wave_Devito

May 20, 2017

0.0.1 Wave Equation

Problem setting: one-dimensional waves on a string of length L

$$\frac{d^2u}{dt^2} = c^2\frac{d^2u}{dx^2} x \in (0, L), t \in (0, T]$$

with two initial conditions:

1. the initial shape of the string

$$u(x, 0) = I(x)$$

2. reflecting that the initial velocity of the string is zero

$$\frac{d}{dt}u(x, 0) = 0$$

and boundary conditions:

$$u(0, t) = 0$$

$$u(L, t) = 0$$

The constant c and the function I(x) must be prescribed
Concrete example

$$u(x, 0) = A\sin(\frac{\pi}{L}x)$$

$$u_e(0, t) = u_e(L, t) = 0$$

analytic solution is

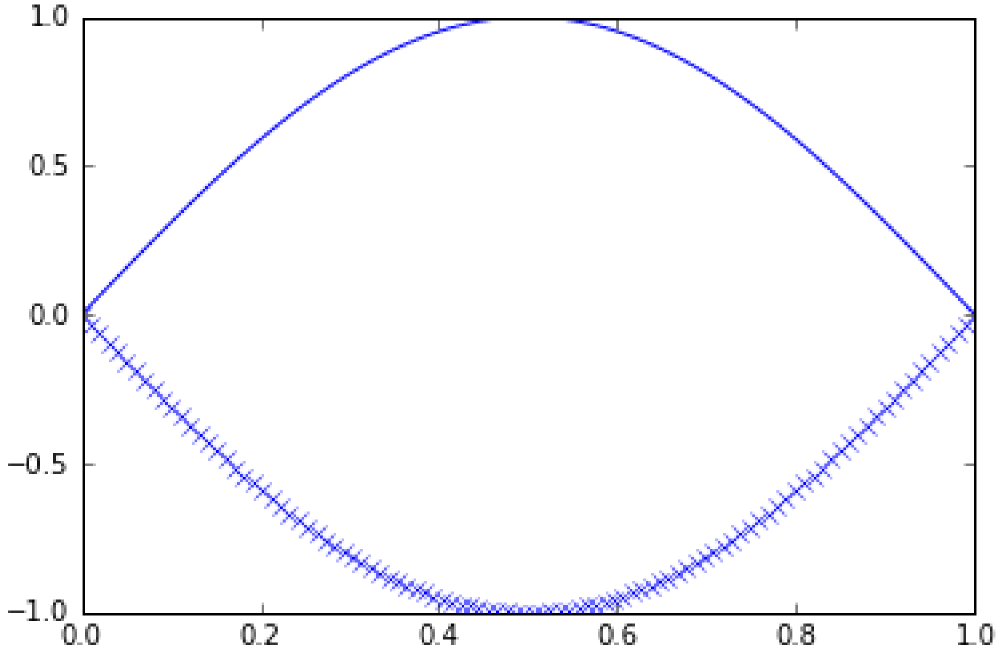
$$u_e(x, y, t) = A\sin(\frac{\pi}{L}x)\cos(\frac{\pi}{L}ct)$$

In [1]:
```
import numpy as np
def initial(A, x):
    return A * np.sin((np.pi/L)*x)

def analytic(A,x,c, t):
    return A * np.sin((np.pi/L)*x)*np.cos((np.pi/L)*c*t)
```

```
A = 1.
L = 1
c = 1
x = np.linspace(0,1,100)
y0 = initial(A, x)
y = analytic(A, x, c, 1)
```

```
In [2]: import matplotlib.pyplot as plt
        %matplotlib inline
        plt.plot(x,y0,'b')
        plt.plot(x,y, 'x')
```

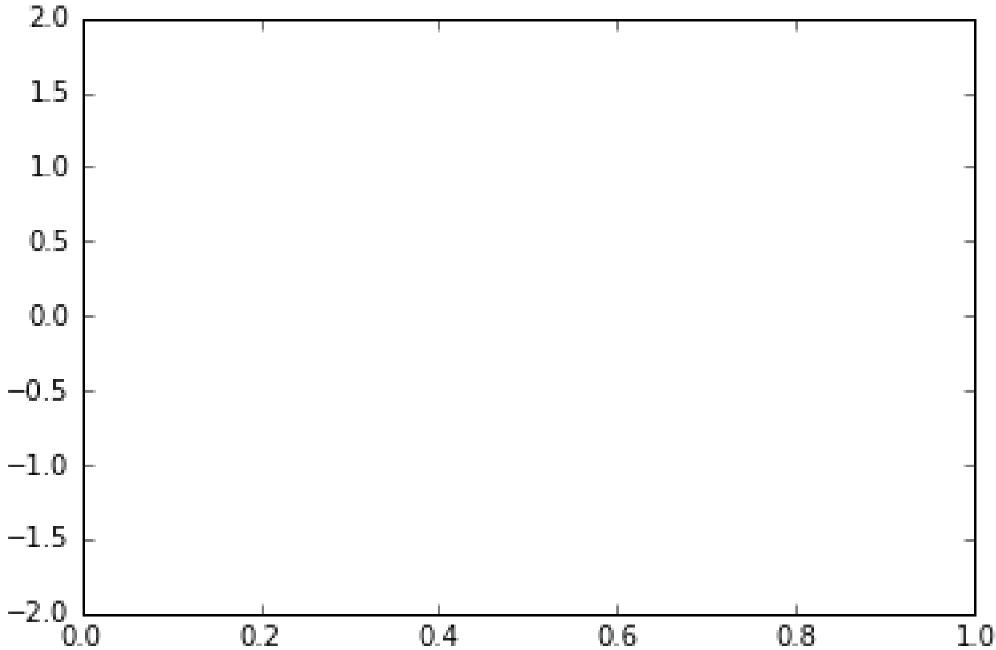
```
Out[2]: [<matplotlib.lines.Line2D at 0x105578908>]
```




```
In [3]: from IPython.core.display import HTML
        #css_file = 'https://raw.githubusercontent.com/ngcm/training-public/master/
        #HTML(url=css_file)
```

```
In [4]: from matplotlib import animation

        # First set up the figure, the axis, and the plot element we want to animat
        fig = plt.figure()
        ax = plt.axes(xlim=(0, 1), ylim=(-2,2))
        line, = ax.plot([], [], lw=2)
```

```
# initialization function: plot the background of each frame
def init():
    line.set_data([], [])
    return line,

# animation function.  This is called sequentially
def animate(i):
    c = 1
    x = np.linspace(0, 1, 500)
    #t = i/100.
    y =  A * np.sin((np.pi/L)*x)*np.cos((np.pi/L)*c*i)
    line.set_data(x, y)
    return line,
```




```
In [5]: HTML(animation.FuncAnimation(fig, animate, init_func=init, interval=200, fr
```

```
Out[5]: <IPython.core.display.HTML object>
```

Discretization

$$\frac{u_i^{n+1} - 2u_i^n + u_i^{n-1}}{\delta t^2} = c^2 \frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{\delta x^2}$$

$$u_i^{n+1} = 2u_i^n - u_i^{n-1} + C^2(u_{i+1}^n - 2u_i^n + u_{i-1}^n)$$

$$C = c\frac{\delta t}{\delta x}$$

```
In [6]: Nt = 100
        Nx = 100
        x, dx = np.linspace(0, 1, Nx, retstep=True)
        t, dt = np.linspace(0,1, Nt, retstep=True)
        c=1
        C = c *dt/dx
```

```
In [42]: from devito import TimeData, Operator
         from sympy import Eq, solve
         from sympy.abc import h, s

         Nt = 500
         L = 1
         dx = 0.01
         x = np.arange(0, L ,dx)
         #dt = 1e-9
         dt =0.009

         c=1

         #initial condition
         u0 = initial(A, x)

         #first step, du/dt = 0 ---> u_i_n-1 & u_i_n+1 are equal, v=0
         u1 = np.zeros_like(u0)
         u1[1:-1] = 2 * u0[1:-1] - u0[1:-1] + C**2 * (u0[2:] -2*u0[1:-1]+u0[0:-2])


         nx = len(x)
         # Allocate the grid and set initial condition
         # Note: This should be made simpler through the use of defaults
         u = TimeData(name='u', shape=(nx, ), time_order=2, space_order=2,time_dim=
         u.data[0, :] = u0[:]
         u.data[1, :] = u1[:]
```

```
In [43]: (0.009**2)/dx**2

Out[43]: 0.8099999999999998
```

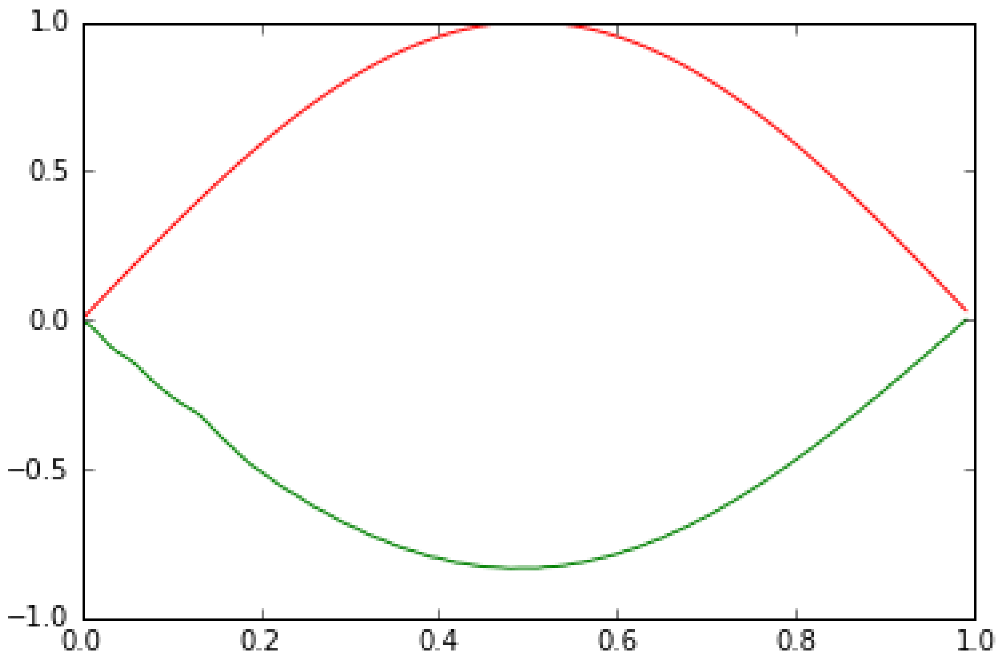
```
In [44]: # Derive the stencil according to devito conventions
         eqn = Eq(u.dt2, c * u.dx2)
         stencil = solve(eqn, u.forward)[0]
         op = Operator([Eq(u.forward, stencil)], subs={h: dx, s: dt})

         # Execute the generated Devito stencil operator
         op.apply(u=u)

DSE: <filter object at 0x114d8c780> [flops: 7, elapsed: 0.00] >>
     <filter object at 0x114dfcb00> [flops: 7, elapsed: 0.00] >>
```

```
     <filter object at 0x114d8c780> [flops: 7, elapsed: 0.00] >>
     <filter object at 0x114dfcb00> [flops: 6, elapsed: 0.01] >>
     <filter object at 0x114d8c780> [flops: 6, elapsed: 0.00]
     [Total elapsed: 0.02 s]
DLE: <filter object at 0x114e26748> [elapsed: 0.00] >>
     <filter object at 0x114e26748> [elapsed: 0.00] >>
     <filter object at 0x114e26748> [elapsed: 0.00] >>
     <filter object at 0x114e26748> [elapsed: 0.00] >>
     <filter object at 0x114e26748> [elapsed: 0.01] >>
     <filter object at 0x114e26710> [elapsed: 0.00]
     [Total elapsed: 0.02 s]
ClangCompiler: compiled /var/folders/f3/c2wg4lkx1ms7_41c3qzz8x7nrdhbjw/T/devito-189
===============================================================================
Section main<499,99> with OI=0.74 computed in 0.000 s [Perf: 2.69 GFlops/s]
===============================================================================
```

```
Out[44]: PerformanceSummary([('main',
                              PerfEntry(time=0.00011, gflopss=2.6946, oi=0.741015,
```

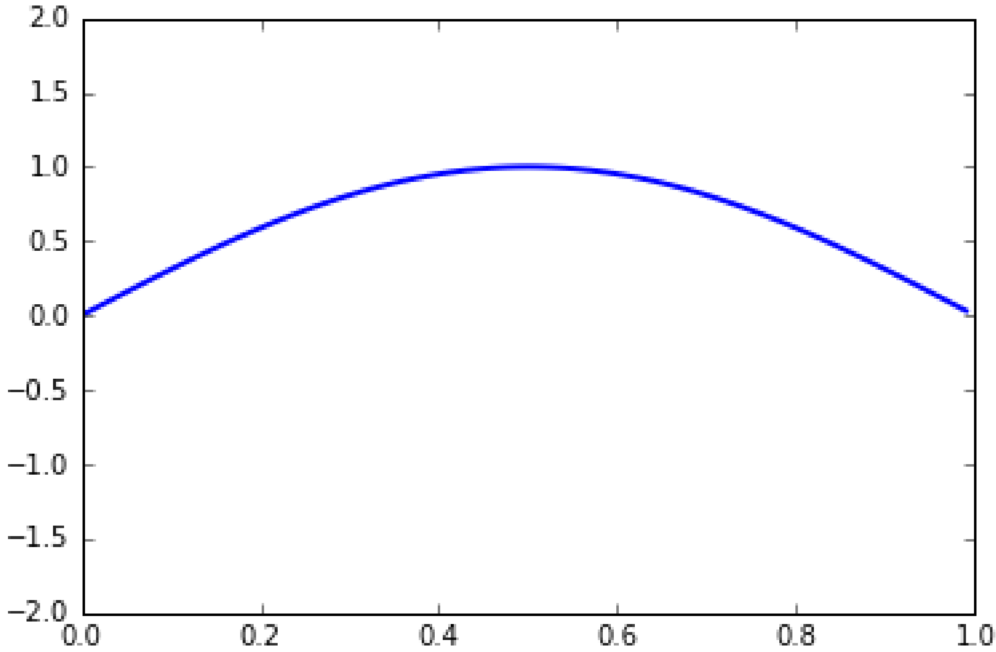
```
In [45]: plt.plot(x,u0, '-r')
         plt.plot(x,u.data[350,:], '-g')
```

```
Out[45]: [<matplotlib.lines.Line2D at 0x114df0550>]
```




```
In [46]: #plot the first step
         fig = plt.figure()
```

```
ax = plt.axes(xlim=(0, 1), ylim=(-2,2))
line, = ax.plot(x, u0, lw=2)

def do_steps(i):
    line.set_ydata(u.data[i,:])     # update data in plot
    fig.canvas.draw()# redraw the canvas
    return line,
```




```
In [47]: HTML(animation.FuncAnimation(fig,do_steps, range(Nt),interval=10).to_html5
```

```
Out[47]: <IPython.core.display.HTML object>
```

```
In [ ]:
```