

# Project Part 1

## Pattern Recognition

### ECE 759

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# 1 Introduction

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## 2 Feature Selection

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### 2.1 Decision Tree Feature Generation

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#### 2.1.1 MNIST

In working with the decision trees, we utilized the SVD of each image in the training set.

### 2.2 LDA vs PCA

---

LDA and PCA linearly transforms the data to reduce the dimensions. The difference is that LDA is supervised whereas PCA is unsupervised. PCA finds directions which maximize the variance along that direction and they are orthogonal to each other. We can view a PCA technique as in Figure(2.1).

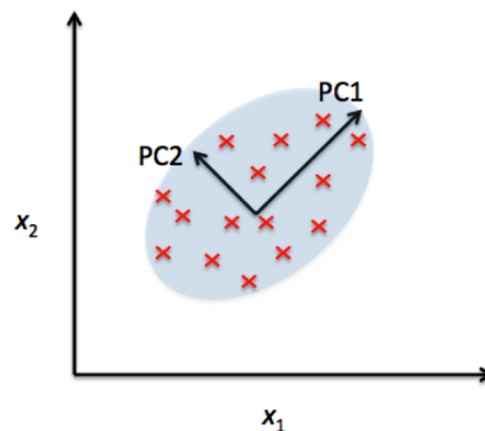


Figure 2.1: PCA implemented on 2-dimensional data

LDA maximizes the class separability and can be viewed as in Figure(2.2).

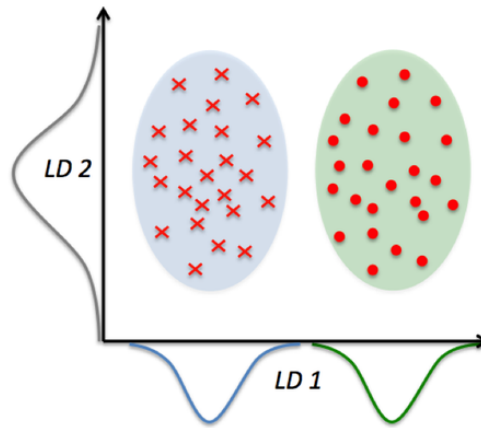


Figure 2.2: LDA visualization

The underlying concept of LDA is taking the eigenvectors of  $\frac{\Sigma_b}{\Sigma}$  where  $\Sigma$  is within-class scatter matrix and  $\Sigma_b$  is between-class scatter matrix. By having this division of matrices, we separate the classes away from each other. On the other hand, since PCA is unsupervised it only tries to capture the data in minimum number of variables.

## 2.3 LDA for MNIST and Extended YaleB

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LDA reduces the data with  $K$  classes with large dimensions to  $K-1$  dimensions which captures the most energy of the data. Implementing LDA as a dimension reduction, MNIST and Extended YaleB datasets are reduced to 9 and 37 variables for each data point. Then we worked with this transformed data to do further classification.

# 3 Algorithm Implementations

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## 3.1 Decision Tree Algorithm

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A binary decision tree is a hierarchical structure that takes input data at its root and propagates it to one of many leaves. Each *leaf* of the tree represents a class designation. To reach a leaf, the features of the data are utilized at *nodes* to make a binary decision: to proceed down the left or right *branch* of the tree? To answer this question, the node also carries a *threshold* that the feature value of the test data is compared against. If the test feature is less than the threshold, we proceed down the left branch. Otherwise, the right. An illustration of a simple decision tree is pictured below.

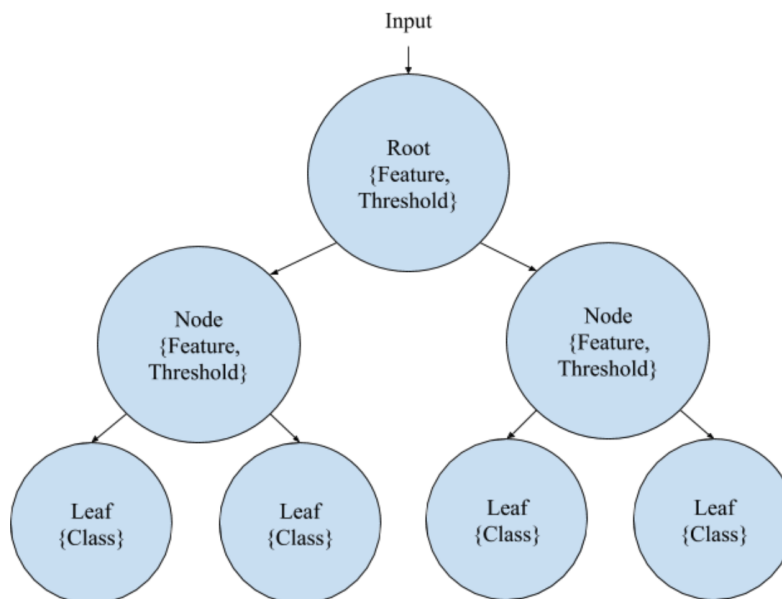


Figure 3.1: An example decision tree.

This decision tree structure needs to be generated before it can be used with test data. To train a decision tree that appropriately classifies our test data according to the features we generated, we employ a recursive function. The function signature is

```
tree = trainDecisionTree(set)
```

Where **set** is the training set, which is a MATLAB structure that contains the raw data (unused), class labels, and generated features. **tree** is the returned structure that can be used during testing. It is essentially a nested structure that contains two types of elements: nodes and leaves. At each node of the tree, a feature and threshold are specified. If a test sample's value at that particular feature is less than the threshold, the sample is passed down the left branch of the node. Similarly, if the sample's feature is greater than the threshold, it goes through the right branch. This is repeated until we reach a leaf node, which specifies a class membership.

The decision tree training algorithm has a few major steps, and proceeds by evaluating a metric called *information gain* at various configurations. For now, suffice it to say that information gain is a scalar that represents the improvement in prediction as we narrow down the set (by growing the tree) to find appropriate leaves. Our implementation most closely follows that of the ID3 and C4.5 algorithms developed by Ross Quinlan in the late 80s and early 90s [1] [2].

1. Check stopping conditions, which generate leaves.
  - If there are no more features to split on, return a leaf with the class mode of the set.
  - The set is smaller than **minLeaf**, which is a tuning parameter that is meant to reduce overfitting of the training data. If this condition is met, return a leaf with the class mode of the set.
  - If all samples in the set belong to the same class, return a leaf with the class.
  - If no feature yields an improvement to the information gain (discussed below), then return a leaf with the class mode of the set. Note that this condition is only evaluated after step 2.
2. Iterate over each feature. Sort the set along the current feature. We utilize a threshold that splits the set between adjacent feature values. Because the information gain across thresholds is convex on the whole (see Fig. 3.2), we use a line search that approximates the highest information gain for each threshold.

Let **attributeBest** and **indBest** be the feature and index that yield the highest information gain. Since the set is sorted, we can simply split the set at the index given by **indBest** for the recursion.

3. Recur over the subsets given by `indBest` to find the next attribute that yields the highest information gain. Note that we exclude the attribute/feature we chose in this execution of `trainDecisionTree(.)`.

The algorithm is reproduced in pseudocode below.

---

**Algorithm 3.1:** `trainDecisionTree`


---

**Data:** *set* of training samples with class labels (*set*(1)) and attributes (features) (*set*(2)).

*minLeaf*, an integer specifying the minimum number of elements in *set* required to make a splitting node. *i.e.* a leaf is generated if the set has fewer elements than *minLeaf*.

**Result:** *tree*, a structure containing nodes and leaves.

```

1 begin
2   check for base cases:
3   if set(2) =  $\emptyset$  then
4     no more attributes (features) to split on.
5     return leaf with mode of set(1) (class labels)
6   if length(set(1)) < minLeaf then
7     return leaf with mode of set(1) (class labels)
8   thisSetEntropy  $\leftarrow$  getEntropy(set)
9   if not thisSetEntropy then
10    all samples are in the same class.
11    return leaf with first element of set(1)
12
13   instantiate tracking variables:
14   attributeBest  $\leftarrow$  0
15   thresholdBest  $\leftarrow$  0
16   infoGainBest  $\leftarrow$  0
17   indexBest  $\leftarrow$  0
18   for attribute  $\in$  range(# of features left in set) do
19     sort set along attribute.
20     perform line search heuristic to approximate max information gain by splitting set at various indices.
21     thisInfoGain  $\leftarrow$  lineSearch(set)
22     if thisInfoGain > infoGainBest then
23       attributeBest  $\leftarrow$  attribute
24       infoGainBest  $\leftarrow$  thisInfoGain
25       indexBest  $\leftarrow$  index given by line search
26       thresholdBest  $\leftarrow$  attribute value midway between indexBest and indexBest + 1
27
28   if not infoGainBest or not attributeBest then
29     no attribute provides an information gain.
30     return leaf with mode of set(1) (class labels)
31
32   re-sort set along attributeBest.
33   subsets  $\leftarrow$  getSubsets(set, attributeBest, indexBest)
34   subtree1  $\leftarrow$  trainDecisionTree(subsets(1), minLeaf)
35   subtree2  $\leftarrow$  trainDecisionTree(subsets(2), minLeaf)
36
37   return node with attributeBest, thresholdBest, subtree1, and subtree2

```

---

Once we have trained the decision tree, we can begin to test it by passing the features generated from the test set

through the tree. The testing algorithm is reproduced in pseudocode below.

---

**Algorithm 3.2:** testDecisionTree
 

---

**Data:** *set* of test samples with class labels (*set*(1)) and attributes (features) (*set*(2)).

*tree*, the struct of structs that represents the trained decision tree.

**Result:** *set*, the input structure modified to include predicted class membership.

```

1 begin
2   for each sample ∈ set do
3     treeWalked ← tree
4     classified ← false
5     attributes ← features corresponding to this sample
6     while not classified do
7       if treeWalked is a node then
8         thisAttribute ← attribute given by treeWalked
9         thisAttributeValue ← value of the sample's feature at thisAttribute
10        thisThreshold ← threshold given by treeWalked
11        remove thisAttribute from the sample's feature vector.
12        compare thisAttributeValue to thisThreshold:
13        if thisAttributeValue < thisThreshold then
14          choose left branch.
15          treeWalked ← left branch given by current treeWalked.
16        else
17          choose right branch.
18          treeWalked ← right branch given by current treeWalked.
19      else
20        treeWalked is a leaf.
21        append predicted label to set(2) at this sample.
22        classified ← true

```

---

### 3.1.1 Entropy and Information Gain

When growing the decision tree, the training algorithm utilizes a metric called *information gain* to optimize the predictive power of the tree. Before we can define information gain, we must first understand *entropy*. Entropy is an information theoretic topic that represents the amount of uncertainty in a given set of data. It is defined as

$$H(X) = - \sum_{i=1}^n P(x_i) \log_2 P(x_i) \quad (3.1)$$

Where  $P(x_i)$  is the proportion of the number of elements with class  $x_i$  to the number of elements in the set  $X$ . there are  $n$  classes in the set. Note that if all samples belong to class  $i$ ,  $P(x_i) = 1$  and  $\log_2 P(x_i) = 0$ , so the entropy is zero.

In calculating the information gain of splitting the set into two subsets, we utilize the entropy of the parent set and subtract from it a weighted entropy of each subset. This is better expressed by the following equation:

$$IG(X) = H(X) - \sum_{i=1}^2 \frac{|S_i|}{|X|} H(S_i) \quad (3.2)$$

Where  $|\cdot|$  is the cardinality, or number of elements in the set, and  $S_i$  are the two subsets composed of the elements of  $X$  partitioned across a given threshold for a given feature. The figure below is a plot of the information gain

across splitting indices. So, after partitioning the set across a single attribute, (all elements of  $S_1$  are less than the threshold, and all elements in  $S_2$  are greater), we have a scalar value of information gain to decide which splitting threshold and feature would most improve the predictive ability of the decision tree. The plot below represents just one feature, but this calculation is required across all currently available features in the set. This quickly becomes computationally taxing for a large data-set, so we utilize a line search heuristic to improve the computational time without sacrificing a significant amount of accuracy.

The line search starts in the middle of the set and splits it, computing an information gain (IG). Then, we compare this middle IG to the IGs obtained by splitting the set at the 25% and 75% indices. If either of these reveals a larger IG, we set it to the new 'middle' index and consider the IGs which emerge from splitting the set half-way between the old 'middle' and half-way between the old 'left' or 'right' (depending on which one yielded a larger IG.) This allows us to approximate the maximum value of the information gain for a given feature without a brute-force technique of considering each threshold. We ensured that the line search incorporates a variety of locations to coax it to approaching the global maximum, rather than getting caught in the minor variations between adjacent indices.

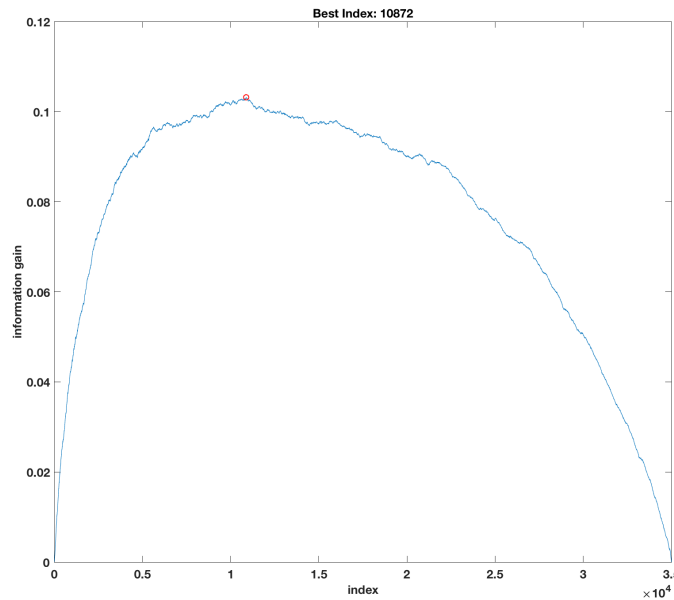


Figure 3.2: Information Gain across all possible thresholds.

## 3.2 Linear Discriminant Analysis

Our classification criterion is to misclassify as small as possible. The rule we employ in classifying the data points is through the use of Bayes Rule. We put the data point to the group with the highest conditional probability (i.e.  $P(w_i|x)$ ). In practice, it is not feasible to get conditional probability for a given point unless we have a huge data. So we should assume the distribution and calculate the probabilities.

LDA relies on the assumption of normal distribution of data for each class. Linear discriminant analysis frequently achieves good performances in the tasks of face and object recognition, even though the assumptions of common covariance matrix among groups and normality are often violated (Duda, et al., 2001) (Tao Li, et al., 2006).

Since MNIST and Yale datasets are high-dimensional, we can not check the normality of variables. Instead, we can reduce the dimensions via a projection. We need to employ multiclass LDA as we have several classes. The class separation in this case will be given by the ratio of  $\frac{\mathbf{w}^T \Sigma_b \mathbf{w}}{\mathbf{w}^T \Sigma_w \mathbf{w}}$ . In the case of two classes, this will reduce just to the



ratio of between-class variance to within-class variance.

### 3.2.1 Algorithm steps

Variable notations:

$n$  is number of classes

$N$  is the total number of training data

$N_i$  is the number of points in each class

$\mu_i$  and  $\mu$  are mean vectors for each class and overall mean for the data

The steps that we follow in our LDA algorithm are:

1. We calculate within-class scatter matrix  $\Sigma_i = \frac{1}{N_i-1} \sum_{\mathbf{x} \in D_i} (\mathbf{x} - \mu_i)(\mathbf{x} - \mu_i)^T$  for each class, then sum them up to get  $\Sigma_W = \sum_{i=1}^c (N_i - 1)\Sigma_i$ .
2. Then we find average within-class scatter matrix by calculating  $\Sigma = \frac{\Sigma_W}{N}$
3. We also calculate the between-class scatter matrix by  $\Sigma_B = \sum_{i=1}^c \frac{N_i}{N} (\mu_i - \mu)(\mu_i - \mu)^T$ .
4. We need to find eigenvectors and eigenvalues of  $\Sigma^{-1}\Sigma_B$ .
5. We then sort the eigenvectors depending on the magnitude of eigenvalues.
6. The number of highest eigenvalues will be  $c-1$  which will be 9 and 37 for MNIST and Extended Yale datasets.
7. Project our data onto the subspace(constructed by the eigenvectors of the highest eigenvalues).
8. Since we have reduced dimensions of our data, we can easily apply a discriminant function for a test vector to see which class it belongs
9. A discriminant function for each class is  $f_i(x_k) = \mu_i w_a^{-1} x_k^T - \frac{1}{2} \mu_i w_a^{-1} \mu_i^T + \ln(P_i)$  is evaluated where  $P_i$  is the probability of each class.
10. Then we select a class with highest discriminant function.

### 3.2.2 Helpful Functions

To accomplish the above pseudo code, we used own created functions.

## 4 Test Results

The table below illustrates the lowest classification errors we were able to achieve with our Decision Tree and LDA classifiers on the MNIST and Yale Extended B datasets.

Algorithm	MNIST	Yale B
LDA		
Decision Tree	18.42%	81.16%

Table 4.1: Lowest classification errors achieved with LDA and Decision Trees.

## 4.1 Preliminary Hyperparameter Optimization

### 4.1.1 Decision Trees

The table below illustrates some results that we obtained during various configurations of the decision tree training algorithm. In an effort to optimize training time vs. test accuracy, we noticed some trends in the algorithm's performance. Notably, utilizing `minLeaf` doesn't seem to contribute to over-fitting, as was noted in other implementations [3]. This may be in part to the type of algorithm we are using. Particularly, the pure C4.5 algorithm moves back up the tree if certain conditions are met, thereby improving a previously-made split. This bore some extra complexity than we were able to achieve at this time, but may be responsible for a more simpler tree generation on our part.

Training Configuration	1	2	2	2	3	3	1	1	1	1	4
numFeatures	30	30	60	200	200	200	30	100	100	20	10
minLeaf	1	1	2	2	3	4	4	4	1	1	1
Error Rate	22	26	26	26	23	24	22.6	23.6	23.5	22.9	18.42
Mins To Train	48	2	2	3	12	12	47	60	102	11	3

Table 4.2: Hyperparameter Configuration of Decision Tree.

The training configurations were primarily modifications to the algorithm's search through the feature's to find the best one to split the set on. They are as follows:

1. All features were considered for the best information gain in a particular set.
2. The decision tree was grown by considering only the first feature in the set. This was an attempt to reduce the computational complexity.
3. If the number of features left in a set was greater than 5, we considered only the first 5. Otherwise, we considered all features left.
4. For configurations 1-3, we utilized features generated via PCA. For this experiment, we utilized features generated with LDA. This yielded our best performance.

## 5 Conclusion

In conclusion, we have shown that utilizing Decision Trees and Linear Discriminant Analysis on the Extended Yale Dataset B and MNIST datasets yields a favorable result. In the next part of this project, we will improve our classification accuracy and partake in a more thorough exploration of the hyperparameter space to optimize and generalize our classifiers.

## 6 References

- [1] Wikipedia contributors. Id3 algorithm — wikipedia, the free encyclopedia, 2017. [Online; accessed 14-March-2018].
- [2] Wikipedia contributors. C4.5 algorithm — wikipedia, the free encyclopedia, 2018. [Online; accessed 14-March-2018].
- [3] Mathworks. Matlab documentation - fitctree. <https://www.mathworks.com/help/stats/fitctree.html>, 2018. [Online; accessed 2010-09-30].

## 7 Code Listings

---

Below are the primary scripts that solve the project. Please see the `code` folder for supporting functions.

Listing 1: Code for 2.35

```
1  %{
2
3  kudiyar orazymbetov
4  n casale
5
6  ECE 759 Project
7  18/03/16
8
9  this script orchestrates the training and testing of
10 the decision tree classifier
11
12 features are generated using principal component analysis
13
14 %}
15
16 clear; close all;
17 addpath('utility', 'MNIST', 'MNIST/data', 'MNIST/loadMNIST');
18
19 fprintf('begin MNIST decision tree script\n');
20
21 % hyper-parameters
22 N_tr = 35e3; % training samples
23 N_te = 35e3; % test samples
24
25 % for feature selection
26 usePCA = false;
27 if usePCA
28     numFeatures = 30;
29 else % use LDA
30     numFeatures = 10;
31 end
32
33 % for decision tree
34 minLeaf = 2; % to prevent overfitting
35
36 seed = 152039828;
37 rng(seed); % for reproducibility
38
39 % partition data
40 % MNIST contains 70k examples
41 [train, test] = loadMNIST(N_tr);
42
43 %% dimensionality reduction / feature generation
44 % via principal component analysis (pca) (svd)
45 st = cputime;
46
47 if usePCA
48     [train, U, V] = pca_(train, numFeatures);
49 else
50     [train, test] = lda_features(train, test, 0:numFeatures-1);
```

```

51 end
52
53 fprintf('Features Generated in %4.2f minutes\n', (cputime - st)/60);
54
55 %% train
56 st = cputime;
57
58 tree = trainDecisionTree({train{2:3}}, minLeaf);
59
60 fprintf('Trained in %4.2f minutes\n', (cputime - st)/60);
61
62 %% test
63 st = cputime;
64
65 if usePCA
66     test{3} = (U'*test{1}.*V)';
67     test{3} = test{3}(1:numFeatures,:);
68 end
69
70 test = testDecisionTree(test, tree);
71
72 fprintf('Tested in %4.2f minutes\n', (cputime - st)/60);
73
74 % Classification Error
75 errors = nnz(test{2}(:,1) ~= test{2}(:,2));
76 errorRate = (errors/N_te)*100;
77
78 filename = sprintf('mnist_tree%2.0f_%d.mat', errorRate, minLeaf);
79 save(filename, 'tree');
80 %filename = sprintf('UV%2.0f.mat', errorRate);
81 %save(filename, 'U', 'V', '-v7.3');
82
83 fprintf('numFeatures: %d, minLeaf: %d, error rate: %2.2f\n', ...
84         numFeatures, minLeaf, errorRate);
85
86 %% Test Results
87 %{
88     , * * * & & , , , , ,
89 numFeatures : 30 30 60 200 200 200 30 100 100 20 10 (lda)
90 minLeaf : 1 1 2 2 3 4 4 4 1 1 1
91 Error Rate : 22 26 26 26 23 24 22.6 23.6 23.5 22.9 18.42
92 Mins To Train : 48 2 2 3 12 12 47 60 102 11 3
93
94 , - all attributes were considered for the best information gain for a
95 particular set.
96
97 * - for these experiments, the decision tree was grown by considering
98 only the first attribute in the set. all things held equal, this was simply
99 an attempt to reduce the computational complexity.
100
101 & - if the number of attributes was > 5, consider only the first 5, else,
102 consider all attributes (1-5)
103
104 %}

```