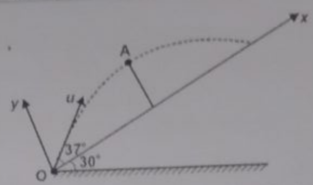


$$v_y^2 = u_y^2 + 2a_y h_{max}$$

or $$0 = (30)^2 + 2(-5\sqrt{3})h_{max}$$

$$\Rightarrow \quad h_{max} = \frac{(30)^2}{2(5\sqrt{3})} = 30\sqrt{3}\text{ m} = 52\text{ m}$$

Let T be time of flight for complete journey from O to B. Displacement in y direction from O to B, $\Delta y = 0$. Applying $\Delta y = u_y t + \frac{1}{2}a_y t^2$ between O and B.

$$0 = 30T + \frac{1}{2}(-5\sqrt{3})T^2$$

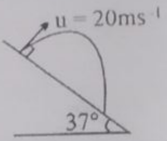
$$\Rightarrow \quad T = \frac{30 \times 2}{5\sqrt{3}} = 4\sqrt{3}\text{ s}$$

Hence Range, $R = OQ = u_x T + \frac{1}{2}a_x T^2 = 40(4\sqrt{3}) + \frac{1}{2}(-5)(4\sqrt{3})^2 = 157\text{ m}$

Section Review 4.4

1. A particle is projected horizontally with a speed u from the top of a plane inclined at angle θ with the horizontal. How far from the point of projection will the particle strike the plane?

2. Find range of projectile on the inclined plane which is projected perpendicular to the incline plane with velocity 20 m / s as shown in figure.



4.8 Circular Motion Kinematics

4.8.1 Angular Variables

Consider a particle P, moving in a circle of radius r and centre O as shown in Fig. 4.15. Let O be the origin and OX the x-axis.

Angular Position (θ)

The position of a particle at a given instant may be discribed by the angle θ between OP and OX. This angle θ is called the angular position of the particle. As the particle moves on circle, its angular position changes.

Angular Displacement ($\Delta\theta$)

The change in angular position i.e. the angle turned by the position vector of the particle in a given time interval is angular displacement. In Fig. 4.15, let P and Q be the positions of particle at time t and $t + \Delta t$ respectively. The angular displacement in time Δt is $\angle POQ = \Delta\theta$.

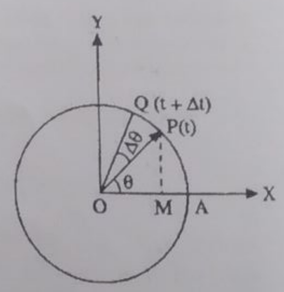


Fig. 4.15 Circular motion in x-y plane.

Average Angular Speed ($\overline{\omega}$) (ω_{av}) $= \frac{\Delta\theta}{\Delta t}$

We define the average angular speed $\overline{\omega}$ (omega) as the ratio of the angular displacement to the time interval Δt. If θ_1 and θ_2 be the angular positions of the moving particle at time t_1 and t_2 respectively then average angular speed

$\Delta x \to \Delta\theta$
$v \to \omega$
$u \to \omega_0$
$a \to \alpha_0$

$(\overline{\omega})$ is,

$$\overline{\omega} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t} \quad \ldots (4.50)$$

Angular Velocity (ω)

The rate of change of angular position is known as the angular velocity (ω). Thus

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} \quad \ldots (4.51)$$

(i) SI unit of angular speed is radian per second (rad / s). Another common descriptive unit for angular speed is revolution per minute (rpm).

(ii) As in the linear case, if the angular speed is constant, then average angular speed $\overline{\omega}$ is equal to instantaneous angular speed.

If position vector turn by equal angle in equal time interval then ω = const.

(iii) If the angular speed ω is constant then angular displacement $\Delta\theta$ in time t is given by, $\omega = \omega_{const}$

$$\Delta\theta = \omega t \quad \ldots (4.52)$$

If ω varies with time then, angular displacement is given by,

$$\Delta\theta = \int_{t_1}^{t_2} \omega \, dt \quad \ldots (4.53)$$

Average Angular Acceleration $(\overline{\alpha})$

The average angular acceleration $\overline{\alpha}$ of a rotating object is defined as the ratio of the change in the angular speed to the time interval Δt,

$$\overline{\alpha} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t} \quad \ldots (4.54)$$

Angular Acceleration (α)

The rate of change of angular velocity is called angular acceleration. Thus, the angular acceleration is

$\alpha = f(\theta)$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} \quad \ldots (4.55)$$

$\alpha = \omega \frac{d\omega}{d\theta}$

The SI units of angular acceleration are rad / s^2.

$\alpha = const$, $\Delta\omega = \alpha \Delta t$; $\alpha = f(t)$, $\Delta\omega = \int \alpha \, dt$

If the angular acceleration α is constant, we can write the following equations of motion as we did in motion in one dimension. Thus,

$\alpha = f(\omega)$

$$\omega = \omega_0 + \alpha t \quad \ldots (4.56)$$

$\alpha(\omega) = \omega \frac{d\omega}{d\theta}$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \quad \ldots (4.57)$$

$\alpha(\omega) = \frac{d\omega}{dt}$

and

$$\omega^2 = \omega_0^2 + 2\alpha\theta \quad \ldots (4.58)$$

where ω_0 and ω are the angular velocities at $t = 0$ and at time t and θ is the angular displacement in time t.

Direction of Angular Variables

Dirⁿ of $\vec{\omega}$ doesn't change so $\vec{\alpha} \parallel \vec{\omega}$

The directions $\vec{\omega}$ and $\vec{\alpha}$ are along the axis given by the right-hand rule shown in Fig. 4.16 When the fingers of the right hand are curled in the direction of the circular motion, the extended right thumb points in the direction of $\vec{\omega}$. The direction of angular acceleration $\vec{\alpha}$ is the same as that of $\vec{\omega}$ if the angular speed (the magnitude of $\vec{\omega}$) is increasing in time and antiparallel to $\vec{\omega}$ if the angular speed is decreasing in time.

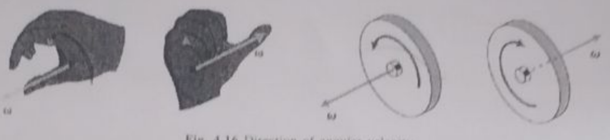


Fig. 4.16 Direction of angular velocity

4.8.2 Relationships Between Angular and Linear Variables

It is important to be able to relate the angular description of circular motion to the orbital or tangential description that is, to relate the angular displacement to the arc lengths. When the particles in Fig. 4.17 moves by an angle $\Delta\theta$ in Δt time, the linear distance Δs, travelled by the particle is equal to the arc length given by, $r\Delta\theta$. Thus,

$$\Delta s = r\,\Delta\theta \quad \ldots (4.59)$$

If v is the linear speed of the particle then,

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = r\frac{d\theta}{dt} = r\omega \quad \ldots (4.60)$$

In vector notation, we can write,

$$\boxed{\vec{v} = \vec{\omega} \times \vec{r}} \quad \ldots (4.61)$$

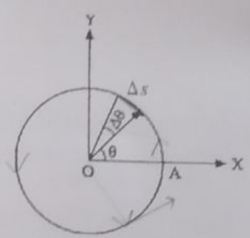


Fig. 4.17 Circular motion in xy plane

A particle moving in a circle has an instantaneous velocity tangential to its circular path. The direction of velocity continuously changes hence circular motion is an accelerated motion.

4.8.3 Acceleration in Circular Motion

We have already seen that in circular motion, direction of velocity continuously changes. Thus, it is an accelerated motion. In circular motion, the acceleration vector has two components one directed toward the centre called *centripetal acceleration* and the other one directed along the tangent called *tangential acceleration*. We discuss the two components in the following section.

Tangential Acceleration (a_t)

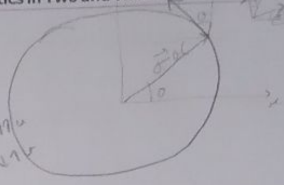
(change in magnitude)

The component of acceleration in the tangential direction is called tangential acceleration. It is rate of change of speed and given by

$$\vec{a}_t = \frac{dv}{dt}\hat{\theta} = r\frac{d\omega}{dt}\hat{\theta} = r\alpha\hat{\theta} \quad \ldots (4.62)$$

The magnitude of tangential acceleration is

$$a_t = \frac{dv}{dt} = r\alpha \ldots (4.63)$$



In vector notation, we can write,

$$\vec{a}_t = \vec{\alpha} \times \vec{r} \ldots (4.64)$$

v↑ then a_t↑

v↓ then a_t↑

(i) Tangential acceleration is zero if the speed of particle remains constant.

(ii) Tangential acceleration is in the direction of velocity if speed increases with time otherwise it is directed opposite to the velocity.

(iii) Tangential acceleration is solely responsible for changing the magnitude of velocity i.e. speed. It has nothing to do with the direction of velocity.

(iv) The direction of tangential acceleration continuously changes.

Centripetal Acceleration (a_c)

(It change the direction of velocity

The component of acceleration directed towards the centre of circle is called *centripetal* or *radial acceleration*. The value of centripetal acceleration is given by,

$$\vec{a}_c = -\omega^2 r(\hat{r}) = \frac{v^2}{r}(-\hat{r}) \ldots (4.65)$$

Thus the acceleration of the particle is in the direction of $-\hat{r}$, that is towards the centre. The magnitude of acceleration is

$$a_c = \frac{v^2}{r} = \omega^2 r \ldots (4.66)$$

In vector notation, we can write, $\vec{a}_c = \vec{\omega} \times \vec{v} \ldots (4.67)$

(i) It is solely responsible for changing the direction of velocity. It has nothing to do with the change in magnitude of velocity.

(ii) The direction of centripetal acceleration continuously changes.

Net Acceleration

The net acceleration $\vec{a}$ is the rate of change of velocity. It is equal to the vector sum of tangential and centripetal acceleration as shown in Fig. 4.18. Thus,

$$\vec{a} = \frac{d\vec{v}}{dt} = \vec{a}_c + \vec{a}_t \cdots (4.68)$$

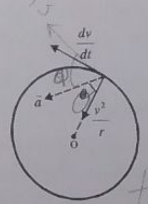


Fig. 4.18 Acceleration in circular motion.

(i) The magnitude of the net acceleration is equal to,

$$a = \sqrt{a_c^2 + a_t^2} = \sqrt{\left(\frac{v^2}{r}\right)^2 + \left(\frac{dv}{dt}\right)^2} \ldots (4.69)$$

The direction of this resultant acceleration makes an angle θ with the radius where,

$$\frac{a_t}{a_c} = \tan\theta = \frac{\left(\frac{dv}{dt}\right)}{\left(\frac{v^2}{r}\right)} \quad \ldots (4.70)$$

4.9 Motion Along A Curved Path

Consider the motion along a two dimensional curved path where the velocity changes both in direction and in magnitude, as described in Fig. 4.19. The velocity vector is always tangent to the path, however in this situation, the acceleration vector $\vec{a}$ is at some angle to the path.

more the curvature lesser is the Radius

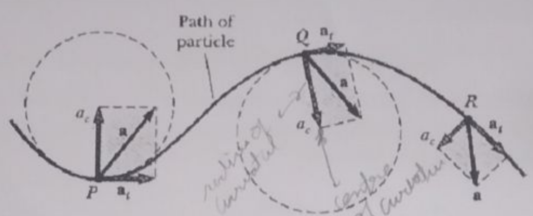


Fig. 4.19 The motion of a particle along the arbitrary curved path lying in the xy plane. If the velocity vector v (always tangent to the path) changes in direction and magnitude, the component vectors of the acceleration $\vec{a}$ are a tangential vector $\vec{a}_t$ and a radial vector $\vec{a}_c$. The acceleration vectors are not necessarily to scale.

As shown in the Fig. 4.19, any small section of the curve can be considered as arc of circle. The radius of the corresponding circle is called radius of curvature and the centre of circle is called centre of curvature. We note that for any section of curved track the greater is the curvature, the lesser is the radius of curvature. As we can see in Fig. 4.19, the curvature at point P is greater that at point Q and hence radius of curvature at P is lesser than that at Q. Thus we can apply the theory of circular motion developed in the above section for any small portion of a curved track. The component of acceleration along the tangent which we call tangential acceleration (a_t) changes speed where as compenent of acceleration normal to the velocity which we call centripetal acceleration (a_c) changes direction of velocity. Following points are worth noting related to motion on a curved track.

(i) As the direction of velocity continuously changes on a curved track the angle between acceleration vector $\vec{a}$ and velocity vector $\vec{v}$ must be other than 0° and 180° i.e. the $\vec{a}$ and $\vec{v}$ can not be parallel or antiparallel.

(ii) If the angle between $\vec{v}$ and $\vec{a}$ is acute i.e. between 0° and 90° then the speed increases with time and if it is obtuse i.e. between 90° and 180° then speed decreases with time.

(iii) For a particle moving with constant speed on a curved track angle between acceleration vector $\vec{a}$ and velocity vector $\vec{v}$ must be 90°. In this case tangential component of acceleration is zero.

(iv) **Radius of curvature at a point:** To find out the radius of curvature at any point P of the curved track we need to follow the following steps:

. Direction of $\vec{v}$ is changing so a_n is indispensable

$a_t = \frac{dv}{dt} = \frac{a \cdot \vec{v}}{v}$

$a_n = a \sin\theta = \sqrt{a^2 - a_t^2} = \frac{|a \times v|}{v}$

(a) find out speed v at point P.

(b) find out component of acceleration perpendicular to velocity vector. i.e. a_c.

(c) If r be the radius of curvature at point P then

$$a_c = \frac{v^2}{r} \text{ or } r = \frac{v^2}{a_c}$$

Note : For a particle, moving on a curved track with constant speed, acceleration continuously varies.

Example 15. A car goes around a curve of radius 40 m. When its velocity points north its speed is changing at 2 m / s^2 and its total acceleration is at 37° N of W.

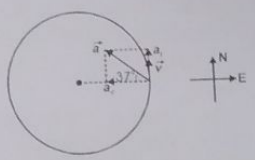
(i) Is the car speeding up or slowing down?

(ii) What is its speed at this instant?

Solution: The given situation is as shown in figure. As at point P for the shown direction of the total acceleration **a**, it has a component in direction of $\vec{v}$ i.e. tangential acceleration a_t is in direction of $\vec{v}$, so speed must increase. From figure,

$$a_t = a_c \tan 37^0$$

$$a_c = \frac{a_t}{\tan 37^0} = \frac{2}{(3/4)} = \frac{8}{3} \text{ m / s}^2$$



and since $a_c = \frac{v^2}{r}$

∴ $v^2 = a_c r$

or $v = \sqrt{a_c r} = \sqrt{\frac{8 \times 40}{3}} = \sqrt{\frac{320}{3}}$ m / s

Example 16. A particle moves in the x-y plane with the velocity $\bar{v} = a\hat{i} + bt\,\hat{j}$. At the instant $t = \frac{a\sqrt{3}}{b}$, Find the magnitude of tangential, normal and total acceleration.

Solution: Given $\bar{v} = a\hat{i} + bt\,\hat{j}$ so the magnitude of velocity at $t = \frac{a\sqrt{3}}{b}$ is

$$\left|\bar{v}\left(t = \frac{a\sqrt{3}}{b}\right)\right| = \sqrt{a^2 + b^2\left(\frac{a\sqrt{3}}{b}\right)^2} = 2a$$

Acceleration of particle, $\bar{a} = \frac{d\bar{v}}{dt} = b\hat{j}$ with $|\bar{a}| = b$

The tangential acceleration is component of net acceleration parallel to the direction of velocity so

$$a_t = \frac{\bar{v}.\bar{a}}{|\bar{v}|}$$

$$\therefore \quad a_t = \frac{\left(a\hat{i} + bt\,\hat{j}\right)\cdot\left(b\hat{j}\right)}{2a} = \frac{b^2 t}{2a} = \frac{b^2}{2a}\left(\frac{a\sqrt{3}}{b}\right) = \frac{\sqrt{3}b}{2}$$

If a_c be the normal acceleration then

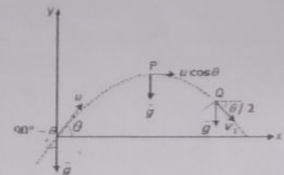
$$a_t^2 + a_c^2 = a^2 \text{ or } a_c^2 = \sqrt{a^2 - a_t^2}$$

$$\therefore \quad a_c\left(t = \frac{a\sqrt{3}}{b}\right) = \sqrt{b^2 - \left(\frac{\sqrt{3}}{2}b\right)^2} = \sqrt{\frac{b^2}{4}} = \frac{b}{2}$$

Example 17. A particle is projected with a speed u at an angle θ with the horizontal.
(a) What is the initial rate of change of speed?
(b) Consider a small part of its path near the highest position and take it approximately to be a circular arc. What is the radius of this circle? This radius is called the radius of curvature of the curve at the point.
(c) What is the radius of curvature of the parabola traced out by the projectile at a point where the particle velocity makes an angle $\theta / 2$ with the horizontal?

Solution: For projectile motion at every point of trajectory the acceleration of particle is

$$\vec{a} = -g\hat{j}$$



The rate of change of speed is equal to the component of acceleration vector parallel to the velocity. Initially the component of $\vec{a} = \vec{g}$ parallel to the velocity is $-g\cos(90^\circ - \theta) = -g\sin\theta$. Thus, Initial rate of change of speed

$$\left.\frac{dv}{dt}\right|_{t=0} = -g\sin\theta$$

(b) At highest point P as the vertical component of velocity is zero, so at this point, velocity will be $v_P = u\cos\theta$ along the horizontal. The acceleration $\vec{g}$ is being normal to the velocity acts as centripetal acceleration. If r_P be the radius of curvature at the point then

$$g = \frac{v_P^2}{r_P}$$

or
$$r_P = \frac{v_P^2}{g} = \frac{u^2\cos^2\theta}{g}$$

(c) Consider point Q (figure) Where velocity $\vec{v}_Q$ (hence a_t) makes angle $\theta / 2$ with horizontal. The angle between the velocity vector $\vec{v}_Q$ and acceleration vector $\vec{g}$ is $90^\circ - \theta / 2$ so component of acceleration normal to the velocity, a_c is

$$a_c = g\sin\left(90^\circ - \frac{\theta}{2}\right) = g\cos\frac{\theta}{2}$$

As at every point on trajectory the horizontal component of velocity is unchanged so

$$v_Q\cos\left(\frac{\theta}{2}\right) = u\cos\theta$$

or
$$v_Q = \frac{u\cos\theta}{\cos\theta / 2}$$

If r_Q be the radius of curvature at point Q then

$$a_c = \frac{v_Q^2}{r_Q} \text{ or } r_Q = \frac{v_Q^2}{a_c} = \frac{\left(\frac{u\cos\theta}{\cos\theta / 2}\right)^2}{g\cos(\theta / 2)} = \frac{u^2\cos^2\theta}{g\cos^3(\theta / 2)}$$

Note : During the flight magnitude of tangential acceleration (rate of change of speed) and radius of curvature first decreases than increases where as normal component of acceleration first increases and than decreases. The radius of curvature of trajectory is lowest at the highest point.

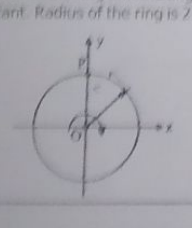
Section Review 4.5

1. A watch has a second hand 2 cm long. Calculate
(a) the angular speed of second hand,
(b) the speed of the tip of the second hand,
(c) magnitude of change in velocity of the tip from $t = 0$ to $t = 15$ s

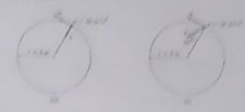
2. The linear velocity of a particle moving on the circumference of a circle is equal to the velocity acquired by a freely falling body through a distance equal to one fourth the diameter of the circle. What is the centripetal acceleration of the particle moving along the circle.

3. Figure shows a body of mass m moving with a uniform speed v along a circle of radius r. Find the magnitude of average acceleration in going from A to B.

4. A ring rotates about z axis as shown in figure. The plane of rotation is xy. At a certain instant the acceleration of a particle P (shown in figure) on the ring is $(6\hat{i} - 8\hat{j})$ m/s^2. Find the angular acceleration of the ring and the angular velocity at that instant. Radius of the ring is 2 m.



5. In figure the particles are traveling counterclockwise in a circle of radius 5 m with speeds which may be varying. The acceleration vectors are indicated at certain times. Find the values of v and dv/dt for each case.



6. A particle is moving in a circular path with speed varying with time as $v = 1.5t^2 + 2t$ m/s. If 2 cm the radius of circular path, find the angular acceleration at $t = 2$ s.

7. A particle travels in a circle of radius 6 m. Its speed increases at 8 m/s^2 and its centripetal acceleration is 6 m/s^2. Find :
(a) the magnitude of its total linear acceleration;
(b) its speed.

8. A particle is moving with a constant angular acceleration of 4 rad/s^2 in a circular path. At time $t = 0$, particle was at rest. Find the time at which the magnitudes of centripetal acceleration and tangential acceleration are equal.

9. A particle is revolving in a circle of radius 1 m with an angular speed of 12 rad/s. At $t = 0$, it was subjected to a constant angular acceleration α and its angular speed increased to $(480/\pi)$ rpm in 2 seconds. Particle then continues to move with attained speed. Calculate
(a) angular acceleration of the particle,
(b) tangential velocity of the particle as a function of time.
(c) acceleration of the particle at $t = 0.5$ second and at $t = 3$ seconds
(d) angular displacement at $t = 3$ second.

4.10 Uniform Circular Motion

If the particle moves in the circle with constant speed, we call it uniform circular motion. In this case tangential acceleration $(dv/dt) = 0$, therefore, angular acceleration α is also zero.

(i) The net acceleration of the particle is directed towards the centre of the circle which has magnitude.

$$a_c = \frac{v^2}{r} = \omega^2 r \quad \ldots (4.71)$$

a is const in magnitude but varying in ~~magnitude~~ direction so non uniform accln

(ii) Since the speed is constant hence, the particle turns by equal angle in equal time interval.

(iii) Speed and magnitude of acceleration are constant but their directions are always changing as in Fig. 4.20(a). So velocity and acceleration are not constant. The angle between velocity and acceleration is always 90° as in Fig. 4.20(b).

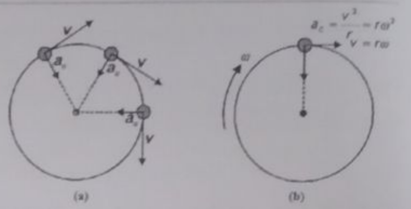


Fig. 4.20 Velocity and acceleration in uniform circular motion

(iv) **Time Period:** The period (T) is the time it takes for an object in circular motion to make one complete revolution, or *cycle*.

$$T = \frac{2\pi}{\omega} = \frac{2\pi r}{v} \quad \ldots (4.72)$$

Closely related to the period is the frequency (f), which is the number of revolutions, or cycles, made per second. Thus unit of frequency is per seconds (s^{-1}), which is called the hertz (Hz) in the SI. Thus,

$$1 \text{ hertz} = 1 \text{ cycle per second} \quad \ldots (4.73)$$

The frequency (f) and time period (T) are related as,

$$f = \frac{1}{T} \quad \ldots (4.74)$$

Miscellaneous Example

Example 1. The position of a particle varies as

$$\vec{r} = t^3\hat{i} - \frac{12}{5}t^{5/2}\hat{j} + 3t^2\hat{k} \text{ cm}$$

where t is in seconds. Find the distance travelled in first 2 seconds. What is the distance of particle from the origin at $t = 1$s?

Solution: To find out the distance travelled, we need to calculate speed as a function of time. So

$$\vec{v} = \frac{d\vec{r}}{dt} = 3t^2\hat{i} - 6t^{3/2}\hat{j} + 6t\hat{k} \text{ m/s}$$

$$\therefore \quad v = |\vec{v}| = 3t\sqrt{t^2 + \left(\sqrt{2t}\right)^2 + (2)^2} = 3t(t+2) = 3t^2 + 6t \text{ cm/s}$$

Thus $\quad v = \frac{ds}{dt} = 3t^2 + 6t$

or $\quad s = \int_0^2 \left(3t^2 + 6t\right)dt = \left(t^3 + 3t^2\right)_0^2 = 20 \text{ cm}$

The position of particle at $t = 1$s is,

$$\vec{r} = (1)^3\hat{i} - \frac{12}{1}(1)^{5/2}\hat{j} + 3\hat{k} = \hat{i} - \frac{12}{5}\hat{j} + 3\hat{k} \text{ cm}$$

Fig. 7.14 Ball bearings

(IV) Use of ball bearings or roller-bearings: The rolling friction is much less than the sliding friction. So, we convert sliding friction into rolling friction. Even the axle is not allowed to move directly in the hub. The friction is further minimised by the use of roller bearings or ball bearings as Fig. 7.14.

7.10 Dynamics of Uniform Circular Motion

If a particle of mass m moves in a circle of radius r with constant speed v, a net non zero force F must act towards the centre of the circle to generate the centripetal acceleration a. Applying Newton's second law,

$$\boxed{F = ma = \frac{mv^2}{r} = m\omega^2 r} \quad \ldots (7.16)$$

In vector notation, $\vec{F} = \dfrac{mv^2}{r}(-\hat{r}) = -m\omega^2\vec{r}$ . . . (7.17)

Here, $\hat{r}$ is a unit vector in radial direction. Since, this resultant force is directed towards the centre, it is called *centripetal force*.

In uniform circular motion, net force on the particle is directed toward the centre of circle & called centripetal force

(i) Circular Motion of a Block Connected With a String.

Consider a disk of mass m connected with a string as shown in Fig. 7.15. The disk is rotated on a frictionless horizontal table with constant speed v in a circle of radius R as shown.

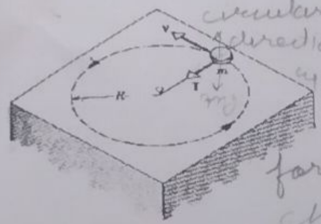


Fig. 7.15 Block connected with a string

* In uniform circular motion, direction of accl & hence direction of net force continuous changes.

As in Fig. 7.15, the necessary centripetal force is provided by tension T in string. If accelerates the disk by constantly changing the direction of its velocity so that the disk moves in a circle. The direction of $\vec{T}$ is always towards the centre. Thus from Newton's second law.

$F < \frac{mv^2}{r}$

$F > \frac{mv^2}{r}$

Net force directed towards centre = Mass times acceleration

or $$T = \frac{mv^2}{r}$$

As we increase the speed of particle, the tension in string also increases.

(ii) Conical Pendulum

Figure 7.16(a) shows a small body of mass m revolving in a horizontal circle with constant speed v at the end of a string of length L. As the body swings around, the string sweeps over the surface of an imaginary cone. This device is called a conical pendulum. Let us find the time required for one complete revolution of the body.

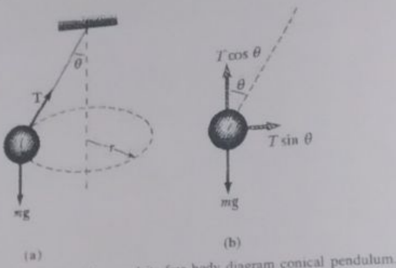


Fig. 7.16 The conical Pendulum and its free body diagram conical pendulum.

If the string makes an angle θ with the vertical, the radius of the circular path is $R = L\sin\theta$. The force acting on the body of mass m are its weight $m\vec{g}$ and the tension $\vec{T}$ of the string, as shown in Fig. 7.16(b). In figure, the vertical component of tension balances the weight while its horizontal component provides the necessary centripetal force. Thus,

$$T\sin\theta = \frac{mv^2}{R} \quad \ldots (i)$$

$$T\cos\theta = mg \quad \ldots (ii)$$

From these two equations, $v = \sqrt{Rg\tan\theta}$ so, the time period is

$$T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{Rg\tan\theta}} = 2\pi\sqrt{\frac{R}{g\tan\theta}}$$

But, $R = L\sin\theta$, so that

$$T = 2\pi\sqrt{\frac{L\cos\theta}{g}} \quad \ldots (7.18)$$

* If a table is rotating with ω then all particle rotating with 'ω' also
* but these particle velocity are not same due to radius

(iii) Circular Motion of a Coin on a Turn Table

A coin of mass m is placed on a circular table rotating with an angular velocity ω about a vertical axis passing through its centre. The distance of the object form the axis is r.

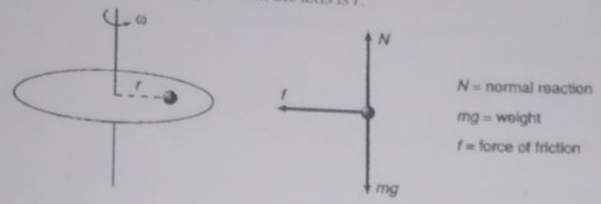


Fig. 7.17 Coin on a rotating turn table

If angular speed of turn table is not too large, the coin does not slip over it and the necessary centripetal force is provided by static friction as shown in Fig. 7.17. Thus,

$$f = m\omega^2 r \dots \text{(i)}$$

As the coin does not slide on the table so it moves in a circle with angular speed ω of observer

For equilibrium in vertical, $N = mg \dots \text{(ii)}$

As angular speed of turn table increases, the requirement of frictional force also increases. We can calculate the maximum angular speed up to which there is no sliding. For no sliding the required static friction should be less than or equal to the limiting friction. Thus,

$$f \le f_{s,max} = \mu_s N$$

The coin has tendency to slip outwards so static friction acts towards centre.

From Eq. (i) and (ii), $\boxed{m\omega^2 r \le \mu_s mg} \Rightarrow \boxed{\omega \le \sqrt{\frac{\mu_s g}{r}}} \dots (7.19)$

Thus the maximum angular speed up to which there is no sliding is $\sqrt{\frac{\mu_s g}{r}}$. If angular speed of turn table is greater than $\sqrt{\frac{\mu g}{r}}$, the coin slips outwards.

(iv) The Rotor

In many amusement parks we find a device called the rotor. The rotor is a hollow cylindrical room that can be set rotating about the central vertical axis of the cylinder. A person enters the rotor closes the door, and stands up against the wall. The rotor gradually increases its rotational speed from rest until, at a predetermined speed, the floor below the person is opened downward, revealing a deep pit. The person does not fall but remains "pinned up" against the wall of the rotor as in Fig. 7.18. What minimum rotational speed is necessary prevent falling?

Let us consider the free body diagram of man (Fig. 7.18). In the figure friction balances the weight of person while normal reaction between the person's body and the wall provides the centripetal force for circular motion. Thus,

$$f_s = mg \ldots \text{(i)}$$

$$N = m\omega^2 r \ldots \text{(ii)}$$

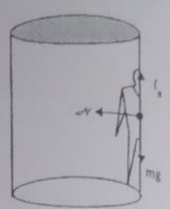


Fig. 7.18 Person standing against inner wall of a rotor

For the person not to slide the requirement of friction of rotor should be less than or equal to the limiting friction. Thus

$$f_s \le f_{s,\max} = \mu_s N$$

Using Eq. (i) and (ii) in above equation

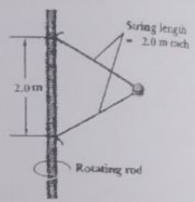
$$mg \le \mu_s (m\omega^2 r)$$

Therefore, $$\sqrt{\frac{g}{\mu_s r}} \le \omega$$

Thus, the minimum speed required to keep the person from falling down is,

$$\omega_{\min} = \sqrt{\frac{g}{\mu_s r}} \ldots (7.20)$$

Note that the result does not depend on the person's weight.

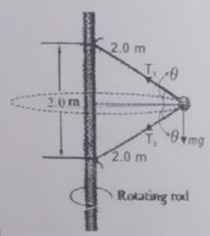


Example 11. As shown in figure, a ball of mass 1.5 kg is connected by means of two massless strings to a vertical, rotating rod. The strings are tied to the rod and are taut. The tension in the upper string is 40 N.

(a) Draw the free body diagram for the ball. What are

(b) the tension in the lower string,

(c) the speed of the ball?



Solution: The forces acting on the bob are:

(i) weight mg downward

(ii) tension force T_1 due to upper string

(iii) tension force T_2 due to lower string

From the geometry of figure,

$$\sin\theta = \frac{1.0}{2.0} = \frac{1}{2} \Rightarrow \theta = 30^\circ$$

The ball moves in a horizontal circle of radius r where,

$$r = 2.0\cos 30^\circ = \sqrt{3}\text{ m}$$

The net force on the ball in vertical direction is zero so for its equilibrium along vertical

$$T_1 \sin 30^\circ = T_2 \sin 30^\circ + mg$$

or $$\frac{40}{2} = \frac{T_2}{2} + (1.5)(10) \Rightarrow T_2 = 10\text{ N}$$

The horizontal components of tension force T_1 and T_2 provide the necessary centripetal force so

$$T_1 \cos 30^\circ + T_2 \cos 30^\circ = \frac{mv^2}{r}$$

or $$(40)\frac{\sqrt{3}}{2}+(10)\frac{\sqrt{3}}{2}=1.5\times\frac{v^2}{\sqrt{3}}$$

or $$v^2=\frac{75}{1.5}=50 \Rightarrow v=\sqrt{50}\,\text{m/s}=7\,\text{m/s}$$

Example 12. A hemispherical bowl of radius R is set rotating about its axis of symmetry which is kept vertical. A small block kept in the bowl rotates with the bowl without slipping on its surface. If the surface of the bowl is smooth and the angle made by the radius through the block with the vertical is q, find the angular speed at which the bowl is rotating.

Solution: The given situation is as shown in figure. The forces acting on the block are
(i) weight mg downward
(ii) normal contact force N normal to the surface of sphere along the radius.
The block rotates along with bowl without slipping in a horizontal circle of radius r where

$$r=R\sin\theta$$

The net force on the block in vertical direction is zero so for vertical equilibrium of block

$$N\cos\theta=mg\ \ldots\text{(i)}$$

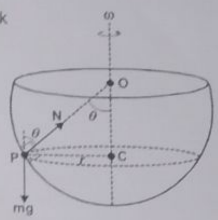
The horizontal component of normal contact force provides the necessary centripetal force. Thus

$$N\sin\theta=m\omega^2 r$$

or $$N\sin\theta=m\omega^2 R\sin\theta \quad\text{or}\quad N=m\omega^2 R\ \ldots\text{(ii)}$$

From equations (i) and (ii)

$$\omega^2 R=\frac{g}{\cos\theta}\Rightarrow\omega=\sqrt{\frac{g}{R\cos\theta}}$$



Example 13. A car moves at a constant speed on a straight but hilly road. One section has a crest and dip of the same 250 m radius; see Fig.

(a) As the car passes over the crest, the normal force on the car is one-half the 16 kN weight of the car. What will be the normal force on the car as it passes through the bottom of the dip?
(b) What is the greatest speed at which the car can move without leaving the road at the top of the hill?
(c) Moving at the speed found in (b), what will be the normal force on the car as it moves through the bottom of the dip?

Solution: (a) The free body diagram of the car at point A (crest) is shown in figure. The forces acting on the car are
(i) weight mg downward
(ii) normal contact force N_A perpendicular to the track in vertical direction.
The net force towards the centre of the circle is $mg-N$ which provides the necessary centripetal force. Applying Newton's second law for the radial direction gives

$$mg-N_A=\frac{mv^2}{R}\ \ldots\text{(i)}$$

Substituting $mg=16\text{N}$ and $N_A=\frac{mg}{2}=8\text{N}$

$$\frac{mv^2}{R}=16-8=8\text{N}\ldots\text{(ii)}$$

when the car passes through point B (the bottom of the dip), the normal contact force N_B acts upward. The net force towards the centre of circle at point B is $N_B - mg$ which provides the required centripetal force so applying Newton's second law in radial direction gives

$$N_B - mg = \frac{mv^2}{R} \quad \dots \text{(iii)}$$

substituting the value of $\frac{mv^2}{R}$ from equation (i), we get

$$N_B = 16 + 8 = 24\,\text{N}$$

(b) For the car not to leave the road at the top of hill, $N_A > 0$. At A

$$mg - N_A = \frac{mv^2}{R} \quad \text{so} \quad N_A = mg - \frac{mv^2}{R} > 0 \quad \text{or} \quad v < \sqrt{Rg}$$

Thus the maximum speed at which the car does not have the road at the top of the hill is

$$v_m = \sqrt{Rg} = \sqrt{250 \times 10} = 50\,\text{m/s}$$

(c) From equation (ii), for $v = v_m$,

$$N_B = mg + \frac{mv_m^2}{R} = mg + \frac{m}{R}(Rg) = 2mg = 32\,\text{N}$$

Section Review 7.4

1. A stone of mass 0.25 kg tied to the end of a string is whirled round in a circle of radius 1.5 m with a speed of $\frac{60}{\pi}$ rpm a horizontal plane.
   (a) What is the tension in the string?
   (b) What is the maximum speed with which the stone can be whirled around if the string can withstand a maximum tension of 150 N?

2. Tarzan (m = 85.0 kg) tries to cross a river by swinging from a vine. The vine is 10.0 m long, and his speed at the bottom of the swing (as he just clears the water) is 7.00 m / s. Tarzen doesn't know that the vine has breaking strength 1000 N. Does he make it safely across the river?

3. A student of weight 667 N rides a steadily rotating Ferris wheel (the student sits upright). At the highest point, the magnitude of the normal force $\vec{N}_1$ on the student from the seat is 556 N.
   (a) Does the student feel "light" or "heavy" there?
   (b) What is the magnitude of $\vec{N}_2$ at the lowest point?
   (c) What is the magnitude N_1 if the wheel's speed is doubled?

4. A ceiling fan has a diameter (of the circle through the outer edges of the three blades) of 120 cm and rpm 1500 at full speed. Consider a particle of mass 1 g sticking at the outer end of a blade. How much force does it experience when the fan runs at full speed ? Who exerts this force on the particle? How much force does the particle exert on the blade along its surface?

5. A small coin is placed on a flat, horizontal turnable. The turntable is observed to make three revolutions in 3.14 s.
   (a) What is the speed of the coin when it rides without slipping at a distance 5.0 cm from the center of the turntable?
   (b) What is the acceleration (magnitude and direction) of the coin?
   (c) What is the magnitude of the firctional force acting on the coin if the coin has a mass of 2.0 g?
   (d) What is the coefficient of static friction between the coin and the turntable if the coin is observed to slide off the turntable when it is more than 10 cm from the center of the turntable?

6. A cylindirical bucket filled with water is whirled around in a vertical circle of radius r. What can be the minimum speed at the top of the path if water does not fall out from the bucket? If it continues with this speed, what normal contact force the bucket exerts on water at the lowest point of the path?

1. By Friction Only

Suppose a car of mass m is moving at a speed v in a horizontal circular arc of radius R. In this case, the necessary centripetal force to the car will be provided by force of static friction f between the tires and the road acting towards centre.

The car has tendency to skid outward therefore static f acts inwards & provides the req. centripetal force

Thus, $$f = \frac{mv^2}{R} \quad \ldots (7.21)$$

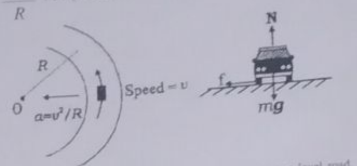


Fig. 7.19 Circular motion of a car on a level road

However, this static friction has a maximum limiting value. If the speed of the car is high enough, the friction will not be sufficient to supply the necessary centripetal acceleration, and the car will skid outward from the center of the curve. Further, limiting value of f is, $f_{s,max} = \mu N = \mu mg(N = mg)$. Therefore, for a safe turn without sliding

$$\frac{mv^2}{R} \le f_{s,max}$$

$$\mu \le \frac{v^2}{Rg} \text{ or } v \le \sqrt{\mu Rg}$$

If the speed of the car is too high, car starts skidding outwards. Thus, the maximum speed up to which safe turning is possible is given by,

$$v_{max} = \sqrt{\mu Rg} \quad \ldots (7.22)$$

$v > \sqrt{\mu gR}$ car skids outward

2. By Banking of Roads

Friction is not always reliable at circular turns if high speeds and sharp turns are involved. To avoid dependence on friction, the roads are banked at the turn so that the outer part of the road is some what lifted compared to the inner part. This reduces the chances of skidding because the normal force exerterd on the car by the road then has a component toward the center of the curve that reduces the need for friction. In fact, for a circular curve with a given banking angle and radius, there is one speed for which no friction is required at all. This condition is used in banking design.

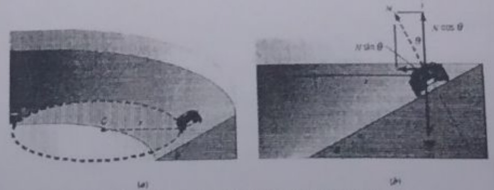


Fig. 7.20 Motion of a vehicle on a banked circular turn

$v < \sqrt{rg\tan\theta}$, static f acts upward along the incline. The min safe speed

$v_{min} = \sqrt{rg\left(\frac{\tan\theta - \mu_s}{1+\tan\theta}\right)}$

Figure 7.20 shows a vehicle on a banked circular turn than and its free body diagram. The horizontal component of normal reaction $N\sin\theta$ provides the necessary centripetal force. Thus,

$$N\sin\theta = \frac{mv^2}{r} \dots \text{(i)}$$

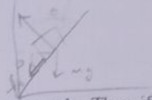
$v > \sqrt{rg\tan\theta}$, the vehicle has tendency to skid outward & therefore static fr acts downwards along the incline. In the limiting situation i.e. when vehicle is at the threshold of sliding $f = f_{s\,max} = \mu_s N$

In vertical direction, the vehicle is in equilibrium. Thus,

$$N\cos\theta = mg \text{ (ii)}$$

From these two equations, we get

$$\tan\theta = \frac{v^2}{rg} \dots (7.23)$$



$\mu_s N\cos\theta + N\sin\theta = \frac{mv^2}{r}$

$N\cos\theta - \mu N\sin\theta = mg$

$v_{max} = \sqrt{rg\left(\frac{\tan\theta + \mu_s}{1-\mu_s\tan\theta}\right)}$

Equation 7.23 gives the angle of banking corresponding to speed v. Thus, if we know the angle of banking, the corresponding speed for which no friction is required for turning can be found from,

& μ_s ... is $\tan\theta$ ... μ_s

$v_{min} = 0$

$$v = \sqrt{rg\tan\theta} \dots (7.24)$$

When speed of vehicle is equal to $\sqrt{rg\tan\theta}$, no friction force acts and the component of normal reaction is sufficient to provide necessary centripetal force. If the speed of vehicle is greater than $\sqrt{rg\tan\theta}$ than it has tendency to skid outwards. Similarly if the speed is less then $\sqrt{rg\tan\theta}$,which will tend to skid downward.

Example 14. A cyclist speeding at 18 km / h on a level road takes a sharp circular turn of radius 9 m without reducing the speed. The co-efficient of static friction between the tyres and the road is 0.4. Will the cyclist slip while taking the turn? The mass of bicycle plus cyclist is 81 kg.

Solution: For turning on a level road the necessary centripetal force is provided by static friction which acts towards the centre of turn.

$$f = \frac{mv^2}{r} = \frac{81\times(5)^2}{9} = 225\text{N} \qquad [v = 18\text{ km / h} = 5\text{ m / s}]$$

The maximum available friction is

$$f_s = \mu_s N = \mu_s mg = 0.4\times 81\times 10 = 320\text{ N}$$

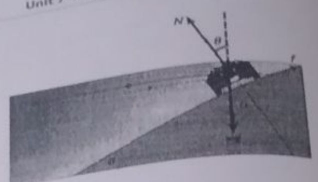
Since $f < f_s$, the force of friction is sufficient to provide the centripetal force so cyclist will not slip.

Example 15. A turn of radius 20 m is banked for the vehicle of mass 200 kg going at a speed of 10 m / s. Find the direction and magnitude of frictional force acting on a vehicle if it moves with a speed 5 m / s. Assume that firction is sufficient to prevent slipping.

Solution: As the road is banked for speed v = 10 m / s. So the angle of banking is

$$\tan\theta = \frac{v^2}{rg} = \frac{(10)^2}{(20)(10)} = \frac{1}{2}$$

When the speed of vehicle is less than the speed corresponding to banking, the vehicle has tendency to skid inward and so force of static friction acts upward (figure).



The forces acting on the vehicle are
(i) weight mg downward,
(ii) normal contact force N perpendicular to the road,
(iii) friction f directed up the turn
The FBD of the vehicle is as shown in the figure.
As the vehicle moves in horizontal circle net force on it towards the centre of the turn. The resultant of horizontal component of normal reaction and frictional force provide the required centripetal force i.e.,

$$N\sin\theta - f\cos\theta = \frac{mv^2}{r} \quad \ldots (i)$$

In the vertical direction net force on the vehicle is zero so

$$N\cos\theta + f\sin\theta = mg \quad \ldots (ii)$$

On multiplying (i) by $\cos\theta$ and , (ii) by $\sin\theta$ and then subtracting the two equations we obtain

$$f = \left[mg\sin\theta - \frac{mv^2}{r}\cos\theta\right]$$

Alternately, we can apply newton second law along the banked road. The acceleration of car is $\frac{v^2}{r}$ along the horizontal so its component parallel to the incline is $\frac{v^2}{r}\cos\theta$. Now from Newton's second law along the inclined road,

$$mg\sin\theta - f_s = m\left(\frac{v^2}{r}\cos\theta\right)$$

or

$$f_s = mg\sin\theta - \frac{mv^2}{r}\cos\theta$$

As $\tan\theta = \frac{1}{2}$ so $\cos\theta = \frac{2}{\sqrt{5}}$ and $\sin\theta = \frac{1}{\sqrt{5}}$

$$\therefore \quad f = \left[200 \times 10\frac{1}{\sqrt{5}} - \frac{200 \times 25}{20} \times \frac{2}{\sqrt{5}}\right] = \frac{1500}{\sqrt{5}} = 300\sqrt{5} \approx 675\,\text{N}$$

Section Review 7.5

1. Keeping the banking angle same, the maximum speed with which a vehicle can travel on a curved road is to be increased by 10%. What should be the new radius of curvature if its initial value is 20 m?

2. If a road is horizontal (no banking), what should be the minimum friction coefficient so that a scooter going at 18 km / h in a circle of radius 10 m does not skid?

3. A train rounds an unbanked circular bend of radius 30 m at a speed of 54 km / h. What is the angle of banking required to prevent wearing out of the rail?

4. The road at a circular turn of radius 4.8 m is banked by an angle of 37°. With what speed should a vehicle move on the turn so that the normal contact force is able to provide the necessary centripetal force?

NCERT Part I: Read 104 - 105 pages;
Exercise : 4.31, 5.31, 5.37, 5.38, 5.39 and 5.40.

Solves Examples : 5.10 and 5.11

7.12 Non-Uniform Circular Motion

In nonuniform circular motion, speed of particle continuously changes. Thus the acceleration has both radial and tangential components as shown in Fig. 7.20(a). Therefore, the force acting on the particle must also have a tangential and a radial component. That is since the total acceleration is $\vec{a} = \vec{a}_c + \vec{a}_t$ the total force exerted on the particle is $\vec{F} = \vec{F}_c + \vec{F}_t$ as shown in Fig. 7.20(b).

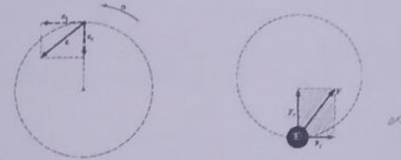


Fig. 7.20 (a) acceleration vector, (b) force vector in nonuniform circular motion

The vector $\vec{F}_t$ is directed along the tangent of the circle and is responsible for the tangential acceleration , which causes the speed of the particle to change time. The vector $\vec{F}_r$ is directed towards the centre and generates radial acceleration which is responsible for changing the direction of velocity. Applying Newton's second law in radial and tangential directions separate by we can write,

Net force towards the centre of the circle, $F_c = \dfrac{mv^2}{r} \ldots (7.25)$

Net force along the tangent $F_t = m\dfrac{dv}{dt} \ldots (7.26)$

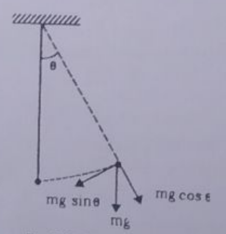


Fig. 7.21 The motion of a simple pendulum

The following example demonstrates this type of motion. Consider a simple pendulum, constructed by attaching a bob of mass m to a string of length L fixed at its upper end as shown in Fig. 7.21. The bob oscillates in a vertical circle. Let the speed of the bob is v when the string makes an angle θ with the vertical. The forces acting on the bob are

(a) the tension T

(b) the weight mg

As the bob moves in a vertical circle with centre at O, the radial acceleration is $\dfrac{v^2}{L}$ towards O. Taking the components along this radius and applying Newton's second law, we get

$$T - mg\cos\theta = \frac{mv^2}{L}$$

or

$$T = m\left(g\cos\theta + \frac{v^2}{L}\right) \ldots (7.27)$$

Similarly, in tangential direction the net force is $mg\sin\theta$. Thus,

The tangential acceleration is, $a_t = g\sin\theta \ldots (7.28)$

At extrem T is min & at mean posi T is maximum

Thus, the bob has both radial and tangential accelerations during the oscillations.

The following figure shows a pendulum oscillating with angular amplitude (maximum angular deflection) θ_0 in three different positions.

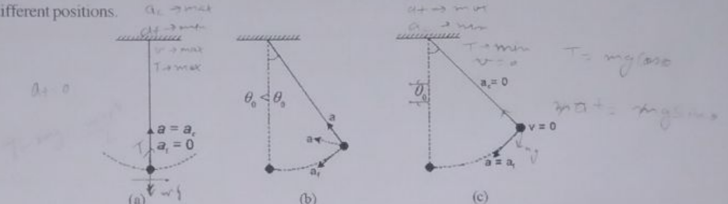
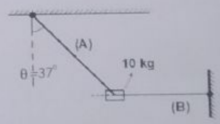


Fig. 7.22 pendulum at (a) ($\theta = 0$) position, (b) intermediate position ($0 < \theta < \theta_0$) and (c) extreme position ($\theta = \theta_0$)

In Fig. 7.22(a), the pendulum bob is at mean position. Its speed is maximum. As there is no force in tangential direction so tangential acceleration a_t is zero. Thus, the net acceleration of pendulum is radial directed towards the centre O of the circular arc.

In Fig. 7.22(b), the pendulum bob is at some intermediate position between mean and extreme. The acceleration vector has both radial and tangential components. The net acceleration is at some angle with string which is between 0° and 90°.

In Fig. 7.22(c) the pendulum is at extreme position. It comes to momentarly rest. At this position radial component of acceleration is zero. Thus the net acceleration is tangential i.e. along the tangent to the circular path at an angle 90° with the string.



Example 16. The 10 kg block is in equilibrium.

(i) Find the tension in string A.

(ii) Find the tension in string A just after the string B is cut?

Solution: (a) The forces acting on the block are figure (a)
(a) weight *mg* downward,
(b) tension force T_1 exerted by string A and
(c) tension force T_2 exerted by string B.
As the block is in equilibrium so net force on it zero
For the vertical equilibrium of block

$$T_1 \cos 37^\circ = Mg$$

$$T_1 = \frac{mg}{\cos 37^0} = \frac{10 \times 10}{4} \times 5 = 125\,\text{N}$$

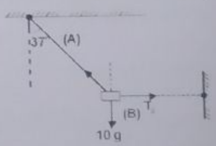


Fig. (a)

(b) Just after the string B is cut the tension T_2 ceases to act and block is on the verge of executing oscillation on a circular arc figure (b).

As the speed of block is zero at this position which means that radial acceleration is zero, at this instant. So the net force along the length of the string is zero. Thus

$$T_1' = Mg \cos 37^\circ = 100 \times \frac{4}{5} = 80\,\text{N}$$

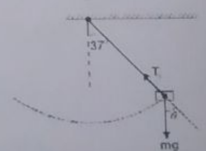


Fig. (b)

7.13 Centrifugal Force

A frame of reference rotating with constant angular velocity ω in a circle of radius r is a non-inertial frame. If we see an object of mass m in this frame then pseudo force of magnitude $m\omega^2 r$ will have to be applied to this object in a direction away from the centre. This pseduo force is called the *centrifugal force*. "Centrifugal means fleeing from the center". Applying this force we can now apply Newton's laws in their usual form. Thus

$$\vec{F}_{cf} = m\omega^2\vec{r} \quad \ldots (7.29)$$

The centrifugal force is an imaginary force. In reality, it does not exist. We need to apply centrifugal force only when we observe motion of a body from a rotating frame. The following example explain the above discussion.

Suppose a block of mass m lying on a horizontal frictionless turntable is connected to a string as in Fig. 7.23. According to an inertial observer, if the block rotates uniformly, it undergoes an acceleration of magnitude v^2/r where v is its tangential speed. The inertial observer concludes that this centripetal acceleration is provided by the force exerted by the string, T and writes Newton's second law.

$$T = \frac{mv^2}{r}$$

According to a noninertial observer attached to the turntable the block is at rest. Therefore in applying Newton second law, this observer introduces a frictions outward force of magnitude $m\omega^2 r$. According to the noninertial observer this outward force balances the force exerted by the string and there- fore $T - mv^2/r = 0$. You should be careful when using fictitious forces to describe physical phenomena. Remember that fictitious forces are used only in noninertial frames of reference. When solving problems, it is often best to use an inertial frame.

If object moves w.r.t. to rotating frame or the frame rotate with varying angular velocity along with C.F we need to incorporate other pseudo force which are called Coriolis & Euler force

$F_c = -2m(\vec{\omega} \times \vec{v}_{rel})$

$F_e = -m(\vec{\alpha} \times \vec{r})$

object is sliding w.r.t. frame of reference

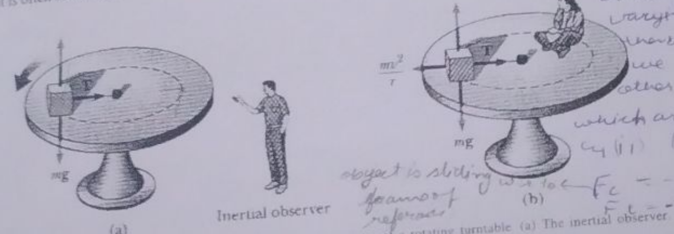


Fig. 7.23 A block of mass m connected to a string tied to the center of a rotating turntable. (a) The inertial observer

Note: Centrifugal force is a sufficient pseudo force, only if we are analysing the particles at rest in a uniformly rotating frame. If we analyse the motion of a particle that moves in the rotating frame, we may have to assume other pseudo forces, together with the centrifugal force. Such forces are called the coriolis forces. Such cases are beyond our scope of coverage we will deal only with that type of problems in which angular velocity of rotating frame will be constant and particle in that frame will be at rest. It is a common misconception among the beginners that centrifugal force acts on a particle because the particle goes on a circle. Centrifugal force acts (or is assumed to act) because we describe the particle from a rotating frame which is noninertial and still use Newton's laws.

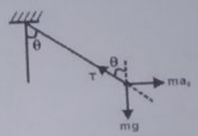
varying angular velocity

Example 17. A simple pendulum is suspended from the ceiling of a car taking a turn of radius 10 m at a speed of 36 km / h. Find the angle made by the string of the pendulum with the vertical if this angle does not change during the turn.

Solution: The acceleration of the car is

$$a_0 = \frac{v^2}{r} = \frac{10^2}{10} = 10 \qquad [v = 36 \text{ km/h} = 10 \text{ m/s}]$$

The car turning on a road constitutes a non inertial frame. In such a frame in addition to real forces, a pseudo force ma_0 must act on the bob in radially outward direction.
The forces acting on the pendulum are
(i) weight mg downward,
(ii) tension force T along the length of string,
(iii) pseudo force ma_0 radialy outward.
The FBD of the pendulum is as shown. As in the rotating frame of car, pendulum remains at rest so net form on it must be zero.



For vertical equilibrium of bob

$$T\cos\theta = mg \quad \ldots (i)$$

while for its horizontal equilibrium

$$T\sin\theta = ma_0 \quad \ldots (ii)$$

from (i) and (ii)

$$\tan\theta = \frac{a_0}{g} = \frac{10}{10} = 1 \Rightarrow \theta = 45^\circ$$

Alternatly we can visualize the motion of pendulum from the frame of ground as well. The pendulum bob is moving in a circle of radius r in horizontal plane w.r.t. ground. The necessary centripetal force is provided by the horizontal component of tension force. Thus applying Newton's second law in the radial direction

$$T\sin\theta = \frac{mv^2}{r} \quad \ldots (iii)$$

In the vertical direction, bob remains in equilibrium so

$$T\cos\theta = mg \quad \ldots (iv)$$

From equation (iii) and (iv) we get the same result

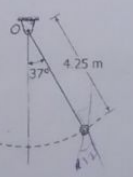
$$\tan\theta = \frac{v^2}{rg} = \frac{(10)^2}{10 \times 10} = 1 \Rightarrow \theta = 45^\circ$$

Note: When a body is dropped from a high tower, its initial velocity is greater than that of the point on the ground vertically below. As a result it will fall toward the east of the expected landing point.
The coriolis effect is a deflection of a moving objects when they are views in rotating reference frame. The corisolis force acts in a direction perpendicular to the rotation axis and to the velocity of the body in the rotating frame.
In reference frame with clockwise rotation, the deflection is to the left of the motion of the object; in one with counter clockwise rotation, the deflection is to the right.
In the ground frame we need not apply any pseudo force.

Section Review 7.6

1. A 40.0 kg child sits in a swing supported by two chains, each 3.00 m long. If the tension in each chain at the lowest point is 350 N, find
   (a) the child's speed at the lowest point and
   (b) the force of the seat on the child at the lowest point.
   (Neglect the mass of the seat.)

2. The bob of the pendulum shown in figure describes an arc of circle in a vertical plane. If the tension in the cord is 2.5 times the weight of the bob for the position shown. Find the velocity and the acceleration of the bob in that position.



3. The blocks are of mass 2 kg shown is in equilibrium. At $t = 0$ right spring in Fig. (i) and right string in Fig. (ii) breaks. Find the ratio of instantaneous acceleration of blocks?

(i) (ii)

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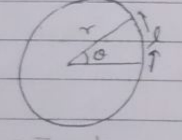
Circular motion

Circular motion is a 2-D motion. The dir'n of velocity (along the tangent to the circle) continuously changes in dir'n so it is an accelerated motion.

NOTE



$180 = \pi$ radin.

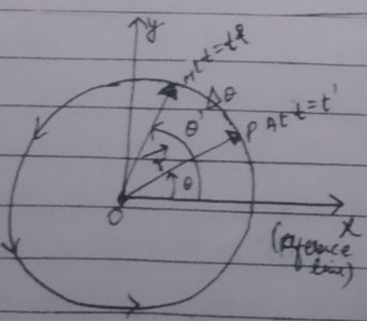
$\theta(rad) = \frac{\pi}{180^\circ}\theta(degree)$

length corresponding 2π angle is $2\pi r$

So, length corres θ is $l = \frac{2\pi r}{2\pi}.\theta$

$l = r\theta$

area of sector $= \frac{r^2\theta}{2}$



angular position (θ)
Angle of posn vector with ref. line ox

Ang. Disp:
Total angle turned by the posn vector

$\Delta\theta = \theta - \theta$

eg in $1\frac{1}{2}$ motion ang disp is 3π rad

eg in 1 rotation ang disp is 2π rad

• Avg. ang speed (ω_{av})

$$\boxed{\omega_{av} = \frac{\Delta\theta}{\Delta t}}$$

• Ang. speed

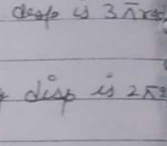
$$\omega = \lim_{\Delta t \to 0}\frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

• Unit is rad/s

(I) If ω is constant i.e. particle covers equal angles in equal time interval then

$\Delta\theta = \omega\Delta t$

or $\Delta t = \frac{\Delta\theta}{\omega}$

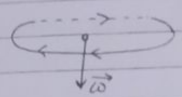


• In this case, $\omega_{av} = \omega$

(II) If ω varies with time then

$$\boxed{\Delta\theta = \int_{t_1}^{t_2}\omega\,dt}$$

- On turning the fingers of rt hand along the sense of motion, thumb pts in the dir'n of ang. velo ($\vec{\omega}$).



(iv) Dir'n of $\vec{\alpha}$

If $\omega\uparrow$ with time then $\vec{\omega}\uparrow\uparrow\vec{\alpha}$

" $\omega\downarrow$ " " " $\vec{\omega}\uparrow\downarrow\vec{\alpha}$

eg find the angular speed of second hand, hour, & minute hand of a wall clock.

$$\omega_s = \frac{2\pi \text{ rad}}{60 s} = \frac{\pi}{30} \text{ rad/s}$$

$$\omega_m = \frac{2\pi \text{ rad}}{3600 s} = \frac{\pi}{1800} \text{ rad/s}$$

$$\omega_H = \frac{2\pi \text{ rad}}{12 hr} = \frac{\pi}{6} \text{ rad/h}$$

- Avg angular accl'n (α_{av})

$$\alpha_{av} = \frac{\Delta\omega}{\Delta t}$$

- Ang. accl'n

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

- unit is rad/s^2

- If $\alpha = 0$ then $\vec{\omega}$ = const.
- If α = const. then ang velocity changes by equal amounts in equal time interval

$u \to \omega_0$, $v \to \omega$, $a \to \alpha$, $s \to \Delta\theta$

(i) $\omega = \omega_0 + \alpha t$

(ii) $\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

(iii) $\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$

(iv) $\Delta\theta = \left(\frac{\omega + \omega_0}{2}\right)t$

(iii) If α varies with time then

$$\Delta\omega = \omega(t_2) - \omega(t_1) = \int_{t_1}^{t_2} \alpha\, dt$$

When a rigid body rotates about a fixed line, all pts on it (expect on axis) have same $\Delta\theta$, ω & α.

eg A particles moves in a ~~circular~~ circle with the angular ~~thread~~ pos'n varying as $\theta(t) = 2t^3 + 4t^2 + t$ rad

find (1) $\omega(t)$, $\alpha(t)$

(ii) ω_{av} b/n $t=0$ & $t=2s$

(iii) α_{av} b/n $t=1$ & $t=3s$.

(i) $\omega = \frac{d\theta}{dt} = 6t^2 + 8t + 1$ rad/s

(b) $\alpha = \frac{d\omega}{dt} = 12t + 8$ rad/s^2

(ii) $\omega_{av} = \frac{\Delta\theta}{\Delta t} = \frac{\theta - \theta}{t_2 - t_1}$

$\omega_{av} = \frac{\theta(t=2s) - \theta(t=0)}{2-0}$

$\omega_{av} = \frac{34-0}{2} = 17$ rad/s

(iii) $\alpha_{av} = \frac{\omega(t=2s) - \omega(t=0)}{t_2 - t_1}$

$\alpha_{av} = \frac{79-15}{2} = 32$ rad/s^2

eg A particle starts circular motion from rest with angular accln $\alpha = \frac{1}{2}$ rad/s^2 find angular speed and angle turned in 4s.
(ii) what is the time taken for two rotations from start (iii) How many rotation will the particle makes when its angular speed is 6 rad/s

$\omega_0 = 0$, $\alpha = \frac{1}{2}$ rad/s^2

(i) $t = 4s$, $\omega = ?$

$\omega = \omega_0 + \alpha t$

$\omega = 0 + \frac{1}{2} \times 4 = 2$ rad/s

$\Delta\theta(t = 4s) = ?$

$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\Delta\theta = 0 + \frac{1}{2}\left(\frac{1}{2}\right)(4)^2 = 4$ rad

(b) $\Delta\theta = 2$ rotation $= 4\pi$ rad

$t = ?$

$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$4\pi = 0 + \frac{1}{2} \times \frac{1}{2} t^2$

$t = 4\sqrt{\pi}$ s

(c) $\omega = 6$ rad/s, $\Delta\theta = ?$

$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$

$36 = 2\left(\frac{1}{2}\right)\Delta\theta$

$\Delta\theta = 36$ rad

No. of rotations

$n = \frac{\Delta\theta}{2\pi} = \frac{18}{\pi}$

eg Find the angular speed at $t = 4s$, if $\omega(t=0) = 2$ rad for a particle executing circular motion with $\alpha = 2t - 3$ rad/s^2

Soln $\omega(t_2) - \omega(t_1) = \int_{t_1}^{t_2} \alpha\, dt$

$\omega(t_2 = 4s) - \omega(t_1 = 2) = \int_2^4 (2t-3)\, dt$

$\omega(t_2 = 4s) - 2\text{ rad} = t^2 - 3t \Big|_0^4$

$= 16 - 12$

$\omega(t = 4s) = 6$ rad/s

$|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$

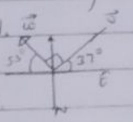
Frequency (no. of revolutions per second) $= \frac{\omega}{2\pi}$

Eg A car is moving due east with decreasing speed. find dir'n of $\vec{\omega}$ & $\vec{\alpha}$ of wheels.

Sol^n ω due north & α due South.

Solve again if car is moving 37° NE with ↑ speed.

ω is 53° north of west. α due N-W.

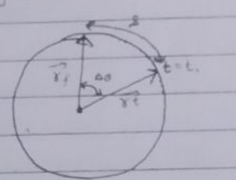


$1\ rpm = \frac{2\pi\ rad}{60\ s} = \frac{\pi}{30}\ rad/s$

Eg $180\ rpm = \ ?\ rad/s$

$\frac{180(2\pi)}{60} = 6\pi\ rad/s$

Relation b/n linear & Angular variables.



$s = r\Delta\theta$

$\frac{s}{\Delta t} = r\frac{\Delta\theta}{\Delta t}$

$\Delta t \to 0 \qquad \Delta t \to 0$

speed

$v = r\omega \qquad$ unit: rad/s · m

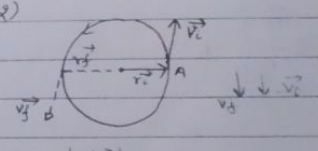
Vectorally $\Rightarrow \vec{v} = \vec{\omega}\times\vec{r}$

Eg A particle moves with const. speed v in a circle of radius r. find the magnitude of avg velocity and avg accl'n in

1) one fourth rotation
2) " half "
3) the interval in which particle turns by θ

NOTE: In circular motion $\vec{r}$ & $\vec{v}$ turn by equal angle in equal time interval

Sol (2)



$|\vec{V}_{av}| = \frac{|\Delta\vec{r}|}{\Delta t} = \frac{2r}{\left(\frac{\pi r}{v}\right)} = \frac{2v}{\pi}$

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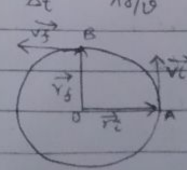
- When particle moves with constant speed v in a circle of radius r, the time period of motion

$T\left\{\begin{matrix}\text{time taken to}\\ \text{complete one}\\ \text{revolution}\end{matrix}\right\} = \frac{2\pi r}{v} = \frac{2\pi}{\omega}$

$|\vec{a}_{av}| = \frac{|\Delta\vec{v}|}{\Delta t} = \frac{2v}{\pi r/v} = \frac{2v^2}{\pi r}$

(1)



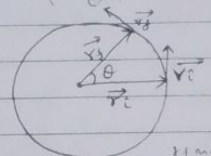
$$|\vec{V}_{av}| = \frac{|\Delta \vec{r}|}{\Delta t} = \frac{|\vec{V}_f - \vec{V}_i|}{\Delta t}$$

$$|\vec{V}_{av}| = \frac{\sqrt{2}r}{\left(\frac{\pi r/2}{v}\right)} = \frac{2\sqrt{2}v}{\pi}$$

$$|\vec{a}_{av}| = \frac{|\Delta \vec{V}|}{\Delta t} = \frac{|\vec{V}_f - \vec{V}_i|}{\Delta t}$$

$$|\vec{a}_{av}| = \frac{\sqrt{2}v}{\left(\frac{\pi r/2}{v}\right)} = \frac{2\sqrt{2}v^2}{\pi r}$$

(3)



r_f and r_i same length

$$|\vec{V}_{av}| = \frac{|\Delta \vec{r}|}{\Delta t} = \frac{2r\sin(\theta/2)}{(r\theta/v)}$$

$$|\vec{V}_{av}| = \frac{2\cancel{r}\sin(\theta/2)}{\theta}$$

$$|\vec{a}_{av}| = \frac{|\Delta \vec{V}|}{\Delta t} = \frac{|\vec{V}_f - \vec{V}_i|}{\Delta t} = \frac{2v\sin(\theta/2)}{r\theta/v}$$

$$|\vec{a}_{av}| = \frac{2v^2\sin(\theta/2)}{r\theta}$$

Accl'n in Circular Motion

In circular motion, accl'n vector vector $\vec{a}$ has two mutually $\perp$ components namely

i) Centrepetal accl'n, $\vec{a}_c$

ii) tangential accl'n $\vec{a}_t$

$$\vec{a} = \frac{d\vec{v}}{dt} = \vec{a}_c + \vec{a}_t$$

i) Centripetal accl'n

The accl'n component directed towards the centre of circle is called centrepetal (centre seeking) accl'n.

$$\boxed{a_c = \frac{v^2}{r} = \omega^2 r = \omega v}$$

$$\boxed{\vec{a}_c = \vec{\omega} \times \vec{v}} = \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

$$\boxed{\vec{a}_c = -\omega^2 \vec{r} = -\frac{v^2}{r}\hat{r}}$$

- Centrepetal accl'n is responsible for changing the dir'n of velocity vector
- $\vec{a}_c$ is always towards centre

ii) Tangential accl'n ($\vec{a}_t$)

The accl'n component directed along the tangent to the circle is called tangential accl'n

$$\boxed{a_t = \frac{dv}{dt} = \alpha r}$$

Rate of change of speed

- It change only the magnitude of velocity.

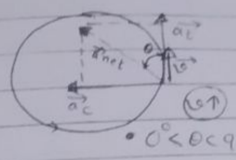
- If speed $\uparrow$ then $\vec{a}_t \uparrow\uparrow \vec{v}$
- If " $\downarrow$ " $\vec{a}_t \uparrow\downarrow \vec{v}$

- If speed is const. then $\{a_t = 0\}$

Note $\boxed{\vec{a}_t = \vec{\alpha} \times \vec{r}}$

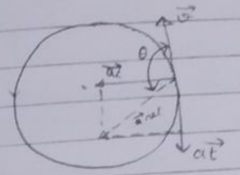
$\frac{d}{dt}(\omega r) = r\frac{d\omega}{dt} = r\alpha$

1.



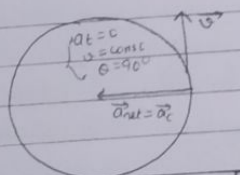
- $0° < \theta < 90°$

2.



- $v\downarrow$, $90° < \theta < 180°$

3.



$$\boxed{a_{net} = \sqrt{a_c^2 + a_t^2} = \sqrt{\left(\frac{v^2}{r}\right)^2 + \left(\frac{dv}{dt}\right)^2}}$$

$$\boxed{\tan\theta = a_c/a_t}$$

For a_t = const. $= \alpha r$

1. $\begin{cases} \omega = \omega_0 + \alpha t \\ v = u + a_t t \end{cases}$ Multiply by r

2. $\begin{cases} \Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \\ s = ut + \frac{1}{2}a_t t^2 \end{cases}$

3. $\begin{cases} \omega^2 = \omega_0^2 + 2\alpha\Delta\theta \\ v^2 = u^2 + 2a_t s \end{cases}$

eg A particle starts moving in a circle of rad $r = 2m$ with speed changing at a constant rate of $2m/s^2$ from rest (i) find net accl'n at $t = 2s$ and (ii) speed and ang speed, $t = 2s$.

$r = 2m$, $a_t = \frac{dv}{dt} = 2m/s^2$
$u = 0$, $t = 2s$ } G.m

i) $v = \cancel{u} + a_t t$

$v = 0 + 2(2) = 4m/s$

$\omega = \frac{v}{r} = \frac{4}{2} = 2\ rad/s$

ii) $a_{net} = \sqrt{a_c^2 + a_t^2}$

$a_{net} = \sqrt{\left(\frac{v^2}{r}\right)^2 + (a_t)^2}$

$a_{net} = \sqrt{\frac{(4)^4}{2^2} + (2)^2} = \sqrt{68}\ m/s^2$

$\tan\theta = \frac{a_c}{a_t} = \frac{v^2/r}{a_t}$

$= \frac{(4^2)/2}{2} = 4$

$\therefore \theta = \tan^{-1}(4)$ with $\vec{v}$

eg Speed of a particle moving in a circular of radius $r = 4cm$ is varying as $v = 2t^2\ m/s$ when t is in seconds find

a) rate of change of speed as a fn of time

b) net accl'n at $t = 2$.

a) $a_t = \frac{dv}{dt} = 4t\ m/s^2$

$a_t(t=2s) = 8\ m/s^2$

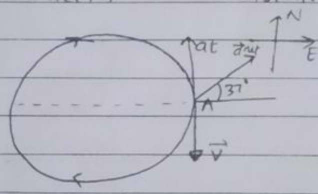
b) $v(t=2) = 2(2)^2 = 8\ m/s$

$a_{net} = \sqrt{\left(\frac{v^2}{R}\right)^2 + a_t^2}$

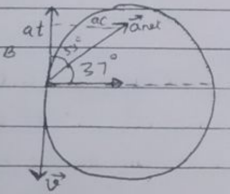
$a_{net} = \sqrt{\frac{8^4}{16} + 8^2} = 8\sqrt{5}\ m/s^2$

Q. A car is moving in a circle of radius 50m. At an instant its velocity is directed due south speed is ↓ at a rate of $4 m/s^2$ and total acc'n is at 37° NE.

1) Centrepetal acc'n
2) Speed
3) net acc'n at this instant.



This situation is wrong.
So we have to chng the p.t of v



(i) $\tan 53^\circ = \frac{a_c}{a_t} \Rightarrow \frac{4}{3} = \frac{a_c}{4}$

$a_c = \frac{16}{3}\ m/s^2$

ii) $a_c = \frac{v^2}{R}$ ∴ $\frac{16}{3} = \frac{v^2}{50}$

$v = \sqrt{\frac{800}{3}}\ m/s$

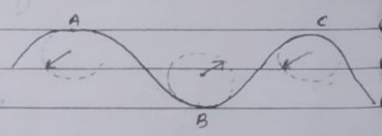
iii) $\frac{a_t}{a_{net}} = \cos 53^\circ$

$a_{net} = \frac{20}{3}\ m/s^2$

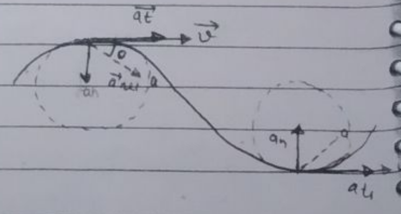
Lecture ~ 46 1-8-18

Motion on a curved track

Any small portion of a curved track can be assumed to be part of a circle whose radius is called radius of curvature.



- The greater is the curvature the lesser is the radius of curvature
- For a flat portion, $r \to \infty$.



• ... $\parallel$ ... tangential ...

$a_t \left(= \frac{dv}{dt}\right) = a\cos\theta = \frac{\vec{a}\cdot\vec{v}}{v}$

• Accl'n comp. $\perp$ ... is normal ... (centripetal) accl'n

$a_n = a\sin\theta = \frac{|\vec{a}\times\vec{v}|}{v}$

$= \sqrt{a^2 - a_t^2}$

Finding radius of curvature

(i) find speed v

(ii) " a_n i.e accl'n comp $\perp$ to $\vec{v}$

(iii) $a_n = \frac{v^2}{r}$

eg At an instant velocity & accl'n of a particle are $\vec{v} = 3\hat{i} - 4\hat{k}$ m/s

$\vec{a} = -\hat{i} + 2\hat{j} + 2\hat{k}$ m/s^2 find

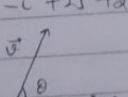
i) rate of change of speed

ii) accl'n compt $\perp$ to $\vec{V}$.

iii) radius of curvature of trajectory.

(i) $\vec{v} = 3\hat{i} - 4\hat{k}$ m/s

$\vec{a} = -\hat{i} + 2\hat{j} + 2\hat{k}$ m/s^2



$a_t = \frac{\vec{a}\cdot\vec{v}}{v}$

$= \frac{a_x v_x + a_y v_y + a_z v_z}{\sqrt{v_x^2 + v_y^2 + v_z^2}}$

$a_t = \frac{(-3) + (0) + (-8)}{\sqrt{3^2 + (-4)^2}} = -\frac{11}{5}$ m/s^2

ii) $a^2 = a_t^2 + a_n^2$

$a_n = \sqrt{a^2 - a_t^2}$

$= \sqrt{(1+4+4) - \frac{121}{25}}$

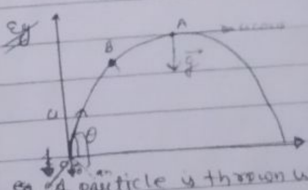
$= \sqrt{9 - \frac{121}{25}}$

$= \frac{\sqrt{104}}{5}$ m/s^2

(c) $a_n = \frac{v^2}{r}$

$r = \frac{v^2}{a_n} = \frac{25}{\sqrt{104}/5} = \frac{125}{\sqrt{104}}$ m

Eg



eg A particle is thrown with speed u at an angle θ above horizontal. Ignoring air effects find the rate of change of speed and radius of curvature

1) when particle is at height pt

2) Just After projection

3) when velocity is $\theta/2$ above horizontal.

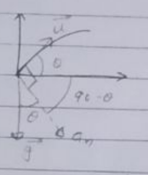
(i) $\frac{dv}{dt} = a_t = 0 \quad [\because \vec{a} \perp \vec{v}]$

$a_n = g, \; v = u\cos\theta$

$a_n = \frac{v^2}{r_A}$

$g = \frac{u^2\cos^2\theta}{r_A} \Rightarrow r_A = \frac{u^2\cos^2\theta}{g}$

ii)



a_t why down

$a_t = -g\sin\theta$ why -ve

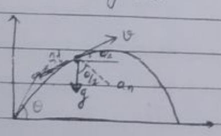
$a_n = g\cos\theta$ why not here -ve

$a_n = \frac{v^2}{r_0}$

$g\cos\theta = \frac{u^2}{r_0}$

$r_0 = \frac{u^2}{g\cos\theta}$

(iii)



$u_x = v_x$

$u\cos\theta = v\cos(\theta/2)$

$v = \frac{u\cos\theta}{\cos(\theta/2)}$

$a_t = -g\sin(\theta/2)$

$a_n = g\cos(\theta/2)$

$a_n = \frac{v^2}{r_B}$

$g\cos(\theta/2) = \frac{u^2\cos^2\theta/\cos^2(\theta/2)}{r_B}$

$r_B = \frac{u^2\cos^2\theta}{g\cos^3(\theta/2)}$

Q. A particle is moving with const speed on a curved track.

i) $\vec{a} \perp \vec{v}$

ii) $\vec{a}$ must be varying

iii) a vector may remain constant in magnitude

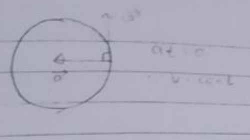
(i) Since $a_t = 0$ it is true

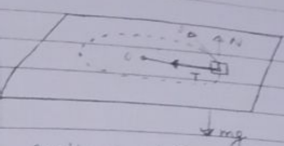
ii) True (dir'n is continuously change)

iii) ~~magnitude~~ uniform circular motion

DYANAMICS of Uniform circular motion

- circular motion with const speed.
- $\boxed{a_t = \alpha r = \frac{dv}{dt} = 0}$
- Accl'n vector is directed towards center $a = \frac{v^2}{r}$ towards center.
- It is non uniformly accl'd motion since $\vec{a}$ is varying in dir'n.



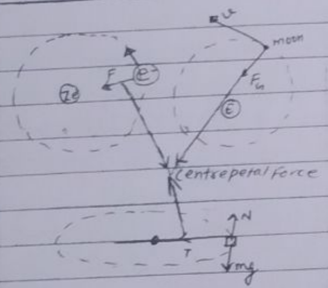


- In uniform circular motion net force is directed towards the center of the circle and called centrepetal Force

$$\boxed{F_{net}\left(\substack{\text{net force}\\ \text{towards centre}}\right) = \frac{mv^2}{r} = m\omega^2 r}$$

Accl'n of bl is $\frac{v^2}{L}$ towards center

$$N = mg$$
$$T = \frac{mv^2}{L}$$



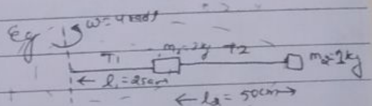
- If T_{max} is breaking strength of string then the max speed with which the block can be moved in $T_{max} = \frac{mv^2_{max}}{l}$
- If string breaks then the bl moves in tangential dir'n

Lecture 47 2-8

Blocks Connected with a String.

Consider a block of mass m connected with an inextensible string of length l moving in horizontal circle on a smooth surface with speed v.

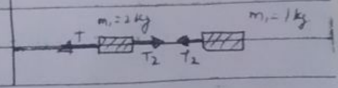
Eg 1



find T_1 & T_2

- accl'n is diff so we can't take it as a system.

FBD



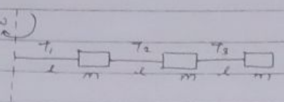
$$\underset{(\text{net force})}{T_1 - T_2} = m_1\omega^2 l_1 \quad -(1)$$
$$= 2(16)\left(\frac{1}{4}\right)$$

$T_2 = m_2 \omega^2 (l_1 + l_2)$

$T_2 = 1(16)(\frac{3}{4})$

$T_2 = 12N$, & $T_1 = 20N$

eg



(A) $T_1 = T_2 = T_3$

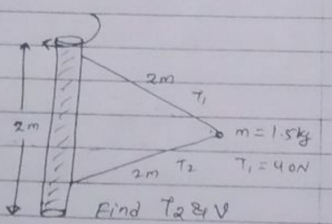
(B) $T_1 > T_2 > T_3$

(C) $T_1 < T_2 < T_3$

(D) $T_1 - T_2 = T_2 - T_3$

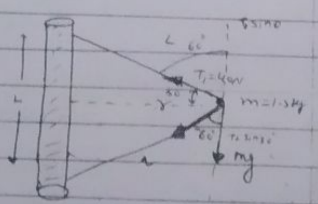
(E) $2(T_1 - T_2) = T_2 - T_3$

eg



Find T_2 & v

Soln



$r = L\cos 30° = 2 \times \frac{\sqrt{3}}{2} = \sqrt{3}m$

$T_1 \sin 30 = T_2 \sin 30 + mg$ — ①

$T_1 \cos 30° + T_2 \cos 30° = \frac{mv^2}{r}$ ②

(More Tension)

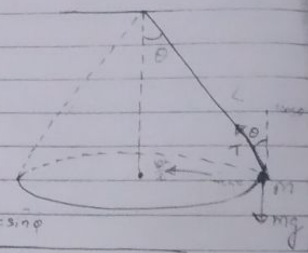
from ① $40(\frac{1}{2}) = \frac{T_2}{2} + 15$

$\boxed{T_2 = 10N}$

from ② $(40+10)\frac{\sqrt{3}}{2} = \frac{1.5v^2}{\sqrt{3}}$

$v = \sqrt{50}$ m/s

Conical Pendulum



$r/L = \sin\theta$

$\boxed{T\cos\theta = mg}$

$\boxed{T\sin\theta = \frac{mv^2}{L\sin\theta}}$

$\frac{\cos\theta}{\sin\theta} = \frac{gL\sin\theta}{v^2}$

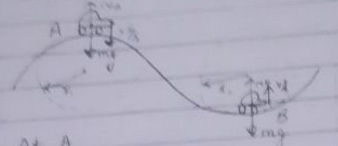
$\boxed{v = \sqrt{\frac{gL\sin^2\theta}{\cos\theta}}}$

T'(Time period) $= \frac{2\pi r}{v}$

$= \frac{2\pi(L\sin\theta)}{(\sqrt{\frac{gL}{\cos\theta}})\sin\theta}$

$\boxed{T' = 2\pi\sqrt{\frac{L\cos\theta}{g}}}$

Motion on a curved road



At A

$mg - N_A = \frac{mv^2}{r_1}$

$$\boxed{N_A = m\left(g - \frac{v^2}{r_1}\right)}$$

- As $v \uparrow$ $N_A \downarrow$
- If v_{min} is the speed at which N_A just becomes zero

$mg - 0 = \frac{mv_{min}^2}{r_1}$

$$v_{min} = \sqrt{r_1 g}$$

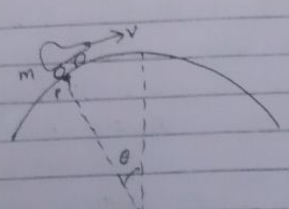
- $v < \sqrt{r_1 g}$ $N_A > 0$
- $v \geq \sqrt{r_1 g}$ $N_A = 0$

At B

$N_B - mg = \frac{mv^2}{r_2}$

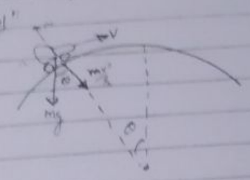
$$\boxed{N_B = m\left(g + \frac{v^2}{r_2}\right)}$$

Eg



A car is moving over a convex hill bridge with speed v find N at pt P.

Sol^n

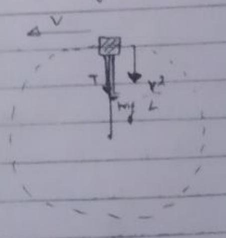


$mg\cos\theta - N = \frac{mv^2}{r}$

$N = m\left(g\cos\theta - \frac{v^2}{r}\right)$

As car is moving to the top with const speed then N is ↑

Eg A block tied to a string of length l is whirled in vertical circle.
Find the min speed at top so that string does not slack



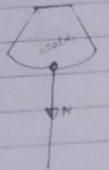
$T + mg = \frac{mv^2}{L}$

for $T > 0$, $\frac{mv^2}{L} > mg$

$v \geq \sqrt{gL}$

$\boxed{v_{min} = \sqrt{gL}}$ critical speed

#



$$N + mg = \frac{mv^2}{l}$$

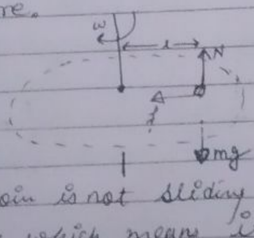
$$N = \frac{mv^2}{l} - mg > 0$$

$$v > \sqrt{gL}$$

Note Even at speed $v = \sqrt{gL}$ at top water will not fall down.

Coin on a rotating Table

- consider a coin of mass m lying on a rough horizontal turn table which is rotating at ang speed ω about its axis at a distance r from centre.



- The coin is not sliding on the table which means it turning with same speed ω along with table
- With respect to table the coin has tendency to slip

outward so static friction acts on the coin radially inward which provides a required centrepetal force

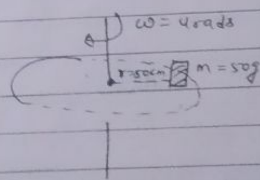
$$\boxed{f_s = m(\omega^2 r)}$$ centripetal

As angular speed of table increases then requirement of static fr^n also increases. To avoid slipping

$$\boxed{f_s = m\omega^2 r \leq \mu_s N = \mu_s mg}$$

$\boxed{\omega^2 r \leq \mu_s g}$

Imp Eg



a) find f_s if block doesn't not slide.

b) For which of the following values(s) of μ_s the block does not slide

(A) 0.3 (B) 0.6 (C) 0.9 (D) 1.2

Sol^n: a) $f_s = m\omega^2 r = 0.4N$

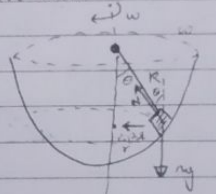
b) $f_s = m\omega^2 r \leq \mu_s mg$

$\mu_s \geq \frac{\omega^2 r}{g} = \frac{(16)(1/2)}{10} = 0.8$

$\therefore$ μ_s should be greater than 0.8

$\therefore$ Ans are, 0.9 & 1.2.

Q A block of mass m is lying inside a smooth hemispherical bowl of radius R, rotating about its axis of symetry which is verticle with ang. speed ω. The radius through the block makes θ with verticle. Find ω if the block does not slide.



$r = R\sin\theta$

$N\sin\theta = m(\omega^2 R\sin\theta)$ -①

$N\cos\theta = mg$ -②

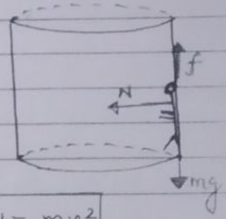
$\frac{1}{\cos\theta} = \frac{\omega^2 R}{g}$

$\omega = \sqrt{\frac{g}{R\cos\theta}}$

Lecture - 48

Rotor

A boy of mass m is made to stand against the rough wall of a cylindrical rotor of radius R, when rotor attains sufficient speed ω, the floor on which boy is standing is removed.



$\boxed{N = \frac{mv^2}{R}}$

$\boxed{f_s = mg}$

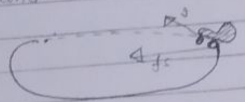
$f_s = mg \leq \mu_s N = \mu_s \frac{mv^2}{R}$

$\boxed{g \leq \frac{\mu_s v^2}{R}}$

• Circular Turning and Banking

A car of mass m is moving in a horizontal (level) in circular turn of

radius small r with speed v.
The required ac is provided by static fr. from the ground



$$f_s = ma = \frac{mv^2}{r}$$

for no skidding,

$$f_s = \frac{mv^2}{r} \leq \mu_s mg$$

$$v \leq \sqrt{\mu_s rg}$$

Max safe speed

$$v_s = \sqrt{\mu_s rg}$$

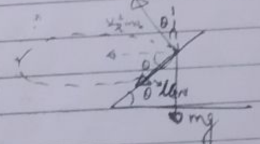
$$\boxed{N\cos\theta = mg} \quad N > mg$$

$$\tan\theta = \frac{v_0^2}{rg}$$

- ~~If v < v0~~
- If $v > v_0$, the car has tendency to skid upward (towards outer edge) so f_s acts in (downward / towards inner edge) and contributes appropriately in centripetal force

- At the threshold of sliding

$$f_s = \mu_s N$$

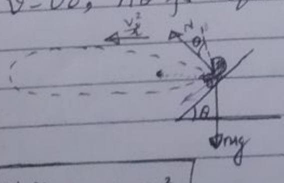


- To minimize dependance on friction the outer ~~edge~~ edges of circular turn are elevated which is called **Banking**. Let θ is the angle of banking and v_0 is the ~~baken~~ banking speed (at $v = v_0$, no f_s required)

$$N\sin\theta + \mu_s N\cos\theta = \frac{mv_{max}^2}{r}$$

$$N\cos\theta - \mu_s N\sin\theta = mg$$

$$\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta} = \frac{v_{max}^2}{rg}$$



$$v_{max} = \sqrt{rg\left(\frac{\tan\theta + \mu_s}{1 - \mu_s\tan\theta}\right)}$$

$$N > \frac{mv_0^2}{r} \qquad \boxed{N\sin\theta = \frac{mv_0^2}{r}}$$

using ...

$N\cos\theta = mg$

$$\boxed{\tan\theta = \frac{v_0^2}{rg}}$$

- If $v < v_0$, the car has tendency to skid ~~upward~~ down ~~towards outer edge~~ so friction acts upwards

- The ~~min~~ safe speed

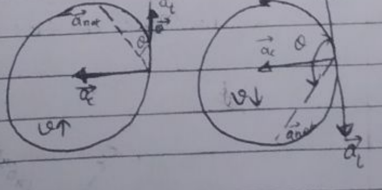
$$v_{min} = \sqrt{\frac{rg(\tan\theta - \mu_s)}{1 + \mu_s \tan\theta}}$$

- If $\tan\theta \le \mu$ then $v_{min} = 0$

Lecture ~ 49

Non-uniform Circular Motion

- Speed is varying with time.
  so $a_t = \alpha r = \frac{dv}{dt} \neq 0$



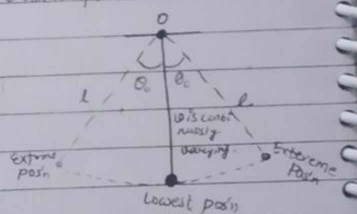
- In non-uniform circular motion we resolve the force along radial & tangential dir'n.

Net force towards centre $= \frac{mv^2}{r} = m\omega^2 r$

Net force towards along the tang $F_t = ma_t = m\frac{dv}{dt} = m\alpha r$

- Simple Pendulum

Consider a pendulum oscillating with angular amplitude θ_0 in vertical plane



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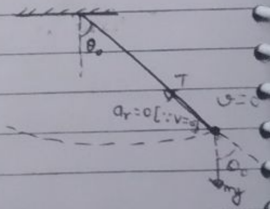

$\because a_r = \frac{v^2}{l} = 0$ so $T = mg\cos\theta_0$

$mg\sin\theta_0 = ma_t$

$\Rightarrow a_t = g\sin\theta$

Net accl'n is along the tangent

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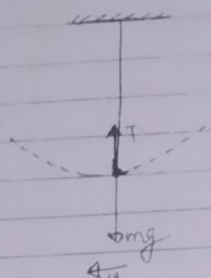

In radial dir'n

$$T - mg = \frac{mv_0^2}{l} \Rightarrow T = m\left(g + \frac{v_0^2}{l}\right)$$

- no force in tangential dir'n so $a_t = 0$

Thus, net accl'n is towards centre

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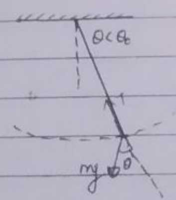


In radial dir'n

$$T - mg\cos\theta = \frac{mv^2}{l}$$

$$T = m\left(g\cos\theta + \frac{v^2}{l}\right)$$

On moving from extreme to lower pos'n both speed & tension ↑ continu

$\Rightarrow$ T is max at lowest pos'n

T " min " extreme.

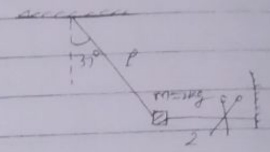
In tangential dir'n

$mg\sin\theta = ma_t$

$\Rightarrow a_t = g\sin\theta$

On moving from extreme to lowest pos'n, a_t is ↓ where as $a_c = \frac{v^2}{l}$ is ↑

Eg
Imp

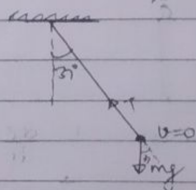


- find the tension in string-1 just after the string 2 is cut

Sol^n

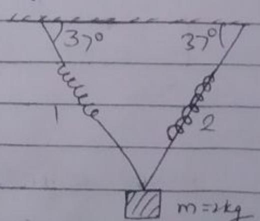
After the st-2 is cut bob executes oscillation

Just after the cut speed is zero so $a_r = \frac{v^2}{l} = 0$



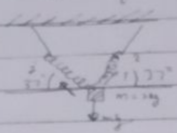
$$T = mg\cos 37° = 16N$$

Eg



#

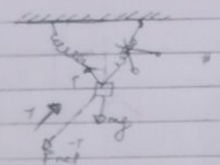
find accl'n just after sp 2 is cut of M



Before cut

$2T\sin 37^\circ = mg$

$T = \frac{20\times5}{2\times3} = \frac{50}{3} N$



$\therefore a = \frac{T}{m} = \frac{50/3}{2} = \frac{25}{3} m/s^2$

- Tension in string does not change abruptly.

an inertial frame

- Since earth is rotating & revolving so it is a non inertial frame
→ for all practicle purposes [explaining set of near by point], we approx treat earth as an inertial frame.

- Any frame which is at rest or in uniform motion (a=0) w.r.t earth is an inertial frame.
We can apply Newtons IInd law only in interial Frame.

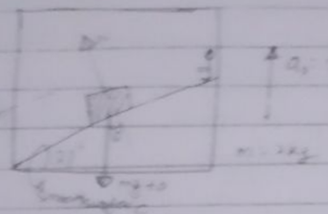
Pseudo force

- A frame of refrence in which Newtons first law is valid i.e in the absence of net force on an object, accl'n recorded by the frame is zero.

- A non accl'ntiny frame w.r.t an inertial frame is also

To apply Newtons IInd law in a non inertial frame which is accl'ting translationally with $\vec{a_0}$, we need to incorporate an Imaginary force $-m\vec{a_0}$ on the object. This force is called Pseudo force.

$*$



find accl'n of bl w.r.t the elevator

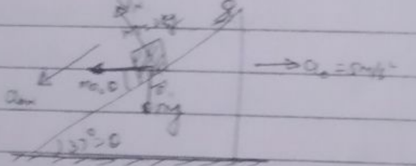
$$m(g+a_0)\sin 37 = m a_{be}$$

$$a_{be} = 9\,m/s^2$$

$$N = m(g+a_0)\cos 37$$

$$= 24 \text{ newton}$$

Q. A block of $m = 2kg$ is lying on a wedge which is accling with $a_0 = 5m/s^2$ as shown. find accl'n of block w.r.t wedge & normal Rn.



Observer is sitting on the wedge so the block will appear to be moving along the wedge.

$$mg\sin\theta + ma_0\cos\theta = ma_{bw}$$

$$a_{bw} = (10\times0.6) + (5\times0.8)$$

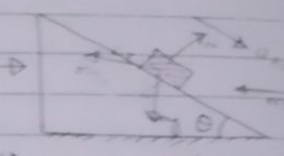
$$= 10\,m/s^2$$

$$N + ma_0\sin\theta = mg\cos\theta$$

$$N = m(g\cos\theta - a_0\sin\theta)$$

$$N = 2[10\times0.8 - (5\times0.6)] = 10N$$

Q. find the value of the accl'n with which wedge should be moved so that block does not slide relative

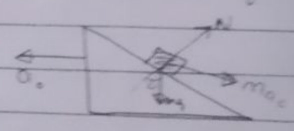


Sol^n

$$mg\sin\theta - ma_0 = ma_{bw}$$

$$\boxed{a_0 = g\tan\theta}$$

Q. find the minimum accl'n that should be given to the wedge so that block falls freely



$\because \vec{a_b} = g\downarrow$ so $N = 0$

$$mg\cos\theta = ma_0\sin\theta + N$$

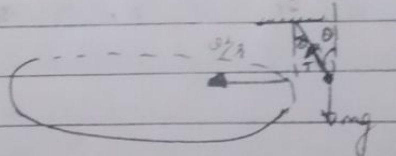
$$\boxed{a_0 = g\cot\theta}$$

CENTRIFUGAL FORCE

A rotating frame of reference is a non inertial frame

- To apply Newton's IInd law on a object of mass m at rest in a frame rotating with const. ang velocity ω, we need to apply psedo force $m\omega^2\vec{r}$ radially outward. This is called centrifugal force

eg A pendulum of length 'l' is hanging from a ceiling of car which is moving in a circle of radius r and with speed v. find the angle that it makes with vertical.



$$T\sin\theta = \frac{mv^2}{r}$$

$$T\cos\theta = mg$$

$$\boxed{\tan\theta = \frac{v^2}{rg}}$$

Lecture
Units and di

Physical

Fundamental
or
Base Quantities

1. Mass
2. Length
3 Time
4. Electric Current
5. Temperature
6. Luminous Inte
7. Quantity of Ma

Supplem

1. Plane Angle
2. Solid Angle

Unit: The u quantity is the quantity which our refrence it.

- The magnitud is given by

$n \times u =$

number of the un
the measurem