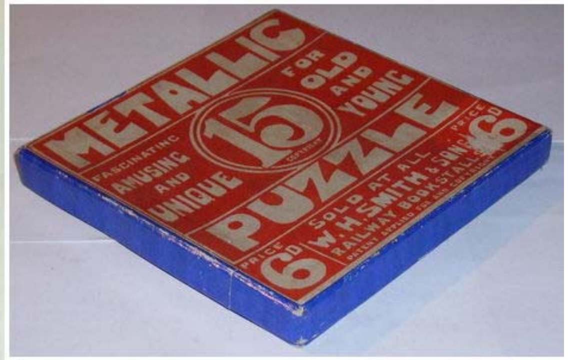
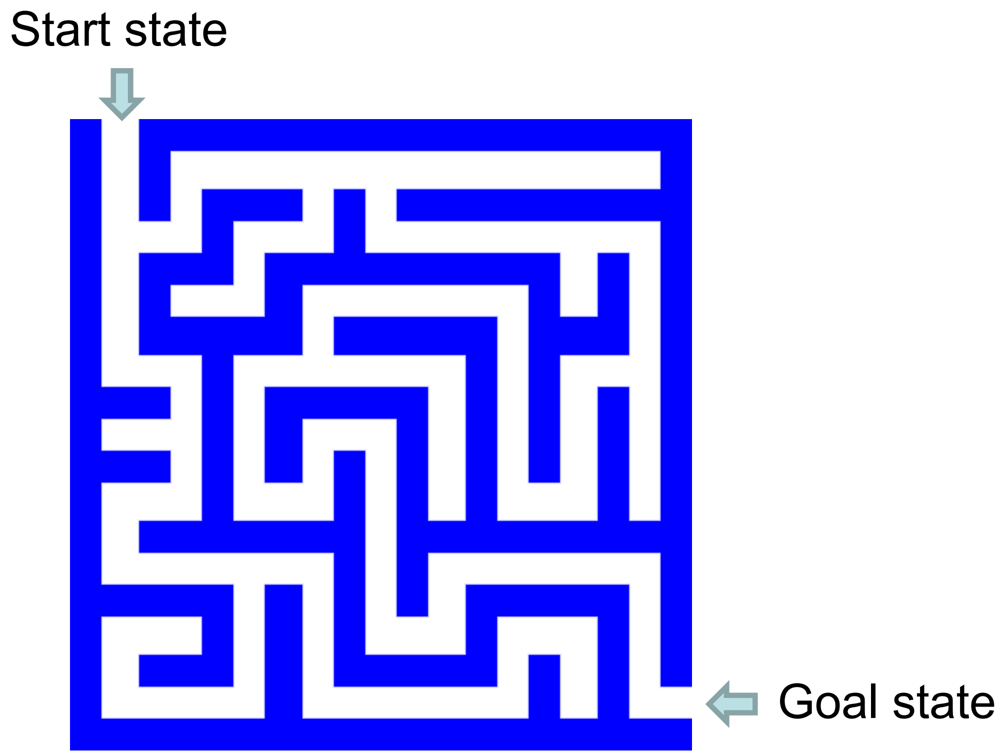
Solving problems by searching

Chapter 3



Search

- We will consider the problem of designing **goal-based agents** in **fully observable**, **deterministic**, **discrete**, **known** environments



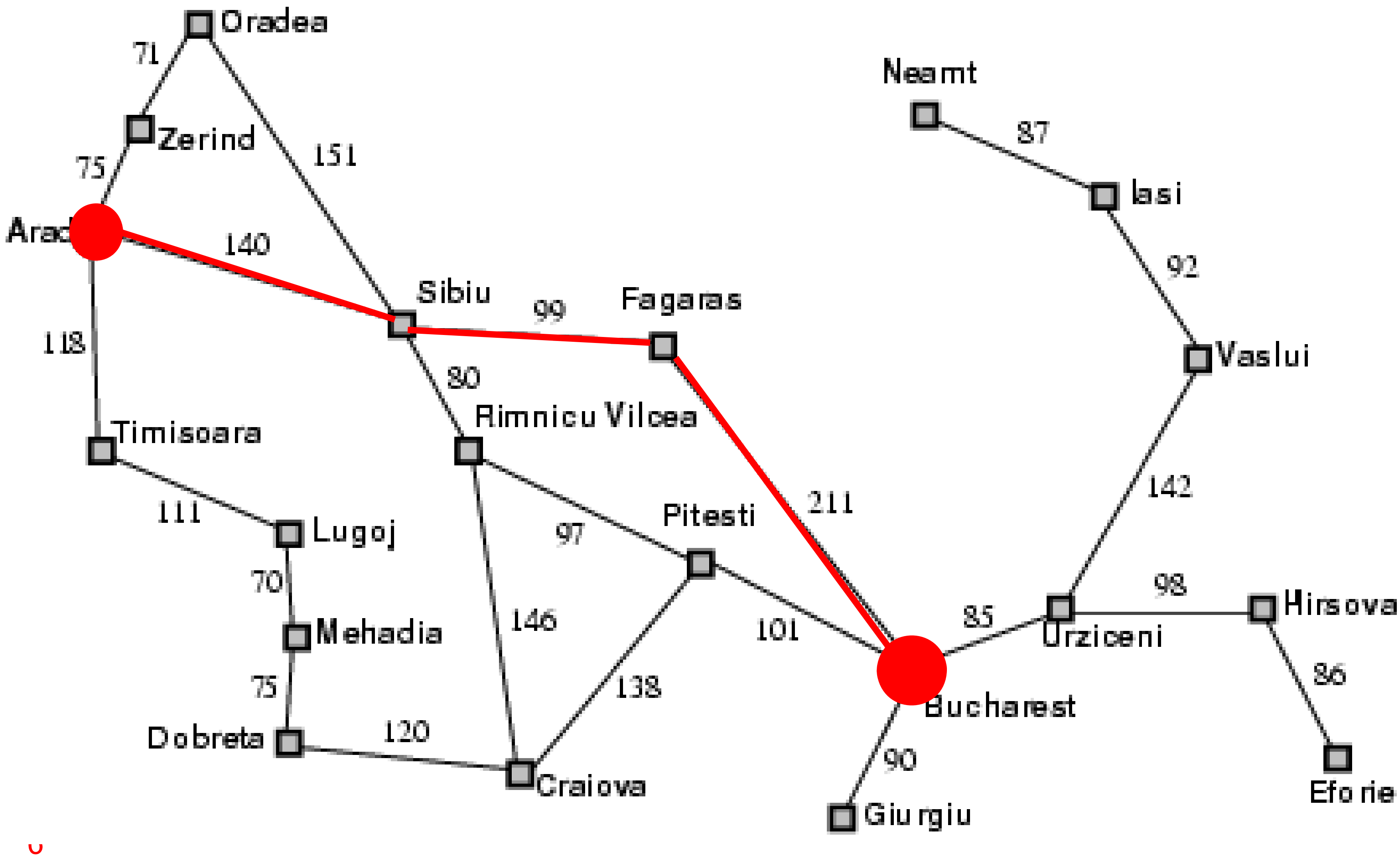
Search

- We will consider the problem of designing **goal-based agents** in **fully observable**, **deterministic**, **discrete**, **known** environments
  - The agent must find a *sequence of actions* that reaches the goal
  - The **performance measure** is defined by (a) reaching the goal and (b) how "expensive" the path to the goal is
  - We are focused on the process of finding the solution; while executing the solution, we assume that the agent can safely ignore its percepts (**open-loop system**)

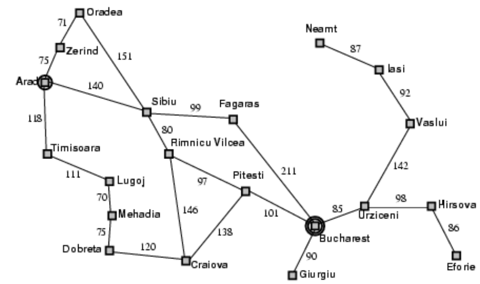
Search Problems

Search Problems Are Models

Example: Romania



Single-state problem formulation



A **problem** is defined by four items:

1. initial state e.g., "at Arad"

2. actions or successor function *S(x)* = set of action–state pairs
   - e.g., *S(Arad)* = {*<Arad → Zerind, Zerind>, … *}

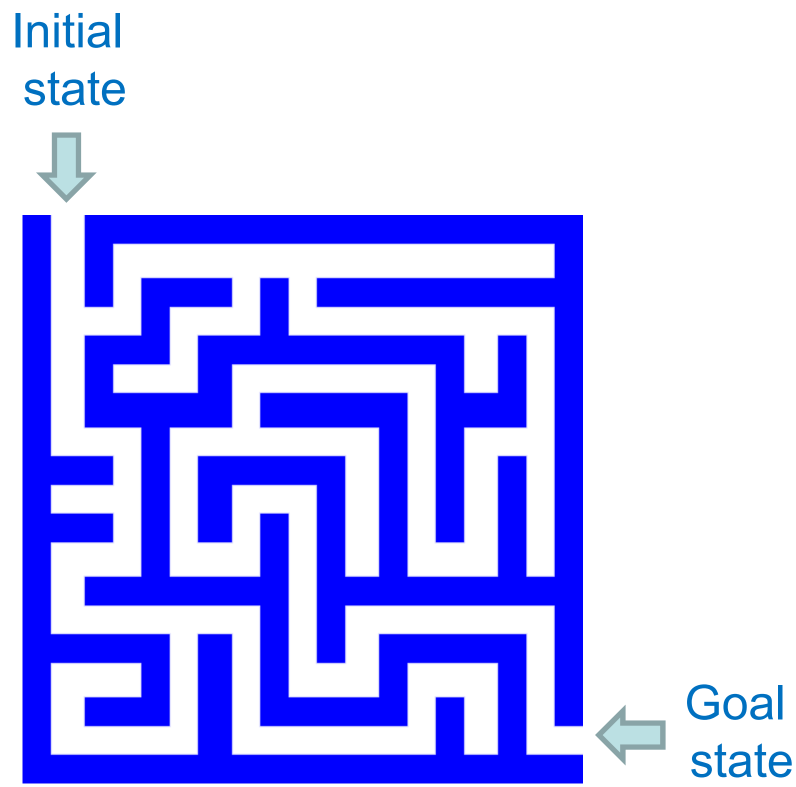
3. goal test, can be
   - explicit, e.g., *x* = "at Bucharest"
   - implicit, e.g., *Checkmate(x)*

4. path cost (additive)
   - e.g., sum of distances, number of actions executed, etc.
   - *c(x,a,y)* is the step cost, assumed to be ≥ 0

A solution is a sequence of actions leading from the initial state to a goal state

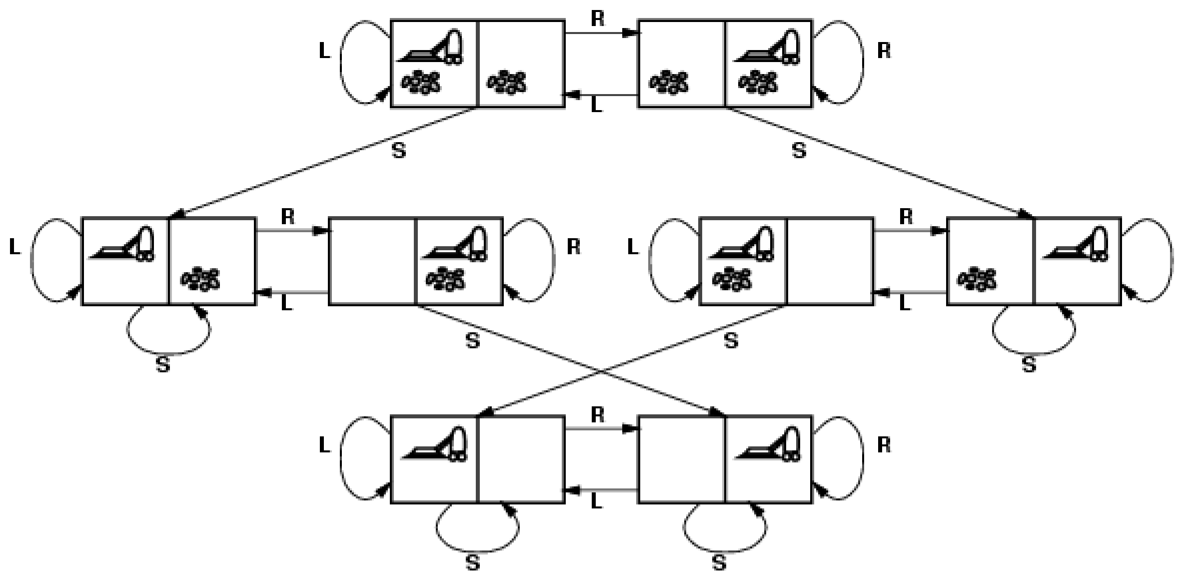
Search problem components

- **Initial state**
- **Actions**
- **Transition model**
  - What state results from performing a given action in a given state?
- **Goal state**
- **Path cost**
  - Assume that it is a sum of nonnegative *step costs*



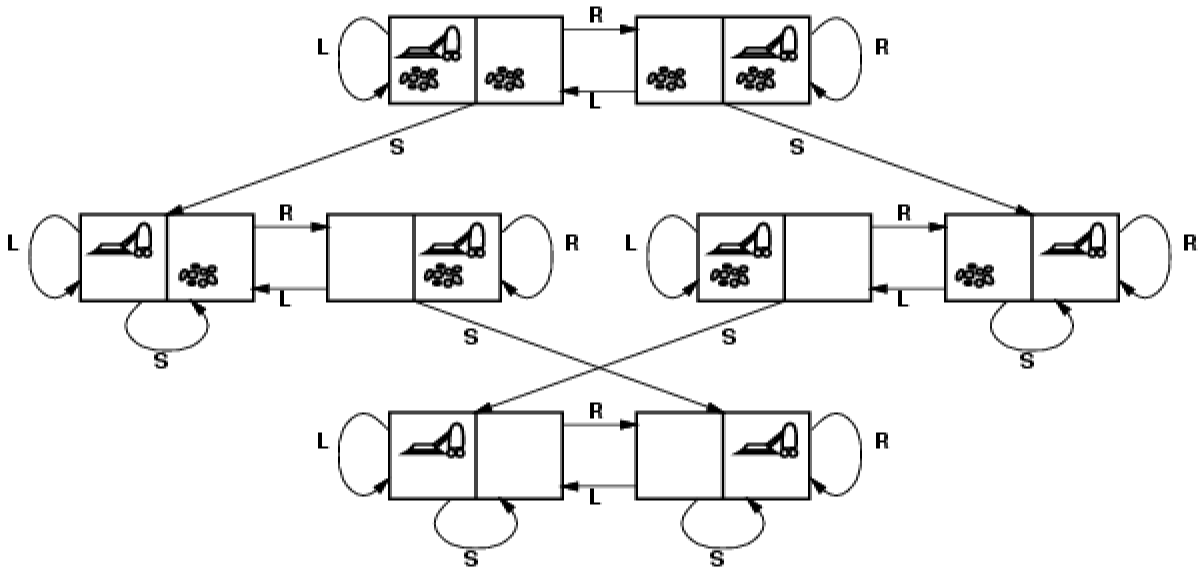
- The **optimal solution** is the sequence of actions that gives the *lowest* path cost for reaching the goal

Vacuum world state space graph



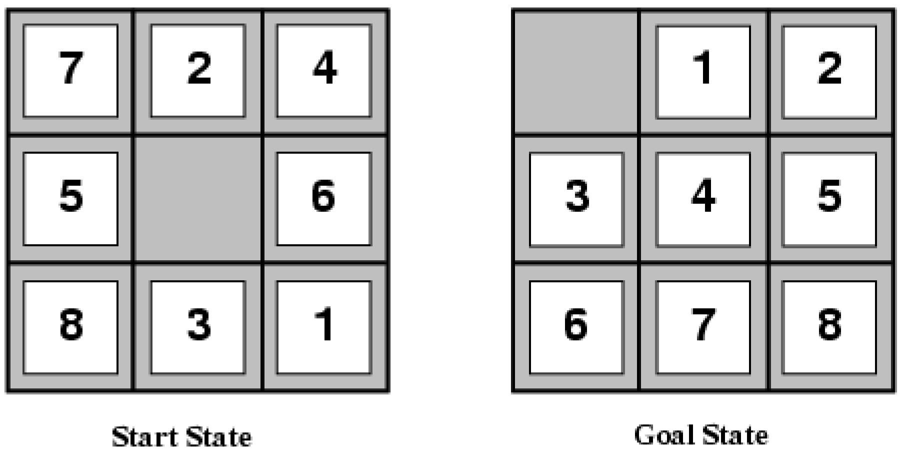
- states?
- actions?
- goal test?
- path cost?

Vacuum world state space graph



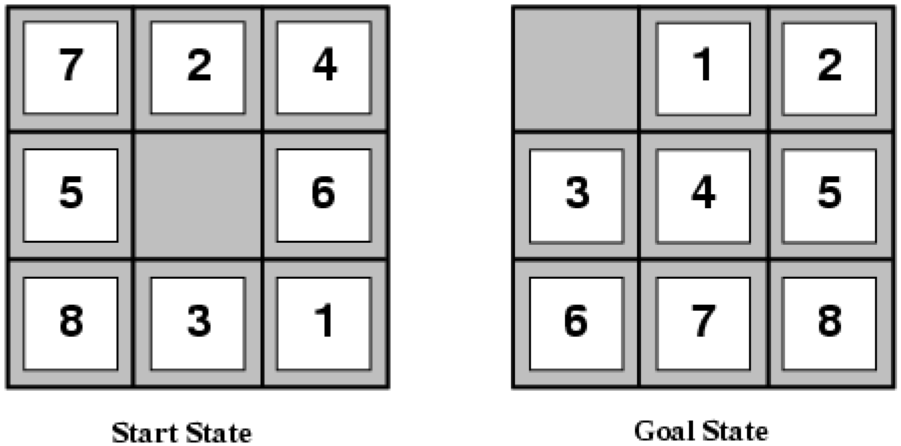
- states? dirt and robot location
- actions? *Left*, *Right*, *Clean*
- goal test? no dirt at all locations
- path cost? 1 per action

Example: The 8-puzzle



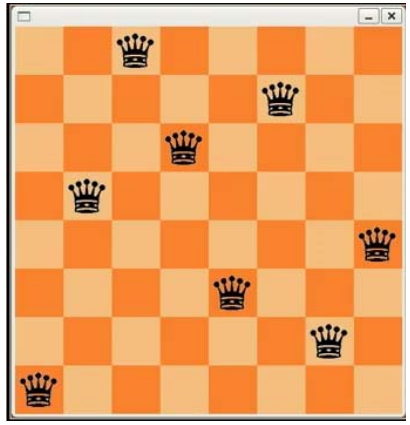
- states?
- actions?
- goal test?
- path cost?

Example: The 8-puzzle



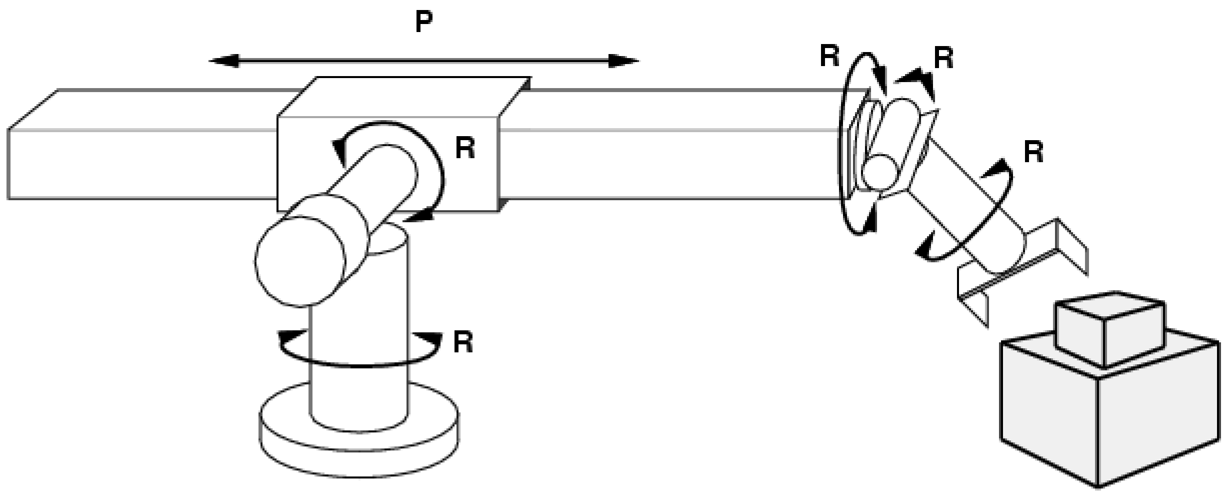
- states? locations of tiles
- actions? move blank left, right, up, down
- goal test? = goal state (given)
- path cost? 1 per move

Example: 8 queen



- states?: Any arrangement of 0 to 8 queens on the board is a state
- Initial state: No queen on the board
- Actions?: Add a queen to any empty square
- goal test?: 8 queens are on the board, none attacked

Example: Robot motion planning



- **States**
  – Real-valued joint parameters (angles, displacements)
- **Actions**
  – Continuous motions of robot joints
- **Goal state**
  – Configuration in which object is grasped
- **Path cost**
  – Time to execute, smoothness of path, etc.

Search

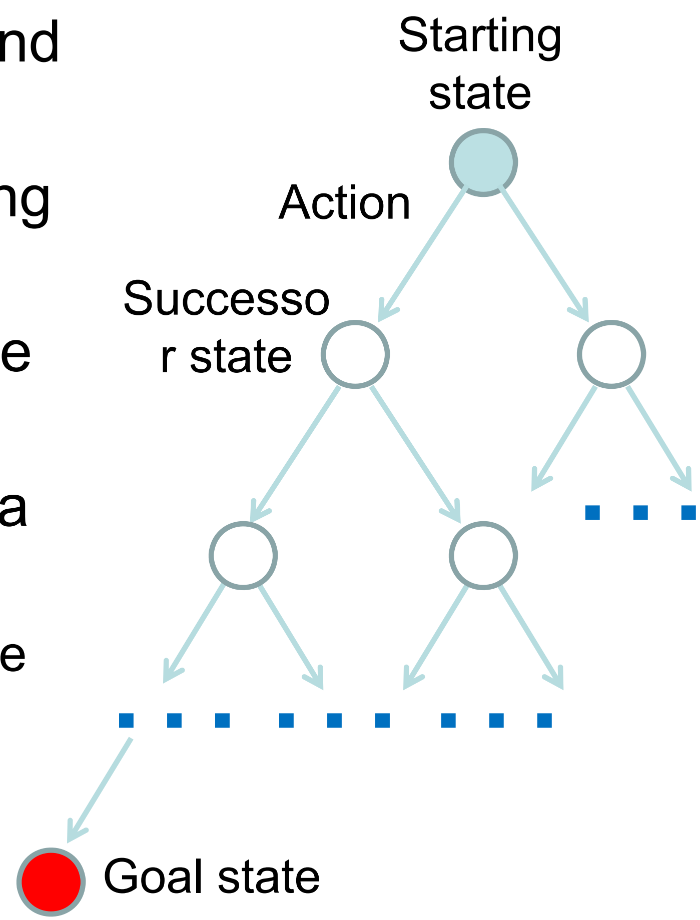
- Given:
  - **Initial state**
  - **Actions**
  - **Transition model**
  - **Goal state**
  - **Path cost**
- How do we find the optimal solution?
  - How about building the state space and then using Dijkstra's shortest path algorithm?
    - Complexity of Dijkstra's is $O(E + V \log V)$, where V is the size of the state space
    - The state space may be huge!

Search: Basic idea

- Let's begin at the start state and **expand** it by making a list of all possible successor states
- Maintain a **frontier** or a list of unexpanded states
- At each step, pick a state from the frontier to expand
- Keep going until you reach a goal state
- Try to expand as few states as possible

Search tree

- “What if” tree of sequences of actions and outcomes
- The root node corresponds to the starting state
- The children of a node correspond to the **successor states** of that node’s state
- A path through the tree corresponds to a sequence of actions
  - A solution is a path ending in the goal state
- Nodes vs. states
  - A state is a representation of the world, while a node is a data structure that is part of the search tree
    - Node has to keep pointer to parent, path cost, possibly other info

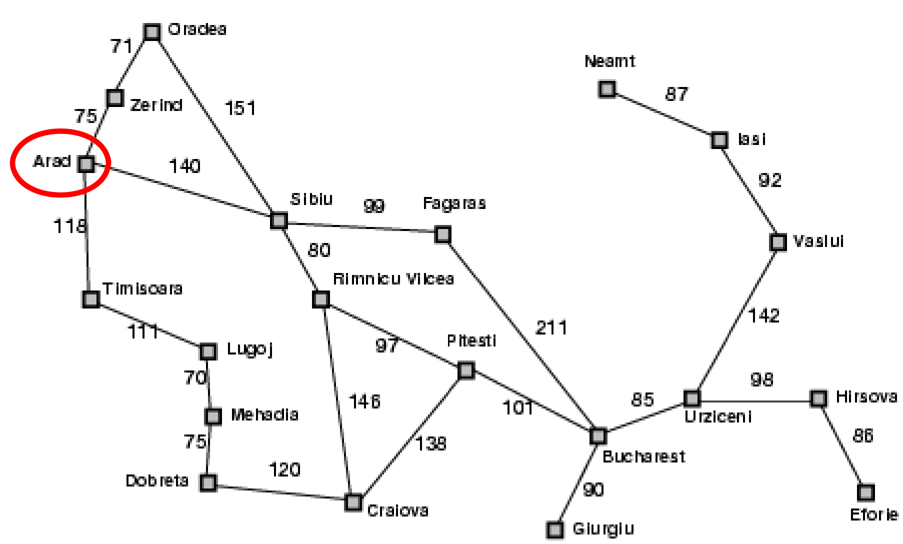


Tree Search Algorithm Outline

- Initialize the **frontier** using the **starting state**
- While the frontier is not empty
  - Choose a frontier node according to **search strategy** and take it off the frontier
  - If the node contains the **goal state**, return solution
  - Else **expand** the node and add its children to the frontier

Tree search example

Start: Arad
Goal: Bucharest



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Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
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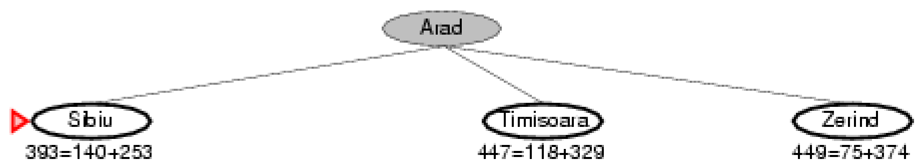
Tree search example



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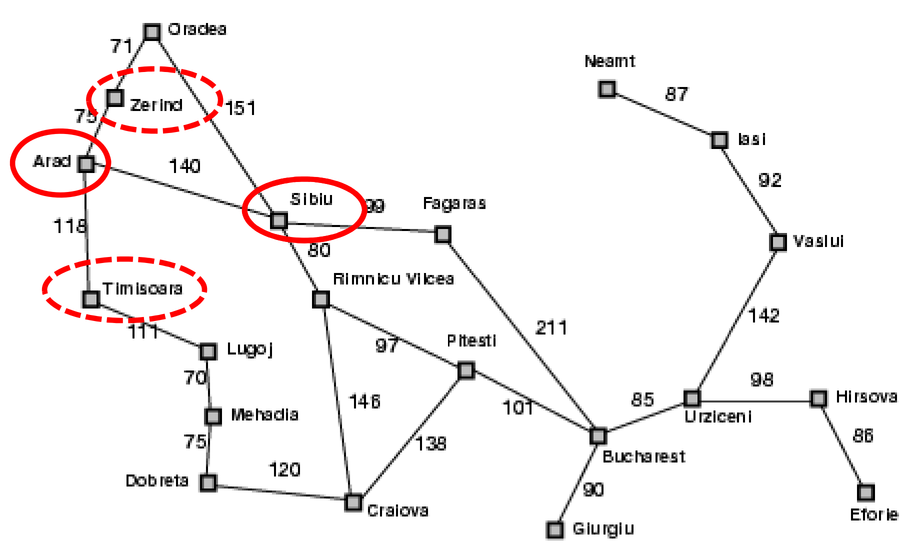

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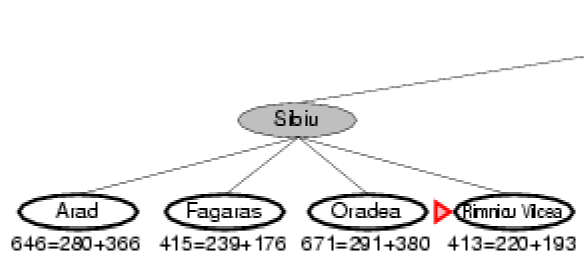

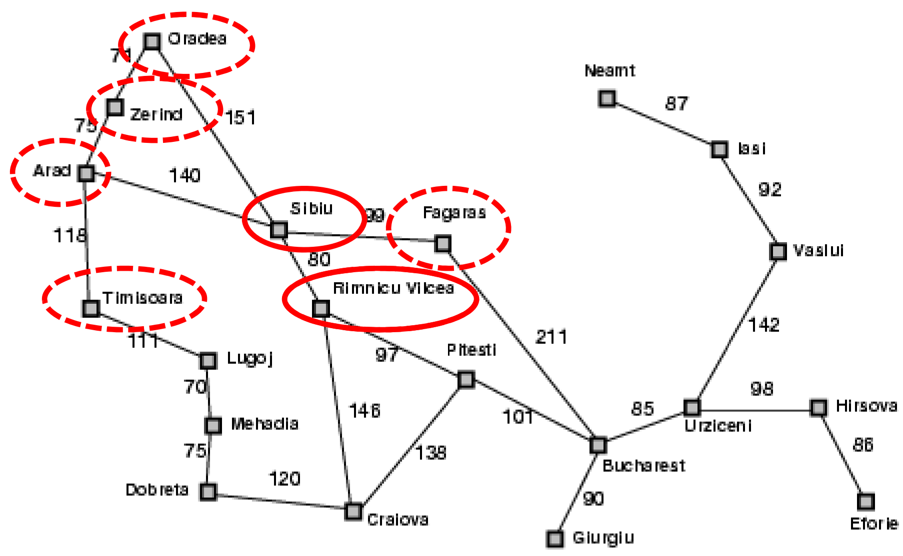

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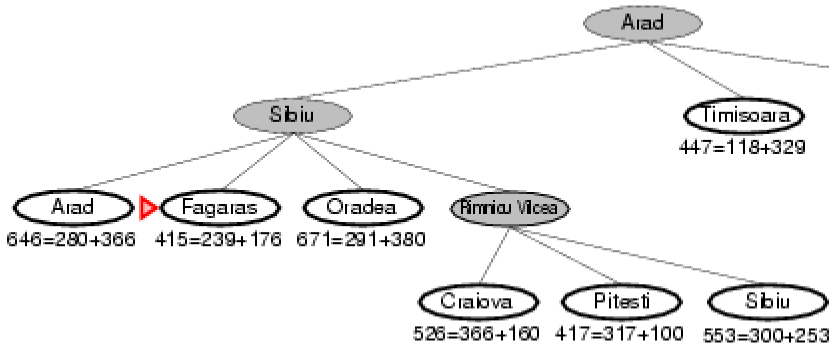
Goal: Bucharest

Tree search example



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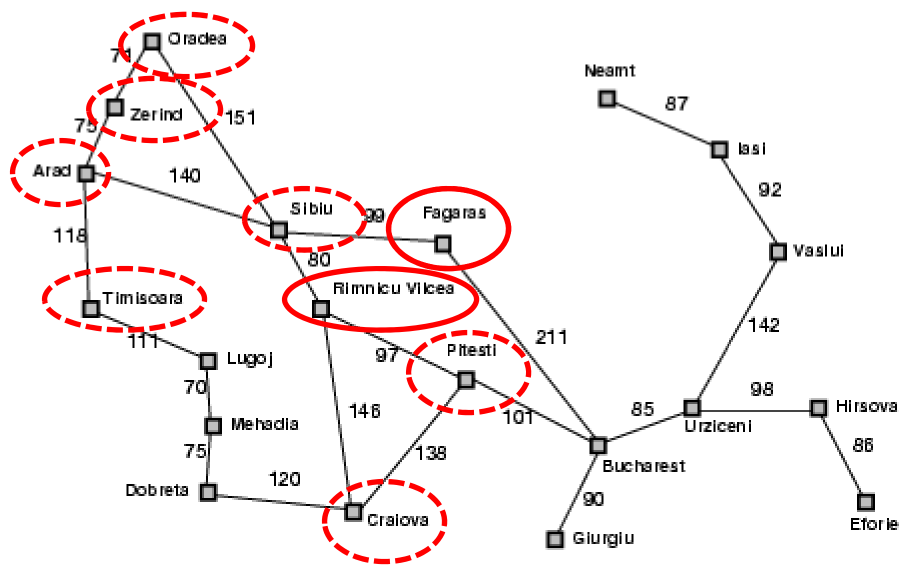
Goal: Bucharest



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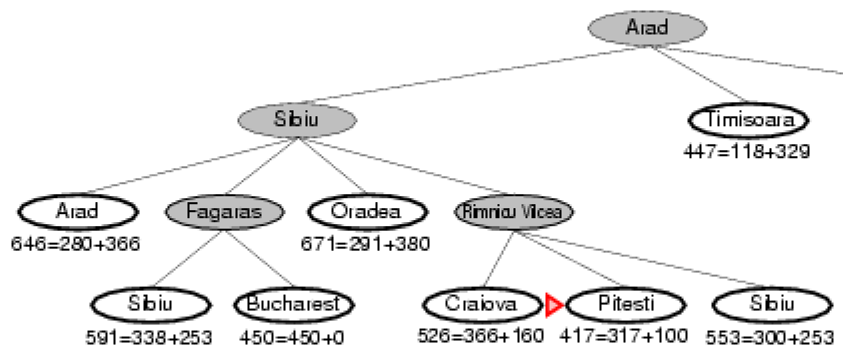

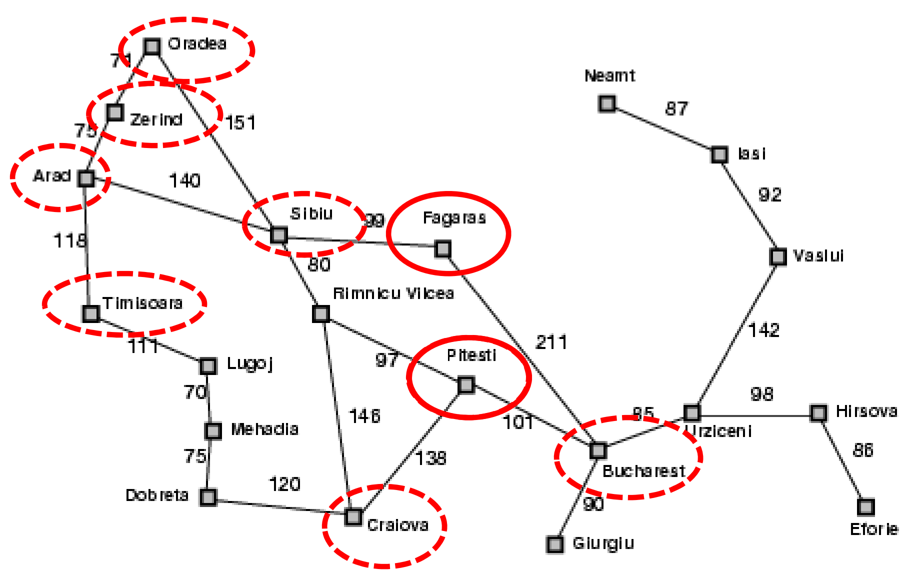

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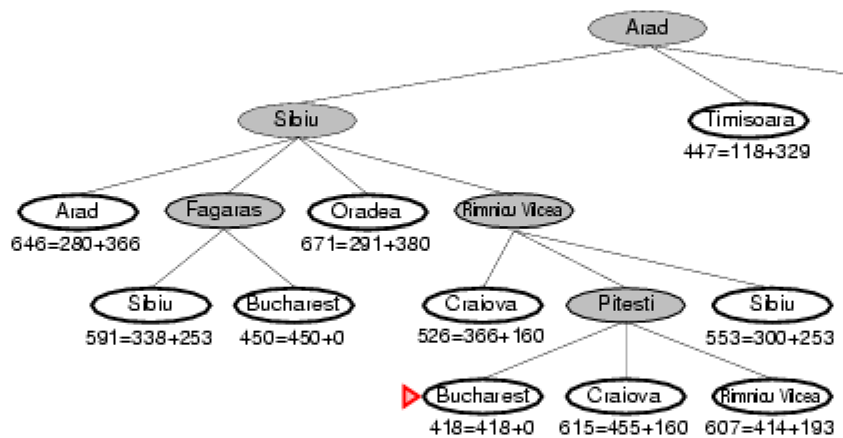

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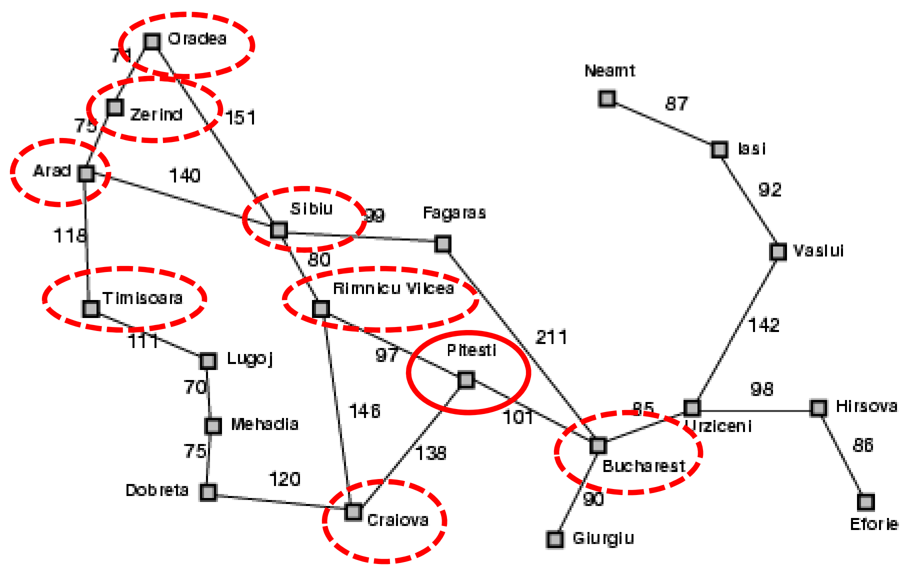

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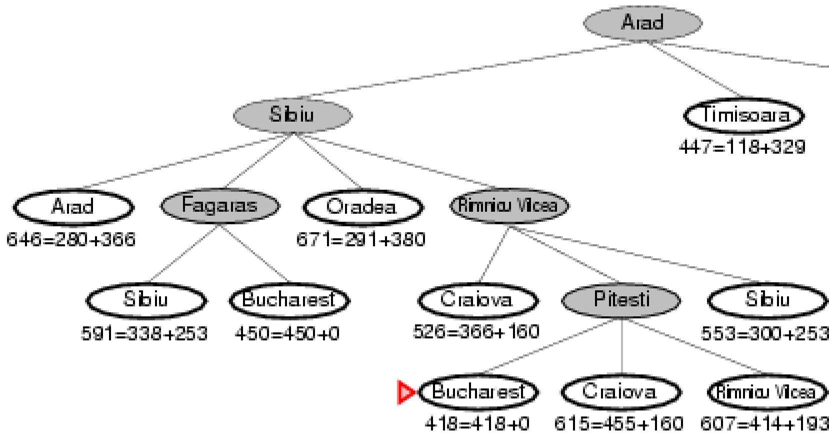

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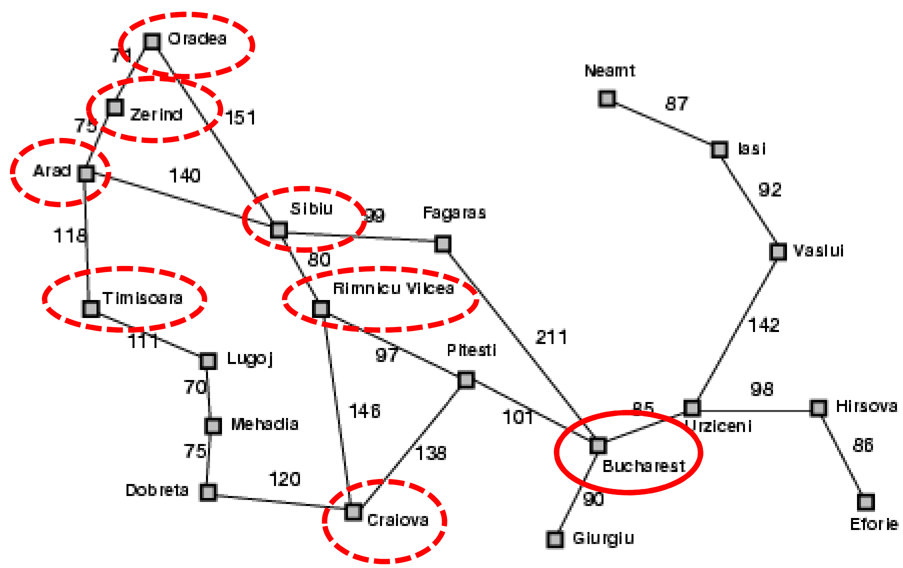

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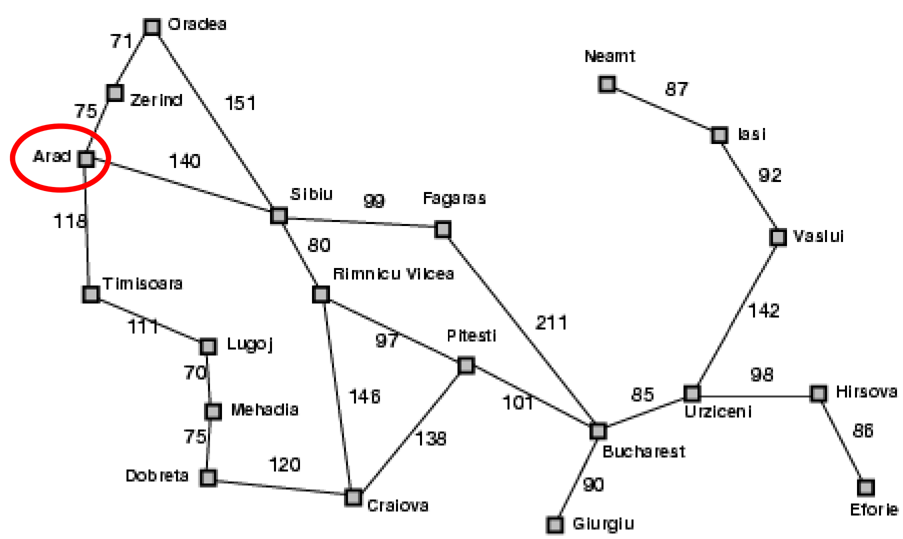
Handling repeated states

- Initialize the **frontier** using the **starting state**
- While the frontier is not empty
  - Choose a frontier node according to **search strategy** and take it off the frontier
  - If the node contains the **goal state**, return solution
  - Else **expand** the node and add its children to the frontier

- To handle repeated states:
  - Every time you expand a node, add that state to the **explored set**; do not put explored states on the frontier again
  - Every time you add a node to the frontier, check whether it already exists in the frontier with a higher path cost, and if yes, replace that node with the new one

Search without repeated states

Start: Arad
Goal: Bucharest



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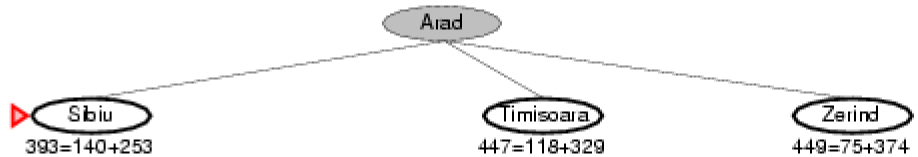

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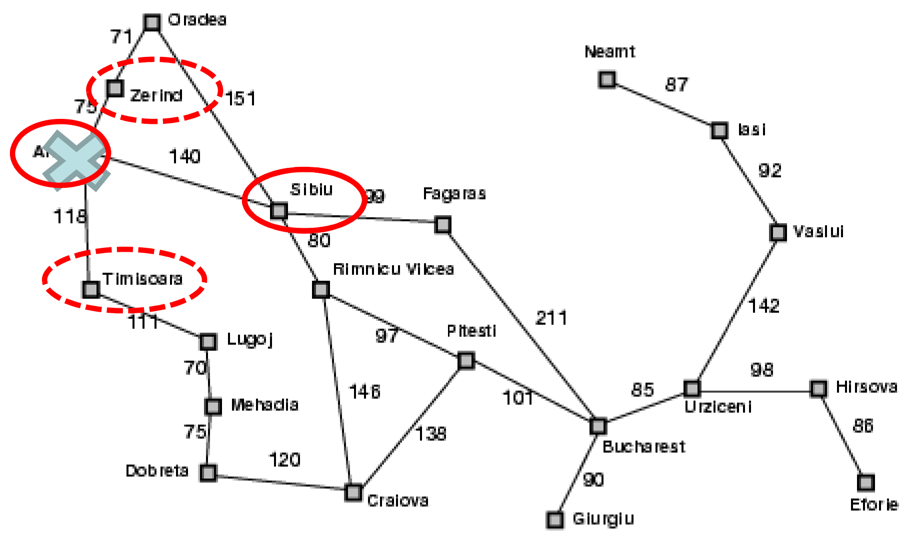

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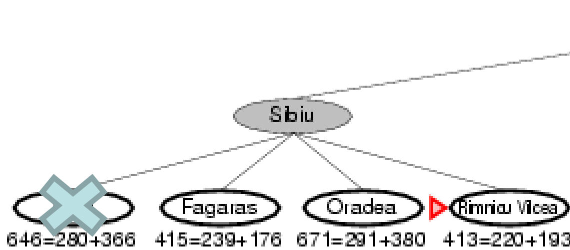

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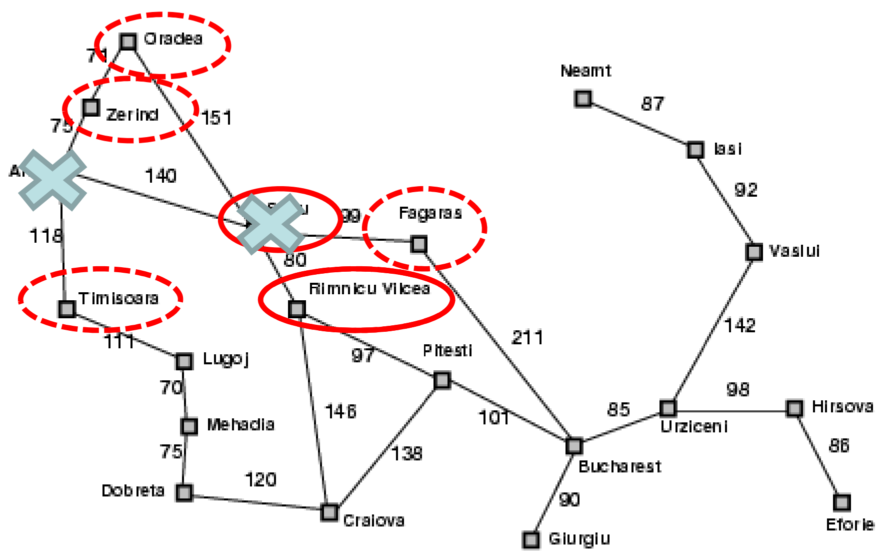

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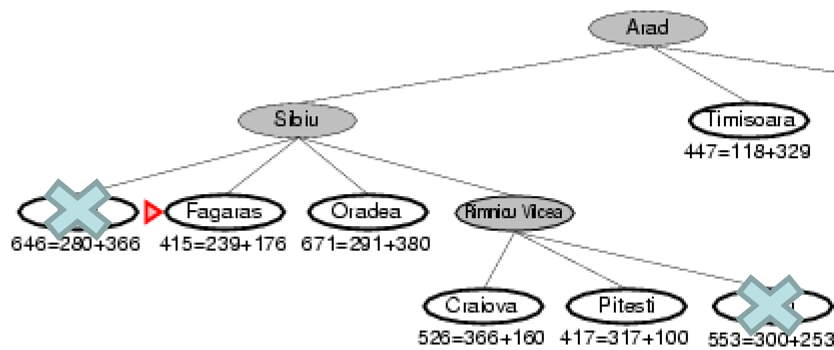

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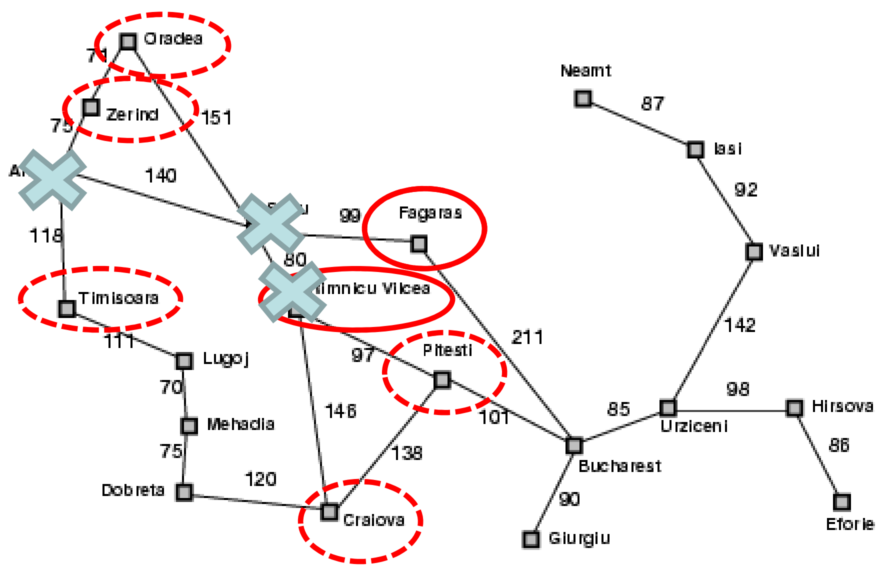

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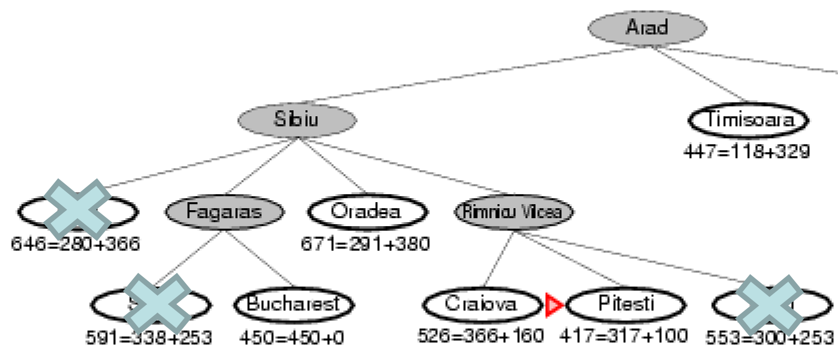

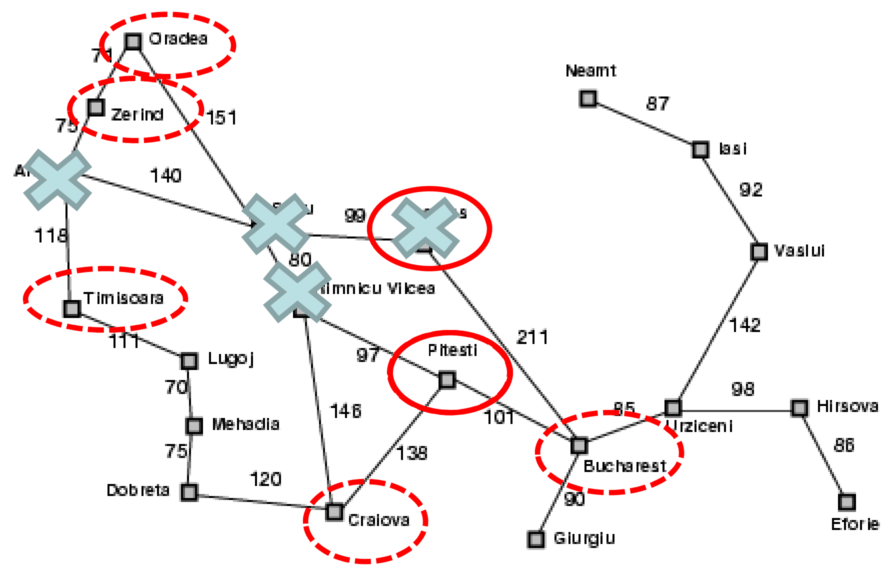

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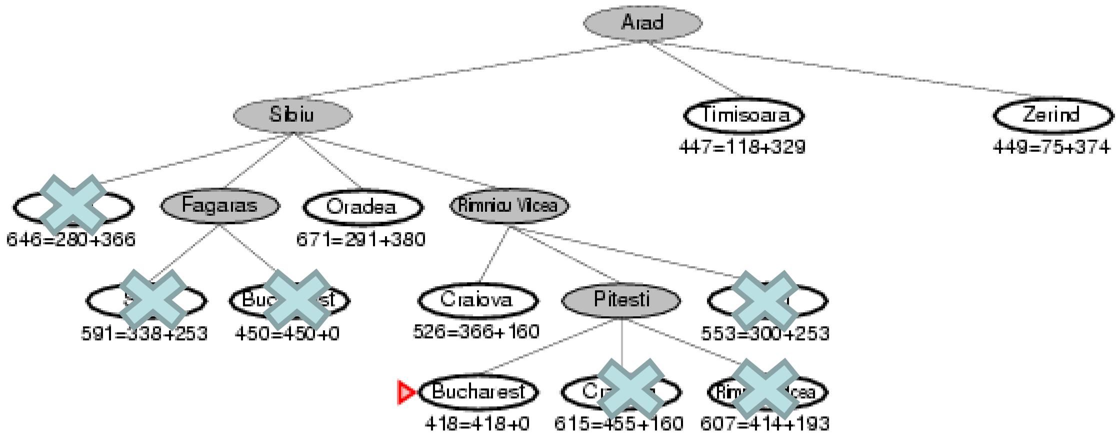

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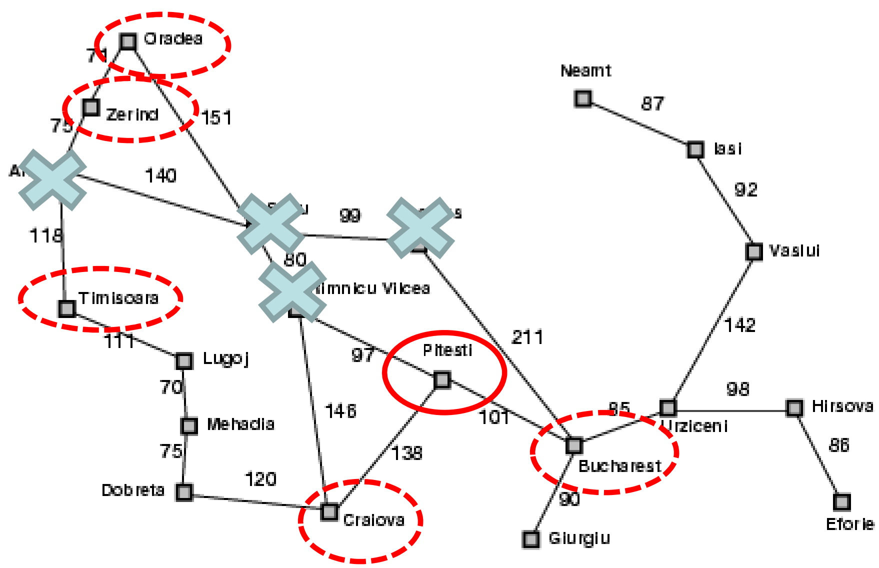

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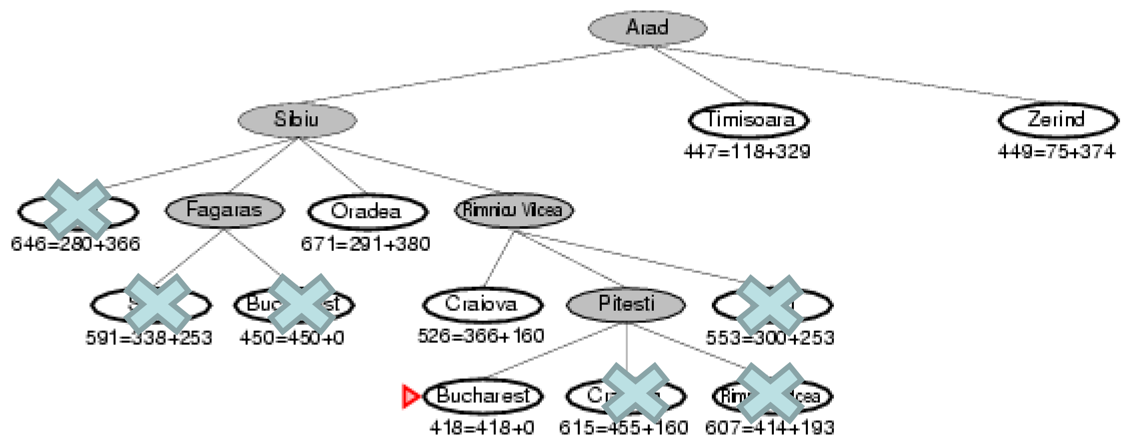

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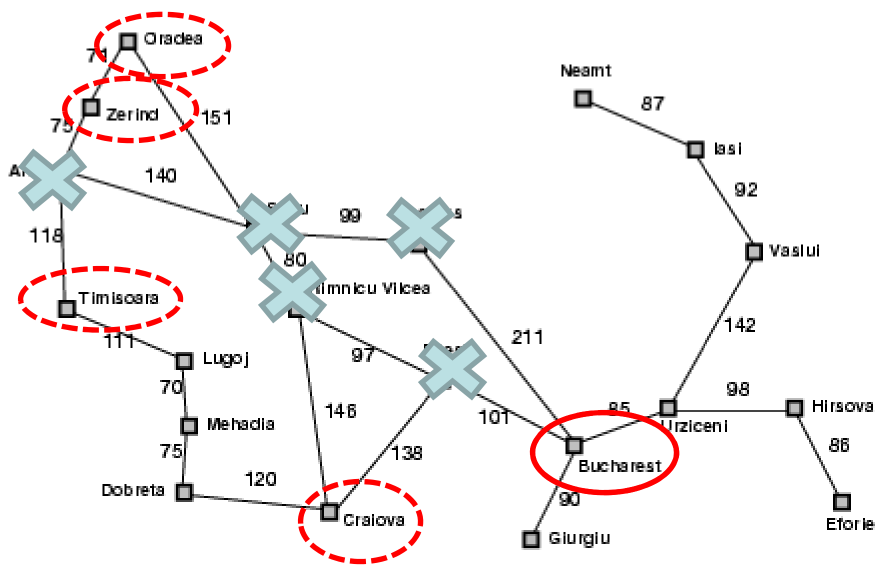

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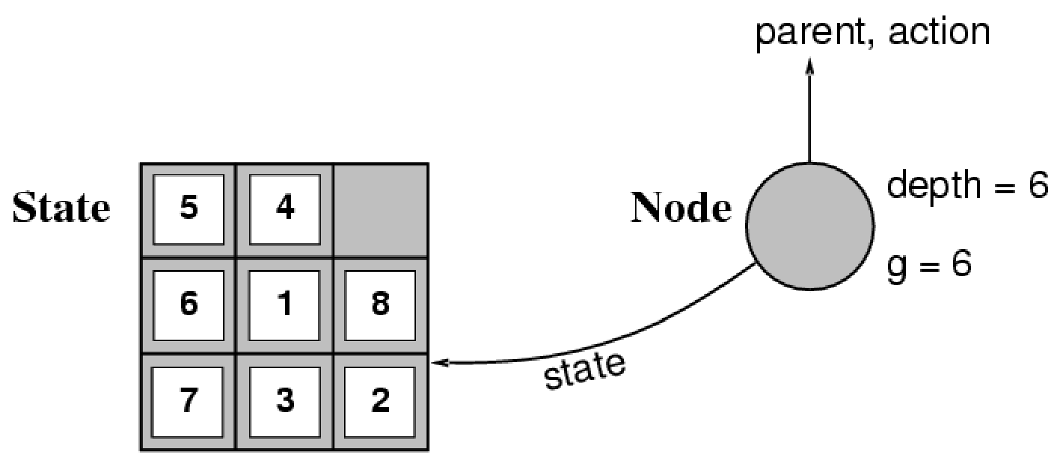
Implementation: states vs. nodes

- A state is a (representation of) a physical configuration
- A node is a data structure constituting part of a search tree includes state, parent node, action, path cost *g(x)*, depth



Search strategies

- A search strategy is defined by picking the order of node expansion
- Strategies are evaluated along the following dimensions:
  - completeness: does it always find a solution if one exists?
  - time complexity: number of nodes generated
  - space complexity: maximum number of nodes in memory
  - optimality: does it always find a least-cost solution?
- Time and space complexity are measured in terms of
  - *b:* maximum branching factor of the search tree
  - *d:* depth of the least-cost solution
  - *m*: maximum depth of the state space (may be ∞)

Uninformed search strategies

- Uninformed (blind) search strategies use only the information available in the problem definition

- Breadth-first search
- Uniform-cost search
- Depth-first search
- Depth-limited search
- Iterative deepening search

Graph & BFS

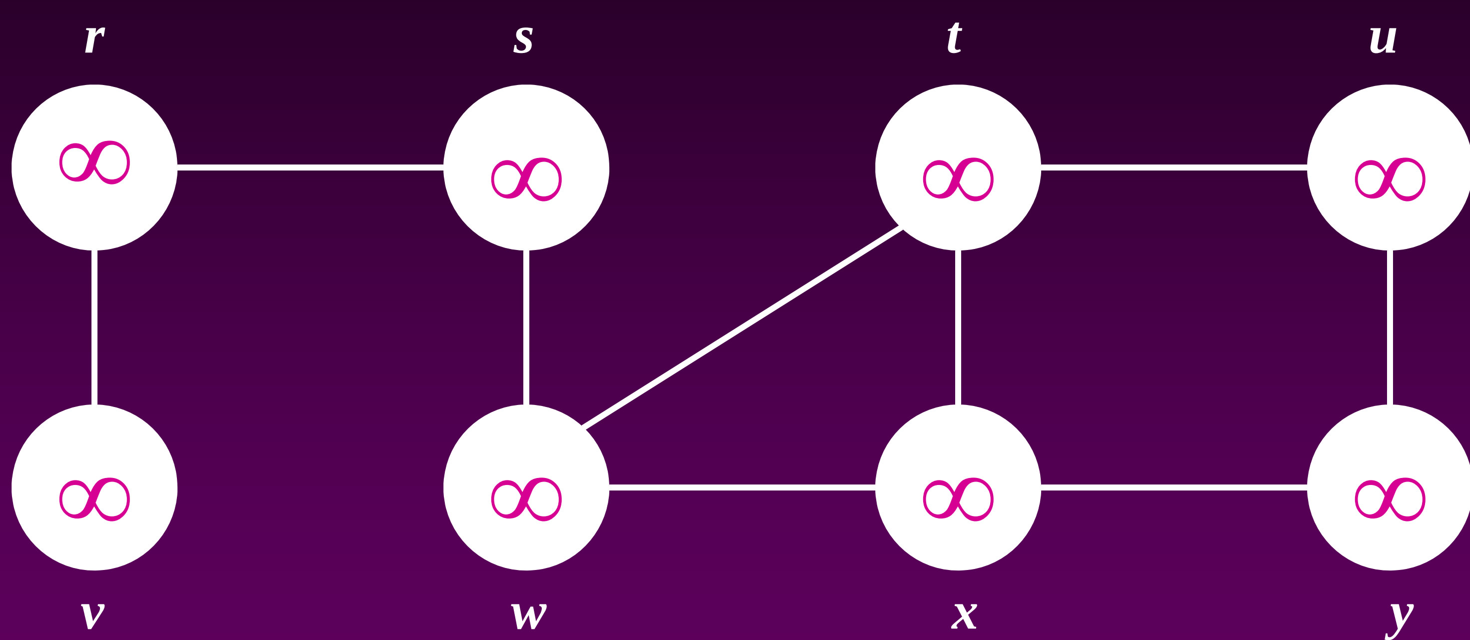
Breadth-First Search: The Code

```
Data: color[V], prev[V],d[V]

BFS(G) // starts from here
{
   for each vertex u ∈ V-{s}
   {
      color[u]=WHITE;
       prev[u]=NIL;
       d[u]=inf;
   }
   color[s]=GRAY;
  d[s]=0; prev[s]=NIL;
  Q=empty;
  ENQUEUE(Q,s);
```

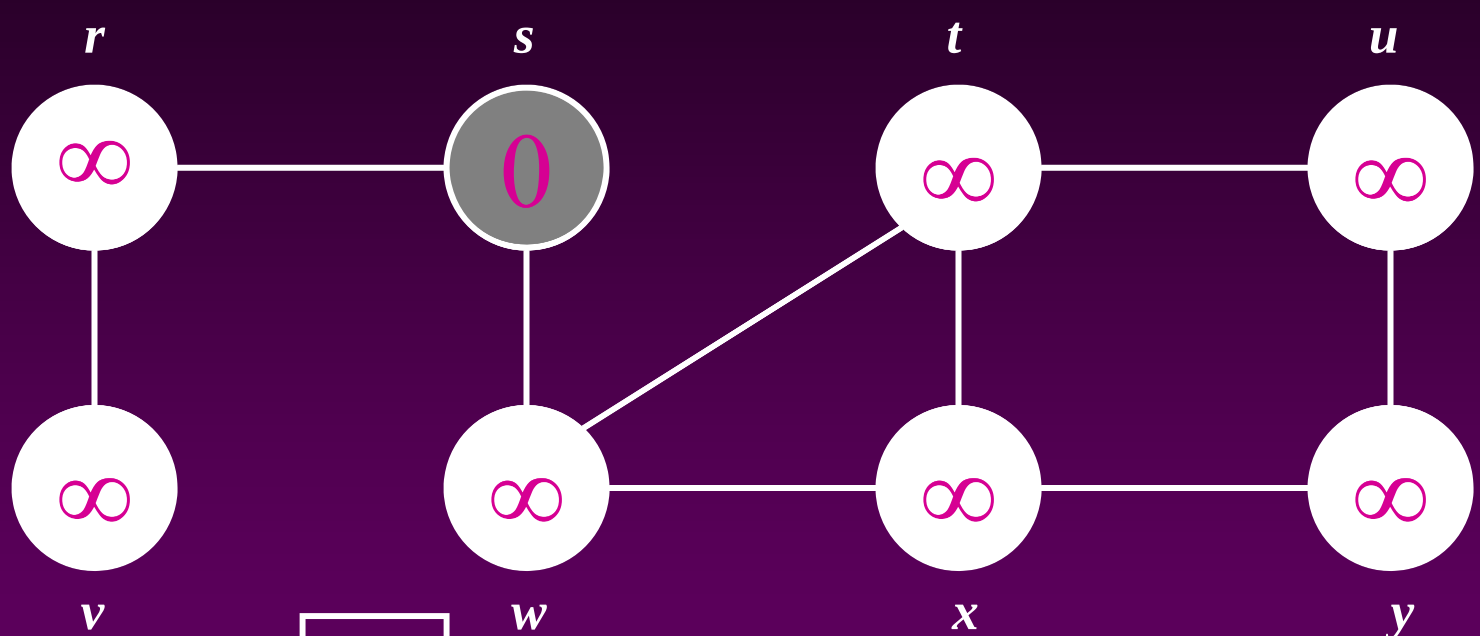
```
While(Q not empty)
{
  u = DEQUEUE(Q);
  for each v ∈ adj[u]{
    if (color[v] ==
 WHITE){
        color[v] = GREY;
        d[v] = d[u] + 1;
        prev[v] = u;
        Enqueue(Q, v);
    }
  }
  color[u] = BLACK;
 }
}
```

Breadth-First Search: Example



Vertex	r	s	t	u	v	w	x	y
color	W	W	W	W	W	W	W	W
d	∞	∞	∞	∞	∞	∞	∞	∞
prev	nil	nil	nil	nil	nil	nil	nil	nil

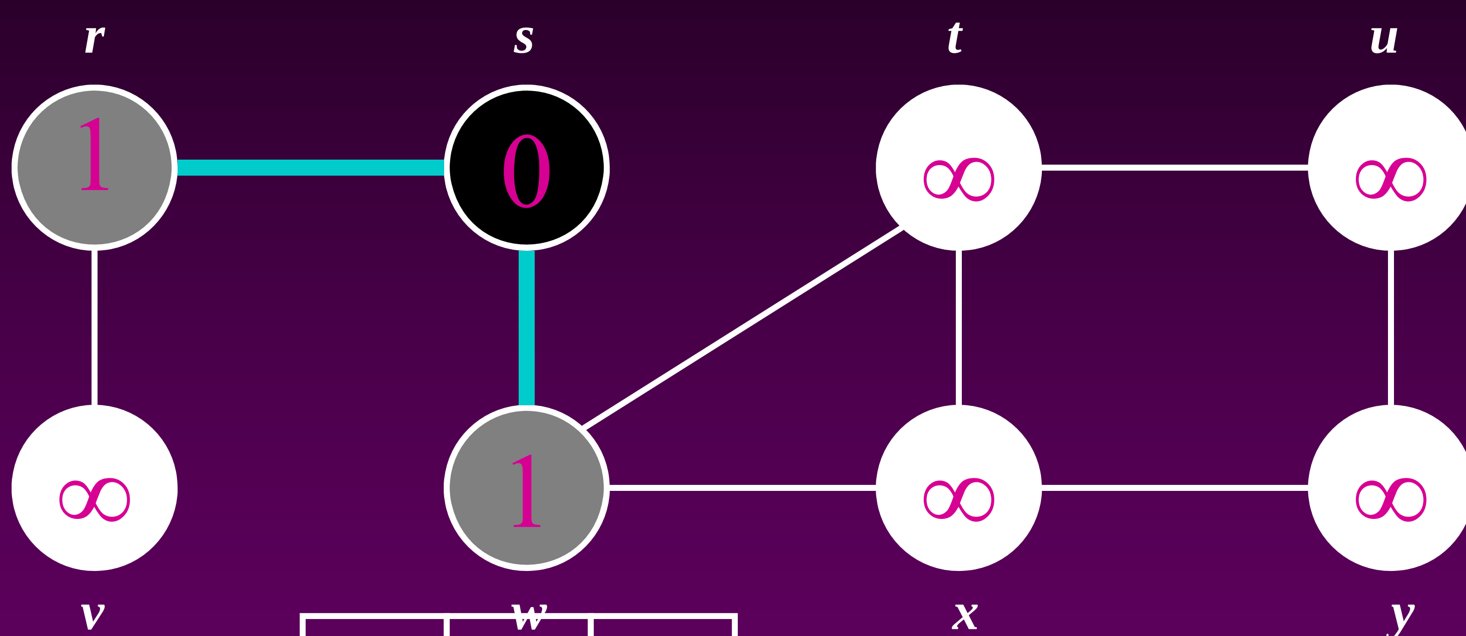
Breadth-First Search: Example



Q: s

vertex	r	s	t	u	v	w	x	y
Color	W	G	W	W	W	W	W	W
d	∞	0	∞	∞	∞	∞	∞	∞
prev	nil	nil	nil	nil	nil	nil	nil	nil

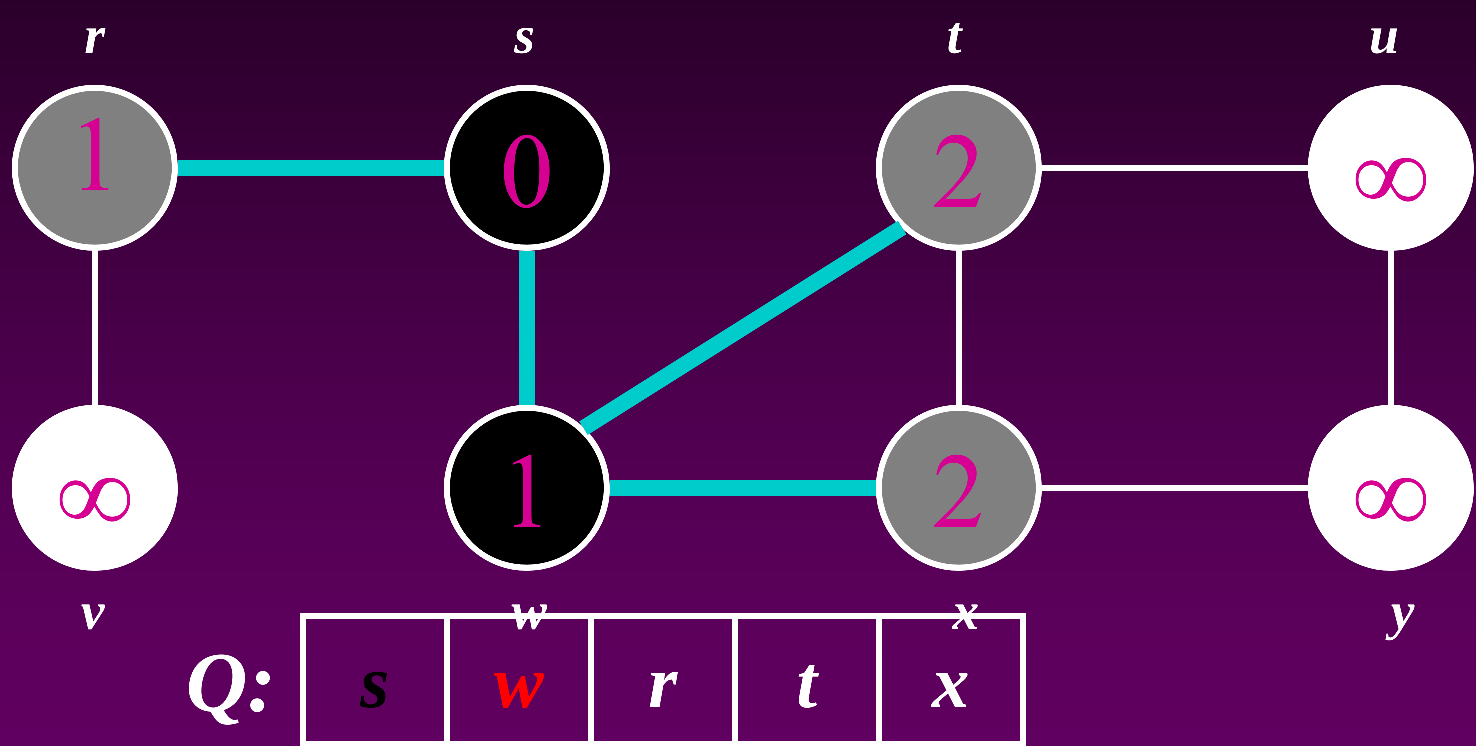
Breadth-First Search: Example



Q: | s | w | r |

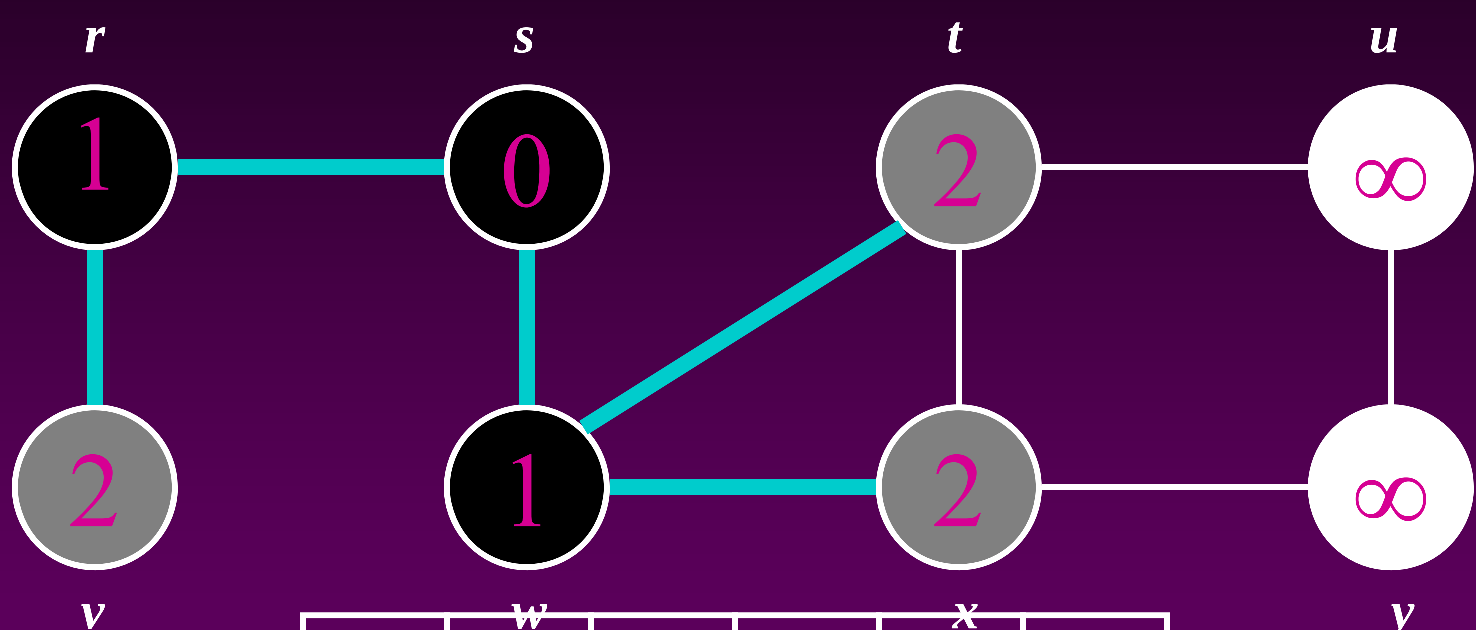
vertex	r	s	t	u	v	w	x	y
Color	G	B	W	W	W	G	W	W
d	1	0	∞	∞	∞	1	∞	∞
prev	s	nil	nil	nil	nil	s	nil	nil

Breadth-First Search: Example



vertex	r	s	t	u	V	w	X	y
Color	G	B	**G**	W	W	**B**	**G**	W
d	1	0	**2**	∞	∞	**1**	**2**	∞
prev	s	nil	**w**	nil	nil	**s**	**w**	nil

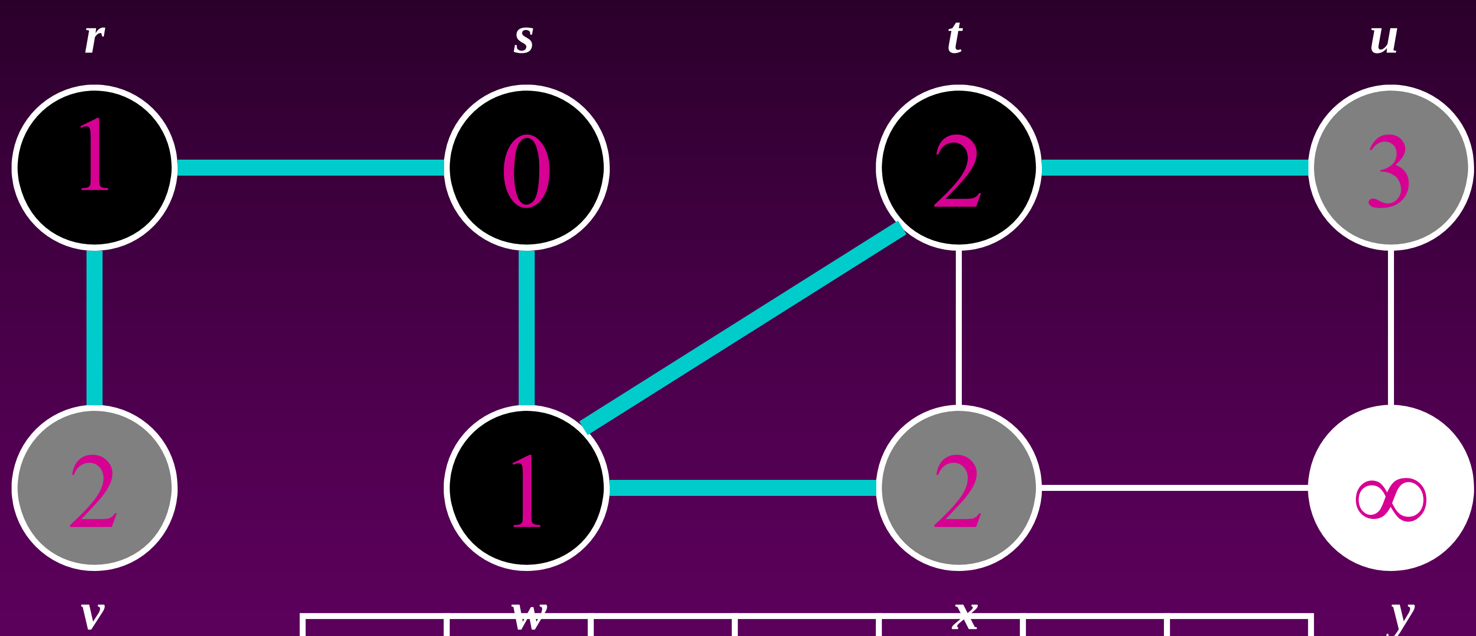
Breadth-First Search: Example



Q: | s | w | r | t | x | v |

vertex	r	s	t	u	v	w	x	y
Color	**B**	B	G	W	**G**	B	G	W
d	**1**	0	2	∞	**2**	1	2	∞
prev	**s**	nil	w	nil	**r**	s	w	nil

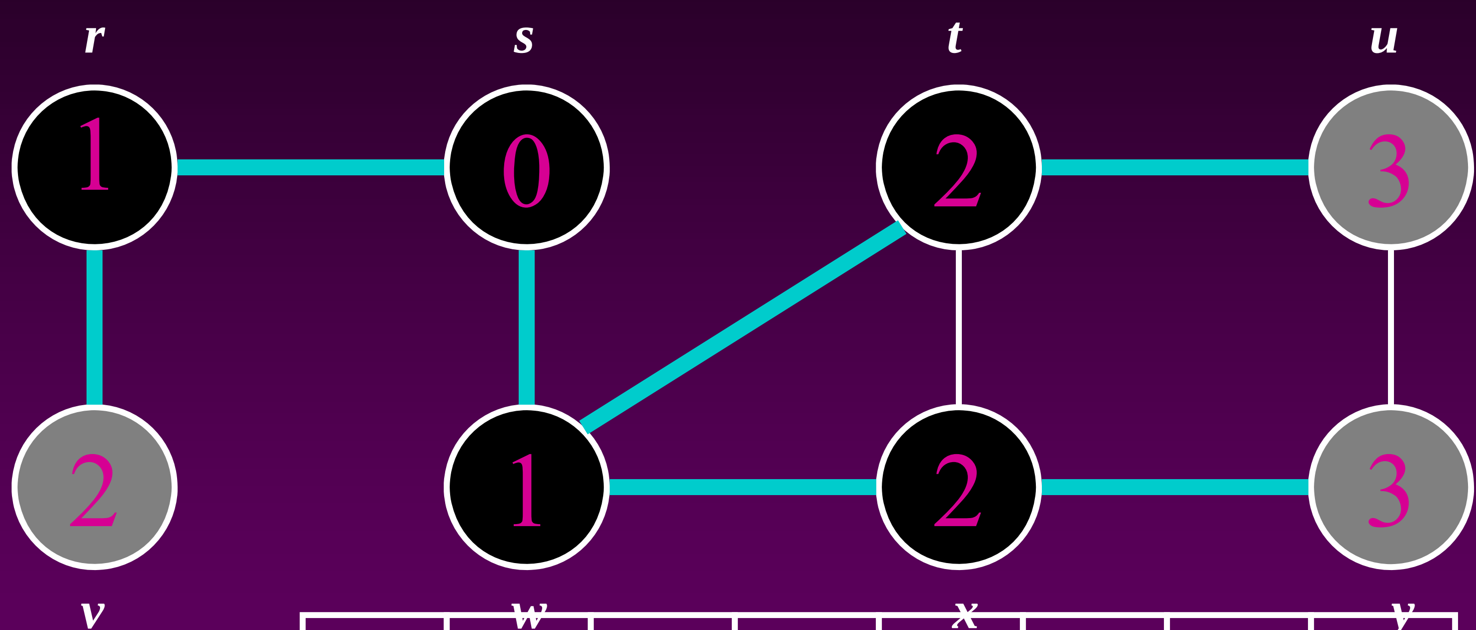
Breadth-First Search: Example



Q:	s	w	r	t	x	v	u

vertex	r	s	t	u	v	w	x	y
Color	B	B	**B**	**G**	G	B	G	W
d	1	0	**2**	**3**	2	1	2	∞
prev	s	nil	**w**	**t**	r	s	w	nil

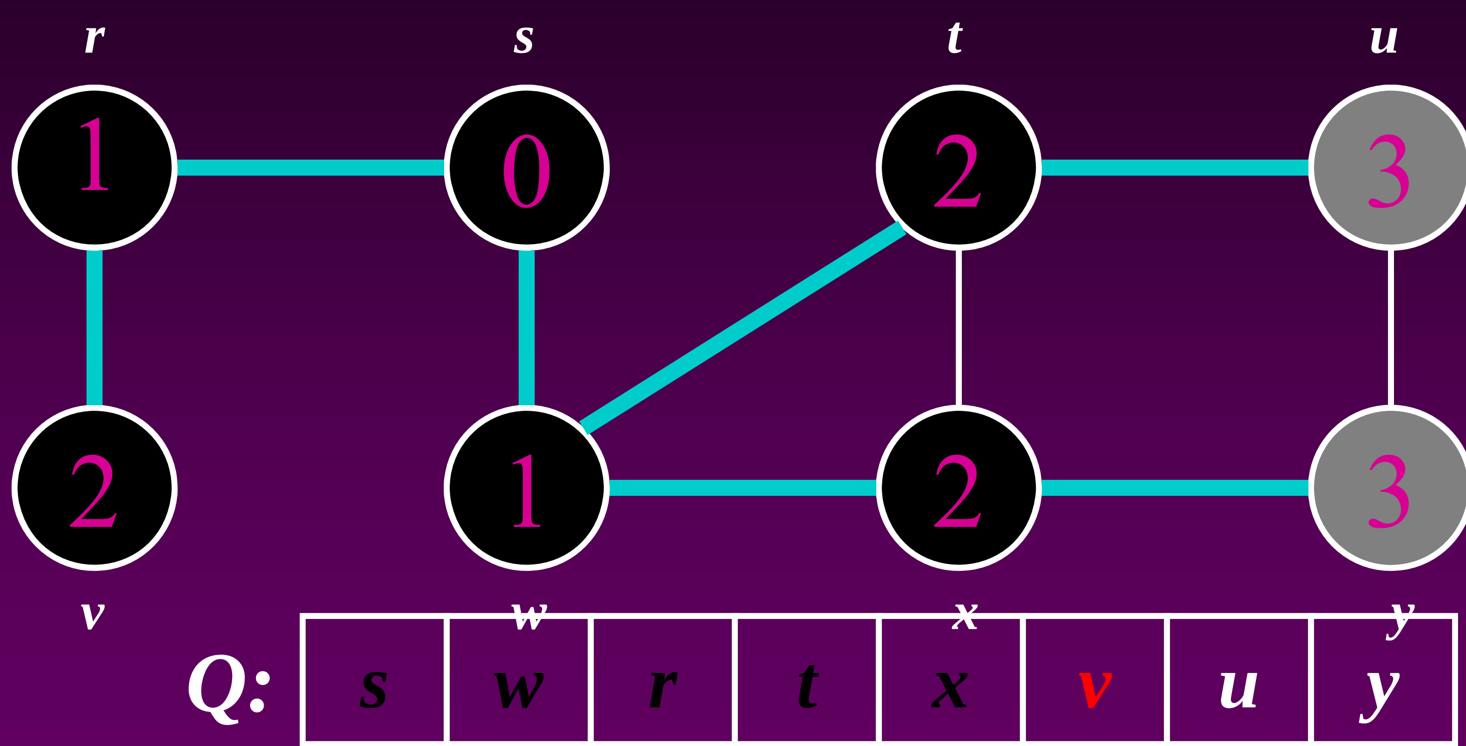
Breadth-First Search: Example



Q:	s	w	r	t	x	v	u	y

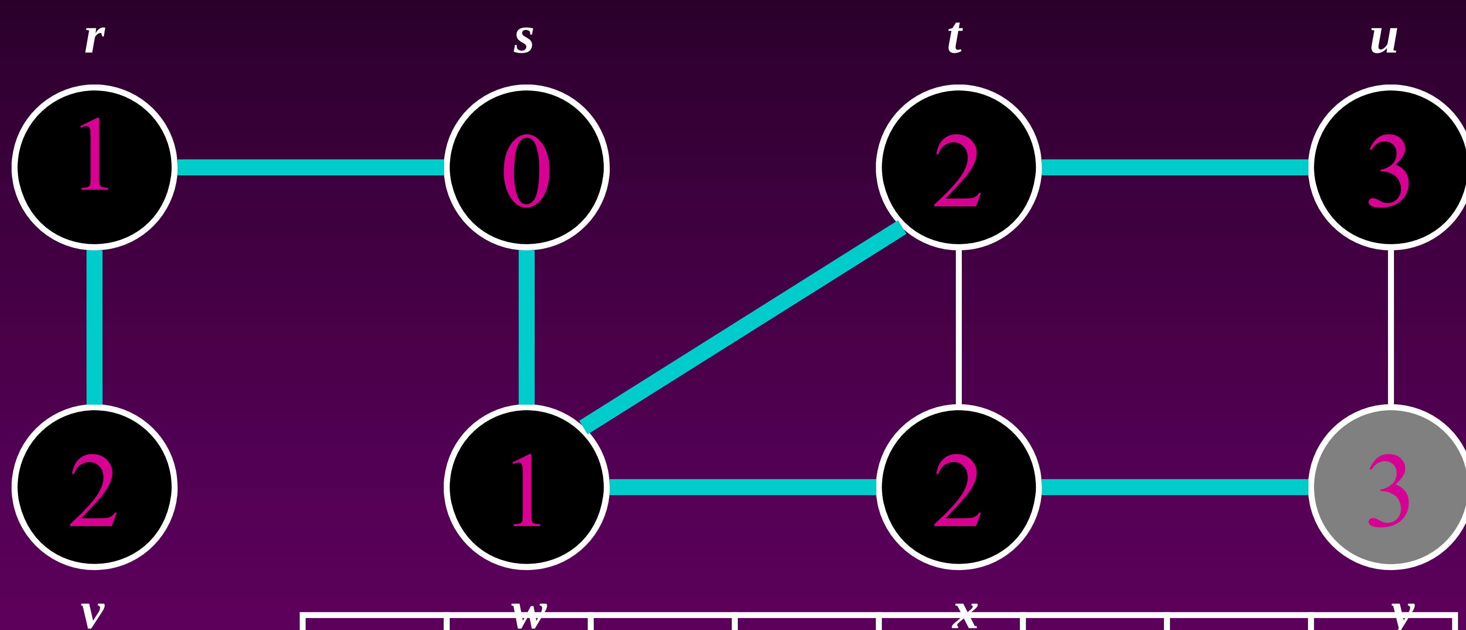
vertex	r	s	t	u	v	w	x	y
Color	B	B	B	G	G	B	B	G
d	1	0	2	3	2	1	2	3
prev	s	nil	w	t	r	s	w	x

Breadth-First Search: Example



vertex	r	s	t	u	v	w	x	y
Color	B	B	B	G	**B**	B	B	G
d	1	0	2	3	**2**	1	2	3
prev	s	nil	w	t	**r**	s	w	x

Breadth-First Search: Example

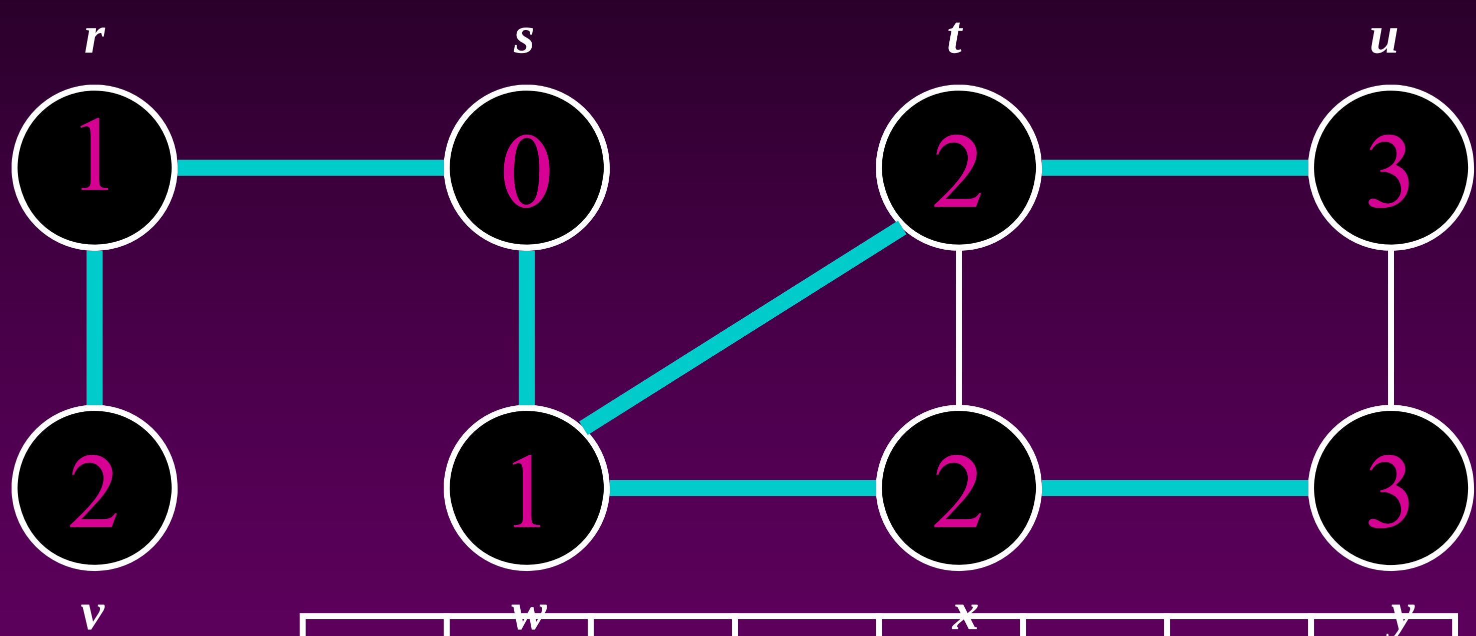


Q:

s	*w*	*r*	*t*	*x*	*v*	*u*	*y*

vertex	r	s	t	u	v	w	x	y
Color	B	B	B	**B**	B	B	B	G
d	1	0	2	**3**	2	1	2	3
prev	s	nil	w	**t**	r	s	w	x

Breadth-First Search: Example



Q: *s* *w* *r* *t* *x* *v* *u* *y*

vertex	r	s	t	u	v	w	x	y
Color	B	B	B	G	B	B	B	**B**
d	1	0	2	3	2	1	2	**3**
prev	s	nil	w	t	r	s	w	**x**

BFS: The Code (again)

```
Data: color[V], prev[V],d[V]

BFS(G) // starts from here
{
   for each vertex u ∊ V-{s}
   {
      color[u]=WHITE;
       prev[u]=NIL;
       d[u]=inf;
   }
   color[s]=GRAY;
  d[s]=0; prev[s]=NIL;
  Q=empty;
  ENQUEUE(Q,s);
```

```
While(Q not empty)
{
  u = DEQUEUE(Q);
  for each v ∊ adj[u]{
    if (color[v] ==
 WHITE){
          color[v] = GREY;
          d[v] = d[u] + 1;
          prev[v] = u;
          Enqueue(Q, v);
    }
  }
  color[u] = BLACK;
}
}
```

Breadth-First Search: Print Path

```
Data: color[V], prev[V],d[V]

Print-Path(G, s, v)
{
   if(v==s)
       print(s)
    else if(prev[v]==NIL)
       print(No path);
   else{
       Print-Path(G,s,prev[v]);
       print(v);
   }
}
```

Thank You