Ex.1 The roots of the equation $x^2 - 2\sqrt{2}x + 1 = 0$ are

(1) Real and different (2) Imaginary and different

(3) Real and equal (4) Rational and different

Sol. The discriminant of the equation

$(-2\sqrt{2})^2 - 4(1)(1)$

$= 8 - 4 = 4 > 0$ and a perfect square

so roots are real and different but we can't say that roots are rational because coefficients are not rational therefore.

$$\frac{\sqrt{2\sqrt{2} \pm \sqrt{(2\sqrt{2})^2 - 4}}}{2} = \frac{2\sqrt{2} \pm 2}{2} = \sqrt{2} \pm 1 \text{ this is irrational}$$

$\therefore$ the roots are real and different. **Ans. [1]**

Ex.2 If the roots of the equation $x^2 + 2x + p = 0$ are real then the value of P is

(1) $P \leq 2$ (2) $P \leq 1$ (3) $P \leq 3$ (4) none of these

Sol. Here a = 1, b = 2, c = P

$\therefore$ discriminant $= (2)^2 - 4(1)(P) \geq 0$ (Since roots are real)

$= 4 - 4P \geq 0$ $= 4 \geq 4P \Rightarrow P \leq 1$ **Ans. [2]**

Ex.3 If the equation $(k-2)x^2-(k-4)x-2=0$ has difference of roots as 3 then the value of k is -

(1) 1, 3 (2) 3, 3/2 (3) 2, 3/2 (4) 3/2, 1

Sol. $(\alpha-\beta)=\sqrt{(\alpha+\beta)^2-4\alpha\beta}$

Now $\alpha+\beta=\dfrac{(k-4)}{(k-2)}$, $\alpha\beta=\dfrac{-2}{k-2}$

$$\therefore(\alpha-\beta)=\sqrt{\left(\frac{k-4}{k-2}\right)^2+\frac{8}{(k-2)}}=\frac{\sqrt{k^2+16-8k+8(k-2)}}{(k-2)}$$

$$3=\frac{\sqrt{k^2+16-8k+8k-16}}{(k-2)}$$

$3k-6=\pm k$

$k=3, 3/2$

Ans. [2]

Ex.4 If α, β are the root of a quadratic equation $x^2 - 3x + 5 = 0$ then the equation whose roots are $(\alpha^2 - 3\alpha + 7)$ and $(\beta^2 - 3\beta + 7)$ is

(1) $x^2 + 4x + 1 = 0$ (2) $x^2 - 4x + 4 = 0$ (3) $x^2 - 4x - 1 = 0$ (4) $x^2 + 2x + 3 = 0$

Sol. Since α, β are the roots of equation $x^2 - 3x + 5 = 0$

So $\alpha^2 - 3\alpha + 5 = 0$

$\beta^2 - 3\beta + 5 = 0$

$\therefore$ $\alpha^2 - 3\alpha = -5$

$\beta^2 - 3\beta = -5$

Putting in $(\alpha^2 - 3\alpha + 7)$ & $(\beta^2 - 3\beta + 7)$ ......(1)

$-5 + 7, -5 + 7$

$\therefore$ 2 and 2 are the roots

$\therefore$ The required equation is $x^2 - 4x + 4 = 0$ **Ans. [2]**

Ex.5 The equation whose roots are 3 and 4 will be –

(1) $x^2 + 7x + 12 = 0$ (2) $x^2 - 7x + 12 = 0$ (3) $x^2 - x + 12 = 0$ (4) $x^2 + 7x - 12 = 0$

Sol. The quadratic equation is given by

x^2 – (sum of the roots) x + (product of roots) = 0

$\therefore$ The required equation

$= x^2 - (3 + 4)x + 3.4 = 0$

$= x^2 - 7x + 12 = 0$ **Ans. [2]**

Ex.6 If α, β are root of the equation $x^2 - 5x + 6 = 0$ then the equation whose roots are $\alpha + 3$ and $\beta + 3$ is

(1) $x^2 - 11x + 30 = 0$

(2) $(x-3)^2 - 5(x-3) + 6 = 0$

(3) Both (1) and (2)

(4) none of these

Sol. Let $\alpha + 3 = x$

$\therefore \alpha = x - 3$ (Replace x by $x - 3$)

So the required equation is

$= (x-3)^2 - 5(x-3) + 6 = 0$

$= x^2 - 6x + 9 - 5x + 15 + 6 = 0$ .... (1)

$= x^2 - 11x + 30 = 0$ ....(2)

Ans. [3]

Ex.7 If equation $\frac{x^2 - bx}{ax - c} = \frac{k-1}{k+1}$ has equal and opposite roots in sign then the value of k is -

(1) $\frac{a+b}{a-b}$ (2) $\frac{a-b}{a+b}$ (3) $\frac{a}{b}+1$ (4) $\frac{a}{b}-1$

Sol. Let the roots are α & $-\alpha$

given equation is

$(x^2 - bx)(k + 1) = (k - 1)(ax - c)$

$\Rightarrow x^2(k+1) - bx(k+1) = ax(k-1) - c(k-1)$

$\Rightarrow x^2(k+1) - bx(k-1) - ax(k-1) + c(k-1) = 0$

Now sum of roots = $(\because \alpha - \alpha = 0)$

$\therefore\ b(k+1) + a(k-1) = 0$

$\Rightarrow \frac{a-b}{a+b}$ **Ans. [2]**

Ex.8 If the equation $2x^2 + x + k = 0$ and $x^2 + x/2 - 1 = 0$ have 2 common roots then the value of k is

(1) 1 (2) 3 (3) -1 (4) -2

Sol. Since the given equation have two roots in common so from the condition

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \frac{2}{1} = \frac{1}{1/2} = \frac{k}{-1}$$

$\therefore\ k = -2$

Ans. [4]

SOLVED EXAMPLES

Ex.1 If r and s are positive, then roots of the equation $x^2 - rx - s = 0$ are –

(1) imaginary
(2) real and both positive
(3) real and of opposite signs
(4) real and both negative

Sol. Here Discriminant

$= r^2 + 4s > 0$ ($\because$ r, s > 0)

$\Rightarrow$ roots are real Again a = 1 > 0 and c = – s < 0

$\Rightarrow$ roots are of opposite signs **Ans. [3]**

Ex.2 If a < b < c < d, then roots of $(x - a)(x - c) + 2(x - b)(x - d) = 0$ are –

(1) real and equal
(2) real and unequal
(3) imaginary
(4) rational

Sol. Here

$3x^2 - (a + c + 2b + 2d)x + (ac + 2bd) = 0$

$\therefore$ Discriminant $= (a + c + 2b + 2d)^2 - 12(ac + 2bd)$

$= [(a + 2d) - (c + 2b)]^2 + 4(a + 2d)(c + 2b) - 12(ac + 2bd)$

$= [(a + 2d) - (c + 2b)]^2 + 8(c - b)(d - a) > 0$ **Ans. [2]**

Ex.3 If p, q, r s are real number and $pr = 2(q + s)$ then for the equation $x^2 + px + q = 0$ and $x^2 + rx + s = 0$ the following statement is true –

(1) Both equation have imaginary roots

(2) Both have equal roots

(3) at least one of the equations has real roots

(4) none of these

Sol. Assuming that both equation have imaginary roots,

$p^2 - 4q < 0,\ r^2 - 4s < 0$

$\Rightarrow \quad p^2 + r^2 - 4(q + s) < 0$

But given $2(q + s) = pr$

$\therefore \quad p^2 + r^2 - 2pr < 0 \Rightarrow (p - r)^2 < 0$

Which is not true, hence our assumption is wrong, therefore at least one of the equations has real roots.

Ans. [3]

Ex.4 If $x^2 - 2px + q = 0$ has real roots then the equation $(1 + y)x^2 - 2(p + y) + (q + y) = 0$ will have its roots real and distinct if and only if –

(1) y is negative (2) p is not unity

(3) y is negative and p is not unity (4) none of these

Sol. Here $4p^2 - 4q = 0 \Rightarrow p^2 = q$

Also $D = 4(p + y)^2 - 4(1 + y)(q + y)$

$= 4[p^2 + 2py + y^2 - q - qy - y - y^2]$

$= +4y(2p - q - 1)$

$= 4y(2p - p^2 - 1)$

$= -4y(p - 1)^2$

Here $D > 0$ if y is negative and p is not one

Hence (3) is correct answer **Ans. [3]**

Ex.5 For the equation $\frac{1}{x+a} - \frac{1}{x+b} = \frac{1}{x+c}$, if the product of roots is zero, then the sum of roots is

(1) 0 (2) $\frac{2ab}{b+c}$ (3) $\frac{2bc}{b+c}$ (4) $\frac{-2bc}{b+c}$

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Sol. $\frac{1}{x+a}-\frac{1}{x+b}=\frac{1}{x+c}$

$$\frac{b-a}{x^2+(b+a)x+ab}=\frac{1}{x+c}$$

or $x^2 + (a + b)\,x + ab = (b - a)\,x + (b - a)\,c$

or $x^2 + 2ax + ab + ca - bc = 0$

Since product of the roots = 0

$ab + ca - bc = 0$

$a = \frac{bc}{b+c}$ Thus sum of roots = $-2a$

$=\frac{-2bc}{b+c}$

Ans. [4]

Ex.6 If p and q are roots of the equation $x^2 - 2x + A = 0$ and r and s be roots of the equation $x^2 - 18x + B = 0$ if $p < q < r < s$ be in A.P., then A and B are respectively –

(1) $-3, 77$ (2) $3, 77$

(3) $3, -77$ (4) none of these

Sol. Here p, q are roots of $x^2 - 2x + A = 0$

$\therefore p + q = 2$ .....(1)

Also r, s are roots of $x^2 - 18x + B = 0$

$\therefore r + s = 18$ .....(2)

Now since p, q, r, s in A.P. say with common difference d.

$\therefore q = p + d, r = p + 2d, s = p + 3d$

From (1) and (2)

$$\left.\begin{aligned} 2p + d &= 2 \\ 2p + 5d &= 18 \end{aligned}\right\} \Rightarrow 4d = 16 \Rightarrow d = 4$$

$\therefore 2p + 4 = 2 \Rightarrow p = -1$

Hence $p = -1, q = -1 + 4 = 3$

$r = -1 + 8 = 7, s = -1 + 12 = 11$

$A = pq = -3, B = rs = 77$ **Ans. [1]**

Ex.7 If α, β are the roots of the equation $\lambda\,(x^2 - x) + x + 5 = 0$ and λ_1 and λ_2 are two values of λ for which the roots α, β are connected by the relation $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{4}{5}$, Then value of $\frac{\lambda_1}{\lambda_2} + \frac{\lambda_2}{\lambda_1} =$

(1) 260 (2) 258 (3) 256 (4) 254

Sol. Here $\alpha + \beta = \frac{\lambda - 1}{\lambda}$, $\alpha\beta = \frac{5}{\lambda}$

Given $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{4}{5} \Rightarrow 5[\alpha^2 + \beta^2] = 4\alpha\beta$

$$\Rightarrow 5\left[\frac{\lambda - 1}{\lambda}\right]^2 = 14 \times \frac{5}{\lambda}$$

$\Rightarrow \lambda^2 - 16\lambda + 1 = 0$

Now $\lambda_1 + \lambda_2 = 16$, $\lambda_1\lambda_2 = 1$

$$\therefore \frac{\lambda_1}{\lambda_2} + \frac{\lambda_2}{\lambda_1} = \frac{\lambda_1^2 + \lambda_2^2}{\lambda_1\lambda_2} = \frac{(-16)^2 - 2}{1} = 254$$

Ans. [4]

Ex.8 If α, β are the roots of the equation $ax^2 + bx + c = 0$ and $S_n = \alpha^n + \beta^n$, then $a\,s_{n+1} + c\,S_{n-1} =$

(1) $b\,S_n$ (2) b^2S_n (3) $2bS_n$ (4) $-bS_n$

Sol. Here $\alpha + \beta$ are roots

$\therefore \quad a\alpha^2 + b\alpha + c = 0$ ...... (1)

$a\beta^2 + b\beta + c = 0$ ....(2)

Now let us consider (Keeping results (1), (2) in mind)

$a\,S_{n+1} + b\,S_n + C\,S_{n-1}$

$= a[\alpha^{n+1} + \beta^{n+1}] + b[\alpha^n + \beta^n] + c[\alpha^{n-1} + \beta^{n-1}]$

$= [a\alpha^{n+1} + b\alpha^n + c\alpha^{n-1}] + [a\beta^{n+1} + b\beta^n + c\beta^{n-1}]$

$= \alpha^{n-1}[a\alpha^2 + b\alpha + c] \mid \beta^{n-1}[a\beta^2 + b\beta + c]$

$= 0 + 0 = 0$

Hence $aS_{n+1} + cS_{n-1} = -bS_n$ **Ans. [4]**

Ex.9 If the roots of equation $x^2 + bx + ac = 0$ are α, β and roots of the equation $x^2 + ax + bc = 0$ are α, γ then the value of α, β, λ respectively -

(1) a, b, c (2) b, c, a (3) c, a, b (4) none of these

Sol. From the given two equation

$\alpha + \beta = -b$ ..... (1) $\alpha\beta = ac$ ...... (2)

$\alpha + \gamma = -a$ ..... (3) $\alpha\gamma = bc$ ...... (4)

$(1) - (3) \Rightarrow \beta - \gamma = a - b$ ...... (5)

$(2) / (4) \Rightarrow \beta/\gamma = a/b$

$$\beta = \frac{a\gamma}{b} \quad \text{...... (6)}$$

putting the value of β in (5)

$$\frac{a\gamma}{b} - \gamma = a - b \Rightarrow \gamma\,\frac{(a-b)}{b} = (a-b)$$

$\therefore \gamma = b$

$\therefore \beta = a$ & $\alpha = c$ **Ans. [3]**

Ex.10 If the quadratic equations $ax^2 + 2cx + b = 0$ and $ax^2 + 2bx + c = 0$ $(b \neq c)$ have a common root, then $a + 4b + 4c$ is equal to –

(1) – 2 (2) – 1 (3) 0 (4) 1

Sol. Let α be the common root of the given equations. Then

$$a\alpha^2 + 2c\alpha + b = 0$$

and $$a\alpha^2 + 2b\alpha + c = 0$$

$$\Rightarrow 2\,\alpha\,(c - b) + (b - c) = 0 \Rightarrow \alpha = \frac{1}{2} \qquad [\therefore b \neq c]$$

Puting $\alpha = 1/2$ in $a\alpha^2 + 2c\alpha + b = 0$, we get $a + 4b + 4c = 0$ **Ans. [3]**

Ex.11 The value of m for which one of the roots of $x^2 - 3x + 2m = 0$ is double of one of the roots of $x^2 - x + m = 0$ is

(1) 0, 2 (2) 0, – 2 (3) 2, – 2 (4) none of these

Sol. Let α be the root of $x^2 - x + m = 0$ and 2α be the root of $x^2 - 3x + 2m = 0$. Then,

$\alpha^2 - \alpha + m = 0$ and $4\alpha^2 - 6\alpha + 2m = 0$

$$\Rightarrow \frac{\alpha^2}{-m} = \frac{\alpha}{-m} = \frac{1}{2} \Rightarrow m^2 = -2m \Rightarrow m = 0,\ m = -2$$ **Ans. [2]**

Ex.12 If the expression $x^2 - 11x + a$ and $x^2 - 14x + 2a$ must have a common factor and $a \neq 0$, then, the common factor is –

(1) $(x - 3)$ (2) $(x - 6)$ (3) $(x - 8)$ (4) none of these

Sol. Here Let $x - \alpha$ is the common factor

then $x = \alpha$ is root of the corresponding equation

$$\therefore \quad \alpha^2 - 11\alpha + a = 0$$

$$\alpha^2 - 14\alpha + 2a = 0$$

Subtracting $3\alpha - a = 0 \Rightarrow \alpha = a/3$

Hence $\frac{a^2}{9} - 11\frac{a}{3} + a = 0$, $a = 0$ or $a = 24$

since $a \neq 0$, $a = 24$

$\therefore$ the common factor of $\begin{cases} x^2 - 11x + 24 \\ x^2 - 14x + 48 \end{cases}$ is clearly $x - 8$ **Ans. [3]**

Ex.13 If x is real then the value of the expression $\frac{x^2+14x+9}{x^2+2x+3}$ lies between

(1) -3 and 3 (2) -4 and 5 (3) -4 and 4 (4) -5 and 4

Sol. Let $\frac{x^2+14x+9}{x^2+2x+3} = y$

$\Rightarrow$ $x^2(1-y) + 2x(7-y) + 3(3-y) = 0$

Hence $4(7-y)^2 - 12(1-y)(3-y) \ge 0$ gives

$-2y^2 - 2y + 40 \ge 0$

$\Rightarrow$ $y^2 + y - 20 \le 0$

$\Rightarrow$ $(y+5)(y-4) \le 0 \Rightarrow -5 \le y \le 4$ **Ans. [4]**

Ex.14 If the eqaution $x^2 + 2(k + 1)x + 9k - 5 = 0$ has only negative roots, then –

(1) $k \leq 0$ (2) $k \geq 0$ (3) $k \geq 6$ (4) $k \leq 6$

Sol. Let $f(x) = x^2 + 2(k+1)x + 9k - 5$. Let α, β be the roots of $f(x) = 0$. The equation $f(x) = 0$ will have both negative roots, if –

(i) Disc. ≥ 0 (ii) $\alpha < 0, \beta < 0$, i.e. $(\alpha + \beta) < 0$ and (iii) $f(0) > 0$

Now, Discriminant $\geq 0 \Rightarrow 4(k+1)^2 - 36k + 20 \geq 0$

$$\Rightarrow k^2 - 7k + 6 \geq 0$$

$$\Rightarrow (k - 1)(k - 6) \geq 0$$

$$\Rightarrow k \leq 1 \text{ or } k \geq 6 \qquad \text{......(i)}$$

$$(\alpha + \beta) < 0 \Rightarrow -2(k+1) < 0 \Rightarrow k+1 > 0 \Rightarrow k > -1 \qquad \text{......(ii)}$$

$$\text{and, } f(0) > 0 \Rightarrow 9k - 5 > 0 \Rightarrow k < \frac{5}{9} \qquad \text{......(iii)}$$

From (i), (ii), (iii), we get $k \geq 6$ **Ans. [3]**

Ex.15 If the roots of the equation $x^2 + 3x + 2 = 0$ and $x^2 - x + \lambda = 0$ are in same ratio then the value of λ is given by-

(1) 2/7 (2) 2/9 (3) 9/2 (4) 7/2

Sol. If roots are in same ratio then

$$\frac{3^2}{(-1)^2} = \frac{(1).(2)}{(1).(\lambda)} \Rightarrow 9 = \frac{(2)}{(\lambda)}$$

$$\Rightarrow \lambda = \frac{2}{9}$$

Ans. [2]

Ex.16 The sum of all real roots of the equation $|x-2|^2 + |x-2| - 2 = 0$, is -

(1) 0 (2) 8 (3) 4 (4) none of these

Sol. **Case I** $x - 2 > 0$, Putting $x - 2 = y,\ y > 0$

$x > 2$

$\therefore Y^2 + Y - 2 = 0 \Rightarrow Y = -2, 1$

$\Rightarrow \quad x = 0, 3$

But $0 < 2$, Hence $x = 3$ is the real root.

Case II $x - 2 < 0 \Rightarrow x < 2,\ y < 0$

$y^2 - y - 2 = 0 \Rightarrow y = 2, -1 \Rightarrow x = 4,\ x = 1$

Since $4 \nless 2$, only $x = 1$ is the real root

Hence the sum of the real roots $= 3 + 1 = 4$

(3) is correct option **Ans. [3]**

Ex.17 If $0 \le x \le \pi$, then solution of the equation $16^{\sin^2 x} + 16^{\cos^2 x} = 10$ is given by x equal to

(1) $\frac{\pi}{6}, \frac{\pi}{3}$ (2) $\frac{\pi}{3}, \frac{\pi}{2}$ (3) $\frac{\pi}{6}, \frac{\pi}{2}$ (4) none of these

Sol. Let $16^{\sin^2 x} = y$, then $16^{\cos^2 x} = 16^{1-\sin^2 x} = \frac{16}{y}$

Hence $y + \frac{16}{y} = 10 \Rightarrow y^2 - 10y + 16 = 0$ or $y = 2, 8$

Now $16^{\sin^2 x} = 2 \Rightarrow (2)^{4\sin^2 x} = (2)^1$

$\Rightarrow 4\sin^2 x = 1$ $\therefore \sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{3}$

Ex.18 If p, q, r be in H.P. and p and r be different having same sign, then the root of the equation $px^2 + 2qx + r = 0$ will be

(1) real (2) equal (3) imaginary (4) none of these

Sol. Here p, q, r in H.P. $\Rightarrow q = \dfrac{2pr}{p+r}$ .....(1)

Now $D = 4q^2 - 4pr$

$= -4\left[pr - \left(\dfrac{2pr}{p+r}\right)^2\right]$ using (1)

$= -(pr)\left[2\left(\dfrac{p-r}{p+r}\right)^2\right]$

Since $pr > 0$, $p \neq r$ given,

$D \neq 0$ and $D < 0$ Hence the roots are imaginary **Ans. [3]**

Note : The students should develope a practice of arguing, using given conditions. The discriminant should be expressed in perfact square form as far as possible

Ex.19 The number of solution of $\frac{\log 5+\log(x^2+1)}{\log(x-2)} = 2$ is/are

(1) 1 (2) 2 (3) 3 (4) none of these

Sol. We have $\log 5\,(x^2+1) = 2\log(x-2)$

$\Rightarrow 5(x^2+1) = (x-2)^2$

$\Rightarrow 4x^2+4x+1 = 0$

$\Rightarrow (2x+1)^2 = 0$

$\therefore x = -\frac{1}{2}$

But for $x = -\frac{1}{2}$, the denominator is imaginary, the equation is not meaningful Hence no root exist

Ans. [4]

Ex.20 If $x = 2 + \sqrt{3}$ then the value of $x^3 - 7x^2 + 13x - 12$ is

(1) 3 (2) 6 (3) -9 (4) 9

Sol. $x = 2 + \sqrt{3}$

$\Rightarrow x - 2 = \sqrt{3}$

$\Rightarrow x^2 + 4 - 4x = 3$

$\Rightarrow x^2 - 4x = -1$

$x^2 - 4x + 1 = 0$

Now we can write the given equation as

$x^3 - 7x^2 + 13x - 12 = x(x^2 - 4x + 1) - 3x^2 + 12x - 12$

$= x(x^2 - 4x + 1) - 3(x^2 + 4x + 1) - 9$

Now putting the value of $x^2 - 4x + 1 = 0$

$= x(0) - 3(0) - 9 = -9$

Ans. [3]