Structuring and Valuation

Lecture - Winter 2014/14

Professor Dr. Rüdiger Kiesel | Chair for Energy Trading and Finance | University of Duisburg-Essen

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Part I

Review of Basic Objects

Derivatives

How to Model Price Movements?

- Price Processes
- Basic Stochastic Calculus

Electricity Trading

A structural model – Barlow (2002)

Options

- An option is a financial instrument giving one the *right, but not the obligation* to make a specified transaction at (or by) a specified date at a specified price.
- *Call* options give one the right to buy. *Put* options give one the right to sell.
- *European* options give one the right to buy/sell on the specified date, the expiry date, on which the option expires or matures. *American* options give one the right to buy/sell at any time prior to or at expiry.

Options - Terminology (1)

- The asset to which the option refers is called the *underlying asset* or the *underlying*.
- The price at which the transaction to buy/sell the underlying, on/by the expiry date (if exercised), is made, is called the *exercise price* or *strike price*.
- We shall usually use K for the strike price, time $t = 0$ for the initial time (when the contract between the buyer and the seller of the option is struck), time $t = T$ for the expiry or final time.

Options - Terminology (2)

Consider, say, a European call option, with strike price K; write $S(t)$ for the value (or price) of the underlying at time t.

- If $S(t) > K$, the option is *in the money*,
- if $S(t) = K$, the option is *at the money*,
- if $S(t) < K$, the option is *out of the money*.

Options - Payoff

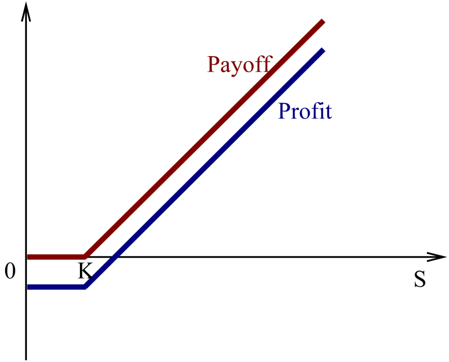
- The payoff from a call option is

$$S(T) - K \text{ if } S(T) > K \quad \text{and} \quad 0 \text{ otherwise}$$

(more briefly written as $(S(T) - K)^+$).
- The profit from a call option is the payoff $(S(T) - K)^+$ minus the call premium c.

Options - Payoff/Profit diagram

Considering only the option payoff, we obtain the payoff diagram, taking into account the initial payment of an investor one obtains the profit diagram below.



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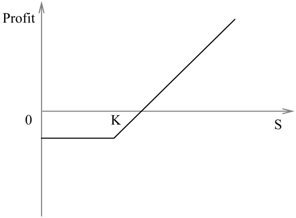
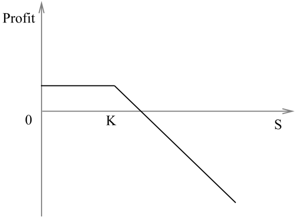
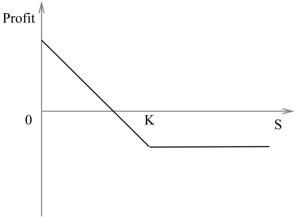
Options - Profit diagrams of vanilla options



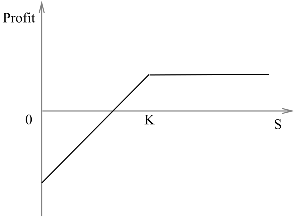
(a) long call



(b) short call



(c) long put



(d) short put

Forwards - Basic Structure

- A *forward contract* is an agreement to buy or sell an asset S at a certain future date T for a certain price K.
- The agent who agrees to buy the underlying asset is said to have a *long* position, the other agent assumes a *short* position.
- The settlement date is called *delivery date* and the specified price is referred to as *delivery price*.

Forwards

- The *forward price* $F(t, T)$ is the delivery price which would make the contract have zero value at time t.
- At the time the contract is set up, $t = 0$, the forward price therefore equals the delivery price, hence $F(0, T) = K$.
- The forward prices $F(t, T)$ need not (and will not) necessarily be equal to the delivery price K during the life-time of the contract.

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Forwards

- The payoff from a long position in a forward contract on one unit of an asset with price $S(T)$ at the maturity of the contract is
$$S(T) - K.$$
- Compared with a call option with the same maturity and strike price K we see that the investor now faces a downside risk, too. He has the obligation to buy the asset for price K.

Spot-Forward Relationship

Under the no-arbitrage assumption we have

	t	T
buy stock	$-S(t)$	delivery
borrow to finance	$S(t)$	$-S(t)e^{r(T-t)}$
sell forward on S		$F(t,T)$

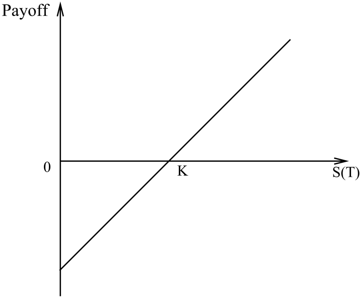
All quantities are known at t, the time t cashflow is zero, so the cashflow at T needs to be zero so we have

$$F(t,T) = S(t)e^{r(T-t)}$$

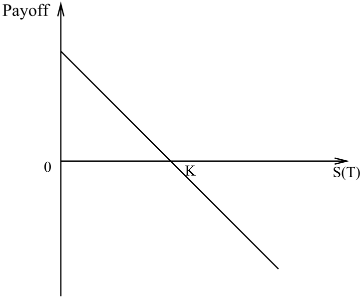
Futures

- Futures can be defined as standardised forward contracts traded at exchanges where a clearing house acts as a central counterparty for all transactions.
- Usually an initial margin is paid as a guarantee.
- Each trading day a settlement price is determined and gains or losses are immediately realized at a margin account.
- Thus credit risk is eliminated, but there is exposure to interest rate risk.

Payoff from a forward/futures contract



(e) long position



(f) short position

Swaps

- A *swap* is an agreement whereby two parties undertake to exchange, at known dates in the future, various financial assets (or cash flows) according to a prearranged formula that depends on the value of one or more underlying assets.
- Examples are currency swaps (exchange currencies) and interest-rate swaps (exchange of fixed for floating set of interest payments).

Stock Price Return

- We wish to model the time evolution of a stock price $S(t)$ and consider how S will change in some small time-interval from the present time t to a time $t + dt$ in the near future.
- Writing $dS(t)$ for the change $S(t + dt) - S(t)$ in S, the *return* on S in this interval is $dS(t)/S(t)$. We decompose the return into two components, a *systematic* part and a *random* part.
- The systematic is modelled by μdt, where μ is some parameter representing the mean rate of return of the stock.
- The random part is modelled by $\sigma dW(t)$, where $dW(t)$ represents the stochastic noise term driving the stock price dynamics, and σ is a second parameter describing how much the stock price fluctuates. Thus σ governs how volatile the price is, and is called the *volatility* of the stock.

Geometric Brownian Motion

Putting this together, we have the stochastic differential equation (SDE)

$$dS(t) = S(t)(\mu dt + \sigma dW(t)), \quad S(0) > 0, \tag{1}$$

due to Itô in 1944.

The economic importance of geometric Brownian motion was recognised by Paul A. Samuelson in his work, for which Samuelson received the Nobel Prize in Economics in 1970, and by Robert Merton, in work for which he was similarly honoured in 1997.

Brownian Motion I

- For the random noise we use Brownian Motion (introduced by the Botanist Robert Brown in 1828. It was introduced into finance by Louis Bachelier in 1900, and developed in physics by Albert Einstein in 1905. A mathematical theory was developed by Norbert Wiener) A stochastic process $X = (X(t))_{t \geq 0}$ is a standard Brownian motion, *BM*, if

(i) $X(0) = 0$ a.s.,

(ii) X has *independent increments*: $X(t+u) - X(t)$ is independent of $\sigma(X(s) : s \leq t)$ for $u \geq 0$,

(iii) X has *stationary increments*: the law of $X(t+u) - X(t)$ depends only on u,

and (iv), (v)

Brownian Motion II

A stochastic process $X = (X(t))_{t \geq 0}$ is a standard Brownian motion, *BM*, if (i) – (iii) and

(iv) X has *Gaussian increments*: $X(t+u) - X(t)$ is normally distributed with mean 0 and variance u,
$X(t+u) - X(t) \sim N(0, u)$,

(v) X has *continuous paths*: $X(t)$ is a continuous function of t.

Itô Processes

- We will use the following type of process expressed in terms of the stochastic differential equation

$$dX(t) = b(t)dt + \sigma(t)dW(t), \quad X(0) = x_0.$$

- For functions f we want to give meaning to the stochastic differential $df(X(t))$ of the process $f(X(t))$.
- This is done by the *Itô Formula*

$$\begin{aligned} df(X(t)) &= f'(X(t))dX(t) \\ &\quad + \frac{1}{2}f''(X(t))\sigma^2 dt. \end{aligned}$$

Multiplication rules

- The second term above corrects for special path properties of Brownian Motion and needs the quadratic variation of the process.
- We find

$$\begin{aligned}(dX)^2 &= (bdt + \sigma dW)^2 \\ &= \sigma^2 dt + 2b\sigma dtdW + b^2(dt)^2 = \sigma^2 dt.\end{aligned}$$

- The quadratic variation of any Itô process can be calculated using the multiplication rules

	dt	dW
dt	0	0
dW	0	dt

General Itô Formula

If $X(t)$ is an Itô process and $f(t,x)$ a function with time and location variable, then $f = f(t, X(t))$ has stochastic differential

$$df = \left(f_t + bf_x + \frac{1}{2}\sigma^2 f_{xx}\right) dt + \sigma f_x dW.$$

Observe, that we left out all function arguments

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Example: Geometric Brownian Motion

The SDE for GBM has the unique solution

$$S(t) = S(0) \exp\left\{ \left(\mu - \frac{1}{2}\sigma^2 \right) t + \sigma W(t) \right\}.$$

Therefore, writing

$$f(t, x) := \exp\left\{ \left(\mu - \frac{1}{2}\sigma^2 \right) t + \sigma x \right\},$$

we have

$$f_t = \left(\mu - \frac{1}{2}\sigma^2 \right) f, \quad f_x = \sigma f, \quad f_{xx} = \sigma^2 f,$$

and with $x = W(t)$, one has

$$dx = dW(t), \quad (dx)^2 = dt.$$

Example: GBM

Thus Itô's lemma gives

$$
\begin{aligned}
df &= f_t dt + f_x dW + \frac{1}{2} f_{xx} (dW)^2 \\
&= f\left(\left(\mu - \frac{1}{2}\sigma^2\right) dt + \sigma dW + \frac{1}{2}\sigma^2 dt\right) \\
&= f(\mu dt + \sigma dW).
\end{aligned}
$$

Electricity Markets

A centralized platform where participants can exchange electricity transparently according to the price they are will to pay or receive, and according to the capacity of the electrical network.

- Fixed Gate Auction
  - Participants submit sell or buy orders for several areas, several hours,
  - the submissions are closed at a pre-specified time (closure)
  - the market is cleared.
  - Example: day-ahead.
- Continuous-time Auction
  - Participants continuously submit orders. Orders are stored,
  - Each time a deal is feasible, it is executed,
  - Example: intra-day.

Electricity Exchanges

Electricity related contracts can be traded at exchanges such as

- the Nord Pool, mainly Northern European countries, `http://www.nordpoolspot.com/`
- the European Energy Exchange (EEX), `http://www.eex.com/en`
- EPEX, located in Paris, founded by EEX and Powernext (French Energy Exchange); Electricity spot market for Germany, Austria, France and Switzerland; `http://www.epexspot.com/en/`
- Amsterdam Power Exchange (APX), covers the Netherlands, Belgium and the UK, `http://www.apxgroup.com`

EEX – traded products

- Futures contracts for Germany and France with delivery periods week, month, quarter, year.
- For Germany single days and weekends are available.
- European style options on futures.

EPEX – traded products

- Auction day-ahead and continuous intra-day market.
- Products are individual hours, baseload, peakload, blocks of contiguous hours.
- Intraday market is open 24 hours a day, 7 days a week and products can be traded until 45 minutes before delivery.
- in Germany 15 minutes contracts can be traded.

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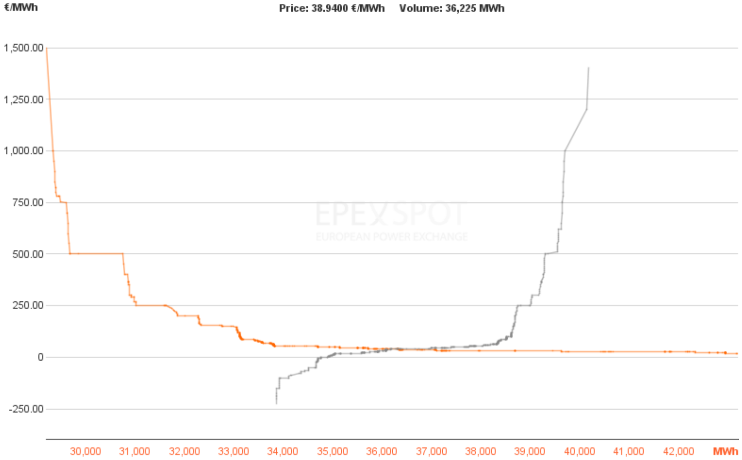
Auction EPEX



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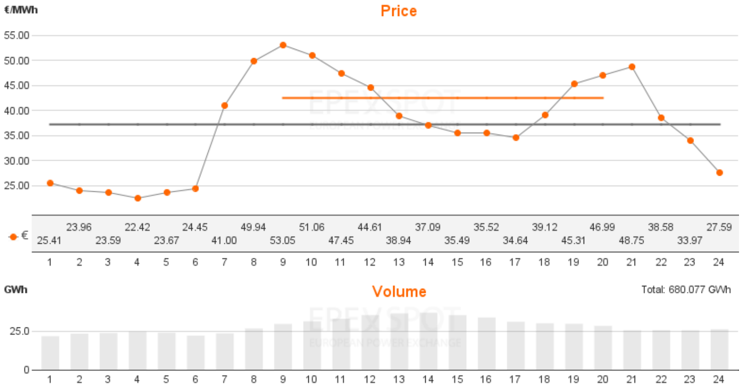
Spot price EPEX



Day-Ahead Market

- Possibility to correct long-term production schedule (build on the forward markets) in terms of hourly production schedule of power plants (Delta Hedging) – sell more expensive hours, buy cheaper hours for flexible power plants.
- Adjust for residual load profiles on an hourly basis
- Market for production from renewable energy sources (wind, solar) as on long-term markets only averages can be traded

Intra-Day Market

- Trading of hours, quarter-hours until 45 min before start of period continuously during the day
- From 15:00 hours of next day, from 16:00 quarter-hours of next day

Motivation for Trading Intra-Day Market

- Correction or optimisation of Day-Ahead position
  - power plant outages
  - optimisation of power plant usage (generator)
  - optimisation of demand (costumer)
  - renewable energy producer – changes of forecasts
- Balancing quarter hour ramps with quarter-hour contracts
- proprietary trading

Structure of Balancing and Reserve Markets

In Europe, the *European Network of Transmission System Operators for Electricity,(ENTSO-E)* coordinates overarching grid topics. The main task of a TSO is to ensure a constant power frequency in the transmission system. The following control actions are applied

- *Primary Reserve* starts within seconds as a joined action of all TSOs in the system.
- *Secondary Reserve* replaces the primary reserve after a few minutes and is put into action by the responsible TSOs only.
- *Tertiary Reserve* frees secondary reserves by rescheduling generation by the responsible TSOs.

The TSO tenders the required products for fulfilling these functions. Reserve products may involve payments for the availability of the reserved capacity.

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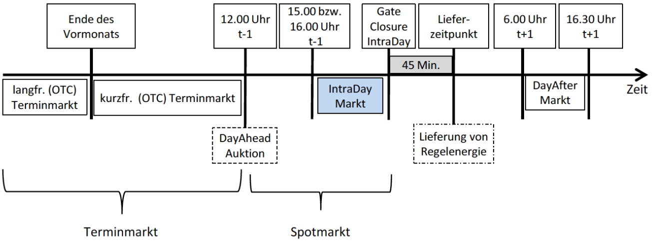
Timing Electricity Trading



Set-up

- $u_t(x)$ is supply at time t, if price is x; an increasing function.
- $d_t(x)$ is demand at time t, if price is x; a decreasing function.
- The electricity price at time t is the unique number S_t such that
$$u_t(S_t) = d_t(S_t)$$
- Need to specify u_t and d_t.

Barlow (2002) Specification

- Supply is non-random, independent of t

$$u_t(x) = g(x).$$

- Demand is inelastic, independent of x

$$d_t(x) = D_t$$

  a random process.
- With

$$f_\alpha(x) = (1 + \alpha x)^{\frac{1}{\alpha}}, \ \alpha \neq 0, \ f_0(x) = \exp(x)$$

  and

$$X_t = -\lambda(X_t - a)dt + \sigma dW_t$$

  Barlow (2002) motivates the model

$$S_t = \begin{cases} f_\alpha(X_t) & 1 + \alpha X_t > \epsilon_0 \\ \epsilon_0^{\frac{1}{\alpha}} & 1 + \alpha X_t \leq \epsilon_0 \end{cases}$$

Mean-reverting Processes

X_t follows a stochastic process which can be described as follows:

$$dX_t = \kappa(\theta - X_t)\,dt + \sigma dW_t.$$

This is a mean reverting diffusion process, the so-called Ornstein-Uhlenbeck-Process. The parameters can be interpreted as

- κ - speed of mean reversion
- θ - level of mean reversion
- σ - volatility of the process

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Part II

Forward Markets

Basic Definitions and Models

- Basic Pricing Relations for Futures
- Black's Formula

Bessembinder - Lemon model

A Dynamic Equilibrium Approach

Information Approach

Storage, Inventory and Convenience Yield

- The theory of storage aims to explain the differences between spot and Futures (Forward) prices by analyzing why agents hold inventories.
- Inventories allow to meet unexpected demand, avoid the cost of frequent revisions in the production schedule and eliminate manufacturing disruption.
- This motivates the concept of convenience yield as a benefit, that accrues to the owner of the physical commodity but not to the holder of a forward contract.
- Thus the convenience yield is comparable to the dividend yield for stocks.
- A modern view is to view storage (inventory) as a timing option, that allows to put the commodity to the market when prices are high and hold it when the prices are low.

Spot-Forward Relationship in Commodity Markets

Under the no-arbitrage assumption we have

$$F(t,T) = S(t)e^{(r-y)(T-t)} \tag{2}$$

where r is the interest rate at time t for maturity T and y is the convenience yield.

Spot-Forward Relationship in Commodity Markets

Observe that (2) implies

- spot and forward are redundant (one can replace the other) and form a linear relationship (unlike options)
- with two forward prices we can derive the value of $S(t)$ and y
- knowledge of $S(t)$ and y allows us to construct the whole forward curve
- for $r - y < 0$ we have backwardation; for $r - y > 0$ we have contango.

Spot-Forward Relationship: Classical theory

- In a stochastic model we use

$$F(t,T) = \mathbb{E}_{\mathbb{Q}}(S(T)|\mathcal{F}_t)$$

  where $\mathcal{F}_t$ is the accumulated available market information (in most models the information generated by the spot price).
- $\mathbb{Q}$ is a risk-neutral probability
  - discounted spot price is a $\mathbb{Q}$-martingale
  - fixed by calibration to market prices or a market price of risk argument

Market Risk Premium

- The *market risk premium* or *forward bias* $\pi(t, T)$ relates forward and expected spot prices.
- It is defined as the difference, calculated at time t, between the forward $F(t, T)$ at time t with delivery at T and expected spot price:

$$\pi(t, T) = F(t, T) - \mathbb{E}^{\mathbb{P}}[S(T)|\mathcal{F}_t]. \tag{3}$$

Here $\mathbb{E}^{\mathbb{P}}$ is the expectation operator, under the historical measure $\mathbb{P}$, with information up until time t and $S(T)$ is the spot price at time T.

Simple Futures Market Model

- We assume for the futures dynamics

$$df(t) = \mu f(t)dt + \sigma f(t)dW(t).$$

- But since it costs nothing to enter a futures position, we can constantly change our futures portfolio. Thus financing for buying a traded asset is not relevant.

Black's Formula

- We use the usual notation - strike K, expiry T, $\tau = T - t$ for time to maturity as in the spot case, and write Φ for the standard normal distribution function.
- The arbitrage price C of a European futures call option is

$$C(t) = e^{-r\tau}(f\Phi(d_1(f,\tau)) - K\Phi(d_2(f,\tau))), \tag{4}$$

where

$$d_{1,2}(f,\tau) := \frac{\log(f/K) \pm \frac{1}{2}\sigma^2\tau}{\sigma\sqrt{\tau}}.$$

Black's Formula

- Observe that the quantities d_1 and d_2 do not depend on the interest rate r.
- This is intuitively clear from the classical Black approach: one sets up a replicating risk-free portfolio consisting of a position in futures options and an offsetting position in the underlying futures contract. The portfolio requires no initial investment and therefore should not earn any interest.

Bessembinder- Lemon Model specification

- One-period model
- Power companies are able to forecast demand in the immediate future with precision
- N_P identical producers; N_R identical retailers that buy power in the wholesale market and sell it to final consumers at fixed unit price
- P_R fixed unit price that consumers pay
- Q_{R_i} an exogenous random variable that denotes the realized demand for retailer i

The cost function

- Each producer i has cost function

$$TC_i = F + \frac{a}{c}(Q_{P_i})^c,$$

  where F are fixed costs, Q_{P_i} is the output of producer i, and $c \geq 2$.
- The cost function implies that the marginal production costs increase with output.
- If $c > 2$ marginal costs increase at an increasing rate with output.
- Moreover, the distribution of power prices will be positively skewed even when the distribution of power demand is symmetric.

Clearing prices

- First, assume that forward prices are gives
- Obtain optimal behaviour in the spot market
- Work back and find optimal positions in the forward market.

The wholesale spot market

- Producers sell to retailers who in turn distribute to power consumers
- P_W denotes the wholesale spot price, $Q^W_{P_i}$ quantity sold by producer i in the wholesale spot market, $Q^F_{P_i}$ quantity that producer i has agreed to deliver (purchase if negative) in the forward market at the fixed forward price P_F.
- The ex-post profit of producer i is given by

$$\pi_{P_i} = P_W Q^W_{P_i} + P_F Q^F_{P_i} - F - \frac{a}{c}(Q_{P_i})^c,$$

where each producer's physical production, Q_{P_i}, is the sum of its spot and forward sales $Q^W_{P_i} + Q^F_{P_i}$.

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The wholesale spot market

- Retailers buy in the real-time wholesale market the difference between realised retail demand and their forward positions
- $Q^F_{R_j}$ quantity sold (purchased if negative) forward by retailer j, P_R fixed retail price per unit
- The ex-post profit for each retailer is

$$\pi_{R_j} = P_R Q_{R_j} + P_F Q^F_{R_j} - P_W (Q_{R_j} + Q^F_{R_j}),$$

- The profit maximising quantity for producer i is (FOC wrt $Q^W_{P_i}$)

$$Q^W_{P_i} = \left(\frac{P_W}{a}\right)^x - Q^F_{P_i}$$

with $x = 1/(c-1)$

The wholesale spot market

- The equilibrium total retail demand is equal to total production and forward contracts are in zero net supply
- Hence we must have that summing over all producers production must equal total demand from retailers

$$N_P \left(\frac{P_W}{a} \right)^x = \sum_{i=1}^{N_R} Q^F_{R_i}$$

The wholesale spot market

- Therefore the market-clearing wholesale price is

$$P_W = a\left(\frac{Q^D}{N_P}\right)^{c-1},$$

where $Q^D = \sum_{j=1}^{N_R} Q_{R_j}$ is total system demand. We see that when $c > 2$ an increase in demand has a disproportionate effect on power prices.

- Each producers sale in the wholesale market is

$$Q^W_{P_i} = \frac{Q^D}{N_P} - Q^F_{P_i}.$$

Demand for forward positions

- Producers profit (with no forwards) is

$$\rho_{P_i} = P_W \frac{Q^D}{N_P} - F - \frac{a}{c}\left(\frac{Q^D}{N_P}\right)^c.$$

- Retailers profit (with no forwards) is

$$\rho_{R_j} = P_R Q_{R_j} - P_W Q_{R_j}.$$

Mean-Variance Analysis for optimal forward position

Assume that market players

$$\max_{Q^F_{\{P_i,R_j\}}} \mathbb{E}[\pi_{\{P_i,R_j\}}] - \frac{A}{2}\mathbb{V}\mathrm{ar}[\pi_{\{P_i,R_j\}}]$$

where, for example, producers have the profit function

$$\pi_{P_i} = \rho_{P_i} + P^F Q^F - P_W Q^F.$$

FOCs imply

$$Q^F_{\{P_i,R_j\}} = \frac{P^F - \mathbb{E}[P_W]}{A\mathbb{V}\mathrm{ar}[P_W]} + \frac{\mathbb{C}\mathrm{ov}[\rho_{\{P_i,R_j\}}, P_W]}{\mathbb{V}\mathrm{ar}[P_W]}.$$

Mean-Variance Analysis for optimal forward position

- The optimal forward position contains two components
  - The first term reflects the position taken in response to the bias $P^F - \mathbb{E}[P_W]$
  - The second term is the quantity sold or bought forward to minimize the variance of profits
- Forward hedging can reduce risk precisely because the covariance term is generally non-zero.

The equilibrium forward price

- One can show that

$$P_F = \mathbb{E}[P_W] - \frac{N_P}{Nca^x}\left[cP_R \mathbb{C}\mathrm{ov}[P_W^x, P_W] - \mathbb{C}\mathrm{ov}[P_W^{x+1}, P_W]\right],$$

  where $N = (N_R + N_P)/A$ reflects the number of firms in the industry and the degree to which they are concerned with risk.
- The forward price will be less than the expected wholesale price, if the first term in brackets, which reflects retail risk, is larger than the second term, which reflects production cost risk.

Equilibrium Approach – Players

- The main motivation for players to engage in forward contracts is that of risk diversification.
- Producers have made large investments with the aim of recouping them over a long period of time as well as making a return on them.
- Retailers (which might be intermediaries and/or use the commodity in their production process) also have an incentive to hedge their positions in the market by contracting forwards that help diversify their risks.
- Exposure to the market will differ both between producers and retailers as well as within their own group. So the need for risk-diversification has a temporal dimension.

Market Risk Premium

- These differences in the desire to hedge positions are employed to explain the market risk premium and its sign.
- Retailers are less incentivized to contract commodity forwards the further out we look into the market. We associate situations where $\pi(t, T) > 0$ with the fact that retailers' desire to cover their positions 'outweighs' those of the producers, resulting in a positive market risk premium.
- In contrast, on the producers' side the need to hedge in the long-term does not fade away as quickly. Now the producers' desire to hedge their positions outweighs that of the retailers resulting in a negative market risk premium.

Representative Agents

- We describe producers' and retailers' preferences via the utility function of two representative agents.
- Agents must decide how to manage their exposure to the spot and forward markets for every future date T.
- A key question for the producer is how much of his future production, which cannot be predicted with total certainty, will he wish to sell on the forward market or, when the time comes, sell it on the spot market.
- Similarly, the retailer must decide how much of her future needs, which cannot be predicted with full certainty either, will be acquired via the forward markets and how much on the spot.

Representative Agents

We approach this financial decision and equilibrium price formation in two steps.

- First, we determine the forward price that makes the agents indifferent between the forward and spot market.
- Second, we discuss how the relative willingness of producers and retailers to hedge their exposures determines market clearing prices.

Representative Agents

We assume that the risk preferences of the representative agents are expressed in terms of an exponential utility function parameterized by the risk aversion constant $\gamma > 0$;

$$U(x) = 1 - \exp(-\gamma x)\,.$$

We let $\gamma := \gamma_p$ for the producer and $\gamma := \gamma_r$ for the retailer.

The Model

- We assume that the electricity spot price follows a mean-reverting multi-factor additive process

$$S_t = \Lambda(t) + \sum_{i=1}^{m} X_i(t) + \sum_{j=1}^{n} Y_j(t) \tag{5}$$

- where
  - $\Lambda(t)$ is the deterministic seasonal spot price level,
  - $X_i(t)$ are zero-mean reverting processes that account for the normal variations in the spot price evolution with lower degree of mean-reversion.
  - $Y_j(t)$ are zero-mean reverting processes responsible for the spikes or large deviations which revert at a fast rate.

Indifference Prices

Assume that the producer will deliver the spot over the time interval $[T_1, T_2]$.

He has the choice to deliver the production in the spot market, where he faces uncertainty in the prices over the delivery period, or to sell a forward contract with delivery over the same period.

The producer takes this decision at time $t \leq T_1$.

Indifference Prices

We determine the forward price that makes the producer indifferent between the two alternatives: denote by $F_{\mathrm{pr}}(t, T_1, T_2)$ the forward price derived from the equation

$$
\begin{aligned}
& 1 - \mathbb{E}^P \left[\exp\left(-\gamma_p \int_{T_1}^{T_2} S(u)\, du \right) \mid \mathcal{F}_t \right] \\
= \quad & 1 - \mathbb{E}^P \left[\exp\left(-\gamma_p (T_2 - T_1) F_{\mathrm{pr}}(t, T_1, T_2) \right) \mid \mathcal{F}_t \right]
\end{aligned}
$$

Indifference Price – Producer

Equivalently,

$$F_{\mathrm{pr}}(t, T_1, T_2) = -\frac{1}{\gamma_p} \frac{1}{T_2 - T_1} \ln \mathbb{E}^P \left[\exp\left(-\gamma_p \int_{T_1}^{T_2} S(u)\, du \right) \mid \mathcal{F}_t \right], \tag{6}$$

where for simplicity we have assumed that the risk-free interest rate is zero.

$\int_{T_1}^{T_2} S(u)\, du$ is what the producer collects from selling the commodity on the spot market over the delivery period $[T_1, T_2]$, while he receives $(T_2 - T_1)F_{\mathrm{pr}}(t, T_1, T_2)$ from selling it on the forward market.

Indifference Price – Retailer

The retailer will derive the indifference price from the incurred expenses in the spot or forward market, which entails

$$\begin{aligned} & 1-\mathbb{E}^P\left[\exp\left(-\gamma_r\left(-\int_{T_1}^{T_2} S(u)\,du\right)\right)\,|\,\mathcal{F}_t\right] \\ = \quad & 1-\mathbb{E}^P\left[\exp\left(-\gamma_r(-(T_2-T_1)F_{\mathrm{r}}(t,T_1,T_2))\right)|\,\mathcal{F}_t\right], \end{aligned}$$

or,

$$F_{\mathrm{r}}(t,T_1,T_2)=\frac{1}{\gamma_r}\frac{1}{T_2-T_1}\ln\mathbb{E}^P\left[\exp\left(\gamma_r\int_{T_1}^{T_2} S(u)\,du\right)\,|\,\mathcal{F}_t\right]. \tag{7}$$

Indifference Price – Bounds

Note that the producer prefers to sell his production in the forward market as long as the market forward price $F(t, T_1, T_2)$ is higher than $F_{\mathrm{pr}}(t, T_1, T_2)$. On the other hand, the retailer prefers the spot market if the market forward price is more expensive than his indifference price $F_{\mathrm{r}}(t, T_1, T_2)$. Thus, we have the bounds

$$F_{\mathrm{pr}}(t, T_1, T_2) \leq F(t, T_1, T_2) \leq F_{\mathrm{r}}(t, T_1, T_2)\,. \tag{8}$$

Market Power

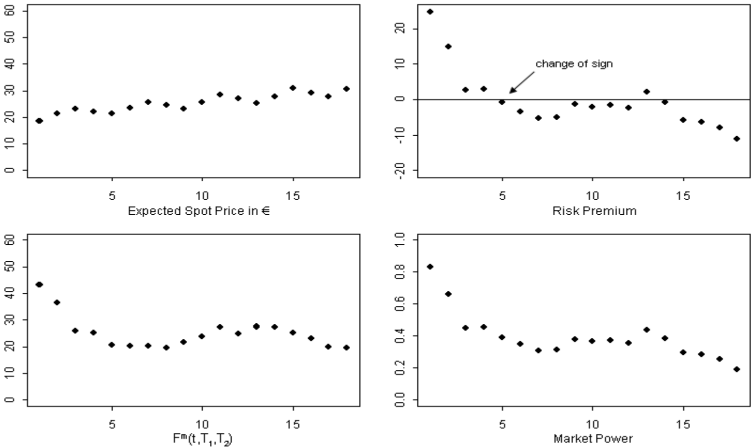
- We introduce the deterministic function $p(t, T_1, T_2) \in [0, 1]$ describing the *market power of the representative producer*.
- For $p(t, T_1, T_2) = 1$ the producer has full market power and can charge the maximum price possible in the forward market (short-term positions), namely $F_r(t, T_1, T_2)$.
- If the retailer has full power, ie $p(t, T_1, T_2) = 0$ (long-term positions), she will drive the forward price as far down as possible which corresponds to $F_{pr}(t, T_1, T_2)$.

Market Power – Forward Price

For any market power $0 < p(t, T_1, T_2) < 1$,
the forward price $F^p(t, T_1, T_2)$ is defined to be

$$\begin{aligned} F^p(t, T_1, T_2) &= p(t, T_1, T_2) F_{\mathrm{r}}(t, T_1, T_2) \\ &\quad + (1 - p(t, T_1, T_2)) F_{\mathrm{pr}}(t, T_1, T_2). \end{aligned} \tag{9}$$

Producer's market power and market risk premium, 18 monthly contracts with t = January 2 2002



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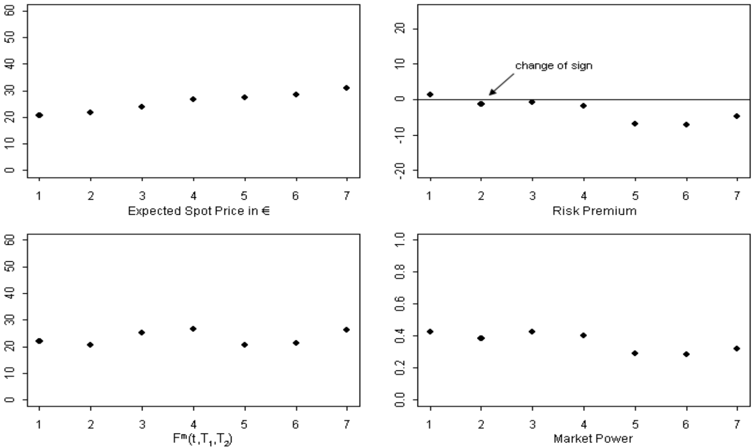
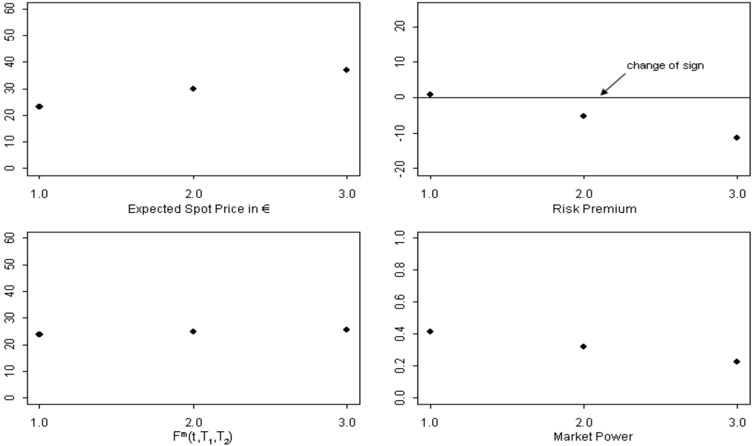
Producer's market power and market risk premium, 7 quarterly contracts with $t =$ second quarter 2002



Producer's market power and market risk premium, 3 yearly contracts with $t = 2002$



Information Approach

- As electricity is non-storable future predictions about the market will not affect the current spot price, but will affect forward prices.
- Stylized example: planned outage of a power plant in one month
- Market example: in 2007 the market knew that in 2008 CO_2 emission costs will be introduced; this had a clearly observable effect on the forward prices!
- German moratorium 2011: shut-down 7 nuclear power plants for 3 months with possible complete shut-down.

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German Moratorium I

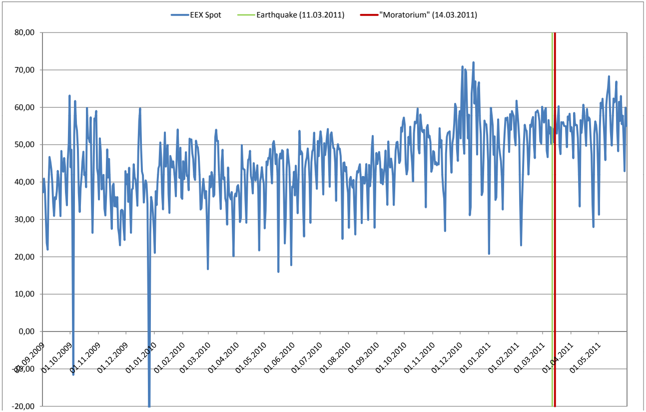

Figure : EEX spot prices

German Moratorium II

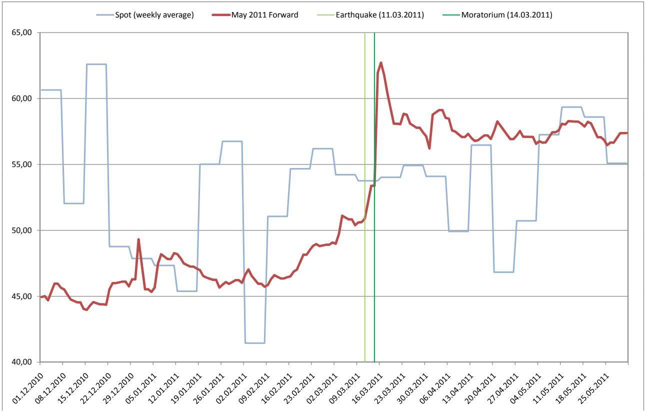

Figure : EEX forward prices delivery May 2011

German Moratorium III

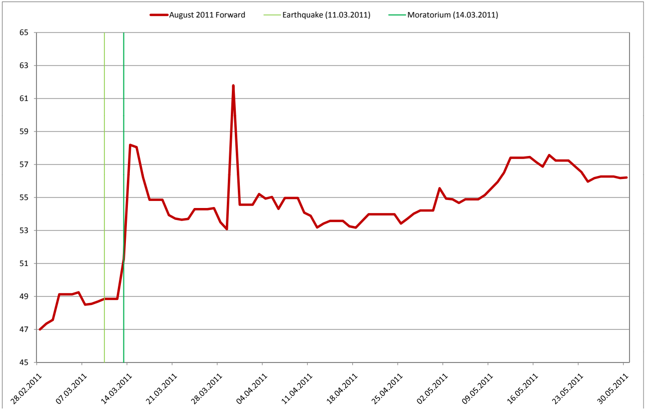

Figure : EEX forward prices delivery August 2011

Example: 2008 CO_2 Emission Costs

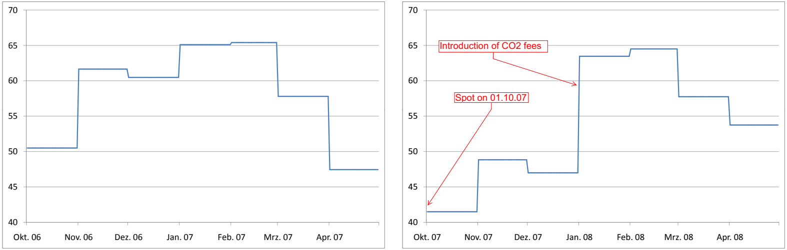

Figure : EEX Forward prices observed on 01/10/06 (left) and 01/10/07 (right)

- Typical winter and bank holidays behaviour in both graphs
- General upward shift in 2008

$\Rightarrow$ 2nd phase of CO_2 certificates

Information Approach

- Future information is incorporated in the forward price
- ... but not necessarily in the spot price due to non-storability
- ... buy-and-hold strategy does not work

Information Approach

- The usual pricing relation between spot and forward:

$$F(t, T) = \mathbb{E}^{\mathbb{Q}}[S_T | \mathcal{F}_t]$$

- Not sufficient: natural filtration $\mathcal{F}_t = \sigma(S_s, s \leq t)$

- Idea: enlarge the information set!

- ... by information about the spot at some future time T_Υ
- Info could be that spot will be in certain interval...
- ... or the value of a correlated process (temperature)

The Information Premium

- Quantify the influence of future information using:

Information Premium

The information premium is defined to be

$$I(t, T) = \mathbb{E}[S_T|\mathcal{G}_t] - \mathbb{E}[S_T|\mathcal{F}_t]$$

i.e. the difference between the prices of the forward under $\mathcal{G}$ and $\mathcal{F}$.