

$$6) \sin \pi t \quad (2 < t < 4)$$

$$f(t) = \sin \pi t [u(t-2) - u(t-4)]$$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{\sin \pi t u(t-2)\} - \mathcal{L}\{\sin \pi t u(t-4)\}$$

$$\mathcal{L}\{\sin \pi t u(t-2)\} = \mathcal{L}\{\sin \pi (t+2-2) u(t-2)\}$$

$$= \mathcal{L}\{\sin(\pi(t+2)+2\pi) u(t-2)\}$$

$$= \mathcal{L}\{[\sin(\pi(t+2)) \cos(2\pi) + \cos(\pi(t+2)) \sin(2\pi)] u(t-2)\}$$

$$= \mathcal{L}\{\sin \pi(t-2) \cdot u(t-2)\}$$

$$= e^{-2s} \left(\frac{\pi}{s^2 + \pi^2} \right)$$

$$\text{Similarly for } \mathcal{L}\{\sin \pi t u(t-4)\} = \mathcal{L}\{\sin \pi (t-4) u(t-4)\}$$

$$= e^{-4s} \left(\frac{\pi}{s^2 + \pi^2} \right)$$

$$\mathcal{L}\{f(t)\} = \frac{\pi}{s^2 + \pi^2} \{e^{-2s} - e^{-4s}\}$$

$$10) \sinh t (0 \leq t < 2)$$

$$f(t) = \sinh t \{ u(t) - u(t-2) \}$$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{\sinh t u(t)\} - \mathcal{L}\{\sinh t u(t-2)\}$$

$$= \frac{1}{s^2-1} - \mathcal{L}\{\sinh(t-2+2) u(t-2)\}$$

$$= \frac{1}{s^2-1} - \mathcal{L}\{\sinh(t-2) \cosh(2) u(t-2) + \cosh(t-2) \sinh(2) u(t-2)\}$$

$$= \frac{1}{s^2-1} - e^{-2s} \left(\frac{\cosh(2)}{s^2-1} \right) + e^{-2s} \frac{\sinh(2)}{s^2-1}$$

$$12) \frac{e^{-3s}}{(s-1)^3}$$

$$F(s) = \frac{1}{(s-1)^3}$$

$$f(t) = e^t \mathcal{L}^{-1}\left(\frac{1}{s^3}\right) = e^t \left(\frac{1}{2} t^2\right)$$

$$f(t) = \frac{e^t t^2}{2}$$

$$\mathcal{L}\{f(t-a) u(t-a)\} = e^{-as} F(s)$$

$$f(t) = \frac{e^t t^2}{2}$$

Then,

$$f(t) = f(t-3) u(t-3)$$

$$= \frac{e^{t-3} (t-3)^2}{2} u(t-3)$$

$$(13) \frac{6(1 - e^{-\pi s})}{s^2 + 9}$$

$$= \frac{6}{s^2 + 9} - \frac{6e^{-\pi s}}{s^2 + 9}$$

$$= (2) \frac{3}{s^2 + 9} - 2e^{-\pi s} (3)$$

$$f(t) = 2 \sin(3t)$$

$$f(t) = 2 \sin(3t)$$

$$\mathcal{L}\{f(t-a)u(t-a)\} = \mathcal{L}\{2 \sin 3t u(t)\} - \mathcal{L}\{2 \sin(t-\pi) u(t-\pi)\}$$

$$f(t) = \mathcal{L}\{2 \sin 3t u(t) - 2 \sin(t-\pi) u(t-\pi)\}$$

$$(15) \frac{e^{-3s}}{s^4}$$

$$F(s) = \frac{1}{s^4}$$

$$f(t) = \mathcal{L}^{-1}\left(\frac{1}{s^4}\right) = \frac{1}{6} \mathcal{L}^{-1}\left(\frac{6}{s^4}\right) = \frac{t^3}{6}$$

$$f(t) = t^3/6$$

$$\mathcal{L}\{f(t-a)u(t-a)\} = e^{-as} F(s)$$

$$a=3$$

Then,

$$f(t) = f(t-3) u(t-3)$$

$$f(t) = \frac{(t-3)^3}{6} u(t-3)$$