Turbulence Modelling for CFD

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Outline

Objective

- Review turbulence modelling practices in modern CFD: assumptions, choice of model and performance

Topics

- Handling turbulent flows
- Vortex dynamics and energy cascade
- Direct numerical simulation
- Reynolds-averaged Navier-Stokes equations
  - Eddy viscosity models
  - Reynolds stress transport models
  - Near-wall effects and low-Re models
  - Special topics: transient RANS and transitional flows
- Large Eddy Simulation (LES)
- Choosing a turbulence model for you application
- Review of two popular turbulence models
  - Spalart-Allmaras model; $k - \omega$ SST model by Menter
- Future of turbulence modelling

Why Model Turbulence?

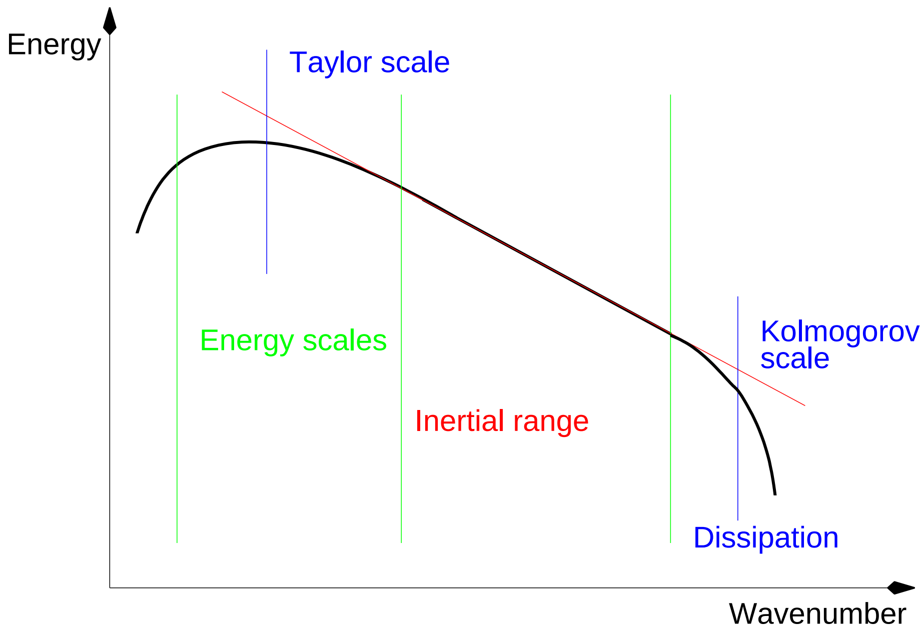
- **The physics of turbulence is completely understood** and described in all its detail: turbulent fluid flow is strictly governed by the Navier-Stokes equations
- . . . but we do not like the answer very much!
  - Turbulence spans wide spatial and temporal scales
  - When described in terms of vortices (= eddies), non-linear interaction is complex
  - Because of non-linear interactions and correlated nature, it cannot be attacked statistically
  - It is not easy to assemble the results of full turbulent interaction and describe them in a way relevant for **engineering simulations**: we are more interested in mean properties of physical relevance
- In spite of its complexity, there is a number of analytical, order-of-magnitude and quantitative result for simple turbulence flows. Some of them are extremely useful in model formulation
- Mathematically, after more than 100 years of trying, we are nowhere near to describing turbulence the way we wish to

Handling Turbulent Flows

- Turbulence is irregular, disorderly, non-stationary, three-dimensional, highly non-linear, irreversible stochastic phenomenon
- Characteristics of turbulent flows (Tennekes and Lumley: First Course in Turbulence)
  - **Randomness**, meaning disorder and no-repeatability
  - **Vorticality**: high concentration and intensity of vorticity
  - **Non-linearity and three-dimensionality**
  - **Continuity of Eddy Structure**, reflected in a continuous spectrum of fluctuations over a range of frequencies
  - **Energy cascade, irreversibility and dissipativeness**
  - **Intermittency**: turbulence can only occupy only parts of the flow domain
  - **High diffusivity** of momentum, energy, species etc.
  - **Self-preservation and self-similarity**: in simple flows, turbulence structure depends only on local environment
- Turbulence is characterised by higher diffusion rates: increased drag, mixing, energy diffusion. In engineering machinery, this is sometimes welcome and sometimes detrimental to the performance
- Laminar-turbulent transition is a process where laminar flow naturally and without external influence becomes turbulent. Example: instability of free shear flows

Vortex Dynamics and Energy Cascade

- A useful way of looking at turbulence is **vortex dynamics**.
  - Large-scale vortices are created by the flow. Through the process of **vortex stretching** vortices are broken up into smaller vortices. This moves the energy from large to smaller scales
  - Energy dissipation in the system scales with the velocity gradient, which is largest in small vortices



- The abscissa of the above is expressed in terms of **wavenumber**: how many vortices fit into the space

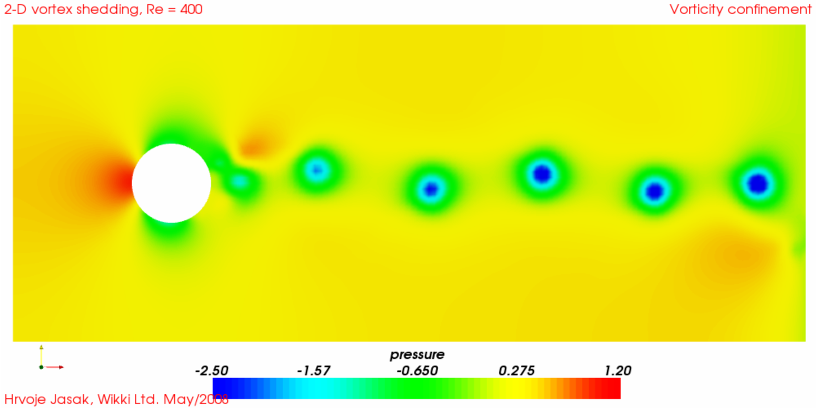
Vortex Dynamics and Energy Cascade

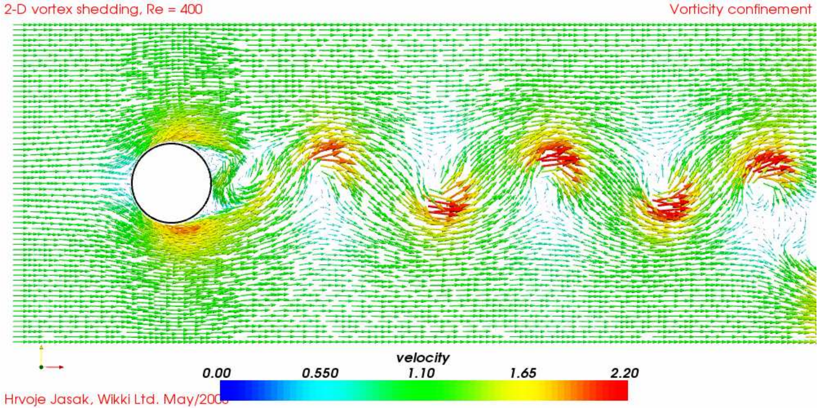
Parts of the Energy Cascade

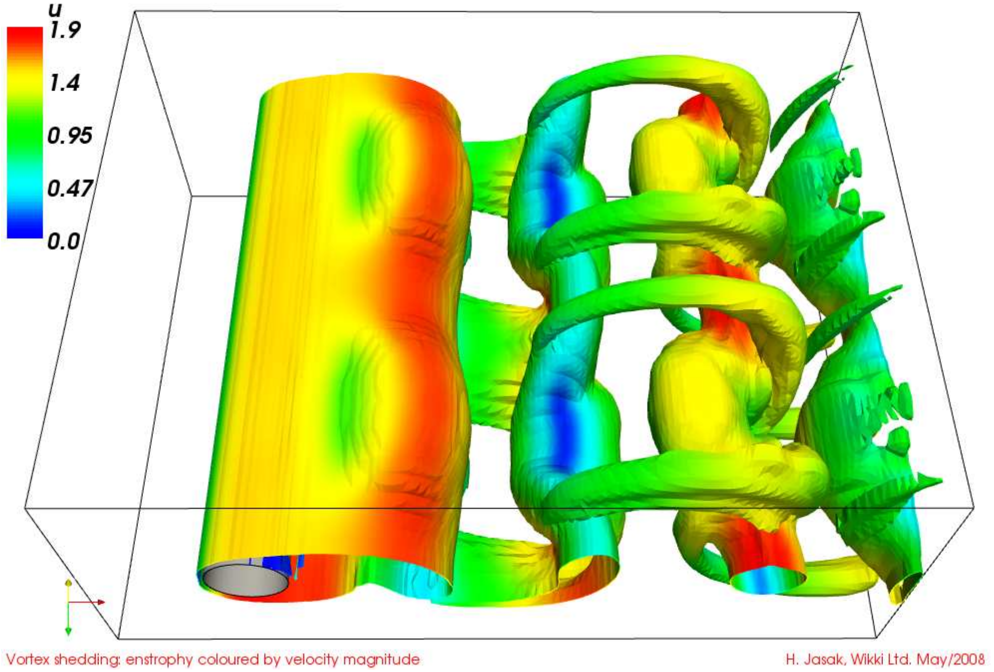
- Large scale vortices, influenced by the shape of flow domain and global flow field. Large scale turbulence is problematic: it is difficult yo decide which of it is a **coherent structure** and which is actually turbulence
- Energy-containing vortices, which contain the highest part of the turbulent kinetic energy. This scale is described by the **Taylor scale**
- Inertial scale, where vortex stretching can be described by inertial effects of vortex breakup. Viscous effects are negligible
- Small vortices, which contain low proportion of overall energy, but contribute most of dissipation. This is also the smallest relevant scale in turbulent flows, characterised by the **Kolmogorov micro-scale**

Vortex Dynamics and Energy Cascade

Example: Vortex Shedding Behind a Cylinder







Turbulence Modelling

Task of Turbulence Modelling

We are trying to find approximate simplified solutions for the Navier-Stokes equations in the manner that either describes turbulence in terms of mean properties or limits the spatial/temporal resolution requirements associated with the full model

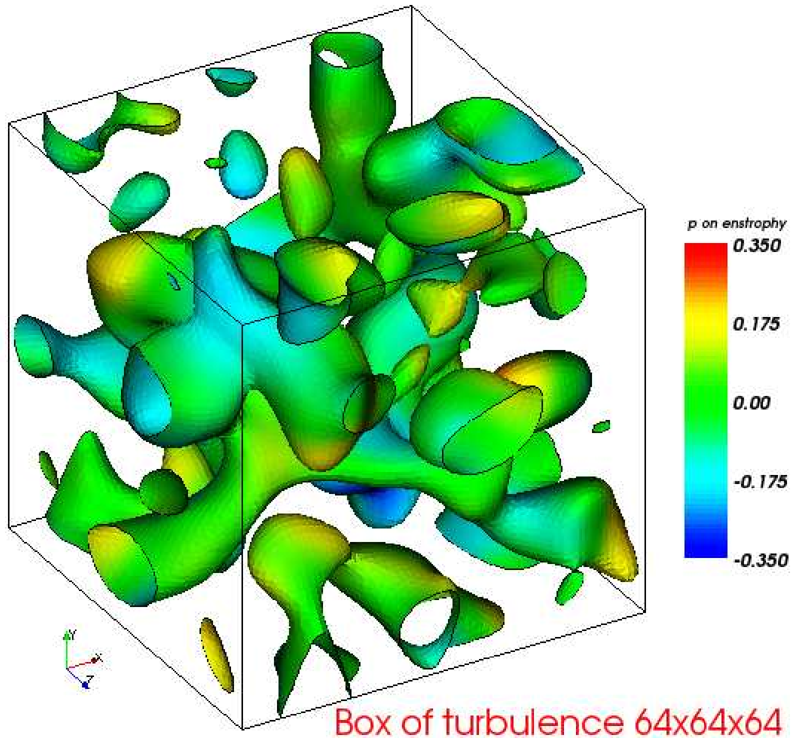
- Turbulence modelling is therefore about manipulating equations and creating closed models in the form that allows us to simulate turbulence interaction under our own conditions. For example, a set of equations describing mean properties would allow us to perform steady-state simulations when only mean properties are of interest
- We shall here examine three modelling frameworks
  - **Direct Numerical Simulation (DNS)**
  - **Reynolds-Averaged Navier-Stokes Equations (RANS)**, including eddy viscosity models and higher moment closure. For compressible flows with significant compressibility effects, the averaging is actually of the **Favre** type
  - **Large Eddy Simulation (LES)**

Direct Numerical Simulation

- DNS is, strictly speaking, not a turbulence model at all: we will simulate all scales of interest in a well-resolved transient mode with sufficient spatial and temporal resolution
- In order to perform the simulation well, it is necessary to ensure sufficient spatial and temporal resolution:
  - Spatial resolution: vortices smaller that Kolmogorov scale will dissipate their energy before a full turn. Smaller flow features are of no interest; Kolmogorov scale is a function of the Re number
  - Temporal resolution is also related to Kolmogorov scale; but may be adjusted for temporal accuracy
- Computer resources are immense: we can really handle relatively modest Re numbers and very simple geometry
- ... but this is the best way of gathering detailed information on turbulent interaction: mean properties, first and second moments, two-point correlations etc. in full fields

Direct Numerical Simulation

- Unrealistically expensive for flows of engineering interest
- . . . but still has a role: gathering data for model evaluation and fundamental understanding of turbulent interaction
- Compared to experiments, much more reliable and complete data sets, including correlations and visualisation
- Today includes scalar mixing, heat transfer, buoyancy effects etc.



Direct Numerical Simulation

- In order to secure accurate higher moments, special numerics is used: *e.g.* sixth order in space and tenth order in space will ensure that higher moments are not polluted numerically. An alternative are spectral models, using Fourier modes or Chebyshev polynomials as a discretisation base
- DNS simulations involve simple geometries and lots of averaging. Data is assembled into large databases and typically used for validation or tuning of “proper” turbulent models
- DNS on engineering geometries is beyond reach: the benefit of more complete fluid flow data is not balanced by the massive cost involved in producing it
- Current research frontier: compressible turbulence with basic chemical reactions, *e.g.* mixing of hydrogen and oxygen with combustion; buoyancy-driven flows

Reynolds-Averaged Models

Reynolds Averaging

- The rationale for Reynolds averaging is that we are not interested in the part of flow solution that can be described as “turbulent fluctuations”: instead, it is the mean (velocity, pressure, lift, drag) that is of interest. Looking at turbulent flow, it may be steady **in the mean** in spite of turbulent fluctuations. If this is so, and we manage to derive the equations for the mean properties directly, we may reduce the cost by orders of magnitude:
  - It is no longer necessary to perform transient simulation and assemble the averages: we are solving for average properties directly
  - Spatial resolution requirement is no longer governed by the Kolmogorov micro-scale! We can tackle high Reynolds numbers and determine the resolution based on required engineering accuracy

Reynolds-Averaged Models

Reynolds Averaged Navier-Stokes Equations

- Repeating from above: decompose $\mathbf{u}$ and p into a mean and fluctuating component:

$$\mathbf{u} = \overline{\mathbf{u}} + \mathbf{u}'$$

$$p = \overline{p} + p'$$

- Substitute the above into original equations. Eliminate all terms containing products of mean and fluctuating values

$$\frac{\partial \overline{\mathbf{u}}}{\partial t} + \nabla\bullet(\overline{\mathbf{u}}\,\overline{\mathbf{u}}) - \nabla\bullet\left(\nu \nabla \overline{\mathbf{u}}\right) = -\nabla \overline{p} + \nabla\bullet(\overline{\mathbf{u}'\mathbf{u}'})$$

$$\nabla\bullet\overline{\mathbf{u}} = 0$$

Reynolds-Averaged Models

Reynolds Averaged Navier-Stokes Equations

- One new term: the **Reynolds stress tensor**:

$$\mathbf{R} = \overline{\mathbf{u}'\mathbf{u}'}$$

  $\mathbf{R}$ is a second rank symmetric tensor. We have seen something similar when the continuum mechanics equations were assembled, but with clear separation of scales: molecular interaction is described as diffusion

Modelling Paradigms

- In order to close the system, we need to describe the unknown value, $\mathbf{R}$ as a function of the solution. Two ways of doing this are:
  1. Write an algebraic function, resulting in **eddy viscosity models**

$$\mathbf{R} = f(\overline{\mathbf{u}}, \overline{p})$$

  2. Add more differential equations, *i.e.* a transport equation for $\mathbf{R}$, producing **Reynolds Transport Models**. As we keep introducing new equations, the above problem will recur. Option 1 will need to be used at some level
- Both options are in use today, but first massively out-weights the second

Eddy Viscosity Models

Dimensional Analysis

- Looking at $\mathbf{R}$, the starting point is to find an appropriate symmetric second rank tensor. Remember that the terms acts as diffusion of momentum, appears in the equation under divergence and appears to act as diffusion
- Based on this, the second rank tensor is the symmetric velocity gradient $\mathbf{S}$:

$$\mathbf{R} = f(\overline{\mathbf{S}})$$

where

$$\overline{\mathbf{S}} = \frac{1}{2}\left[\nabla\overline{\mathbf{u}} + (\nabla\overline{\mathbf{u}})^T\right]$$

Under divergence, this will produce a $\nabla\bullet(\nabla\overline{\mathbf{u}})$ kind of term, which makes physical sense and is numerically well behaved

- Using dimensional analysis, it turns out that we need a pre-factor of dimensions of viscosity: for laminar flows, this will be $[\mathrm{m}^2/\mathrm{s}]$ and because of its equivalence with laminar viscosity we may call it **turbulent viscosity** ν_t

Eddy Viscosity Models

Dimensional Analysis

- The problem reduces to finding ν_t as a function of the solution. Looking at dimensions, we need a length and time-scale, either postulated or calculated. On second thought, it makes more sense to use **velocity scale** U and **length-scale** Δ
- We can think of the velocity scale as the size of $\mathbf{u}'$ and length-scale as the size of energy-containing vortices. Thus:

$$\mathbf{R} = \nu_t \frac{1}{2} \left[\nabla \overline{\mathbf{u}} + (\nabla \overline{\mathbf{u}})^T \right]$$

and

$$\nu_t = A\, U\, \Delta$$

where A is a dimensionless constant allowing us to tune the model to the actual physical behaviour

Eddy Viscosity Models

Velocity and Length Scale

- Velocity scale is relatively easy: it represents the strength of turbulent fluctuations. Thus, $U \approx |\mathbf{u}'|$. Additionally, it is easy to derive the equation for turbulence kinetic energy k:

$$k = \frac{3}{2}\mathbf{u}'^2$$

directly from the momentum equation in the following form:

$$\frac{\partial k}{\partial t} + \nabla\bullet(\overline{\mathbf{u}}k) - \nabla\bullet[(\nu_{eff})\nabla k] = \nu_t \left[\frac{1}{2}(\nabla\overline{\mathbf{u}} + \nabla\overline{\mathbf{u}}^T)\right]^2 - \epsilon$$

Here ϵ is turbulent dissipation which contains the length scale:

$$\epsilon = C_\epsilon \frac{k^{\frac{3}{2}}}{\Delta}$$

Eddy Viscosity Models

Zero and One-Equation Models

- **Zero equation model**: assume local equilibrium above: $k = \epsilon$, with no transport. The problem reduces to the specification of length-scale. Example: Smagorinsky model

$$\nu_t = (C_S \Delta)^2 |\overline{\mathbf{S}}|$$

where C_S is the Smagorinsky "constant". The model is actually in active use (!) but not in this manner – see below

- **One equation model**: solve the k equation and use an algebraic equation for the length scale. Example: length-scale for airfoil simulations can be determined form the distance to the wall

Two-Equation Model

- Two-equation models are the work-horse of engineering simulations today. Using the k equation from above, the system is closed by forming an equation for turbulent dissipation ϵ and modelling its generation and destruction terms
- Other choices also exist. For example, the Wilcox model uses **eddy turnover time** ω as the second variable, claiming better behaviour near the wall and easier modelling
- Two-equation models are popular because it accounts for transport of both the velocity and length-scale and can be tuned to return several canonical results

Eddy Viscosity Models

Standard $k - \epsilon$ Model

- This is the most popular 2-equation model, now on its way out. There exists a number of minor variants, but the basic idea is the same
- Turbulence kinetic energy equation

$$\frac{\partial k}{\partial t} + \nabla\bullet(\overline{\mathbf{u}} k) - \nabla\bullet[(\nu_{eff})\nabla k] = G - \epsilon$$

  where

$$G = \nu_t \left[\frac{1}{2}(\nabla\overline{\mathbf{u}} + \nabla\overline{\mathbf{u}}^T)\right]^2$$

- Dissipation of turbulence kinetic energy equation

$$\frac{\partial \epsilon}{\partial t} + \nabla\bullet(\overline{\mathbf{u}} \epsilon) - \nabla\bullet[(\nu_{eff})\nabla \epsilon] = C_1 \, G \frac{\epsilon}{k} - C_2 \frac{\epsilon^2}{k}$$

Eddy Viscosity Models

Standard $k - \epsilon$ Model

- Turbulent viscosity

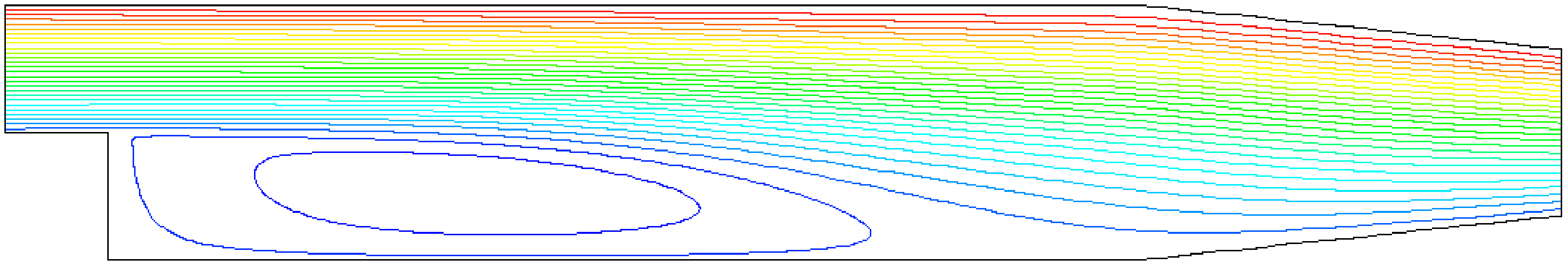
$$\nu_t = C_\mu \frac{k^2}{\epsilon}$$

- Reynolds stress

$$\mathbf{R} = \nu_t \left[\frac{1}{2} (\nabla \overline{\mathbf{u}} + \nabla \overline{\mathbf{u}}^T) \right]$$

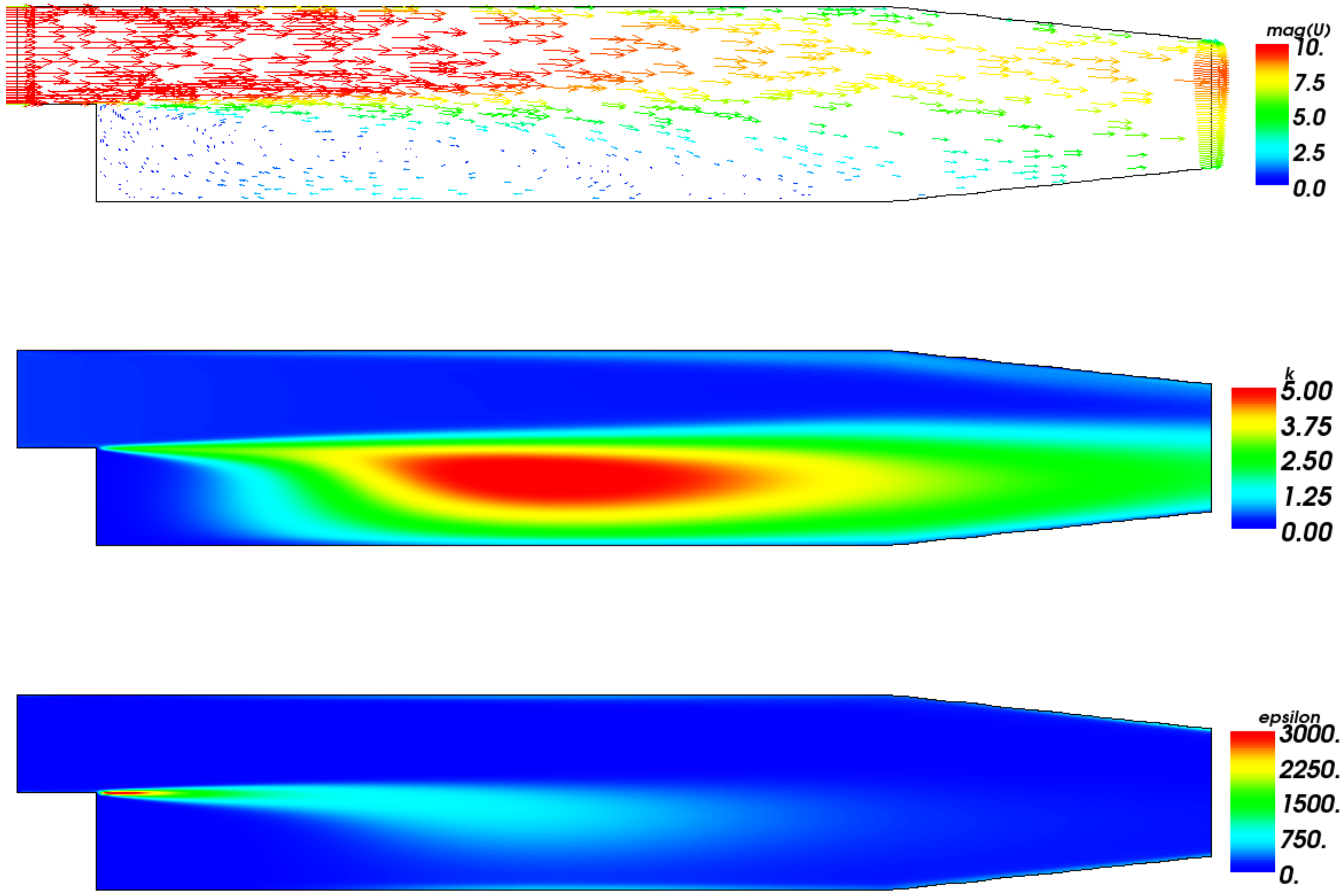
- Model constants are tuned to canonical flows. Which?

Example: Backward-facing step



The $k - \epsilon$ Model

Example: Backward-facing step



Reynolds Transport Models

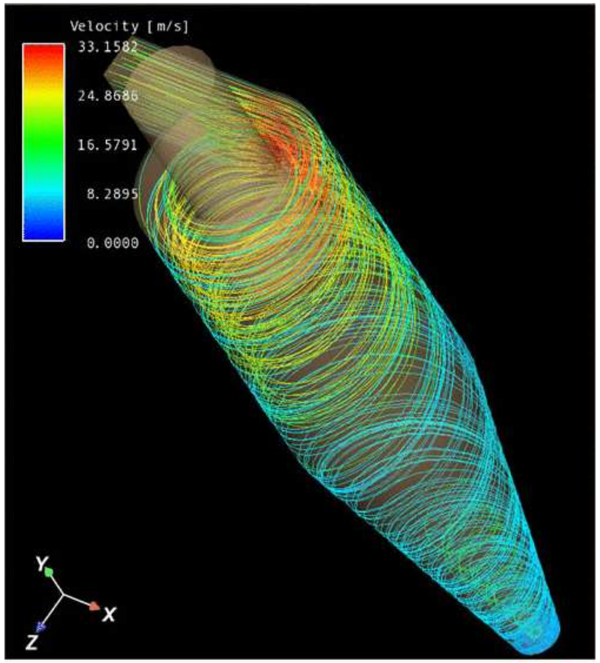
- Transport equation for Reynolds stress $\mathbf{R} = f(\overline{\mathbf{u}}, \overline{p})$ is derived in a manner similar to the derivation of the Reynolds-averaged Navier-Stokes equation. We encounter a number of terms which are physically difficult to understand (a pre-requisite for the modelling)
- Again the most difficult term is the destruction of $\mathbf{R}$, which will ba handled by solving its own equation: it is unreasonable to expect a postulated or equilibrium length-scale to be satisfactory
- Analytical form of the (scalar) turbulence destruction equation is even more complex: in full compressible form it contains over 70 terms
- The closure problem can be further extended by writing out equations for higher moments etc. but “natural” closure is never achieved: the number of new terms expands much faster that the number of equations
- Original closure dates from 1970s and in spite of considerable research efforts, it always contained problems
- Currently, Reynolds transport models are used only in situations where it is **a-priori** known that eddy viscosity models fails. Example: cyclone simulations

Reynolds Transport Models

Modelling Reynolds Stress Equation

- Briefly looking at the modelling of the $\mathbf{R}$ and ϵ equations, physical understanding of various terms is relatively weak and uninteresting. As a result, terms are grouped into three categories
  - Generation terms
  - Redistribution terms
  - Destruction terms

  Each category is then modelled as a whole



Reynolds Transport Models

Standard Closure

- Reynolds stress transport equation

$$\frac{\partial \mathbf{R}}{\partial t} + \nabla\bullet(\mathbf{u}\mathbf{R}) - \nabla\bullet[(\alpha_R \nu_t + \nu_l)\nabla\mathbf{R}] = \mathbf{P} - C_1 \frac{\epsilon}{k}\mathbf{R} + \frac{2}{3}(C_1 - 1)\mathbf{I}\epsilon - C_2 dev(\mathbf{P}) + \mathbf{W}$$

  where

  - $\mathbf{P}$ is the production term

$$\mathbf{P} = -\mathbf{R}\bullet[\nabla\mathbf{u} + (\nabla\mathbf{u})^T]$$

  - $\mathbf{W}$ is the wall reflection term(s)

$$G = \frac{1}{2} tr(\mathbf{P})$$

Reynolds-Averaged Models

Comparing Reynolds Closure with Eddy Viscosity Models

- Eddy viscosity implies that the Reynolds stress tensor is aligned with the velocity gradient

$$\mathbf{R} = \nu_t \frac{1}{2} \left[\nabla \overline{\mathbf{u}} + (\nabla \overline{\mathbf{u}})^T \right]$$

  This would represent **local equilibrium**: compare with equilibrium assumptions for k and ϵ above

- In cases where the two tensors are not aligned, Reynolds closure results are considerably better
- . . . but at a considerable cost increase: more turbulence equations, more serious coupling with the momentum equation

Near-Wall Effects

Turbulence Near the Wall

- Principal problem of turbulence next to the wall is the **inverted energy cascade**: small vortices are rolled up and ejected from the wall. Here, small vortices create big ones, which is not accounted in the standard modelling approach
- Presence of the wall constrains the vortices, giving them orientation: effect on turbulent length-scales
- Most seriously of all, both velocity and turbulence properties contain very steep gradients near the wall. Boundary layers on high Re are extremely thin. Additionally, turbulent length-scale exhibits complex behaviour: in order for the model to work well, all of this needs to be resolved in the simulation

Resolved Boundary Layers

- **Low-Re Turbulence Models** are based on the idea that all details of turbulent flow (in the mean: this is still RANS!) will be resolved
- In order to achieve this, damping functions are introduced in the near-wall region and tuned to actual (measured, DNS) near-wall behaviours
- Examples of such models are: Launder-Sharma, Lam-Bremhorst $k - \epsilon$
- Near-wall resolution requirements and boundary conditions depend on the actual model, but range from $y^+ = 0.01 - 0.1$ for the first node, with grading away from the wall. This is a massive resolution requirement!

Near-Wall Effects

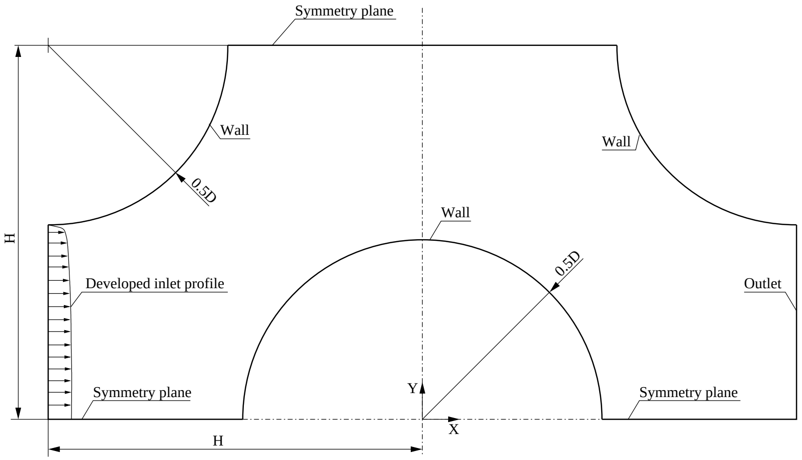
Wall Functions

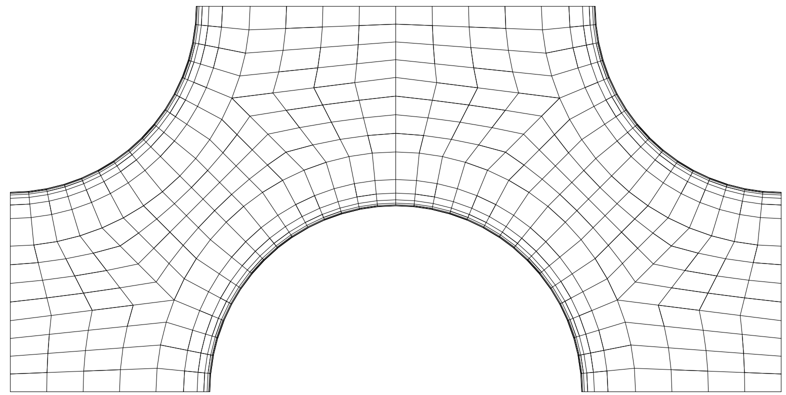
- In engineering simulations, we are typically not interested in the details of the near-wall region. Instead, we need to know the drag
- This allows us to bridge the troublesome region near the wall with a coarse mesh and replace it with an equilibrium model for attached flows: **wall functions**
- Wall functions bridge the problematic near-wall region, accounting for drag increase and turbulence. A typical resolution requirement is $y^+ = 30 - 50$, but coarser meshes can also be used
- This is a simple equilibrium model for fully developed attached boundary layer. It will cause loss of accuracy in non-equilibrium boundary layers, but it will still produce a result
- Wall functions split the region of operation below and above $y^+ = 11.6$ and revert to laminar flow for below it. Here, increased mesh resolution may result in less accurate drag prediction – this is not a well-behaved model
- Advanced wall functions may include effects of adverse pressure gradient and similar but are still a very crude model
- Note that wall functions are used with high-Re bulk turbulence models, reducing the need for high resolution next to the wall

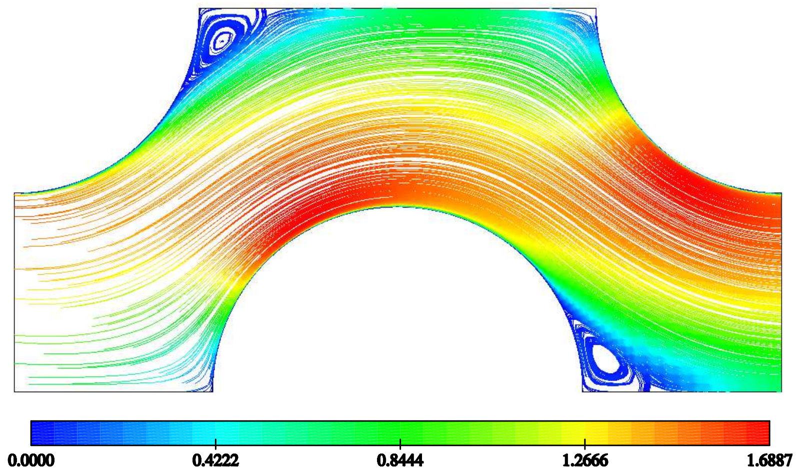
Low-Re Model

Example: Tube Bundle

- Repeating geometry: modelling only one segment with periodic boundary conditions

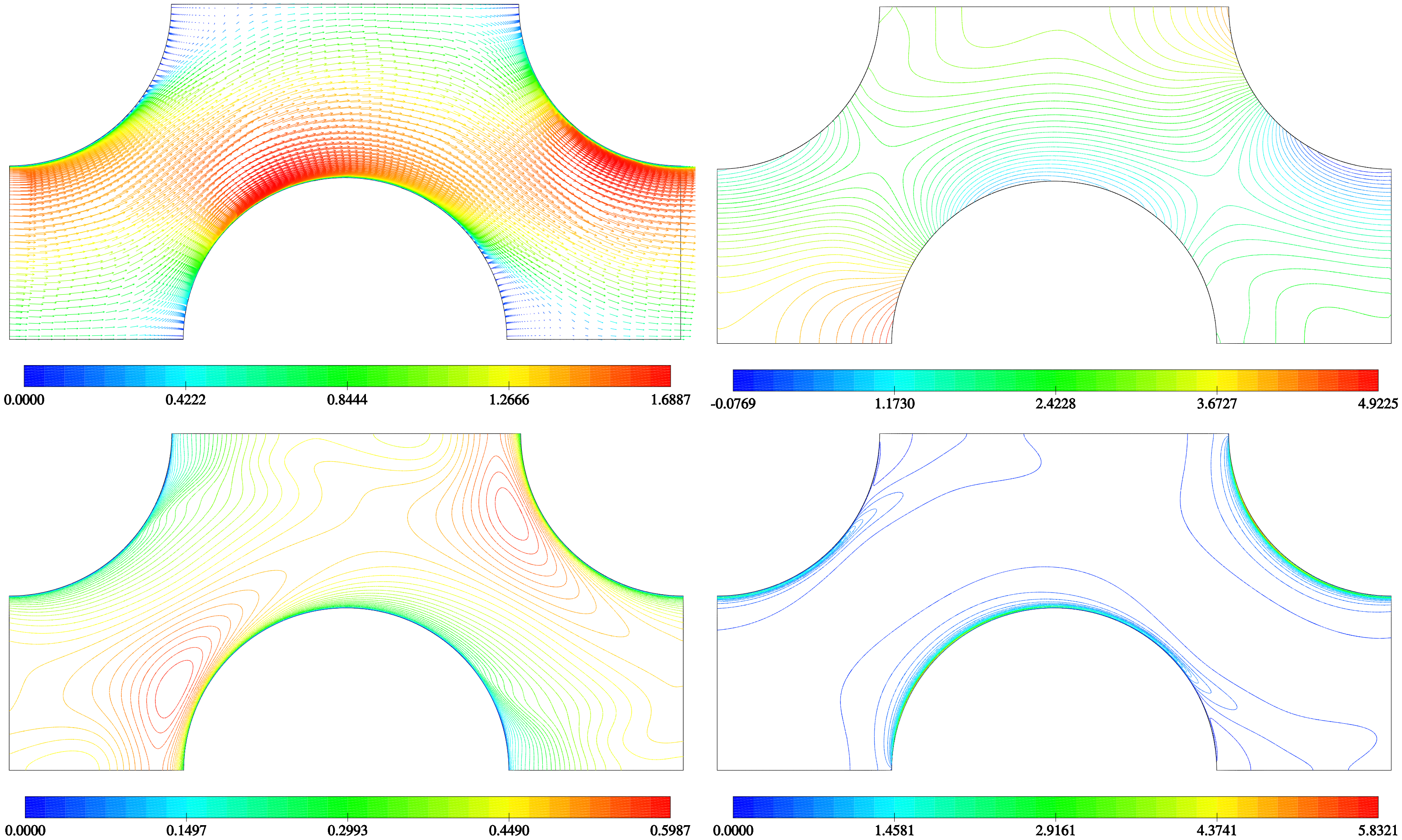






Low-Re Model

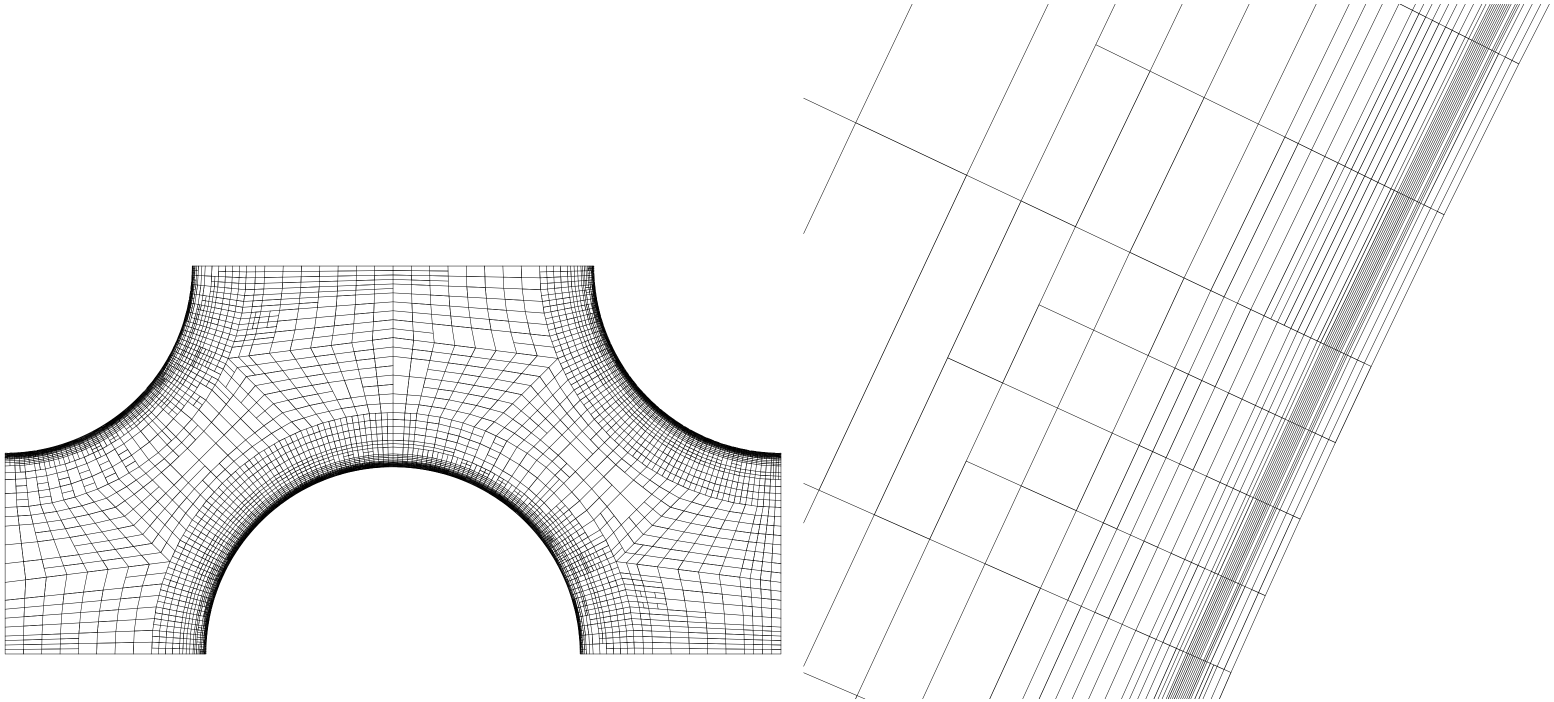
- In order to simplify the mesh resolution requirements, a special form of 2-equation model is used: $q = \sqrt{k}$, and ζ is its dissipation.



Low-Re Model

Example: Tube Bundle

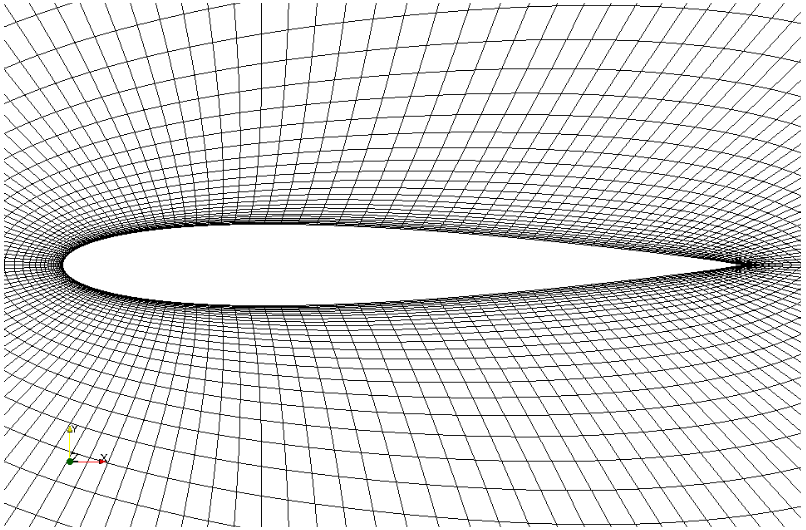
- Mesh resolution near the wall captures the high gradients
- Note elongated cells and possible high discretisation errors

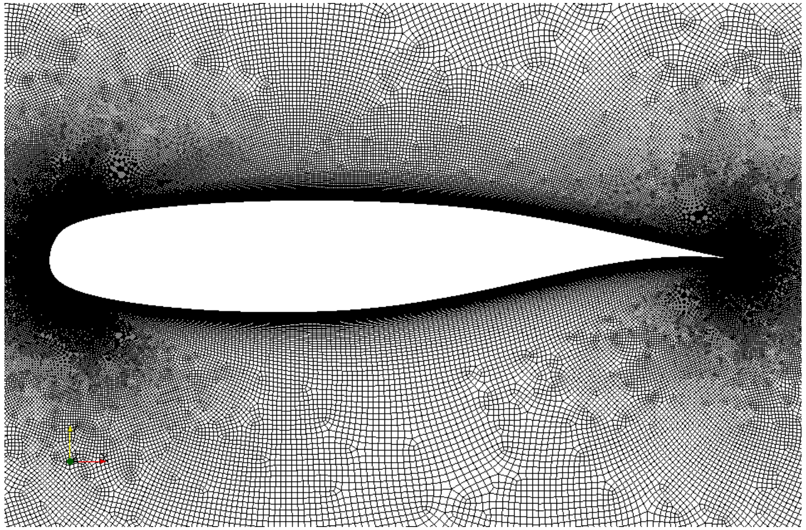


Near-Wall Effects

What Can a Low-Re Model Do For Me?

- With decreasing Re number, turbulence energy spectrum loses its inertial range and regularity: energy is not moved smoothly from larger scales to smaller; importance of dissipation spreads to lower wavenumber
- Low-Re models are aimed at capturing the **details of the near-wall flow**, characterised by lower Re
- However, near-wall turbulence is nothing like low-Re bulk flow: this is to do with the presence and effect of the wall, not the loss of turbulence structure
- A low-Re turbulence model is **not appropriate** for low-Re flows away from the wall: the results will be wrong!





Transient RANS Simulations

Concept of Transient RANS

- RANS equations are derived by separating the variable into the mean and fluctuation around it. In simple situations, this implies a well-defined meaning: mean is (well,) mean – implying time-independence and the fluctuation carries the transient component
- In many physical simulations, having a time-independent mean makes no sense: consider a flow simulation in an internal combustion engine. Here, we will change the mean into a **ensemble average** (over a number of identical experiments) and allow the mean to be time-dependent
- In other cases, the difference between the mean and fluctuation may become even more complex: consider a vortex shedding behind a cylinder at high Re, where large shed vortices break up into turbulence further downstream
- Idea of RANS here is recovered through **separation of scales**, where large scales are included in the time-dependence of the mean and turbulence is modelled as before. It is postulated that there exists **separation of scales** between the mean (= **coherent structures**) and turbulence

Transient RANS Simulations

Using Transient RANS

- Transient RANS is a great step forward in the fidelity of modelling. Consider a flow behind an automobile, with counter-rotating vortices in the wake and various other unsteady effects. Treating it as “steady” implies excessive damping, typically done through first-order numerics because the simulation does not naturally converge to steady-state
- Simulations can still be 2-D where appropriate and the answer is typically analysed in terms of a mean and coherent structure behaviour
- RANS equations are assembled as before, using a transient Navier-Stokes simulations. Usually, no averaging is involved

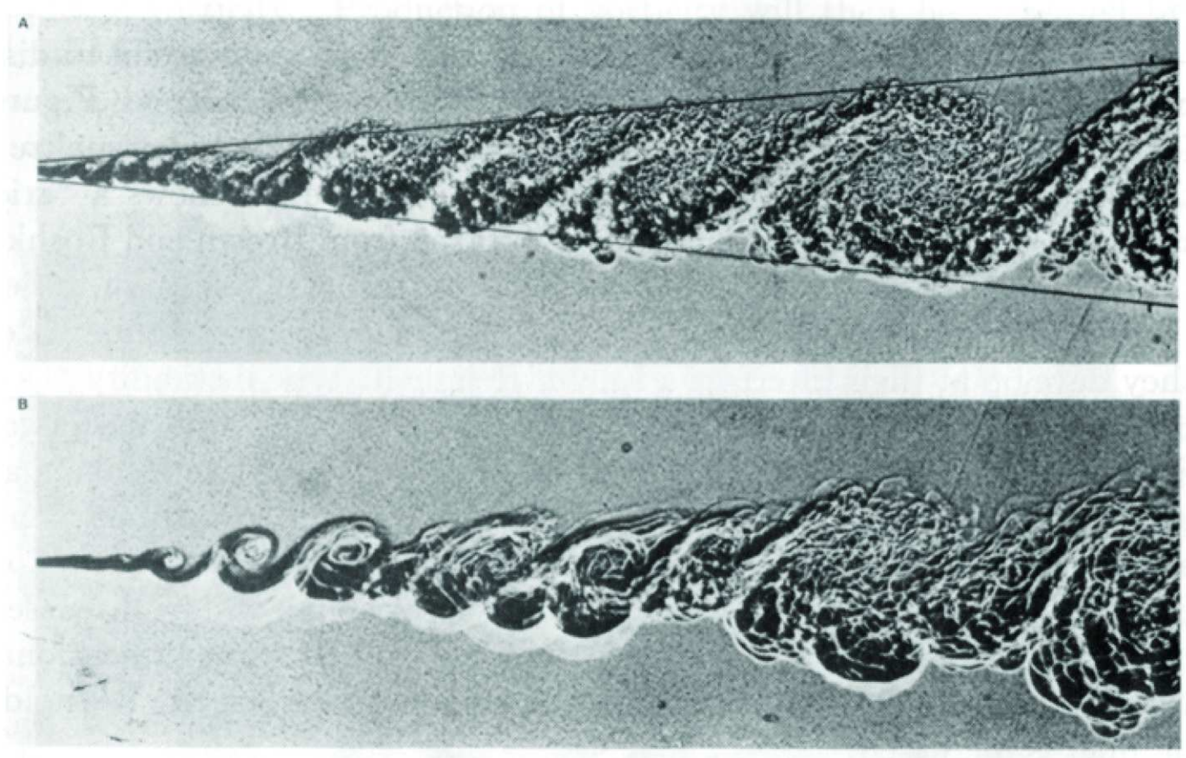
Transitional Flows

- Phenomena of transition are extremely difficult to model: as shown before, a low-Re turbulence model would be a particularly bad choice
- The flow consists of a mixture of laminar pockets and various levels of turbulence, with laminar-to-turbulent transition within it
- Apart from the fact that a low-Re flow is difficult to model in RANS, additional problem stems from the fact that $k = \epsilon = 0$ is the solution to the model: thus if no initial or boundary turbulence is given, transition will not take place
- Introducing intermittency equation: to handle this a RANS model is augmented by an equation marking presence of turbulence
- Transition models are hardly available: basically, a set of correlations is packed as transport equations. Details of proper boundary conditions, posedness of the model, user-controlled parameters and model limitations are badly understood. A better approach is needed!

Large Eddy Simulation

Filtered Navier-Stokes Equations

- Reynolds averaging is not always appropriate: some physics may be lost when the fluctuation magnitude is substantial relative to the mean
- Idea: filter the Navier-Stokes equations to some scale. Vortices smaller than the filter scale are resolved and larger-scale turbulence and coherent structures will be simulated



Large Eddy Simulation

Deriving LES Equations

- Idea of LES comes from the fact that large-scale turbulence strongly depends on the mean, geometry and boundary conditions, making it case-dependent and difficult to model. Small-scale turbulence is close to homogenous and isotropic, its main role is energy removal from the system, it is almost universal (Re dependence) and generally not of interest
- Mesh resolution requirements are imposed by the **small scales**, which are not of interest anyway
- In LES we shall therefore **simulate the coherent structures and large-scale turbulence and model small-scale effects**
- For this purpose, we need to make the equations understand scale, using **equation filtering**: a variable is decomposed into large scales which are solved for and modelled small scales. To help with the modelling, we wish to capture a part of the inertial range and model the (universal) high wavenumber part of the spectrum
- Unlike transient RANS, a LES simulation still captures a part of turbulence dynamics: a simulation must be 3-D and transient, with the results obtained by averaging

Large Eddy Simulation

Filtered Navier-Stokes Equations

- Equation averaging is mathematically defined as:

$$\overline{\mathbf{u}} = \int G(\mathbf{x}, \mathbf{x}')\, \mathbf{u}(\mathbf{x}') d\mathbf{x}'$$

  where $G(\mathbf{x}, \mathbf{x}')$ is the localised **filter function**

- Various forms of the filter functions can be used: local Gaussian distribution, top-hat etc. with minimal differences. The important principle is localisation
- After filtering, the equation set looks very similar to RANS, but the meaning is considerably different

$$\frac{\partial \overline{\mathbf{u}}}{\partial t} + \nabla \bullet (\overline{\mathbf{u}}\, \overline{\mathbf{u}}) - \nabla \bullet \left(\nu \nabla \overline{\mathbf{u}}\right) = -\nabla \overline{p} + \nabla \bullet \boldsymbol{\tau}$$

$$\nabla \bullet \overline{\mathbf{u}} = 0$$

with

$$\boldsymbol{\tau} = (\overline{\overline{\mathbf{u}}\, \overline{\mathbf{u}}} - \overline{\mathbf{u}}\, \overline{\mathbf{u}}) + (\overline{\overline{\mathbf{u}}\, \mathbf{u}'} + \overline{\mathbf{u}',\ \overline{\mathbf{u}}}) + \overline{\mathbf{u}'\, \mathbf{u}'} = \mathbf{L} + \mathbf{C} + \mathbf{B}$$

Large Eddy Simulation

Filtered Navier-Stokes Equations

- The first term, $\mathbf{L}$ is called the **Leonard stress**. It represents the interaction between two resolved scale eddies to produce small scale turbulence
- The second term, $\mathbf{C}$ (**cross term**), contains the interaction between resolved and small scale eddies. It can transfer energy in either direction but on average follows the energy cascade
- The third term represents interaction between two small eddies to create a resolved eddy. $\mathbf{B}$ (**backscatter**) represents energy transfer from small to large scales

Large Eddy Simulation

Sub-Grid Scale (SGS) Modelling

- The scene in LES has been set to ensure that single turbulence models work well: small-scale turbulence is close to homogenous and isotropic
- The length-scale is related to the separation between resolved and unresolved scales: therefore, it is related to the filter width
- In LES, **implicit filtering** is used: separation between resolved and unresolved scales depends on mesh resolution. Filter size is therefore calculated as a measure of mesh resolution and results are interpreted accordingly
- Typical models in use are of **Smagorinsky model** type, with the fixed or dynamic coefficients. In most models, all three terms are handled together
- Advanced models introduce some transport effects by solving a sub-grid k-equation, use double filtering to find out more about sub-grid scale or create a “structural picture” of sub-grid turbulence from resolved scales
- Amazingly, most models work very well: it is only important to remove the correct amount of energy from resolved scales

Large Eddy Simulation

LES Inlet and Boundary Conditions

- In the past, research on LES has centred on sub-grid scale modelling and the problem can be considered to be resolved
- Two problematic areas in LES are the inlet conditions and near-wall treatment
- **Modelling near-wall turbulence** A basic assumption of LES is energy transfer from large towards smaller scales, with the bulk of dissipation taking place in small vortices. Near the wall, the situation is reversed: small vortices and streaks are rolled up on the wall and ejected into the bulk
  - Reversed direction of the energy cascade violates the modelling paradigm. In principle, the near-wall region should be resolved in full detail, with massive resolution requirements
  - A number of modelling approaches to overcome the problem exists: structural SGS models (guessing the sun-grid scale flow structure), dynamic SGS models, approaches inspired by the wall function treatment and **Detached Eddy Simulation**
- **Inlet boundary condition**. On inlet boundaries, flow conditions are typically known in the mean, or (if we are lucky) with $\mathbf{u}'$ and turbulence length-scale. An important property of turbulence in the energy cascade: correlation between various scales and vortex structures. The inlet condition should contain the “real” turbulence interaction and it is not immediately clear how to do this

Large Eddy Simulation

Energy-Conserving Numerics

- For accurate LES simulation, it is critical to correctly predict the amount of energy removal from resolved scale into sub-grid. This is the role of a SGS model
- In order for the SGS model to perform its job, it is critical that the rest of implementation does not introduce dissipative errors: we need **energy-conserving numerics**
- Errors introduced by spatial and temporal discretisation must not interfere with the modelling
- In short, good RANS numerics is not necessarily sufficient for LES simulations. In RANS, a desire for steady-state and performance of RANS models masks poor numerics; in LES this is clearly not the case

Large Eddy Simulation

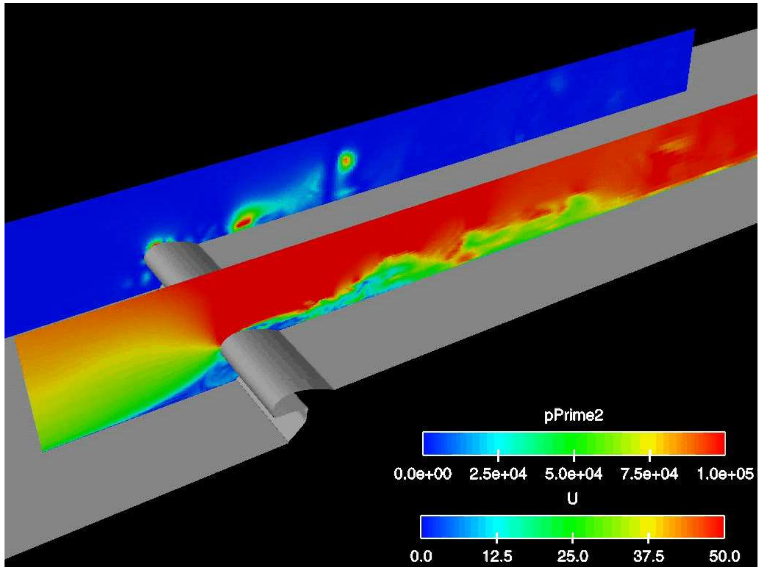
Averaging and Post-Processing

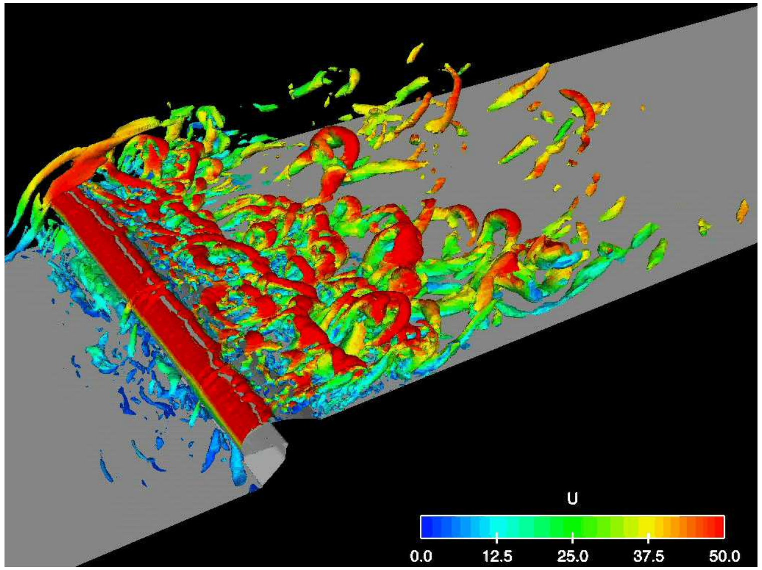
- Understanding LES results is different than looking at steady or transient RANS: we have at disposal a combination of instantaneous fields and averaged results
- Resolved LES fields contain a combination of mean (in the RANS sense) and large-scale turbulence. Therefore, it is extremely useful in studying the details of flow structure
- The length of simulation, number of averaging steps etc. is studied in terms of **converging averages**: for statistically steady simulations, averages must converge!
- A good LES code will provide a set of on-the-fly averaging tools to assemble data of interest during the run
- Flow instability and actual vortex dynamics will be more visible in the instantaneous field
- Data post-processing
    - It is no longer trivial to look and understand the LES results, especially in terms of **vortex interaction**: we typically use special derived fields, *e.g.* enstrophy (magnitude of curl of velocity), invariants of the strain tensor etc.
    - Looking at LES results takes some experience and patience: data sets will be very large

Example: Large Eddy Simulation

LES for Aero-Acoustic Noise Generation on a Rain Gutter

- 3-D and transient simulation, incompressible Navier-Stokes model, segregated pressure-velocity solver
- Sufficient mesh resolution to capture energy-containing scales of turbulence. Efficient linear equation solvers and massive parallelism
- Second-order energy-conserving numerics in space and time
- Sub-grid scale model library: 15 models on run-time selection
- Field mapping and data analysis tools, *e.g.* inlet data mapping for realistic inlet turbulence; on-the-fly sampling and averaging

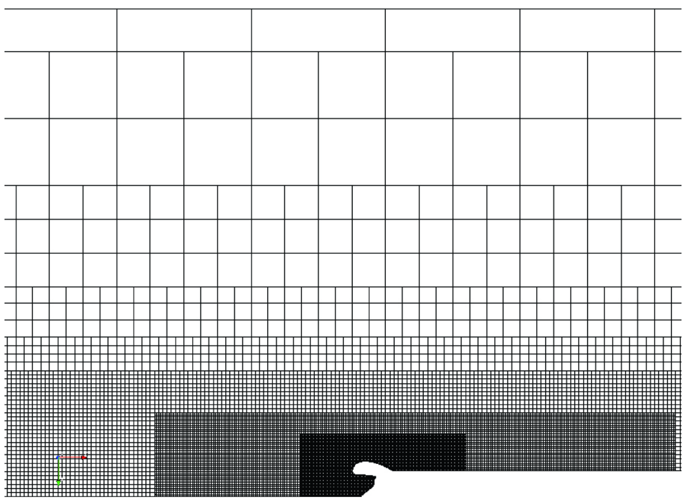


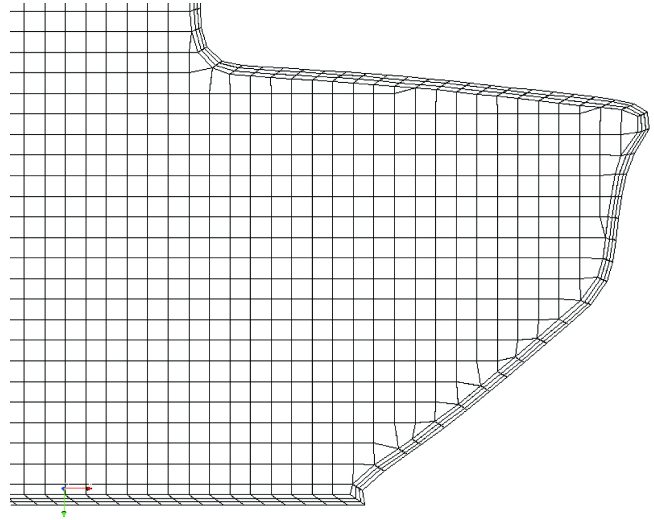


Example: Large Eddy Simulation

LES for Aero-Acoustic Noise Generation on a Rain Gutter

- Small-scale turbulence is much easier to model than the large scale: isotropic and homogenous. Its main role is removal of energy from the mean flow
- Mesh size features in the turbulence model: we cannot resolve vortices smaller that the mesh size. In modern LES, cell size is not necessarily equal to filter width, to ensure “mesh independent solution”
- Mesh resolution should be adjusted to expected size of local flow features





Choosing a Turbulence Model

Turbulence Models in Airfoil Simulations

- Simulations typically done in steady-state and 2-D
- Objective of simulation is mainly lift/drag and stall characteristics
- This automatically implies 2-D steady-state RANS. Moreover, region of interest is close to the surface of the airfoil; the bulk flow is simple
- Presence of the wall allows for simple prescription of length-scale

New Challenges

- Laminar-to turbulent transition occurs along the airfoil; in multiple airfoil configuration, upstream components trigger transition downstream
- In order to handle transition, new models are being developed (currently: useless!)
- Problematic region is also found around the trailing edge: flow detachment
- LES is very expensive: from steady-state 2-D RANS to unsteady 3-D + averaging

Bluff-Body Aerodynamics

- Bluff body flows (*e.g.* complete aircraft, automobile, submarine) are considerably more complex, both in the structure of boundary layers and in the wake
- Standard choice: 2-equation RANS with wall functions. Currently moving to transient RANS

Choosing a Turbulence Model

Choice of Models

- $k - \epsilon$ model and its variants; $k - \omega$ model represent normal industrial choice. There are still issues with mesh resolution for full car/aeroplane aerodynamics: meshes for steady RANS with wall functions can be of the order of 100 million cells and larger
- Low-Re formulations wall-bounded flows is not popular: excessive mesh resolution for realistic geometric shapes
- Study of instabilities and aero-acoustic effects in moving steadily to LES. Typically, only a part of the geometry is modelled and coupled to the global (RANS) model for boundary conditions. Examples: bomb bay in aeroplanes or wing mirrors in automobiles

Current Fashion in Turbulence Modelling

- Spalart-Allmaras 1-equation model
- $k - \omega$ Shear Stress Transport (SST) 2-equation eddy viscosity model

Turbulence Model Review

Spalart-Allmaras Model

- Spalart-Allmaras model is a one equation model which solves a transport equation for a viscosity-like variable $\tilde{\nu}$

$$\nu_t = \tilde{\nu} f_{v1} \quad f_{v1} = \frac{\chi^3}{\chi^3 + C_{v1}^3} \quad \chi = \frac{\tilde{\nu}}{\nu}$$

$$\frac{\partial \tilde{\nu}}{\partial t} + \nabla \bullet (\mathbf{u} \tilde{\nu}) - \frac{1}{\sigma} \nabla \bullet [(\nu + \tilde{\nu}) \nabla \tilde{\nu}] = C_{b1} [1 - f_{t2}] \tilde{S} \tilde{\nu} + \frac{1}{\sigma} C_{b2} |\nabla \nu|^2 - \left[C_{w1} f_w - \frac{C_{b1}}{\kappa^2} f_{t2} \right] \left(\frac{\tilde{\nu}}{d} \right)^2 + f_{t1} \Delta U^2$$

$$\tilde{S} = S + \frac{\tilde{\nu}}{\kappa^2 d^2} f_{v2} \quad f_{v2} = 1 - \frac{\chi}{1 + \chi f_{v1}}$$

$$S = \sqrt{2 \Omega \bullet \Omega} \quad \Omega = \frac{1}{2} [\nabla \mathbf{u} - (\nabla \mathbf{u})^T] \quad f_w = g \left[\frac{1 + C_{w3}^6}{g^6 + C_{w3}^6} \right]^{1/6}$$

$$g = r + C_{w2}(r^6 - r) \quad r = \frac{\tilde{\nu}}{\tilde{S} \kappa^2 d^2}$$

Turbulence Model Review

Spalart-Allmaras Model

- Model is originally written as an airfoil-specific RANS model
- Solution method is compact and numerically stable
- Originally written as a RANS model; can be used in LES by manipulating d: instead of using distance to the nearest wall, a LES-type filtering is prescribed
- Since the model can be used both in RANS and LES mode, it is ideal for hybrid simulations

Turbulence Model Review

$k - \omega$ SST Model

- Developed by Menter (1993-) it is a two-equation eddy-viscosity model
- In the inner parts of the boundary layer it uses original formulation by Wilcox
- Eliminates sensitivity to free-stream conditions by “blending” into standard $k - \epsilon$ model, reformulated in terms of ω

$$\nu_T = \frac{a_1 k}{max(a_1 \omega, S\, F_2)}$$

$$\frac{\partial k}{\partial t} + \nabla\bullet(\mathbf{u}k) - \nabla\bullet[(\nu + \sigma_k \nu_T)\nabla k] = P_k - \beta^* k\omega$$

$$\frac{\partial \omega}{\partial t} + \nabla\bullet(\mathbf{u}\omega) - \nabla\bullet[(\nu + \sigma_\omega \nu_T)\nabla \omega] = \alpha S^2 - \beta\omega^2 + 2(1 - F_1)\sigma_{\omega,2}\frac{1}{\omega}\nabla k\bullet\nabla\omega$$

$$F_2 = tanh\left[\left[max\left(\frac{2\sqrt{k}}{\beta^*\omega y}, \frac{500\nu}{y^2\omega}\right)\right]^2\right] \qquad P_k = min\left(\tau : \nabla\mathbf{u}, 10\beta^* k\omega\right)$$

Turbulence Model Review

$k - \omega$ SST Model

$$F_1 = tanh \left\{ \left\{ min \left[max \left(\frac{\sqrt{k}}{\beta * \omega y}, \frac{500\nu}{y^2 \omega} \right), \frac{4\sigma_{\omega 2} k}{CD_{k\omega} y^2} \right] \right\}^4 \right\}$$

$$CD_{k\omega} = max \left(2\rho\sigma_{\omega 2} \frac{1}{\omega} \nabla k \,\mathbf{:}\, \nabla \omega, 10^{-10} \right)$$

- In the inner parts of the boundary layer it uses original formulation by Wilcox
- Eliminates sensitivity to free-stream conditions by “blending” into standard $k - \epsilon$ model, reformulated in terms of ω
- Using the ω equation next to the wall means the model can (“should be able to”) operate without wall functions, in the low-Re regime

Future of Turbulence Modelling

Future of Turbulence Modelling in Industrial Applications

- Future trends are quite clear: moving from the RANS modelling to LES on a case-by-case basis and depending on problems with current models and available computer resources
- RANS is recognised as insufficient in principle because the decomposition into mean and fluctuation. Also, models are too diffusive to capture detailed flow dynamics. Research in RANS is scaled down to industrial support; everything else is moving to LES
- Transient RANS is a stop-gap solution until LES is not available at reasonable cost
- DNS remains out of reach for all engineering use, but provides a very good base for model development and testing