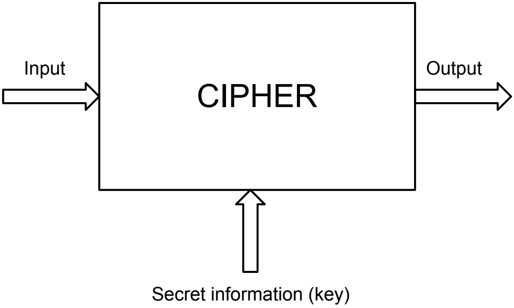
AES Timing Attacks

Hardware and Software Design for Cryptographic Applications

April 10, 2013

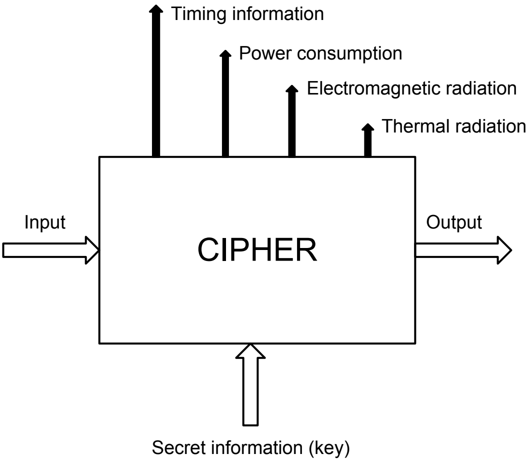
Ciphers as a Black Box

In theory, encryption (and decryption) implementations operate as black boxes.



Information Leakage

In reality, it's hard to prevent additional information from being leaked at runtime.



Side Channel Attacks

Definition: Any attack on a cryptosystem using information leaked given off as a byproduct of the physical implementation of the cryptosystem, rather than a theoretical weakness (TODO: CITE jbonneau), is a *side channel attack*. We focus on **timing attacks** for **software implementations** of AES.

History of Timing Attacks on Cryptographic Primitives

- LIST OF PAPERS FROM BONNEAU HISTORY

Timing Attacks on AES

- Rijndael was deemed not susceptible to timing attacks in the AES contest
- AES targeted attacks can be *statistical* (BERNSTEIN)
  - Observation: The entire encryption time can be affected
  - Step 1:
  - Step 2:
- Or they can be more targeted
  - Exploit relationships between secret information of the primitive and known data.

Cache Memory

TODO: how it's arranged, lines, and whatnot... TODO: image of memory layout for the T_boxes side by side..., and how cache lines may spread across all of them...
$\langle l_i \rangle = \langle l_j \rangle$ are the lower bits of the data entry (data is put into cache based on its MSbits)

Cache Collisions Reveal Weaknesses

Let $T_E(K, P)$ be the encryption time for a plaintext P using key K. Let $\bar{T}_E(K)$ be the $\frac{1}{n}\sum_{i=1}^{n} T_E(K, P_i)$, where P_i is a random plaintext from $\{0|1\}^{128}$.

Cache-Collision Assumption [?]. For any pair of table lookups i, j, given a large enough number of *random* AES encryption that use the *same key*, $\bar{T}_E(K)$ will be lower when $\langle I_i \rangle = \langle I_j \rangle$ than when $\langle I_i \rangle \neq \langle I_j \rangle$

Note: The table lookup indices must be *independent* for random plaintexts.

Cache Collisions (cont'd)

Let a and b be two memory addresses looked up in memory. Let $\langle a \rangle$ and $\langle b \rangle$ denote the MSBs of a and b, respectively.

- Cache memory is organized into *lines*
  - MSB is mapped to the cache line index, LSB is mapped to the line (block) offset
- Reads on a and b cause a collision if $\langle a \rangle = \langle b \rangle$ (assuming other memory reads have not evicted (or invalidated) a or b from the cache.
- If $\langle a \rangle \neq \langle b \rangle$ then a cache collision *might* occur.
- We cannot say for certain whether or not the lower LSBs are equivalent...

Attacks from Cache Collisions

That's it! We may now build an attack based on this result.

LUT-Based Implementations

Let X^i be the state of AES at round i. With the exception of $i = 10$, we have:

$$
\begin{aligned}
X^{i+1} = \{ & T_0[x^i_0] \oplus T_1[x^i_5] \oplus T_2[x^i_{10}] \oplus T_3[x^i_{15}] \oplus \{k^i_0, k^i_1, k^i_2, k^i_3\}, \\
& T_0[x^i_4] \oplus T_1[x^i_9] \oplus T_2[x^i_{14}] \oplus T_3[x^i_3] \oplus \{k^i_4, k^i_5, k^i_6, k^i_7\}, \\
& T_0[x^i_8] \oplus T_1[x^i_{13}] \oplus T_2[x^i_2] \oplus T_3[x^i_7] \oplus \{k^i_8, k^i_9, k^i_{10}, k^i_{11}\}, \\
& T_0[x^i_{12}] \oplus T_1[x^i_1] \oplus T_2[x^i_6] \oplus T_3[x^i_{11}] \oplus \{k^i_{12}, k^i_{13}, k^i_{14}, k^i_{15}\}\}
\end{aligned}
$$

First Round

TODO: image of AES lookup after first round

- First round: $x_i^0 = p_i \oplus k_i$
- With the T-box implementation, x_0^0, x_4^0, x_8^0, and x_{12}^0 are used as indices into T_0
- If we are looking for cache collisions, we must consider input bytes of the same “family” (i.e. incides into the same T-box)

$$\begin{aligned} \langle x_i^0 \rangle = \langle x_j^0 \rangle &\Rightarrow \langle p_i \rangle \oplus \langle k_i \rangle = \langle p_j \rangle \oplus \langle k_j \rangle \\ &\Rightarrow \langle p_i \rangle \oplus \langle p_j \rangle = \langle k_i \rangle \oplus \langle k_j \rangle \end{aligned}$$

First Round Attack Algorithm

ALGORITHM 1: FirstRoundAttack(N_s)

$n \leftarrow 2^8 - 1$
$T \leftarrow \text{array}[0 \ldots n, 1 \ldots n, 0 \ldots n]$
for $i = 0 \rightarrow N_s$ **do**
    $P \leftarrow RandomBytes(16)$
    $start \leftarrow time()$
    $C \leftarrow E_K(P)$
    $end \leftarrow time()$
    $tt \leftarrow (start - end)$
    $T[i, j, \langle p_i \rangle \oplus \langle p_j \rangle] \leftarrow T[i, j, \langle p_i \rangle \oplus \langle p_j \rangle] + tt$
end for
$T[i, j, \langle p_i \rangle \oplus \langle p_j \rangle] \leftarrow T[i, j, \langle p_i \rangle \oplus \langle p_j \rangle] / N_s$
$mi, mj \leftarrow min(t)$ ▷ Use t-test if necessary
$\langle k_{mi} \rangle \oplus \langle k_{mj} \rangle \leftarrow \langle p_{mi} \rangle \oplus \langle p_{mj} \rangle$

First Round Attack Algorithm (cont'd)

$$
\begin{aligned}
X^{i+1} = \{ & T_0[x_0^i] \oplus T_1[x_5^i] \oplus T_2[x_{10}^i] \oplus T_3[x_{15}^i] \oplus \{k_0^i, k_1^i, k_2^i, k_3^i\}, \\
& T_0[x_4^i] \oplus T_1[x_9^i] \oplus T_2[x_{14}^i] \oplus T_3[x_3^i] \oplus \{k_4^i, k_5^i, k_6^i, k_7^i\}, \\
& T_0[x_8^i] \oplus T_1[x_{13}^i] \oplus T_2[x_2^i] \oplus T_3[x_7^i] \oplus \{k_8^i, k_9^i, k_{10}^i, k_{11}^i\}, \\
& T_0[x_{12}^i] \oplus T_1[x_1^i] \oplus T_2[x_6^i] \oplus T_3[x_{11}^i] \oplus \{k_{12}^i, k_{13}^i, k_{14}^i, k_{15}^i\}\}
\end{aligned}
$$

First Round Attack Algorithm (cont'd)

$$
\begin{aligned}
X^{i+1} = \{ & T_0[x_0^i] \oplus T_1[x_5^i] \oplus T_2[x_{10}^i] \oplus T_3[x_{15}^i] \oplus \{k_0^i, k_1^i, k_2^i, k_3^i\}, \\
& T_0[x_4^i] \oplus T_1[x_9^i] \oplus T_2[x_{14}^i] \oplus T_3[x_3^i] \oplus \{k_4^i, k_5^i, k_6^i, k_7^i\}, \\
& T_0[x_8^i] \oplus T_1[x_{13}^i] \oplus T_2[x_2^i] \oplus T_3[x_7^i] \oplus \{k_8^i, k_9^i, k_{10}^i, k_{11}^i\}, \\
& T_0[x_{12}^i] \oplus T_1[x_1^i] \oplus T_2[x_6^i] \oplus T_3[x_{11}^i] \oplus \{k_{12}^i, k_{13}^i, k_{14}^i, k_{15}^i\}\}
\end{aligned}
$$

First Round Limitation

We only know that $\langle k_0 \rangle \oplus \langle k_4 \rangle = \Delta_1$, $\langle k_0 \rangle \oplus \langle k_8 \rangle = \Delta_2$, $\langle k_0 \rangle \oplus \langle k_{12} \rangle = \Delta_3$, $\langle k_4 \rangle \oplus \langle k_8 \rangle = \Delta_4$, $\langle k_4 \rangle \oplus \langle k_{12} \rangle = \Delta_5$, and $\langle k_8 \rangle \oplus \langle k_{12} \rangle = \Delta_6$.

Of course, there exists **18** other equations we can derive for T_1, T_2, and T_3.

We cannot determine the lower $\log_2$ bits of each key... What to do now?

The Last Round

When $i = 10$, the lookup table is just the S-box S. At this point, the ciphertext C is:

$$\begin{aligned} C = \{ & S[x_0^{10}] \oplus k_0^{10}, S[x_5^{10}] \oplus k_1^{10}, S[x_{10}^{10}] \oplus k_2^{10}, S[x_{15}^{10}] \oplus k_3^{10}, \\ & S[x_4^{10}] \oplus k_5^{10}, S[x_9^{10}] \oplus k_6^{10}, S[x_{14}^{10}] \oplus k_7^{10}, S[x_3^{10}] \oplus k_7^{10}, \\ & S[x_8^{10}] \oplus k_8^{10}, S[x_{13}^{10}] \oplus k_9^{10}, S[x_2^{10}] \oplus k_{10}^{10}, S[x_7^{10}] \oplus k_{11}^{10}, \\ & S[x_{12}^{10}] \oplus k_{12}^{10}, S[x_1^{10}] \oplus k_{13}^{10}, S[x_6^{10}] \oplus k_{14}^{10}, S[x_{11}^{10}] \oplus k_{15}^{10} \} \end{aligned}$$

Final Round Collisions

Let x_s and x_t be two random bytes in the last round.

We will always have that $c_i = k_i^{10} \oplus S[x_s]$ and $c_j = k_j^{10} \oplus S[x_t]$. If $x_s = x_t$, then a collision *will usually* occur and $c_i = k_i^{10} \oplus \alpha$ and $c_j = k_j^{10} \oplus \alpha$.

Therefore, $c_i \oplus c_j = k_i^{10} \oplus k_j^{10}$

Final Round Misses

What if $x_s \neq x_t$?

$c_i \oplus c_j \neq k_i^{10} \oplus k_j^{10} \Rightarrow$ two different values came out of the LUT!

The *S-box nonlinearity* means that the difference between $S[x_s]$ and $S[x_t]$ *does not* imply a similar difference between x_s and x_t.

If $c_i \oplus c_j \neq k_i^{10} \oplus k_j^{10}$, then x_s and x_t are two *random* values.

Final Round Attack Algorithm

ALGORITHM 2: LastRoundAttack(N_s)

$diffs \leftarrow [0,\ldots,2^n-1,0,\ldots,2^n-1]$
$T \leftarrow \text{array}[0\ldots2^n-1,0\ldots2^n-1,0\ldots2^n-1]$
for $i = 0 \rightarrow N_s$ **do**
    $P \leftarrow RandomBytes(16)$
    $start \leftarrow time()$
    $C \leftarrow E_K(P)$
    $end \leftarrow time()$
    $tt \leftarrow (start - end)$
    for i, j in C **do**
        $T[i,j,C_i \oplus C_j] \leftarrow T[i,j,C_i \oplus C_j] + tt$
    end for
end for
$T[i,j,\Delta_{i,j}] \leftarrow T[i,j,\Delta_{i,j}]/N_s$
$\Delta'_{i,j} \leftarrow min(T,i,j)$ ▷ Use t-test if necessary **return** all $\Delta'_{i,j}$

Final Round Attack Algorithm (cont'd)

With knowledge of $\Delta_{i,j}$ for all i, j such that $\Delta_{i,j} = k_i^{10} \oplus k_j^{10}$ the attacker can now make informed guesses at the key

TODO: example of how they would recover it here...

Thanks to Rijndael's invertible key schedule, $[k^{10}]$ can be reverted back to $[k^0]$.

Done.

Timing Attack Countermeasures

- Masking...