A Collective Decision Model Involving Vague Concepts and Linguistic Expressions

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***Abstract*—In linguistic collective decision, the main objective is to select the best alternatives using linguistic evaluations provided by multiple experts. This paper presents a collective decision model, which is able to deal with complex linguistic evaluations. In this decision model, the linguistic evaluations are represented by linguistic expressions which are the logic formulas obtained by applying logic connectives to the set of basic linguistic labels. The vagueness of each linguistic expression is implicitly captured by a semantic similarity relation rather than a fuzzy set, since each linguistic expression determines a semantic similarity distribution on the set of basic linguistic labels. The basic idea of this collective decision model is to convert the semantic similarity distributions determined by linguistic expressions into probability distributions of the corresponding linguistic expressions. The main advantage of this proposed model is its capability to deal with complex linguistic evaluations and partial semantic overlapping among neighboring linguistic labels.**

***Index Terms*—Collective decision, computing with words, linguistic evaluation, linguistic expression, linguistic information fusion, semantic similarity.**

I. Introduction

ONE REMARKABLE human capability is to manipulate perceptions in recognition, decision, and execution processes. Humans perceive physical and mental objects, extract images of these objects, and describe and convey these images using linguistic descriptions or words rather than numbers. As a methodology, computing with words or linguistic modeling provides a foundation for a computational theory of perceptions or linguistic descriptions [1], [2]. A key aspect of computing with words is that it involves a fusion of natural languages and computation with vague concepts. Such linguistic modeling is of central importance for many emerging information technologies, such as linguistic information fusion [3]–[6].

To date, most applications of linguistic modeling [7]–[10] use fuzzy sets or membership functions to describe the semantics of words. The use of fuzzy sets as the foundation of linguistic modeling is due to Zadeh [11]–[13]. Zadeh introduced the concept of linguistic variable as a model of how words or linguistic labels can represent the vague concepts in natural language. Although the use of fuzzy sets is central to computing with words as they provide a means of modeling vagueness underlying most natural language terms [1], [2], [14], its use is not compulsive. Lawry [15], [16] recently introduced a new framework for label semantics where the semantics of linguistic labels are described by appropriateness measures. The main difference between these two kinds of frameworks is the interpretation of linguistic uncertainty. In Lawry's framework, the appropriateness measure $\mu_L(x)$ means the belief that linguistic label L is appropriate for describing x. But in Zadeh's framework, the membership function $\mu_L(x)$ means the degree of x belonging to the extensions of vague concept L.

Both fuzzy sets and appropriateness measures are suitable mathematical tools for modeling linguistic information. But in some situations, the semantic relation between vague concepts is more important than the concrete semantics of underlying concepts. In fact, the linguistic uncertainty can be directly modeled by a similarity relation on the set of linguistic labels. This similarity relation can be interpreted as the degrees of semantic similarities among linguistic labels. One linguistic model based on a semantic relation between linguistic labels was proposed recently by Tang and Zheng [17]. Using the same idea as the label semantic framework proposed by Lawry, this linguistic model presents an inference mechanism for computing the degree of similarity between any linguistic expressions from a similarity relation among linguistic labels. We think that using a semantic similarity relation to capture the vagueness of concepts is natural in human natural languages. This paper mainly discusses the application of Tang and Zheng's linguistic model in linguistic collective decision.

In linguistic collective decision, a collection of linguistic expressions is provided to evaluate the underlying objects; each one corresponds to an information source. The so-called linguistic expressions are the linguistic logic formulas generated by applying logic connectives to the linguistic labels [15]. The main objective of collective decision is to select the best alternatives using the combined linguistic information. In our proposed collective decision model, we allow that any neighboring basic linguistic labels have partial semantic overlapping. In order to fuse all linguistic expressions provided by information sources, first, we convert each linguistic expression into an equivalent representation—a probability distribution defined on the set of basic linguistic labels by using the semantic similarity relation among basic linguistic labels. We then use weighted summation to combine all probability distributions derived from all linguistic evaluations for each underlying alternative. We finally introduce two possible decision methods in our proposed model to make collective decision.

This paper is organized as follows. Section II introduces the basic process of collective decision and reviews two

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well-known linguistic information fusion models. Section III outlines some basic concepts and properties on a semantic similarity relation among linguistic labels proposed by Tang and Zheng [17]. Section IV presents our collective decision model using the semantic similarity relation outlined in Section III. Section V illustrates the main characteristics of our proposed model by some numerical examples. The last section summarizes this paper and gives some conclusions.

II. Two Linguistic Information Fusion Models

We often face the problem of collective decision, e.g., one best product should be selected from a set of candidates in terms of linguistic evaluations from different aspects. In this situation, linguistic evaluations may be represented by linguistic expressions to capture the fuzziness or vagueness of concepts. Then, one basic issue is how to fuse multiple linguistic evaluations.

For convenience in discussing the collective decision models, first, we introduce some notations and assumptions. The finite set of alternatives is denoted as $X = \{x_1, \ldots, x_n\}$, the set of basic linguistic labels is denoted as $\mathrm{LA} = \{L_1, \ldots, L_m\}$, in which there exists an ordering relation "$<$" such that $L_i < L_j$ for $i < j$. In addition, more complex linguistic expressions could be generated from LA by applying logic connectives $\neg$, $\wedge$, $\vee$. The formal definition of a set of linguistic expressions LE shall be given in the following (see Definition 2). Moreover, assume that there are N different information sources (experts) $\{\mathcal{S}_i\}$, which provide linguistic expressions in LE for all alternatives in X. Suppose that the linguistic expression provided by $\mathcal{S}_i$ for alternative x_j is $\theta_{ij} \in \mathrm{LE}$, and each information source $\mathcal{S}_i$ has a weight $w_i \in [0, 1]$, such that $\sum_{i=1}^{N} w_i = 1$, where w_i represents the importance of $\mathcal{S}_i$.

To resolve the problem of collective decision involving linguistic evaluations, two kinds of linguistic information fusion models have been proposed: the first one uses the associated membership functions to fuse linguistic labels [11], [18], [19], and the second one acts by direct computation on linguistic labels [3], [5], [6], [20]–[24]. The models in the first kind usually involve the linguistic approximation process, since the final fuzzy set may not correspond to any linguistic label in the original label set. Clearly, this linguistic approximation process usually causes the loss of information. In Section II-A, the fusion model based on the extension principle in fuzzy logic, one representative in the first kind, is briefly introduced.

On the other hand, the models in the second kind operate directly the linguistic labels. Some operators are proposed to combine the linguistic labels without reference to the associated membership functions, such as the ordered weighted averaging (OWA) operator [25], the ordinal OWA operator [26], the linguistic OWA operator [23], [27] based on OWA [25], the convex combination of linguistic labels [20], the linguistic weighted averaging (LWA) operators [22], and the linguistic hybrid geometric averaging operator [24]. However, these methods only deal with the linguistic labels but not complex linguistic expressions. In Section II-B, we briefly introduce the symbolic method which is initially introduced in [21] and [22] and improved later in [6] by proposing a representation of linguistic information based on a 2-tuple. The symbolic method is essentially based on the LWA operator, which performs operations on the indexes of linguistic labels. However, in this method, there is an implicit assumption: any neighboring linguistic labels have no semantic overlapping; this point will be manifested in Section IV.

A. Linguistic Information Fusion Based on the Extension Principle

In this linguistic information fusion model, the semantics of linguistic labels are represented by fuzzy sets. By applying fuzzy arithmetic based on the extension principle to the set of linguistic labels, we obtain a new fuzzy set having more vagueness which does not match the basic linguistic labels [18]. Therefore, a linguistic approximation process is needed to represent the fusion result in LA. The fusion process in this model can be outlined as follows.

1) Aggregate the collective linguistic labels represented by fuzzy sets for each alternative under consideration and achieve an aggregated fuzzy set.
2) Transform the aggregated fuzzy set to one of basic linguistic label for each alternative and select the alternatives having the biggest linguistic labels as the best.

The above fusion process can also be described as follows:

$$\mathrm{LA}^N \xrightarrow{\widetilde{F}} F(R) \xrightarrow{app(\cdot)} \mathrm{LA}$$

where LA^N represents N Cartesian product of LA, $\widetilde{F}$ is an aggregation operator based on the extension principle, $F(R)$ is the set of all fuzzy sets over real numbers R, $app(\cdot) : F(R) \rightarrow \mathrm{LA}$ is a linguistic approximate function that returns a linguistic label in LA, whose meaning is the closest to the input fuzzy set.

In this model, the linguistic labels L_i's are often assumed to be of the triangular type $C_i = (a_i, b_i, c_i)$. The arithmetic mean is often selected as an aggregation operator to fuse N linguistic information $y = \{L_{i_1}, \ldots, L_{i_N}\}$, obtaining a new fuzzy set $\overline{y}$

$$\overline{y} = \left(\sum_{k=1}^{N} a_{i_k} w_k, \sum_{k=1}^{N} b_{i_k} w_k, \sum_{k=1}^{N} c_{i_k} w_k \right).$$

Since fuzzy set $\overline{y}$ does not exactly match the linguistic label in LA, an approximation process is needed. The approximate function $app(\cdot)$ based on Euclidean distance between fuzzy sets is often used in practical applications.

B. Linguistic Information Fusion Based on the Symbolic Method

Herrera and Martínez [6] proposed a fusion model by introducing a new representation of linguistic information on the basis of the symbolic method initially presented in [21] and [22]. The fusion process in this linguistic fusion model can be illustrated as follows:

$$\mathrm{LA}^N \xrightarrow{\Psi} [0, m] \xrightarrow{\Delta} \mathrm{LA} \times [-0.5, 0.5).$$

The aggregation operator Ψ is a weighted average operator (see Definition 1) operating on the indexes of linguistic labels. That is, for a set of linguistic evaluations represented by linguistic labels $y = \{L_{i_1}, \ldots, L_{i_N}\}$, where $i_k \in \{1, \ldots, m\}$, we have $\Psi(y) = \sum_{k=1}^{N} i_k w_k \in [0, m]$. In particular, the symbolic

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translation function Δ has the following formal definition:

$$\Delta : [0, m] \longrightarrow \text{LA} \times [-0.5, 0.5) \tag{1}$$

$$\Delta(\beta) = (L_i, \alpha), \text{with} \begin{cases} L_i, & i = \text{round}(\beta) \\ \alpha = \beta - i, & \alpha \in [-0.5, 0.5) \end{cases} \tag{2}$$

where round$(\cdot)$ is the usual round operator, L_i means the linguistic label having the closest index to "β."

The advantage of this symbolic method is that no loss of information is involved when fusing multiple linguistic labels. In fact, there exists a function $\Delta^{-1} : \text{LA} \times [-0.5, 0.5) \longrightarrow [0, m]$ such that

$$\Delta^{-1}(L_i, \alpha) = i + \alpha.$$

Therefore, in this model, the fused linguistic information is expressed by a 2-tuple $(L_i, \alpha) \in \text{LA} \times [-0.5, 0.5)$, and two fused linguistic information (L_k, α_1) and (L_l, α_2) are compared by comparing $k + \alpha_1$ and $l + \alpha_2$. In particular, each linguistic label $L \in \text{LA}$ can be expressed as a 2-tuple $(L, 0)$.

In general, when multiple linguistic information expressed by 2-tuples is available, the fusion result can be derived by using weighted average operator as follows.

Definition 1: Let $y = \{(L_{i_1}, \alpha_1), \ldots, (L_{i_N}, \alpha_N)\}$ be the set of 2-tuples provided by N information sources. The 2-tuple weighted average $\overline{y}$ is

$$\Delta\left(\sum_{k=1}^{N} (i_k + \alpha_k) \cdot w_k\right). \tag{3}$$

Since no approximation process is involved in the symbolic method, the fusion model based on the symbolic method is superior to the fusion model based on the extension principle. But the symbolic method still has one drawback, it only fuses the linguistic information represented by basic linguistic labels rather than complex linguistic expressions. In addition, the symbolic method assumes that any neighboring linguistic labels have no semantic overlapping; this point will be manifested in Section IV. Clearly, this assumption is impractical. In order to overcome these drawbacks, in Section IV, one new collective decision model based on a semantic similarity relation among linguistic labels is proposed. This new model can deal with linguistic labels and linguistic expressions in a natural way. Before discussing this new model, some basic concepts and properties on the semantic similarity relation are introduced in the following section.

III. Semantic Similarity Relation $\langle R, \text{LA} \rangle$

In linguistic information fusion, humans often use the similarity relation among linguistic expressions for decision making but not the concrete semantics of vague concepts itself. In this section, we outline some basic concepts and properties on a semantic similarity relation $\langle R, \text{LA} \rangle$ proposed by Tang and Zheng [17], where $\text{LA} = \{L_1, \ldots, L_m\}$ is a set of basic linguistic labels, and the similarity relation $R = (r(L_i, L_j))_{m \times m}$ is defined on LA such that $r(L_i, L_j) = r(L_j, L_i) \in [0, 1]$ and $r(L_i, L_i) = 1$ for $i, j = 1, \ldots, m$. Intuitively, $r(L_i, L_j)$ represents the degree of semantic overlapping or similarity between linguistic labels L_i and L_j. Then, one natural question is how to compute the degree of similarity $r(\theta, L_i)$ for any linguistic expression $\theta \in \text{LE}$ and linguistic label $L_i \in \text{LA}$. Before we continue this topic, we introduce the formal definition of linguistic expressions and related definitions.

Definition 2 (Linguistic Expressions): The set of linguistic expressions LE is defined recursively as follows.

1) $L_i \in \text{LE}$ for $i = 1, \ldots, m$.
2) If $\theta, \phi \in \text{LE}$, then $\neg\theta, \theta \wedge \phi, \theta \vee \phi \in \text{LE}$.

For any general linguistic expression, we can define a λ mapping to convert it into a set of subsets of LA [15].

Definition 3 (λ Sets): Every linguistic expression $\theta \in \text{LE}$ is associated with a set of subsets of LA, denoted as $\lambda(\theta)$ and defined recursively as follows.

1) $\lambda(L_i) = \{S \subseteq \text{LA} | L_i \in S\}$ for $i = 1, \ldots, m$.
2) $\lambda(\theta \wedge \phi) = \lambda(\theta) \cap \lambda(\phi)$.
3) $\lambda(\theta \vee \phi) = \lambda(\theta) \cup \lambda(\phi)$.
4) $\lambda(\neg\theta) = \overline{\lambda(\theta)}$.

The λ-set $\lambda(\theta)$ means all possible sets of linguistic labels which are appropriate for describing the object x when we evaluate x using linguistic expression θ [15], [16]. In other words, $\lambda(\theta)$ means all possible sets of linguistic labels which can be considered as the extensions of vague concepts $\theta \in \text{LE}$.

Note that similarity relation R on LA reflects the semantic similarities among linguistic labels. In particular, we interpret the value $r(L_j, L_i)$ as the degree of possibility that L_j belongs to the extensions of linguistic label L_i. According to this semantic interpretation, when we evaluate the object x using linguistic label L_i, other linguistic labels $L_j (j \neq i)$ may also be possible for describing x, but which of these linguistic labels are possible is uncertain. Then, for each $L_i \in \text{LA}$, a consonant mass assignment m_{L_i} defined on 2^{LA} can be derived from the similarity distribution $[r(L_1, L_i), \ldots, r(L_m, L_i)]$.

Definition 4 (Consonant Mass Selection Function): Given the degrees of similarities $r(L_1, L_i), \ldots, r(L_m, L_i)$, ordered such that $r(L_j, L_i) \geq r(L_{j+1}, L_i)$ for $j = 1, \ldots, m - 1$, then the consonant mass selection function identifies the mass assignment

$$m_{L_i}(\{L_1, \ldots, L_m\}) = r(L_m, L_i)$$
$$m_{L_i}(\{L_1 \ldots L_j\}) = r(L_j, L_i) - r(L_{j+1}, L_i) : j = 1 \ldots m-1$$
$$m_{L_i}(\emptyset) = 1 - r(L_1, L_i).$$

In general, a mass assignment function is able to represent a piece of evidence in Dempster–Shafer evidence theory [28], [29] and transferable belief model [30], [31]. In Definition 4, the mass $m_{L_i}(T)$ means one's belief that T is the extensions of linguistic label L_i. Note that $\lambda(\theta)$ means all possible sets which can be considered as the extensions of linguistic expression θ, hence evaluating the degree of possibility that $L_i \in \text{LA}$ belongs to the extensions of $\theta \in \text{LE}$ is to aggregate the values of m_{L_i} across $\lambda(\theta)$. Therefore, we have the following definition on the degree of similarity $r(\theta, L_i)$ [17], where $\theta \in \text{LE}$ and $L_i \in \text{LA}$.

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Definition 5: For the linguistic expression $\theta \in \mathrm{LE}$, the degree of similarity between θ and $L_i \in \mathrm{LA}$ is defined as follows:

$$r(\theta, L_i) = \sum_{S \in \lambda(\theta)} m_{L_i}(S) \tag{4}$$

where m_{L_i} is a consonant mass assignment induced from $[r(L_1, L_i), \ldots, r(L_m, L_i)]$.

This definition is motivated by Lawry's work [15]. Moreover, Tang and Zheng [17] gave a definition for generalizing the similarity relation R on LA to the similarity relation R on LE. But in this paper, we only need to compute $r(\theta, L_i)$ for any $\theta \in \mathrm{LE}$ and $L_i \in \mathrm{LA}$. It should be pointed out that in Dempster–Shafer evidence theory or transferable belief model the belief or plausibility measure can also be generated from a mass assignment function; however, in general, $r(\theta, L_i)$ in Definition 5 is not a belief or plausibility measure.

Two properties of linguistic model $\langle \mathrm{LA}, R\rangle$ are given in the following [17]. By applying these properties, the inference process in $\langle \mathrm{LA}, R\rangle$ can be simplified significantly.

Proposition 1: For any $\theta \in \mathrm{LE}$ and $L_i \in \mathrm{LA}$, the following holds:

$$r(\neg\theta, L_i) = 1 - r(\theta, L_i). \tag{5}$$

Proposition 2: For any linguistic expressions $\theta, \phi \in \mathrm{LE}$ and $L_i \in \mathrm{LA}$, where θ, ϕ are the linguistic expressions generated by connectives $\wedge$, $\vee$ and the symbols in LA, the following hold:

$$r(\theta \wedge \phi, L_i) = \min\{r(\theta, L_i), r(\phi, L_i)\} \tag{6}$$

$$r(\theta \vee \phi, L_i) = \max\{r(\theta, L_i), r(\phi, L_i)\} \tag{7}$$

$$r(\neg\theta \wedge \phi, L_i) = \max\{0, r(\phi, L_i) - r(\theta, L_i)\} \tag{8}$$

$$r(\neg\theta \vee \phi, L_i) = \min\{1, 1 - r(\theta, L_i) + r(\phi, L_i)\}. \tag{9}$$

IV. Collective Decision Model Based on $\langle R, \mathrm{LA}\rangle$

In this section, we propose a collective decision model based on a semantic similarity relation $\langle R, \mathrm{LA}\rangle$. The basic idea of this model is to convert each linguistic expression into a probability distribution on LA using semantic similarity relation R.

Assume that each information source (expert) $\mathcal{S}_i$ provides a linguistic expression $\theta_{ij} \in \mathrm{LE}$ for alternative x_j. Our objective is to fuse the set of linguistic expressions $y_j = \{\theta_{1j}, \ldots, \theta_{Nj}\}$ for each alternative x_j and select the best alternatives according to the fused results.

Note that, for any $\theta \in \mathrm{LE}$, we can derive a semantic similarity distribution $[r(\theta, L_1), \ldots, r(\theta, L_m)]$ according to the semantic similarity relation R and Definition 5. Then, according to the definition of consonant mass selection function (see Definition 4), there is a consonant mass assignment m_θ corresponding to this similarity distribution $[r(\theta, L_1), \ldots, r(\theta, L_m)]$. For any focal element $F \subseteq \mathrm{LA}$, its mass $m_\theta(F)$ means one's belief that F can be considered as the extensions of linguistic expression θ. The notion of mass assignment m_θ suggests a definition of probability distribution Pr_θ for any linguistic expression $\theta \in \mathrm{LE}$ as follows.

Definition 6: For $\theta \in \mathrm{LE}$, then the probability distribution of θ is given by

$$\forall L \in \mathrm{LA}, Pr_{\theta,R}(L) = \sum_{F: L \in F} \frac{m_\theta(F)}{(1 - m_\theta(\emptyset))|F|'} \tag{10}$$

where m_θ is the consonant mass assignment of θ, and $\{F\}$ is the corresponding set of focal elements. In the sequel, we shall drop the subscript R (meaning the semantic similarity relation on LA) and write simply $Pr_\theta(L)$.

Note that, when $m_\theta(\emptyset) = 0$, the probability distribution is the pignistic distribution introduced by Smets and Kennes [31]. The mass $m_\theta(\emptyset)$ can be interpreted as the degree of inconsistency conveyed by linguistic expression θ or the belief committed exactly to other hypotheses which are not included in LA. In this paper, we adopt the former interpretation on $m_\theta(\emptyset)$. The idea underlying the definition of probability distribution in Definition 6 is that, for each focal set F containing linguistic label L, a uniform proportion $1/|F|$ of mass $m_\theta(F)/(1 - m_\theta(\emptyset))$ is reallocated to L. The probability of L is then taken to be the sum across the focal sets of the reallocated masses. In other words, the value $Pr_\theta(L)$ reflects the probability that $L \in \mathrm{LA}$ belongs to the extensions of linguistic expression $\theta \in \mathrm{LE}$.

The notion of probability distribution of linguistic expression provides a mechanism by which we can, in a sense, convert a linguistic expression into a probability distribution on the underlying extensions. That is, in the absence of any prior knowledge, we might on being told linguistic expression θ naturally infer the distribution $Pr_\theta(\cdot)$ on its extensions. On the other hand, given a probability distribution on the set of basic linguistic labels, there is a unique similarity distribution which can be converted into this probability distribution using Definition 6. This transformation process is similar to that of fuzzy set introduced by Lawry [14]. In this sense, we might view a linguistic expression as a conceptual description of a probability distribution on LA. However, given a probability distribution on LA, it is very hard to derive an explicit and equivalent linguistic expression in LE corresponding to the underlying probability distribution.

For convenience of decision making, we can define a score for each linguistic expression $\theta \in \mathrm{LE}$

$$G(\theta) = \sum_{i=1}^{m} i \cdot Pr_\theta(L_i) \tag{11}$$

where G is a score function. In general, $G(L_i) \neq i$ for $i = 1, \ldots, m$. However, if any neighboring linguistic labels have no semantic overlapping, we have $G(L_i) = i$ for $i = 1, \ldots, m$.

We now consider the issue of multiple linguistic information fusion. Assume that each information source $\mathcal{S}_i$ provides a linguistic expression $\theta_{ij} \in \mathrm{LE}$ for alternative x_j. Then, for the set of linguistic expressions $y_j = \{\theta_{1j}, \ldots, \theta_{Nj}\}$ for each alternative x_j, we can also determine a probability distribution $Pr_{y_j}(\cdot)$ as follows:

$$Pr_{y_j}(L) = \sum_{i=1}^{N} Pr_{\theta_{ij}}(L) \cdot w_i \tag{12}$$

where $L \in \mathrm{LA}$, and w_i is the weight of information source $\mathcal{S}_i$. $Pr_{y_j}(L)$ is interpreted as the probability that $L \in \mathrm{LA}$ belongs to the extensions of fused implicit linguistic evaluation $\overline{y}_j$, but we cannot derive the concrete form of the fused linguistic evaluation $\overline{y}_j$. Therefore, this probability distribution $Pr_{y_j}(\cdot)$ can be considered as a linguistic description for the alternative

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x_j. We can also define a score function for the set of linguistic expressions y_j

$$G(y_j) = \sum_{i=1}^{m} i \cdot Pr_{y_j}(L_i) = \sum_{i=1}^{N} G(\theta_{ij}) \cdot w_i. \tag{13}$$

Therefore, we can compare any two linguistic information y_j and y_k by comparing $G(y_j)$ and $G(y_k)$. This decision result is a compromise by considering all linguistic evaluations given by all information sources. This decision method is referred to as the least prejudiced decision method. Moreover, if one most appropriate linguistic label in LA is required to approximate the fused linguistic information $\overline{y}_j$, we can use the following formula to determine the linguistic label having the closest score to the fused result $\overline{y}_j$:

$$\arg\min_{L_i \in \mathrm{LA}} |G(y_j) - G(L_i)|. \tag{14}$$

In total, the collective decision model based on $\langle R, \mathrm{LA}\rangle$ has the following steps.

1) For each linguistic expression $\theta_{ij} \in y_j$ given by expert $\mathcal{S}_i$ for alternative x_j, compute the similarity distribution $r(\theta_{ij}, \cdot)$ using (4) in Definition 5.
2) Convert each similarity distribution $r(\theta_{ij}, \cdot)$ into a probability distribution $Pr_{\theta_{ij}}(\cdot)$ of linguistic expression θ_{ij} using (10) in Definition 6.
3) For each alternative x_j, obtain the probability distribution $Pr_{y_j}(\cdot)$ by aggregating probability values $Pr_{\theta_{ij}}(\cdot)$ for $i = 1, \ldots, N$ using (12).
4) Obtain the score values $G(y_j)$ for each alternative x_j using (13). Moreover, select the best alternatives having the maximum score values.
5) Assign the most appropriate linguistic labels to each alternative x_j using (14) if necessary.

In the aforementioned least prejudiced decision method, we introduce a score function $G(\cdot)$ for the collective linguistic evaluations. The function $G(\cdot)$ provides a mechanism to distinguish linguistic evaluations and assign the most appropriate linguistic labels to each alternative. However, this mechanism still involves the linguistic approximation process in the earlier final step. In addition, in our proposed collective decision model, no explicit linguistic expression in LE is returned to represent the fused linguistic evaluation. These are some drawbacks of our proposed model. However, in general, it is very hard to return a linguistic expression in LE, which is an equivalent representation of the corresponding collective linguistic evaluations for the underlying alternative.

Without reference to the score function $G(\cdot)$, we can define another decision method to compare any two alternatives (my thanks to one of referees for pointing this out to me). Note that, in general, the semantic similarity relation R is a quantitative description of a qualitative ordering relation "$>$" on LA. Therefore, we can provide an alternative ranking mechanism to compare any two linguistic information y_i and y_j by estimating the probability that $y_i > y_j$ according to the following:

$$Pr(y_i > y_j) = \sum_{L \in \mathrm{LA}} \sum_{L' > L} Pr_{y_j}(L) Pr_{y_i}(L'). \tag{15}$$

This decision method has the advantage of providing a measure of uncertainty for a given ranking. Therefore, after obtaining the probability distributions $Pr_{y_j}(\cdot)$ for all alternatives, we can compare any two alternative x_i and x_j according to the following criterion:

$$x_i > x_j \Leftrightarrow Pr(y_i > y_j) > Pr(y_j > y_i). \tag{16}$$

Using the notions introduced in the least prejudiced decision method, we find that the 2-tuple fusion model based on the symbolic method presented in [6] is a special case of our proposed model. We can now rewrite the 2-tuple fusion model. Aforementioned in the earlier discussion, in the symbolic method, there is an implicit assumption: $r(L_i, L_j) = 1$ if $j = i$ and $r(L_i, L_j) = 0$ if $j \neq i$ for $i, j = 1, \ldots, m$. That is, any two neighboring linguistic labels have no semantic overlapping. Then, it is clear that $G(L_i) = i$, and for any set of linguistic evaluations $y = \{L_{i_1}, \ldots, L_{i_N}\}$, where $L_{i_k} \in \mathrm{LA}$ for $k = 1, \ldots, N$, we have $G(y) = \sum_{k=1}^{N} i_k w_k$. We can see that this result is consistent with Definition 1 in Section II-B. Then, from the score $G(y)$, we can derive a 2-tuple (L, α) corresponding to the fused result $G(y)$, where L is linguistic label having the closest index to the fused result.

In this paper, we briefly outline the difference between representation methods of linguistic information in the aforementioned three collective decision or fusion models. In the fusion model based on the extension principle, the linguistic information is represented by basic linguistic labels with fuzzy sets (usually having triangle-shaped membership functions). In the fusion model based on the symbolic method, the linguistic information is also represented by basic linguistic labels. In this model, however, there is an implicit assumption that all neighboring linguistic labels have no semantic overlapping. In our proposed model, the linguistic information is represented by complex linguistic expressions. Moreover, all neighboring linguistic labels may have partial semantic overlapping. At this point, we can say that the collective decision model proposed in this section has strong ability to deal with linguistic uncertainty. A comparison between our proposed collective decision model and fusion model based on the symbolic method are illustrated in the following section.

In conclusion, we can say that our proposed collective decision model is a generalization of the symbolic method proposed by Herrera and Martínez [6]. Our proposed model has prominent advantage in two aspects. The first aspect is that this model can deal with linguistic labels having partial overlapping semantics. That is, any two neighboring linguistic labels may have nonnull semantic similarity degree. The second aspect is that this model can deal with complex linguistic expressions.

V. Numerical Examples

In this section, we present some collective decision examples involving linguistic information. The main goal of this section is to show the decision methods presented in the collective decision model based on $\langle R, \mathrm{LA}\rangle$.

Four experts $\mathcal{S}_1$, $\mathcal{S}_2$, $\mathcal{S}_3$, and $\mathcal{S}_4$ are evaluating the performances of five same type of products x_1, x_2, x_3, x_4, and x_5 according to some criteria and to decide which is the best alternative.

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TABLE I
PERFORMANCE EVALUATIONS FOR FIVE ALTERNATIVES USING LINGUISTIC LABELS IN LA

	Alternatives				
	x_1	x_2	x_3	x_4	x_5
$\mathcal{S}_1$	M	M	L	L	VL
$\mathcal{S}_2$	M	L	VL	VH	M
$\mathcal{S}_3$	H	H	M	M	M
$\mathcal{S}_4$	H	VH	L	L	H

Each expert have the same weight and provides a performance vector expressing its performance evaluations for all alternatives. These evaluations are assessed based on the linguistic label set LA $= \{VL, L, M, H, VH\}$.

To obtain the collective performance evaluation for each alternative and make a decision using linguistic information, we should define a semantic relation among linguistic labels LA. First, we define an ordering relation "$<$" on LA such that $VL < L < M < H < VH$. We then assume that the degree of semantic similarity between any two neighboring linguistic labels is 0.5; therefore, we have the following similarity relation R on LA:

$$R = \begin{array}{c} \\ VL \\ L \\ M \\ H \\ VH \end{array} \begin{array}{c} \begin{array}{ccccc} VL & L & M & H & VH \end{array} \\ \begin{bmatrix} 1 & 0.5 & 0 & 0 & 0 \\ 0.5 & 1 & 0.5 & 0 & 0 \\ 0 & 0.5 & 1 & 0.5 & 0 \\ 0 & 0 & 0.5 & 1 & 0.5 \\ 0 & 0 & 0 & 0.5 & 1 \end{bmatrix} \end{array}. \quad (17)$$

A. Comparison Between Symbolic Method and $\langle R, LA\rangle$

In this section, we focus on the comparison between the symbolic method and $\langle R, \text{LA}\rangle$. In order to compare the decision results of symbolic method and $\langle R, \text{LA}\rangle$, we assume that each expert only use the linguistic label in LA to provide the performance evaluations for all alternatives. Assume that their performance evaluations are given in Table I.

When using the fusion model based on the symbolic method, the linguistic labels given by experts are firstly transformed into a value in [0, m]. Then, using (3), we can compute the fused results for all alternatives

$$\overline{y}_1 = (H, -0.5)$$
$$\overline{y}_2 = (H, -0.5)$$
$$\overline{y}_3 = (L, 0)$$
$$\overline{y}_4 = (M, 0)$$
$$\overline{y}_5 = (M, -0.25).$$

Therefore, according to the collective performance evaluations, we can arrange all alternatives in a decreasing ordering: x_1 or $x_2 > x_4 > x_5 > x_3$. But we cannot distinguish x_1 and x_2.

Now, we consider the model based on $\langle R, \text{LA}\rangle$ to make a decision. Denote the collective performance evaluation for alternative x_j as y_j. All y_j's are shown in Table II.

According to Definition 6, each linguistic label can be converted into a probability distribution on LA. These five probability distributions on LA are shown in Table III.

TABLE II
COLLECTIVE PERFORMANCE EVALUATIONS FOR FIVE ALTERNATIVES CORRESPONDING TO TABLE I

	Collective Performance Evaluation
y_1	$\{M, M, H, H\}$
y_2	$\{M, L, H, VH\}$
y_3	$\{L, VL, M, L\}$
y_4	$\{L, VH, M, L\}$
y_5	$\{VL, M, M, H\}$

TABLE III
PROBABILITY REPRESENTATIONS OF LINGUISTIC LABELS IN LA

$Pr_\theta(\cdot)$	VL	L	M	H	VH
VL	3/4	1/4	0	0	0
L	1/6	4/6	1/6	0	0
M	0	1/6	4/6	1/6	0
H	0	0	1/6	4/6	1/6
VH	0	0	0	1/4	3/4

TABLE IV
PROBABILITY REPRESENTATIONS OF COLLECTIVE PERFORMANCE EVALUATIONS GIVEN IN TABLE II

$Pr_{y_i}(\cdot)$	VL	L	M	H	VH
y_1	0	1/12	5/12	5/12	1/12
y_2	1/24	5/24	1/4	13/48	11/48
y_3	13/48	21/48	1/4	1/24	0
y_4	1/12	9/24	1/4	5/48	3/16
y_5	3/16	7/48	9/24	1/4	1/24

Therefore, each collective performance evaluation given in Table II can be converted into a probability distribution on LA using weighted summation. The final results are shown in Table IV.

Then, according to (13), we have

$$G(y_1) = 3.5$$
$$G(y_2) = 3.4375$$
$$G(y_3) = 2.0625$$
$$G(y_4) = 2.9375$$
$$G(y_5) = 2.8125.$$

Notice that $G(y_1) > G(y_2) > G(y_4) > G(y_5) > G(y_3)$, so we have the following ordering on all alternatives: $x_1 > x_2 > x_4 > x_5 > x_3$. All alternatives are distinguished completely, and this decision result is consistent with that of fusion model based on the symbolic method. However, using the fusion model based on the symbolic method, we cannot distinguish alternatives x_1 and x_2. This comparison shows that we can distinguish all alternatives by assuming that any two neighboring linguistic labels have partial semantic overlapping.

If one linguistic label should be assigned to each alternative based on linguistic information given by experts from the above score values and (14), we can say that the most appropriate linguistic labels for x_1, x_2, x_3, x_4, and x_5 are H (or M), M, L, M, and M, respectively.

We now use another decision method to rank all five alternatives by estimating the probabilities $Pr(y_i > y_j)(i, j = 1, \ldots, 5)$. Using (15) and the probability distributions in Table IV, we can derive the probability values $Pr(y_i > y_j)$ given in Table V. From Table V, it is easy to derive that $x_1 > x_2 > x_4 > x_5 > x_3$ since we have $Pr(y_1 > y_2) > Pr(y_2 > y_1)$, $Pr(y_2 > y_4) > Pr(y_4 > y_2)$, $Pr(y_4 > y_5) > Pr(y_5 > y_4)$,

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TABLE V
PROBABILITY VALUES $Pr(y_i > y_j)$, WHERE y_i AND y_j FOR i, $j = 1, \ldots, 5$ ARE GIVEN IN TABLE II

$Pr(y_i > y_j)$	y_1	y_2	y_3	y_4	y_5
y_1	0	0.3802	0.8003	0.5608	0.5295
y_2	0.3663	0	0.7222	0.51	0.5339
y_3	0.0417	0.1016	0	0.1806	0.1949
y_4	0.2448	0.2747	0.566	0	0.4071
y_5	0.1944	0.2569	0.5864	0.395	0

TABLE VI
PERFORMANCE EVALUATIONS FOR FIVE ALTERNATIVES USING LINGUISTIC EXPRESSIONS IN LE

Alternatives				
x_1	x_2	x_3	x_4	x_5
$\neg L$	M	L	$\neg H$	$\neg H$
$\neg VL \wedge \neg VH$	$VL \vee L$	VL	L	M
H	$\neg VL$	M	M	$\neg VL \wedge \neg VH$
$\neg VH$	$\neg H \wedge \neg VH$	$\neg VH$	$\neg VH$	L

TABLE VII
COLLECTIVE PERFORMANCE EVALUATIONS FOR FIVE ALTERNATIVES CORRESPONDING TO TABLE VI

	Collective Performance Evaluation
y_1	$\{\neg L, \neg VL \wedge \neg VH, H, \neg VH\}$
y_2	$\{M, VL \vee L, \neg VL, \neg H \wedge \neg VH\}$
y_3	$\{L, VL, M, \neg VH\}$
y_4	$\{\neg H, L, M, \neg VH\}$
y_5	$\{\neg H, M, \neg VL \wedge \neg VH, L\}$

and $Pr(y_5 > y_3) > Pr(y_3 > y_5)$. This ranking result is consistent with that of the least prejudiced decision method.

B. Using Linguistic Expressions for Decision Making in $\langle R, LA\rangle$

In natural language, humans often use linguistic expressions to evaluate the objects. Therefore, we think that any collective decision model should have the capability to deal with linguistic expressions. One characteristic of collective decision based on $\langle R, \mathrm{LA}\rangle$ is its capability to deal with linguistic expressions in a natural manner.

We continue to discuss the aforementioned example. In this subsection, we assume that the performance evaluations for all alternatives given by four experts are given in Table VI.

We still denote the collective performance evaluation for alternative x_j as y_j. Obviously, some linguistic descriptions provided by experts include more vagueness than that of linguistic labels, but other linguistic expressions provided by experts are more precise than the linguistic labels. The collective performance evaluations for all alternatives are shown in Table VII.

By considering the properties presented in Propositions 1 and 2, we can derive the similarity distributions for all complex linguistic expressions involved in the evaluations given by experts. These similarity distributions are shown in Table VIII.

Then, according to the definition of probability distribution of linguistic expressions (see Definition 6), these similarity distributions correspond to the probability distributions of linguistic expressions given in Table IX.

Finally, by combining all probability distributions on LA corresponding to all linguistic expressions involved in the linguistic evaluations, we can derive five probability distributions on LA for $y_j (j = 1, \ldots, 5)$ (see Table X).

TABLE VIII
SIMILARITY DISTRIBUTIONS DETERMINED BY LINGUISTIC EXPRESSIONS INVOLVED IN TABLE VII

$r(\cdot,\cdot)$	VL	L	M	H	VH
$\neg L$	0.5	0	0.5	1	1
$\neg VL \wedge \neg VH$	0	0.5	1	0.5	0
$\neg VH$	1	1	1	0.5	0
$VL \vee L$	1	1	0.5	0	0
$\neg VL$	0	0.5	1	1	1
$\neg H \wedge \neg VH$	1	1	0.5	0	0
$\neg H$	1	1	0.5	0	0.5

TABLE IX
PROBABILITY REPRESENTATIONS OF LINGUISTIC EXPRESSIONS INVOLVED IN TABLE VII

$Pr_\theta(\cdot)$	VL	L	M	H	VH
$\neg L$	1/8	0	1/8	3/8	3/8
$\neg VL \wedge \neg VH$	0	1/6	2/3	1/6	0
$\neg VH$	7/24	7/24	7/24	1/8	0
$VL \vee L$	5/12	5/12	1/6	0	0
$\neg VL$	0	1/8	7/24	7/24	7/24
$\neg H \wedge \neg VH$	5/12	5/12	1/6	0	0
$\neg H$	3/8	3/8	1/8	0	1/8

TABLE X
PROBABILITY REPRESENTATIONS OF COLLECTIVE PERFORMANCE EVALUATIONS GIVEN IN TABLE VII

$Pr_{y_i}(\cdot)$	VL	L	M	H	VH
y_1	5/48	11/96	30/96	1/3	13/96
y_2	5/24	27/96	31/96	11/96	7/96
y_3	29/96	33/96	27/96	7/96	0
y_4	20/96	36/96	30/96	7/96	1/32
y_5	13/96	33/96	39/96	1/12	1/32

Now, it is easy to deduce the scores for these five linguistic evaluations according to (13)

$$G(y_1) = 3.28125$$

$$G(y_2) = 2.5625$$

$$G(y_3) = 2.125$$

$$G(y_4) = 2.34375$$

$$G(y_5) = 2.53125.$$

Therefore, we have the following ordering on all alternatives: $x_1 > x_2 > x_5 > x_4 > x_3$ since $G(y_1) > G(y_2) > G(y_5) > G(y_4) > G(y_3)$. If we consider all linguistic evaluations provided by experts for each alternative as the constraints on the underlying linguistic value assigned to this alternative, we can see that the decision result derived from the least prejudiced decision method is consistent with this interpretation in this example.

Of course, we can select the linguistic labels having the closest scores to the fused results as the appropriate linguistic labels for the alternatives. According to (14), we can derive that linguistic label M is the most appropriate evaluation for the alternative x_1 and x_2; L is the most appropriate evaluation for other alternatives.

We now check another ranking mechanism by estimating the probabilities $Pr(y_i > y_j)$ for $i, j = 1, \ldots, 5$. According to (15) and probability distributions in Table X, we can obtain the probabilities $Pr(y_i > y_j)$ for i, $j = 1, \ldots, 5$ (see Table XI). From Table XI, we have the following ranking relation such that $x_1 > x_5 > x_2 > x_4 > x_3$ since $Pr(y_1 > y_5) > Pr(y_5 >$

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TABLE XI
PROBABILITY VALUES $Pr(y_i < y_j)$, WHERE y_i AND y_j FOR i, $j = 1, \ldots, 5$ ARE GIVEN IN TABLE VII

$Pr(y_i > y_j)$	y_1	y_2	y_3	y_4	y_5
y_1	0	0.5732	0.6809	0.636	0.5916
y_2	0.2238	0	0.4673	0.4202	0.365
y_3	0.1361	0.2686	0	0.301	0.2459
y_4	0.1732	0.3193	0.414	0	0.2954
y_5	0.196	0.3672	0.4747	0.4135	0

y_1), $Pr(y_5 > y_2) > Pr(y_2 > y_5)$, $Pr(y_2 > y_4) > Pr(y_4 > y_2)$, and $Pr(y_4 > y_3) > Pr(y_3 > y_4)$. Note that, according to the least prejudiced decision method, we have $x_2 > x_5$, which is inconsistent with the current decision result. However, we cannot say which decision method is more superior from one another, although the different decision results may be inferred from these two decision methods. In fact, we can see that x_2 and x_5 are hard to be distinguished, since $|G(x_2) - G(x_5)| = 0.03125$ is close to zero, and $|Pr(y_2 > y_5) - Pr(y_5 > y_2)| = 0.0022$ is also close to zero.

VI. CONCLUSION

In linguistic information fusion, the semantic relation among linguistic labels plays an important role. In our proposed collective decision model, we use an explicit fuzzy relation to capture the semantic similarities among linguistic labels. The inference mechanism in this fuzzy relation, however, is not based on fuzzy logic but the inference method proposed by Tang and Zheng [17]. The main advantage of our proposed model is that it can deal with linguistic labels having partial semantic overlapping and deal with the complex linguistic expressions—the logic formulas generated from linguistic labels and some logic connectives. Traditional linguistic information fusion models are not able to deal with linguistic expressions.

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