WS-1 (DAA)

20PW31
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1.

a) $5n^6 + n^3 + n^2\log n - n$

Its correct. Given complexity is Omega.
So take highest power of n to be complexity
So $O(n^6)$.

$$f(n) \le c.g(n), \forall n > n_0$$

$$(5n^6 + n^3 + n^2\log n - n) \le 6n^6$$

For $n = 1$

$$5 + 1 + \log(1) - 1 \le 6$$

$$6 \le 6$$

So $O(n^6)$ is complexity

b) $10n^3 + 5n\log n = O(n^3)$

$$f(n) \le c.g(n)$$

$$(10n^3 + 5n\log n) \le 11(n^3) \quad \forall n > n_0$$

$n = 1$

$$10 + \le 11 \rightarrow \text{ so } O(n^6).$$

c) $\log(n^3) = O(\log n)$

$f(n) \leq c\,g(n) \quad \forall n > n_0$

$\log(n^3) \leq 2 \log n$

$n = 1$

$0 \leq 0$ so $O(\log n)$

2. $T(1) = 1$

a) $T(n) = 2T(n/2) + n^3$

Master's theorem

$= A\,T(n/B) + f(n)$

$A = 2,\ B = 2,\ d = 3$

$= O(n^d)$ because $2 < 2^3$

$= O(n^3)$

b) $B = 10/9$, $A = 1$, $d = 1$

$1 < 1.11$, so $a < b^d$

so $O(n^1)$.

3. $T(n) = T(n/4) + 1$. $T(1) = 1$

~~$T(n/2) = T(n/8)$~~

$T(n) = 4 \times \frac{\log n}{\log 4}$

$T(n) = \frac{\log_2(n)}{2}$

~~Substitution~~

~~let $T(n) = \log n$~~

~~$T(n) \geq c \log n$~~

~~$T(c) \geq 0$~~

$T(m) \geq c \log n$

$T(n) = T(\frac{n}{4}) + 1$

$= \cancel{cn/4 \log n/4}$

$T(n) = O(n \log n)$

$T(n) \leq T(n/4) + 1$

$\leq cn/\log(n/4) + 1$

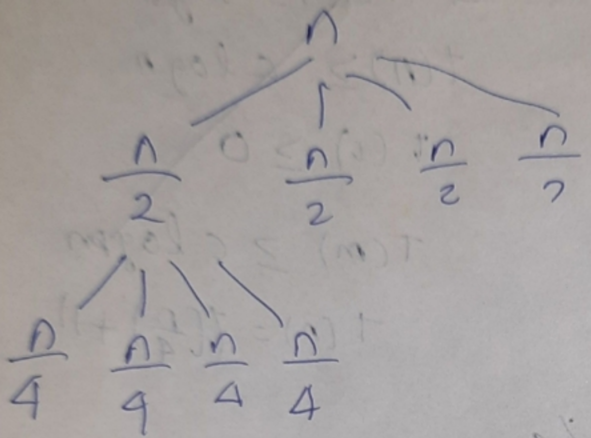
$\leq cn \log n - cn \log_2 2^2 + 1$

$\leq cn \log n - 2cn + 1$

$O(\log n)$

4) $T(n) = 4T(n/2) + cn$

i) Recursion Tree



$: \frac{4n}{2}$

$: \left(\frac{4}{2}\right)^2 n$

$$T(n) = n + \frac{4n}{2} + \left(\frac{4}{2}\right)^2 + \dots$$

$$= n\left(1 + \frac{4}{2} + \left(\frac{4}{2}\right)^2 + \dots\right)$$

$$= n\left(\frac{\left(\frac{4}{2}\right)^{i+1} - 1}{\left(\frac{4}{2}\right) - 1}\right)$$

$$\frac{n}{2^i} = 1$$

$$n = 2^i$$

$$i = \log_2 n$$

$$= n\left(\frac{\left(\frac{4}{2}\right)^{\log_2 n + 1} - 1}{1}\right)$$

$$= n\left(4^{\log_2 n + 1} - 2^{\log_2 n}(2)\right)$$

$$= n\left(4^{\log_2 n} 4 - 2^{\log_2 n}(2)\right)$$

$= n\left(4\left(n^{\log_2 4}\right) - 2n\right)$

$= 4n^{2\lg_2 2} - 2n$

$= O(n^2)$

ii) Subsum tree

$T(n) = O(n \log n)$

$= 4\,c\,n/4 \log(n/2) + cn$

$= cn(\log n) - cn \log 2 + cn$

$T(n) \leq cn \log n$

5.

a) ~~[illegible]~~ Upper bound - Lower bound + 1

$= n - 0 + 1 = n + 1$

$O(n)$

b) $\sum_{i=0}^{n} \sum_{j=0}^{n} (1) = (n+1)(n+1) = n^2 + 2n + 1$

$O(n^2)$

$= n\left(4\left(n^{\log_2 4}\right) - 2n\right)$

$= 4n^{2\log_2 2} - 2n$

$= O(n^2)$

ii) Subsum tree

$T(n) = O(n \log n)$

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$O(n^2)$

c. $\sum_{i=0}^{n} \sum_{j=0}^{i} (1) \Rightarrow \sum_{i=0}^{n} \frac{i(i+1)}{2}$

$\frac{1}{2} \sum i^2 + 1$

$\boxed{\frac{1}{2} \sum_{i=0}^{n} i^2} + \frac{1}{2} \sum_{i=0}^{n} i$

$\frac{1}{2} [1^2 + 2^2 + \cdots + n^2]$

$\frac{1}{2} \frac{n(n+1)(2n+1)}{6}$

$O(n^3)$,,

d) i = 1

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while (i<n) {
    tot += i;
    i = i * 2;
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$\sum_{i=0}^{n} \log n \Rightarrow \log n \sum_{i=0}^{n} (1)$

$= n \log n$

f. $\sum_{i=0}^{n} \sum_{j=0}^{n} \sum_{0}^{n} (1)$

$= O(n^3)$

g. $\sum_{i=0}^{n} \sum_{j=0}^{n} (1)$

$= (O(n^2))$

h. $\sum_{i=0}^{n} \sum_{j=0}^{\sqrt{n}} (1)$

$= \sum_{i=0}^{n} (\sqrt{n} + 1) \rightarrow \sum_{i=0}^{n} \sqrt{n}$

$= \sqrt{n} \sum_{i=0}^{n} 1$

$= n\sqrt{n} = n^{3/2}$

i = $\sum_{i=0}^{n-2} \sum_{j=i+1}^{n-1} (1)$

$= O(n^2)$

6. $T(n) = 2T(n/2 + 17) + n$

$= \cancel{2}\, c(n)/\cancel{2} \log(n/2 + 17) + n$

$= c(n) \log(n/2) - c(n) \cdot \log(17) +$

$T(n) \le cn \log(n/2)$

7) a) $f(n) = 3x^2 \quad g(n) = x^2 + 5$

$$\lim_{x\to\infty} \frac{f(n)}{g(n)} = \lim_{x\to\infty} \frac{3x^2}{x^2+5} = \frac{6\cancel{x}}{2\cancel{x}} = c$$

Both are good

b) $f(n) = x \quad , \quad g(n) = \ln x$

$$\lim_{x\to\infty} \frac{f(n)}{g(n)} = \lim_{x\to\infty} \frac{x}{\ln x} = \lim_{x\to\infty} \frac{1}{1/x}$$

$= \infty$

$f(n)$ is better.

8. $5n^2 + 6n \Rightarrow$

•) Worst case

$f(n) = 5n^2 - 6n$

$5n^2 - 6n \le 6n^2$

$(n = 1) \Rightarrow \quad 5 - 6 \le 6$

$-1 \le 6 \qquad n \ge n_0$

$O(n^2)$

-) Best case

$5n^2 - 6n \ge 4n^2$

$n = 1 \Rightarrow \quad 5 - 6 \ge 4$ ✗

$-1 \ge 4$

n^2

$n = 2 \Rightarrow \quad 20 - 12 \ge 16$

$8 \ge 16$ ✗

It is not $\Theta(n^2)$

It's $O(n^2)$

ITS

9) i) $T(n) = 3T(n/2) + n^2$

$A = 3, B = 2, D = 2$

$3 < 4$

$\Theta(n^2)$

ii) $T(n) = 4T(n/2) + n^2$

$A = 4, B = 2, D = 2$

$4 = 4$

$\Theta(n^d \log_b n) = O(n^2 \log_2 n)$

iii) $T(n) = T(n/2) + n^n$

$A = 1, B = 2, d = n$

$1 < 2^n$

$\cancel{\Theta(n^n)}$ It cant be done by recursion. power should be constant

1 iv) $T(n) = 2T(n/2) + n/\log n$

Can t done by recursion It should be power of n

v) $T(n) = 0.5T(n/2) + 1/n$

~~It can't be done by recursion~~

$= 0.5T(n/2) + n^{-1}$

$A = 0.5,\ B = 2,\ d = -1$

$0.5 = 2^{-1}$

$= O(n^{-1}\log_2 n)$ → Time Comple

Can be negative)

$= O\left(\frac{\log_2 n}{n}\right)$

10)

i) $T(n) = 2T(n/2) + n^2$

$$cn \quad - \quad n$$

$$\frac{cn}{2} \quad \frac{cn}{2} \quad - \quad \frac{1}{2} n$$

$$\frac{cn^2}{4} \quad \frac{cn^2}{4} \quad - \quad \left(\frac{1}{2}\right)^2 n$$

$$T(n) = n\left(1 + \left(\tfrac{1}{2}\right)^2 + \cdots\right.$$

$$= n\left(\frac{\left(\tfrac{1}{2}\right)^{\log_2 n+1} - 1}{\tfrac{1}{2} - 1}\right)$$

$$\frac{n}{2^i} = 1$$

$$n = 2^i$$

$$\boxed{i = \log_2 n}$$

$$= n\frac{(\log_2 n(1) - 2^{\log_2 n}(2))}{2(-1/2)}$$

$$= -\frac{n(\log_2 n(1) - 2(n^{\log_2 2})}{-1}$$

$$= 2n - n\log_2 n$$

$O(n \log_2 n)$ //