A Novel Approach of an Absolute Encoder Coding Pattern

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***Abstract*— This paper presents a novel approach of an absolute rotary and linear optical encoder codding patterns. The concept is based on the analysis of 2-D images to find a unique sequence that allows to unambiguously determine an angular shaft position. The adopted coding method allows to achieve a very high data density for specified number of tracks on the code pattern disc. Encoders based on the proposed solution enable the production from readily available and inexpensive components. Nevertheless, encoders retain high measuring accuracy and high dependability obtained in the classical approach. The optical device and a design of processing system is proposed. Finally, the feasibility of the pattern coding method is further proved with encoder prototype.**

***Index Terms*— Absolute encoder, CCD sensor, Hamiltonian cycle, rotary encoder, 2D image.**

I. Introduction

ROTARY and linear encoders play an extremely important role in automation systems, provide feedback about the current position of the controlled object. Accurate information about the position with a small operating noise level can then be used for precise positioning of robotic manipulators, in flow production lines, surveying instruments, astronomical apparatus and many others.

Currently, most of available absolute rotary encoders use one line of heads (binary detectors) to read a linear code placed on the rotary disc. Classical absolute encoders have a binary code pattern based on the Gray code – a type of binary pattern in which there is only a single bit changing during a transition from one position to the next. This sequential order benefits for the reliability, minimizing the maximum error, that can occur while reading during transition. However, to encode 2^n positions, n separate tracks and sensor channels are needed. Many improvements of the Gray code based pattern have been published in recent years [1]–[4]. There also exist absolute encoders based on De Bruijn [5] sequences or pseudorandom code patterns [6], [7], which consist of only a single track used to find the position. These kind of sensors require to have n heads facing in parallel to the track direction or use only one head and analyze the sequence with microprocessor unit. There are known applications based on image recognition algorithms, that achieve angle or linear displacement measurements [8].

In the article [9] authors propose to use a optical mouse sensor to build an absolute rotary encoder. They try to take advantage of the two-dimensional image matrix. However, contrary to the claims that have been made, they do not use all the available bits, that are achievable on the image in all strategies proposed by the authors.

In this paper the author proposes a novel coding method that allows to encode 2^{n^2} positions with n separate tracks and 2^{n^2} columns. The code design is based on finding a solution for Hamiltonian cycle problem. This will enable the two dimensional (2D) image sensor to reach its full potential. By further improving coding method, the increase the reliability of measurements by introducing a checksum can be achieved, despite the impairing of effective code density [10].

II. Operating Principle of an Optical Encoder

An optical encoder is an electromechanical device allowing to represent the angular or linear position in the form of a digital signal. The following description will focus on the rotary encoder, but the description also applies by analogy with linear encoders. The proposed absolute rotary encoder is composed of light emitter diode, a transparent plastic covering lens, a disc code-pattern mounted on a encoder shaft, a Charge Coupled Device (CCD) matrix sensor and the processor unit with code memory. The high-speed CCD sensor captures the fragment of the pattern disc in respect to the current shafts position. The shaft speed can not exceed the value for which the measurement by the sensor becomes unavailable or non-unique. The correct reading of binary code on the pattern disc allows the processor unit to assign an appropriate angle of the shafts rotation. The assignment between the angle value and the code matrix is stored in non-volatile memory. Appropriate sorting of code patterns in the memory allows direct readout of angular value, without the use of search algorithms.

Numbers of unique codes on the pattern disc determine the encoders resolution. Dependent on the number of output signal wires and the digital representation of angles, encoders may provide either a serial or a parallel output. A common trend in industrial automation are encoders with serial output interfaces, i.a. PROFIBUS, DeviceNet or Synchronous Serial Interface (SSI).

The functional diagram of the discussed above rotary encoder is illustrated in Fig. 1.

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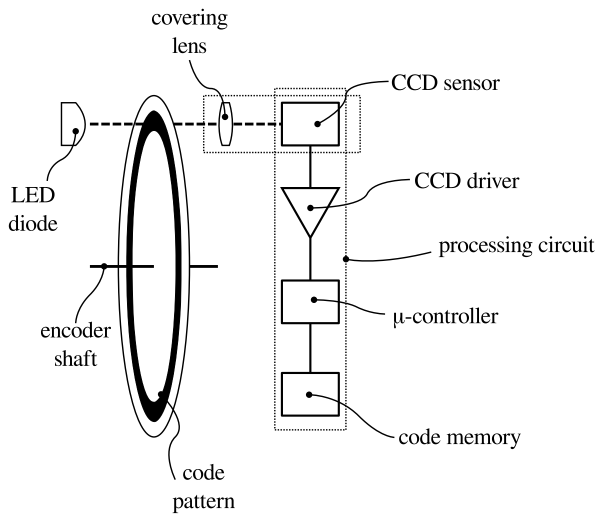

Fig. 1. Functional diagram of an absolute optical rotary encoder.

III. Design of the Pattern Disc

The operation of an absolute encoder is based on the reading and unambiguous decoding of the position from the code marked on the pattern disc. Therefore the code needs to be bijective. Most preferred is the high density of the code, thereby to achieve a high measurement angular resolution. The structure of the pattern code can be represented as cycles in the directed graph (digraph). The goal is to design the cyclic sequence, where all $n \times n$-squares of possible sequences appears exactly once. The analysis of $n \times n$ cell array allows to uniquely identify the angular position of the disk.

Definition 1 (Left Submatrix): A left submatrix of u is a matrix formed by selecting a subset of all columns except last of matrix u in the same relative position.

Definition 2 (Right Submatrix): A right submatrix of u is a matrix formed by selecting a subset of all columns except first of matrix u in the same relative position.

Definition 3: Let $G = (V, E)$ be the simple directed graph composed of a set $V(G)$ of vertices $\{0, 1\}^n$ ($n \in \mathbb{N}, n > 1$) of all n-dimensional, binary square matrices and a set $E(G)$ of edges defined by the following rule: There is an arc between different matrices u and v, if the right submatrix of u is equal to the left submatrix of v

$$(u, v) \in E(G) : u \neq v, \quad \begin{bmatrix} u_{1,2} & u_{1,3} & \cdots & u_{1,n} \\ u_{2,2} & u_{2,3} & \cdots & u_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ u_{n,2} & u_{n,3} & \cdots & u_{n,n} \end{bmatrix} = \begin{bmatrix} v_{1,1} & v_{1,2} & \cdots & v_{1,n-1} \\ v_{2,1} & v_{2,2} & \cdots & v_{2,n-1} \\ \vdots & \vdots & \ddots & \vdots \\ v_{n,1} & v_{n,2} & \cdots & v_{n,n-1} \end{bmatrix} \tag{1}$$

In other words vertex u has a successor v if and only if the right submatrix of u corresponds to the left submatrix of v.

A. Properties of G

The properties of digraph G from Def. 3 are as follows:

TABLE I
Translation Between Cells of the Proposed Code and the Angle

position	code	angle [°]
1	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0
2	$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$	22.5
3	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$	45
4	$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$	67.5
⋮	⋮	⋮
16	$\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$	337.5

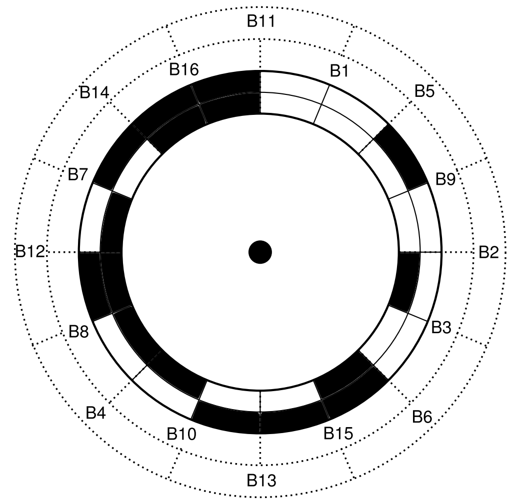

Fig. 2. The 2-track, 4-bit code pattern disc.

- order (number of vertices): $|G| = 2^{n^2}$
- out-degree and in-degree of $V_1 : |V_1| = 2^n$, $d_G^+(V_1) = d_G^-(V_1) = 2^n - 1$
- out-degree and in-degree of $V_2 = V \setminus V_1$: $d_G^+(V_2) = d_G^-(V_2) = 2^n$
- minimum out-degree and in-degree: $\delta^+(G) = \delta^-(G) = 2^n - 1$
- minimum semi-degree: $\delta^0(G) = 2^n - 1$
- maximum out-degree and in-degree: $\Delta^+(G) = \Delta^-(G) = 2^n$
- maximum semi-degree: $\Delta^0(G) = 2^n$.

Further analysis of the digraph G based on definitions from [11] allows to formulate the following theorem.

Theorem 1: Digraph G from def. 3 is strongly connected. The diameter of G is finite and equal to $\operatorname{diam}(G) = \operatorname{dist}(V, V) = n - 1$. □

B. Hamiltonian Cycle

Hamiltonian cycle in digraph G is a cycle that goes through each vertex in the graph exactly once. A graph which has at least one Hamiltonian cycle is called the Hamiltonian graph. The problem of finding high-density coding patterns can be reduced to a problem of finding the Hamiltonian digraph. The formal proof for Hamiltonicity of the digraph G from

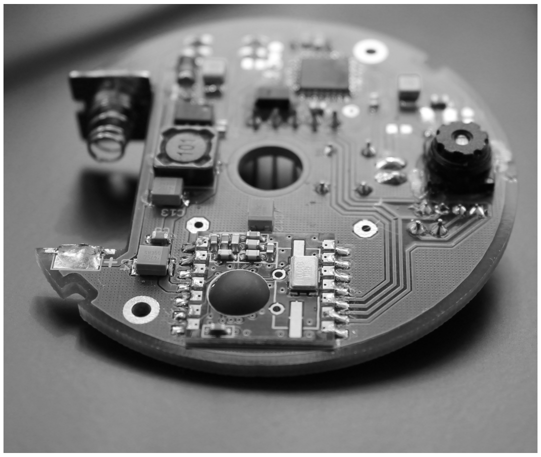

Fig. 3. The prototype of absolute encoder PCB board.

Fig. 4. The prototype of pattern disc mounted on an encoder shaft.

Def. 3 is introduced by theorem 2. For $n = 2$ there are up to 20736 different Hamiltonian cycles.

Each Hamiltonian cycle allows to design a complete and unique pattern disk. Matrices are ordered on a disk in respect to the Hamiltonian cycle and each matrix overlaps the previous by exactly all columns except the last one.

Lemma 1: Euler's Theorem A digraph has an Euler cycle if and only if it is connected and the in-degree of each vertex equals its out-degree.

Lemma 2: If a digraph F is Eulerian, than the line digraph $L(F)$ is Hamiltonian.

Theorem 2: Let's consider the simple directed graph $F = (V, E)$, where a set $V(F)$ of vertices represents of all n-by-$n+1$ binary matrices where $n \in \mathbb{N}, n > 1$ and a set $E(F)$ of edges defined by the following rule: There is an arc between different matrices u and v, if the right submatrix of u is equal to the left submatrix of v. Because F is Eulerian from Lemma 1, the line digraph $L(F) = G$ which is equal to digraph G is Hamiltonian from Lemma 2. $\square$

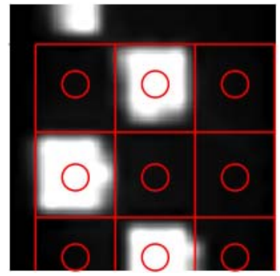

Fig. 5. The image captures of matrix code (n = 3).

IV. Example

Let's consider the graph G for $n = 2$. The set of matrices B represents seed codes for $2^{n^2} = 16$ positions and constitutes the set $V(G)$.

$$B_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \quad B_3 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$
$$B_4 = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \quad B_5 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad B_6 = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$
$$B_7 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad B_8 = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \quad B_9 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$
$$B_{10} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad B_{11} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \quad B_{12} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$
$$B_{13} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \quad B_{14} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \quad B_{15} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$
$$B_{16} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}.$$

The adjacency matrix A is the most convenient way in this case to present a set of edges $E(G)$ in the graph G for a fixed set of vertices. Each row of the adjacency matrix shows a set of successors for a node corresponding with the current row. Analogically, each column of the adjacency matrix shows the set of predecessors for node corresponding with the current column. The following matrix A describes the set of edges for a set of vertices $V(G) = \{B_1 \ldots B_{16}\}$

$$A = \begin{bmatrix}
0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\
1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0
\end{bmatrix}$$

As it is said in subsection III-A, for $n = 2$ there can be found numbers of Hamiltonian cycle in graph G.

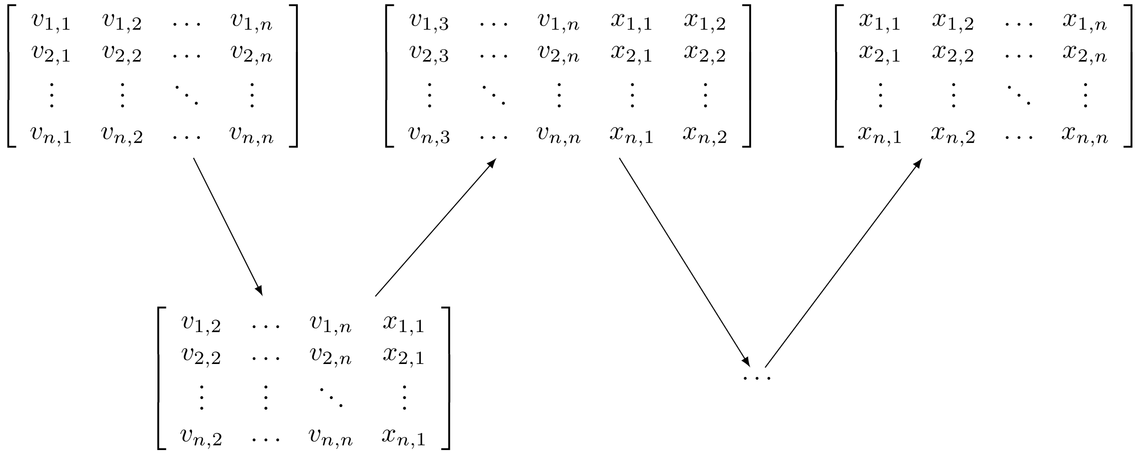

Fig. 6. The illustration of the maximum distance between given vertex v and arbitrary chosen vertex x.

In this example one of Hamiltonian cycles is arbitrarily chosen:

B_1, B_5, B_9, B_2, B_3, B_6, B_{15}, B_{13}, B_{10}, B_4, B_8, B_{12}, B_7, B_{14}, B_{16}, B_{11}, B_1.

Based on above-mentioned sequence, the angular values have been assigned to respective matrices. The selected elements are presented in Table I and the pattern disc for the sequence is shown in Fig. 2 has been designed. The above example only illustrates the concept of the coding pattern and the purpose of this is to make it easier to imagine the design of a higher resolution case.

V. Prototype

The prototype was built to validate applicability of the presented method. The electronic circuit is basically composed of a miniature CCD sensor with array of 15×15 pixels, Light-Emitting Diode (LED), Atmel AVR microcontroller and radio transmitter for wireless data acquisition. The electronic circuit is assembled in two-layer round printed circuit board with a diameter of 60 mm which is shown in Fig. 3.

The pattern code has been printed directly on the polyester film with Computer to Film (CTF) print workflow. The polyester film has been attached to a shaft encoder with the aid of appropriately selected washers (Fig. 4). Light emitted by LED is absorbed by the black fragments of the pattern on the disc. The remaining part of the light passes through the disc and is focused on the CCD sensor.

The sample image captures of the proposed matrix code obtained with the optical CCD sensor, which is shown in Fig. 5. The red circle represents the center pixel of a single cell. The brightness of this pixel determines the logic state. The image shows the part of the pattern disc with 3 tracks and 512 positions.

VI. Conclusion

It seems that the proposed coding method has important practical significance for the production of optical encoders. Power-law increases in code density (2^{n^2}) compared to the classical encoder based on Gray code (2^n) is a significant step towards miniaturization and increasing accuracy. The concept based on searching the Hamiltonian cycle in the set of 2D matrices has not been published so far.

The new method of the pattern-code disc is presented to improve resolution and reduce the costs associated with the production of absolute optical encoders. The binary code presented in the paper allows to obtain the highest data compression. The concept has been finally proved, verified and tested with laboratory prototype.

Appendix A
Proof of Theorem 1

The distance from vertex v to x is the minimum length of a (v, x)-walk, if x is reachable from v, otherwise $\operatorname{dist}(v, x) = \infty$. The diameter of G is distance from a set V to a set V, where the distance from a set is defined as follows:

$$\operatorname{dist}(X, Y) = \max \{\operatorname{dist}(x, y) : x \in X, y \in Y\}.$$

The path between vertex v and x must pass through maximum $n - 1$ nodes, therefore $\operatorname{diam}(G) = n - 1$, which is shown in Fig. 6. ■

Intellectual Property

Part of this work is an invention that is the subject of patent application number P.405218.

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