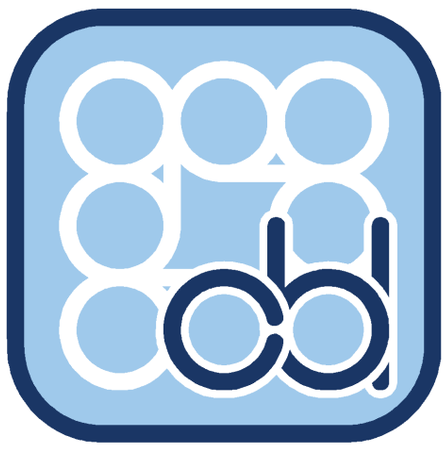
Do Deep Generative Models Know What They Don't Know?

Eric Nalisnick

LTL Seminar
14.2.19



Computational and Biological Learning
University of Cambridge

Outline

1 **Motivation:** Protecting Predictive Models

2 **An Experiment:** Chicken or Seven?

3 **Flow-Based Generative Models:** Background

4 **More Experiments**

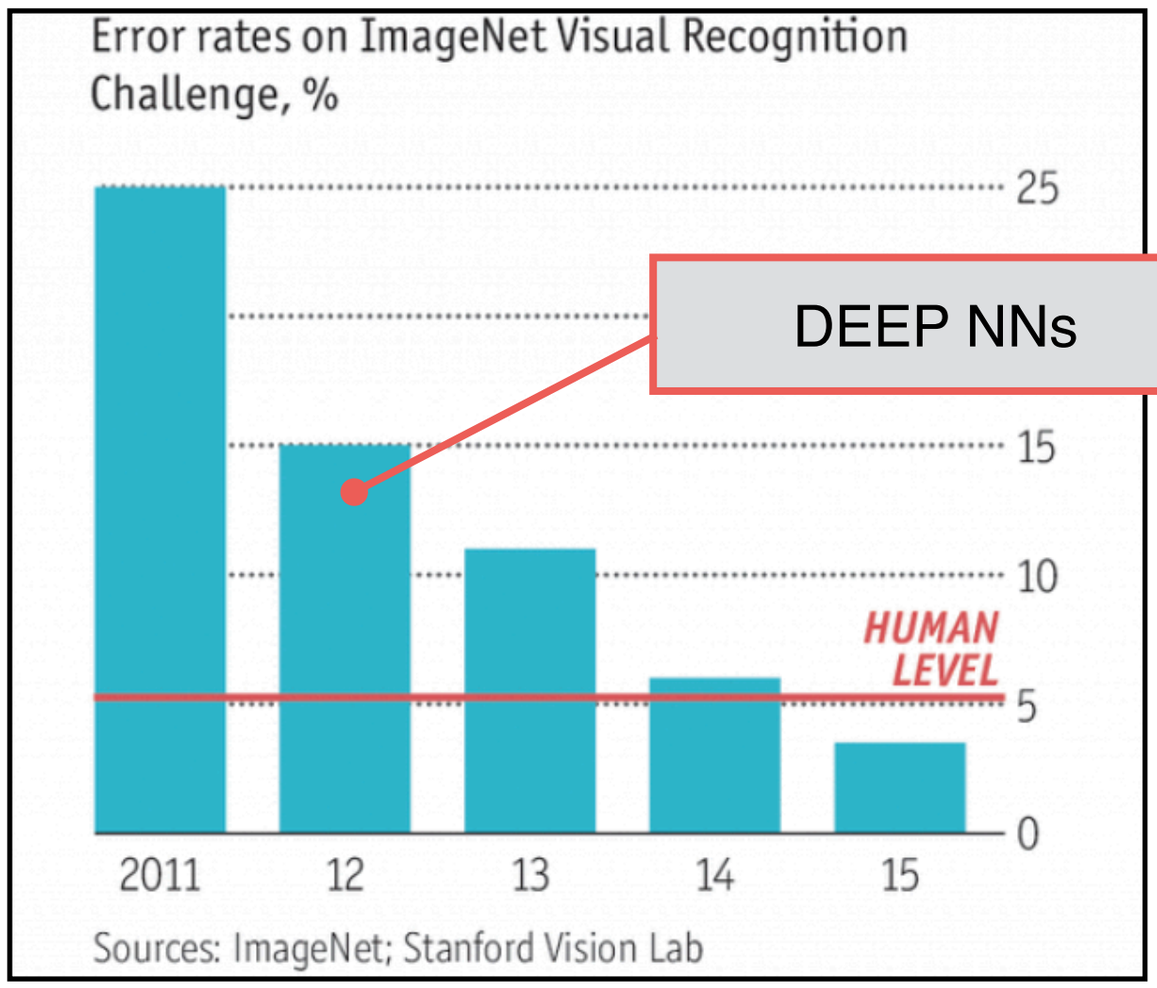
5 **A Mathematical Characterization**

6 **Parting Thoughts and Future Work**

Protecting Predictive Models

Deep Learning Results

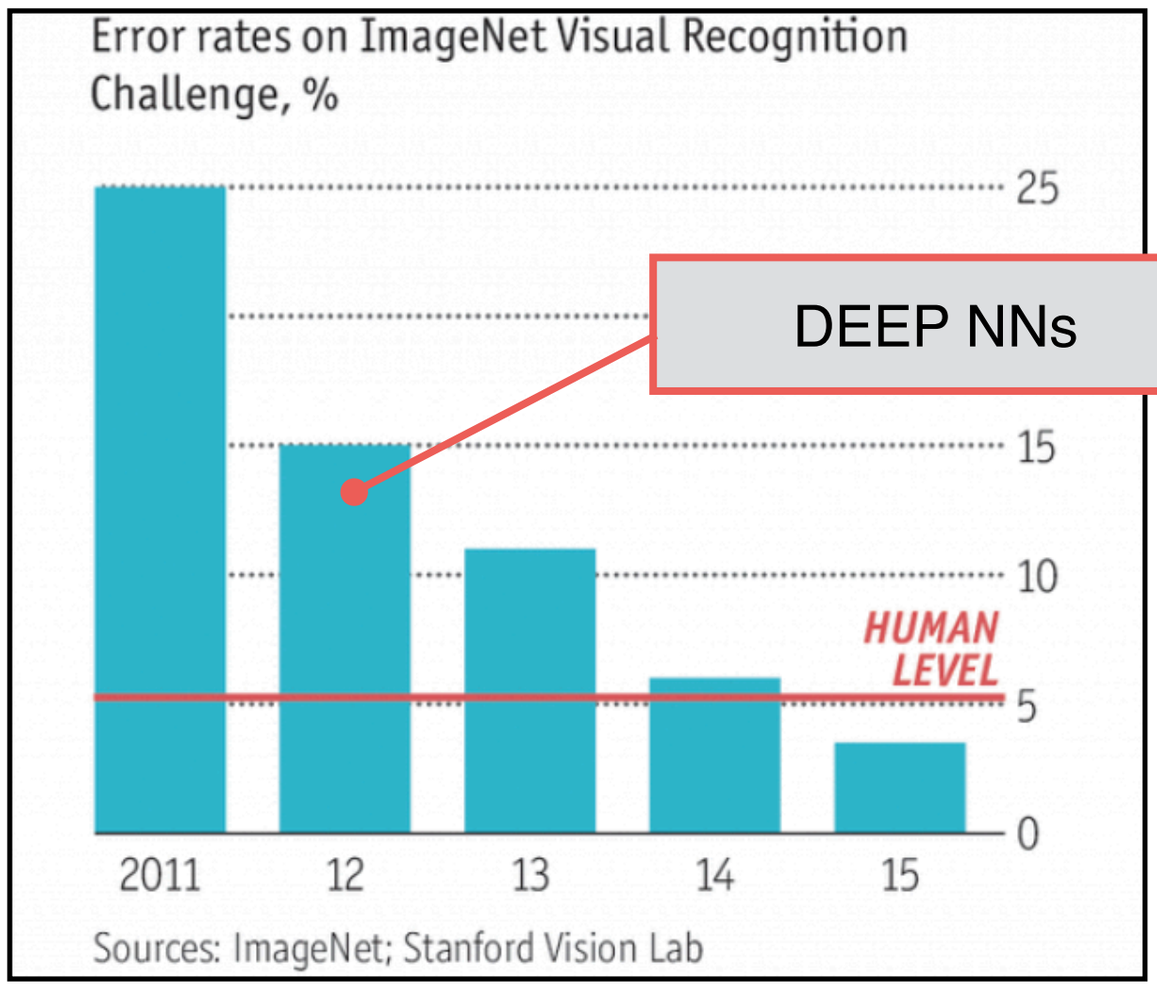
- **Computer Vision**: Results on ImageNet object classification dataset.



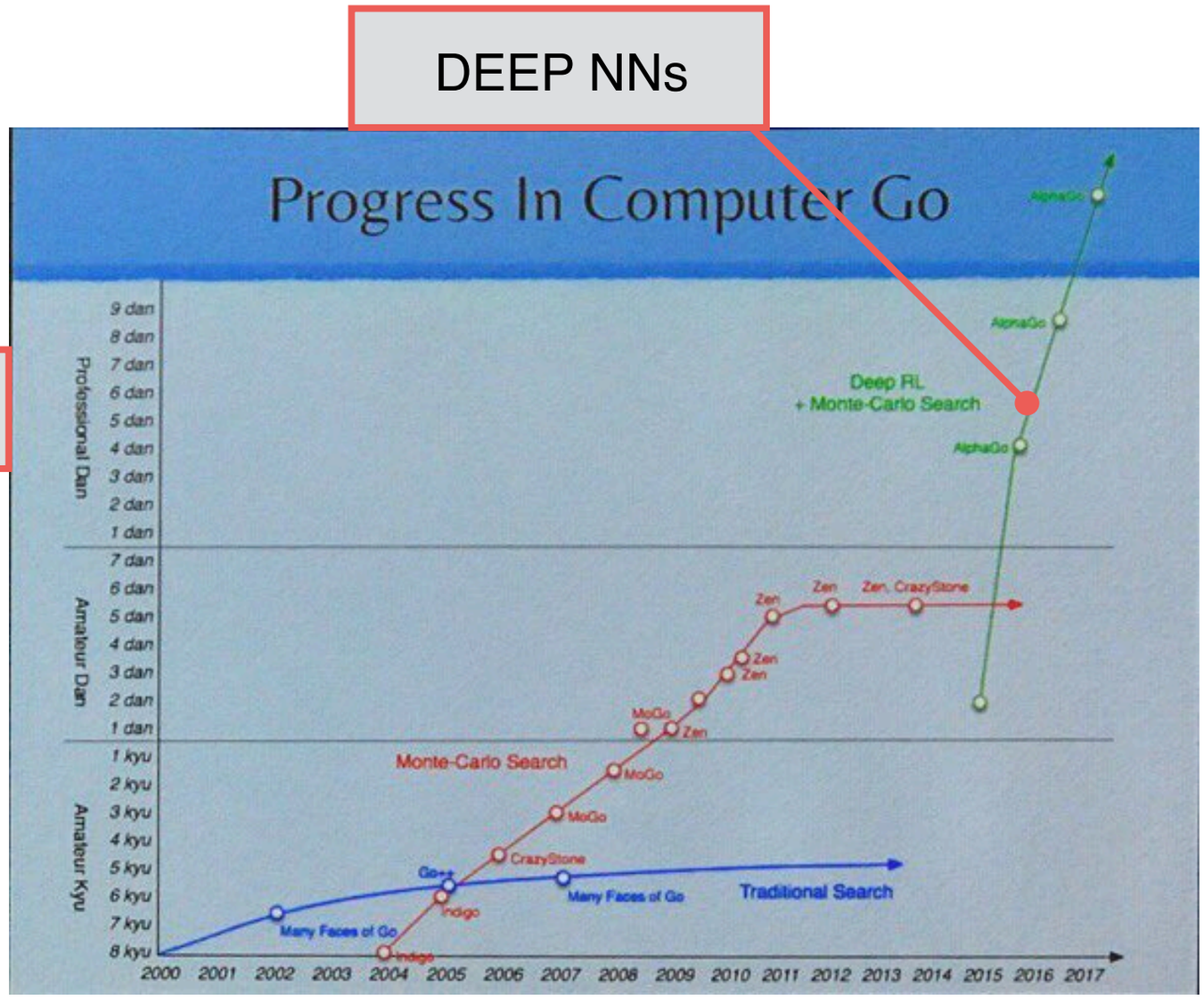
https://www.economist.com/news/special-report/21700756-artificial-intelligence-boom-based-old-idea-modern-twist-not

Deep Learning Results

- **Computer Vision**: Results on ImageNet object classification dataset.



- **Reinforcement Learning**: Results in playing Go.



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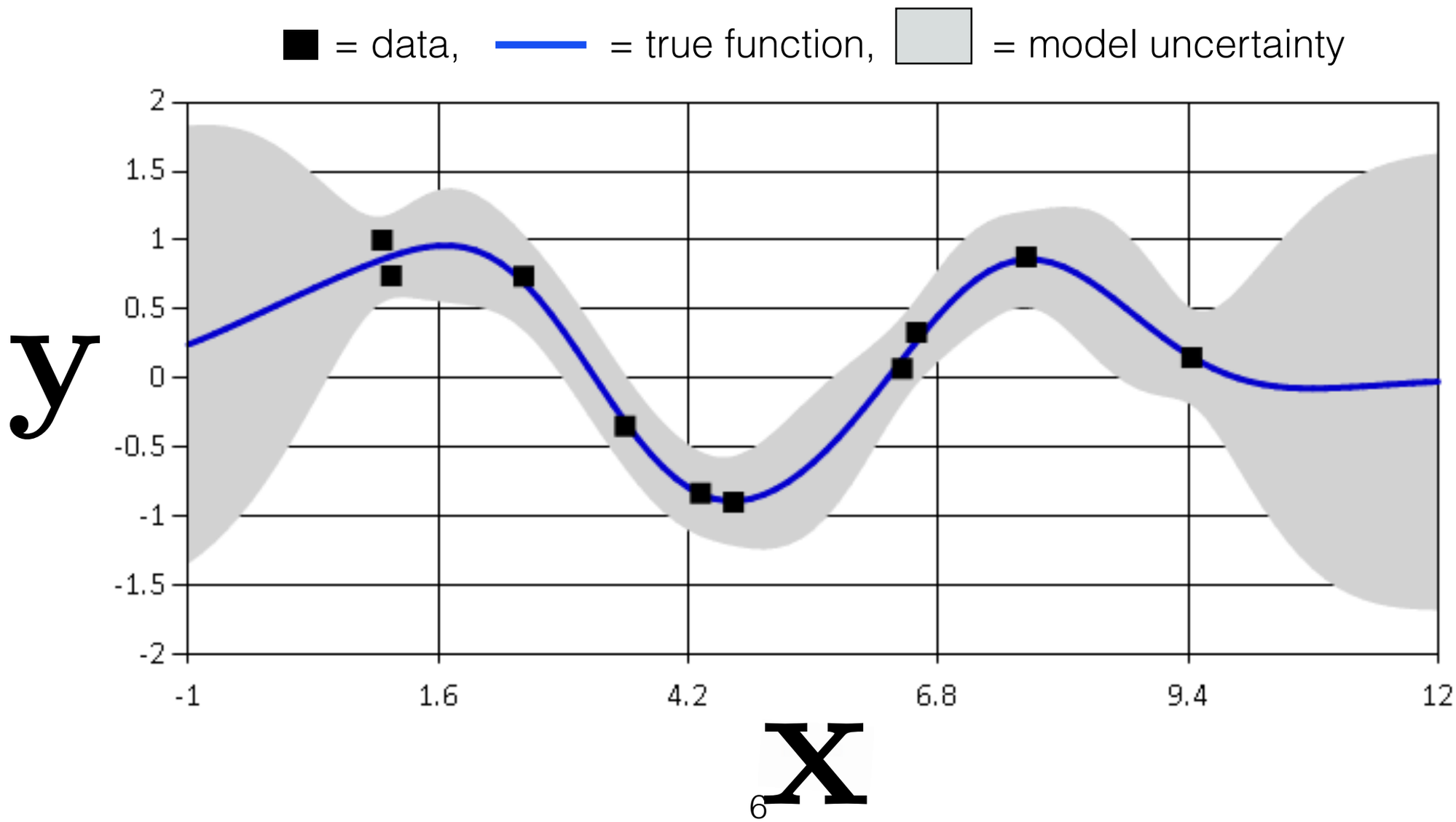
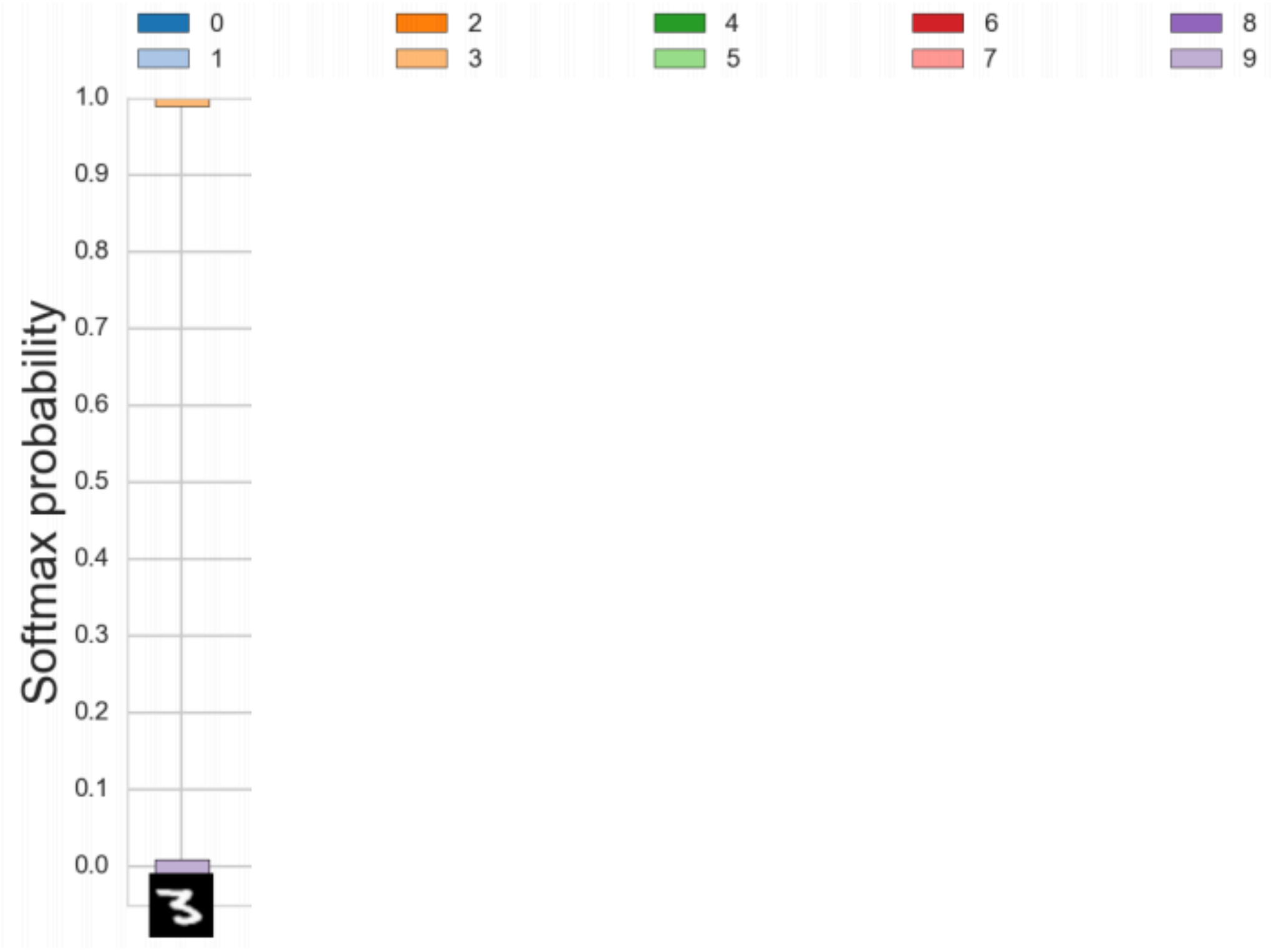
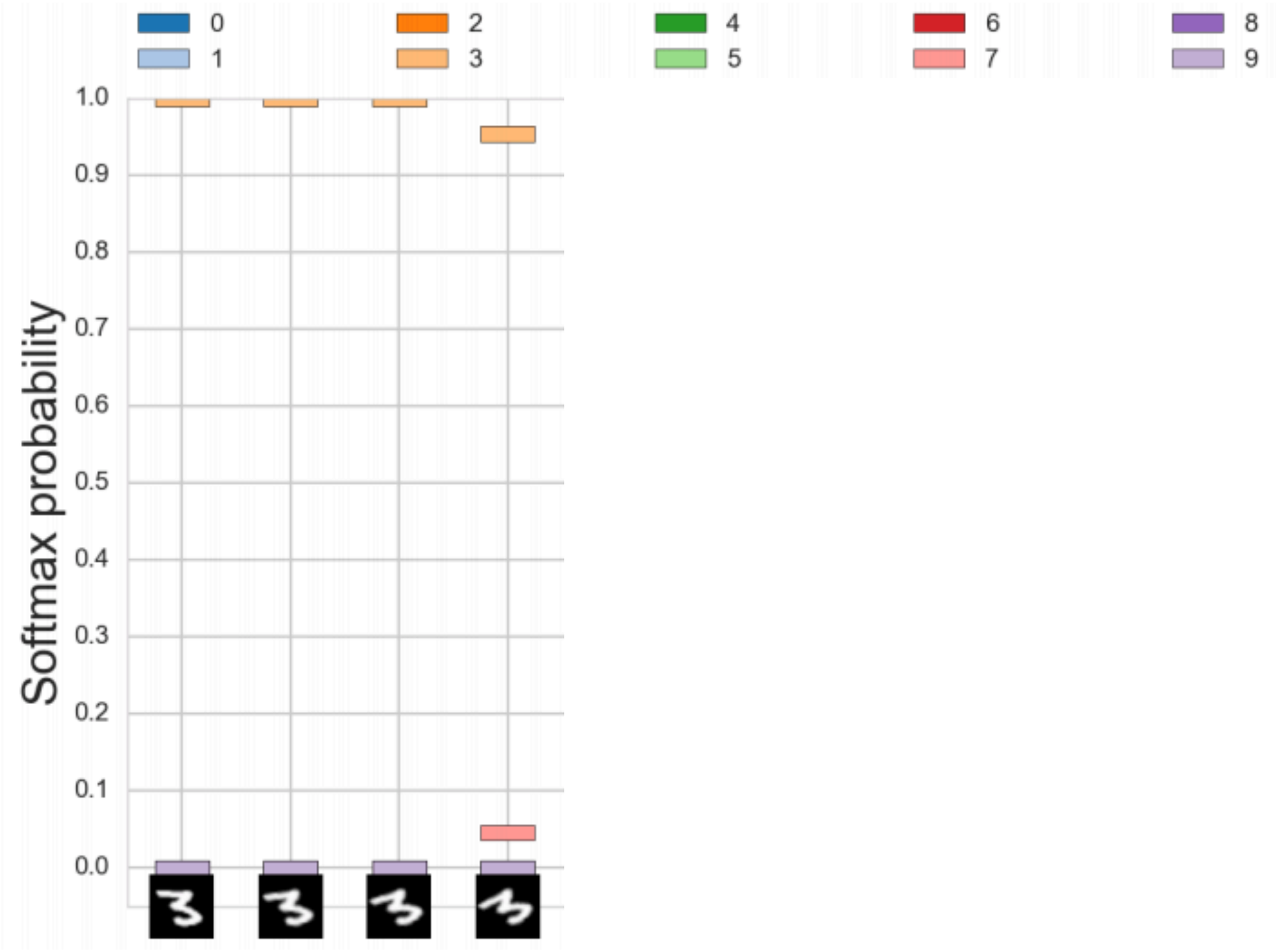
$$p(\mathbf{y}|\mathbf{X};\boldsymbol{\theta})$$

Labels **Features** **Parameters**





[Louizos & Welling, ICML 2017]



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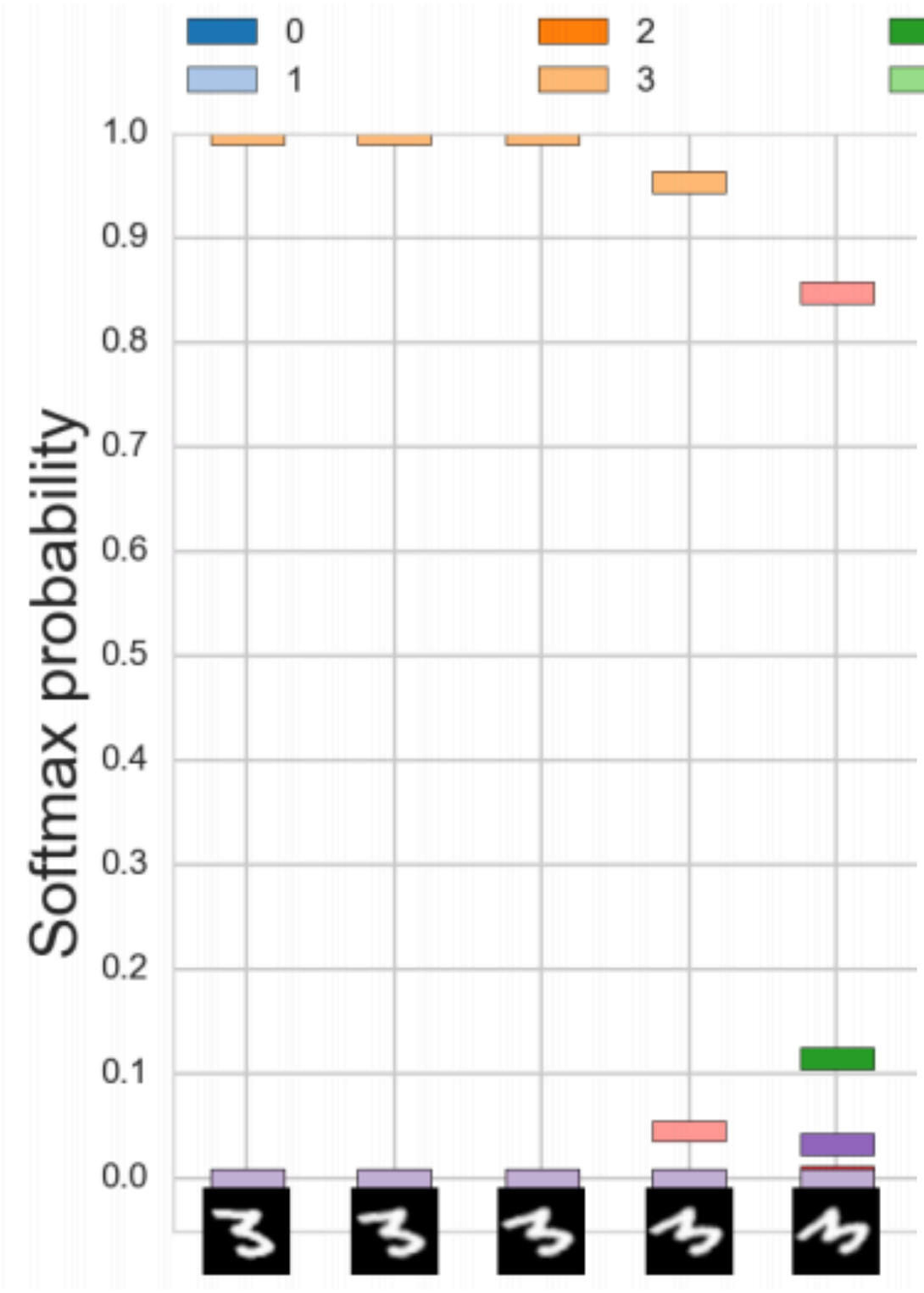

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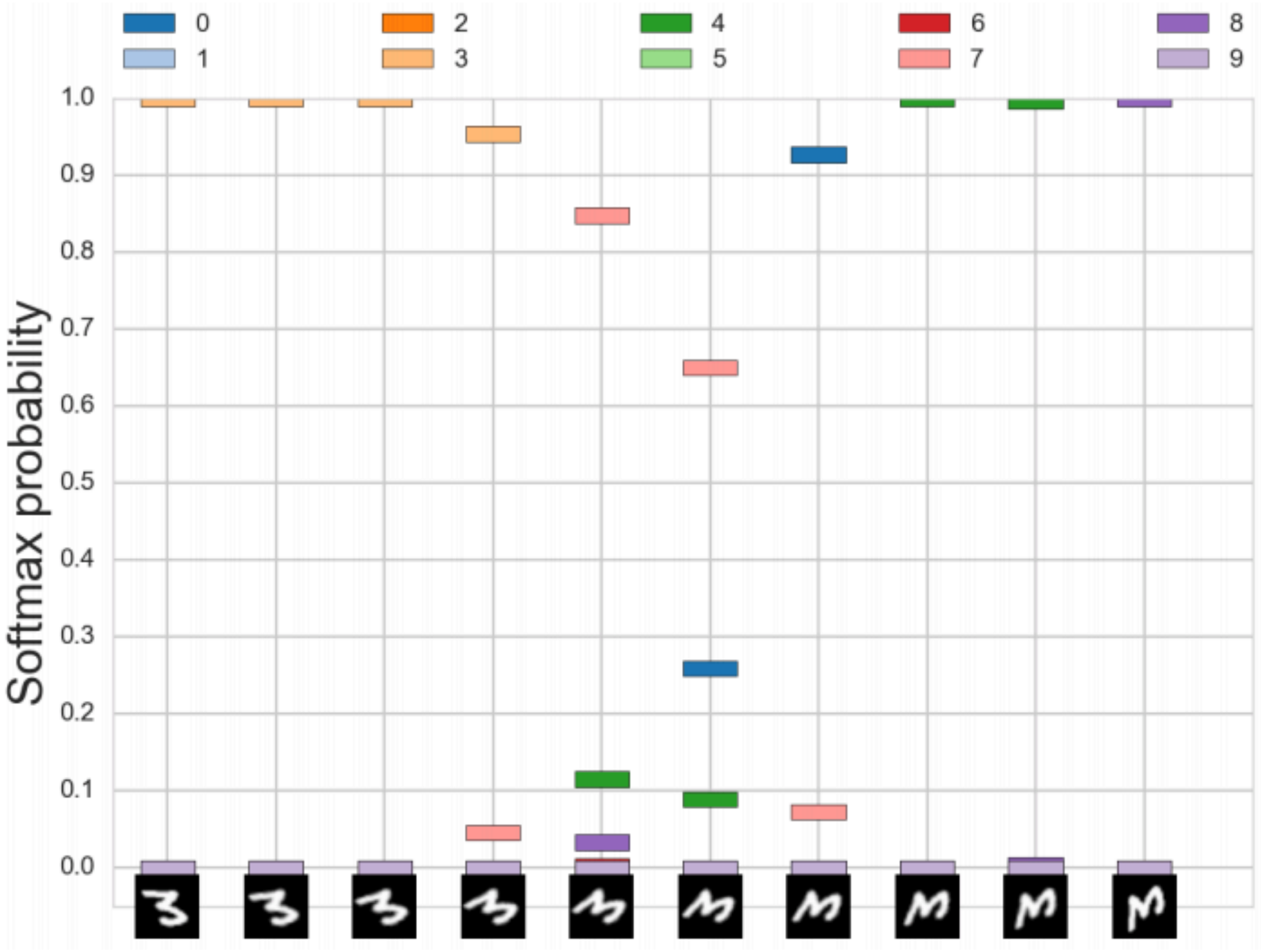

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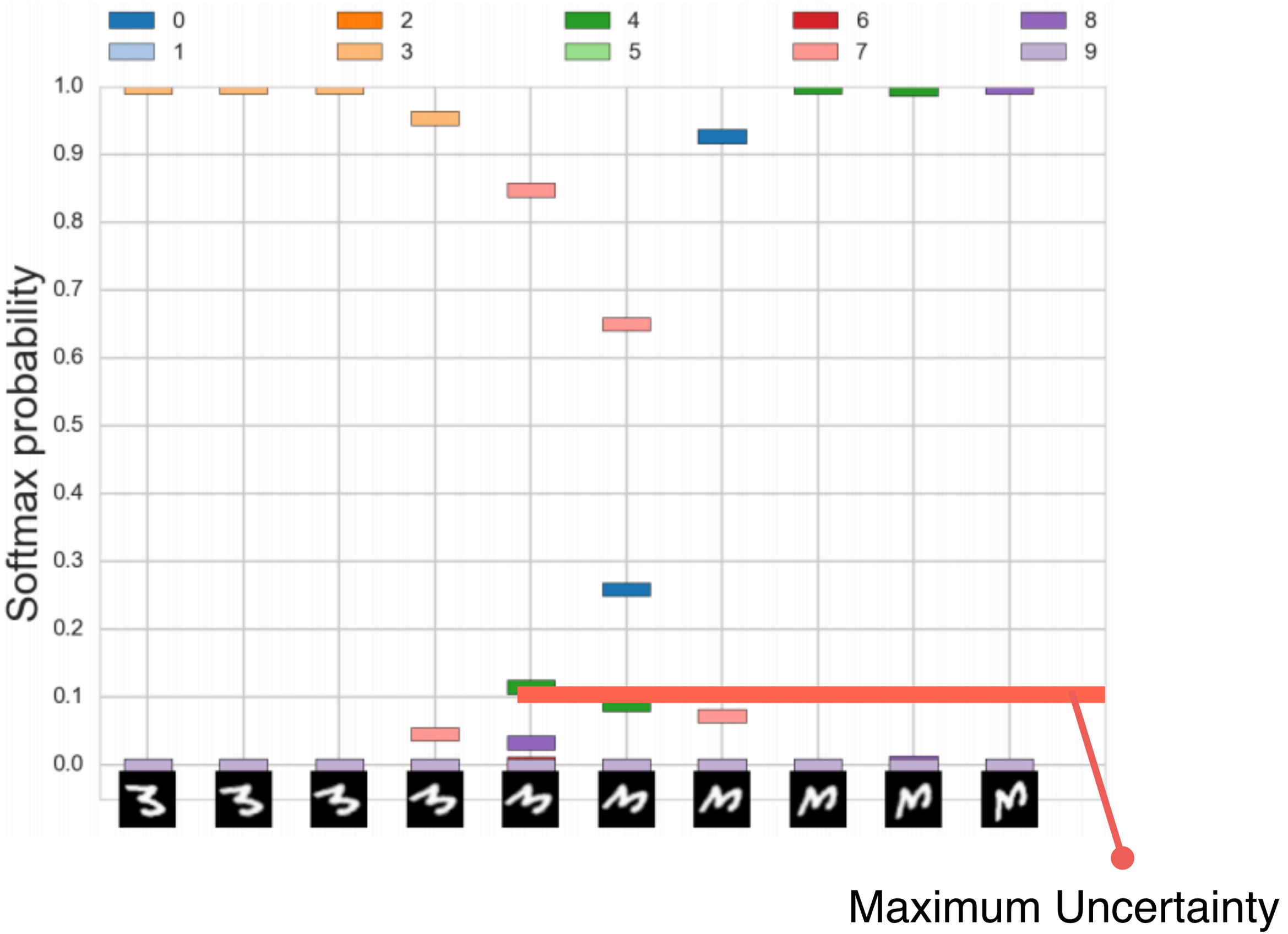
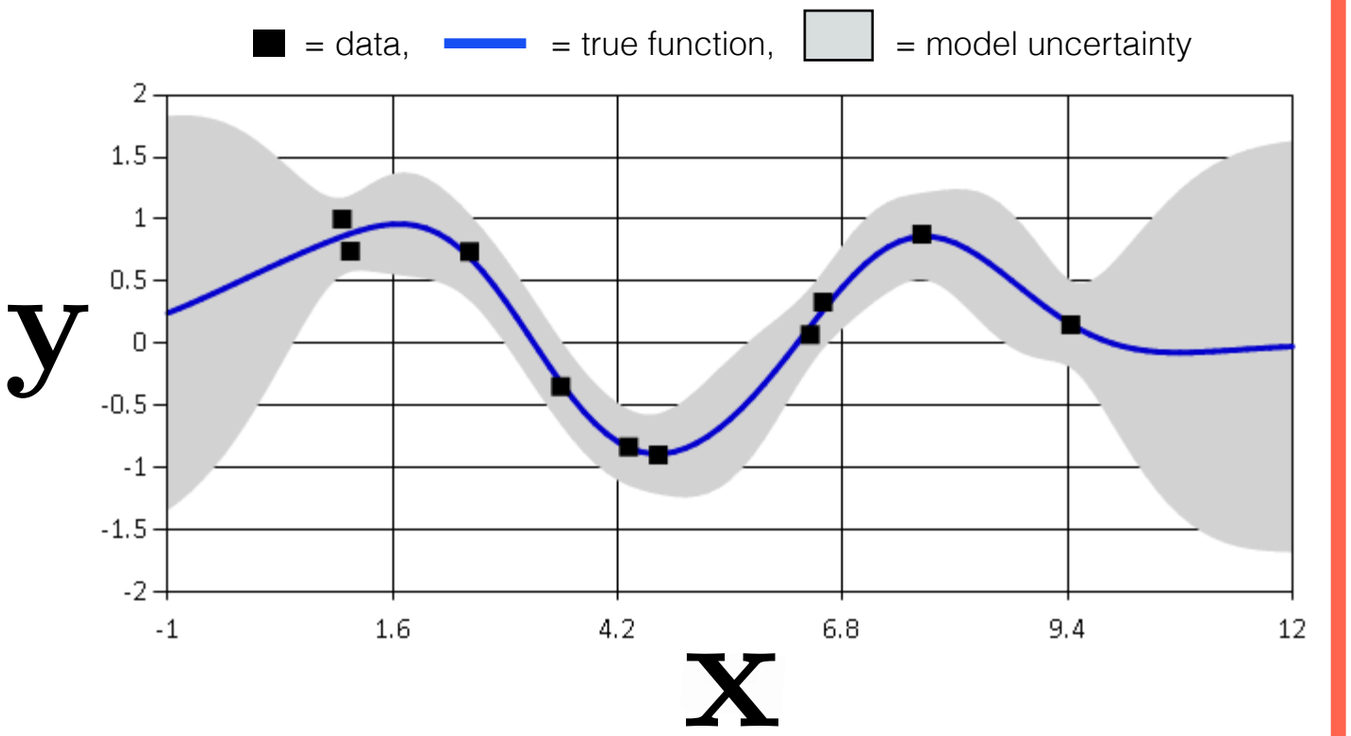


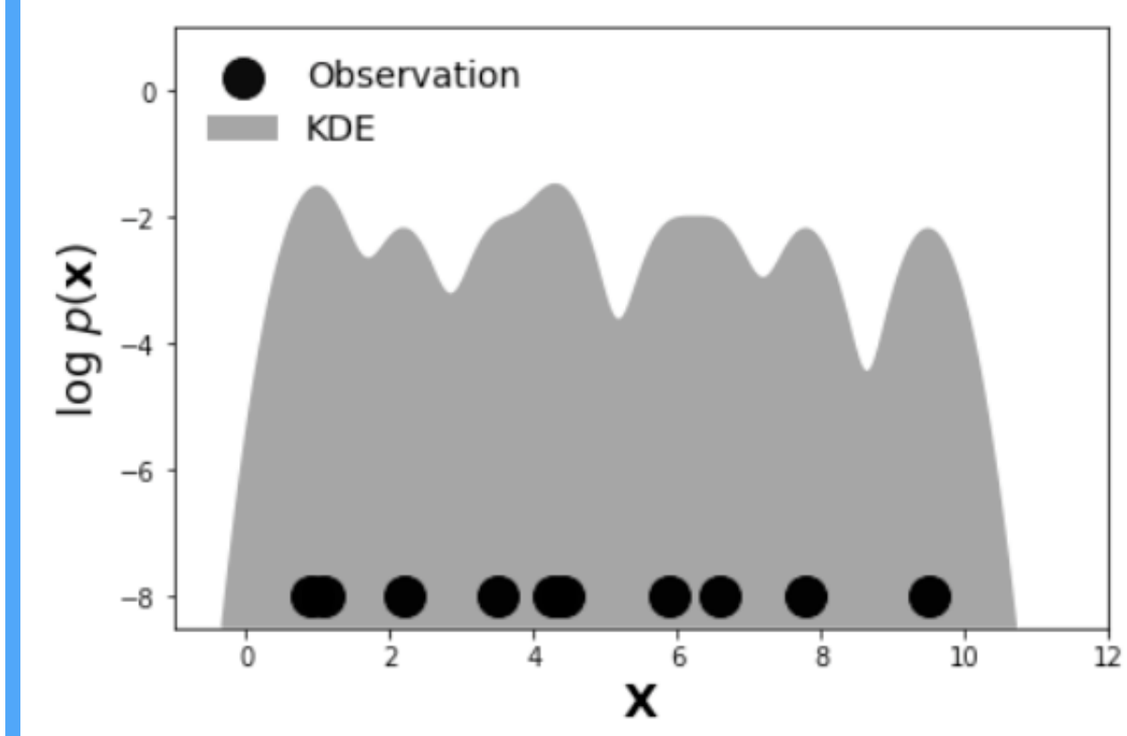
[Louizos & Welling, ICML 2017]

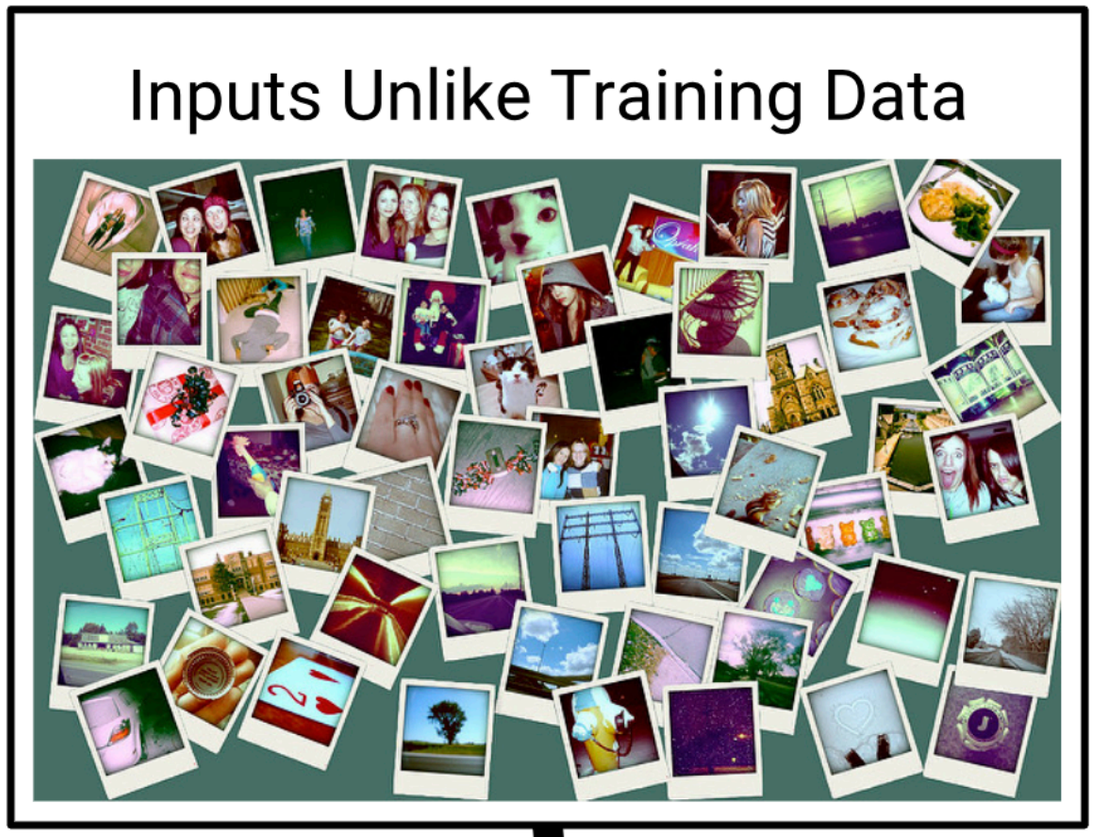
$$p(\mathbf{y}, \mathbf{X}) = p(\mathbf{y}|\mathbf{X}; \boldsymbol{\theta})\ p(\mathbf{X}; \boldsymbol{\phi})$$

Predictive Model

Generative Model







$$\text{if } p(\boldsymbol{x}^*; \boldsymbol{\phi}) < \tau,$$

$$\text{then } \texttt{reject } \boldsymbol{x}^*$$

Use p(**X**) model to reject inputs with density below some threshold [Bishop, 1994].

Novelty Detection and Neural Network Validation

Chris M. Bishop (May 1994)

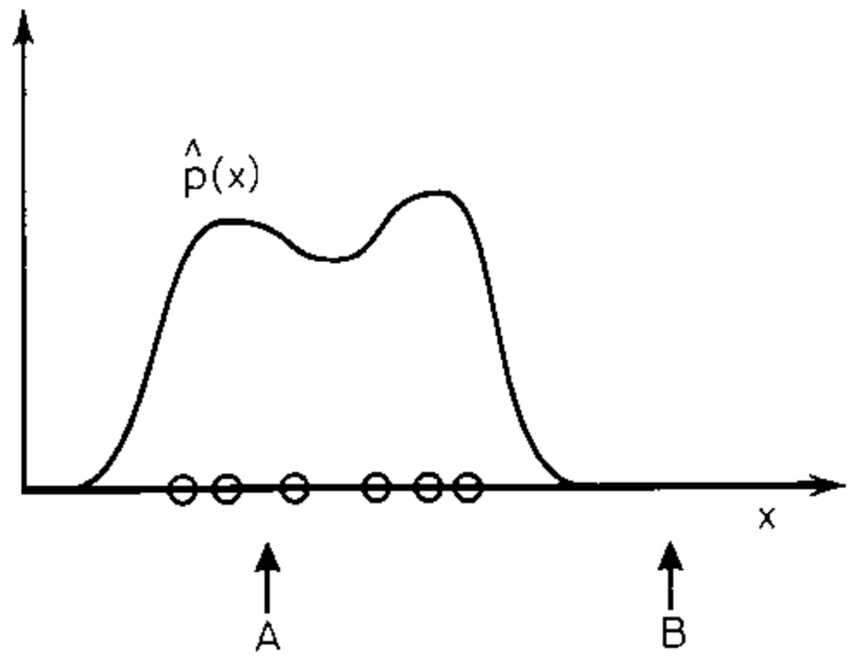


Fig. 1 *Use of the unconditional probability density to measure the degree of novelty of new input vectors*

Novelty Detection and Neural Network Validation

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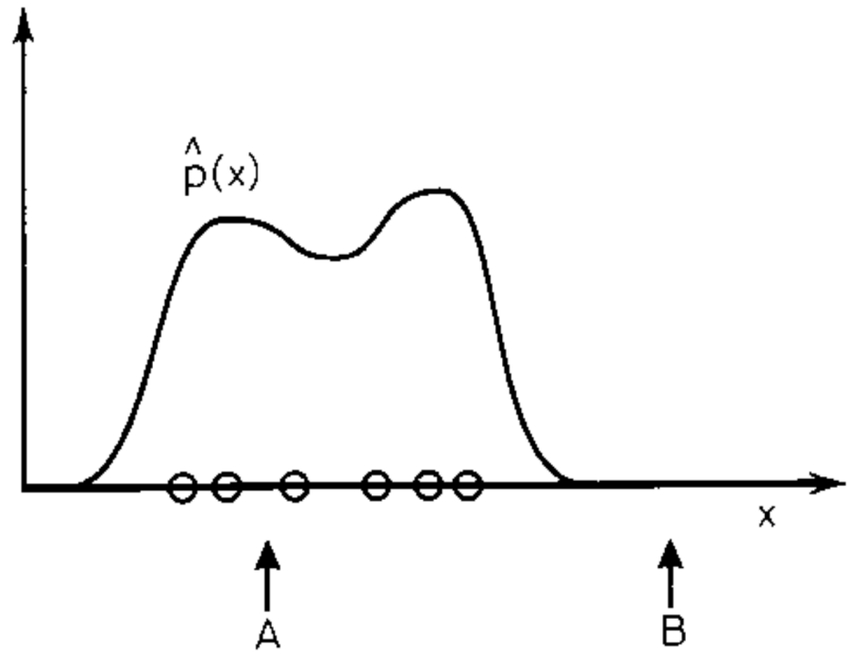

Fig. 1 *Use of the unconditional probability density to measure the degree of novelty of new input vectors*

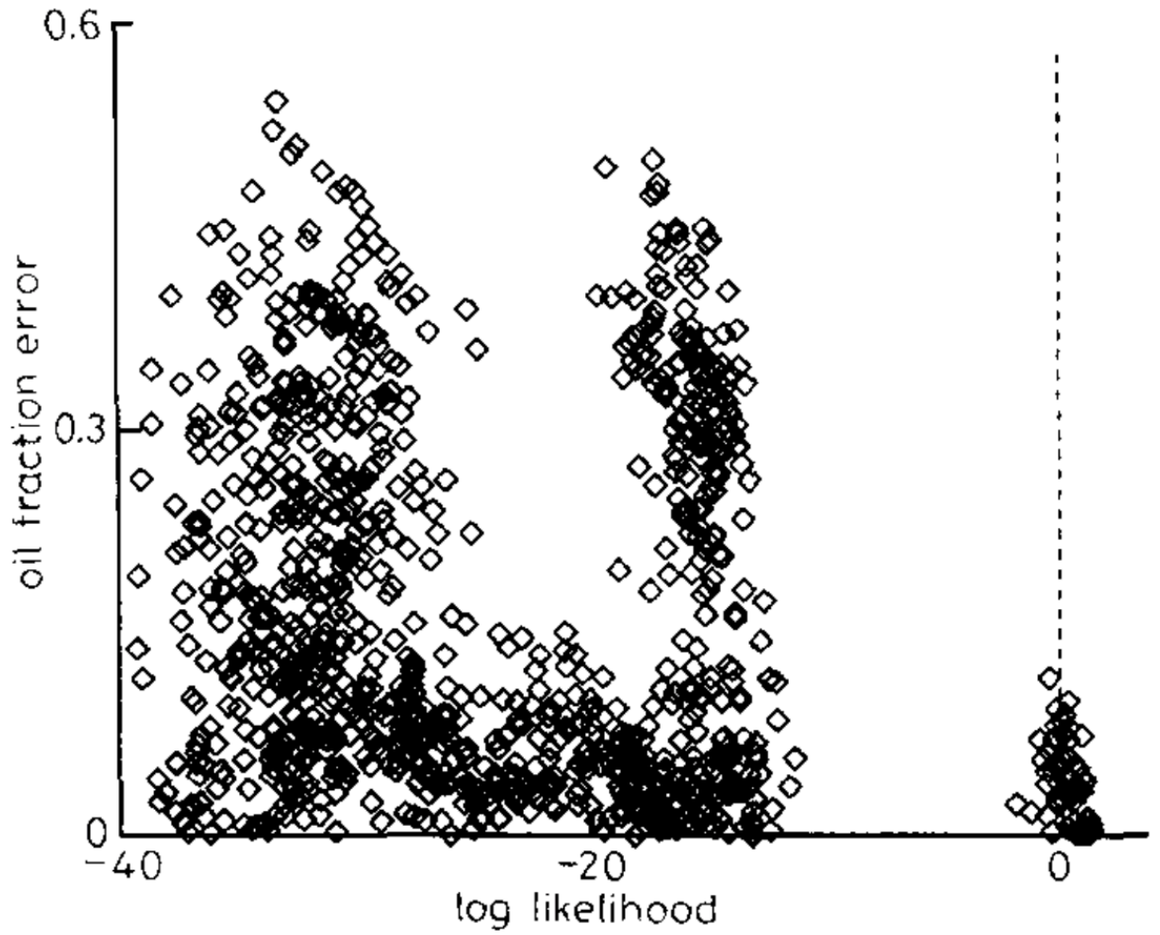

Fig. 7 *Magnitude of the oil fraction error from the neural network prediction against the log likelihood from the novelty detector*

Panel Discussion

Advances in Approximate Bayesian Inference, Dec 2017

Panel Discussion

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Deep Generative Models

- [ ] **Autoregressive Models** (e.g. WaveNet)

  [Larochelle & Murray, AISTATS 2011; van den Oord, ICLM 2016; van den Oord, NeurIPS 2016]

- [ ] **Variational Autoencoders** (VAEs)

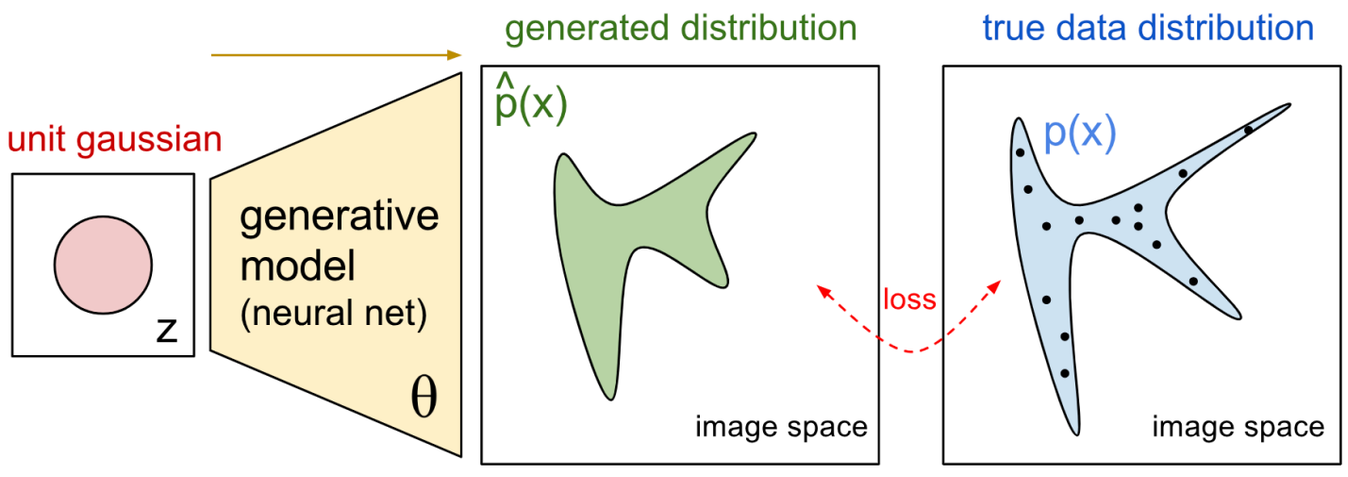
  [Kingma & Welling, ICLR 2014; Rezende et al., ICML 2014]

- [ ] **Generative Adversarial Networks** (GANs)

  [Goodfellow et al., NeurIPS 2014]

- [ ] **Normalizing Flows**

  [Tabak & Turner, 2013; Rezende & Mohamed, ICML 2015; Dinh et al., ICLR 2017]



https://blog.openai.com/generative-models/

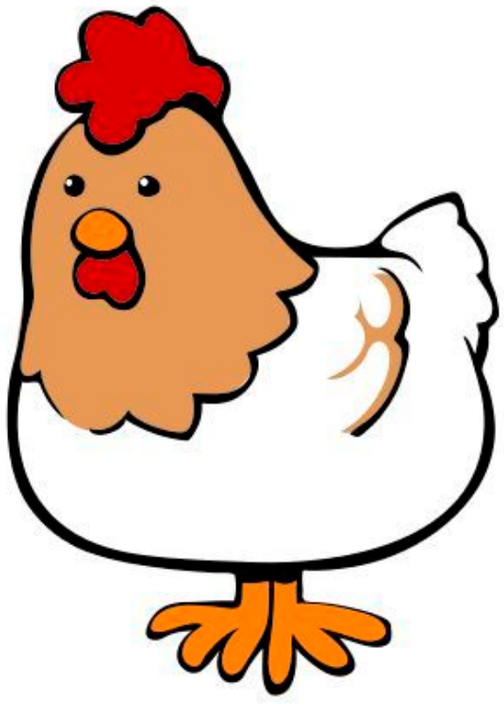
[Kingma & Dhariwal, NeurIPS 2018]

Other Applications

- [ ] **Active Learning**
- [ ] **Semi-Supervised Learning**
- [ ] **Generation and Super-Resolution**
- [ ] **Simulation for Reinforcement Learning**
- [ ] **Transfer Learning**
- [ ] **Open-Set Recognition**

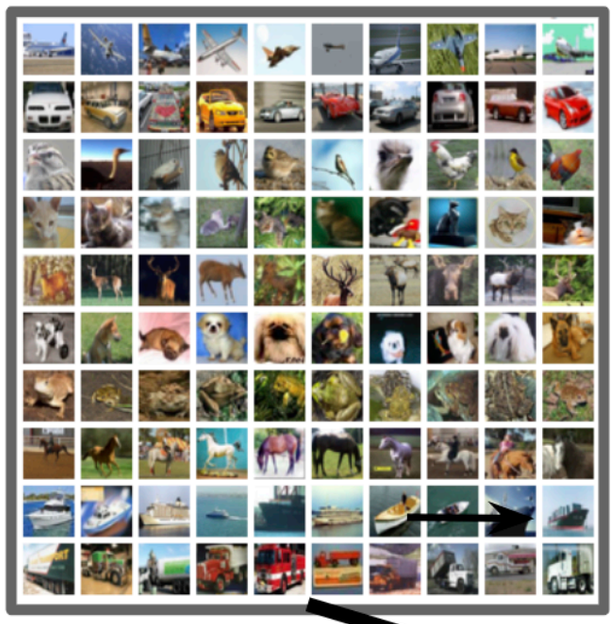
EXPERIMENT

Chicken or Seven?



Chicken or Seven?: Simulating the Scenario

Training: *CIFAR-10*



Testing: *SVHN*

GENERATIVE MODEL

$$p(\mathbf{x}_{\text{CIFAR-10}}) \overset{?}{>} p(\mathbf{x}_{\text{SVHN}})$$

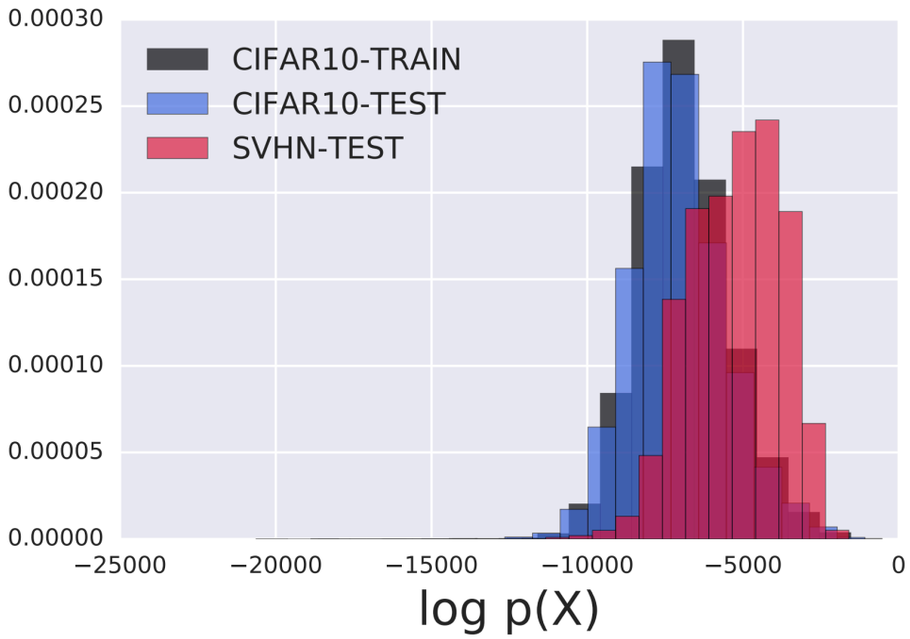
Experiment: CIFAR-10 vs SVHN

Variational Autoencoder

PixelCNN

Glow (Normalizing Flow)

Experiment: CIFAR-10 vs SVHN



Variational Autoencoder

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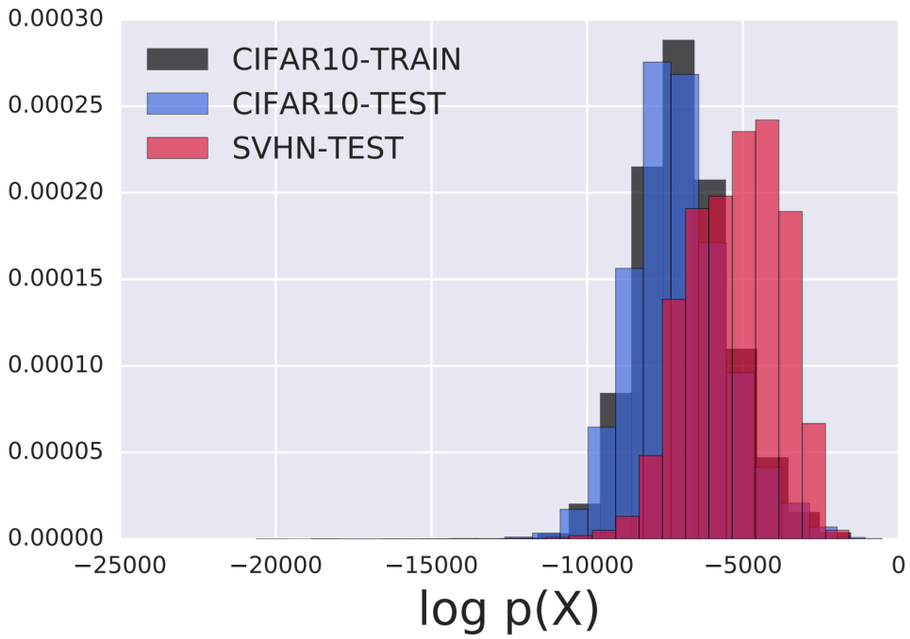
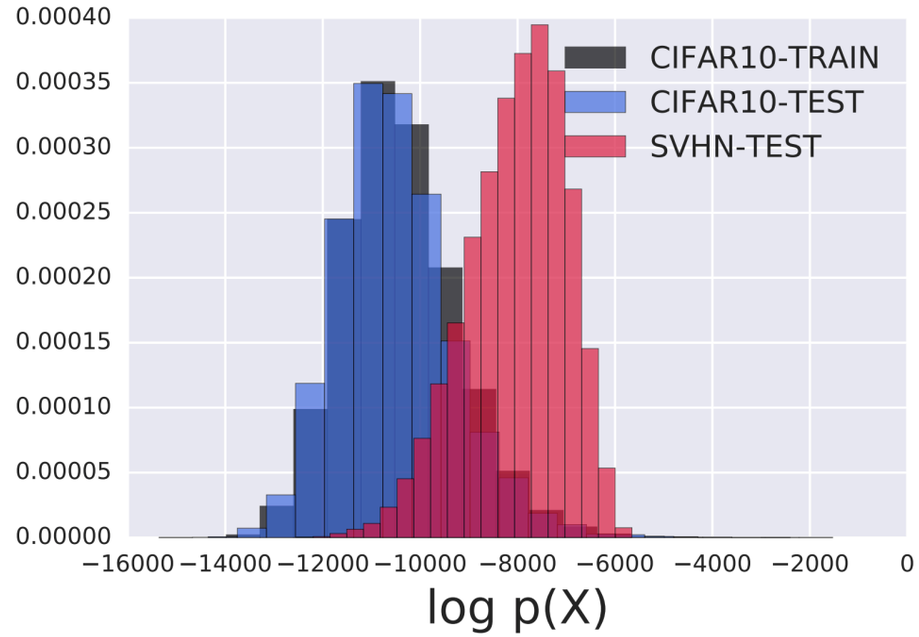


Variational Autoencoder



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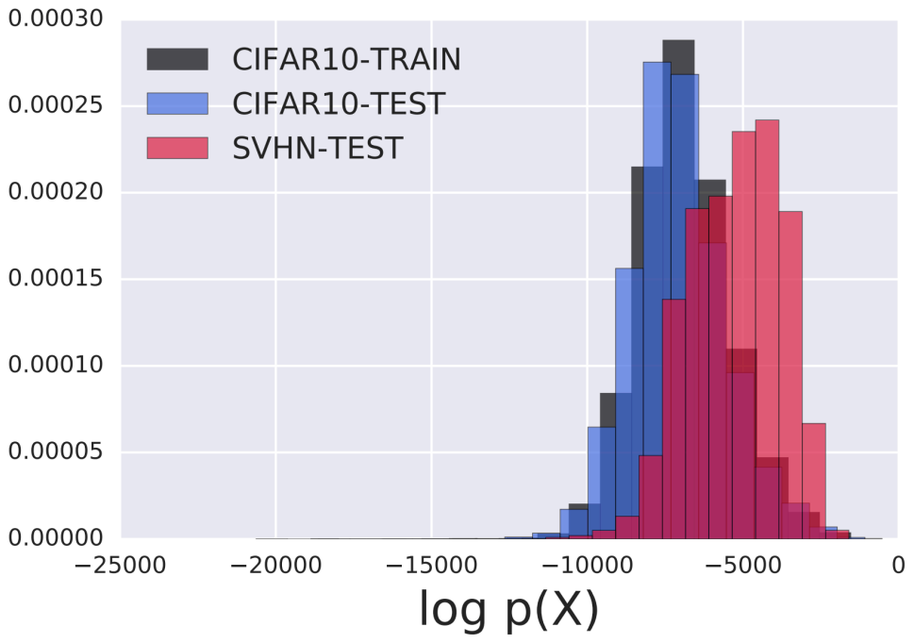

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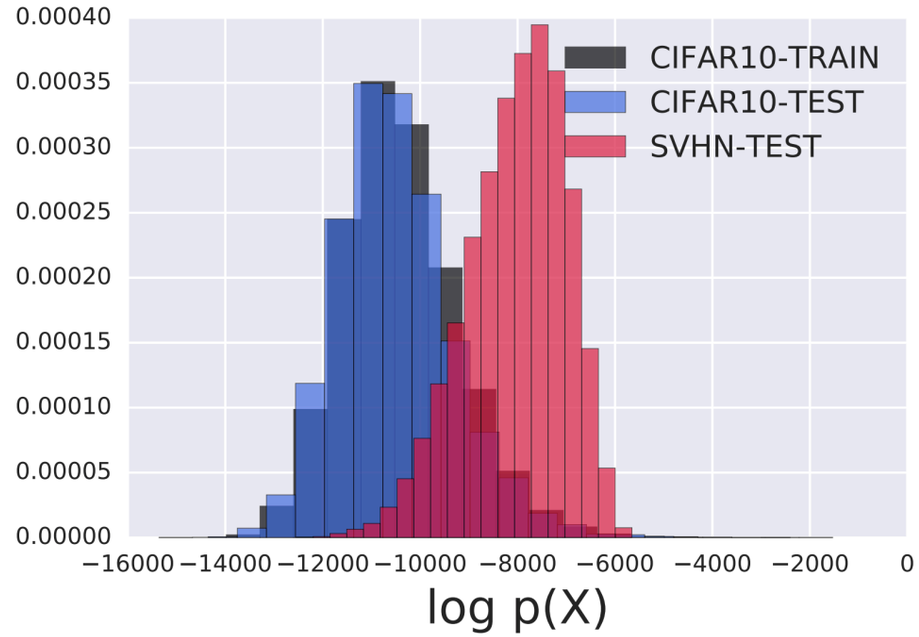

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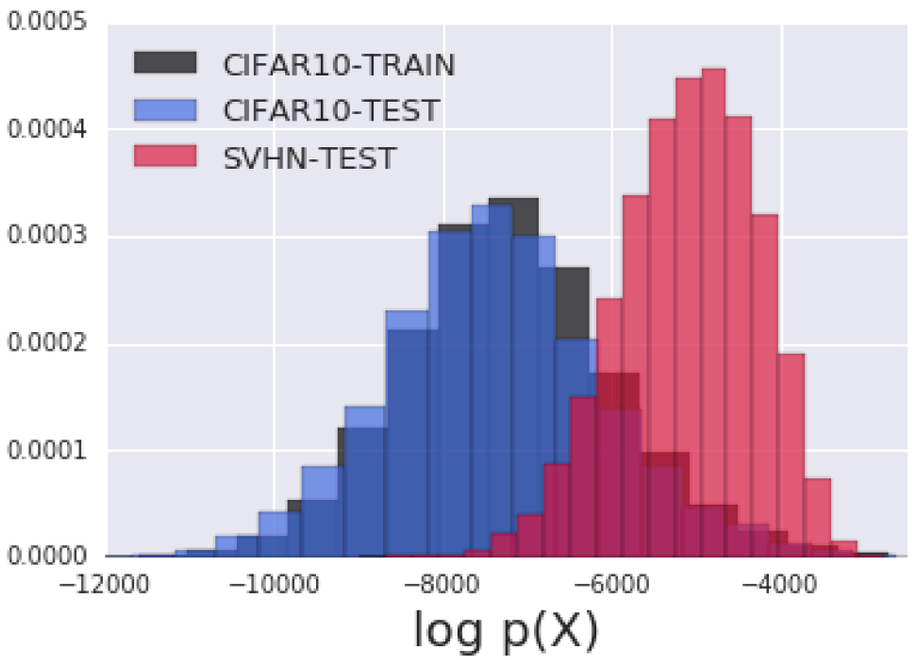

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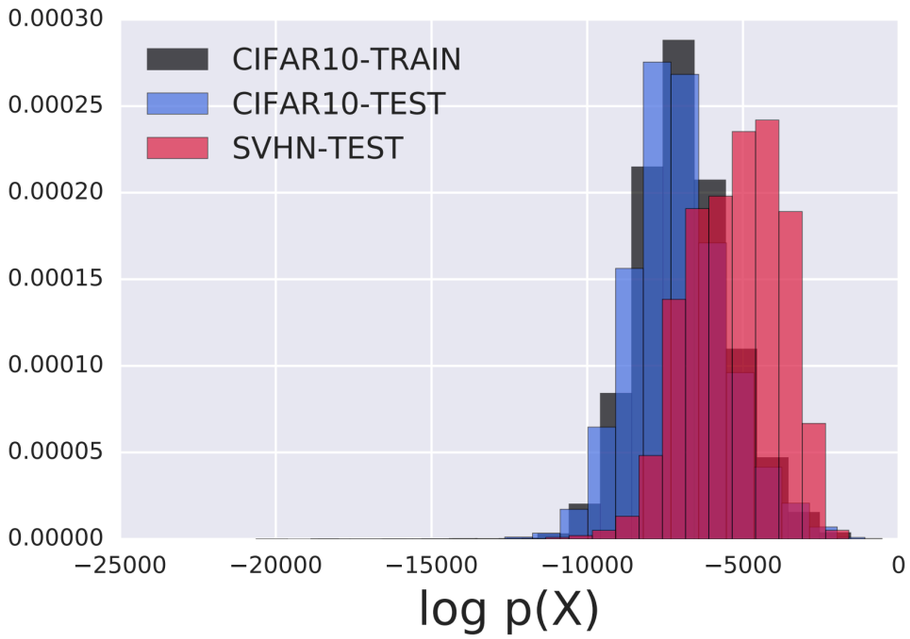

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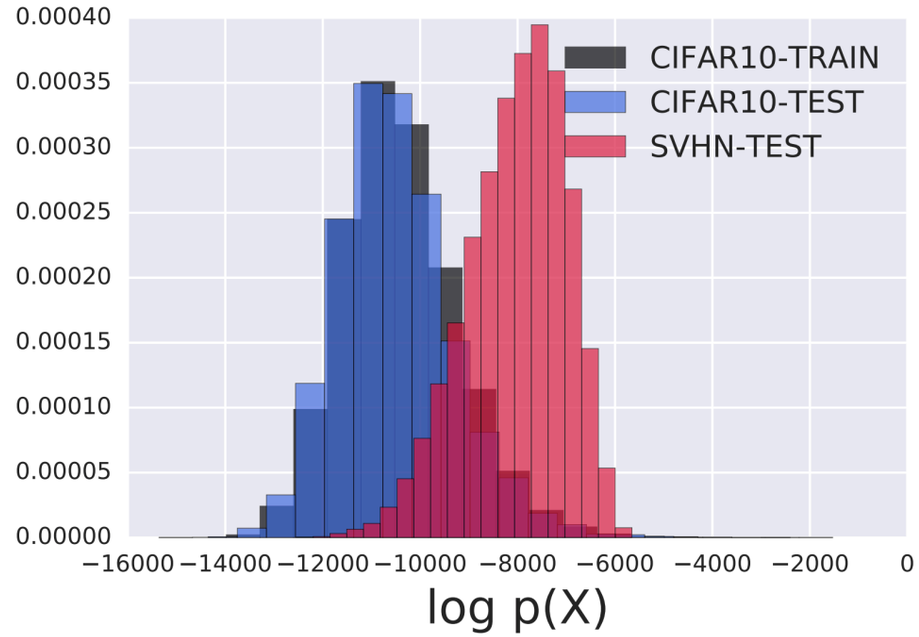
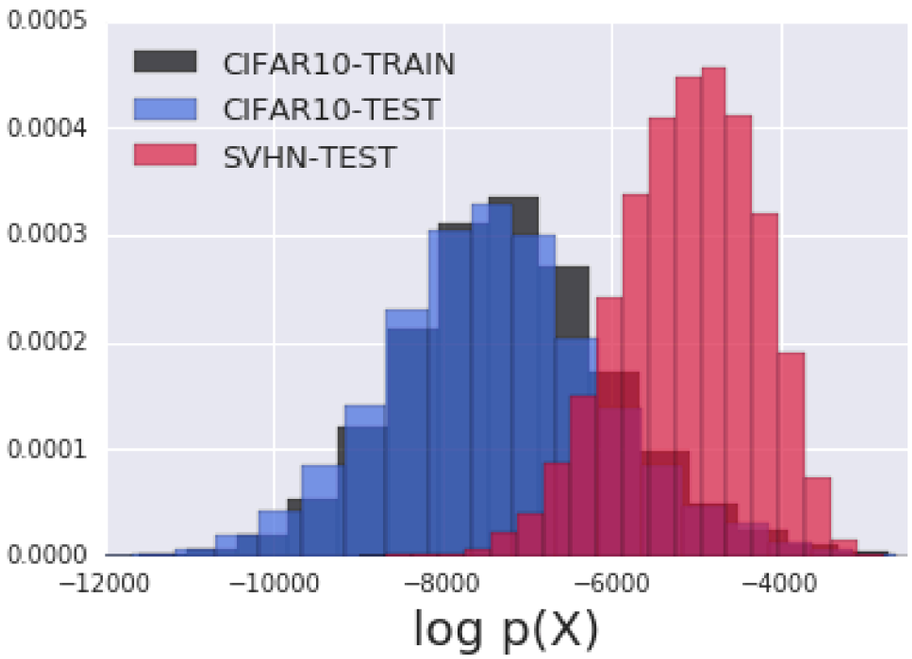


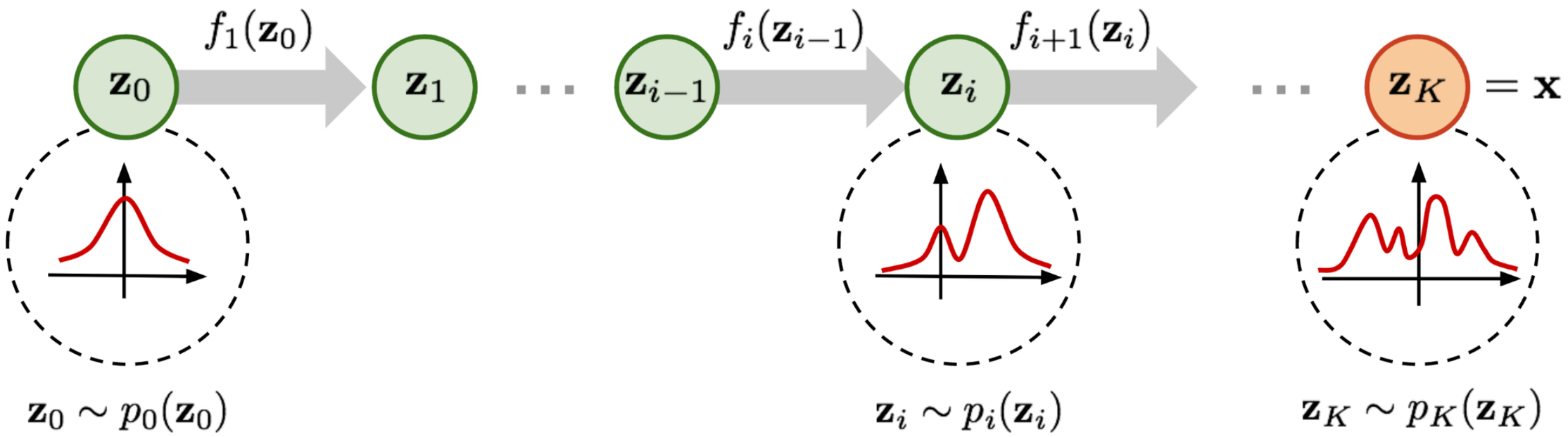
PixelCNN

- Exact likelihood calculations
- Additional structure (e.g. invertibility)



Glow (Normalizing Flow)

Digging Deeper into Flow-Based Models



Deriving Flow-Based Generative Models

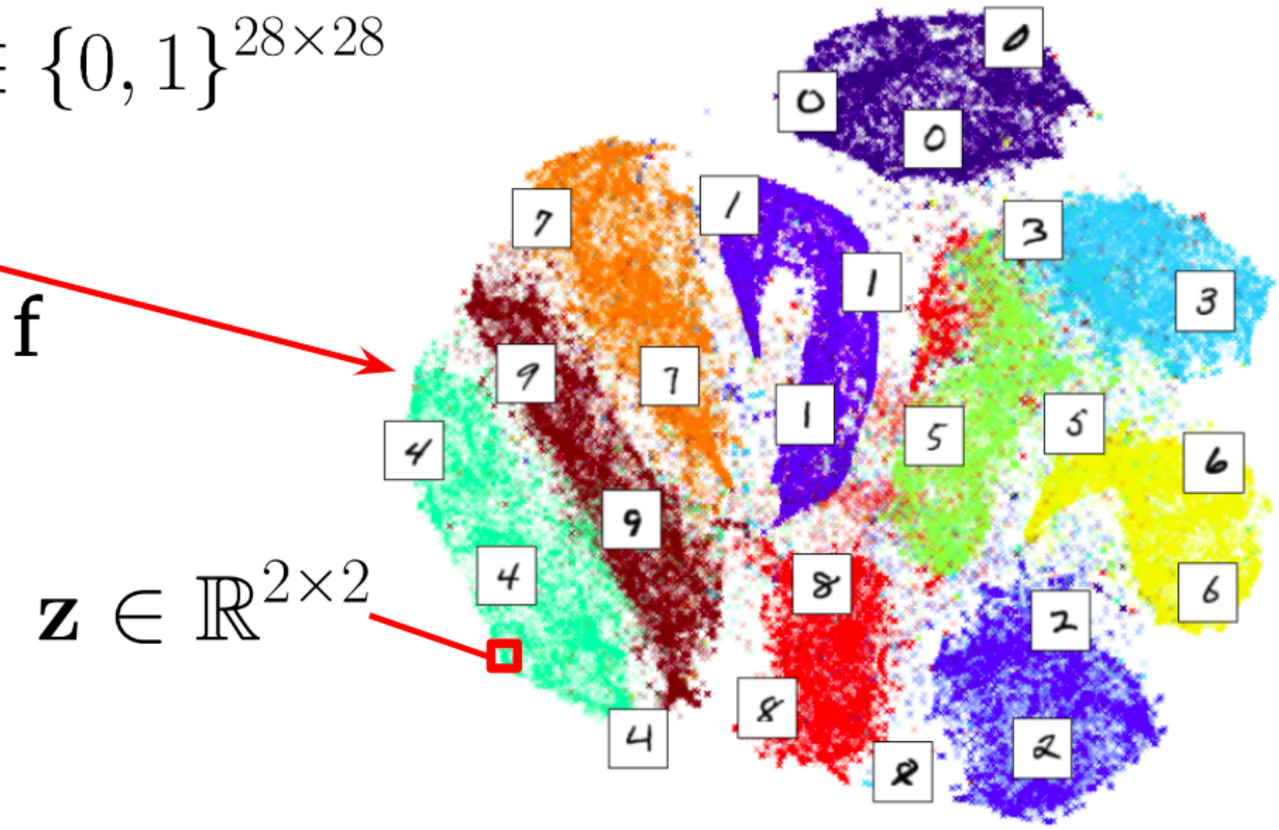
Latent variable models allow data to be represented in some auxiliary space that hopefully better captures the underlying semantics.

MNIST Dataset

$\mathbf{x} \in \{0, 1\}^{28 \times 28}$

$\mathbf{f}$

$\mathbf{z} \in \mathbb{R}^{2 \times 2}$



MNIST Latent Space

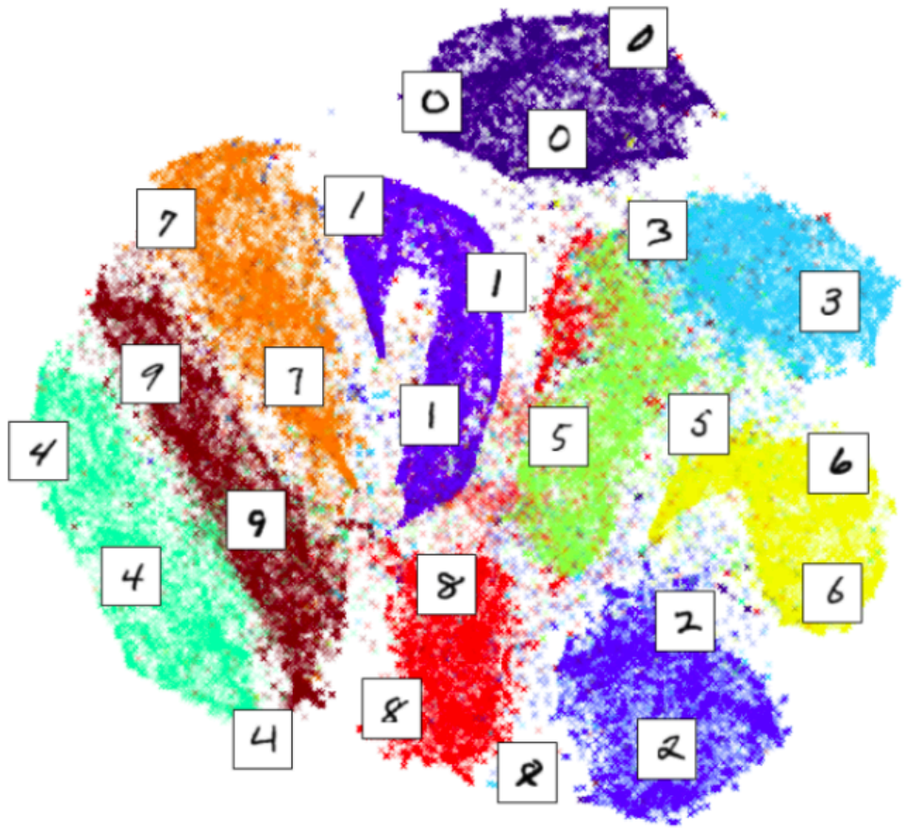
Deriving Flow-Based Generative Models

Latent variable models allow data to be represented in some auxiliary space that hopefully better captures the underlying semantics.

Most latent variable models define this latent / auxiliary space in terms of joint probabilities:

$$p(\mathbf{x}) = \int p(\mathbf{x}, \mathbf{z}) \, d\mathbf{z} = \int p(\mathbf{x}|\boldsymbol{\mu} = f(\mathbf{z})) \, p(\mathbf{z}) \, d\mathbf{z}$$

Examples:
(Probabilistic) *Principal Components Analysis* (PCA)
Deep Belief Networks
Topic Models (LDA)
Variational Autoencoders (VAEs)



Deriving Flow-Based Generative Models

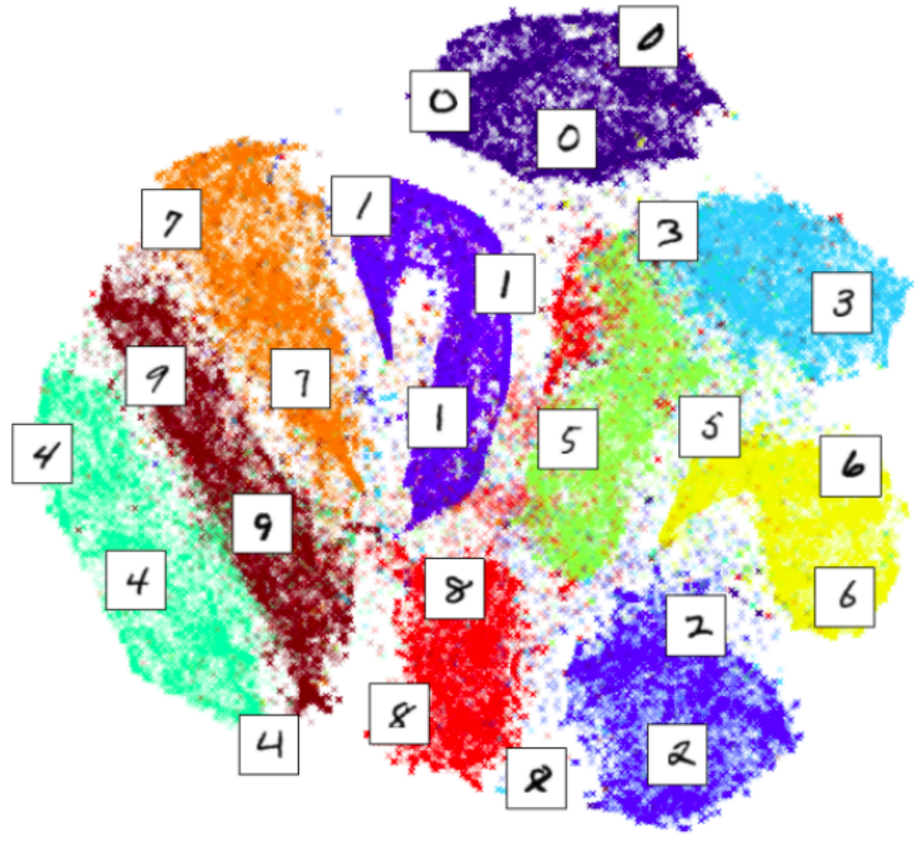
Latent variable models allow data to be represented in some auxiliary space that hopefully better captures the underlying semantics.

However, that's not the only way to define a latent variables model. We can define ***X*** as the transformation itself:

$$p(\mathbf{x}|\boldsymbol{\mu} = f(\mathbf{z})) \qquad \mathbf{x} = f(\mathbf{z})$$

Examples:

Standard Normal Transform (i.e. **(x-μ)/σ**)
Independent Component Analysis (ICA)
Generative Adversarial Networks (GANs)
Copulas
Flow-Based Generative Models



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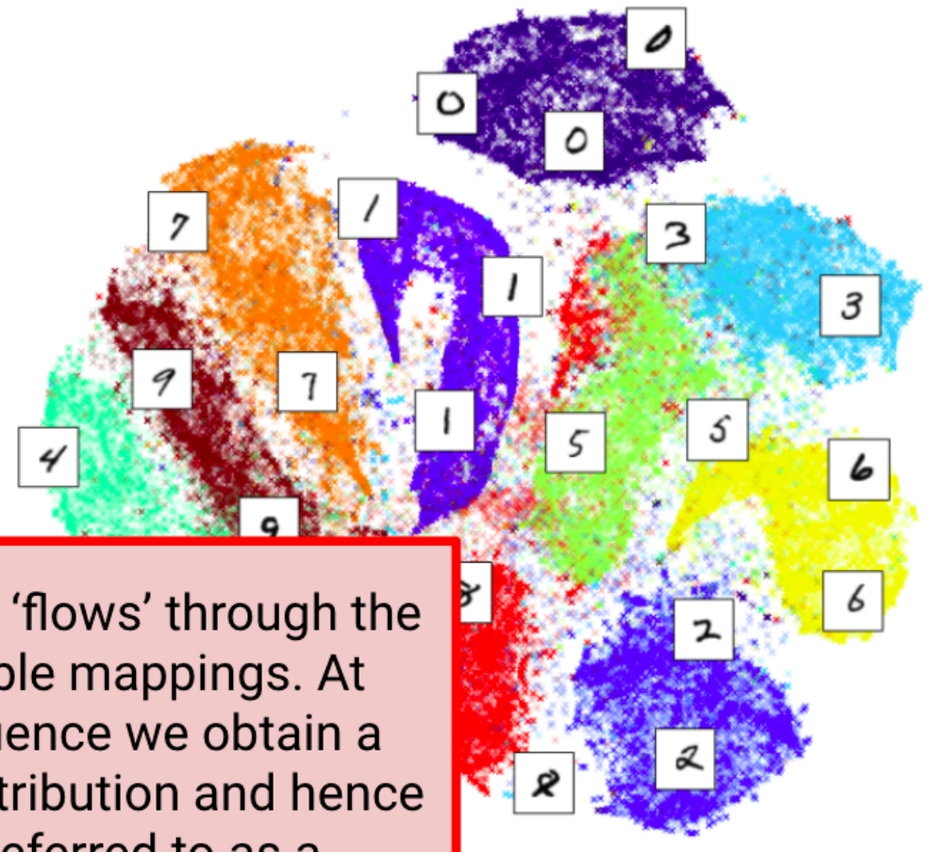

"...the initial density 'flows' through the sequence of invertible mappings. At the end of this sequence we obtain a valid probability distribution and hence this type of flow is referred to as a normalizing flow." [Rezende & Mohamed, 2016]

Deriving Flow-Based Generative Models

Definition of a probability density function (PDF):

$$\frac{d}{dZ}P(Z) = p(Z)$$

Deriving Flow-Based Generative Models

Definition of a probability density function (PDF):

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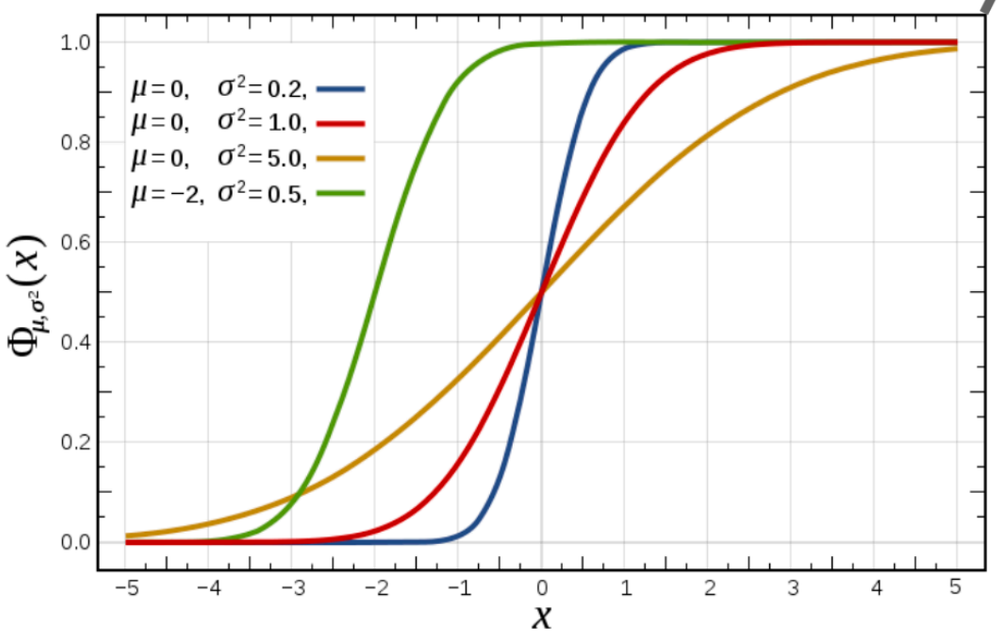
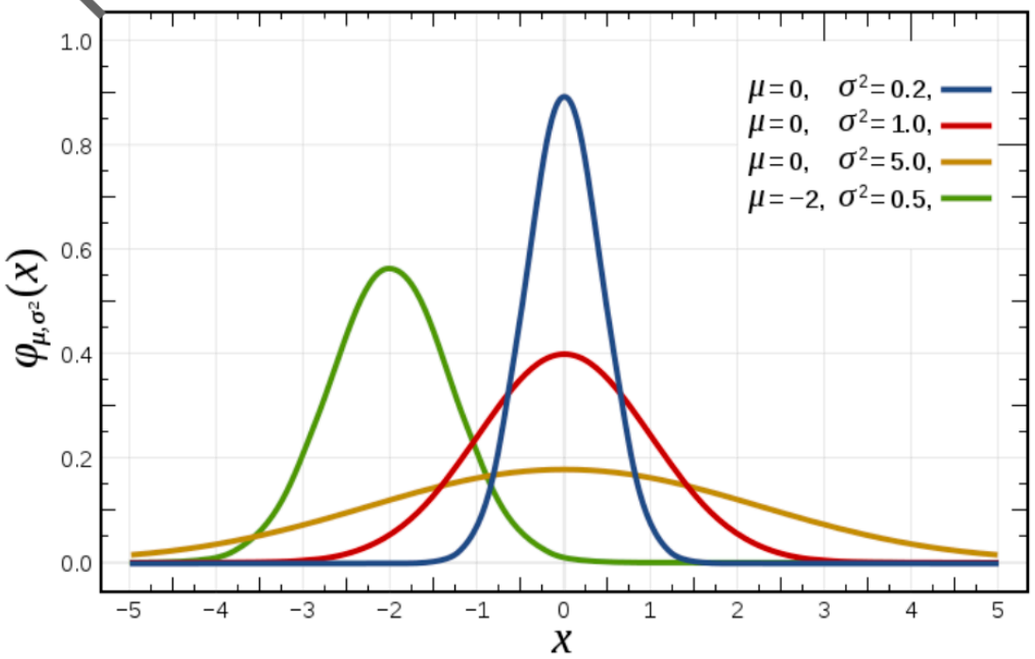
Gaussian CDF



Gaussian PDF



Deriving Flow-Based Generative Models

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Consider defining Z by a transformation of another variable X:

$$Z = f(X)$$

Deriving Flow-Based Generative Models

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Consider defining Z by a transformation of another variable X:

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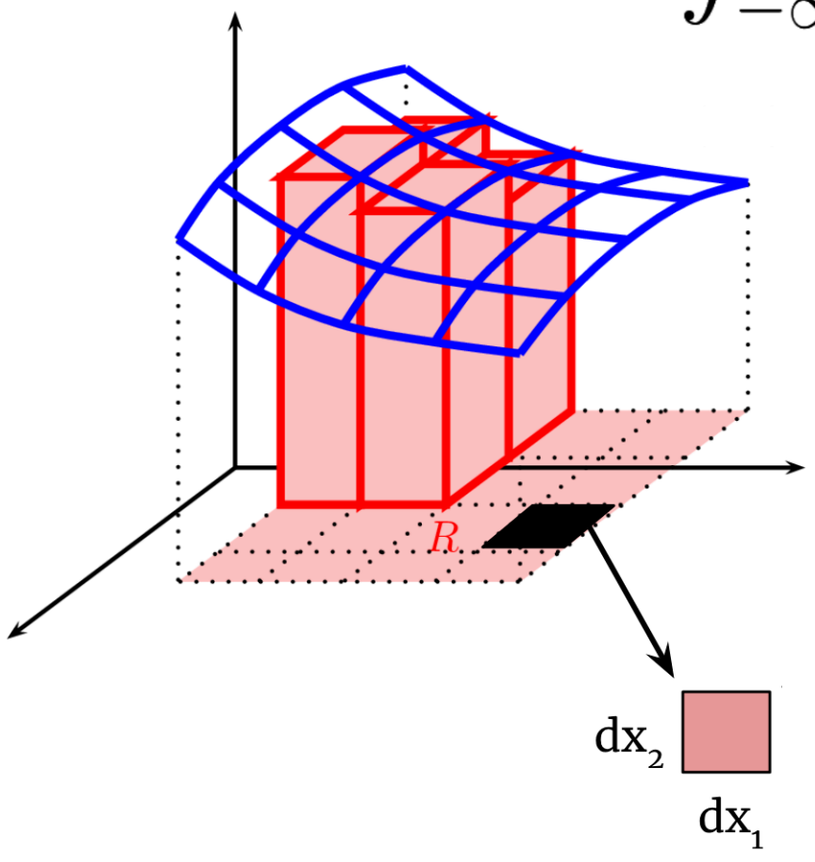
Change of Variables Formula (X → Z):

$$\frac{d}{dX}P_z(f(X)) = p_z(f(X))\left|\frac{df(X)}{dX}\right| = p(X)$$

Deriving Flow-Based Generative Models

$$P(\mathbf{x}_0) = \int_{-\infty}^{\mathbf{x}_0} p(\mathbf{x})\ d\mathbf{x}$$



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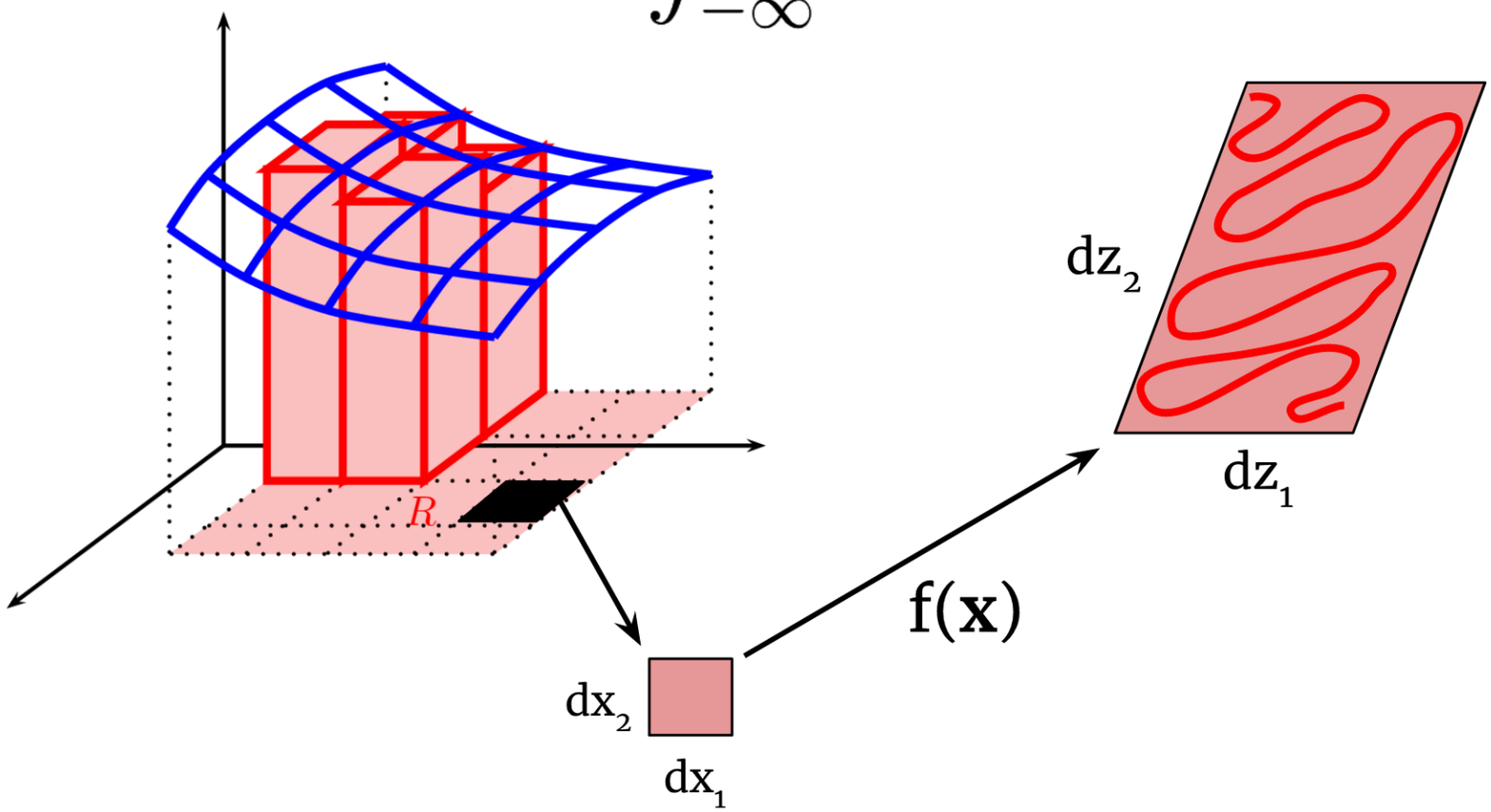
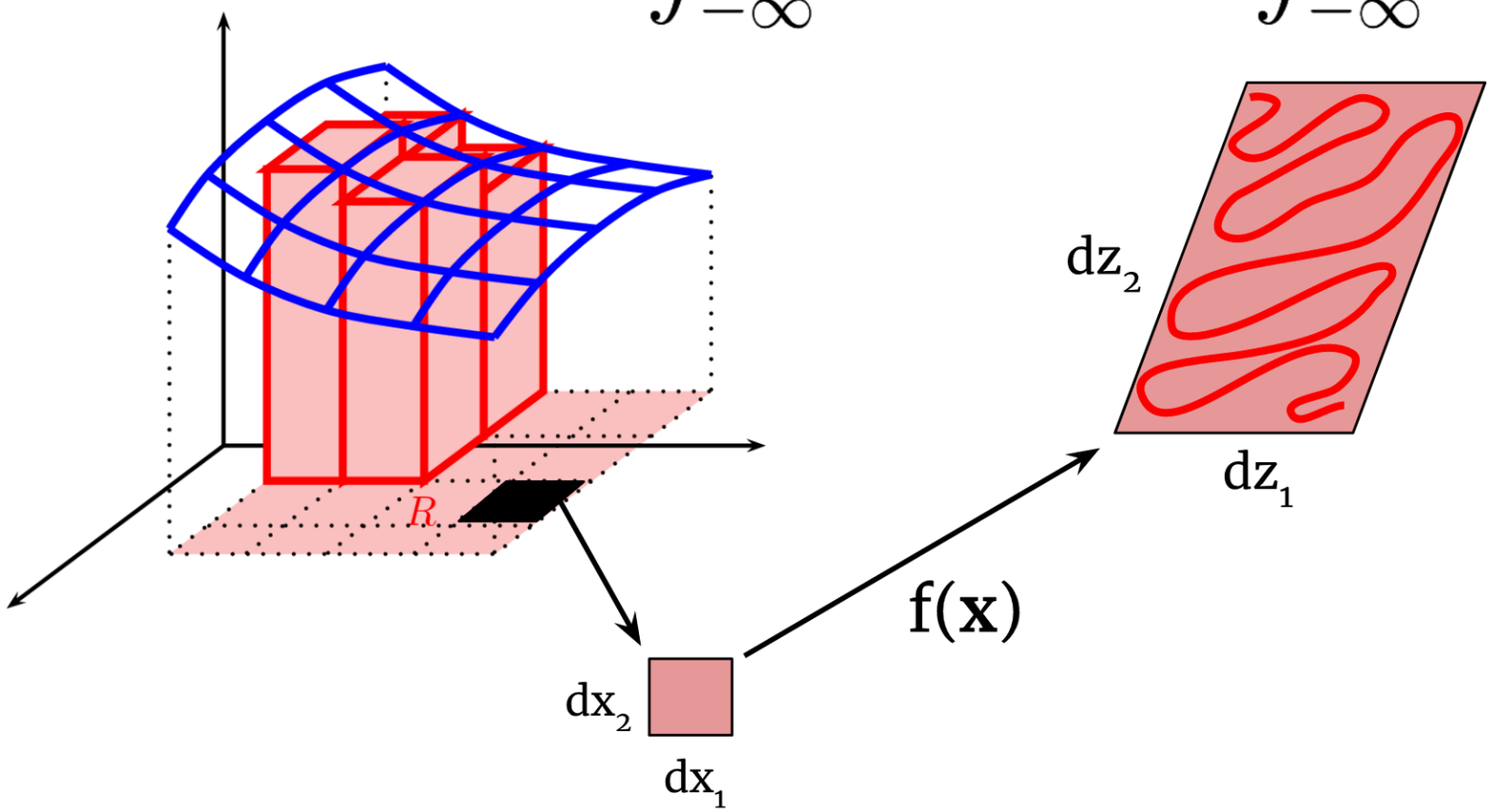


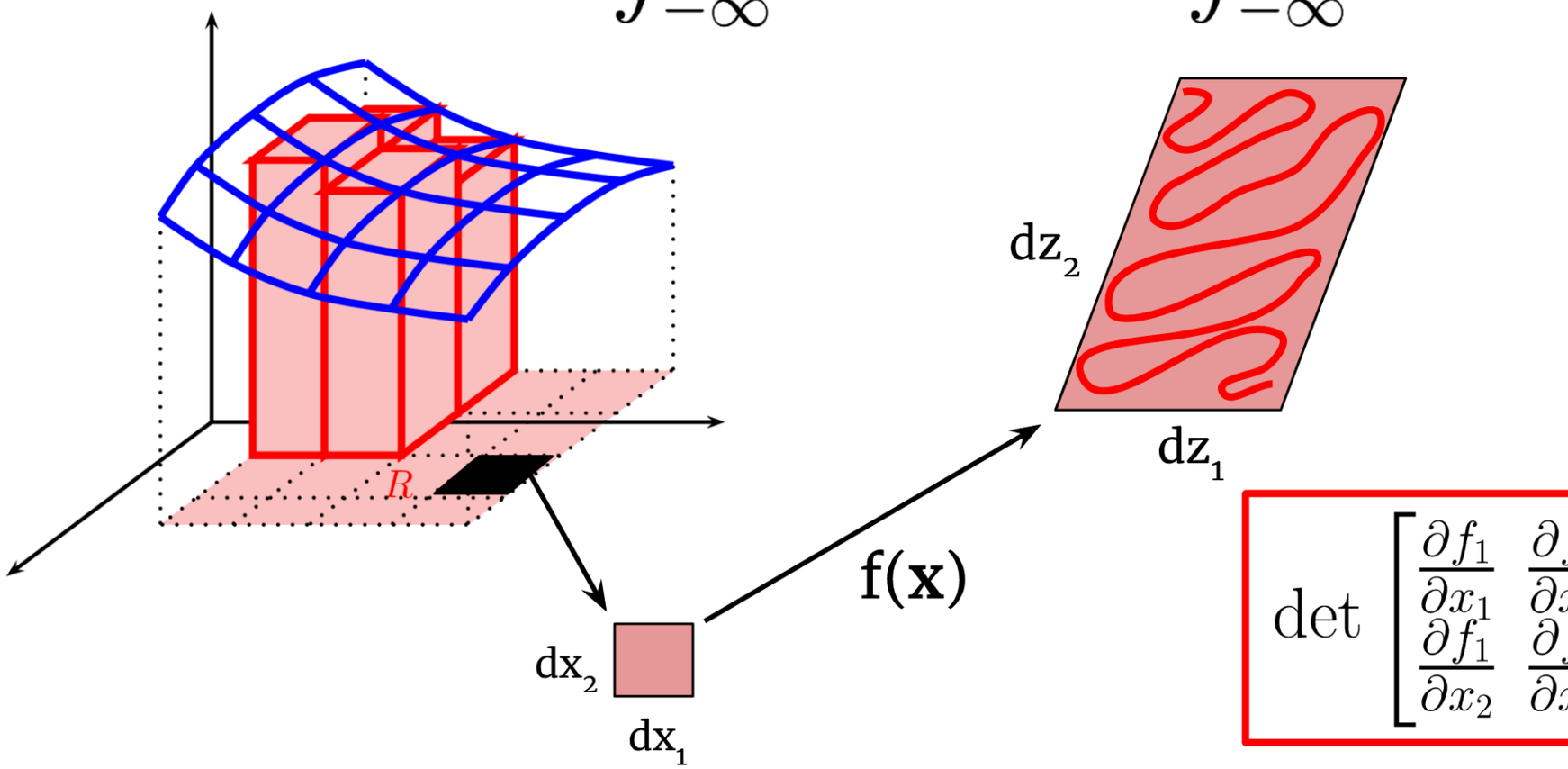
Deriving Flow-Based Generative Models

$$P(\mathbf{x}_0) = \int_{-\infty}^{\mathbf{x}_0} p(\mathbf{x})\, d\mathbf{x} = \int_{-\infty}^{\mathbf{x}_0} p_Z(f(\mathbf{x})) \underbrace{\left|\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right|}_{\text{Volume Correction}} d\mathbf{x}$$



Deriving Flow-Based Generative Models

$$P(\mathbf{x}_0) = \int_{-\infty}^{\mathbf{x}_0} p(\mathbf{x})\, d\mathbf{x} = \int_{-\infty}^{\mathbf{x}_0} p_Z(f(\mathbf{x})) \underbrace{\left|\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right|}_{\text{Volume Correction}} d\mathbf{x}$$



$$\det \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_2}{\partial x_1} \\ \frac{\partial f_1}{\partial x_2} & \frac{\partial f_2}{\partial x_2} \end{bmatrix} = \frac{\partial f_1}{\partial x_1}\frac{\partial f_2}{\partial x_2} - \frac{\partial f_2}{\partial x_1}\frac{\partial f_1}{\partial x_2}$$

Deriving Flow-Based Generative Models

Change of Variables Formula (X → Z):

$$p_z(f(X)) \left| \frac{df(X)}{dX} \right| = p(X)$$

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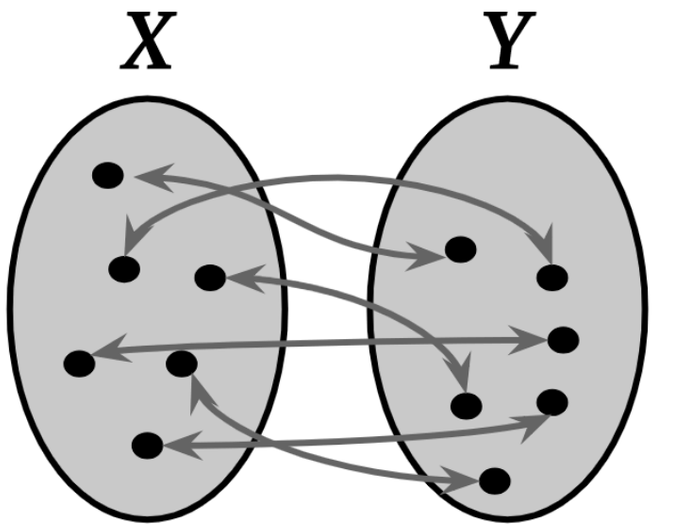
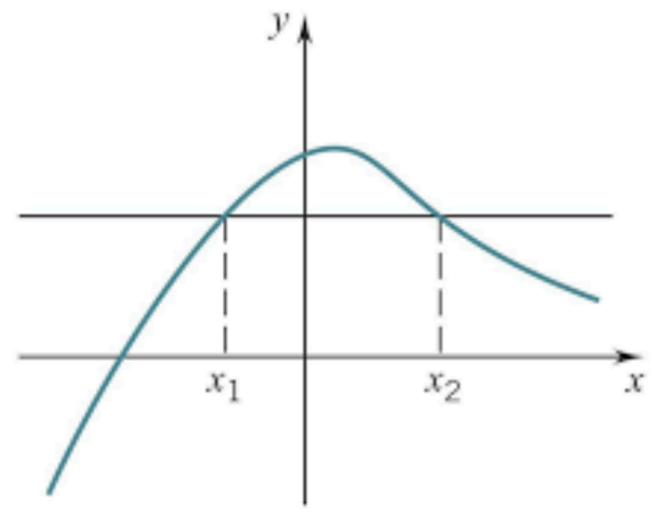
So what's the catch?

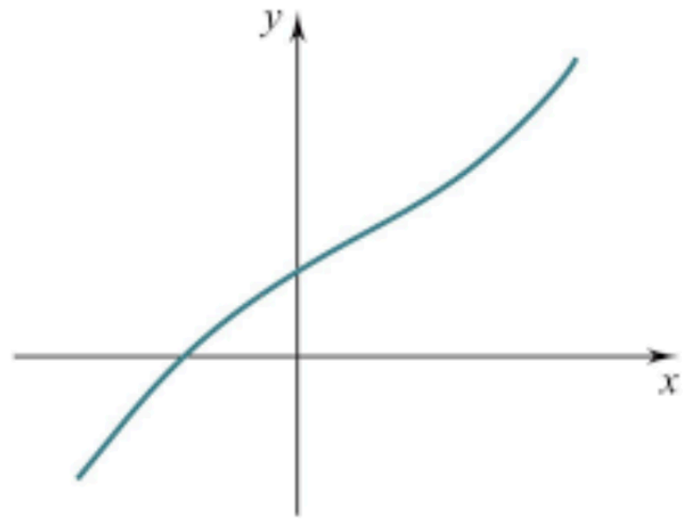
f(**x**) must be a *bijection*



$\mathbf{x} = \mathbf{f}^{-1}(\mathbf{y})$ $\mathbf{y} = \mathbf{f}(\mathbf{x})$



f is not one-to-one: $f(x_1) = f(x_2)$



f is one-to-one:

Deriving Flow-Based Generative Models

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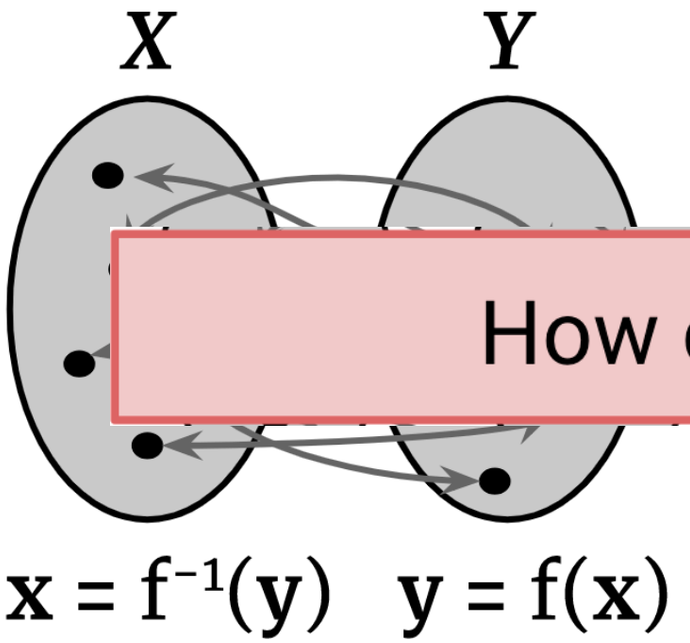
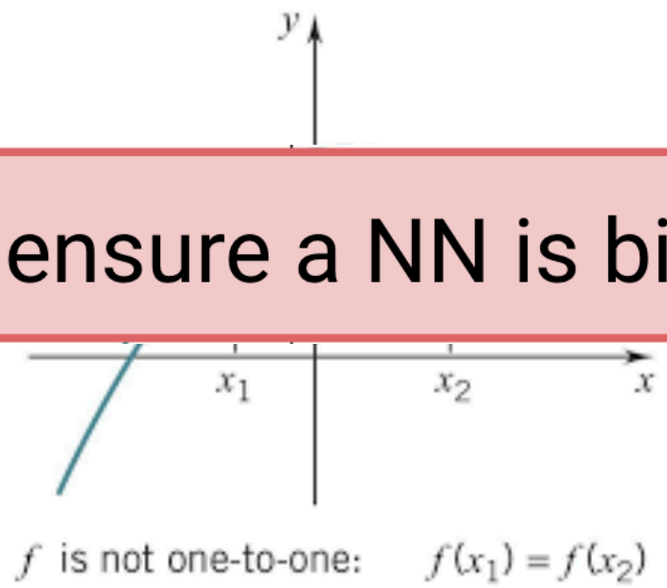
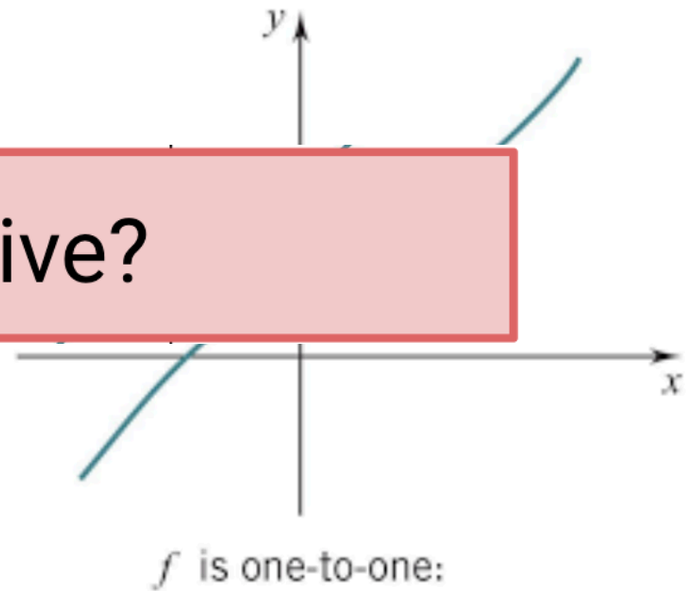


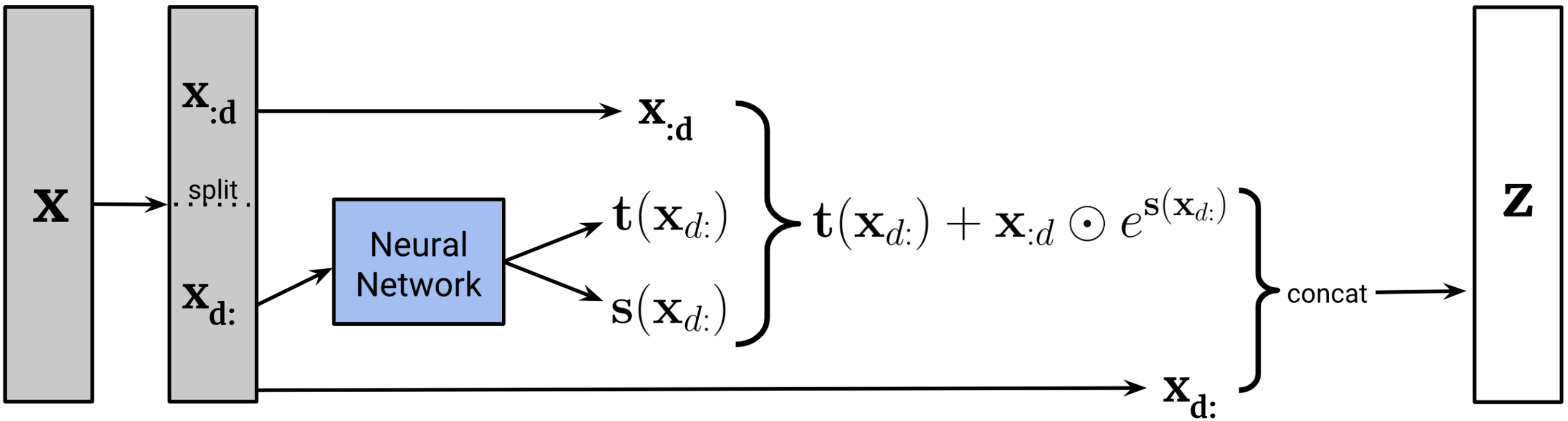
How can we ensure a NN is bijective?





The Affine Coupling Layer

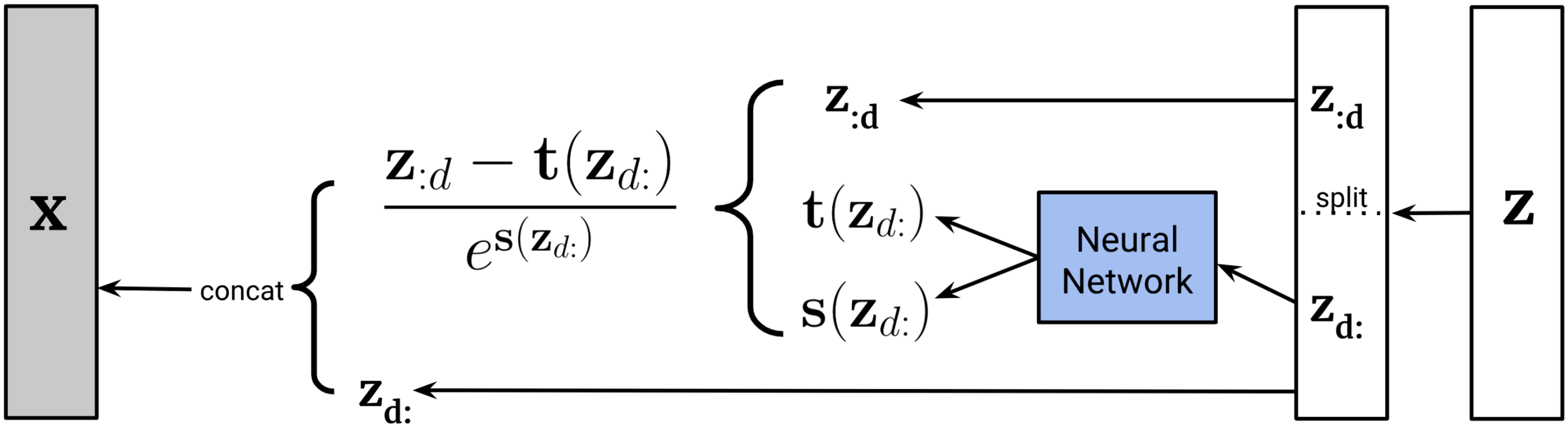
[Dinh et al., ICLR 2017]



$$\mathbf{z} = \left[\mathbf{t}(\mathbf{x}_{d:}) + \mathbf{x}_{:d} \odot e^{\mathbf{s}(\mathbf{x}_{d:})}, \mathbf{x}_{d:}\right]$$

The Affine Coupling Layer: Inverse

[Dinh et al., ICLR 2017]



$$\mathbf{x} = \left[\left(\mathbf{z}_{:d} - \mathbf{t}(\mathbf{z}_{d:}) \right) \odot e^{-\mathbf{s}(\mathbf{z}_{d:})}, \mathbf{z}_{d:} \right]$$

The Affine Coupling Layer: Volume Change

[Dinh et al., ICLR 2017]

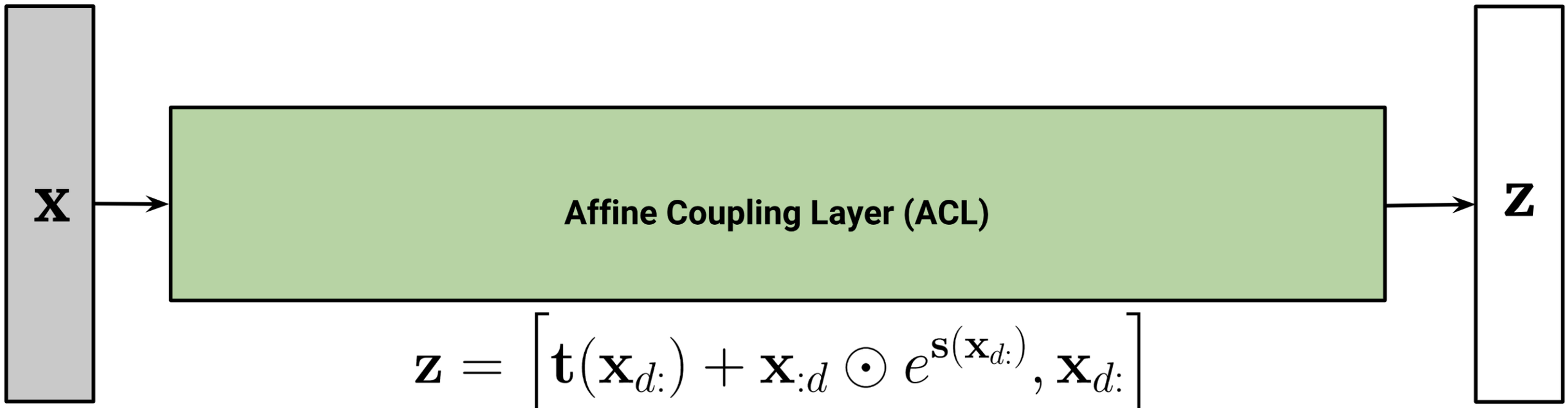
$$\mathbf{z} = \left[\mathbf{t}(\mathbf{x}_{d:}) + \mathbf{x}_{:d} \odot e^{\mathbf{s}(\mathbf{x}_{d:})}, \mathbf{x}_{d:}\right]$$

$$\begin{matrix} & \mathbf{Z}_{:d} & \mathbf{Z}_{d:} \\ \frac{\partial \mathbf{f}}{\partial \mathbf{x}} = & \mathbf{X}_{:d} & \begin{bmatrix} e^{s(\mathbf{x}_{d:})} & \mathbf{0} \\ \frac{\partial f(\mathbf{x}_{:})}{\partial \mathbf{x}_{d:}} & \mathbf{1} \end{bmatrix} \\ & \mathbf{X}_{d:} & \end{matrix}$$

$$\left|\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right| = \prod_j e^{s_j(\mathbf{x}_{d:})} \cdot 1 = e^{\sum_j s_j(\mathbf{x}_{d:})}$$

The Affine Coupling Layer: Stacking

[Dinh et al., ICLR 2017]



$$\mathbf{z} = \left[\mathbf{t}(\mathbf{x}_{d:}) + \mathbf{x}_{:d} \odot e^{\mathbf{s}(\mathbf{x}_{d:})}, \mathbf{x}_{d:}\right]$$

The Affine Coupling Layer: Stacking

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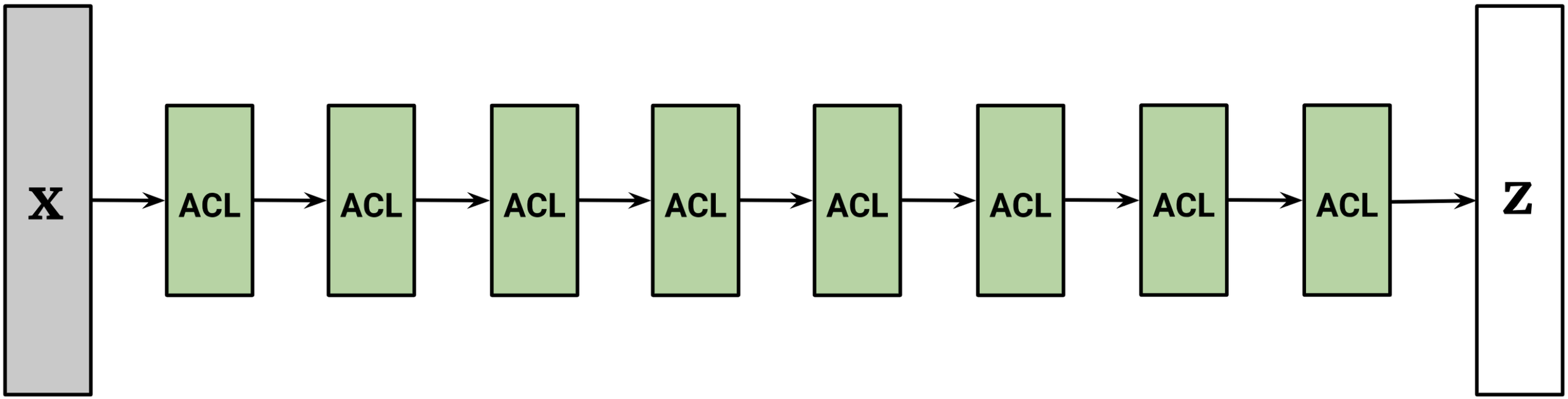
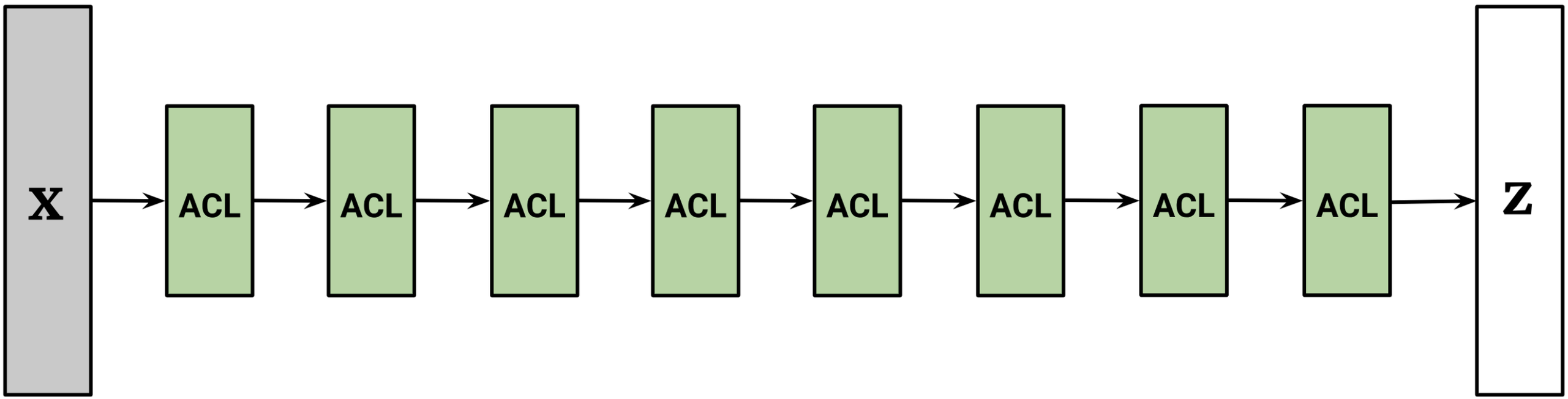


The Affine Coupling Layer: Stacking

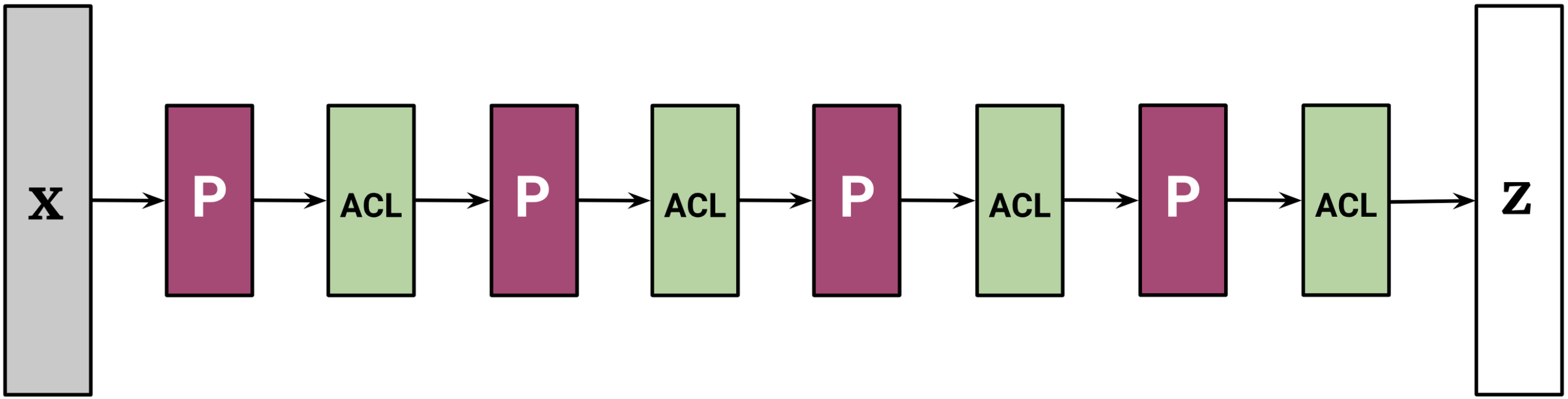
[Dinh et al., ICLR 2017]



Problem!: $\mathbf{x}_{:d} = \mathbf{z}_{:d}$

The Affine Coupling Layer: Stacking

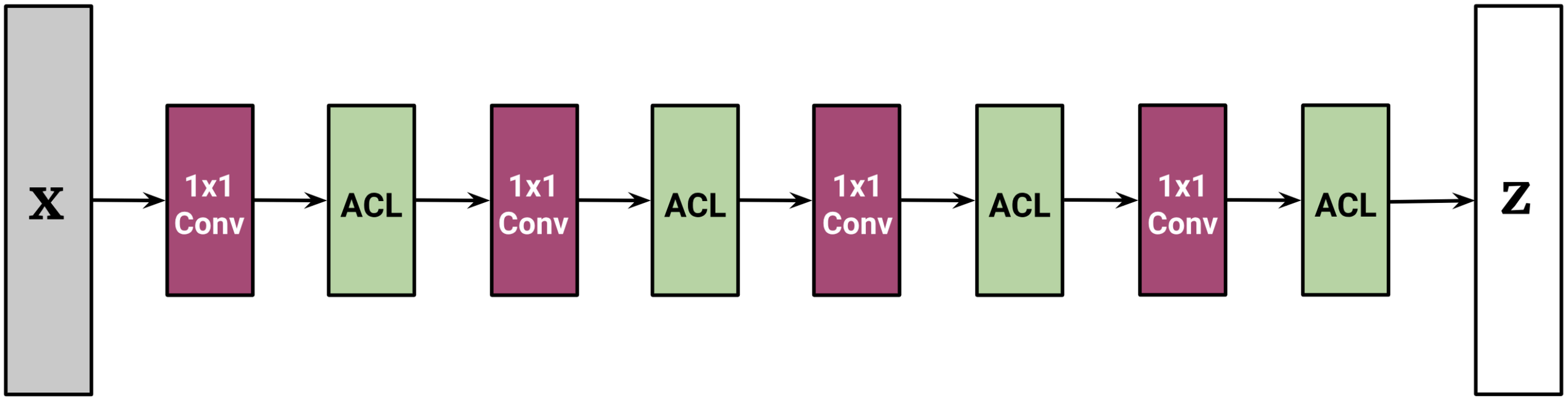
[Dinh et al., ICLR 2017]



Solution: permutation layers

The Affine Coupling Layer: Stacking

[Kingma & Dhariwal, NeurIPS 2018]



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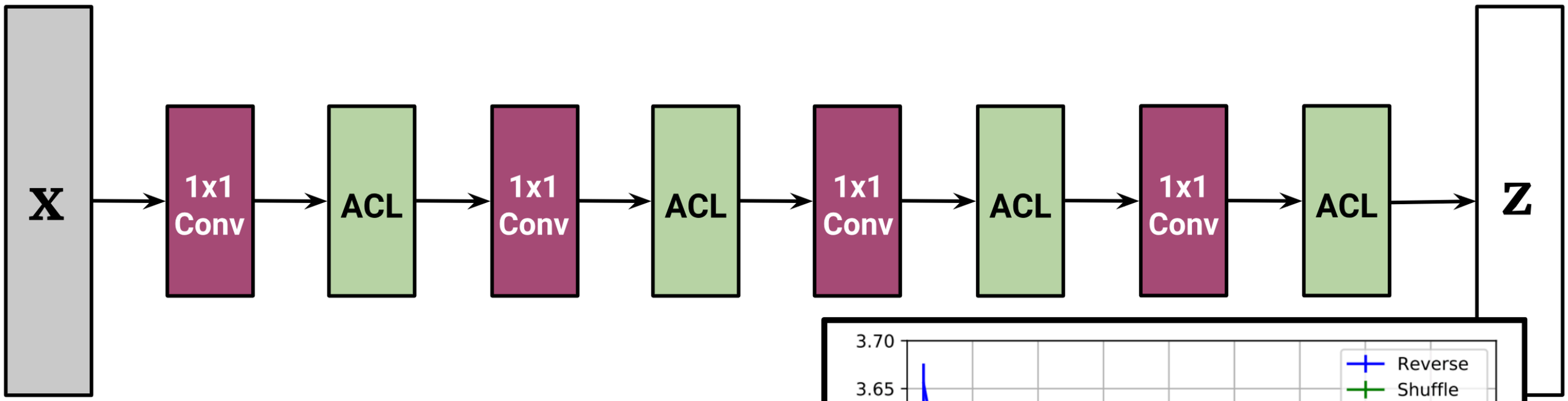
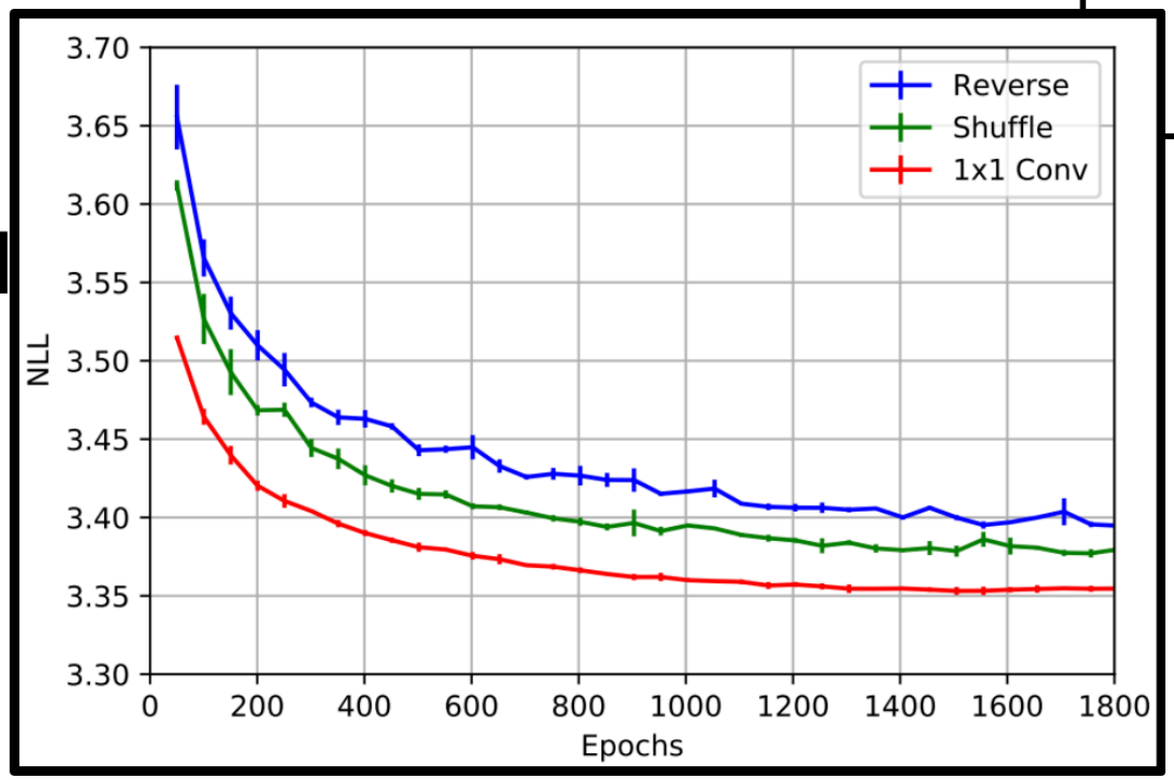
The Affine Coupling Layer: Stacking

[Kingma & Dhariwal, NeurIPS 2018]



Solution: permu

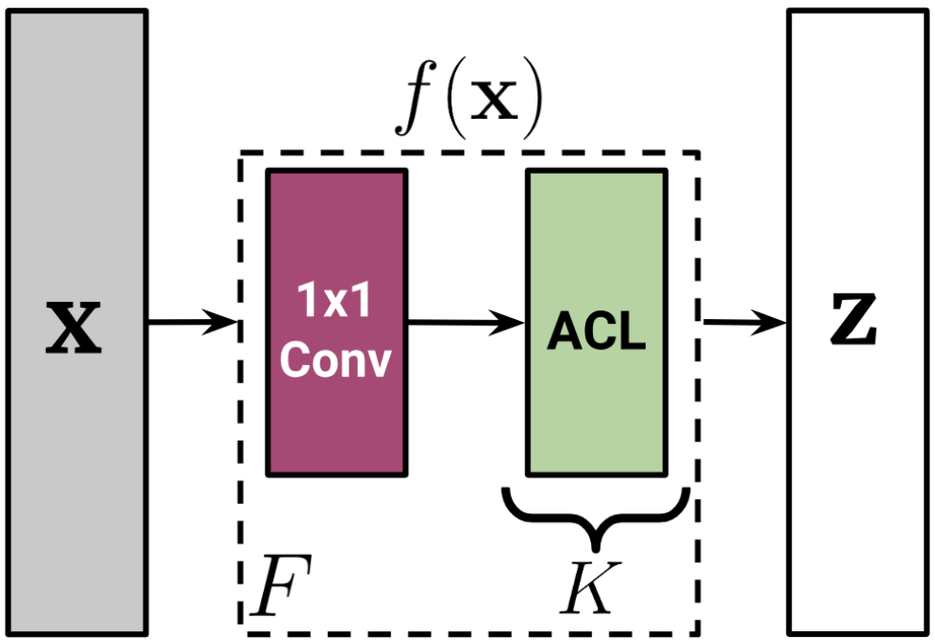


[Kingma & Dhariwal, NeurIPS 2018]

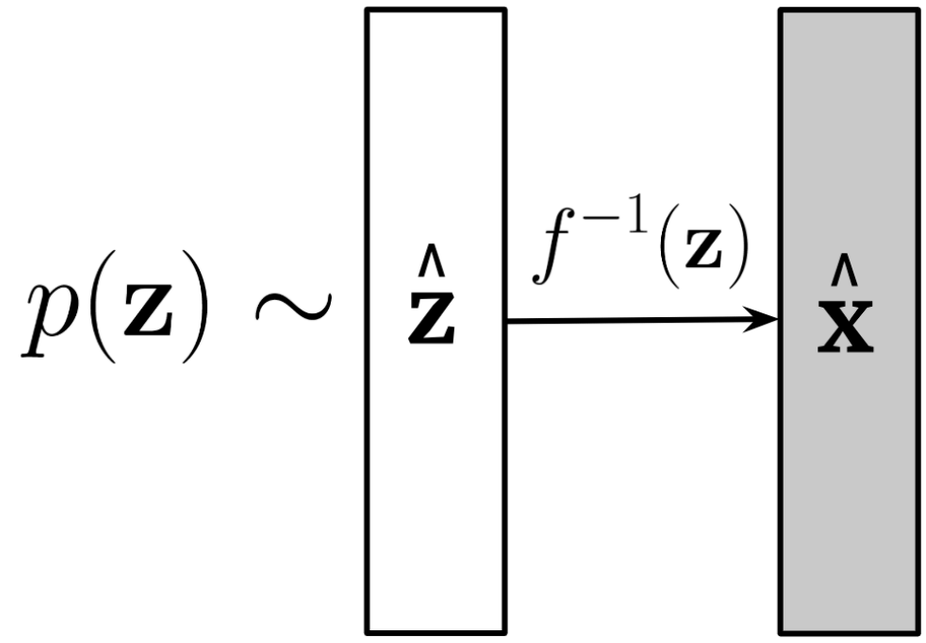
Flow-Based Generative Model

[Kingma & Dhariwal, NeurIPS 2018]

Architecture:



Sampling:



Training Objective:

$$\boldsymbol{\phi}^* = \text{argmax}_{\boldsymbol{\phi}} \sum_{i=1}^{N} \log p(\mathbf{x}_i; \boldsymbol{\phi}) = \text{argmax}_{\boldsymbol{\phi}} \sum_{i=1}^{N} \log p_z(f(\mathbf{x}_i; \boldsymbol{\phi})) + \log \left| \frac{\partial \mathbf{f}_{\boldsymbol{\phi}}}{\partial \mathbf{x}_i} \right|$$

Flow-Based Generative Model

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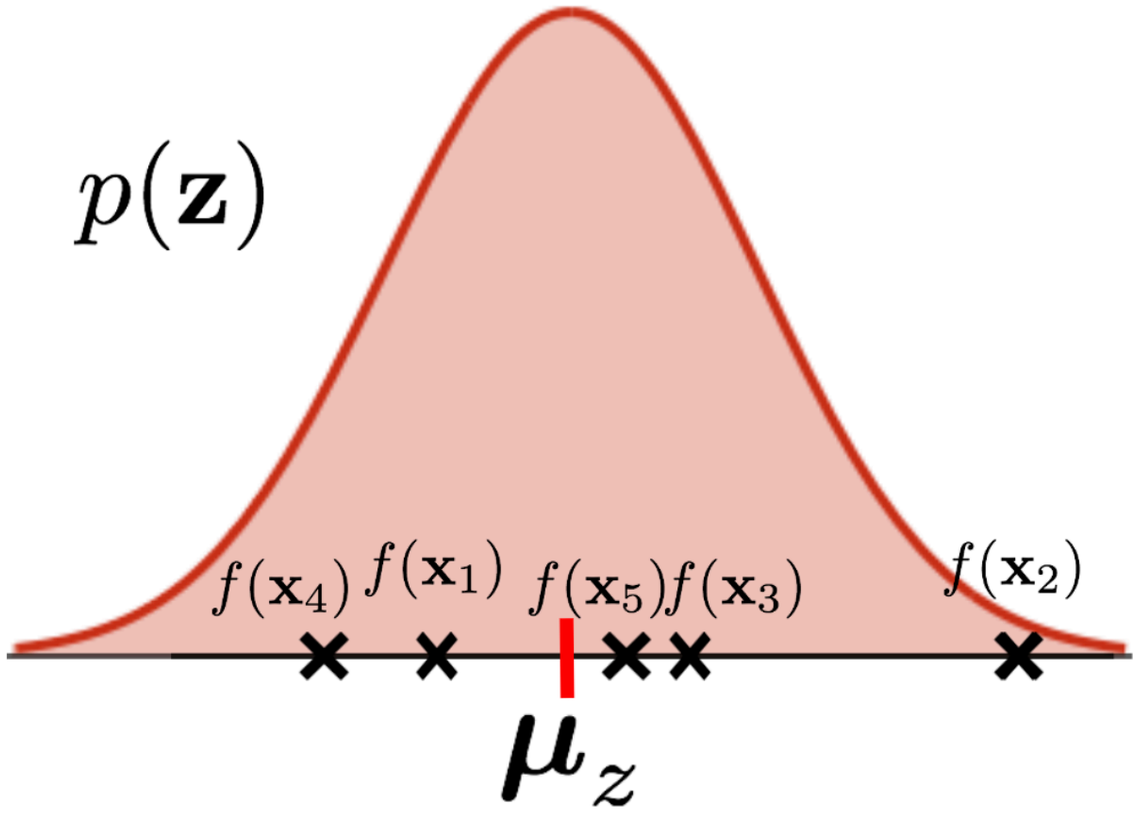
Encourages f(x)'s to distribute according to p(z)



Flow-Based Generative Model

Training Objective:

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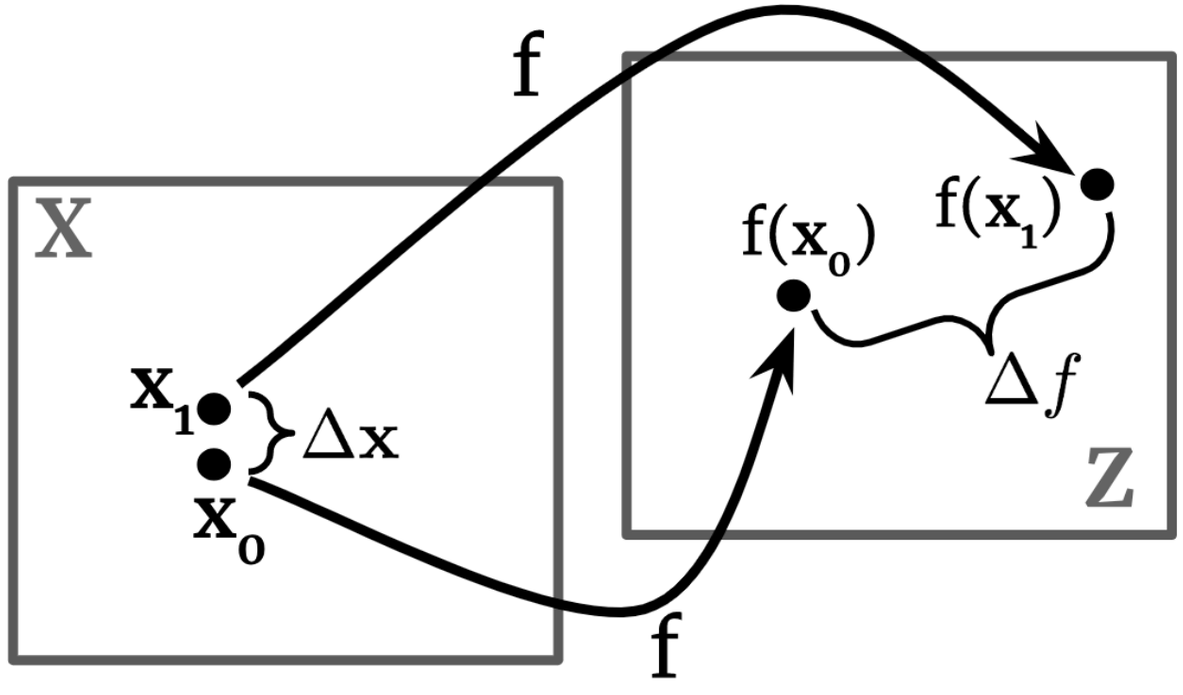

Encourages an increase in the ‘sensitivity’ of f(x)

Flow-Based Generative Model

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$$\boldsymbol{\phi}^* = \text{argmax}_{\boldsymbol{\phi}} \sum_{i=1}^{N} \log p(\mathbf{x}_i; \boldsymbol{\phi}) = \text{argmax}_{\boldsymbol{\phi}} \sum_{i=1}^{N} \log p_z(f(\mathbf{x}_i; \boldsymbol{\phi})) + \underbrace{\log \left| \frac{\partial \mathbf{f}_{\boldsymbol{\phi}}}{\partial \mathbf{x}_i} \right|}$$

Encourages an increase in the 'sensitivity' of f(x)

Penalties on NN derivatives have been used for decades:

- *Regularization Theory and Neural Network Architectures*, Girosi et al., 1990
- *Training with Noise is Equivalent to Tikhonov Regularization*, C. Bishop., 1995
- *Contractive Autoencoders: Explicit Invariance During Feature Extraction*, Rifai et al., 2011

$$\mathcal{R} = \left|\left| \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right|\right|_F^2$$

More Experiments with Flows

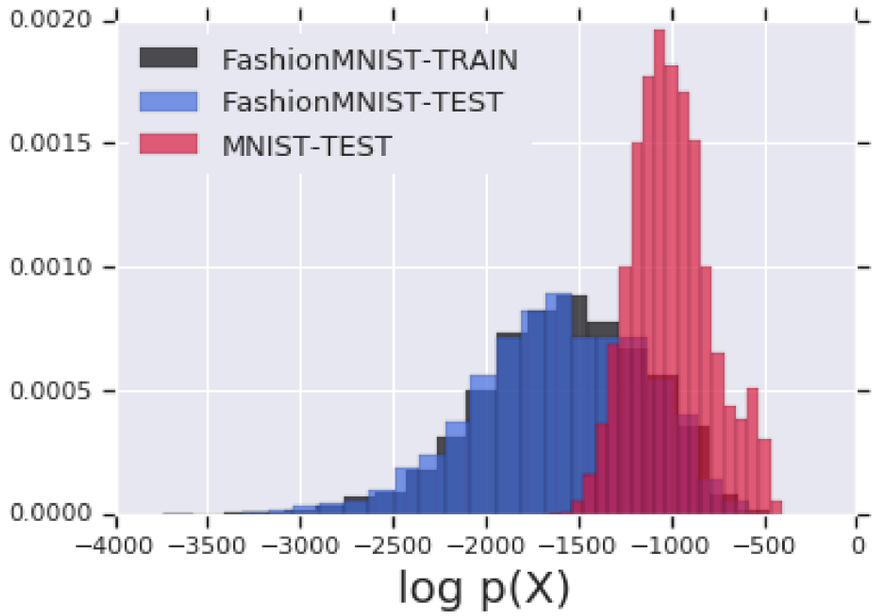
Additional Out-of-Distribution Tests

FashionMNIST vs **MNIST**

CelebA vs **SVHN**

ImageNet vs **CIFAR-10** vs **SVHN**

Additional Out-of-Distribution Tests

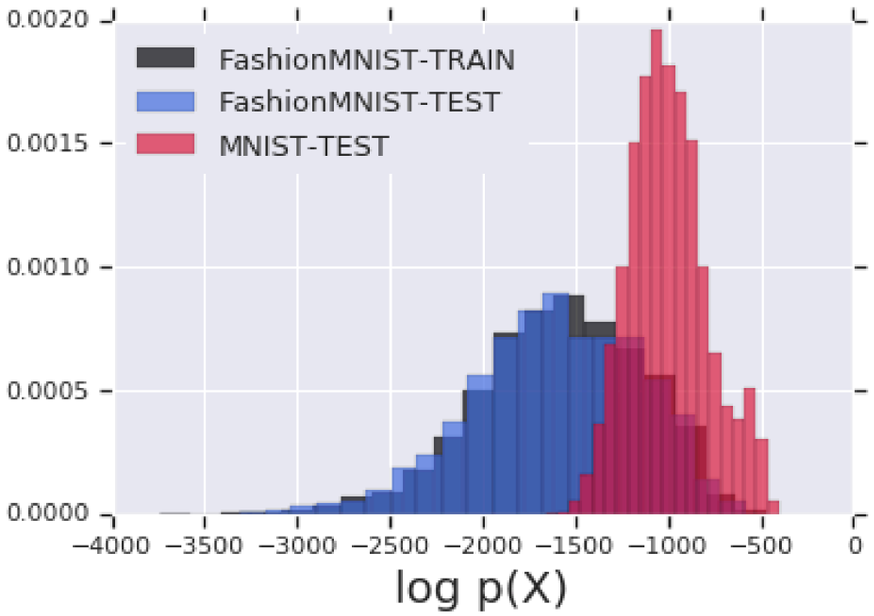


FashionMNIST vs MNIST

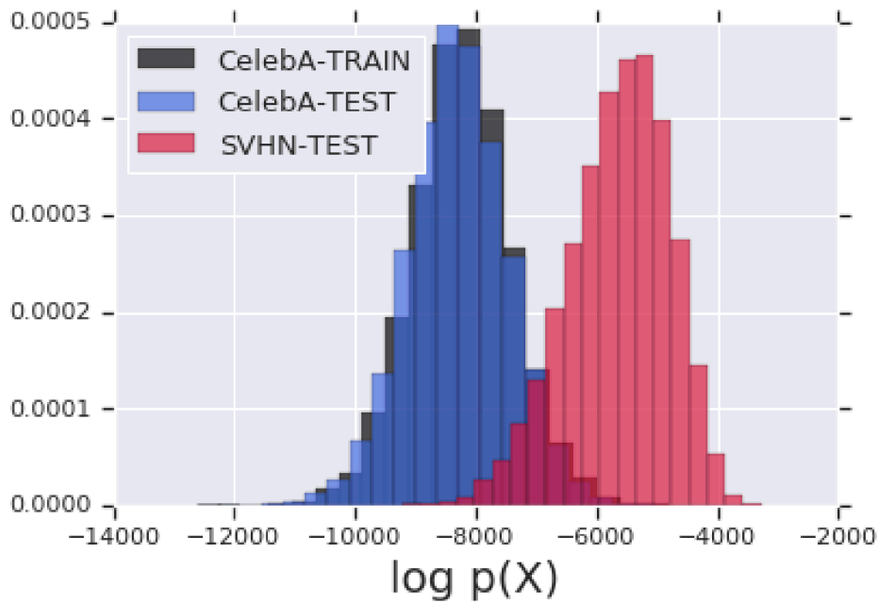
CelebA vs SVHN

ImageNet vs CIFAR-10 vs SVHN

Additional Out-of-Distribution Tests



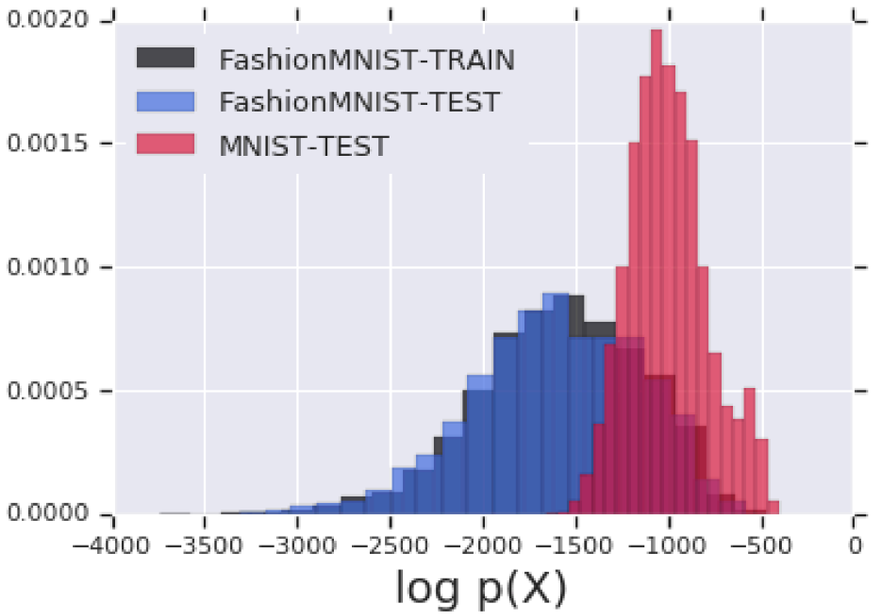
FashionMNIST vs MNIST



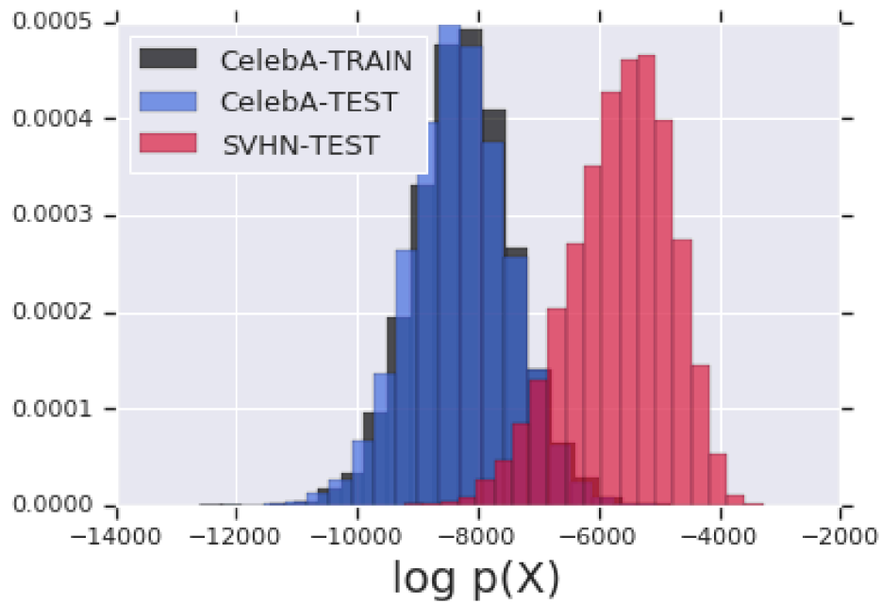
CelebA vs SVHN

ImageNet vs CIFAR-10 vs SVHN

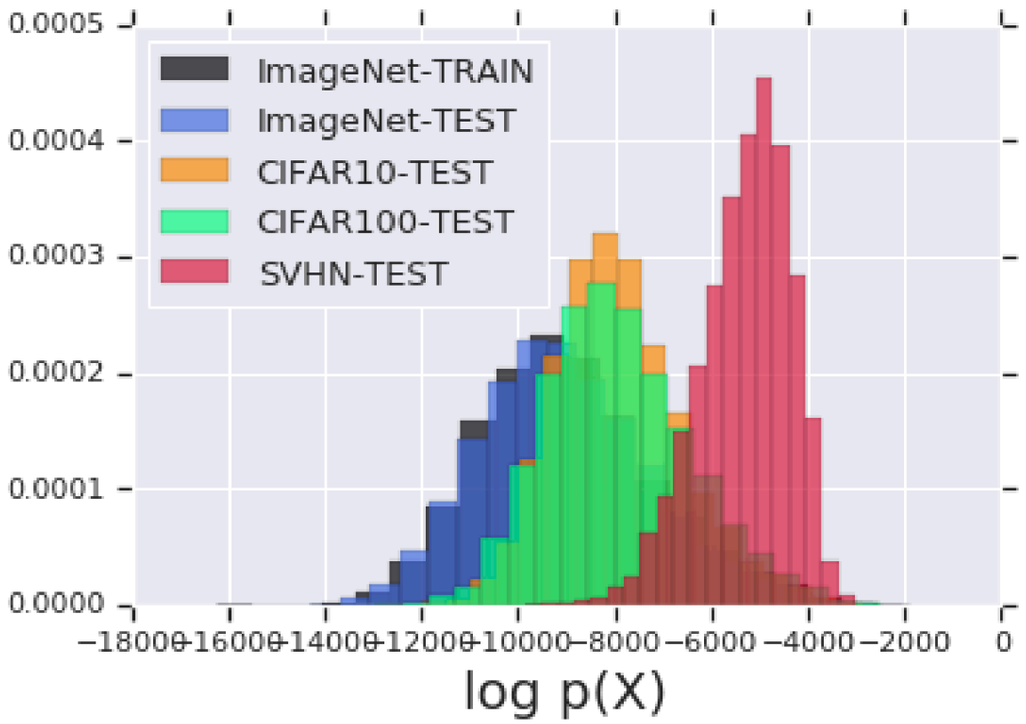
Additional Out-of-Distribution Tests



FashionMNIST vs MNIST

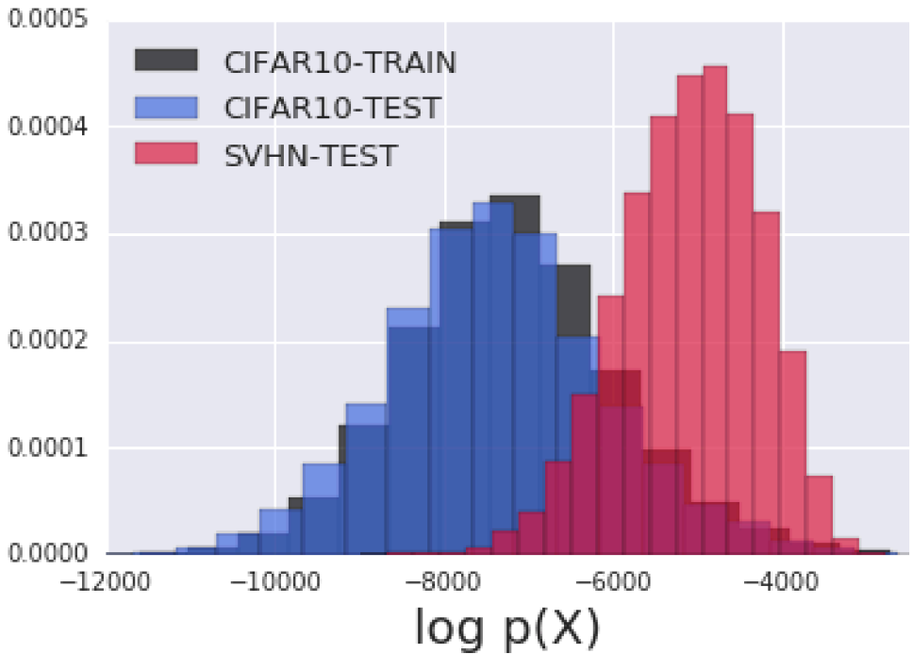


CelebA vs SVHN

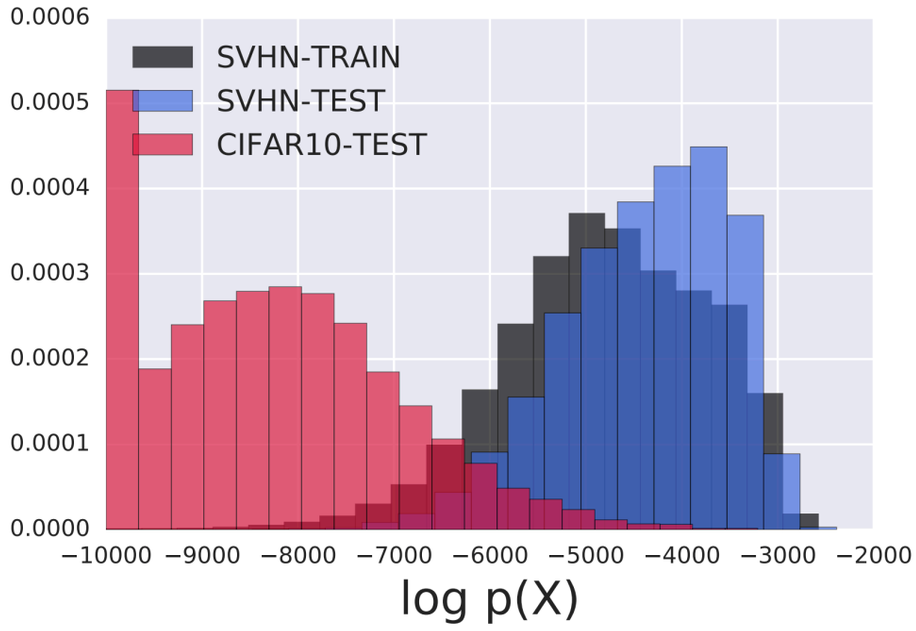


ImageNet vs CIFAR-10 vs SVHN

The Phenomenon is not Symmetric

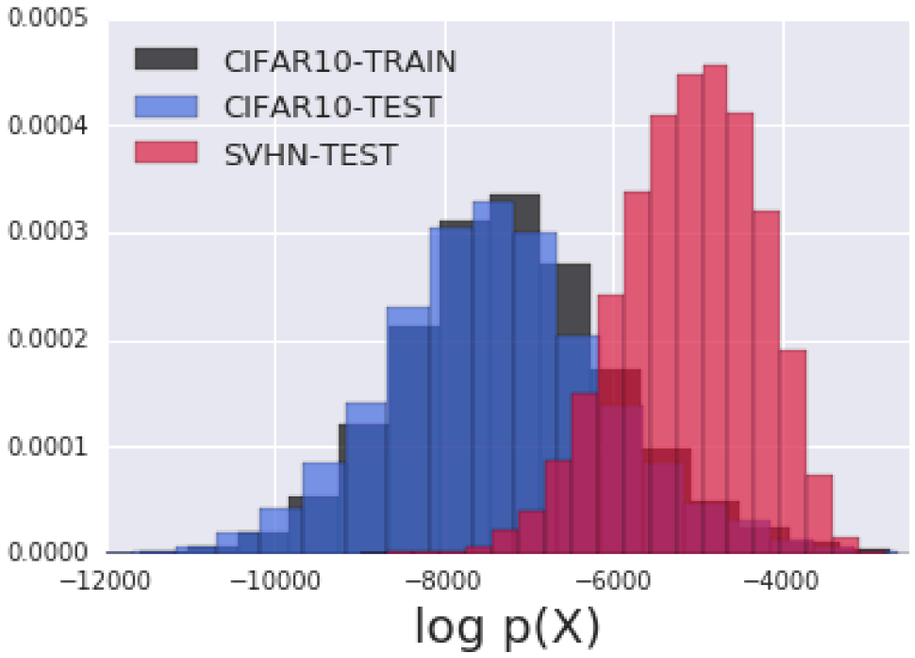


CIFAR-10 vs SVHN

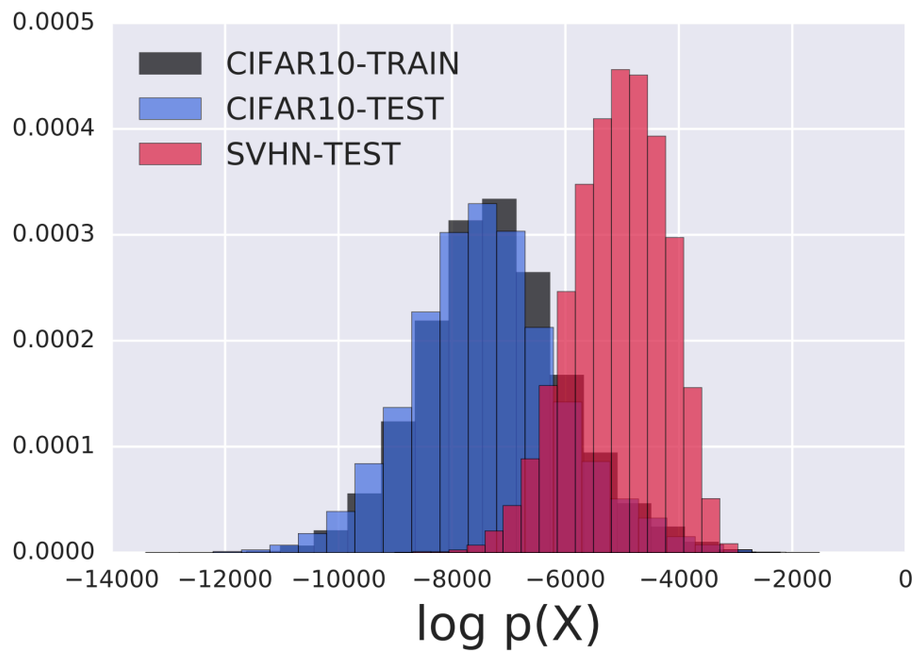


SVHN vs CIFAR-10

Ensembles Do Not Help



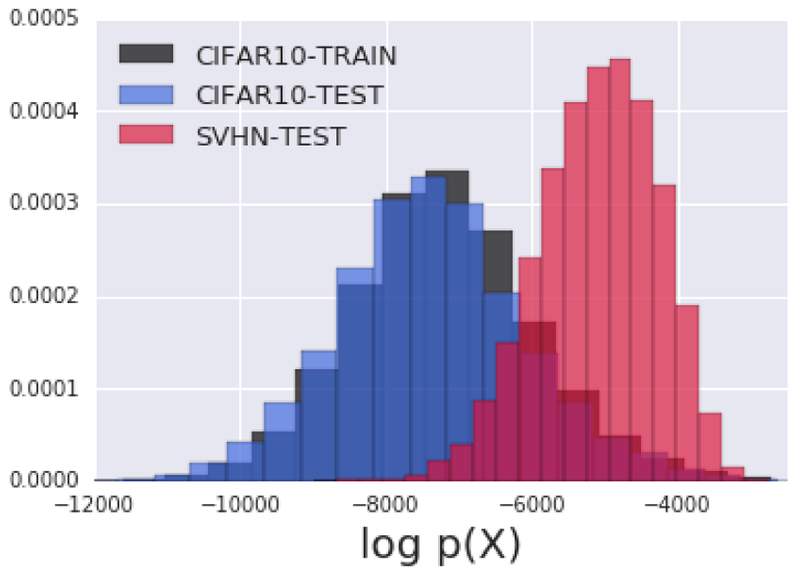
CIFAR-10 vs SVHN



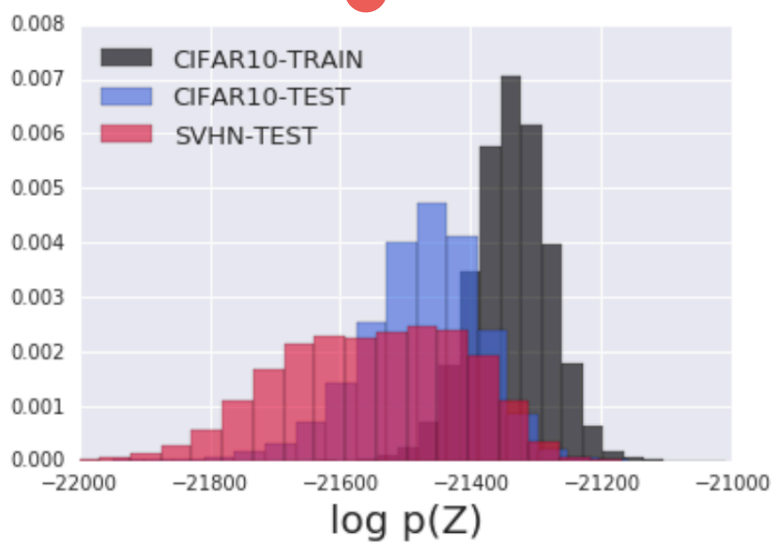
CIFAR-10 vs SVHN

Ensemble of 10 Glows

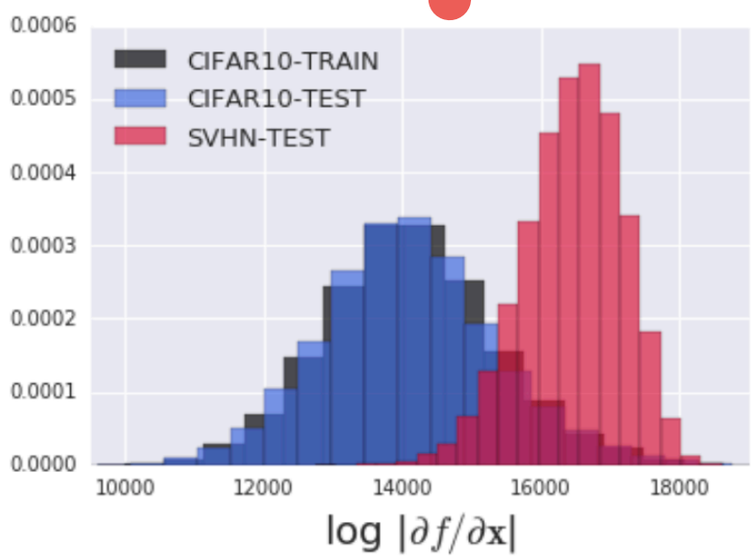
Decomposing the Change-of-Variables



CIFAR-10 vs SVHN

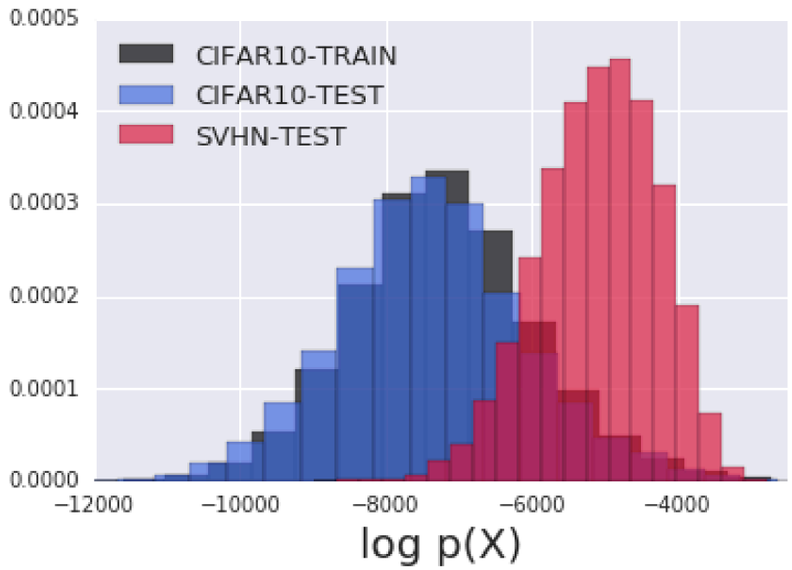


Distribution Term



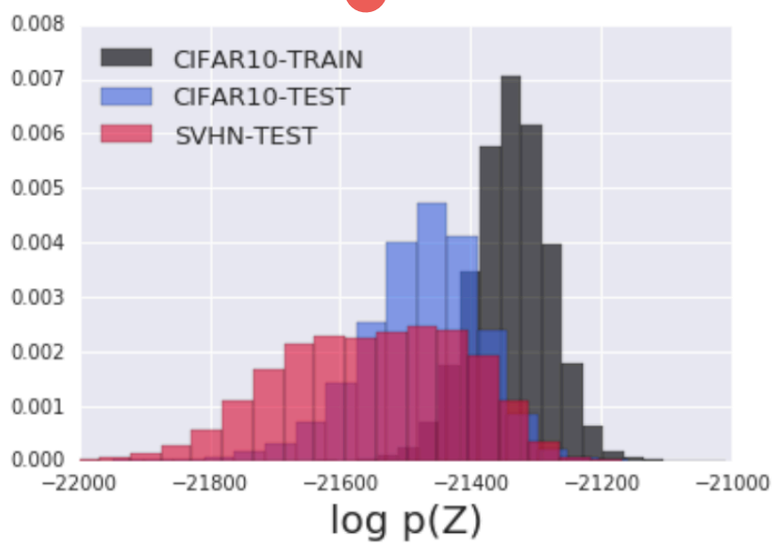
Volume Term

Decomposing the Change-of-Variables

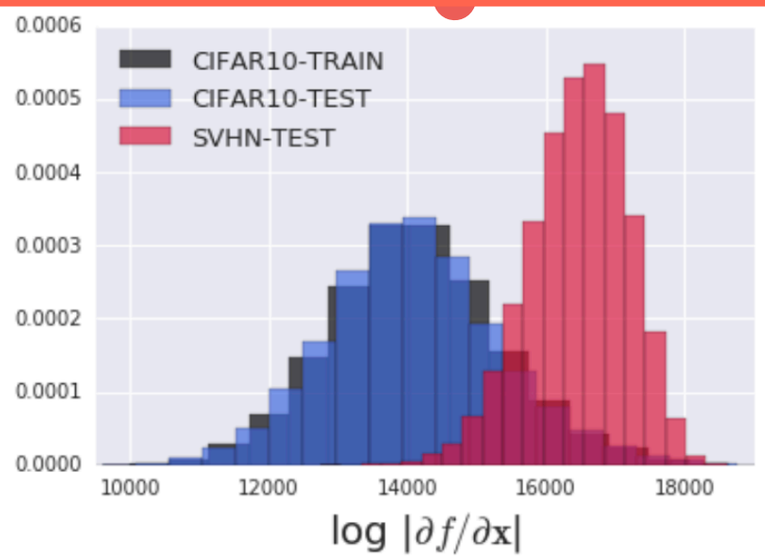


CIFAR-10 vs SVHN

- **Looks to be the cause of the phenomenon**



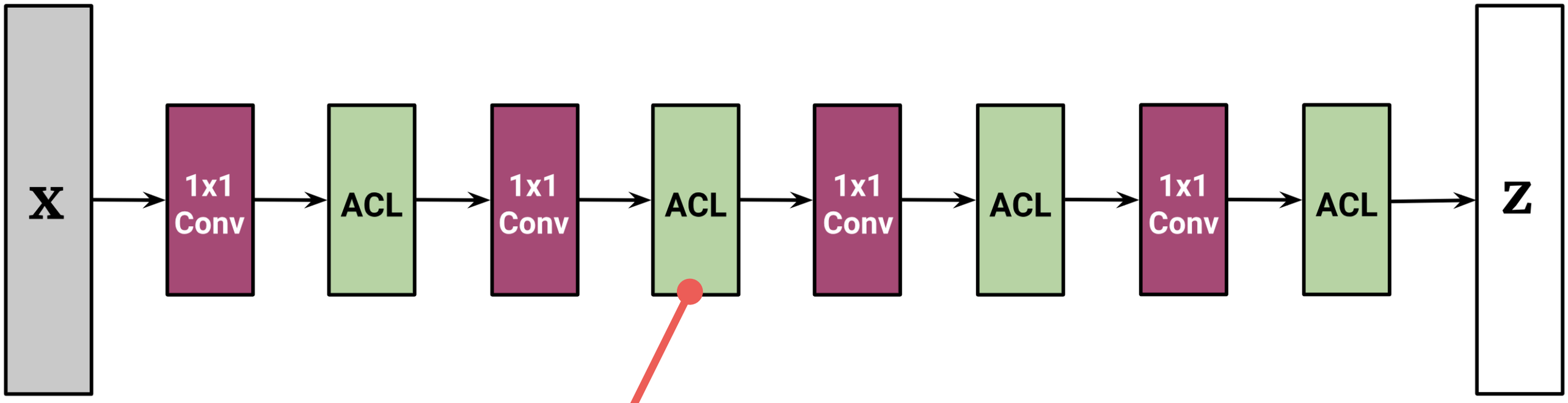
Distribution Term



Volume Term

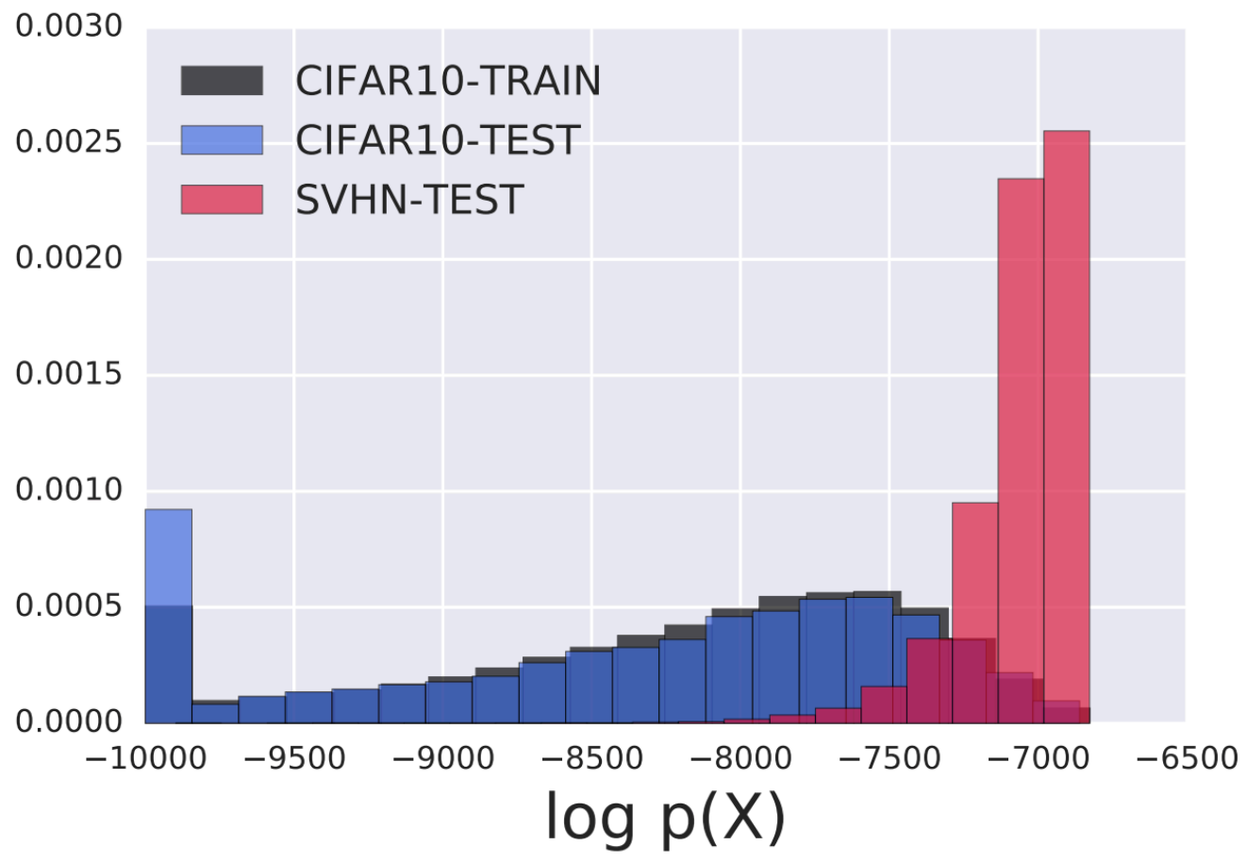
Constant-Volume Flows

To isolate the effect of the volume term, we define *constant-volume* (w.r.t. input) flows.



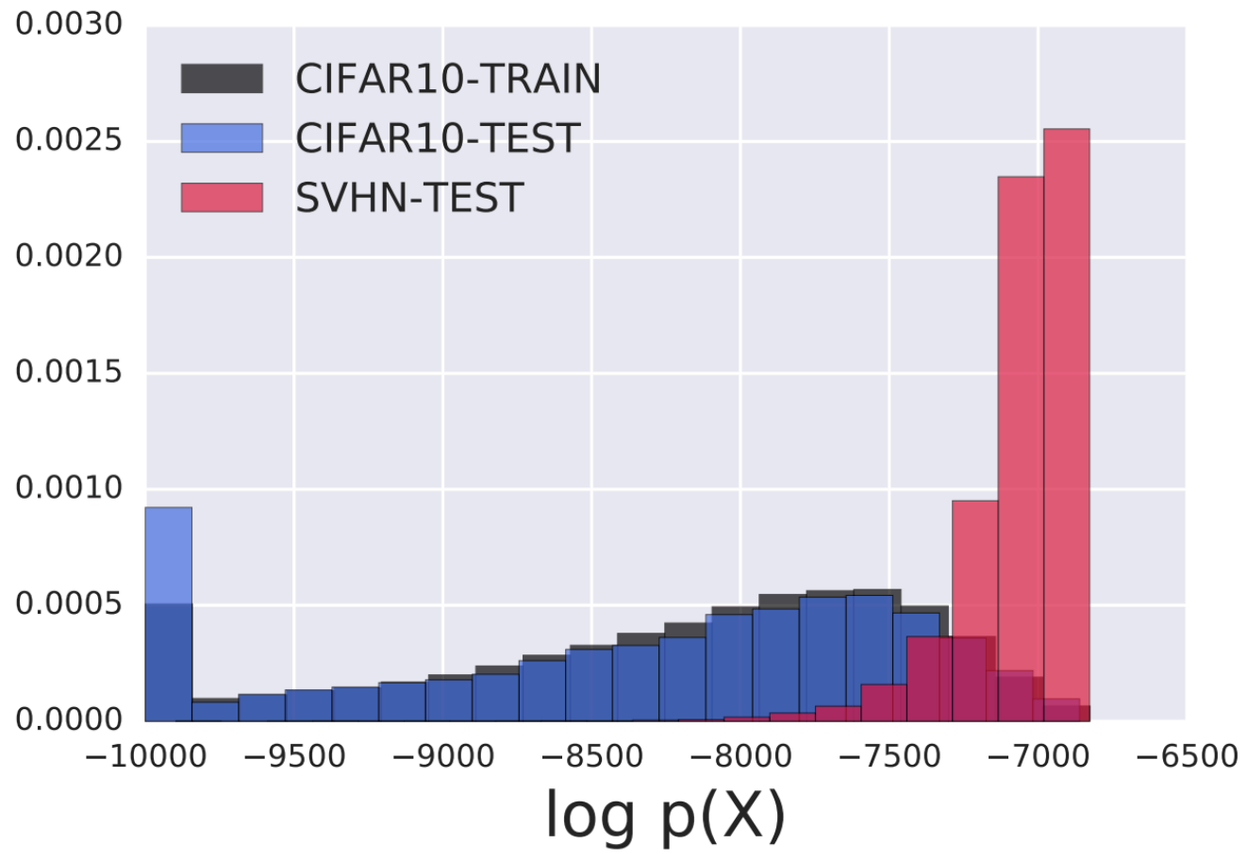
- **Use only translation operations.**

Constant-Volume Experiments

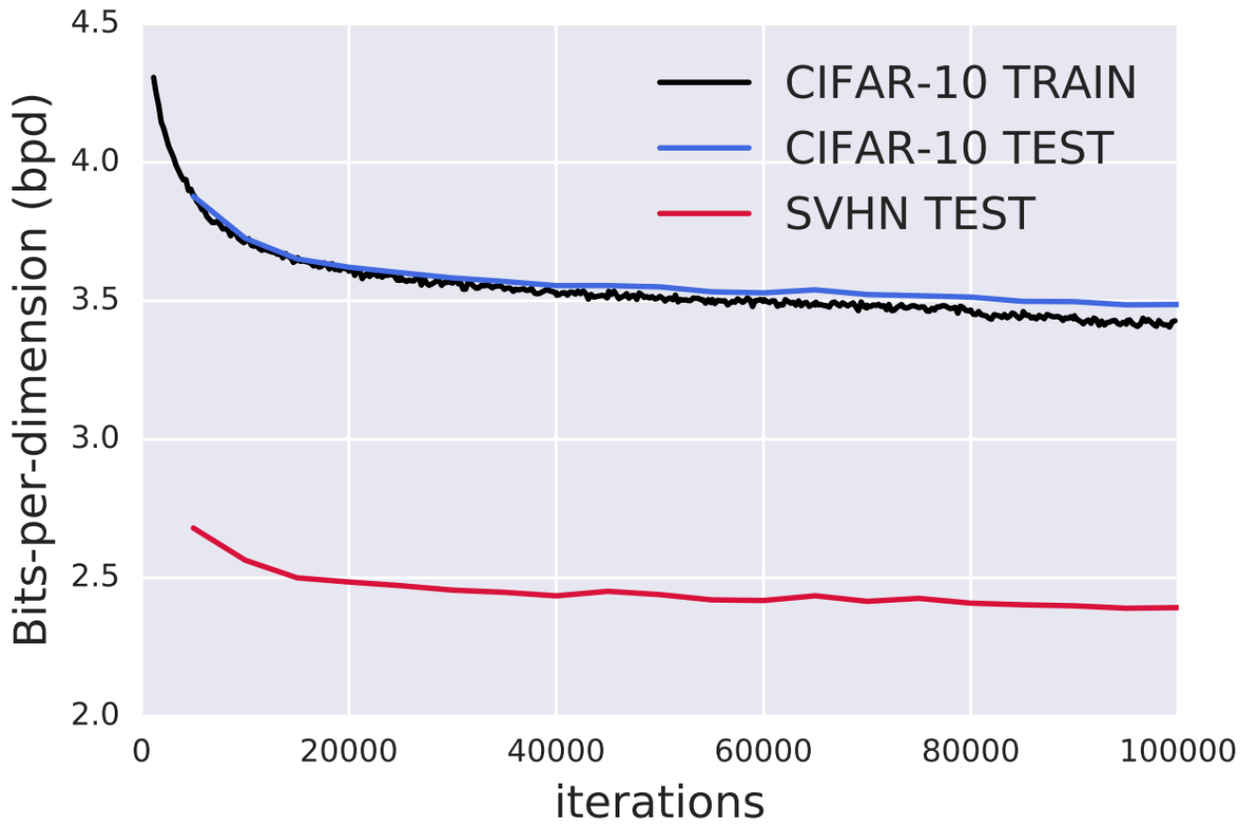


CIFAR-10 vs SVHN

Constant-Volume Experiments



CIFAR-10 vs SVHN



During Optimization

Analytical Investigation

Constant-Volume Flow Analysis

Mathematical characterization:

$$0 < \underbrace{\mathbb{E}_q[\log p(\boldsymbol{x};\boldsymbol{\theta})]}_{\text{Non-Training Distribution}} - \underbrace{\mathbb{E}_{p^*}[\log p(\boldsymbol{x};\boldsymbol{\theta})]}_{\text{Training Distribution}}$$

Constant-Volume Flow Analysis

Mathematical characterization:

$$0 < \underbrace{\mathbb{E}_q}_{\text{Non-Training Distribution}}[\log p(\boldsymbol{x};\boldsymbol{\theta})] - \underbrace{\mathbb{E}_{p^*}}_{\text{Training Distribution}}[\log p(\boldsymbol{x};\boldsymbol{\theta})]$$

$$\approx \frac{1}{2}\operatorname{Tr}\left\{\left[\nabla^2_{\boldsymbol{x}_0}\log p_z(f(\boldsymbol{x}_0;\boldsymbol{\phi})) + \nabla^2_{\boldsymbol{x}_0}\log\left|\frac{\partial \boldsymbol{f}_{\boldsymbol{\phi}}}{\partial \boldsymbol{x}_0}\right|\right](\boldsymbol{\Sigma}_q - \boldsymbol{\Sigma}_{p^*})\right\}$$

Constant-Volume Flow Analysis

Mathematical characterization:

$$0 < \underline{\mathbb{E}_q}[\log p(\boldsymbol{x};\boldsymbol{\theta})] - \underline{\mathbb{E}_{p^*}}[\log p(\boldsymbol{x};\boldsymbol{\theta})]$$

Non-Training Distribution

Training Distribution

Second Moment of Training Distribution

$$\approx \frac{1}{2}\operatorname{Tr}\left\{\left[\nabla^2_{\boldsymbol{x}_0}\log p_z(f(\boldsymbol{x}_0;\boldsymbol{\phi})) + \nabla^2_{\boldsymbol{x}_0}\log\left|\frac{\partial \boldsymbol{f}_{\boldsymbol{\phi}}}{\partial \boldsymbol{x}_0}\right|\right](\underline{\boldsymbol{\Sigma}_q} - \overline{\boldsymbol{\Sigma}_{p^*}})\right\}$$

Second Moment of Non-Training Distribution

Constant-Volume Flow Analysis

Plugging in the CV-Glow transform:

$$\mathrm{Tr}\left\{\left[\nabla^2_{\boldsymbol{x}_0}\log p(\boldsymbol{x}_0;\boldsymbol{\theta})\right](\boldsymbol{\Sigma}_q-\boldsymbol{\Sigma}_{p^*})\right\}$$

$$=\frac{\partial^2}{\partial z^2}\log p(\boldsymbol{z};\boldsymbol{\psi})\sum_{c=1}^{C}\left(\prod_{k=1}^{K}\sum_{j=1}^{C}u_{k,c,j}\right)^2\sum_{h,w}(\sigma^2_{q,h,w,c}-\sigma^2_{p^*,h,w,c})$$

Constant-Volume Flow Analysis

Plugging in the CV-Glow transform:

$$\mathrm{Tr}\left\{\left[\nabla^2_{\boldsymbol{x}_0}\log p(\boldsymbol{x}_0;\boldsymbol{\theta})\right](\boldsymbol{\Sigma}_q-\boldsymbol{\Sigma}_{p^*})\right\}$$

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1x1 Conv. Params

Constant-Volume Flow Analysis

Plugging in the CV-Glow transform:

$$\mathrm{Tr}\left\{\left[\nabla^2_{\boldsymbol{x}_0} \log p(\boldsymbol{x}_0;\boldsymbol{\theta})\right](\boldsymbol{\Sigma}_q - \boldsymbol{\Sigma}_{p^*})\right\}$$

$$= \frac{\partial^2}{\partial z^2} \log p(\boldsymbol{z};\boldsymbol{\psi}) \sum_{c=1}^{C} \left(\prod_{k=1}^{K} \sum_{j=1}^{C} u_{k,c,j} \right)^2 \sum_{h,w} (\sigma^2_{q,h,w,c} - \sigma^2_{p^*,h,w,c})$$

1x1 Conv. Params

Sums over channel dimensions

Product over steps in flow

Sum over spatial dimensions

Constant-Volume Flow Analysis

Plugging in the CV-Glow transform:

$$\mathrm{Tr}\left\{\left[\nabla^2_{\boldsymbol{x}_0} \log p(\boldsymbol{x}_0;\boldsymbol{\theta})\right](\boldsymbol{\Sigma}_q - \boldsymbol{\Sigma}_{p^*})\right\}$$

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< 0 for all log-concave densities (e.g. Gaussian)

Non-negative due to square

Constant-Volume Flow Analysis

Plugging in the CIFAR-10 and SVHN statistics:

$$\mathbb{E}_{\text{SVHN}}[\log p(\boldsymbol{x};\boldsymbol{\theta})] - \mathbb{E}_{\text{CIFAR10}}[\log p(\boldsymbol{x};\boldsymbol{\theta})]$$

$$\approx \frac{1}{2\sigma_{\boldsymbol{\psi}}^2}\left[\alpha_1^2 \cdot 12.3 + \alpha_2^2 \cdot 6.5 + \alpha_3^2 \cdot 14.5\right] \geq 0 \quad \text{where } \alpha_c = \prod_{k=1}^{K}\sum_{j=1}^{C} u_{k,c,j}$$

Differences in variances in the three spatial dimensions

Constant-Volume Flow Analysis

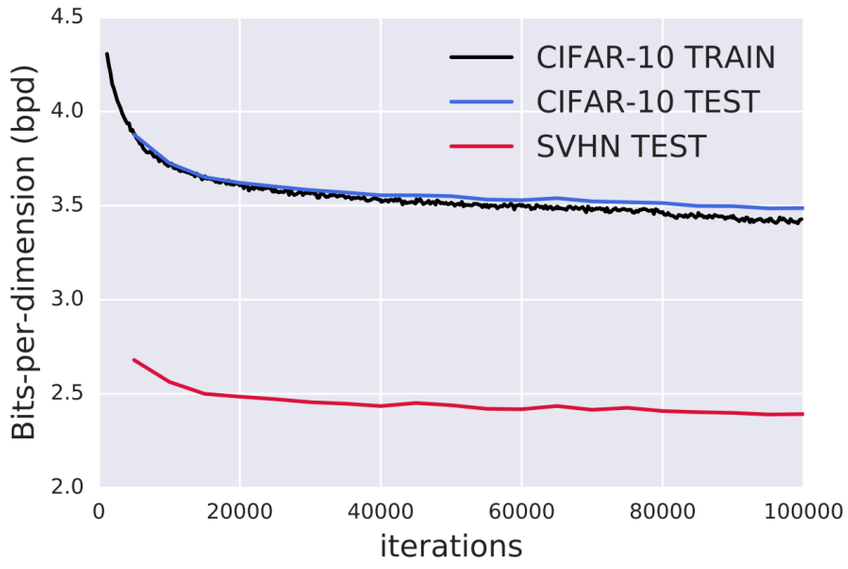
Plugging in the CIFAR-10 and SVHN statistics:

$$\mathbb{E}_{\text{SVHN}}[\log p(\boldsymbol{x};\boldsymbol{\theta})] - \mathbb{E}_{\text{CIFAR10}}[\log p(\boldsymbol{x};\boldsymbol{\theta})]$$

$$\approx \frac{1}{2\sigma_{\boldsymbol{\psi}}^2}\left[\alpha_1^2 \cdot 12.3 + \alpha_2^2 \cdot 6.5 + \alpha_3^2 \cdot 14.5\right] \geq 0 \quad \text{where } \alpha_c = \prod_{k=1}^{K}\sum_{j=1}^{C} u_{k,c,j}$$

Differences in variances in the three spatial dimensions

The expression will be non-negative **for any** parameter setting of the CV flow….



Constant-Volume Flow Analysis

Plugging in the CIFAR-10 and SVHN statistics:

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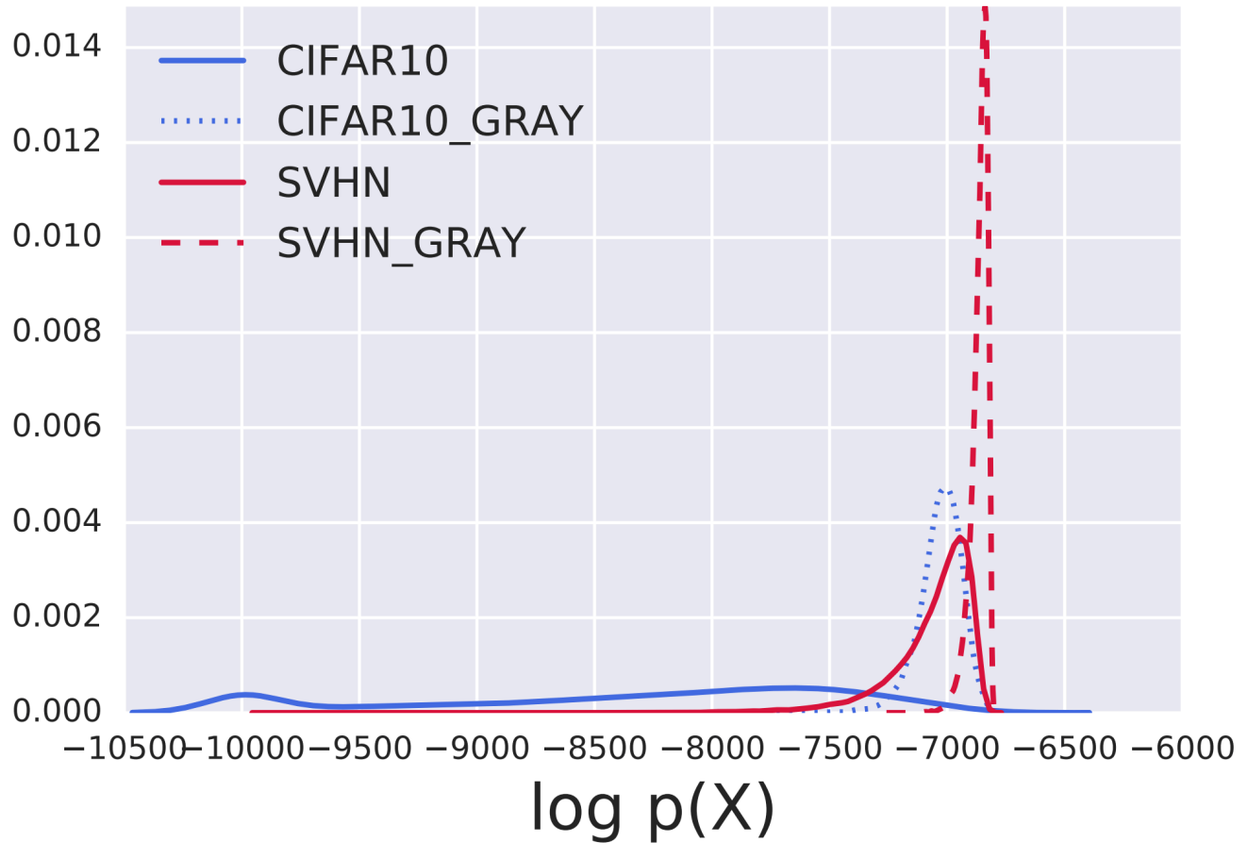
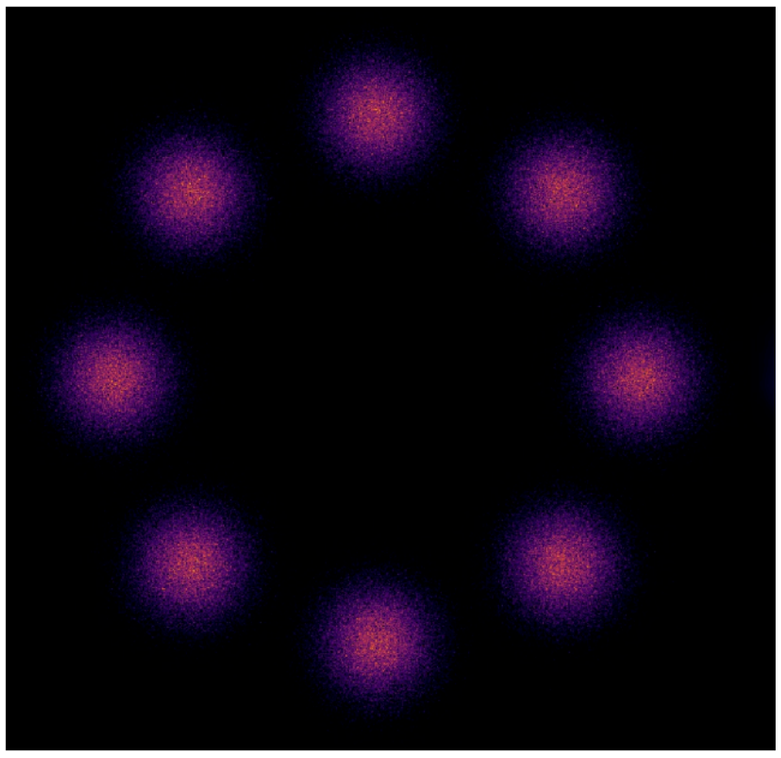
This also means that we can manipulate the relative log likelihoods just by changing the variance of the data. For natural images, this amounts to **graying**…



Parting Thoughts

Parting Thoughts

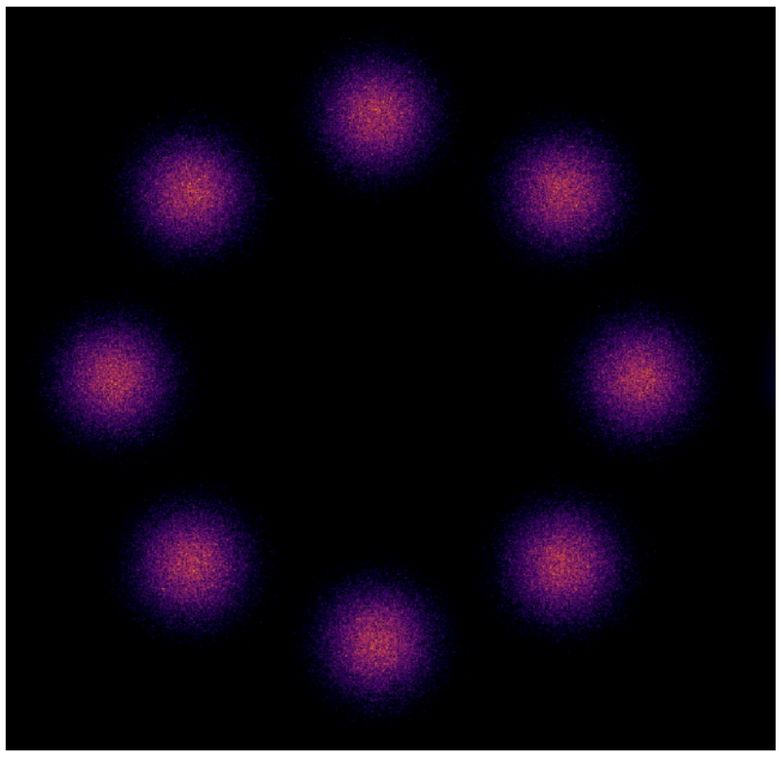
What does it mean for a deep generative model to *generalize*?



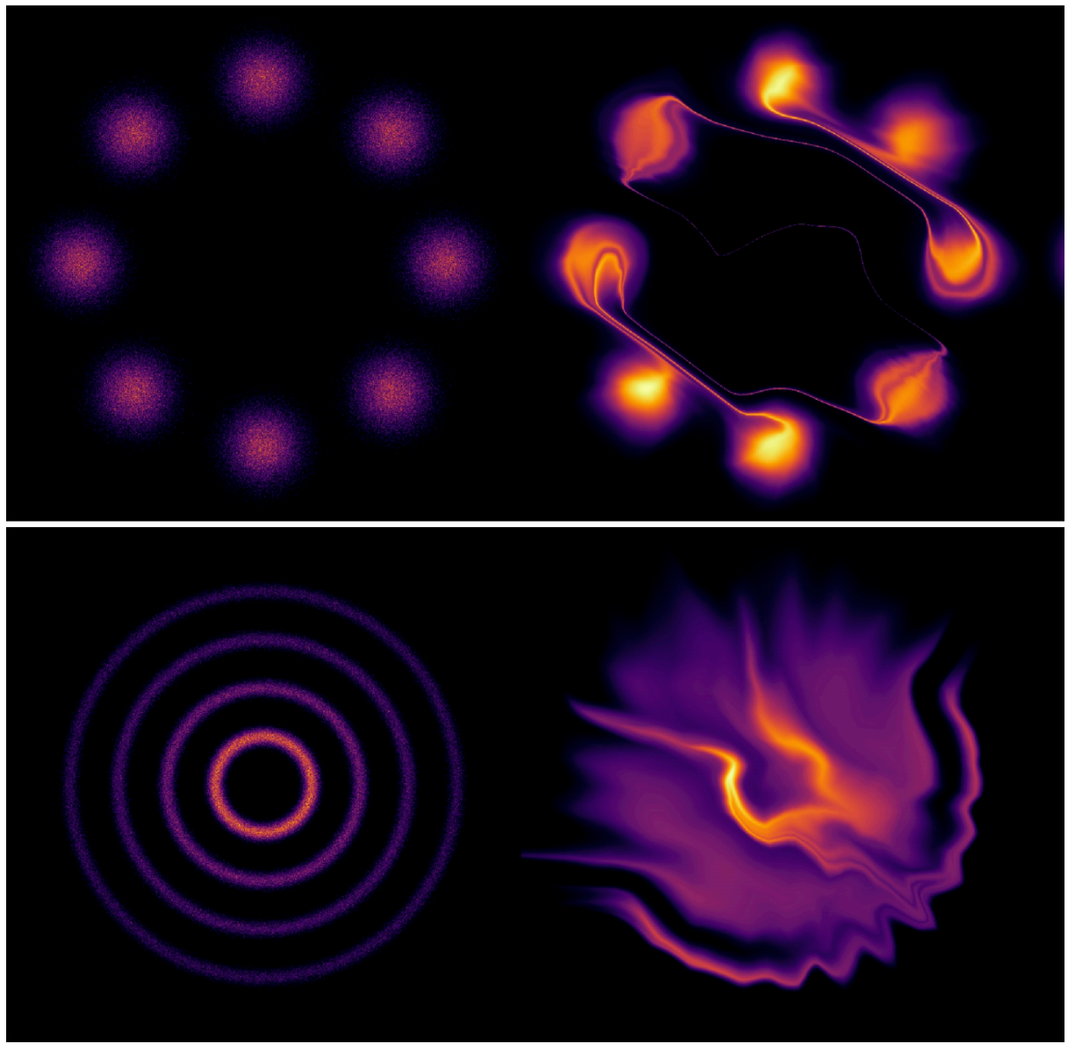
Data Samples

Parting Thoughts

What does it mean for a deep generative model to *generalize*?



Data Samples



Data Samples Glow

[Behrmann et al., ArXiv 2019]

How do we get Glow (and other deep gen. models) to ‘smear’ in the right directions?

Parting Thoughts

Do deep generative models need a better notion of semantics?

	CIFAR-10 TEST (BITS/DIM)	SVHN TEST (BITS/DIM)
BIVA $\mathcal{L}_1^{>13}$	79.36	121.04
BIVA $\mathcal{L}_{100}^{>13}$	54.91	71.38
BIVA $\mathcal{L}_1^{>11}$	35.34	58.82
BIVA $\mathcal{L}_1^{>9}$	20.93	26.76
BIVA $\mathcal{L}_1^{>0}$	3.12	2.28

Table 6. The $\mathcal{L}^{>k}$ for $k = 13, 11, 9, 0$ evaluated in bits/dim by a model trained on the CIFAR-10 train set and evaluated on the CIFAR-10 test set and the SVHN test set. It is possible to use BIVA for anomaly detection of high-level semantics in complex datasets when using a high k.

[Maaløe et al., ArXiv 2019]

Parting Thoughts

Does there exist analogous behavior for language?

Parting Thoughts

Conclusion: We should temper the enthusiasm with which we preach the benefits of deep generative models until their out-of-distribution behavior is better understood.

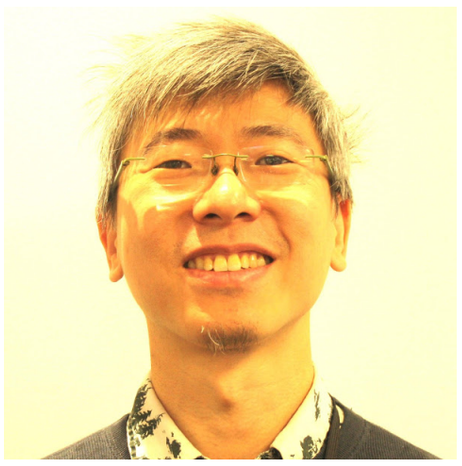
Thank you. Questions?

In collaboration with…

Aki Matsukawa

Balaji Lakshminarayanan

Dilan Gorur



Yee Whye Teh