Energy Trading

Lecture - Summer 2014

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Agenda

Liberalisation

The German Electricity market went into Liberalization in April 1998.
The Pre - Liberalisation system was based on calculatory costs: the price was according to the 'cost-plus' rule

- Integrated value-chain: production, grid, distribution
- Electricity production to secure supply within a regional monopole
- Long-term supply contracts
- No liquid market on the whole sale market
- Regulated consumer prices, regulated investments

Liberalisation

Post - Liberalisation system based on forces of market: higher volatility of prices, flexibility has value.

- Unbundling of value-chain
- Power plants are used optimally – no obligation to secure supply
- New players and products
- Trading in Long- and Short-positions on a liquid whole sale market
- Investments based on market expectations

Forwards and Futures

- A *forward contract* is an agreement to buy or sell an asset S at a certain future date T for a certain price K.
- The agent who agrees to buy the underlying asset is said to have a *long* position, the other agent assumes a *short* position.
- The settlement date is called *delivery date* and the specified price is referred to as *delivery price*.

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Forwards

- The *forward price* $F(t, T)$ is the delivery price which would make the contract have zero value at time t.
- At the time the contract is set up, $t = 0$, the forward price therefore equals the delivery price, hence $F(0, T) = K$.
- The forward prices $F(t, T)$ need not (and will not) necessarily be equal to the delivery price K during the life-time of the contract.

Forwards

- The payoff from a long position in a forward contract on one unit of an asset with price $S(T)$ at the maturity of the contract is
$$S(T) - K.$$
- The investor now faces a downside risk as well as an upside opportunity. He has the obligation to buy the asset for price K.

Futures

- Futures can be defined as standardized forward contracts traded at exchanges where a clearing house acts as a central counterparty for all transactions.
- Usually an initial margin is paid as a guarantee.
- Each trading day a settlement price is determined and gains or losses are immediately realized at a margin account.
- Thus credit risk is eliminated, but there is exposure to interest rate risk.

Options

- An option is a financial instrument giving one the *right but not the obligation* to make a specified transaction at (or by) a specified date at a specified price.
- *Call* options give one the right to buy an underlying. *Put* options give one the right to sell.
- *European* options give one the right to buy/sell on the specified date, the expiry date, on which the option expires or matures. *American* options give one the right to buy/sell at any time prior to or at expiry.

European Call Price

For a European call $X = (S(T) - K)^+$ with a spot underlying S we have in the Black-Scholes model the explicit price process of a European call given by

$$
\begin{aligned}
C^S(t) &= S(t)\Phi(d_1(S(t), T-t)) \\
&\quad - Ke^{-r(T-t)}\Phi(d_2(S(t), T-t)).
\end{aligned}
$$

The functions $d_1(s, \tau)$ and $d_2(s, \tau)$ are given by

$$
\begin{aligned}
d_1(s,\tau) &= \frac{\log(s/K) + (r + \frac{\sigma^2}{2})\tau}{\sigma\sqrt{\tau}}, \\
d_2(s,\tau) &= \frac{\log(s/K) + (r - \frac{\sigma^2}{2})\tau}{\sigma\sqrt{\tau}}
\end{aligned}
$$

Φ is the standard normal distribution function.

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Black's Formula

In Black's futures model the price C^f of a European call option with a futures (forward) as underlying is given by Black's futures options formula:

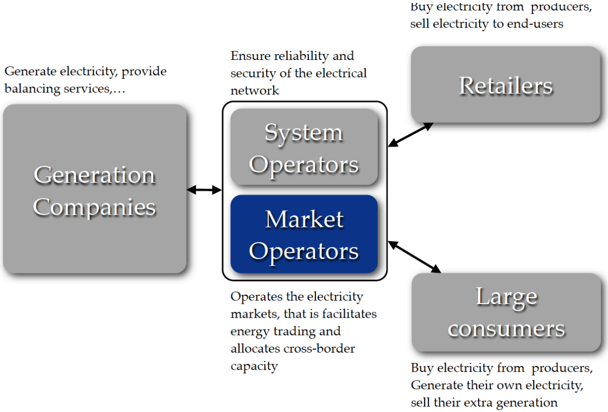
$$C^f(f, T-t) = e^{-r(T-t)}(f\Phi(d_1^f(f, T-t)) - K\Phi(d_2^f(f, T-t))),$$

where

$$d_{1,2}^f(f, \tau) := \frac{\log(f/K) \pm \frac{1}{2}\sigma^2\tau}{\sigma\sqrt{\tau}}.$$

We may use $C_{\mathrm{BS}}(S, K, \sigma_S, \tau)$ as a general reference for the Black-Scholes formula.

Organisation of the Power System



System Balancing

- The transmission system operator (TSO) has the task to match demand and supply (to balance the system).
- The TSO defines a balancing period (15 minutes in Germany), which is the granularity of the measured electric energy supply and during which a constant power supply takes place (by the energy merchant).
- The power balancing during the balancing period (not smaller than the granularity, in Germany an hour) is the task of the TSO.
- The TSO usually has no own generation capacities and has to act on the reserve market to compensate imbalances.

Balancing and Reserve Markets

We use the following definitions

- Reserve Market: allows the TSO to purchase the products needed for compensating imbalances between supply and demand
- Balancing Market: allows merchants to purchase or sell additional energy for balancing their accounting grid. Typically, the only market partner is the TSO.

Balancing and Reserve Markets

In Europe, the *European Network of Transmission System Operators for Electricity,(ENTSO-E)* coordinates overarching grid topics. The main task of a TSO is to ensure a constant power frequency in the transmission system. The following control actions are applied

- *Primary Reserve* starts within seconds as a joined action of all TSOs in the system.
- *Secondary Reserve* replaces the primary reserve after a few minutes and is put into action by the responsible TSOs only.
- *Tertiary Reserve* frees secondary reserves by rescheduling generation by the responsible TSOs.

The TSO tenders the required products for fulfilling these functions. Reserve products may involve payments for the availability of the reserved capacity.

Market Coupling

- Neighbouring electricity markets are typically coupled via transmission capacities owned by the TSOs.
- Transmission capacities can be integrated in the price finding algorithm of cooperating exchanges via implicit auctioning.
- With implicit auctions players do not receive allocations of cross-border capacity themselves but bid for energy on their Exchange. The Exchanges then use the available cross-border transmission capacity to minimize the price difference between two or more areas.
- Currently, the Central Western Europe (CWE) initiative couples Belgium, France, the Netherlands, Germany and Luxemburg.

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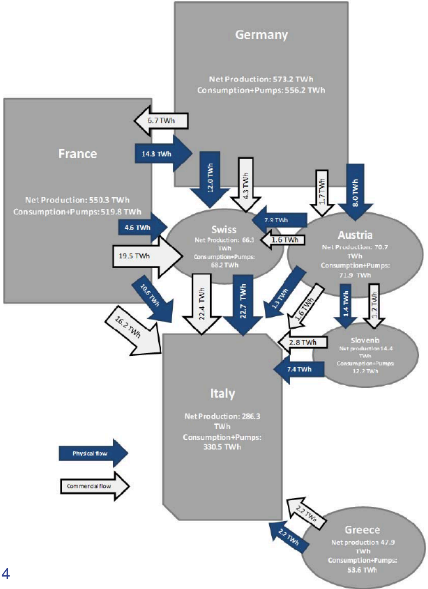
Market Coupling



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Electricity Markets

A centralized platform where participants can exchange electricity transparently according to the price they are will to pay or receive, and according to the capacity of the electrical network.

- Fixed Gate Auction
  - Participants submit sell or buy orders for several areas, several hours,
  - the submissions are closed at a pre-specified time (closure)
  - the market is cleared.
  - Example: day-ahead.
- Continuous-time Auction
  - Participants continuously submit orders. Orders are stored,
  - Each time a deal is feasible, it is executed,
  - Example: intra-day.

Electricity Exchanges

Electricity related contracts can be traded at exchanges such as

- the Nord Pool, mainly Northern European countries, `http://www.nordpoolspot.com/`
- the European Energy Exchange (EEX), `http://www.eex.com/en`
- EPEX, located in Paris, founded by EEX and Powernext (French Energy Exchange); Electricity spot market for Germany, Austria, France and Switzerland; `http://www.epexspot.com/en/`
- Amsterdam Power Exchange (APX), covers the Netherlands, Belgium and the UK, `http://www.apxgroup.com`

EPEX – traded products

- Auction day-ahead and continuous intra-day market.
- Products are individual hours, baseload, peakload, blocks of contiguous hours.
- Intraday market is open 24 hours a day, 7 days a week and products can be traded until 45 minutes before delivery.
- in Germany 15 minutes contracts can be traded.

EEX – traded products

- Futures contracts for Germany and France with delivery periods week, month, quarter, year.
- For Germany single days and weekends are available.
- European style options on futures.

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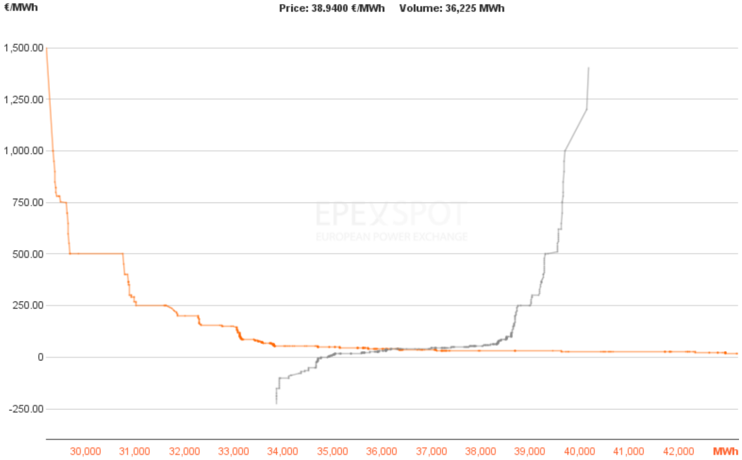
Auction EPEX



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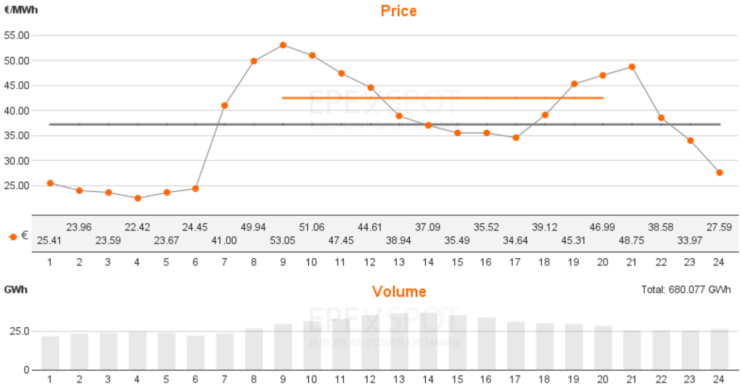
Spot price EPEX



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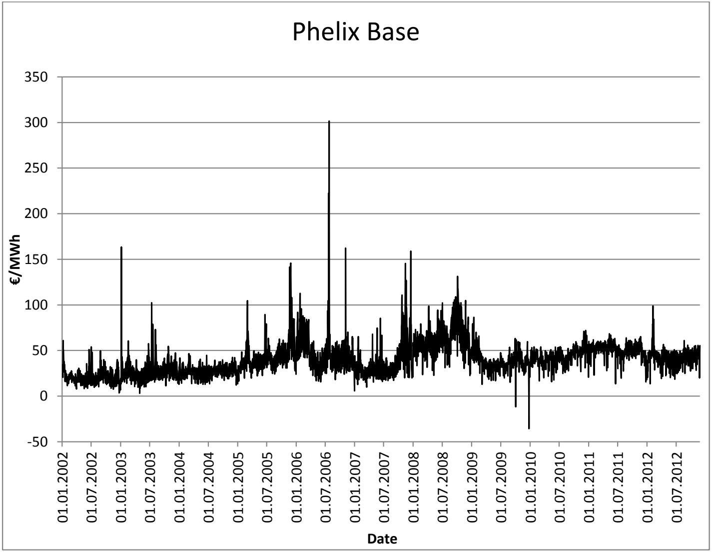
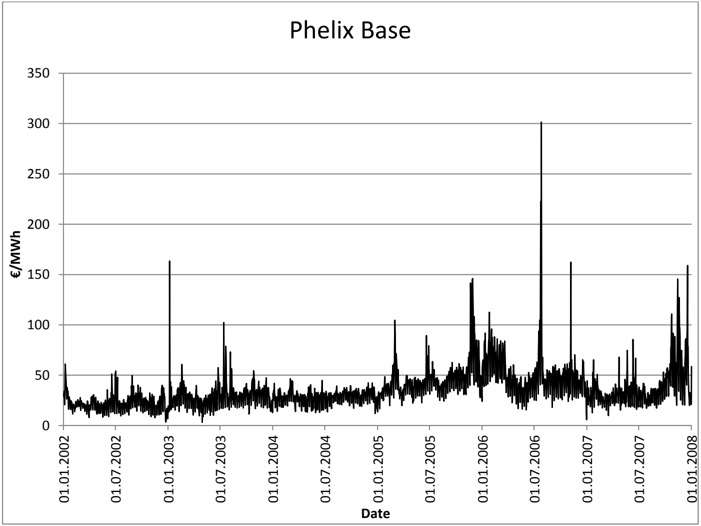
Spot prices



Spot prices


Phelix Base
€/MWh
350
300
250
200
150
100
50
0
01.01.2002
01.07.2002
01.01.2003
01.07.2003
01.01.2004
01.07.2004
01.01.2005
01.07.2005
01.01.2006
01.07.2006
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01.07.2007
01.01.2008
Date

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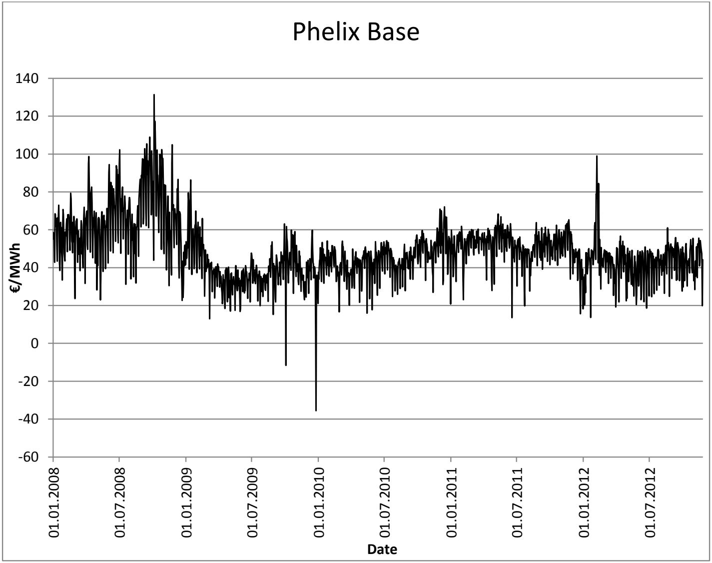
Spot prices



Swaps

- A *swap* is an agreement whereby two parties undertake to exchange, at known dates in the future, various financial assets (or cash flows) according to a prearranged formula that depends on the value of one or more underlying assets or a (commodity price) index.
- Swaps are very popular instruments and often offered by banks with a high liquidity for many commodities.
- If used for hedging the underlying index should have a close relation to the actual price of a physical commodity in question to keep the basis risk small.

Example: Fixed for Floating Swap

Consider the case of a *forward swap settled in arrears* characterized by:

- a fixed time t, the contract time,
- dates $T_0 < T_1 < \ldots < T_n$, equally distanced $T_{i+1} - T_i = \delta$,
- R, a prespecified fixed price,
- K, a nominal amount.

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Example: Fixed for Floating Swap

A swap contract S with K and R fixed for the period $T_0, \dots T_n$ is a sequence of payments, where the amount of money paid out at $T_{i+1},\ i = 0, \dots, n-1$ is defined by

$$X_{i+1} = K\delta(L(T_i, T_{i+1}) - R).$$

$L(T_i, T_{i+1})$ is the value of the index over $[T_i, T_{i+1}]$ observed at T_i.

Spread Options

Spread options can be used by owners of corresponding plants to manage market risk.
The pay off of a typical spread is

$$C^{(T)}_{\text{spread}} = \max(S_1(T) - S_2(T) - K, 0)$$

with S_i the underlyings, K the strike.

Spread Options

For $K = 0$ (exchange option) there is an analytic formula due to Margrabe (1978).

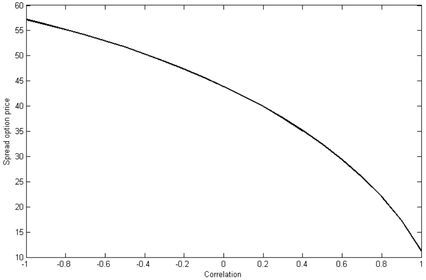
$$
\begin{aligned}
C_{\text{spread}}(t) &= e^{-r(T-t)}(S_1(t)\Phi(d_1) - S_2(t)\Phi(d_2)) \\
\text{where} \quad d_1 &= \frac{\log(S_1(t)/S_2(t)) + \sigma^2(T-t)/2}{\sqrt{\sigma^2(T-t)}} \quad , d_2 = d_1 - \sqrt{\sigma^2(T-t)} \\
\text{and} \quad \sigma &= \sqrt{\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2}
\end{aligned}
$$

where ρ is the correlation between the two underlyings.
For $K \neq 0$ no easy analytic formula is available.

Spread Option Value and Correlation

The value of a spread option depends strongly on the correlation between the two underlyings.

$S_1 = S_2 = 100$, $T = 3$, $r = 0.02$, $\sigma_1 = 0.6$, $\sigma_2 = 0.4$.



- The higher the correlation between the two underlyings the lower is the volatility of the spread and hence the value of the spread option.

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Approximation by Kirk's Formula (3 Assets)

An accurate approximation formula for the three asset case is also given in E.Alos, A.Eydeland and P.Laurence, Energy Risk, (2011). Again for $r = 0$ we have for τ small the formula

$$C_{\mathrm{K3}}(S_1(t), S_2(t), S_3(t), K, \tau) \approx C_{\mathrm{BS}}(S_1(t), S_2(t) + S_3(t) + K, \sigma_S, \tau) \tag{1}$$

with

$$\begin{aligned} \sigma_S &= \sqrt{\sigma_1^2 + b_2^2\sigma_2^2 + b_3^2\sigma_3^2 - 2\rho_{12}\sigma_1\sigma_2 b_2 - 2\rho_{13}\sigma_1\sigma_3 b_3 + 2\rho_{23}\sigma_2\sigma_3 b_2 b_3} \\ b_2 &= \frac{S_2(t)}{S_2(t)+S_3(t)+K} \quad \text{and} \quad b_3 = \frac{S_3(t)}{S_2(t)+S_3(t)+K} \end{aligned}$$

and ρ_{ij} is the correlation between the underlying i, j.

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Basket Options

Assume underlyings

$$dS_i(t) = \mu_i S_i(t)dt + \sigma_i S_i(t)dW_i(t)$$

with $dW_i(t)dW_j(t) = \rho_{ij}dt$.
Here a basket of commodities is the underlying

$$B(t) = \sum_{i=1}^{m} w_i S_i(t).$$

In this model the forward price can be calculated as $F_i(0, T) = \mathbb{E}(S(T))$.

Basket Options – Example

Basket of energy prices related to power production

$$B(t) = 100 \times \left(30\% \frac{\mathrm{oil}(t)}{\mathrm{oil}(0)} + 30\% \frac{\mathrm{coal}(t)}{\mathrm{coal}(0)} + 40\% \frac{CO_2(T)}{CO_2(0)} \right).$$

At time 0 the basket is nomalized to 100. If the oil price increases by 100% the basket value increases by 30% to 130.

Basket Options

Pricing is not straightforward even in a BS-framework, since the sum of lognormals is not lognormal.
Typically a lognormal approximation is used, i.e.

$$\tilde{B}(T) = F_B \cdot \exp\left\{-\frac{1}{2}\beta^2 + \beta N)\right\} \quad N \sim \mathcal{N}(0,1).$$

Now

$$\begin{aligned}
\mathbb{E}(B(T)) &= \sum_{i=1}^{n} w_i F_i(0,T) \\
\mathbb{E}[B^2(T)] &= \sum_{i,j=1}^{n} w_i w_j F_i(0,T) F_j(0,T) \exp\{\rho_{ij}\sigma_i\sigma_j T\}
\end{aligned}$$

Basket Options

On the other hand

$$\mathbb{E}(\tilde{B}(T)) = F_B \quad \mathbb{E}(\tilde{B}^2(T)) = F_B^2 \exp(\beta^2).$$

Setting the first two moments equal

$$\begin{aligned} F_B &= \sum_{i=1}^{n} w_i F_i(0, T) \\ \beta^2 &= \log\left(\frac{\sum_{i,j=1}^{n} w_i w_j F_i(0,T) F_j(o,T) \exp\{\rho_{ij}\sigma_i\sigma_j T\}}{F_B^2}\right) \end{aligned}$$

Now Black's formula can be use on $\tilde{B}$.

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Basket Options- Example(continued)

With volatilities 0.3, 0.2 and 0.4 for Oil, Coal, CO2 and correlations $\varphi_{0C} = 0.1$, $\varphi_{0CO_2} = 0.6$, $\varphi_{CCO_2} = -0.2$ one gets for one year at the money calls

	price	imp vol.
MC	8,81	22.85%
Approx	8,95	23.16%

This implies a small pricing error, which s acceptable given the parameter uncertainty for σ_i and ρ_{ij}.

Agenda

Markets and Objects

Stochastic Processes for Price Movements
- How to Model Price Movements?
- Basic Stochastic Calculus
- A Mean-Reversion Diffusion Model

Forward Models

Energy Derivatives

Stock Price Return

- We wish to model the time evolution of a stock price $S(t)$ and consider how S will change in some small time-interval from the present time t to a time $t + dt$ in the near future.
- Writing $dS(t)$ for the change $S(t + dt) - S(t)$ in S, the *return* on S in this interval is $dS(t)/S(t)$. We decompose the return into two components, a *systematic* part and a *random* part.
- The systematic is modelled by μdt, where μ is some parameter representing the mean rate of return of the stock.
- The random part is modelled by $\sigma dW(t)$, where $dW(t)$ represents the stochastic noise term driving the stock price dynamics, and σ is a second parameter describing how much the stock price fluctuates. Thus σ governs how volatile the price is, and is called the *volatility* of the stock.

Geometric Brownian Motion

Putting this together, we have the stochastic differential equation (SDE)

$$dS(t) = S(t)(\mu dt + \sigma dW(t)), \quad S(0) > 0, \tag{2}$$

due to Itô in 1944.

The economic importance of geometric Brownian motion was recognised by Paul A. Samuelson in his work, for which Samuelson received the Nobel Prize in Economics in 1970, and by Robert Merton, in work for which he was similarly honoured in 1997.

Brownian Motion I

- For the random noise we use Brownian Motion (introduced by the Botanist Robert Brown in 1828. It was introduced into finance by Louis Bachelier in 1900, and developed in physics by Albert Einstein in 1905. A mathematical theory was developed by Norbert Wiener) A stochastic process $X = (X(t))_{t \geq 0}$ is a standard Brownian motion, *BM*, if

(i) $X(0) = 0$ a.s.,

(ii) X has *independent increments*: $X(t+u) - X(t)$ is independent of $\sigma(X(s) : s \leq t)$ for $u \geq 0$,

(iii) X has *stationary increments*: the law of $X(t+u) - X(t)$ depends only on u,

and (iv), (v)

Brownian Motion II

A stochastic process $X = (X(t))_{t \geq 0}$ is a standard Brownian motion, *BM*, if (i) – (iii) and

(iv) X has *Gaussian increments*: $X(t+u) - X(t)$ is normally distributed with mean 0 and variance u, $X(t+u) - X(t) \sim N(0, u)$,

(v) X has *continuous paths*: $X(t)$ is a continuous function of t.

Itô Processes

- We will use the following type of process expressed in terms of the stochastic differential equation

$$dX(t) = b(t)dt + \sigma(t)dW(t), \quad X(0) = x_0.$$

- For functions f we want to give meaning to the stochastic differential $df(X(t))$ of the process $f(X(t))$.
- This is done by the *Itô Formula*

$$\begin{aligned} df(X(t)) &= f'(X(t))dX(t) \\ &\quad + \frac{1}{2} f''(X(t))\sigma^2 dt. \end{aligned}$$

Multiplication rules

- The second term above corrects for special path properties of Brownian Motion and needs the quadratic variation of the process.
- We find

$$\begin{aligned}(dX)^2 &= (bdt + \sigma dW)^2 \\ &= \sigma^2 dt + 2b\sigma dtdW + b^2(dt)^2 = \sigma^2 dt.\end{aligned}$$

- The quadratic variation of any Itô process can be calculated using the multiplication rules

	dt	dW
dt	0	0
dW	0	dt

General Itô Formula

If $X(t)$ is an Itô process and $f(t,x)$ a function with time and location variable, then $f = f(t, X(t))$ has stochastic differential

$$df = \left(f_t + bf_x + \frac{1}{2}\sigma^2 f_{xx} \right) dt + \sigma f_x dW.$$

Observe, that we left out all function arguments

Example: Geometric Brownian Motion

The SDE for GBM has the unique solution

$$S(t) = S(0)\exp\left\{\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma W(t)\right\}.$$

Therefore, writing

$$f(t,x) := \exp\left\{\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma x\right\},$$

we have

$$f_t = \left(\mu - \frac{1}{2}\sigma^2\right)f, \quad f_x = \sigma f, \quad f_{xx} = \sigma^2 f,$$

and with $x = W(t)$, one has

$$dx = dW(t), \quad (dx)^2 = dt.$$

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Example: GBM

Thus Itô's lemma gives

$$
\begin{aligned}
df &= f_t dt + f_x dW + \frac{1}{2} f_{xx} (dW)^2 \\
&= f\left(\left(\mu - \frac{1}{2}\sigma^2\right) dt + \sigma dW + \frac{1}{2}\sigma^2 dt\right) \\
&= f(\mu dt + \sigma dW).
\end{aligned}
$$

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Pricing Derivatives: Risk-Neutral Valuation

- A financial derivative is a function $f(S_T)$ of some underlying stock (or other financial asset) and as such a random variable.
- In a standard financial market model we can calculate the price of a derivative with the the risk-neutral valuation formula

$$\Pi(0) = e^{-rT}\mathbb{E}^*\left[f(S(T))\right],$$

  where r is the interest rate and
- a special probability measure (or distribution for the underlying) has been used.

Black-Scholes Model

The classical Black-Scholes model is

$$\begin{aligned} dB(t) &= rB(t)dt, & B(0) &= 1, \\ dS(t) &= S(t)\left(bdt + \sigma dW(t)\right), & S(0) &= p, \end{aligned}$$

with constant coefficients $b \in \mathbb{R},\ r, \sigma \in \mathbb{R}_+$.

Pricing a European Call

For a European call $C(T) = (S(T) - K)^+$ and we can evaluate the above expected value to obtain its Black-Scholes price process

$$
\begin{aligned}
C(t) &= S(t)\Phi(d_1(S(t), T-t)) \\
&\quad -Ke^{-r(T-t)}\Phi(d_2(S(t), T-t)).
\end{aligned}
$$

The functions $d_1(s,t)$ and $d_2(s,t)$ are given by

$$
\begin{aligned}
d_1(s,t) &= \frac{\log(s/K) + (r + \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}, \\
d_2(s,t) &= \frac{\log(s/K) + (r - \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}
\end{aligned}
$$

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Definition of the model

We assume a simple market model in which the price of the underlying commodity, S_t, follows a stochastic process which can be described as follows: Let $X_t = \ln S_t$ and

$$dX_t = \kappa(\ln\theta - X_t)dt + \sigma dW_t \quad , \quad X_0 = \ln(S_0)$$

Thus, the logarithm of the prices follow a mean reverting diffusion process, the so-called Ornstein-Uhlenbeck-Process.

- κ - Speed of mean reversion
- θ - Level of mean reversion
- σ - Volatility of the process
- dW_t - Brownian increments

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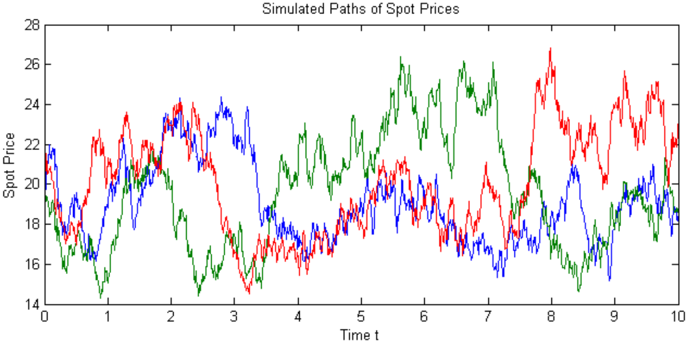
Sample paths of the model

We simulate S_t (with $S_0 = 20, \theta = 20, \kappa = 1, \sigma = 0.2$) and get sample paths



Properties of the model

- Mean reverting
- Bounded volatility
- Continuous paths
- Relative price changes are normally distributed
- Analytic results for the forward-curve and option prices exist
- Calibration easily possible

Spot prices in the model

The spot price at any time t is

$$S_t = \exp\left(e^{-\kappa t}\ln S_0 + (1 - e^{-\kappa t})\ln\theta + \int_0^t \sigma e^{-\kappa(t-s)} dW_s\right).$$

Forward prices in the model

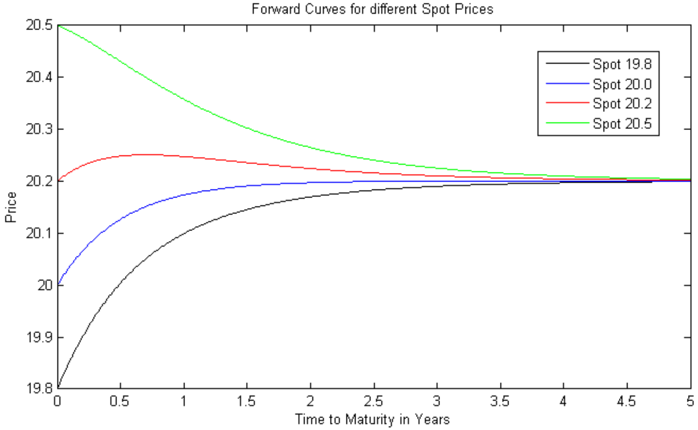
Using that the forward price is the expected value of future spot prices, $F(t, T) = \mathbb{E}^Q[S_T|\mathcal{F}_t]$, we get the formula for the forward price at time t for the forward expiring in T as

$$F(t, T) =$$
$$\exp\left(e^{-\kappa(T-t)} \ln S_t + (1 - e^{-\kappa(T-t)}) \ln \theta + \frac{\sigma^2}{4\kappa}(1 - e^{-2\kappa(T-t)})\right).$$

We can see that the forward prices converge to the spot price with time to maturity tending to zero.

Forward curves in the model

Assuming the same parameters as before and varying S_t we get the forward curves:



Properties of the forward curve in the model

- $F(t,T) \to \exp(\ln\theta + \frac{\sigma^2}{4\kappa}) = \theta \exp\left(\frac{\sigma^2}{4\kappa}\right)$ as $T \to \infty$.
- If the spot price is low compared to the long term mean, the forward curve is upward sloping (contango).
- If the spot price is high compared to the long term mean, the forward curve is downward sloping (backwardation).
- If the spot price is close to the long term mean, the forward curve might be humped-shaped.

Agenda

Storage, Inventory and Convenience Yield

- The theory of storage aims to explain the differences between spot and Futures (Forward) prices by analyzing why agents hold inventories.
- Inventories allow to meet unexpected demand, avoid the cost of frequent revisions in the production schedule and eliminate manufacturing disruption.
- This motivates the concept of convenience yield as a benefit, that accrues to the owner of the physical commodity but not to the holder of a forward contract.
- Thus the convenience yield is comparable to the dividend yield for stocks.
- A modern view is to view storage (inventory) as a timing option, that allows to put the commodity to the market when prices are high and hold it when the prices are low.

Spot-Forward Relationship in Commodity Markets

Under the no-arbitrage assumption we have

$$F(t, T) = S(t)e^{(r-y)(T-t)} \tag{3}$$

where r is the interest rate at time t for maturity T and y is the convenience yield.

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Spot-Forward Relationship in Commodity Markets

Observe that (3) implies

- spot and forward are redundant (one can replace the other) and form a linear relationship (unlike options)
- with two forward prices we can derive the value of $S(t)$ and y
- knowledge of $S(t)$ and y allows us to construct the whole forward curve
- for $r - y < 0$ we have backwardation; for $r - y > 0$ we have contango.

Spot-Forward Relationship: Classical theory

- In a stochastic model we use

  $$F(t, T) = \mathbb{E}_{\mathbb{Q}}[S(T)|\mathcal{F}_t]$$

  where $\mathcal{F}_t$ is the accumulated available market information (in most models the information generated by the spot price).
- $\mathbb{Q}$ is a risk-neutral probability
  - discounted spot price is a $\mathbb{Q}$-martingale
  - fixed by calibration to market prices or a market price of risk argument

Forward Prices and Expectation of Future Spot Prices

The rational expectation hypothesis (REH) states that the current forward price $F(t, T)$ for a commodity with delivery a time $T > t$ is the best estimator for the price $S(T)$ of the commodity. In mathematical terms

$$F(t, T) = \mathbb{E}_{\mathbb{P}}[S(T)|\mathcal{F}_t]. \tag{4}$$

where $\mathcal{F}_t$ represents the information available at time t. The REH has been statistically tested in many studies for a wide range of commodities.

Futures Prices and Expectation of Future Spot Prices

When equality in (4) does not hold forward prices are biased estimators of future spot prices. If

- $>$ holds, then $F(t,T)$ is an up-ward biased estimate, then risk-aversion among market participants is such that buyers are willing to pay more than the expected spot price in order to secure access to the commodity at time T (political unrest);
- $<$ holds, then $F(t,T)$ is an down-ward biased estimate, this may reflect a perception of excess supply in the future.

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Market Risk Premium

- The *market risk premium* or *forward bias* $\pi(t, T)$ relates forward and expected spot prices.
- It is defined as the difference, calculated at time t, between the forward $F(t, T)$ at time t with delivery at T and expected spot price:

$$\pi(t, T) = F(t, T) - \mathbb{E}_{\mathbb{P}}[S(T)|\mathcal{F}_t]. \tag{5}$$

Here $\mathbb{E}_{\mathbb{P}}$ is the expectation operator, under the historical measure $\mathbb{P}$, with information up until time t and $S(T)$ is the spot price at time T.

Heath-Jarrow-Morton (HJM) model

The Heath-Jarrow-Morton model uses the entire forward rate curve as (infinite-dimensional) state variable. The dynamics of the forward rates $F(t, T)$ are *exogenously* given by

$$dF(t, T) = \alpha(t, T)dt + \sigma(t, T)dW(t).$$

For any fixed maturity T, the initial condition of the stochastic differential equation is determined by the current value of the empirical (observed) forward rate for the future date T which prevails at time 0.

One-Factor GBM Specification

Here the volatility is

$$\sigma_1(t, T) = e^{-\kappa(T-t)}\sigma$$

and

$$dF(t, T) = F(t, T)\sigma_1(t, T)dW(t)$$

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Two-Factor GBM Specification

Here the volatilities are

$$\sigma_1(t,T) = e^{-\kappa(T-t)}\sigma_1 \ \text{ and } \ \sigma_2 > 0$$

and

$$\frac{dF(t,T)}{F(t,T)} = \sigma_1(t,T)dW_1(t) + \sigma_2 dW_2(t)$$

Modelling Approach

- We use the HJM-framework to model the forward dynamics directly.
- We distinguish between forward contracts with a fixed time delivery and forward contracts with a delivery period, called *swaps*.
- Since the HJM-framework cannot be applied to the swap dynamics literally, we differentiate between the *decomposable* swaps and the *atomic* swaps and apply the framework to the atomic swaps.

Modelling Approach

The dynamics of a decomposable swap with delivery period $[T_1, T_N]$ can than be obtained from $N-1$ atomic swaps by

$$F(t, T_1, T_N) = \sum_{i=1}^{N-1} \frac{T_{i+1} - T_i}{T_N - T_1} F(t, T_i, T_{i+1}) \tag{6}$$

Modelling Approach

We discuss several lognormal dynamics of the swap price,

$$dF(t, T_1, T_2) = \Sigma(t, T_1, T_2)F(t, T_1, T_2)\, dW(t). \tag{7}$$

The only parameter in this model is the volatility function Σ which has to capture all movements of the swap price and especially the time to maturity effect.

Volatility Functions

We assume that the swap price dynamics *for all atomic swaps* is given by (7) where $\Sigma(t, T_1, T_2)$ is a continuously differentiable and positive function.

Starting out with a given volatility function for a fixed time forward contract we see that the volatility function Σ for the swap contract is given by

$$\Sigma(t, T_1, T_2) = \int_{T_1}^{T_2} \hat{w}(u, T_1, T_2)\sigma(t, u)\, du. \tag{8}$$

Schwartz

For the volatility function of the forward we use

$$\sigma(t, u) = ae^{-b(u-t)} \tag{9}$$

where $a, b > 0$ are constant.

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Schwartz

The time to maturity effect is modeled by a negative exponential function.

- When the time to maturity tends to infinity the volatility function converges to zero.
- The exponential function causes that the volatility increases as the time to maturity decreases which leads to an increased volatility when the contract approaches the maturity.

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Schwartz

Applying this forward volatility to (8) the swap volatility is:

$$\Sigma(t, T_1, T_2) = a\,\varphi(T_1, T_2) \tag{10}$$

where

$$\varphi(T_1, T_2) = \frac{e^{-b(T_1 - t)} - e^{-b(T_2 - t)}}{b(T_2 - T_1)} \tag{11}$$

The Black-76 specification of the forward volatility can be obtained if $\varphi(T_1, T_2) = 1$, that is $b = 0$ in (9).
The associated swap price volatility is then given by $\Sigma(t, T_1, T_2) = a$.

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Fackler-Tian

Fackler and Tian replaced the factor a by a seasonality function $a(t)$.

$$\sigma(t,u) = a(t)e^{-b(u-t)} \tag{12}$$

where $a(t)$ is a seasonality function modeled by

$$a(t) = a + \sum_{j=1}^{7}(\alpha_j \cos(2\pi jt) + \beta_j sin(2\pi jt)) \tag{13}$$

with t measured in years. The parameters α_j and β_j are real numbers and $a \geq 0, b > 0$.

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Fackler-Tian

The volatility function of the swap price is therefore

$$\Sigma(t, T_1, T_2) = a(t)\,\varphi(T_1, T_2) \tag{14}$$

where $\varphi(T_1, T_2)$ is given by (11).

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Clewlow - Strickland

- Empirical observations show that in energy markets we can observe that the volatility increases strictly as the contract approaches maturity.
- Such a sharp raise can be modeled by a high value for b.
- The drawback of a high value for b is that for contracts with long time to maturity the volatility decreases fast and thus becomes very small.

Clewlow - Strickland

For this reason *Clewlow and Strickland* suggested

$$\sigma(t,u) = a((1-c)e^{-b(u-t)} + c) \tag{15}$$

where $a \geq 0, b > 0$ and $0 \leq c \leq 1$.

Clewlow - Strickland

The associated swap volatility model becomes

$$\Sigma(t, T_1, T_2) = a((1 - c)\,\varphi(T_1, T_2) + c) \tag{16}$$

where $\varphi(T_1, T_2)$ is defined as in (11).

Koekenbakker – Lien

Koekenbakker and Lien suggested to use a seasonality function in the *Strickland* model (15) and proposed a volatility which is given by

$$\sigma(t,u) = a(t)\left((1-c)e^{-b(u-t)} + c\right) \tag{17}$$

where $a(t)$ is defined as in (13) and with parameters $a \geq 0, b > 0$, $0 \leq c \leq 1,\ \alpha_j,\ \beta_j$ are real constants.

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Koekenbakker – Lien

- Observe that $\lim_{u \to t} \sigma(t, u)$ exists and $\sigma(t, t) = a(t)$.
- Hence the spot volatility is modeled by the seasonal function.
- As u approaches infinity we obtain that $\sigma(t, u) = a(t)c$.
- Thus, the volatility is bounded within the interval $[a(t)c, a(t)]$, where $ca(t) \leq a(t)$, since $0 \leq c \leq 1$.

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Koekenbakker – Lien

We obtain the swap volatility model

$$\Sigma(t, T_1, T_2) = a(t)((1 - c)\,\varphi(T_1, T_2) + c). \tag{18}$$

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Benth – Koekenbakker

Benth and Koekebakker suggested the following forward curve model

$$\sigma(t,u) = \hat{a}e^{-b(u-t)} + a(t) \tag{19}$$

where $\hat{a} \geq 0, b > 0$ and $a(t)$ is given by (13).
In this model the seasonality effect is separated from the maturity effect. For the short term volatility we have $\hat{a} + a(t)$ and for the long term volatility we obtain $a(t)$.

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Benth – Koekenbakker

The associated swap volatility is given by

$$\Sigma(t, T_1, T_2) = \hat{a}\,\varphi(T_1, T_2) + a(t). \tag{20}$$

Model Summary

A summary of the different models is listed in Table 1.

Model	$\Sigma(t, T_1, T_2)$
Black-76	a
Schwartz	$a\,\varphi(T_1, T_2)$
Fackler and Tien	$a(t)\,\varphi(T_1, T_2)$
Clewlow and Strickland	$a((1-c)\,\varphi(T_1, T_2) + c)$
Koekebakker and Lien	$a(t)((1-c)\,\varphi(T_1, T_2) + c)$
Benth and Koekebakker	$\hat{a}\,\varphi(T_1, T_2) + a(t)$

Table : The associated swap volatility models generated by (8) with $a \geq 0, b > 0$ and $0 \leq c \leq 1$ constants, $a(t)$ defined in (13) and $\varphi(T_1, T_2)$ is given by (11).

Agenda

Markets and Objects

Stochastic Processes for Price Movements

Forward Models

Energy Derivatives

- Caps and Floors
- Swing Options
- Spread Options
- Energy Swaps
- Options on Electricity Swaps

Caps

Buying a cap, the option holder has the right (but not the obligation) to buy a certain amount of energy at stipulated times $t_1, \ldots, t_N$ during the delivery period at a fixed strike price K. It can be viewed as a strip of independent call options, for each time t_i the holder of the cap holds call options with maturity t_i and Strike K.

The static factors describing the cap are:

- times $t_1, \ldots, t_N$ (how often? when?)
- strike K (price?)
- amount of the underlying (how much?)

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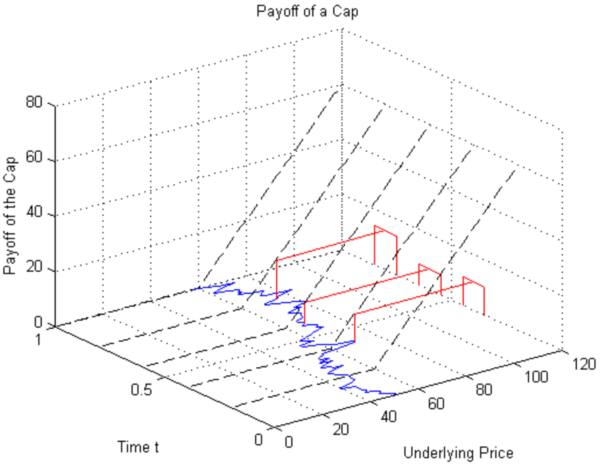
Cap - Payoff



Caps - Pricing

Whenever the price of the underlying exceeds the strike K at one of the dates $t_1, \ldots, t_N$, the seller of the cap pays the holder of the cap the difference between the price of the underlying and the strike K or - in case one agreed on physical delivery - the underlying is delivered for the price K. Typically, the price of a cap is quoted as price per delivery hours to make different delivery periods comparable. In this case we get a price per MWh. The formula is

$$U_c(t) = \frac{1}{N} \sum_{i=1}^{N} e^{-r(t_i - t)} \mathbb{E}[\max(S(t_i) - K, 0)].$$

Caps - Hedging

The strike price K secures a maximum price for which the option holder is able to buy energy. A cap is used to cover a short position in the underlying (energy) against increasing market prices not only at a certain point in time but over the whole period covered by the exercising times $t_1, \ldots, t_N$. On the other hand, the option holder is still able to profit from low energy prices as he has the right but not the obligation to exercise the option at each time point.

Caps - Example

Assume you need 100 units of the underlying per day to run your business. Today it costs 100 Euro/unit. You can accept resource cost of up to 110 Euro/unit in order to beneficially run your business. You are afraid of rising prises and want to hedge against this risk but still have the chance to profit from low prices.
Thus, you ask for a cap with daily exercise up to the business horizon of 8 days with volume 100 units and, say, strike 108 Euro/unit which might cost 1600 Euro (2 Euro/unit). Then, your total cost is at most 108 Euro + 2 Euro = 110 Euro/unit but you still participate on low prices.

Caps - Example

The table shows one possible result of the cap on the profit of the company.

Day	Underlying	Cost without Cap	Cost with Cap
1	100	100	102
2	111	111	110
3	116	116	110
4	120	120	110
5	109	109	110
6	97	97	99
7	85	85	87
8	78	78	80
Average		102	101
S.d.		14.9	11.7

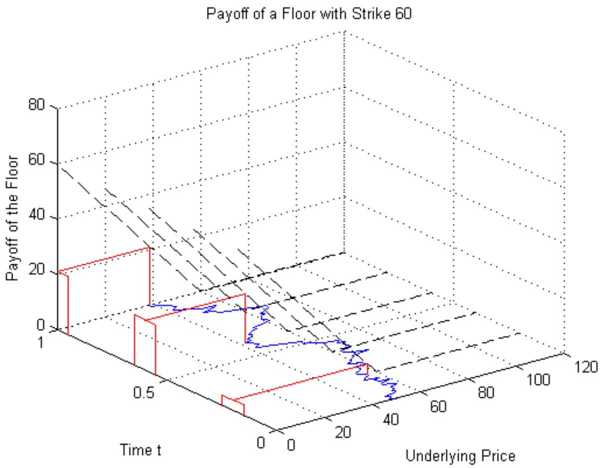
Floors

Buying a floor, the option holder has the right (but not the obligation) to sell a certain amount of energy at stipulated times $t_1, \ldots, t_N$ during the delivery period at a fixed strike price K. It can be viewed as a strip of independent put options, for each time t_i the holder of the floor holds put options with maturity t_i and Strike K.
Similar to the case of a cap, the pricing formula is

$$U_f(t) = \frac{1}{N} \sum_{i=1}^{N} e^{-r(t_i - t)} \mathbb{E}[\max(K - S(t_i), 0)].$$

As with the cap, the price is quoted in Euro/MWh.

Floor - Payoff

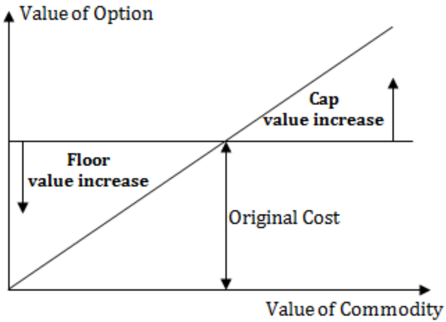


Floors - Hedging

The strike price K secures a minimum price for which the option holder is able to sell energy. A floor is used to cover a long position in the underlying (energy) against decreasing market prices not only at a certain point in time but over the whole period covered by the exercising times $t_1, \dots, t_N$. On the other hand, the option holder is still able to profit from high energy prices as he has the right but not the obligation to exercise the option at each time point.
The holder of a short position might write a floor to produce liquidity upfront. The maximum gain from the short position is then limited to the strike K.

Example: Caps and Floors

For a fixed premium, a buyer of a cap (call) is protected on the market price becoming stronger, while a buyer of a floor (put) is protected on the market price becoming weaker.

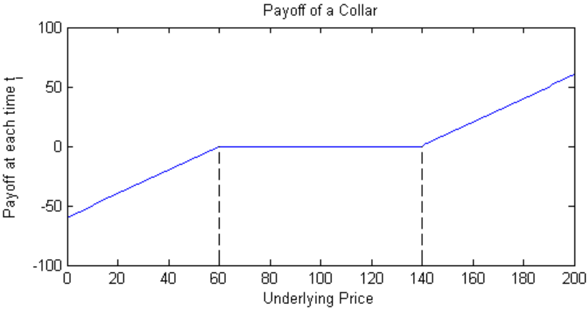


Collars

A collar is a combination of a cap and a floor such that variable prices are limited to a certain corridor. A long collar position consists of long one cap (with high strike K_2) and short one floor (with low strike K_1) - a short collar position is short one cap and long one floor. As long as the price of the underlying is between K_1 and K_2 at one of the dates t_i, no cash flows are exchanged. If the underlying is above K_2, the holder of the long collar position receives the difference of the actual price and K_2. If the underlying is below K_1, the short collar position receives the difference between K_1 and the actual price.

Collar - Payoff

As a long collar position is a strip of call options minus a strip of put options, the payoff of a collar at each time point t_i is the following:



Collar - Pricing

Collars might be seen as a strip of bear/bull spreads, or as a strip of call options minus a strip of put options in the case of a long collar position. Consequently, the pricing formula is just the combination of the formulas for the cap and the floor:

$$\begin{aligned} U_{collar}^{K_1,K_2}(t) &= U_{cap}^{K_2}(t) - U_{floor}^{K_1}(t) \\ &= \frac{1}{N}\sum_{i=1}^{N} e^{-r(t_i-t)}\mathbb{E}[(S(t_i)-K_2)^+ - (K_1 - S(t_i))^+] \end{aligned}$$

The price of a collar might be positive or negative - or even zero. In case the price is zero, the collar is called zero-cost collar.

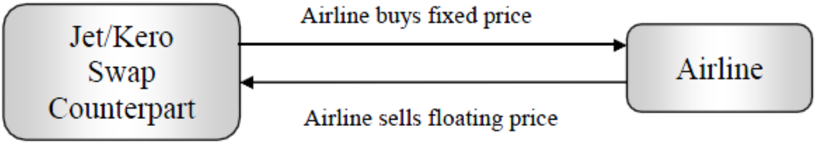
Collars - Hedging

The holder of a long position in a collar is protected against increases in the underlying price above K_2, but does not profit from falling underlying prices below K_1. Thus he is protected against rising prices with limited participation on downside prices. Having a short position in the underlying, a long collar ensures the ability to cover the short position for prices in the range of $[K_1, K_2]$. A short collar protects against falling prices. At the same time, the ability to participate on rising prices is limited to K_2. Having a long position in the underlying, a short collar ensures that the position can be closed for prices in the range of $[K_1, K_2]$.

Collars - Example

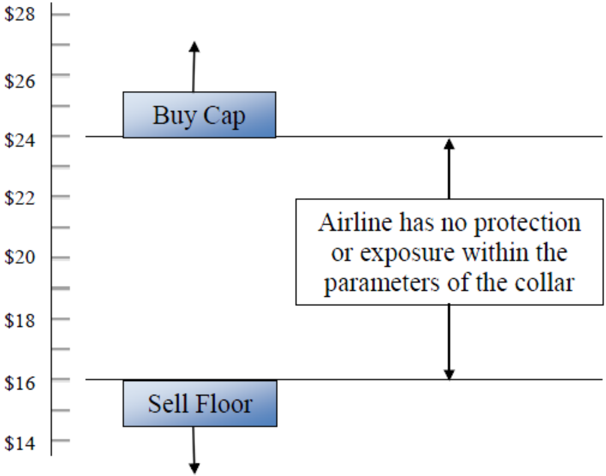
An energy consuming manufacturer bought the energy needed on the futures market. As its competitors did not, the manufacturer is now concerned about falling energy prices which would lead to a competitive disadvantage. Thus, the manufacturer tries to enter a short collar, protecting him against falling prices but leaving the risk of rising prices above K_2. This risk might be acceptable for the manufacturer as if prices rise too much, the manufacturer is able to stop its production and selling the energy already bought on the spot market - offsetting the losses of the collar.

Example: Jet Fuel Hedge by an Airline



- An airline buys a fixed-price swap from a bank or trader against its jet-fuel price exposure.
- Buying a swap it must lock in its minimum net price receivable at the current perceived swap value.

Jet Fuel Hedge by an Airline: Collar



Jet Fuel Hedge by an Airline: Collar

- Using a collar structure the airline can still protect itself from a price increase, but can keep its minimum net price receivable locked in at a lower rate than the current swap price.
- The purchase of the cap protects against jet-fuel prices rising above the strike of the cap.
- The sale of the floor reduces the cost of the premium in the purchase of the cap.
- A popular strategy is a zero-cost collar.

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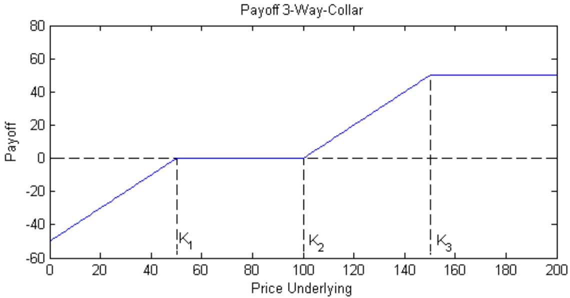
Collars - 3-way-collars

A long collar is short one floor with strike K_1, long one cap with higher strike K_2. A possible extension is to include a short position in one cap with strike $K_3 >> K_2$ in order to reduce the cost of the collar. This extension is called 3-way-collar. The price of a 3-way-collar is thus:

$$\begin{aligned} U_{3-way}^{K_1,K_2,K_3}(t) &= U_{cap}^{K_2}(t) - U_{cap}^{K_3}(t) - U_{floor}^{K_1}(t) \\ &= \frac{1}{N}\sum_{i=1}^{N} e^{-r(t_i-t)}\mathbb{E}[(S(t_i) - K_2)^+ \\ &\quad - (S(t_i) - K_3)^+ - (K_1 - S(t_i))^+] \end{aligned}$$

3-Way-Collar - Payoff

The holder of the 3-way-collar is protected against increases in the underlying price above K_2, but only till K_3. Afterwards, no protection exists anymore. This strategy might be a good choice if one wants to protect its buying costs but is able to stop its business if prices rally unexpectedly high (above K_3).



Swing Options

A swing option is similar to a cap or floor except that we have additional restrictions on the number of option exercises. Let $\phi_i \in \{0,1\}$ be the decision whether to exercise ($\phi_i = 1$) or not to exercise ($\phi_i = 0$) the option at time t_i. The option's payoff at time t_i is given by

$$\phi_i(S(t_i) - K) \quad \text{call resp.} \quad \phi_i(K - S(t_i)) \quad \text{put.}$$

We now require that the number of exercises is between $E_{\min}$ and $E_{\max}$.

Swing Options

To determine the swing option value, we have to find an optimal exercise strategy $\Phi = (\phi_1, \ldots, \phi_N)$ maximising the expected payoff

$$\sum_{i=1}^{N} e^{-r(t_i-t)}\mathbb{E}[\phi_i(S(t_i) - K)] \quad \rightarrow \max$$

subject to

$$E_{\min} \leq \sum_{i=1}^{N} \phi_i \leq E_{\max}.$$

To calculate the option value various mathematical techniques are used.

Bounds for Swing Options

Strategy

For deterministic spot prices, we

- Calculate the discounted payoffs $P(t_i) = e^{-r(t_i - t)}(S(t_i) - K)$.
- Sort the discounted payoffs $P(t_i)$ in descending order.
- Take the first $E_{\min}$ payoffs regardless of their value and subsequent payoffs up to $E_{\max}$ until their sign become negative.

Bounds for Swing Options

For stochastic spot prices the MC-approach gives an upper bound, since information on the whole path is used, but in reality only information up to time t is available when deciding at time t.

A lower bound is given by the intrinsic value

$$\sum_{i=1}^{N} e^{-r(t_i-t)} \phi_i^F (F(t,t_i) - K) \quad \to \max$$

subject to

$$E_{\min} \leq \sum_{i=1}^{N} \phi_i^F \leq E_{\max}$$

where $\phi_i^F = \mathbf{1}_{\{F(t,t_i)>K\}}$, unless the restriction on $E_{\min}$ is in force.

Spread Options

Some market participants are exposed to the difference of commodity prices. Examples are

- the dark spread between power and coal (model for a coal-fired power plant)
- the spark spread between power and gas (model for a gas-fired power plant)
- the crack spread between different refinements of oil (model for a refinement plant)

Spread Trading

Spreads are used to describe power plants, refineries, storage facilities and transmission lines. Spread positions may be initiated in futures contracts

- for different, but related commodities,
- for different delivery month of the same commodity,
- for same commodity traded on different exchanges.

Spread Trading

Spread trading involves taking a long position in one futures contract and simultaneously taking a short position in another, related futures contract.

- Spread position neutralizes price risk.
- A profit or loss results only if the relative prices of the two contracts change.
- If spreads are expected to narrow, buy the lower-priced contract and sell the higher-priced contract.
- If spreads are expected to widen, buy the higher-priced contract and sell the lower-priced contract.

Spark Spread

- Differential between the price of electricity (output) and the price of natural gas (input).
- Can be used to financially replicate the physical reality of a gas-fired power plant: Short position in fuels and long position in electricity.
- Spark spreads are traded OTC.

Spark Spread

$$\text{Spark\_Spread} = \text{Power\_Price} - \text{Heat\_Rate} \cdot \text{Fuel\_Price}.$$

- Heat rate provides a conversion factor between fuels used to generate power and the power itself.
- Heat rate is the number of Btus needed to make 1kWh of electricity.
- In the absence of any inefficiency it takes 3412Btu to produce 1kWh of electricity.

Example: Spark Spread

The price of electricity is currently 42.69EUR/MWh, the price of natural gas is 4.86EUR/MMBtu and the heat rate is 8152Btu/kWh. The spark spread quoted in EUR/MWh is

$$
\begin{aligned}
\mathit{Spread} &= 42.69\text{EUR/MWh} - 0.001 * 8152\text{Btu/kWh} * 4.86\text{EUR/MMBtu} \\
&= 3.07\text{EUR/MWh}.
\end{aligned}
$$

The positive spark spread means that it is economical to run the plant (without taking into account additional generating costs).

Clean Spreads

In countries covered by the European Union Emissions Trading Scheme, utilities have to consider also the cost of carbon dioxide emission allowances. Emission trading has started in the EU in January 2005.

- Clean spark spread represents the net revenue a gas-fired power plant makes from selling power, having bought gas and the required number of carbon allowances.
- Clean dark spread represents the net revenue a coal-fired power plant makes from selling power, having bought coal and the required number of carbon allowances.
- The difference between the clean dark spread and the clean spark spread is known as the climate spread.

Clean Spark Spread

Clean Spark Spread = Power Price - Heat Rate · Gas Price -
- Gas Emission Intensity Factor · Carbon Price

Clean Spark Spread reflects the cost of generating power from gas after taking into account gas and carbon allowance costs. A positive spread effectively means that it is profitable to generate electricity, while a negative spread means that generation would be a loss-making activity. However, it is important to note that the Clean Spark Spreads do not take into account additional generating charges beyond gas and carbon, such as operational costs.

Clean Dark Spread

Clean Dark Spread = Power Price - Heat Rate · Coal Price -
- Coal Emission Intensity Factor · Carbon Price

Clean Dark Spread reflects the cost of generating power from coal after taking into account coal and carbon allowance costs. A positive spread effectively means that it is profitable to generate electricity for the period in question, while a negative spread means that generation would not be profitable. Clean Dark Spreads do not account for additional generating charges beyond coal and carbon.

Power Plant as a Clean Dark Spread

A coal-fired power plant can be viewed as a call option on the clean dark spread with the variable cost of running the plant (beyond coal and carbon) being the strike and the payoff equal to

$$\Pi = max\{P - HR \cdot Coal - I \cdot Carbon - V\}.$$

P: Power Price
HR: Heat Rate
Coal: Coal Price
I: Coal Emission Intensity Factor
Carbon: Carbon Price
V: Variable cost of running the plant (beyond coal and carbon)

Power Plant as a Clean Dark Spread

Indeed, the decision to run or not to run the power plant can be described as follows:

- If $P - HR \cdot Coal - I \cdot Carbon - V \geq 0$, then run the plant. In this case buying fuel and paying variable costs ($HR \cdot Coal + I \cdot Carbon + V$) to run the plant and then selling the generated power for P results in the positive gain.
- If $P - HR \cdot Coal - I \cdot Carbon - V < 0$, then do not run the plant. In this case buying fuel and paying variable costs ($HR \cdot Coal + I \cdot Carbon + V$) to run the plant will not be compensated by sold power.

Climate Spread

Climate Spread = Clean Dark Spread - Clean Spark Spread

In a carbon constrained economy a power producer in a geographic area where coal is currently the preferred method by which electricity is generated may eventually encounter a negative climate spread if carbon credit prices rise. This would mean that when taking into consideration the cost to produce (coal is on average 2.5 times as polluting as natural gas for the same MWh of electricity) the natural gas would be a better decision.

Example: Clean Spark Spread

The price of electricity is currently 42.69EUR/MWh, the price of natural gas is 4.86EUR/MMBtu, the carbon price is 12EUR/tCO_2, the heat rate is 8152Btu/kWh and the gas emission intensity factor is 0.11tCO_2/MWh. The clean spark spread quoted in EUR/MWh is

$$\begin{aligned}\text{Clean Spark Spread} &= 42.69\text{EUR/MWh} \\ &-0.001 * 8152\text{Btu/kWh} * 4.86\text{EUR/MMBtu} \\ &-0.11tCO_2/\text{MWh} * 12\text{EUR/t}CO_2 \\ &= 1.75\text{EUR/MWh}.\end{aligned}$$

It is profitable to generate electricity, if additional generating charges beyond gas and carbon are lower than 1.75EUR/MWh.

Example: Clean Dark Spread

The price of electricity is currently 42.69EUR/MWh, the coal price is 95.04EUR/t or 3.96EUR/MMBtu (with heat content of 24MMBtu/t), the carbon price is 12EUR/tCO_2, the heat rate is 9500Btu/kWh and the coal emission intensity factor is 0.26tCO_2/MWh. The clean dark spread quoted in EUR/MWh is

$$
\begin{aligned}
\text{Clean Dark Spread} &= 42.69\text{EUR/MWh} \\
&\quad -0.001 * 9500\text{Btu/kWh} * 3.96\text{EUR/MMBtu} \\
&\quad -0.26tCO_2/\text{MWh} * 12\text{EUR/t}CO_2 \\
&= 1.95\text{EUR/MWh}.
\end{aligned}
$$

It is profitable to generate electricity, if additional generating charges beyond coal and carbon are lower than 1.95EUR/MWh.

Example: Climate Spread

Suppose that the price of carbon rises to 19.6EUR/tCO_2.

$$
\begin{aligned}
\text{Climate Spread} = \text{Clean Dark Spread - Clean Spark Spread} \\
&= -0.026\text{EUR/MWh} - 0.915\text{EUR/MWh} \\
&= -0.941\text{EUR/MWh}.
\end{aligned}
$$

The clean dark spread becomes negative (-0.026EUR/MWh), implying that electricity generation by a coal-fired power plant would be a loss-making activity, whereas the clean spark spread remains positive (0.915EUR/MWh), meaning that it is profitable to generate electricity by a gas-fired power plant, if additional generating charges beyond gas and carbon are lower than 0.915EUR/MWh.

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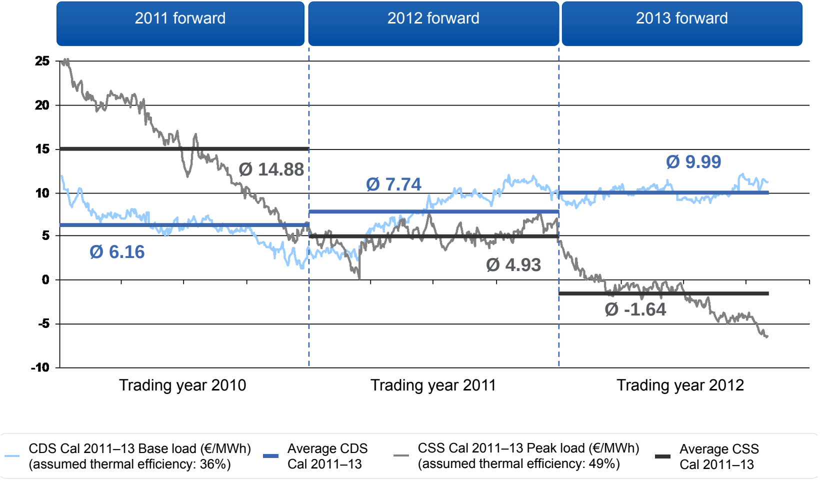
Clean Spark Spread Forward



Gas Power Plant

Political Risk

- Installed capacity: 876 MW
- Variable cost ca. 60 EUR/MWh
- Profitable hours per year
  - 2010 (993),
  - 2011 (2309),
  - 2012 (737),

  with average profit 6.9 EUR per MWh.
- Typical assumption on investing
  - 3500 profitable hours
  - 10 EUR per MWh profit
- loss per year
  - 2010: (3500-993)*876*10=21961320 EUR,
  - 2011: (3500-2309)*876*10 = 10433160 EUR,
  - 2012: (3500-737)*876*10= 24203880 EUR.

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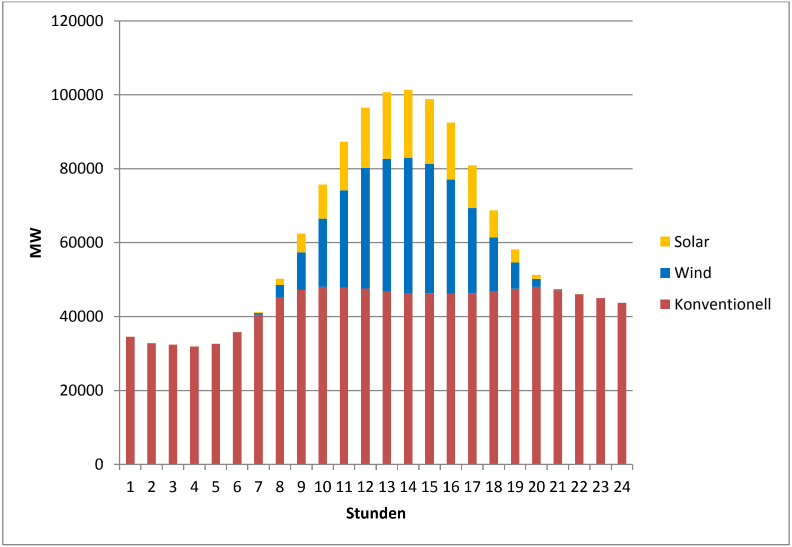
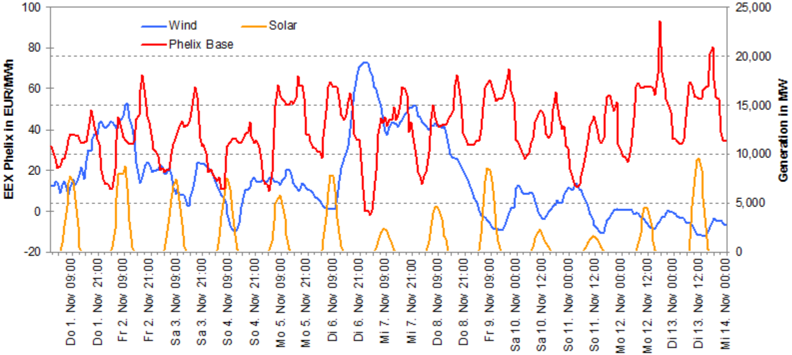
A day in august



Wind, sun and electricity



Standard Specification of Energy Swap

- An Energy Swaps is a contract between two parties to exchange - or swap - cash flows, one of which is a fixed price normally agreed at execution; the other is based on the average of a floating price index during the contract period.
- Thus the two counterparties of the deal, the buyer and the seller, exchange fixed cash-flows agreed at the contract time for unknown floating cash-flows in the future.
- No physical delivery of the underlying energy takes place; there is only financial settlement.

Specification of a Swap Contract

When traders are negotiating a swap contact (OTC deal) they have to define

- the fixed price
- the floating-price reference
- the pricing period (e.g. one month, quarterly, calender year)
- the start date (effective date)
- the end date (termination)
- the payment-due date

Pricing Period

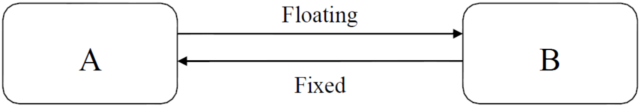
- In energy and general commodity markets, OTC derivatives are priced monthly.
- So even if a quarterly contract is traded, after each month during the pricing period, one-third of the volume will be priced out and a settlement will become due or a payment will be received by the organization.

Payment-due Date

- For a swap priced against an American or European floating price reference, the payment due date is normally the 5. business day after the last pricing day of each pricing period.
- For contracts priced against an Asian-based floating-price reference, payment for settlement is generally due 10 business days, sometimes up to 14 business days, after each pricing period.

Plain Vanilla Swap

- Simple monthly averaging swap in which a fixed price is exchanged against a floating price in the future.
- Extensively used in oil, LPG (liquified petrolium/propane gas), and LNG (liquified natural gas)-related trading and hedging.
- When executing the deal, the counterparties agree on the fixed price for that day, and which floating price reference they will use to calculate the settlement.
-

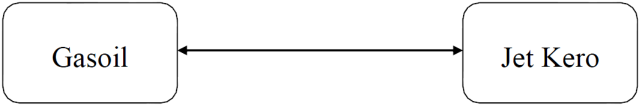


Plain Vanilla Swap

Cash-flow example of plain vanilla swap:

- A buys fixed price $15.00 (buys fixed, sells floating)
- B sells fixed price $15.00 (sells fixed, buys floating)
- Floating price reference (e.g. Platts) average during the pricing period $16.00
- Net result: Counterparty A = +$1.00 (floating-fixed)
  Counterparty B = -$1.00
- ⇒ B pays A $1.00. Only the difference is exchanged, NOT the
  principal national amount.

Differential Swap

- Instead of having one fixed price against a floating price, it is based on the difference between a fixed price in two products.
- In the oil sector, the most popular differential swap is the jet kero against gasoil (regrade swap).
- Use across the whole energy spectrum, e.g. spark spread, dark spread.
- 

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Differential Swap

Cash-flow example of differential swap:

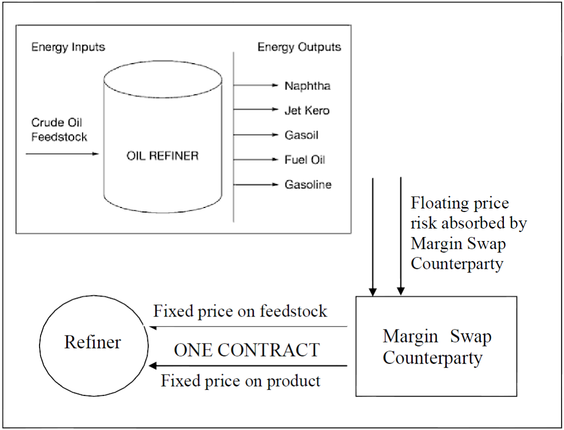
- A buys fixed-price kero and sells fixed-price gasoil at a difference of $0.50 per barrel kero premium
- B sells fixed-price kero and buys fixed-price gasoil at a difference of $0.50 per barrel kero premium
- Floating price reference (Platts) kero and gasoil average difference during the pricing period $0.60 per barrel kero premium
- Net result: Counterparty A = +$0.10 per barrel (floating-fixed)
  Counterparty B = -$0.10 per barrel
- $\Rightarrow$ B pays A $0.10 per barrel. Only the difference is exchanged,
  NOT the principal national amount.

Margin Swap

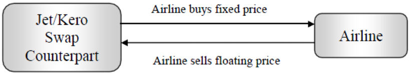
- An organization can take its overall price risks from several energy inputs and outputs of the business process and get a complete swap structure that guarantees its profit margin.
- Instead of managing many individual positions with several counterparties it can be more cost-efficient to enter into a margin swap with one counterparty that is willing to provide a contract that covers all the price risk.
- Single swap contract protects the net overall margin.

Margin Swap

Margin Swap for an oil refiner:



Jet Fuel Swap



- An airline buys a fixed-price swap from a bank or trader against its jet-fuel price exposure.
- Buying a swap it must lock in its minimum net price receivable at the current perceived swap value.

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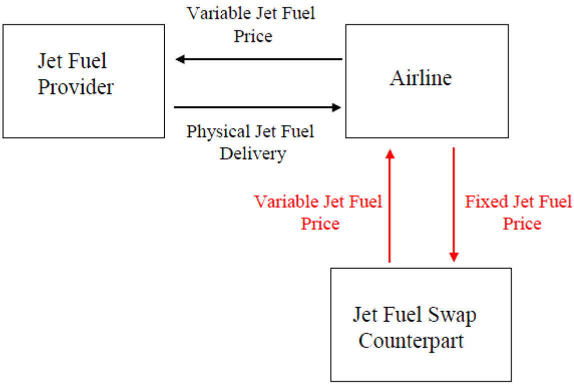
Example: Jet Fuel Swap

An airline expects that the oil price will rise in the future. In order to fix the oil price it buys on 15. June 2010 a fixed-price swap.



Example: Jet Fuel Swap

Contract specification/Fixed Jet Fuel Price:

Seller:	Trading Services GmbH
Buyer:	Airline AG
Fixed price:	600 USD/mt
Floating-price reference:	Platts PJAAU00 Jet Cargoes CIF NEW
Pricing period:	1. October 2011 (start/effective date) till 31. October 2011 (end date/termination)
Contract volume:	2000 mt, spread over all the days in October 2011 for which the reference index is published

Example: Jet Fuel Swap

Contract specification/Floating Jet Fuel Price:

Seller:	Airline AG
Buyer:	Trading Services GmbH
Fixed price:	600 USD/mt
Floating-price reference:	Platts PJAAU00 Jet Cargoes CIF NEW
Pricing period:	1. October 2011 (start/effective date) till 31. October 2011 (end date/termination)
Contract volume:	2000 mt, spread over all the days in October 2011 for which the reference index is published

Example: Jet Fuel Swap

Clearing:

- The payment settlement date is the 1. bank business day (1. November 2011) after the last pricing day (31. October 2011) of the pricing period (1.-31. October 2011).
- The payment due date is the 5. bank business day (7. November 2011) after the last pricing day of the pricing period.
- Bank holidays according to `http://www.chicagofed.org/webpages/utilities/about_us/bank_holidays.cfm`.

Example: Jet Fuel Swap

Payment:

- Payment $=(\text{Float-Fix})\times\text{Volume}$, if $\text{Float}\geq\text{Fix}$.
- Payment$=(\text{Fix-Float})\times\text{Volume}$, if $\text{Float}<\text{Fix}$.

Payment in USD
Floating/fixed price in USD/mt
Volume in mt (metric tones)

Example: Jet Fuel Swap

- Suppose that the average Jet Fuel reference index for October 2011 is 700USD/mt. The airline buys the fuel for 600USD/mt (fixed price) and sells for 700USD/mt (floating price reference for October 2011). Thus the airline's profit is

$$(700\text{USD/mt} - 600\text{USD/mt}) \times 2000\text{mt} = 200\,000\text{USD}.$$

- Suppose that the average Jet Fuel reference index for October 2011 is 550USD/mt. The airline buys the fuel for 600USD/mt (fixed price) and sells for 550USD/mt (floating price reference for October 2011). Thus the airline's loss is

$$(600\text{USD/mt} - 550\text{USD/mt}) \times 2000\text{mt} = 100\,000\text{USD}.$$

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Two-Factor GBM Specification

The basic HJM model for the dynamics of the forward rates $f(t,T)$ is given by

$$df(t,T) = \alpha(t,T)dt + \sigma(t,T)dW(t).$$

We consider a two-factor model

$$\frac{dF(t,T)}{F(t,T)} = \sigma_1(t,T)dW_1(t) + \sigma_2 dW_2(t)$$

where the volatilities are

$$\sigma_1(t,T) = e^{-\kappa(T-t)}\sigma_1 \text{ and } \sigma_2 > 0$$

The Model Framework for Electricity Swaps

- Use observable products, e.g. month futures as building blocks,
- Under a risk-neutral measure month forward prices $F(t, T, T+m) = F(t, T)$ have to be martingales,
- Assume the dynamics

  $$dF(t, T) = \sigma(t, T)F(t, T)dW(t),$$

  where $\sigma(t, T)$ is an adapted d-dimensional deterministic function and $W(t)$ a d-dimensional Brownian motion.
- Initial value of this SDE is the initial forward curve observed at the market.

Options on Building Blocks

A European call option on $F(t, T)$ with maturity T_0 and strike K can be easily evaluated by the Black-formula

$$V^{option}(0) = e^{-rT_0}\left(F(0, T)\Phi(d_1) - K\Phi(d_2)\right), \tag{21}$$

where Φ denotes the normal distribution, $\Sigma(T_0, T) = \int_0^{T_0} ||\sigma(s, T)||^2 ds$ and

$$\begin{aligned} d_1 &= \frac{\log \frac{F(0,T)}{K} + \frac{1}{2}\Sigma(T_0, T)}{\sqrt{\Sigma(T_0, T)}} \\ d_2 &= d_1 - \sqrt{\Sigma(T_0, T)} \end{aligned}$$

The Model Framework – *n*-period futures

- Use observable products, e.g. month futures as building blocks,
- Express an n-period delivery futures as

$$Y_{T_1,\ldots,T_n}(t) = \frac{\sum_{i=1}^{n} e^{-r(T_i-t)} F(t,T_i)}{\sum_{i=1}^{n} e^{-r(T_i-t)}}.$$

  (Compare modelling of forward swap rates in terms of forward LIBOR rates)
- In case of 1-year-futures, the swap rate is the forward price of the 1-year-futures, which can be also observed in the market.

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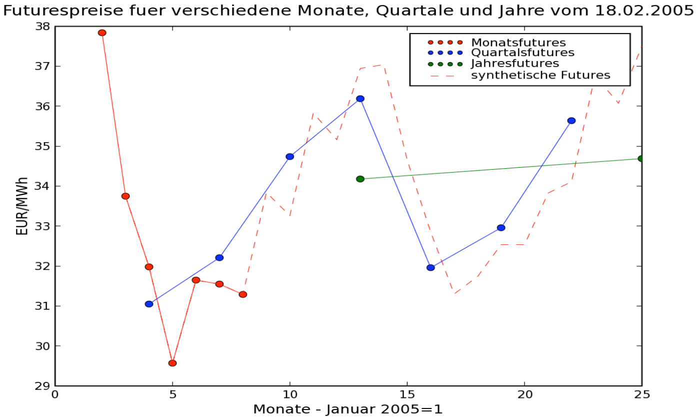
Implied Prices of Futures



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Options on *n*-period futures

- We need to compute

$$e^{-rT_0}\mathbb{E}\left[(Y(T_0)-K)^+\right],$$

where the distribution of Y as a sum of lognormals is unknown.

- We approximate Y by a random variable $\hat{Y}$, which is lognormal and matches Y in mean and variance.
- Then,

$$\log \hat{Y} \sim \Phi(m,s)$$

with s^2 depending on the choice of the volatility functions $\sigma(t,T_i)$.

- An analysis of the goodness of the approximation can be found in Brigo-Mercurio (2003).

Options on *n*-period futures

Using this approximation, it is possible to apply a Black-Option formula again to obtain the option value as

$$\begin{aligned} V^{option} &= e^{-rT_0}\mathbb{E}\left[(Y(T_0)-K)^+\right] \\ &\approx e^{-rT_0}\mathbb{E}\left[\left(\hat{Y}(T_0)-K\right)^+\right] \\ &= e^{-rT_0}\left(Y(0)\Phi(d_1)-K\Phi(d_2)\right) \end{aligned} \tag{22}$$

with

$$\begin{aligned} d_1 &= \frac{\log\frac{Y(0)}{K}+\frac{1}{2}s^2}{s} \\ d_2 &= d_1 - s \end{aligned}$$

Specific Two-Factor Model

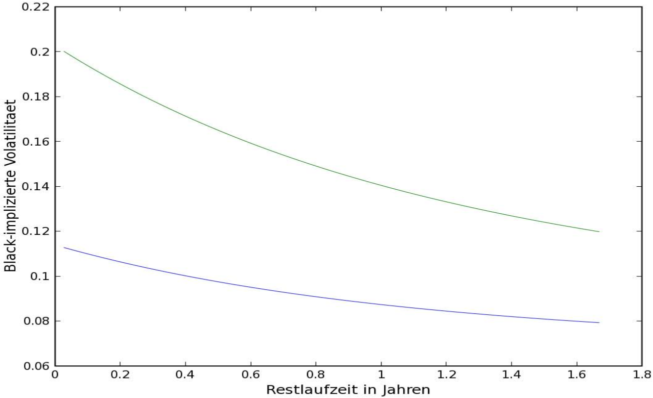
- For a fixed delivery start T and delivery period 1 month, let the dynamics of a Future $F_{t,T}$ be given by the two factor model:

$$F(t,T) = F(0,T)\exp\left\{\mu(t,T) + \int_0^t \hat{\sigma_1}(s,T)dW_s^{(1)} + \sigma_2 W_t^{(2)}\right\}$$

- $W^{(1)}$ and $W^{(2)}$ independent Brownian motions
- $\hat{\sigma_1}(s,T) = \sigma_1 e^{-\kappa(T-s)}$
- $\sigma_1, \sigma_2, \kappa > 0$ constants
- $\mu(t,T)$ being the risk-neutral martingale drift

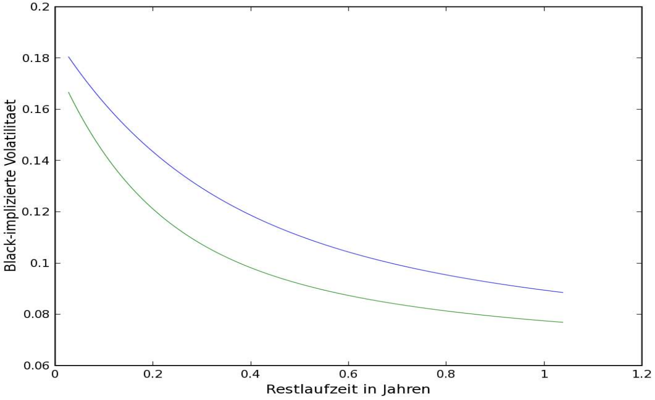
Model Parameters

σ_1 affects the level at the short end of the volatility curve



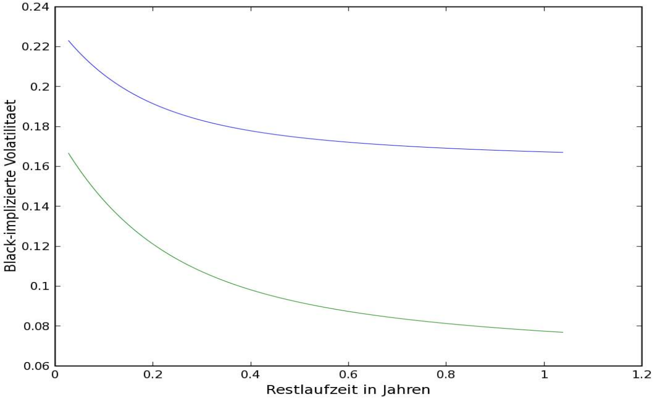
Model Parameters

κ affects the slope of the volatility curve at the short end



Model Parameters

σ_2 affects the level at the long end of the volatility curve



Pricing of Futures

- In this model, all products are expressed using Month-Futures
- Prices of quarterly and yearly Futures are given as an average of the n corresponding monthly Futures.
- $Y_{t,T_1,\ldots T_n} = Y = \frac{\sum e^{-rT_i} F_{t,T_i}}{\sum e^{-rT_i}}$ is the forward price of a n-month forward quoted in the market (cp. swap rate)

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Pricing of Options on Month-Futures

- At time $t = 0$, the price of a Call-Option with strike K and maturity T_0 on a Month-Future $F_{t,T}$ is given by

$$e^{-rT_0}\mathbb{E}\left[\left(F_{T_0,T} - K\right)^+\right]$$

- Within the model, $F_{T_0,T}$ is log-normally distributed with known variance

$$\Sigma(T_0, T) = \frac{\sigma_1^2}{2\kappa}(e^{-2\kappa(T-T_0)} - e^{-2\kappa T}) + \sigma_2^2 T_0$$

- Thus the option's value is given by the formula (Black 76):

$$e^{-rT_0}\mathbb{E}\left[\left(F_{T_0,T} - K\right)^+\right] \quad = \quad e^{-rT_0}\left(F_{0,T}\Phi(d_1) - K\Phi(d_2)\right)$$

with $d_{1,2}$ depending on the parameters $\sigma_1, \sigma_2, \kappa$.

Pricing of Options on quarterly and yearly Futures

- At time $t = 0$, the price of a Call-Option with strike K and maturity T_0 on a n-Month-Future Y is given by

$$e^{-rT_0}\mathbb{E}\left[(Y-K)^+\right] = e^{-rT_0}\mathbb{E}\left[\left(\frac{\sum e^{-rT_i}F_{t,T_i}}{\sum e^{-rT_i}} - K\right)^+\right]$$

- The distribution of the sum is not known within the model. There is no explicit solution to this integral.
- Approximate the random variable Y by a log-normal random variable $\hat{Y}$ with same mean and variance (depending on the model parameters)

Matching the Variance

Using the moment-generating function of a normal random variable, we get

$$\exp(s^2) = \frac{\mathrm{Var}(Y)}{(\mathbb{E}(Y))^2} + 1 = \frac{\mathbb{E}(Y^2)}{\mathbb{E}(Y)^2}$$

From the martingale property $\mathbb{E}(F_{T_0,T_i}) = F_{0,T_i}$ and

$$\mathbb{E}(Y_{T_1,\dots T_n}(T_0)) = \frac{\sum e^{-r(T_i - T_0)} F_{0,T_i}}{\sum e^{-r(T_i - T_0)}}.$$

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Matching the Variance

So

$$\mathbb{E}(Y_{T_1,\dots T_n}(T_0)^2) = \frac{\sum_{i,j} e^{-r(T_i+T_j-2T_0)} F_{0,T_i} F_{0,T_j} \cdot \exp Cov_{ij}}{\left(\sum e^{-r(T_i-T_0)}\right)^2}$$

with $Cov_{ij} = \mathrm{Cov}(\log F(T_0, T_i), \log F(T_0, T_j))$.
The covariance can be computed directly from the explicit solution of the SDE

$$\begin{aligned} & \mathrm{Cov}(\log F(T_0, T_i), \log F(T_0, T_j)) \\ = \quad & e^{-\kappa(T_i+T_j-2T_0)} \frac{\sigma_1^2}{2\kappa}(1 - e^{-2\kappa T_0}) + \sigma_2^2 T_0 \end{aligned}$$

Pricing of Options on quarterly and yearly Futures

The option value can be computed by Black's formula

$$\begin{aligned} e^{-rT_0}\mathbb{E}\left[(Y-K)^+\right] &\approx e^{-rT_0}\mathbb{E}\left[\left(\hat{Y}-K\right)^+\right] \\ &= e^{-rT_0}\left(Y(0)\Phi(d_1)-K\Phi(d_2)\right) \end{aligned}$$

with $d_{1,2}$ depending on the parameters $\sigma_1, \sigma_2, \kappa$.

Parameter Estimation

- Use the approximating Black-formula

$$\begin{aligned} \text{Option value} &= e^{-rT_0}\left(Y(0)\Phi(d_1) - K\Phi(d_2)\right) \\ d_{1,2} &= d_{1,2}\left(Y(0), K, Var(\log \hat{Y}(T_0))\right) \end{aligned}$$

  Only the variance $Var(\log \hat{Y}(T_0))$ depends on the unknown parameters

- Compute the variances $Var(\log \hat{Y}(T_0))$ for products observable in the market
- Choose parameter σ_1, σ_2 and κ to minimize the distance of the model variances to the market variances in a given metric (in the least-square sense)

Data

Product	Delivery Start	Strike	Forward	Market Price	Implied Vola
Month	October 05	48	48.90	2.023	43.80%
Month	November 05	49	50.00	3.064	37.66%
Month	December 05	49	49.45	3.244	34.72%
Quarter	October 05	48	49.44	2.086	35.15%
Quarter	January 06	47	48.59	3.637	28.43%
Quarter	April 06	40	40.71	3.421	26.84%
Quarter	July 06	42	41.80	3.758	27.19%
Quarter	October 06	43	43.71	4.566	25.35%
Year	January 06	44	43.68	1.521	20.19%
Year	January 07	43	42.62	3.228	19.14%
Year	January 08	42	42.70	4.286	17.46%

Table : ATM calls and implied Black-volatility, Sep 14

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Parameter Estimates

Method	Constraints	σ_1	σ_2	κ	Time
Function calls and numerical gradient	yes	0.37	0.15	1.40	$<$1min
Least Square Algorithm	no	0.37	0.15	1.41	$<$1min

Table : Parameter estimates with different optimizers, market data as of Sep 14

Options, which are far away from maturity, will have a volatility of about 15%, which can add up to more than 50%, when time to maturity decreases.
A κ value of 1.40 indicates, that disturbances in the futures market halve in $\frac{1}{\kappa} \cdot \log 2 \approx 0.69$ years.