

Relational
Algebra
Operators

$\pi \sigma \rho$
 $\bowtie \cup \cap$

Presentation by
Dr. Jenila Livingston L.M.
VIT Chennai

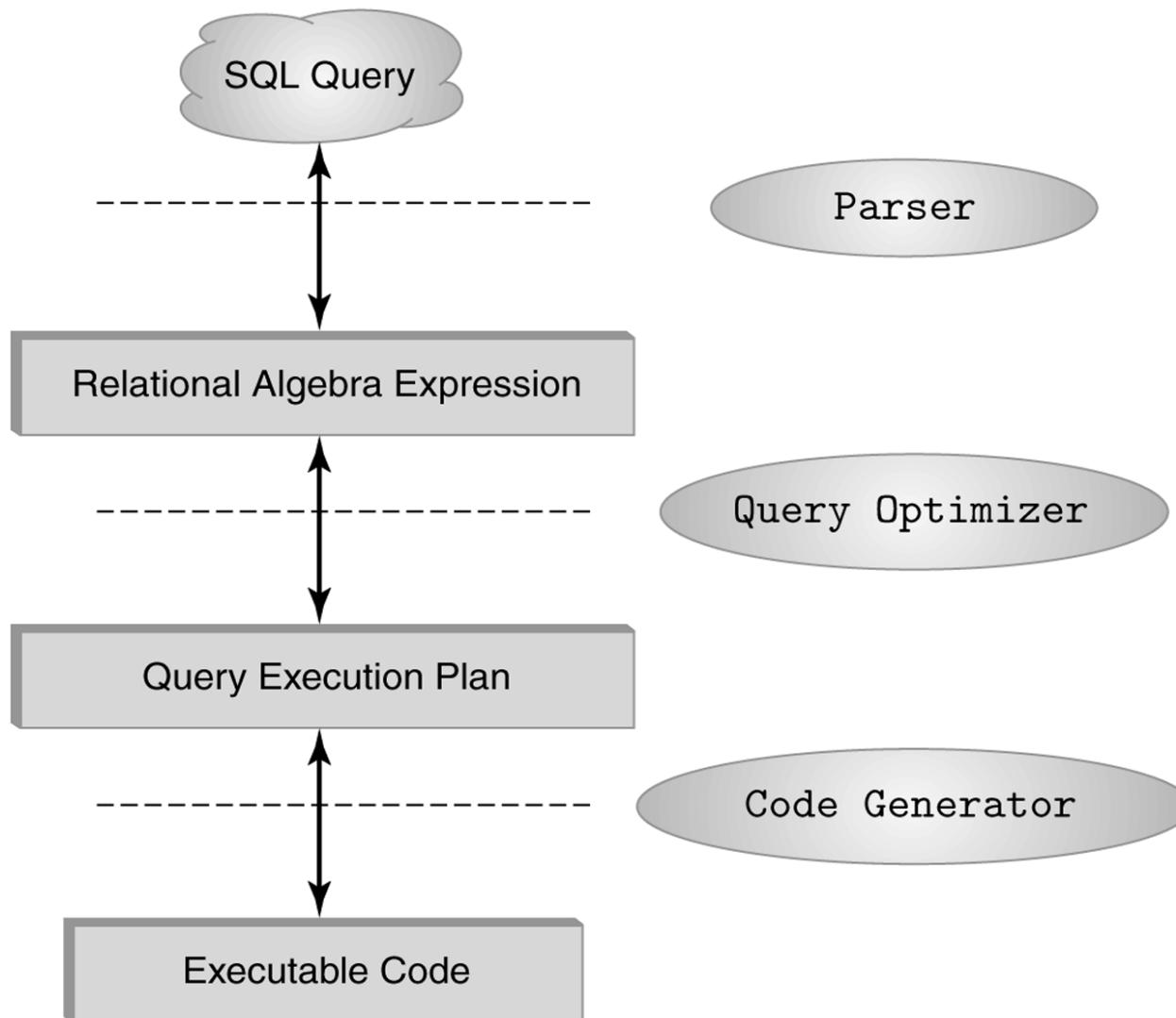
Formal Query Languages

- Specialized languages for asking questions or queries
- Theoretical languages
- Developed before SQL. SQL is based on these languages
 - Relational Algebra
 - Relational Calculus

Relational Algebra

- Introduced by E. F. Codd in 1970.
- More operational, very useful for representing execution plans.
- relational algebra, a procedural language that defines database operations in terms of algebraic expressions.
- Similar to normal algebra (as in $2+3*x-y$), except we use relations as values instead of numbers/ operands , and the operations and operators are different.
- We need to know about relational algebra to understand query execution and optimization in a relational DBMS.

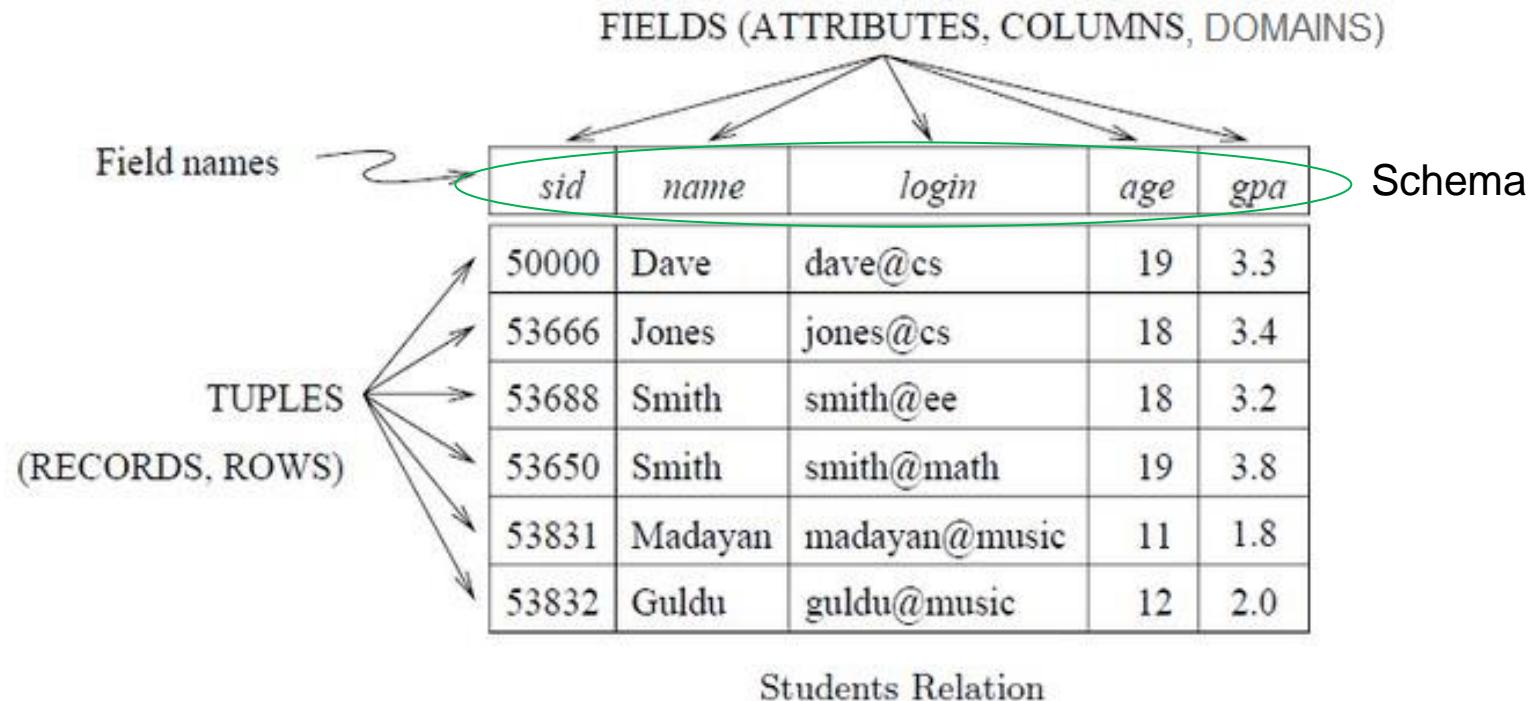
The Role of Relational Algebra in a DBMS



Relational Algebra

1. Selection of Tuples
2. Selection of Attributes
3. Set Operations
4. Join operations
5. Rename
6. Assignment
7. Aggregation

Table or Relation



Degree: No. of attributes = 5
Cardinality: No. of tuples = 6

1. Selection of tuples

Returns all tuples which satisfy a condition

- Relation r
- Selection (σ)

Notation: $\sigma_c(r)$

The condition c can be $=, <, \leq, >, \geq, \neq$

- Select all attributes/ tuples from r

$\sigma(r)$

- Select tuples with $A=B$ and $D > 5$

$\sigma_{A=B \text{ and } D > 5}(r)$

| A | B | C | D |
|----------|----------|----|----|
| α | α | 1 | 7 |
| α | β | 5 | 7 |
| β | β | 12 | 3 |
| β | β | 23 | 10 |

| A | B | C | D |
|----------|----------|----|----|
| α | α | 1 | 7 |
| β | β | 23 | 10 |

Select Operator

- Produce table containing subset of rows of argument table satisfying condition

$\sigma_{condition}(relation)$

- Example: List the persons where hobby is stamp collection

$\sigma_{Hobby='stamps'}(Person)$

| <i>Id</i> | <i>Name</i> | <i>Address</i> | <i>Hobby</i> |
|-----------|-------------|----------------|--------------|
| 1123 | John | 123 Main | stamps |
| 1123 | John | 123 Main | coins |
| 5556 | Mary | 7 Lake Dr | hiking |
| 9876 | Bart | 5 Pine St | stamps |

Person

Selection Condition - Examples

- $\sigma_{Id > 3000 \text{ OR } Hobby = 'hiking'}(\text{Person})$
- $\sigma_{Id > 3000 \text{ AND } Id < 3999}(\text{Person})$
- $\sigma_{\text{NOT}(Hobby = 'hiking')}(\text{Person})$
- $\sigma_{Hobby \neq 'hiking'}(\text{Person})$

2. Selection of Columns (Attributes)

Retains only wanted **columns** from relation (vertical).

- Relation r :
- **Projection (Π)**

Notation: $\Pi_{A_1, \dots, A_n}(r)$

| A | B | C |
|----------|----|---|
| α | 10 | 1 |
| α | 20 | 1 |
| β | 30 | 1 |
| β | 40 | 2 |

- Select A and C

■ $\Pi_{A, C}(r)$

$$\begin{array}{c} \begin{array}{|c|c|} \hline A & C \\ \hline \alpha & 1 \\ \alpha & 1 \\ \beta & 1 \\ \beta & 2 \\ \hline \end{array} & = & \begin{array}{|c|c|} \hline A & C \\ \hline \alpha & 1 \\ \beta & 1 \\ \beta & 2 \\ \hline \end{array} \end{array}$$

Project Operator

- Produces table containing subset of columns of argument table

$\pi_{attribute\ list}(relation)$

- Example: Display name and hobby of persons

$\pi_{Name,Hobby}(\text{Person})$

| <i>Id</i> | <i>Name</i> | <i>Address</i> | <i>Hobby</i> |
|-----------|-------------|----------------|--------------|
| 1123 | John | 123 Main | stamps |
| 1123 | John | 123 Main | coins |
| 5556 | Mary | 7 Lake Dr | hiking |
| 9876 | Bart | 5 Pine St | stamps |

Person

| <i>Name</i> | <i>Hobby</i> |
|-------------|--------------|
| John | stamps |
| John | coins |
| Mary | hiking |
| Bart | stamps |

Project Operator

- Example:

$$\pi_{Name,Address}(\text{Person})$$

| <i>Id</i> | <i>Name</i> | <i>Address</i> | <i>Hobby</i> |
|-----------|-------------|----------------|--------------|
| 1123 | John | 123 Main | stamps |
| 1123 | John | 123 Main | coins |
| 5556 | Mary | 7 Lake Dr | hiking |
| 9876 | Bart | 5 Pine St | stamps |

Person

| <i>Name</i> | <i>Address</i> |
|-------------|----------------|
| John | 123 Main |
| Mary | 7 Lake Dr |
| Bart | 5 Pine St |

Result is a table (no duplicates); can have fewer tuples than the original

Expressions

$$\pi_{Id, Name} (\sigma_{Hobby='stamps' \text{ OR } Hobby='coins'} (\text{Person}))$$

| <i>Id</i> | <i>Name</i> | <i>Address</i> | <i>Hobby</i> |
|-----------|-------------|----------------|--------------|
| 1123 | John | 123 Main | stamps |
| 1123 | John | 123 Main | coins |
| 5556 | Mary | 7 Lake Dr | hiking |
| 9876 | Bart | 5 Pine St | stamps |

Person

| <i>Id</i> | <i>Name</i> |
|-----------|-------------|
| 1123 | John |
| 9876 | Bart |

Result

3. Set Operations

- Union (\cup)
- Intersection (\cap)
- Set Difference ($-$)
- Division ($/$)
- Cartesian Product / Cross Product

Union of two relations

- Relations r, s :

| A | B |
|----------|-----|
| α | 1 |
| α | 2 |
| β | 1 |

r

| A | B |
|----------|-----|
| α | 2 |
| β | 3 |

s

- $r \cup s$:

| A | B |
|----------|-----|
| α | 1 |
| α | 2 |
| β | 1 |
| β | 3 |

Union

| <u>sid</u> | sname | rating | age |
|------------|--------|--------|------|
| 22 | dustin | 7 | 45.0 |
| 31 | lubber | 8 | 55.5 |
| 58 | rusty | 10 | 35.0 |

s1

| <u>sid</u> | sname | rating | age |
|------------|--------|--------|------|
| 22 | dustin | 7 | 45.0 |
| 31 | lubber | 8 | 55.5 |
| 58 | rusty | 10 | 35.0 |
| 44 | guppy | 5 | 35.0 |
| 28 | yuppy | 9 | 35.0 |

$S1 \cup S2$

| <u>sid</u> | sname | rating | age |
|------------|--------|--------|------|
| 28 | yuppy | 9 | 35.0 |
| 31 | lubber | 8 | 55.5 |
| 44 | guppy | 5 | 35.0 |
| 58 | rusty | 10 | 35.0 |

s2

Set Intersection of two relations

- Relation r, s :

| A | B |
|----------|-----|
| α | 1 |
| α | 2 |
| β | 1 |

r

| A | B |
|----------|-----|
| α | 2 |
| β | 3 |

s

- $r \cap s$

| A | B |
|----------|-----|
| α | 2 |

Intersection

| sid | sname | rating | age |
|-----|--------|--------|------|
| 22 | dustin | 7 | 45.0 |
| 31 | lubber | 8 | 55.5 |
| 58 | rusty | 10 | 35.0 |

S1

| sid | sname | rating | age |
|-----|--------|--------|------|
| 31 | lubber | 8 | 55.5 |
| 58 | rusty | 10 | 35.0 |

| sid | sname | rating | age |
|-----|--------|--------|------|
| 28 | yuppy | 9 | 35.0 |
| 31 | lubber | 8 | 55.5 |
| 44 | guppy | 5 | 35.0 |
| 58 | rusty | 10 | 35.0 |

S2

$$S1 \cap S2$$

Set difference of two relations

- Relations r, s :

| A | B |
|----------|-----|
| α | 1 |
| α | 2 |
| β | 1 |

r

| A | B |
|----------|-----|
| α | 2 |
| β | 3 |

s

- $r - s$:

| A | B |
|----------|-----|
| α | 1 |
| β | 1 |

$r - s$ means Tuples in r , but not in s .

Set Difference

| <u>sid</u> | sname | rating | age |
|------------|--------|--------|------|
| 22 | dustin | 7 | 45.0 |
| 31 | lubber | 8 | 55.5 |
| 58 | rusty | 10 | 35.0 |

S1

| <u>sid</u> | sname | rating | age |
|------------|--------|--------|------|
| 22 | dustin | 7 | 45.0 |

$S1 - S2$

| <u>sid</u> | sname | rating | age |
|------------|--------|--------|------|
| 28 | yuppy | 9 | 35.0 |
| 31 | lubber | 8 | 55.5 |
| 44 | guppy | 5 | 35.0 |
| 58 | rusty | 10 | 35.0 |

S2

| <u>sid</u> | sname | rating | age |
|------------|-------|--------|------|
| 28 | yuppy | 9 | 35.0 |
| 44 | guppy | 5 | 35.0 |

$S2 - S1$

4. Division

- Denoted by \div
- Binary Operation
- Used in queries that include the phrase “for all”.

EXAMPLES OF DIVISION

A

| <i>sno</i> | <i>pno</i> |
|------------|------------|
| s1 | p1 |
| s1 | p2 |
| s1 | p3 |
| s1 | p4 |
| s2 | p1 |
| s2 | p2 |
| s3 | p2 |
| s4 | p2 |
| s4 | p4 |

B1

| |
|------------|
| <i>pno</i> |
| p2 |

A/B1

| <i>sno</i> |
|------------|
| s1 |
| s2 |
| s3 |
| s4 |

B2

| |
|------------|
| <i>pno</i> |
| p2 |
| p4 |

A/B2

| <i>sno</i> |
|------------|
| s1 |
| s4 |

B3

| |
|------------|
| <i>pno</i> |
| p1 |
| p2 |
| p4 |

A/B3

| <i>sno</i> |
|------------|
| s1 |

Exercise

- Selects tuples from books where subject is 'database'.
- Selects tuples from books where subject is 'database' and 'price' is 450.
- Selects tuples from books where subject is 'database' and 'price' is 450 or those books published after 2010
- Selects subject and author from the relation Books.

- Projects the names of the authors who have either written a book or an article or both
- Provides the name of authors who have written books but not articles.
- Yields a relation, which shows all the books and articles written by bala.

- $\sigma_{\text{subject} = \text{"database"}}(\text{Books})$
- $\sigma_{\text{subject} = \text{"database"} \text{ and } \text{price} = \text{"450"}}(\text{Books})$
- $\sigma_{\text{subject} = \text{"database"} \text{ and } \text{price} = \text{"450"} \text{ or } \text{year} > \text{"2010}}(\text{Books})$
- $\prod_{\text{subject, author}} (\text{Books})$

- $\Pi_{\text{author}} (\text{Books}) \cup \Pi_{\text{author}} (\text{Articles})$
- $\Pi_{\text{author}} (\text{Books}) - \Pi_{\text{author}} (\text{Articles})$
- $\sigma_{\text{author} = \text{'bala}}} (\text{Books} \times \text{Articles})$

Group by and Order by

- Grouping γ
- Sorting τ

4. Joining two relations

Cartesian Product

■ Relations r, s :

| A | B |
|----------|---|
| α | 1 |
| β | 2 |

r

| C | D | E |
|----------|----|---|
| α | 10 | a |
| β | 10 | a |
| β | 20 | b |
| γ | 10 | b |

s

■ $r \times s$:

| A | B | C | D | E |
|----------|---|----------|----|---|
| α | 1 | α | 10 | a |
| α | 1 | β | 10 | a |
| α | 1 | β | 20 | b |
| α | 1 | γ | 10 | b |
| β | 2 | α | 10 | a |
| β | 2 | β | 10 | a |
| β | 2 | β | 20 | b |
| β | 2 | γ | 10 | b |

Cartesian Product - Example

r

| A | B |
|----|---|
| a1 | 2 |
| a2 | 4 |

s

| C | D | E |
|---|----|----|
| 4 | d1 | e1 |
| 3 | d2 | e1 |
| 5 | d3 | e2 |

r x s

| A | B | C | D | E |
|----|---|---|----|----|
| a1 | 2 | 4 | d1 | e1 |
| a1 | 2 | 3 | d2 | e1 |
| a1 | 2 | 5 | d3 | e2 |
| a2 | 4 | 4 | d1 | e1 |
| a2 | 4 | 3 | d2 | e1 |
| a2 | 4 | 5 | d3 | e2 |

Cartesian Product: $R \times S$

R

| A | B | C |
|----|----|----|
| a1 | b1 | c3 |
| a2 | b1 | c5 |
| a3 | b4 | c7 |

S

| E | F |
|----|----|
| e1 | f1 |
| e2 | f5 |

$R \times S$

| A | B | C | E | F |
|----|----|----|----|----|
| a1 | b1 | c3 | e1 | f1 |
| a1 | b1 | c3 | e2 | f5 |
| a2 | b1 | c5 | e1 | f1 |
| a2 | b1 | c5 | e2 | f5 |
| a3 | b4 | c7 | e1 | f1 |
| a3 | b4 | c7 | e2 | f5 |

Inner Join

- Relations r, s:

| A | B | C | D |
|----------|---|----------|---|
| α | 1 | α | a |
| β | 2 | γ | a |
| γ | 4 | β | b |
| α | 1 | γ | a |
| δ | 2 | β | b |

r

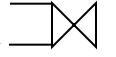
| B | D | E |
|---|---|---------------|
| 1 | a | α |
| 3 | a | β |
| 1 | a | γ |
| 2 | b | δ |
| 3 | b | ε |

s

- Natural Join
 - $r \bowtie s$

| A | B | C | D | E |
|----------|---|----------|---|----------|
| α | 1 | α | a | α |
| α | 1 | α | a | γ |
| α | 1 | γ | a | α |
| α | 1 | γ | a | γ |
| δ | 2 | β | b | δ |

Outer Join

- Left Outer Join ()
- Right Outer Join ()
- Full Outer Join ()

Join – Example

■ Relation *loan*

| <i>loan_number</i> | <i>branch_name</i> | <i>amount</i> |
|--------------------|--------------------|---------------|
| L-170 | Downtown | 3000 |
| L-230 | Redwood | 4000 |
| L-260 | Perryridge | 1700 |

■ Relation *borrower*

| <i>customer_name</i> | <i>loan_number</i> |
|----------------------|--------------------|
| Jones | L-170 |
| Smith | L-230 |
| Hayes | L-155 |

Outer Join – Example

- Inner Join

loan \bowtie *Borrower*

| <i>loan_number</i> | <i>branch_name</i> | <i>amount</i> | <i>customer_name</i> |
|--------------------|--------------------|---------------|----------------------|
| L-170 | Downtown | 3000 | Jones |
| L-230 | Redwood | 4000 | Smith |

- Left Outer Join

loan  *Borrower*

| <i>loan_number</i> | <i>branch_name</i> | <i>amount</i> | <i>customer_name</i> |
|--------------------|--------------------|---------------|----------------------|
| L-170 | Downtown | 3000 | Jones |
| L-230 | Redwood | 4000 | Smith |
| L-260 | Perryridge | 1700 | null |

Outer Join – Example

■ Right Outer Join

loan \bowtie *borrower*

| <i>loan_number</i> | <i>branch_name</i> | <i>amount</i> | <i>customer_name</i> |
|--------------------|--------------------|---------------|----------------------|
| L-170 | Downtown | 3000 | Jones |
| L-230 | Redwood | 4000 | Smith |
| L-155 | <i>null</i> | <i>null</i> | Hayes |

■ Full Outer Join

loan $\bowtie\bowtie$ *borrower*

| <i>loan_number</i> | <i>branch_name</i> | <i>amount</i> | <i>customer_name</i> |
|--------------------|--------------------|---------------|----------------------|
| L-170 | Downtown | 3000 | Jones |
| L-230 | Redwood | 4000 | Smith |
| L-260 | Perryridge | 1700 | <i>null</i> |
| L-155 | <i>null</i> | <i>null</i> | Hayes |

5. Rename operation - ρ

The operation $\rho_x(r)$

- means relation r renamed to x.

6. Assignment

- The assignment operation (\leftarrow) provides a convenient way to express complex queries.
 - Write query as a sequential program consisting of
 - a series of assignments
 - followed by an expression whose value is displayed as a result of the query.
 - Assignment must always be made to a temporary relation variable.
- Example: Write $r \div s$ as

```
temp1  $\leftarrow \Pi_{R-S}(r)$ 
temp2  $\leftarrow \Pi_{R-S}((temp1 \times s) - \Pi_{R-S,S}(r))$ 
result = temp1 - temp2
```

- The result to the right of the \leftarrow is assigned to the relation variable on the left of the \leftarrow .
- May use variable in subsequent expressions

7. Aggregate

- Functions on more than one tuple
- Samples:

- Sum
- Count-distinct
- Max
- Min
- Count
- Avg

- Use “as” to rename

$\prod_{branchname} \ g \ sum(balance) \ as \ totalbalance \ (account)$

Aggregate Operation – Example

■ Relation *account* grouped by *branch-name*:

| <i>branch_name</i> | <i>account_number</i> | <i>balance</i> |
|--------------------|-----------------------|----------------|
| Perryridge | A-102 | 400 |
| Perryridge | A-201 | 900 |
| Brighton | A-217 | 750 |
| Brighton | A-215 | 750 |
| Redwood | A-222 | 700 |

$\Pi \text{ branch_name } g \text{ sum(balance)} (\text{account})$

| <i>branch_name</i> | sum (<i>balance</i>) |
|--------------------|-------------------------------|
| Perryridge | 1300 |
| Brighton | 1500 |
| Redwood | 700 |

Summary

| Symbol (Name) | Example of Use |
|---------------------------------|---|
| σ (Selection) | $\sigma_{\text{salary} \geq 85000}(\text{instructor})$ Return rows of the input relation that satisfy the predicate. |
| Π (Projection) | $\Pi_{ID, \text{salary}}(\text{instructor})$ Output specified attributes from all rows of the input relation. Remove duplicate tuples from the output. |
| \bowtie (Natural Join) | $\text{instructor} \bowtie \text{department}$ Output pairs of rows from the two input relations that have the same value on all attributes that have the same name. |
| \times (Cartesian Product) | $\text{instructor} \times \text{department}$ Output all pairs of rows from the two input relations (regardless of whether or not they have the same values on common attributes) |
| \cup (Union) | $\Pi_{name}(\text{instructor}) \cup \Pi_{name}(\text{student})$ Output the union of tuples from the two input relations. |

EXAMPLES OF ALGEBRA QUERIES

In the rest of this section we shall illustrate queries using the following relations

Sailors, Reserves and Boats.

| <i>sid</i> | <i>sname</i> | <i>rating</i> | <i>age</i> |
|------------|--------------|---------------|------------|
| 22 | Dustin | 7 | 45.0 |
| 29 | Brutus | 1 | 33.0 |
| 31 | Lubber | 8 | 55.5 |
| 32 | Andy | 8 | 25.5 |
| 58 | Rusty | 10 | 35.0 |
| 64 | Horatio | 7 | 35.0 |
| 71 | Zorba | 10 | 16.0 |
| 74 | Horatio | 9 | 35.0 |
| 85 | Art | 3 | 25.5 |
| 95 | Bob | 3 | 63.5 |

Figure 4.15 An Instance S_3 of Sailors

| <i>sid</i> | <i>bid</i> | <i>day</i> |
|------------|------------|------------|
| 22 | 101 | 10/10/98 |
| 22 | 102 | 10/10/98 |
| 22 | 103 | 10/8/98 |
| 22 | 104 | 10/7/98 |
| 31 | 102 | 11/10/98 |
| 31 | 103 | 11/6/98 |
| 31 | 104 | 11/12/98 |
| 64 | 101 | 9/5/98 |
| 64 | 102 | 9/8/98 |
| 74 | 103 | 9/8/98 |

Figure 4.16 An Instance R_2 of Reserves

| <i>bid</i> | <i>bname</i> | <i>color</i> |
|------------|--------------|--------------|
| 101 | Interlake | blue |
| 102 | Interlake | red |
| 103 | Clipper | green |
| 104 | Marine | red |

Figure 4.17 An Instance B_1 of Boats

QUERY Q1

(Q1) Find the names of sailors who have reserved boat 103

Two relations involved here are

| <i>sid</i> | <i>sname</i> | <i>rating</i> | <i>age</i> |
|------------|--------------|---------------|------------|
| 22 | Dustin | 7 | 45.0 |
| 29 | Brutus | 1 | 33.0 |
| 31 | Lubber | 8 | 55.5 |
| 32 | Andy | 8 | 25.5 |
| 58 | Rusty | 10 | 35.0 |
| 64 | Horatio | 7 | 35.0 |
| 71 | Zorba | 10 | 16.0 |
| 74 | Horatio | 9 | 35.0 |
| 85 | Art | 3 | 25.5 |
| 95 | Bob | 3 | 63.5 |

Figure 4.15 An Instance *S3* of Sailors

| <i>sid</i> | <i>bid</i> | <i>day</i> |
|------------|------------|------------|
| 22 | 101 | 10/10/98 |
| 22 | 102 | 10/10/98 |
| 22 | 103 | 10/8/98 |
| 22 | 104 | 10/7/98 |
| 31 | 102 | 11/10/98 |
| 31 | 103 | 11/6/98 |
| 31 | 104 | 11/12/98 |
| 64 | 101 | 9/5/98 |
| 64 | 102 | 9/8/98 |
| 74 | 103 | 9/8/98 |

Figure 4.16 An Instance *R2* of Reserves

$$\pi_{sname}((\sigma_{bid=103} \text{ Reserves}) \bowtie \text{Sailors})$$

The answer is thus the following relational instance

{<Dustin>, <Lubber>, <Horatio>}

QUERY Q1 (cont'd)

There are of course several ways to express Q1 in relational algebra.

Here is another:

$$\pi_{sname}(\sigma_{bid=103}(\text{Reserves} \bowtie \text{Sailors}))$$

Which of these expressions should we use?

That is a question of optimization. Indeed, when we describe how to state queries in SQL, we can leave it to the optimizer in the DBMS to select the best approach.

QUERY Q2

(Q2) Find the names of sailors who have reserved a red boat.

| <i>sid</i> | <i>sname</i> | <i>rating</i> | <i>age</i> |
|------------|--------------|---------------|------------|
| 22 | Dustin | 7 | 45.0 |
| 29 | Brutus | 1 | 33.0 |
| 31 | Lubber | 8 | 55.5 |
| 32 | Andy | 8 | 25.5 |
| 58 | Rusty | 10 | 35.0 |
| 64 | Horatio | 7 | 35.0 |
| 71 | Zorba | 10 | 16.0 |
| 74 | Horatio | 9 | 35.0 |
| 85 | Art | 3 | 25.5 |
| 95 | Bob | 3 | 63.5 |

Figure 4.15 An Instance *S3* of Sailors

| <i>sid</i> | <i>bid</i> | <i>day</i> |
|------------|------------|------------|
| 22 | 101 | 10/10/98 |
| 22 | 102 | 10/10/98 |
| 22 | 103 | 10/8/98 |
| 22 | 104 | 10/7/98 |
| 31 | 102 | 11/10/98 |
| 31 | 103 | 11/6/98 |
| 31 | 104 | 11/12/98 |
| 64 | 101 | 9/5/98 |
| 64 | 102 | 9/8/98 |
| 74 | 103 | 9/8/98 |

Figure 4.16 An Instance *R2* of Reserves

| <i>bid</i> | <i>bname</i> | <i>color</i> |
|------------|--------------|--------------|
| 101 | Interlake | blue |
| 102 | Interlake | red |
| 103 | Clipper | green |
| 104 | Marine | red |

Figure 4.17 An Instance *B1* of Boats

$$\pi_{sname}((\sigma_{color='red'} \text{Boats}) \bowtie \text{Reserves} \bowtie \text{Sailors})$$

QUERY Q3

(Q3) Find the colors of boats reserved by Lubber.

| <i>sid</i> | <i>sname</i> | <i>rating</i> | <i>age</i> |
|------------|--------------|---------------|------------|
| 22 | Dustin | 7 | 45.0 |
| 29 | Brutus | 1 | 33.0 |
| 31 | Lubber | 8 | 55.5 |
| 32 | Andy | 8 | 25.5 |
| 58 | Rusty | 10 | 35.0 |
| 64 | Horatio | 7 | 35.0 |
| 71 | Zorba | 10 | 16.0 |
| 74 | Horatio | 9 | 35.0 |
| 85 | Art | 3 | 25.5 |
| 95 | Bob | 3 | 63.5 |

Figure 4.15 An Instance *S3* of Sailors

| <i>sid</i> | <i>bid</i> | <i>day</i> |
|------------|------------|------------|
| 22 | 101 | 10/10/98 |
| 22 | 102 | 10/10/98 |
| 22 | 103 | 10/8/98 |
| 22 | 104 | 10/7/98 |
| 31 | 102 | 11/10/98 |
| 31 | 103 | 11/6/98 |
| 31 | 104 | 11/12/98 |
| 64 | 101 | 9/5/98 |
| 64 | 102 | 9/8/98 |
| 74 | 103 | 9/8/98 |

Figure 4.16 An Instance *R2* of Reserves

| <i>bid</i> | <i>bname</i> | <i>color</i> |
|------------|--------------|--------------|
| 101 | Interlake | blue |
| 102 | Interlake | red |
| 103 | Clipper | green |
| 104 | Marine | red |

Figure 4.17 An Instance *B1* of Boats

QUERY Q3

(Q3) Find the colors of boats reserved by Lubber.

| <i>sid</i> | <i>sname</i> | <i>rating</i> | <i>age</i> | <i>bid</i> | <i>day</i> |
|------------|--------------|---------------|------------|------------|------------|
| 22 | Dustin | 7 | 45.0 | 22 | 101 |
| 29 | Brutus | 1 | 33.0 | 22 | 102 |
| 31 | Lubber | 8 | 55.5 | 22 | 103 |
| 32 | Andy | 8 | 25.5 | 22 | 104 |
| 58 | Rusty | 10 | 35.0 | 31 | 102 |
| 64 | Horatio | 7 | 35.0 | 31 | 103 |
| 71 | Zorba | 10 | 16.0 | 31 | 104 |
| 74 | Horatio | 9 | 35.0 | 64 | 101 |
| 85 | Art | 3 | 25.5 | 64 | 102 |
| 95 | Bob | 3 | 63.5 | 74 | 103 |

Figure 4.15 An Instance *S3* of Sailors

Figure 4.16 An Instance *R2* of Reserves

| <i>bid</i> | <i>bname</i> | <i>color</i> |
|------------|--------------|--------------|
| 101 | Interlake | blue |
| 102 | Interlake | red |
| 103 | Clipper | green |
| 104 | Marine | red |

Figure 4.17 An Instance *B1* of Boats

$\pi_{color}((\sigma_{sname='Lubber'} \text{Sailors}) \bowtie \text{Reserves} \bowtie \text{Boats})$

QUERY Q4

(Q4) Find the names of Sailors who have reserved at least one boat

| <i>sid</i> | <i>sname</i> | <i>rating</i> | <i>age</i> |
|------------|--------------|---------------|------------|
| 22 | Dustin | 7 | 45.0 |
| 29 | Brutus | 1 | 33.0 |
| 31 | Lubber | 8 | 55.5 |
| 32 | Andy | 8 | 25.5 |
| 58 | Rusty | 10 | 35.0 |
| 64 | Horatio | 7 | 35.0 |
| 71 | Zorba | 10 | 16.0 |
| 74 | Horatio | 9 | 35.0 |
| 85 | Art | 3 | 25.5 |
| 95 | Bob | 3 | 63.5 |

Figure 4.15 An Instance *S3* of Sailors

| <i>sid</i> | <i>bid</i> | <i>day</i> |
|------------|------------|------------|
| 22 | 101 | 10/10/98 |
| 22 | 102 | 10/10/98 |
| 22 | 103 | 10/8/98 |
| 22 | 104 | 10/7/98 |
| 31 | 102 | 11/10/98 |
| 31 | 103 | 11/6/98 |
| 31 | 104 | 11/12/98 |
| 64 | 101 | 9/5/98 |
| 64 | 102 | 9/8/98 |
| 74 | 103 | 9/8/98 |

Figure 4.16 An Instance *R2* of Reserves

| <i>bid</i> | <i>bname</i> | <i>color</i> |
|------------|--------------|--------------|
| 101 | Interlake | blue |
| 102 | Interlake | red |
| 103 | Clipper | green |
| 104 | Marine | red |

Figure 4.17 An Instance *B1* of Boats

QUERY Q4

(Q4) Find the names of Sailors who have reserved at least one boat

| <i>sid</i> | <i>sname</i> | <i>rating</i> | <i>age</i> |
|------------|--------------|---------------|------------|
| 22 | Dustin | 7 | 45.0 |
| 29 | Brutus | 1 | 33.0 |
| 31 | Lubber | 8 | 55.5 |
| 32 | Andy | 8 | 25.5 |
| 58 | Rusty | 10 | 35.0 |
| 64 | Horatio | 7 | 35.0 |
| 71 | Zorba | 10 | 16.0 |
| 74 | Horatio | 9 | 35.0 |
| 85 | Art | 3 | 25.5 |
| 95 | Bob | 3 | 63.5 |

Figure 4.15 An Instance *S3* of Sailors

| <i>sid</i> | <i>bid</i> | <i>day</i> |
|------------|------------|------------|
| 22 | 101 | 10/10/98 |
| 22 | 102 | 10/10/98 |
| 22 | 103 | 10/8/98 |
| 22 | 104 | 10/7/98 |
| 31 | 102 | 11/10/98 |
| 31 | 103 | 11/6/98 |
| 31 | 104 | 11/12/98 |
| 64 | 101 | 9/5/98 |
| 64 | 102 | 9/8/98 |
| 74 | 103 | 9/8/98 |

Figure 4.16 An Instance *R2* of Reserves

| <i>bid</i> | <i>bname</i> | <i>color</i> |
|------------|--------------|--------------|
| 101 | Interlake | blue |
| 102 | Interlake | red |
| 103 | Clipper | green |
| 104 | Marine | red |

Figure 4.17 An Instance *B1* of Boats

$$\pi_{\text{sname}}(\text{Sailors} \bowtie \text{Reserves})$$

QUERY Q5

(Q5) Find the names of sailors who have reserved a red or a green boat.

| <i>sid</i> | <i>sname</i> | <i>rating</i> | <i>age</i> |
|------------|--------------|---------------|------------|
| 22 | Dustin | 7 | 45.0 |
| 29 | Brutus | 1 | 33.0 |
| 31 | Lubber | 8 | 55.5 |
| 32 | Andy | 8 | 25.5 |
| 58 | Rusty | 10 | 35.0 |
| 64 | Horatio | 7 | 35.0 |
| 71 | Zorba | 10 | 16.0 |
| 74 | Horatio | 9 | 35.0 |
| 85 | Art | 3 | 25.5 |
| 95 | Bob | 3 | 63.5 |

Figure 4.15 An Instance *S3* of Sailors

| <i>sid</i> | <i>bid</i> | <i>day</i> |
|------------|------------|------------|
| 22 | 101 | 10/10/98 |
| 22 | 102 | 10/10/98 |
| 22 | 103 | 10/8/98 |
| 22 | 104 | 10/7/98 |
| 31 | 102 | 11/10/98 |
| 31 | 103 | 11/6/98 |
| 31 | 104 | 11/12/98 |
| 64 | 101 | 9/5/98 |
| 64 | 102 | 9/8/98 |
| 74 | 103 | 9/8/98 |

Figure 4.16 An Instance *R2* of Reserves

| <i>bid</i> | <i>bname</i> | <i>color</i> |
|------------|--------------|--------------|
| 101 | Interlake | blue |
| 102 | Interlake | red |
| 103 | Clipper | green |
| 104 | Marine | red |

Figure 4.17 An Instance *B1* of Boats

QUERY Q5

(Q5) Find the names of sailors who have reserved a red or a green boat.

| <i>sid</i> | <i>sname</i> | <i>rating</i> | <i>age</i> |
|------------|--------------|---------------|------------|
| 22 | Dustin | 7 | 45.0 |
| 29 | Brutus | 1 | 33.0 |
| 31 | Lubber | 8 | 55.5 |
| 32 | Andy | 8 | 25.5 |
| 58 | Rusty | 10 | 35.0 |
| 64 | Horatio | 7 | 35.0 |
| 71 | Zorba | 10 | 16.0 |
| 74 | Horatio | 9 | 35.0 |
| 85 | Art | 3 | 25.5 |
| 95 | Bob | 3 | 63.5 |

Figure 4.15 An Instance *S3* of Sailors

| <i>sid</i> | <i>bid</i> | <i>day</i> |
|------------|------------|------------|
| 22 | 101 | 10/10/98 |
| 22 | 102 | 10/10/98 |
| 22 | 103 | 10/8/98 |
| 22 | 104 | 10/7/98 |
| 31 | 102 | 11/10/98 |
| 31 | 103 | 11/6/98 |
| 31 | 104 | 11/12/98 |
| 64 | 101 | 9/5/98 |
| 64 | 102 | 9/8/98 |
| 74 | 103 | 9/8/98 |

Figure 4.16 An Instance *R2* of Reserves

| <i>bid</i> | <i>bname</i> | <i>color</i> |
|------------|--------------|--------------|
| 101 | Interlake | blue |
| 102 | Interlake | red |
| 103 | Clipper | green |
| 104 | Marine | red |

Figure 4.17 An Instance *B1* of Boats

$$\rho(\mathbf{Tempboats}, (\sigma_{\text{color}=\text{'red'}} \mathbf{Boats}) \cup (\sigma_{\text{color}=\text{'green'}} \mathbf{Boats}))$$

$$\pi_{\text{sname}}(\mathbf{Tempboats} \bowtie \mathbf{Reserves} \bowtie \mathbf{Sailors})$$

QUERY Q6

(Q6) *Find the names of Sailors who have reserved a red and a green boat.*

It seems tempting to use the expression used in Q5, replacing simply \cup by \cap . However, this won't work, for such an expression is requesting the names of sailors who have requested a boat that is both red and green! The correct expression is as follows:

$$\rho(\text{Tempred}, \pi_{sid}((\sigma_{color='red'}.\text{Boats}) \bowtie \text{Reserves}))$$
$$\rho(\text{Tempgreen}, \pi_{sid}((\sigma_{color='green'}.\text{Boats}) \bowtie \text{Reserves}))$$
$$\pi_{sname} ((\text{Tempred} \cap \text{Tempgreen}) \bowtie \text{Sailors})$$

QUERY 7

(Q7) Find the sids of sailors with age over 20 who have not reserved a red boat.

| <i>sid</i> | <i>sname</i> | <i>rating</i> | <i>age</i> |
|------------|--------------|---------------|------------|
| 22 | Dustin | 7 | 45.0 |
| 29 | Brutus | 1 | 33.0 |
| 31 | Lubber | 8 | 55.5 |
| 32 | Andy | 8 | 25.5 |
| 58 | Rusty | 10 | 35.0 |
| 64 | Horatio | 7 | 35.0 |
| 71 | Zorba | 10 | 16.0 |
| 74 | Horatio | 9 | 35.0 |
| 85 | Art | 3 | 25.5 |
| 95 | Bob | 3 | 63.5 |

Figure 4.15 An Instance *S3* of Sailors

| <i>sid</i> | <i>bid</i> | <i>day</i> |
|------------|------------|------------|
| 22 | 101 | 10/10/98 |
| 22 | 102 | 10/10/98 |
| 22 | 103 | 10/8/98 |
| 22 | 104 | 10/7/98 |
| 31 | 102 | 11/10/98 |
| 31 | 103 | 11/6/98 |
| 31 | 104 | 11/12/98 |
| 64 | 101 | 9/5/98 |
| 64 | 102 | 9/8/98 |
| 74 | 103 | 9/8/98 |

Figure 4.16 An Instance *R2* of Reserves

| <i>bid</i> | <i>bname</i> | <i>color</i> |
|------------|--------------|--------------|
| 101 | Interlake | blue |
| 102 | Interlake | red |
| 103 | Clipper | green |
| 104 | Marine | red |

Figure 4.17 An Instance *B1* of Boats

$$\pi_{\text{sid}}(\sigma_{\text{age} > 20} \text{Sailors}) - \pi_{\text{sid}}((\sigma_{\text{color} = \text{'red'}} \text{Boats}) \bowtie \text{Reserves} \bowtie \text{Sailors})$$

QUERY 8

(Q) Find the names of sailors who have reserved all boats.

| <i>sid</i> | <i>sname</i> | <i>rating</i> | <i>age</i> |
|------------|--------------|---------------|------------|
| 22 | Dustin | 7 | 45.0 |
| 29 | Brutus | 1 | 33.0 |
| 31 | Lubber | 8 | 55.5 |
| 32 | Andy | 8 | 25.5 |
| 58 | Rusty | 10 | 35.0 |
| 64 | Horatio | 7 | 35.0 |
| 71 | Zorba | 10 | 16.0 |
| 74 | Horatio | 9 | 35.0 |
| 85 | Art | 3 | 25.5 |
| 95 | Bob | 3 | 63.5 |

Figure 4.15 An Instance *S3* of Sailors

| <i>sid</i> | <i>bid</i> | <i>day</i> |
|------------|------------|------------|
| 22 | 101 | 10/10/98 |
| 22 | 102 | 10/10/98 |
| 22 | 103 | 10/8/98 |
| 22 | 104 | 10/7/98 |
| 31 | 102 | 11/10/98 |
| 31 | 103 | 11/6/98 |
| 31 | 104 | 11/12/98 |
| 64 | 101 | 9/5/98 |
| 64 | 102 | 9/8/98 |
| 74 | 103 | 9/8/98 |

Figure 4.16 An Instance *R2* of Reserves

| <i>bid</i> | <i>bname</i> | <i>color</i> |
|------------|--------------|--------------|
| 101 | Interlake | blue |
| 102 | Interlake | red |
| 103 | Clipper | green |
| 104 | Marine | red |

Figure 4.17 An Instance *B1* of Boats

$\rho(\text{Tempsids}, (\pi_{\text{sid}, \text{bid}} \text{Reserves}) / (\pi_{\text{bid}} \text{Boats}))$

$\pi_{\text{sname}}(\text{Tempsids} \bowtie \text{Sailors}$

QUERY Q9

(Q9) Find the names of sailors who have reserved all boats called Interlake.

| <i>sid</i> | <i>sname</i> | <i>rating</i> | <i>age</i> |
|------------|--------------|---------------|------------|
| 22 | Dustin | 7 | 45.0 |
| 29 | Brutus | 1 | 33.0 |
| 31 | Lubber | 8 | 55.5 |
| 32 | Andy | 8 | 25.5 |
| 58 | Rusty | 10 | 35.0 |
| 64 | Horatio | 7 | 35.0 |
| 71 | Zorba | 10 | 16.0 |
| 74 | Horatio | 9 | 35.0 |
| 85 | Art | 3 | 25.5 |
| 95 | Bob | 3 | 63.5 |

Figure 4.15 An Instance *S3* of Sailors

| <i>sid</i> | <i>bid</i> | <i>day</i> |
|------------|------------|------------|
| 22 | 101 | 10/10/98 |
| 22 | 102 | 10/10/98 |
| 22 | 103 | 10/8/98 |
| 22 | 104 | 10/7/98 |
| 31 | 102 | 11/10/98 |
| 31 | 103 | 11/6/98 |
| 31 | 104 | 11/12/98 |
| 64 | 101 | 9/5/98 |
| 64 | 102 | 9/8/98 |
| 74 | 103 | 9/8/98 |

Figure 4.16 An Instance *R2* of Reserves

| <i>bid</i> | <i>bname</i> | <i>color</i> |
|------------|--------------|--------------|
| 101 | Interlake | blue |
| 102 | Interlake | red |
| 103 | Clipper | green |
| 104 | Marine | red |

Figure 4.17 An Instance *B1* of Boats

$$\begin{aligned}
 & \rho(\mathbf{Temp} \mathbf{sids}, (\pi_{\mathbf{sid}, \mathbf{bid}} \mathbf{Reserves}) / (\pi_{\mathbf{bid}} (\sigma_{\mathbf{bname} = 'Interlake'} \mathbf{Boats}))) \\
 & \pi_{\mathbf{sname}} (\mathbf{Temp} \mathbf{sids} \bowtie \mathbf{Sailors})
 \end{aligned}$$

Examples

- *Person (name, age, gender)*
Frequents (name, pizzeria)
Eats (name, pizza)
Serves (pizzeria, pizza, price)

Examples

- a. Find all pizzerias frequented by at least one person under the age of 18.
- b. Find the names of all females who eat either mushroom or pepperoni pizza (or both).
- c. Find the names of all females who eat both mushroom and pepperoni pizza.

- a. $\pi_{pizzeria}(\sigma_{age < 18}(Person) \bowtie Frequent)$
- b. $\pi_{name}\left(\sigma_{gender = 'female'} \wedge (pizza = 'mushroom' \vee pizza = 'pepperoni') (Person \bowtie Eats)\right)$
- c. $\pi_{name}(\sigma_{gender = 'female'} \wedge pizza = 'mushroom' (Person \bowtie Eats)) \cap$
 $\pi_{name}(\sigma_{gender = 'female'} \wedge pizza = 'pepperoni' (Person \bowtie Eats))$

- d. Find all pizzerias that serve at least one pizza that Amy eats for less than \$10.00.
- e. Find all pizzerias that are frequented by only females or only males.
- f. For each person, find all pizzas the person eats that are not served by any pizzeria the person frequents. Return all such person (name) / pizza pairs.

d. $\pi_{\text{pizzeria}}(\sigma_{\text{name}=\text{'Amy'}}(Eats) \bowtie \sigma_{\text{price} < 10}(Serves))$

e.
$$\left(\begin{array}{l} \pi_{\text{pizzeria}}(\sigma_{\text{gender}=\text{'female'}}(\text{Person}) \bowtie \text{Frequents}) - \\ \pi_{\text{pizzeria}}(\sigma_{\text{gender}=\text{'male'}}(\text{Person}) \bowtie \text{Frequents}) \end{array} \right) \cup$$
$$\left(\begin{array}{l} \pi_{\text{pizzeria}}(\sigma_{\text{gender}=\text{'male'}}(\text{Person}) \bowtie \text{Frequents}) - \\ \pi_{\text{pizzeria}}(\sigma_{\text{gender}=\text{'female'}}(\text{Person}) \bowtie \text{Frequents}) \end{array} \right)$$

f. $Eats - \pi_{\text{name}, \text{pizza}}(\text{Frequents} \bowtie \text{Serves})$

- g. Find the names of all people who frequent only pizzerias serving at least one pizza they eat.
- h. Find the names of all people who frequent every pizzeria serving at least one pizza they eat.
- i. Find the pizzeria serving the cheapest pepperoni pizza. In the case of ties, return all of the cheapest-pepperoni pizzerias.

- g. $\pi_{name}(Person) - \pi_{name}(Frequent - \pi_{name,pizzeria}(Eats \bowtie Serves))$
 h. $\pi_{name}(Person) - \pi_{name}(\pi_{name,pizzeria}(Eats \bowtie Serves) - Frequent)$
 i. $\pi_{pizza}(\sigma_{pizza='pepperoni'} Serves) -$
 $\pi_{pizza} \left(\sigma_{price > price2} \left(\begin{array}{c} \pi_{pizza,price}(\sigma_{pizza='pepperoni'} Serves) \\ \times \\ \rho_{pizza,price2}(\pi_{pizza,price}(\sigma_{pizza='pepperoni'} Serves)) \end{array} \right) \right)$

Thank You!