H|W - 3, 7, 11, 6

3. $y'' - y' - 6y = 0$ $\quad y(0) = 11 \quad y'(0) = 28$

$$L\{y'' - y' - 6y\} = 0$$

$$L\{y''\} - L\{y'\} - 6L\{y(t)\} = 0$$

$$\left[s^2 L\{y(t)\} - s y'(0) - y'(0)\right] - \left[s L\{y(t)\} - y(0)\right] - 6L\{y(t)\} = 0$$

$$\left[s^2 L\{y(t)\} - 11s - 28\right] - \left[s L\{y(t)\} - 11\right] - 6L\{y(t)\} = 0$$

$$L\{y(t)\}\left[s^2 - s - 6\right] - 11s - 28 + 11 = 0$$

$$L\{y(t)\}(s^2 - s - 6) = 11s + 17$$

$$L\{y(t)\} = \frac{11s}{(s-3)(s+2)} + \frac{17}{(s-3)(s+2)}$$

$$\frac{11s + 17}{(s-3)(s+2)} = \frac{A}{s-3} + \frac{B}{s+2}$$

$$11s + 17 = A(s+2) + B(s-3)$$

$$\begin{array}{ll} S = -2 & S = 3 \\ -5 = -5B & 50 = 5A \\ \boxed{B = 1} & \boxed{A = 10} \end{array}$$

$$Y(t) = L^{-1}\left\{\frac{10}{s-3} + \frac{1}{s+2}\right\} = 10e^{3t} + e^{-2t}$$

6. $y'' - 6y' + 5y = 29\cos 2t$ $y(0) = 3.2$ $y'(0) = 6.2$

$L\{y'' - 6y' + 5y\} = L\{29\cos 2t\}$

$L\{y''\} - 6L\{y'\} + 5L\{y(t)\} = 29 L\{\cos 2t\}$

$\left[s^2 L\{y(t)\} - sy(0) - y'(0)\right] - 6\left[sL\{y(t)\} - y(0)\right] + 5 = 29\left(\dfrac{s}{s^2+4}\right)$
$\underbrace{}_{L\{y(t)\}}$ $\underbrace{}_{L\{y(t)\}}$

$\left[s^2 L\{y(t)\} - s(3.2) - 6.2\right] - 6\left[sL\{y(t)\} - 3.2\right] + 5 = \dfrac{29\,s}{s^2+4}$
$\underbrace{}_{L\{y(t)\}}$ $\underbrace{}_{L\{y(t)\}}$

$L\{y(t)\}\left[s^2 - 6s + 5\right] - 3.2s - 6.2 + 19.2 = \dfrac{29s}{s^2+4}$

$L\{y(t)\} = \dfrac{29s}{s^2+2^2} + 3.2s - 13$
$(s^2 - 6s + 5)$

$\dfrac{\quad 5 \quad}{-6}$
$-5 \qquad -1$

$L\{y(t)\} = \dfrac{29s}{(s^2+2^2)(s^2-6s+5)} + \dfrac{3.2s}{(s-5)(s-1)} - \dfrac{13}{(s-5)(s-1)}$

$L\{y(t)\} = \dfrac{29s + 3.2s(s^2+2^2) - 13(s^2+2^2)}{(s^2+2^2)(s-5)(s-1)}$

$\dfrac{29s + 3.2s(s^2+2^2) - 13(s^2+2^2)}{(s^2+4)(s-5)(s-1)} = \dfrac{Ax+B}{s^2+4} + \dfrac{C}{s-5} + \dfrac{D}{s-1}$

$$29S + 3.2S(s^2+4) - 13/s^2+4) = AS + B(s-5)(s-1) + C(s^2+4) + D(s^2+4)(s-5)$$

with superscript $(S-1)$ over the $A S$ term area.

* $S = 1$

$$29 + 3.2(5) - 13(5) = A + 5D(-4)$$

$$-20 = -20D$$

$$\boxed{D = 1}$$

* $S = 5$

$$145 + 3.2(5)(29) - 13(29) = C(4)(29)$$

$$+232 = 116C$$

$$\boxed{C = +2}$$

* $S = 0$

$$-52 = B(-5)(-1) + C(-1)(4) + D(4)(-5)$$

$$-52 = 5B + 8 - 20D$$

$$-24 = 5B \qquad \boxed{B = 48}$$

* $S = 2$

$$58 + 3.2(2)(8) - 13(8) = (2A+B)(-3)(1) + C(8) + D(-3)(8)$$

$$5.2 = -6A - 3B + 16 - 24$$

$$= -6A - 14.4 + 16 - 24$$

$$27.6 = -6A$$

$$\boxed{A = -4.6}$$

$$L\{Y(t)\} = \frac{-3.5S - 8}{S^2 + 4} + \frac{2}{S-5} + \frac{1}{S-1}$$

$$Y(t) = -3.5 \cos 2t + 48 \sin 2t + 2e^{5t} + e^{t}$$

7. $Y'' + 7Y' + 12Y = 21e^{3t}$ $Y(0) = 3.5$ $Y'(0) = -10$

$$L\{Y''\} + 7L\{Y'\} + 12 L\{Y(t)\} = L\{21e^{3t}\}$$

$$\left[S^2 L\{Y(t)\} - SY(0) - Y'(0)\right] + 7\left[SL\{Y(t)\} - Y(0)\right] + 12L\{Y(t)\}$$

$$= 21\left(\frac{1}{S-3}\right)$$

$$L\{Y(t)\}\left[S^2 + 7S + 12\right] - S(3.5) + 10 - 7(3.5) = \frac{21}{S-3}$$

$$L\{Y(t)\}(S^2 + 7S + 12) = \frac{21}{S-3} + 3.5S + 14.5$$

$$\overset{12}{\underset{4 \quad 3}{\wedge}}$$

$$L\{Y(t)\} = \frac{21}{(S-3)(S+4)(S+3)} + \frac{3.5S + 14.5}{(S+4)(S+3)}$$

$$21 + 3.5S(S+4)(S-3) + 14.5(S+4)(S-3) = A(S+4)(S+3) + B(S-3)(S+3)$$
$$+ C(S-3)(S+4)$$

$S = 3$ $S = -4$ $S = 0$

$21 = A(7)(6)$ $-101.5 + 21 = B(-7)(-1)$ $21 + 14.5(4)(-3) =$

$\boxed{A = \dfrac{1}{2}}$ $\boxed{B = 2.5}$ -22.5 $12A - 9B - 12C$

                                         $6 - 22.5 - 12C$

                                           $C = \dfrac{1}{2}$

$$L\{y(t)\} = \frac{1}{2(s-3)} + \frac{3}{s+4} + \frac{12.75}{s+3}$$

$$Y(t) = \frac{1}{2}e^{3t} + \frac{5}{2}e^{-4t} + 12.75 \, e^{-3t}$$

II. $y'' + 3y' + 2.25y = 9t^3 + 64 \qquad y(0) = 1 \quad y'(0) = 31.5$

$$L\{y''\} + 3L\{y'\} + 2.25 L\{y(t)\} = L\{9t^3 + 64\}$$

$$\left[s^2 L\{y(t)\} - sy(0) - y'(0)\right] + 3\left[sL\{y(t)\} - y(0)\right]$$

$$+ 2.25 L\{y(t)\} = \frac{9(6)}{s^4} + \frac{64}{s}$$

$$\left(s^2 L\{y(t)\} - s - 31.5\right) + 3\left[sL\{y(t)\} - 1\right]$$

$$+ 2.25 L\{y(t)\} = \frac{54}{s^4} + \frac{64}{s}$$

2.25

3

1.5   1.5

$$L\{y(t)\}\left[s^2 + 3s + 2.25\right] - s - 31.5 - 3 = \frac{54}{s^4} + \frac{64}{s}$$

$$L\{y(t)\}(s^2 + 3s + 2.25) = \frac{54}{s^4} + \frac{64}{s} + s + 34.5$$

$$L\{y(t)\}(s+1.5)^2 = \frac{54 + 64s^3 + s^5 + 34.5s^4}{s^4}$$

$$L\{y(t)\} = \frac{54 + 64s^3 + s^5 + 34.5s^4}{s^4 (s+1.5)^2}$$

$$\frac{54 + 64s^3 + s^5 + 34.5s^4}{s^4(s+1.5)^2} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^2} + \frac{D}{s^4} + \frac{E}{s+1.5} + \frac{F}{(s+1.5)}$$

$$54 + 64s^3 + s^5 + 34.5s^4 = As^3(s+1.5)^2 + Bs^2(s+1.5)^2$$
$$+ Cs(s+1.5)^2 + D(s+1.5)^2 + Es(s+1.5) + F(s^4)$$

* $\underline{S = 0}$

$$54 = D(1.5)^2$$

$$\boxed{D = 24}$$

* $\underline{S = -1.5}$

$$54 + 64(-3.4) - 7.6 + 174.7 = F(5.06)$$

$$54 - 216 - 7.6 + 174.7 = F(5.06)$$

$$\boxed{F = 1}$$

$$54 + 64S^3 + S^5 + 34 \cdot 5S^4 = AS^3(S^2 + 2 \cdot 25 + 3S)$$

$$+ B(S^2)(S^2 + 2 \cdot 25 + 3S) + CS(S^2 + 2 \cdot 25 + 3S) +$$

$$24(S^2 + 2 \cdot 25 + 3S) + ES^4(S + 1 \cdot 5) + S^4$$

$$= (E + A)S^5 + S^4(3A + B + 1 \cdot 5E + 1) + S^3(2 \cdot 25A + 3B + C)$$

$$+ S^2(2 \cdot 25B + 3C + 24) + S(2 \cdot 25C + 72) + 54$$

$E + A = 1$	$2 \cdot 25C + 72 = 0$	$2 \cdot 25B - 96 + 24 = 0$
	$C = \dfrac{-72}{2 \cdot 25}$	$2 \cdot 25B = 72$
		$\boxed{B = 32}$

$$\boxed{C = -32}$$

$$2 \cdot 25A + 3(32) - 32 = 64$$

$$2 \cdot 25A + 64 = 64 \qquad \boxed{E = +1}$$

$$\boxed{A = 0}$$

$$Y(t) = \mathcal{L}^{-1}\left\{\frac{32}{S^2} + \frac{-32}{S^3} + \frac{24}{S^4} + \frac{1}{S + 1 \cdot 5} + \frac{1}{(S + 1 \cdot 5)^2}\right\}$$

$$= 32t - 16t^2 + 4t^3 + e^{-1 \cdot 5t} + te^{-1 \cdot 5t}$$