

# A Gentle Adventure Mechanising Message Passing Concurrency Systems

## An Experience/Walkthrough Report

David Castro-Perez, Lorenzo Gheri, Francisco Ferreira, Martin Vassor, and Nobuko Yoshida

DisCoTec22

Imperial College  
London

University of  
**Kent**



# Roadmap

## 1. Binders and Linearity

**Act I:** Binary Session Types

## 2. Multiparty Processes and Coinduction

**Act II:** Mechanising Multiparty Processes

**Act III:** Mechanising Multiparty Session Types

# SmolEMTST: Tutorial Repository

The screenshot shows the GitHub repository page for `emtst/GentleAdventure`. The repository is public and has 0 forks and 0 stars. The main branch is `main` with 1 branch and 0 tags. The repository contains 92 commits, with the latest commit `8efff16` made 6 days ago. The repository is titled "A Gentle Adventure Mechanising Message Passing Concurrency Systems".

The repository structure is as follows:

File	Description	Commit Date
<code>act1</code>	powerpoint format for the intro too	28 days ago
<code>act2</code>	Detail a bit more <code>step</code> and the local types.	7 days ago
<code>act3</code>	more	6 days ago
<code>doc</code>	updated it to support latex generation	16 months ago
<code>.gitignore</code>	started act1 slides from David's talk on Zoid, to keep the format (s...	11 months ago
<code>LICENSE</code>	Initial commit	16 months ago
<code>Readme.txt</code>	changed the name of a file	11 months ago

The `Readme.txt` file content is:

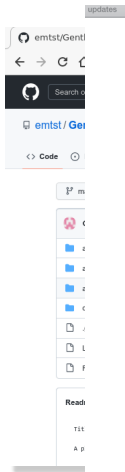
```
Title: A Gentle Adventure Mechanising Message Passing Concurrency Systems

A play in three acts.
```

The right sidebar contains the following sections:




- About**: A Gentle Adventure Mechanising Message Passing Concurrency Systems. Includes links to Readme, MIT license, 0 stars, 3 watching, and 0 forks.
- Releases**: No releases published. [Create a new release](#).
- Packages**: No packages published. [Publish your first package](#).

# SmolEMTST: Tutorial Repository




updates


## EMTST: Engineering the Meta-theory of Session Types

David Castro , Francisco Ferreira , and Nobuko Yoshida 

Imperial College London,  
{d.castro-perez, f.ferreira-ruiz, n.yoshida}  
@imperial.ac.uk



**Abstract** Session types provide a principled programming discipline for structured interactions. They represent a wide spectrum of type-systems for concurrency. Their type safety is thus extremely important. EMTST is a tool to aid in representing and validating theorems about session types in the Coq proof assistant. On paper, these proofs are often tricky, and error prone. In proof assistants, they are typically long and difficult to prove. In this work, we propose a library that helps validate the theory of session types calculi in proof assistants. As a case study, we study two of the most used binary session types systems: we show the impossibility of representing the first system in  $\alpha$ -equivalent representations, and we prove type preservation for the revisited system. We develop our tool in the Coq proof assistant, using locally nameless for binders and small scale reflection to simplify the handling of linear typing environments.



# SmolEMTST: Tutorial Repository



## Zooid: A DSL for Certified Multiparty Computation

From Mechanised Metatheory to Certified Multiparty Processes

David Castro-Perez  
Imperial College London and University of Kent, UK

Lorenzo Gheri  
Imperial College London, UK

Francisco Ferreira  
Imperial College London, UK

Nobuko Yoshida  
Imperial College London, UK

### Abstract

We design and implement Zooid, a domain specific language for certified multiparty communication, embedded in Coq and implemented atop our mechanisation framework of asynchronous multiparty session types (the first of its kind). Zooid provides a fully mechanised metatheory for the semantics of global and local types, and a fully verified end-point process language that faithfully reflects the type-level behaviours and thus inherits the global types properties such as deadlock freedom, protocol compliance, and liveness guarantees.

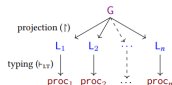


Figure 1. MPST in a nutshell

compliance for processes. Session types consist of actions for sending and receiving, sequencing, choices, and recur-

and error prone. In proof assistants, they are typically long and difficult to prove. In this work, we propose a library that helps validate the theory of session types calculi in proof assistants. As a case study, we study two of the most used binary session types systems: we show the impossibility of representing the first system in  $\alpha$ -equivalent representations, and we prove type preservation for the revisited system. We develop our tool in the Coq proof assistant, using locally nameless for binders and small scale reflection to simplify the handling of linear typing environments.

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# Mechanising the **Honda, Vasconcelos and Kubo's binary session type system**

Honda, K., Vasconcelos, V. T., and Kubo, M. (1998). [Language primitives and type discipline for structured communication-based programming](#). In Hankin, C., editor, Programming Languages and Systems, pages 122–138, Berlin, Heidelberg. Springer Berlin Heidelberg

# Processes: Key Features

$P, Q, R ::=$	
$k![e]; P$	data sending
$k?(x).P$	data receiving
$\text{throw } k[k']; P$	channel sending
$\text{catch } k(k').P$	channel receive
$P \mid Q$	parallel composition
$\nu_c(k).P$	channel hiding
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# Binder Mechanisation: DeBruijn Indices

Terms  $M, N ::= n \mid M N \mid \lambda. M$

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$\lambda. (\lambda. 0) (\lambda. 0 1)$



# Binder Mechanisation: Locally Nameless

Natural number

Name that represents a free variable

Terms:  $M, N ::= n \mid x \mid M N \mid \lambda. M$

# Binder Mechanisation: Locally Nameless

Terms:  $M, N ::= n \mid x \mid M N \mid \lambda. M$

$M^x \equiv \{0 \rightarrow x\}M$     Open a term

$$\frac{\Gamma(x) = T}{\Gamma \vdash x : T}$$

$$\frac{\forall x \notin L \quad \Gamma, x : S \vdash M^x : T}{\Gamma \vdash \lambda. M : S \rightarrow T}$$

$\backslash^x M \equiv \{0 \leftarrow x\}M$     Close a term

$$\frac{\Gamma \vdash M : S \rightarrow T \quad \Gamma \vdash N : S}{\Gamma \vdash M N : T}$$

$\text{lc}(M)$     Locally closed term

# Process Mechanisation

```
Inductive name : Set :=
| fnm : atom → name
| bnm : nat → name
.

Definition channel := name.

Inductive proc : Set :=
| send : channel → exp → proc → proc
| receive : channel → proc → proc

| throw : channel → channel → proc → proc
| catch : channel → proc → proc

| nu_ch : proc → proc (* hides a channel *)
...
```

```
Inductive lc : proc → Prop :=
| lc_send : forall k e P,
  lc_nm k →
  lc_exp e →
  lc P →
  lc (send k e P)

| lc_receive : forall (L : seq atom) k P,
  lc_nm k →
  (forall x, x \notin L → lc (open P x)) →
  lc (receive k P)

| lc_throw : forall k k' P,
  lc_nm k → lc_nm k' →
  lc P →
  lc (throw k k' P)

| lc_catch : forall (L : seq atom) k P,
  lc_nm k →
  (forall x, x \notin L → lc (open P x)) →
  lc (catch k P)
...
```

## Semantics (Excerpt as in Paper)

R-PASS      $\text{throw } k [k']; P \mid \text{catch } k (k').Q \rightarrow P \mid Q$

R-COM      $k![e]; P \mid k?(x).Q \rightarrow P \mid \{x \rightarrow e\}Q$

R-CONG      $P \equiv P' \text{ and } P' \rightarrow Q' \text{ and } Q' \equiv Q \Rightarrow P \rightarrow Q$

R-SCOP      $P \rightarrow Q \Rightarrow \nu_c(k).P \rightarrow \nu_c(k).Q$

R-PAR      $P \rightarrow P' \Rightarrow P \mid Q \rightarrow P' \mid Q$

# Mechanising the Semantics

Using Locally Nameless,  $\alpha$ -equivalent terms are **syntactically equal**

How do we mechanise  $R_{\text{PASS}}$ ?

$$\text{throw } k [k']; P \mid \text{catch } k (k').Q \rightarrow P \mid Q$$



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A naive attempt:

$$\text{throw } k [k']; P \mid \text{catch } k ().Q \rightarrow P \mid Q^{k'}$$

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How do we mechanise  $R_{\text{PASS}}$ ?

$$\text{throw } k [k']; P \mid \text{catch } k (k').Q \rightarrow P \mid Q$$

A **WRONG** attempt:

$$\text{throw } k [k']; P \mid \text{catch } k ().Q \rightarrow P \mid Q^{k'}$$

## Why is our mechanisation of $r\text{-pass}$ wrong?

Type     $\alpha, \beta ::= ![S]; \alpha \mid ?[S]; \alpha \mid \text{end} \mid \perp$   
Typing    $\Delta ::= \cdot \mid \Delta, k : \alpha$

**Subject Reduction:** if  $\Gamma \vdash P \triangleright \Delta$  with  $\Delta$  balanced, and  $P \rightarrow Q$ , then there exists  $\Delta'$  s.t.  $\Gamma \vdash Q \triangleright \Delta'$ , with  $\Delta'$  balanced.

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Rule  $\text{throw } k[k']; P \mid \text{catch } k().Q \rightarrow P \mid Q^{k'}$   
breaks subject reduction.

# The Problem with Equating $\alpha$ -equivalent Terms

The idea behind  $\text{R-PASS}$ :

$$\begin{array}{l} \text{throw } k [k_0]; P \mid \text{catch } k (k_1).Q \rightarrow P \mid Q' \\ \text{if } \text{catch } k (k_1).Q \equiv_{\alpha} \text{catch } k (k_0).Q' \end{array}$$

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$k_0$  cannot be free in  $Q$

We used a **standard** representation of binders, to mechanise a **non-standard, but correct** use of binders.



# Mechanising the Revised System

Yoshida, N. and Vasconcelos, V. T. (2007). [Language primitives and type discipline for structured communication-based programming revisited: Two systems for higher-order session communication](#). Electronic Notes in Theoretical Computer Science, 171(4):73 – 93. [Proceedings of the First International Workshop on Security and Rewriting Techniques \(SecReT 2006\)](#)

# Processes, Channels, and Polarities

A solution to the naive R-PASS is to distinguish **channel polarities** ([Gay and Hole, 2005])

$$p, q ::= + \mid -$$
$$\text{throw } k^p [k'^q]; P \mid \text{catch } k^{\bar{p}} ().Q \rightarrow P \mid Q^{k'^q}$$

```
Inductive channel :=  
| Ch of (kvar * polarity)  
| Var of cvar.
```

```
Inductive proc : Set :=  
| send : channel → exp → proc → proc  
| receive : channel → proc → proc  
  
| throw : channel → channel → proc → proc  
| catch : channel → proc → proc  
  
| nu_ch : proc → proc  
...
```

# Atoms: Separating Namespaces

```
Module CA := AtomScope Atom.Atom. (* Module of the atoms for channels *)
Module KA := AtomScope Atom.Atom. (* Module of the atoms for channel name *)
Module EA := AtomScope Atom.Atom. (* Module of the atoms for expressions *)

Notation cvar := (CA.var).
Notation kvar := (KA.var).
Notation evar := (EA.var).
```

# Typing

Type  $\alpha, \beta ::= ![S]; \alpha \mid ?[S]; \alpha \mid \text{end} \mid \dots$

Typing  $\Delta ::= \cdot \mid \Delta, k : \alpha$

Judgement  $\Gamma \vdash P : \Delta$

```
Inductive tp : Set :=
| input : sort → tp → tp
| output : sort → tp → tp
| ended : tp
| ...
```

```
Inductive oft : sort_env → proc → tp_env → Prop :=
| t_send : forall G kt e P D S T,
  oft_exp G e S →
  oft G P (add kt T D) →
  oft G (send (chan_of_entry kt) e P) (add kt (output S T) D)
| ...
```

# Linear Environments

$$\Delta ::= \cdot \mid \Delta, k : \alpha$$

```
Inductive env := Undef | Def of {finMap K → V}.
```

```
Definition add x t E :=  
  if x \in dom E then Undef else upd x t E.
```

We defined operations on environments that contain **linear** channels.

Adding a channel that is already in the environment results in an **undefined** environment.

We use this fact pervasively in our mechanised proofs.

# Subject Reduction

**Theorem** SubjectReductionStep  $G \ P \ Q \ D$ :  
 $\text{oft } G \ P \ D \rightarrow P \longrightarrow Q \rightarrow \text{exists } D', D \rightsquigarrow D' \wedge \text{oft } G \ Q \ D'.$

**Theorem** SubjectReduction  $G \ P \ Q \ D$ :  
 $\text{oft } G \ P \ D \rightarrow P \longrightarrow * \ Q \rightarrow \text{exists } D', \text{oft } G \ Q \ D'.$

# Mechanising a Proof of Subject Reduction

Using separate namespaces requires us to prove distinct **substitution lemmas** for every different kind of binder (expression, channel, shared channel).




Separate namespaces helps us avoid errors (e.g. using a channel instead of an expression), and simplifies proofs.

Linear environments allow us to make simplifying assumptions about defined environments.

# Summary of Act I

- Deep Embedding binders allows us to fully control the calculus.
- LN requires a number of theorems and lemmas to prove our basic safety properties.
- EMTST (our tool) helps with nominal sets and environments.
- Next, we will explore what do we gain if we give up control (using shallow embeddings).



-  Gay, S. and Hole, M. (2005). Subtyping for Session Types in the Pi Calculus. [Acta Informatica](#), 42(2):191–225.
-  Honda, K., Vasconcelos, V. T., and Kubo, M. (1998). Language primitives and type discipline for structured communication-based programming. In Hankin, C., editor, [Programming Languages and Systems](#), pages 122–138, Berlin, Heidelberg. Springer Berlin Heidelberg.
-  Yoshida, N. and Vasconcelos, V. T. (2007). Language primitives and type discipline for structured communication-based programming revisited: Two systems for higher-order session communication. [Electronic Notes in Theoretical Computer Science](#), 171(4):73 – 93. Proceedings of the First International Workshop on Security and Rewriting Techniques (SecReT 2006).