Markov Chain Model of Genetic Evolution: EE 126 Project 1

Ajeya Cotra, Leah Dickstein, Davis Foote, Ena Hariyoshi

October 23, 2014

Our project models the evolution of the peppered moth (*Biston betularia*) over the last two hundred years. The peppered moth initially started off mostly white, to camouflage with the light trees and lichen in its environment. However, with the advent of the industrial revolution, increased pollution in the moths' habitats killed off most lichen and stained light-colored trees with soot, causing

	WW, WW	WW, Wg	WW, gg	Wg, Wg	Wg, gg	gg, gg
WW	1	1/2	0	1/4	0	0
Wg	0	1/2	1	1/2	1/2	0
gg	0	0	0	1/4	1/2	1
	$(\frac{N(WW)}{N(all)})^2$	$2(\frac{N(WW)}{N(all)})(\frac{N(Wg)}{N(all)})$	$2(\frac{N(WW)}{N(all)})(\frac{N(gg)}{N(all)})$	$(\frac{N(Wg)}{N(all)})^2$	$2(\frac{N(WW)}{N(all)})(\frac{N(gg)}{N(all)})$	$(\frac{N(gg)}{N(all)})^2$

So the probabilities of birth for each genotype should be:

$$b_{WW} = \left(\frac{WW}{all}\right)^2 + \frac{WW * gg}{all^2} + \frac{1}{4}\left(\frac{Wg}{all}\right)^2$$
$$b_{Wg} = \frac{WW * Wg}{all^2} + \frac{1}{2}\left(\frac{Wg * gg}{all^2}\right) + \frac{1}{2}\left(\frac{Wg}{all}\right)^2 + 2 * \frac{WW * gg}{all^2}$$
$$b_{gg} = \left(\frac{gg}{all}\right)^2 + \frac{gg * Wg}{all^2} + \frac{1}{4}\left(\frac{Wg}{all}\right)^2$$

In addition, at each timestep we certainly kill a moth, but we randomly choose which moth to kill. The probability is greater for white instead of gray.

$$d_{WW} = \frac{d(WW)}{WW + Wg}$$
$$d_{Wg} = \frac{d(Wg)}{WW + Wg}$$
$$d_{gg} = 1 - d$$

Now, let

$$x = \text{Number of WW}$$
$$y = \text{Number of Wg}$$
$$z = \text{Number of gg}$$