The nonconforming problem minimizes the discrete energy $E_{\mathrm{NC}}$ in $\mathrm{CR}_0^1(\mathcal{T})$ with

$$E_{\mathrm{NC}}(v_{\mathrm{CR}}) := \frac{\alpha}{2}\|v_{\mathrm{CR}}\|_{L^2(\Omega)}^2 + |v_{\mathrm{CR}}|_{1,1,\mathrm{NC}} - \int_\Omega fv\,\mathrm{d}x.$$

The adaptive algorithm employs the following error estimator

$$\eta^2(T) := \underbrace{|T|^{1/2}\|f - \alpha u_{\mathrm{CR}}\|_{L^2(\Omega)}^2}_{\mu(T)} + \underbrace{|T|^{\beta/2}\sum_{F\in\mathcal{F}(T)}\|[u_{\mathrm{CR}}]_F\|_{L^1(F)}}_{\xi(T)}$$
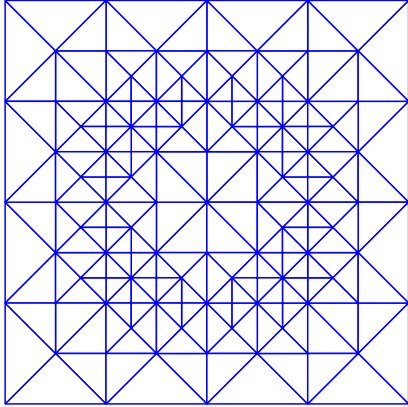
for any $T \in \mathcal{T}$. The stopping criteria is set to $\epsilon = 10^{-5}$ in the following examples.
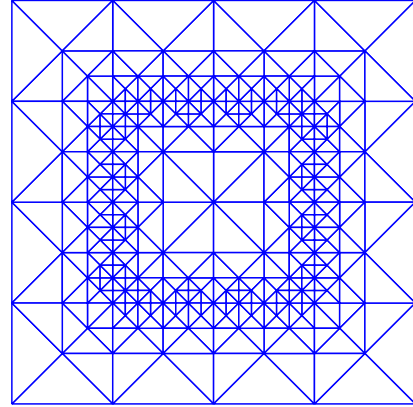
# 1 Example with exact solution

# 2 Example with discontinuity set

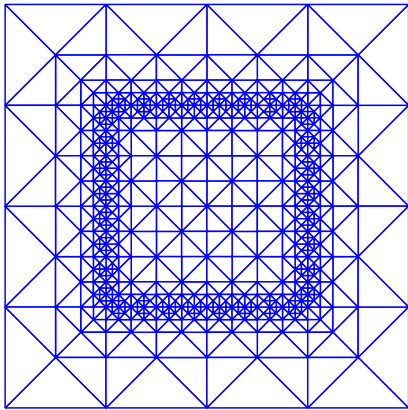Let $\Omega = (-1,1)^2$, $\alpha = 100$ and $f \in L^2(\Omega)$ with

$$f(x) = \begin{cases} 100 & \text{if } |x|_\infty < 1/2, \\ 0 & \text{otherwise.} \end{cases}$$
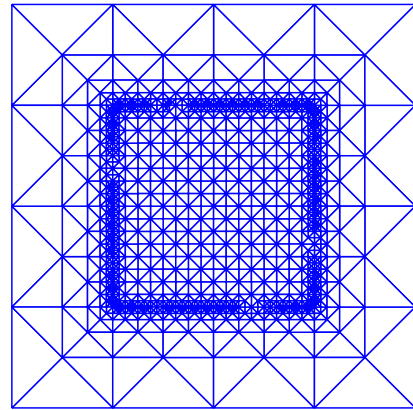


(a) 117 nodes

(b) 256 nodes

(c) 550 nodes

(d) 1138 nodes
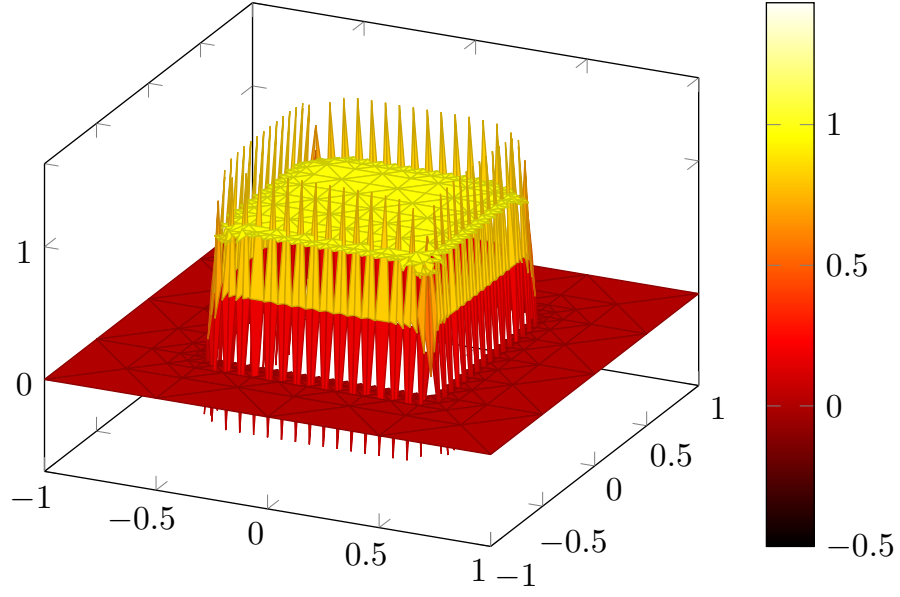
Figure 1: successive adaptively generated meshes with $\beta = 1$

Figure 2: discrete solution on an adaptively refined mesh ($\beta = 1$) with 550 nodes
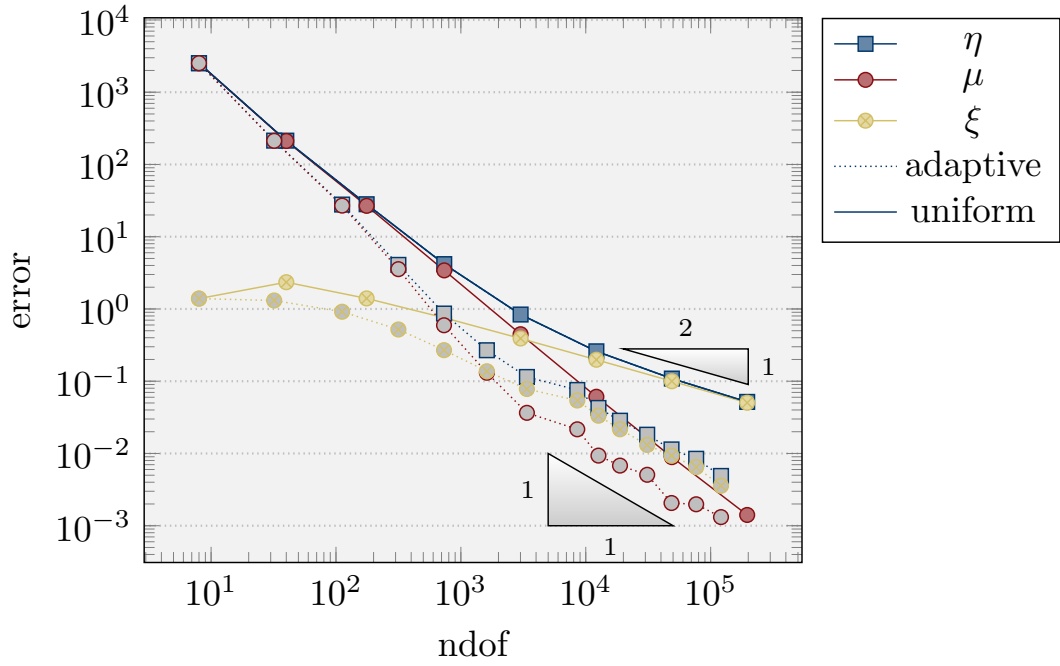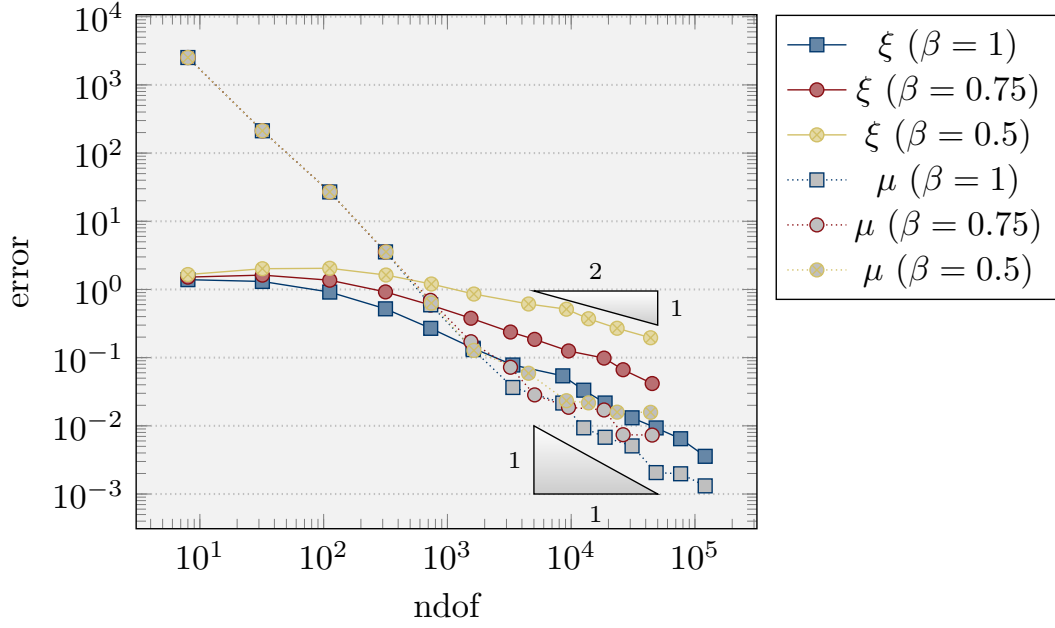


Figure 3: convergence history plot for $\eta$, $\mu$ and $\xi$ with $\beta = 1$

2

Figure 4: convergence history plot for $\mu$ and $\xi$ with different $\beta$

# 3 Application to image processing

The right hand side $f$ represents the greyscale of the image in the left plot of times the factor $\alpha = 10000$ where the greyscale is some real number in $[0, 1]$. The original image has a size of 256x256 pixels and is displayed in . The image is scaled to the domain $\Omega = (0, 1)^2$.



Figure 5: original image (left) on a uniform mesh with 66049 nodes and generated image (right) on a uniform mesh with 16641 nodes
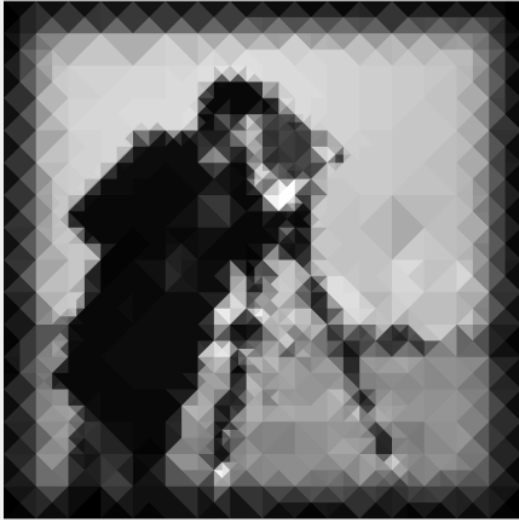
(a) 850 nodes



(b) 2091 nodes



(c) 5527 nodes



(d) 13955 nodes

Figure 6: generated images on adaptive meshes after 12 (upper left), 14 (upper right), 16 (bottom left) and 18 (bottom right) iterations of the adaptive algorithm
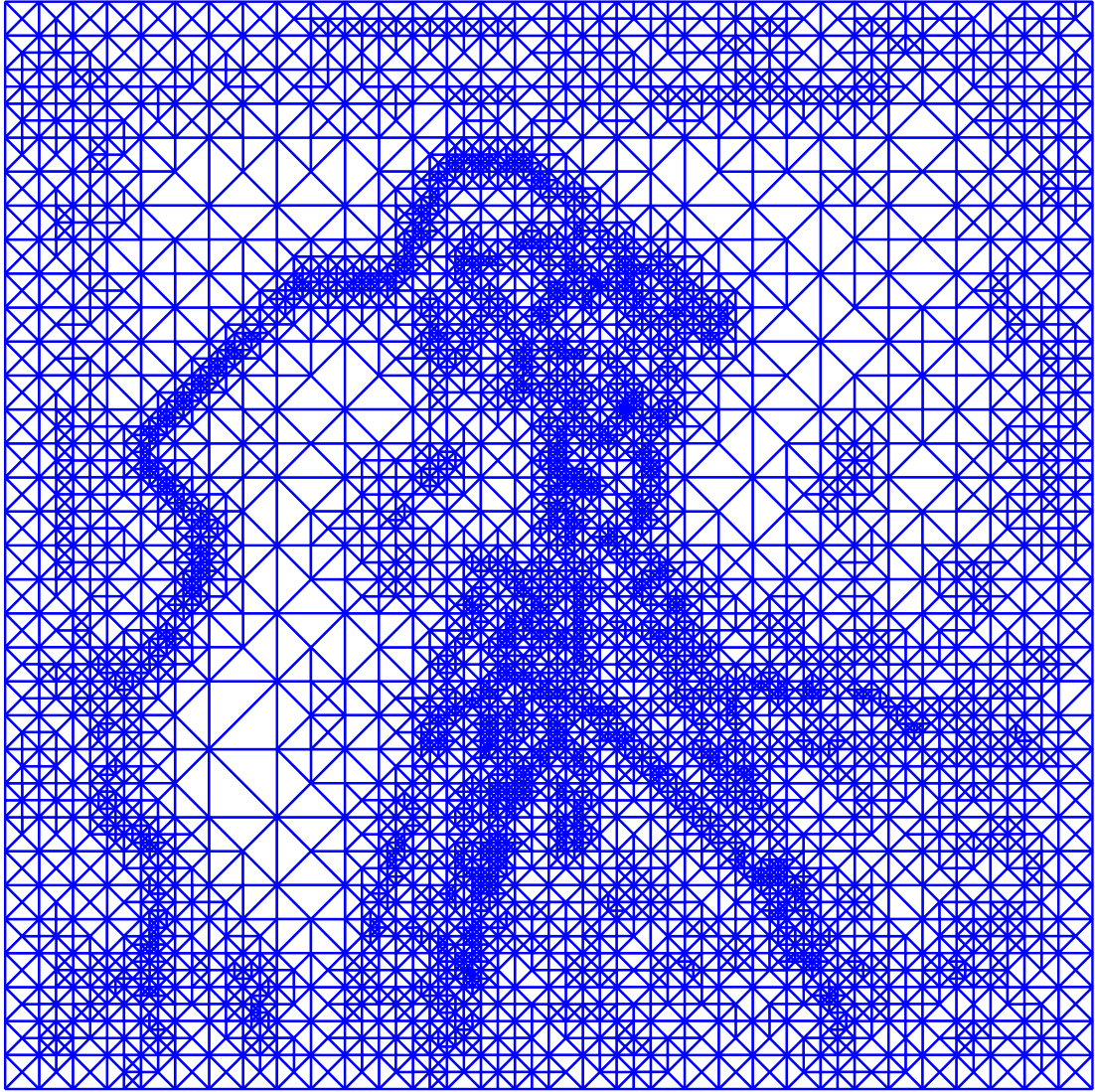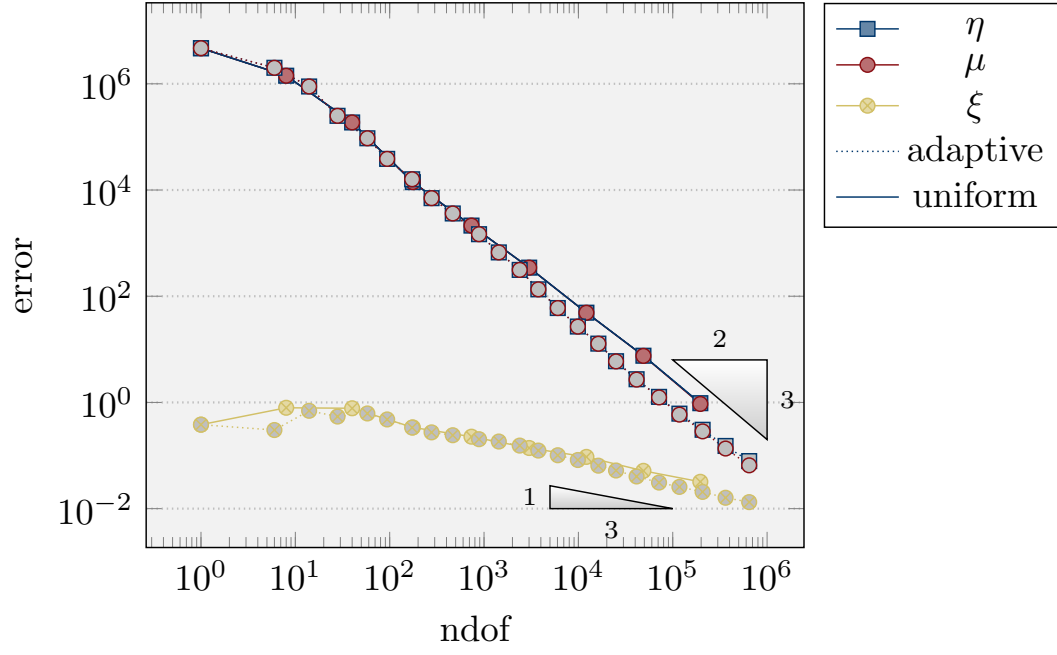
Figure 7: adaptive mesh with 5527 nodes

Figure 8: convergence history plot for $\eta$, $\mu$ and $\xi$

# 4   Example with a noisy image