An Efficient Score Alignment Algorithm and its Applications

by

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Abstract

String alignment and comparison in Computer Science is a well-explored space with classic problems such as Longest Common Subsequence that have practical application in bioinformatic genomic sequencing and data comparison in revision control systems. In the field of musicology, score alignment and comparison is a problem with many similarities to string comparison and alignment but also vast differences. In particular we can use ideas in string alignment and comparison to compare a music score in the MIDI format with a music score generated from Optical Musical Recognition (OMR), both of which have incomplete or wrong information, and correct errors that were introduced in the OMR process to create an improved third score. My thesis focuses on creating a set of algorithms that align and compare MIDI and OMR music scores to produce a corrected version of the OMR score that borrows ideas from classic computer science string comparison and alignment algorithm but also incorporates optimizations and heuristics from music theory.

Thesis Supervisor: Prof. Michael S. Cuthbert
Title: Associate Professor

Acknowledgments

This is the acknowledgements section. You should replace this with your own acknowledgements.

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Chapter 1

Introduction

1.1 Motivation

Comparing music is seminal to the field of Music Information Retrieval (MIR). Questions such as "Is this piece of music plagiarized from Bach?", "Is this rhythmically more similar to Philip Glass or Scott Joplin? Melodically?", "Is this newly discovered musical transcription an entirely new folk song or just a variation of an old one?" can be answered by comparing the pieces in question and applying a quantitative metric to determine that, "Yes, this piece quotes Bach more than other pieces of similar time period and style", or, "This piece has 74% melodic similarity to Joplin's repertoire, 10% similarity to Glass, but rhythmically shares 64% similarity with Glass and only 12% with Joplin."

Computational music comparison, a subset of classical computer science string comparison, comes in many forms and each domain has slightly different specifications for alignment and comparison.

The domain of music comparison that I will be studying and building tools for will be comparison between a MIDI score, which represents music in a restricted manner and its OMR counterpart, possibly a scanned score. Each representation is imperfect on

its own.

MIDI represents music in a very limited manner. This format encodes enharmonic notes as the "same"; note lengths are relative to each other, not necessarily in relation to a defined time signature; there is freedom in representing duration, as long as the total length is correct; and many other design decisions that make MIDI very universally compatible but also lacking in definition. Furthermore, since MIDI scores are generally created with some kind of human input, they tend to be correct, pitch-wise and rhythmically.

OMR takes a score that likely to be correct (the idea being this is a printed and published work, and also likely proofed for errors by the composer or publisher), scans its and tries to recreate what it scans in a digital notation format. While the more commonplace OCR has been optimized and whittled to an extremely high standard, OMR is still at present only 90% correct [1]. One can imagine that the "alphabet" that OMR deals with is much larger and less well-defined than the 52 capital and lowercase letters and punctuation that OCR has been optimized for. Therefore, much of the optimizations for OCR do not directly apply to OMR. Some common errors in OMR include incorrectly recognizing a sharp as a natural and vice versa, mistaking rests as notes and vice versa, and other errors that any musician trained in the basics of music theory could identify as "wrong". Simple rules such as: all measures must sum to the same duration, only notes with existing accidentals can be made natural, should be able to great improve the quality of OMR if taken into account.

The idea here is that we can compare MIDI scores and OMR scores, properly align them for as many points in the music as possible, and then use the correct metadata present in the MIDI to score to correct for scanning errors in the OMR score.

1.2 Project Overview

1.3 Chapter Summary

Chapter 2 will discuss background information that is important to understanding my work, including basic musical terms, an overview of canonical sequence alignment methods, as well as related work in musical sequence representation, alignment, and correction. Chapter 3 will summarize the work I did previously that is used as a building block for my thesis work. It also briefly discusses some ideas that were not as promising to explore. Chapter 4 will be a deep dive into the final implementations of the three main modules of my work and provides a concrete example of how the entire system would work. Performance and testing metrics are discussed in Chapter 5. Finally, Chapter 6 concludes with areas to explore in future work.

Chapter 2

Background and Related Work

2.1 Musical Terminology

2.2 Non-musical Work in Sequence Alignment

It is far easier to understand sequence alignment in a non-musical context (here the context is plain string alignment), understand the rules and assumptions made in the non-musical context, generalize them, and then recreate music-specific rules and assumptions.

If you are familiar with the canonical representation of the string alignment problem as a dynamic programming matrix, you can skip this section.

2.2.1 An Overview of Sequence Alignment

Sequence alignment in bioinformatics is an application of well-studied problem. In this field, researchers want to identify similar regions of DNA, RNA, or protein sequences to study functional, structural, or evolutionary relationships between the two sequences.

```
Hsa_TMEM66   ALTLHYDRYTTSRRLDPIPQLKCVGGTAGCDSYTPKVIQCQNKGWDGYDVQWECKTDLDI 103
Ptr_TMEM66   ALTLHYDRYTTSRRLDPIPQLKCVGGTAGCDSYTPKVIQCQNKGWDGYDVQWECKTDLDI 103
Ppy_TMEM66   ALTLHYDRYTTSRRLDPIPQLKCVGGTAGCDSYTPKVIQCQNKGWDGYDVQWECKTDLDI 103
Mml_TMEM66   ALTLHYDRYTTSRRLDPIPQLKCVGGTAGCDSYTPKVIQCQNKGWDGYDVQWECKTDLDI 103
Mfa_TMEM66   ALTLHYDRYTTSRRLDPIPQLKCVGGTAGCDSYTPKVIQCQNKGWDGYDVQWECKTDLDI 103
Mne_TMEM66   ALTLHYDRYTTSRRLDPIPQLKCVGGTAGCDSYTPKVIQCQNKGWDGYDVQWECKTDLDI 103
Ssc_TMEM66   ALTLHYDRYTTSRRLDPIPQLKCVGGTAGCDSYTPKVIQCQNKGWDGYDVQWECKTDLDV 103
Bta_TMEM66   ALTLYYDRYTTSRRLEPIPQLKCVGGTAGCDSYTPKVIQCQNKGWDGYDVQWECKTDLDV 103
Cfa_TMEM66   ALTLHHDRYTTSRRLDPIPQLKCVGGTAGCDSYTPKVIQCQNKGWDGYDVQWECKTDLDI 103
Mmu_TMEM66   ALTLYSDRYTTSRRLDPIPQLKCVGGTAGCEAYTPRVIQCQNKGWDGYDVQWECKTDLDI 104
Rno_TMEM66   ALTLYSDRYTTSRRLDPIPQLKCVGGTAGCDAYTPKVVQCQNKGWDGYDVQWECKTDLDI 104
Ocu_TMEM66   ALTLHYDRYTTSRRLEPIPQLKCVGGTAGCDAYTPKVVQCQNKGWDGYDVQWECKTDLDV 97
Laf_TMEM66   ALTLHYNRYTTSRRLDPVPQLKCIGGTAGCNSYTPKVIQCQNKGWDGYDVQWECKTDLDI 89
Mdo_TMEM66   ALTLHRDRFTTARRTAPIPQLQCLGGSAGCPAHIPEIVQCRNKGWDGFDVQWECKAELDT 119
Gga_TMEM66   VLTLHRGRYTTARRTAAVPQLQCIGGSAGCS-DIPEVVQCYNRGWDGYDVQWQCKADLEN 94
Xla_TMEM66   TITLYADRYTNARRSAPVPQLKCIGGNAGCHAMVPQVVQCHNRGWDGLDVQWECRVDMDN 93
Xtr_TMEM66   AITLYADRYTNARRSAPVPQLKCIGGSAGCHTMVPQVVQCHNRGWDGFDVQWECKVDMDN 93
Dre_TMEM66   VLTLYRGRYTTARRSSPVPQLQCIGGSAGCGSFTPEVVQCYNRGSDGIDAQWECKADMDN 93
Ssa_TMEM66   VLTLYKGKYTTARRSSAVPQLQCVGGSAGCGSFIPEVVQCKNKGWDGVDAQWECKTDMDN 93
Tru_TMEM66   VLTLYRGLYTTARRSSPVPQLQCVGGSAGCHAFVPEVVQCQNKGWDGMDIQWECRTDMDN 99
Tni_TMEM66   TLTLYRGRYTTARRSSPVPQLRCVGGSAGCQAFVPEVVQCQNRGWDGVDVQWECKTDMDN 89
Gac_TMEM66   ALTLYKNRYTTARRASPVPQLQCVGGSAGCQAFVPEVVQCQNKGWDGVDVQWECRTDMDN 92
Ppr_TMEM66   VLTLYKGRYTTARRSSPVLQLQCAGGTAGCGSFVPEVVQCYNRGSDGIDTQWECKADMDN 93
Cel_TMEM66   AITLHKGKMTTGRRVSPTFQLKCVGG-SAKGAFTPKVVQCANQGFDGSDVQWRCDADLPH 96
Cre_TMEM66   AITLNKGKMTTGRRVAPTLQLKCVGG-SAKGAFTPKVVQCSNQGFDGSDVQWRCDADLPH 96
Cbr_TMEM66   AITLHKGKMTTGRRVAPALQLKCVGG-SAKGQFSPKVVQCANQGFDGSDVQWRCDADLPH 96
             .:**   .  *..**   .  **:* **  :.     *.::** *:* ** * **.* .::
```

Figure 2-1: An example of protein alignment

[[figure out how to cite this image: `https://commons.wikimedia.org/wiki/File:An_excerpt_of_a_mu`

There are various computational algorithms that can be used to solve sequence alignment including slower but more accurate dynamic programming algorithms and faster but less accurate probabilistic or heuristic-based algorithms. In my work, it makes more sense to use the slower but more accurate methods, as our datasets are not so large and small mistakes that might be permissible and overlook-able in larger data could be much more egregious in smaller contexts.

The specific algorithm I chose to implement for solving the musical sequence alignment problem is a version of the Needleman-Wunsch dynamic programming algorithm for global sequence alignment. [1]

At a high level, the Needleman-Wunsch algorithm and its family of dynamic programming sequence alignment algorithms calculate the edit distance between two sequences by producing a least-cost alignment of the sequences. An alignment is an

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[1]This algorithm has a history of multiple invention. Needleman-Wunsch was first published in the context of protein alignment in bioinformatics. People with a more theoretical computer science bent might know the same algorithm by a different name, Wagner-Fisher.

assignment of pairwise characters and edit operations. Overall cost of an alignment (i.e. its **edit distance**) is determined by summing the individual costs of insertion, deletion, substitution, or no-change operations between pairwise characters in order to morph one sequence into the other.

The next sections discuss coming up with edit distance calculations and then the method that Needleman-Wunsch uses to find the best alignment.

2.2.1.1 Edit Distance

The term edit distance is a way of quantifying how dissimilar two sequences are to each other. If you think of sequences as being made up of characters, you can also think of one sequence transforming into the other through a series of insertion, deletion, substitution, and no-change operations, with each operation "costing" a certain amount. Edit distance is the smallest possible total cost associated with the transformation that is a series of operations that turns one sequence into another. As an example, consider the calculation of the edit distance between the two strings **MUSICAL** and **JUSTICE** that uses this series of operations for transforming **MUSICAL** into **JUSTICE**:

1. M → J (substitution)

2. ϵ → T (insertion)

3. A → E (substitution)

4. L → ϵ (deletion)

Note that implicitly I am excluding all the no-change operations that look like this:

1. U → U (no-change)

This corresponds to an alignment that looks like this. The green pair of characters indicate an insertion operation, red is deletion, and purple is substitution. These

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Operation	Cost
Insertion	1
Deletion	1
Substitution	1
No-change	0

Table 2.1: Naive Operation Cost Function Table

colors are also used in the visualization of OMR/MIDI stream alignment.

seq1 M U S   I C A L
seq2 J U S T̄ I C E _

Figure 2-2: Alignment of MUSICAL and JUSTICE

You would say that the two strings have an edit distance of 4, because it requires four operations to completely morph one string into the other (discounting no-change operations).

However, you also might think that a substitution should "cost" more than either an insertion or deletion; after all, a substitution can be thought of as a consecutive insertion and deletion operation.

A slightly different cost function that assigns more cost to substitution changes could look like this. Using this cost function, our edit distance of changing **MUSICAL** into **JUSTICE** is 5.

Operation	Cost
Insertion	1
Deletion	1
Substitution	1.5
No-change	0

Table 2.2: Operation Cost Table With Costlier Substitutions

But wait, should certain subtypes of a single type of operation have differently weights? Consider if you were trying to run a spell checker (one application of string alignment and correction algorithms) on a word that wasn't recognized by the dictionary. In your particular spell check model, maybe substitution of a vowel for a vowel isn't

as costly as any other types of substitution. Let's say that substituting a vowel for a vowel should only have a cost of 1.2. Using this new cost function, our new edit distance is 4.7.

Operation	Cost
Insertion	1
Deletion	1
Substitution - vowel for vowel	1.2
Substitution - all others	1.5
No-change	0

Table 2.3: Operation Cost Table With Different Substitution Costs

2.2.1.2 Alignment

Recall that the edit distance between two sequences come from the least-cost alignment. The figure below shows a valid alignment, but it is certainly not the least-cost alignment, and therefore does not correspond to the edit distance between between the two sequences.

seq1   _   M   U   S   I   C   A   L
seq2   J   U   S   T   I   C   E   _

Figure 2-3: A bad alignment of MUSICAL and JUSTICE

If we have two sequences, of length n and m, then we have $\binom{n+m}{m}$ possible alignments. Put into context, consider a two melodies of 10 notes each. Naively aligning just the note pitches, we have 184,765 different possible alignments! Clearly this isn't scalable, which is why we rely heavily on this piece of insight: A globally optimal alignment contains locally optimal alignments! This means that we can reuse the computations of subproblems to solve larger problems. What follows from this piece of insight is that for two strings split at (i, j), the best alignment is:

best alignment of `string1`$[: i]$ and `string2`$[: j]$ + best alignment of `string1`$[i :]$ and

$$\text{string2}[j :]$$

$$i$$
$$\downarrow$$

```
    seq1  M   U   S   I   C   A   L
    seq2  J   U   S   T   I   C   E
```

$$\uparrow$$
$$j$$
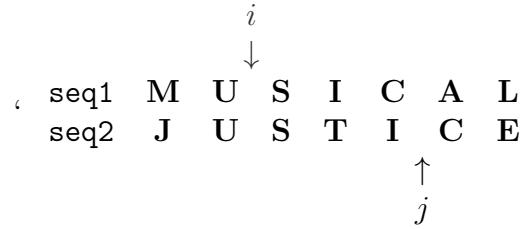
Figure 2-4: The alignment of MUSICAL split at i and JUSTICE split at j

If we can keep track of all the possible alignment subproblems, we can recursively build an optimal solution using the answers from the subproblems. One way to represent this is with a scoring matrix indexed by i and j. For our problem of aligning **MUSICAL** and **JUSTICE**, the setup would look something like this:

[[much more here on the steps of alignment. 2) populating distance matrix 3) back-tracing from bottom right corner using the cost function we have previously defined]]

	-	J	U	S	T	I	C	E
-								
M								
U								
S								
I								
C								
A								
L								

Figure 2-5: Initial distance matrix setup for aligning MUSICAL and JUSTICE

Recall that we have defined our problem in three different ways:

1. Aligning **MUSICAL** and **JUSTICE**

2. Finding the edit distance between **MUSICAL** and **JUSTICE**

3. Finding a series of operations that transforms **MUSICAL** into **JUSTICE**

All of these problem definitions are (almost) one and the same. Calculating the edit distance will also provide a good alignment of the two words and a good alignment necessitates a series of character change operations. It is important to know that that

(3) has an implicit directionality built into the problem statement. It won't always be the case that alignments and transformations are symmetric (i.e. cost the same and have the same series of operations), and having a simple cost operation functions, such as in tables and will make such problems symmetric.

In our particular sample problem, we will always be transforming **MUSICAL** into **JUSTICE**. In the context of my thesis, we will always be transforming OMR sequences in MIDI sequences.

After we've set up is the scoring matrix, we will populate it with initial values:

1. 0 in $(0, 0)$

2. $i - 1 \cdot insertioncost$ for i in $(i, 0)$ (this is the first column)

3. $j - 1 \cdot deletioncost$ for j in $(0, j)$ (this is the first column)

[[somehow there needs to be an explanation that insertions are vertical movements, deletions are horizontal movements, and this is all relative to how you define the problem of transforming X->Y or Y->X]]

After we've set up the initial matrix, we can go through and fill in all of the remaining slots using this update rule:

[[update rules here– find the min (moving up + insertion, right + deletion, diagonal + subst)]]

2.2.2 Formal Definitions in Sequence Alignment

This section is a more mathematical and formal guide to sequence alignment in a matrix representation that uses the cost functions we have been slowly refining.

[[this section might not be necessary if the previous sections are rigorous enough...]]

[[no, there's a really cool way of explaining the set up of the problem in space transformation language... see notes!]]

2.3 Turning Sequence Alignment into a Musical Problem

Finding the edit distance of a musical sequence as opposed to a string of ASCII characters is a little trickier. First, there isn?t an intuitive "space" of music elements the way that there is a "space" of all characters. Second, even after identifying the musical elements, there isn?t an intuitive metric of distance between elements. For example, how would the distance between [Ab quarter note, 4th octave] and [Bb quarter note, 4th octave] compare with [Ab either note, 4th octave] and [Ab quarter note, 4th octave]? [pictures, maybe an even more ambiguous example]

Going back to [original model] and the questions we posed and answered for string alignment,

- What sequences are and what makes them up: We can think of musical sequences as an ordering of notes, rests, and chords

- The space that sequences occupy: This question can roughly be approximated as, what properties of notes, rests, chords do we care about? Some immediate answers are pitch, duration, rhythm.

- Coming up with an appropriate metric for finding distance: In the previous section we had two different substitution functions. What kind of and how many ways do we want to calculate distance in musical elements?

2.4 Related Work

2.4.1 Typke: Music Retrieval based on Melodic Similarity

[[re-edit to talk about feature extraction as an important step, even though i don't necessarily call it feature extraction]] Another instance in which music is transformed and compressed into its most relevant features before comparison is in Typke's thesis on Music Retrieval based on Melodic Similarity [2]. Here, Typke proposes encoding music into what he calls a "weighted point set" in a two-dimensional space of time and pitch before measuring any sort of melodic similarity between music. A note in a piece is represented as point set of pitch (in the Hewlett 40 system [2]) and an offset in quarter notes from the beginning of the piece.

2.4.2 Church and Cuthbert: Rhythmic Comparison and Alignment

2.4.3 Viro: Peachnote, Musical Score Search and Analysis and IMSLP

2.4.4 White: Statistical Properties of Music and MIDI

White's dissertation [4] describes properties of music that can be turned into heuristics for refining algorithmic analysis of music. Specifically, he builds upon Temperley's 1997 [?] paper's details to harmonic analysis of MIDI and refers to other research that has been constructive towards Temperley's model. White does say that Temperley's algorithm fails when faced with ambiguity, such as in MIDI differentiating between enharmonic tones.

The hope is that by combining the ambiguous parts of MIDI with the probabilistic

and rigorous details of OMR scores, ambiguities in MIDI tonality can be resolved. In addition, White also cites a large repository of high quality MIDI files.

2.4.5 Rebelo et al: Optical Music Recognition - State-of-the-Art and Open Issues

[3]

Chapter 3

Previous Work

The work of my thesis uses the universal Hasher for digital representations of music tool. This work was largely done in the Spring of 2015 as part of my 6.UAP. Additionally, experimental results in a related project focused on speeding up musical comparison by converting Python objects to C objects provide evidence that the bottleneck in musical sequence alignment is not in the comparison step. Both subjects are described in more detail below but only to the extent necessary to understand the work of my thesis.

3.1   A Modular Universal Hasher

Prior to my work on building a musical sequence aligner and fixer, I built a hasher with the specific intention that it would be able to adapt according to different specification settings decided by the programmer. The programmer decides which kinds of music21 stream elements and properties are relevant to their hash, and selects them to build a specific instance of a Hasher object. This Hasher instance can then be used to produce a hash from any music21 stream.

3.1.1 Why Musical Comparison and Alignment Needs a Modular Hashing Function

A hash function maps pieces of data of arbitrary size into data of fixed size and standard representation. If musical sequences could be discretized into elements and if these elements could be hashed, then the processes of comparison and alignment could be made deterministic and quick.

Depending upon the problem statement that calls for a hash in the first place, the details of this two step process:

1. discretizing a musical sequence into a list of relevant elements

2. hashing those elements

can vary For instance, in determining the the rhythmic similarity between two musical sequences, only the musical elements with durations (e.g. notes, chords, rests) in each sequence would be relevant. In hashing these elements, only properties such as duration and offset might only be relevant.

Figure 3-1: The hash of rhythmic features of this short melody would be *NoteHash(offset=0, duration=0.5), NoteHash(offset=0.5, duration=0.5), NoteHash(offset=1.0, duration=2.0) NoteHash(offset=4, duration=1.0)*

Another example instance in determining whether one piece was an approximate transposition of another, only the elements with some relation to pitch would be relevant (e.g. notes and chords) would be relevant.

Figure 3-2: The hash of features related to transposition of this short melody would be *NoteHash(intervalFromLastNote=0), NoteHash(intervalFromLastNote=-2), NoteHash(intervalFromLastNote=-5)*

[[check to things are signed]] Once musical sequences are discretized and represented in a hash sequence, each individual tuple in a hash sequence can be thought of as a discrete point in the space of all combinations of possible element properties. Then, canonical edit distance or string alignment algorithms could be used to come up with a measure of similarity or an alignment between the two original musical sequences.

3.1.2   Previous Work on Modular Hashing Functions

The idea of extracting qualities of music specific to the needs of distance analyses and encoding that information in a lower-dimensional and representation is not new. Typke in his thesis proposes what he calls a weighted point set representation of music [2]. His method of creating a weighted point set for a stream of music involves only taking melodic information (i.e. notes) and representing each discrete note as a three-tuple of *(*time, pitch, weight). The time component encodes information about the note's offset from the beginning. The pitch component encodes the note's pitch using the Hewlett's base-40 system (this is a numerical encoding similar to MIDI pitch but is also able to encode more information than just absolute frequency). The weight component is an assignment of how important the particular note is to the melody of the music stream.

3.1.3   Overview of System

The `Hasher` object is found in `hasher.py`. Its purpose is to identify the relevant features of relevant elements in any music stream and to represent them in a hashed format that is easy to compute with. Relevant elements here means objects such as notes, rests, chords. Relevant features here means qualities of these elements such as pitch, duration, offset. I represent these qualities in the hash as lightweight integers, floats, and strings, which makes future computations such as comparisons, and numerical operations easy and fast.

Upon instantiation, `Hasher` does not take in any parameters. After an instance of `Hasher` is created, however, the user can set specific parameters in the `Hasher` to fit their own needs. The user must specify which relevant stream elements (notes, rests, chords) should be hashed, as well as which relevant qualities of those elements should be hashed (pitch, duration, offset, etc.).

After the hashing parameters are set, the user calls the `hashStream` method on a stream. This method strips the stream only to the relevant elements, creates a set of hash functions based on the user's selection of relevant element qualities, and individually hashes each element with the entire set of specified hashing functions.

For each element, the Hasher will create either a `NoteHash` or a `NoteHashWithReference` object and add it to the end of the `FinalHash` list. Both `NoteHash` and `NoteHashWithReference` are objects that hold the hashed qualities of a single stream element. The only difference is that a `NoteHashWithReference` stores a reference back to the original stream element. Even though this has a lot of overhead, it is necessary if ever the original streams need to referenced for context or if they need to be changed.

The resulting hashes of the entire process are stored in `finalHash`.

3.1.3.1 Pre-Hashing: High Level Hasher Settings

There are several system-wide high level settings that must first be set.

- `includeReference` - By default this is set to `False`, but if set to `True`, each element hash will be stored in a `NoteHashWithReference` object that includes a reference to the original element in its original stream. For many purposes, having this reference is not necessary and takes up extra overhead, so it is set to `False` by default.

- `validTypes` - This is a list of what types stream elements should be hashed. By default this is `[note.Note, note.Rest, chord.Chord]` and can be changed to be any subset of those three stream elements.

- `stateVars` - This is a dictionary that keep track of state between consecutive hashes. By default it doesn't contain anything, but can be programmed to contain *intervalFromLastNote* and *KeySignature* during the later process of preprocessing the streams and setting the appropriate hashing functions if the user-set paramets call for their use.

- `hashingFunctions` - This is the dictionary that stored which hashing functions will be used for what qualities. For example, if the user chose to hash notes and their MIDI pitch values, then the dictionary value for the key *Pitch* would be _*hashMIDIPitchName*, the name of the function that takes a note and returns its MIDI pitch.

3.1.3.2 Pre-Hashing: Low Level General Settings

These settings affect more than one of notes, rests, chords.

- `_hashOffset` - If `True`, this will include in the hash a number that is associated with the offset of a musical element from the beginning of the stream.

- `_hashDuration` - If `True`, this will include in the hash a number that is associated with its duration.

- `_roundDurationAndOffset` - This is a boolean for determining whether offsets and durations should be rounded. This is useful for files (e.g. MIDI recordings without rounding) where beginnings of notes, rests, and chords may not fall exactly on the beat (e.g. beat 2.985 instead of 3)

- `granularity` - A number that determines how precise we round. If duration and offset are rounded, they will be rounded to this granularity of beat division. For example, a granularity of 32 will round notes to the nearest 32nd beat division and no further.

3.1.3.3 Pre-Hashing: Low Level Note Settings

- `_hashPitch` and `_hashMIDI` - These settings determine whether pitch of a note will be hashed, and if it is, how it will be represented. If `hashMIDI` is `True`, then the MIDI pitch value is hashed. Otherwise, its name representation is hashed (e.g.“C$\#\#$"). This setting is useful depending upon whether the user would want to differentiate between B$\#$ and C.

- `_hashOctave` - If True, this includes in the hash the number octave that a note is in. This could be useful for an envelope-like following of pitches.

- `_hashIntervalFromLastNote` - If `True`, this returns a number corresponding to the interval between the current note and the last note when applicable. This setting is a useful alternative to hashing pitch when trying to identify transpositions of pieces.

- `_hashIsAccidental` $=$ If `True`, this will return a boolean of whether the pitch of the hashed element is outside of the key signature. This setting is currently not fully supported.

3.1.3.4 Pre-Hashing: Low Level Chord Hashing Settings

- `_hashPrimeFormString` - If `True`, this includes in the hash of chord a string representation of its prime form.

- `_hashChordNormalOrderString` - If `True`, this includes in the hash of a chord a string representation of its normal order.

3.1.3.5 Hashing: `hashStream`, the engine

Once the settings of a `Hasher` instance have been set, we can call its `hashStreams` method on any stream. The following sections describe the what happens within a single `hashStreams` call.

3.1.3.6 Hashing: Setting up `ValidTypes` and `StateVars`

First, the method `setupValidTypesAndStateVars` is called to set up the state variables and the valid hashing types. Recall that by default `stateVars` is empty and `validTypes` is `[note.Note, note.Rest, chord.Chord]` (of course, the user has already had the option of changing `validTypes` to contain any subset of the default types). This method checks whether the user has set either of these two settings:

1. `hashIntervalFromLastNote` - if this is set, then `IntervalFromLastNote` is added to `stateVars` to keep track of

2. `hashIsAccidental` - if this is set, then `KeySignature` is added to `stateVars` to keep track of, and `key.KeySignature` is added to the list of `validTypes` to hash.

3.1.4   Hashing: Preprocessing the Stream

The stream that the `hashStream` method acts upon is stripped of all note ties in the `preprocessStream` if the `stripTies` setting is `True`. Additionally, this method returns the stream in a generator form by calling `recurse()` on the stream.

3.1.4.1   Hashing: Creating a List of Elements to be Hashed

Next the `hashStream` method builds a list of elements that are to be hashed by filtering the recursed stream of any types that are not in `validTypes`. This is stored in `finalEltsToBeHashed`.

3.1.4.2   Hashing: Building a Set of Hashing Functions

The method `setupTupleList` is called, and it sets up `tupleList`, `hashingFunctions` and `tupleClass`, all related to each other. `tupleList` is a list of all the element properties that should be hashed. `hashingFunctions` is a dictionary of which hashing function should be used for each property. `tupleClass` is a `NamedTuple` that is constructed ad hoc based on which properties are to be hashed. The ad hoc construction accommodates for the fact that different `Hasher` instances will contain different things to hash.

3.1.4.3   Hashing: Creating a `NoteHash` for Every Element

For each of the elements in `finalEltsToBeHashed`, the `Hasher` hashes every one of its properties that are to be hashed using the hashing function listed in `hashingFunctions`. It creates a single `NamedTuple` called a `NoteHash` for each element from `finalEltsToBeHashed`.

3.1.4.4 Hashing: Building `finalHash`

If `includeReference` is set to `False`, then each new `NoteHash` object is directly added to `finalHash`. If `includeReference` is `True`, then each `NoteHash` gets replaced with a `NoteHashWithReference` object that includes a reference to the original stream element, and that gets added to `finalHash` instead.

3.2 Low-level Object Comparison Optimizations

Work during the fall of 2015 focused on optimizing the comparison process with variable-length data. The underlying idea was that Python objects tend to have more overhead than C objects, and if comparison and alignment algorithms were going to be doing a lot of comparisons between NoteHash objects produced by the Hasher, then converting these Python objects into C objects of appropriate size could save a lot of time. Here, "appropriate size" refers to the idea that NoteHash objects with only a few hashed properties would correspond to smaller C objects, and NoteHash objects with more hashed properties would correspond to larger C objects.

A smart implementation of the most efficient method, Yeti Levenshtein, already exists in C, so my work was to cleverly convert variable-length NoteHash Python objects to C objects to use the Yeti Levenshtein library, which I hoped would be faster than using any Python edit distance algorithm.

I observed that the speedup we gained in converting variable-length NoteHash Python objects to C objects to use this library was a negligible cost compared to the run time of the comparison algorithm. This empirical evidence suggested that massive parallelization and speedup of the comparison process is difficult and that optimizations in alignment would provide non-negligible impact of speedup.

Thus, I concluded that musical comparison was already near the boundaries of its optimization and my work turned to focus on creating a robust alignment algorithm.

Chapter 4

Final Implementation

This chapter describes the final implementation of the entire OMR/MIDI musical stream alignment and correction system. First there will be an overview of the entire system. Then, there will be discussion on the final implementation of each of the large modules that is divorced from the context of the `OMRMIDICorrector` class. Afterwards, I will present an example use case of the system. I will conclude with a discussion of design tradeoffs and choices that I made in implementing the system.

The final implementation of the OMR/MIDI musical stream alignment and correction system. can be found in `omrMidiCorrector.py`. The OMR system is a combination of one or more of each of the `Hasher`, `Aligner`, and `Fixer` modules with specific parameters set.

The inputs to the system is are two `music21` streams (one OMR, one MIDI). The output of the system is a corrected OMR stream.

4.1 OMRMIDICorrector

4.1.1 System Overview

The entire system resides within the `OMRMIDICorrector` class in `omrMidiCorrector.py`.
This class is initialized with a `midiStream`, an `omrStream` and an optional `Hasher`.
The `Hasher` parameter is optional because the system provides a default `Hasher` with
some all-purpose settings if no `Hasher` is passed in. The method `processRunner` is
the main method of this class that preprocesses the streams, sets up the appropriate
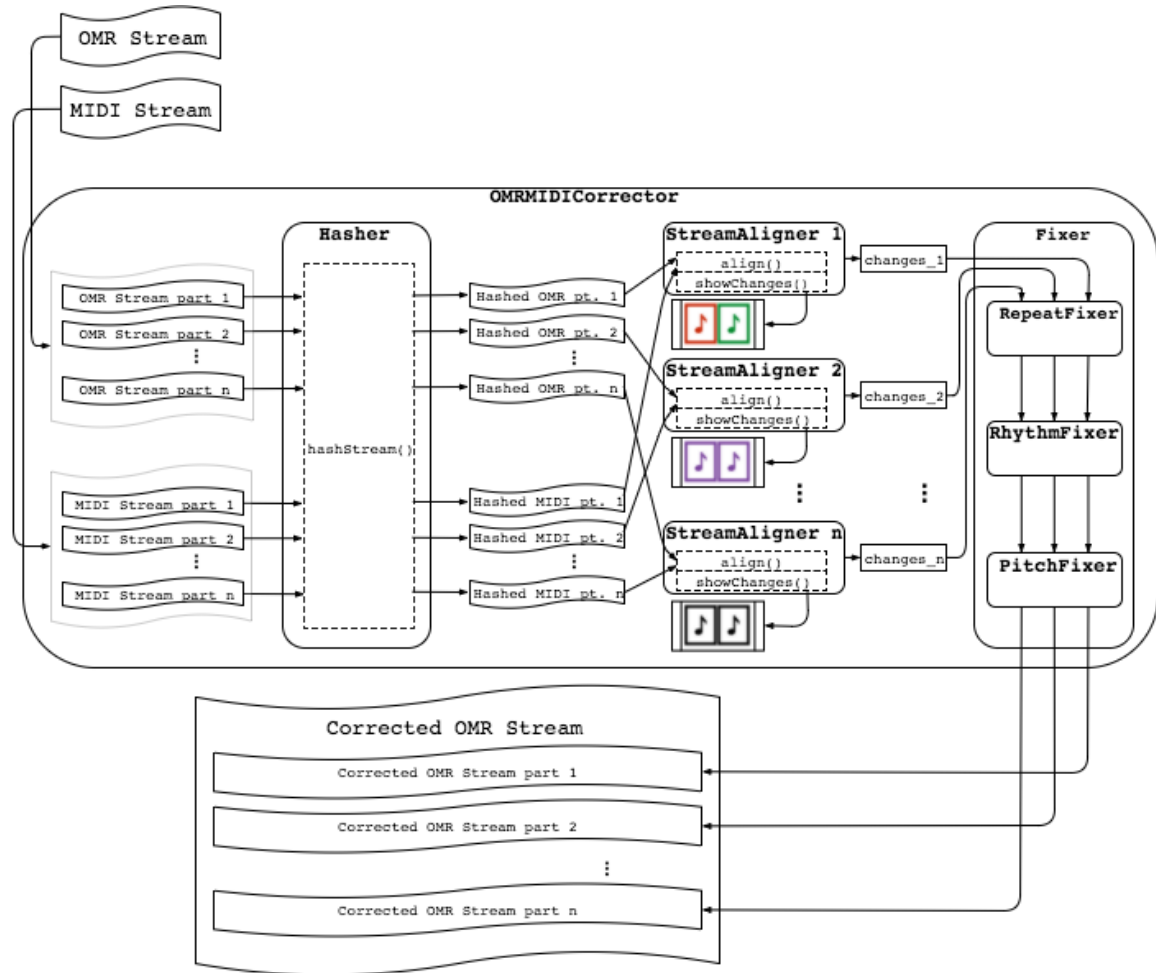`Hasher`, and calls the aligning and fixing methods.

Figure 4-1: System diagram. The inputs to the system are OMR and MIDI streams.
The output of the system is a corrected OMR stream.

4.1.1.1 processRunner: preprocessStreams

The `processRunner` first verifies the streams that are passed in and preprocess them before being hashed, aligned, and fixed in the method `preprocessStreams`

The first thing that `preprocessStreams` does is discretize the parts of the stream if the user has set `discretizeParts` to be `True`. Discretizing parts means to split the input streams into their constituent parts, so that the input streams are aligned part-wise. Since music is often comprised of multiple parts, this preprocessing step allows for input streams that contain nested streams. Take for example, an OMR/MIDI alignment of a string quartet. Both the OMR and MIDI inputs are stream in and of themselves, and so are all individual instrumental parts within both of the streams.

A good alignment of the piece would involve aligning corresponding instrumental parts with each other to ensure that the beginnings and ends of corresponding parts aligned with each other. However, if we treat the inputs both as one long stream composed of four serial streams instead of four parallel streams, only the beginnings of the first instrumental parts and the ends of the last instrumental parts would be guaranteed to be aligned with each other. This guarantee comes from the global alignment method that I chose to implement. The figures below illustrate the phenomenon.

By default, `discretizeParts` is set to `True`. However, the user ultimately has the choice to align input streams part-wise or After a stream is split up into its constituent parts, the parts are stored in the lists `midiParts` and `omrParts`. The aligners and fixers work on all of the elements in the list, and so the entire system is agnostic to the number of parts; a stream comprised of a single part is processed in the same way as a multi-instrumental score.

Then, `preprocessStreams` checks if the number of parts in MIDI and the OMR stream are the same. If they are off by more than one, then a `OMRMIDICorrectorException` is raised and the process halts. This is a cursory check to see if the two input streams can even reasonably be aligned.
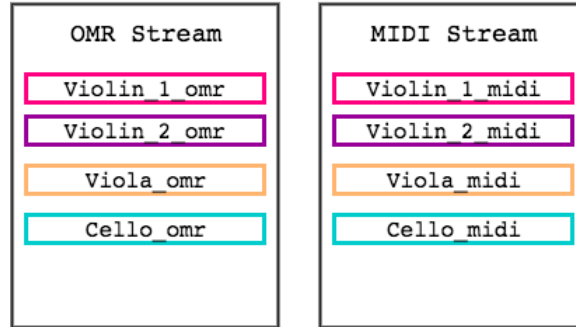
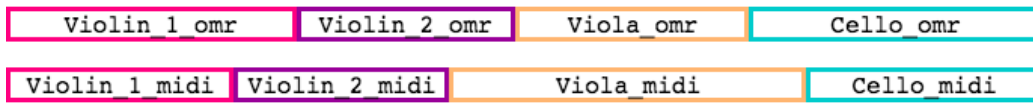Figure 4-2: An example string quartet stream and its four parts.

Figure 4-3: A bad alignment setup of the quartet would treat the entire piece as four serial streams and only the only guaranteed alignment would be between the beginning of the first part and the end of the last part.
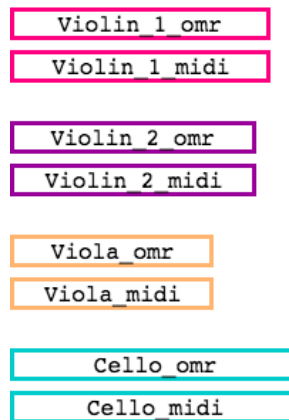
Figure 4-4: A better alignment setup of the quartet would do part-wise alignment instead of treating the entire piece as a sequence of parts. This methodology guarantees that the beginnings and ends of each part are aligned.

If the numbers of parts are off by exactly one, then `preprocessStreams` spins up a lightweight `StreamAligner` object to try and see if there is an implicit bassline, where the bass doubles the cello part but an octave lower. In such a scenario, the OMR score would have one fewer part than the MIDI score because a bassist would

read the cello line while reading sheet music, but the MIDI score would explicitly list all the instrument lines. If the similarity of the two streams is above a certain threshold, then I conclude that that bass part doubles the cello part and we disregard it in the alignment process. The `checkBassDoublesCello` method and its use of an `StreamAligner` object to determine whether two streams are an octave apart is a incidental example of a different use case of the `StreamAligner`.

4.1.1.2   processRunner: setupHasher

The next step in the `processRunner` method is setting up the `Hasher` object that will be used to hash both input streams. This is done in the `setupHasher` method. If user has passed in a `Hasher` object upon instantiation of an `OMRMIDICorrector` object, then that is used. Otherwise this method sets up a default `Hasher` with settings we have found to be generally effective in the `setDefaultHasher` method. The default `Hasher` hashes only pitch and duration and includes references to the original stream elements.

4.1.1.3   processRunner: alignStreams

This method creates a `StreamAligner` object for each pairwise MIDI part and OMR part found in `midiParts` and `omrParts` and aligns the two parts, calculates their similarity metrics, and visually shows the changes and alignment if `debugShow` is `True`. There is more detail about how the `StreamAligner` object works in section 4.3

4.1.1.4   processRunner: fixStreams

The last step is of the entire process is to fix the OMR stream. While the `Fixer` still remains in development, the description of this method and the `Fixer` in section will remain at a high level.

In its final implementation, `fixStreams` should

4.2   Hasher

The `Hasher` object used in the final implementation is a specific instance of the `Hasher` object described in section 3.1. The user can choose to pass in their own instance of `Hasher` or use the default `Hasher` provided.

4.2.1   The Default Hasher

The default `Hasher`, as described in section 4.1.1.2, hashes only pitch and duration of stream elements and set `includeReference` to be `True`. This setting is general enough to provide for a reasonable alignment for most generic alignment problems. But for more niche problems (for example, a rhythmic alignment), the user ought to define their own `Hasher` and the default `Hasher` should not be used.

4.2.2   NoteHashWithReference

One update I made to the original `Hasher` was the addition of a new type of `NamedTuple` that includes a reference to the original element that is hashed and stored within the tuple. This new type provided access to the original stream elements which was necessary for three reasons:

1. It allows the user to visually see the alignment using the `showChanges` method in the `Aligner`. A list of the changes between two streams can be hard to parse and see patterns within, so this method provides a more visually appealing way of interpreting the list of changes.

2. It allows the user to access other elements in the context of the original stream

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element. For example, it might be helpful to be able to access the most recent key signature change in the stream for some fixing purpose.

3. In order for the `Fixer` to fix the OMR stream, it must be able to change the original elements.

4.3 Aligner

4.3.1 System Overview

`Aligner.py` contains the `StreamAligner` class that is the engine of the Alignment step. `StreamAligner` accepts as input two `music21` streams, one specified as a source stream, one as a target stream. The main functionality of `StreamAligner` is that it produces a global alignment between the source and the target streams in the form of a list of Change Tuples that maps each element in the source stream to an element in the target stream via one of the four change operations (insertion, deletion, substitution, no change). Additionally, `StreamAligner` also outputs a basic measurement of similarity between the two stream inputs `similarityScore`. Lastly `StreamAligner` has a `show` function that visually displays the alignment between the source and target streams.

4.3.2 Producing a Global Alignment

The main objective of the Aligner is to produce a global alignment of two streams. In order to do so, it must hash the two streams with an instance of a `Hasher` (either passed in as a parameter during instantiation of a `StreamAligner` or using a default Hasher built into the `StreamAligner` class). After the two streams are hashed, `StreamAligner` sets up a distance matrix, a la the method described in Chapter 2 for classic string alignment, populates the matrix, and then performs a backtrace of the

matrix starting the lower right corner to produce the alignment.

4.3.2.1 Pre-Alignment: Setting Appropriate Parameters

In the interest of being a modular system, I chose to give the programmer the option of setting their own Hasher. Upon instantiation of a `StreamAligner`, the programmer can choose to pass in an instance of a Hasher. If no Hasher is set initially, then `StreamAligner` uses the default Hasher that is set to hash Notes, Rests, and Chords, and their MIDI pitches and durations. The default Hasher is generic enough to be able to work with general alignment, but a more niche problem would require the programmer to use a Hasher more specific to their problem

I also chose to leave the cost of Insertion, Deletion, and Substitution easily substitutable as well. By default, the cost of both Insertion and Deletion is equal to the length of one of the `NoteHashWithReference` tuples, which is exactly the number of properties that are hashed of any musical element. (So in the case of the default Hasher, Insertion and Deletion both have cost 2.) The cost of Substitution between two `NoteHashWithReference` tuples is equal to the number of properties they don't have in common with each other. For example, the cost of substituting

        NoteHashWithReference(pitch=59, offset=1.0, duration=2.0)

with either

        NoteHashWithReference(pitch=59, offset=1.0, duration=3.0)

or

        NoteHashWithReference(pitch=60, offset=1.0, duration=2.0)

would have a cost of 1.

4.3.2.2   Pre-Alignment: Hashing the Streams

The alignment algorithm can only be applied after the streams are represented in a hashed format as a list of `NoteHashWithReference` tuples. Both input streams will be hashed and stored as the `hashedTargetStream` and the `hashedSourceStream`. The hashed format is the analog to a string and each `NoteHashWithReference` tuple is the analog to a character in classic string alignment.

It recommended that the hasher used to the hash the streams be set to include references to the original stream by using `NoteHashWithReference` tuples in the creation of the hash (as opposed to the `NoteHash` tuple that has much less overhead), and in the default hasher, This ensures that the `show` function and future fixers have access to the original objects that each hash came from. This makes it easier to color the original objects in the case of the show function and to extract any necessary metadata that wasn't encoded in the hash for future fixers.

There is also an option to indicate that the streams passed in are already hashed, but in the context of my thesis work, this should never be the case.

4.3.2.3   Alignment: Initializing the Distance Matrix

The first step in the alignment process is initializing the distance matrix. We set up an empty $n+1 \times m+1$ matrix, where n is the length of the hashed target stream and the m is the length of the hashed source. The extra column and extra row are populated with initial costs that correspond to just Insertion or just Deletion operations. In the leftmost column, the value $i \times insertCost$ is put into entries $(i, 0)$. In the topmost row, the value $j \times deleteCost$ is put into entries $(0, j)$.

4.3.2.4 Alignment: Populating the Distance Matrix

The next step in the alignment process is to fill in the remainder of the distance matrix with values generated with these update rules.

$$D[i][j] = \min \{$$
$$D[i\text{-}1][j] + insCost,$$
$$D[i][j\text{-}1] + delCost,$$
$$D[i\text{-}1][j\text{-}1] + subCost$$
$$\}$$

where

$$insCost = insertCost(hashedSourceStream[0])$$
$$delCost = deleteCost(hashedSourceStream[0])$$
$$subCost = substitutionCost(hashedTargetStream[i\text{-}1], hashedSourceStream[j\text{-}1])$$

Each entry A[i][j] in the distance matrix stores the lowest cost for aligning `hashedTargetStream[i]` with `hashedSourceStream[j]`. There are 4 possible ways of relating the two tuples.

1. *hashedTargetStream[i] is an insertion* - in the case of insertion, the cost of aligning `hashedTargetStream[i]` with `hashedSourceStream[j]` is equal to the cost of aligning `hashedTargetStream[i-i]` with `hashedSourceStream[j]` plus the cost of insertion.

2. *hashedTargetStream[i] is a deletion* - similar to the insertion case, for deletion, the cost of aligning `hashedTargetStream[i]` with `hashedSourceStream[j]` is

equal to the cost of the subproblem of aligning `hashedTargetStream[i]` with `hashedSourceStream[j-i]` plus the cost of deletion.

3. *`hashedTargetStream[i]` is a substitution of `hashedSourceStream[j]`* - in the case of substitution, the cost of aligning `hashedTargetStream[i]` with `hashedSourceStream[j]` is equal to the cost of the subproblem of aligning `hashedTargetStream[i-i]` with `hashedSourceStream[j-1]` plus the cost of a substitution of `hashedSourceStream[j]` for `hashedTargetStream[i]`.

4. *No change between the two tuples i.e. the two tuples are the same* - same as above, where substitutionCost(`hashedTargetStream[i]`, `hashedSourceStream[j]`) is 0.

The method `populateDistanceMatrix` fills in all the entries of the distance matrix using the rules and calculations listed above. It is important to note that in my work, all changes are made relative to `TargetStream`. That is, whenever possible, `SourceStream` is the stream that is being transformed into `TargetStream`. This invariant holds because in OMR/MIDI correction, the approach and direction I take is to change OMR into MIDI. Therefore, the MIDI stream is always the `TargetStream` and the OMR stream is always the `SourceStream`. It is certainly possible to go in either direction, but this paper will be done this way.

4.3.2.5 Alignment: Backtrace to Find the Best Alignment and Create the Changes List

Once the distance matrix has been completely filled in, we use a backtrace starting from the bottom right hand corner of the matrix (i.e. A[i][j]) to find the path of least of cost.

Starting from the value found at A[i][j], we look at the values directly above, to the left, and diagonal in the up-left direction. That is, we look at the values in A[i-][j], A[i][j-1], and A[i-1][j-1]. Among these three values, we choose the minimum value. The

combination of direction and value that we moved in tells us what kind of operation was performed to align NoteHashWithReference tuples `hashedSourceStream[i]` and `hashedSourceStream[j]`:

1. if the direction is up, then regardless of the value, this corresponds to an insertion operation.

2. if the direction is left, then regardless of the value, this corresponds to an insertion operation.

3. if the direction is diagonal up-left, and the value at A[i-1][j-1] is *different* from the value at A[i][j], then this corresponds to a substitution operation.

4. if the direction is diagonal up-left, and the value at A[i-1][j-1] is the *same* as the value at A[i][j], then this corresponds to a no-change operation.

We continue this backtrace until we end up back at A[1][1]. Since we are using a global alignment technique, we should always end up back at this entry. If at the end of backtrace we end up in another part of the distance matrix, the method throws an `AlignmentException`.

Additionally, at every step of the backtrace, we create a `ChangeTuple` that holds references to the original musical elements that are represented in the original `SourceStream` and `TargetStream` and the kind of operation (insertion, deletion, substitution, no-change) that links them. Then we insert this `ChangeTuple` into the beginning of the `changes` list.

This is all performed in the `calculateChangesList` method.

4.3.2.6   Post-Alignment: Measures of Similarity

After the backtrace to find the alignment, there is enough data to calculate some basic metrics of similarity.

`changesCount` is Counter object that provides a count of how many of each of the four different change operations appear in the `changes` list.

`similarityScore` is a float between 0 and 1.0 that is the ratio of `NoChange` operations to the total number operations in the `changes` list.

4.3.2.7 Post-Alignment: Visual Display of Alignment

The `showChanges` method provides a visual display of how the two streams are related via the `changes` list. For each `ChangeTuple` in the list, this method goes back to the initial reference and changes the color of the element in reference and adds in an id number as a lyric to that element if the change operation is not a `NoChange`. The color is determined by the type of change operation. Green corresponds to an insertion, red to deletion, and purple to substitution. The id number is the index of the `ChangeTuple` in the list.

The figures below shows the visual display of alignment between a scanned score of an excerpt of *String Quartet No.7 in E-flat major, K.160: I. Allegro* and a MIDI recording of the same piece as well as the original score that the OMR came from.
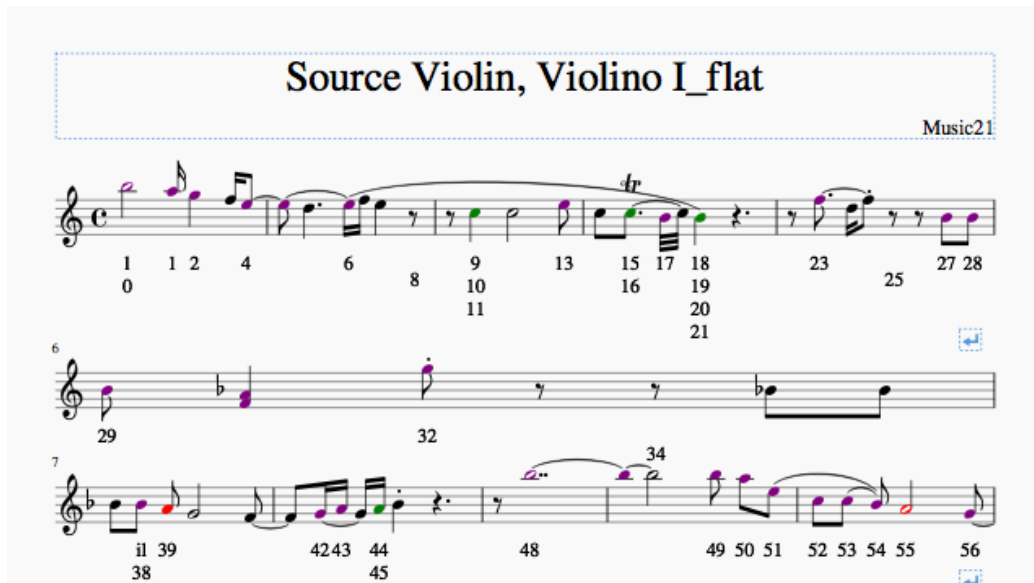

Figure 4-5: Visual display of changes in source (OMR) stream

Figure 4-6: Visual display of changes in target (MIDI) stream

Figure 4-7: Excerpt of original score taken from IMSLP

Since this method has a lot of overhead, it is by default set not to run. It can be set to run for every alignment by passing in the argument `show=True`.

4.4 Fixer

4.5 Tradeoffs

4.6 A Use Case

Chapter 5

Testing

5.1 Existing Music

5.2 Timing

Chapter 6

Future Work

6.1 Future Fixers

Appendix A

Tables

Table A.1:

Appendix B

Figures

Figure B-1:

Figure B-2:

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