

# Implementation of the Higher Order Spectral Method for Nonlinear Wave Propagation in FOAM

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## Abstract

Higher Order Spectral (HOS) method for nonlinear surface wave propagation is implemented in OpenFOAM. HOS is a pseudo-spectral method for solving nonlinear free surface boundary conditions up to arbitrary order of nonlinearity. The code is validated on both regular and irregular wave propagation. Coupling with CFD allows nonlinear wave initialisation in the far field or SWENSE.

## 1. Introduction

The problem of assessing wave loads on marine objects is getting more important as offshore objects get installed further away in the waters of open ocean. Freak waves are especially dangerous due to their steep wave slope, causing high dynamic loads on offshore structures. HOS coupled with CFD provides a capability of simulating a realistic three-dimensional freak wave with relatively low CPU expense. HOS develops a fully nonlinear sea state, where a freak wave can emerge naturally.

## 2. HOS Mathematical Model

Dynamic and kinematic free surface boundary conditions are the governing equations for surface waves. Expressed in terms of surface velocity potential  $\psi(x, y, t) = \phi(x, y, z = \eta, t)$  and vertical velocity  $W = \partial\phi/\partial z$ :

$$\begin{aligned} \frac{\partial\psi}{\partial t} + g\eta + \frac{1}{2} \left( \frac{\partial\psi}{\partial x}, \frac{\partial\psi}{\partial y} \right)^2 - \frac{1}{2} W^2 \left( 1 + \left( \frac{\partial\eta}{\partial x}, \frac{\partial\eta}{\partial y} \right)^2 \right) &= 0, \\ \frac{\partial\eta}{\partial t} + \left( \frac{\partial\psi}{\partial x}, \frac{\partial\psi}{\partial y} \right) \cdot \left( \frac{\partial\eta}{\partial x}, \frac{\partial\eta}{\partial y} \right) - W \left( 1 + \left( \frac{\partial\eta}{\partial x}, \frac{\partial\eta}{\partial y} \right)^2 \right) &= 0. \end{aligned}$$

In HOS,  $W$  is evaluated by expanding  $\psi$  in perturbation order regarding wave steepness  $\epsilon = ka$ , where each order is expanded into Taylor series around  $z = 0$ :

$$\psi(x, y, t) = \sum_{m=1}^M \sum_{i=0}^{M-m} \frac{\eta^i}{i!} \frac{\partial^i}{\partial z^i} \phi^{(m)}(x, y, 0, t).$$

Spatial derivatives are evaluated in Fourier space, while nonlinear products are calculated in spatial space. FFT is used to project from spatial to Fourier space and vice versa.

## 3. Validation

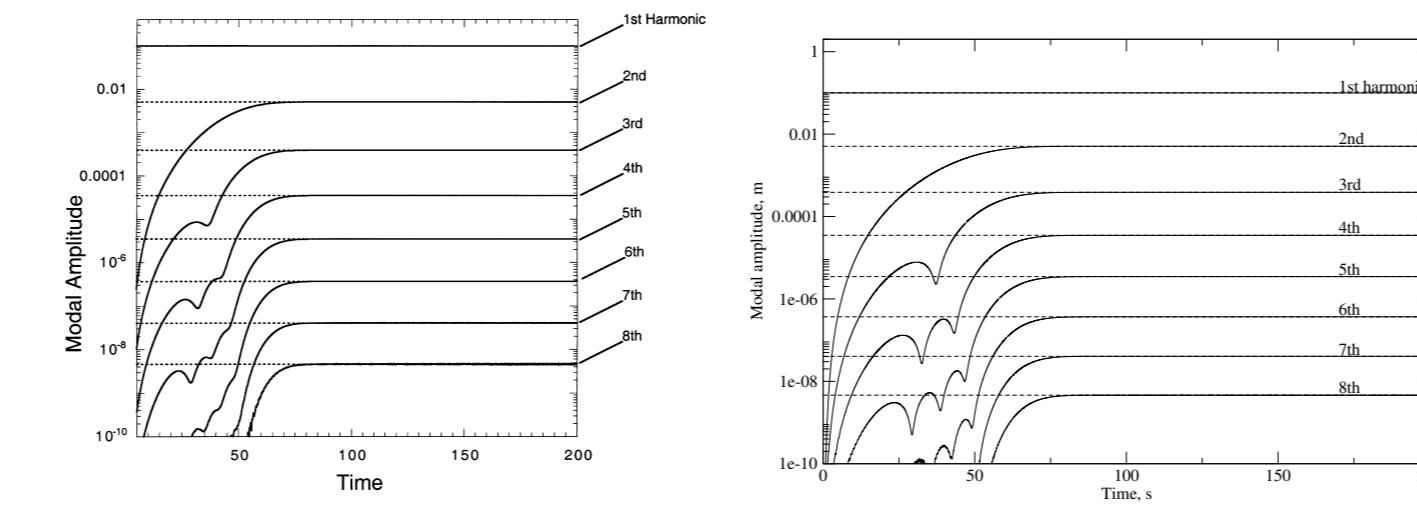
### 3.1 Monochromatic Wave Propagation

Long time HOS propagation of monochromatic regular wave is performed to allow nonlinear higher order modes to develop. Magnitudes of higher order modes are compared to exact nonlinear Stokes solution, Table 1. Convergence is also tested by comparing to Dommermuth 2000, Fig. 1.

Table 1: Comparison of HOS results and exact Stokes solution.

| Order | Modal amplitude, m<br>Exact value | Modal amplitude, m<br>HOS result | Relative Err, % |
|-------|-----------------------------------|----------------------------------|-----------------|
| 1     | 9.987052e-2                       | 9.987052e-2                      | 4.34e-6         |
| 2     | 5.059412e-3                       | 5.059419e-3                      | 1.43e-4         |
| 3     | 3.858423e-4                       | 3.858434e-4                      | 2.78e-4         |
| 4     | 3.492969e-5                       | 3.492983e-5                      | 4.20e-4         |
| 5     | 3.476967e-6                       | 3.476967e-6                      | -3.26e-6        |
| 6     | 3.676395e-7                       | 3.676318e-7                      | -2.07e-3        |
| 7     | 4.053174e-8                       | 4.053083e-8                      | -2.24e-3        |
| 8     | 4.607693e-9                       | 4.602681e-9                      | -1.09e-1        |

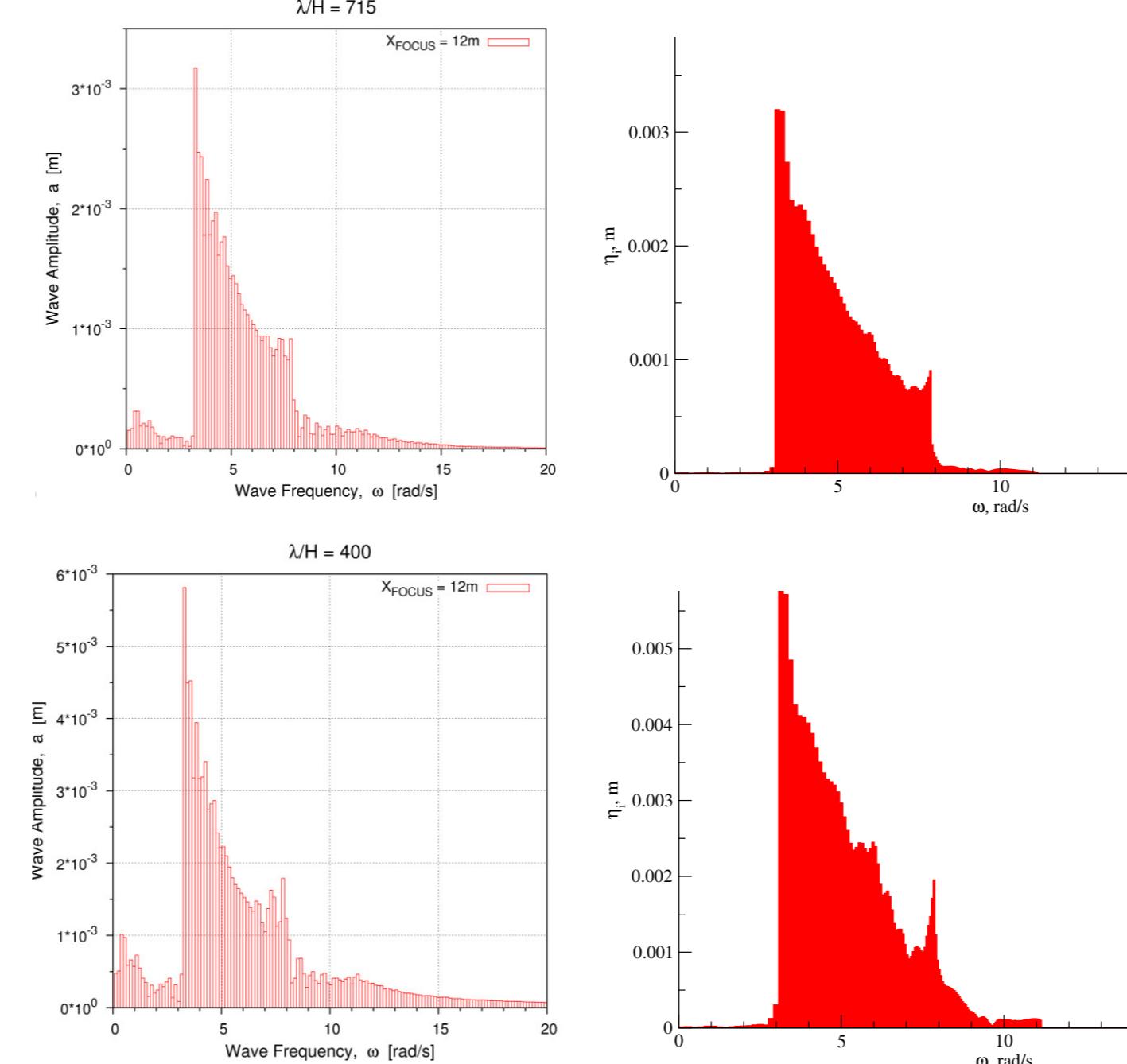
Figure 1: Higher order modes convergence; left: Dommermuth 2000, right: Present model.



### 3.2 Propagation of a Wave Spectrum

Propagation of wave spectra with HOS is compared to a viscous study performed in OpenFOAM by Lupieri et al. (2014). Input spectra are composed of wave components with equal steepness  $H_i/\lambda_i$ . Spectra comparison is shown in Fig. 2.

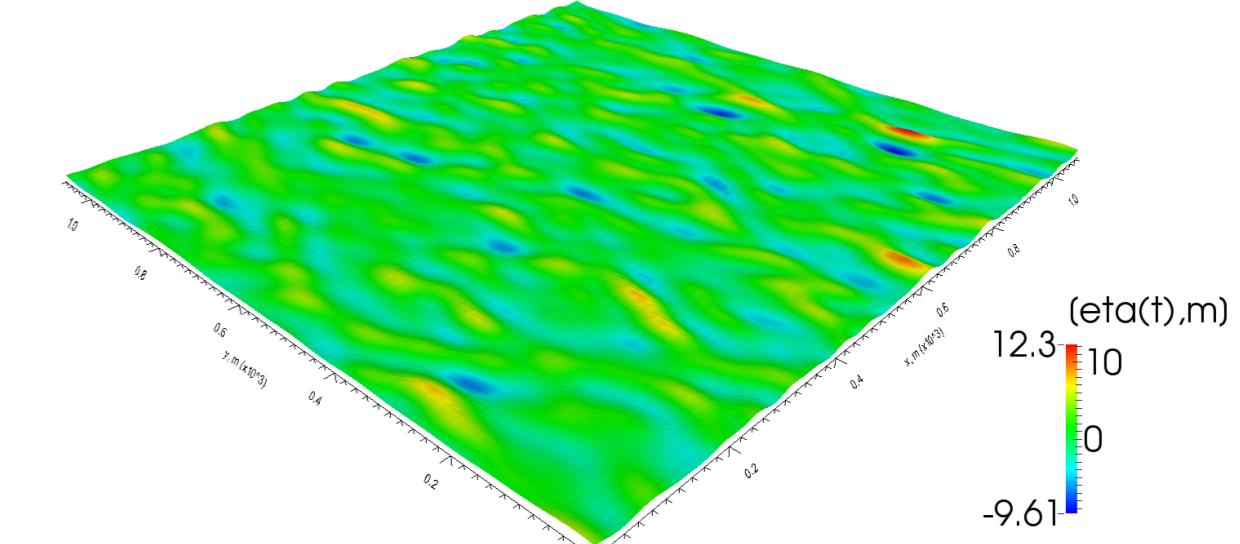
Figure 2: Spectra comparison; left: Lupieri et al. (2014), right: Present model.



## 4. CFD Simulations of a Three-Dimensional Freak Wave

Example simulations of a directional freak wave are conducted to demonstrate the capability of HOS-CFD coupling. Freak wave used in the simulations is obtained by long time directional HOS simulation, Fig. 3. Significant wave height is 10.5 m while the freak wave height is 21.9 m.

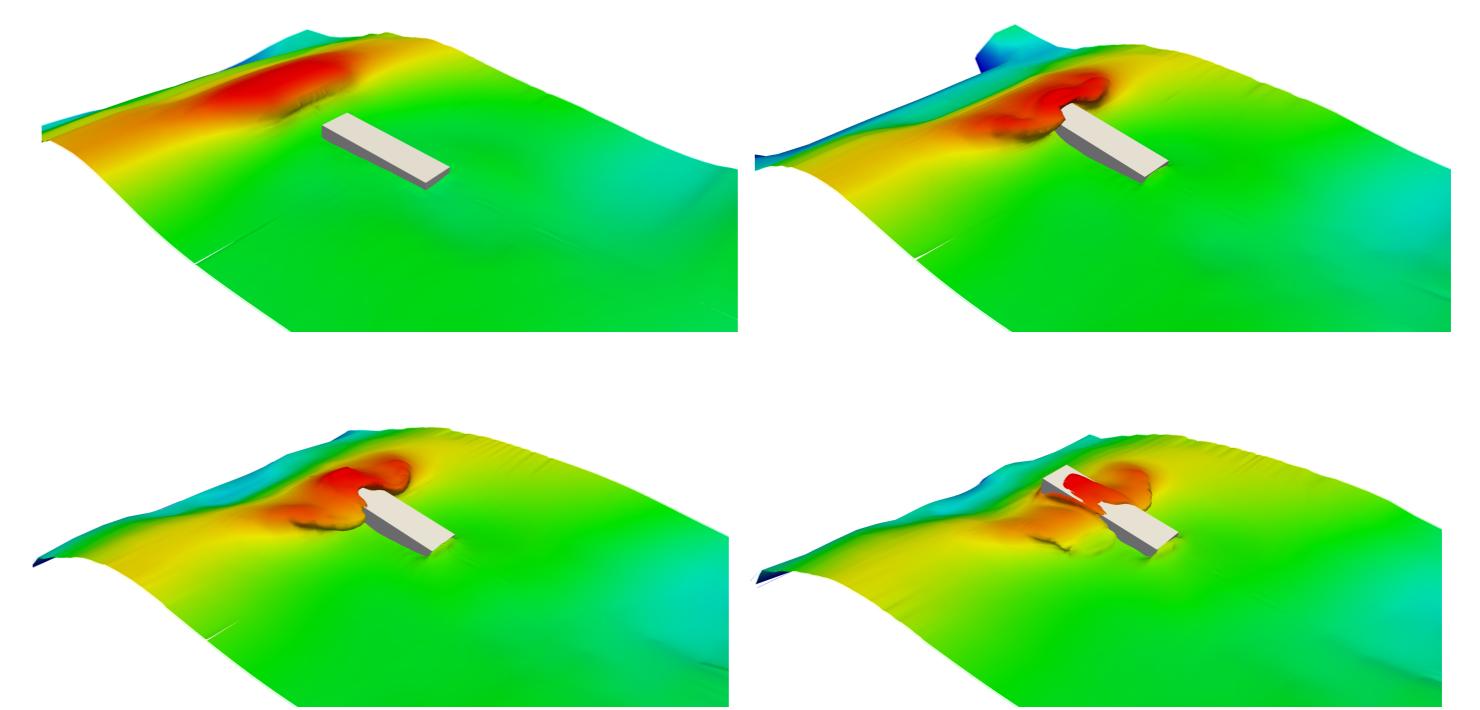
Figure 3: Freak wave emergence in HOS simulation.



### 4.1 Freak Wave Encountering a Barge

Simulation of a freak wave encountering a full scale barge ( $L = 27$  m,  $B = 7$  m,  $H = 5$  m) is conducted. Extremely coarse mesh is used with only 110 000 cells. Simulation screen shots are shown in Fig. 4.

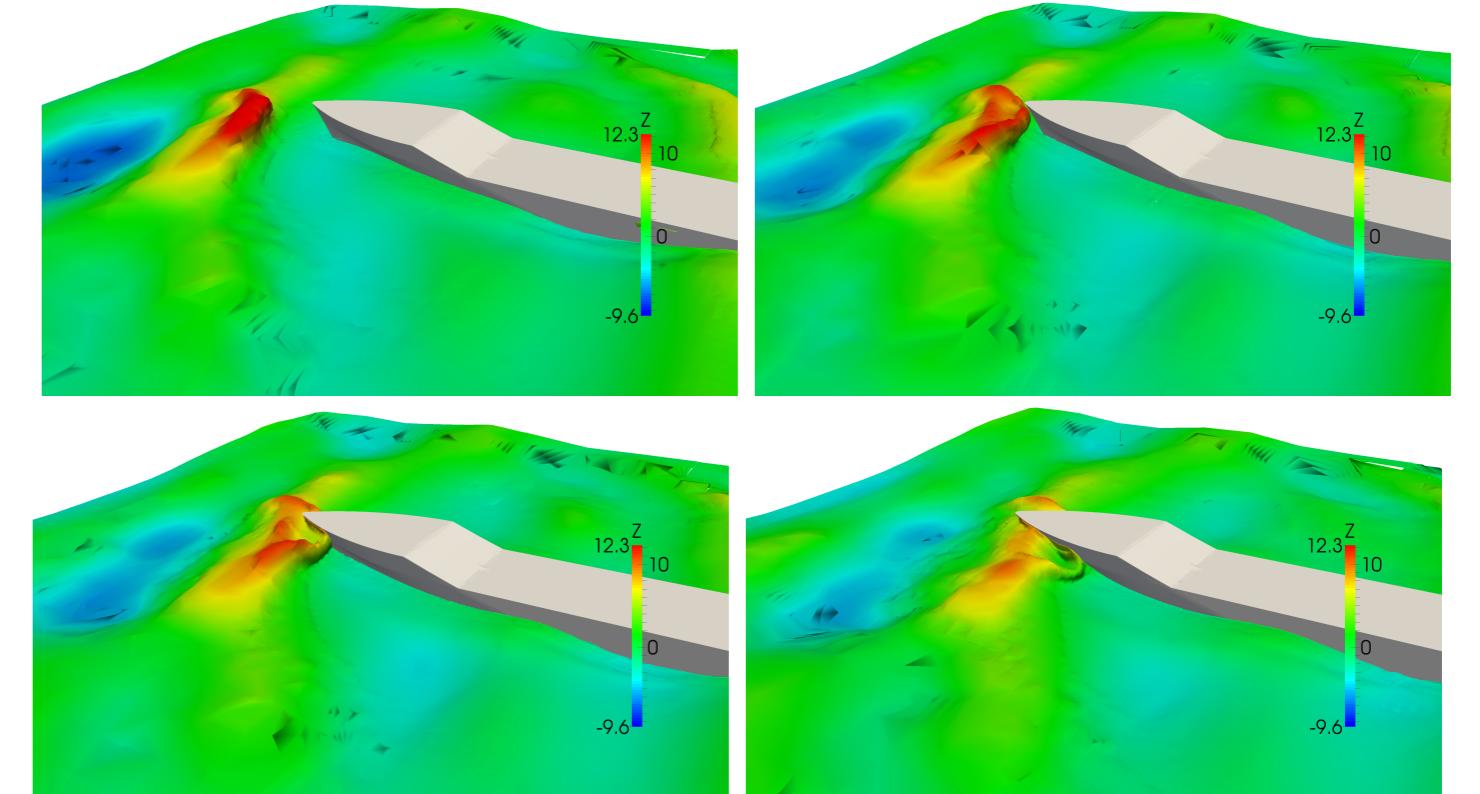
Figure 4: Freak wave encountering a barge.



### 4.2 Freak Wave Encountering a KCS Hull

Full scale KCS ( $L = 230$ ) encountered by a freak wave was simulated. Mesh size is 1.2 million cells, which is relatively coarse for this type of simulation. Freak wave encountering the KCS is shown in Fig. 5.

Figure 5: Freak wave encountering a KCS hull.



## 5. Conclusion

Validity of the implemented HOS method is confirmed on three test cases. First test case showed excellent agreement of developed higher order modes with exact Stokes solution. Convergence also compares well with Dommermuth 2000. Nonlinear propagation of wave spectra is confirmed in the second test case, where similar changes of wave spectra are noticed in HOS and viscous study of Lupieri et al. (2014). Finally, two example three-dimensional CFD simulations of a freak wave are performed to show the capability of HOS-CFD coupling.