Video Communications Homework 1

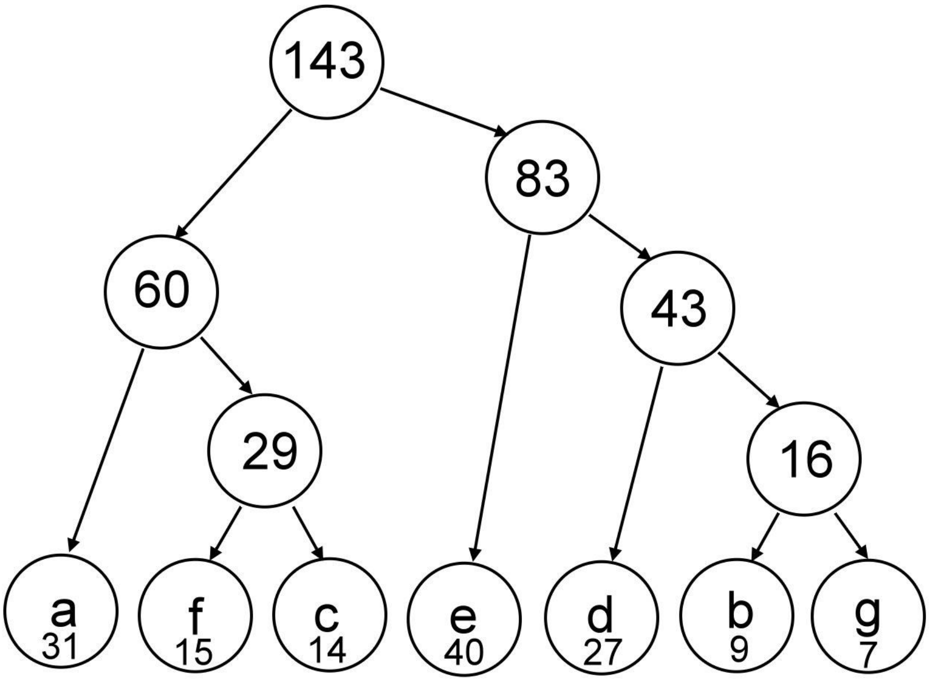
B04902028　資工三　洪浩翔

1. Problem 1: Huffman Coding

Given a text with characters a, b, ..., g occurring with the following frequencies:

A	B	C	D	E	F	G
31	9	14	27	40	15	7

Please design a Huffman tree to encode those alphabets and compare the average length of codes per symbol with the entropy.



The Huffman Tree

A	B	C	D	E	F	G
00	1110	011	110	10	010	1111

The encoded bits

The entropy is 2.58, while the average length of codes is 2.6153846. These two values are closed in fact. That is why we use Huffman tree to encode the characters through frequencies.

Problem 2: Hamming Code

(a) Given a received bit stream, find which bit is corrupted.

1 0 1 0 0 1 0 1 0 0 1

After calculated, only P8 = (0+0+1)%2 = 1 is correct, while P1 = (1+0+0+0+1)%2, P2 = (1+1+0+0+1)%2 = 1, and P4 = (0+1+0)%2 = 1 is different with the original number. Therefore, the error bit is 0111, which means bit 7 is corrupted in fact.

(b)Please generate a hamming code of the following bit stream.

1 0 1 1 0 1 0

P1 = (1+0+1+0+0)%2 is 0, P2 = (1+1+1+1+0)%2 is 0, P4 = (0+1+1)%2 is 0, and P8 = (0+1+0)%2 is 1. As a result, the bit stream is 00100111010 in the end.

Problem 3: JPEG

(a) Please find Y, Cb and Cr values of the following 4x4 image with RGB channels as:

$$R = \begin{bmatrix} 217 & 217 & 217 & 219 \\ 220 & 221 & 217 & 218 \\ 222 & 218 & 223 & 218 \\ 224 & 220 & 221 & 218 \end{bmatrix}, G = \begin{bmatrix} 141 & 137 & 143 & 142 \\ 137 & 146 & 147 & 143 \\ 148 & 146 & 159 & 153 \\ 159 & 155 & 166 & 163 \end{bmatrix}, B = \begin{bmatrix} 119 & 133 & 132 & 127 \\ 125 & 133 & 141 & 139 \\ 135 & 134 & 140 & 142 \\ 140 & 136 & 138 & 154 \end{bmatrix}.$$

.

Please explain why the color space should be converted to YCbCr before applying DCT.

We can find the YCbCr values with the equations:

$$Y = 0.299*R + 0.587*G + 0.114*B$$
$$Cb = - 0.299*R - 0.587*G + 0.886*B$$
$$Cr = 0.701*R - 0.587*G - 0.114*B$$

Therefore, the YCbCr values should be:

y				Cb				Cr			
161.2160	160.4640	163.8720	163.3130	-42.2160	-27.4640	-31.8720	-36.3130	55.7840	56.5360	53.1280	55.6870
160.4490	166.9430	167.2460	164.9690	-35.4490	-33.9430	-26.2460	-25.9690	59.5510	54.0570	49.7540	53.0310
168.6440	166.1600	175.9700	171.1810	-33.6440	-32.1600	-35.9700	-29.1810	53.3560	51.8400	47.0300	46.8190
176.2690	172.2690	179.2530	178.4190	-36.2690	-36.2690	-41.2530	-24.4190	47.7310	47.7310	41.7470	39.5810

The reason why using YCbCr to do DCT instead of RGB is that YCbCr, not like RGB who represents color with 3 different color values directly, represents a value with luminance and chrominance. This makes it easy to be subsampled on the chrominance channels. In addition, this method also makes the channel be quantized more coarsely. The last one is that the chrominance channels are rather uncorrelated with the luminance channels, which makes it yield better compression.

(b) Please compute the DCT coefficients of the Y component obtained in part(a) and the quantized coefficients (please round to integers) by using the following quantization matrix Q.

$$Q = \begin{bmatrix} 16 & 10 & 24 & 51 \\ 14 & 16 & 40 & 69 \\ 18 & 37 & 68 & 103 \\ 49 & 78 & 103 & 120 \end{bmatrix}$$

To compute the DCT coefficients, we can use the equations:

$$F(u,v) = \frac{4C(u)C(v)}{n^2} \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} f(j,k) \cos[\frac{(2j+1)u\pi}{2n}] \cos[\frac{(2k+1)v\pi}{2n}]$$

Then, we can get the result of DCT coefficients:

674.1592	-6.4667	-1.9293	5.1683
-21.7549	1.0004	-1.8459	-3.3426
3.3782	0.8801	3.6088	0.4713
-0.4590	-0.5631	1.7098	3.7796

To get the quantized result, we divide every element in DCT coefficients with corresponding elements in matrix Q:

42	-1	0	0
-2	0	0	0
0	0	0	0
0	0	0	0

This is the resulted quantized coefficients of quantization.

(c) Encode the quantized coefficients in part (b) as a run-length code.

DC is 42, and AC starts with -1. All we have to do is encode the AC part. Therefore, the encoded stream start with (0 , -1), and the next one is (0 , -2) because of zigzag. All the left elements are 0, so we can put EOB here. As a result, the encoded code is (0 , -1) (0 , -2) EOB.