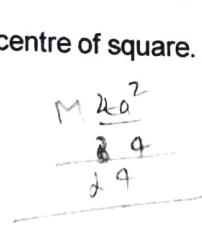
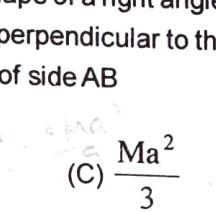
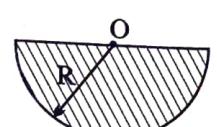
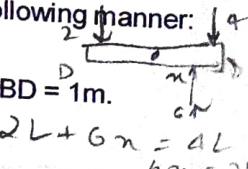
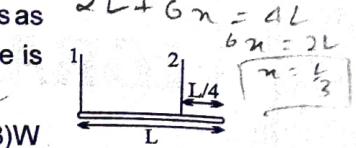


# EXERCISE-I

**Only one option is correct**

- Three bodies have equal masses  $m$ . Body A is solid cylinder of radius  $R$ , body B is a square lamina of side  $R$ , and body C is a solid sphere of radius  $R$ . Which body has the smallest moment of inertia about an axis passing through their centre of mass and perpendicular to the plane (in case of lamina)?
   
 (A) A   
 (B) B   
 (C) C   
 (D) A and C both
- For the same total mass which of the following will have the largest moment of inertia about an axis passing through its centre of mass and perpendicular to the plane of the body?
   
 (A) a disc of radius  $a$    
 (B) a ring of radius  $a$    
 (C) a square lamina of side  $2a$    
 (D) four rods forming a square of side  $2a$
- Let  $I_1$  and  $I_2$  be the moment of inertia of a uniform square plate about axes APC and OPO' respectively as shown in the figure. P is centre of square. The ratio  $\frac{I_1}{I_2}$  of moment of inertia is
   
 (A)  $\frac{1}{\sqrt{2}}$    
 (B) 2   
 (C)  $\frac{1}{2}$    
 (D) 1
   

- Find the moment of inertia of a plate cut in shape of a right angled triangle of mass  $M$ , side  $AC = BC = a$  about an axis perpendicular to the plane of the plate and passing through the mid point of side AB
   
 (A)  $\frac{Ma^2}{12}$    
 (B)  $\frac{Ma^2}{6}$    
 (C)  $\frac{Ma^2}{3}$    
 (D)  $\frac{2Ma^2}{3}$ 
  

- A thin uniform rod of mass  $M$  and length  $L$  has its moment of inertia  $I_1$  about its perpendicular bisector. The rod is bent in the form of a semicircular arc. Now its moment of inertia through the centre of the semi circular arc and perpendicular to its plane is  $I_2$ . The ratio of  $I_1 : I_2$  will be \_\_\_\_\_
   
 (A)  $< 1$    
 (B)  $> 1$    
 (C) = 1   
 (D) can't be said
- Moment of inertia of a thin semicircular disc (mass =  $M$  & radius =  $R$ ) about an axis through point O and perpendicular to plane of disc, is given by :
   
 (A)  $\frac{1}{4} MR^2$    
 (B)  $\frac{1}{2} MR^2$    
 (C)  $\frac{1}{8} MR^2$    
 (D)  $MR^2$ 
  

- A weightless rod is acted on by upward parallel forces of  $2N$  and  $4N$  ends A and B respectively. The total length of the rod AB =  $3m$ . To keep the rod in equilibrium a force of  $6N$  should act in the following manner:
   
 (A) Downwards at any point between A and B.   
 (B) Downwards at mid point of AB.   
 (C) Downwards at a point C such that  $AC = 1m$ .   
 (D) Downwards at a point D such that  $BD = 1m$ .
   

- A heavy rod of length  $L$  and weight  $W$  is suspended horizontally by two vertical ropes as shown. The first rope is attached to the left end of the rod while the second rope is attached a distance  $L/4$  from right end. The tension in the second rope is:
   
 (A)  $(1/2)W$    
 (B)  $(1/4)W$    
 (C)  $(1/3)W$    
 (D)  $(2/3)W$ 
  
 (E)  $W$ 
  


9. A uniform cube of side 'b' and mass M rest on a rough horizontal table. A horizontal force F is applied normal to one of the faces at a point, at a height  $3b/4$  above the base. What should be the coefficient of friction ( $\mu$ ) between cube and table so that it will tip about an edge before it starts slipping?

(A)  $\mu > \frac{2}{3}$

(B)  $\mu > \frac{1}{3}$

(C)  $\mu > \frac{3}{2}$

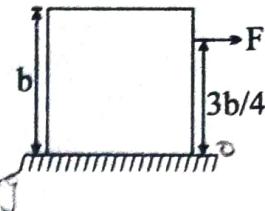
(D) none

$$f = f_s$$

$$F = \mu Mg$$

$$\rightarrow F(\frac{3b}{4}) = mg \frac{b}{2}$$

$$F = \frac{2Mg}{3} = \mu Mg$$



$$Mg \frac{3}{4} > M \cdot \frac{2}{3} b$$

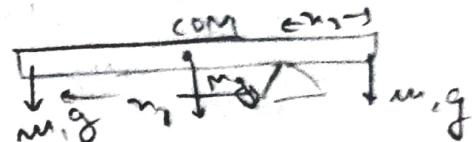
10. A heavy seesaw (i.e., not massless) is out of balance. A light girl sits on the end that is tilted downward, and a heavy body sits on the other side so that the seesaw now balances. If they both move forward so that they are one-half their original distance from the pivot point (the fulcrum) what will happen to the seesaw?

(A) The side the body is sitting on will tilt downward

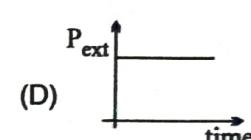
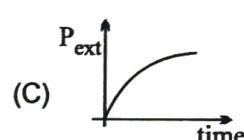
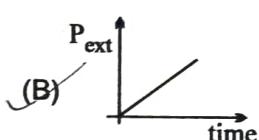
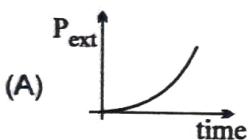
(B) The side the girl is sitting on will once again tilt downward

(C) Nothing ; the seesaw will still be balanced

(D) It is impossible to say without knowing the masses and the distances



11. A rod is hinged at its centre and rotated by applying a constant torque starting from rest. The power developed by the external torque as a function of time is :



12. A pulley is hinged at the centre and a massless thread is wrapped around it. The thread is pulled with a constant force F starting from rest. As the time increases,

(A) its angular velocity increases, but force on hinge remains constant

(B) its angular velocity remains same, but force on hinge increases

(C) its angular velocity increases and force on hinge increases

(D) its angular velocity remains same and force on hinge is constant



13. A man, sitting firmly over a rotating stool has his arms stretched. If he folds his arms, the work done by the man is

(A) zero

(B) positive

(C) negative

(D) may be positive or negative.

$$T_{\text{ext}} = \frac{1}{2} I \omega^2 = \left( \frac{1}{2} I \omega \right)^2 \text{ const}$$

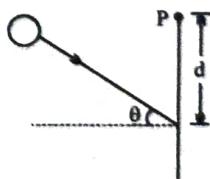
14. A ball of mass m moving with velocity v, collide with the wall elastically as shown in the figure. After impact the change in angular momentum about P is:

(A)  $2mvd$

(B)  $2mv d \cos\theta$

(C)  $2mv d \sin\theta$

(D) zero



15. Two uniform spheres of mass M have radii R and  $2R$ . Each sphere is rotating about a fixed axis through a diameter. the rotational kinetic energies of the spheres are identical. What is the ratio of the magnitude of the

use  
 $I = \frac{1}{2} I^2$   
angular momenta of these spheres? That is,  $\frac{L_{2R}}{L_R} =$

$$\frac{I_{2R} \omega_2}{I_R \omega_1} = \frac{\frac{1}{2} (2R)^2 \omega_2}{\frac{1}{2} R^2 \omega_1}$$

(A) 4

(B)  $2\sqrt{2}$

(C) 2

(D)  $\sqrt{2}$

(E) 1

16. A spinning ice skater can increase his rate of rotation by bringing his arms and free leg closer to his body. How does this procedure affect the skater's angular momentum and kinetic energy?

- (A) angular momentum remains the same while kinetic energy increases  
 (B) angular momentum remains the same while kinetic energy decreases  
 (C) both angular momentum and kinetic energy remain the same  
 (D) angular momentum increases while kinetic energy remains the same  
 (E) both angular momentum and kinetic energy increase

$$I \downarrow \omega \uparrow I = \text{const}$$

$$K_E = \frac{1}{2} I \omega^2$$

$$= \frac{1}{2} (I + \omega) \omega^2$$

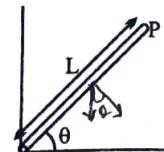
$$\uparrow \quad \text{const}$$

17. A uniform flag pole of length L and mass M is pivoted on the ground with a frictionless hinge. The flag pole makes an angle  $\theta$  with the horizontal. The moment of inertia of the flag pole about one end is  $(1/3)ML^2$ . If it starts falling from the position shown in the accompanying figure, the linear acceleration of the free end of the flag pole — labeled P — would be:

- (A)  $(2/3)g\cos\theta$   
 (B)  $(2/3)g$   
 (C)  $g$   
 (D)  $(3/2)g\cos\theta$   
 (E)  $(3/2)g$

$$mg \cos\theta \frac{L}{2} = \frac{1}{3} I \omega^2 L$$

$$3g \cos\theta = d \omega^2$$



$$a_{P\perp} = d\omega^2 = \frac{3g \cos\theta}{2}$$

18. A mass  $m$  is moving at speed  $v$  perpendicular to a rod of length  $d$  and mass  $M = 6m$  which pivots around a frictionless axle running through its centre. It strikes and sticks to the end of the rod. The moment of inertia of the rod about its centre is  $Md^2/12$ . Then the angular speed of the system right after the collision is

- (A)  $2v/d$       (B)  $2v/(3d)$       (C)  $v/d$       (D)  $3v/(2d)$

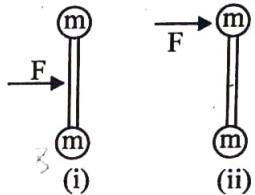
19. The moment of inertia of a solid cylinder about its axis is given by  $(1/2)MR^2$ . If this cylinder rolls without slipping, the ratio of its rotational kinetic energy to its translational kinetic energy is

- (A)  $1 : 1$       (B)  $2 : 2$       (C)  $1 : 2$       (D)  $1 : 3$

$$\frac{\frac{1}{2}(I + M R^2) v^2}{\frac{1}{2} M v^2} = \frac{1}{2}$$

20. A force  $F$  is applied to a dumbbell for a time interval,  $t$ , first as in (i) and then as in (ii). In which case does the dumbbell acquire the greater centre-of-mass speed?

- (A) (i)  
 (B) (ii)  
 (C) there is no difference  
 (D) the answer depends on the rotational inertia of the dumbbell



21. A hoop and a solid cylinder have the same mass and radius. They both roll, without slipping, on a horizontal surface. If their kinetic energies are equal

- (A) the hoop has a greater translational speed than the cylinder  
 (B) the cylinder has a greater translational speed than the hoop  
 (C) the hoop and the cylinder have the same translational speed  
 (D) the hoop has a greater rotational speed than the cylinder

$$T_H = \frac{m v^2}{R} \quad T_S = \frac{m v^2}{2}$$

$$T_H > T_S \quad \text{for equal final } K_E$$

$$T_K = L^2 / R$$

22. A wheel of radius  $r$  rolling on a straight line, the velocity of its centre being  $v$ . At a certain instant the point of contact of the wheel with the ground is M and N is the highest point on the wheel (diametrically opposite to M). The incorrect statement is:

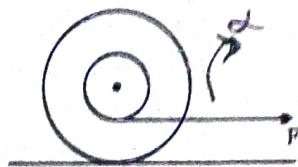
- (A) The velocity of any point P of the wheel is proportional to MP.  
 (B) Points of the wheel moving with velocity greater than  $v$  form a larger area of the wheel than points moving with velocity less than  $v$ .  
 (C) The point of contact M is instantaneously at rest.  
 (D) The velocities of any two parts of the wheel which are equidistant from centre are equal.



23. Inner and outer radii of a spool are  $r$  and  $R$  respectively. A thread is wound over its inner surface and placed over a rough horizontal surface. Thread is pulled by a force  $F$  as shown in fig. then in case of pure rolling

(A) Thread unwinds, spool rotates anticlockwise and friction act leftwards  
 (B) Thread winds, spool rotates clockwise and friction acts leftwards  
 (C) Thread winds, spool moves to the right and friction act rightwards  
 (D) Thread winds, spool moves to the right and friction does not come into existence

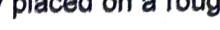






25. A solid sphere with a velocity (of centre of mass)  $v$  and angular velocity  $\omega$  is gently placed on a rough horizontal surface. The frictional force on the sphere:

(A) must be forward (in direction of  $v$ )      (B) must be backward (opposite to  $v$ )  
 (C) cannot be zero      (D) none of the above



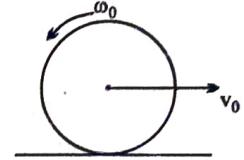


26. A cylinder is pure rolling up an incline plane. It stops momentarily and then rolls back. The force of friction  
(A) on the cylinder is zero throughout the journey  
(B) is directed opposite to the velocity of the centre of mass throughout the journey  
 (C) is directed up the plane throughout the journey  
(D) is directed down the plane throughout the journey

27. A uniform circular disc placed on a rough horizontal surface has initially a velocity  $v_0$  and an angular velocity  $\omega_0$  as shown in the figure. The disc comes to rest after moving some distance in the direction of motion. Then

$$\frac{v_0}{r\omega_0}$$

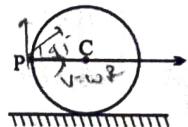
- (A)  $\frac{1}{2}$       (B) 1      (C)  $\frac{3}{2}$



28. A disc of radius  $R$  is rolling purely on a flat horizontal surface, with a constant angular velocity. The angle between the velocity and acceleration vectors of point P is

(A) zero  
 (B)  $45^\circ$   
 (C)  $135^\circ$   
 (D)  $\tan^{-1}(1/2)$





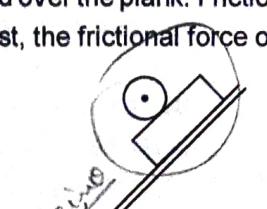
29. A solid uniform disk of mass  $m$  rolls without slipping down a fixed inclined plane with an acceleration  $a$ . The frictional force on the disk due to surface of the plane is :

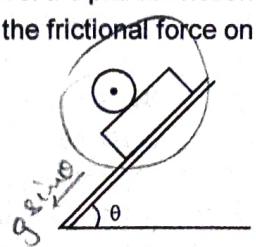
$$(A) 2 \text{ ma} \quad N(F_s) = \frac{Mg^2 \alpha}{2} \\ (C) \text{ ma } T_{\text{COM}} \Rightarrow F_s = \frac{Ma}{2}$$

- (B)  $\frac{3}{2}$  ma  
(D)  ~~$\frac{1}{2}$~~  ma

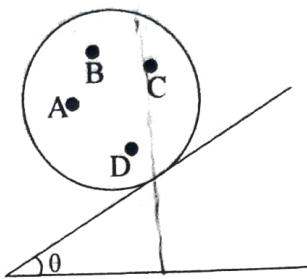
- ~~30.~~ A plank of mass  $M$  is placed over smooth inclined plane and a sphere is also placed over the plank. Friction is sufficient between sphere and plank. If plank and sphere are released from rest, the frictional force on sphere is

(A) up the plane  
 (B) down the plane  
 (C) horizontal  
 (D) zero





31. When the speed of a front wheel drive car is increasing on a horizontal road the direction of the frictional force on the tires is :  
 (A) forward for all tires  
 (B) backward for all tires  
 (C) forward for the front tires and backward for the rear tires  
 (D) backward for the front tires and forward for the rear tires
32. A non uniform sphere can be kept on a rough inclined plane so that it is in equilibrium. In the figure below the dots represents location of centre of mass. In which one of the positions can sphere be in equilibrium?



33. **Statement-1 :** The moment of inertia of a rigid body reduces to its minimum value as compared to any other parallel axis when the axis of rotation passes through its centre of mass.  
**Statement-2 :** The weight of a rigid body always acts through its centre of mass in uniform gravitational field. Of these statements:  
 (A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.  
 (B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.  
 (C) Statement-1 is true, statement-2 is false.  
 (D) Statement-1 is false, statement-2 is true.
34. **Statement-1 :** A body cannot roll on smooth horizontal surface.  
**Statement-2 :** When a body rolls purely, the point of contact should be at rest with respect to surface.  
 (A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.  
 (B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.  
 (C) Statement-1 is true, statement-2 is false.  
 (D) Statement-1 is false, statement-2 is true.
35. **Statement-1 :** The angular velocity of all the points on the rigid body as seen from any other point on it is the same.  
**Statement-2 :** The distance between any 2 points on the rigid body remains constant.  
 (A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.  
 (B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.  
 (C) Statement-1 is true, statement-2 is false.  
 (D) Statement-1 is false, statement-2 is true.
36. **Statement-1 :** KE of rotating rigid body in CM frame is  $\frac{1}{2} I_{cm} \omega^2$ , where symbols have usual meaning.  
**Statement-2 :** In CM frame rigid body can have only pure rotational motion.  
 (A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.  
 (B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.  
 (C) Statement-1 is true, statement-2 is false.  
 (D) Statement-1 is false, statement-2 is true.

37.

- Statement-1 :** A rigid disc rolls without slipping on a fixed rough horizontal surface with uniform angular velocity. Then the acceleration of lowest point on the disc is zero.  
**Statement-2 :** For a rigid disc rolling without slipping on a fixed rough horizontal surface, the velocity of the lowest point on the disc is always zero.

- (A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.  
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.  
(C) Statement-1 is true, statement-2 is false.  
(D) Statement-1 is false, statement-2 is true.

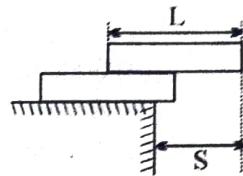
38. Two identical bricks of length L are piled one on top of the other on a table as shown in the figure. The maximum distance S the top brick can overhang the table with the system still balanced is

(A)  $\frac{1}{2}L$

(B)  $\frac{2}{3}L$

(C)  $\frac{3}{4}L$

(D)  $\frac{7}{8}L$



39. A thin hoop of weight 500 N and radius 1 m rests on a rough inclined plane as shown in the figure. The minimum coefficient of friction needed for this configuration is

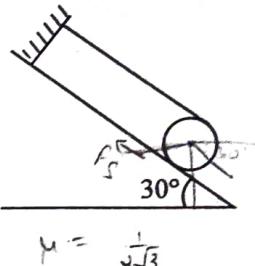
(A)  $\frac{1}{3\sqrt{3}}$

$F_S R = TR$   
 $F_S = T$

(C)  $\frac{1}{2}$

(D)  $\frac{1}{2\sqrt{3}}$

(B)  $\frac{1}{\sqrt{3}}$   
 $T + F_S = 500 \sin 30^\circ$   
 $T + F_S = 250$   
 $2F_S = 250$   
 $F_S = 125$   
 $\mu mg \cos 30^\circ = 125$   
 $\mu \cdot 4\sqrt{3} = 125$   
 $\mu = \frac{1}{\sqrt{3}}$



40. A uniform 2 kg cylinder rests on a laboratory cart as shown. The coefficient of static friction between the cylinder and the cart is 0.5. If the cylinder is 4 cm in diameter and 10 cm in height, which of the following is closest to the minimum acceleration of the cart needed to cause the cylinder to tip over?

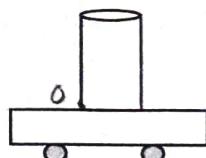
(A)  $2 \text{ m/s}^2$

(B)  $4 \text{ m/s}^2$

(C)  $5 \text{ m/s}^2$

(D)  $6 \text{ m/s}^2$

(E) the cylinder would slide at all of these accelerations



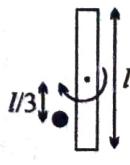
41. A uniform rod of length l and mass M rotating about a fixed vertical axis on a smooth horizontal table. It elastically strikes a particle placed at a distance  $l/3$  from its axis and stops. Mass of the particle is

(A)  $3M$

(B)  $\frac{3M}{4}$

(C)  $\frac{3M}{2}$

(D)  $\frac{4M}{3}$



42. A yo-yo is released from your hand with the string wrapped around your finger. If you hold your hand still, the acceleration of the yo-yo is

(A) downward, much greater than g

(C) upward, much less than g

(E) downward, at g

(B) downward, much less than g

(D) upward, much greater than g

$Mg - T = ma_{\text{yo-yo}}$

43. Statement-1 : A cyclist always bends inwards while negotiating a curve  
 Statement-2 : By bending he lowers his centre of gravity  
 Of these statements, *Not such big difference will be made by it* hence N will provide more centripetal force  
 (A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.  
 (B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.  
 (C) Statement-1 is true, statement-2 is false.  
 (D) Statement-1 is false, statement-2 is true.

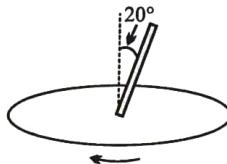
44. Statement-1 :  $\bar{L} = I\bar{\omega}$  is always true for bodies of all shapes.

Statement-2 :  $\bar{\tau} = \frac{d\bar{L}}{dt}$  is always true in inertial frames.

- (A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.  
 (B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.  
 (C) Statement-1 is true, statement-2 is false.  
 (D) Statement-1 is false, statement-2 is true.

#### Question No. 45 & 46 (2 questions)

A uniform rod is fixed to a rotating turntable so that its lower end is on the axis of the turntable and it makes an angle of  $20^\circ$  to the vertical. (The rod is thus rotating with uniform angular velocity about a vertical axis passing through one end.) If the turntable is rotating clockwise as seen from above.



45. What is the direction of the rod's angular momentum vector (calculated about its lower end)?  
 (A) vertically downwards  
 (B) down at  $20^\circ$  to the horizontal  
 (C) up at  $20^\circ$  to the horizontal  
 (D) vertically upwards

46. Is there a torque acting on it, and if so in what direction?  
 (A) yes, vertically  
 (B) yes, horizontally  
 (C) yes at  $20^\circ$  to the horizontal  
 (D) no

#### One or More than one option may be correct

47. A ring rolls without slipping on the ground. Its centre C moves with a constant speed u. P is any point on the ring. The speed of P with respect to the ground is v.  
 (A)  $0 \leq v \leq 2u$   
 (B)  $v = u$ , if CP is horizontal  
 (C)  $v = u$ , if CP makes an angle of  $30^\circ$  with the horizontal and P is below the horizontal level of C.  
 (D)  $v = \sqrt{2}u$ , if CP is horizontal
48. A plank with a uniform sphere placed on it rests on a smooth horizontal plane. Plank is pulled to right by a constant force F. If sphere does not slip over the plank. Which of the following is correct.  
 (A) Acceleration of the centre of sphere is less than that of the plank.  
 (B) Work done by friction acting on the sphere is equal to its total kinetic energy.  
 (C) Total kinetic energy of the system is equal to work done by the force F  
 (D) None of the above

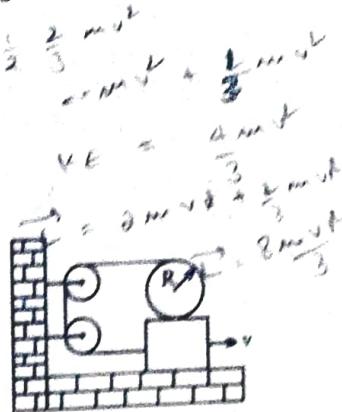


49.

- A hollow sphere of radius  $R$  and mass  $m$  is fully filled with non viscous liquid of mass  $m$ . It is rolled down a horizontal plane such that its centre of mass moves with a velocity  $v$ . If it purely rolls
- Kinetic energy of the sphere is  $5/6 mv^2$
  - Kinetic energy of the sphere is  $4/5 mv^2$
  - Angular momentum of the sphere about a fixed point on ground is  $8/3 mvR$
  - Angular momentum of the sphere about a fixed point on ground is  $14/5 mvR$

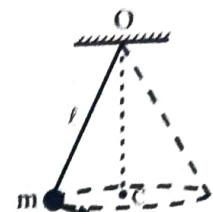
50.

- In the figure shown, the plank is being pulled to the right with a constant speed  $v$ . If the cylinder does not slip then:
- the speed of the centre of mass of the cylinder is  $2v$ .
  - the speed of the centre of mass of the cylinder is zero.
  - the angular velocity of the cylinder is  $v/R$ .
  - the angular velocity of the cylinder is zero.

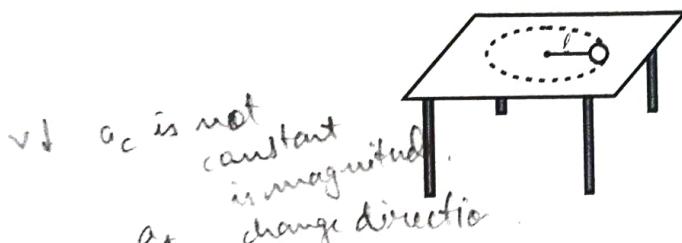


51.

- A small ball of mass  $m$  suspended from the ceiling at a point  $O$  by a thread of length  $\ell$  moves along a horizontal circle with a constant angular velocity  $\omega$ .
- angular momentum is constant about  $O$
  - angular momentum is constant about  $C$
  - vertical component of angular momentum about  $O$  is constant
  - Magnitude of angular momentum about  $O$  is constant



52. A disc is moving on a rough horizontal table in a circular path being tied to a nail by a string. While the disc is in motion, match the quantities in column-I with their description in column-II.



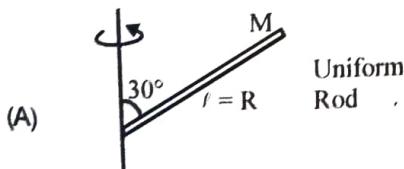
**Column-I**

- Acceleration of disc  $R$
- Friction force on disc  $P$
- Angular momentum of the particle  $\alpha$
- Normal force by table  $S$

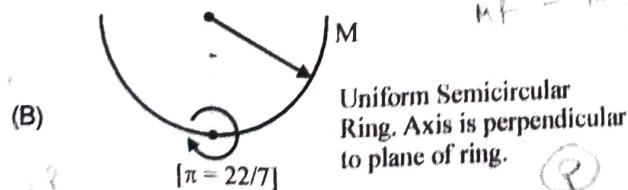
**Column-II**

- Changes direction but constant in magnitude
- Changes magnitude but constant in direction
- Changes direction as well as magnitude
- Direction as well as the magnitude is constant

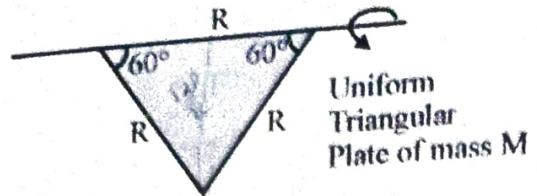
53. **Column-I**



$$(P) \frac{8MR^2}{11}$$



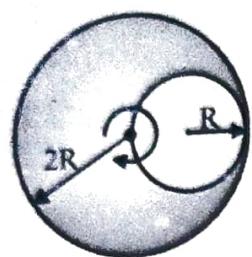
$$(Q) \frac{MR^2}{12}$$



(C)

(R)

$$\frac{13MR^2}{8}$$



(D)

Uniform disk  
of initial mass  $M$  from  
which circular  
portion of radius  $R$   
is then removed.  
M.I. of remaining mass  
about axis which is  
perpendicular  
to plane of plate

(S)

$$\frac{MR^2}{8}$$

54. An object is allowed to roll down the incline starting from rest. All are uniform and have same mass and radius

**Column-I**

- (A) The object which has largest rotational inertia about its axes of symmetry
- (B) The object which will experience the largest net torque about CM
- (C) The object which will have the largest speed at the bottom of the incline
- (D) The object which will reach the bottom of incline in the shortest time.

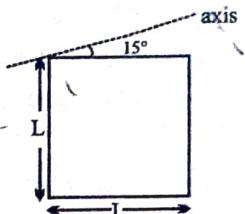
 $S \sqrt{\frac{I}{2}} + m$ **Column-II**

- (P) Solid sphere
- (Q) Spherical shell
- (R) Solid disk
- (S) Thin hollow cylinder

## EXERCISE-II

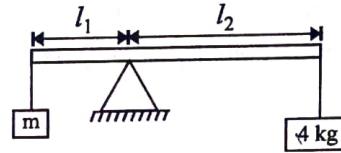
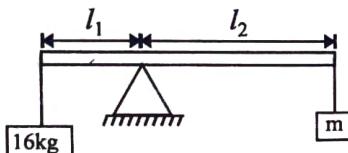
1. Two rods of equal mass  $m$  and length  $l$  lie along the  $x$  axis and  $y$  axis with their centres origin. What is the moment of inertia of both about the line  $x = y$  :

2. A square plate of mass  $M$  and edge  $L$  is shown in figure. Find the moment of inertia of the plate about the axis in the plane of plate passing through one of its vertex making an angle  $15^\circ$  from horizontal.



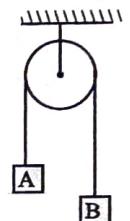
3. A cube is in equilibrium on an inclined plane forming an angle of  $30^\circ$  with the horizontal. What is distance of line of action of the normal reaction of the plane on the cube from the centre.

4. In an experiment with a beam balance on unknown mass  $m$  is balanced by two known masses of  $16\text{ kg}$  and  $4\text{ kg}$  as shown in figure. (massless beam)



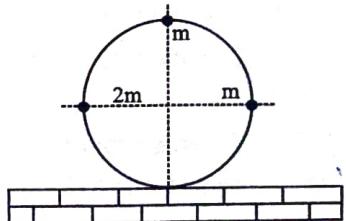
Find the value of the unknown mass  $m$ .

5. In the figure A & B are two blocks of mass  $4\text{ kg}$  &  $2\text{ kg}$  respectively attached to the two ends of a light string passing over a disc C of mass  $40\text{ kg}$  and radius  $0.1\text{ m}$ . The disc is free to rotate about a fixed horizontal axes, coinciding with its own axis. The system is released from rest and the string does not slip over the disc. Find :

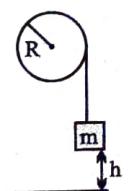


- (i) the linear acceleration of mass B .
- (ii) the number of revolutions made by the disc at the end of  $10\text{ sec}$ . from the start.
- (iii) the tension in the string segment supporting the block A.

6. A ring of mass  $m$  and radius  $R$  has three particles attached to the ring as shown in the figure. The centre of the ring has a speed  $v_0$ . Find the kinetic energy of the system. (Slipping is absent)



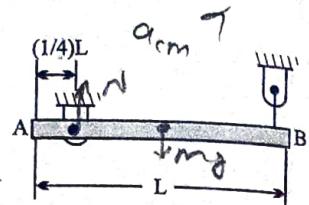
7. A mass  $m$  is attached to a pulley through a cord as shown in the fig. The pulley is a solid disk with radius  $R$ . The cord does not slip on the disk. The mass is released from rest at a height  $h$  from the ground and at the instant the mass reaches the ground, the disk is rotating with angular velocity  $\omega$ . Find the mass of the disk.



8. A solid homogeneous cylinder of height  $h$  and base radius  $r$  is kept vertically on a conveyor belt moving horizontally with an increasing velocity  $v = a + bt^2$ . If the cylinder is not allowed to slip find the time when the cylinder is about to topple.

9. A uniform beam of length  $L$  and mass  $m$  is supported as shown. If the cable suddenly breaks, determine;

- (a) the acceleration of end B.
- (b) the reaction at the pin support.



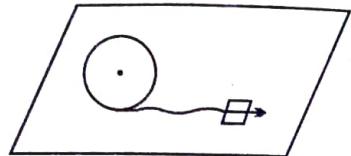
$$Mg - N = ma_{cm}$$

10. A solid uniform disk of mass  $m$  rolls without slipping down a fixed inclined plane with an acceleration  $a$ . Find the frictional force on the disk due to surface of the plane.

11. A ring of radius  $R$  rolls without sliding with a constant velocity. Find the radius of curvature of the path followed by any particle of the ring at the highest point of its path.

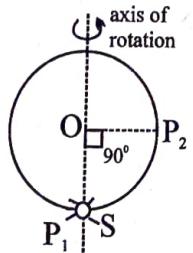
$$R_0$$

12. A block of mass  $m$  is attached to a pulley disc of equal mass  $m$ , radius  $r$  by means of a slack string as shown. The pulley is hinged about its centre on a horizontal table and the block is projected with an initial velocity of  $5 \text{ m/s}$ . Find the velocity when the string becomes taut.



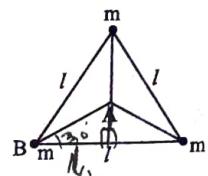
13. The angular momentum of a flywheel having a moment of inertia of  $0.4 \text{ kg m}^2$  decreases from  $30$  to  $20 \text{ kg m}^2/\text{s}$  in a period of  $2$  second. Find the average torque acting on the flywheel during this period.

14. A uniform ring is rotating about vertical axis with angular velocity  $\omega$  initially. A point insect (S) having the same mass as that of the ring starts walking from the lowest point  $P_1$  and finally reaches the point  $P_2$  (as shown in figure). What is the final angular velocity of the ring?

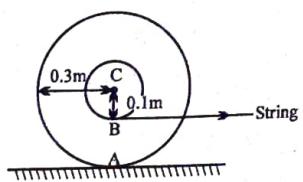


15. A particle of mass  $10 \text{ kg}$  is moving with a uniform speed of  $6 \text{ m/sec}$ . in  $x-y$  plane along the line  $3y = 4x + 10$ , what is the magnitude of its angular momentum about the origin in  $\text{kg-m}^2/\text{s}$ ?

16. Three equal masses  $m$  are rigidly connected to each other by massless rods of length  $l$  forming an equilateral triangle, as shown above. The assembly is to be given an angular velocity  $\omega$  about an axis perpendicular to the triangle. For fixed  $\omega$ , what is the ratio of the kinetic energy of the assembly for an axis through B compared with that for an axis through A.

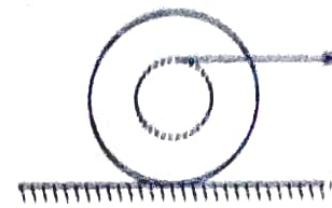


17. A wheel is made to roll without slipping, towards right, by pulling a string wrapped around a coaxial spool as shown in figure. With what velocity the string should be pulled so that the centre of the wheel moves with a velocity of  $3 \text{ m/s}$ ?



18.

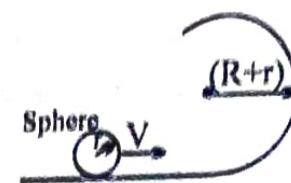
A spool of inner radius  $R$  and outer radius  $3R$  has a moment of inertia  $= MR^2$  about an axis passing through its geometric centre, where  $M$  is the mass of the spool. A thread wound on the inner surface of the spool is pulled horizontally with a constant force  $= Mg$ .



Find the acceleration of the point on the thread which is being pulled assuming that the spool rolls purely on the floor.

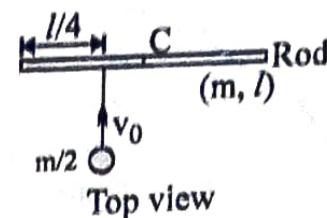
19.

A sphere of mass  $m$  and radius  $r$  is pushed onto the fixed horizontal surface such that it rolls without slipping from the beginning. Determine the minimum speed  $v$  of its mass centre at the bottom so that it rolls completely around the loop of radius  $(R + r)$  without leaving the track in between.



20.

A uniform rod of mass  $m$  and length  $l$  is resting on a smooth horizontal surface. A particle of mass  $m/2$  travelling with a speed  $v_0$  hits the rod normally and elastically. Find final velocity of particle and the angular velocity of the rod.



# EXERCISE-III

1. Initial angular velocity of a circular disc of mass  $M$  is  $\omega_1$ . Then two small spheres of mass  $m$  are attached gently to diametrically opposite points on the edge of the disc. What is the final angular velocity of the disc?

(A)  $\left(\frac{M+m}{M}\right)\omega_1$       (B)  $\left(\frac{M+m}{m}\right)\omega_1$       (C)  $\left(\frac{M}{M+4m}\right)\omega_1$       (D)  $\left(\frac{M}{M+2m}\right)\omega_1$  [AIEEE-2002]

2. A solid sphere, a hollow sphere and a ring are released from top of an inclined plane (frictionless) so that they slide down the plane. Then maximum acceleration down the plane is for (no rolling) [AIEEE-2002]

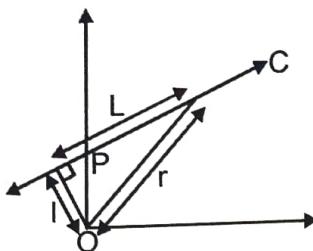
(A) solid sphere      (B) hollow sphere      (C) ring      (D) all same

[AIEEE-2002]

3. Moment of inertia of a circular wire of mass  $M$  and radius  $R$  about its diameter is

(A)  $\frac{MR^2}{2}$       (B)  $MR^2$       (C)  $2MR^2$       (D)  $\frac{MR^2}{4}$

4. A particle of mass  $m$  moves along line PC with velocity  $v$  as shown. What is the angular momentum of the particle about P? [AIEEE-2002]



(A)  $mVL$       (B)  $mvL$       (C)  $mvr$       (D) zero

5. Let  $\vec{F}$  be the force acting on a particle having position vector  $\vec{r}$  and  $\vec{T}$  be the torque of this force about the origin. Then [AIEEE-2003]

(A)  $\vec{r} \cdot \vec{T} = 0$  and  $\vec{F} \cdot \vec{T} \neq 0$       (B)  $\vec{r} \cdot \vec{T} \neq 0$  and  $\vec{F} \cdot \vec{T} = 0$   
 (C)  $\vec{r} \cdot \vec{T} \neq 0$  and  $\vec{F} \cdot \vec{T} \neq 0$       (D)  $\vec{r} \cdot \vec{T} = 0$  and  $\vec{F} \cdot \vec{T} = 0$

6. A solid sphere is rotating in free space. If the radius of the sphere is increased keeping mass same which one of the following will not be affected? [AIEEE-2004]

(A) Moment of inertia      (B) Angular momentum  
 (C) Angular velocity      (D) rotational kinetic energy

7. One solid sphere A and another hollow sphere B are of same mass and same outer radii. Their moment of inertia about their diameters are respectively  $I_A$  and  $I_B$  such that [AIEEE-2004]

(A)  $I_A = I_B$       (B)  $I_A > I_B$       (C)  $I_A < I_B$       (D)  $\frac{I_A}{I_B} = \frac{d_A}{d_B}$

(where  $d_A$  and  $d_B$  are their densities)

An annular ring with inner and outer radii  $R_1$  and  $R_2$  is rolling without slipping with a uniform angular speed. The ratio of the forces experienced by the two particles situated on the inner and outer parts of the ring,  $F_1/F_2$  is

[AIEEE-2005]

(A)  $\frac{R_2}{R_1}$

(B)  $\left(\frac{R_1}{R_2}\right)^2$

(C) 1

(D)  $\frac{R_1}{R_2}$

The moment of inertia of a uniform semicircular disc of mass  $M$  and radius  $r$  about a line perpendicular to the plane of the disc through the centre is

(A)  $\frac{1}{4}Mr^2$

(B)  $\frac{2}{5}Mr^2$

(C)  $Mr^2$

(D)  $\frac{1}{2}Mr^2$

A 'T' shaped object with dimensions shown in the figure, is lying on a smooth floor. A force  $F$  is applied at the point  $P$  parallel to  $AB$ , such that the object has only the translational motion without rotation. Find the location of  $P$  with respect to  $C$

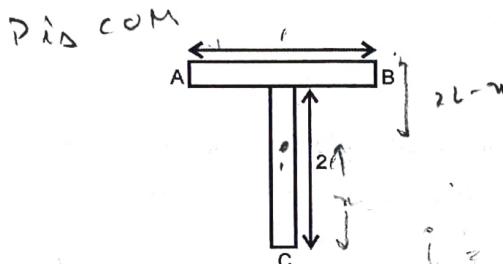
[AIEEE-2005]

(A)  $\frac{2}{3}\ell$

(B)  $\frac{3}{2}\ell$

(C)  $\frac{4}{3}\ell$

(D)  $\ell$



A force of  $-F\hat{k}$  acts on  $O$ , the origin of the coordinate system. The torque about the point  $(1, -1)$  is

(A)  $-F(\hat{i} - \hat{j})$

(B)  $F(\hat{i} - \hat{j})$

(C)  $-F(\hat{i} + \hat{j})$

(D)  $F(\hat{i} + \hat{j})$

[AIEEE-2006]

A thin circular ring of mass  $m$  and radius  $R$  is rotating about its axis with a constant angular velocity  $\omega$ . Two objects each of mass  $M$  are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with an angular velocity  $\omega'$  =

[AIEEE-2006]

(A)  $\frac{\omega m}{(m+2M)}$

(B)  $\frac{\omega(m+2M)}{m}$

(C)  $\frac{\omega(m-2M)}{(m+2M)}$

(D)  $\frac{\omega m}{(m+M)}$

Four point masses, each of value  $m$ , are placed at the corners of a square ABCD of side  $\ell$ . The moment of inertia through A and parallel to BD is

[AIEEE-2006]

(A)  $m\ell^2$

(B)  $2m\ell^2$

(C)  $\sqrt{3}m\ell^2$

(D)  $3m\ell^2$

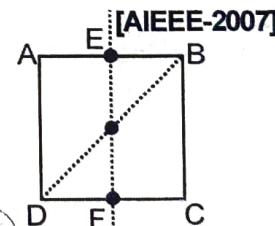
For the given uniform square lamina ABCD, whose centre is O,

(A)  $\sqrt{2}I_{AC} = I_{EF}$

(B)  $I_{AD} = 3I_{EF}$

(C)  $I_{AC} = I_{EF}$

(D)  $I_{AC} = \sqrt{2}I_{EF}$



A round uniform body of radius  $R$ , mass  $M$  and moment of inertia ' $I$ ', rolls down (without slipping) an inclined plane making an angle  $\theta$  with the horizontal. Then its acceleration is :

[AIEEE-2007]

(A)  $\frac{gsin\theta}{1+\frac{I}{MR^2}}$

(B)  $\frac{gsin\theta}{1+\frac{MR^2}{I}}$

(C)  $\frac{gsin\theta}{1-\frac{I}{MR^2}}$

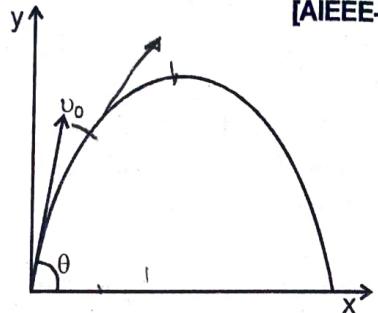
(D)  $\frac{gsin\theta}{1-\frac{MR^2}{I}}$

16. Angular momentum of the particle rotating with a central force is constant due to [AIEEE-2007]  
 (A) Constant Force (B) Constant linear momentum  
 (C) Zero Torque (D) Constant Torque
17. Consider a uniform square plate of side 'a' and mass 'm'. The moment of inertia of this plate about an axis perpendicular to its plane and passing through one of its corners is [AIEEE-2008]

- (A)  $\frac{5}{6}ma^2$  (B)  $\frac{1}{12}ma^2$  (C)  $\frac{7}{12}ma^2$  (D)  $\frac{2}{3}ma^2$

18. A small particle of mass  $m$  is projected at an angle  $\theta$  with the x-axis with an initial velocity  $v_0$  in the x-y plane as shown in the figure. At a time  $t < \frac{v_0 \sin \theta}{g}$ , the angular momentum of the particle about point of projection is [AIEEE-2010]

- (A)  $-\frac{1}{2}mg v_0 t^2 \cos \theta \hat{k}$   
 (B)  $\frac{1}{2}mg v_0 t^2 \cos \theta \hat{i}$   
 (C)  $-mg v_0 t^2 \cos \theta \hat{j}$   
 (D)  $mg v_0 t \cos \theta \hat{k}$



where  $\hat{i}, \hat{j}$  and  $\hat{k}$  are unit vectors along x, y and z-axis respectively.

19. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. During the journey of the insect, the angular speed of the disc : [AIEEE-2011]  
 (A) remains unchanged (B) continuously decreases  
 (C) continuously increases (D) first increases and then decreases

20. A mass  $m$  hangs with the help of a string wrapped around a pulley on a frictionless bearing. The pulley has mass  $m$  and radius  $R$ . Assuming pulley to be a perfect uniform circular disc, the acceleration of the mass  $m$ , if the string does not slip on the pulley, is : [AIEEE-2011]

- (A)  $\frac{3}{2}g$  (B)  $g$  (C)  $\frac{2}{3}g$  (D)  $\frac{g}{3}$

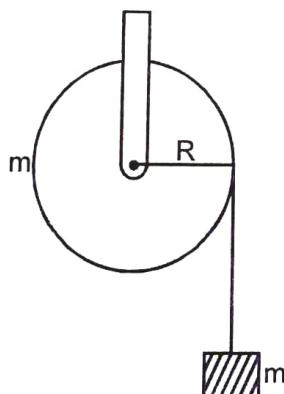


21. A pulley of radius 2 m is rotated about its axis by a force  $F = (20t - 5t^2)$  N (where  $t$  is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is  $10 \text{ kg m}^2$ , the number of rotations made by the pulley before its direction of motion is reversed, is : [AIEEE-2011]  
 (A) less than 3 (B) more than 3 but less than 6  
 (C) more than 6 but less than 9 (D) more than 9

22. A hoop of radius  $r$  and mass  $m$  rotating with an angular velocity  $\omega_0$  is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it ceases to slip? [JEE (Mains)-2013]

- (A)  $r\omega_0$  (B)  $\frac{r\omega_0}{4}$  (C)  $\frac{r\omega_0}{3}$  (D)  $\frac{r\omega_0}{2}$

23. A mass 'm' is supported by a massless string wound around a uniform hollow cylinder of mass  $m$  and radius  $R$ . If the string does not slip on the cylinder, with what acceleration will the mass fall on release?



$$\begin{aligned} mg - T &= ma \quad \text{(1)} \\ T(R) &= mR\alpha \quad \text{(2)} \\ T &= mR\alpha \quad \text{(3)} \\ mg - mR\alpha &= ma \quad \text{(4)} \\ \alpha &= g/R \end{aligned}$$

- (A)  $\frac{g}{2}$  (B)  $\frac{5g}{6}$  (C)  $g$  (D)  $\frac{2g}{3}$  [JEE Mains 2014]

24.

A smooth sphere A is moving on a frictionless horizontal plane with angular speed  $\omega$  and centre of mass velocity  $v$ . It collides elastically and head on with an identical sphere B at rest. Neglect friction everywhere. After the collision, their angular speeds are  $\omega_A$  and  $\omega_B$ , respectively. Then

(A)  $\omega_A < \omega_B$

(B)  $\omega_A = \omega_B$

(C)  $\omega_A = \omega$

(D)  $\omega_B = \omega$

[JEE' 99]

25.

A disc of mass M and radius R is rolling with angular speed  $\omega$  on a horizontal as shown. The magnitude of angular momentum of the disc about the origin O is:

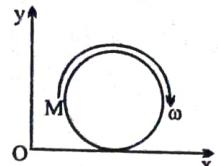
(A)  $(1/2)MR^2\omega$

(B)  $MR^2\omega$

(C)  $(3/2)MR^2\omega$

(D)  $2MR^2\omega$

[JEE' 99]

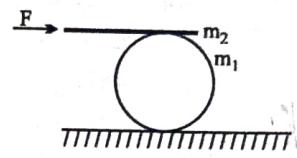


26.

A man pushes a cylinder of mass  $m_1$  with the help of a plank of mass  $m_2$  as shown. There is no slipping at any contact. The horizontal component of the force applied by the man is F. Find

(a) the accelerations of the plank and the center of mass of the cylinder, and

(b) the magnitudes and directions of frictional forces at contact points.



[JEE' 99]

27.

A cubical block of side L rests on a rough horizontal surface with coefficient of friction  $\mu$ . A horizontal force F is applied on the block as shown. If the coefficient of friction is sufficiently high so that the block does not slide before toppling, the minimum force required to topple the block is:

[JEE'(Scr)'2000]

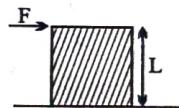
(A) infinitesimal

(B)  $mg/4$

(C)  $mg/2$

(D)  $mg(1-\mu)$

$$F_k = mg \frac{L}{2}$$



28.

A thin wire of length L and uniform linear mass density  $\rho$  is bent into a circular loop with centre at O as shown. The moment of inertia of the loop about the axis XX' is:

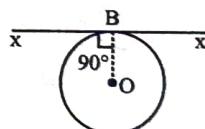
[JEE'(Scr)'2000]

(A)  $\rho L^3/8\pi^2$

(B)  $\rho L^3/16\pi^2$

(C)  $5\rho L^3/16\pi^2$

(D)  $3\rho L^3/8\pi^2$



29.

A rod AB of mass M and length L is lying on a horizontal frictionless surface. A particle of mass m travelling along the surface hits the end 'A' of the rod with a velocity  $v_0$  in the direction perpendicular to AB. The collision is completely elastic. After the collision the particle comes to rest.

(a) Find the ratio  $m/M$ .

(b) A point P on the rod is at rest immediately after the collision. Find the distance AP.

(c) Find the linear speed of the point P at a time  $\pi L/(3v_0)$  after the collision.

[JEE' 2000]

30.

A particle is moving in a horizontal uniform circular motion. The angular momentum of the particle is conserved about the point :

(A) Centre of the circle

(B) Outside the circle

(C) Inside the circle

(D) Point on circumference

[JEE'(Scr)2003]

31. Two particles each of mass  $M$  are connected by a massless rod of length  $l$ . The rod is lying on the smooth surface. If one of the particle is given an impulse  $Mv$  as shown in the figure then angular velocity of the rod would be:

- (A)  $v/l$   
 (B)  $2v/l$   
 (C)  $v/2l$   
 (D) None

$$Mv = 2Mw \quad w = v/2$$

[JEE (Scr) 2003]



32. A child is standing with folded hands at the center of a platform rotating about its central axis. The kinetic energy of the system is  $K$ . The child now stretches his arms so that the moment of inertia of the system doubles. The kinetic energy of the system now is

- (A)  $2K$       (B)  $K/2$       (C)  $K/4$       (D)  $4K$

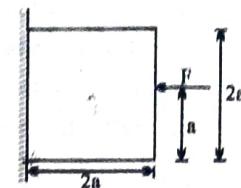
[JEE' 2004 (Scr)]

33. A block of mass  $m$  is held fixed against a wall by applying a horizontal force  $F$ . Which of the following option is incorrect:

- (A) friction force  $= mg$   
 (B)  $F$  will not produce torque  
 (C) normal will not produce torque  
 (D) normal reaction  $= F$

$$\text{mg} \cancel{F} = F\cancel{a}$$

Friction  $\mu$

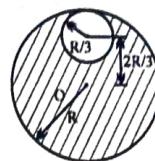


[JEE' 2005 (Scr)]

34. A disc has mass  $9m$ . A hole of radius  $R/3$  is cut from it as shown in the figure. The moment of inertia of remaining part about an axis passing through the centre 'O' of the disc and perpendicular to the plane of the disc is:

[JEE' 2005 (Scr)]

- (A)  $8mR^2$   
 (B)  $4mR^2$   
 (C)  $\frac{40}{9}mR^2$   
 (D)  $\frac{37}{9}mR^2$



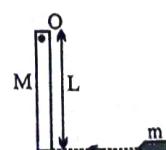
35. A particle moves in circular path with decreasing speed. Which of the following is correct

- (A)  $\vec{L}$  is constant  
 (B) only direction of  $\vec{L}$  is constant  
 (C) acceleration  $\vec{a}$  is towards the centre  
 (D) it will move in a spiral and finally reach the centre

[JEE' 2005 (Scr)]

36. A wooden log of mass  $M$  and length  $L$  is hinged by a frictionless nail at O. A bullet of mass  $m$  strikes with velocity  $v$  and sticks to it. Find angular velocity of the system immediately after the collision about O.

$$mv_0 = [Mv + mv] \omega$$

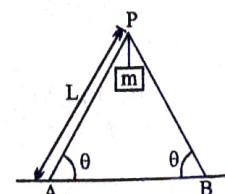


[JEE 2005]

37. A cylinder of mass  $m$  and radius  $R$  rolls down an inclined plane of inclination  $\theta$ . Calculate the linear acceleration of the axis of cylinder.

[JEE 2005]

38. Two identical ladders, each of mass  $M$  and length  $L$  are resting on the rough horizontal surface as shown in the figure. A block of mass  $m$  hangs from P. If the system is in equilibrium, find the magnitude and the direction of frictional force at A and B.



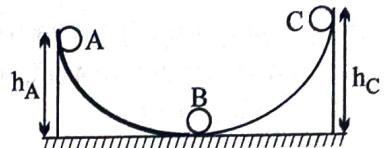
[JEE 2005]

39. A solid sphere of mass  $M$ , radius  $R$  and having moment of inertia about an axis passing through the centre of mass as  $I$ , is recast into a disc of thickness  $t$ , whose moment of inertia about an axis passing through its edge and perpendicular to its plane remains  $I$ . Then, radius of the disc will be

- (A)  $2R/\sqrt{15}$       (B)  $R\sqrt{2/15}$       (C)  $4R/\sqrt{15}$       (D)  $R/4$       [JEE 2006]

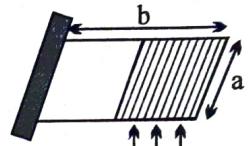
40. A ball moves over a fixed track as shown in the figure. From A to B the ball rolls without slipping. Surface BC is frictionless.  $K_A$ ,  $K_B$  and  $K_C$  are kinetic energies of the ball at A, B and C, respectively. Then (C is highest point reached on smooth track)

- (A)  $h_A > h_C$ ;  $K_B > K_C$   
 (B)  $h_A > h_C$ ;  $K_C > K_A$   
 (C)  $h_A = h_C$ ;  $K_B = K_C$   
 (D)  $h_A < h_C$ ;  $K_B > K_C$



[JEE 2006]

41. There is a rectangular plate of mass  $M$  kg of dimensions  $(a \times b)$ . The plate is held in horizontal position by striking  $n$  small balls each of mass  $m$  kg per unit area per unit time. These are striking in the shaded half region of the plate. The balls are colliding elastically with velocity  $v$ . What is  $v$ ? It is given  $n = 100$ ,  $M = 3$  kg,  $m = 0.01$  kg;  $b = 2$  m;  $a = 1$  m;  $g = 10$  m/s $^2$ .



[JEE 2006]

42. STATEMENT-1 : Two cylinders, one hollow (metal) and the other solid (wood) with the same mass and identical dimensions are simultaneously allowed to roll without slipping down an inclined plane from the same height. The hollow cylinder will reach the bottom of the inclined plane first.

and

STATEMENT-2 : By the principle of conservation of energy, the total kinetic energies of both the cylinders are identical when they reach the bottom of the incline.

- (A) STATEMENT-1 is True, STATEMENT-2 is True ; STATEMENT-2 is a correct explanation for STATEMENT-1  
 (B) STATEMENT-1 is True, STATEMENT-2 is True ; STATEMENT-2 is NOT a correct explanation for STATEMENT-1  
 (C) STATEMENT-1 is True, STATEMENT-2 is False  
 (D) STATEMENT-1 is False, STATEMENT-2 is True

[JEE 2008]

43. If the resultant of all the external forces acting on a system of particles is zero, then from an inertial frame, one can surely say that

- (A) linear momentum of the system does not change in time  
 (B) kinetic energy of the system does not change in time  
 (C) angular momentum of the system does not change in time  
 (D) potential energy of the system does not change in time

[JEE 2009]

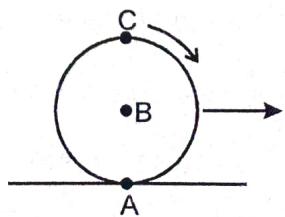
44. A sphere is rolling without slipping on a fixed horizontal plane surface. In the figure, A is the point of contact, B is the centre of the sphere and C is its topmost point. Then,

(A)  $\bar{V}_C - \bar{V}_A = 2(\bar{V}_B - \bar{V}_C)$

(B)  $\bar{V}_C - \bar{V}_B = \bar{V}_B - \bar{V}_A$

(C)  $|\bar{V}_C - \bar{V}_A| = 2|\bar{V}_B - \bar{V}_C|$

(D)  $|\bar{V}_C - \bar{V}_A| = 4|\bar{V}_B|$



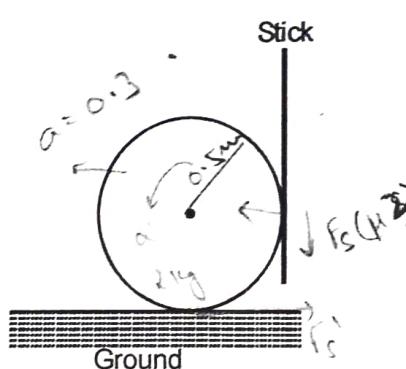
[JEE 2009]

45. A block of base  $10\text{ cm} \times 10\text{ cm}$  and height  $15\text{ cm}$  is kept on an inclined plane. The coefficient of friction between them is  $\sqrt{3}$ . The inclination  $\theta$  of this inclined plane from the horizontal plane is gradually increased from  $0^\circ$ . Then [JEE 2009]

- (A) at  $\theta = 30^\circ$ , the block will start sliding down the plane
- (B) the block will remain at rest on the plane up to certain  $\theta$  and then it will topple
- (C) at  $\theta = 60^\circ$ , the block will start sliding down the plane and continue to do so at higher angles
- (D) at  $\theta = 60^\circ$ , the block will start sliding down the plane and on further increasing  $\theta$ , it will topple at certain  $\theta$

46. Four solid spheres each of diameter  $\sqrt{5}\text{ cm}$  and mass  $0.5\text{ kg}$  are placed with their centers at the corners of a square of side  $4\text{ cm}$ . The moment of inertia of the system about the diagonal of the square is  $N \times 10^{-4}\text{ kgm}^2$ , then  $N$  is : [JEE 2011]

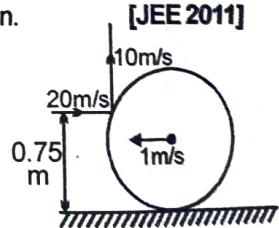
47. A boy is pushing a ring of mass  $2\text{ kg}$  and radius  $0.5\text{ m}$  with a stick as shown in the figure. The stick applies a force of  $2N$  on the ring and rolls it without slipping with an acceleration of  $0.3\text{ m/s}^2$ . The coefficient of friction between the ground and the ring is large enough that rolling always occurs and the coefficient of friction between the stick and the ring is  $(P/10)$ . The value of  $P$  is : [JEE 2011]



$$\begin{aligned} F &= 2 \text{ N} \\ F &= F_s \\ 2 - F_s &= 2(0.3a) \\ (F_s)0.5 - 2M(\frac{1}{2}) &= 2(0.3a) \\ 0.7 - \mu = 0.3 & \\ \mu &= 0.4 \end{aligned}$$

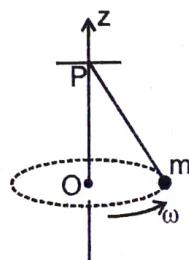
48. A thin ring of mass  $2\text{ kg}$  and radius  $0.5\text{ m}$  is rolling without slipping on a horizontal plane with velocity  $1\text{ m/s}$ . A small ball of mass  $0.1\text{ kg}$ , moving with velocity  $20\text{ m/s}$  in the opposite direction, hits the ring at a height of  $0.75\text{ m}$  and goes vertically up with velocity  $10\text{ m/s}$ . Immediately after the collision. [JEE 2011]

- (A) the ring has pure rotation about its stationary CM.
- (B) the ring comes to a complete stop.
- (C) friction between the ring and the ground is to the left.
- (D) there is no friction between the ring and the ground.



49. A small mass  $m$  is attached to a massless string whose other end is fixed at  $P$  as shown in the figure. The mass is undergoing circular motion in the  $x-y$  plane with centre at  $O$  and constant angular speed  $\omega$ . If the angular momentum of the system, calculated about  $O$  and  $P$  are denoted by  $\vec{L}_O$  and  $\vec{L}_P$  respectively, then

- (A)  $\vec{L}_O$  and  $\vec{L}_P$  do not vary in time
- (B)  $\vec{L}_O$  varies with time while  $\vec{L}_P$  remains constant
- (C)  $\vec{L}_O$  remains constant while  $\vec{L}_P$  varies with time
- (D)  $\vec{L}_O$  and  $\vec{L}_P$  both vary with time



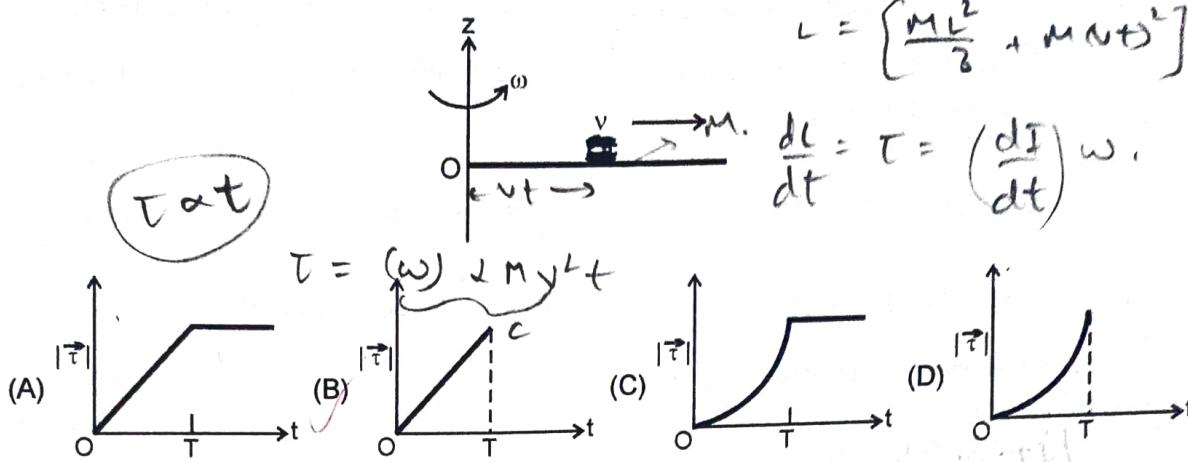
[JEE 2012]

50.

A thin uniform rod, pivoted at O, is rotating in the horizontal plane with constant angular speed  $\omega$ , as shown in the figure. At time  $t = 0$ , a small insect starts from O and moves with constant speed  $v$  with respect to the rod towards the other end. It reaches the end of the rod at  $t = T$  and stops. The angular speed of the system remains  $\omega$  throughout. The magnitude of the torque ( $|\tau|$ ) on the system about O, as a function of time is best represented by which plot?

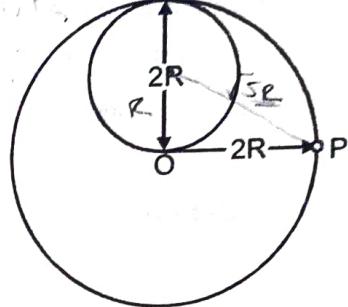
$$\tau = I\omega$$

$$\tau = \left[ \frac{ML^2}{3} + Mv^2t \right]$$



51.

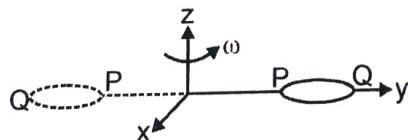
A lamina is made by removing a small disc of diameter  $2R$  from a bigger disc of uniform mass density and radius  $2R$ , as shown in the figure. The moment of inertia of this lamina about axes passing through O and P is  $I_O$  and  $I_P$ , respectively. Both these axes are perpendicular to the plane of the lamina. [JEE 2012]



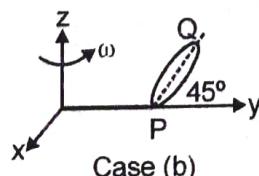
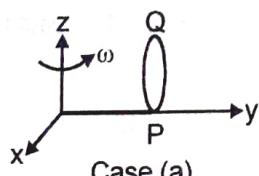
The ratio  $\frac{I_P}{I_O}$  to the nearest integer is

#### Paragraph for Questions 52 and 53

The general motion of a rigid body can be considered to be a combination of (i) a motion of its centre of mass about an axis, and (ii) its motion about an instantaneous axis passing through the centre of mass. These axes need not be stationary. Consider, for example, a thin uniform disc welded (rigidly fixed) horizontally at its rim to a massless stick, as shown in the figure. When the disc-stick system is rotated about the origin on a horizontal frictionless plane with angular speed  $\omega$ , the motion at any instant can be taken as a combination of (i) a rotation of the centre of mass of the disc about the z-axis, and (ii) a rotation of the disc through an instantaneous vertical axis passing through its centre of mass (as is seen from the changed orientation of points P and Q). Both these motions have the same angular speed  $\omega$  in this case.



Now consider two similar systems as shown in the figure : Case (a) the disc with its face vertical and parallel to x-z plane; Case (b) the disc with its face making an angle of  $45^\circ$  with x-y plane and its horizontal diameter parallel to x-axis. In both the cases, the disc is welded at point P, and the systems are rotated with constant angular speed  $\omega$  about the z-axis.



[JEE 2012]

52.

Which of the following statements about the instantaneous axis (passing through the centre of mass) is correct?

- (A) It is vertical for both the cases (a) and (b).
- (B) It is vertical for case (a); and is at  $45^\circ$  to the x-z plane and lies in the plane of the disc for case (b).
- (C) It is horizontal for case (a); & is at  $45^\circ$  to the x-z plane and is normal to the plane of the disc for case (b).
- (D) It is vertical for case (a); and is at  $45^\circ$  to the x-z plane and is normal to the plane of the disc for case (b).

53.

Which of the following statements regarding the angular speed about the instantaneous axis (passing through the centre of mass) is correct?

- (A) It is  $\sqrt{2}\omega$  for both the cases.
- (B) It is  $\omega$  for case (a); and  $\frac{\omega}{\sqrt{2}}$  for case (b).
- (C) it is  $\omega$  for case (a); and  $\sqrt{2}\omega$  for case (b)
- (D) It is  $\omega$  for both the cases.

54.

The figure shows a system consisting of (i) a ring of outer radius  $3R$  rolling clockwise without slipping on a horizontal surface with angular speed  $\omega$  and (ii) an inner disc of radius  $2R$  rotating anti-clockwise with angular speed  $\omega/2$ . The ring and disc are separated by frictionless ball bearings. The system is in the x-z plane. The point P on the inner disc is at a distance  $R$  from the origin, where OP makes an angle of  $30^\circ$  with the horizontal. Then with respect to the horizontal surface,

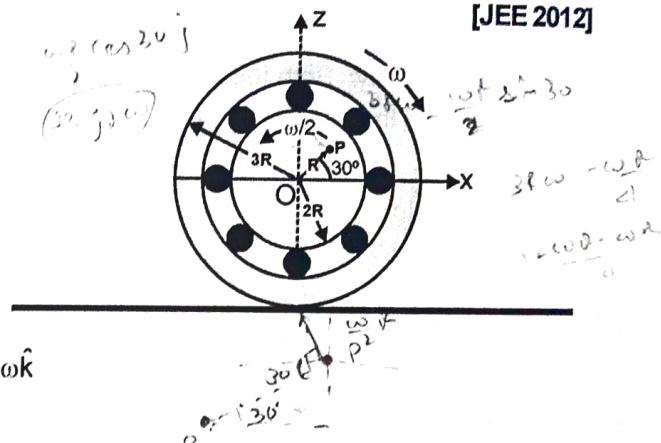
- (A) The point O has a linear velocity  $3R\omega\hat{i}$

- (B) The point P has a linear velocity  $\frac{11}{4}R\omega\hat{i} + \frac{\sqrt{3}}{4}R\omega\hat{k}$ .

- (C) The point P has a linear velocity  $\frac{13}{4}R\omega\hat{i} - \frac{\sqrt{3}}{4}R\omega\hat{k}$

- (D) The point P has a linear velocity  $\left(3 - \frac{\sqrt{3}}{4}\right)R\omega\hat{i} + \frac{1}{4}R\omega\hat{k}$

[JEE 2012]



55.

Two solid cylinders P and Q of same mass and same radius start rolling down a fixed inclined plane from the same height at the same time. Cylinder P has most of its mass concentrated near its surface, while Q has most of its mass concentrated near the axis. Which statement(s) is(are) correct?

- (A) Both cylinders P and Q reach the ground at the same time.

[JEE 2012]

- (B) Cylinder P has larger linear acceleration than cylinder Q.

- (C) Both cylinders reach the ground with same translational kinetic energy.

- (D) Cylinder Q reaches the ground with larger angular speed.

56.

A uniform circular disc of mass 50 kg and radius 0.4 m is rotating with an angular velocity of  $10 \text{ rad s}^{-1}$  about its own axis, which is vertical. Two uniform circular rings, each of mass 6.25 kg and radius 0.2 m, are gently placed symmetrically on the disc in such a manner that they are touching each other along the axis of the disc and are horizontal. Assume that the friction is large enough such that the rings are at rest relative to the disc and the system rotates about the original axis. The new angular velocity (in  $\text{rad s}^{-1}$ ) of the system is

[JEE (Advanced)-2013]

57.

In the figure, a ladder of mass  $m$  is shown leaning against a wall. It is in static equilibrium making an angle  $\theta$  with the horizontal floor. The coefficient of friction between the wall and the ladder is  $\mu_1$ , and that between the floor and the ladder is  $\mu_2$ . The normal reaction of the wall on the ladder is  $N_1$ , and that of the floor is  $N_2$ . If the ladder is about to slip, then

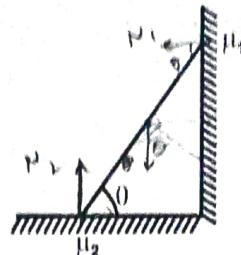
[JEE (Advanced)-2014]

(A)  $\mu_1 = 0; \mu_2 = 0$  and  $N_2 \tan \theta = \frac{mg}{2}$

(B)  $\mu_1 = 0; \mu_2 = 0$  and  $N_1 \tan \theta = \frac{mg}{2}$

(C)  $\mu_1 = 0; \mu_2 \neq 0$  and  $N_2 = \frac{mg}{1 + \mu_1 \mu_2}$

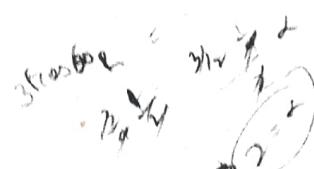
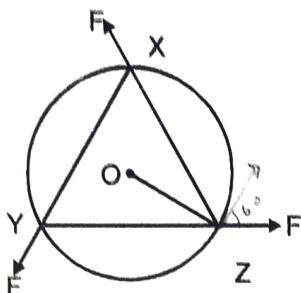
(D)  $\mu_1 = 0; \mu_2 = 0$  and  $N_1 \tan \theta = \frac{mg}{2}$



58.

A uniform circular disc of mass 1.5 kg and radius 0.5 m is initially at rest on a horizontal frictionless surface. Three forces of equal magnitude  $F = 0.5$  N are applied simultaneously along the three sides of an equilateral triangle XYZ with its vertices on the perimeter of the disc (see figure). One second after applying the forces, the angular speed of the disc in  $\text{rad s}^{-1}$  is

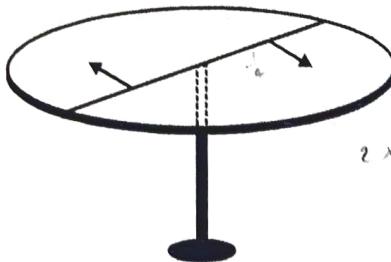
[JEE (Advanced)-2014]



59.

A horizontal circular platform of radius 0.5 m and mass 0.45 kg is free to rotate about its axis. Two massless spring toy-guns, each carrying a steel ball of mass 0.05 kg are attached to the platform at a distance 0.25 m from the centre on its either sides along its diameter (see figure). Each gun simultaneously fires the balls horizontally and perpendicular to the diameter in opposite directions. After leaving the platform the balls have horizontal speed of  $9 \text{ ms}^{-1}$  with respect to the ground. The rotational speed of the platform in  $\text{rad s}^{-1}$  after the balls leave the platform is

[JEE (Advanced)-2014]

 $\omega$ :

$$2 \times 0.05 \times 9 \times \frac{1}{0.25}$$

$$2 \times 9 \times 9 \times \frac{1}{0.25} = \frac{1}{0.25} \omega$$

$$\omega = 24$$

60.

From a solid sphere of mass  $M$  and radius  $R$  a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its center and perpendicular to one of its faces is [JEE (Mains)-2015]

(A)  $\frac{4MR^2}{3\sqrt{3}\pi}$

(B)  $\frac{MR^2}{32\sqrt{2}\pi}$

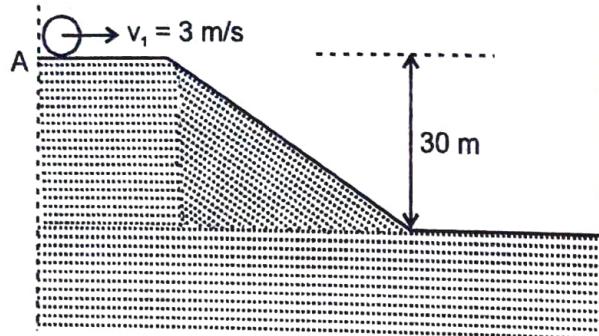
(C)  $\frac{MR^2}{16\sqrt{2}\pi}$

(D)  $\frac{4MR^2}{9\sqrt{3}\pi}$

61.

Two identical uniform discs roll without slipping on two different surfaces AB and CD (see figure) starting at A and C with linear speeds  $v_1$  and  $v_2$ , respectively, and always remain in contact with the surfaces. If they reach B and D with the same linear speed and  $v_1 = 3 \text{ m/s}$ , then  $v_2$  in m/s is ( $g = 10 \text{ m/s}^2$ ) [JEE (Advanced)-2015]

$$\frac{3}{4}mv_0^2 + mg(27) \\ = \frac{3}{4}mv_1^2$$



$$\frac{1}{2}mv_0^2 + \frac{1}{2}(\frac{1}{2}MR^2)\omega^2 \\ + mg(30) \\ = \frac{1}{2}mv_1^2 + \frac{1}{2}(\frac{1}{2}MR^2)\omega^2$$

$$\frac{3}{4}mv_0^2 + mg(30) = \frac{3}{4}mv_1^2 \\ \frac{3}{4}m(b)^2 + mg(30) = \frac{3}{4}mv_1^2$$

$$\frac{3}{4}m + 30mg = 27mg + \frac{3}{4}mv_1^2$$

$$3mg + \frac{27}{4}mg = \frac{27}{4}mgv_2^2$$

$$8D + 9 = \frac{27}{4}v_2^2$$

62.

The densities of two solid spheres A and B of the same radii R vary with radial distances  $r$  as  $\rho_A(r) = k\left(\frac{r}{R}\right)$

and  $\rho_B(r) = k\left(\frac{r}{R}\right)^5$ , respectively, where  $k$  is a constant. The moments of inertia of the individual spheres

about axes passing through their centres are  $I_A$  and  $I_B$  respectively. If  $\frac{I_B}{I_A} = \frac{n}{10}$ , the value of  $n$  is

[JEE (Advanced)-2015]

63.

A ring of mass  $M$  and radius  $R$  is rotating with angular speed  $\omega$  about a fixed vertical axis passing through its centre O with two point masses each of mass  $M/8$  at rest at O. These masses can move radially outwards along two massless rods fixed on the ring as shown in the figure. At some instant the angular speed of the

system is  $\frac{8}{9}\omega$  and one of the masses is at a distance of  $\frac{3}{5}R$  from O. At this instant the distance of the other

mass from O is

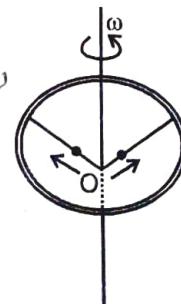
[JEE (Advanced)-2015]

(A)  $\frac{2}{3}R$

(B)  $\frac{1}{3}R$

(C)  $\frac{3}{5}R$

(D)  $\frac{4}{5}R$

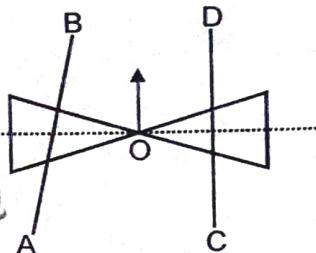


64.

A roller is made by joining together two cones at their vertices O. It is kept on two rails AB and CD which are placed asymmetrically (see figure), with its axis perpendicular to CD and its centre O at the centre of line joining AB and CD (see figure). It is given a light push so that it starts rolling with its centre O moving parallel to CD in the direction shown. As it moves, the roller will tend to

[JEE (Mains)-2016]

*\* As distance of A from O is continuously decrease then speed of cone on A will decrease hence it turn left*



- (A) Turn left  
(C) Go straight

- (B) Turn right  
(D) Turn left and right alternately

65.

The moment of inertia of a uniform cylinder of length  $\ell$  and radius R about its perpendicular bisector is I. What is the ratio  $\ell/R$  such that the moment of inertia is minimum

[JEE Main 2017]

(A)  $\frac{\sqrt{3}}{2}$

(B) 1

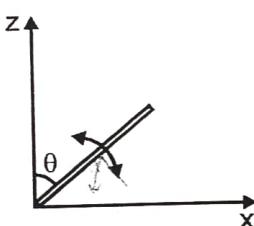
(C)  $\frac{3}{\sqrt{2}}$

(D)  $\sqrt{\frac{3}{2}}$

66.

A slender uniform rod of mass M and length  $\ell$  is pivoted at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle  $\theta$  with the vertical is

[JEE Main 2017]



$$\text{Ans} = g \sin \theta$$

(A)  $\frac{2g}{3\ell} \sin \theta$

(B)  $\frac{3g}{2\ell} \cos \theta$

(C)  $\frac{2g}{3\ell} \cos \theta$

(D)  $\frac{3g}{2\ell} \sin \theta$

67.

A uniform wooden stick of mass 1.6 kg and length  $\ell$  rests in an inclined manner on a smooth, vertical wall of height  $h (< \ell)$  such that a small portion of the stick extends beyond the wall. The reaction force of the wall on the stick is perpendicular to the stick. The stick makes an angle of  $30^\circ$  with the wall and the bottom of the stick is on a rough floor. The reaction of the wall on the stick is equal in magnitude to the reaction of the floor on the stick. The ratio  $h/\ell$  and the frictional force  $f$  at the bottom of the stick are ( $g = 10 \text{ ms}^{-2}$ )

[JEE (Advanced)-2016]

(A)  $\frac{h}{\ell} = \frac{\sqrt{3}}{16}, f = \frac{16\sqrt{3}}{3} \text{ N}$

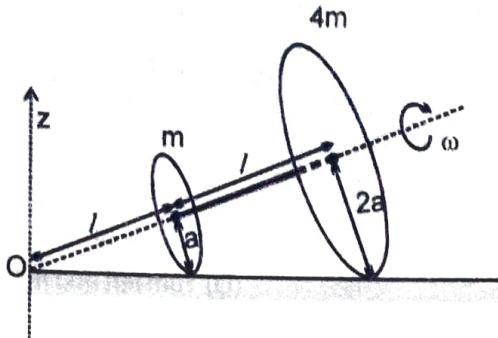
(B)  $\frac{h}{\ell} = \frac{3}{16}, f = \frac{16\sqrt{3}}{3} \text{ N}$

(C)  $\frac{h}{\ell} = \frac{3\sqrt{3}}{16}, f = \frac{8\sqrt{3}}{3} \text{ N}$

(D)  $\frac{h}{\ell} = \frac{3\sqrt{3}}{16}, f = \frac{16\sqrt{3}}{3} \text{ N}$

68. Two thin circular discs of mass  $m$  and  $4m$ , having radii of  $a$  and  $2a$ , respectively, are rigidly fixed by a massless, rigid rod of length  $l = \sqrt{24}a$  through their centers. This assembly is laid on a firm and flat surface, and set rolling without slipping on the surface so that the angular speed about the axis of the rod is  $\omega$ . The angular momentum of the entire assembly about the point 'O' is  $\vec{L}$  (see the figure). Which of the following statement(s) is(are) true ?

[JEE (Advanced)-2016]



- (A) The magnitude of angular momentum of the assembly about its center of mass is  $17ma^2\omega/2$ .  
 (B) The magnitude of the z-component of  $\vec{L}$  is  $55ma^2\omega$ .  
 (C) The magnitude of angular momentum of center of mass of the assembly about the point O is  $81ma^2\omega$ .  
 (D) The center of mass of the assembly rotates about the z-axis with an angular speed of  $\omega/5$ .
69. The position vector  $\vec{r}$  of a particle of mass  $m$  is given by the following equation  $\vec{r}(t) = \alpha t^3 \hat{i} + \beta t^2 \hat{j}$ , where  $\alpha = 10/3 \text{ ms}^{-3}$ ,  $\beta = 5 \text{ ms}^{-2}$  and  $m = 0.1 \text{ kg}$ . At  $t = 1 \text{ s}$ , which of the following statement(s) is(are) true about the particle?

[JEE (Advanced)-2016]

- (A) The velocity  $\vec{v}$  is given by  $\vec{v} = (10\hat{i} + 10\hat{j}) \text{ ms}^{-1}$   
 (B) The angular momentum  $\vec{L}$  with respect to the origin is given by  $\vec{L} = -(5/3)\hat{k} \text{ Nms}$   
 (C) The force  $\vec{F}$  is given by  $\vec{F} = (\hat{i} + 2\hat{j}) \text{ N}$   
 (D) The torque  $\vec{\tau}$  with respect to the origin is given by  $\vec{\tau} = -(20/3)\hat{k} \text{ Nm}$

#### Comprehension (Q.70 to Q.71) :

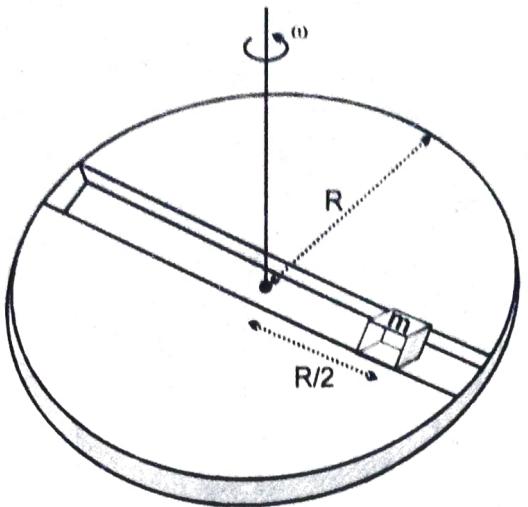
A frame of reference that is accelerated with respect to an inertial frame of reference is called a non-inertial frame of reference. A coordinate system fixed on a circular disc rotating about a fixed axis with a constant angular velocity  $\omega$  is an example of a non-inertial frame of reference. The relationship between the force  $\vec{F}_{\text{rot}}$  experienced by a particle of mass  $m$  moving on the rotating disc and the force  $\vec{F}_{\text{in}}$  experienced by the particle in an inertial frame of reference is

$$\vec{F}_{\text{rot}} = \vec{F}_{\text{in}} + 2m(\vec{v}_{\text{rot}} \times \vec{\omega}) + m(\vec{\omega} \times \vec{r}) \times \vec{\omega},$$

where  $\vec{v}_{\text{rot}}$  is the velocity of the particle in the rotating frame of reference and  $\vec{r}$  is the position vector of the particle with respect to the centre of the disc.

Now consider a smooth slot along a diameter of a disc of radius  $R$  rotating counter-clockwise with a constant angular speed  $\omega$  about its vertical axis through its center. We assign a coordinate system with the origin at the center of the disc, the x-axis along the slot, the y-axis perpendicular to the slot and the z-axis along the rotation axis ( $\vec{\omega} = \omega\hat{k}$ ). A small block of mass  $m$  is gently placed in the slot at  $\vec{r} = (R/2)\hat{i}$  at  $t = 0$  and is constrained to move only along the slot.

[JEE (Advanced)-2016]



70. The distance  $r$  of the block at time  $t$  is

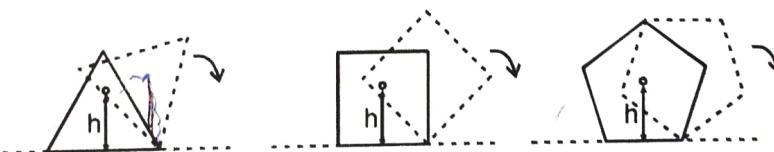
(A)  $\frac{R}{4}(e^{2\omega t} + e^{-2\omega t})$       (B)  $\frac{R}{2}\cos 2\omega t$       (C)  $\frac{R}{2}\cos \omega t$       (D)  $\frac{R}{4}(e^{\omega t} + e^{-\omega t})$

71. The net reaction of the disc on the block is

(A)  $-m\omega^2 R \cos \omega t \hat{j} - mg \hat{k}$       (B)  $m\omega^2 R \sin \omega t \hat{j} - mg \hat{k}$   
 (C)  $\frac{1}{2}m\omega^2 R(e^{\omega t} - e^{-\omega t}) \hat{j} + mg \hat{k}$       (D)  $\frac{1}{2}m\omega^2 R(e^{2\omega t} - e^{-2\omega t}) \hat{j} + mg \hat{k}$

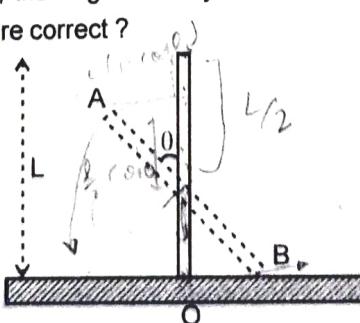
72. Consider regular polygons with number of sides  $n = 3, 4, 5, \dots$  as shown in the figure. The center of mass of all the polygons is at height  $h$  from the ground. They roll on a horizontal surface about the leading vertex without slipping and sliding as depicted. The maximum increase in height of the locus of the center of mass for each polygon is  $\Delta$ . Then  $\Delta$  depends on  $n$  and  $h$  as

[JEE Advance 2017]



(A)  $\Delta = h \sin^2\left(\frac{\pi}{n}\right)$       (B)  $\Delta = h \sin\left(\frac{2\pi}{n}\right)$       (C)  $\Delta = h \left( \frac{1}{\cos\left(\frac{\pi}{n}\right)} - 1 \right)$       (D)  $\Delta = h \tan^2\left(\frac{\pi}{2n}\right)$

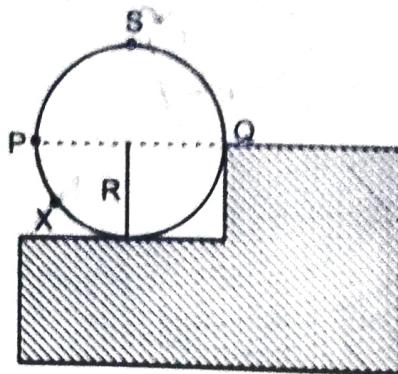
73. A rigid uniform bar AB of length L is slipping from its vertical position on a frictionless floor (as shown in the figure). At some instant of time, the angle made by the bar with the vertical is  $\theta$ . Which of the following statements about its motion is/are correct?



- (A) When the bar makes an angle  $\theta$  with the vertical, the displacement of its midpoint from the initial position is proportional to  $(1 - \cos \theta)$   
 (B) The midpoint of the bar will fall vertically downward  
 (C) Instantaneous torque about the point in contact with the floor is proportional to  $\sin \theta$  at any time only  
 (D) The trajectory of the point A is a parabola

74. A wheel of radius  $R$  and mass  $M$  is placed at the bottom of a fixed step of height  $R$  as shown in the figure. A constant force is continuously applied on the surface of the wheel so that it just climbs the step without slipping. Consider the torque  $\tau$  about an axis normal to the plane of the paper passing through the point Q. Which of the following options is/are correct?

[JEE Advance 2017]



If direction of any force is not change then D is correct.

If direction of any force is also changing then A and C is climbing then A

- (A) If the force is applied normal to the circumference at point X then  $\tau$  is constant
- (B) If the force is applied tangentially at point S then  $\tau \neq 0$  but the wheel never climbs the step
- (C) If the force is applied normal to the circumference at point P then  $\tau$  is zero
- (D) If the force is applied at point P tangentially then  $\tau$  decreases continuously as the wheel climbs

#### Comprehension (Q.75 to Q.76):

[JEE Advance 2017]

One twirls a circular ring (of mass  $M$  and radius  $R$ ) near the tip of one's finger as shown in Figure 1. In the process the finger never loses contact with the inner rim of the ring. The finger traces out the surface of a cone, shown by the dotted line. The radius of the path traced out by the point where the ring and the finger is in contact is  $r$ . The finger rotates with an angular velocity  $\omega_0$ . The rotating ring rolls without slipping on the outside of a smaller circle described by the point where the ring and the finger is in contact (Figure 2). The coefficient of friction between the ring and the finger is  $\mu$  and the acceleration due to gravity is  $g$ .

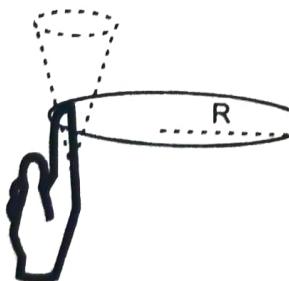


Figure 1

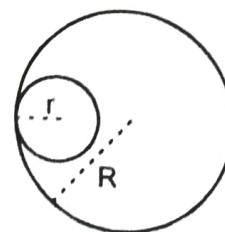


Figure 2

75. The total kinetic energy of the ring is

- (A)  $M\omega_0^2 R^2$
- (B)  $M\omega_0^2(R - r)^2$
- (C)  $\frac{1}{2}M\omega_0^2(R - r)^2$
- (D)  $\frac{3}{2}M\omega_0^2(R - r)^2$

76. The minimum value of  $\omega_0$  below which the ring will drop down is

- (A)  $\sqrt{\frac{3g}{2\mu(R - r)}}$
- (B)  $\sqrt{\frac{g}{\mu(R - r)}}$
- (C)  $\sqrt{\frac{2g}{\mu(R - r)}}$
- (D)  $\sqrt{\frac{g}{2\mu(R - r)}}$

77. Seven identical circular planar disks, each of mass  $M$  and radius  $R$  are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point  $P$  is:

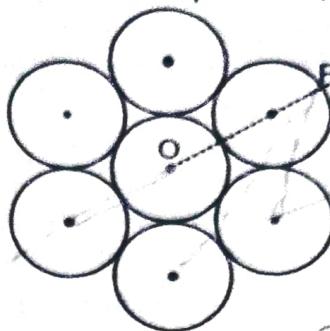
[JEE-Mains-2018]

(A)  $\frac{181}{2} MR^2$

(B)  $\frac{19}{2} MR^2$

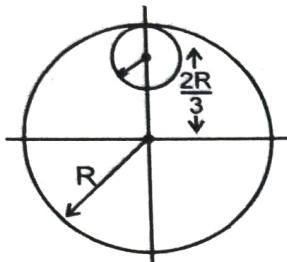
(C)  $\frac{55}{2} MR^2$

(D)  $\frac{73}{2} MR^2$



78. From a uniform circular disc of radius  $R$  and mass  $9M$ , a small disc of radius  $\frac{R}{3}$  is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is :

[JEE-Mains-2018]



(A)  $\frac{37}{9} MR^2$

(B)  $4 MR^2$

(C)  $\frac{40}{9} MR^2$

(D)  $10 MR^2$

79. The potential energy of a particle of mass  $m$  at a distance  $r$  from a fixed point  $O$  is given by  $V(r) = kr^2/2$ , where  $k$  is a positive constant of appropriate dimensions. This particle is moving in a circular orbit of radius  $R$  about the point  $O$ . If  $v$  is the speed of the particle and  $L$  is the magnitude of its angular momentum about  $O$ , which of the following statements is (are) true?

[JEE-Advanced-2018]

(A)  $v = \sqrt{\frac{k}{2m}} R$

(B)  $v = \sqrt{\frac{k}{m}} R$

(C)  $L = \sqrt{mk} R^2$

(D)  $L = \sqrt{\frac{mk}{2}} R^2$

80. Consider a body of mass  $1.0\text{ kg}$  at rest at the origin at time  $t = 0$ . A force  $\vec{F} = (\alpha t \hat{i} + \beta \hat{j})$  is applied on the body, where  $\alpha = 1.0\text{ N s}^{-1}$  and  $\beta = 1.0\text{ N}$ . The torque acting on the body about the origin at time  $t = 1.0\text{ s}$  is  $\vec{\tau}$ . Which of the following statement is(are) true?

[JEE-Advanced-2018]

(A)  $|\vec{\tau}| = \frac{1}{3}\text{ N m}$

(B) The torque  $\vec{\tau}$  is in the direction of the unit vector  $+\hat{k}$

(C) The velocity of the body at  $t = 1$  is  $\vec{v} = \frac{1}{2}(\hat{i} + 2\hat{j})\text{ ms}^{-1}$

(D) The magnitude of displacement of the body at  $t = 1\text{ s}$  is  $\frac{1}{6}\text{ m}$

81. A ring and a disc are initially at rest, side by side, at the top of an inclined plane which makes an angle  $60^\circ$  with the horizontal. They start to roll without slipping at the same instant of time along the shortest path. If the time difference between their reaching the ground is  $\frac{(2 - \sqrt{3})}{\sqrt{10}}$  s, then the height of the top of the inclined plane, in metres, is ..... Take  $g = 10 \text{ m s}^{-2}$ . [JEE-Advanced-2018]

82. In the List-I below, four different paths of a particle are given as functions of time. In these functions,  $\alpha$  and  $\beta$  are positive constants of appropriate dimensions and  $\alpha \neq \beta$ . In each case, the force acting on the particle is either zero or conservative. In List-II, five physical quantities of the particle are mentioned :  $\vec{p}$  is the linear momentum,  $\vec{L}$  is the angular momentum about the origin,  $K$  is the kinetic energy,  $U$  is the potential energy and  $E$  is the total energy. Match each path in List-I with those quantities in List-II, which are conserved for that path. [JEE-Advanced-2018]

**List-I**

- (P)  $\vec{r}(t) = \alpha t \hat{i} + \beta t \hat{j}$
- (Q)  $\vec{r}(t) = \alpha \cos \omega t \hat{i} + \beta \sin \omega t \hat{j}$
- (R)  $\vec{r}(t) = \alpha(\cos \omega t \hat{i} + \sin \omega t \hat{j})$
- (S)  $\vec{r}(t) = \alpha t \hat{i} + \frac{\beta}{2} t^2 \hat{j}$

**List-II**

- (1)  $\vec{p}$
- (2)  $\vec{L}$
- (3)  $K$
- (4)  $U$
- (5)  $E$

- (A) P  $\rightarrow$  1,2,3,4,5 ; Q  $\rightarrow$  2,5 ; R  $\rightarrow$  2,3,4,5 ; S  $\rightarrow$  5  
 (B) P  $\rightarrow$  1,2,3,4,5 ; Q  $\rightarrow$  3,5 ; R  $\rightarrow$  2,3,4,5 ; S  $\rightarrow$  2,5  
 (C) P  $\rightarrow$  2,3,4 ; Q  $\rightarrow$  5 ; R  $\rightarrow$  1,2,4 ; S  $\rightarrow$  2,5  
 (D) P  $\rightarrow$  1,2,3,5 ; Q  $\rightarrow$  2,5 ; R  $\rightarrow$  2,3,4,5 ; S  $\rightarrow$  2,5

## Hints for Difficult Problems of Exercise-II

2. Draw a line parallel to the shown axis through centre of the plate. Draw another line perpendicular to first line and through the centre of plate. Apply perpendicular and parallel axes theorems.
8. Static friction on cylinder increases with acceleration.
12. Conserve angular momentum about centre of the disc as jerk gives internal impulse.

17.  $\omega = \frac{3}{0.3} = 10 \text{ rad/s}$

$$v_B = 3 - (10)(0.1) = 2 \text{ m/s}$$

18.  $Mg - f = Ma$  ( $f$  is leftward friction)

$$MgR + f(3R) = I\alpha = MR^2\alpha$$

$$a = \alpha(3R)$$

$$f = \frac{-Mg}{5} \quad (\text{negative sign shows friction is rightward})$$

Find  $\alpha$

20. Let velocity of particle be  $v_1$  after collision and in the same direction as that of  $v_0$ . Let velocity of COM of rod be  $v$  (in the direction of  $v_0$ ) and angular velocity be  $\omega$  clockwise after collision.

$$\frac{m}{2}v_0 = \frac{m}{2}v_1 + mv \quad (\text{Linear mom. conservation})$$

$$v_0 = \left(v + \omega \frac{\ell}{4}\right) - v_1 \quad (\text{elastic collision})$$

$$\frac{m}{2}v_0 \frac{\ell}{4} = \frac{m}{2}(v_1) \frac{\ell}{4} + I\omega$$

$$I = \frac{M\ell^2}{12}$$

(Angular mom conservation about initial position of mid point of the rod)

---

## ANSWER KEY EXERCISE-I

Only one option is correct

1. B	2. D	3. D	4. B	5. A	6. B	7. D
8. D	9. A	10. B	11. B	12. A	13. B	14. B
15. C	16. A	17. D	18. B	19. C	20. C	21. B
22. D	23. B	24. A	25. D	26. C	27. A	28. B
29. D	30. D	31. C	32. C	33. B	34. D	35. A
36. A	37. D	38. C	39. D	40. B	41. B	42. B
43. B	44. D	45. B	46. B			

One or more than one option may be correct

47. ACD    48. ABC    49. C    50. BC    51. BCD
52. (A) → R ; (B) → P ; (C) → Q ; (D) → S    53. (A) → Q ; (B) → P ; (C) → S ; (D) → R
54. (A) → S ; (B) → S ; (C) → P ; (D) → P

## EXERCISE-II

1.  $\frac{ml^2}{12}$       2.  $\frac{11ML^2}{24}$       3. At a distance  $a/2\sqrt{3}$  from the centre down the plane.  
 4. 8 kg      5. (i)  $10/13 m/s^2$ , (ii)  $5000/26\pi$ , (iii)  $480/13 N$       6.  $6mv_0^2$
7.  $M = 2m \left( \frac{2gh}{R^2\omega^2} - 1 \right)$       8. gr/bh      9. (a)  $\frac{9g}{7} \downarrow$  (b)  $\frac{4mg}{7} \uparrow$       10.  $1/2 ma$       11. 4R
12.  $10/3 m/s$       13.  $5 N.m$       14.  $\frac{\omega}{3}$       15. 120      16. 2
17. 2 m/s      18.  $16 m/s^2$       19.  $v = \sqrt{\frac{27}{7} gR}$       20.  $-\frac{1}{15} v_0$ ,  $\frac{8v_0}{5\ell}$

## EXERCISE-III

### Old AIEEE Questions

- |     |   |     |   |     |   |     |   |     |   |     |   |     |   |
|-----|---|-----|---|-----|---|-----|---|-----|---|-----|---|-----|---|
| 1.  | C | 2.  | D | 3.  | A | 4.  | D | 5.  | D | 6.  | B | 7.  | C |
| 8.  | D | 9.  | D | 10. | C | 11. | C | 12. | A | 13. | D | 14. | C |
| 15. | A | 16. | C | 17. | D | 18. | A | 19. | D | 20. | C | 21. | B |
| 22. | D | 23. | A | 24. | C | 25. | C |     |   |     |   |     |   |
26.  $a_c = \frac{4F}{(3m_1 + 8m_2)}$ ,  $a_p = \frac{8F}{(3m_1 + 8m_2)}$ ;  $f_1 = \frac{3m_1 F}{(3m_1 + 8m_2)}$ ,  $f_2 = \frac{m_1 F}{(3m_1 + 8m_2)}$       27. C
28. D      29. (a)  $\frac{m}{M} = \frac{1}{4}$ ; (b)  $x = \frac{2L}{3}$ ; (c)  $\frac{v_0}{2\sqrt{2}}$       30. A      31. A      32. B
33. C      34. B      35. B      36.  $\omega = \frac{3mv}{(3m+M)L}$       37.  $a_{axis} = \frac{2g \sin \theta}{3}$
38.  $f = (M+m) g \frac{\cot \theta}{2}$       39. A      40. A,B      41. 10 m/s      42. D      43. A
44. B,C      45. B      46. 9      47. 4      48. A, C      49. C      50. B
51. 3      52. A      53. D      54. AB      55. D      56. 8      57. C,D
58. 2      59. 4      60. D      61. 7      62. 6      63. D      64. A
65. D      66. D      67. D      68. D      69. A, B, D      70. D      71. C
72. C      73. ABC      74. AC or CD      75. Bonus      76. B      77. A
78. B      79. BC      80. AC      81. 0.75      82. A