a;c;x)$ by an iterative procedure described in Luke, Algorithms for the Computation of Mathematical Functions.

template<typename _Tp >

This routine returns the confluent hypergeometric function by series expansion.

template<typename
 Tp >

Return the hyperbolic cosine integral li(x).

template<typename _Tp >

$$std::complex<_Tp>__cyl_bessel \ (std::complex<_Tp>__nu, std::complex<_Tp>__z)$$

Return the complex cylindrical Bessel function.

template<typename _Tp >

Return the regular modified Bessel function of order ν : $I_{\nu}(x)$.

• template<typename $_{\rm Tp}>$

This routine returns the cylindrical Bessel functions of order ν : J_{ν} or I_{ν} by series expansion.

template<typename
 Tp >

Return the modified cylindrical Bessel functions and their derivatives of order ν by various means.

template<typenameTp >

This routine computes the asymptotic modified cylindrical Bessel and functions of order nu: $I_{\nu}(x)$, $N_{\nu}(x)$. Use this for $x >> nu^2 + 1$.

template<typenameTp >

```
void <u>cyl_bessel_ik_steed</u> (_Tp __nu, _Tp __x, _Tp &_Inu, _Tp &_Knu, _Tp &_Ipnu, _Tp &_Kpnu)
```

Compute the modified Bessel functions $I_{\nu}(x)$ and $K_{\nu}(x)$ and their first derivatives $I'_{\nu}(x)$ and $K'_{\nu}(x)$ respectively. These four functions are computed together for numerical stability.

```
template<typename _Tp >
  _Tp __cyl_bessel_j (_Tp __nu, _Tp __x)
      Return the Bessel function of order \nu: J_{\nu}(x).
template<typename _Tp >
  void __cyl_bessel_jn (_Tp __nu, _Tp __x, _Tp &_Jnu, _Tp &_Nnu, _Tp &_Jpnu, _Tp &_Npnu)
      Return the cylindrical Bessel functions and their derivatives of order \nu by various means.

    template<typename</li>
    Tp >

  void __cyl_bessel_jn_asymp (_Tp __nu, _Tp __x, _Tp &_Jnu, _Tp &_Nnu, _Tp &_Jpnu, _Tp &_Npnu)
      This routine computes the asymptotic cylindrical Bessel and Neumann functions of order nu: J_{\nu}(x), N_{\nu}(x). Use this for
      x >> nu^2 + 1.
template<typename _Tp >
  void cyl bessel jn steed (Tp nu, Tp x, Tp & Jnu, Tp & Nnu, Tp & Jpnu, Tp & Npnu)
      Compute the Bessel J_{\nu}(x) and Neumann N_{\nu}(x) functions and their first derivatives J'_{\nu}(x) and N'_{\nu}(x) respectively. These
      four functions are computed together for numerical stability.

    template<typename</li>
    Tp >

  _Tp __cyl_bessel_k (_Tp __nu, _Tp __x)
      Return the irregular modified Bessel function K_{\nu}(x) of order \nu.
template<typename_Tp>
  std::complex< _Tp > __cyl_hankel_1 (_Tp __nu, _Tp __x)
      Return the cylindrical Hankel function of the first kind H_{\nu}^{(1)}(x).
template<typename _Tp >
  std::complex < Tp > cyl hankel 1 (std::complex < Tp > nu, std::complex < Tp > z)
      Return the complex cylindrical Hankel function of the first kind.
template<typename</li>Tp >
  std::complex < _Tp > \__cyl_hankel_2 (_Tp \__nu, _Tp x)
      Return the cylindrical Hankel function of the second kind H_n^{(2)}u(x).
template<typename_Tp>
  std::complex < _Tp > \underline{_cyl\_hankel\_2} (std::complex < _Tp > \underline__nu, std::complex < _Tp > \underline__z)
      Return the complex cylindrical Hankel function of the second kind.
template<typename_Tp>
                          cyl neumann (std::complex< Tp > nu, std::complex< Tp > z)
  std::complex< Tp >
      Return the complex cylindrical Neumann function.
template<typename _Tp >
  _Tp __cyl_neumann_n (_Tp __nu, _Tp _ x)
      Return the Neumann function of order \nu: N_{\nu}(x).
template<typename</li>Tp >
  _Tp __dawson (_Tp __x)
      Return the Dawson integral, F(x), for real argument x.
template<typename _Tp >
  Tp dawson const frac (Tp x)
      Compute the Dawson integral using a sampling theorem representation.

    template<typename</li>
    Tp >

  _Tp __dawson_series (_Tp __x)
      Compute the Dawson integral using the series expansion.
template<typename _Tp >
  void <u>debye_region</u> (std::complex< _Tp > __alpha, int &__indexr, char &__aorb)
template<typename _Tp >
  _Tp <u>__dilog</u> (_Tp __x)
      Compute the dilogarithm function Li_2(x) by summation for x \le 1.
```

```
template<typename _Tp >
  Tp dirichlet beta (std::complex < Tp > w)

    template<typename</li>
    Tp >

  _Tp __dirichlet_beta (_Tp __w)
template<typename _Tp >
  std::complex< Tp > dirichlet eta (std::complex< Tp > w)
template<typename Tp >
  _Tp __dirichlet_eta (_Tp __w)
template<typename_Tp>
  GLIBCXX14 CONSTEXPR Tp double factorial (int n)
      Return the double factorial of the integer n.
• template<typename_Tp>
  _Tp __ellint_1 (_Tp __k, _Tp __phi)
      Return the incomplete elliptic integral of the first kind F(k,\phi) using the Carlson formulation.
• template<typename _{\rm Tp}>
  _Tp <u>__ellint_2</u> (_Tp __k, _Tp __phi)
      Return the incomplete elliptic integral of the second kind E(k,\phi) using the Carlson formulation.
\bullet \ \ \mathsf{template} \!<\! \mathsf{typename} \ \_\mathsf{Tp} >
  _Tp __ellint_3 (_Tp __k, _Tp __nu, _Tp __phi)
      Return the incomplete elliptic integral of the third kind \Pi(k, \nu, \phi) using the Carlson formulation.
template<typename_Tp>
  _Tp <u>__ellint_cel</u> (_Tp <u>__k_c, _Tp __p, _Tp __a, _Tp __b)</u>
template<typename _Tp >
  _Tp __ellint_d (_Tp __k, _Tp __phi)
template<typename_Tp>
  Tp ellint el1 (Tp x, Tp k c)

    template<typename _Tp >

  _Tp <u>__ellint_el2</u> (_Tp __x, _Tp __k_c, _Tp __a, _Tp __b)
template<typename Tp >
  _Tp <u>__ellint_el3</u> (_Tp __x, _Tp __k_c, _Tp __p)
template<typename _Tp >
  _Tp __ellint_rc (_Tp __x, _Tp __y)
      Return the Carlson elliptic function R_C(x,y) = R_F(x,y,y) where R_F(x,y,z) is the Carlson elliptic function of the first
template<typename _Tp >
  _Tp __ellint_rd (_Tp __x, _Tp __y, _Tp __z)
      Return the Carlson elliptic function of the second kind R_D(x,y,z) = R_J(x,y,z,z) where R_J(x,y,z,p) is the Carlson
      elliptic function of the third kind.

 template<typename _Tp >

  _Tp __ellint_rf (_Tp __x, _Tp __y, _Tp __z)
      Return the Carlson elliptic function R_F(x, y, z) of the first kind.

    template<typename</li>
    Tp >

  _Tp __ellint_rg (_Tp __x, _Tp __y, _Tp __z)
      Return the symmetric Carlson elliptic function of the second kind R_G(x, y, z).
template<typename_Tp>
  _Tp __ellint_rj (_Tp __x, _Tp __y, _Tp __z, _Tp __p)
      Return the Carlson elliptic function R_J(x, y, z, p) of the third kind.
template<typename_Tp>
  _Tp __ellnome (_Tp __k)

 template<typename _Tp >

  _Tp __ellnome_k (_Tp __k)
```

```
template<typename _Tp >
  Tp ellnome series (Tp k)
template<typename_Tp>
  _Tp __expint (unsigned int __n, _Tp __x)
      Return the exponential integral E_n(x).
template<typename _Tp >
  Tp expint (Tp x)
      Return the exponential integral Ei(x).
template<typename _Tp >
  _Tp __expint_asymp (unsigned int __n, _Tp __x)
      Return the exponential integral E_n(x) for large argument.

    template<typename</li>
    Tp >

  _Tp __expint_E1 (_Tp __x)
      Return the exponential integral E_1(x).
template<typename _Tp >
  _Tp __expint_E1_asymp (_Tp __x)
      Return the exponential integral E_1(x) by asymptotic expansion.
template<typename _Tp >
  _Tp __expint_E1_series (_Tp __x)
      Return the exponential integral E_1(x) by series summation. This should be good for x < 1.
template<typename_Tp>
  _Tp <u>expint_Ei</u> (_Tp __x)
      Return the exponential integral Ei(x).
template<typename_Tp>
  _Tp __expint_Ei_asymp (_Tp __x)
      Return the exponential integral Ei(x) by asymptotic expansion.
template<typename_Tp>
  _Tp __expint_Ei_series (_Tp __x)
      Return the exponential integral Ei(x) by series summation.
template<typename_Tp>
  Tp expint En cont frac (unsigned int n, Tp x)
      Return the exponential integral E_n(x) by continued fractions.
template<typename _Tp >
  _Tp __expint_En_recursion (unsigned int __n, _Tp __x)
      Return the exponential integral E_n(x) by recursion. Use upward recursion for x < n and downward recursion (Miller's
     algorithm) otherwise.

    template<typename</li>
    Tp >

  _Tp __expint_En_series (unsigned int __n, _Tp __x)
      Return the exponential integral E_n(x) by series summation.
template<typename _Tp >
  Tp expint large n (unsigned int n, Tp x)
      Return the exponential integral E_n(x) for large order.
template<typename</li>Tp >
  _GLIBCXX14_CONSTEXPR _Tp __factorial (unsigned int __n)
      Return the factorial of the integer n.
template<typename</li>Tp >
  _Tp __fermi_dirac (_Tp __s, _Tp __x)
template<typename _Tp >
  GLIBCXX14 CONSTEXPR Tp fisher f cdf (Tp F, unsigned int nu1, unsigned int nu2)
```

Return the F-distribution propability function. This returns the probability that the observed chi-square for a correct model exceeds the value χ^2 .

template<typename _Tp >

Return the F-distribution propability function. This returns the probability that the observed chi-square for a correct model exceeds the value χ^2 .

template<typename _Tp >

$$\label{local_complex} $$\operatorname{\mathsf{w1}}, \operatorname{\mathsf{std::complex}} \subset \operatorname{\mathsf{Tp}} > \&_{w1}, \operatorname{\mathsf{std::complex}} \subset \operatorname{\mathsf{Tp}} > \&_{w2}, \operatorname{\mathsf{std::complex}} \subset \operatorname{\mathsf{Tp}} > \&_{w1}, \operatorname{\mathsf{std::complex}} \subset \operatorname{\mathsf{Tp}} > \&_{w2}, \operatorname{\mathsf{std::complex}} \subset \operatorname{\mathsf{Tp}} \subset \operatorname{\mathsf{Tp}} = \&_{w2}, \operatorname{\mathsf{Tp}} \subset \operatorname{\mathsf{Tp}} \subset \operatorname{\mathsf{Tp}} \subset \operatorname{\mathsf{Tp}} = \&_{w2}, \operatorname{\mathsf{Tp}} \subset \operatorname{\mathsf{Tp}}$$

Compute the Fock-type Airy functions $w_1(x)$ and $w_2(x)$ and their first derivatives $w_1'(x)$ and $w_2'(x)$ respectively.

$$w_1(x) = \sqrt{\pi}(Ai(x) + iBi(x))$$

$$w_2(x) = \sqrt{\pi}(Ai(x) - iBi(x))$$

.

template<typename _Tp >

• template<typename $_{\rm Tp}>$

bool $__$ fpimag (const std::complex< $_$ Tp > & $__$ w)

template<typename _Tp >

template<typename _Tp >

template<typename_Tp>

template<typename _Tp >

Return the Fresnel cosine and sine integrals as a complex number f(C(x) + iS(x)).

template<typename _Tp >

This function computes the Fresnel cosine and sine integrals by continued fractions for positive argument.

• template<typename _Tp >

This function returns the Fresnel cosine and sine integrals as a pair by series expansion for positive argument.

template<typename_Tp>

Return $\Gamma(x)$.

• template<typename_Tp>

template<typename _Tp >

Return the lower incomplete gamma function. The lower incomplete gamma function is defined by

$$\gamma(a, x) = \int_0^x e^{-t} t^{a-1} dt (a > 0)$$

.

template<typename Tp >

Return the regularized lower incomplete gamma function. The regularized lower incomplete gamma function is defined by

$$P(a,x) = \frac{\gamma(a,x)}{\Gamma(a)}$$

where $\Gamma(a)$ is the gamma function and

$$\gamma(a, x) = \int_0^x e^{-t} t^{a-1} dt (a > 0)$$

is the lower incomplete gamma function.

template<typename
 Tp >

Return the regularized upper incomplete gamma function. The regularized upper incomplete gamma function is defined by

$$Q(a,x) = \frac{\Gamma(a,x)}{\Gamma(a)}$$

where $\Gamma(a)$ is the gamma function and

$$\Gamma(a,x) = \int_{x}^{\infty} e^{-t} t^{a-1} dt (a > 0)$$

is the upper incomplete gamma function.

- template < typename _Tp >
 std::pair < _Tp, _Tp > __gamma_series (_Tp __a, _Tp __x)
- template<typename _Tp >
 void __gamma_temme (_Tp __mu, _Tp &__gam1, _Tp &__gam2, _Tp &__gampl, _Tp &__gammi)

Compute the gamma functions required by the Temme series expansions of $N_{\nu}(x)$ and $K_{\nu}(x)$.

$$\Gamma_1 = \frac{1}{2\mu} \left[\frac{1}{\Gamma(1-\mu)} - \frac{1}{\Gamma(1+\mu)} \right]$$

and

$$\Gamma_2 = \frac{1}{2} \left[\frac{1}{\Gamma(1-\mu)} + \frac{1}{\Gamma(1+\mu)} \right]$$

where $-1/2 <= \mu <= 1/2$ is $\mu = \nu - N$ and N. is the nearest integer to ν . The values of $\Gamma(1 + \mu)$ and $\Gamma(1 - \mu)$ are returned as well.

• template<typename_Tp>

Return the upper incomplete gamma function. The lower incomplete gamma function is defined by

$$\Gamma(a,x) = \int_{x}^{\infty} e^{-t} t^{a-1} dt (a > 0)$$

• template<typename_Tp>

• template<typename $_{\rm Tp}>$

_Tp __gegenbauer_poly (unsigned int __n, _Tp __alpha, _Tp __x)

template<typename _Tp >

 $\label{lem:complex} \begin{tabular}{lll} void $_$-hankel (std::complex< $_$Tp > $_$nu, std::complex< $_$Tp > $_$z, std::complex< $_$Tp > &_$H1, std$ $$::complex< $_$Tp > &_$H2, std::complex< $_$Tp > &_$H2p) \\ \end{tabular}$

- template<typename _Tp >
- void __hankel_debye (std::complex< _Tp > __nu, std::complex< _Tp > __z, std::complex< _Tp > __alpha, int __indexr, char &__aorb, int &__morn, std::complex< _Tp > &_H1, std::complex< _Tp > &_H2, std::complex< _Tp > &_H1p, std::complex< _Tp > &_H2p)
- template<typename _Tp >

 $\label{lem:complex} $$\operatorname{\sc omplex} = Tp > \underline{\nu}, \ std:: complex < \underline{\nu} > \underline{\nu}$

Compute parameters depending on z and nu that appear in the uniform asymptotic expansions of the Hankel functions and their derivatives, except the arguments to the Airy functions.

template<typenameTp >

```
\label{lem:complex} $$\operatorname{\begin{tabular}{l} void $\_$ hankel\_uniform (std::complex< $\_$Tp > $\_$nu, std::complex< $\_$Tp > $\_$z, std::complex< $\_$Tp > &_$H1, std::complex< $\_$Tp > &_$H2p, std::complex< $\_$Tp > &_$H2p}$}
```

This routine computes the uniform asymptotic approximations of the Hankel functions and their derivatives including a patch for the case when the order equals or nearly equals the argument. At such points, Olver's expressions have zero denominators (and numerators) resulting in numerical problems. This routine averages results from four surrounding points in the complex plane to obtain the result in such cases.

template<typename _Tp >

```
void __hankel_uniform_olver (std::complex < _Tp > __nu, std::complex < _Tp > __z, std::complex < _Tp > & \leftarrow H1, std::complex < _Tp > & H2p, std::complex < _Tp > & H2p)
```

Compute approximate values for the Hankel functions of the first and second kinds using Olver's uniform asymptotic expansion to of order nu along with their derivatives.

template<typename_Tp>

```
\label{lem:complex} $$\operatorname{void}_{\operatorname{hankel\_uniform\_outer}}(\operatorname{std::complex} < Tp > \underline{\quad} \operatorname{nu}, \operatorname{std::complex} < Tp > \underline{\quad} \operatorname{z}, Tp \underline{\quad} \operatorname{eps}, \operatorname{std::complex} < Tp > \underline{\quad} \operatorname{nu}, \operatorname{std::complex} < Tp > \underline{\quad} \operatorname{num1d3}, \operatorname{std::complex} < Tp > \underline{\quad} \operatorname{num1d3}, \operatorname{std::complex} < Tp > \underline{\quad} \operatorname{num2d3}, \operatorname
```

Compute outer factors and associated functions of z and nu appearing in Olver's uniform asymptotic expansions of the Hankel functions of the first and second kinds and their derivatives. The various functions of z and nu returned by $hankel_uniform_outer$ are available for use in computing further terms in the expansions.

template<typename _Tp >

```
void __hankel_uniform_sum (std::complex< _Tp > __p, std::complex< _Tp > __p2, std::complex< _Tp > __num2, std::complex< _Tp > __zetam3hf, std::complex< _Tp > __ahp, std::complex< _Tp > __o4dp, std \leftrightarrow ::complex< _Tp > __ahm, std::complex< _Tp > __o4dm, std::complex< _Tp > __o4dp, std::complex< _Tp > __o4dp,
```

Compute the sums in appropriate linear combinations appearing in Olver's uniform asymptotic expansions for the Hankel functions of the first and second kinds and their derivatives, using up to nterms (less than 5) to achieve relative error eps.

• template<typename $_{\mathrm{Tp}}$ >

```
_Tp __heuman_lambda (_Tp __k, _Tp __phi)
```

template<typename _Tp >

Return the Hurwitz zeta function $\zeta(s,a)$ for all s = 1 and a > -1.

template<typename
 Tp >

```
std::complex< _Tp > __hurwitz_zeta (_Tp __s, std::complex< _Tp > __a)
```

template<typename_Tp>

Return the Hurwitz zeta function $\zeta(s,a)$ for all s = 1 and a > -1.

template<typename
 Tp >

```
std::complex< _Tp > __hydrogen (const unsigned int __n, const unsigned int __l, const unsigned int __m, const Tp Z, const Tp r, const Tp theta, const Tp phi)
```

• template<typename _Tp >

Return the hypergeometric function ${}_{2}F_{1}(a,b;c;x)$.

template<typename
 Tp >

Return the hypergeometric function ${}_2F_1(a,b;c;x)$ by an iterative procedure described in Luke, Algorithms for the Computation of Mathematical Functions.

• template<typename $_{\rm Tp}>$

Return the hypergeometric function ${}_2F_1(a,b;c;x)$ by the reflection formulae in Abramowitz & Stegun formula 15.3.6 for d=c-a - b not integral and formula 15.3.11 for d=c-a - b integral. This assumes a, b, c != negative integer.

template<typename _Tp >

Return the hypergeometric function ${}_2F_1(a,b;c;x)$ by series expansion.

template<typename_Tp>

$$std::tuple < _Tp, _Tp, _Tp > \underline{_jacobi_sncndn} (_Tp \underline{_k}, _Tp \underline{_u})$$

template<typename _Tp >

• template<typename _Tp >

This routine returns the Laguerre polynomial of order n: $L_n(x)$.

template<typename _Tp >

Return the logarithm of the binomial coefficient. The binomial coefficient is given by:

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

•

template<typename
 Tp >

template<typename _Tp >

Return the logarithm of the double factorial of the integer n.

ullet template<typename _Tp >

Return the logarithm of the factorial of the integer n.

template<typename_Tp>

Return $log(|\Gamma(x)|)$. This will return values even for x < 0. To recover the sign of $\Gamma(x)$ for any argument use $_log_{\leftarrow}$ gamma_sign.

template<typename _Tp >

Return $log(\Gamma(x))$ by asymptotic expansion with Bernoulli number coefficients. This is like Sterling's approximation.

template<typename_Tp>

Return $log(\Gamma(x))$ by the Lanczos method. This method dominates all others on the positive axis I think.

• template<typename _Tp >

Return the sign of $\Gamma(x)$. At nonpositive integers zero is returned.

template<typename _Tp >

Return $\Gamma(z)$ by the Spouge algorithm:

$$\Gamma(z+1) = (z+a)^{z+1/2} e^{-z-a} \left[\sqrt{2\pi} \sum_{k=1}^{\lceil a \rceil + 1} \frac{c_k(a)}{z+k} \right]$$

where

$$c_k(a) = \frac{(-1)^{k-1}}{(k-1)!} (a-k)^{k-1/2} e^{a-k}$$

and the error is bounded by

$$\epsilon(a) < a^{-1/2} (2\pi)^{-a-1/2}$$

•

template<typename _Tp >

Return the logarithm of the lower Pochhammer symbol or the falling factorial function. The lower Pochammer symbol is defined by

$$(a)_n = \prod_{k=0}^{n-1} (a-k), (a)_0 = 1 = \Gamma(a+1)/\Gamma(a-n+1)$$

In particular, f(n) = n! f. Thus this function returns

$$ln[(a)_n] = \Gamma(a+1) - \Gamma(a-n+1), ln[(a)_0] = 0$$

Many notations exist:

 $a^{\underline{n}}$

,

 $\left\{\begin{array}{c} a \\ n \end{array}\right\}$

, and others.

• template<typename _Tp >

Return the logarithm of the (upper) Pochhammer symbol or the rising factorial function. The Pochammer symbol is defined by

$$(a)_n = \prod_{k=0}^{n-1} (a+k), (a)_0 = 1 = \Gamma(a+n)/\Gamma(n)$$

Thus this function returns

$$ln[(a)_n] = \Gamma(a+n) - \Gamma(n), ln[(a)_0] = 0$$

Many notations exist:

 $a^{\overline{n}}$

,

 $\left[\begin{array}{c} a \\ n \end{array}\right]$

, and others.

template<typename
 Tp >

Return the logarithmic integral li(x).

template<typename _Tp >

template<typename _Tp >

Return the logarithm of the lower Pochhammer symbol or the falling factorial function. The lower Pochammer symbol is defined by

$$(a)_n = \prod_{k=0}^{n-1} (a-k), (a)_0 = 1 = \Gamma(a+1)/\Gamma(a-n+1)$$

In particular, $f[(n)_n = n! f]$.

template<typename _Tp >

Return the (upper) Pochhammer function or the rising factorial function. The Pochammer symbol is defined by

$$(a)_n = \prod_{k=0}^{n-1} (a+k), (a)_0 = 1 = \Gamma(a+n)/\Gamma(n)$$

```
Many notations exist:
      , and others.
• template<typename _Tp >
  _Tp __poly_hermite (unsigned int __n, _Tp __x)
      This routine returns the Hermite polynomial of order n: H_n(x).
template<typename Tp >
  _Tp __poly_hermite_asymp (unsigned int __n, _Tp __x)
      This routine returns the Hermite polynomial of large order n: H_n(x). We assume here that x >= 0.
template<typename Tp >
  _Tp __poly_hermite_recursion (unsigned int __n, _Tp __x)
      This routine returns the Hermite polynomial of order n: H_n(x) by recursion on n.
template<typename _Tp >
  Tp poly jacobi (unsigned int n, Tp alpha, Tp beta, Tp x)

    template<typename _Tpa , typename _Tp >

  _Tp __poly_laguerre (unsigned int __n, _Tpa __alpha1, _Tp __x)
      This routine returns the associated Laguerre polynomial of order n, degree \alpha: L_n^a lpha(x).
• template<typename _{\rm Tpa}, typename _{\rm Tp} >
  _Tp __poly_laguerre_hyperg (unsigned int __n, _Tpa __alpha1, _Tp __x)
      Evaluate the polynomial based on the confluent hypergeometric function in a safe way, with no restriction on the arguments.
• template<typename _Tpa , typename _Tp >
  _Tp __poly_laguerre_large_n (unsigned __n, _Tpa __alpha1, _Tp __x)
      This routine returns the associated Laguerre polynomial of order n, degree \alpha for large n. Abramowitz & Stegun, 13.5.21.

    template<typename _Tpa , typename _Tp >

  _Tp __poly_laguerre_recursion (unsigned int __n, _Tpa __alpha1, _Tp __x)
      This routine returns the associated Laguerre polynomial of order n, degree \alpha: L_n^{\alpha}(x) by recursion.

 template<typename _Tp >

  _Tp __poly_legendre_p (unsigned int __l, _Tp __x)
      Return the Legendre polynomial by upward recursion on order l.

    template<typename</li>
    Tp >

  _Tp __poly_legendre_q (unsigned int __l, _Tp __x)
      Return the Legendre function of the second kind by upward recursion on order l.

    template<typename</li>
    Tp >

  _Tp __poly_radial_jacobi (unsigned int __n, unsigned int __m, _Tp __rho)

    template<typename</li>
    Tp >

  _Tp <u>__polylog</u> (_Tp <u>__s, _</u>Tp <u>__</u>x)
template<typename _Tp >
  std::complex< _Tp > __polylog (_Tp __s, std::complex< _Tp > __w)

    template<typename</li>
    Tp , typename
    ArgType >

   _gnu_cxx::__promote_num_t< std::complex< _Tp >, ArgType > __polylog_exp (_Tp __s, ArgType __w)

    template<typename _Tp >

  std::complex < _Tp > __polylog_exp_asymp (const _Tp __s, std::complex < _Tp > __w)
template<typename _Tp >
  std::complex < _Tp > __polylog_exp_int_neg (const int __s, std::complex < _Tp > __w)
template<typename_Tp>
  std::complex< _Tp > __polylog_exp_int_neg (const int __s, _Tp __w)

 template<typename _Tp >

  std::complex< Tp > polylog exp int pos (const unsigned int s, std::complex< Tp > w)
```

```
template<typename _Tp >
  std::complex< _Tp > __polylog_exp_int_pos (const unsigned int __s, _Tp __w)
template<typename</li>Tp >
  std::complex < _Tp > \__polylog_exp_neg (_Tp \__s, std::complex < _Tp > \__w)
template<typename Tp >
  std::complex < Tp > polylog exp neg (int s, std::complex < Tp > w)
• template<typename _Tp , int __sigma>
  std::complex< _Tp > __polylog_exp_neg_even (unsigned int __n, std::complex< _Tp > __w)
• template<typename _Tp , int __sigma>
  std::complex< _Tp > __polylog_exp_neg_odd (unsigned int __n, std::complex< _Tp > __w)
• template<typename _PowTp , typename _Tp >
  _Tp __polylog_exp_negative_real_part (_PowTp __s, _Tp __w)
template<typename</li>Tp >
  std::complex< _Tp > __polylog_exp_pos (unsigned int __s, std::complex< _Tp > __w)
template<typename _Tp >
  std::complex< _Tp > __polylog_exp_pos (unsigned int __s, _Tp __w)
template<typename _Tp >
  std::complex< _Tp > __polylog_exp_pos (_Tp __s, std::complex< _Tp > __w)

    template<typename</li>
    Tp >

  std::complex< _Tp > __polylog_exp_real_neg (_Tp __s, std::complex< _Tp > __w)
template<typename _Tp >
  std::complex< _Tp > __polylog_exp_real_neg (_Tp __s, _Tp __w)
template<typename _Tp >
  std::complex< _Tp > __polylog_exp_real_pos (_Tp __s, std::complex< _Tp > __w)
• template<typename _Tp >
  std::complex< Tp > polylog exp real pos (Tp s, Tp w)
template<typename_Tp>
 _Tp __psi (_Tp __x)
```

Return the digamma function. The digamma or $\psi(x)$ function is defined by

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

For negative argument the reflection formula is used:

$$\psi(x) = \psi(1-x) - \pi \cot(\pi x)$$

. . . .

template<typename _Tp > _Tp __psi (unsigned int __n, _Tp __x)

Return the polygamma function $\psi^{(n)}(x)$.

• template<typename $_{\rm Tp}>$

Return the digamma function for large argument. The digamma or $\psi(x)$ function is defined by

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

• template<typename_Tp>

Return the digamma function by series expansion. The digamma or $\psi(x)$ function is defined by

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

.

```
template<typename _Tp > 
_Tp __riemann_zeta (_Tp __s)
```

Return the Riemann zeta function $\zeta(s)$.

• template<typename $_{\mathrm{Tp}}$ >

Evaluate the Riemann zeta function $\zeta(s)$ by an alternate series for s > 0.

template<typename_Tp>

Evaluate the Riemann zeta function $\zeta(s)$ by an alternate series for s > 0.

template<typename _Tp >

Evaluate the Riemann zeta function by series for all $s \neq 1$. Convergence is great until largish negative numbers. Then the convergence of the > 0 sum gets better.

template<typename _Tp >

Return the Riemann zeta function $\zeta(s) - 1$.

template<typename _Tp >

Return the Riemann zeta function $\zeta(s)-1$ by summation for s>1. This is a small remainder for large s.

template<typename _Tp >

Compute the Riemann zeta function $\zeta(s)$ using the product over prime factors.

template<typenameTp >

Compute the Riemann zeta function $\zeta(s)$ by summation for s > 1.

template<typename
 Tp >

$$_$$
gnu_cxx:: $_$ promote_num_t< $_$ Tp $>$ $_$ sinc ($_$ Tp $_$ a, $_$ Tp $_$ x)

Return the generalized sinus cardinal function

$$sinc_a(x) = \frac{\sin(\pi x/a)}{(\pi x/a)}$$

template<typename_Tp>

gnu cxx:: promote num t
$$<$$
 Tp $>$ sinc (Tp x)

Return the normalized sinus cardinal function

$$sinc(x) = \frac{\sin(\pi x)}{\pi x}$$

• template<typename $_{\rm Tp}>$

$$_$$
gnu_cxx:: $_$ promote_num_t< $_$ Tp $>$ $_$ sinc_pi ($_$ Tp $_$ x)

Return the unnormalized sinus cardinal function

$$sinc_{\pi}(x) = \frac{\sin(x)}{x}$$

.

• template<typename $_{\mathrm{Tp}}>$

This function returns the sine Si(x) and cosine Ci(x) integrals as a pair.

• template<typename _Tp >

This function computes the sine Si(x) and cosine Ci(x) integrals by asymptotic series summation for positive argument.

template < typename _Tp >
 void sincosint cont frac (Tp t, Tp & Si, Tp & Ci)

This function computes the sine Si(x) and cosine Ci(x) integrals by continued fraction for positive argument.

template<typename _Tp >

This function computes the sine Si(x) and cosine Ci(x) integrals by series summation for positive argument.

template<typename _Tp >

Return the generalized hyperbolic sinus cardinal function

$$sinhc_a(x) = \frac{\sinh(\pi x/a)}{\pi x/a}$$

•

template<typename_Tp>

Return the normalized hyperbolic sinus cardinal function

$$sinhc(x) = \frac{\sinh(\pi x)}{\pi x}$$

.

template<typename
 Tp >

Return the unnormalized hyperbolic sinus cardinal function

$$sinhc_{\pi}(x) = \frac{\sinh(x)}{x}$$

.

• template<typename _Tp >

Return the hyperbolic sine integral li(x).

template<typename_Tp>

Return the spherical Bessel function $j_n(x)$ of order n and non-negative real argument x.

template<typenameTp >

Return the complex spherical Bessel function.

• template<typename _Tp >

Compute the spherical modified Bessel functions $i_n(x)$ and $k_n(x)$ and their first derivatives $i'_n(x)$ and $k'_n(x)$ respectively.

template<typename _Tp >

Compute the spherical Bessel $j_n(x)$ and Neumann $n_n(x)$ functions and their first derivatives $j_n(x)$ and $n'_n(x)$ respectively.

template<typename_Tp>

void
$$_$$
sph_hankel (unsigned int $_$ n, std::complex< $_$ Tp $>$ $_$ z, std::complex< $_$ Tp $>$ & $_$ H1, std::complex< $_$ Tp $>$ & $_$ H2, std::complex< $_$ Tp $>$ & $_$ H2p)

Helper to compute complex spherical Hankel functions and their derivatives.

template<typename _Tp >

Return the spherical Hankel function of the first kind $h_n^{(1)}(x)$.

template<typename_Tp>

```
std::complex< Tp > sph hankel 1 (unsigned int n, std::complex< Tp > z)
```

```
Return the complex spherical Hankel function of the first kind.
template<typename _Tp >
  std::complex < Tp > sph hankel 2 (unsigned int n, Tp x)
      Return the spherical Hankel function of the second kind h_n^{(2)}(x).
template<typename _Tp >
  std::complex < _Tp > __sph_hankel_2 (unsigned int __n, std::complex < _Tp > __z)
      Return the complex spherical Hankel function of the second kind.
template<typename _Tp >
  std::complex < _Tp > __sph_harmonic (unsigned int __l, int __m, _Tp __theta, _Tp __phi)
      Return the spherical harmonic function.

    template<typename</li>
    Tp >

  _Tp __sph_legendre (unsigned int __l, unsigned int __m, _Tp __theta)
      Return the spherical associated Legendre function.
template<typename _Tp >
  _Tp __sph_neumann (unsigned int __n, _Tp __x)
      Return the spherical Neumann function n_n(x) of order n and non-negative real argument x.
  std::complex < _Tp > __sph_neumann (unsigned int __n, std::complex < _Tp > __z)
      Return the complex spherical Neumann function.
template<typename_Tp>
  _GLIBCXX14_CONSTEXPR _Tp __student_t_cdf (_Tp __t, unsigned int __nu)
      Return the Students T probability function.
template<typename_Tp>
  _GLIBCXX14_CONSTEXPR _Tp __student_t_cdfc (_Tp __t, unsigned int __nu)
      Return the complement of the Students T probability function.
template<typename</li>Tp >
  _Tp <u>__theta_</u>1 (_Tp __nu, _Tp __x)
template<typename Tp >
  _Tp <u>__theta_2</u> (_Tp __nu, _Tp __x)
template<typename</li>Tp >
  _Tp __theta_2_asymp (_Tp __nu, _Tp __x)

    template<typename</li>
    Tp >

  _Tp <u>__theta_2_sum</u> (_Tp __nu, _Tp __x)
template<typename _Tp >
  _Tp <u>__theta_3</u> (_Tp __nu, _Tp __x)
template<typename _Tp >
  _Tp __theta_3_asymp (_Tp __nu, _Tp __x)

 template<typename _Tp >

  _Tp <u>__theta_3_sum</u> (_Tp __nu, _Tp __x)
template<typename _Tp >
  _Tp <u>__theta_4</u> (_Tp __nu, _Tp __x)
template<typename _Tp >
  _Tp <u>__theta_</u>c (_Tp __k, _Tp __x)
template<typename _Tp >
  _Tp <u>__theta_</u>d (_Tp __k, _Tp __x)
template<typename _Tp >
  _Tp <u>__theta_</u>n (_Tp __k, _Tp __x)
• template<typename _{\mathrm{Tp}} >
  _Tp <u>__theta_s</u> (_Tp __k, _Tp __x)

 template<typename _Tp >

   __gnu_cxx::__promote_num_t< _Tp > __zernike (unsigned int __n, int __m, _Tp __rho, _Tp __phi)
```

```
    template < typename _Tp >
        _Tp __znorm1 (_Tp __x)
    template < typename _Tp >
        _Tp __znorm2 (_Tp __x)
    template < typename _Tp = double >
        _Tp evenzeta (unsigned int __k)
```

Variables

```
    constexpr size t Num Euler Maclaurin zeta = 100

    constexpr Factorial table < long double > S double factorial table [301]

    constexpr long double _S_Euler_Maclaurin_zeta [_Num_Euler_Maclaurin_zeta]

    constexpr _Factorial_table < long double > _S_factorial_table [171]

    constexpr_Factorial_table< long double > _S_neg_double_factorial_table [999]

template<typename _Tp >
  constexpr std::size_t _S_num_double_factorials = 0
template<>
  constexpr std::size t S num double factorials < double > = 301
• template<>
  constexpr std::size t S num double factorials < float > = 57
template<>
  constexpr std::size_t _S_num_double_factorials< long double > = 301
template<typename _Tp >
  constexpr std::size_t _S_num_factorials = 0
template<>
  constexpr std::size_t _S_num_factorials< double > = 171
template<>
  constexpr std::size_t _S_num_factorials< float > = 35
template<>
  constexpr std::size_t _S_num_factorials< long double > = 171
template<typename</li>Tp >
  constexpr std::size_t _S_num_neg_double_factorials = 0
template<>
  constexpr std::size_t _S_num_neg_double_factorials< double > = 150
  constexpr std::size t S num neg double factorials < float > = 27
template<>
  constexpr std::size_t _S_num_neg_double_factorials< long double > = 999
constexpr size_t _S_num_zetam1 = 33

    constexpr long double S zetam1 [S num zetam1]
```

8.3.1 Enumeration Type Documentation

8.3.1.1 anonymous enum

Enumerator

SININT

COSINT

Definition at line 42 of file sf trigint.tcc.

8.3.2 Function Documentation

8.3.2.1 template<typename _Tp > void std::__detail::__airy(_Tp __z, _Tp & _*Ai,* _Tp & _*Bi,* _Tp & _*Aip,* _Tp & _*Bip*)

Compute the Airy functions Ai(x) and Bi(x) and their first derivatives Ai'(x) and Bi(x) respectively.

Parameters

Z	The argument of the Airy functions.
_Ai	The output Airy function of the first kind.
_Bi	The output Airy function of the second kind.
_Aip	The output derivative of the Airy function of the first kind.
_Bip	The output derivative of the Airy function of the second kind.

Definition at line 486 of file sf mod bessel.tcc.

References __cyl_bessel_ik(), and __cyl_bessel_jn().

8.3.2.2 template<typename _Tp > void std::__detail::__airy (const std::complex< _Tp > & __z, _Tp __eps, std::complex< _Tp > & _Ai, std::complex< _Tp > & _Aip, std::complex< _Tp > & _Bi, std::complex< _Tp > & _Bip)

This function computes the Airy function Ai(z) and its first derivative in the complex z-plane.

The algorithm used exploits numerous representations of the Airy function and its derivative. The representations are recorded here for reference:

$$(1a)Ai(z) = \frac{\sqrt{z}}{3}(I_{-1/3}(\zeta) - I_{1/3}(\zeta))$$

$$(1b)Bi(z) = \sqrt{\frac{z}{3}}(I_{-1/3}(\zeta) + I_{1/3}(\zeta))$$

$$(2)Ai(z) = \frac{\sqrt{z/3}}{\pi}K_{1/3}(\zeta) = \frac{2^{2/3}3^{-5/6}}{\sqrt{(\pi)}}z\exp(-\zeta)U(5/6;5/3;2\zeta)$$

$$(3a)Ai(-z) = \frac{\sqrt{z}}{3}(J_{-1/3}(\zeta) + J_{1/3}(\zeta))$$

$$(3b)Bi(-z) = \sqrt{\frac{z}{3}}(J_{-1/3}(\zeta) - J_{1/3}(\zeta))$$

$$(4a)Ai'(z) = \frac{z}{3}(I_{2/3}(\zeta) - I_{-2/3}(\zeta))$$

$$(4b)Bi'(z) = \frac{z}{\sqrt{3}}(I_{-2/3}(\zeta) + I_{2/3}(\zeta))$$

$$(5a)Ai'(z) = -\frac{z}{\pi\sqrt{(3)}}K_{(2/3)}(zeta) = -\frac{4^{2/3}3^{-7/6}}{\sqrt{(\pi)}}z^2\exp(-\zeta)U(7/6;7/3;2\zeta)$$

$$(6a)Ai'(-z) = \frac{z}{3}(J_{2/3}(\zeta) - J_{-2/3}(\zeta)),$$

$$(6b)Bi'(-z) = \frac{z}{\sqrt{3}}(J_{-2/3}(\zeta) + J_{2/3}(\zeta)),$$

Where $\zeta=-\frac{2}{3}z^{3/2}$ and U(a;b;z) is the confluent hypergeometric function defined in

See also

Stegun, I. A. and Abramowitz, M., Handbook of Mathematical Functions, Natl. Bureau of Standards, AMS 55, pp 504-515, 1964.

The asymptotic expansions derivable from these representations and Hankel's asymptotic expansions for the Bessel functions are used for large modulus of z. The implementation has taken advantage of the error bounds given in

See also

Olver, F. W. J., Error Bounds for Asymptotic Expansions, with an Application to Cylinder Functions of Large Argument, in Asymptotic Solutions of Differential Equations (Wilcox, Ed.), Wiley and Sons, pp 163-183, 1964

and

See also

Olver, F. W. J., Asymptotics and Special Functions, Academic Press, pp 266-268, 1974.

For small modulus of z, a rational approximation is used. This approximant is derived from

Luke, Y. L., Mathematical Functions and their Approximations, Academic Press, pp 361-363, 1975.

The identities given below are for Bessel functions of the first kind in terms of modified Bessel functions of the first kind are also used with the rational approximant.

For moderate modulus of z, three techniques are used. Two use a backward recursion algorithm with (1), (3), (4), and (6). The third uses the confluent hypergeometric representations given by (2) and (5). The backward recursion algorithm generates values of the modified Bessel functions of the first kind of orders + or - 1/3 and + or - 2/3 for z in the right half plane. Values for the corresponding Bessel functions of the first kind are recovered via the identities

$$J_{\nu}(z) = exp(\nu \pi i/2)I_{\nu}(zexp(-\pi i/2)), 0 \le arg(z) \le \pi/2$$

and

$$J_{\nu}(z) = exp(-\nu\pi i/2)I_{\nu}(zexp(\pi i/2)), -\pi/2 \le arg(z) < 0.$$

The particular backward recursion algorithm used is discussed in

See also

Olver, F. W. J, Numerical solution of second-order linear difference equations, NBS J. Res., Series B, VOL 71B, pp 111-129, 1967.

Olver, F. W. J. and Sookne, D. J., Note on backward recurrence algorithms, Math. Comp. Vol 26, No. 120, pp 941-947, Oct. 1972

Sookne, D. J., Bessel Functions I and J of Complex Argument and Integer Order, NBS J. Res., Series B, Vol 77B, Nos 3& 4, pp 111-113, July-December, 1973.

The following paper was also useful

See also

Cody, W. J., Preliminary report on software for the modified Bessel functions of the first kind, Applied Mathematics Division, Argonne National Laboratory, Tech. Memo. no. 357.

A backward recursion algorithm is also used to compute the confluent hypergeometric function. The recursion relations and a convergence theorem are given in

See also

Wimp, J., On the computation of Tricomi's psi function, Computing, Vol 13, pp 195-203, 1974.

in	z	The argument at which the Airy function and its derivative are computed.
in	eps	Relative error required. Currently, eps is used only in the backward recursion
		algorithms.
out	_Ai	The value computed for Ai(z).
out	_Aip	The value computed for Ai'(z).
out	_Bi	The value computed for Bi(z).
out	_Bip	The value computed for Bi'(z).

Definition at line 1008 of file sf_airy.tcc.

References __airy_asymp_absarg_ge_pio3(), __airy_asymp_absarg_lt_pio3(), __airy_bessel_i(), __airy_bessel_k(), and __airy_hyperg_rational().

Referenced by __airy_ai(), __airy_bi(), __hankel_uniform_outer(), and __poly_hermite_asymp().

8.3.2.3 template<typename _Tp > std::complex< _Tp> std::__detail::__airy_ai (std::complex< _Tp > __z)

Return the complex Airy Ai function.

Definition at line 1145 of file sf airy.tcc.

References __airy().

8.3.2.4 template < typename _Tp > void std::__detail::__airy_arg (std::complex < _Tp > __num2d3, std::complex < _Tp > __zeta, std::complex < _Tp > & __argp, std::complex < _Tp > & __argm)

Compute the arguments for the Airy function evaluations carefully to prevent premature overflow. Note that the major work here is in $safe_div$. A faster, but less safe implementation can be obtained without use of safe_div.

Parameters

in	num2d3	$nu^{(-2/3)}$ - output from hankel_params.
in	zeta	zeta in the uniform asymptotic expansions - output from hankel_params.
out	argp	$\exp(+2*pi*i/3)*nu^{(2/3)}*zeta.$
out	argm	$\exp(-2*pi*i/3)*nu^{\wedge}(2/3)*zeta.$

Exceptions

std::runtime_error.	

Definition at line 241 of file sf_hankel.tcc.

Referenced by __hankel_uniform_outer().

8.3.2.5 template < typename _Tp > void std::__detail::__airy_asymp_absarg_ge_pio3 (std::complex < _Tp > __z, std::complex < _Tp > & _Ai, std::complex < _Tp > & _Aip, int __sign = -1)

This function evaluates Ai(z) and Ai'(z) from their asymptotic expansions for $|arg(z)| < 2 * \pi/3$. For speed, the number of terms needed to achieve about 16 decimals accuracy is tabled and determined from abs(z).

Note that for the sake of speed and the fact that this function is to be called by another, checks for valid arguments are not made.

See also

Digital Library of Mathematical Finctions Sec. 9.7 Asymptotic Expansions http://dlmf.nist.gov/9.7

Parameters

in	z	Complex input variable set equal to the value at which $Ai(z)$ and $Bi(z)$ and their
		derivative are evaluated. This function assumes $ z >15$ and $ arg(z) <2\pi/3$.
in,out	_Ai	The value computed for $Ai(z)$.
in,out	_Aip	The value computed for $Ai'(z)$.
in	sign	The sign of the series terms amd exponent. The default (-1) gives the Airy
		Ai functions for $ arg(z) < \pi$. The value +1 gives the Airy Bi functions for
		$ arg(z) < \pi/3.$

Definition at line 71 of file sf airy.tcc.

Referenced by __airy().

This function evaluates Ai(z) and Ai'(z) from their asymptotic expansions for |arg(-z)| < pi/3. For speed, the number of terms needed to achieve about 16 decimals accuracy is tabled and determined from |z|.

Note that for the sake of speed and the fact that this function is to be called by another, checks for valid arguments are not made. Hence, an error return is not needed. This function assumes |z| > 15 and |arg(-z)| < pi/3.

Parameters

in	Z	The value at which the Airy function and its derivative are evaluated.
out	_Ai	The computed value of the Airy function $Ai(z)$.
out	_Aip	The computed value of the Airy function derivative $Ai'(z)$.

Definition at line 187 of file sf airy.tcc.

Referenced by __airy().

Compute the modified Bessel functions of the first kind of orders +-1/3 and +-2/3 needed to compute the Airy functions and their derivatives from their representation in terms of the modified Bessel functions. This function is only used for z less than two in modulus and in the closed right half plane. This stems from the fact that the values of the modified Bessel functions occuring in the representations of the Airy functions and their derivatives are almost equal for z large in the right half plane. This means that loss of significance occurs if these representations are used for z to large in magnitude. This algorithm is also not used for z too small, since a low order rational approximation can be used instead.

This routine is an implementation of a modified version of Miller's backward recurrence algorithm for computation by from the recurrence relation

$$I_{\nu-1} = (2\nu/z)I_{\nu} + I_{\nu+1}$$

satisfied by the modified Bessel functions of the first kind. the normalization relationship used is

$$\frac{z/2)^{\nu}e^{z}}{\Gamma(\nu+1)} = I_{\nu}(z) + 2\sum_{k=1}^{\infty} \frac{(k+\nu)\Gamma(2\nu+k)}{k!\Gamma(1+2\nu)} I_{\nu+k}(z).$$

This modification of the algorithm is given in part in

Olver, F. W. J. and Sookne, D. J., Note on Backward Recurrence Algorithms, Math. of Comp., Vol. 26, no. 120, Oct. 1972.

And further elaborated for the Bessel functions in

Sookne, D. J., Bessel Functions I and J of Complex Argument and Integer Order, J. Res. NBS - Series B, Vol 77B, Nos. 3 & 4, July-December, 1973.

Insight was also gained from

Cody, W. J., Preliminary Report on Software for the Modified Bessel Functions of the First Kind, Argonne National Laboratory, Applied Mathematics Division, Tech. Memo. No. 357, August, 1980.

Cody implements the algorithm of Sookne for fractional order and nonnegative real argument. Like Cody, we do not change the convergence testing mechanism in any substantial way. However, we do trim the overhead by making the additional assumption that performing the convergence test for the functions of order 2/3 will suffice for order 1/3 as well. This assumption has not been established by rigourous analysis at this time. For the sake of speed the convergence tests are performed in the 1-norm instead of the usual Euclidean norm used in the complex plane using the inequality

$$|x| + |y| \le \sqrt{(2)}\sqrt{(x^2 + y^2)}$$

Parameters

in	z	The argument of the modified Bessel functions.
in	eps	The maximum relative error required in the results.
out	_lp1d3	The value of $I_{(}+1/3)(z)$.
out	_lm1d3	The value of $I_{(}-1/3)(z)$.
out	_lp2d3	The value of $I_{(}+2/3)(z)$.
out	_Im2d3	The value of $I_{(}-2/3)(z)$.

Definition at line 393 of file sf_airy.tcc.

Referenced by airy().

Compute approximations to the modified Bessel functions of the second kind of orders 1/3 and 2/3 for moderate arguments.

This routine computes

$$E_{\nu}(z) = \exp z \sqrt{2z/\pi} K_{\nu}(z), for \nu = 1/3 and \nu = 2/3$$

using a rational approximation given in

Luke, Y. L., Mathematical functions and their approximations, Academic Press, pp 366-367, 1975.

Though the approximation converges in $|\arg(z)| <= pi$, The convergence weakens as abs(arg(z)) increases. Also, in the case of real order between 0 and 1, convergence weakens as the order approaches 1. For these reasons, we only use this code for $|\arg(z)| <= 3pi/4$ and the convergence test is performed only for order 2/3.

The coding of this function is also influenced by the fact that it will only be used for about 2 <= |z| <= 15. Hence, certain considerations of overflow, underflow, and loss of significance are unimportant for our purpose.

in	z	The value for which the quantity E_nu is to be computed. it is recommended that
		abs(z) not be too small and that $ \arg(z) <= 3pi/4$.
in	eps	The maximum relative error allowable in the computed results. The relative error
		test is based on the comparison of successive iterates.
out	_Kp1d3	The value computed for $E_{+1/3}(z)$.
out	_Kp2d3	The value computed for $E_{+2/3}(z)$.

Note

In the worst case, say, z=2 and $\arg(z)=3pi/4$, 20 iterations should give 7 or 8 decimals of accuracy.

Definition at line 607 of file sf airy.tcc.

Referenced by airy().

8.3.2.9 template> std::complex<_Tp
$$>$$
 std::__detail::__airy_bi (std::complex< _Tp $>$ __z)

Return the complex Airy Bi function.

Definition at line 1158 of file sf airy.tcc.

References airy().

This function computes rational approximations to the hypergeometric functions related to the modified Bessel functions of orders $\nu=+-1/3$ and $\nu+-2/3$. That is, A(z)/B(z), Where A(z) and B(z) are cubic polynomials with real coefficients, approximates

$$\frac{\Gamma(\nu+1)}{(z/2)^n u} I_{\nu}(z) =_0 F_1(;\nu+1;z^2/4),$$

where the function on the right is a confluent hypergeometric limit function. For |z| <= 1/4 and |arg(z)| <= pi/2, the approximations are accurate to about 16 decimals.

For further details including the four term recurrence relation satisfied by the numerator and denominator poly-nomials in the higher order approximants, see

Luke, Y.L., Mathematical Functions and their Approximations, Academic Press, pp 361-363, 1975.

An asymptotic expression for the error is given as well as other useful expressions in the event one wants to extend this function to incorporate higher order approximants.

Note also that for the sake of speed and the fact that this function will be driven by another, checks that are not absolutely necessary are not made.

Parameters

in	Z	The argument at which the hypergeometric given above is to be evaluated. Since
		the approximation is of fixed order, $\left z\right $ must be small to ensure sufficient accuracy
		of the computed results.

out	_Ai	The Airy function $Ai(z)$.
out	_Aip	The Airy function derivative $Ai'(z)$.
out	_Bi	The Airy function $Bi(z)$.
out	_Bip	The Airy function derivative $Bi'(z)$.

Definition at line 791 of file sf airy.tcc.

Referenced by airy().

8.3.2.11 template < typename _Tp > _Tp std::__detail::__assoc_laguerre (unsigned int $_n$, unsigned int $_m$, _Tp $_x$)

This routine returns the associated Laguerre polynomial of order n, degree m: $L_n^m(x)$.

The associated Laguerre polynomial is defined for integral $\alpha=m$ by:

$$L_n^m(x) = (-1)^m \frac{d^m}{dx^m} L_{n+m}(x)$$

where the Laguerre polynomial is defined by:

$$L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n} (x^n e^{-x})$$

Parameters

n	The order
m	The degree
X	The argument

Returns

The value of the associated Laguerre polynomial of order n, degree m, and argument x.

Definition at line 292 of file sf_laguerre.tcc.

Referenced by __hydrogen().

8.3.2.12 template<typename _Tp > _Tp std::__detail::__assoc_legendre_p (unsigned int __I, unsigned int __m, _Tp __x)

Return the associated Legendre function by recursion on l and downward recursion on m.

The associated Legendre function is derived from the Legendre function $P_l(x)$ by the Rodrigues formula:

$$P_l^m(x) = (1 - x^2)^{m/2} \frac{d^m}{dx^m} P_l(x)$$

Parameters

	The order of the associated Legendre function. $l>=0$.
m	The order of the associated Legendre function. $m <= l$.
x	The argument of the associated Legendre function. $ x <= 1$.

Definition at line 175 of file sf_legendre.tcc.

References __poly_legendre_p().

8.3.2.13 template < typename $_{Tp} > _{GLIBCXX14}_{CONSTEXPR}_{Tp}$ std:: __detail:: __bernoulli (int $_{n}$)

This returns Bernoulli number B_n .

n	the order n of the Bernoulli number.

Returns

The Bernoulli number of order n.

Definition at line 1673 of file sf gamma.tcc.

This returns Bernoulli number B_n .

Parameters

_	the ender a of the Democritic acceptant
n n	the order n of the Bernoulli number.
	and diddin in an and Donnoun indination

Returns

The Bernoulli number of order n.

Definition at line 1685 of file sf gamma.tcc.

This returns Bernoulli numbers from a table or by summation for larger values.

Upward recursion is unstable.

Parameters

n the order n of the Bernoulli number.
--

Returns

The Bernoulli number of order n.

Definition at line 1608 of file sf_gamma.tcc.

Return the beta function B(a, b).

The beta function is defined by

$$B(a,b) = \int_0^1 t^{a-1} (1-t)^{b-1} dt = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

Parameters

a	The first argument of the beta function.
b	The second argument of the beta function.

Returns

The beta function.

Definition at line 173 of file sf_beta.tcc.

References __beta_lgamma().

Referenced by __poly_jacobi(), __gnu_cxx::jacobi(), __gnu_cxx::jacobif(), and __gnu_cxx::jacobil().

8.3.2.17 template<typename _Tp > _Tp std::__detail::__beta_gamma (_Tp $_a$, _Tp $_b$)

Return the beta function: B(a,b).

The beta function is defined by

$$B(a,b) = \int_0^1 t^{a-1} (1-t)^{b-1} dt = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

Parameters

a	The first argument of the beta function.
b	The second argument of the beta function.

Returns

The beta function.

Definition at line 75 of file sf_beta.tcc.

References __gamma().

8.3.2.18 template < typename _Tp > _Tp std::__detail::__beta_inc (_Tp __a, _Tp __b, _Tp __x)

Return the regularized incomplete beta function, $I_x(a,b)$, of arguments a, b, and x.

The regularized incomplete beta function is defined by:

$$I_x(a,b) = \frac{B_x(a,b)}{B(a,b)}$$

where

$$B_x(a,b) = \int_0^x t^{a-1} (1-t)^{b-1} dt$$

is the non-regularized beta function and B(a,b) is the usual beta function.

Parameters

a	The first parameter
---	---------------------

b	The second parameter
X	The argument

Definition at line 262 of file sf_beta.tcc.

References __beta_inc_cont_frac().

Referenced by $_$ binomial_cdf(), $_$ binomial_cdfc(), $_$ fisher_f_cdf(), $_$ fisher_f_cdfc(), $_$ student_t_cdfc(), and $_$ \leftarrow student_t_cdfc().

8.3.2.19 template < typename _Tp > _Tp std::__detail::__beta_inc_cont_frac (_Tp $_a$, _Tp $_b$, _Tp $_x$)

Return the regularized incomplete beta function, $I_x(a, b)$, of arguments a, b, and x.

Parameters

a	The first parameter
b	The second parameter
X	The argument

Definition at line 193 of file sf beta.tcc.

Referenced by __beta_inc().

8.3.2.20 template<typename _Tp > _Tp std::__detail::__beta_lgamma (_Tp *__a,* _Tp *__b*)

Return the beta function B(a,b) using the log gamma functions.

The beta function is defined by

$$B(a,b) = \int_0^1 t^{a-1} (1-t)^{b-1} dt = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

Parameters

a	The first argument of the beta function.
b	The second argument of the beta function.

Returns

The beta function.

Definition at line 109 of file sf beta.tcc.

References __log_gamma().

Referenced by __beta().

8.3.2.21 template < typename _Tp > _Tp std::__detail::__beta_product (_Tp $_a$, _Tp $_b$)

Return the beta function B(x, y) using the product form.

The beta function is defined by

$$B(a,b) = \int_0^1 t^{a-1} (1-t)^{b-1} dt = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

Here, we employ the product form:

$$B(a,b) = \frac{a+b}{ab} \prod_{k=1}^{\infty} \frac{1 + (a+b)/k}{(1+a/k)(1+b/k)}$$

a	The first argument of the beta function.
b	The second argument of the beta function.

Returns

The beta function.

Definition at line 140 of file sf beta.tcc.

8.3.2.22 template < typename $_{\rm Tp} > _{\rm Tp}$ std::__detail::__bincoef (unsigned int $_{\rm n}$, unsigned int $_{\rm k}$)

Return the binomial coefficient. The binomial coefficient is given by:

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

Parameters

n	The first argument of the binomial coefficient.
k	The second argument of the binomial coefficient.

Returns

The binomial coefficient.

Definition at line 1888 of file sf_gamma.tcc.

8.3.2.23 template < typename _Tp > _GLIBCXX14_CONSTEXPR _Tp std::__detail::__binomial_cdf (_Tp __p, unsigned int __n, unsigned int __k)

Return the binomial cumulative distribution function.

The binomial cumulative distribution function is related to the incomplete beta function:

$$P(p|n,k) = I_p(k, n - k + 1)$$

Parameters

p	
n	
k	

Definition at line 405 of file sf beta.tcc.

References beta inc().

8.3.2.24 template < typename _Tp > _GLIBCXX14_CONSTEXPR _Tp std::__detail::__binomial_cdfc (_Tp __p, unsigned int __n, unsigned int __k)

Return the complementary binomial cumulative distribution function.

The binomial cumulative distribution function is related to the incomplete beta function:

$$Q(p|n,k) = I_{1-p}(n-k+1,k)$$

p	
n	
k	

Definition at line 435 of file sf_beta.tcc.

References __beta_inc().

```
8.3.2.25 template < typename _Tp > _Tp std::__detail::__bose_einstein ( _Tp __s, _Tp __x )
```

Return the Bose-Einstein integral of real order s and real argument x.

See also

```
https://en.wikipedia.org/wiki/Clausen_function
http://dlmf.nist.gov/25.12#iii
```

Parameters

s	The order $s \ge 0$.
x	The real argument.

Returns

The real Fermi-Dirac cosine sum $G_s(x)$,

Definition at line 1401 of file sf_polylog.tcc.

References __polylog_exp().

Definition at line 44 of file sf chebyshev.tcc.

Referenced by __chebyshev_t(), __chebyshev_u(), __chebyshev_v(), and __chebyshev_w().

```
8.3.2.27 template < typename _Tp > _Tp std::__detail::__chebyshev_t ( unsigned int __n, _Tp __x )
```

Definition at line 58 of file sf chebyshev.tcc.

References __chebyshev_recur().

```
8.3.2.28 template < typename _Tp > _Tp std::__detail::__chebyshev_u ( unsigned int __n, _Tp __x )
```

Definition at line 73 of file sf chebyshev.tcc.

References __chebyshev_recur().

```
8.3.2.29 template<typename _Tp > _Tp std::__chebyshev_v ( unsigned int __n, _Tp __x )
```

Definition at line 88 of file sf_chebyshev.tcc.

References chebyshev recur().

8.3.2.30 template<typename _Tp > _Tp std::__detail::__chebyshev_w (unsigned int __n, _Tp __x)

Definition at line 103 of file sf_chebyshev.tcc.

References __chebyshev_recur().

8.3.2.31 template < typename _Tp > _GLIBCXX14_CONSTEXPR _Tp std::__detail::__chi_squared_pdf (_Tp __chi2, unsigned int __nu)

Return the chi-squared propability function. This returns the probability that the observed chi-squared for a correct model is less than the value χ^2 .

The chi-squared propability function is related to the normalized lower incomplete gamma function:

$$P(\chi^2|\nu) = \Gamma_P(\frac{\nu}{2}, \frac{\chi^2}{2})$$

Definition at line 2544 of file sf gamma.tcc.

References __gamma_p().

8.3.2.32 template < typename _Tp > _GLIBCXX14_CONSTEXPR _Tp std::__detail::__chi_squared_pdfc (_Tp __chi2, unsigned int __nu)

Return the complementary chi-squared propability function. This returns the probability that the observed chi-squared for a correct model is greater than the value χ^2 .

The complementary chi-squared propability function is related to the normalized upper incomplete gamma function:

$$Q(\chi^2|\nu) = \Gamma_Q(\frac{\nu}{2}, \frac{\chi^2}{2})$$

Definition at line 2568 of file sf gamma.tcc.

References __gamma_q().

8.3.2.33 template < typename _Tp > std::pair < _Tp, _Tp> std::__detail::__chshint (_Tp __x, _Tp & _Chi, _Tp & _Shi)

This function returns the hyperbolic cosine Ci(x) and hyperbolic sine Si(x) integrals as a pair.

The hyperbolic cosine integral is defined by:

$$Chi(x) = \gamma_E + \log(x) + \int_0^x dt \frac{\cosh(t) - 1}{t}$$

The hyperbolic sine integral is defined by:

$$Shi(x) = \int_0^x dt \frac{\sinh(t)}{t}$$

Definition at line 162 of file sf hypint.tcc.

References chshint cont frac(), and chshint series().

```
8.3.2.34 template < typename _Tp > void std::__detail::__chshint_cont_frac ( _Tp __t, _Tp & _Chi, _Tp & _Shi )
```

This function computes the hyperbolic cosine Chi(x) and hyperbolic sine Shi(x) integrals by continued fraction for positive argument.

Definition at line 50 of file sf_hypint.tcc.

Referenced by chshint().

```
8.3.2.35 template<typename_Tp > void std::__detail::__chshint_series ( _Tp __t, _Tp & _Chi, _Tp & _Shi )
```

This function computes the hyperbolic cosine Chi(x) and hyperbolic sine Shi(x) integrals by series summation for positive argument.

Definition at line 93 of file sf hypint.tcc.

Referenced by __chshint().

```
8.3.2.36 template < typename _Tp > std::complex < _Tp > std::__detail::__clamp_0_m2pi ( std::complex < _Tp > __w )
```

Definition at line 136 of file sf polylog.tcc.

Referenced by __polylog_exp_int_neg(), __polylog_exp_int_pos(), __polylog_exp_real_neg(), and __polylog_exp_\times real_pos().

```
8.3.2.37 template<typename_Tp > std::complex<_Tp> std::__detail::__clamp_pi ( std::complex<_Tp > __w )
```

Definition at line 123 of file sf polylog.tcc.

Referenced by $_$ polylog_exp_int_neg(), $_$ polylog_exp_int_pos(), $_$ polylog_exp_real_neg(), and $_$ polylog_exp_ \leftarrow real_pos().

```
8.3.2.38 template < typename _Tp > std::complex < _Tp > std::__detail::__clausen ( unsigned int __m, std::complex < _Tp > __w )
```

Return Clausen's function of integer order m and complex argument w. The notation and connection to polylog is from Wikipedia

Parameters

m	The non-negative integral order.
w	The complex argument.

Returns

The complex Clausen function.

Definition at line 1230 of file sf polylog.tcc.

References polylog exp().

```
8.3.2.39 template<typename _Tp > _Tp std::__detail::__clausen ( unsigned int __m, _Tp __w )
```

Return Clausen's function of integer order m and real argument w. The notation and connection to polylog is from Wikipedia

m	The integer order $m \ge 1$.
w	The real argument.

Returns

The Clausen function.

Definition at line 1254 of file sf_polylog.tcc.

References __polylog_exp().

Return Clausen's cosine sum Cl_m for positive integer order m and complex argument w.

See also

```
https://en.wikipedia.org/wiki/Clausen_function
```

Parameters

m	The integer order $m \ge 1$.
w	The real argument.

Returns

The Clausen cosine sum Cl_m(w),

Definition at line 1329 of file sf_polylog.tcc.

References __polylog_exp().

8.3.2.41 template < typename _Tp > _Tp std::__detail::__clausen_c (unsigned int
$$_m$$
, _Tp $_w$)

Return Clausen's cosine sum Cl_m for positive integer order m and real argument w.

See also

```
https://en.wikipedia.org/wiki/Clausen_function
```

Parameters

m	The integer order $m \ge 1$.
w	The real argument.

Returns

The real Clausen cosine sum Cl m(w),

Definition at line 1354 of file sf_polylog.tcc.

References __polylog_exp().

8.3.2.42 template < typename $_{\rm Tp} > _{\rm Tp}$ std::__detail::__clausen_s (unsigned int $_{\rm m}$, std::complex < $_{\rm Tp} > _{\rm w}$)

Return Clausen's sine sum SI m for positive integer order m and complex argument w.

See also

https://en.wikipedia.org/wiki/Clausen_function

Parameters

m	The integer order $m \ge 1$.
и	The complex argument.

Returns

The Clausen sine sum SI m(w),

Definition at line 1279 of file sf_polylog.tcc.

References polylog exp().

Return Clausen's sine sum SI m for positive integer order m and real argument w.

See also

https://en.wikipedia.org/wiki/Clausen_function

Parameters

m	The integer order $m \ge 1$.
w	The complex argument.

Returns

The Clausen sine sum SI_m(w),

Definition at line 1304 of file sf_polylog.tcc.

References polylog exp().

8.3.2.44 template _Tp std::__detail::__comp_ellint_1 (_Tp
$$_k$$
)

Return the complete elliptic integral of the first kind K(k) using the Carlson formulation.

The complete elliptic integral of the first kind is defined as

$$K(k) = F(k, \pi/2) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - k^2 sin^2 \theta}}$$

where $F(k, \phi)$ is the incomplete elliptic integral of the first kind.

k	The modulus of the complete elliptic function.

Returns

The complete elliptic function of the first kind.

Definition at line 565 of file sf ellint.tcc.

References __comp_ellint_rf().

Referenced by __ellint_1(), __ellnome_k(), __jacobi_zeta(), __theta_c(), __theta_d(), __theta_n(), and __theta_s().

8.3.2.45 template<typename _Tp > _Tp std::__detail::__comp_ellint_2 (_Tp $_k$)

Return the complete elliptic integral of the second kind E(k) using the Carlson formulation.

The complete elliptic integral of the second kind is defined as

$$E(k, \pi/2) = \int_0^{\pi/2} \sqrt{1 - k^2 sin^2 \theta}$$

Parameters

_		
	k	The modulus of the complete elliptic function.
		-

Returns

The complete elliptic function of the second kind.

Definition at line 638 of file sf ellint.tcc.

References __ellint_rd(), and __ellint_rf().

Referenced by __ellint_2().

8.3.2.46 template < typename _Tp > _Tp std::__detail::__comp_ellint_3 (_Tp $_k$, _Tp $_nu$)

Return the complete elliptic integral of the third kind $\Pi(k,\nu)=\Pi(k,\nu,\pi/2)$ using the Carlson formulation.

The complete elliptic integral of the third kind is defined as

$$\Pi(k,\nu) = \int_0^{\pi/2} \frac{d\theta}{(1-\nu\sin^2\theta)\sqrt{1-k^2\sin^2\theta}}$$

Parameters

k	The argument of the elliptic function.
nu	The second argument of the elliptic function.

Returns

The complete elliptic function of the third kind.

Definition at line 727 of file sf_ellint.tcc.

References __ellint_rf(), and __ellint_rj().

Referenced by __ellint_3().

```
8.3.2.47 template<typename_Tp > _Tp std::__detail::__comp_ellint_d ( _Tp __k )
```

Return the complete Legendre elliptic integral D.

Definition at line 832 of file sf_ellint.tcc.

References __ellint_rd().

Definition at line 235 of file sf_ellint.tcc.

Referenced by __comp_ellint_1(), and __ellint_rf().

Definition at line 346 of file sf_ellint.tcc.

Referenced by __ellint_rg().

Return the confluent hypergeometric function ${}_{1}F_{1}(a;c;x)$.

Parameters

a	The numerator parameter.
c	The denominator parameter.
X	The argument of the confluent hypergeometric function.

Returns

The confluent hypergeometric function.

Definition at line 281 of file sf_hyperg.tcc.

References __conf_hyperg_luke(), and __conf_hyperg_series().

8.3.2.51 template < typename
$$_{\rm Tp} > _{\rm Tp}$$
 std::__conf_hyperg_lim ($_{\rm Tp}$ __c, $_{\rm Tp}$ __x)

Return the confluent hypergeometric limit function ${}_{0}F_{1}(-;c;x)$.

Parameters

c	The denominator parameter.
X	The argument of the confluent hypergeometric limit function.

Returns

The confluent limit hypergeometric function.

Definition at line 109 of file sf hyperg.tcc.

References __conf_hyperg_lim_series().

This routine returns the confluent hypergeometric limit function by series expansion.

$$_{0}F_{1}(-;c;x) = \Gamma(c) \sum_{n=0}^{\infty} \frac{1}{\Gamma(c+n)} \frac{x^{n}}{n!}$$

If a and b are integers and a < 0 and either b > 0 or b < a then the series is a polynomial with a finite number of terms.

Parameters

c	The "denominator" parameter.
X	The argument of the confluent hypergeometric limit function.

Returns

The confluent hypergeometric limit function.

Definition at line 76 of file sf_hyperg.tcc.

Referenced by __conf_hyperg_lim().

Return the hypergeometric function ${}_1F_1(a;c;x)$ by an iterative procedure described in Luke, Algorithms for the Computation of Mathematical Functions.

Like the case of the 2F1 rational approximations, these are probably guaranteed to converge for x < 0, barring gross numerical instability in the pre-asymptotic regime.

Definition at line 176 of file sf_hyperg.tcc.

Referenced by __conf_hyperg().

This routine returns the confluent hypergeometric function by series expansion.

$$_1F_1(a;c;x) = \frac{\Gamma(c)}{\Gamma(a)} \sum_{n=0}^{\infty} \frac{\Gamma(a+n)}{\Gamma(c+n)} \frac{x^n}{n!}$$

Parameters

a	The "numerator" parameter.
c	The "denominator" parameter.
X	The argument of the confluent hypergeometric function.

Returns

The confluent hypergeometric function.

Definition at line 141 of file sf hyperg.tcc.

Referenced by __conf_hyperg().

8.3.2.55 template<typename _Tp > _Tp std::__detail::__coshint (const _Tp __x)

Return the hyperbolic cosine integral li(x).

The hyperbolic cosine integral is given by

$$Chi(x) = (Ei(x) - E_1(x))/2$$

Parameters

Returns

The hyperbolic cosine integral.

Definition at line 558 of file sf_expint.tcc.

References __expint_E1(), and __expint_Ei().

8.3.2.56 template<typename_Tp > std::complex<_Tp> std::__detail::__cyl_bessel (std::complex< _Tp > __nu, std::complex< _Tp > __z)

Return the complex cylindrical Bessel function.

Parameters

in	nu	The order for which the cylindrical Bessel function is evaluated.
in	Z	The argument at which the cylindrical Bessel function is evaluated.

Returns

The complex cylindrical Bessel function.

Definition at line 1222 of file sf hankel.tcc.

References __hankel().

8.3.2.57 template < typename _Tp > _Tp std::__detail::__cyl_bessel_i (_Tp __nu, _Tp __x)

Return the regular modified Bessel function of order ν : $I_{\nu}(x)$.

The regular modified cylindrical Bessel function is:

$$I_{\nu}(x) = \sum_{k=0}^{\infty} \frac{(x/2)^{\nu+2k}}{k!\Gamma(\nu+k+1)}$$

Parameters

nu	The order of the regular modified Bessel function.
----	--

	The argument of the regular modified Bessel function.

Returns

The output regular modified Bessel function.

Definition at line 375 of file sf mod bessel.tcc.

References __cyl_bessel_ij_series(), and __cyl_bessel_ik().

This routine returns the cylindrical Bessel functions of order ν : J_{ν} or I_{ν} by series expansion.

The modified cylindrical Bessel function is:

$$Z_{\nu}(x) = \sum_{k=0}^{\infty} \frac{\sigma^k (x/2)^{\nu+2k}}{k!\Gamma(\nu+k+1)}$$

where $\sigma = +1$ or -1 for Z = I or J respectively.

See Abramowitz & Stegun, 9.1.10 Abramowitz & Stegun, 9.6.7 (1) Handbook of Mathematical Functions, ed. Milton Abramowitz and Irene A. Stegun, Dover Publications, Equation 9.1.10 p. 360 and Equation 9.6.10 p. 375

Parameters

nu	The order of the Bessel function.
X	The argument of the Bessel function.
sgn	The sign of the alternate terms -1 for the Bessel function of the first kind. +1 for the modified
	Bessel function of the first kind.
max_iter	The maximum number of iterations for sum.

Returns

The output Bessel function.

Definition at line 413 of file sf_bessel.tcc.

References log gamma().

Referenced by __cyl_bessel_i(), and __cyl_bessel_j().

Return the modified cylindrical Bessel functions and their derivatives of order ν by various means.

Parameters

nu	The order of the Bessel functions.
X	The argument of the Bessel functions.

_Inu	The output regular modified Bessel function.
_Knu	The output irregular modified Bessel function.
_lpnu	The output derivative of the regular modified Bessel function.
_Kpnu	The output derivative of the irregular modified Bessel function.

Definition at line 316 of file sf_mod_bessel.tcc.

References cyl bessel ik asymp(), and cyl bessel ik steed().

Referenced by __airy(), __cyl_bessel_i(), __cyl_bessel_k(), and __sph_bessel_ik().

This routine computes the asymptotic modified cylindrical Bessel and functions of order nu: $I_{\nu}(x)$, $N_{\nu}(x)$. Use this for $x >> nu^2 + 1$.

References: (1) Handbook of Mathematical Functions, ed. Milton Abramowitz and Irene A. Stegun, Dover Publications, Section 9 p. 364, Equations 9.2.5-9.2.10

Parameters

nu	The order of the Bessel functions.
X	The argument of the Bessel functions.
_Inu	The output regular modified Bessel function.
_Knu	The output irregular modified Bessel function.
_lpnu	The output derivative of the regular modified Bessel function.
_Kpnu	The output derivative of the irregular modified Bessel function.

Definition at line 81 of file sf_mod_bessel.tcc.

Referenced by __cyl_bessel_ik(), and __cyl_bessel_ik_steed().

Compute the modified Bessel functions $I_{\nu}(x)$ and $K_{\nu}(x)$ and their first derivatives $I'_{\nu}(x)$ and $K'_{\nu}(x)$ respectively. These four functions are computed together for numerical stability.

Parameters

nu	The order of the Bessel functions.
X	The argument of the Bessel functions.
_Inu	The output regular modified Bessel function.
_Knu	The output irregular modified Bessel function.
_lpnu	The output derivative of the regular modified Bessel function.
_Kpnu	The output derivative of the irregular modified Bessel function.

Definition at line 152 of file sf_mod_bessel.tcc.

References __cyl_bessel_ik_asymp(), and __gamma_temme().

Referenced by __cyl_bessel_ik().

8.3.2.62 template < typename _Tp > _Tp std::__detail::__cyl_bessel_j (_Tp __nu, _Tp __x)

Return the Bessel function of order ν : $J_{\nu}(x)$.

The cylindrical Bessel function is:

$$J_{\nu}(x) = \sum_{k=0}^{\infty} \frac{(-1)^k (x/2)^{\nu+2k}}{k! \Gamma(\nu+k+1)}$$

Parameters

nu	The order of the Bessel function.
X	The argument of the Bessel function.

Returns

The output Bessel function.

Definition at line 503 of file sf bessel.tcc.

References __cyl_bessel_ij_series(), and __cyl_bessel_jn().

Return the cylindrical Bessel functions and their derivatives of order ν by various means.

Definition at line 442 of file sf bessel.tcc.

References __cyl_bessel_jn_asymp(), and __cyl_bessel_jn_steed().

Referenced by $_airy()$, $_cyl_bessel_j()$, $_cyl_hankel_1()$, $_cyl_hankel_2()$, $_cyl_neumann_n()$, and $_sph_\leftrightarrow bessel_jn()$.

This routine computes the asymptotic cylindrical Bessel and Neumann functions of order nu: $J_{\nu}(x)$, $N_{\nu}(x)$. Use this for $x >> nu^2 + 1$.

References: (1) Handbook of Mathematical Functions, ed. Milton Abramowitz and Irene A. Stegun, Dover Publications, Section 9 p. 364, Equations 9.2.5-9.2.10

Parameters

	nu	The order of the Bessel functions.
	x	The argument of the Bessel functions.
out	_Jnu	The Bessel function of the first kind.
out	_Nnu	The Neumann function (Bessel function of the second kind).
out	_Jpnu	The Bessel function of the first kind.
out	_Npnu	The Neumann function (Bessel function of the second kind).

Definition at line 79 of file sf bessel.tcc.

Referenced by __cyl_bessel_jn(), and __cyl_bessel_jn_steed().

Compute the Bessel $J_{\nu}(x)$ and Neumann $N_{\nu}(x)$ functions and their first derivatives $J'_{\nu}(x)$ and $N'_{\nu}(x)$ respectively. These four functions are computed together for numerical stability.

	nu	The order of the Bessel functions.
	x	The argument of the Bessel functions.
out	_Jnu	The output Bessel function of the first kind.
out	_Nnu	The output Neumann function (Bessel function of the second kind).
out	_Jpnu	The output derivative of the Bessel function of the first kind.
out	_Npnu	The output derivative of the Neumann function.

Definition at line 197 of file sf bessel.tcc.

References __cyl_bessel_jn_asymp(), and __gamma_temme().

Referenced by __cyl_bessel_in().

8.3.2.66 template < typename _Tp > _Tp std::__detail::__cyl_bessel_k (_Tp __nu, _Tp __x)

Return the irregular modified Bessel function $K_{\nu}(x)$ of order ν .

The irregular modified Bessel function is defined by:

$$K_{\nu}(x) = \frac{\pi}{2} \frac{I_{-\nu}(x) - I_{\nu}(x)}{\sin \nu \pi}$$

where for integral $\nu=n$ a limit is taken: $lim_{\nu\to n}$. For negative argument we have simply:

$$K_{-\nu}(x) = K_{\nu}(x)$$

Parameters

nu	The order of the irregular modified Bessel function.
X	The argument of the irregular modified Bessel function.

Returns

The output irregular modified Bessel function.

Definition at line 413 of file sf mod bessel.tcc.

References __cyl_bessel_ik().

 $8.3.2.67 \quad template < typename _Tp > std::_detail::_cyl_hankel_1 \left(\ _Tp __nu, \ _Tp __x \ \right)$

Return the cylindrical Hankel function of the first kind $H^{(1)}_{\nu}(x)$.

The cylindrical Hankel function of the first kind is defined by:

$$H_{\nu}^{(1)}(x) = J_{\nu}(x) + iN_{\nu}(x)$$

Parameters

nu	The order of the spherical Neumann function.

X	The argument of the spherical Neumann function.

Returns

The output spherical Neumann function.

Definition at line 568 of file sf bessel.tcc.

References __cyl_bessel_jn().

Return the complex cylindrical Hankel function of the first kind.

Parameters

in	nu	The order for which the cylindrical Hankel function of the first kind is evaluated.
in	z	The argument at which the cylindrical Hankel function of the first kind is evaluated.

Returns

The complex cylindrical Hankel function of the first kind.

Definition at line 1190 of file sf_hankel.tcc.

References hankel().

Return the cylindrical Hankel function of the second kind $H_n^{(2)}u(x)$.

The cylindrical Hankel function of the second kind is defined by:

$$H_{\nu}^{(2)}(x) = J_{\nu}(x) - iN_{\nu}(x)$$

Parameters

nu	The order of the spherical Neumann function.
X	The argument of the spherical Neumann function.

Returns

The output spherical Neumann function.

Definition at line 604 of file sf bessel.tcc.

References __cyl_bessel_jn().

Return the complex cylindrical Hankel function of the second kind.

in	nu	The order for which the cylindrical Hankel function of the second kind is evaluated.
in	z	The argument at which the cylindrical Hankel function of the second kind is eval-
		uated.

Returns

The complex cylindrical Hankel function of the second kind.

Definition at line 1206 of file sf_hankel.tcc.

References __hankel().

Return the complex cylindrical Neumann function.

Parameters

in	nu	The order for which the cylindrical Neumann function is evaluated.
in	z	The argument at which the cylindrical Neumann function is evaluated.

Returns

The complex cylindrical Neumann function.

Definition at line 1238 of file sf_hankel.tcc.

References __hankel().

Return the Neumann function of order ν : $N_{\nu}(x)$.

The Neumann function is defined by:

$$N_{\nu}(x) = \frac{J_{\nu}(x)\cos\nu\pi - J_{-\nu}(x)}{\sin\nu\pi}$$

where for integral $\nu = n$ a limit is taken: $\lim_{\nu \to n}$.

Parameters

nu	The order of the Neumann function.
X	The argument of the Neumann function.

Returns

The output Neumann function.

Definition at line 538 of file sf_bessel.tcc.

References __cyl_bessel_jn().

8.3.2.73 template<typename _Tp > _Tp std::__dawson (_Tp __x)

Return the Dawson integral, F(x), for real argument x.

The Dawson integral is defined by:

$$F(x) = e^{-x^2} \int_0^x e^{y^2} dy$$

and it's derivative is:

$$F'(x) = 1 - 2xF(x)$$

Parameters

X	The argument $-inf < x < inf$.
---	---------------------------------

Definition at line 233 of file sf dawson.tcc.

References __dawson_const_frac(), and __dawson_series().

8.3.2.74 template<typename _Tp > _Tp std::__dawson_const_frac (_Tp __x)

Compute the Dawson integral using a sampling theorem representation.

Todo this needs some compile-time construction!

Definition at line 71 of file sf dawson.tcc.

Referenced by dawson().

8.3.2.75 template<typename _Tp > _Tp std::__detail::__dawson_series (_Tp __x)

Compute the Dawson integral using the series expansion.

Definition at line 47 of file sf dawson.tcc.

Referenced by __dawson().

8.3.2.76 template < typename _Tp > void std::__detail::__debye_region (std::complex < _Tp > __alpha, int & __indexr, char & __aorb)

Compute the Debye region in te complex plane.

Definition at line 54 of file sf_hankel.tcc.

Referenced by __hankel().

8.3.2.77 template<typename _Tp > _Tp std::__detail::__dilog (_Tp __x)

Compute the dilogarithm function $Li_2(x)$ by summation for x <= 1.

The Riemann zeta function is defined by:

$$Li_2(x) = \sum_{k=1}^{\infty} \frac{1}{k^s} fors > 1$$

For |x| near 1 use the reflection formulae:

$$Li_2(-x) + Li_2(1-x) = \frac{\pi^2}{6} - \ln(x)\ln(1-x)$$

$$Li_2(-x) - Li_2(1-x) - \frac{1}{2}Li_2(1-x^2) = -\frac{\pi^2}{12} - \ln(x)\ln(1-x)$$

For x < 1 use the reflection formula:

$$Li_2(1-x) - Li_2(1-\frac{1}{1-x}) - \frac{1}{2}(\ln(x))^2$$

Definition at line 194 of file sf zeta.tcc.

 $8.3.2.78 \quad template < typename _Tp > _Tp \ std::_detail::_dirichlet_beta \ (\ std::complex < _Tp > _w \)$

Return the Dirichlet beta function. Currently, w must be real (complex type but negligible imaginary part.) Otherwise std::domain error is thrown.

Parameters

__w The complex (but on-real-axis) argument.

Returns

The Dirichlet Beta function of real argument.

Exceptions

std::domain_error if the argument has a significant imaginary part.

Definition at line 1192 of file sf_polylog.tcc.

References __fpequal(), and __polylog().

8.3.2.79 template<typename _Tp > _Tp std::__detail::__dirichlet_beta (_Tp __w)

Return the Dirichlet beta function for real argument.

Parameters

__w The real argument.

Returns

The Dirichlet Beta function of real argument.

Definition at line 1211 of file sf_polylog.tcc.

References __polylog().

 $8.3.2.80 \quad template < typename _Tp > std::complex < _Tp > std::_detail::_dirichlet_eta \ (\ std::complex < _Tp > _w \)$

Return the Dirichlet eta function. Currently, w must be real (complex type but negligible imaginary part.) Otherwise std::domain_error is thrown.

w	The complex (but on-real-axis) argument.

Returns

The complex Dirichlet eta function.

Exceptions

std::domain_error	if the argument has a significant imaginary part.

Definition at line 1155 of file sf polylog.tcc.

References __fpequal(), and __polylog().

Return the Dirichlet eta function for real argument.

Parameters

W	The real argument.

Returns

The Dirichlet eta function.

Definition at line 1173 of file sf polylog.tcc.

References __polylog().

Return the double factorial of the integer n.

The double factorial is defined for integral n by:

$$n!! = 135...(n-2)n, noddn!! = 246...(n-2)n, neven - 1!! = 10!! = 1$$

The double factorial is defined for odd negative integers in the obvious way:

$$(-2m-1)!! = 1/(1(-1)(-3)...(-2m+1)(-2m-1)) = \frac{(-1)^m}{(2m-1)!!}$$

for f[n = -2m - 1 f].

Definition at line 2480 of file sf gamma.tcc.

References factorial(), log double factorial(), S double factorial table, and S neg double factorial table.

Return the incomplete elliptic integral of the first kind $F(k,\phi)$ using the Carlson formulation.

The incomplete elliptic integral of the first kind is defined as

$$F(k,\phi) = \int_0^\phi \frac{d\theta}{\sqrt{1 - k^2 sin^2 \theta}}$$

k	The argument of the elliptic function.
phi	The integral limit argument of the elliptic function.

Returns

The elliptic function of the first kind.

Definition at line 594 of file sf ellint.tcc.

References __comp_ellint_1(), and __ellint_rf().

8.3.2.84 template> _Tp std::__detail::__ellint_2 (_Tp
$$_k$$
, _Tp $_phi$)

Return the incomplete elliptic integral of the second kind $E(k,\phi)$ using the Carlson formulation.

The incomplete elliptic integral of the second kind is defined as

$$E(k,\phi) = \int_0^{\phi} \sqrt{1 - k^2 sin^2 \theta}$$

Parameters

k	The argument of the elliptic function.
phi	The integral limit argument of the elliptic function.

Returns

The elliptic function of the second kind.

Definition at line 673 of file sf_ellint.tcc.

References __comp_ellint_2(), __ellint_rd(), and __ellint_rf().

8.3.2.85 template < typename _Tp > _Tp std::__detail::__ellint_3 (_Tp
$$_k$$
, _Tp $_nu$, _Tp $_phi$)

Return the incomplete elliptic integral of the third kind $\Pi(k,\nu,\phi)$ using the Carlson formulation.

The incomplete elliptic integral of the third kind is defined as

$$\Pi(k,\nu,\phi) = \int_0^\phi \frac{d\theta}{(1-\nu\sin^2\theta)\sqrt{1-k^2\sin^2\theta}}$$

Parameters

	k	The argument of the elliptic function.
	nu	The second argument of the elliptic function.
ĺ	phi	The integral limit argument of the elliptic function.

Returns

The elliptic function of the third kind.

Definition at line 768 of file sf ellint.tcc.

References __comp_ellint_3(), __ellint_rf(), and __ellint_rj().

8.3.2.86 template < typename $_{\rm Tp} > _{\rm Tp}$ std::__detail::__ellint_cel ($_{\rm Tp} _k_c, _{\rm Tp} _p, _{\rm Tp} _a, _{\rm Tp} _b$)

Return the Bulirsch complete elliptic integrals.

Definition at line 920 of file sf_ellint.tcc.

References __ellint_rf(), and __ellint_rj().

8.3.2.87 template < typename _Tp > _Tp std::__detail::__ellint_d (_Tp $_k$, _Tp $_phi$)

Return the Legendre elliptic integral D.

Definition at line 809 of file sf_ellint.tcc.

References __ellint_rd().

8.3.2.88 template> _Tp std::__detail::__ellint_el1 (_Tp
$$_x$$
, _Tp $_k_c$)

Return the Bulirsch elliptic integrals of the first kind.

Definition at line 848 of file sf ellint.tcc.

References __ellint_rf().

Return the Bulirsch elliptic integrals of the second kind.

Definition at line 869 of file sf ellint.tcc.

References __ellint_rd(), and __ellint_rf().

Return the Bulirsch elliptic integrals of the third kind.

Definition at line 894 of file sf ellint.tcc.

References __ellint_rf(), and __ellint_rj().

8.3.2.91 template < typename
$$_{\rm Tp} > _{\rm Tp}$$
 std::__ellint_rc ($_{\rm Tp}$ __x, $_{\rm Tp}$ __y)

Return the Carlson elliptic function $R_C(x,y) = R_F(x,y,y)$ where $R_F(x,y,z)$ is the Carlson elliptic function of the first kind.

The Carlson elliptic function is defined by:

$$R_C(x,y) = \frac{1}{2} \int_0^\infty \frac{dt}{(t+x)^{1/2}(t+y)}$$

Based on Carlson's algorithms:

- B. C. Carlson Numer. Math. 33, 1 (1979)
- B. C. Carlson, Special Functions of Applied Mathematics (1977)
- Numerical Recipes in C, 2nd ed, pp. 261-269, by Press, Teukolsky, Vetterling, Flannery (1992)

X	The first argument.
у	The second argument.

Returns

The Carlson elliptic function.

Definition at line 81 of file sf ellint.tcc.

Referenced by __ellint_rf(), and __ellint_rj().

Return the Carlson elliptic function of the second kind $R_D(x,y,z) = R_J(x,y,z,z)$ where $R_J(x,y,z,p)$ is the Carlson elliptic function of the third kind.

The Carlson elliptic function of the second kind is defined by:

$$R_D(x,y,z) = \frac{3}{2} \int_0^\infty \frac{dt}{(t+x)^{1/2}(t+y)^{1/2}(t+z)^{3/2}}$$

Based on Carlson's algorithms:

- B. C. Carlson Numer. Math. 33, 1 (1979)
- B. C. Carlson, Special Functions of Applied Mathematics (1977)
- Numerical Recipes in C, 2nd ed, pp. 261-269, by Press, Teukolsky, Vetterling, Flannery (1992)

Parameters

X	The first of two symmetric arguments.
y	The second of two symmetric arguments.
Z	The third argument.

Returns

The Carlson elliptic function of the second kind.

Definition at line 163 of file sf ellint.tcc.

Referenced by $_$ comp_ellint_2(), $_$ comp_ellint_d(), $_$ ellint_2(), $_$ ellint_d(), $_$ ellint_el2(), $_$ ellint_rg(), and $_$ \leftarrow ellint_rj().

Return the Carlson elliptic function $R_F(x, y, z)$ of the first kind.

The Carlson elliptic function of the first kind is defined by:

$$R_F(x,y,z) = \frac{1}{2} \int_0^\infty \frac{dt}{(t+x)^{1/2}(t+y)^{1/2}(t+z)^{1/2}}$$

X	The first of three symmetric arguments.
у	The second of three symmetric arguments.
z	The third of three symmetric arguments.

Returns

The Carlson elliptic function of the first kind.

Definition at line 277 of file sf ellint.tcc.

References __comp_ellint_rf(), and __ellint_rc().

Referenced by __comp_ellint_2(), __comp_ellint_3(), __ellint_1(), __ellint_2(), __ellint_3(), __ellint_cel(), __ellint_el1(), __ellint_el2(), __ellint_el3(), and __heuman_lambda().

Return the symmetric Carlson elliptic function of the second kind $R_G(x, y, z)$.

The Carlson symmetric elliptic function of the second kind is defined by:

$$R_G(x,y,z) = \frac{1}{4} \int_0^\infty dt t [(t+x)(t+y)(t+z)]^{-1/2} \left(\frac{x}{t+x} + \frac{y}{t+y} + \frac{z}{t+z}\right)$$

Based on Carlson's algorithms:

- B. C. Carlson Numer. Math. 33, 1 (1979)
- B. C. Carlson, Special Functions of Applied Mathematics (1977)
- · Numerical Recipes in C, 2nd ed, pp. 261-269, by Press, Teukolsky, Vetterling, Flannery (1992)

Parameters

X	The first of three symmetric arguments.
у	The second of three symmetric arguments.
z	The third of three symmetric arguments.

Returns

The Carlson symmetric elliptic function of the second kind.

Definition at line 408 of file sf_ellint.tcc.

References comp ellint rg(), and ellint rd().

$$8.3.2.95 \quad template < typename _Tp > _Tp \ std::_detail::_ellint_rj \ (\ _Tp _x, \ _Tp _y, \ _Tp _z, \ _Tp _p \)$$

Return the Carlson elliptic function $R_J(x, y, z, p)$ of the third kind.

The Carlson elliptic function of the third kind is defined by:

$$R_J(x,y,z,p) = \frac{3}{2} \int_0^\infty \frac{dt}{(t+x)^{1/2}(t+y)^{1/2}(t+z)^{1/2}(t+p)}$$

Based on Carlson's algorithms:

- B. C. Carlson Numer. Math. 33, 1 (1979)
- B. C. Carlson, Special Functions of Applied Mathematics (1977)
- Numerical Recipes in C, 2nd ed, pp. 261-269, by Press, Teukolsky, Vetterling, Flannery (1992)

X	The first of three symmetric arguments.
y	The second of three symmetric arguments.
Z	The third of three symmetric arguments.
p	The fourth argument.

Returns

The Carlson elliptic function of the fourth kind.

Definition at line 456 of file sf_ellint.tcc.

References __ellint_rc(), and __ellint_rd().

Referenced by __comp_ellint_3(), __ellint_cel(), __ellint_el3(), __heuman_lambda(), and __jacobi_zeta().

8.3.2.96 template < typename _Tp > _Tp std::__detail::__ellnome (_Tp $\underline{\hspace{0.1cm}}$)

Return the elliptic nome given the modulus k.

Definition at line 292 of file sf theta.tcc.

References __ellnome_k(), and __ellnome_series().

Referenced by theta c(), theta d(), theta n(), and theta s().

8.3.2.97 template < typename $_{\rm Tp} > _{\rm Tp}$ std::__detail::__ellnome_k ($_{\rm Tp}$ __k)

Use the arithmetic-geometric mean to calculate the elliptic nome given the , k.

Definition at line 278 of file sf theta.tcc.

References __comp_ellint_1().

Referenced by __ellnome().

8.3.2.98 template < typename $_{\rm Tp} > _{\rm Tp}$ std::__detail::__ellnome_series ($_{\rm Tp} _{\it k}$)

Use MacLaurin series to calculate the elliptic nome given the , k.

Definition at line 262 of file sf theta.tcc.

Referenced by __ellnome().

8.3.2.99 template<typename _Tp > _Tp std::__detail::__expint (unsigned int __n, _Tp __x)

Return the exponential integral $E_n(x)$.

The exponential integral is given by

$$E_n(x) = \int_1^\infty \frac{e^{-xt}}{t^n} dt$$

This is something of an extension.

n	The order of the exponential integral function.
X	The argument of the exponential integral function.

Returns

The exponential integral.

Definition at line 474 of file sf_expint.tcc.

References __expint_E1(), and __expint_En_recursion().

Referenced by __logint().

8.3.2.100 template<typename _Tp > _Tp std::__detail::__expint (_Tp __x)

Return the exponential integral Ei(x).

The exponential integral is given by

$$Ei(x) = -\int_{-x}^{\infty} \frac{e^t}{t} dt$$

Parameters

Х	The argument of the exponential integral function.
	The digament of the experiental integral fallottern

Returns

The exponential integral.

Definition at line 514 of file sf_expint.tcc.

References __expint_Ei().

8.3.2.101 template<typename _Tp > _Tp std::__expint_asymp (unsigned int __n, _Tp __x)

Return the exponential integral $E_n(x)$ for large argument.

The exponential integral is given by

$$E_n(x) = \int_1^\infty \frac{e^{-xt}}{t^n} dt$$

This is something of an extension.

Parameters

n	The order of the exponential integral function.
X	The argument of the exponential integral function.

Returns

The exponential integral.

Definition at line 406 of file sf expint.tcc.

8.3.2.102 template<typename _Tp > _Tp std::__detail::__expint_E1 (_Tp __x)

Return the exponential integral $E_1(x)$.

The exponential integral is given by

$$E_1(x) = \int_1^\infty \frac{e^{-xt}}{t} dt$$

Parameters

__x The argument of the exponential integral function.

Returns

The exponential integral.

Todo Find a good asymptotic switch point in $E_1(x)$.

Todo Find a good asymptotic switch point in $E_1(x)$.

Definition at line 375 of file sf expint.tcc.

References __expint_E1_asymp(), __expint_E1_series(), __expint_Ei(), and __expint_En_cont_frac().

Referenced by __coshint(), __expint_Ei(), __expint_Ei(), __expint_En_recursion(), and __sinhint().

8.3.2.103 template<typename _Tp > _Tp std::__detail::__expint_E1_asymp (_Tp __x)

Return the exponential integral $E_1(x)$ by asymptotic expansion.

The exponential integral is given by

$$E_1(x) = \int_1^\infty \frac{e^{-xt}}{t} dt$$

Parameters

__x The argument of the exponential integral function.

Returns

The exponential integral.

Definition at line 112 of file sf expint.tcc.

Referenced by __expint_E1().

8.3.2.104 template < typename _Tp > _Tp std::__detail::__expint_E1_series (_Tp __x)

Return the exponential integral $E_1(x)$ by series summation. This should be good for x < 1.

The exponential integral is given by

$$E_1(x) = \int_1^\infty \frac{e^{-xt}}{t} dt$$

__x The argument of the exponential integral function.

Returns

The exponential integral.

Definition at line 75 of file sf expint.tcc.

Referenced by __expint_E1().

8.3.2.105 template<typename _Tp > _Tp std::__detail::__expint_Ei (_Tp __x)

Return the exponential integral Ei(x).

The exponential integral is given by

$$Ei(x) = -\int_{-x}^{\infty} \frac{e^t}{t} dt$$

Parameters

__x The argument of the exponential integral function.

Returns

The exponential integral.

Definition at line 351 of file sf expint.tcc.

References __expint_E1(), __expint_Ei_asymp(), and __expint_Ei_series().

Referenced by __coshint(), __expint(), __expint_E1(), and __sinhint().

8.3.2.106 template < typename _Tp > _Tp std::__detail::__expint_Ei_asymp (_Tp __x)

Return the exponential integral Ei(x) by asymptotic expansion.

The exponential integral is given by

$$Ei(x) = -\int_{-x}^{\infty} \frac{e^t}{t} dt$$

Parameters

__x The argument of the exponential integral function.

Returns

The exponential integral.

Definition at line 318 of file sf expint.tcc.

Referenced by __expint_Ei().

8.3.2.107 template<typename _Tp > _Tp std::__detail::__expint_Ei_series (_Tp __x)

Return the exponential integral Ei(x) by series summation.

The exponential integral is given by

$$Ei(x) = -\int_{-x}^{\infty} \frac{e^t}{t} dt$$

Parameters

X	The argument of the exponential integral function.

Returns

The exponential integral.

Definition at line 286 of file sf_expint.tcc.

Referenced by expint Ei().

 $8.3.2.108 \quad template < typename _Tp > _Tp \ std:: _detail:: _expint_En_cont_frac \ (\ unsigned \ int __n, \ _Tp __x \)$

Return the exponential integral $E_n(x)$ by continued fractions.

The exponential integral is given by

$$E_n(x) = \int_1^\infty \frac{e^{-xt}}{t^n} dt$$

Parameters

n	The order of the exponential integral function.
x	The argument of the exponential integral function.

Returns

The exponential integral.

Definition at line 195 of file sf expint.tcc.

Referenced by __expint_E1().

 $8.3.2.109 \quad template < typename _Tp > _Tp \ std:: _detail:: _expint_En_recursion \ (\ unsigned \ int __n, \ _Tp __x \)$

Return the exponential integral $E_n(x)$ by recursion. Use upward recursion for x < n and downward recursion (Miller's algorithm) otherwise.

The exponential integral is given by

$$E_n(x) = \int_1^\infty \frac{e^{-xt}}{t^n} dt$$

n	The order of the exponential integral function.
X	The argument of the exponential integral function.

Returns

The exponential integral.

Todo Find a principled starting number for the $E_n(\boldsymbol{x})$ downward recursion.

Definition at line 240 of file sf_expint.tcc.

References __expint_E1().

Referenced by __expint().

 $8.3.2.110 \quad template < typename _Tp > _Tp \ std:: _detail:: _expint_En_series \ (\ unsigned \ int __n, \ _Tp __x \)$

Return the exponential integral $E_n(x)$ by series summation.

The exponential integral is given by

$$E_n(x) = \int_1^\infty \frac{e^{-xt}}{t^n} dt$$

Parameters

n	The order of the exponential integral function.
X	The argument of the exponential integral function.

Returns

The exponential integral.

Definition at line 149 of file sf_expint.tcc.

References psi().

8.3.2.111 template<typename _Tp > _Tp std::__expint_large_n (unsigned int __n, _Tp __x)

Return the exponential integral $E_n(x)$ for large order.

The exponential integral is given by

$$E_n(x) = \int_1^\infty \frac{e^{-xt}}{t^n} dt$$

This is something of an extension.

Parameters

n	The order of the exponential integral function.
X	The argument of the exponential integral function.

Returns

The exponential integral.

Definition at line 440 of file sf expint.tcc.

8.3.2.112 template < typename _Tp > _GLIBCXX14_CONSTEXPR _Tp std::__detail::__factorial (unsigned int $_n$)

Return the factorial of the integer n.

The factorial is:

$$n! = 12...(n-1)n, 0! = 1$$

Definition at line 2422 of file sf_gamma.tcc.

References _S_factorial_table.

Referenced by __double_factorial().

8.3.2.113 template<typename _Tp > _Tp std::__detail::__fermi_dirac (_Tp __s, _Tp __x)

Return the Fermi-Dirac integral of real order s and real argument x.

See also

```
https://en.wikipedia.org/wiki/Clausen_function
http://dlmf.nist.gov/25.12#iii
```

Parameters

s	The order $s \ge 0$.
X	The real argument.

Returns

The real Fermi-Dirac cosine sum $F_s(x)$,

Definition at line 1379 of file sf_polylog.tcc.

References polylog exp().

8.3.2.114 template<typename _Tp > _GLIBCXX14_CONSTEXPR _Tp std::__detail::__fisher_f_cdf (_Tp __F, unsigned int __nu1, unsigned int __nu2)

Return the F-distribution propability function. This returns the probability that the observed chi-square for a correct model exceeds the value χ^2 .

The f-distribution propability function is related to the incomplete beta function:

$$Q(F|\nu_1,\nu_2) = I_{\frac{\nu_2}{\nu_2 + \nu_1 F}}(\frac{\nu_2}{2}, \frac{\nu_1}{2})$$

Parameters

nu1	The number of degrees of freedom of sample 1
nu2	The number of degrees of freedom of sample 2
F	The F statistic

Definition at line 350 of file sf beta.tcc.

References beta inc().

8.3.2.115 template < typename _Tp > _GLIBCXX14_CONSTEXPR _Tp std::__detail::__fisher_f_cdfc (_Tp __F, unsigned int __nu1, unsigned int __nu2)

Return the F-distribution propability function. This returns the probability that the observed chi-square for a correct model exceeds the value χ^2 .

The f-distribution propability function is related to the incomplete beta function:

$$P(F|\nu_1, \nu_2) = 1 - I_{\frac{\nu_2}{\nu_2 + \nu_1 F}}(\frac{\nu_2}{2}, \frac{\nu_1}{2}) = 1 - Q(F|\nu_1, \nu_2)$$

Parameters

F	
nu1	
nu2	

Definition at line 379 of file sf_beta.tcc.

References __beta_inc().

8.3.2.116 template<typename _Tp > void std::__detail::__fock_airy (_Tp __x, std::complex< _Tp > & __w1, std::complex< _Tp > & __w2, std::complex< _Tp > & __w2p)

Compute the Fock-type Airy functions $w_1(x)$ and $w_2(x)$ and their first derivatives $w_1'(x)$ and $w_2'(x)$ respectively.

$$w_1(x) = \sqrt{\pi}(Ai(x) + iBi(x))$$
$$w_2(x) = \sqrt{\pi}(Ai(x) - iBi(x))$$

Parameters

X	The argument of the Airy functions.
w1	The output Fock-type Airy function of the first kind.
w2	The output Fock-type Airy function of the second kind.
w1p	The output derivative of the Fock-type Airy function of the first kind.
w2p	The output derivative of the Fock-type Airy function of the second kind.

Definition at line 569 of file sf mod bessel.tcc.

8.3.2.117 template<typename _Tp > bool std::__detail::__fpequal (const _Tp & __a, const _Tp & __b)

A function to reliably compare two floating point numbers.

Parameters

a	the left hand side.
b	the right hand side

Returns

returns true if a and b are equal to zero or differ only by max(a,b) * 5 * eps

Definition at line 62 of file sf_polylog.tcc.

Referenced by $_$ dirichlet_beta(), $_$ dirichlet_eta(), $_$ fpimag(), $_$ polylog(), $_$ polylog(), $_$ polylog_exp_asymp(), $_$ \hookrightarrow polylog_exp_int_neg(), $_$ polylog_exp_int_pos(), $_$ polylog_exp_neg(), $_$ polylog_exp_neg_even(), $_$ polylog_exp_ \hookleftarrow neg_odd(), $_$ polylog_exp_negative_real_part(), $_$ polylog_exp_pos(), and $_$ polylog_exp_real_pos().

8.3.2.118 template<typename _Tp > bool std::__detail::__fpimag (const std::complex < _Tp > & __w)

A function to reliably test a complex number for imaginaryness [?].

Parameters

__w The complex argument.

Returns

true if Re(w) is zero within 5*epsilon, false otherwize.

Definition at line 107 of file sf polylog.tcc.

References __fpequal().

8.3.2.119 template<typename _Tp > bool std::__detail::__fpimag (const _Tp)

Definition at line 117 of file sf polylog.tcc.

8.3.2.120 template<typename_Tp > bool std::__detail::__fpreal (const std::complex< _Tp > & __w)

A function to reliably test a complex number for realness.

Parameters

W	The complex argument.

Returns

true if Im(w) is zero within 5*epsilon, false otherwize.

Definition at line 84 of file sf_polylog.tcc.

References __fpequal().

Referenced by __polylog_exp_int_pos(), and __polylog_exp_real_pos().

8.3.2.121 template<typename _Tp > bool std::__detail::__fpreal (const _Tp)

Definition at line 94 of file sf polylog.tcc.

8.3.2.122 template < typename _Tp > std::complex < _Tp > std::__detail::__fresnel (const _Tp __x)

Return the Fresnel cosine and sine integrals as a complex number f[C(x) + iS(x)].

The Fresnel cosine integral is defined by:

$$C(x) = \int_0^x \cos(\frac{\pi}{2}t^2)dt$$

The Fresnel sine integral is defined by:

$$S(x) = \int_0^x \sin(\frac{\pi}{2}t^2)dt$$

	The amount of the second of th
X	The argument
	· · · · · · · · · · · · · · · · · · ·

Definition at line 166 of file sf fresnel.tcc.

References __fresnel_cont_frac(), and __fresnel_series().

8.3.2.123 template < typename _Tp > void std::__detail::__fresnel_cont_frac (const _Tp __ax, _Tp & _Cf, _Tp & _Sf)

This function computes the Fresnel cosine and sine integrals by continued fractions for positive argument.

Definition at line 105 of file sf fresnel.tcc.

Referenced by __fresnel().

8.3.2.124 template < typename _Tp > void std::__detail::__fresnel_series (const _Tp __ax, _Tp & _Cf, _Tp & _Sf)

This function returns the Fresnel cosine and sine integrals as a pair by series expansion for positive argument.

Definition at line 48 of file sf fresnel.tcc.

Referenced by __fresnel().

8.3.2.125 template<typename _Tp > _Tp std::__detail::__gamma (_Tp $_x$)

Return $\Gamma(x)$.

Parameters

__x The argument of the gamma function.

Returns

The gamma function.

Definition at line 1918 of file sf_gamma.tcc.

References log gamma().

Referenced by __beta_gamma(), and __riemann_zeta().

8.3.2.126 template < typename _Tp > std::pair < _Tp, _Tp > std::__detail::__gamma_cont_frac (_Tp __a, _Tp __x)

Definition at line 1964 of file sf gamma.tcc.

Referenced by __gamma_l(), __gamma_p(), __gamma_q(), and __gamma_u().

8.3.2.127 template<typename _Tp > _Tp std::__detail::__gamma_I (_Tp __a, _Tp __x)

Return the lower incomplete gamma function. The lower incomplete gamma function is defined by

$$\gamma(a,x) = \int_0^x e^{-t} t^{a-1} dt (a > 0)$$

.

Definition at line 2070 of file sf_gamma.tcc.

References __gamma_cont_frac(), and __gamma_series().

8.3.2.128 template < typename _Tp > _Tp std::__detail::__gamma_p (_Tp __a, _Tp __x)

Return the regularized lower incomplete gamma function. The regularized lower incomplete gamma function is defined by

$$P(a,x) = \frac{\gamma(a,x)}{\Gamma(a)}$$

where $\Gamma(a)$ is the gamma function and

$$\gamma(a,x) = \int_0^x e^{-t} t^{a-1} dt (a > 0)$$

is the lower incomplete gamma function.

Definition at line 2013 of file sf gamma.tcc.

References __gamma_cont_frac(), and __gamma_series().

Referenced by __chi_squared_pdf().

8.3.2.129 template<typename _Tp > _Tp std::__detail::__gamma_q (_Tp __a, _Tp __x)

Return the regularized upper incomplete gamma function. The regularized upper incomplete gamma function is defined by

$$Q(a,x) = \frac{\Gamma(a,x)}{\Gamma(a)}$$

where $\Gamma(a)$ is the gamma function and

$$\Gamma(a,x) = \int_{x}^{\infty} e^{-t} t^{a-1} dt (a > 0)$$

is the upper incomplete gamma function.

Definition at line 2044 of file sf_gamma.tcc.

References __gamma_cont_frac(), and __gamma_series().

Referenced by __chi_squared_pdfc().

8.3.2.130 template<typename _Tp > std::pair<_Tp, _Tp> std::__detail::__gamma_series (_Tp __a, _Tp __x)

Definition at line 1930 of file sf gamma.tcc.

Referenced by __gamma_l(), __gamma_p(), __gamma_q(), and __gamma_u().

8.3.2.131 template<typename _Tp > void std::__detail::__gamma_temme (_Tp __mu, _Tp & __gam1, _Tp & __gam2, _Tp & __gammi)

Compute the gamma functions required by the Temme series expansions of $N_{\nu}(x)$ and $K_{\nu}(x)$.

$$\Gamma_1 = \frac{1}{2\mu} \left[\frac{1}{\Gamma(1-\mu)} - \frac{1}{\Gamma(1+\mu)} \right]$$

and

$$\Gamma_2 = \frac{1}{2} \left[\frac{1}{\Gamma(1-\mu)} + \frac{1}{\Gamma(1+\mu)} \right]$$

where $-1/2 <= \mu <= 1/2$ is $\mu = \nu - N$ and N. is the nearest integer to ν . The values of $\Gamma(1+\mu)$ and $\Gamma(1-\mu)$ are returned as well.

The accuracy requirements on this are exquisite.

Parameters

	mu	The input parameter of the gamma functions.
out	gam1	The output function $\Gamma_1(\mu)$
out	gam2	The output function $\Gamma_2(\mu)$
out	gampl	The output function $\Gamma(1+\mu)$
out	gammi	The output function $\Gamma(1-\mu)$

Definition at line 163 of file sf bessel.tcc.

Referenced by __cyl_bessel_ik_steed(), and __cyl_bessel_jn_steed().

Return the upper incomplete gamma function. The lower incomplete gamma function is defined by

$$\Gamma(a,x) = \int_{x}^{\infty} e^{-t} t^{a-1} dt (a > 0)$$

.

Definition at line 2102 of file sf gamma.tcc.

References __gamma_cont_frac(), and __gamma_series().

8.3.2.133 template> _Tp std::__detail::__gauss (_Tp
$$_x$$
)

The CDF of the normal distribution. i.e. the integrated lower tail of the normal PDF.

Definition at line 70 of file sf owens t.tcc.

Definition at line 44 of file sf_gegenbauer.tcc.

Parameters

	in	nu	The order for which the Hankel functions are evaluated.
Ī	in	Z	The argument at which the Hankel functions are evaluated.

out	_H1	The Hankel function of the first kind.
out	_H1p	The derivative of the Hankel function of the first kind.
out	_H2	The Hankel function of the second kind.
out	_H2p	The derivative of the Hankel function of the second kind.

Definition at line 1127 of file sf hankel.tcc.

References debye region(), hankel debye(), and hankel uniform().

Referenced by __cyl_bessel(), __cyl_hankel_1(), __cyl_hankel_2(), __cyl_neumann(), and __sph_hankel().

8.3.2.136 template < typename _Tp > void std::__detail::__hankel_debye (std::complex < _Tp > __nu, std::complex < _Tp > __z, std::complex < _Tp > __alpha, int __indexr, char & __aorb, int & __morn, std::complex < _Tp > & _H1, std::complex < _Tp > & _H2, std::complex < _Tp > & _H2p, std::complex < _Tp > & _H2p)

Parameters

in	nu	The order for which the Hankel functions are evaluated.
in	z	The argument at which the Hankel functions are evaluated.
in	alpha	
in	indexr	
out	aorb	
out	morn	
out	_H1	The Hankel function of the first kind.
out	_H1p	The derivative of the Hankel function of the first kind.
out	_H2	The Hankel function of the second kind.
out	_H2p	The derivative of the Hankel function of the second kind.

Definition at line 959 of file sf_hankel.tcc.

Referenced by __hankel().

```
8.3.2.137 template<typename _Tp > void std::__detail::__hankel_params ( std::complex< _Tp > __nu, std::complex< _Tp > __zhat, std::complex< _Tp > & __p, std::complex< _Tp > & __nup2, std::complex< _Tp > & __nup2, std::complex< _Tp > & __num2d3, std::complex< _Tp > & __num2d3, std::complex< _Tp > & __num2d3, std::complex< _Tp > & __zetamhf, std::complex< _Tp > &
```

Compute parameters depending on z and nu that appear in the uniform asymptotic expansions of the Hankel functions and their derivatives, except the arguments to the Airy functions.

Definition at line 110 of file sf hankel.tcc.

Referenced by hankel uniform outer().

```
8.3.2.138 template < typename _Tp > void std::__detail::__hankel_uniform ( std::complex < _Tp > __nu, std::complex < _Tp > __nu, std::complex < _Tp > & _H1, std::complex < _Tp > & _H2, std::complex < _Tp > & _H1p, std::complex < _Tp > & _H2p )
```

This routine computes the uniform asymptotic approximations of the Hankel functions and their derivatives including a patch for the case when the order equals or nearly equals the argument. At such points, Olver's expressions have zero denominators (and numerators) resulting in numerical problems. This routine averages results from four surrounding points in the complex plane to obtain the result in such cases.

in	nu	The order for which the Hankel functions are evaluated.
in	Z	The argument at which the Hankel functions are evaluated.
out	_H1	The Hankel function of the first kind.
out	_H1p	The derivative of the Hankel function of the first kind.
out	_H2	The Hankel function of the second kind.
out	_H2p	The derivative of the Hankel function of the second kind.

Definition at line 904 of file sf hankel.tcc.

References __hankel_uniform_olver().

Referenced by __hankel().

8.3.2.139 template < typename _Tp > void std::__detail::__hankel_uniform_olver (std::complex < _Tp > __nu, std::complex < _Tp > __z, std::complex < _Tp > & _H1, std::complex < _Tp > & _H2, std::complex < _Tp > & _H1p, std::complex < _Tp > & _H2p)

Compute approximate values for the Hankel functions of the first and second kinds using Olver's uniform asymptotic expansion to of order nu along with their derivatives.

Parameters

in	nu	The order for which the Hankel functions are evaluated.
in	z	The argument at which the Hankel functions are evaluated.
out	_H1	The Hankel function of the first kind.
out	_H1p	The derivative of the Hankel function of the first kind.
out	_H2	The Hankel function of the second kind.
out	_H2p	The derivative of the Hankel function of the second kind.

Definition at line 818 of file sf hankel.tcc.

References __hankel_uniform_outer(), and __hankel_uniform_sum().

Referenced by __hankel_uniform().

8.3.2.140 template<typename _Tp > void std::__detail::__hankel_uniform_outer (std::complex< _Tp > __nu, std::complex < _Tp > __z, _Tp __eps, std::complex < _Tp > & __zhat, std::complex < _Tp > & __num1d3, std::complex < _Tp > & __num2d3, std::complex < _Tp > & __p, std::complex < _Tp > & __p2, std::complex < _Tp > & __etrat, std::complex < _Tp > & __aip, std::complex < _Tp > & __aip, std::complex < _Tp > & __o4dp, std::complex < _Tp > & __o4dm, std::complex < _Tp > & __o4dm, std::complex < _Tp > & __o4dp, std::complex < _Tp > & __o4dm, std::complex < _Tp > & __o4ddn) }

Compute outer factors and associated functions of z and nu appearing in Olver's uniform asymptotic expansions of the Hankel functions of the first and second kinds and their derivatives. The various functions of z and nu returned by $hankel_uniform_outer$ are available for use in computing further terms in the expansions.

Definition at line 273 of file sf hankel.tcc.

References __airy(), __airy_arg(), and __hankel_params().

Referenced by __hankel_uniform_olver().

8.3.2.141 template < typename _Tp > void std::__detail::__hankel_uniform_sum (std::complex < _Tp > __p, std::complex < _Tp > __aip, std::complex < _Tp > __o4dp, std::complex < _Tp > __o4dm, _Tp __eps, std::complex < _Tp > __o4dm, std::complex < _Tp > __o4dm, std::complex < _Tp > __o4dm, _Tp __eps, std::complex < _Tp > & __H1sum, std::complex < _Tp > & __H2sum, std::complex < _Tp > & __H2sum)

Compute the sums in appropriate linear combinations appearing in Olver's uniform asymptotic expansions for the Hankel functions of the first and second kinds and their derivatives, using up to nterms (less than 5) to achieve relative error eps.

Parameters

in	p	
in	p2	
in	num2	
in	zetam3hf	
in	_Aip	The Airy function value $Ai()$.
in	o4dp	
in	_Aim	The Airy function value $Ai()$.
in	o4dm	
in	od2p	
in	od0dp	
in	od2m	
in	od0dm	
in	eps	The error tolerance
out	_H1sum	The Hankel function of the first kind.
out	_H1psum	The derivative of the Hankel function of the first kind.
out	_H2sum	The Hankel function of the second kind.
out	_H2psum	The derivative of the Hankel function of the second kind.

Definition at line 351 of file sf_hankel.tcc.

Referenced by hankel uniform olver().

8.3.2.142 template<typename _Tp > _Tp std::__detail::__heuman_lambda (_Tp __k, _Tp __phi)

Return the Heuman lambda function.

Definition at line 941 of file sf_ellint.tcc.

References ellint rf(), and ellint rj().

8.3.2.143 template<typename _Tp > _Tp std::__detail::__hurwitz_zeta (_Tp __s, _Tp __a)

Return the Hurwitz zeta function $\zeta(s,a)$ for all s = 1 and a > -1.

The Hurwitz zeta function is defined by:

$$\zeta(s,a) = \sum_{n=0}^{\infty} \frac{1}{(n+a)^s}$$

The Riemann zeta function is a special case:

$$\zeta(s) = \zeta(s, 1)$$

s	The argument $s! = 1$	
a	The scale parameter $a>-1$	

Definition at line 702 of file sf zeta.tcc.

References __hurwitz_zeta_euler_maclaurin().

 $8.3.2.144 \quad template < typename _Tp > std::_detail::_hurwitz_zeta (_Tp __s, std::complex < _Tp > _a)$

Return the Hurwitz Zeta function for real s and complex a.

Parameters

s	The real argument	
a	The complex parameter	

Todo This __hurwitz_zeta prefactor is prone to overflow. positive integer orders s?

Definition at line 1119 of file sf polylog.tcc.

References __polylog_exp().

Referenced by __psi().

 $8.3.2.145 \quad template < typename _Tp > _Tp \ std::_detail::_hurwitz_zeta_euler_maclaurin \ (\ _Tp __s, \ _Tp __a \)$

Return the Hurwitz zeta function $\zeta(s,a)$ for all s != 1 and a > -1.

See also

An efficient algorithm for accelerating the convergence of oscillatory series, useful for computing the polylogarithm and Hurwitz zeta functions, Linas Vep

Parameters

s	The argument $s! = 1$
a	The scale parameter $a > -1$

Definition at line 560 of file sf zeta.tcc.

References _S_Euler_Maclaurin_zeta.

Referenced by __hurwitz_zeta().

8.3.2.146 template<typename _Tp > std::complex<_Tp> std::__detail::__hydrogen (const unsigned int __n, const unsigned int __n, const unsigned int __n, const _Tp __r, const _Tp __theta, const _Tp __phi)

Definition at line 44 of file sf_hydrogen.tcc.

References __assoc_laguerre(), __psi(), and __sph_legendre().

8.3.2.147 template<typename_Tp > _Tp std::__detail::__hyperg (_Tp __a, _Tp __b, _Tp __c, _Tp __x)

Return the hypergeometric function ${}_{2}F_{1}(a,b;c;x)$.

The hypergeometric function is defined by

$${}_{2}F_{1}(a,b;c;x) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)} \sum_{n=0}^{\infty} \frac{\Gamma(a+n)\Gamma(b+n)}{\Gamma(c+n)} \frac{x^{n}}{n!}$$

Parameters

a	The first <i>numerator</i> parameter.
b	The second <i>numerator</i> parameter.
c	The denominator parameter.
X	The argument of the confluent hypergeometric function.

Returns

The confluent hypergeometric function.

Definition at line 776 of file sf hyperg.tcc.

References __hyperg_luke(), __hyperg_reflect(), __hyperg_series(), __log_gamma(), and __log_gamma_sign().

Return the hypergeometric function ${}_2F_1(a,b;c;x)$ by an iterative procedure described in Luke, Algorithms for the Computation of Mathematical Functions.

Definition at line 352 of file sf hyperg.tcc.

Referenced by __hyperg().

Return the hypergeometric function ${}_2F_1(a,b;c;x)$ by the reflection formulae in Abramowitz & Stegun formula 15.3.6 for d=c-a-b not integral and formula 15.3.11 for d=c-a-b integral. This assumes a,b,c!= negative integer.

The hypergeometric function is defined by

$$_{2}F_{1}(a,b;c;x) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)} \sum_{n=0}^{\infty} \frac{\Gamma(a+n)\Gamma(b+n)}{\Gamma(c+n)} \frac{x^{n}}{n!}$$

The reflection formula for nonintegral d = c - a - b is:

$${}_{2}F_{1}(a,b;c;x) = \frac{\Gamma(c)\Gamma(d)}{\Gamma(c-a)\Gamma(c-b)} {}_{2}F_{1}(a,b;1-d;1-x) + \frac{\Gamma(c)\Gamma(-d)}{\Gamma(a)\Gamma(b)} {}_{2}F_{1}(c-a,c-b;1+d;1-x)$$

The reflection formula for integral m=c-a-b is:

$${}_{2}F_{1}(a,b;a+b+m;x) = \frac{\Gamma(m)\Gamma(a+b+m)}{\Gamma(a+m)\Gamma(b+m)} \sum_{k=0}^{m-1} \frac{(m+a)_{k}(m+b)_{k}}{k!(1-m)_{k}} -$$

Definition at line 486 of file sf_hyperg.tcc.

References __hyperg_series(), __log_gamma(), __log_gamma_sign(), and __psi().

Referenced by hyperg().

8.3.2.150 template<typename_Tp > _Tp std::__detail::__hyperg_series (_Tp __a, _Tp __b, _Tp __c, _Tp __x)

Return the hypergeometric function ${}_{2}F_{1}(a,b;c;x)$ by series expansion.

The hypergeometric function is defined by

$$_{2}F_{1}(a,b;c;x) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)} \sum_{n=0}^{\infty} \frac{\Gamma(a+n)\Gamma(b+n)}{\Gamma(c+n)} \frac{x^{n}}{n!}$$

This works and it's pretty fast.

Parameters

a	The first <i>numerator</i> parameter.	
b	The second <i>numerator</i> parameter.	
c	The <i>denominator</i> parameter.	
X	The argument of the confluent hypergeometric function.	

Returns

The confluent hypergeometric function.

Definition at line 321 of file sf_hyperg.tcc.

Referenced by hyperg(), and hyperg reflect().

Return a tuple of the three primary Jacobi elliptic functions: sn(k, u), cn(k, u), dn(k, u).

Definition at line 414 of file sf_theta.tcc.

Return the Jacobi zeta function.

Definition at line 971 of file sf_ellint.tcc.

References __comp_ellint_1(), and __ellint_rj().

This routine returns the Laguerre polynomial of order n: $L_n(x)$.

The Laguerre polynomial is defined by:

$$L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n} (x^n e^{-x})$$

Parameters

n	The order of the Laguerre polynomial.

X	The argument of the	Laguerre polynomial.

Returns

The value of the Laguerre polynomial of order n and argument x.

Definition at line 312 of file sf laguerre.tcc.

Return the logarithm of the binomial coefficient. The binomial coefficient is given by:

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

Parameters

n	The first argument of the binomial coefficient.
k	The second argument of the binomial coefficient.

Returns

The logarithm of the binomial coefficient.

Definition at line 1862 of file sf gamma.tcc.

Definition at line 2450 of file sf gamma.tcc.

References __log_gamma().

Referenced by __double_factorial(), and __log_double_factorial().

Return the logarithm of the double factorial of the integer n.

The double factorial is defined for integral n by:

$$n!! = 135...(n-2)n, noddn!! = 246...(n-2)n, neven - 1!! = 10!! = 1$$

The double factorial is defined for odd negative integers in the obvious way:

$$(-2m-1)!! = 1/(1(-1)(-3)...(-2m+1)(-2m-1)) = \frac{(-1)^m}{(2m-1)!!}$$

for f[n = -2m - 1 f].

Definition at line 2516 of file sf gamma.tcc.

References __log_double_factorial(), __log_factorial(), _S_double_factorial_table, and _S_neg_double_factorial_table.

8.3.2.157 template < typename _Tp > _GLIBCXX14_CONSTEXPR _Tp std::__detail::__log_factorial (unsigned int __n)

Return the logarithm of the factorial of the integer n.

The factorial is:

$$n! = 12...(n-1)n, 0! = 1$$

Definition at line 2440 of file sf gamma.tcc.

References __log_gamma(), and _S_factorial_table.

Referenced by __log_double_factorial().

8.3.2.158 template<typename $_{Tp} > _{Tp}$ std::__detail::__log_gamma ($_{Tp} _{x}$)

Return $log(|\Gamma(x)|)$. This will return values even for x < 0. To recover the sign of $\Gamma(x)$ for any argument use $\underline{\hspace{0.5cm}}log_{\hookleftarrow}$ $gamma_sign$.

Parameters

__x The argument of the log of the gamma function.

Returns

The logarithm of the gamma function.

Definition at line 1800 of file sf_gamma.tcc.

References __log_gamma_lanczos().

8.3.2.159 template<typename_Tp > _GLIBCXX14_CONSTEXPR _Tp std::__detail::__log_gamma_bernoulli (_Tp __x)

Return $log(\Gamma(x))$ by asymptotic expansion with Bernoulli number coefficients. This is like Sterling's approximation.

Parameters

__x The argument of the log of the gamma function.

Returns

The logarithm of the gamma function.

Definition at line 1699 of file sf_gamma.tcc.

8.3.2.160 template < typename _Tp > _GLIBCXX14_CONSTEXPR _Tp std::__detail::__log_gamma_lanczos (_Tp __x)

Return $log(\Gamma(x))$ by the Lanczos method. This method dominates all others on the positive axis I think.

__x The argument of the log of the gamma function.

Returns

The logarithm of the gamma function.

Definition at line 1755 of file sf gamma.tcc.

Referenced by __log_gamma().

8.3.2.161 template<typename _Tp > _Tp std::__detail::__log_gamma_sign (_Tp __x)

Return the sign of $\Gamma(x)$. At nonpositive integers zero is returned.

Parameters

__x The argument of the gamma function.

Returns

The sign of the gamma function.

Definition at line 1831 of file sf_gamma.tcc.

Referenced by __hyperg(), __hyperg_reflect(), and __pochhammer_l().

8.3.2.162 template < typename _Tp > _GLIBCXX14_CONSTEXPR _Tp std::__detail::__log_gamma_spouge (_Tp __z)

Return $\Gamma(z)$ by the Spouge algorithm:

$$\Gamma(z+1) = (z+a)^{z+1/2} e^{-z-a} \left[\sqrt{2\pi} \sum_{k=1}^{\lceil a \rceil + 1} \frac{c_k(a)}{z+k} \right]$$

where

$$c_k(a) = \frac{(-1)^{k-1}}{(k-1)!} (a-k)^{k-1/2} e^{a-k}$$

and the error is bounded by

$$\epsilon(a) < a^{-1/2} (2\pi)^{-a-1/2}$$

See also

Spouge, J.L., Computation of the gamma, digamma, and trigamma functions. SIAM Journal on Numerical Analysis 31, 3 (1994), pp. 931-944

Parameters

 \underline{z} The argument of the gamma function.

Returns

The the gamma function.

Definition at line 1739 of file sf_gamma.tcc.

Return the logarithm of the lower Pochhammer symbol or the falling factorial function. The lower Pochammer symbol is defined by

$$(a)_n = \prod_{k=0}^{n-1} (a-k), (a)_0 = 1 = \Gamma(a+1)/\Gamma(a-n+1)$$

In particular, f(n) = n! f. Thus this function returns

$$ln[(a)_n] = \Gamma(a+1) - \Gamma(a-n+1), ln[(a)_0] = 0$$

Many notations exist:

 $a^{\underline{n}}$

,

 $\left\{\begin{array}{c} a \\ n \end{array}\right\}$

, and others.

Definition at line 2209 of file sf_gamma.tcc.

8.3.2.164 template < typename
$$_{\rm Tp} > _{\rm Tp}$$
 std::__detail::__log_pochhammer_u ($_{\rm Tp}$ __a, $_{\rm Tp}$ __n)

Return the logarithm of the (upper) Pochhammer symbol or the rising factorial function. The Pochammer symbol is defined by

$$(a)_n = \prod_{k=0}^{n-1} (a+k), (a)_0 = 1 = \Gamma(a+n)/\Gamma(n)$$

Thus this function returns

$$ln[(a)_n] = \Gamma(a+n) - \Gamma(n), ln[(a)_0] = 0$$

Many notations exist:

 $a^{\overline{n}}$

,

 $\begin{vmatrix} a \\ n \end{vmatrix}$

, and others.

Definition at line 2144 of file sf_gamma.tcc.

References log gamma().

Return the logarithmic integral li(x).

The logarithmic integral is given by

$$li(x) = Ei(\log(x))$$

	T
V	I he argument of the logarithmic integral function.
Λ	
	1

Returns

The logarithmic integral.

Definition at line 535 of file sf_expint.tcc.

References expint().

8.3.2.166 template<typename _Tp > _Tp std::__detail::__owens_t (_Tp __h, _Tp __a)

Return the Owens T function:

$$T(h,a) = \frac{1}{2\pi} \int_0^a \frac{\exp[-\frac{1}{2}h^2(1+x^2)]}{1+x^2} dx$$

This implementation is a translation of the Fortran implementation in

See also

Patefield, M. and Tandy, D. "Fast and accurate Calculation of Owen's T-Function", Journal of Statistical Software, 5 (5), 1 - 25 (2000)

Parameters

in	h	The scale parameter.
in	a	The integration limit.

Returns

The owens T function.

Definition at line 92 of file sf owens t.tcc.

References __znorm1(), and __znorm2().

8.3.2.167 template < typename _Tp > _Tp std::__detail::__pochhammer_I (_Tp $_a$, _Tp $_n$)

Return the logarithm of the lower Pochhammer symbol or the falling factorial function. The lower Pochammer symbol is defined by

$$(a)_n = \prod_{k=0}^{n-1} (a-k), (a)_0 = 1 = \Gamma(a+1)/\Gamma(a-n+1)$$

In particular, $f(n)_n = n! f$.

Definition at line 2232 of file sf_gamma.tcc.

References __log_gamma_sign().

8.3.2.168 template<typename_Tp > _Tp std::__detail::__pochhammer_u (_Tp __a, _Tp __n)

Return the (upper) Pochhammer function or the rising factorial function. The Pochammer symbol is defined by

$$(a)_n = \prod_{k=0}^{n-1} (a+k), (a)_0 = 1 = \Gamma(a+n)/\Gamma(n)$$

Many notations exist:

$$a^{\overline{n}}$$

,

$$\begin{bmatrix} a \\ n \end{bmatrix}$$

, and others.

Definition at line 2170 of file sf gamma.tcc.

8.3.2.169 template<typename _Tp > _Tp std::__detail::__poly_hermite (unsigned int __n, _Tp __x)

This routine returns the Hermite polynomial of order n: $H_n(x)$.

The Hermite polynomial is defined by:

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2}$$

The Hermite polynomial obeys a reflection formula:

$$H_n(-x) = (-1)^n H_n(x)$$

Parameters

n	The order of the Hermite polynomial.
x	The argument of the Hermite polynomial.

Returns

The value of the Hermite polynomial of order n and argument x.

Definition at line 179 of file sf hermite.tcc.

References poly hermite asymp(), and poly hermite recursion().

8.3.2.170 template<typename_Tp > _Tp std::__detail::__poly_hermite_asymp (unsigned int __n, _Tp __x)

This routine returns the Hermite polynomial of large order n: $H_n(x)$. We assume here that $x \ge 0$.

The Hermite polynomial is defined by:

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2}$$

see "Asymptotic analysis of the Hermite polynomials from their differential-difference equation", Diego Dominici, arXiv← :math/0601078v1 [math.CA] 4 Jan 2006

Parameters

n	The order of the Hermite polynomial.
x	The argument of the Hermite polynomial.

Returns

The value of the Hermite polynomial of order n and argument x.

Definition at line 113 of file sf_hermite.tcc.

References __airy().

Referenced by __poly_hermite().

8.3.2.171 template<typename_Tp > _Tp std::__detail::__poly_hermite_recursion (unsigned int __n, _Tp __x)

This routine returns the Hermite polynomial of order n: $H_n(x)$ by recursion on n.

The Hermite polynomial is defined by:

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2}$$

Parameters

n	The order of the Hermite polynomial.
X	The argument of the Hermite polynomial.

Returns

The value of the Hermite polynomial of order n and argument x.

Definition at line 69 of file sf hermite.tcc.

Referenced by __poly_hermite().

8.3.2.172 template<typename_Tp > _Tp std::__detail::__poly_jacobi (unsigned int __n, _Tp __alpha, _Tp __beta, _Tp __x)

Compute the Jacobi polynomial by recursion on x:

$$2k(\alpha+\beta+k)(\alpha+\beta+2k-2)P_k^{(\alpha,\beta)}(x) = (\alpha+\beta+2k-1)((\alpha^2-\beta^2) + x(\alpha+\beta+2k-2)(\alpha+\beta+2k))P_{k-1}^{(\alpha,\beta)}(x) - 2(\alpha+k-1)(\beta+k-1)(\alpha+\beta+2k-2)P_{k-1}^{(\alpha,\beta)}(x) = (\alpha+\beta+2k-1)((\alpha^2-\beta^2) + x(\alpha+\beta+2k-2)(\alpha+\beta+2k))P_{k-1}^{(\alpha,\beta)}(x) = (\alpha+\beta+2k-1)((\alpha^2-\beta^2) + x(\alpha+\beta+2k-2)(\alpha+\beta+2k))P_{k-1}^{(\alpha,\beta)}(x) = (\alpha+\beta+2k-1)(\alpha+\beta+2k-2)(\alpha+\beta+2k)P_{k-1}^{(\alpha,\beta)}(x) = (\alpha+\beta+2k-1)(\alpha+\beta+2k-2)(\alpha+\beta+2k)P_{k-1}^{(\alpha,\beta)}(x) = (\alpha+\beta+2k-1)(\alpha+\beta+2k-2)(\alpha+2k-2)$$

Definition at line 57 of file sf jacobi.tcc.

References __beta().

Referenced by __poly_radial_jacobi().

8.3.2.173 template<typename _Tpa , typename _Tp > _Tp std::__detail::__poly_laguerre (unsigned int __n, _Tpa __alpha1, _Tp __x)

This routine returns the associated Laguerre polynomial of order n, degree α : $L_n^a lpha(x)$.

The associated Laguerre function is defined by

$$L_n^{\alpha}(x) = \frac{(\alpha+1)_n}{n!} F_1(-n; \alpha+1; x)$$

where $(\alpha)_n$ is the Pochhammer symbol and ${}_1F_1(a;c;x)$ is the confluent hypergeometric function.

The associated Laguerre polynomial is defined for integral $\alpha=m$ by:

$$L_n^m(x) = (-1)^m \frac{d^m}{dx^m} L_{n+m}(x)$$

where the Laguerre polynomial is defined by:

$$L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n} (x^n e^{-x})$$

n	n The order of the Laguerre function.	
alpha1	The degree of the Laguerre function.	
X	The argument of the Laguerre function.	

Returns

The value of the Laguerre function of order n, degree α , and argument x.

Definition at line 240 of file sf_laguerre.tcc.

References __poly_laguerre_hyperg(), __poly_laguerre_large_n(), and __poly_laguerre_recursion().

Evaluate the polynomial based on the confluent hypergeometric function in a safe way, with no restriction on the arguments.

The associated Laguerre function is defined by

$$L_n^{\alpha}(x) = \frac{(\alpha+1)_n}{n!} {}_1F_1(-n; \alpha+1; x)$$

where $(\alpha)_n$ is the Pochhammer symbol and ${}_1F_1(a;c;x)$ is the confluent hypergeometric function.

This function assumes x = 0.

This is from the GNU Scientific Library.

Parameters

n	The order of the Laguerre function.
alpha1	The degree of the Laguerre function.
X	The argument of the Laguerre function.

Returns

The value of the Laguerre function of order n, degree α , and argument x.

Definition at line 125 of file sf_laguerre.tcc.

Referenced by __poly_laguerre().

This routine returns the associated Laguerre polynomial of order n, degree α for large n. Abramowitz & Stegun, 13.5.21.

Parameters

n The order of the Laguerre function.

alpha1	The degree of the Laguerre function.
X	The argument of the Laguerre function.

Returns

The value of the Laguerre function of order n, degree α , and argument x.

This is from the GNU Scientific Library.

Definition at line 70 of file sf laguerre.tcc.

References __log_gamma().

Referenced by __poly_laguerre().

This routine returns the associated Laguerre polynomial of order n, degree α : $L_n^{\alpha}(x)$ by recursion.

The associated Laguerre function is defined by

$$L_n^{\alpha}(x) = \frac{(\alpha+1)_n}{n!} F_1(-n; \alpha+1; x)$$

where $(\alpha)_n$ is the Pochhammer symbol and ${}_1F_1(a;c;x)$ is the confluent hypergeometric function.

The associated Laguerre polynomial is defined for integral $\alpha=m$ by:

$$L_n^m(x) = (-1)^m \frac{d^m}{dx^m} L_{n+m}(x)$$

where the Laguerre polynomial is defined by:

$$L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n} (x^n e^{-x})$$

Parameters

n	The order of the Laguerre function.
alpha1	The degree of the Laguerre function.
X	The argument of the Laguerre function.

Returns

The value of the Laguerre function of order n, degree α , and argument x.

Definition at line 181 of file sf laguerre.tcc.

Referenced by __poly_laguerre().

8.3.2.177 template<typename _Tp > _Tp std::__detail::__poly_legendre_p (unsigned int __I, _Tp __x)

Return the Legendre polynomial by upward recursion on order l.

The Legendre function of order l and argument x, $P_l(x)$, is defined by:

$$P_l(x) = \frac{1}{2^l l!} \frac{d^l}{dx^l} (x^2 - 1)^l$$

	The order of the Legendre polynomial. $l>=0$.
x	The argument of the Legendre polynomial. $ x <= 1$.

Definition at line 73 of file sf legendre.tcc.

Referenced by __assoc_legendre_p(), and __sph_legendre().

8.3.2.178 template<typename _Tp > _Tp std::__detail::__poly_legendre_q (unsigned int __I, _Tp __x)

Return the Legendre function of the second kind by upward recursion on order l.

The Legendre function of order l and argument x, $Q_l(x)$, is defined by:

$$Q_l(x) = \frac{1}{2^l l!} \frac{d^l}{dx^l} (x^2 - 1)^l$$

Parameters

	The order of the Legendre polynomial. $l>=0$.
X	The argument of the Legendre polynomial. $ x <= 1$.

Definition at line 123 of file sf_legendre.tcc.

8.3.2.179 template<typename_Tp > _Tp std::__detail::__poly_radial_jacobi (unsigned int __n, unsigned int __n, _Tp __rho)

Definition at line 111 of file sf jacobi.tcc.

References __poly_jacobi().

Referenced by __zernike(), __gnu_cxx::radpolyf(), and __gnu_cxx::radpolyl().

8.3.2.180 template<typename _Tp > _Tp std::__detail::__polylog (_Tp __s, _Tp __x)

Return the polylog Li s(x) for two real arguments.

Parameters

s	The real index.
X	The real argument.

Returns

The complex value of the polylogarithm.

Definition at line 1072 of file sf_polylog.tcc.

References __fpequal(), and __polylog_exp().

Referenced by __dirichlet_beta(), __dirichlet_eta(), and __polylog().

8.3.2.181 template < typename _Tp > std::complex < _Tp > std::__detail::__polylog (_Tp __s, std::complex < _Tp > __w)

Return the polylog in those cases where we can calculate it.

s	The real index.
w	The complex argument.

Returns

The complex value of the polylogarithm.

Definition at line 1102 of file sf polylog.tcc.

References __fpequal(), __polylog(), and __polylog_exp().

8.3.2.182 template<typename _Tp , typename ArgType > __gnu_cxx::__promote_num_t<std::complex<_Tp>, ArgType> std::__detail::__polylog_exp (_Tp __s, ArgType __w)

This is the frontend function which calculates $Li_s(e^w)$ First we branch into different parts depending on the properties of s. This function is the same irrespective of a real or complex w, hence the template parameter ArgType.

Note

: I really wish we could return a variant<Tp, std::complex<Tp>>.

Parameters

s	The real order.
w	The real or complex argument.

Returns

The real or complex value of $Li_s(e^w)$.

Definition at line 1039 of file sf polylog.tcc.

References __polylog_exp_int_neg(), __polylog_exp_int_pos(), __polylog_exp_negative_real_part(), __polylog_exp← _real_neg(), and __polylog_exp_real_pos().

Referenced by __bose_einstein(), __clausen(), __clausen_c(), __clausen_s(), __fermi_dirac(), __hurwitz_zeta(), and __polylog().

8.3.2.183 template<typename _Tp > std::complex<_Tp> std::__detail::__polylog_exp_asymp (const _Tp __s, std::complex< _Tp > __w)

This function implements the asymptotic series for the polylog. It is given by

$$2\sum_{k=0}^{\infty} \zeta(2k)w^{s-2k}/\Gamma(s-2k+1) - i\pi w^{(s-1)}/\Gamma(s)$$

for Re(w) >> 1

Don't check this against Mathematica 8. For real u the imaginary part of the polylog is given by $Im(Li_s(e^u)) = -\pi u^{s-1}/\Gamma(s)$ Check this relation for any benchmark that you use. The use of evenzeta leads to a speedup of about 1000.

s	the real index s.
W	the large complex argument w.

Returns

the value of the polylogarithm.

Definition at line 686 of file sf_polylog.tcc.

References __fpequal().

Referenced by __polylog_exp_int_neg(), __polylog_exp_int_pos(), __polylog_exp_real_neg(), and __polylog_exp_\times real_pos().

8.3.2.184 template<typename _Tp > std::complex<_Tp> std::__detail::__polylog_exp_int_neg (const int __s, std::complex< _Tp > __w)

This treats the case where s is a negative integer.

Parameters

s	a negative integer.
w	an arbitrary complex number

Returns

the value of the polylogarith,.

Definition at line 856 of file sf polylog.tcc.

 $References \ __clamp_0_m2pi(), \ __clamp_pi(), \ __polylog_exp_asymp(), \ __polylog_exp_neg(), \ and \ __\hookleftarrow polylog_exp_negative_real_part().$

Referenced by __polylog_exp().

8.3.2.185 template<typename_Tp > std::complex<_Tp> std::__detail::__polylog_exp_int_neg (const int __s, _Tp __w)

This treats the case where s is a negative integer and w is a real.

Parameters

s	a negative integer.
w	the argument.

Returns

the value of the polylogarithm.

Definition at line 898 of file sf_polylog.tcc.

References __fpequal(), __polylog_exp_asymp(), __polylog_exp_neg(), and __polylog_exp_negative_real_part().

 $8.3.2.186 \quad template < typename _Tp > std::complex < _Tp > std::__detail::__polylog_exp_int_pos \ (\ const \ unsigned \ int __s, \\ std::complex < _Tp > __w \)$

Here s is a positive integer and the function descends into the different kernels depending on w.

s	a positive integer.
W	an arbitrary complex number.

Returns

The value of the polylogarithm.

Definition at line 767 of file sf polylog.tcc.

Referenced by __polylog_exp().

8.3.2.187 template<typename _Tp > std::complex<_Tp> std::__detail::__polylog_exp_int_pos (const unsigned int __s, _Tp __w)

Here s is a positive integer and the function descends into the different kernels depending on w.

Parameters

s	a positive integer
w	an arbitrary real argument w

Returns

the value of the polylogarithm.

Definition at line 815 of file sf_polylog.tcc.

References $_$ fpequal(), $_$ polylog_exp_asymp(), $_$ polylog_exp_negative_real_part(), $_$ polylog_exp_pos(), and $_$ \leftarrow riemann_zeta().

8.3.2.188 template<typename _Tp > std::complex<_Tp> std::__detail::__polylog_exp_neg (_Tp __s, std::complex< _Tp > __w)

This function treats the cases of negative real index s. Theoretical convergence is present for $|w| < 2\pi$. We use an optimized version of

$$Li_s(e^w) = \Gamma(1-s)(-w)^{(s-1)} + (2\pi)^{(-s)}/\pi A_p(w)$$
$$A_p(w) = \sum_k \Gamma(1+k-s)/k! \sin(\pi/2*(s-k))(w/2/\pi)^k \zeta(1+k-s)$$

Parameters

s	The real index
w	The complex argument

Returns

The value of the polylogarithm.

Definition at line 346 of file sf_polylog.tcc.

References __fpequal(), __riemann_zeta(), and __riemann_zeta_m_1().

Referenced by polylog exp int neg(), and polylog exp real neg().

8.3.2.189 template<typename_Tp > std::complex < _Tp > std::__detail::__polylog_exp_neg (int __s, std::complex < _Tp > __w)

This function treats the cases of negative integer index s and branches accordingly

Parameters

s	the integer index s.
W	The Argument w

Returns

The value of the Polylogarithm evaluated by a suitable function.

Definition at line 564 of file sf polylog.tcc.

References __polylog_exp_neg_even(), and __polylog_exp_neg_odd().

8.3.2.190 template<typename _Tp , int __sigma> std::complex<_Tp> std::__detail::__polylog_exp_neg_even (unsigned int __n, std::complex< _Tp > __w)

This function treats the cases of negative integer index s which are multiples of two.

In that case the sine occurring in the expansion occasionally takes on the value zero. We use that to provide an optimized series for p = 2n:

In the template parameter sigma we transport whether p = 4k(sigma = 1) or p = 4k + 2(sigma = -1)

$$Li_p(e^w) = Gamma(1-p)(-w)^{p-1} - A_p(w) - \sigma * B_p(w)$$

with

$$A_p(w) = 2(2\pi)^{(p-1)(-p)!/(2\pi)^{(-p/2)(1+w^2/(4pi^2))^{-1/2+p/2}}\cos((1-p)ArcTan(2pi/w))$$

and

$$B_p(w) = -2(2pi)^{\ell}(p-1)\sum_{k=0}^{\infty} \Gamma(2+2k-p)/(2k+1)!(-1)^k(w/2\pi)^{\ell}(2k+1)(\zeta(2+2k-p)-T)$$

This is suitable for $|w| < 2\pi$ The original series is (This might be worthwhile if we use the already present table of the Bernoullis)

$$Li_p(e^w) = \Gamma(1-p)(-w)^{p-1} - \sigma(2\pi)^p/pi \sum_{k=0}^{\infty} \Gamma(2+2k-p)/(2k+1)!(-1)^k (w/2\pi)^{(2k+1)} \zeta(2+2k-p)$$

Parameters

n	the integral index $n=4k$.
W	The complex argument w

Returns

the value of the Polylogarithm.

Definition at line 450 of file sf_polylog.tcc.

References fpequal().

Referenced by __polylog_exp_neg().

8.3.2.191 template<typename _Tp , int __sigma> std::complex<_Tp> std::__detail::__polylog_exp_neg_odd (unsigned int __n, std::complex< _Tp > __w)

This function treats the cases of negative integer index s which are odd.

In that case the sine occurring in the expansion occasionally vanishes. We use that to provide an optimized series for p=1+2k: In the template parameter sigma we transport whether p=1+4k(sigma=1) or p=3+4k(sigma=-1)

$$Li_p(e^w) = \Gamma(1-p) * (-w)^{p-1} + \sigma * A_p(w) - \sigma * B_p(w)$$

with

$$A_p(w) = 2(2\pi)^{(p-1)} * \Gamma(1-p)(1+w^2/(4pi^2))^{-1/2+p/2} \cos((1-p)ArcTan(2pi/w))$$

and

$$B_p(w) = 2(2pi)^{(p-1)} * \sum_{k=0}^{\infty} \Gamma(1+2k-p)/(2k)! (-w^2/4/\pi^2)^k (\zeta(1+2k-p) -_T p1)$$

This is suitable for $|w| < 2\pi$. The use of evenzeta gives a speedup of about 50 The original series is (This might be worthwhile if we use the already present table of the Bernoullis)

$$Li_{p}(e^{w}) = Gamma(1-p)*(-w)^{p-1} - \sigma*2*(2pi)^{(}p-1)*\sum_{k=0}^{\infty}\Gamma(1+2k-p)/(2k)!(-1)^{k}(w/2/\pi)^{(}2k)\zeta(1+2k-p)$$

Parameters

n	the integral index n = 4k.
w	The complex argument w.

Returns

The value of the Polylogarithm.

Definition at line 517 of file sf_polylog.tcc.

References __fpequal().

Referenced by __polylog_exp_neg().

8.3.2.192 template < typename _PowTp , typename _Tp > _Tp std::__detail::__polylog_exp_negative_real_part (_PowTp __s, _Tp __w)

Theoretical convergence for Re(w) < 0.

Seems to beat the other expansions for $Re(w) < -\pi/2 - \pi/5$. Note that this is an implementation of the basic series:

$$Li_s(e^z) = \sum_{k=1}^{\infty} e^(k * z) * k^(-s)$$

Parameters

s	is an arbitrary type, integral or float.
---	--

w someth	ing with a negative real part.
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Returns

the value of the polylogarithm.

Definition at line 737 of file sf polylog.tcc.

References fpequal().

Referenced by __polylog_exp(), __polylog_exp_int_neg(), __polylog_exp_int_pos(), __polylog_exp_real_neg(), and \cdot __polylog_exp_real_pos().

8.3.2.193 template<typename _Tp > std::complex<_Tp> std::__detail::__polylog_exp_pos (unsigned int __s, std::complex< _Tp > __w)

This function treats the cases of positive integer index s.

$$Li_s(e^w) = \sum_{k=0, k!=s-1} \zeta(s-k)w^k/k! + (H_{s-1} - \log(-w))w^(s-1)/(s-1)!$$

The radius of convergence is |w| < 2pi. Note that this series involves a $\log(-x)$. gcc and Mathematica differ in their implementation of $\log(e^(i\pi))$: gcc: $\log(e^(+-i*\pi)) = +-i\pi$ whereas Mathematica doesn't preserve the sign in this case: $\log(e^(+-i\pi)) = +i\pi$

Parameters

s	the index s.
w	the argument w.

Returns

the value of the polylogarithm.

Definition at line 206 of file sf_polylog.tcc.

References fpequal(), and riemann zeta().

Referenced by __polylog_exp_int_pos(), and __polylog_exp_real_pos().

8.3.2.194 template < typename $_{
m Tp} >$ std::complex < $_{
m Tp} >$ std:: $_{
m detail}$:: $_{
m polylog}$ exp $_{
m pos}$ (unsigned int $_{
m cs}$, $_{
m Tp}$ $_{
m w}$)

This function treats the cases of positive integer index s for real w.

This specialization is worthwhile to catch the differing behaviour of log(x).

$$Li_s(e^w) = \sum_{k=0, k!=s-1} \zeta(s-k)w^k/k! + (H_{s-1} - \log(-w))w^(s-1)/(s-1)!$$

The radius of convergence is $|w| < 2\pi$. Note that this series involves a $\log(-x)$. The use of evenzeta yields a speedup of about 2.5. gcc and Mathematica differ in their implementation of $\log(e^{(i\pi)})$: gcc: $\log(e^{(i\pi)}) = -i\pi$ whereas Mathematica doesn't preserve the sign in this case: $\log(e^{(i\pi)}) = +i\pi$

s	the index.
w	the argument

Returns

the value of the Polylogarithm

Definition at line 279 of file sf polylog.tcc.

References fpequal(), and riemann zeta().

8.3.2.195 template<typename _Tp > std::complex<_Tp> std::__detail::__polylog_exp_pos (_Tp __s, std::complex<_Tp > __w)

This function treats the cases of positive real index s.

The defining series is

$$Li_s(e^w) = A_s(w) + B_s(w) + \Gamma(1-s)(-w)(s-1)$$

with

$$A_s(w) = \sum_{k=0}^{m} \zeta(s-k)w^k/k!$$

$$B_s(w) = \sum_{k=m+1}^{\infty} \sin(\pi/2(s-k))\Gamma(1-s+k)\zeta(1-s+k)(w/2/\pi)^k/k!$$

Parameters

s	the positive real index s.
W	The complex argument w.

Returns

the value of the polylogarithm.

Definition at line 603 of file sf_polylog.tcc.

References __fpequal(), and __riemann_zeta().

8.3.2.196 template<typename _Tp > std::complex<_Tp> std::__detail::__polylog_exp_real_neg (_Tp __s, std::complex< _Tp > __w)

Return the polylog where s is a negative real value and for complex argument. Now we branch depending on the properties of w in the specific functions

Parameters

s	A negative real value that does not reduce to a negative integer.
w	The complex argument.

Returns

The value of the polylogarithm.

Definition at line 985 of file sf_polylog.tcc.

References $_$ clamp $_$ 0 $_$ m2pi(), $_$ polylog $_$ exp $_$ asymp(), $_$ polylog $_$ exp $_$ neg(), and $_$ polylog $_$ exp $_$ exp $_$ exp $_$ negative $_$ real $_$ part().

Referenced by __polylog_exp().

```
8.3.2.197 template<typename_Tp > std::complex<_Tp> std::__detail::__polylog_exp_real_neg ( _Tp __s, _Tp __w )
```

Return the polylog where s is a negative real value and for real argument. Now we branch depending on the properties of w in the specific functions.

Parameters

s	A negative real value.
w	A real argument.

Returns

The value of the polylogarithm.

Definition at line 1013 of file sf polylog.tcc.

References __polylog_exp_asymp(), __polylog_exp_neg(), and __polylog_exp_negative_real_part().

Return the polylog where s is a positive real value and for complex argument.

Parameters

s	A positive real number.
w	the complex argument.

Returns

The value of the polylogarithm.

Definition at line 922 of file sf_polylog.tcc.

```
References \_clamp_0m2pi(), \_clamp_pi(), \_fpequal(), \_fpreal(), \_polylog_exp_asymp(), \_polylog_exp_evolved negative_ereal_part(), \_polylog_exp_pos(), and \_riemann_pzeta().
```

Referenced by __polylog_exp().

```
8.3.2.199 template<typename_Tp > std::complex<_Tp> std::__detail::__polylog_exp_real_pos ( _Tp __s, _Tp __w )
```

Return the polylog where s is a positive real value and the argument is real.

s	A positive real number tht does not reduce to an integer.
w	The real argument w.

Returns

The value of the polylogarithm.

Definition at line 956 of file sf polylog.tcc.

References $_$ fpequal(), $_$ polylog_exp_asymp(), $_$ polylog_exp_negative_real_part(), $_$ polylog_exp_pos(), and $_$ \leftarrow riemann_zeta().

8.3.2.200 template<typename _Tp > _Tp std::__detail::__psi(_Tp __x)

Return the digamma function. The digamma or $\psi(x)$ function is defined by

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

For negative argument the reflection formula is used:

$$\psi(x) = \psi(1-x) - \pi \cot(\pi x)$$

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Definition at line 2330 of file sf_gamma.tcc.

References __psi_asymp().

Referenced by __expint_En_series(), __hydrogen(), __hyperg_reflect(), and __psi().

8.3.2.201 template<typename _Tp > _Tp std::__detail::__psi (unsigned int __n, _Tp __x)

Return the polygamma function $\psi^{(n)}(x)$.

The polygamma function is related to the Hurwitz zeta function:

$$\psi^{(n)}(x) = (-1)^{n+1} m! \zeta(m+1, x)$$

Definition at line 2395 of file sf gamma.tcc.

References __hurwitz_zeta(), __log_gamma(), and __psi().

8.3.2.202 template<typename _Tp > _Tp std::__detail::__psi_asymp (_Tp __x)

Return the digamma function for large argument. The digamma or $\psi(x)$ function is defined by

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

The asymptotic series is given by:

$$\psi(x) = \ln(x) - \frac{1}{2x} - \sum_{n=1}^{\infty} \frac{B_{2n}}{2nx^{2n}}$$

Definition at line 2299 of file sf gamma.tcc.

Referenced by __psi().

8.3.2.203 template<typename _Tp > _Tp std::__detail::__psi_series (_Tp __x)

Return the digamma function by series expansion. The digamma or $\psi(x)$ function is defined by

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

The series is given by:

$$\psi(x) = -\gamma_E - \frac{1}{x} \sum_{k=1}^{\infty} \frac{x-1}{(k+1)(x+k)}$$

Definition at line 2268 of file sf_gamma.tcc.

8.3.2.204 template<typename _Tp > _Tp std::__detail::__riemann_zeta (_Tp $_s$)

Return the Riemann zeta function $\zeta(s)$.

The Riemann zeta function is defined by:

$$\zeta(s) = \sum_{k=1}^\infty k^{-s} \text{ for } s > 1 \frac{(2\pi)^s}{\pi} \sin(\frac{\pi s}{2}) \Gamma(1-s) \zeta(1-s) \text{ for } s < 1$$

For s < 1 use the reflection formula:

$$\zeta(s) = 2^s \pi^{s-1} \Gamma(1-s) \zeta(1-s)$$

Parameters

s	The argument
---	--------------

Definition at line 505 of file sf zeta.tcc.

References __gamma(), __log_gamma(), __riemann_zeta_glob(), __riemann_zeta_product(), and __riemann_zeta_common sum().

Referenced by __polylog_exp_int_pos(), __polylog_exp_neg(), __polylog_exp_pos(), __polylog_exp_real_pos(), and evenzeta().

8.3.2.205 template<typename _Tp > _Tp std::__detail::__riemann_zeta_alt (_Tp __s)

Evaluate the Riemann zeta function $\zeta(s)$ by an alternate series for s > 0.

The Riemann zeta function is defined by:

$$\zeta(s) = \sum_{k=1}^{\infty} \frac{1}{k^s} fors > 1$$

For s < 1 use the reflection formula:

$$\zeta(s) = 2^s \pi^{s-1} \Gamma(1-s) \zeta(1-s)$$

Definition at line 329 of file sf_zeta.tcc.

8.3.2.206 template < typename _Tp > _Tp std::__detail::__riemann_zeta_euler_maclaurin (_Tp $_s$)

Evaluate the Riemann zeta function $\zeta(s)$ by an alternate series for s > 0.

This is a specialization of the code for the Hurwitz zeta function.

Definition at line 282 of file sf_zeta.tcc.

References S Euler Maclaurin zeta.

8.3.2.207 template < typename _Tp > _Tp std::__detail::__riemann_zeta_glob (_Tp $_s$)

Evaluate the Riemann zeta function by series for all s != 1. Convergence is great until largish negative numbers. Then the convergence of the > 0 sum gets better.

The series is:

$$\zeta(s) = \frac{1}{1 - 2^{1 - s}} \sum_{n=0}^{\infty} \frac{1}{2^{n+1}} \sum_{k=0}^{n} (-1)^k \frac{n!}{(n-k)!k!} (k+1)^{-s}$$

Havil 2003, p. 206.

The Riemann zeta function is defined by:

$$\zeta(s) = \sum_{k=1}^{\infty} \frac{1}{k^s} fors > 1$$

For s < 1 use the reflection formula:

$$\zeta(s) = 2^s \pi^{s-1} \Gamma(1-s) \zeta(1-s)$$

Definition at line 374 of file sf_zeta.tcc.

References __log_gamma().

Referenced by __riemann_zeta().

8.3.2.208 template<typename _Tp > _Tp std::__detail::__riemann_zeta_m_1 (_Tp __s)

Return the Riemann zeta function $\zeta(s) - 1$.

Parameters

$$__s$$
 The argument $s!=1$

Definition at line 672 of file sf_zeta.tcc.

References __riemann_zeta_m_1_sum(), _S_num_zetam1, and _S_zetam1.

Referenced by __polylog_exp_neg().

8.3.2.209 template<typename _Tp > _Tp std::__detail::__riemann_zeta_m_1_sum (_Tp $_s$)

Return the Riemann zeta function $\zeta(s)-1$ by summation for s>1. This is a small remainder for large s.

The Riemann zeta function is defined by:

$$\zeta(s) = \sum_{k=1}^{\infty} \frac{1}{k^s} fors > 1$$

Parameters

Definition at line 645 of file sf zeta.tcc.

Referenced by ___riemann_zeta_m_1().

8.3.2.210 template<typename _Tp > _Tp std::__detail::__riemann_zeta_product (_Tp __s)

Compute the Riemann zeta function $\zeta(s)$ using the product over prime factors.

$$\zeta(s) = \prod_{i=1}^{\infty} \frac{1}{1 - p_i^{-s}}$$

where p_i are the prime numbers.

The Riemann zeta function is defined by:

$$\zeta(s) = \sum_{k=1}^{\infty} \frac{1}{k^s} fors > 1$$

For s < 1 use the reflection formula:

$$\zeta(s) = 2^s \pi^{s-1} \Gamma(1-s) \zeta(1-s)$$

Parameters

__s The argument

Definition at line 463 of file sf zeta.tcc.

Referenced by __riemann_zeta().

8.3.2.211 template < typename _Tp > _Tp std::__detail::__riemann_zeta_sum (_Tp __s)

Compute the Riemann zeta function $\zeta(s)$ by summation for s > 1.

The Riemann zeta function is defined by:

$$\zeta(s) = \sum_{k=1}^{\infty} \frac{1}{k^s} fors > 1$$

For s < 1 use the reflection formula:

$$\zeta(s) = 2^s \pi^{s-1} \Gamma(1-s) \zeta(1-s)$$

Definition at line 254 of file sf zeta.tcc.

Referenced by __riemann_zeta().

 $8.3.2.212 \quad template < typename _Tp > __gnu_cxx::_promote_num_t < _Tp > std::_detail::_sinc (_Tp __a, _Tp __x)$

Return the generalized sinus cardinal function

$$sinc_a(x) = \frac{\sin(\pi x/a)}{(\pi x/a)}$$

Definition at line 51 of file sf cardinal.tcc.

8.3.2.213 template<typename _Tp > __gnu_cxx::__promote_num_t<_Tp> std::__detail::__sinc (_Tp __x)

Return the normalized sinus cardinal function

$$sinc(x) = \frac{\sin(\pi x)}{\pi x}$$

Definition at line 98 of file sf cardinal.tcc.

 $8.3.2.214 \quad template < typename_Tp > _gnu_cxx::_promote_num_t < _Tp > std::__detail::_sinc_pi \left(\ _Tp \ _x \ \right)$

Return the unnormalized sinus cardinal function

$$sinc_{\pi}(x) = \frac{\sin(x)}{x}$$

.

Definition at line 78 of file sf_cardinal.tcc.

8.3.2.215 template<typename _Tp > std::pair<_Tp, _Tp> std::__detail::__sincosint(_Tp __x)

This function returns the sine Si(x) and cosine Ci(x) integrals as a pair.

The sine integral is defined by:

$$Si(x) = \int_0^x dt \frac{\sin(t)}{t}$$

The cosine integral is defined by:

$$Ci(x) = \gamma_E + \log(x) + \int_0^x dt \frac{\cos(t) - 1}{t}$$

Definition at line 227 of file sf_trigint.tcc.

References __sincosint_asymp(), __sincosint_cont_frac(), and __sincosint_series().

8.3.2.216 template<typename _Tp > void std::__detail::__sincosint_asymp (_Tp __t, _Tp & _Si, _Tp & _Ci)

This function computes the sine Si(x) and cosine Ci(x) integrals by asymptotic series summation for positive argument.

The asymptotic series is very good for x > 50.

Definition at line 163 of file sf trigint.tcc.

Referenced by sincosint().

8.3.2.217 template<typename _Tp > void std::__detail::__sincosint_cont_frac (_Tp __t, _Tp & _Si, _Tp & _Ci)

This function computes the sine Si(x) and cosine Ci(x) integrals by continued fraction for positive argument.

Definition at line 55 of file sf_trigint.tcc.

Referenced by __sincosint().

8.3.2.218 template<typename _Tp > void std::__detail::__sincosint_series (_Tp __t, _Tp & _Si, _Tp & _Ci)

This function computes the sine Si(x) and cosine Ci(x) integrals by series summation for positive argument.

Definition at line 98 of file sf trigint.tcc.

Referenced by __sincosint().

8.3.2.219 template<typename_Tp > __gnu_cxx::__promote_num_t<_Tp> std::__detail::__sinhc (_Tp __a, _Tp __x)

Return the generalized hyperbolic sinus cardinal function

$$sinhc_a(x) = \frac{\sinh(\pi x/a)}{\pi x/a}$$

.

Definition at line 124 of file sf cardinal.tcc.

8.3.2.220 template<typename_Tp > __gnu_cxx::_promote_num_t<_Tp> std::__detail::__sinhc (_Tp __x)

Return the normalized hyperbolic sinus cardinal function

$$sinhc(x) = \frac{\sinh(\pi x)}{\pi x}$$

.

Definition at line 167 of file sf_cardinal.tcc.

 $8.3.2.221 \quad template < typename _Tp > _gnu_cxx::_promote_num_t < _Tp > std::_detail::_sinhc_pi (_Tp _x)$

Return the unnormalized hyperbolic sinus cardinal function

$$sinhc_{\pi}(x) = \frac{\sinh(x)}{x}$$

.

Definition at line 149 of file sf_cardinal.tcc.

8.3.2.222 template < typename _Tp > _Tp std::__detail::__sinhint (const _Tp __x)

Return the hyperbolic sine integral li(x).

The hyperbolic sine integral is given by

$$Shi(x) = (Ei(x) - E_1(x))/2$$

Parameters

__X

The argument of the hyperbolic sine integral function.

Returns

The hyperbolic sine integral.

Definition at line 581 of file sf_expint.tcc.

References __expint_E1(), and __expint_Ei().

8.3.2.223 template < typename $_{\rm Tp}$ > $_{\rm Tp}$ std::__detail::__sph_bessel (unsigned int $_{\rm m}$, $_{\rm Tp}$ $_{\rm m}$)

Return the spherical Bessel function $j_n(x)$ of order n and non-negative real argument x.

The spherical Bessel function is defined by:

$$j_n(x) = \left(\frac{\pi}{2x}\right)^{1/2} J_{n+1/2}(x)$$

Parameters

n	The non-negative integral order
x	The non-negative real argument

Returns

The output spherical Bessel function.

Definition at line 675 of file sf bessel.tcc.

References __sph_bessel_jn().

Return the complex spherical Bessel function.

Parameters

in	n	The order for which the spherical Bessel function is evaluated.
in	z	The argument at which the spherical Bessel function is evaluated.

Returns

The complex spherical Bessel function.

Definition at line 1314 of file sf hankel.tcc.

References sph hankel().

Compute the spherical modified Bessel functions $i_n(x)$ and $k_n(x)$ and their first derivatives $i_n'(x)$ and $k_n'(x)$ respectively.

Parameters

n	The order of the modified spherical Bessel function.
X	The argument of the modified spherical Bessel function.
i_n	The output regular modified spherical Bessel function.
k_n	The output irregular modified spherical Bessel function.
ip_n	The output derivative of the regular modified spherical Bessel function.
kp_n	The output derivative of the irregular modified spherical Bessel function.

Definition at line 445 of file sf_mod_bessel.tcc.

References __cyl_bessel_ik().

Compute the spherical Bessel $j_n(x)$ and Neumann $n_n(x)$ functions and their first derivatives $j_n(x)$ and $n'_n(x)$ respectively.

Parameters

	n	The order of the spherical Bessel function.
	x	The argument of the spherical Bessel function.
out	j_n	The output spherical Bessel function.
out	n_n	The output spherical Neumann function.
out	jp_n	The output derivative of the spherical Bessel function.
out	np_n	The output derivative of the spherical Neumann function.

Definition at line 640 of file sf_bessel.tcc.

References __cyl_bessel_jn().

Referenced by __sph_bessel(), __sph_hankel_1(), __sph_hankel_2(), and __sph_neumann().

Helper to compute complex spherical Hankel functions and their derivatives.

Parameters

in	n	The order for which the spherical Hankel functions are evaluated.
in	z	The argument at which the spherical Hankel functions are evaluated.
out	_H1	The spherical Hankel function of the first kind.
out	_H1p	The derivative of the spherical Hankel function of the first kind.
out	_H2	The spherical Hankel function of the second kind.
out	_H2p	The derivative of the spherical Hankel function of the second kind.

Definition at line 1258 of file sf hankel.tcc.

References __hankel().

Referenced by __sph_bessel(), __sph_hankel_1(), __sph_hankel_2(), and __sph_neumann().

8.3.2.228 template<typename_Tp > std::complex<_Tp> std::__detail::__sph_hankel_1 (unsigned int __n, _Tp __x)

Return the spherical Hankel function of the first kind $h_n^{(1)}(x)$.

The spherical Hankel function of the first kind is defined by:

$$h_n^{(1)}(x) = j_n(x) + i n_n(x)$$

Parameters

n	The order of the spherical Neumann function.
X	The argument of the spherical Neumann function.

Returns

The output spherical Neumann function.

Definition at line 744 of file sf bessel.tcc.

References sph bessel in().

8.3.2.229 template<typename _Tp > std::complex<_Tp> std::__detail::__sph_hankel_1 (unsigned int __n, std::complex< _Tp > _z)

Return the complex spherical Hankel function of the first kind.

Parameters

in	n	The order for which the spherical Hankel function of the first kind is evaluated.
in	z	The argument at which the spherical Hankel function of the first kind is evaluated.

Returns

The complex spherical Hankel function of the first kind.

Definition at line 1282 of file sf_hankel.tcc.

References __sph_hankel().

8.3.2.230 template<typename _Tp > std::complex<_Tp> std::__detail::__sph_hankel_2 (unsigned int __n, _Tp __x)

Return the spherical Hankel function of the second kind $h_n^{(2)}(x)$.

The spherical Hankel function of the second kind is defined by:

$$h_n^{(2)}(x) = j_n(x) - in_n(x)$$

Parameters

n	The non-negative integral order
X	The non-negative real argument

Returns

The output spherical Neumann function.

Definition at line 777 of file sf_bessel.tcc.

References __sph_bessel_in().

Return the complex spherical Hankel function of the second kind.

Parameters

in	n	The order for which the spherical Hankel function of the second kind is evaluated.
in	z	The argument at which the spherical Hankel function of the second kind is evalu-
		ated.

Returns

The complex spherical Hankel function of the second kind.

Definition at line 1298 of file sf_hankel.tcc.

References __sph_hankel().

8.3.2.232 template<typename _Tp > std::complex<_Tp> std::__detail::__sph_harmonic (unsigned int __I, int __m, _Tp __theta, __Tp __phi)

Return the spherical harmonic function.

The spherical harmonic function of l, m, and θ , ϕ is defined by:

$$Y_l^m(\theta,\phi) = (-1)^m \left[\frac{(2l+1)}{4\pi} \frac{(l-m)!}{(l+m)!} \right] P_l^{|m|}(\cos\theta) \exp^{im\phi}$$

Parameters

	The order of the spherical harmonic function. $l>=0$.
m	The order of the spherical harmonic function. $m <= l$.
theta	The radian polar angle argument of the spherical harmonic function.
phi	The radian azimuthal angle argument of the spherical harmonic function.

Definition at line 350 of file sf legendre.tcc.

References __sph_legendre().

8.3.2.233 template<typename_Tp > _Tp std::__detail::__sph_legendre (unsigned int __I, unsigned int __m, _Tp __theta)

Return the spherical associated Legendre function.

The spherical associated Legendre function of l,m, and θ is defined as $Y_l^m(\theta,0)$ where

$$Y_l^m(\theta,\phi) = (-1)^m \left[\frac{(2l+1)}{4\pi} \frac{(l-m)!}{(l+m)!} \right] P_l^m(\cos\theta) \exp^{im\phi}$$

is the spherical harmonic function and $P_l^m(x)$ is the associated Legendre function.

This function differs from the associated Legendre function by argument ($x = \cos(\theta)$) and by a normalization factor but this factor is rather large for large l and m and so this function is stable for larger differences of l and m.

Parameters

	The order of the spherical associated Legendre function. $l>=0$.
m	The order of the spherical associated Legendre function. $m <= l$.
theta	The radian polar angle argument of the spherical associated Legendre function.

Definition at line 253 of file sf_legendre.tcc.

References __log_gamma(), and __poly_legendre_p().

Referenced by __hydrogen(), and __sph_harmonic().

8.3.2.234 template < typename _Tp > _Tp std::__detail::__sph_neumann (unsigned int __n, _Tp __x)

Return the spherical Neumann function $n_n(x)$ of order n and non-negative real argument x.

The spherical Neumann function is defined by:

$$n_n(x) = \left(\frac{\pi}{2x}\right)^{1/2} N_{n+1/2}(x)$$

Parameters

n	The order of the spherical Neumann function.	
x The argument of the spherical Neumann function.		

Returns

The output spherical Neumann function.

Definition at line 712 of file sf_bessel.tcc.

References __sph_bessel_in().

8.3.2.235 template<typename _Tp > std::complex<_Tp> std::__detail::__sph_neumann (unsigned int __n, std::complex< _Tp > __z)

Return the complex spherical Neumann function.

Parameters

in	n	The order for which the spherical Neumann function is evaluated.	
in	z	The argument at which the spherical Neumann function is evaluated.	

Returns

The complex spherical Neumann function.

Definition at line 1330 of file sf hankel.tcc.

References __sph_hankel().

8.3.2.236 template < typename _Tp > _GLIBCXX14_CONSTEXPR _Tp std::__detail::__student_t_cdf (_Tp __t, unsigned int __nu)

Return the Students T probability function.

The students T propability function is related to the incomplete beta function:

$$A(t|\nu) = 1 - I_{\frac{\nu}{\nu + t^2}}(\frac{\nu}{2}, \frac{1}{2})A(t|\nu) =$$

Parameters

t	
nu	

Definition at line 301 of file sf_beta.tcc.

References __beta_inc().

 $8.3.2.237 \quad template < typename _Tp > _GLIBCXX14_CONSTEXPR _Tp \ std::__detail::__student _t_cdfc \ (\ _Tp __t, \ unsigned \ int __nu \)$

Return the complement of the Students T probability function.

The complement of the students T propability function is:

$$A_c(t|\nu) = I_{\frac{\nu}{\nu+t^2}}(\frac{\nu}{2}, \frac{1}{2}) = 1 - A(t|\nu)$$

Parameters

t	
nu	

Definition at line 324 of file sf beta.tcc.

References __beta_inc().

8.3.2.238 template<typename _Tp > _Tp std::__detail::__theta_1 (_Tp __nu, _Tp __x)

Return the exponential theta-1 function of period nu and argument x.

The Neville theta-1 function is defined by

$$\theta_1(\nu, x) = \frac{1}{\sqrt{\pi x}} \sum_{j=-\infty}^{+\infty} (-1)^j \exp\left(\frac{-(\nu + j - 1/2)^2}{x}\right)$$

Parameters

nu	The periodic (period = 2) argument	
x	x The argument	

Definition at line 190 of file sf_theta.tcc.

References __theta_2().

Referenced by __theta_s().

8.3.2.239 template<typename _Tp > _Tp std::__detail::__theta_2 (_Tp $_nu$, _Tp $_x$)

Return the exponential theta-2 function of period nu and argument x.

The exponential theta-2 function is defined by

$$\theta_2(\nu, x) = \frac{1}{\sqrt{\pi x}} \sum_{j=-\infty}^{+\infty} (-1)^j \exp\left(\frac{-(\nu+j)^2}{x}\right)$$

Parameters

nu	The periodic (period = 2) argument
x The argument	

Definition at line 162 of file sf_theta.tcc.

References __theta_2_asymp(), and __theta_2_sum().

Referenced by __theta_1(), and __theta_c().

8.3.2.240 template<typename _Tp > _Tp std::__detail::__theta_2_asymp (_Tp __nu, _Tp __x)

Compute and return the θ_2 function by series expansion.

Definition at line 103 of file sf theta.tcc.

Referenced by __theta_2().

8.3.2.241 template<typename _Tp > _Tp std::__detail::__theta_2_sum (_Tp __nu, _Tp __x)

Compute and return the θ_1 function by series expansion.

Definition at line 49 of file sf_theta.tcc.

Referenced by __theta_2().

8.3.2.242 template<typename_Tp > _Tp std::__detail::__theta_3 (_Tp __nu, _Tp __x)

Return the exponential theta-3 function of period nu and argument x.

The exponential theta-3 function is defined by

$$\theta_3(\nu, x) = \frac{1}{\sqrt{\pi x}} \sum_{j=-\infty}^{+\infty} \exp\left(\frac{-(\nu+j)^2}{x}\right)$$

Parameters

nu	The periodic (period = 1) argument	
x	x The argument	

Definition at line 216 of file sf_theta.tcc.

References theta 3 asymp(), and theta 3 sum().

Referenced by __theta_4(), and __theta_d().

8.3.2.243 template<typename _Tp > _Tp std::__detail::__theta_3_asymp (_Tp $_$ nu, _Tp $_$ x)

Compute and return the θ_3 function by asymptotic series expansion.

Definition at line 128 of file sf theta.tcc.

Referenced by __theta_3().

8.3.2.244 template<typename _Tp > _Tp std::__detail::__theta_3_sum (_Tp __nu, _Tp __x)

Compute and return the θ_3 function by series expansion.

Definition at line 77 of file sf theta.tcc.

Referenced by __theta_3().

8.3.2.245 template < typename $_{Tp} > _{Tp}$ std::__detail::__theta_4 ($_{Tp}$ __nu, $_{Tp}$ __x)

Return the exponential theta-2 function of period nu and argument x.

The exponential theta-2 function is defined by

$$\theta_2(\nu, x) = \frac{1}{\sqrt{\pi x}} \sum_{j=-\infty}^{+\infty} (-1)^j \exp\left(\frac{-(\nu+j)^2}{x}\right)$$

Parameters

nu	The periodic (period = 2) argument
X	The argument

Definition at line 244 of file sf theta.tcc.

References __theta_3().

Referenced by __theta_n().

8.3.2.246 template<typename _Tp > _Tp std::__detail::__theta_c (_Tp __k, _Tp __x)

Return the Neville θ_c function

Definition at line 337 of file sf_theta.tcc.

References __comp_ellint_1(), __ellnome(), and __theta_2().

8.3.2.247 template<typename _Tp > _Tp std::__detail::__theta_d (_Tp $_$ k, _Tp $_$ x)

Return the Neville θ_d function

Definition at line 362 of file sf_theta.tcc.

References __comp_ellint_1(), __ellnome(), and __theta_3().

8.3.2.248 template<typename _Tp > _Tp std::__detail::__theta_n (_Tp __k, _Tp __x)

Return the Neville θ_n function

Definition at line 387 of file sf theta.tcc.

References __comp_ellint_1(), __ellnome(), and __theta_4().

8.3.2.249 template<typename _Tp > _Tp std::__detail::__theta_s (_Tp $_k$, _Tp $_x$)

Return the Neville θ_s function

Definition at line 311 of file sf_theta.tcc.

References __comp_ellint_1(), __ellnome(), and __theta_1().

8.3.2.250 template<typename _Tp > __gnu_cxx::__promote_num_t<_Tp> std::__detail::__zernike (unsigned int __n, int __m, __Tp __rho, _Tp __phi)

Definition at line 133 of file sf_jacobi.tcc.

References __poly_radial_jacobi().

8.3.2.251 template<typename _Tp > _Tp std::__detail::__znorm1 (_Tp __x)

Definition at line 58 of file sf owens t.tcc.

Referenced by __owens_t().

```
8.3.2.252 template<typename _Tp > _Tp std::__detail::__znorm2 ( _Tp \_x )
```

Definition at line 47 of file sf_owens_t.tcc.

Referenced by __owens_t().

```
8.3.2.253 template < typename _{Tp} = double > _{Tp} std::__detail::evenzeta (unsigned int <math>_{k})
```

A function to calculate the values of zeta at even positive integers. For values smaller than thirty a table is used.

Parameters

```
\underline{\phantom{a}} an integer at which we evaluate the Riemann zeta function.
```

Returns

Definition at line 156 of file sf_polylog.tcc.

References riemann zeta().

8.3.3 Variable Documentation

8.3.3.1 constexpr size_t std::__detail::_Num_Euler_Maclaurin_zeta = 100

Coefficients for Euler-Maclaurin summation of zeta functions.

$$B_{2j}/(2j)!$$

where B_k are the Bernoulli numbers.

Definition at line 65 of file sf_zeta.tcc.

8.3.3.2 constexpr Factorial_table < long double > std::__detail::_S_double_factorial_table[301]

Definition at line 274 of file sf_gamma.tcc.

Referenced by __double_factorial(), and __log_double_factorial().

8.3.3.3 constexpr long double std::__detail::_S_Euler_Maclaurin_zeta[_Num_Euler_Maclaurin_zeta]

Definition at line 68 of file sf zeta.tcc.

Referenced by __hurwitz_zeta_euler_maclaurin(), and __riemann_zeta_euler_maclaurin().

8.3.3.4 constexpr _Factorial_table<long double> std::__detail::_S_factorial_table[171]

Definition at line 84 of file sf_gamma.tcc.

Referenced by __factorial(), and __log_factorial().

```
8.3.3.5 constexpr_Factorial_table<long double> std::__detail::_S_neg_double_factorial_table[999]
Definition at line 595 of file sf gamma.tcc.
Referenced by __double_factorial(), and __log_double_factorial().
8.3.3.6 template<typename Tp > constexpr std::size t std:: detail:: S num double factorials = 0
Definition at line 259 of file sf_gamma.tcc.
8.3.3.7 template <> constexpr std::size_t std::__detail::_S_num_double_factorials < double >= 301
Definition at line 264 of file sf gamma.tcc.
8.3.3.8 template <> constexpr std::size_t std::__detail::_S_num_double_factorials < float > = 57
Definition at line 262 of file sf gamma.tcc.
8.3.3.9 template <> constexpr std::size_t std::__detail::_S_num_double_factorials < long double >= 301
Definition at line 266 of file sf gamma.tcc.
8.3.3.10 template < typename _Tp > constexpr std::size_t std::__detail::_S_num_factorials = 0
Definition at line 69 of file sf gamma.tcc.
8.3.3.11 template <> constexpr std::size_t std:: detail:: S num factorials < double > = 171
Definition at line 74 of file sf_gamma.tcc.
8.3.3.12 template <> constexpr std::size_t std:: detail:: S num_factorials < float > = 35
Definition at line 72 of file sf gamma.tcc.
8.3.3.13 template<> constexpr std::size_t std::__detail::_S_num_factorials< long double > = 171
Definition at line 76 of file sf_gamma.tcc.
8.3.3.14 template < typename _Tp > constexpr std::size_t std::__detail::_S_num_neg_double_factorials = 0
Definition at line 579 of file sf_gamma.tcc.
```

8.3.3.15 template <> constexpr std::size_t std::__detail::_S_num_neg_double_factorials < double >= 150

Definition at line 584 of file sf gamma.tcc.

Definition at line 596 of file sf_zeta.tcc.

Referenced by __riemann_zeta_m_1().

8.3.3.16 template <> constexpr std::size_t std::__detail::_S_num_neg_double_factorials < float > = 27

Definition at line 582 of file sf_gamma.tcc.

8.3.3.17 template <> constexpr std::size_t std::__detail::_S_num_neg_double_factorials < long double > = 999

Definition at line 586 of file sf_gamma.tcc.

8.3.3.18 constexpr size_t std::__detail::_S_num_zetam1 = 33

Table of zeta(n) - 1 from 2 - 32. MPFR - 128 bits.

Definition at line 592 of file sf_zeta.tcc.

Referenced by __riemann_zeta_m_1().

8.3.3.19 constexpr long double std::__detail::_S_zetam1[_S_num_zetam1]

Namespace I	Documentation
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Chapter 9

Class Documentation

9.1 std::__detail::_Factorial_table< _Tp > Struct Template Reference

Public Attributes

- _Tp __factorial
- _Tp __log_factorial
- unsigned int __n

9.1.1 Detailed Description

template<typename _Tp>struct std::__detail::_Factorial_table< _Tp>

Definition at line 61 of file sf_gamma.tcc.

9.1.2 Member Data Documentation

 $9.1.2.1 \quad template < typename _Tp > _Tp \ std::__detail::_Factorial_table < _Tp > ::__factorial$

Definition at line 64 of file sf_gamma.tcc.

 $9.1.2.2 \quad template < typename _Tp > _Tp \ std::__detail::_Factorial_table < _Tp > ::__log_factorial$

Definition at line 65 of file sf_gamma.tcc.

9.1.2.3 template < typename $_{Tp} >$ unsigned int std:: $_{detail}$:: $_{factorial}$ template < $_{Tp} >$:: $_{n}$

Definition at line 63 of file sf_gamma.tcc.

The documentation for this struct was generated from the following file:

• bits/sf_gamma.tcc

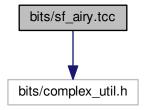
212 **Class Documentation**

Chapter 10

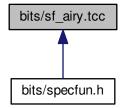
File Documentation

10.1 bits/sf_airy.tcc File Reference

#include <bits/complex_util.h>
Include dependency graph for sf_airy.tcc:



This graph shows which files directly or indirectly include this file:



Namespaces

- std
- std:: detail

Macros

#define _GLIBCXX_BITS_SF_AIRY_TCC 1

Functions

template<typename _Tp >
 void std::__detail::__airy (const std::complex< _Tp > &__z, _Tp __eps, std::complex< _Tp > &_Ai, std
 ::complex< _Tp > &_Aip, std::complex< _Tp > &_Bi, std::complex< _Tp > &_Bip)

This function computes the Airy function Ai(z) and its first derivative in the complex z-plane.

template<typename _Tp >
 std::complex< _Tp > std::__detail::__airy_ai (std::complex< _Tp > __z)

Return the complex Airy Ai function.

template<typename _Tp >
 void std::__detail::__airy_asymp_absarg_ge_pio3 (std::complex< _Tp > __z, std::complex< _Tp > &_Ai, std
 ::complex< _Tp > &_Aip, int __sign=-1)

This function evaluates Ai(z) and Ai'(z) from their asymptotic expansions for $|arg(z)| < 2 * \pi/3$. For speed, the number of terms needed to achieve about 16 decimals accuracy is tabled and determined from abs(z).

template<typename _Tp >
 void std::__detail::__airy_asymp_absarg_lt_pio3 (std::complex< _Tp > __z, std::complex< _Tp > &_Ai, std
 ::complex< Tp > & Aip)

This function evaluates Ai(z) and Ai'(z) from their asymptotic expansions for |arg(-z)| < pi/3. For speed, the number of terms needed to achieve about 16 decimals accuracy is tabled and determined from |z|.

- template<typename _Tp >
 void std::__detail::__airy_bessel_i (const std::complex< _Tp > &__z, _Tp __eps, std::complex< _Tp > &_lp1d3, std::complex< _Tp > &_lm1d3, std::complex< _Tp > &_lm2d3)
- template<typename _Tp >
 void std::__detail::__airy_bessel_k (const std::complex< _Tp > &__z, _Tp __eps, std::complex< _Tp > &_
 Kp1d3, std::complex< _Tp > &_Kp2d3)

Compute approximations to the modified Bessel functions of the second kind of orders 1/3 and 2/3 for moderate arguments.

template<typename _Tp >
 std::complex< _Tp > std::__detail::__airy_bi (std::complex< _Tp > __z)
 Return the complex Airy Bi function.

template<typename _Tp >
 void std::__detail::__airy_hyperg_rational (const std::complex< _Tp > &__z, std::complex< _Tp > &_Ai, std↔
 ::complex< Tp > & Aip, std::complex< Tp > & Bi, std::complex< Tp > & Bip)

This function computes rational approximations to the hypergeometric functions related to the modified Bessel functions of orders $\nu = + -1/3$ and $\nu + -2/3$. That is, A(z)/B(z), Where A(z) and B(z) are cubic polynomials with real coefficients, approximates

$$\frac{\Gamma(\nu+1)}{(z/2)^n u} I_{\nu}(z) =_0 F_1(;\nu+1;z^2/4),$$

where the function on the right is a confluent hypergeometric limit function. For |z| <= 1/4 and |arg(z)| <= pi/2, the approximations are accurate to about 16 decimals.

10.1.1 Detailed Description

This is an internal header file, included by other library headers. You should not attempt to use it directly.

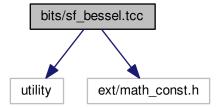
10.1.2 Macro Definition Documentation

10.1.2.1 #define _GLIBCXX_BITS_SF_AIRY_TCC 1

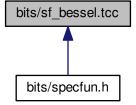
Definition at line 31 of file sf_airy.tcc.

10.2 bits/sf_bessel.tcc File Reference

```
#include <utility>
#include <ext/math_const.h>
Include dependency graph for sf bessel.tcc:
```



This graph shows which files directly or indirectly include this file:



Namespaces

std

• std::__detail

Macros

#define _GLIBCXX_BITS_SF_BESSEL_TCC 1

Functions

template<typename _Tp >
 _Tp std::__detail::__cyl_bessel_ij_series (_Tp __nu, _Tp __x, _Tp __sgn, unsigned int __max_iter)

This routine returns the cylindrical Bessel functions of order ν : J_{ν} or I_{ν} by series expansion.

template<typename_Tp>

```
_Tp std::__detail::__cyl_bessel_j (_Tp __nu, _Tp __x)
```

Return the Bessel function of order ν : $J_{\nu}(x)$.

template<typename_Tp>

```
void std::__detail::__cyl_bessel_jn (_Tp __nu, _Tp __x, _Tp &_Jnu, _Tp &_Nnu, _Tp &_Jpnu, _Tp &_Npnu)
```

Return the cylindrical Bessel functions and their derivatives of order ν by various means.

template<typename_Tp>

void std::__detail::__cyl_bessel_jn_asymp (_Tp __nu, _Tp __x, _Tp &_Jnu, _Tp &_Nnu, _Tp &_Jpnu, _Tp &_↔ Npnu)

This routine computes the asymptotic cylindrical Bessel and Neumann functions of order nu: $J_{\nu}(x)$, $N_{\nu}(x)$. Use this for $x >> nu^2 + 1$.

template<typename _Tp >

void std::__detail::__cyl_bessel_jn_steed (_Tp __nu, _Tp __x, _Tp &_Jnu, _Tp &_Nnu, _Tp &_Jpnu, _Tp &_↔ Npnu)

Compute the Bessel $J_{\nu}(x)$ and Neumann $N_{\nu}(x)$ functions and their first derivatives $J'_{\nu}(x)$ and $N'_{\nu}(x)$ respectively. These four functions are computed together for numerical stability.

template<typename _Tp >

Return the cylindrical Hankel function of the first kind $H_{\nu}^{(1)}(x)$.

• template<typename $_{\rm Tp}>$

Return the cylindrical Hankel function of the second kind $H_n^{(2)}u(x)$.

• template<typename _Tp >

Return the Neumann function of order ν : $N_{\nu}(x)$.

template<typename_Tp>

Compute the gamma functions required by the Temme series expansions of $N_{\nu}(x)$ and $K_{\nu}(x)$.

$$\Gamma_1 = \frac{1}{2\mu} \left[\frac{1}{\Gamma(1-\mu)} - \frac{1}{\Gamma(1+\mu)} \right]$$

and

$$\Gamma_2 = \frac{1}{2} \left[\frac{1}{\Gamma(1-\mu)} + \frac{1}{\Gamma(1+\mu)} \right]$$

where $-1/2 <= \mu <= 1/2$ is $\mu = \nu - N$ and N. is the nearest integer to ν . The values of $\Gamma(1+\mu)$ and $\Gamma(1-\mu)$ are returned as well.

• template<typename $_{\rm Tp}>$

Return the spherical Bessel function $j_n(x)$ of order n and non-negative real argument x.

template<typename _Tp >
 void std::__detail::__sph_bessel_jn (unsigned int __n, _Tp __x, _Tp &__j_n, _Tp &__n_n, _Tp &__jp_n, _Tp &__np_n)

Compute the spherical Bessel $j_n(x)$ and Neumann $n_n(x)$ functions and their first derivatives $j_n(x)$ and $n'_n(x)$ respectively.

template < typename _Tp >
 std::complex < _Tp > std::__detail::__sph_hankel_1 (unsigned int __n, _Tp __x)

Return the spherical Hankel function of the first kind $h_n^{(1)}(x)$.

template < typename _Tp >
 std::complex < _Tp > std::__detail::__sph_hankel_2 (unsigned int __n, _Tp __x)

Return the spherical Hankel function of the second kind $h_n^{(2)}(x)$.

template<typename _Tp >
 _Tp std::__detail::__sph_neumann (unsigned int __n, _Tp __x)

Return the spherical Neumann function $n_n(x)$ of order n and non-negative real argument x.

10.2.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <cmath>.

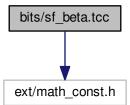
10.2.2 Macro Definition Documentation

10.2.2.1 #define GLIBCXX BITS SF BESSEL TCC 1

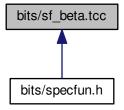
Definition at line 47 of file sf_bessel.tcc.

10.3 bits/sf beta.tcc File Reference

#include <ext/math_const.h>
Include dependency graph for sf_beta.tcc:



This graph shows which files directly or indirectly include this file:



Namespaces

- std
- std::__detail

Macros

• #define GLIBCXX BITS SF BETA TCC 1

Functions

```
template<typename _Tp >
 _Tp std::__detail::__beta (_Tp __a, _Tp __b)
     Return the beta function B(a,b).
template<typename _Tp >
  _Tp std::__detail::__beta_gamma (_Tp __a, _Tp __b)
     Return the beta function: B(a, b).
template<typename _Tp >
  _Tp std::__detail::__beta_inc (_Tp __a, _Tp __b, _Tp __x)
template<typename _Tp >
  _Tp std::__detail::__beta_inc_cont_frac (_Tp __a, _Tp __b, _Tp __x)
template<typename _Tp >
 _Tp std::__detail::__beta_lgamma (_Tp __a, _Tp __b)
      Return the beta function B(a,b) using the log gamma functions.

    template<typename</li>
    Tp >

 _Tp std::__detail::__beta_product (_Tp __a, _Tp __b)
      Return the beta function B(x, y) using the product form.

    template<typename</li>
    Tp >

  _GLIBCXX14_CONSTEXPR _Tp std::__detail::__binomial_cdf (_Tp __p, unsigned int __n, unsigned int __k)
      Return the binomial cumulative distribution function.

    template<typename</li>
    Tp >

  _GLIBCXX14_CONSTEXPR _Tp std::__detail::__binomial_cdfc (_Tp __p, unsigned int __n, unsigned int __k)
     Return the complementary binomial cumulative distribution function.
```

template<typename _Tp >
 _GLIBCXX14_CONSTEXPR _Tp std::__detail::__fisher_f_cdf (_Tp __F, unsigned int __nu1, unsigned int __nu2)

Return the F-distribution propability function. This returns the probability that the observed chi-square for a correct model exceeds the value χ^2 .

template<typename_Tp >
 _GLIBCXX14_CONSTEXPR _Tp std::__detail::__fisher_f_cdfc (_Tp __F, unsigned int __nu1, unsigned int __
 nu2)

Return the F-distribution propability function. This returns the probability that the observed chi-square for a correct model exceeds the value χ^2 .

template<typename_Tp >
 _GLIBCXX14_CONSTEXPR _Tp std::__detail::__student_t_cdf (_Tp __t, unsigned int __nu)

Return the Students T probability function.

template<typename_Tp >
 _GLIBCXX14_CONSTEXPR _Tp std::__detail::__student_t_cdfc (_Tp __t, unsigned int __nu)

Return the complement of the Students T probability function.

10.3.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <cmath>.

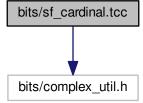
10.3.2 Macro Definition Documentation

10.3.2.1 #define _GLIBCXX_BITS_SF_BETA_TCC 1

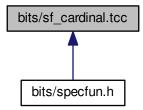
Definition at line 49 of file sf_beta.tcc.

10.4 bits/sf cardinal.tcc File Reference

#include <bits/complex_util.h>
Include dependency graph for sf cardinal.tcc:



This graph shows which files directly or indirectly include this file:



Namespaces

- std
- std:: detail

Macros

• #define _GLIBCXX_BITS_SF_CARDINAL_TCC 1

Functions

template < typename _Tp >
 __gnu_cxx::__promote_num_t < _Tp > std::__detail::__sinc (_Tp __a, _Tp __x)

 Return the generalized sinus cardinal function

• /

$$sinc_a(x) = \frac{\sin(\pi x/a)}{(\pi x/a)}$$

template<typename _Tp >

Return the normalized sinus cardinal function

$$sinc(x) = \frac{\sin(\pi x)}{\pi x}$$

template<typename _Tp >

Return the unnormalized sinus cardinal function

$$sinc_{\pi}(x) = \frac{\sin(x)}{x}$$

• template<typename_Tp>

Return the generalized hyperbolic sinus cardinal function

$$sinhc_a(x) = \frac{\sinh(\pi x/a)}{\pi x/a}$$

.

 $\bullet \ \ template {<} typename \ _Tp >$

Return the normalized hyperbolic sinus cardinal function

$$sinhc(x) = \frac{\sinh(\pi x)}{\pi x}$$

.

 $\bullet \ \ template {<} typename \ _Tp >$

Return the unnormalized hyperbolic sinus cardinal function

$$sinhc_{\pi}(x) = \frac{\sinh(x)}{x}$$

.

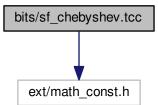
10.4.1 Macro Definition Documentation

10.4.1.1 #define _GLIBCXX_BITS_SF_CARDINAL_TCC 1

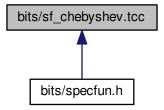
Definition at line 30 of file sf_cardinal.tcc.

10.5 bits/sf_chebyshev.tcc File Reference

#include <ext/math_const.h>
Include dependency graph for sf_chebyshev.tcc:



This graph shows which files directly or indirectly include this file:



Namespaces

- std
- std::__detail

Macros

#define _GLIBCXX_SF_CHEBYSHEV_TCC 1

Functions

```
template<typename _Tp >
    _Tp std::__detail::__chebyshev_recur (unsigned int __n, _Tp __x, _Tp _C0, _Tp _C1)
template<typename _Tp >
    _Tp std::__detail::__chebyshev_t (unsigned int __n, _Tp __x)
template<typename _Tp >
    _Tp std::__detail::__chebyshev_u (unsigned int __n, _Tp __x)
template<typename _Tp >
    _Tp std::__detail::__chebyshev_v (unsigned int __n, _Tp __x)
template<typename _Tp >
    _Tp std::__detail::__chebyshev_w (unsigned int __n, _Tp __x)
```

10.5.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

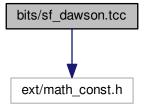
10.5.2 Macro Definition Documentation

10.5.2.1 #define _GLIBCXX_SF_CHEBYSHEV_TCC 1

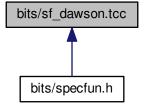
Definition at line 31 of file sf chebyshev.tcc.

10.6 bits/sf_dawson.tcc File Reference

#include <ext/math_const.h>
Include dependency graph for sf_dawson.tcc:



This graph shows which files directly or indirectly include this file:



Namespaces

- std
- std::__detail

Macros

#define _GLIBCXX_SF_DAWSON_TCC 1

Functions

```
• template<typename _Tp > 
 _Tp std::__detail::__dawson (_Tp __x) 
 Return the Dawson integral, F(x), for real argument x.
```

```
    template<typename_Tp >
        _Tp std::__detail::__dawson_const_frac (_Tp __x)
```

Compute the Dawson integral using a sampling theorem representation.

```
template<typename _Tp >
    _Tp std::__detail::__dawson_series (_Tp __x)
```

Compute the Dawson integral using the series expansion.

10.6.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

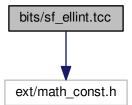
10.6.2 Macro Definition Documentation

10.6.2.1 #define _GLIBCXX_SF_DAWSON_TCC 1

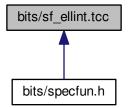
Definition at line 31 of file sf_dawson.tcc.

10.7 bits/sf_ellint.tcc File Reference

#include <ext/math_const.h>
Include dependency graph for sf_ellint.tcc:



This graph shows which files directly or indirectly include this file:



Namespaces

- std
- std:: detail

Macros

#define _GLIBCXX_BITS_SF_ELLINT_TCC 1

Functions

```
ullet template<typename _Tp >
  _Tp std::__detail::__comp_ellint_1 (_Tp __k)
      Return the complete elliptic integral of the first kind K(k) using the Carlson formulation.
template<typename _Tp >
  _Tp std::__detail::__comp_ellint_2 (_Tp __k)
      Return the complete elliptic integral of the second kind E(k) using the Carlson formulation.
template<typename _Tp >
  _Tp std::__detail::__comp_ellint_3 (_Tp __k, _Tp __nu)
      Return the complete elliptic integral of the third kind \Pi(k,\nu)=\Pi(k,\nu,\pi/2) using the Carlson formulation.
• template<typename _{\mathrm{Tp}} >
  _Tp std::__detail::__comp_ellint_d (_Tp __k)
template<typename _Tp >
  _Tp std::__detail::__comp_ellint_rf (_Tp __x, _Tp __y)

    template<typename _Tp >

  _Tp std::__detail::__comp_ellint_rg (_Tp __x, _Tp __y)
template<typename _Tp >
  _Tp std::__detail::__ellint_1 (_Tp __k, _Tp __phi)
      Return the incomplete elliptic integral of the first kind F(k, \phi) using the Carlson formulation.

    template<typename</li>
    Tp >

  _Tp std::__detail::__ellint_2 (_Tp __k, _Tp __phi)
      Return the incomplete elliptic integral of the second kind E(k, \phi) using the Carlson formulation.
```

```
template<typename _Tp >
  _Tp std::__detail::__ellint_3 (_Tp __k, _Tp __nu, _Tp __phi)
      Return the incomplete elliptic integral of the third kind \Pi(k, \nu, \phi) using the Carlson formulation.
template<typename</li>Tp >
  _Tp std::__detail::__ellint_cel (_Tp __k_c, _Tp __p, _Tp __a, _Tp __b)

    template<typename</li>
    Tp >

  _Tp std::__detail::__ellint_d (_Tp __k, _Tp __phi)

    template<typename _Tp >

  _Tp std::__detail::__ellint_el1 (_Tp __x, _Tp __k_c)
template<typename _Tp >
  _Tp std::__detail::__ellint_el2 (_Tp __x, _Tp __k_c, _Tp __a, _Tp __b)
• template<typename _{\rm Tp}>
  _Tp std::__detail::__ellint_el3 (_Tp __x, _Tp __k_c, _Tp __p)
template<typename _Tp >
  _Tp std::__detail::__ellint_rc (_Tp __x, _Tp __y)
      Return the Carlson elliptic function R_C(x,y) = R_F(x,y,y) where R_F(x,y,z) is the Carlson elliptic function of the first
      kind.
template<typename _Tp >
  _Tp std::__detail::__ellint_rd (_Tp __x, _Tp __y, _Tp __z)
      Return the Carlson elliptic function of the second kind R_D(x,y,z) = R_J(x,y,z,z) where R_J(x,y,z,p) is the Carlson
      elliptic function of the third kind.
template<typename_Tp>
  _Tp std::__detail::__ellint_rf (_Tp __x, _Tp __y, _Tp __z)
      Return the Carlson elliptic function R_F(x,y,z) of the first kind.
template<typename _Tp >
  _Tp std::__detail::__ellint_rg (_Tp __x, _Tp __y, _Tp __z)
      Return the symmetric Carlson elliptic function of the second kind R_G(x, y, z).
template<typename _Tp >
  _Tp std::__detail::__ellint_rj (_Tp __x, _Tp __y, _Tp __z, _Tp __p)
      Return the Carlson elliptic function R_J(x, y, z, p) of the third kind.
template<typename _Tp >
  _Tp std::__detail::__heuman_lambda (_Tp __k, _Tp __phi)
template<typename</li>Tp >
  _Tp std::__detail::__jacobi_zeta (_Tp __k, _Tp __phi)
```

10.7.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <cmath>.

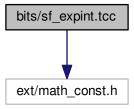
10.7.2 Macro Definition Documentation

10.7.2.1 #define _GLIBCXX_BITS_SF_ELLINT_TCC 1

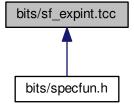
Definition at line 47 of file sf ellint.tcc.

10.8 bits/sf_expint.tcc File Reference

#include <ext/math_const.h>
Include dependency graph for sf_expint.tcc:



This graph shows which files directly or indirectly include this file:



Namespaces

- std
- std::__detail

Macros

#define _GLIBCXX_BITS_SF_EXPINT_TCC 1

Functions

```
• template<typename _Tp > 
 _Tp std::__detail::__coshint (const _Tp __x) 
 Return the hyperbolic cosine integral li(x).
```

```
template<typename _Tp >
  Tp std:: detail:: expint (unsigned int n, Tp x)
      Return the exponential integral E_n(x).
template<typename _Tp >
  _Tp std::__detail::__expint (_Tp __x)
      Return the exponential integral Ei(x).

    template<typename</li>
    Tp >

  _Tp std::__detail::__expint_asymp (unsigned int __n, _Tp __x)
      Return the exponential integral E_n(x) for large argument.
template<typename _Tp >
  Tp std:: detail:: expint E1 (Tp x)
      Return the exponential integral E_1(x).
template<typename _Tp >
  _Tp std::__detail::__expint_E1_asymp (_Tp __x)
      Return the exponential integral E_1(x) by asymptotic expansion.
template<typename _Tp >
  _Tp std::__detail::__expint_E1_series (_Tp __x)
      Return the exponential integral E_1(x) by series summation. This should be good for x < 1.

    template<typename</li>
    Tp >

  _Tp std::__detail::__expint_Ei (_Tp __x)
      Return the exponential integral Ei(x).
template<typename_Tp>
  _Tp std::__detail::__expint_Ei_asymp (_Tp __x)
      Return the exponential integral Ei(x) by asymptotic expansion.
template<typename _Tp >
  _Tp std::__detail::__expint_Ei_series (_Tp __x)
      Return the exponential integral Ei(x) by series summation.

    template<typename</li>
    Tp >

  _Tp std::__detail::__expint_En_cont_frac (unsigned int __n, _Tp __x)
      Return the exponential integral E_n(x) by continued fractions.

    template<typename</li>
    Tp >

  _Tp std::__detail::__expint_En_recursion (unsigned int __n, _Tp __x)
      Return the exponential integral E_n(x) by recursion. Use upward recursion for x < n and downward recursion (Miller's
      algorithm) otherwise.
template<typename _Tp >
  _Tp std::__detail::__expint_En_series (unsigned int __n, _Tp __x)
      Return the exponential integral E_n(x) by series summation.

    template<typename</li>
    Tp >

  _Tp std::__detail::__expint_large_n (unsigned int __n, _Tp __x)
      Return the exponential integral E_n(x) for large order.

    template<typename</li>
    Tp >

  _Tp std::__detail::__logint (const _Tp __x)
      Return the logarithmic integral li(x).
template<typename _Tp >
  _Tp std::__detail::__sinhint (const _Tp __x)
      Return the hyperbolic sine integral li(x).
```

10.8.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

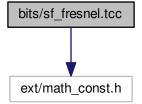
10.8.2 Macro Definition Documentation

10.8.2.1 #define _GLIBCXX_BITS_SF_EXPINT_TCC 1

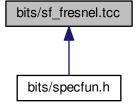
Definition at line 47 of file sf_expint.tcc.

10.9 bits/sf_fresnel.tcc File Reference

#include <ext/math_const.h>
Include dependency graph for sf fresnel.tcc:



This graph shows which files directly or indirectly include this file:



Namespaces

std

• std::__detail

Macros

#define _GLIBCXX_SF_FRESNEL_TCC 1

Functions

```
    template<typename _Tp >
    std::complex< _Tp > std::__detail::__fresnel (const _Tp __x)
```

Return the Fresnel cosine and sine integrals as a complex number f(C(x) + iS(x))

```
    template<typename _Tp >
        void std::__detail::__fresnel_cont_frac (const _Tp __ax, _Tp &_Cf, _Tp &_Sf)
```

This function computes the Fresnel cosine and sine integrals by continued fractions for positive argument.

```
    template<typename _Tp >
        void std::__detail::__fresnel_series (const _Tp __ax, _Tp &_Cf, _Tp &_Sf)
```

This function returns the Fresnel cosine and sine integrals as a pair by series expansion for positive argument.

10.9.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

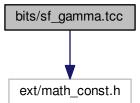
10.9.2 Macro Definition Documentation

```
10.9.2.1 #define _GLIBCXX_SF_FRESNEL_TCC 1
```

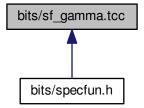
Definition at line 31 of file sf_fresnel.tcc.

10.10 bits/sf gamma.tcc File Reference

```
#include <ext/math_const.h>
Include dependency graph for sf_gamma.tcc:
```



This graph shows which files directly or indirectly include this file:



Classes

struct std::__detail::_Factorial_table< _Tp >

Namespaces

- std
- std:: detail

Macros

#define _GLIBCXX_BITS_SF_GAMMA_TCC 1

Functions

_GLIBCXX14_CONSTEXPR _Tp std:: _detail:: _chi_squared_pdf (_Tp __chi2, unsigned int __nu)

template<typename _Tp >

Return the chi-squared propability function. This returns the probability that the observed chi-squared for a correct model is less than the value χ^2 .

template<typename
 Tp >

Return the complementary chi-squared propability function. This returns the probability that the observed chi-squared for a correct model is greater than the value χ^2 .

template<typename_Tp>

Return the double factorial of the integer n.

template<typename
 Tp >

Return the factorial of the integer n.

template<typename _Tp >

Return $\Gamma(x)$.

• template<typename $_{\mathrm{Tp}}>$

template<typename _Tp >

Return the lower incomplete gamma function. The lower incomplete gamma function is defined by

$$\gamma(a, x) = \int_0^x e^{-t} t^{a-1} dt (a > 0)$$

•

template<typename _Tp >

Return the regularized lower incomplete gamma function. The regularized lower incomplete gamma function is defined by

$$P(a,x) = \frac{\gamma(a,x)}{\Gamma(a)}$$

where $\Gamma(a)$ is the gamma function and

$$\gamma(a,x) = \int_0^x e^{-t} t^{a-1} dt (a > 0)$$

is the lower incomplete gamma function.

• template<typename $_{\rm Tp}>$

Return the regularized upper incomplete gamma function. The regularized upper incomplete gamma function is defined by

$$Q(a,x) = \frac{\Gamma(a,x)}{\Gamma(a)}$$

where $\Gamma(a)$ is the gamma function and

$$\Gamma(a,x) = \int_{-\infty}^{\infty} e^{-t} t^{a-1} dt (a > 0)$$

is the upper incomplete gamma function.

• template<typename_Tp>

$$std::pair < _Tp, _Tp > std:: __detail:: __gamma_series (_Tp __a, _Tp __x)$$

template<typename _Tp >

Return the upper incomplete gamma function. The lower incomplete gamma function is defined by

$$\Gamma(a,x) = \int_{-\infty}^{\infty} e^{-t} t^{a-1} dt (a > 0)$$

.

template<typename _Tp >

Return the logarithm of the binomial coefficient. The binomial coefficient is given by:

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

template<typename_Tp>

template<typename_Tp>

Return the logarithm of the double factorial of the integer n.

template<typename _Tp >

Return the logarithm of the factorial of the integer n.

template<typename _Tp >

Return $log(|\Gamma(x)|)$. This will return values even for x < 0. To recover the sign of $\Gamma(x)$ for any argument use $_log_ \leftrightarrow gamma_sign$.

template<typename _Tp >

Return $log(\Gamma(x))$ by asymptotic expansion with Bernoulli number coefficients. This is like Sterling's approximation.

template<typename_Tp>

Return $log(\Gamma(x))$ by the Lanczos method. This method dominates all others on the positive axis I think.

template<typename _Tp >

Return the sign of $\Gamma(x)$. At nonpositive integers zero is returned.

template<typename _Tp >

Return $\Gamma(z)$ by the Spouge algorithm:

$$\Gamma(z+1) = (z+a)^{z+1/2} e^{-z-a} \left[\sqrt{2\pi} \sum_{k=1}^{\lceil a \rceil + 1} \frac{c_k(a)}{z+k} \right]$$

where

$$c_k(a) = \frac{(-1)^{k-1}}{(k-1)!} (a-k)^{k-1/2} e^{a-k}$$

and the error is bounded by

$$\epsilon(a) < a^{-1/2} (2\pi)^{-a-1/2}$$

template<typename_Tp>

Return the logarithm of the lower Pochhammer symbol or the falling factorial function. The lower Pochammer symbol is defined by

$$(a)_n = \prod_{k=0}^{n-1} (a-k), (a)_0 = 1 = \Gamma(a+1)/\Gamma(a-n+1)$$

In particular, $f(n)_n = n! f$. Thus this function returns

$$ln[(a)_n] = \Gamma(a+1) - \Gamma(a-n+1), ln[(a)_0] = 0$$

Many notations exist:

 $a^{\underline{n}}$

$$\left\{\begin{array}{c} a \\ n \end{array}\right\}$$

, and others.

• template<typename $_{\mathrm{Tp}}$ >

Return the logarithm of the (upper) Pochhammer symbol or the rising factorial function. The Pochammer symbol is defined by

$$(a)_n = \prod_{k=0}^{n-1} (a+k), (a)_0 = 1 = \Gamma(a+n)/\Gamma(n)$$

Thus this function returns

$$ln[(a)_n] = \Gamma(a+n) - \Gamma(n), ln[(a)_0] = 0$$

Many notations exist:

 $a^{\overline{n}}$

,

 $\begin{bmatrix} a \\ n \end{bmatrix}$

, and others.

template<typename _Tp >

Return the logarithm of the lower Pochhammer symbol or the falling factorial function. The lower Pochammer symbol is defined by

$$(a)_n = \prod_{k=0}^{n-1} (a-k), (a)_0 = 1 = \Gamma(a+1)/\Gamma(a-n+1)$$

In particular, $f[(n)_n = n! f]$.

template<typename _Tp >

Return the (upper) Pochhammer function or the rising factorial function. The Pochammer symbol is defined by

$$(a)_n = \prod_{k=0}^{n-1} (a+k), (a)_0 = 1 = \Gamma(a+n)/\Gamma(n)$$

Many notations exist:

 $a^{\bar{\imath}}$

,

 $\left[\begin{array}{c} a \\ n \end{array}\right]$

, and others.

template<typename_Tp>

Return the digamma function. The digamma or $\psi(x)$ function is defined by

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

For negative argument the reflection formula is used:

$$\psi(x) = \psi(1-x) - \pi \cot(\pi x)$$

template<typename_Tp>

Return the polygamma function $\psi^{(n)}(x)$.

template<typename _Tp >
 _Tp std::__detail::__psi_asymp (_Tp __x)

Return the digamma function for large argument. The digamma or $\psi(x)$ function is defined by

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

•

ullet template<typename_Tp>

Return the digamma function by series expansion. The digamma or $\psi(x)$ function is defined by

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

.

Variables

```
• constexpr Factorial table < long double > std:: detail:: S double factorial table [301]
```

- constexpr_Factorial_table < long double > std::__detail::_S_factorial_table [171]
- constexpr_Factorial_table < long double > std::__detail:: S_neg_double_factorial_table [999]
- template<typename_Tp >
 constexpr std::size_t std::__detail::_S_num_double_factorials = 0
- template<>

constexpr std::size_t std::__detail::_S_num_double_factorials< double > = 301

template<

constexpr std::size_t std::__detail::_S_num_double_factorials< float > = 57

• template<>

constexpr std::size_t std::__detail::_S_num_double_factorials< long double > = 301

ullet template<typename_Tp>

constexpr std::size_t std::__detail::_S_num_factorials = 0

• template<>

constexpr std::size_t std::__detail::_S_num_factorials< double > = 171

template<>

constexpr std::size t std:: detail:: S num factorials < float > = 35

template<>

constexpr std::size_t std::__detail::_S_num_factorials< long double > = 171

template<typename_Tp>

constexpr std::size_t std::__detail::_S_num_neg_double_factorials = 0

template<>

constexpr std::size_t std::__detail::_S_num_neg_double_factorials< double > = 150

template<>

constexpr std::size_t std::__detail::_S_num_neg_double_factorials< float > = 27

template<>

constexpr std::size_t std::__detail::_S_num_neg_double_factorials< long double > = 999

10.10.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <cmath>.

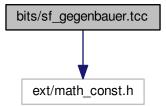
10.10.2 Macro Definition Documentation

10.10.2.1 #define _GLIBCXX_BITS_SF_GAMMA_TCC 1

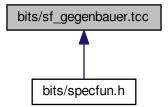
Definition at line 49 of file sf_gamma.tcc.

10.11 bits/sf_gegenbauer.tcc File Reference

#include <ext/math_const.h>
Include dependency graph for sf_gegenbauer.tcc:



This graph shows which files directly or indirectly include this file:



Namespaces

- std
- std::__detail

Macros

#define _GLIBCXX_SF_GEGENBAUER_TCC 1

Functions

```
    template<typename _Tp >
        _Tp std::__detail::__gegenbauer_poly (unsigned int __n, _Tp __alpha, _Tp __x)
```

10.11.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

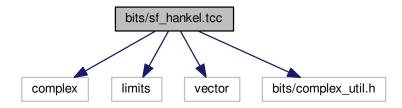
10.11.2 Macro Definition Documentation

10.11.2.1 #define _GLIBCXX_SF_GEGENBAUER_TCC 1

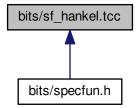
Definition at line 31 of file sf_gegenbauer.tcc.

10.12 bits/sf hankel.tcc File Reference

```
#include <complex>
#include <limits>
#include <vector>
#include <bits/complex_util.h>
Include dependency graph for sf_hankel.tcc:
```



This graph shows which files directly or indirectly include this file:



Namespaces

- std
- std::__detail

Macros

• #define GLIBCXX BITS SF HANKEL TCC 1

```
template<typename _Tp >
  void std::__detail::__airy_arg (std::complex< _Tp > __num2d3, std::complex< _Tp > __zeta, std::complex<
  _Tp > &__argp, std::complex< _Tp > &__argm)
      Compute the arguments for the Airy function evaluations carefully to prevent premature overflow. Note that the major work
      here is in safe_div. A faster, but less safe implementation can be obtained without use of safe_div.
template<typename _Tp >
  std::complex< _Tp > std::__detail::__cyl_bessel (std::complex< _Tp > __nu, std::complex< _Tp > __z)
      Return the complex cylindrical Bessel function.
template<typename</li>Tp >
  std::complex< _Tp > std::__detail::__cyl_hankel_1 (std::complex< _Tp > __nu, std::complex< _Tp > __z)
      Return the complex cylindrical Hankel function of the first kind.
template<typename _Tp >
  std::complex< _Tp > std::__detail::__cyl_hankel_2 (std::complex< _Tp > __nu, std::complex< _Tp > __z)
      Return the complex cylindrical Hankel function of the second kind.
template<typename Tp >
  std::complex< _Tp > std::__detail::__cyl_neumann (std::complex< _Tp > __nu, std::complex< _Tp > __z)
      Return the complex cylindrical Neumann function.
template<typename_Tp>
  void std:: __detail:: __debye_region (std::complex < _Tp > __alpha, int &__indexr, char &__aorb)
template<typename _Tp >
  void std::__detail::__hankel (std::complex< _Tp > __nu, std::complex< _Tp > __z, std::complex< _Tp > &_H1,
  std::complex< _Tp > &_H2, std::complex< _Tp > &_H1p, std::complex< _Tp > &_H2p)
```

- template<typename _Tp >
 void std::__detail::__hankel_debye (std::complex< _Tp > __nu, std::complex< _Tp > __z, std::complex< _Tp
 > _alpha, int __indexr, char &__aorb, int &__morn, std::complex< _Tp > &_H1, std::complex< _Tp > &_H2, std::complex< _Tp > &_H1p, std::complex< _Tp > &_H2p)

Compute parameters depending on z and nu that appear in the uniform asymptotic expansions of the Hankel functions and their derivatives, except the arguments to the Airy functions.

template<typename_Tp >
 void std::__detail::__hankel_uniform (std::complex< _Tp > __nu, std::complex< _Tp > __z, std::complex< _Tp > &_H1, std::complex< _Tp > &_H2, std::complex< _Tp > &_H1p, std::complex< _Tp > &_H2p)

This routine computes the uniform asymptotic approximations of the Hankel functions and their derivatives including a patch for the case when the order equals or nearly equals the argument. At such points, Olver's expressions have zero denominators (and numerators) resulting in numerical problems. This routine averages results from four surrounding points in the complex plane to obtain the result in such cases.

• template<typename _Tp > void std:: __detail:: __hankel_uniform_olver (std::complex< _Tp > __nu, std::complex< _Tp > __z, std \leftarrow ::complex< _Tp > &_H1, std::complex< _Tp > &_H1p, std::complex< _Tp > &_H2p)

Compute approximate values for the Hankel functions of the first and second kinds using Olver's uniform asymptotic expansion to of order nu along with their derivatives.

Compute outer factors and associated functions of z and nu appearing in Olver's uniform asymptotic expansions of the Hankel functions of the first and second kinds and their derivatives. The various functions of z and nu returned by $hankel_uniform_outer$ are available for use in computing further terms in the expansions.

Compute the sums in appropriate linear combinations appearing in Olver's uniform asymptotic expansions for the Hankel functions of the first and second kinds and their derivatives, using up to nterms (less than 5) to achieve relative error eps.

template<typename_Tp >
 std::complex< _Tp > std::__detail::__sph_bessel (unsigned int __n, std::complex< _Tp > __z)
 Return the complex spherical Bessel function.

• template<typename _Tp > void std::__detail::__sph_hankel (unsigned int __n, std::complex< _Tp > __z, std::complex< _Tp > &_H1, std \leftarrow ::complex< _Tp > &_H1p, std::complex< _Tp > &_H2p)

Helper to compute complex spherical Hankel functions and their derivatives.

template<typename _Tp >
 std::complex< Tp > std:: detail:: sph hankel 1 (unsigned int n, std::complex< Tp > z)

Return the complex spherical Hankel function of the first kind.

```
    template<typename _Tp >
        std::complex< _Tp > std::__detail::__sph_hankel_2 (unsigned int __n, std::complex< _Tp > __z)
        Return the complex spherical Hankel function of the second kind.
    template<typename _Tp >
        std::complex< _Tp > std::__detail::__sph_neumann (unsigned int __n, std::complex< _Tp > __z)
        Return the complex spherical Neumann function.
```

10.12.1 Detailed Description

This is an internal header file, included by other library headers. You should not attempt to use it directly.

10.12.2 Macro Definition Documentation

```
10.12.2.1 #define _GLIBCXX_BITS_SF_HANKEL_TCC 1
```

Definition at line 31 of file sf hankel.tcc.

10.13 bits/sf_hankel_new.tcc File Reference

```
#include <complex>
#include <limits>
#include <vector>
#include "specfun_util.h"
#include "complex_util.h"
Include dependency graph for sf_hankel_new.tcc:
```

bits/sf_hankel_new.tcc

vector

specfun_util.h

Macros

#define _GLIBCXX_BITS_SF_HANKEL_NEW_TCC 1

limits

10.13.1 Macro Definition Documentation

10.13.1.1 #define _GLIBCXX_BITS_SF_HANKEL_NEW_TCC 1

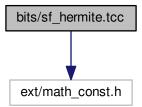
complex

Definition at line 31 of file sf hankel new.tcc.

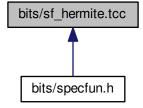
complex_util.h

10.14 bits/sf_hermite.tcc File Reference

#include <ext/math_const.h>
Include dependency graph for sf_hermite.tcc:



This graph shows which files directly or indirectly include this file:



Namespaces

- std
- std::__detail

Macros

#define _GLIBCXX_BITS_SF_HERMITE_TCC 1

```
• template<typename _Tp > 
 _Tp std::__detail::__poly_hermite (unsigned int __n, _Tp __x) 
 This routine returns the Hermite polynomial of order n: H_n(x).
```

```
    template<typename _Tp >
        _Tp std::__detail::__poly_hermite_asymp (unsigned int __n, _Tp __x)
        This routine returns the Hermite polynomial of large order n: H<sub>n</sub>(x). We assume here that x >= 0.
    template<typename _Tp >
        _Tp std::__detail::__poly_hermite_recursion (unsigned int __n, _Tp __x)
        This routine returns the Hermite polynomial of order n: H<sub>n</sub>(x) by recursion on n.
```

10.14.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

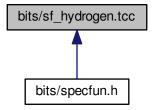
10.14.2 Macro Definition Documentation

```
10.14.2.1 #define _GLIBCXX_BITS_SF_HERMITE_TCC 1
```

Definition at line 42 of file sf_hermite.tcc.

10.15 bits/sf_hydrogen.tcc File Reference

This graph shows which files directly or indirectly include this file:



Namespaces

- std
- std::__detail

Macros

#define _GLIBCXX_BITS_SF_HYDROGEN_TCC 1

Functions

template<typename _Tp >
 std::complex< _Tp > std::__detail::__hydrogen (const unsigned int __n, const unsigned int __l, const unsigned int __m, const _Tp _Z, const _Tp __r, const _Tp __theta, const _Tp __phi)

10.15.1 Detailed Description

This is an internal header file, included by other library headers. You should not attempt to use it directly.

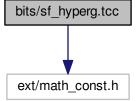
10.15.2 Macro Definition Documentation

10.15.2.1 #define _GLIBCXX_BITS_SF_HYDROGEN_TCC 1

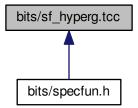
Definition at line 31 of file sf_hydrogen.tcc.

10.16 bits/sf_hyperg.tcc File Reference

#include <ext/math_const.h>
Include dependency graph for sf_hyperg.tcc:



This graph shows which files directly or indirectly include this file:



Namespaces

- std
- std:: detail

Macros

#define _GLIBCXX_BITS_SF_HYPERG_TCC 1

```
template<typename _Tp >
  _Tp std::__detail::__conf_hyperg (_Tp __a, _Tp __c, _Tp __x)
      Return the confluent hypergeometric function _1F_1(a;c;x).
• template<typename _Tp >
  _Tp std::__detail::__conf_hyperg_lim (_Tp __c, _Tp __x)
      Return the confluent hypergeometric limit function {}_{0}F_{1}(-;c;x).
• template<typename _Tp >
  _Tp std::__detail::__conf_hyperg_lim_series (_Tp __c, _Tp __x)
      This routine returns the confluent hypergeometric limit function by series expansion.
template<typename _Tp >
  _Tp std::__detail::__conf_hyperg_luke (_Tp __a, _Tp __c, _Tp __xin)
      Return the hypergeometric function _1F_1(a;c;x) by an iterative procedure described in Luke, Algorithms for the Compu-
      tation of Mathematical Functions.
template<typename _Tp >
  _Tp std::__detail::__conf_hyperg_series (_Tp __a, _Tp __c, _Tp __x)
      This routine returns the confluent hypergeometric function by series expansion.

    template<typename</li>
    Tp >

  _Tp std::__detail::__hyperg (_Tp __a, _Tp __b, _Tp __c, _Tp __x)
      Return the hypergeometric function _2F_1(a,b;c;x).
template<typename _Tp >
  _Tp std::__detail::__hyperg_luke (_Tp __a, _Tp __b, _Tp __c, _Tp __xin)
```

Return the hypergeometric function $_2F_1(a,b;c;x)$ by an iterative procedure described in Luke, Algorithms for the Computation of Mathematical Functions.

```
    template<typename _Tp >
        _Tp std::__detail::__hyperg_reflect (_Tp __a, _Tp __b, _Tp __c, _Tp __x)
```

Return the hypergeometric function ${}_2F_1(a,b;c;x)$ by the reflection formulae in Abramowitz & Stegun formula 15.3.6 for d=c-a - b not integral and formula 15.3.11 for d=c - a - b integral. This assumes a, b, c != negative integer.

```
    template<typename _Tp >
        _Tp std::__detail::__hyperg_series (_Tp __a, _Tp __b, _Tp __c, _Tp __x)
```

Return the hypergeometric function ${}_2F_1(a,b;c;x)$ by series expansion.

10.16.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

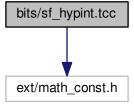
10.16.2 Macro Definition Documentation

10.16.2.1 #define _GLIBCXX_BITS_SF_HYPERG_TCC 1

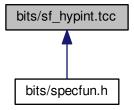
Definition at line 44 of file sf_hyperg.tcc.

10.17 bits/sf_hypint.tcc File Reference

#include <ext/math_const.h>
Include dependency graph for sf hypint.tcc:



This graph shows which files directly or indirectly include this file:



Namespaces

- std
- std:: detail

Macros

#define _GLIBCXX_SF_HYPINT_TCC 1

Functions

```
    template<typename _Tp >
        std::pair< _Tp, _Tp > std::__detail::__chshint (_Tp __x, _Tp &_Chi, _Tp &_Shi)
```

This function returns the hyperbolic cosine Ci(x) and hyperbolic sine Si(x) integrals as a pair.

• template<typename _Tp >

```
void std::__detail::__chshint_cont_frac (_Tp __t, _Tp &_Chi, _Tp &_Shi)
```

This function computes the hyperbolic cosine Chi(x) and hyperbolic sine Shi(x) integrals by continued fraction for positive argument.

```
    template<typename _Tp >
        void std::__detail::__chshint_series (_Tp __t, _Tp &_Chi, _Tp &_Shi)
```

This function computes the hyperbolic cosine Chi(x) and hyperbolic sine Shi(x) integrals by series summation for positive argument.

10.17.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

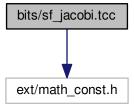
10.17.2 Macro Definition Documentation

10.17.2.1 #define _GLIBCXX_SF_HYPINT_TCC 1

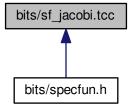
Definition at line 31 of file sf hypint.tcc.

10.18 bits/sf_jacobi.tcc File Reference

#include <ext/math_const.h>
Include dependency graph for sf_jacobi.tcc:



This graph shows which files directly or indirectly include this file:



Namespaces

- std
- std::__detail

Macros

• #define _GLIBCXX_SF_JACOBI_TCC 1

```
    template<typename _Tp >
    _Tp std::__detail::__poly_jacobi (unsigned int __n, _Tp __alpha, _Tp __beta, _Tp __x)
```

```
    template<typename _Tp >
        _Tp std::__detail::__poly_radial_jacobi (unsigned int __n, unsigned int __m, _Tp __rho)
```

```
    template<typename _Tp >
        __gnu_cxx::__promote_num_t< _Tp > std::__detail::__zernike (unsigned int __n, int __m, _Tp __rho, _Tp __phi)
```

10.18.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

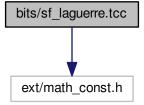
10.18.2 Macro Definition Documentation

10.18.2.1 #define GLIBCXX_SF_JACOBI_TCC 1

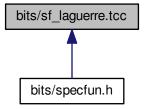
Definition at line 31 of file sf_jacobi.tcc.

10.19 bits/sf_laguerre.tcc File Reference

#include <ext/math_const.h>
Include dependency graph for sf_laguerre.tcc:



This graph shows which files directly or indirectly include this file:



Namespaces

- std
- std:: detail

Macros

#define _GLIBCXX_BITS_SF_LAGUERRE_TCC 1

```
template<typename _Tp >
  _Tp std::__detail::__assoc_laguerre (unsigned int __n, unsigned int __m, _Tp __x)
      This routine returns the associated Laguerre polynomial of order n, degree m: L_n^m(x).

    template<typename</li>
    Tp >

  _Tp std::__detail::__laguerre (unsigned int __n, _Tp __x)
      This routine returns the Laguerre polynomial of order n: L_n(x).
• template<typename _{\rm Tpa}, typename _{\rm Tp} >
  _Tp std::__detail::__poly_laguerre (unsigned int __n, _Tpa __alpha1, _Tp __x)
      This routine returns the associated Laguerre polynomial of order n, degree \alpha: L_n^a lpha(x).

    template<typename _Tpa , typename _Tp >

  _Tp std::__detail::__poly_laguerre_hyperg (unsigned int __n, _Tpa __alpha1, _Tp __x)
      Evaluate the polynomial based on the confluent hypergeometric function in a safe way, with no restriction on the arguments.

    template<typename _Tpa , typename _Tp >

  _Tp std::__detail::__poly_laguerre_large_n (unsigned __n, _Tpa __alpha1, _Tp __x)
      This routine returns the associated Laguerre polynomial of order n, degree \alpha for large n. Abramowitz & Stegun, 13.5.21.
• template<typename Tpa, typename Tp>
  _Tp std::__detail::__poly_laguerre_recursion (unsigned int __n, _Tpa __alpha1, _Tp __x)
      This routine returns the associated Laguerre polynomial of order n, degree \alpha: L_n^n(x) by recursion.
```

10.19.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

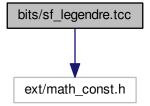
10.19.2 Macro Definition Documentation

10.19.2.1 #define _GLIBCXX_BITS_SF_LAGUERRE_TCC 1

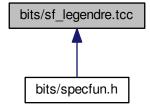
Definition at line 44 of file sf_laguerre.tcc.

10.20 bits/sf_legendre.tcc File Reference

#include <ext/math_const.h>
Include dependency graph for sf legendre.tcc:



This graph shows which files directly or indirectly include this file:



Namespaces

std

• std::__detail

Macros

• #define _GLIBCXX_BITS_SF_LEGENDRE_TCC 1

Functions

```
template<typename _Tp >
    _Tp std::__detail::__assoc_legendre_p (unsigned int __l, unsigned int __m, _Tp __x)
    Return the associated Legendre function by recursion on l and downward recursion on m.
template<typename _Tp >
    _Tp std::__detail::__poly_legendre_p (unsigned int __l, _Tp __x)
    Return the Legendre polynomial by upward recursion on order l.
template<typename _Tp >
    _Tp std::__detail::__poly_legendre_q (unsigned int __l, _Tp __x)
    Return the Legendre function of the second kind by upward recursion on order l.
template<typename _Tp >
    std::complex < _Tp > std::__detail::__sph_harmonic (unsigned int __l, int __m, _Tp __theta, _Tp __phi)
    Return the spherical harmonic function.
template<typename _Tp >
    _Tp std::__detail::__sph_legendre (unsigned int __l, unsigned int __m, _Tp __theta)
```

10.20.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

10.20.2 Macro Definition Documentation

```
10.20.2.1 #define _GLIBCXX_BITS_SF_LEGENDRE_TCC 1
```

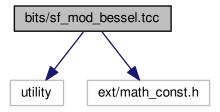
Return the spherical associated Legendre function.

Definition at line 47 of file sf_legendre.tcc.

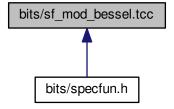
10.21 bits/sf mod bessel.tcc File Reference

```
#include <utility>
#include <ext/math_const.h>
```

Include dependency graph for sf_mod_bessel.tcc:



This graph shows which files directly or indirectly include this file:



Namespaces

- std
- std::__detail

Macros

#define _GLIBCXX_BITS_SF_MOD_BESSEL_TCC 1

```
• template<typename _Tp > void std::__detail::__airy (_Tp __z, _Tp &_Ai, _Tp &_Bi, _Tp &_Aip, _Tp &_Bip) 
 Compute the Airy functions Ai(x) and Bi(x) and their first derivatives Ai'(x) and Bi(x) respectively. 
• template<typename _Tp > 
 _Tp std::__detail::__cyl_bessel_i (_Tp __nu, _Tp __x) 
 Return the regular modified Bessel function of order \nu: I_{\nu}(x).
```

template < typename _Tp >
 void std:: detail:: cyl bessel ik (Tp nu, Tp x, Tp & Inu, Tp & Knu, Tp & Ipnu, Tp & Kpnu)

Return the modified cylindrical Bessel functions and their derivatives of order ν by various means.

template<typename _Tp >
 void std::__detail::__cyl_bessel_ik_asymp (_Tp __nu, _Tp __x, _Tp &_Inu, _Tp &_Knu, _Tp &_Ipnu, _Tp &_
 Kpnu)

This routine computes the asymptotic modified cylindrical Bessel and functions of order nu: $I_{\nu}(x)$, $N_{\nu}(x)$. Use this for $x >> nu^2 + 1$.

template<typename _Tp >
 void std::__detail::__cyl_bessel_ik_steed (_Tp __nu, _Tp __x, _Tp &_Inu, _Tp &_Knu, _Tp &_Ipnu, _Tp &_Kpnu)

Compute the modified Bessel functions $I_{\nu}(x)$ and $K_{\nu}(x)$ and their first derivatives $I'_{\nu}(x)$ and $K'_{\nu}(x)$ respectively. These four functions are computed together for numerical stability.

template < typename _Tp >
 _Tp std::__detail::__cyl_bessel_k (_Tp __nu, _Tp __x)

Return the irregular modified Bessel function $K_{\nu}(x)$ of order ν .

template<typename _Tp >
 void std::__detail::__fock_airy (_Tp __x, std::complex< _Tp > &__w1, std::complex< _Tp > &__w2, std
 ::complex< _Tp > &__w1p, std::complex< _Tp > &__w2p)

Compute the Fock-type Airy functions $w_1(x)$ and $w_2(x)$ and their first derivatives $w_1'(x)$ and $w_2'(x)$ respectively.

$$w_1(x) = \sqrt{\pi}(Ai(x) + iBi(x))$$

$$w_2(x) = \sqrt{\pi}(Ai(x) - iBi(x))$$

template<typename _Tp >
 void std::__detail::__sph_bessel_ik (unsigned int __n, _Tp __x, _Tp &__i_n, _Tp &__k_n, _Tp &__ip_n, _Tp &_ kp n)

Compute the spherical modified Bessel functions $i_n(x)$ and $k_n(x)$ and their first derivatives $i'_n(x)$ and $k'_n(x)$ respectively.

10.21.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <cmath>.

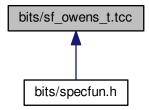
10.21.2 Macro Definition Documentation

10.21.2.1 #define _GLIBCXX_BITS_SF_MOD_BESSEL_TCC 1

Definition at line 47 of file sf mod bessel.tcc.

10.22 bits/sf_owens_t.tcc File Reference

This graph shows which files directly or indirectly include this file:



Namespaces

- std
- std:: detail

Macros

#define _GLIBCXX_BITS_SF_OWENS_T_TCC 1

Functions

```
template<typename _Tp >
    _Tp std::__detail::__gauss (_Tp __x)
template<typename _Tp >
    _Tp std::__detail::__owens_t (_Tp __h, _Tp __a)
template<typename _Tp >
    _Tp std::__detail::__znorm1 (_Tp __x)
template<typename _Tp >
    _Tp std::__detail::__znorm2 (_Tp __x)
```

10.22.1 Detailed Description

This is an internal header file, included by other library headers. You should not attempt to use it directly.

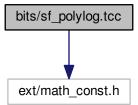
10.22.2 Macro Definition Documentation

10.22.2.1 #define _GLIBCXX_BITS_SF_OWENS_T_TCC 1

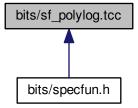
Definition at line 31 of file sf_owens_t.tcc.

10.23 bits/sf_polylog.tcc File Reference

#include <ext/math_const.h>
Include dependency graph for sf_polylog.tcc:



This graph shows which files directly or indirectly include this file:



Namespaces

- std
- std::__detail

Macros

• #define _GLIBCXX_BITS_SF_POLYLOG_TCC 1

```
template<typename _Tp >
_Tp std::__detail::__bose_einstein (_Tp __s, _Tp __x)
```

```
template<typename _Tp >
  std::complex< _Tp > std::__detail::__clamp_0_m2pi (std::complex< _Tp > __w)
template<typename_Tp>
  std::complex< _Tp > std::__detail::__clamp_pi (std::complex< _Tp > __w)
template<typename _Tp >
  std::complex< Tp > std:: detail:: clausen (unsigned int m, std::complex< Tp > w)
template<typename _Tp >
  _Tp std::__detail::__clausen (unsigned int __m, _Tp __w)
template<typename _Tp >
  _Tp std::__detail::__clausen_c (unsigned int __m, std::complex< _Tp > w)
template<typename</li>Tp >
  _Tp std::__detail::__clausen_c (unsigned int __m, _Tp __w)

    template<typename _Tp >

  _Tp std::__detail::__clausen_s (unsigned int __m, std::complex< Tp > w)

    template<typename</li>
    Tp >

  _Tp std::__detail::__clausen_s (unsigned int __m, _Tp __w)
template<typename _Tp >
  Tp std:: detail:: dirichlet beta (std::complex < Tp > w)
template<typename _Tp >
  _Tp std::__detail::__dirichlet_beta (_Tp __w)
template<typename_Tp>
  std::complex< Tp > std:: detail:: dirichlet eta (std::complex< Tp > w)
template<typename _Tp >
  Tp std:: detail:: dirichlet eta (Tp w)
template<typename _Tp >
  _Tp std::__detail::__fermi_dirac (_Tp __s, _Tp __x)

    template<typename</li>
    Tp >

  bool std::__detail::__fpequal (const _Tp &__a, const _Tp &__b)
template<typename _Tp >
  bool std::__detail::__fpimag (const std::complex < _Tp > &__w)
template<typename</li>Tp >
  bool std::__detail::__fpimag (const _Tp)

    template<typename</li>
    Tp >

  bool std::__detail::__fpreal (const std::complex < _Tp > &__w)
template<typename _Tp >
  bool std:: detail:: fpreal (const Tp)
template<typename _Tp >
  std::complex< _Tp > std::__detail::__hurwitz_zeta (_Tp __s, std::complex< _Tp > __a)

    template<typename</li>
    Tp >

  _Tp std::__detail::__polylog (_Tp __s, _Tp __x)
template<typename _Tp >
  std::complex< _Tp > std::__detail::__polylog (_Tp __s, std::complex< _Tp > __w)

    template<typename</li>
    Tp , typename
    ArgType >

    _gnu_cxx::__promote_num_t< std::complex< _Tp >, ArgType > std::__detail::__polylog_exp (_Tp __s, Arg↔
  Type __w)
template<typename _Tp >
  std::complex < \_Tp > std::\_\_detail::\_\_polylog\_exp\_asymp \ (const \_Tp \_\_s, std::complex < \ Tp > \ \ w)
template<typename</li>Tp >
  std::complex < _Tp > std::__detail::__polylog_exp_int_neg (const int __s, std::complex < _Tp > __w)
template<typename_Tp>
  std::complex < _Tp > std::__detail::__polylog_exp_int_neg (const int __s, _Tp __w)
template<typename _Tp >
  std::complex< Tp > std:: detail:: polylog exp int pos (const unsigned int s, std::complex< Tp > w)
```

```
template<typename _Tp >
  std::complex< _Tp > std::__detail::__polylog_exp_int_pos (const unsigned int __s, _Tp __w)
template<typename _Tp >
  std::complex < \_Tp > std::\__detail::\__polylog\_exp\_neg \ (\_Tp \_\_s, \ std::complex < \_Tp > \_\_w)
• template<typename _{\mathrm{Tp}}>
  std::complex < Tp > std:: detail:: polylog exp neg (int s, std::complex < Tp > w)
• template<typename _Tp , int __sigma>
  std::complex< Tp > std:: detail:: polylog exp neg even (unsigned int n, std::complex< Tp > w)
• template<typename _Tp , int __sigma>
  std::complex< Tp > std:: detail:: polylog exp neg odd (unsigned int n, std::complex< Tp > w)
• template<typename _PowTp , typename _Tp >
  _Tp std::__detail::__polylog_exp_negative_real_part (_PowTp __s, _Tp __w)
template<typename _Tp >
  std::complex < Tp > std:: detail:: polylog exp pos (unsigned int s, std::complex < Tp > w)
template<typename _Tp >
  std::complex< Tp > std:: detail:: polylog exp pos (unsigned int s, Tp w)
template<typename _Tp >
  std::complex< _Tp > std::__detail::__polylog_exp_pos (_Tp __s, std::complex< _Tp > __w)
template<typename _Tp >
  std::complex < Tp > std:: detail:: polylog exp real neg ( Tp s, std::complex < Tp > w)
template<typename _Tp >
  std::complex< _Tp > std::__detail::__polylog_exp_real_neg (_Tp __s, _Tp __w)
template<typename _Tp >
  std::complex< _Tp > std::__detail::__polylog_exp_real_pos (_Tp __s, std::complex< _Tp > __w)
template<typename _Tp >
  std::complex< _Tp > std::__detail::__polylog_exp_real_pos (_Tp __s, _Tp __w)
• template<typename _Tp = double>
  _Tp std::__detail::evenzeta (unsigned int __k)
```

10.23.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <cmath>.

10.23.2 Macro Definition Documentation

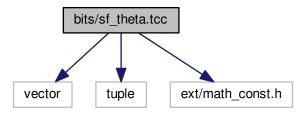
```
10.23.2.1 #define _GLIBCXX_BITS_SF_POLYLOG_TCC 1
```

Definition at line 41 of file sf_polylog.tcc.

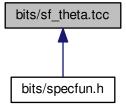
10.24 bits/sf theta.tcc File Reference

```
#include <vector>
#include <tuple>
#include <ext/math_const.h>
```

Include dependency graph for sf_theta.tcc:



This graph shows which files directly or indirectly include this file:



Namespaces

- std
- std::__detail

Macros

• #define _GLIBCXX_SF_THETA_TCC 1

```
template<typename _Tp >
    _Tp std::__detail::__ellnome (_Tp __k)
template<typename _Tp >
    _Tp std::__detail::__ellnome_k (_Tp __k)
template<typename _Tp >
    _Tp std::__detail::__ellnome_series (_Tp __k)
```

```
template<typename _Tp >
  std::tuple < _Tp, _Tp, _Tp > std::__detail::__jacobi_sncndn (_Tp __k, _Tp __u)
template<typename _Tp >
  _Tp std::__detail::__theta_1 (_Tp __nu, _Tp __x)
• template<typename _{\mathrm{Tp}} >
  _Tp std::__detail::__theta_2 (_Tp __nu, _Tp __x)
ullet template<typename _Tp >
  _Tp std::__detail::__theta_2_asymp (_Tp __nu, _Tp __x)
template<typename _Tp >
  _Tp std::__detail::__theta_2_sum (_Tp __nu, _Tp __x)
template<typename _Tp >
  _Tp std::__detail::__theta_3 (_Tp __nu, _Tp __x)
template<typename _Tp >
  _Tp std::__detail::__theta_3_asymp (_Tp __nu, _Tp __x)
template<typename _Tp >
  _Tp std::__detail::__theta_3_sum (_Tp __nu, _Tp __x)
template<typename_Tp>
  _Tp std::__detail::__theta_4 (_Tp __nu, _Tp __x)
template<typename _Tp >
  _Tp std::__detail::__theta_c (_Tp __k, _Tp __x)
template<typename _Tp >
  _Tp std::__detail::__theta_d (_Tp __k, _Tp __x)
template<typename</li>Tp >
  _Tp std::__detail::__theta_n (_Tp __k, _Tp __x)
template<typename_Tp>
  _Tp std::__detail::__theta_s (_Tp __k, _Tp __x)
```

10.24.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

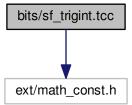
10.24.2 Macro Definition Documentation

```
10.24.2.1 #define _GLIBCXX_SF_THETA_TCC 1
```

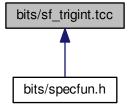
Definition at line 31 of file sf_theta.tcc.

10.25 bits/sf_trigint.tcc File Reference

#include <ext/math_const.h>
Include dependency graph for sf_trigint.tcc:



This graph shows which files directly or indirectly include this file:



Namespaces

- std
- std::__detail

Macros

• #define _GLIBCXX_SF_TRIGINT_TCC 1

Enumerations

enum { std::__detail::SININT, std::__detail::COSINT }

Functions

template < typename _Tp >
 std::pair < _Tp, _Tp > std::__detail::__sincosint (_Tp __x)

This function returns the sine Si(x) and cosine Ci(x) integrals as a pair.

template<typename _Tp >
 void std:: __detail:: __sincosint_asymp (_Tp __t, _Tp &_Si, _Tp &_Ci)

This function computes the sine Si(x) and cosine Ci(x) integrals by asymptotic series summation for positive argument.

template < typename _Tp >
 void std::__detail::__sincosint_cont_frac (_Tp __t, _Tp &_Si, _Tp &_Ci)

This function computes the sine Si(x) and cosine Ci(x) integrals by continued fraction for positive argument.

template < typename _Tp >
 void std:: __detail:: __sincosint_series (_Tp __t, _Tp &_Si, _Tp &_Ci)

This function computes the sine Si(x) and cosine Ci(x) integrals by series summation for positive argument.

10.25.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

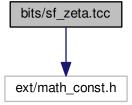
10.25.2 Macro Definition Documentation

10.25.2.1 #define _GLIBCXX_SF_TRIGINT_TCC 1

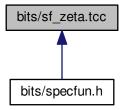
Definition at line 31 of file sf_trigint.tcc.

10.26 bits/sf_zeta.tcc File Reference

#include <ext/math_const.h>
Include dependency graph for sf_zeta.tcc:



This graph shows which files directly or indirectly include this file:



Namespaces

- std
- std::__detail

Macros

• #define GLIBCXX BITS SF ZETA TCC 1

convergence of the > 0 sum gets better.

```
template<typename</li>Tp >
  _Tp std::__detail::__dilog (_Tp __x)
      Compute the dilogarithm function Li_2(x) by summation for x \le 1.
template<typename _Tp >
  _Tp std::__detail::__hurwitz_zeta (_Tp __s, _Tp __a)
      Return the Hurwitz zeta function \zeta(s,a) for all s = 1 and a > -1.
template<typename _Tp >
  _Tp std::__detail::__hurwitz_zeta_euler_maclaurin (_Tp __s, _Tp __a)
      Return the Hurwitz zeta function \zeta(s,a) for all s \neq 1 and a > -1.
template<typename _Tp >
  _Tp std::__detail::__riemann_zeta (_Tp __s)
      Return the Riemann zeta function \zeta(s).

    template<typename</li>
    Tp >

  _Tp std::__detail::__riemann_zeta_alt (_Tp __s)
      Evaluate the Riemann zeta function \zeta(s) by an alternate series for s > 0.
template<typename _Tp >
  _Tp std::__detail::__riemann_zeta_euler_maclaurin (_Tp __s)
      Evaluate the Riemann zeta function \zeta(s) by an alternate series for s > 0.
template<typename _Tp >
  _Tp std::__detail::__riemann_zeta_glob (_Tp __s)
      Evaluate the Riemann zeta function by series for all s != 1. Convergence is great until largish negative numbers. Then the
```

```
template<typename _Tp >
    _Tp std::__detail::__riemann_zeta_m_1 (_Tp __s)
Return the Riemann zeta function ζ(s) - 1.
template<typename _Tp >
    _Tp std::__detail::__riemann_zeta_m_1_sum (_Tp __s)
Return the Riemann zeta function ζ(s) - 1 by summation for s > 1. This is a small remainder for large s.
template<typename _Tp >
    _Tp std::__detail::__riemann_zeta_product (_Tp __s)
Compute the Riemann zeta function ζ(s) using the product over prime factors.
template<typename _Tp >
    _Tp std::__detail::__riemann_zeta_sum (_Tp __s)
Compute the Riemann zeta function ζ(s) by summation for s > 1.
```

Variables

- constexpr size_t std::__detail::_Num_Euler_Maclaurin_zeta = 100
- constexpr long double std:: __detail:: S _Euler _Maclaurin _zeta [Num _Euler _Maclaurin _zeta]
- constexpr size_t std::__detail::_S_num_zetam1 = 33
- constexpr long double std::__detail::_S_zetam1 [_S_num_zetam1]

10.26.1 Detailed Description

This is an internal header file, included by other library headers. Do not attempt to use it directly. Instead, include <math>.

10.26.2 Macro Definition Documentation

10.26.2.1 #define _GLIBCXX_BITS_SF_ZETA_TCC 1

Definition at line 46 of file sf_zeta.tcc.

10.27 bits/specfun.h File Reference

```
#include <bits/c++config.h>
#include <limits>
#include <bits/stl_algobase.h>
#include <bits/specfun_util.h>
#include <type_traits>
#include <bits/numeric_limits.h>
#include <bits/complex_util.h>
#include <bits/sf_gamma.tcc>
#include <bits/sf_bessel.tcc>
#include <bits/sf_beta.tcc>
#include <bits/sf_cardinal.tcc>
#include <bits/sf_chebyshev.tcc>
#include <bits/sf dawson.tcc>
#include <bits/sf_ellint.tcc>
#include <bits/sf_expint.tcc>
#include <bits/sf_fresnel.tcc>
#include <bits/sf_gegenbauer.tcc>
#include <bits/sf_hyperg.tcc>
#include <bits/sf_hypint.tcc>
#include <bits/sf_jacobi.tcc>
#include <bits/sf_laguerre.tcc>
#include <bits/sf_legendre.tcc>
#include <bits/sf_hydrogen.tcc>
#include <bits/sf_mod_bessel.tcc>
#include <bits/sf_hermite.tcc>
#include <bits/sf_theta.tcc>
#include <bits/sf_trigint.tcc>
#include <bits/sf_zeta.tcc>
#include <bits/sf_owens_t.tcc>
#include <bits/sf_polylog.tcc>
#include <bits/sf_airy.tcc>
#include <bits/sf hankel.tcc>
Include dependency graph for specfun.h:
```



Namespaces

- __gnu_cxx
- std

Macros

- #define __cpp_lib_math_special_functions 201603L
- #define STDCPP MATH SPEC FUNCS 201003L

Enumerations

Functions

```
template<typename _Tp >
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::airy_ai (_Tp __x)

    float __gnu_cxx::airy_aif (float __x)

    long double <u>__gnu_cxx::airy_ail</u> (long double <u>__x</u>)

    template<typename</li>
    Tp >

  __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::airy_bi (_Tp __x)

    float __gnu_cxx::airy_bif (float __x)

• long double gnu cxx::airy bil (long double x)
template<typename _Tp >
    gnu cxx:: promote< Tp >:: type std::assoc laguerre (unsigned int n, unsigned int m, Tp x)

    float std::assoc_laguerref (unsigned int __n, unsigned int __m, float __x)

    long double std::assoc_laguerrel (unsigned int __n, unsigned int __m, long double __x)

template<typename_Tp>
    _gnu_cxx::__promote< _Tp >::__type std::assoc_legendre (unsigned int __l, unsigned int __m, _Tp __x)
• float std::assoc legendref (unsigned int I, unsigned int m, float x)

    long double std::assoc legendrel (unsigned int I, unsigned int m, long double x)

template<typename _Tp >
    gnu cxx:: promote num t< Tp > gnu cxx::bernoulli (unsigned int n)

    float gnu cxx::bernoullif (unsigned int n)

    long double __gnu_cxx::bernoullil (unsigned int __n)

    template<typename _Tpa , typename _Tpb >

   gnu cxx:: promote 2< Tpa, Tpb >:: type std::beta (Tpa a, Tpb b)

    float std::betaf (float a, float b)

    long double std::betal (long double __a, long double __b)

template<typename_Tp>
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::bincoef (unsigned int __n, unsigned int __k)
• float gnu cxx::bincoeff (unsigned int n, unsigned int k)

    long double gnu cxx::bincoefl (unsigned int n, unsigned int k)

template<typename</li>Tp >
  __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::chebyshev_t (unsigned int __n, _Tp __x)

    float gnu cxx::chebyshev tf (unsigned int n, float x)

    long double __gnu_cxx::chebyshev_tl (unsigned int __n, long double __x)

    template<typename</li>
    Tp >

    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::chebyshev_u (unsigned int __n, _Tp __x)

    float gnu cxx::chebyshev uf (unsigned int n, float x)

    long double __gnu_cxx::chebyshev_ul (unsigned int __n, long double __x)

template<typename _Tp >
    gnu cxx:: promote num t< Tp > gnu cxx::chebyshev v (unsigned int n, Tp x)

    float gnu cxx::chebyshev vf (unsigned int n, float x)

    long double gnu cxx::chebyshev vl (unsigned int n, long double x)

    template<typename</li>
    Tp >

   __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::chebyshev_w (unsigned int __n, _Tp __x)

    float gnu cxx::chebyshev wf (unsigned int n, float x)

    long double gnu cxx::chebyshev wl (unsigned int n, long double x)
```

```
template<typename _Tp >
   gnu cxx:: promote num t < Tp > gnu cxx::clausen (unsigned int m, Tp w)

    template<typename</li>
    Tp >

  std::complex< __gnu_cxx::__promote_num_t< _Tp >> __gnu_cxx::clausen (unsigned int __m, std::complex<
  _{\mathsf{Tp}} > _{\mathsf{w}}

    template<typename</li>
    Tp >

   _gnu_cxx::_ promote_num_t< _Tp > __gnu_cxx::clausen_c (unsigned int __m, _Tp __w)
• float <u>gnu_cxx::clausen_cf</u> (unsigned int <u>m</u>, float <u>w</u>)

    long double __gnu_cxx::clausen_cl (unsigned int __m, long double __w)

template<typename</li>Tp >
  __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::clausen_s (unsigned int __m, _Tp __w)

    float gnu cxx::clausen sf (unsigned int m, float w)

    long double __gnu_cxx::clausen_sl (unsigned int __m, long double __w)

    float __gnu_cxx::clausenf (unsigned int __m, float __w)

• std::complex < float > gnu cxx::clausenf (unsigned int m, std::complex < float > w)

    long double __gnu_cxx::clausenl (unsigned int __m, long double __w)

    std::complex < long double > __gnu_cxx::clausenl (unsigned int __m, std::complex < long double > __w)

template<typename _Tp >
    gnu cxx:: promote < Tp >:: type std::comp ellint 1 (Tp k)

    float std::comp ellint 1f (float k)

    long double std::comp ellint 11 (long double k)

template<typename_Tp>
    _gnu_cxx::__promote< _Tp >::__type std::comp_ellint_2 (_Tp __k)

    float std::comp ellint 2f (float k)

    long double std::comp ellint 2l (long double k)

    template<typename _Tp , typename _Tpn >

   _gnu_cxx::__promote_2< _Tp, _Tpn >::__type std::comp_ellint_3 (_Tp __k, _Tpn __nu)

    float std::comp ellint 3f (float k, float nu)

      Return the complete elliptic integral of the third kind \Pi(k,\nu) for float modulus k.

    long double std::comp ellint 3l (long double k, long double nu)

      Return the complete elliptic integral of the third kind \Pi(k,\nu) for long double modulus k.

    template<typename Tk >

    gnu cxx:: promote num t < Tk > gnu cxx::comp ellint d (Tk k)

    float gnu cxx::comp ellint df (float k)

    long double gnu cxx::comp ellint dl (long double k)

    float __gnu_cxx::comp_ellint_rf (float __x, float __y)

• long double gnu cxx::comp ellint rf (long double x, long double y)
• template<typename _{\rm Tx}, typename _{\rm Ty} >
   _gnu_cxx::__promote_num_t< _Tx, _Ty > __gnu_cxx::comp_ellint_rf (_Tx __x, _Ty __y)

    float __gnu_cxx::comp_ellint_rg (float __x, float __y)

    long double gnu cxx::comp ellint rg (long double x, long double y)

• template<typename _Tx , typename _Ty >
    _gnu_cxx::__promote_num_t< _Tx, _Ty > __gnu_cxx::comp_ellint_rg (_Tx __x, _Ty __y)
template<typename _Tpa , typename _Tpc , typename _Tp >
   _gnu_cxx::__promote_3< _Tpa, _Tpc, _Tp >::__type __gnu_cxx::conf_hyperg (_Tpa __a, _Tpc __c, _Tp __x)

    template<typename _Tpc , typename _Tp >

  __gnu_cxx::_promote_2< _Tpc, _Tp >::_type __gnu_cxx::conf_hyperg_lim (_Tpc __c, _Tp __x)

    float gnu cxx::conf hyperg limf (float c, float x)

    long double __gnu_cxx::conf_hyperg_liml (long double __c, long double __x)

    float __gnu_cxx::conf_hypergf (float __a, float __c, float __x)

    long double gnu cxx::conf hypergl (long double a, long double c, long double x)
```

```
template<typename _Tp >
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::coshint (_Tp __x)

    float gnu cxx::coshintf (float x)

    long double <u>gnu_cxx::coshintl</u> (long double <u>x</u>)

template<typename</li>Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::cosint (_Tp __x)

    float gnu cxx::cosintf (float x)

    long double gnu cxx::cosintl (long double x)

    template<typename _Tpnu , typename _Tp >

   _gnu_cxx::__promote_2< _Tpnu, _Tp >::__type std::cyl_bessel_i (_Tpnu __nu, _Tp __x)

    float std::cyl_bessel_if (float __nu, float __x)

    long double std::cyl bessel il (long double nu, long double x)

• template<typename Tpnu, typename Tp>
    _gnu_cxx::__promote_2< _Tpnu, _Tp >::__type std::cyl_bessel_j (_Tpnu __nu, _Tp __x)

    float std::cyl bessel if (float nu, float x)

    long double std::cyl_bessel_jl (long double __nu, long double __x)

• template<typename _Tpnu , typename _Tp >
    _gnu_cxx::__promote_2< _Tpnu, _Tp >::__type std::cyl_bessel_k (_Tpnu __nu, _Tp __x)

    float std::cyl bessel kf (float nu, float x)

    long double std::cyl_bessel_kl (long double __nu, long double __x)

• template<typename _Tpnu , typename _Tp >
  std::complex< __gnu_cxx::__promote_num_t< _Tpnu, _Tp >> __gnu_cxx::cyl_hankel_1 (_Tpnu __nu, _Tp
  __z)
• template<typename _{\rm Tpnu}, typename _{\rm Tp} >
  std::complex < \underline{gnu\_cxx::\_promote\_num\_t} < \underline{Tpnu, \_Tp} > \underline{gnu\_cxx::cyl\_hankel\_1} (std::complex < \underline{\leftarrow}
  Tpnu > __nu, std::complex< _Tp > __x)

    std::complex< float > __gnu_cxx::cyl_hankel_1f (float __nu, float __z)

    std::complex < float > __gnu_cxx::cyl_hankel_1f (std::complex < float > __nu, std::complex < float > __x)

• std::complex < long double > gnu cxx::cyl hankel 1l (long double nu, long double z)

    std::complex < long double > gnu cxx::cyl hankel 1l (std::complex < long double > nu, std::complex < long</li>

  double > x)
• template<typename _Tpnu , typename _Tp >
  std::complex< __gnu_cxx::_promote_num_t< _Tpnu, _Tp >> __gnu_cxx::cyl_hankel_2 (_Tpnu __nu, _Tp
  __z)
• template<typename _Tpnu , typename _Tp >
  std::complex < \underline{gnu\_cxx::\_promote\_num\_t < \underline{Tpnu}, \underline{Tp} > \underline{gnu\_cxx::cyl\_hankel\_2} (std::complex < \underline{\leftarrow}
  Tpnu > __nu, std::complex< _Tp > __x)

    std::complex< float > __gnu_cxx::cyl_hankel_2f (float __nu, float __z)

    std::complex < float > __nu, std::complex < float > __nu, std::complex < float > __x)

• std::complex < long double > gnu cxx::cyl hankel 2l (long double nu, long double z)

    std::complex < long double > gnu cxx::cyl hankel 2l (std::complex < long double > nu, std::complex < long</li>

  double > x)
• template<typename _Tpnu , typename _Tp >
   gnu cxx:: promote 2< Tpnu, Tp >:: type std::cyl neumann (Tpnu nu, Tp x)

    float std::cyl neumannf (float nu, float x)

    long double std::cyl_neumannl (long double __nu, long double __x)

template<typename _Tp >
   gnu cxx:: promote num t < Tp > gnu cxx::dawson (Tp x)

    float __gnu_cxx::dawsonf (float __x)

    long double <u>__gnu_cxx::dawsonl</u> (long double <u>__x</u>)

template<typename _Tp >
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::digamma (_Tp __z)
```

```
    float __gnu_cxx::digammaf (float __z)

    long double __gnu_cxx::digammal (long double __z)

template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::dilog (_Tp __x)

    float gnu cxx::dilogf (float x)

    long double __gnu_cxx::dilogl (long double __x)

• template<typename _Tp >
  _Tp __gnu_cxx::dirichlet_beta (_Tp __x)

    float gnu cxx::dirichlet betaf (float x)

    long double gnu cxx::dirichlet betal (long double x)

template<typename _Tp >
  Tp gnu cxx::dirichlet eta (Tp x)

    float gnu cxx::dirichlet etaf (float x)

    long double <u>__gnu_cxx::dirichlet_etal</u> (long double <u>__x)</u>

template<typename</li>Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::double_factorial (int __n)

    float gnu cxx::double factorialf (int n)

    long double __gnu_cxx::double_factoriall (int __n)

• template<typename _Tp , typename _Tpp >
   _gnu_cxx::__promote_2< _Tp, _Tpp >::__type std::ellint_1 (_Tp __k, _Tpp __phi)

    float std::ellint_1f (float __k, float __phi)

    long double std::ellint 11 (long double k, long double phi)

    template<typename _Tp , typename _Tpp >

    _gnu_cxx::__promote_2< _Tp, _Tpp >::__type std::ellint_2 (_Tp __k, _Tpp __phi)

    float std::ellint 2f (float k, float phi)

      Return the incomplete elliptic integral of the second kind E(k, \phi) for float argument.

    long double std::ellint 2l (long double k, long double phi)

      Return the incomplete elliptic integral of the second kind E(k, \phi).

    template<typename Tp, typename Tpn, typename Tpp>

   _gnu_cxx::__promote_3< _Tp, _Tpn, _Tpp >::__type std::ellint_3 (_Tp __k, _Tpn __nu, _Tpp __phi)
      Return the incomplete elliptic integral of the third kind \Pi(k, \nu, \phi).

    float std::ellint 3f (float k, float nu, float phi)

      Return the incomplete elliptic integral of the third kind \Pi(k, \nu, \phi) for float argument.

    long double std::ellint_3l (long double __k, long double __nu, long double __phi)

      Return the incomplete elliptic integral of the third kind \Pi(k, \nu, \phi).

    template<typename _Tk , typename _Tp , typename _Ta , typename _Tb >

    _gnu_cxx::__promote_num_t< _Tk, _Tp, _Ta, _Tb > <u>__gnu_cxx::ellint_cel</u> (_Tk <u>__k_c, _</u>Tp <u>__p, _</u>Ta <u>__a, _</u>Tb
   __b)

    float __gnu_cxx::ellint_celf (float __k_c, float __p, float _ a, float _ b)

    long double gnu cxx::ellint cell (long double k c, long double p, long double a, long double b)

• template<typename _Tk , typename _Tphi >
    _gnu_cxx::__promote_num_t< _Tk, _Tphi > __gnu_cxx::ellint_d (_Tk __k, _Tphi _ phi)

    float __gnu_cxx::ellint_df (float __k, float __phi)

    long double gnu cxx::ellint dl (long double k, long double phi)

    template<typename _Tp , typename _Tk >

    _gnu_cxx::__promote_num_t< _Tp, _Tk > __gnu_cxx::ellint_el1 (_Tp __x, _Tk __k_c)

    float __gnu_cxx::ellint_el1f (float __x, float __k_c)

    long double __gnu_cxx::ellint_el1l (long double __x, long double __k_c)

    template<typename _Tp , typename _Tk , typename _Ta , typename _Tb >

    _gnu_cxx::__promote_num_t< _Tp, _Tk, _Ta, _Tb > __gnu_cxx::ellint_el2 (_Tp __x, _Tk __k_c, _Ta __a, _Tb
  __b)
```

```
    float __gnu_cxx::ellint_el2f (float __x, float __k_c, float __a, float __b)

    long double __gnu_cxx::ellint_el2l (long double __x, long double __k_c, long double __a, long double __b)

• template<typename \_Tx, typename \_Tk, typename \_Tp>
    _gnu_cxx::__promote_num_t< _Tx, _Tk, _Tp > __gnu_cxx::ellint_el3 (_Tx __x, _Tk __k_c, _Tp __p)
• float gnu cxx::ellint el3f (float x, float k c, float p)

    long double gnu cxx::ellint el3l (long double x, long double b, c, long double p)

• template<typename Tp, typename Up>
    _gnu_cxx::__promote_num_t< _Tp, _Up > __gnu_cxx::ellint_rc (_Tp __x, _Up __y)

    float __gnu_cxx::ellint_rcf (float __x, float __y)

    long double __gnu_cxx::ellint_rcl (long double __x, long double __y)

• template<typename _Tp , typename _Up , typename _Vp >
    gnu cxx:: promote num t< Tp, Up, Vp > gnu cxx::ellint rd (Tp x, Up y, Vp z)

    float __gnu_cxx::ellint_rdf (float __x, float __y, float __z)

    long double gnu cxx::ellint rdl (long double x, long double y, long double z)

ullet template<typename _Tp , typename _Up , typename _Vp >
   _gnu_cxx::_promote_num_t< _Tp, _Up, _Vp > __gnu_cxx::ellint_rf (_Tp __x, _Up __y, _Vp __z)

    float __gnu_cxx::ellint_rff (float __x, float __y, float __z)

    long double gnu cxx::ellint rfl (long double x, long double y, long double z)

• template<typename _Tp , typename _Up , typename _Vp >
    _gnu_cxx::__promote_num_t< _Tp, _Up, _Vp > __gnu_cxx::ellint_rg (_Tp __x, _Up __y, _Vp __z)

    float __gnu_cxx::ellint_rgf (float __x, float __y, float __z)

    long double __gnu_cxx::ellint_rgl (long double __x, long double __y, long double __z)

template<typename _Tp , typename _Up , typename _Vp , typename _Wp >
   _gnu_cxx::__promote_num_t< _Tp, _Up, _Vp, _Wp > __gnu_cxx::ellint_rj (_Tp __x, _Up __y, _Vp __z, _Wp
  __p)
• float gnu cxx::ellint rif (float x, float y, float z, float p)

    long double __gnu_cxx::ellint_rjl (long double __x, long double __y, long double __z, long double __p)

template<typename _Tp >
  Tp gnu cxx::ellnome (Tp k)

    float gnu cxx::ellnomef (float k)

    long double gnu cxx::ellnomel (long double k)

template<typename</li>Tp >
   __gnu_cxx::__promote< _Tp >::__type std::expint (_Tp __x)
template<typename _Tp >
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::expint_e1 (_Tp __x)

    float gnu cxx::expint e1f (float x)

    long double gnu cxx::expint e1l (long double x)

template<typename</li>Tp >
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::expint_en (unsigned int __n, _Tp __x)
• float __gnu_cxx::expint_enf (unsigned int __n, float __x)

    long double __gnu_cxx::expint_enl (unsigned int __n, long double __x)

    float std::expintf (float x)

    long double std::expintl (long double x)

template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::factorial (unsigned int __n)

    float gnu cxx::factorialf (unsigned int n)

    long double gnu cxx::factoriall (unsigned int n)

template<typename _Tp >
   \_gnu_cxx::\_promote_num_t< _Tp > \_gnu_cxx::fresnel_c (_Tp \_x)

    float <u>gnu_cxx::fresnel_cf</u> (float <u>x</u>)

    long double gnu cxx::fresnel cl (long double x)
```

```
template<typename _Tp >
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::fresnel_s (_Tp __x)

    float gnu cxx::fresnel sf (float x)

    long double __gnu_cxx::fresnel_sl (long double __x)

• template<typename Tn, typename Tp>
    _gnu_cxx::__promote_num_t< _Tn, _Tp > __gnu_cxx::gamma_l (_Tn __n, _Tp __x)
• float gnu cxx::gamma If (float n, float x)

    long double __gnu_cxx::gamma_ll (long double __n, long double __x)

• template<typename Ta, typename Tp>
   _gnu_cxx::__promote_num_t< _Ta, _Tp > __gnu_cxx::gamma_p (_Ta __a, _Tp __x)

    float gnu cxx::gamma pf (float a, float x)

    long double gnu cxx::gamma pl (long double a, long double x)

• template<typename _Ta , typename _Tp >
    _gnu_cxx::__promote_num_t< _Ta, _Tp > __gnu_cxx::gamma_q (_Ta __a, _Tp __x)

    float gnu cxx::gamma qf (float a, float x)

    long double gnu cxx::gamma ql (long double a, long double x)

    template<typename _Tn , typename _Tp >

   _gnu_cxx::__promote_num_t< _Tn, _Tp > __gnu_cxx::gamma_u (_Tn __n, _Tp __x)

    float gnu cxx::gamma uf (float n, float x)

    long double gnu cxx::gamma ul (long double n, long double x)

    template<typename _Talpha , typename _Tp >

    _gnu_cxx::__promote_num_t< _Talpha, _Tp > __gnu_cxx::gegenbauer (unsigned int __n, _Talpha __alpha,
  Tp x)
• float gnu cxx::gegenbauerf (unsigned int n, float alpha, float x)

    long double __gnu_cxx::gegenbauerl (unsigned int __n, long double __alpha, long double __x)

template<typename _Tp >
    gnu cxx:: promote < Tp >:: type std::hermite (unsigned int n, Tp x)

    float std::hermitef (unsigned int n, float x)

    long double std::hermitel (unsigned int __n, long double __x)

• template<typename Tk, typename Tphi >
    _gnu_cxx::__promote_num_t< _Tk, _Tphi > __gnu_cxx::heuman_lambda (_Tk __k, _Tphi __phi)

    float gnu cxx::heuman lambdaf (float k, float phi)

    long double gnu cxx::heuman lambdal (long double k, long double phi)

• template<typename _Tp , typename _Up >
   _gnu_cxx::__promote_num_t< _Tp, _Up > __gnu_cxx::hurwitz_zeta (_Tp __s, _Up __a)
• template<typename Tp, typename Up>
  std::complex < _Tp > __gnu_cxx::hurwitz_zeta (_Tp __s, std::complex < _Up > __a)

    float __gnu_cxx::hurwitz_zetaf (float __s, float __a)

    long double __gnu_cxx::hurwitz_zetal (long double __s, long double __a)

• template<typename Tpa, typename Tpb, typename Tpc, typename Tp>
    _gnu_cxx::__promote_4< _Tpa, _Tpb, _Tpc, _Tp >::__type __gnu_cxx::hyperg (_Tpa __a, _Tpb __b, _Tpc
   __c, _Tp ___x)
• float gnu cxx::hypergf (float a, float b, float c, float x)
• long double gnu cxx::hypergl (long double a, long double b, long double c, long double x)
template<typename _Ta , typename _Tb , typename _Tp >
    gnu_cxx::__promote_num_t< _Ta, _Tb, _Tp > __gnu_cxx::ibeta (_Ta __a, _Tb __b, _Tp __x)
- template<typename _Ta , typename _Tb , typename _Tp >
  __gnu_cxx::_promote_num_t< _Ta, _Tb, _Tp > __gnu_cxx::ibetac (_Ta __a, _Tb __b, _Tp __x)
• float gnu cxx::ibetacf (float a, float b, float x)

    long double __gnu_cxx::ibetacl (long double __a, long double __b, long double __x)

• float gnu cxx::ibetaf (float a, float b, float x)

    long double gnu cxx::ibetal (long double a, long double b, long double x)
```

```
    template<typename _Talpha , typename _Tbeta , typename _Tp >

   _gnu_cxx::__promote_num_t< _Talpha, _Tbeta, _Tp > __gnu_cxx::jacobi (unsigned __n, _Talpha __alpha,
  Tbeta beta, Tp x)
• template<typename _Kp , typename _Up >
   _gnu_cxx::__promote_num_t< _Kp, _Up > __gnu_cxx::jacobi_cn (_Kp __k, _Up __u)
• float gnu cxx::jacobi cnf (float k, float u)

    long double gnu cxx::jacobi cnl (long double k, long double u)

• template<typename _Kp , typename _Up >
    _gnu_cxx::__promote_num_t< _Kp, _Up > __gnu_cxx::jacobi_dn (_Kp __k, _Up __u)
• float gnu cxx::jacobi dnf (float k, float u)

    long double __gnu_cxx::jacobi_dnl (long double __k, long double __u)

• template<typename _Kp , typename _Up >
    _gnu_cxx::__promote_num_t< _Kp, _Up > __gnu_cxx::jacobi_sn (_Kp __k, _Up __u)

    float gnu cxx::jacobi snf (float k, float u)

    long double __gnu_cxx::jacobi_snl (long double __k, long double __u)

• template<typename Tk, typename Tphi >
    _gnu_cxx::__promote_num_t< _Tk, _Tphi > __gnu_cxx::jacobi_zeta (_Tk __k, _Tphi __phi)

    float gnu cxx::jacobi zetaf (float k, float phi)

    long double __gnu_cxx::jacobi_zetal (long double __k, long double __phi)

    float gnu cxx::jacobif (unsigned n, float alpha, float beta, float x)

    long double __gnu_cxx::jacobil (unsigned __n, long double __alpha, long double __beta, long double __x)

template<typename _Tp >
   gnu cxx:: promote < Tp >:: type std::laguerre (unsigned int n, Tp x)

    float std::laguerref (unsigned int n, float x)

    long double std::laguerrel (unsigned int __n, long double __x)

template<typename _Tp >
   gnu cxx:: promote num t < Tp > gnu cxx::lbincoef (unsigned int n, unsigned int k)
• float gnu cxx::lbincoeff (unsigned int n, unsigned int k)

    long double __gnu_cxx::lbincoefl (unsigned int __n, unsigned int __k)

    template<typename</li>
    Tp >

    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::ldouble_factorial (int __n)

    float __gnu_cxx::ldouble_factorialf (int __n)

    long double __gnu_cxx::ldouble_factoriall (int __n)

template<typename_Tp>
    _gnu_cxx::__promote< _Tp >::__type std::legendre (unsigned int __l, _Tp __x)
template<typename _Tp >
   __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::legendre_q (unsigned int __n, Tp x)

    float gnu cxx::legendre qf (unsigned int n, float x)

    long double gnu cxx::legendre ql (unsigned int n, long double x)

    float std::legendref (unsigned int I, float x)

    long double std::legendrel (unsigned int I, long double x)

template<typename_Tp>
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::lfactorial (unsigned int __n)

    float gnu cxx::lfactorialf (unsigned int

    long double gnu cxx::lfactoriall (unsigned int n)

template<typename_Tp>
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::logint (_Tp __x)

    float gnu cxx::logintf (float x)

    long double gnu cxx::logintl (long double x)

• template<typename _Tp , typename _Tn >
    _gnu_cxx::__promote_num_t< _Tp, _Tn > __gnu_cxx::lpochhammer_l (_Tp __a, _Tn __n)

    float gnu cxx::lpochhammer lf (float a, float n)
```

```
    long double __gnu_cxx::lpochhammer_ll (long double __a, long double __n)

• template<typename _Tp , typename _Tn >
    _gnu_cxx::__promote_num_t< _Tp, _Tn > __gnu_cxx::lpochhammer_u (_Tp __a, _Tn __n)

    float __gnu_cxx::lpochhammer_uf (float __a, float __n)

• long double gnu cxx::lpochhammer ul (long double a, long double n)
• template<typename _Tph , typename _Tpa >
    _gnu_cxx::__promote_num_t< _Tph, _Tpa > __gnu_cxx::owens_t (_Tph __h, _Tpa __a)

    float __gnu_cxx::owens_tf (float __h, float __a)

• long double gnu cxx::owens tl (long double h, long double a)
• template<typename _Tp , typename _Tn >
    _gnu_cxx::__promote_num_t< _Tp, _Tn > __gnu_cxx::pochhammer_l (_Tp __a, _Tn __n)

    float gnu cxx::pochhammer lf (float a, float n)

• long double __gnu_cxx::pochhammer_ll (long double __a, long double __n)
• template<typename _Tp , typename _Tn >
    _gnu_cxx::__promote_num_t< _Tp, _Tn > __gnu_cxx::pochhammer_u (_Tp __a, _Tn _ n)

    float gnu cxx::pochhammer uf (float a, float n)

    long double __gnu_cxx::pochhammer_ul (long double __a, long double __n)

template<typename _Tp >
  std::complex< __gnu_cxx::_promote_num_t< _Tp >> __gnu_cxx::polylog (_Tp __s, std::complex< _Tp >

    std::complex < float > __gnu_cxx::polylogf (float __s, std::complex < float > __w)

    std::complex < long double > __gnu_cxx::polylogl (long double __s, std::complex < long double > __w)

template<typename Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::psi (_Tp __x)

 float __gnu_cxx::psif (float __x)

    long double __gnu_cxx::psil (long double __x)

    template<typename</li>
    Tp >

  gnu_cxx::_ promote_num_t< _Tp > __gnu_cxx::radpoly (unsigned int __n, unsigned int __m, _Tp __rho)

    float __gnu_cxx::radpolyf (unsigned int __n, unsigned int __m, float __rho)

    long double __gnu_cxx::radpolyl (unsigned int __n, unsigned int __m, long double __rho)

template<typename Tp >
   __gnu_cxx::__promote< _Tp >::__type std::riemann_zeta (_Tp __s)

    float std::riemann_zetaf (float __s)

    long double std::riemann zetal (long double s)

template<typename_Tp>
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::sinc (_Tp __x)
template<typename_Tp>
   _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::sinc_pi (_Tp __x)

    float gnu cxx::sinc pif (float x)

    long double gnu cxx::sinc pil (long double x)

    float gnu cxx::sincf (float x)

    long double gnu cxx::sincl (long double x)

template<typename _Tp >
    gnu cxx:: promote num t< Tp> gnu cxx::sinhc (Tpx)
template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::sinhc_pi (_Tp __x)

    float __gnu_cxx::sinhc_pif (float __x)

    long double gnu cxx::sinhc pil (long double x)

    float gnu cxx::sinhcf (float x)

    long double <u>__gnu_cxx::sinhcl</u> (long double <u>__x</u>)

template<typename_Tp>
  __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::sinhint (_Tp __x)
```

```
    float __gnu_cxx::sinhintf (float __x)

    long double __gnu_cxx::sinhintl (long double __x)

template<typename _Tp >
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::sinint (_Tp __x)

    float gnu cxx::sinintf (float x)

    long double <u>gnu_cxx::sinintl</u> (long double <u>x</u>)

template<typename _Tp >
   _gnu_cxx::__promote< _Tp >::__type std::sph_bessel (unsigned int __n, _Tp __x)
template<typename _Tp >
   __gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::sph_bessel_i (unsigned int __n, _Tp __x)

    float gnu cxx::sph bessel if (unsigned int n, float x)

    long double gnu cxx::sph bessel il (unsigned int n, long double x)

template<typename_Tp>
    _gnu_cxx::__promote_num_t< _Tp > __gnu_cxx::sph_bessel_k (unsigned int __n, _Tp __x)

    float gnu cxx::sph bessel kf (unsigned int n, float x)

    long double gnu cxx::sph bessel kl (unsigned int n, long double x)

    float std::sph besself (unsigned int n, float x)

    long double std::sph_bessell (unsigned int __n, long double __x)

template<typename</li>Tp >
  std::complex< __gnu_cxx::__promote_num_t< _Tp >> __gnu_cxx::sph_hankel_1 (unsigned int __n, _Tp __z)

    template<typename</li>
    Tp >

  std::complex< __gnu_cxx::_promote_num_t< _Tp >> __gnu_cxx::sph_hankel_1 (unsigned int __n, std↔
  ::complex< _Tp> __x)

    std::complex < float > gnu cxx::sph hankel 1f (unsigned int n, float z)

    std::complex < float > __gnu_cxx::sph_hankel_1f (unsigned int __n, std::complex < float > __x)

    std::complex < long double > gnu cxx::sph hankel 1l (unsigned int n, long double z)

    std::complex < long double > __gnu_cxx::sph_hankel_1I (unsigned int __n, std::complex < long double > __x)

template<typename_Tp>
  std::complex< gnu cxx:: promote num t< Tp>> gnu cxx::sph hankel 2 (unsigned int n, Tp z)
template<typename _Tp >
  std::complex< __gnu_cxx::_promote_num_t< _Tp >> __gnu_cxx::sph_hankel_2 (unsigned int __n, std↔
  ::complex < Tp > x)

    std::complex< float > gnu cxx::sph hankel 2f (unsigned int n, float z)

    std::complex < float > gnu cxx::sph hankel 2f (unsigned int n, std::complex < float > x)

    std::complex < long double > __gnu_cxx::sph_hankel_2l (unsigned int __n, long double __z)

• std::complex < long double > gnu cxx::sph hankel 2l (unsigned int n, std::complex < long double > x)
• template<typename Ttheta, typename Tphi >
  std::complex< __gnu_cxx::__promote_num_t< _Ttheta, _Tphi >> __gnu_cxx::sph_harmonic (unsigned int ←
  __l, int __m, _Ttheta __theta, _Tphi __phi)
• std::complex < float > gnu cxx::sph harmonicf (unsigned int I, int m, float theta, float phi)

    std::complex < long double > __gnu_cxx::sph_harmonicl (unsigned int __l, int __m, long double __theta, long

  double __phi)
template<typename _Tp >
   gnu cxx:: promote < Tp >:: type std::sph legendre (unsigned int I, unsigned int m, Tp theta)

    float std::sph legendref (unsigned int I, unsigned int m, float theta)

    long double std::sph_legendrel (unsigned int __l, unsigned int __m, long double __theta)

template<typename _Tp >
   gnu cxx:: promote < Tp >:: type std::sph neumann (unsigned int n, Tp x)

    float std::sph neumannf (unsigned int n, float x)

    long double std::sph_neumannl (unsigned int __n, long double __x)

    template<typename _Tpnu , typename _Tp >

   __gnu_cxx::__promote_num_t< _Tpnu, _Tp > __gnu_cxx::theta_1 (_Tpnu __nu, _Tp __x)
```

```
    float __gnu_cxx::theta_1f (float __nu, float __x)

    long double gnu cxx::theta 1l (long double nu, long double x)

• template<typename _Tpnu , typename _Tp >
   _gnu_cxx::__promote_num_t< _Tpnu, _Tp > __gnu_cxx::theta_2 (_Tpnu __nu, _Tp __x)
• float gnu cxx::theta 2f (float nu, float x)

    long double gnu cxx::theta 2l (long double nu, long double x)

• template<typename _Tpnu , typename _Tp >
    _gnu_cxx::__promote_num_t< _Tpnu, _Tp > __gnu_cxx::theta_3 (_Tpnu __nu, _Tp __x)
• float gnu cxx::theta 3f (float nu, float x)

    long double __gnu_cxx::theta_3l (long double __nu, long double __x)

• template<typename _Tpnu , typename _Tp >
   gnu cxx:: promote num t< Tpnu, Tp > gnu cxx::theta 4 ( Tpnu nu, Tp x)
• float gnu cxx::theta 4f (float nu, float x)

    long double __gnu_cxx::theta_4l (long double __nu, long double __x)

• template<typename _Tpk , typename _Tp >
    gnu cxx:: promote num t < Tpk, Tp > gnu cxx::theta c ( Tpk k, Tp x)

    float __gnu_cxx::theta_cf (float __k, float __x)

    long double __gnu_cxx::theta_cl (long double __k, long double __x)

    template<typename Tpk, typename Tp >

    _gnu_cxx::__promote_num_t< _Tpk, _Tp > __gnu_cxx::theta_d (_Tpk __k, _Tp __x)

    float __gnu_cxx::theta_df (float __k, float __x)

    long double __gnu_cxx::theta_dl (long double __k, long double __x)

    template<typename Tpk, typename Tp >

    _gnu_cxx::__promote_num_t< _Tpk, _Tp > __gnu_cxx::theta_n (_Tpk __k, _Tp __x)

    float __gnu_cxx::theta_nf (float __k, float __x)

    long double __gnu_cxx::theta_nl (long double __k, long double __x)

    template<typename Tpk, typename Tp >

  __gnu_cxx::__promote_num_t< _Tpk, _Tp > __gnu_cxx::theta_s (_Tpk __k, _Tp __x)

    float __gnu_cxx::theta_sf (float __k, float __x)

• long double __gnu_cxx::theta_sl (long double __k, long double __x)
• template<typename Trho, typename Tphi >
    _gnu_cxx::__promote_num_t< _Trho, _Tphi > __gnu_cxx::zernike (unsigned int __n, int __m, _Trho __rho,
  Tphi phi)

    float gnu cxx::zernikef (unsigned int n, int m, float rho, float phi)

    long double gnu cxx::zernikel (unsigned int n, int m, long double rho, long double phi)
```

10.27.1 Detailed Description

This is an internal header file, included by other library headers. You should not attempt to use it directly. Instead, include <cmath>.

10.27.2 Macro Definition Documentation

10.27.2.1 #define __cpp_lib_math_special_functions 201603L

Definition at line 39 of file specfun.h.

10.27.2.2 #define __STDCPP_MATH_SPEC_FUNCS__ 201003L

Definition at line 37 of file specfun.h.

Index

_GLIBCXX_BITS_SF_AIRY_TCC	sf_dawson.tcc, 224
sf_airy.tcc, 215	_GLIBCXX_SF_FRESNEL_TCC
_GLIBCXX_BITS_SF_BESSEL_TCC	sf_fresnel.tcc, 230
sf_bessel.tcc, 217	_GLIBCXX_SF_GEGENBAUER_TCC
_GLIBCXX_BITS_SF_BETA_TCC	sf_gegenbauer.tcc, 237
sf_beta.tcc, 219	_GLIBCXX_SF_HYPINT_TCC
_GLIBCXX_BITS_SF_CARDINAL_TCC	sf_hypint.tcc, 246
sf_cardinal.tcc, 221	_GLIBCXX_SF_JACOBI_TCC
_GLIBCXX_BITS_SF_ELLINT_TCC	sf_jacobi.tcc, 248
sf_ellint.tcc, 226	_GLIBCXX_SF_THETA_TCC
_GLIBCXX_BITS_SF_EXPINT_TCC	sf_theta.tcc, 259
sf_expint.tcc, 229	_GLIBCXX_SF_TRIGINT_TCC
_GLIBCXX_BITS_SF_GAMMA_TCC	sf_trigint.tcc, 261
sf_gamma.tcc, 236	_Num_Euler_Maclaurin_zeta
_GLIBCXX_BITS_SF_HANKEL_NEW_TCC	std::detail, 207
sf_hankel_new.tcc, 240	_S_Euler_Maclaurin_zeta
_GLIBCXX_BITS_SF_HANKEL_TCC	std:: detail, 207
sf hankel.tcc, 240	_S_double_factorial_table
GLIBCXX BITS SF HERMITE TCC	std:: detail, 207
sf hermite.tcc, 242	S factorial table
GLIBCXX BITS SF HYDROGEN TCC	std::detail, 207
sf_hydrogen.tcc, 243	_S_neg_double_factorial_table
_GLIBCXX_BITS_SF_HYPERG_TCC	std::detail, 207
sf hyperg.tcc, 245	_S_num_double_factorials
_GLIBCXX_BITS_SF_LAGUERRE_TCC	std::detail, 208
sf_laguerre.tcc, 250	_S_num_double_factorials< double >
_GLIBCXX_BITS_SF_LEGENDRE_TCC	std::detail, 208
sf_legendre.tcc, 251	_S_num_double_factorials< float >
_GLIBCXX_BITS_SF_MOD_BESSEL_TCC	std::detail, 208
sf_mod_bessel.tcc, 253	_S_num_double_factorials< long double >
_GLIBCXX_BITS_SF_OWENS_T_TCC	std::detail, 208
sf_owens_t.tcc, 254	_S_num_factorials
_GLIBCXX_BITS_SF_POLYLOG_TCC	std::detail, 208
sf_polylog.tcc, 257	_S_num_factorials< double >
_GLIBCXX_BITS_SF_ZETA_TCC	std::detail, 208
sf_zeta.tcc, 263	_S_num_factorials< float >
_GLIBCXX_JACOBI_CN	std::detail, 208
GNU Extended Mathematical Special Functions, 46	_S_num_factorials< long double >
_GLIBCXX_JACOBI_DN	std::detail, 208
GNU Extended Mathematical Special Functions, 46	_S_num_neg_double_factorials
_GLIBCXX_JACOBI_SN	std:: detail, 208
GNU Extended Mathematical Special Functions, 46	_S_num_neg_double_factorials< double >
_GLIBCXX_SF_CHEBYSHEV_TCC	std:: detail, 208
sf_chebyshev.tcc, 222	_S_num_neg_double_factorials< float >
GLIBCXX SF DAWSON TCC	std:: detail, 208

_S_num_neg_double_factorials< long double >	bose_einstein
std::detail, 209	std::detail, 134
_S_num_zetam1	chebyshev_recur
std::detail, 209	std::detail, 134
S_zetam1	chebyshev_t
std::detail, 209	std::detail, 134
STDCPP_MATH_SPEC_FUNCS	chebyshev_u
specfun.h, 274	std::detail, 134
airy	chebyshev_v
std:: detail, 123	std:: detail, 134
airy_ai	chebyshev_w
std::detail, 125	std::detail, 134
airy_arg	chi_squared_pdf
std::detail, 125	std::detail, 135
airy_asymp_absarg_ge_pio3	chi_squared_pdfc
std::detail, 125	std::detail, 135
airy_asymp_absarg_lt_pio3	chshint
	
std::detail, 126	std::detail, 135
airy_bessel_i	chshint_cont_frac
std::detail, 126	std::detail, 135
airy_bessel_k	chshint_series
std::detail, 127	std::detail, 136
airy_bi	clamp_0_m2pi
std::detail, 128	std::detail, 136
airy_hyperg_rational	clamp_pi
std::detail, 128	std::detail, 136
assoc_laguerre	clausen
std::detail, 129	std::detail, 136
assoc_legendre_p	clausen_c
std::detail, 129	std::detail, 137
bernoulli	clausen_s
std::detail, 129	std::detail, 137, 138
bernoulli_2n	comp_ellint_1
std::detail, 130	std::detail, 138
bernoulli_series	comp_ellint_2
std::detail, 130	std::detail, 139
beta	comp_ellint_3
std::detail, 130	std::detail, 139
beta_gamma	comp_ellint_d
std::detail, 131	std::detail, 139
beta_inc	comp_ellint_rf
std::detail, 131	std::detail, 140
beta_inc_cont_frac	comp_ellint_rg
std::detail, 132	std::detail, 140
beta_lgamma	conf hyperg
std::detail, 132	std::detail, 140
beta_product	conf_hyperg_lim
std::detail, 132	std::detail, 140
bincoef	conf_hyperg_lim_series
std::detail, 133	std::detail, 140
binomial cdf	conf_hyperg_luke
std::detail, 133	std::detail, 141
binomial cdfc	conf_hyperg_series
-	
std::detail, 133	std::detail, 141

	W + 0
coshint	ellint_3
std::detail, 141	std::detail, 152
cpp_lib_math_special_functions	ellint_cel
specfun.h, 274	std::detail, 152
cyl_bessel	ellint_d
std::detail, 142	std::detail, 153
cyl_bessel_i std::detail, 142	ellint_el1
	std::detail, 153 ellint_el2
cyl_bessel_ij_series std::detail, 143	std:: detail, 153
cyl_bessel_ik	ellint el3
cyi_bessei_ik std:: detail, 143	std::detail, 153
cyl_bessel_ik_asymp	ellint rc
std::detail, 144	std::detail, 153
cyl_bessel_ik_steed	ellint rd
std:: detail, 144	std::detail, 154
cyl bessel j	ellint rf
std::_detail, 144	std::detail, 154
cyl bessel jn	ellint_rg
std::detail, 145	std::detail, 155
cyl_bessel_jn_asymp	ellint_rj
std::detail, 145	std::detail, 155
cyl_bessel_in_steed	ellnome
std:: detail, 145	std:: detail, 156
cyl_bessel_k	ellnome k
std::detail, 146	std::detail, 156
cyl_hankel_1	ellnome_series
std::detail, 146, 147	std:: detail, 156
cyl_hankel_2	expint
std::detail, 147	std::detail, 156, 157
cyl_neumann	expint_E1
std::detail, 148	std::detail, 157
cyl_neumann_n	expint_E1_asymp
std:: detail, 148	std:: detail, 158
dawson	expint_E1_series
std::detail, 148	std::detail, 158
dawson_const_frac	expint_Ei
std:: detail, 149	std:: detail, 159
dawson_series	expint_Ei_asymp
std:: detail, 149	std::detail, 159
debye region	expint_Ei_series
std:: detail, 149	std::detail, 159
dilog	expint_En_cont_frac
std::detail, 149	std::detail, 160
dirichlet beta	expint_En_recursion
std::detail, 150	std:: detail, 160
dirichlet eta	expint En series
std::detail, 150, 151	std::detail, 161
double factorial	expint_asymp
std::detail, 151	std::detail, 157
ellint_1	expint_large_n
 std::detail, 151	std::detail, 161
ellint_2	factorial
 std::detail, 152	 std::detail, 161

std::detail::_Factorial_table, 211	hankel_uniform_sum
fermi_dirac	std::detail, 169
std::detail, 162	heuman_lambda
fisher_f_cdf	std::detail, 170
std::detail, 162	hurwitz_zeta
fisher_f_cdfc	std::detail, 170, 171
std::detail, 162	hurwitz_zeta_euler_maclaurin
fock_airy	std::detail, 171
std::detail, 163	hydrogen
fpequal	std::detail, 171
std::detail, 163	hyperg
fpimag	std::detail, 171
std::detail, 163, 164	hyperg_luke
fpreal	std::detail, 172
std::detail, 164	hyperg_reflect
fresnel	std::detail, 172
std:: detail, 164	hyperg series
fresnel_cont_frac	std:: detail, 172
std:: detail, 165	jacobi_sncndn
fresnel series	std::detail, 173
std::detail, 165	jacobi_zeta
gamma	std::detail, 173
std::detail, 165	laguerre
gamma_cont_frac	std:: detail, 173
std::detail, 165	_log_bincoef
gamma_I	std::detail, 174
std::detail, 165	log_double_factorial
gamma_p	std::detail, 174
std::detail, 166	log_factorial
gamma q	std:: detail, 174
std::detail, 166	std::detail::_Factorial_table, 211
gamma_series	log gamma
std::detail, 166	std::detail, 175
	log gamma bernoulli
gamma_temme	std:: detail, 175
std::detail, 166	
gamma_u std::detail, 167	log_gamma_lanczos std::detail, 175
gauss std:: detail, 167	log_gamma_sign
gegenbauer poly	std::detail, 176
	log_gamma_spouge
std::detail, 167 gnu cxx, 95	std::detail, 176
——• — ·	log_pochhammer_l
hankel	std::detail, 177
std::detail, 167	log_pochhammer_u std:: detail, 177
hankel_debye	
std::detail, 168	logint
hankel_params	std::detail, 177
std::detail, 168	n
hankel_uniform	std::detail::_Factorial_table, 211
std::detail, 168	owens_t
hankel_uniform_olver	std::detail, 178
std::detail, 169	pochhammer_l
hankel_uniform_outer	std::detail, 178
std::detail, 169	pochhammer_u

std::detail, 178	std::detail, 193
poly_hermite	riemann_zeta_alt
std:: detail, 179	std:: detail, 193
poly_hermite_asymp	riemann_zeta_euler_maclaurir
std::detail, 179	std::detail, 193
poly_hermite_recursion	riemann_zeta_glob
std::detail, 179	std:: detail, 194
poly_jacobi	riemann_zeta_m_1
std::detail, 180	std::detail, 194
poly_laguerre	riemann zeta m 1 sum
std::detail, 180	std:: detail, 194
poly laguerre hyperg	riemann zeta product
	std::detail, 194
std::detail, 181	
poly_laguerre_large_n	riemann_zeta_sum
std::detail, 181	std::detail, 195
poly_laguerre_recursion	sinc
std::detail, 182	std::detail, 195
poly_legendre_p	sinc_pi
std::detail, 182	std::detail, 195
poly_legendre_q	sincosint
std::detail, 183	std::detail, 196
poly_radial_jacobi	sincosint_asymp
std::detail, 183	std::detail, 196
polylog	sincosint_cont_frac
std::detail, 183	std::detail, 196
polylog_exp	sincosint_series
std::detail, 184	std::detail, 196
polylog_exp_asymp	sinhc
std::detail, 184	std::detail, 196, 197
polylog_exp_int_neg	sinhc_pi
std:: detail, 185	std::detail, 197
polylog_exp_int_pos	sinhint
std::detail, 185, 186	std::detail, 197
polylog_exp_neg	sph_bessel
std:: detail, 186	std::detail, 197, 198
polylog_exp_neg_even	sph_bessel_ik
std:: detail, 187	std:: detail, 198
polylog_exp_neg_odd	sph bessel in
std::detail, 187	std::detail, 198
polylog_exp_negative_real_part	sph_hankel
std::detail, 188	std::detail, 199
polylog_exp_pos	sph_hankel_1
std::detail, 189, 190	std::detail, 199
polylog_exp_real_neg	sph_hankel_2
std::detail, 190, 191	std:: detail, 201
	sph_harmonic
polylog_exp_real_pos std:: detail, 191	std:: detail, 201
	
psi	sph_legendre
std::detail, 192	std::detail, 202
psi_asymp	sph_neumann
std::detail, 192	std::detail, 202, 203
psi_series	student_t_cdf
std::detail, 192	std::detail, 203
riemann_zeta	student_t_cdfc

std::detail, 203	assoc_legendrel
theta_1	C++17/IS29124 Mathematical Special Functions, 20
std::detail, 204	
theta_2	bernoulli
std::detail, 204	GNU Extended Mathematical Special Functions, 47
theta_2_asymp	bernoullif
std::detail, 204	GNU Extended Mathematical Special Functions, 47
theta_2_sum	bernoullil
std::detail, 204	GNU Extended Mathematical Special Functions, 47
theta_3	beta
std::detail, 205	C++17/IS29124 Mathematical Special Functions, 20
theta_3_asymp	betaf
std::detail, 205	C++17/IS29124 Mathematical Special Functions, 20
theta_3_sum	betal
std::detail, 205	C++17/IS29124 Mathematical Special Functions, 21
theta_4	CNU Futended Methometical Special Functions 47
std::detail, 205	GNU Extended Mathematical Special Functions, 47
theta_c	CNUL Extended Mathematical Special Functions 47
std::detail, 206	GNU Extended Mathematical Special Functions, 47
theta_d	CNUL Extended Mathematical Special Functions 47
std::detail, 206	GNU Extended Mathematical Special Functions, 47 bits/sf airy.tcc, 213
theta_n	bits/sf_bessel.tcc, 215
std::detail, 206	bits/sf_beta.tcc, 217
theta_s	bits/sf_cardinal.tcc, 219
std::detail, 206	bits/sf_chebyshev.tcc, 221
zernike	bits/sf_dawson.tcc, 223
std::detail, 206	bits/sf_ellint.tcc, 224
znorm1	bits/sf_expint.tcc, 227
std::detail, 206	bits/sf_fresnel.tcc, 229
znorm2	bits/sf_gamma.tcc, 230
std::detail, 206	bits/sf_gegenbauer.tcc, 236
	bits/sf_hankel.tcc, 237
airy_ai	bits/sf_hankel_new.tcc, 240
GNU Extended Mathematical Special Functions, 46	bits/sf_hermite.tcc, 241
airy_aif	bits/sf_hydrogen.tcc, 242
GNU Extended Mathematical Special Functions, 46	bits/sf_hyperg.tcc, 243
airy_ail	bits/sf_hypint.tcc, 245
GNU Extended Mathematical Special Functions, 46	bits/sf_jacobi.tcc, 247
airy_bi	bits/sf_laguerre.tcc, 248
GNU Extended Mathematical Special Functions, 46	bits/sf_legendre.tcc, 250
airy_bif GNU Extended Mathematical Special Functions, 46	bits/sf_mod_bessel.tcc, 251
•	bits/sf_owens_t.tcc, 254
airy_bil CNU Extended Mathematical Special Functions 47	bits/sf_polylog.tcc, 255
GNU Extended Mathematical Special Functions, 47	bits/sf_theta.tcc, 257
assoc_laguerre	bits/sf_trigint.tcc, 260
C++17/IS29124 Mathematical Special Functions, 18	bits/sf_zeta.tcc, 261
assoc_laguerref C++17/IS29124 Mathematical Special Functions, 19	bits/specfun.h, 264
assoc_laguerrel	C++ Mathematical Special Functions, 15
C++17/IS29124 Mathematical Special Functions, 19	C++17/IS29124 Mathematical Special Functions, 16
assoc_legendre	assoc_laguerre, 18
C++17/IS29124 Mathematical Special Functions, 19	assoc_laguerref, 19
assoc_legendref	assoc_laguerrel, 19
C++17/IS29124 Mathematical Special Functions, 20	assoc_legendre, 19
5 17/102012 1 Mathematical openial 1 anotherio, 20	2000_10g011d10, 10

assoc_legendref, 20	sph_legendref, 36
assoc_legendrel, 20	sph_legendrel, 36
beta, 20	sph_neumann, 36
betaf, 20	sph_neumannf, 37
betal, 21	sph_neumannl, 37
comp_ellint_1, 21	COSINT
comp_ellint_1f, 21	std::detail, 122
comp_ellint_1I, 21	chebyshev_t
comp_ellint_2, 22	GNU Extended Mathematical Special Functions, 47
comp_ellint_2f, 22	chebyshev_tf
comp_ellint_2l, 22	GNU Extended Mathematical Special Functions, 48
comp_ellint_3, 22	chebyshev_tl
comp_ellint_3f, 23	GNU Extended Mathematical Special Functions, 48
comp_ellint_3l, 23	chebyshev_u
cyl_bessel_i, 23	GNU Extended Mathematical Special Functions, 48
cyl_bessel_if, 24	chebyshev_uf
cyl_bessel_il, 24	GNU Extended Mathematical Special Functions, 48
cyl_bessel_j, 24	chebyshev_ul
cyl_bessel_jf, 25	GNU Extended Mathematical Special Functions, 48
cyl_bessel_jl, 25	chebyshev_v
cyl_bessel_k, 25	GNU Extended Mathematical Special Functions, 49
cyl_bessel_kf, 26	chebyshev_vf
cyl_bessel_kl, 26	GNU Extended Mathematical Special Functions, 49
cyl neumann, 26	chebyshev_vl
cyl_neumannf, 27	GNU Extended Mathematical Special Functions, 49
cyl_neumannl, 27	chebyshev_w
ellint_1, 27	GNU Extended Mathematical Special Functions, 49
ellint_1f, 28	chebyshev_wf
ellint_1I, 28	GNU Extended Mathematical Special Functions, 50
ellint_2, 28	chebyshev_wl
ellint_2f, 29	GNU Extended Mathematical Special Functions, 50
ellint_2l, 29	clausen
ellint_3, 29	GNU Extended Mathematical Special Functions, 50
ellint_3f, 30	clausen_c
ellint_3I, 30	GNU Extended Mathematical Special Functions, 50
expint, 30	clausen_cf
expintf, 31	GNU Extended Mathematical Special Functions, 51
expintl, 31	clausen_cl
hermite, 31	GNU Extended Mathematical Special Functions, 51
hermitef, 32	clausen_s
hermitel, 32	GNU Extended Mathematical Special Functions, 51
laguerre, 32	clausen_sf
laguerref, 32	GNU Extended Mathematical Special Functions, 51
laguerrel, 33	clausen_sl
legendre, 33	GNU Extended Mathematical Special Functions, 51
legendref, 33	clausenf
legendrel, 33	GNU Extended Mathematical Special Functions, 52
riemann zeta, 34	clausenl
riemann_zetaf, 34	GNU Extended Mathematical Special Functions, 52
riemann_zetal, 34	comp_ellint_1
sph_bessel, 34	C++17/IS29124 Mathematical Special Functions, 21
sph_besself, 35	comp_ellint_1f
sph bessell, 35	C++17/IS29124 Mathematical Special Functions, 21
sph_legendre, 35	comp_ellint_1I
	00.11P_011111_11

- C++17/IS29124 Mathematical Special Functions, 21 comp ellint 2 C++17/IS29124 Mathematical Special Functions, 22 comp ellint 2f C++17/IS29124 Mathematical Special Functions, 22 comp ellint 2l C++17/IS29124 Mathematical Special Functions, 22 comp ellint 3 C++17/IS29124 Mathematical Special Functions, 22 comp ellint 3f C++17/IS29124 Mathematical Special Functions, 23 comp ellint 31 C++17/IS29124 Mathematical Special Functions, 23 comp ellint d GNU Extended Mathematical Special Functions, 52 comp ellint df GNU Extended Mathematical Special Functions, 53 comp ellint dl GNU Extended Mathematical Special Functions, 53 comp ellint rf GNU Extended Mathematical Special Functions, 53 comp ellint rg GNU Extended Mathematical Special Functions, 54 conf hyperg GNU Extended Mathematical Special Functions, 54 conf hyperg lim GNU Extended Mathematical Special Functions, 55 conf hyperg limf GNU Extended Mathematical Special Functions, 55 conf_hyperg_liml GNU Extended Mathematical Special Functions, 55 conf hypergf
- GNU Extended Mathematical Special Functions, 55

 conf_hypergl
 GNU Extended Mathematical Special Functions, 56

 coshint
 GNU Extended Mathematical Special Functions, 56

 coshintf
 GNU Extended Mathematical Special Functions, 56

 coshintl
 GNU Extended Mathematical Special Functions, 56

 coshintl
 GNU Extended Mathematical Special Functions, 56

 cosint

cosintf
GNU Extended Mathematical Special Functions, 57
cosintl
GNU Extended Mathematical Special Functions 57

GNU Extended Mathematical Special Functions, 56

GNU Extended Mathematical Special Functions, 57 cyl_bessel_i

C++17/IS29124 Mathematical Special Functions, 23 cyl_bessel_if

C++17/IS29124 Mathematical Special Functions, 24 cyl_bessel_il

C++17/IS29124 Mathematical Special Functions, 24 cyl bessel j

- C++17/IS29124 Mathematical Special Functions, 24 cyl_bessel_if
- C++17/IS29124 Mathematical Special Functions, 25 cyl_bessel_jl
- C++17/IS29124 Mathematical Special Functions, 25 cyl bessel k
- C++17/IS29124 Mathematical Special Functions, 25 cyl bessel kf
- C++17/IS29124 Mathematical Special Functions, 26 cyl bessel kl
- C++17/IS29124 Mathematical Special Functions, 26 cyl hankel 1
- GNU Extended Mathematical Special Functions, 57 cyl hankel 1f
- GNU Extended Mathematical Special Functions, 57 cyl hankel 1l
- GNU Extended Mathematical Special Functions, 58 cyl hankel 2
- GNU Extended Mathematical Special Functions, 58 cyl_hankel_2f
- GNU Extended Mathematical Special Functions, 58 cyl_hankel_2l
- GNU Extended Mathematical Special Functions, 59 cyl neumann
- C++17/IS29124 Mathematical Special Functions, 26 cyl neumannf
- C++17/IS29124 Mathematical Special Functions, 27 cyl_neumannl
 - C++17/IS29124 Mathematical Special Functions, 27

dawson

- GNU Extended Mathematical Special Functions, 59 dawsonf
- GNU Extended Mathematical Special Functions, 59 dawsonl
- GNU Extended Mathematical Special Functions, 59 digamma
- GNU Extended Mathematical Special Functions, 60 digammaf
- GNU Extended Mathematical Special Functions, 60 digammal
- GNU Extended Mathematical Special Functions, 60 dilog
- GNU Extended Mathematical Special Functions, 60 dilogf
- GNU Extended Mathematical Special Functions, 60 dilogl
- GNU Extended Mathematical Special Functions, 60 dirichlet_beta
- GNU Extended Mathematical Special Functions, 61 dirichlet_betaf
- GNU Extended Mathematical Special Functions, 61 dirichlet betal
 - GNU Extended Mathematical Special Functions, 61

dirichlet_eta GNU Extended Mathematical Special Functions, 61 dirichlet etaf GNU Extended Mathematical Special Functions, 61 dirichlet etal GNU Extended Mathematical Special Functions, 61 double factorial GNU Extended Mathematical Special Functions, 62 double factorialf GNU Extended Mathematical Special Functions, 62 double factoriall GNU Extended Mathematical Special Functions, 62 ellint 1 C++17/IS29124 Mathematical Special Functions, 27 ellint 1f C++17/IS29124 Mathematical Special Functions, 28 ellint 11 C++17/IS29124 Mathematical Special Functions, 28 ellint 2 C++17/IS29124 Mathematical Special Functions, 28 ellint 2f C++17/IS29124 Mathematical Special Functions, 29 ellint 2l C++17/IS29124 Mathematical Special Functions, 29 ellint 3 C++17/IS29124 Mathematical Special Functions, 29 ellint 3f C++17/IS29124 Mathematical Special Functions, 30 ellint 3I C++17/IS29124 Mathematical Special Functions, 30 ellint cel GNU Extended Mathematical Special Functions, 62 ellint celf GNU Extended Mathematical Special Functions, 62 ellint cell GNU Extended Mathematical Special Functions, 62 ellint d GNU Extended Mathematical Special Functions, 62 ellint df GNU Extended Mathematical Special Functions, 63 ellint dl GNU Extended Mathematical Special Functions, 63 ellint el1 GNU Extended Mathematical Special Functions, 63 ellint el1f GNU Extended Mathematical Special Functions, 63 ellint el1l GNU Extended Mathematical Special Functions, 63 ellint el2 GNU Extended Mathematical Special Functions, 63

GNU Extended Mathematical Special Functions, 64 ellint el3 GNU Extended Mathematical Special Functions, 64 ellint el3f GNU Extended Mathematical Special Functions, 64 ellint el3l GNU Extended Mathematical Special Functions, 64 ellint rc GNU Extended Mathematical Special Functions, 64 ellint rcf GNU Extended Mathematical Special Functions, 65 ellint rcl GNU Extended Mathematical Special Functions, 65 ellint rd GNU Extended Mathematical Special Functions, 65 ellint rdf GNU Extended Mathematical Special Functions, 66 ellint rdl GNU Extended Mathematical Special Functions, 66 ellint rf GNU Extended Mathematical Special Functions, 66 ellint rff GNU Extended Mathematical Special Functions, 67 ellint rfl GNU Extended Mathematical Special Functions, 67 ellint rg GNU Extended Mathematical Special Functions, 67 ellint raf GNU Extended Mathematical Special Functions, 67 ellint_rgl GNU Extended Mathematical Special Functions, 68 ellint rj GNU Extended Mathematical Special Functions, 68 ellint rjf GNU Extended Mathematical Special Functions, 68 ellint rjl GNU Extended Mathematical Special Functions, 68 ellnome GNU Extended Mathematical Special Functions, 69 ellnomef GNU Extended Mathematical Special Functions, 69 ellnomel GNU Extended Mathematical Special Functions, 69 evenzeta std:: detail, 207 expint C++17/IS29124 Mathematical Special Functions, 30 expint e1 GNU Extended Mathematical Special Functions, 69 expint e1f GNU Extended Mathematical Special Functions, 69 expint e1l

GNU Extended Mathematical Special Functions, 69

expint en

GNU Extended Mathematical Special Functions, 64

ellint el2l

GNU Extended Mathematical Special Functions, 69	chebyshev_wf, 50
expint_enf	chebyshev_wl, 50
GNU Extended Mathematical Special Functions, 69	clausen, 50
expint_enl	clausen_c, 50
GNU Extended Mathematical Special Functions, 70	clausen_cf, 51
expintf	clausen_cl, 51
C++17/IS29124 Mathematical Special Functions, 31	clausen_s, 51
expintl	clausen_sf, 51
C++17/IS29124 Mathematical Special Functions, 31	clausen_sl, 51
	clausenf, 52
factorial	clausenl, 52
GNU Extended Mathematical Special Functions, 70	comp ellint d, 52
factorialf	comp_ellint_df, 53
GNU Extended Mathematical Special Functions, 70	comp_ellint_dl, 53
factoriall	comp_ellint_rf, 53
GNU Extended Mathematical Special Functions, 70	comp_ellint_rg, 54
fresnel_c	conf_hyperg, 54
GNU Extended Mathematical Special Functions, 70	conf_hyperg_lim, 55
fresnel_cf	
GNU Extended Mathematical Special Functions, 70	conf_hyperg_limf, 55
fresnel_cl	conf_hyperg_liml, 55
GNU Extended Mathematical Special Functions, 70	conf_hypergf, 55
fresnel_s	conf_hypergl, 56
GNU Extended Mathematical Special Functions, 70	coshint, 56
fresnel_sf	coshintf, 56
GNU Extended Mathematical Special Functions, 71	coshintl, 56
fresnel_sl	cosint, 56
GNU Extended Mathematical Special Functions, 71	cosintf, 57
and Extended Mathematical opeolar randitions, 71	cosintl, 57
GNU Extended Mathematical Special Functions, 38	cyl_hankel_1, 57
_GLIBCXX_JACOBI_CN, 46	cyl_hankel_1f, 57
_GLIBCXX_JACOBI_DN, 46	cyl_hankel_1I, 58
_GLIBCXX_JACOBI_SN, 46	cyl_hankel_2, 58
airy_ai, 46	cyl_hankel_2f, 58
airy aif, 46	cyl_hankel_2l, 59
airy ail, 46	dawson, 59
airy_bi, 46	dawsonf, 59
airy bif, 46	dawsonl, 59
airy_bil, 47	digamma, 60
bernoulli, 47	digammaf, 60
bernoullif, 47	digammal, 60
bernoullil, 47	dilog, 60
bincoef, 47	dilogf, 60
	dilogl, 60
bincoeff, 47 bincoefl, 47	dirichlet beta, 61
	dirichlet_betaf, 61
chebyshev_t, 47	dirichlet_betal, 61
chebyshev_tf, 48	
chebyshev_tl, 48	dirichlet_eta, 61
chebyshev_u, 48	dirichlet_etaf, 61
chebyshev_uf, 48	dirichlet_etal, 61
chebyshev_ul, 48	double_factorial, 62
chebyshev_v, 49	double_factorialf, 62
chebyshev_vf, 49	double_factoriall, 62
chebyshev_vl, 49	ellint_cel, 62
chebyshev_w, 49	ellint_celf, 62

ellint_cell, 62	gamma_ql, 72
ellint_d, 62	gamma_u, <mark>72</mark>
ellint_df, 63	gamma_uf, 72
ellint_dl, 63	gamma_ul, 72
ellint_el1, 63	gegenbauer, 72
ellint_el1f, 63	gegenbauerf, 72
ellint_el1I, 63	gegenbauerl, 72
ellint_el2, 63	heuman_lambda, 72
ellint_el2f, 64	heuman_lambdaf, 73
ellint_el2l, 64	heuman_lambdal, 73
ellint_el3, 64	hurwitz_zeta, 73
ellint_el3f, 64	hurwitz_zetaf, 73
ellint_el3l, 64	hurwitz_zetal, 73
ellint_rc, 64	hyperg, 74
ellint_rcf, 65	hypergf, 74
ellint_rcl, 65	hypergl, 74
ellint_rd, 65	ibeta, 74
ellint_rdf, 66	ibetac, 75
ellint_rdl, 66	ibetacf, 75
ellint_rf, 66	ibetacl, 75
ellint_rff, 67	ibetaf, 75
ellint_rfl, 67	ibetal, 76
ellint_rg, 67	jacobi, 76
ellint_rgf, 67	jacobi_cn, 76
ellint_rgl, 68	jacobi_cnf, 76
ellint_rj, 68	jacobi_cnl, 76
ellint_rjf, 68	jacobi_dn, 76
ellint_rjl, 68	jacobi_dnf, 77 jacobi_dnl, 77
ellnome, 69 ellnomef, 69	jacobi_sn, 77
ellnomel, 69	jacobi_snf, 77
expint_e1, 69	jacobi_snl, 78
expirit_e1f, 69	jacobi_stri, 78
expint_e1l, 69	jacobi_zetaf, 78
expint_en, 69	jacobi_zetal, 78
expint_enf, 69	jacobif, 78
expint_enl, 70	jacobil, 78
factorial, 70	lbincoef, 78
factorialf, 70	lbincoeff, 79
factoriall, 70	lbincoefl, 79
fresnel_c, 70	Idouble_factorial, 79
fresnel cf, 70	Idouble_factorialf, 79
fresnel cl, 70	Idouble_factoriall, 79
fresnel s, 70	legendre_q, 79
fresnel_sf, 71	legendre_qf, 79
fresnel_sl, 71	legendre_ql, 79
gamma_l, 71	Ifactorial, 79
gamma_lf, 71	lfactorialf, 79
gamma_II, 71	lfactoriall, 79
gamma_p, 71	logint, 80
gamma_pf, 71	logintf, 80
gamma_pl, 71	logintl, 80
gamma_q, 71	lpochhammer_l, 80
gamma_qf, 71	lpochhammer_lf, 80

lpochhammer_II, 80	sph_harmonicl, 88
lpochhammer_u, 80	theta_1, 89
lpochhammer_uf, 81	theta_1f, 89
lpochhammer_ul, 81	theta_1I, 89
owens_t, 81	theta_2, 89
owens_tf, 81	theta_2f, 90
owens_tl, 81	theta_2I, 90
pochhammer_I, 81	theta_3, 90
pochhammer_lf, 82	theta_3f, 90
pochhammer_II, 82	theta_3I, 90
pochhammer_u, 82	theta_4, 91
pochhammer_uf, 82	theta_4f, 91
pochhammer_ul, 82	theta_4I, 91
polylog, 82	theta_c, 91
polylogf, 82	theta_cf, 92
polylogl, 82	theta_cl, 92
psi, 83	theta_d, 92
psif, 83	theta_df, 92
psil, 83	theta_dl, 92
radpoly, 83	theta_n, 93
radpolyf, 83	theta_nf, 93
radpolyl, 83	theta_nl, 93
sinc, 83	theta_s, 93
sinc_pi, 83	theta_sf, 94
sinc_pif, 84	theta_sl, 94
sinc_pil, 84	zernike, 94
sincf, 84	zernikef, 94
sincl, 84	zernikel, 94
sinhc, 84	gamma_l
sinhc_pi, 84	GNU Extended Mathematical Special Functions, 71
sinhc_pif, 84	gamma_lf
sinhc_pil, 84	GNU Extended Mathematical Special Functions, 71
sinhcf, 84	gamma_II
sinhcl, 84	GNU Extended Mathematical Special Functions, 71
sinhint, 84	gamma_p
sinhintf, 85	GNU Extended Mathematical Special Functions, 71
sinhintl, 85	gamma_pf
sinint, 85	GNU Extended Mathematical Special Functions, 71
sinintf, 85	gamma_pl
sinintl, 85	GNU Extended Mathematical Special Functions, 71
sph_bessel_i, 86	gamma_q
sph_bessel_if, 86	GNU Extended Mathematical Special Functions, 71
sph_bessel_il, 86	gamma_qf
sph_bessel_k, 86	GNU Extended Mathematical Special Functions, 71
sph_bessel_kf, 86	gamma_ql
sph_bessel_kl, 86	GNU Extended Mathematical Special Functions, 72
sph_hankel_1, 86	gamma_u
sph_hankel_1f, 87	GNU Extended Mathematical Special Functions, 72
sph_hankel_1I, 87	gamma_uf
sph_hankel_2, 87	GNU Extended Mathematical Special Functions, 72
sph_hankel_2f, 87	gamma_ul
sph_hankel_2l, 88	GNU Extended Mathematical Special Functions, 72
sph_harmonic, 88	gegenbauer
sph_harmonicf, 88	GNU Extended Mathematical Special Functions, 72

gegenbauerf	jacobi_dnl
GNU Extended Mathematical Special Functions, 72	GNU Extended Mathematical Special Functions, 77
gegenbauerl	jacobi_sn
GNU Extended Mathematical Special Functions, 72	GNU Extended Mathematical Special Functions, 77
	jacobi_snf
hermite	GNU Extended Mathematical Special Functions, 77
C++17/IS29124 Mathematical Special Functions, 31	jacobi_snl
hermitef	GNU Extended Mathematical Special Functions, 78
C++17/IS29124 Mathematical Special Functions, 32	jacobi_zeta
hermitel	GNU Extended Mathematical Special Functions, 78
C++17/IS29124 Mathematical Special Functions, 32	jacobi zetaf
heuman lambda	GNU Extended Mathematical Special Functions, 78
GNU Extended Mathematical Special Functions, 72	•
heuman lambdaf	jacobi_zetal
GNU Extended Mathematical Special Functions, 73	GNU Extended Mathematical Special Functions, 78
heuman lambdal	jacobif
GNU Extended Mathematical Special Functions, 73	GNU Extended Mathematical Special Functions, 78
hurwitz zeta	jacobil
GNU Extended Mathematical Special Functions, 73	GNU Extended Mathematical Special Functions, 78
hurwitz_zetaf	laguerre
GNU Extended Mathematical Special Functions, 73	C++17/IS29124 Mathematical Special Functions, 32
hurwitz_zetal	laguerref
GNU Extended Mathematical Special Functions, 73	C++17/IS29124 Mathematical Special Functions, 32
hyperg	laguerrel
GNU Extended Mathematical Special Functions, 74	C++17/IS29124 Mathematical Special Functions, 33
hypergf	lbincoef
GNU Extended Mathematical Special Functions, 74	GNU Extended Mathematical Special Functions, 78
hypergl	Ibincoeff
GNU Extended Mathematical Special Functions, 74	GNU Extended Mathematical Special Functions, 79
	Ibincoefl
ibeta	GNU Extended Mathematical Special Functions, 79
GNU Extended Mathematical Special Functions, 74	Idouble factorial
ibetac	GNU Extended Mathematical Special Functions, 79
GNU Extended Mathematical Special Functions, 75	·
ibetacf	Idouble_factorialf
GNU Extended Mathematical Special Functions, 75	GNU Extended Mathematical Special Functions, 79
ibetacl	Idouble_factoriall
GNU Extended Mathematical Special Functions, 75	GNU Extended Mathematical Special Functions, 79
ibetaf	legendre
GNU Extended Mathematical Special Functions, 75	C++17/IS29124 Mathematical Special Functions, 33
ibetal	legendre_q
GNU Extended Mathematical Special Functions, 76	GNU Extended Mathematical Special Functions, 79
	legendre_qf
jacobi	GNU Extended Mathematical Special Functions, 79
GNU Extended Mathematical Special Functions, 76	legendre_ql
jacobi_cn	GNU Extended Mathematical Special Functions, 79
GNU Extended Mathematical Special Functions, 76	legendref
jacobi_cnf	C++17/IS29124 Mathematical Special Functions, 33
GNU Extended Mathematical Special Functions, 76	legendrel
jacobi_cnl	C++17/IS29124 Mathematical Special Functions, 33
GNU Extended Mathematical Special Functions, 76	Ifactorial
· · · · · · · · · · · · · · · · · · ·	GNU Extended Mathematical Special Functions, 79
jacobi_dn GNU Extended Mathematical Special Functions 76	
GNU Extended Mathematical Special Functions, 76	Ifactorialf CNIL Extended Methamatical Special Eurotions 70
jacobi_dnf	GNU Extended Mathematical Special Functions, 79
GNU Extended Mathematical Special Functions, 77	Ifactoriall

GNU Extended Mathematical Special Functions, 79 logint	GNU Extended Mathematical Special Functions, 83 radpolyl
GNU Extended Mathematical Special Functions, 80	GNU Extended Mathematical Special Functions, 83
logintf	riemann_zeta
GNU Extended Mathematical Special Functions, 80	C++17/IS29124 Mathematical Special Functions, 34
logintl	riemann zetaf
GNU Extended Mathematical Special Functions, 80	C++17/IS29124 Mathematical Special Functions, 34
lpochhammer_I	riemann zetal
GNU Extended Mathematical Special Functions, 80	C++17/IS29124 Mathematical Special Functions, 34
lpochhammer_lf	OTT 17/1023124 Mathematical opecial Functions, 3-
GNU Extended Mathematical Special Functions, 80	SININT
lpochhammer_II	
GNU Extended Mathematical Special Functions, 80	std::detail, 122
lpochhammer_u	sf_airy.tcc
GNU Extended Mathematical Special Functions, 80	_GLIBCXX_BITS_SF_AIRY_TCC, 215
lpochhammer_uf	sf_bessel.tcc
GNU Extended Mathematical Special Functions, 81	_GLIBCXX_BITS_SF_BESSEL_TCC, 217
·	sf_beta.tcc
lpochhammer_ul	_GLIBCXX_BITS_SF_BETA_TCC, 219
GNU Extended Mathematical Special Functions, 81	sf_cardinal.tcc
owens t	_GLIBCXX_BITS_SF_CARDINAL_TCC, 221
GNU Extended Mathematical Special Functions, 81	sf_chebyshev.tcc
	_GLIBCXX_SF_CHEBYSHEV_TCC, 222
owens_tf	sf_dawson.tcc
GNU Extended Mathematical Special Functions, 81	_GLIBCXX_SF_DAWSON_TCC, 224
owens_tl	sf_ellint.tcc
GNU Extended Mathematical Special Functions, 81	_GLIBCXX_BITS_SF_ELLINT_TCC, 226
pochhammer I	sf_expint.tcc
GNU Extended Mathematical Special Functions, 81	_GLIBCXX_BITS_SF_EXPINT_TCC, 229
pochhammer If	sf_fresnel.tcc
GNU Extended Mathematical Special Functions, 82	_GLIBCXX_SF_FRESNEL_TCC, 230
	sf_gamma.tcc
pochhammer_II	_GLIBCXX_BITS_SF_GAMMA_TCC, 236
GNU Extended Mathematical Special Functions, 82	sf_gegenbauer.tcc
pochhammer_u	_GLIBCXX_SF_GEGENBAUER_TCC, 237
GNU Extended Mathematical Special Functions, 82	sf_hankel.tcc
pochhammer_uf	
GNU Extended Mathematical Special Functions, 82	sf_hankel_new.tcc
pochhammer_ul	GLIBCXX BITS SF HANKEL NEW TCC, 240
GNU Extended Mathematical Special Functions, 82	sf hermite.tcc
polylog	_GLIBCXX_BITS_SF_HERMITE_TCC, 242
GNU Extended Mathematical Special Functions, 82	sf hydrogen.tcc
polylogf	_GLIBCXX_BITS_SF_HYDROGEN_TCC, 243
GNU Extended Mathematical Special Functions, 82	sf hyperg.tcc
polylogl	GLIBCXX BITS SF HYPERG TCC, 245
GNU Extended Mathematical Special Functions, 82	
psi	sf_hypint.tcc
GNU Extended Mathematical Special Functions, 83	_GLIBCXX_SF_HYPINT_TCC, 246
psif	sf_jacobi.tcc
GNU Extended Mathematical Special Functions, 83	_GLIBCXX_SF_JACOBI_TCC, 248
psil	sf_laguerre.tcc
GNU Extended Mathematical Special Functions, 83	_GLIBCXX_BITS_SF_LAGUERRE_TCC, 250
	sf_legendre.tcc
radpoly	_GLIBCXX_BITS_SF_LEGENDRE_TCC, 251
GNU Extended Mathematical Special Functions, 83	sf_mod_bessel.tcc
radpolyf	_GLIBCXX_BITS_SF_MOD_BESSEL_TCC, 253

sf_owens_t.tcc	GNU Extended Mathematical Special Functions, 86
_GLIBCXX_BITS_SF_OWENS_T_TCC, 254	sph_bessel_il
sf_polylog.tcc	GNU Extended Mathematical Special Functions, 86
_GLIBCXX_BITS_SF_POLYLOG_TCC, 257	sph_bessel_k
sf_theta.tcc	GNU Extended Mathematical Special Functions, 86
_GLIBCXX_SF_THETA_TCC, 259	sph_bessel_kf
sf_trigint.tcc	GNU Extended Mathematical Special Functions, 86
_GLIBCXX_SF_TRIGINT_TCC, 261	sph_bessel_kl
sf_zeta.tcc	GNU Extended Mathematical Special Functions, 86
_GLIBCXX_BITS_SF_ZETA_TCC, 263	sph_besself
sinc	C++17/IS29124 Mathematical Special Functions, 35
GNU Extended Mathematical Special Functions, 83	sph_bessell
sinc_pi	C++17/IS29124 Mathematical Special Functions, 35
GNU Extended Mathematical Special Functions, 83	sph_hankel_1
sinc pif	GNU Extended Mathematical Special Functions, 86
GNU Extended Mathematical Special Functions, 84	sph_hankel_1f
sinc pil	GNU Extended Mathematical Special Functions, 87
GNU Extended Mathematical Special Functions, 84	sph_hankel_1l
sincf	GNU Extended Mathematical Special Functions, 87
GNU Extended Mathematical Special Functions, 84	sph_hankel_2
sincl	GNU Extended Mathematical Special Functions, 87
GNU Extended Mathematical Special Functions, 84	sph_hankel_2f
sinhc	GNU Extended Mathematical Special Functions, 87
GNU Extended Mathematical Special Functions, 84	sph_hankel_2l
sinhc_pi	GNU Extended Mathematical Special Functions, 88
GNU Extended Mathematical Special Functions, 84	sph_harmonic
•	GNU Extended Mathematical Special Functions, 88
sinhc_pif GNU Extended Mathematical Special Functions, 84	sph_harmonicf
•	• —
sinhc_pil	GNU Extended Mathematical Special Functions, 88
GNU Extended Mathematical Special Functions, 84	sph_harmonicl
Sinhof CNUL Futured ad Mathematical Consider Functions 24	GNU Extended Mathematical Special Functions, 88
GNU Extended Mathematical Special Functions, 84	sph_legendre
Sinhol CNU Fister ded Mathematical Consider Figure 1999	C++17/IS29124 Mathematical Special Functions, 35
GNU Extended Mathematical Special Functions, 84	sph_legendref
sinhint	C++17/IS29124 Mathematical Special Functions, 36
GNU Extended Mathematical Special Functions, 84	sph_legendrel
sinhintf	C++17/IS29124 Mathematical Special Functions, 36
GNU Extended Mathematical Special Functions, 85	sph_neumann
sinhintl	C++17/IS29124 Mathematical Special Functions, 36
GNU Extended Mathematical Special Functions, 85	sph_neumannf
sinint	C++17/IS29124 Mathematical Special Functions, 37
GNU Extended Mathematical Special Functions, 85	sph_neumannl
sinintf	C++17/IS29124 Mathematical Special Functions, 37
GNU Extended Mathematical Special Functions, 85	std, 103
sinintl	std::detail, 105
GNU Extended Mathematical Special Functions, 85	_Num_Euler_Maclaurin_zeta, 207
specfun.h	_S_Euler_Maclaurin_zeta, 207
STDCPP_MATH_SPEC_FUNCS, 274	_S_double_factorial_table, 207
cpp_lib_math_special_functions, 274	_S_factorial_table, 207
sph_bessel	_S_neg_double_factorial_table, 207
C++17/IS29124 Mathematical Special Functions, 34	_S_num_double_factorials, 208
sph_bessel_i	_S_num_double_factorials< double >, 208
GNU Extended Mathematical Special Functions, 86	_S_num_double_factorials< float >, 208
sph bessel if	S num double factorials < long double >, 208

_S_num_factorials, 208	comp_ellint_rg, 140
_S_num_factorials< double >, 208	conf_hyperg, 140
_S_num_factorials< float >, 208	conf_hyperg_lim, 140
_S_num_factorials< long double >, 208	conf_hyperg_lim_series, 140
_S_num_neg_double_factorials, 208	conf_hyperg_luke, 141
_S_num_neg_double_factorials< double >, 208	conf_hyperg_series, 141
_S_num_neg_double_factorials< float >, 208	coshint, 141
_S_num_neg_double_factorials< long double >, 209	cyl_bessel, 142
_S_num_zetam1, 209	cyl_bessel_i, 142
_S_zetam1, 209	cyl_bessel_ij_series, 143
airy, 123	cyl_bessel_ik, 143
airy_ai, 125	cyl_bessel_ik_asymp, 144
airy_arg, 125	cyl_bessel_ik_steed, 144
airy_asymp_absarg_ge_pio3, 125	cyl_bessel_j, 144
airy_asymp_absarg_lt_pio3, 126	cyl_bessel_jn, 145
airy_bessel_i, 126	cyl_bessel_jn_asymp, 145
airy_bessel_k, 127	cyl_bessel_jn_steed, 145
airy_bi, 128	cyl_bessel_k, 146
airy_hyperg_rational, 128	cyl_hankel_1, 146, 147
assoc_laguerre, 129	cyl_hankel_2, 147
assoc_legendre_p, 129	cyl_neumann, 148
bernoulli, 129	cyl_neumann_n, 148
bernoulli_2n, 130	dawson, 148
bernoulli_series, 130	dawson_const_frac, 149
beta, 130	dawson_series, 149
beta_gamma, 131	debye_region, 149
beta_inc, 131	dilog, 149
beta_inc_cont_frac, 132	dirichlet_beta, 150
beta_lgamma, 132	dirichlet_eta, 150, 151
beta_product, 132	double_factorial, 151
bincoef, 133	ellint_1, 151
binomial_cdf, 133	ellint_2, 152
binomial_cdfc, 133	ellint_3, 152
bose_einstein, 134	ellint_cel, 152
chebyshev_recur, 134	ellint_d, 153
chebyshev_t, 134	ellint_el1, 153
chebyshev_u, 134	ellint_el2, 153
chebyshev_v, 134	ellint_el3, 153
chebyshev_w, 134	ellint_rc, 153
chi_squared_pdf, 135	ellint_rd, 154
chi_squared_pdfc, 135	ellint_rf, 154
chshint, 135	ellint_rg, 155
chshint_cont_frac, 135	ellint_rj, 155
chshint_series, 136	ellnome, 156
clamp_0_m2pi, 136	ellnome_k, 156
clamp_pi, 136	ellnome_series, 156
clausen, 136	expint, 156, 157
clausen_c, 137	expint_E1, 157
clausen_s, 137, 138	expint_E1_asymp, 158
comp_ellint_1, 138	expint_E1_series, 158
comp_ellint_2, 139	expint_Ei, 159
comp_ellint_3, 139	expint_Ei_asymp, 159
comp_ellint_d, 139	expint_Ei_series, 159
comp_ellint_rf, 140	expint_En_cont_frac, 160

expint_En_recursion, 160	owens_t, 178
expint_En_series, 161	pochhammer_I, 178
expint_asymp, 157	pochhammer_u, 178
expint_large_n, 161	poly_hermite, 179
factorial, 161	poly_hermite_asymp, 179
fermi_dirac, 162	poly_hermite_recursion, 179
fisher_f_cdf, 162	poly_jacobi, 180
fisher_f_cdfc, 162	poly_laguerre, 180
fock_airy, 163	poly_laguerre_hyperg, 181
fpequal, 163	poly_laguerre_large_n, 181
fpimag, 163, 164	poly_laguerre_recursion, 182
fpreal, 164	poly_legendre_p, 182
fresnel, 164	poly_legendre_q, 183
fresnel_cont_frac, 165	poly_radial_jacobi, 183
fresnel_series, 165	polylog, 183
gamma, 165	polylog_exp, 184
gamma_cont_frac, 165	polylog_exp_asymp, 184
gamma_I, 165	polylog_exp_int_neg, 185
gamma_p, 166	polylog_exp_int_pos, 185, 186
gamma_q, 166	polylog_exp_neg, 186
gamma_series, 166	polylog_exp_neg_even, 187
gamma_temme, 166	polylog_exp_neg_odd, 187
	polylog_exp_negative_real_part, 188
gauss, 167	polylog_exp_pos, 189, 190
gegenbauer_poly, 167	polylog_exp_real_neg, 190, 191
hankel, 167	polylog_exp_real_pos, 191
hankel_debye, 168	psi, 192
hankel_params, 168	psi_asymp, 192
hankel uniform, 168	psi_series, 192
hankel_uniform_olver, 169	riemann_zeta, 193
hankel_uniform_outer, 169	riemann_zeta_alt, 193
hankel_uniform_sum, 169	riemann_zeta_euler_maclaurin, 193
heuman lambda, 170	riemann_zeta_glob, 194
hurwitz_zeta, 170, 171	riemann_zeta_m_1, 194
hurwitz_zeta_euler_maclaurin, 171	riemann_zeta_m_1_sum, 194
hydrogen, 171	riemann_zeta_product, 194
hyperg, 171	riemann_zeta_sum, 195
hyperg luke, 172	sinc, 195
hyperg_reflect, 172	sinc_pi, 195
hyperg_series, 172	sincosint, 196
iacobi_sncndn, 173	sincosint_asymp, 196
jacobi_zeta, 173	sincosint_cont_frac, 196
laguerre, 173	sincosint series, 196
log bincoef, 174	sinhc, 196, 197
log double factorial, 174	sinhc_pi, 197
log factorial, 174	sinhint, 197
log_gamma, 175	sph_bessel, 197, 198
log_gamma_bernoulli, 175	sph_bessel_ik, 198
log_gamma_lanczos, 175	sph_bessel_in, 198
log_gamma_sign, 176	sph_hankel, 199
log_gamma_spouge, 176	sph_hankel_1, 199
log_pathma_speage, 776log_pochhammer_I, 177	sph_hankel_2, 201
log_pochhammer_u, 177	sph_harmonic, 201
logint, 177	sph_legendre, 202

sph_neumann, 202, 203	GNU Extended Mathematical Special Functions, 92
student_t_cdf, 203	theta_cl
student_t_cdfc, 203	GNU Extended Mathematical Special Functions, 92
theta_1, 204	theta_d
theta_2, 204	GNU Extended Mathematical Special Functions, 92
theta_2_asymp, 204	theta_df
theta_2_sum, 204	GNU Extended Mathematical Special Functions, 92
theta_3, 205	theta_dl
theta_3_asymp, 205	GNU Extended Mathematical Special Functions, 92
theta_3_sum, 205	theta_n
theta_4, 205	GNU Extended Mathematical Special Functions, 93
theta_c, 206	theta_nf
theta_d, 206	GNU Extended Mathematical Special Functions, 93
theta_n, 206	theta_nl GNU Extended Mathematical Special Functions, 93
theta_s, 206	theta_s
zernike, 206	GNU Extended Mathematical Special Functions, 93
znorm1, 206	theta_sf
znorm2, 206	GNU Extended Mathematical Special Functions, 94
COSINT, 122	theta_sl
evenzeta, 207 SININT, 122	GNU Extended Mathematical Special Functions, 94
std::detail::_Factorial_table	
factorial, 211	zernike
log_factorial, 211	GNU Extended Mathematical Special Functions, 94
iog_lactorial, 211 n, 211	zernikef
std::detail::_Factorial_table< _Tp >, 211	GNU Extended Mathematical Special Functions, 94
otadotar dotonal_table < _ 1p > , 211	zernikel
theta 1	GNU Extended Mathematical Special Functions, 94
GNU Extended Mathematical Special Functions, 89	
theta 1f	
GNU Extended Mathematical Special Functions, 89	
theta_1I	
GNU Extended Mathematical Special Functions, 89	
theta_2	
GNU Extended Mathematical Special Functions, 89	
theta_2f	
GNU Extended Mathematical Special Functions, 90	
theta_2l	
GNU Extended Mathematical Special Functions, 90	
theta_3	
GNU Extended Mathematical Special Functions, 90	
theta_3f	
GNU Extended Mathematical Special Functions, 90	
theta_3I	
GNU Extended Mathematical Special Functions, 90	
theta_4	
GNU Extended Mathematical Special Functions, 91	
theta_4f	
GNU Extended Mathematical Special Functions, 91	
theta_4l GNU Extended Mathematical Special Functions, 01	
GNU Extended Mathematical Special Functions, 91	
theta_c GNU Extended Mathematical Special Functions, 91	
theta cf	