Coq-Metatheory for Smol-Zooid

A Gentle Adventure Mechanising Message Passing Concurrency Systems, Act 3

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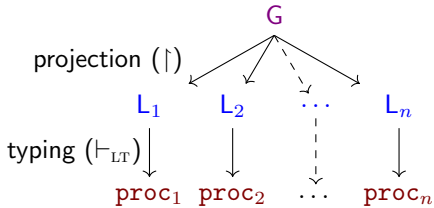
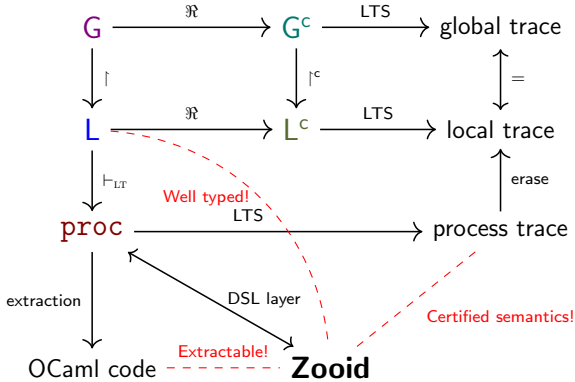
The MPST World, as We Know It



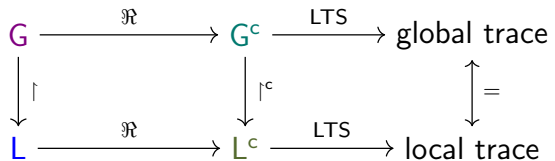
K. Honda, N. Yoshida, and M. Carbone. Multiparty asynchronous session types. POPL '08

Zooid



D. Castro-Perez, F. Ferreira, L. Gheri, and N. Yoshida. Zooid: a DSL for certified multiparty computation: from mechanised metatheory to certified multiparty processes. PLDI 2021

Introducing the Metatheory of Smol-Zooid Types



- unravelling preserves projection; focus on coinduction (1^{st} square)
- trace equivalence; focus on soundness (2^{nd} square)

`https://github.com/emtst/GentleAdventure/act3`

Formalisation of Global and Local Types

Inductively Defined Datatypes

$$
\begin{aligned}
\mathsf{G} \;::=\; & \mathtt{end} \\
& \mid X \\
& \mid \mu X.\mathsf{G} \\
& \mid \mathsf{p} \to \mathsf{q} :(\mathbb{S}).\mathsf{G}
\end{aligned}
$$

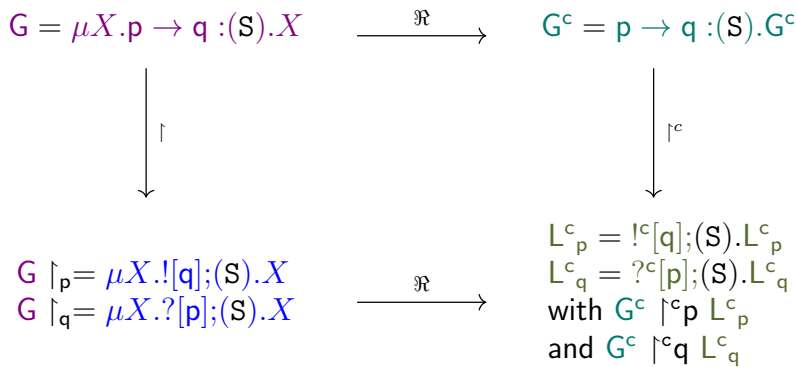
$$
\begin{aligned}
\mathsf{L} \;:=\; & \mathtt{end} \\
& \mid X \\
& \mid \mu X.\mathsf{L} \\
& \mid \,!\,[\mathsf{q}];(\mathbb{S}).\mathsf{L} \\
& \mid \,?\,[\mathsf{p}];(\mathbb{S}).\mathsf{L}
\end{aligned}
$$

Coinductively Defined Datatypes

$$
\begin{aligned}
\mathsf{G}^{\mathsf{c}} ::= \; & \mathtt{end}^{\mathsf{c}} \\
& \mid \mathsf{p} \to \mathsf{q} :(\mathbb{S}).\mathsf{G}^{\mathsf{c}} \\
& \mid \mathsf{p} \rightsquigarrow \mathsf{q} :(\mathbb{S}).\mathsf{G}^{\mathsf{c}}
\end{aligned}
$$

$$
\begin{aligned}
\mathsf{L}^{\mathsf{c}} ::= \; & \mathtt{end}^{\mathsf{c}} \\
& \mid \,!^{\mathsf{c}}[\mathsf{p}];(\mathbb{S}).\mathsf{L}^{\mathsf{c}} \\
& \mid \,?^{\mathsf{c}}[\mathsf{q}];(\mathbb{S}).\mathsf{L}^{\mathsf{c}}
\end{aligned}
$$

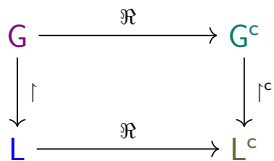
Formalisation of Global and Local Types



Abandoning Inductive Datatypes

Theorem (Unravelling preserves projections)

Given G, L, G^c *and* L^c, *such that* (a) $\mathsf{G}\restriction_{\mathsf{r}} = \mathsf{L}$, (b) $\mathsf{G}\mathrel{\Re}\mathsf{G}^c$, *and* (c) $\mathsf{L}\mathrel{\Re}\mathsf{L}^c$, *then* $\mathsf{G}^c \restriction^c_{\mathsf{r}} \mathsf{L}^c$.



Proof.

By coinduction. :) □

The Paco Library for Coq: https://plv.mpi-sws.org/paco/

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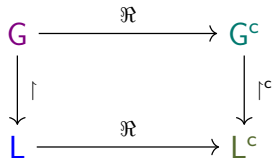

⟶ Coq!

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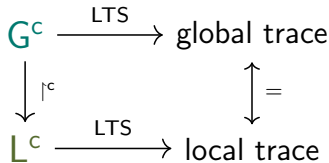
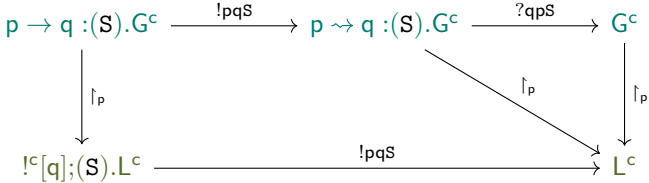
Type Semantics for Zooid

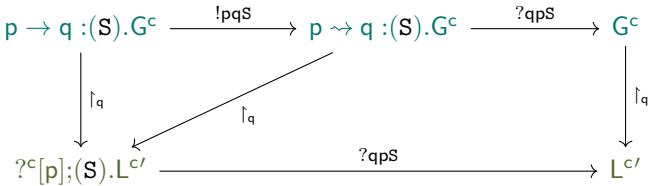


With Love, from p to q

p sends:



q receives:



Tools for our LTS

Actions. $!\mathsf{pqS}$ and $?\mathsf{qpS}$

(Local) Environments. E such that, $E(\mathsf{p}) = \mathsf{L}^{\mathsf{c}}{}_{\mathsf{p}}$ where $\mathsf{G}^{\mathsf{c}} \upharpoonright^{\mathsf{c}} \mathsf{p}\ \mathsf{L}^{\mathsf{c}}{}_{\mathsf{p}}$

Queues and Queue Environments. Q, buffers for asynchronous communication.

$$!^{\mathsf{c}}[\mathsf{q}];(\mathsf{S}).\mathsf{L}^{\mathsf{c}} \xrightarrow{\text{step}} \mathsf{L}^{\mathsf{c}}$$

$$Q(\mathsf{p},\mathsf{q}) = [] \xrightarrow{\text{enqueue}} Q(\mathsf{p},\mathsf{q}) = [\mathsf{S}] \xrightarrow{\text{dequeue}} Q(\mathsf{p},\mathsf{q}) = []$$

$$?^{\mathsf{c}}[\mathsf{p}];(S).\mathsf{L}^{\mathsf{c}\prime} \xrightarrow{\text{step}} \mathsf{L}^{\mathsf{c}\prime}$$

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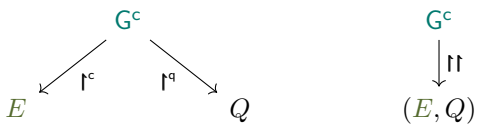


Theorems

Theorem (Step Soundness)

If $\mathsf{G}^c \xrightarrow{a} \mathsf{G}^{c\prime}$ *and* $\mathsf{G}^c \upharpoonright \lceil(E, Q)$*, there exist* E' *and* Q' *such that* $\mathsf{G}^{c\prime} \upharpoonright \lceil(E', Q')$ *and* $(E, Q) \xrightarrow{a} (E', Q')$*.*

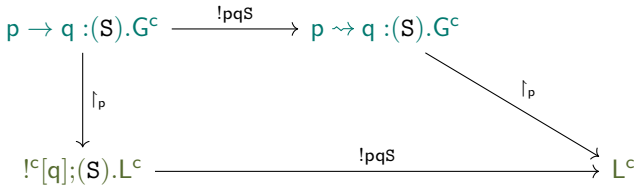
Theorem (Step Completeness)

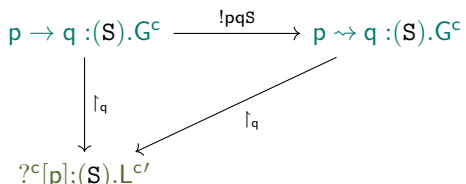
If $(E, Q) \xrightarrow{a} (E', Q')$ *and* $\mathsf{G}^c \upharpoonright \lceil(E, Q)$*, there exist* $\mathsf{G}^{c\prime}$ *such that* $\mathsf{G}^{c\prime} \upharpoonright \lceil(E', Q')$ *and* $\mathsf{G}^c \xrightarrow{a} \mathsf{G}^{c\prime}$*.*

Theorem (Trace equivalence)

If $\mathsf{G}^c \upharpoonright \lceil(E, Q)$*, then* $\mathtt{tr}^g t \mathsf{G}^c$ *if and only if* $\mathtt{tr}^l t(E, Q)$ *.*

Lemma, to give the flavour





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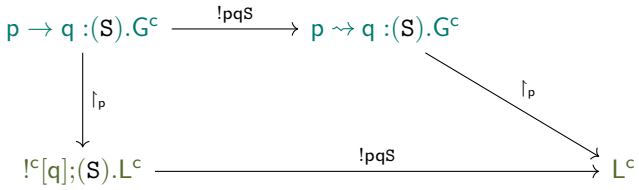

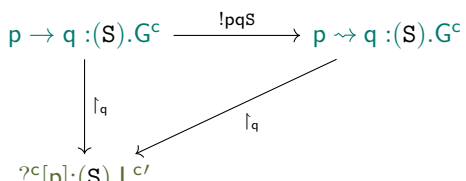

$\longrightarrow$ Coq!

You Suffer...

- Formal proofs are not easy.
- Proof design is the key.
- Proof techniques are to be taken seriously: (co)induction, functions VS relations, treatment of bindings...

→ D. Castro-Perez, F. Ferreira, L. Gheri, and N. Yoshida. "Zooid: a DSL for certified multiparty computation: from mechanised metatheory to certified multiparty processes". PLDI 2021.
DOI: `https://doi.org/10.1145/3453483.3454041`
website: `http://mrg.doc.ic.ac.uk/publications/zooid-paper/`

→ This tutorial is available at `https://github.com/emtst/GentleAdventure`

... but Why?

Formal proofs are not easy

[1] Aydemir et al. "Mechanized Metatheory for the Masses: The POPLmark challenge." 2005

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Formal proofs are not easy, but useful and fun!

As witnessed, e.g., by the influential POPLmark Challenge[1]...

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Towards a **Concurrent Calculi Formalisation Benchmark**

Challenge problems:

- name passing and scope extrusion
- linearity and behavioural type systems
- coinduction and reasoning about process algebras

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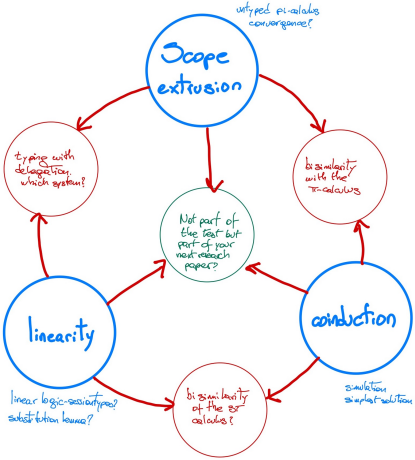
The Future is Unwritten... But Sketched!



- Concurrent Benchmark website: `https://concurrentbenchmark.github.io/`
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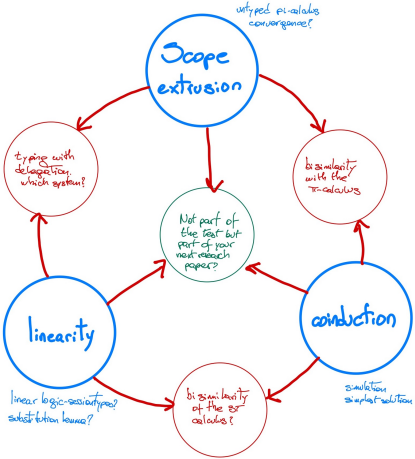

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THANK YOU!