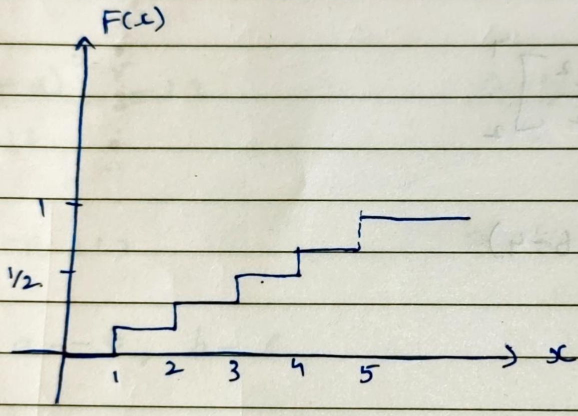
162

1. The pmf of x: $p(x) = \frac{x}{15}$, $x = 1, 2, 3, 4, 5$

The distribution of x

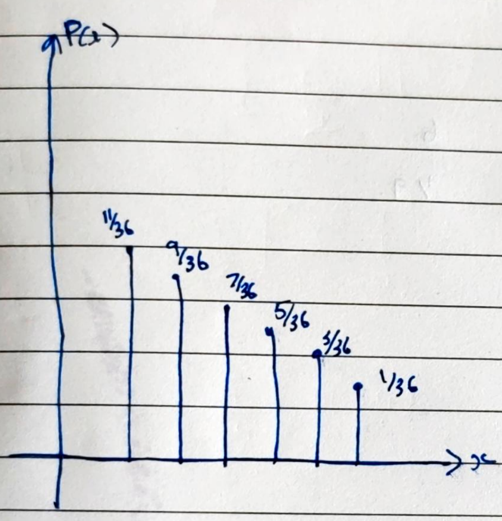
$$F(x) = \begin{cases} 0 & x < 1 \\ 1/15 & 1 \le x < 2 \\ 3/15 & 2 \le x < 3 \\ 6/15 & 3 \le x < 4 \\ 10/15 & 4 \le x < 5 \\ 1 & x \ge 5 \end{cases}$$



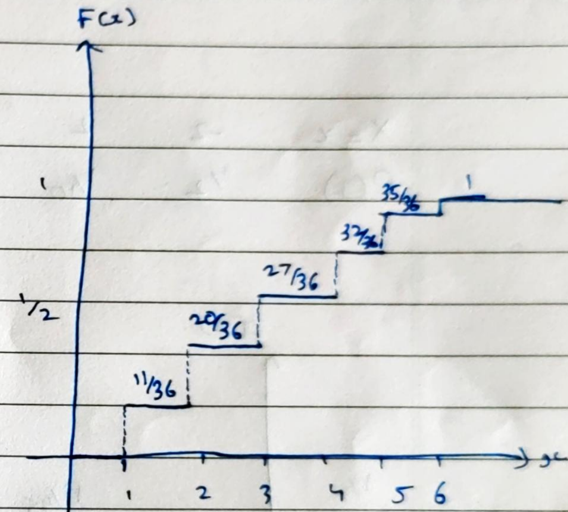
2) X: minimum of the 2 numbers obtained when rolling a die twice

$X = x$	1	2	3	4	5	6
$P(x)$	11/36	9/36	7/36	5/36	3/36	1/36.

$$F(x) = \begin{cases} 0 & x < 1 \\ 11/36 & 1 \le x < 2 \\ 20/36 & 2 \le x < 3 \\ 27/36 & 3 \le x < 4 \\ 32/36 & 4 \le x < 5 \\ 35/36 & 5 \le x < 6 \\ 1 & x \ge 6 \end{cases}$$



PMF



CDF

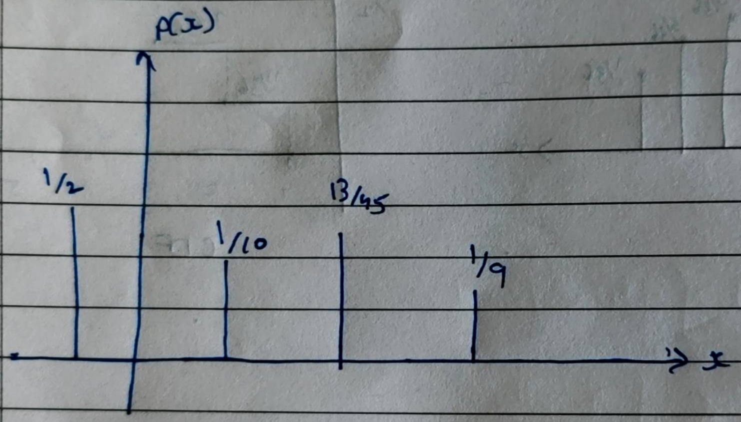
3) X: Sum of the 2 numbers obtained when rolling a dice twice

$X=x$	2	3	4	5	6	7	8	9	10	11	12
$f(x)$	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36.

4 The distribution function of a random variable

X is $F(x) = \begin{cases} 0 & x < -2 \\ 1/2 & -2 \leq x < 2 \\ 3/5 & 2 \leq x < 4 \\ 8/9 & 4 \leq x < 6 \\ 1 & x \geq 6 \end{cases}$

$X = x$	-2	2	4	6
$P(x)$	$1/2$	$1/10$	$13/45$	$1/9$.



5) x: no of random numbers selected from $\{0, 1, 2, \dots 9\}$ independently until 0 is chosen.

a) $P_X(x) = \left(\frac{9}{10}\right)^{x-1} \left(\frac{1}{10}\right)$, $x = 1, 2, 3, \dots$

papergrid

Date: / /

b) $P_Y(y) = P(Y=y)$

$= P(2x+1=y)$

$P\left(x = \frac{y-1}{2}\right)$

$= \left(\frac{9}{10}\right)^{\frac{y-1}{2}} \left(\frac{1}{10}\right)$, $y = 3, 5, 7 \ldots$

papergrid

Date: / /

Pg175

2) X: money charged

PMF: $f(x) = \begin{cases} 0.4 & \text{if } x = 0 \\ 0.6 & \text{if } x = 25 \end{cases}$

$E(x) = \sum x f(x)$

$= 0(0.4) + 25(0.6)$

$= 15$

$\because 15 > 7$, the commuter should park at a lot

3) $E[2(x-1)(3-x)] = 2E[(x-1)(3-x)]$

$= 2\sum(x-1)(3-x)p(x)$

$= 2\left(\frac{-15}{3} - \frac{3}{4} + \frac{1}{4} - \frac{3}{6}\right)$

$= 2\left(\frac{-60-9+3-6}{12}\right)$

$= -12$

5) X: money that a player gains per game

$X = \{4, 9, -1\}$

$P(X=4) = \frac{{}^4C_3\,{}^6C_1}{{}^{10}C_4} = 0.1143$

$P(x=9) = \frac{^4C_4}{^{10}C_4} = 0.0048$

$P(x=-1) = 1 - 0.1143 - 0.0048 = 0.8809$

$E(x) = \Sigma x\, p(x)$
$= -1(0.8809) + 4(0.1143) + 9(0.0048)$
$= -0.3805$

on average, player loses $0.3805

6) X: money that a player gains.

$X=x$	$(-1,1)$	$(1,5)$	$(1,15)$	$(5,5)$	$(5,15)$
$p(x)$	$\frac{^2C_2}{^5C_2}$	$\frac{2\times2}{^5C_2}$	$\frac{2\times1}{^5C_2}$	$\frac{^2C_2}{^5C_2}$	$\frac{2\times1}{^5C_2}$
	$=1/10$	$4/10$	$2/10$	$1/10$	$2/10$

Expected gain $E(x) = \Sigma x\, P(x)$
$= -\frac{8}{10} - \frac{16}{10} + \frac{12}{10} + 0 + \frac{20}{10}$

$\frac{-8}{10} = 0.8$

on an average a player gains \$0.8. Hence this is considered a 'fair' game.

8. X: no. of defective fuses among three

$X = \{0, 1, 2, 3\}$

$P(0) = \frac{5C_0 \; 15C_3}{20C_3} = \frac{1 \times 455}{1140} = \frac{445}{1140}$

$P(1) = \frac{5C_1 \; 15C_2}{20C_3} = \frac{525}{1140}$

$P(2) = \frac{5C_2 \; 15C_1}{20C_3} = \frac{150}{1140}$

$P(3) = \frac{5C_3 \; 15C_0}{20C_3} = \frac{10}{1140}$

$E(x) = \Sigma x P(x)$

$= 0 + \frac{525}{1140} + \frac{300}{1140} + \frac{30}{1140}$

$= \frac{855}{1140} = \frac{171}{228} = \frac{3}{4}$

papergrid

Date: / /

a) $P(i) = \frac{10-i}{18}$ $i = 4, 5, 6, 7$

$X_i \rightarrow$ profit when 'i' maga. are bought

$E(X_4) = \frac{4a}{3} \times 1 = \frac{4a}{3}$

$E(X_5) = \frac{2a}{3} \cdot \frac{\cancel{8}6}{18} + \frac{5a}{3} \cdot \frac{12}{18} = \frac{12a}{9} = \frac{4a}{3}$

$E(X_6) = 0 \cdot \frac{6}{18} + a \cdot \frac{5}{18} + \frac{6a}{3} \cdot \frac{7}{18} = \frac{19a}{18}$

$E(X_7) = -\frac{2a}{3} \cdot \frac{6}{18} + \frac{a}{3} \cdot \frac{5}{18} + \frac{4a}{3} \cdot \frac{4}{18} + \frac{7a}{3} \cdot \frac{3}{18} = \frac{10a}{18}$

we see that X_4 & X_5 have largest expected value
Thus 4 or 5 mag. should be ordered to max. profit

Pg 184

1. Mr. Jones should choose the first business if he is interested in a steady income as the standard deviation of daily expected profit is less when compared to second business.

2. The second device measures the temperature more precisely as its standard deviation is less than that of first device.

papergrid

Date: / /

9) $E(x) = 1$

$E[x(x-2)] = 3$

$E[x^2 - 2x] = 3$

$E(x^2) - 2E(x) = 3$

$E(x^2) = 3 + 2E(x)$

$E(x^2) = 3 + 2(1) = 5$

$Var(x) = E(x^2) - E(x)^2$

$= 5 - 1 = 4$

$Var(-3x+5) = Var(-3x)$

$= 9\,Var(x)$

$= 9(4)$

$= 36$

10) X: Harry's net gain

	HHH	HHT	HTH	THH	HTT	THT	TTH	TTT
X:	0.75	0.5	0.5	0.5	0.25	0.25	0.25	-2

Probability mass function

x	0.25	0.5	0.75	-2
P(x)	3/8	3/8	1/8	1/8

$E(x) = \Sigma x P(x)$

$= 0.25 \times \frac{3}{8} + 0.5 \times \frac{3}{8} + 0.75 \times \frac{1}{8} - \frac{2}{8}$

$= \frac{3}{8} - \frac{2}{8} = 1/8$

$E(x^2) = \Sigma x^2 P(x)$

$= \frac{1}{16}\left(\frac{3}{8}\right) + \frac{1}{4}\left(\frac{3}{8}\right) + \frac{9}{16}\left(\frac{1}{8}\right) + \frac{4}{8}$

$= \frac{3+12+9+64}{128} = \frac{88}{128} = \frac{11}{16}$

$Var(x) = E(x^2) - E(x)^2$

$= \frac{11}{16} - \frac{1}{64} = \frac{43}{64}$

11 $P(x=-1) = P(x=1) = 1/2$

$E(x) = -1/2 + 1/2 = 0$

$P(Y=-10) = P(Y=10) = 1/2$

$\therefore E(Y) = -5 + 5 = 0$

$E(x^2) = \frac{1}{2} + \frac{1}{2} = 1$ $\quad$ $E(Y^2) = 50 + 50 = 100$

$Var(x) = E(x^2) - E(x)^2$ $\quad$ $Var(Y) = E(Y^2) - E(Y)^2$

$= 1 - 0$ $\quad$ $= 100 - 0$

$= 1$ $\quad$ $= 100$

$SD(x) = 1$ $\quad$ $SD(Y) = 10$

$SD(x) < SD(Y)$, x is more concentrated about the mean, the is 0

Pg 190

1 X: total of number drawn.

Total no. of outcomes for the random experiment $= {}^{10}C_2 = \frac{10 \times 9}{2} = 45$

$P(X=1) = 1/45$

$P(X=2) = 1/45$

$P(X=3) = 2/45$

$P(X=4) = 2/45$

$P(X=5) = 3/45$

$P(X=6) = 3/45$

$P(X=7) = 4/45$

$P(X=8) = 4/45$

$P(X=9) = 5/45$

$P(X=10) = 45/45$

$P(X=11) = 4/45$

$P(X=12) = 3/45$

$P(X=13) = 3/45$

$P(X=14) = 2/45$

$P(X=15) = 2/45$

$P(X=16) = 1/45$

$P(X=17) = 1/45$

papergrid

Date: / /

4) X : time in hrs untill the system fails

$$P(X>t) = \left(1+\frac{t}{200}\right) e^{-t/200} \quad , t \geq 0$$

$$P(200 < x < 300) = P(X < 300) - P(X < 200)$$

$$= 1 - \left(1+\frac{3}{2}\right)^{e^{-3/2}} - 1 + 2e^{-1}$$

$$= 0.178$$

6) X: annual amount of Rainfall in cms.

$$F(x) = \begin{cases} 0 & x < 5 \\ 1 - \frac{5}{x^2} & x \geq 0 \end{cases}$$

a) $P(X \geq 6) = 1 - F(6)$

$= 5/36$

b) $P(X \leq 9) = F(9) = 1 - 5/81 = 76/81$

c) $P(2 \leq x \leq 7) = F(7) - F(2)$

$= 1 - \frac{5}{49} - 6$

$= \frac{44}{49}$

7) $E(x) = 15.85 \times 0.15 + 15.9 \times 0.21 + 16 \times 0.35 + 16.1 \times 0.15 +$
16.2×0.14
$= 16$

$Var(x) = E(x-\mu)^2$
$= (15-85-16)^2 0.15 + (15.9-16)^2 0.21$
$+ (16.1-16)^2 0.15 + (16.2-16)^2 0.14$
$= 0.013$

$E(Y) = 15.85 \times 14 + 15.9 \times 0.05 + 16 \times 0.04 + 16.1 \times 0.08 +$
16.2×0.09
$= 16$

$Var(Y) = (15.85-16)^2 0.14 + (15.9-16)^2 0.05 + (16.1-16)^2 0.08$
$+ (16.2-16)^2 0.09$
$= 0.08$

9 $p(i) = \frac{k(2t)^i}{i!}, \quad i = 0,1,2$

a) $P(x<4) = \sum_{i=0}^{3} P(x=i)$

$= e^{-2t}\left(1+2t+\frac{4t^2}{2}+\frac{8t^3}{6}\right)$

$= e^{-2t}\left(1+2t+2t^2+\frac{4t^3}{3}\right)$

$P(x>1) = 1 - P(x=0) - P(x=1)$
$= 1 - e^{-2t}(1+2t)$

papergrid

Date: / /

Pg 204

1 $n=8$

$P = 1/4$ $\therefore q = 3/4$

$P(x=4) = {}^8C_4 \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right)^4 = \frac{87}{1000}$

3 $n=6$

$p = \frac{2}{12} = \frac{1}{6}$ $\therefore q = \frac{5}{6}$

$P(x=3) = {}^6C_3 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^3 = 0.054$

5) $n=6$

$P = 1/10$ $\therefore q = 9/10$

$P(x=2) = {}^6C_2 \left(\frac{1}{10}\right)^2 \left(\frac{9}{10}\right)^4 = 0.098$

6) $n=5$

$P = \frac{10}{30} = \frac{1}{3}$ $\therefore q = \frac{2}{3}$

$P(x=2) = {}^5C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^2$

$= 0.329$

papergrid

Date: / /

Pg 218-220

3) $\lambda = 2.5\%$ of 80

$= 0.025 \times 80$

$= 2$

$X =$ no. of illegal immigrants $\sim$ Poison(2)

$\therefore P(X \geq 2) = 1 - P(X < 2)$

$= 1 - P(X=0) - P(X=1)$

$= 1 - \frac{e^{-2} 2^0}{0!} - \frac{e^{-2} 2!}{1!}$

$= 1 - 3e^{-2}$

6. $\lambda = 35 \times \frac{3}{10} = 10.5$

P(10 misprints in a given chapter)

$= \frac{e^{-10.5}(10.5)^{10}}{10!}$

$= 0.1236$

$\therefore$ P(Chapter 1 & 5 have 10 misprints each)

$= 0.1236 \times 0.1236$

$= 0.0153$

papergrid

Date: / /

7) $P(X=1) = P(X=3)$

$\lambda^2 = 6$

$\lambda = \sqrt{6}$

$P(X=5) = \dfrac{e^{-\sqrt{6}}(\sqrt{6})^5}{}$

0.0634

12) $\lambda = 1/12 \times 30 = 30/12 = 2.5$

$P(X=3) = \dfrac{e^{-2.5}(2.5)^3}{3!} = 0.2138$

15 $\lambda = 3$

$\therefore$ P(no earthquake in a week) $= e^{-3}$

$\therefore$ P(earth quake after 2 weeks) $= (e^{-3})^2 = 0.00248$

19) $\lambda = 5$

$P(N(1)=0) = e^{-5}$

required probability

$12C_2\,(e^{-5})\,(1-e^{-5})^{10}$

$= 0.0028$

papergrid

Date: / /

26) $\lambda = 1.5 \times 6 = 9$

A: event that the earthquake will not damage the bridge

$$P(\text{required}) = \sum_{i=0}^{\infty} P(A/X=i)\, P(X=i)$$

$$= \sum_{i=0}^{\infty} (1-0.015)^i\, e^{-9} \cdot \frac{9^i}{i!}$$

$$= \sum_{i=0}^{\infty} (0.985)^i e^{-9} \frac{9^i}{i!}$$

$$= e^{-9} \sum_{i=0}^{\infty} \frac{(0.985 \times 9)^i}{i!}$$

$$= e^{-9} \cdot e^{0.985 \times 9}$$

$$= 0.8737$$

Pg. 233

3.

a) $\text{mean} = \frac{1}{p} = \frac{1}{1/12} = 12$

b) $\left(\frac{11}{12}\right)^2 \frac{1}{12} = 0.07$

papergrid

Date: / /

5) $p = 20\% = 0.2$

$P(\text{required}) = {}^7C_2 \; (0.2)^3 \; (0.8)^5$

$= 0.0551$

6 $p = 0.45$

a) $q^5 p$

$= (0.55)^5 (0.45)$

$= 0.0226$

b) $q^3 p q^3 p$

$= q^6 p^2$

$= (0.55)^6 (0.45)^2$

$= 0.0056$

8) $P(\text{atleast } n \text{ bulbs are required}) = p^{n-1}$

Pg 238

2 $\lambda = 1/3$

$P(N(30) < 7) = \sum_{i=0}^{6} P(N(30) = i)$

$= \sum_{i=0}^{6} \frac{e^{-30/3} \; (30/3)^i}{i!}$

papergrid

Date: / /

$$= e^{-10}\left(1 + 10 + \frac{100}{2} + \frac{1000}{6} + \frac{10^4}{24} + \frac{10^5}{120} + \frac{10^6}{720}\right)$$

$$= 0.13014$$

5) $n = 5$

$p = 18\% = 0.18$

$\therefore q = 0.82$

$P(X = 2) = {}^5C_2 (0.18)^2 (0.82)^3$

$= 0.179$

8) $n = 10$

$p = \frac{28-15}{60} = \frac{13}{60}$

$E(X) = np = 10 \times \frac{13}{60} = 2.167$

$Var(X) = npq = 10 \times \frac{13}{60} \times \frac{47}{60} = 1.697$

9) $\lambda = 3$

$P(N(2) \leq 4) = \sum_{i=0}^{4} \frac{e^{-3(2)}[3(2)]^i}{i!}$

$= \sum_{i=0}^{4} \frac{e^{-6} 6^i}{i!}$

$= 0.285$

papergrid

Date: / /

24) $p = \frac{b}{w+b}$ $\therefore q = \frac{w}{w+b}$

a) $P(X=n) = q^{n-1} p$

$= \left(\frac{w}{w+b}\right)^{n-1} \left(\frac{b}{w+b}\right)$

b) $P(X \geq n) = \left(\frac{w}{w+b}\right)^{n-1}$

25) $p = 0.75$
$q = 0.25$
$n = ?$

$P(X \geq 5) \geq \frac{90}{100}$

$\therefore P(X < 5) \leq \frac{10}{100}$

$\sum_{i=0}^{4} {}^nC_i (0.75)^i (0.25)^{n-i} \leq 0.1$

n	P(X<5)
5	0.7627
6	0.4661
7	0.2436
8	0.1139
9	0.0489 ≤ 0.1

papergrid

Date: / /

Pg 284

2) 15 points are selected at random & independently from $(0,1)$

$p = 1/4$

$n = 15$

no. of points expected to be greater than $3/4$

$= 15 \times \frac{1}{4}$

$= 3.75$

3) $\frac{(b-a)^2}{12} = 12$ $\frac{a+b}{2} = 0$

$b - a = 12$ $a + b = 0$

$\therefore a = -6, b = 6$

If mean is 2:00 pm then (1:54pm, 2:06 pm)

4 $\Delta \geq 0$

$b^2 - 4 \geq 0$

$b^2 \geq 4$

$|b| \geq 2$

& b is from $(-3, 3)$

$\therefore b \in (-3, -2] \cup [2, 3)$

Thus probability is $\frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$

papergrid

Date: / /

5) X: radius of sphere
$X \sim$ uniform $(2,4)$

$$f(x) = \begin{cases} \frac{1}{4-2} = \frac{1}{2} & 2 < x < 4 \\ 0 & \text{otherwise} \end{cases}$$

$$E(x) = \int x f(x)\, dx$$

$$= \int_2^4 \frac{x}{\cancel{4}2}\, dx$$

$$= \frac{1}{2}\left[\frac{x^2}{2}\right]_2^4$$

$$= \frac{1}{4}(16-4)$$

$$= 3$$

$$E(\text{volume}) = \int \frac{4}{3}\pi x^3 f(x)\, dx$$

$$= \frac{6}{4}\int_2^4 \pi\, x^3 dx$$

$$= \frac{4}{6}\pi\left[\frac{x^4}{4}\right]_2^4$$

$$= \frac{\pi}{6}[256-16]$$

$$= \frac{240\pi}{6} = 40\pi$$