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Structure Function in Lieu of Correlation Function

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Abstract. Using experimental data subject to noise and drift, we find the structure function can be computed to higher accuracy, yet using less data, than the correlation function. While this tendency is in line with theoretical reasoning, we seem to be the first to report on quantitative aspects. Taking wall pressure data from a transsonic wind tunnel, our structure functions are obtained with one to two orders less of data points than correlation functions of comparable information content. These advantages apply to auto- and cross-structure functions alike when compared to auto- and cross-correlation functions, respectively. Some comments are added on the possibility of designing digital "structurators" similar to existing digital correlators, either as software products using the FFT and recursive algorithms, or as hardware products in the form of fast special purpose parallel processors.

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Structure functions were first considered by Kolmogorov in 1941 in his work on locally isotropic and homogeneous turbulence [1–3], where he derived relations between certain longitudinal and transverse "moments of second order", as he called them then. The term structure function as such seems to have been coined by Obuchov and Yaglom [4, 5, 3]. Later on, the definition of structure functions and derivations of the physical laws formulated with their help, have found their way into monograph and textbook literature [6–8].

As pointed out by Tatarski [6], a question of concern in the analysis of fluctuating experimental data $x(t)$ is the following: which of the changes of $x(t)$ with time are those whose statistical properties including variance, correlation time, shape of correlation function, spectral density etc. are of interest to the investigator, and which other slower changes are considered as uninteresting, albeit unavoidable, drift of mean value or slow fluctuations? In practice, different answers may be given, depending on experimental aims and circumstances.

A useful mathematical idealization is a random function with stationary first (or linear) increments, $x(t)$. Then the difference $x(t)-x(t+\tau)$ is a stationary random function and, as such, has finite expectation values for the mean, the square, and higher powers. To the extent that experimental data approximate random functions with stationary increments, if only piecewise for finite periods of time, forming the difference $x(t)-x(t+\tau)$ prior to further statistical evaluations is a reasonable approach. Here the lag time τ should be extended only up to a limit given by those slow fluctuations which are no longer of interest. A similar argument applies to the difference $x(t)-y(t+\tau)$ where x and y are like quantities, e.g. pressure, however measured at two separate locations such that they are subject to correlated slow, long-range drifts and fluctuations.

The elimination of data drifts by subtraction indicates that the structure function tends to converge to its ultimate shape more rapidly than the correlation function. As Tatarski [6] suggested, if one begins to study a random process and is uncertain whether or not it is a stationary one, the structure function is preferable. In fact, for a stationary process the structure and correlation function are fully equivalent, so that at least no disadvantage can be obtained. Similar

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advice was given by Panchev [7]. Nevertheless, little has been done so far to evaluate experimental data by means of structure functions [9], to our knowledge only with respect to meteorological data.

Several likely reasons may be given for this hesitation. One is the central importance of the Wiener-Khintchine theorem which relates correlation function and spectrum. It confers such importance on the correlation function that indirect ways of obtaining it, via the structure function, were not considered. Another reason is the highly developed technology of computing correlation functions. It includes optimized, efficient software based on the fast Fourier transform (FFT) algorithm [10] which runs both on general purpose computers and on more efficient array processors. Alternatively, correlation functions are computed by special purpose parallel processors, i.e. by digital correlators, whose design and applications have attained a high technological standard [11, 12].

Perhaps the most important reason for hesitation, however, is the fact that so far there was no quantitative comparison of the number of data points necessary to obtain correlation and structure functions with similar accuracy. In fact, it is the main purpose of this paper to give such a comparison, based on typical experimental data.

The remainder of the paper begins with definitions and a comparative discussion of correlation and structure functions. We then present experimental data obtained from wall pressure transducers in a transsonic wind tunnel [13] and relate our experiences in computing auto- and cross-, -correlation and -structure functions, respectively.

These functions were obtained by straightforward, unsophisticated programs on a general purpose computer. We finally offer some view-points on how structure functions may be obtained more efficiently. In terms of software for standard computers, this could be done by adding a further recursive algorithm to the existing correlation program using FFT. In terms of hardware, this can be done by a specially designed parallel processor similar to, although different in some aspects from, existing digital correlators.

1. Comparison of Correlation and Structure Functions

The autocorrelation function is defined by

$$G_x(\tau)=\lim_{T\to\infty}\frac{1}{T}\int_0^T x(t)x(t+\tau)dt, \tag{1}$$

similarly the crosscorrelation function by

$$G_{xy}(\tau)=\lim_{T\to\infty}\frac{1}{T}\int_0^T x(t)y(t+\tau)dt. \tag{2}$$

The autostructure function is defined by

$$S_x(\tau)=\lim_{T\to\infty}\frac{1}{T}\int_0^T [x(t)-x(t+\tau)]^2dt, \tag{3}$$

and similarly the cross-structure function by

$$S_{xy}(\tau)=\lim_{T\to\infty}\frac{1}{T}\int_0^T [x(t)-y(t+\tau)]^2dt. \tag{4}$$

In these equations x and y are processes, or functions, of time, t is running time, τ is lag or delay, and T is the integration time. In experimental practice, the processes x and y are sampled at equidistant time increments, so that the integrals are replaced by similar summations. Also, T is necessarily finite. For economic reasons one would like to integrate only as long as is necessary to obtain the limiting functions with prescribed accuracy.

The limes operations in the definitions (1) to (4) deserve some comment. As is shown in the theory of random functions [14], the correlation functions (1) and (2) exist only, if $x(t)$ and $y(t)$ are stationary random functions. Alternatively, if $x(t)$ and $y(t)$ show significant drift in $0<t<T$, convergence of the correlation function is not to be expected. The well-known statements that the autocorrelation (1) is an even function of τ, $G(\tau)=G(-\tau)$, and that it has its maximum at $\tau=0$ where therefore $dG_x/d\tau=0$ and $d^2G_x/d\tau^2<0$, are true only under even more restrictive conditions: $x(t)$ has to be a stationary continuous random function, and the statements are true only in a limiting sense for $T\to\infty$. In other words, for finite T one may well find $G(\tau)\neq G(-\tau)$ and finite slope $dG_x/d\tau$ at $\tau=0$.

By comparison, the structure functions (3) and (4) exist for a more general class of processes $x(t)$ and $y(t)$, namely random functions with stationary first increments. By definition, the autostructure function (3) is non-negative everywhere and it vanishes for $\tau=0$. If $x(t)$ is a continuous function, $dS_x/d\tau|_{\tau=0}=0$ and $d^2S_x/d\tau^2|_{\tau=0}>0$, and this is true for any finite T, not just in the limiting case $T\to\infty$.

According to the definitions (1) to (4), there exist simple relations between correlation and structure functions. By carrying out the square under the integral, one has

$$S_x(\tau)=\lim_{T\to\infty}\frac{1}{T}\int_0^T [x(t)^2+x(t+\tau)^2-2x(t)x(t+\tau)]dt. \tag{5}$$

For a stationary random function the first two terms are identical and equal to the correlation function at zero lag so that then

$$S_x(\tau)=2G_x(0)-2G_x(\tau). \tag{6}$$

In a similar fashion, for stationary random functions $x(t)$ and $y(t)$, one has

$$S_{xy}(\tau)=G_x(0)+G_y(0)-2G_{xy}(\tau). \tag{7}$$

Since the zero lag terms are constants, the structure function offers the same information as the correlation function, the only differences being those constants, a negative sign and a factor of 2. The distinction may become significant, on the other hand, if the random processes under considerations are not stationary or contain slow fluctuations.

Such differences may be illustrated by a simple model function

$$x(t)=at, \tag{8}$$

which represents drift. It fits the definition of a random function with stationary increments. One finds for the autocorrelation function

$$G_x(\tau)=a^2(T^2/3+\tau T/2), \tag{9}$$

which in the limit $T\to\infty$ does not exist. Besides the constant background increasing with T^2 which in most applications is not harmful, a tilted baseline is indicated by the term linear in τ. Also note that this result is independent of the sign of a, hence much the same tilt is to be expected if the drift changes in magnitude and sign on a slow time scale. For the structure function one finds

$$S_x(\tau)=a^2\tau^2, \tag{10}$$

a parabola in τ, independent of T, so that $S_x(\tau)$ exists. It shows the expected features at $\tau=0$, namely $S_x(\tau)=0$, $dS_x/d\tau=0$, $d^2S_x/d\tau^2>0$.

Similar conclusions are derived by considering the model function

$$x(t)=\sin\omega t. \tag{11}$$

If the integration time involves an integral number of half cycles,

$$\omega T=n\pi, \tag{12}$$

then one obtains the "ideal" autocorrelation

$$G_x(\tau)=\tfrac{1}{2}\cos\omega\tau \tag{13}$$

and autostructure function

$$S_x(\tau)=1-\cos\omega\tau. \tag{14}$$

Condition (12) is difficult to realize in practice. A lengthy but trivial calculation yields additive "error" terms for $\omega T\neq n\pi$. For $\omega\tau<1$, i.e. near the origin, their leading term is

$$\Delta G_x(\tau)\approx-\frac{1}{4\omega T}\sin 2\omega T \tag{15}$$

or

$$\Delta S_x(\tau)\approx\frac{\omega^2\tau^2}{4\omega T}\sin 2\omega T, \tag{16}$$

respectively. This shows that for $\omega\tau<1$, the structure function is less distorted than the correlation function. Thus the structure function is less suspectible to low frequency noise.

We note a further distinction between both functions due to Yaglom [15]. Let $f(\omega)$ be the spectrum of the random process $x(t)$. Then, by definition

$$G_x(\tau)=\int_{-\infty}^{+\infty} f(\omega)\cos\omega\tau d\omega \tag{17}$$

and

$$S_x(\tau)=2\int_{-\infty}^{+\infty} f(\omega)(1-\cos\omega\tau)d\omega. \tag{18}$$

Considering these expressions for $\omega\to 0$ and $\omega\to\infty$, Yaglom has shown, and it is not difficult to see, that the correlation function exists only if

$$f(\omega)\propto\omega^{-1-\varepsilon} \quad \text{for} \quad \omega\to\infty, \tag{19a}$$

$$f(\omega)\propto\omega^{-1+\varepsilon} \quad \text{for} \quad \omega\to 0, \tag{19b}$$

while the structure function exists as long as

$$f(\omega)\propto\omega^{-1-\varepsilon} \quad \text{for} \quad \omega\to\infty, \tag{20a}$$

$$f(\omega)\propto\omega^{-3+\varepsilon} \quad \text{for} \quad \omega\to 0, \tag{20b}$$

where ε is a small positive number. Proportionality (20b) shows that the structure function can tolerate much more low frequency noise. Furthermore he has shown that for

$$f(\omega)=A|\omega|^{-p} \qquad 1+\varepsilon<p<3-\varepsilon, \tag{21}$$

$$S(\tau)=C\tau^{p-1} \tag{22}$$

with a constant

$$C=2A\pi\Big/\left[\sin\frac{\pi(p-1)}{2}\Gamma(p)\right]. \tag{22a}$$

Our previous example, see (8) and (10), then corresponds to a spectrum with $p=3$. Interestingly the energy contained in this spectrum diverges, i.e. is infinite.

Panchev [7] quoted another case in which the structure function proves to be less sensitive against manipulations. He considered a random process $x'(t)$ which is derived from a drifting random process $x(t)$ by a continuing adjustment of zero level,

$$x'(t)=x(t)-\langle x(t)\rangle_T, \tag{23}$$

where the last term indicates a running average of run length T. He found the structure functions of $x'(t)$ and

$x(t)$ to be identical. The correlation function becomes distorted, however, if one replaces $x(t)$ by $x'(t)$, and he gave an explicit formula for this distortion.
All of the arguments discussed so far indicate that the structure function is able to tolerate larger drift and low frequency noise. It remains to be seen what reduction in the number of measured data can be tolerated when doing the structure function. Obviously this depends on the quality of the experimental data and the spectral details of interest.

2. Correlation and Structure Functions of Experimental Data

For a comparative, empirical study of correlation and structure functions we used wall pressure data from a transsonic wind tunnel described previously by Meier [13]. The data shown were taken by flush mounted piezoelectric transducers, one being located near the point of smallest cross section of a halved two-dimensional Laval nozzle, the other in the plenum chamber which is joined to the channel of rectangular cross section in the downstream direction and is in the shape of a parallel epiped. Typical pressure data are shown in Fig. 1. Traces C and D show a short sample of 1024 measured data each, corresponding to a period of 128 ms, for both transducers. Traces A and B show the spectra of the data of traces C and D with frequencies between 0 and 2 kHz.

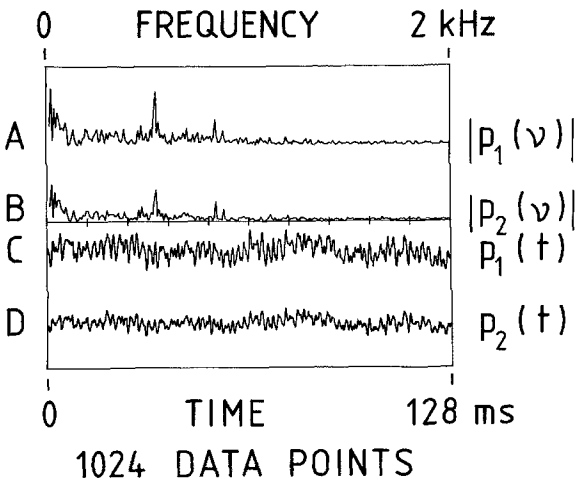

Fig. 1. Sample of experimental data from pressure transducers. Traces C and D represent 1024 data points each gathered within a time span of 128 ms from transducers 1 and 2. Traces A and B are the corresponding amplitude spectra on a frequency scale from 0 to 2.0 kHz

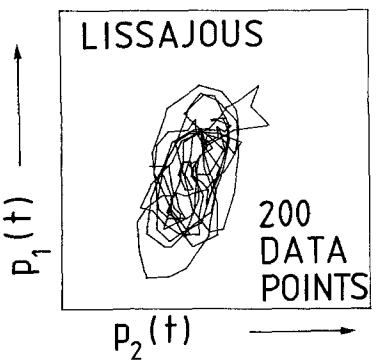

Fig. 2. Lissajous representation of pressure data from transducer 1 versus those of transducer 2 for 200 data points

The curves represent amplitude spectra, i.e. the square root of the sum of the squares of real and imaginary part of the Fourier transforms obtained by FFT. Besides low frequency peaks, both spectra show a prominent peak at 530 Hz. This peak is consistently found in the spectra of consecutive data sets. We tentatively associate it with the lowest acoustic resonance of the plenum chamber, in which the transverse dimension is half a wavelength. Two smaller peaks at 820 and 870 Hz, however, are not a consistent feature. In other, consecutive data sets they appear at frequencies varying between 700 and 1000 Hz and with varying amplitude. Comparison of traces C and D shows close similarity of virtually all features, from slow level changes to details in the shape of individual peaks. This similarity – and the degree to which it is obscured by noiselike fluctuations – is further illustrated by Fig. 2, a Lissajous plot of the data of trace C versus those of trace D, however only for the first 200 data points. One recognizes a tendency to form an ellipse accounting for phase shift and amplitude ratio of both data sets.
Numerically computed autocorrelation and autostructure functions of the pressure data from the first transducer are shown in Figs. 3, 4, and 5. In these graphs, the sums accumulated in 401 channels – in the language of correlator technology – are plotted. The center of the graphs refers to channel number zero, representing zero lag. Along the abscissa lag increases from -25 to $+25$ ms, corresponding to channel numbers -200 to $+200$. In Figs. 3, 4, and 5 every value plotted is the sum of 200, 2000, or 20,000 individual products (for the correlation functions) or squares (for the structure functions).
Examination of these graphs shows that for the autocorrelation function one needs around 20,000 data points, or terms in the sum, to obtain a satisfactory result. Only then the maximum occurs at zero lag, and only then the predominant 530 Hz frequency is clearly discernible. However, even with 20,000 data points, the autocorrelation is not an even function of lag as predicted by theory for the limiting case. For 2000 or 200 data points the baseline is severely distorted to the point that the zero lag maximum is distorted or

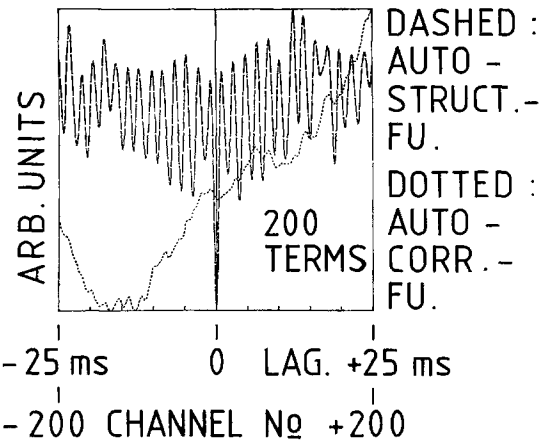

Fig. 3. Computed autocorrelation (dotted curve) and autostructure (dashed curve) function of experimental pressure data. Every value plotted represents the sum of 200 terms each (products for the correlation, squares for the structure function). Lag time shown on the abscissa ranges from −25 ms to +25 ms which is split up into 401 channels

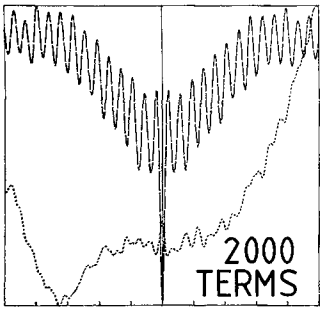

Fig. 4. As Fig. 3, however with values representing the sum of 2000 terms each

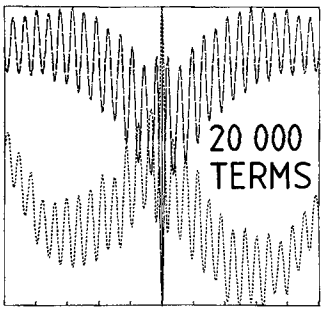

Fig. 5. As Fig. 3, however with values representing the sum of 20,000 terms each

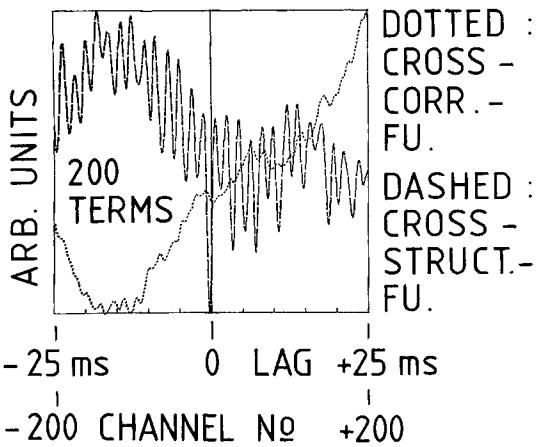

Fig. 6. Similar to Fig. 3, however referring to crosscorrelation (dotted curve) and cross-structure (dashed curve) function of experimental data from both pressure transducers. Every value plotted represents the sum of 200 terms each

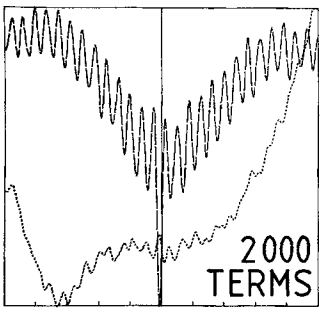

Fig. 7. As Fig. 6, however with values representing the sum of 2000 terms each

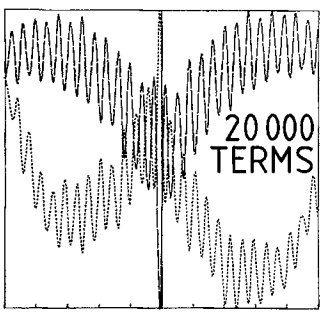

Fig. 8. As Fig. 6, however with values representing the sum of 20,000 terms each

obscured, and the dominant frequency is difficult to see.

By comparison, the autostructure function yields the dominant frequency with good accuracy with as little as 200 data points. Its minimum is symmetric and at zero lag, a feature which is inherent in the definition. With 2000 data points, the autostructure function assumes a rather regular and symmetric shape whose envelope contains information on low-frequency processes. Note that such information can be obtained with better accuracy from the autostructure function with 2000 data points than from the autocorrelation function with 20,000 data points.

Cross correlation and cross-structure functions using data from both transducers are shown, in the same format, in Figs. 6, 7, and 8. Compared to the auto-

functions, the cross-functions are somewhat more erratic. This is not surprising since two physically distant transducers may well experience some unrelated drifts or fluctuations. Nevertheless, the same comments apply as before. The functions shown in Fig. 6, 7, and 8 show a displacement of the central peak amounting to 0.5 ms relative to zero lag. However, it is not visible in the cross correlation of Fig. 6 and the lag appears larger in that of Fig. 7. These lags are interpreted as propagation delays of disturbances. They indicate propagation with the velocity of sound in the upstream direction, from the plenum chamber back to the Laval nozzle.

In either case, information may be gathered more quickly through the structure function. For the data reported here, the saving is between one and two orders of magnitude, referring to the number of terms in the summation. This, of course, depends on the quality of the input signal. As an examination of Fig. 1 shows, in the present case the amplitudes of high-frequency and low-frequency details are comparable. This is apparent in both the frequency spectrum and in the signal as function of time. It is to be expected, therefore, that the odds are even more in favor of the structure function if low-frequency noise amplitudes exceed a high-frequency random signal.

One may be tempted to argue that similar advantages may be obtained by highpass filtering the signal prior to correlating. This is true if one is prepared to accept irrevocable loss of signal information. By contrast, forming the structure function does not imply loss of information. All spectral details are retained in the structure function and can be extracted by Fourier transformation. In particular, low frequency details can be obtained provided, of course, the structure function has been determined up to correspondingly large values of lag.

3. Feasibility of Efficient Structurators

In this section we wish to show that the structure function of experimental data can be obtained with comparable demands on computer storage space and running time (for the software implementation) or by equipment of comparable complexity and speed (for the hardware realization) as the correlation function. Taking this for granted, the faster convergence of the structure function as demonstrated in the figures of this paper results in considerable overall advantage.

For computation by the fast Fourier transform (FFT) algorithm, the (cross) correlation function is defined by

$$G_{xy}(\tau)=\frac{1}{T-\tau}\sum_{t=0}^{T-\tau-1}x(t)y(t+\tau). \tag{24}$$

Here, dropping a multiplying time increment, t, τ, and T are redefined as integers standing, as before, for running time, lag, and integration time, respectively. T may be chosen as a power of 2 for efficiency. Note that the number of terms in the sum decreases with τ so that $G_{xy}(\tau)$ tends to decrease in accuracy for larger τ[1]. The computation of (24) using FFT is well documented [10]. Appropriate program products are commercially available from many computer and software firms.

The corresponding (cross-) structure function is

$$S_{xy}(\tau)=\frac{1}{T-\tau}\sum_{t=0}^{T-\tau-1}[x(t)^2+y(t+\tau)^2]-2G_{xy}(\tau), \tag{25}$$

which we rewrite as

$$S_{xy}(\tau)=\frac{1}{T-\tau}R_{xy}(\tau)-2G_{xy}(\tau). \tag{26}$$

We propose to compute $G_{xy}(\tau)$ by FFT as usually and to obtain R_{xy} recursively. From (25) and (26) one finds the recursive relation

$$R_{xy}(\tau)-R_{xy}(\tau+1)=x(T-\tau-1)^2+y(\tau)^2 \tag{27}$$

and the recursion may start either at $\tau=0$ where

$$R_{xy}(0)=\sum_{t=0}^{T-1}[x(t)^2+y(t)^2] \tag{28a}$$

or at $\tau=T-1$ where

$$R_{xy}(T-1)=x(0)^2+y(T-1)^2. \tag{28b}$$

A little reflection shows that the number of additional computing steps to obtain R_{xy} and divide it by $T-\tau$, is directly proportional to T. Thus it increases less steeply than the FFT where the increase is proportional to $T\log_2 T$. In this sense, the proposed computation of the structure function is a fast algorithm. Estimates show that the total number of computing steps increases by a factor smaller than 2. In addition, some extra storage is required for $x(t)$, $y(t)$, and R_{xy}. Note that our arguments apply also to the case treated by Rader in [10] where the maximum lag of interest is finite whereas the data sequence is essentially unterminated.

The point of departure for hardware implementation is the digital correlator whose technology was pioneered by Pike and his associates at the Royal Radar and Signals Establishment, Malvern, UK. Detailed documentation is available in the literature [11, 12] or may be obtained from manufacturers of correlators[2]. These

[1] Note, however, that the FFT procedure due to Rader [10], involving a fixed upper limit of summation (24), is free of this disadvantage. It is a simple matter to give a recursive algorithm similar to (27) and (28) for this case.

[2] Malvern Instruments, Malvern, UK; Langley-Ford Instruments, Amherst, Mass., USA; Spectron Development Laboratories, Costa Mesa, California, USA.

instruments compute the function

$$G_{xy}(\tau)=\sum_{t=1}^{T} x(t)y(t-\tau). \tag{29}$$

If, at the starting moment $t=0$, the shift register contains previous $y(t)$ data belonging to $-\tau_{max}\leqq t\leqq 0$, then the sum has exactly T terms for all values of τ. Thus, in contrast to implementation by FFT, all values of G_{xy} have the same statistical weight and accuracy. The running time signal, $x(t)$, is handled differently from the delayed one, $y(t-\tau)$. The delayed signal has one-bit or four-bit format suitable for shift registers. The running time signal is accepted in the form of a pulse sequence whose instantaneous rate is a measure of $x(t)$.

The digital structurator should compute, in real time, the function

$$G_{xy}(\tau)=\sum_{t=1}^{T} [x(t)-y(t-\tau)]^2. \tag{30}$$

The form of this expression makes it advisable that both signals are handled on equal footing. We propose to have both in the form of 4-bit numbers, for example, so that shift register operation is possible. Forming the difference and then the square seem "expensive" operations, requiring considerable hardware per channel and being slow in execution time. We therefore propose to do both subtraction and squaring simultaneously by table look-up. To this end, the 4-bit signals $x(t)$ and $y(t-\tau)$ are combined to form an 8-bit address to a read-only memory (ROM) in which square numbers are stored. This operation tends to be fast and the expenditure of one ROM per channel seems economical. Other parts of the structurator may essentially be the same as those of the correlator. We suggest, therefore, that the structurator can be built at comparable cost and that it will perform at comparable speed.

4. Summary and Conclusion

Using experimental data subject to moderate drift, we have found that forming the structure function is a superior way of getting spectral information. Doing the correlation and structure functions side by side, the former required data sets that are one to two orders of magnitude larger until a similar degree of convergence and accuracy was obtained. Furthermore, assuming data sets of equal size, we have shown that both functions can be obtained at comparable expenditure of computing time or hardware investment. It seems probable to us that forming the structure function of raw data is a faster converging, more economical procedure than filtering the data by high-pass filter or trend-elimination schemes prior to forming the correlation function. However, more empirical work on this question is desirable. While the structure function has its place in the theory of turbulence as shown by Kolmogorov and his school, its potential usefulness with respect to computer evaluation of experimental data has yet to be exploited.

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