GESAR

The Group for Environmental Studies in Reservoirs

FIRST FLUID SOLVER: 2D STREAM-VORTICITY FORMULATION

3rd Workshop on Advances in CFD and LB Modelling of Interface Dynamics in Capillary Two-Phase Flows

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Kobe - Japan

October 10th, 2018

GOVERNING EQUATIONS

vorticity transport:

$$\frac{\partial \omega_z}{\partial t} + \mathbf{v} \cdot \nabla \omega_z = \nu \nabla^2 \omega_z$$

stream function:

$$\nabla^2 \psi = -\omega_z$$

auxiliary:

$$\frac{\partial \psi}{\partial y} = v_x$$

$$\frac{\partial \psi}{\partial x} = -v_y$$

$$\omega_z = \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}$$

Required 2 boundary conditions for ψ and ω_z

MATRICIAL EQUATIONS

vorticity transport:

$$\left(\frac{\mathbf{M}}{\Delta t} + \nu\mathbf{K} + \mathbf{v}\cdot\mathbf{G}\right)\omega_z{}^{n+1} = \frac{\mathbf{M}}{\Delta t}\omega_z{}^{n} + \mathbf{b.c.}$$

stream function:

$$\mathbf{K}\psi = \mathbf{M}\omega_z + \mathbf{b.c.}$$

boundary conditions:

ψ ⟶ constant

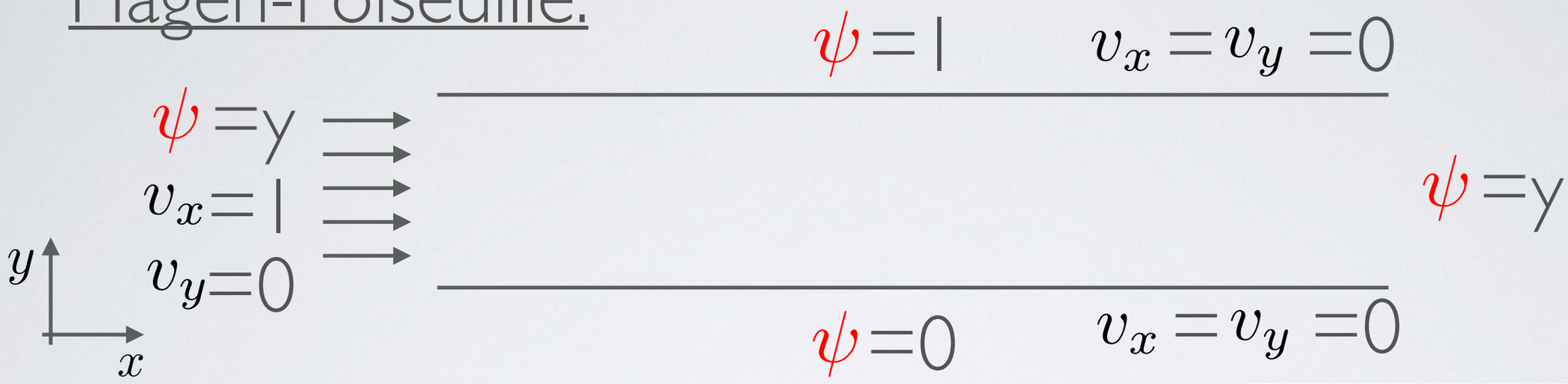
ω_z ⟶ variable

auxiliary:

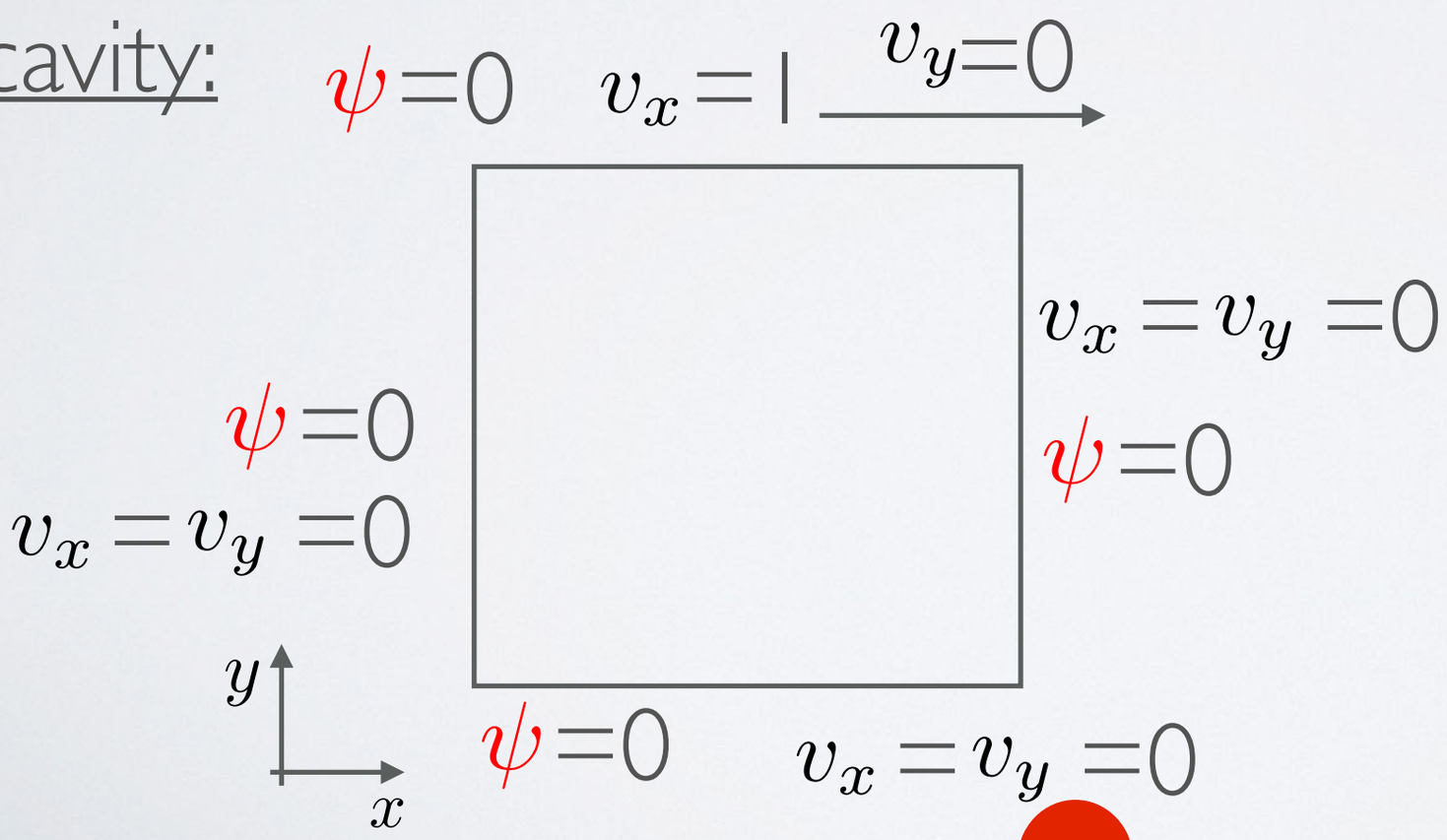
$$\begin{cases} \mathbf{M}v_x = \mathbf{G_y}\psi \\ \mathbf{M}v_y = -\mathbf{G_x}\psi \\ \mathbf{M}\omega_z = \mathbf{G_x}v_y - \mathbf{G_y}v_x \end{cases}$$

EXAMPLES

Hagen-Poiseuille:



cavity:



streamfunction b.c.

$$\psi = \int (v_x dy - v_y dx)$$

vorticity b.c.
solve ω_z every dt:

$$\mathbf{M}\omega_z = \mathbf{G_x} v_y - \mathbf{G_y} v_x$$