1 The continuous and discrete problem

Let $\alpha > 0$, $\Omega \subset \mathbb{R}^n$ bounded polyhedral Lipschitz domain, and $f \in L^2(\Omega)$.

The continuous problem minimizes

$$E(v) := \frac{\alpha}{2}\|v\|^2_{L^2(\Omega)} + |v|_{\mathrm{BV}(\Omega)} + \|v\|_{L^1(\Omega)} - \int_\Omega f\, v \,\mathrm{d}x \tag{1.1}$$

amongst all $v \in V := \mathrm{BV}(\Omega) \cap L^2(\Omega)$ where the BVseminorm $|v|_{\mathrm{BV}(\Omega)}$ is equals to the $W^{1,1}$ seminorm for any $v \in W^{1,1}(\Omega)$.

The nonconforming problem minimizes

$$E_{\mathrm{NC}}(v_{\mathrm{CR}}) := \frac{\alpha}{2}\|v_{\mathrm{CR}}\|^2_{L^2(\Omega)} + |v_{\mathrm{CR}}|_{1,1,\mathrm{NC}} - \int_\Omega f\, v_{\mathrm{CR}} \,\mathrm{d}x \tag{1.2}$$

amongst all $v_{\mathrm{CR}} \in \mathrm{CR}^1_0(\mathcal{T})$ where $|\bullet|_{1,1,\mathrm{NC}} := \|\nabla_{\mathrm{NC}}\|_{L^1(\Omega)}$.

2 Estimator and guaranteed lower energy bound

For $f \in H_0^1(\Omega)$ and $u \in H_0^1(\Omega)$ ($u_{\mathrm{CR}} \in \mathrm{CR}_0^1(\Omega)$) continuous (discrete) minimzer with minimal energy $E(u)$ ($E_{\mathrm{NC}}(u_{\mathrm{CR}})$) it holds

$$E_{\mathrm{NC}}(u_{\mathrm{CR}}) + \frac{\alpha}{2}\|u - u_{\mathrm{CR}}\|^2_{L^2(\Omega)} - \tag{2.1}$$

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3 Example with exact solution

Funcions considered are
the esimator

4 Application to an image