2007-11-29

One Source Shortest Path: The Bellman-Ford Algorithm

Posted by scvalex under Algorithms, Graphs | Tags: algorithm, bellman-ford, C, explanation, graph, Graphs, programming, shortest path, sourcecode, tutorial | [88] Comments

In this article, I describe the Bellman-Ford algorithm for finding the one-source shortest paths in a graph, give an informal proof and provide the source code in C for a simple implementation.

To understand this you should know what a graph is, and how to store one in memory. If in doubt check this and this.

Another solution to this problem is Dijkstra's algorithm.

The Problem

Given the following graph, calculate the length of the shortest path from **node 1** to **node 2**.

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2007-11-29 at 17:50
Category :
Algorithms, Graphs
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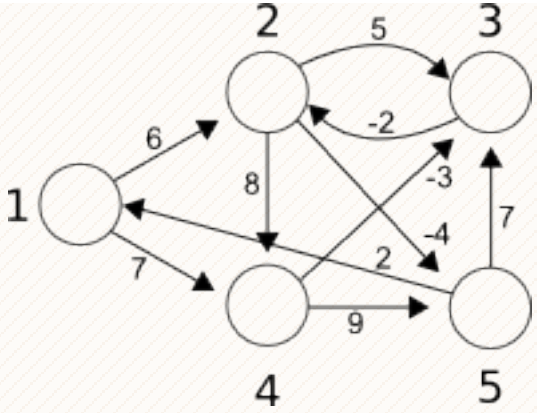


It's obvious that there's a direct route of length *6*, but take a look at path: *1* -> *4* -> *3* -> *2*. The length of the path is *7 – 3 – 2 = 2*, which is less than *6*. BTW, you don't need negative edge weights to get such a situation, but they do clarify the problem.

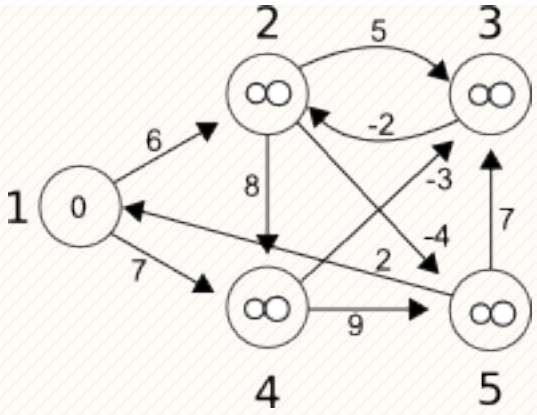
This also suggests a property of shortest path algorithms: to find the shortest path form *x* to *y*, you need to know, beforehand, the shortest paths to *y*'s neighbours. For this, you need to know the paths to *y*'s neighbours' neighbours... In the end, you must calculate the shortest path to the connected component of the graph in which *x* and *y* are found.

That said, you usually calculate **the shortest path to all nodes** and then pick the ones you're intrested in.

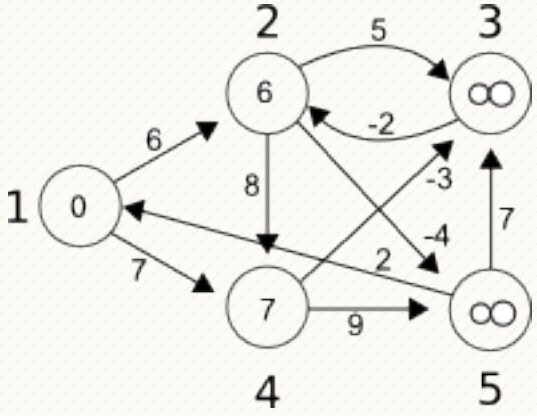
The Algorithm

The Bellman-Ford algorithm is one of the classic solutions to this problem. It calculates the shortest path to all nodes in the graph from a single source.

The basic idea is simple:
Start by considering that the shortest path to all nodes, less the source, is infinity. Mark the length of the path to the source as *0*:

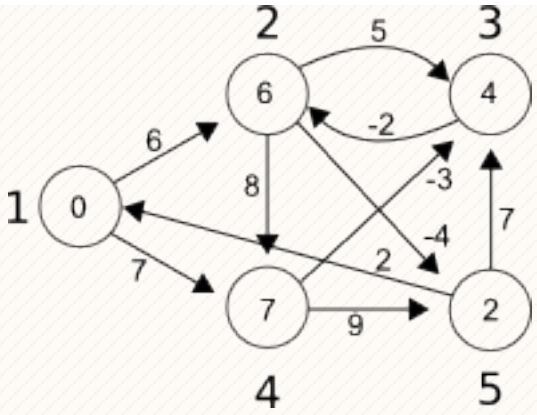


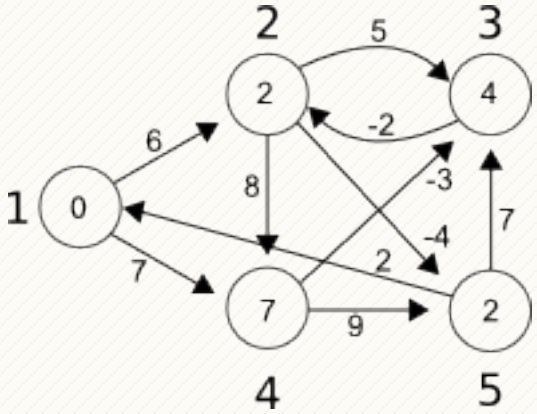
Take every edge and try to *relax* it:

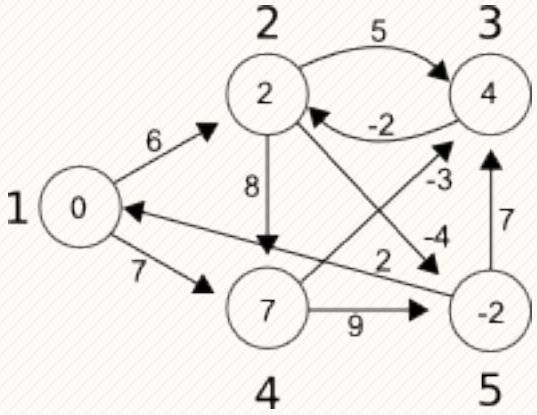


Relaxing an edge means checking to see if the path to the node the edge is pointing to can't be shortened, and if so, doing it. In the above graph, by checking the **edge 1 -> 2** of length *6*, you find that the length of the shortest path to **node 1** plus the length of the **edge 1 -> 2** is less then infinity. So, you replace infinity in **node 2** with *6*. The same can be said for edge *1 -> 4* of length *7*. It's also worth noting that, practically, you can't relax the edges whose start has the shortest path of length infinity to it.

Now, you apply the previous step $n - 1$ times, where n is the number of nodes in the graph. In this example, you have to apply it *4* times (that's *3* more times).







That's it, here's the algorithm in a condensed form:

```
01 void bellman_ford(int s) {
02         int i, j;
03
```

```
        for (i = 0; i < n; ++i)
                d[i] = INFINITY;

        d[s] = 0;

        for (i = 0; i < n - 1; ++i)
                for (j = 0; j < e; ++j)
                        if (d[edges[j].u] + edges[j].w < d[edges[j].v])
                                d[edges[j].v] = d[edges[j].u] + edges[j].w;
}
```

Here, **d[i]** is the shortest path to node **i**, **e** is the number of edges and **edges[i]** is the **i**-th edge.

It may not be obvious why this works, but take a look at what is certain after each step. After the first step, any path made up of at most *2* nodes will be optimal. After the step *2*, any path made up of at most *3* nodes will be optimal... After the *(n – 1)*-th step, any path made up of at most *n* nodes will be optimal.

The Programme

The following programme just puts the **bellman_ford** function into context. It runs in **O(VE)** time, so for the example graph it will do something on the lines of **5 * 9 = 45** relaxations. Keep in mind that this algorithm works quite well on graphs with few edges, but is very slow for dense graphs (graphs with almost n^2 edges). For graphs with lots of edges, you're better off with Dijkstra's algorithm.

Here's the source code in C (bellmanford.c):

```
#include <stdio.h>

typedef struct {
    int u, v, w;
} Edge;

```

```c
int n; /* the number of nodes */
int e; /* the number of edges */
Edge edges[1024]; /* large enough for n <= 2^5=32 */
int d[32]; /* d[i] is the minimum distance from node s to node i
*/

#define INFINITY 10000

void printDist() {
    int i;

    printf("Distances:\n");

    for (i = 0; i < n; ++i)
        printf("to %d\t", i + 1);
    printf("\n");

    for (i = 0; i < n; ++i)
        printf("%d\t", d[i]);

    printf("\n\n");
}

void bellman_ford(int s) {
    int i, j;

    for (i = 0; i < n; ++i)
        d[i] = INFINITY;

    d[s] = 0;

    for (i = 0; i < n - 1; ++i)
        for (j = 0; j < e; ++j)
            if (d[edges[j].u] + edges[j].w < d[edges[j].v])
                d[edges[j].v] = d[edges[j].u] + edges[j].w;
}

int main(int argc, char *argv[]) {
    int i, j;
    int w;
```

```
46
47      FILE *fin = fopen("dist.txt", "r");
48      fscanf(fin, "%d", &n);
49      e = 0;
50
51      for (i = 0; i < n; ++i)
52          for (j = 0; j < n; ++j) {
53              fscanf(fin, "%d", &w);
54              if (w != 0) {
55                  edges[e].u = i;
56                  edges[e].v = j;
57                  edges[e].w = w;
58                  ++e;
59              }
60          }
61      fclose(fin);
62
63      /* printDist(); */
64
65      bellman_ford(0);
66
67      printDist();
68
69      return 0;
70  }
```

And here's the input file used in the example (dist.txt):

```
5
0 6 0 7 0
0 0 5 8 -4
0 -2 0 0 0
0 0 -3 9 0
2 0 7 0 0
```

That's an adjacency matrix.

That's it. Have fun. Always open to comments.

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88 RESPONSES TO "ONE SOURCE SHORTEST PATH: THE BELLMAN-FORD ALGORITHM"

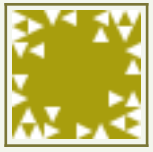
1. Raghavendra Says:

2008-02-08 at 12:58

Thanks a lot for explanation.I have a doubt it detects negative edge cycles.What about the shortest will that be found.Will that include that negative edge also.Will that negative edge be skipped while calculating the shortest path.

Reply

DavivMcD Says:



2009-12-08 at 1:23

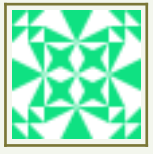
You can use this algorithm to detect negative edge cycles by going 1 step further (for a total of n loops). If a change occurs between the (n-1)th and nth loop, there is a negative edge cycle.

Unfortunately, leaving this vertex out in any shortest path decisions is a much more difficult thing to do.

To put it simply: The Bellman-Ford algorithm can detect negative loops, but it can't work around them.

Reply

Sumit Says:



2011-02-22 at 10:53

To be specific, any negative weigthed cycle reachable from source only not each!!

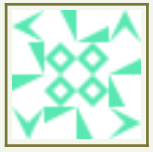
2. papoo Says:

2008-02-29 at 15:27

thank you :)

Reply

3. Rara Says:

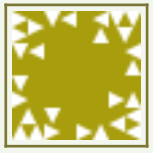


2008-03-12 at 6:01

what's the difference between Dijkstra and Bellman-Ford? i think it's similar. would u like to differ them by their each steps?

Reply

DavivMcD Says:



2009-12-08 at 1:31

Basically the Dijkstra's method marks each node as 'visited' once it relaxes all edges from that node, and will not try to relax any edges leading back to that node after that point. It then chooses the node with the lowest value from the initial node, as the next 'current' node.

Since at each step i out of n nodes, you are relaxing at most n-i edges, (instead of n-1 like Bellman-Ford's), this algorithm completes much quicker than Bellman-Ford's, and scales better with a larger number of nodes. The main drawback is that it will not work with negative edge weights.

Reply

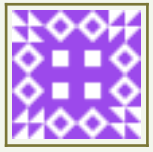
sangamesh Says:

2012-02-12 at 7:33

dijkstra algorithm can be applied to only positive weights, but bellman ford algorithm is applied for both positive and negative weights….

Reply

4. suvo Says:



2008-03-18 at 18:17

I think there is a simple difference btween Dijkstra and Bellman-Ford….like – Bellman-Ford work with negetive weight cycles and relax many time….but Dijkstra not work with negetive cycle and relax only one time…

Reply

LuVar Says:

2010-03-18 at 16:38

Dijkstra cannot work with negative weights at all. If you try it, result will not be shortest path. In JUNG implementation of dijkstra it throws exception if some edge has negative weight.

Reply

DEEPAK UNIYAL Says:

2011-07-06 at 20:07

No algorithm works with negative weight cycle.
Every algorithm fails in case of negative weight cycle.
Bellman ford works with negative weight edges whereas Dijkstra doesn't.

Reply

5. chyrel Says:

2008-05-07 at 21:28

Thanks for the explanation. By the way, can you show me how to find the shortest path in sudoku problem?

Reply

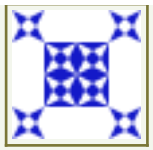
6. george Says:

2008-05-27 at 22:17

can you tell us how to print also the path of the shortest distance.thanks a lot.

Reply

7. Jolmash Says:



2008-09-09 at 5:53

Yeees, how to print also the path of the shortest distance? thanks in advance.

Reply

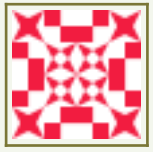
8. Max Says:

2008-10-21 at 16:12

Very good explanations. Much better than my textbook (and my professor). Thanks

Reply

9. Miro Says:



2008-11-24 at 22:21

thanx a lot for explaining, you explained it better than my teacher :)

Reply

10. Abid Says:

2009-02-28 at 22:56

This tutorial is excellent.It has helped me a lot.More tutorial should be published.

Reply

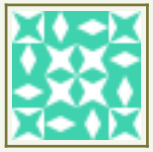
11. Varuna Says:

2009-03-01 at 12:21

thanx… lot of help to the assignment, the dijkstras and bellman ford does arises some complexities but they do differ from negative weights…. thanx again……. keep it up

Reply

12. pavlos Says:



2009-03-23 at 21:57

Bellman-Ford's algorithm returns TRUE if the graph contains no negative weight cycles that are reachable from the source.
So the above source needs some additions to check the above requirement.
For pseudo code you may check this link
http://www.cs.rpi.edu/~musser/gp/algorithm-concepts/bellman-ford-screen.pdf

Reply

13. m@q Says:

2009-03-25 at 15:55

Thanks a lot!! :)

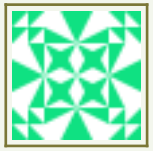
Reply

14. harro Says:

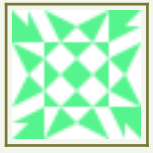
2009-03-26 at 17:06

is there any way to print out the actual path? i.e. node 1 to 3 to 5

Reply

> Sumit Says:
>
> 
>
> 2011-02-22 at 11:02
>
> yes but in reverse. reminding u’ve set parent to the node.. use that..
>
> or u can use recursion to print in sequence
>
> Reply

15. andre Says:



2009-03-27 at 13:30

Ford the best car :)

Reply

16. Hela Says:

2009-03-29 at 4:10

Hi … Does anyone have an example of a real situation where negative weights are used?? Thanks !

Reply

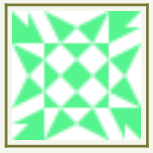
17. sarang s sane Says:

2009-03-31 at 17:09

If i have to laydown the telephone network around the 50 houses around my apartment …Which shortest path method should i use to get it work for me …please explane..

Reply

18. [illegible] Says:

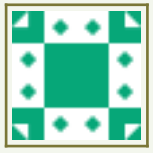


2009-04-02 at 17:17

Ford is a good car :)

Reply

19. JP Says:



2009-04-19 at 16:07

Thanks a lot.

Reply

20. Stephen Says:

2009-05-06 at 0:37

Can the source ever become negative?

Reply

> ramen Says:
>
> 
>
> 2011-02-03 at 14:26
>
> If the source were to ever become negative, then it would be due to a negative weight cycle.
>
> Reply

21. muslima Says:

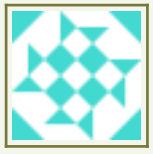
2009-05-18 at 19:11

Thanks alot..

This is simple and helpful :)

Reply

22. rohit Says:



2009-07-01 at 15:47

Great article! However, I was interested in knowing how we could modify

your algorithm to find the Kth shortest path from a source to a node,ie, for K=2,how to find the second shortest path..?

Reply

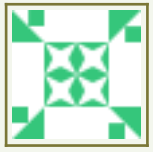
Durjay Says:

2009-09-07 at 17:08

how i find shortest distance when source node 2,3,4,5.

Reply

23. Alyse Says:



2009-07-25 at 12:38

thanks alot...

Reply

24. coolguy Says:

2009-10-21 at 4:12

Very lucid explanation! Thanks for your efforts!

Reply

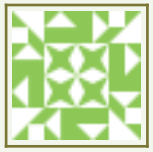
25. ishika jain Says:

2009-11-11 at 10:05

thanks a lot…

Reply

26. Anil Gupta Says:

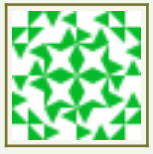


2009-11-16 at 1:20

awesome tuto…it really helps in understanding this algo.Thanks for your efforts!

Reply

27. xxas2axx Says:



2009-11-17 at 23:59

**** you nerds

Reply

28. Osmar Says:

2009-11-22 at 18:26

Excelente este manual…..
Me sirvio para entender mi clase

Gracias

Reply

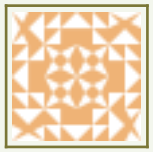
29. akther Says:

2009-12-05 at 7:53

thanks a lot…but is it detect negative cycle??

Reply

30. himanshu Says:

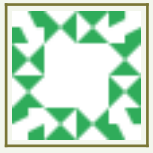


2009-12-08 at 23:12

good explation

Reply

31. Shw05 Says:



2009-12-12 at 7:33

can you please tell me how to print Path in Bellman algorithm ..like the way its done in Dijsktra.
Please provide the code sample if you have..Thanks in advance.

Reply

32. Determinant Says:

2009-12-21 at 22:30

Thank you very much, excellent explanation (Y) really thank you.

I have only one question kindly answer it: what u,v,w represent in this code? it seems to be a silly question.

Thanks in advance

Reply

33. Nelson Says:

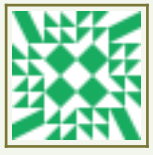
2010-01-02 at 15:21

Hi,

thank you very much.
I think you adjecency matrix has a little mistake: the nine ought to be one spot to the right?
To detect negative circles:
Just let the first for loop go one further, then make an if statement in the existing if statement like if (n == n-1) … an then stop the function and print “negative circle”

Reply

34. yuri Says:



2010-01-10 at 16:18

thank you so much , but the result of this program , is only a matrice , how can’i found the shortest path from onr node to a destination , for example : from 1 to 5.
thank you

Reply

35. Murat Özen Says:

2010-03-27 at 15:48

Thanks for your clear explanation. I have a question that how can I improve this code to find K-th shortest paths.

thank you

Reply

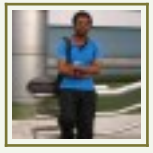
36. Ramkumar Says:

2010-04-01 at 9:31

Cool Dude !!!

Reply

37. Xerxes Says:



2010-04-22 at 7:08

wow… who are you buddy ??? This article is incredibly good, clear and simple. Thank you very much for your article. I'll must recommend my pals to read your article.

Reply

38. Bayan Says:

2010-05-06 at 19:03

thaks very much

Reply

39. ketut nadha Says:

2010-06-05 at 5:22

bagus!!

Reply

40. Martin Says:

2010-07-23 at 3:53

Thanks! The best explanation of Bellman-Ford I have encountered.

Reply

41. saad Says:

2010-08-12 at 19:52

Thanks a lot…nice tutorial..

Reply

42. UVa : 558 (Wormholes) « Solved Programming Problems : Python's Blog Says:

2010-09-01 at 17:11

[...] algorithm (http://compprog.wordpress.com/2007/11/29/one-source-shortest-path-the-bellman-ford-algorithm/ [...]

Reply

43. Nikita Vadher Says:

2010-10-19 at 16:02

Thankx a lot……..

Reply

44. Liviu Says:

2010-12-03 at 15:21

So bellman-ford detects if there’s a negative cycle and tops there?

Doesn’t it find the shortest route by skipping the negative cycle also?

Reply

45. Liviu Says:

2010-12-03 at 15:22

So bellman-ford detects if there’s a negative cycle and tops there?

Doesn’t it find the shortest route by skipping the negative cycle also?

liviu.c.doro@gmail.com

Reply

46. Fahad Says:

2010-12-18 at 5:27

has anyone executed this algorithm by taking the source instead of 0?

Reply

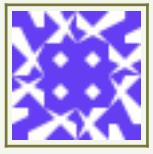
47. Patrick Says:

2011-02-03 at 15:24

Can someone please tell me how the algorithm publishes the shortest path?

Reply

48. elif Says:



2011-03-13 at 18:27

hey i have a question for u all? Can i use Dijsktra algoritm instead of Bellmans for megative weighted graphs, if I add a number for every arc in graph that makes every arc non-negative?

Reply

49. revendera Says:

2011-04-17 at 7:27

thanks

Reply

50. Bellman-Ford Algorithm « Me, when programming Says:

2011-05-15 at 0:35

[...] For example Click here [...]

Reply

51. ájem Says:

2011-05-28 at 20:56

In belmann_ford()

for (j = 0; j < e; ++j)
if (d[edges[j].u] + edges[j].w<d[edges[j].v])
printf("Negativeee Circle\n\n");

hm?

otherwise(){thanks:)}

Reply

52. Pravin Says:

2011-08-05 at 8:37

can u any1 tell me what's the meaning of negative significance(negative weight cycle) in bellman algoritm

Reply

53. Reena Says:

2011-08-22 at 1:19

It was helpful. Thank u so much:-)

Reply

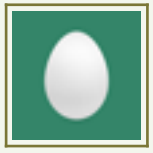
54. John Oke Says:

2011-10-02 at 9:19

So what is thew shortst path and how do you find/work it out
Thanking you in advance
Indiana

Reply

55. sheetal (@sheetal0807) Says:



2011-10-08 at 18:41

Can You plz provide the matlab code for Bellman ford algorithm.

Sonal

Reply

56. Pankaj Nayak Says:

2011-10-11 at 13:19

lalala pila

Reply

57. abnerself Says:

2011-10-15 at 20:57

hola quiero descargar(aquí está el archivo de entrada utilizado en el ejemplo (DIST.TXT):) pero no lo se como soy nuevo usuario, porfavor si pueden me lo embian mi cuenta es abnerself…, o abner.systems@hotmail.com

Reply

58. taman negara pahang Says:

2011-11-05 at 1:56

taman negara pahang…

[...]One Source Shortest Path: The Bellman-Ford Algorithm « Computer programming[...]…

Reply

59. Andy Carloff Says:

2011-11-13 at 0:23

Very well done. The description of “relaxing an edge” appears in Wikipedia, but they don’t give any explanation of what that means. Thanks.

Reply

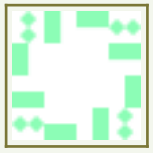
60. Shahal Tharique Says:

2011-11-21 at 19:30

I still don’t get this. In the first loop iteration of the outer loop, let’s say by ur example, u first modify edge 1->2 and edge 1->4, what’s the problem in modifying the edge 2->3, 2->5, 4->3, 4->5, since we have d[2] and d[4] now.

Reply

61. medichismes Says:



2011-11-25 at 8:12

Thanks!!! =D

Reply

62. Dhritisundar Maity Says:

2011-11-26 at 16:03

Given a weighted, directed graph G = (V, E) with no negative-weight cycles, let m be the
maximum over all pairs of vertices u, v V of the minimum number of edges in a shortest
path from u to v. (Here, the shortest path is by weight, not the number of edges.) Suggest a
simple change to the Bellman-Ford algorithm that allows it to terminate in m + 1 passes.plz solve this…

Reply

63. wsnets Says:

2011-12-04 at 6:13

Is there any Matlab code for the Bellman-ford algorithm. I will appreciate if someone can point me to one please. Thanks.

Reply

64. wsnets Says:

2011-12-04 at 6:14

Is there any Matlab code for the Bellman-ford algorithm. Thanks.

Reply

65. Jhonatam Says:

2011-12-15 at 4:17

Hello I wonder if it is possible to print the nodes where he found the best way would be and how to do this?

Reply

66. sara Says:

2012-01-01 at 15:29

thamks alot This really helped me….

Reply

67. maternity fashion Says:

2012-01-09 at 21:05

maternity fashion…

[...]One Source Shortest Path: The Bellman-Ford Algorithm « Computer programming[...]…

Reply

68. Samir Says:

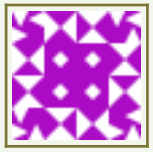
2012-01-11 at 2:02

Samir…

[...]One Source Shortest Path: The Bellman-Ford Algorithm « Computer programming[...]...

Reply

69. venu] Says:

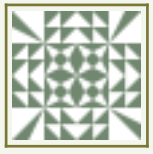


2012-03-03 at 16:41

its not clear

Reply

70. abhi Says:

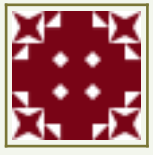


2012-03-06 at 6:25

can u explain the problem in stepwise..

Reply

71. akhileshazad Says:



2012-04-22 at 13:41

i am not be able to open dist.txt file in Bellman-Ford algorithm example plz help me

thanks ;

Reply

72. ameenos2000 Says:

2012-04-22 at 18:50

Reblogged this on ameenos2000.

Reply

73. n1ght3 Says:

2012-04-26 at 8:04

Simple. Concise. I liked it.

Reply

74. Gos Says:

2012-04-26 at 16:18

Thanks ! For an awesome explanation! was struggling for it….
but I needed a java program for this….thanks anyways!

Reply

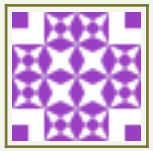
75. ricky... Says:

2012-05-14 at 1:01

nice

Reply

76. Hussnain Says:



2012-05-16 at 18:05

thanks a lottttttttttttttttt

Reply

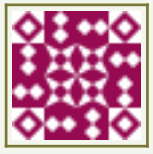
77. nimisha Says:

2012-06-18 at 15:47

when there is positive weights than dijikstras will be applied and for negative weights bellmanford is applied

Reply

78. foufidoom Says:



2012-06-25 at 15:23

Hi there! I am trying to do some kind of programming exercise here on Bellman-Ford algorithm, I'm good with algorithms but I don't have much experience with program languages, not even c. I understood a lot from the way you built your program but unfortunately I can't open the text you were using to import your data, it takes me to a page where it says that should be logged in (I am) and a member of your blog (following is not enough? :( ). I'd really appreciate it if you could help me see that text. Thank you very much.

Reply

79. nilesh Says:

2012-07-31 at 17:40

Can anyone tell me the application of Bellman ford algo and where these negative edge are used?

Reply

LEAVE A REPLY

Enter your comment here...