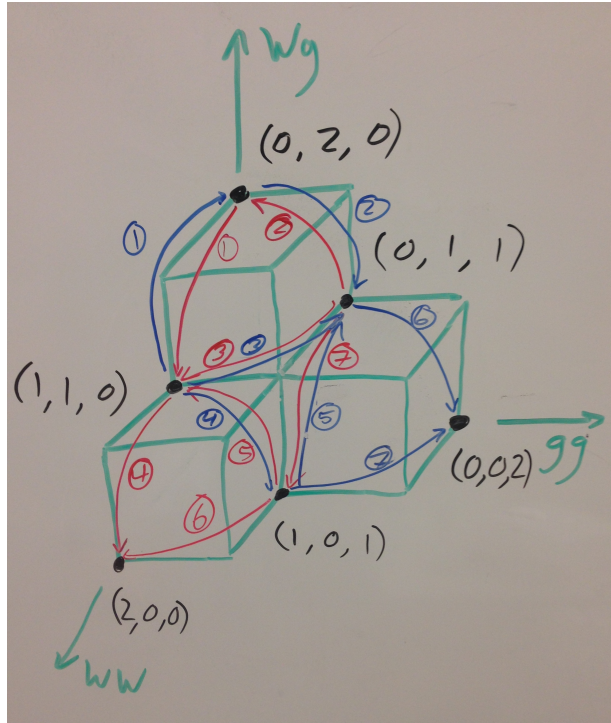


EE126–Fall 2014 — Mini-Project Extras

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In this document you will see the fruits of an attempt to analytically solve for the probability that all the moths will turn grey given a specified starting state. Using the notation specified in our write-up, we drew the smallest possible example that would still be interesting:



We then defined, for each state in the M.c., a probability that the state $(0,0,2)$ would eventually be reached. The arrows are drawn in red and blue to differentiate the ones going left and right. The transition probabilities corresponding to the red probabilities are referred to with b_i s, and those corresponding to the blue transitions are referred to with a_i s. The resulting system of equations is as follows:

$$\begin{aligned}
 p(2, 0, 0) &= 0 \\
 p(0, 0, 2) &= 1 \\
 p(1, 0, 1) &= a_5 p(0, 1, 1) + a_7 + b_5 p(1, 1, 0) \\
 p(1, 1, 0) &= a_1 p(0, 2, 0) + a_3 p(0, 1, 1) + a_4 p(1, 0, 1) \\
 p(0, 2, 0) &= a_2 p(0, 1, 1) + b_1 p(1, 1, 0) \\
 p(0, 1, 1) &= a_6 + b_2 p(0, 2, 0) + b_3 p(1, 1, 0) + b_7 p(1, 0, 1)
 \end{aligned}$$

We then set up the augmented matrix:

$$\begin{array}{cccc|c}
 (0, 2, 0) & (1, 1, 0) & (0, 1, 1) & (1, 0, 1) & \\
 \hline
 1 & -b_1 & -a_2 & 0 & 0 \\
 -a_1 & 1 & -a_3 & -a_4 & 0 \\
 -b_2 & -b_3 & 1 & -b_7 & a_6 \\
 0 & -b_5 & -a_5 & 1 & a_7
 \end{array}$$

Once we have a solution in terms of the transition probabilities a_i and b_i , we can solve for the transition probabilities using the method described in our project write-up to get a final analytical solution for the probabilities of all the moths turning grey from a given starting position. We enter the matrix into Wolfram Alpha to be row-reduced:

$$\left(\begin{array}{ccccc}
 1 & 0 & 0 & 0 & \frac{-a_3 a_6 b_1 - a_4 a_5 a_6 b_1 + a_2 a_4 a_6 b_5 - a_4 a_7 b_1 - a_2 a_4 a_7 b_3 - a_2 a_7 b_7 - a_3 a_7 b_1 b_7 - a_2 a_6}{a_1 b_1 + a_3 b_2 b_1 + a_4 a_5 b_2 b_1 - a_1 a_5 b_7 b_1 + a_2 b_2 + a_1 a_2 b_3 + a_3 b_3 + a_4 a_5 b_3 + a_4 b_5 - a_2 a_4 b_2 b_5 + a_5 b_7 + a_1 a_2 b_5 b_7 + a_3 b_5 b_7 - 1} \\
 0 & 1 & 0 & 0 & \frac{a_2 a_4 a_7 b_2 - a_1 a_2 a_7 b_7 - a_3 a_7 b_7 - a_1 a_2 a_6 - a_3 a_6 - a_4 a_5 a_6 - a_4 a_7}{a_1 b_1 + a_3 b_2 b_1 + a_4 a_5 b_2 b_1 - a_1 a_5 b_7 b_1 + a_2 b_2 + a_1 a_2 b_3 + a_3 b_3 + a_4 a_5 b_3 + a_4 b_5 - a_2 a_4 b_2 b_5 + a_5 b_7 + a_1 a_2 b_5 b_7 + a_3 b_5 b_7 - 1} \\
 0 & 0 & 1 & 0 & \frac{-a_1 a_6 b_1 - a_4 a_6 b_5 + a_4 a_7 b_1 b_2 + a_4 a_7 b_3 + a_7 b_7 - a_1 a_7 b_1 b_7 + a_6}{-a_1 b_1 - a_3 b_2 b_1 - a_4 a_5 b_2 b_1 + a_1 a_5 b_7 b_1 - a_2 b_2 - a_1 a_2 b_3 - a_3 b_3 - a_4 a_5 b_3 - a_4 b_5 + a_2 a_4 b_2 b_5 - a_5 b_7 - a_1 a_2 b_5 b_7 - a_3 b_5 b_7 + 1} \\
 0 & 0 & 0 & 1 & \frac{(a_5 (1 - a_1 b_1) - (-a_1 a_2 - a_3) b_5) (-a_7 (-b_1 b_2 - b_3) - a_6 b_5) + a_7 (1 - a_1 b_1) (a_5 (-b_1 b_2 - b_3) - b_5 (1 - a_2 b_2))}{(b_1 b_2 + b_3 + b_5 b_7) (a_5 (1 - a_1 b_1) - (-a_1 a_2 - a_3) b_5) - (a_1 b_1 + a_4 b_5 - 1) (a_5 (-b_1 b_2 - b_3) - b_5 (1 - a_2 b_2))}
 \end{array} \right)$$

And thus concludes the story of why we did not solve for the probability analytically.

The End